Spring017

Dr. Deefallah Aldalabeeh

BY:

Mohammed Abuhashieh

Powerunit-ju.com

Scanned by CamScanner

Electrical Machines (I)

Second Semester (2017)

Dr. Daifallah Dalabeih.

* Mohammad Abu Hashia. *

* * *

Scanned by CamScanner

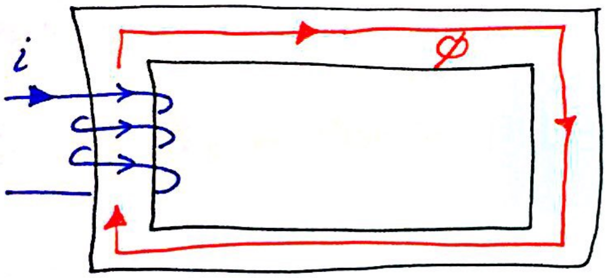
※ Magnetic Fields:

Why ?? → Because it is fundemental for operation of Transformers & Machines as follow:

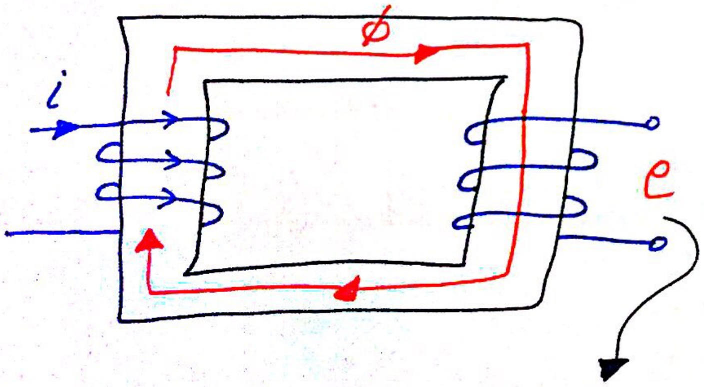
Principle $\boxed{1}$: Any current (i), generates Magnetic Field (ϕ) where

$\phi \propto i$ ⟹ That is for DC current ϕ will not change with time.
also for AC current ϕ will change with time.

* where ϕ is represented by closed lines.
⟹ its direction can be found by mean of RHR.
↳ "Right Hand Rule"



Principle $\boxed{2}$: If the generated ϕ links or cut another conductor it will induce a voltage (e) in it.

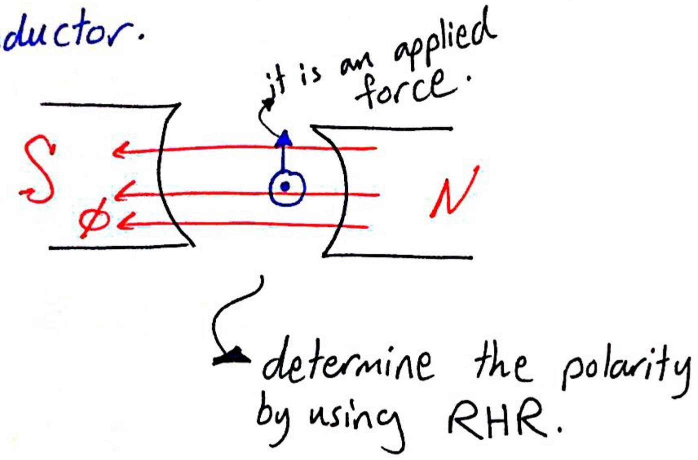


" This is the basic principle of transformer action. "

Determine the polarity by using Dot Convention.

Scanned by CamScanner

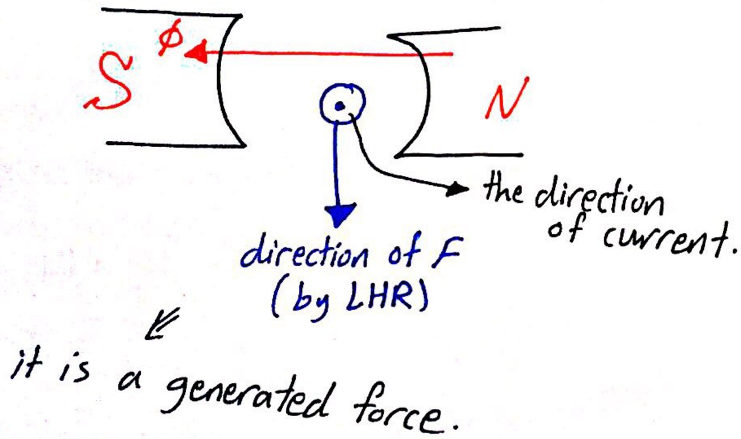
Principle $\boxed{3}$: If the conductor cut or move through a magnetic field then a voltage will be induced in the conductor.



"This is the generator" action.

Principle $\boxed{4}$: the current carrying conductor in a magnetic field will experience a force (F).

* Determine the direction of (F) by using LHR
↳ Left Hand Rule.



"This is the" motor action.

※ Basic Mathematical Laws & Concepts:

$\boxed{1}$ Amper's Law:

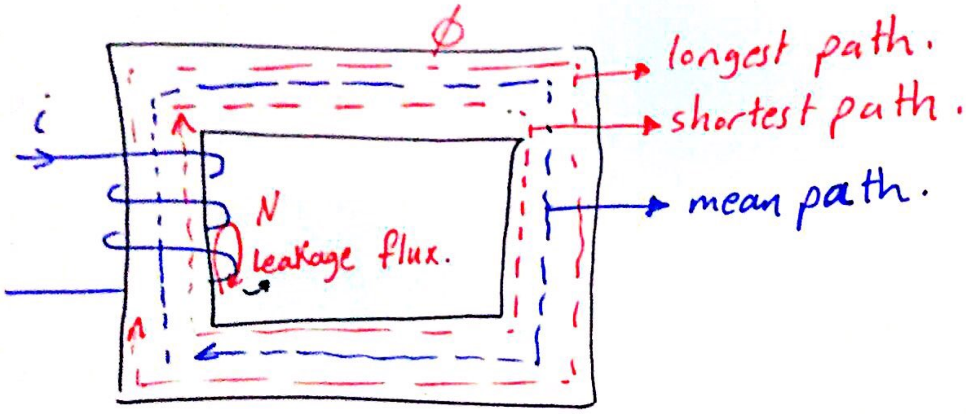
$$\oint \vec{H} \cdot d\vec{l} = Ni$$

H: Magnetic field intensity.
l: length of a closed Loop.
Ni: Total current enclosed by l:
$i \equiv$ Applied Current.
$N \equiv$ Number of Turns.

* for the mean length (l_c) of flux path:

$$H l_c = Ni$$

Scanned by CamScanner



2 Flux Density (B):

$\int \vec{B} \cdot d\vec{A} = \phi$ A: Cross section Area on which ϕ is $\perp$.

$BA = \phi \Rightarrow$ unit of B is Wb/m^2 or Tesla.

$\Rightarrow \boxed{B = \mu H}$ $\mu \equiv$ Permeability.

$\mu = \mu_o \mu_r$ $\mu_r \equiv$ Relative permeability of magnetic material being used.

$\mu_o \equiv$ free space permeability $= 4\pi * 10^{-7}$.

***

$Ni = Hl = \frac{B}{\mu} l$

$\therefore Ni = \frac{\phi}{A\mu} l = \phi \underbrace{\frac{l}{A\mu}}_{R} \Rightarrow \boxed{Ni = \phi R}$

where: $R = \frac{l}{A\mu}$ called Reluctance.

* Typical values of μ_r: $(2000 \to 6000)$.

* $\boxed{\phi = \frac{Ni}{R}}$ $\Rightarrow$ Hence, for a given mmf (i.e Ni) the higher the value of R the smaller the value of produced ϕ.

Note: since air has a very high R then in electrical Machines $\Rightarrow$ the air gap between stator & Rotor should be small as possible (i.e just a clearance to make rotation possible.

Scanned by CamScanner

※ Analogy between electrical & magnetic circuits:

electrical	magnetic
emf, e	mmf, Ni
i	ϕ
R	$\mathcal{R}$

∴ KCL & KVL: series & parallel combination Laws are applicable to magnetic cKts.

* However: the solution obtained by magnetic cKt concept is approximate.

why??

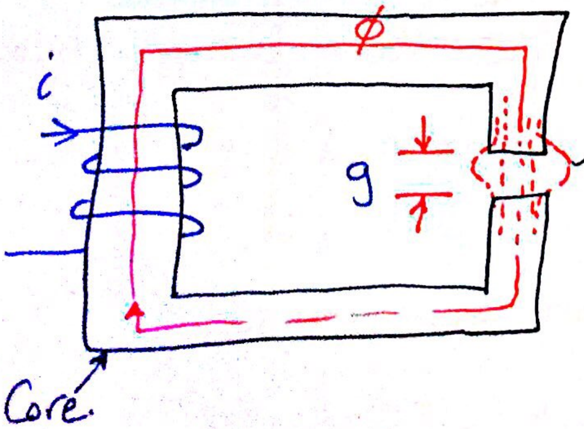
Because:

① Not all the generated flux flow in the core. There is some flux flow in the surrounding air called "Leakage flux".

② Mean length of flux path is used.

③ In solving one use $B = \mu H$ which assumes that μ is constant which is not in the real case as will be shown.

※ Magnetic Circuits with Airgap:



This called: Fringing field.

* Due to fringing field, then:

$$B_g = \frac{\phi}{A_g}$$

$B_g, A_g \equiv$ flux density and Area of the air gap.

$$\Rightarrow B_c = \frac{\phi}{A_c}$$

$B_c, A_c \equiv$ flux density and Area of the core.

$\Rightarrow g \equiv$ length of Airgap.

Scanned by CamScanner

* One of the emprical methods of calculating A_g is to add g to the length of sides which make A_c.

$\Rightarrow$ if $A_c = a * b$ then $A_g = ([a+g] * [b+g])$

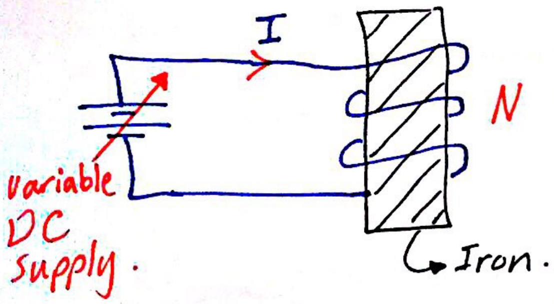
OR: $\{A_g = \text{a factor} * A_c\}$

* For e.g assume that A_g is increased by 5% :

$\therefore A_g = 1.05 * A_c$

※ Characteristic of Magnetic Material:

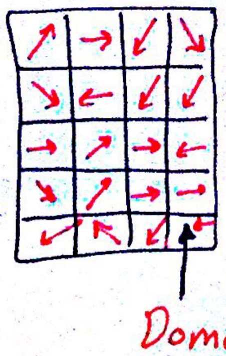
Consider the following structure:



* Gradually Increase I, then measure the generated flux. Assuming initially there is No flux in the Iron.

The result will be as follows:

$\Rightarrow$ This curve called: "Magnetization Curve."



* Each Domain can be considered as a permenant magnet.

↗ ≡ Direction of magnetic field in each domain.

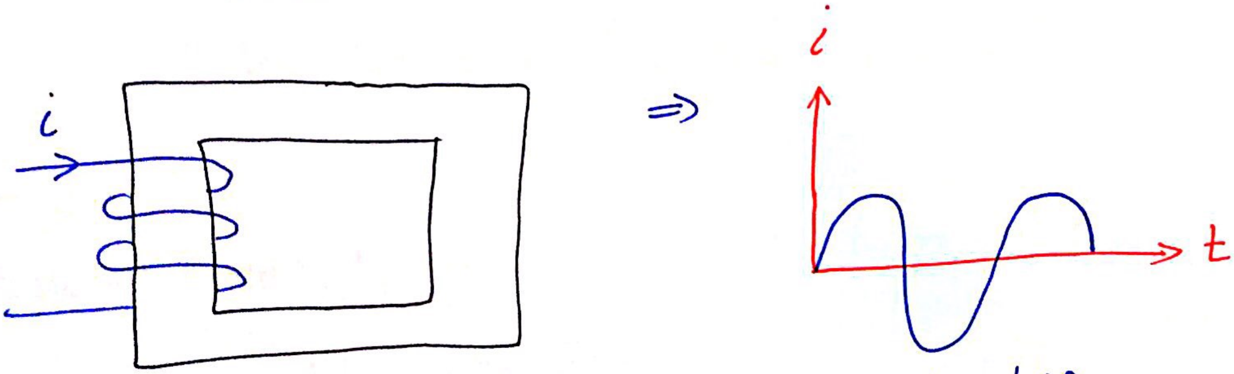
since $\phi = BA$, $\therefore B = \frac{\phi}{A}$ $\Rightarrow Ni = Hl$
$\Rightarrow \therefore H = \frac{Ni}{l}$

so the curve could be for:

ϕ OR B & Ni OR H

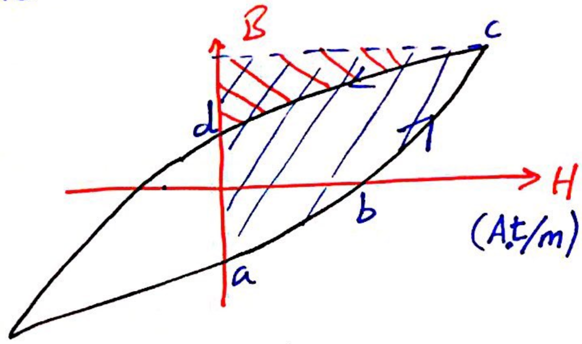
Consequantly, The curve is called "B-H curve."

※ Losses in Magnetic Materials: [Due to Magnetic Effect].



in the case of sinusoidal current the relationship between B and it is expressed in terms of Hysteresis Loop as follow:

$\therefore$ For each cycle of i there is one hysteresis Loop.



Area of H. loop $\iint H\,dB$

$\Rightarrow \int_{B_a}^{B_c} H\,dB \equiv$ Energy supplied by the source to magnetic material. .... $\boxed{1}$
Area

↳ it is the blue dashed area.

$\Rightarrow \int_{B_c}^{B_d} H\,dB \equiv$ Energy Returned to the supply. .... $\boxed{2}$

↳ it is the red dashed area.

Scanned by CamScanner

⇒ ∴ Difference between 1 & 2 gives the Area of $\frac{1}{2}$ H.loop which is equivalent to $\frac{1}{2}$ cycle of applied current.

* This difference represent the energy lost in the magnetic material due to the Domain's orientation.

∴ the Area of H.loop represent the energy lost due to one cycle applied current.

⇒ This called "Hysteresis Losses."

※ Analysis :

Area of H.loop : $A_h = HB$ --- 1 $\boxed{H \equiv A.t/m}$ --- 2

$$F = BIL \Longrightarrow B = \frac{F}{IL} \equiv \boxed{\frac{N}{A.m}} \text{ --- } 3$$

substitute 2 & 3 into 1, to find unit or dimension of A_h:

$$A_h \equiv \frac{A.t}{m} \cdot \frac{N}{A.m} = \frac{N}{m^2} \text{ --- } 4$$

⇒ But $J \equiv N.m$.

so $N = \frac{J}{m}$ --- 5

Now sub. 5 into 4 :

$$A_h = \frac{J}{m^3} \text{ --- } 6$$

∴ i.e Area of H.loop represent:

"Energy density"

Now 6 can be expressed as follows:

$$A_h = \frac{\text{Total Energy Lost}}{\text{Volume of ferro-magnetic material} * \text{Number of cycles}}$$

Scanned by CamScanner

$$\therefore \boxed{A_h = \frac{P_h * \text{Time (seconds)}}{V * \text{No. of cycles}}}$$

where:

$P_h \equiv$ hysteresis losses (Watt).

$$\hookrightarrow A_h = \frac{P_h}{V * \underbrace{\left(\frac{\text{No. of cycles}}{\text{seconds}}\right)}_{f}}$$

f → it is the frequancy of the applied current.

$$\Rightarrow \boxed{P_h = A_h V f}$$

* By performing various experiements A_h is expressed as:

$$\boxed{A_h = K_h B_m^n}$$

where:

$B_m \equiv$ Maximum or Peak value of flux density.

n & $K_h \equiv$ Constants depend on the type of ferromagnetic material.

* typically: $1.5 \leq n \leq 2.5$

$$\therefore \boxed{P_h = K_h B_m^n V f}$$

※ Eddey Current Losses:

if i is sinusoidal ⟹ generates sinusoidal ϕ in the core

⟹ ϕ_i induces a voltage e in the core, where $e \propto B_m f$

⟹ since the core is made of Conducting material Then

a current i_e will flow within the core, it flows in circular direction, hence it is called "Eddey Current."

Scanned by CamScanner

$\Rightarrow i_e \propto e \propto B_m f.$

Hence the so called Eddey current, losses P_e will be generated within the core $\Rightarrow P_e \propto i_e^2, B_m^2 f^2$

* P_e can be reduced by manufacturing the core of Laminations between which there is an insulating material in order to reduce i_e.

$\Rightarrow$ where the laminations are installed parallel to flux direction. $\rightarrow$ Consequantly the stacking factor is introduced and defined as:

$$\text{Stacking factor} = \frac{\text{volume of ferromagnetic material}}{\text{Total volume of the core}}$$

$\therefore P_e$ can be expressed in terms of:

$\tau \equiv$ Lamination Thickness.

$K_e \equiv$ Constant depend on the material.

$V \equiv$ volume of ferromagnetic material.

$B_m, f \equiv$ max. flux & frequancy.

* Eddey current losses P_e can be expressed as:

$$\boxed{P_e = K_e \tau^2 V B_m^2 f^2}$$

* The sum of P_e & P_h is called "Core Losses".

$$P_{core} = P_h + P_e$$

Scanned by CamScanner

ex. A given magnetic material has a core losses of 1800 W when the frequancy is 60 Hz. if the frequancy is increased by 50% and at the same flux density core losses become 3000 W Find P_e & P_h at each frequancy?

Solution:

since B_m is Kept constant and for the same core.

$\therefore P_h \propto f \qquad P_e \propto f^2$

$\therefore P_h = K_1 f \;,\; P_e = K_2 f^2$

for $f = 60$ Hz: $K_1 60 + K_2 3600 = 1800$ --- (1)

for $f = 90$ Hz: $K_1 90 + K_2 8100 = 3000$ --- (2)

⇒ solving: $\boxed{K_2 = \frac{1}{9}}$

f	P_e	P_h
60	$\frac{1}{9} * 60 * 60 = 400$	$1800 - 400 = 1400$
90	$\frac{1}{9} * 90 * 90 = 900$	$3000 - 900 = 2100$

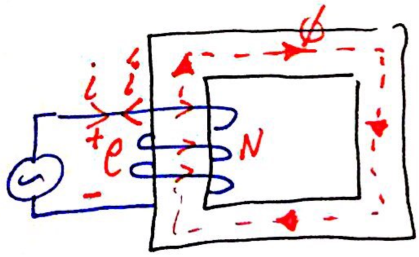
※ Self Inductance:

due to ϕ which is generated by i a voltage "e" is induced in the coil.

⇒ By Faraday's Law: $e = -N \frac{d\phi}{dt}$

Introduced to find the polarity of e.

Number of turns of the coil.



Scanned by CamScanner

$$\therefore e = N \frac{d\phi}{dt} \quad \text{---} \boxed{1}$$

$$Ni = \mathcal{R}\phi \qquad \therefore \phi = \frac{Ni}{\mathcal{R}} \quad \text{---} \boxed{2}$$

substitute $\boxed{2}$ into $\boxed{1}$:

$$e = N \frac{N}{\mathcal{R}} \frac{\partial i}{\partial t} = \frac{N^2}{\mathcal{R}} \frac{\partial i}{\partial t} = L \frac{\partial i}{\partial t}$$

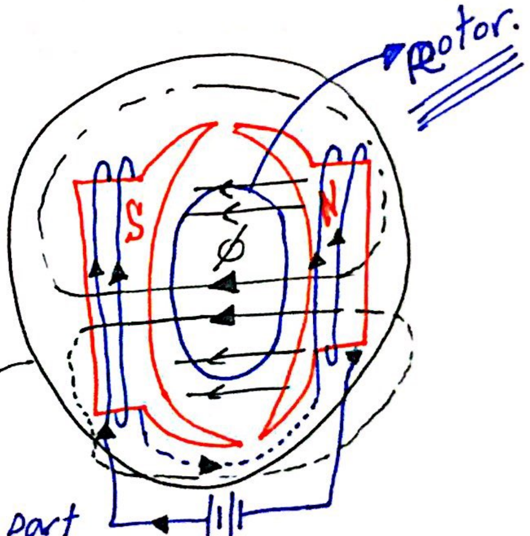
$$L = \frac{N^2}{\mathcal{R}} = \frac{N^2 \mu A}{\ell}$$ $\Rightarrow$ self inductance.

※ Application of Magnetic ccts to Electrical Machines:

* In Electrical Machines, Magnetic field is produced by Electromagnets as follow:

The polarity of the poles can be identified by mean of RHR.

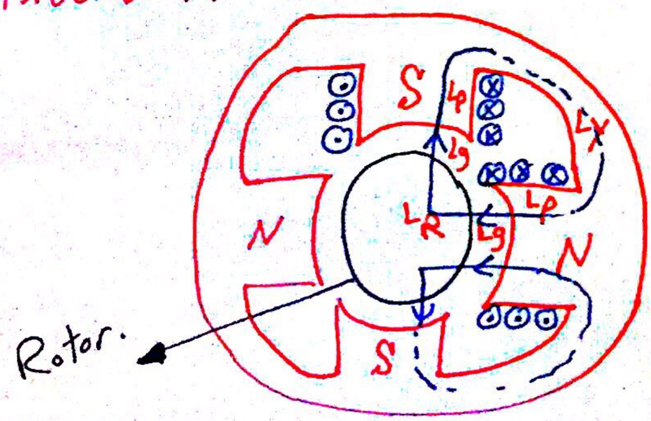
remember:
* Always flux leaving the (N) and entering the (S).



Yoke: It is the part through which magnetic flux Return.

$N \equiv$ number of turns per pole.

* Consider a 4 poles Machine:



$L \equiv$ length of flux path.

subscripts:

$P \equiv$ Pole

$g \equiv$ Airgap.

$R \equiv$ Rotor.

$Y \equiv$ Yoke.

Scanned by CamScanner

⇒ Apply Amper's Law:

$$\oint H \cdot dl = NI \Rightarrow H_p L_p + H_g L_g + H_R L_R + H_g L_g + H_p L_p + H_y L_y = NI + NI$$

$$\Rightarrow 2H_p L_p + 2H_g L_g + H_R L_R + H_y L_y = 2NI$$

$$\therefore \underline{NI} = H_p L_p + H_g L_g + \frac{1}{2} H_R L_R + \frac{1}{2} H_y L_y \quad \text{---- (1)}$$

↳ Amper-turns per pole.

**Procedure:

For the given specified flux density, then one from the given magnetization curve, then one can determine from the curve H_p, H_R and H_y.
Also given B_g since $B_g = \mu_o H_g$, then H_g can be calculated.
Knowing L_p, L_g, L_R and L_y ∴ From (1), Required $\underline{\underline{NI}}$ can be calculated.

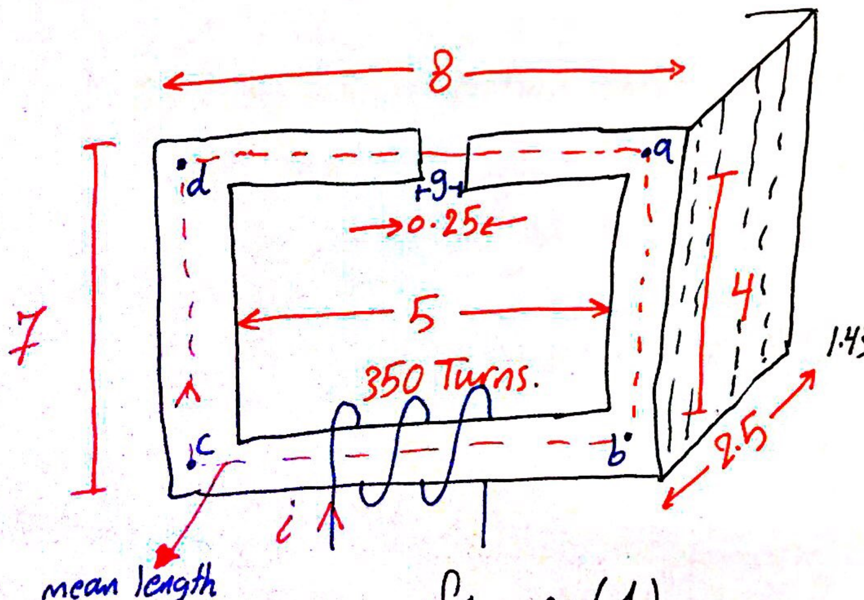


figure (1)

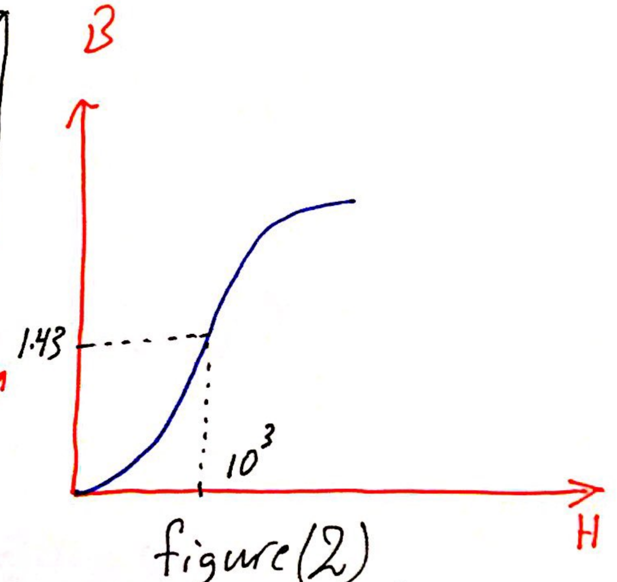


figure (2)

⇒

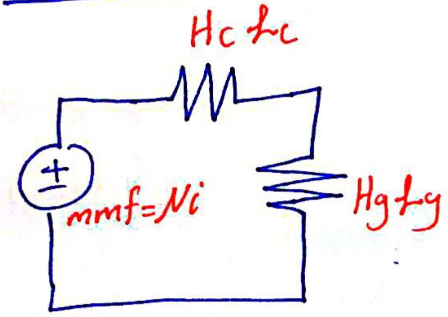
ex.) For the magnetic cct. shown in Figure (1) is made of alaminated steel core. It has the given magnetization curve shown in figure (2)

The stacking factor $= 0.93$

All dimensions in cm.

* Calculate the required exciting current i, To produce a flux of $5*10^{-4}$ Wb in the core? [Neglect leakage flux].

Solution:



* Net Area of the core $= (1.5*10^{-2} * 2.5*10^{-2} * 0.93)$

$$= \boxed{3.49*10^{-4}}\ m^2.$$

* mean length of flux path in steel. $= (ab+cd) + cb + (ad-g)$

$$= 2\left(4+\frac{1.5}{2}+\frac{1.5}{2}\right)+\left(5+\frac{1.5}{2}+\frac{1.5}{2}\right) + \left(5+\frac{1.5}{2}+\frac{1.5}{2}-0.25\right)$$

$$= \boxed{0.2375}\ m.$$

* flux density in the steel. $= \dfrac{\phi}{A}$

$$= \frac{5*10^{-4}}{3.49*10^{-4}} = \boxed{1.43}\ \text{Tesla}.$$

* ∴ from the magnetization curve, for $B = 1.43$ T. ⇒ it can be found that:

$$\boxed{H = 10^3}\ \text{A.t/m}.$$

$$\Rightarrow B_g = \frac{\phi}{A_g}\ ;\quad A_g = (1.5+0.25)*(2.5+0.25)*10^{-4} = \boxed{4.81*10^{-4}}\ m^2.$$

* $B_g = \dfrac{5*10^{-4}}{4.81*10^{-4}} = \boxed{1.04}\ T.$

* $B_g = \mu_o H_g \Rightarrow \boxed{H_g = 8.28*10^5}\ \text{A.t/m}.$

⟹ Continue.

⇒ ∴ By substitution:

$$\text{Total mmf} = Ni = \mathcal{F}_{steel} + \mathcal{F}_{gap} = H_c \ell_c + H_g g$$

$$\Rightarrow \quad Ni = 2307.5$$

so $\boxed{i = 6.6}$ Amp.

End of CH1.

CHAPTER(2):

※ Transformer:

What ?! ⇒ it is a device or Equipment which can be used to step-up or step-down a voltage or current.
(Voltage/Potential transformer) or (current transformer)

Why ?! *Objective:

→ 1) It can be used in the process of transmitting and distributing electrical energy with minimum losses. In this case it is called: "Power Transformer".

⇒ 2) It can be used in the process of measuring High currents & voltages. In this case it is called: "Instrument Transformer".

* There are 2 types: ① Voltage or Potential transformer (VT or PT).
② Current transformer (CT).

⇒ Here we are concerned with: "Power Transformer".

Scanned by CamScanner

* Power Transformer:

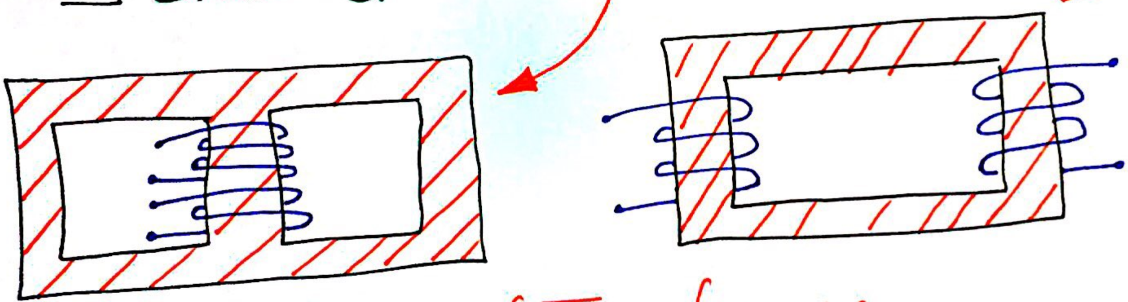
Construction:

It consists of:

1. Core.
2. A number of windings.

** Core: Basically There are:

1. Core Type: This is a rectangular core.
2. Shell Type: This is a 3-Leg. Core.



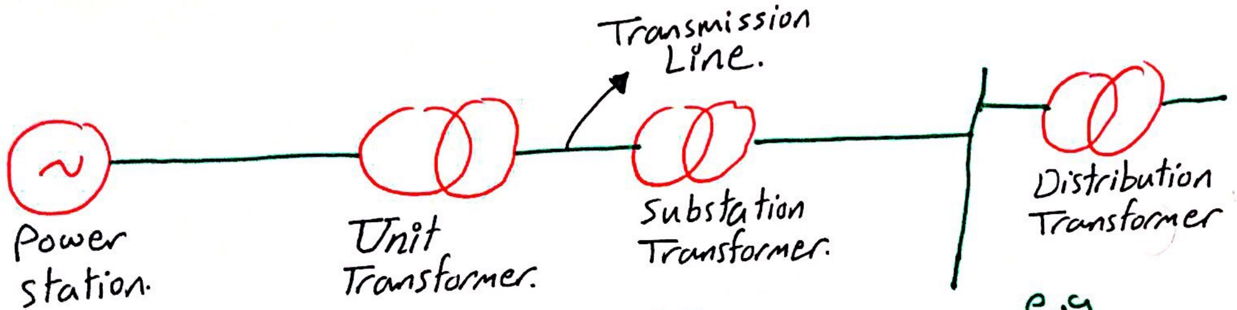
* Classification of Transformers:

1) Unit Transformer: This is at the power station used to step-up voltage for Transmision.

2) Substation Transformer: This is at the end of a transmission line and used to step-down voltage.

3) Distribution Transformer: This is used to step-down the voltage for various consumers.

continue for the figure.



Power station	Unit Transformer	Substation Transformer	Distribution Transformer
e.g	e.g	e.g	e.g
$V = 13.8\,KV$.	$(13.8/132)\,KV$.	$(132/33)\,KV$.	$(33/6.6)\,KV$.

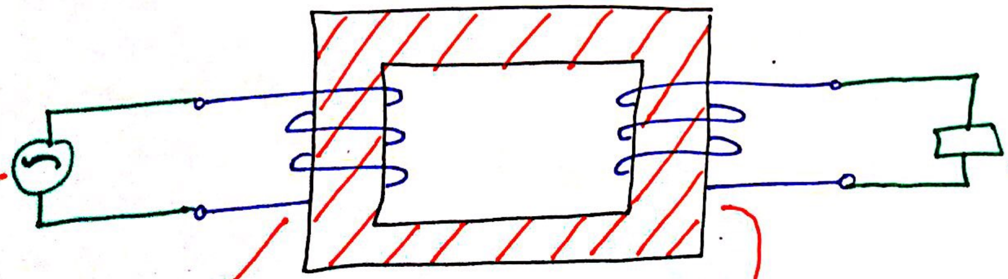
This figure Called: [Single Line Diagram Representation.]

Note:

In Jordan 2-transmission Levels:
[132 and 400] KVolt.

How?! (Analysis)

* Single-phase Transformer:



*Analysis of 1-ph Transformer:

※ Ideal Transformer:

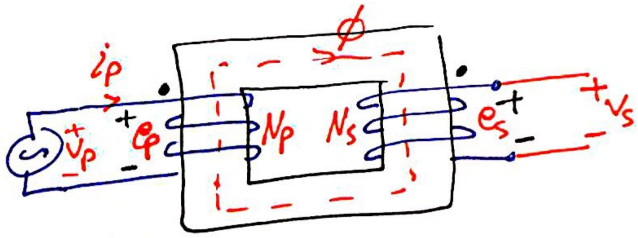
⇒ Assumptions:

1. Lossless Transformer. (i.e. No core Losses & No Electric Losses.)
2. No Leakage Flux.

⇒ Objectives:

- To find voltage, current and power Relationship for this transformer.

⇒ Procedure:

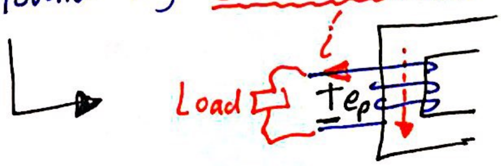


$v_p \equiv$ Primary Applied Voltage.
$i_p \equiv$ Primary Current.

* i_p generates ϕ.

$e_p, e_s \equiv$ The induced voltage in the primary & the secondary due to ϕ.

*** Polarity of e_p can be found by "Lenz's Law."



*** Polarity of e_s can be found by "Dot Convention."

* if the current enter the left dot will make the other dot (+).
* if the curren enter the undotted it will make the the other undotted (+).

⟹

$N_p, N_s \equiv$ Number of turns of primary & secondary windings.

by Faraday's Law $\boxed{e \triangleq N \dfrac{d\phi}{dt}}$

$V_s \equiv$ Called secondary Terminal voltage.

$\therefore \; V_p = e_p = N_p \dfrac{d\phi}{dt}$ ..... [1] $V_s = e_s = N_s \dfrac{d\phi}{dt}$ ..... [2]

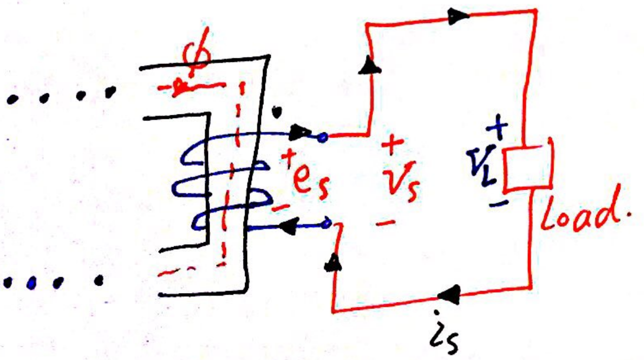
$\therefore$ divide [1] over [2]:

$$\frac{V_p}{V_s} = \frac{e_p}{e_s} = \frac{N_p}{N_s}$$

* Up to Know i_p is called "No-load Primary Current".

* Loading Condition:

⇒ Connect a load to the secondary.



↳ Then i_s is going to flow according to the convention of current flow in a source.

(i.e. The current is going to flow out from the Dot.)

⇒

⇒ i_s is going to generate (ϕ_s) → This ϕ_s opposes the main ϕ → Consequantly, Resultant flux in the core is going to be Reduced

→ Consequantly, $e_p \downarrow$ Hence i_p is going to increase by an amount (i_{pL}) in such away to produce additional mmf to oppose that produced by i_s

⇒ in order to maintain the same flux in the core.

$i_{pL} \equiv$ Load component of primary current.

$$\therefore \; i_{pL} N_p = i_s N_s \quad \cdots \boxed{3}$$

∴ Ingeneral: $i_p \equiv$ No Load Component (i_o) + Load Component (i_{pL}).

However: $i_{pL} >> i_o$

$$\therefore \; i_p \simeq i_{pL} \quad \cdots \boxed{4}$$

⇒ Substitute $\boxed{4}$ in $\boxed{3}$: $\cdots \boxed{5}$

$$\therefore \; i_p N_p = i_s N_s$$

$$\Rightarrow \frac{i_p}{i_s} = \frac{N_s}{N_p}.$$

$$\therefore \; \frac{e_p}{e_s} = \frac{i_s}{i_p} = \underbrace{\frac{N_p}{N_s}}_{a}$$

This called: Current-Voltage Relation.

$$\Rightarrow \boxed{a = \frac{N_p}{N_s}}$$

"Turns Ratio"

* $a = \frac{V_p}{V_s} = \frac{e_p}{e_s} = \frac{i_s}{i_p} = \frac{N_p}{N_s}$

since a is Real, Then e_p & e_s are "in-phase".
Also i_p & i_s are "in-phase".

∴ phase angle of Primary & Secondary ccts are equal.

$\therefore \frac{V_p}{V_s} = \frac{I_s}{I_p} = a$ where $V, I \equiv$ phasor voltages & currents.

⇒ we can reach that:

$|V_p||I_p| = |V_s||I_s|$ Then you can reach that primary & secondary have the Same apparent power (VA).

* $\begin{bmatrix} P_p = V_p I_p \cos\theta_p \\ P_s = V_s I_s \cos\theta_s \end{bmatrix}$

and we Know $\theta_p = \theta_s$

$\Rightarrow P_p = (aV_s) * \left(\frac{I_s}{a}\right) \cos\theta_s = P_s$

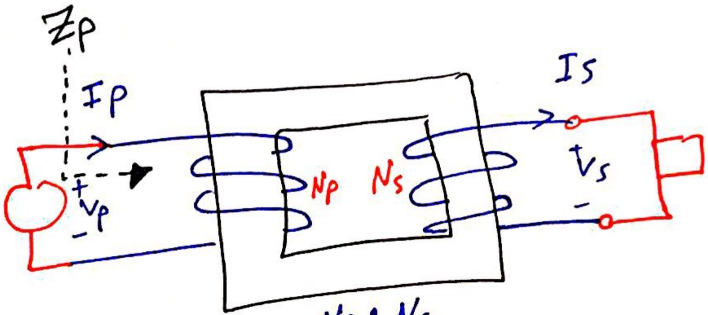
∴ For Ideal Transformer, $\eta = P_{out}/P_{in} = P_s/P_p = \frac{P_p}{P_s} = 1 = 100\%$

effeciency.

Scanned by CamScanner

※ Impedance Reflection:

$Z_P \equiv$ Impedance of Transformer seen from Primary Terminal.



$N_P : N_S$

$1 : \frac{N_S}{N_P} = a_1$

$a_2 = \frac{N_P}{N_S} : 1$

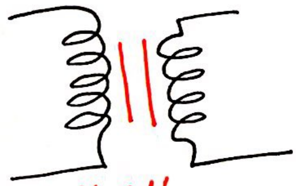
$$\boxed{Z_P = \frac{V_P}{I_P}} , \boxed{Z_S = \frac{V_S}{I_S}}$$

$$\Rightarrow Z_P = \frac{V_P}{I_P} = \frac{aV_S}{\frac{I_S}{a}} = a^2 \frac{V_S}{I_S} = a^2 Z_S$$

$$\Longrightarrow Z_P = a^2 Z_S$$

OR

$$Z_S = \frac{Z_P}{a^2}$$



$N_P : N_S$

$a = \frac{N_P}{N_S} : 1$

This called: (a) side

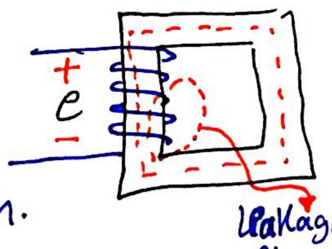
This called: (one) side.

** To Reflect an Impedance to a-side multiply it by $\underline{\underline{a^2}}$.

** To Reflect an Impedance to 1-side divide it by $\underline{\underline{a^2}}$.

※ Real/Practical Transformer:

* since each turn doesn't have the same flux, Then the concept of Flux Linkage (λ) and defined as follow:



$$\lambda = \sum_{i=1}^{N} \phi_i$$

$\phi_i \equiv$ flux passing through i^{th} turn.

$N \equiv$ Number of turns.

∴ Average flux Per turn ($\bar{\phi}$):

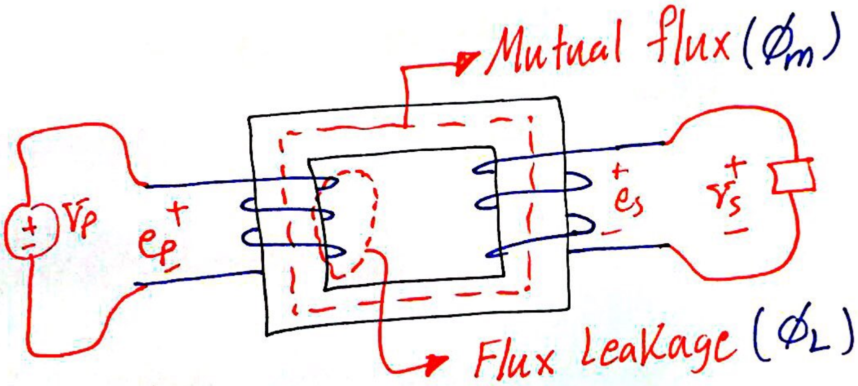
$$\bar{\phi} = \frac{\lambda}{N}$$

∴ Voltage induced (e):

by Faraday's Law: $$e = \frac{\partial \lambda}{\partial t}$$

$$\therefore e = \frac{\partial \lambda}{\partial t} = \frac{\partial (N\bar{\phi})}{\partial t}$$

$$\boxed{e = N\frac{d\bar{\phi}}{dt}}$$



* For the primary:

$$\therefore \bar{\phi}_p = \phi_m + \phi_{LP}$$

$\bar{\phi}_p \equiv$ Average flux in the primary.
$\phi_m \equiv$ Mutual flux.
$\phi_{LP} \equiv$ primary leakage flux.

* For the secondary:

$$\therefore \bar{\phi}_s = \phi_m + \phi_{LS}$$

$\bar{\phi}_s \equiv$ Average flux in the secondary.
$\phi_{LS} \equiv$ Leakge flux in the secondary.

$$\Rightarrow \boxed{\begin{aligned} e_p &= N_p \frac{d\bar{\phi}_p}{dt} = \underbrace{N_p \frac{d\phi_m}{dt}}_{e_{pm}} + \underbrace{N_p \frac{d\phi_{LP}}{dt}}_{e_{PL}} \\ e_s &= N_s \frac{d\bar{\phi}_s}{dt} = \underbrace{N_s \frac{d\phi_m}{dt}}_{e_{sm}} + \underbrace{N_s \frac{d\phi_{LS}}{dt}}_{e_{sL}}. \end{aligned}}$$

$$\therefore \begin{aligned} e_p &= e_{pm} + e_{pL} \\ e_s &= e_{sm} + e_{sL} \end{aligned}$$

$$\therefore \boxed{\frac{e_{pm}}{e_{sm}} = \frac{N_p}{N_s}}$$

Turns Ratio is equal to the ratio of induced voltage in secondary & primary due to mutual flux.

If:

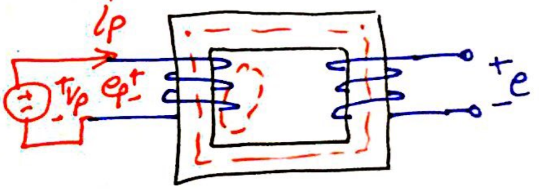
1. Resistance of the windings are neglected.
2. For a well designed Iron Transformer, ϕ_{LP} & ϕ_{LS} are very small, Then:

$$v_p = e_p = N_p \frac{d\bar{\phi}}{dt}$$
$$v_s = e_s = N_s \frac{d\bar{\phi}}{dt}$$
$$\Rightarrow \therefore \boxed{\frac{v_p}{v_s} = \frac{e_p}{e_s} = \frac{N_p}{N_i}}$$

※ Current Relationship:

* Consider No-load Condition:

i_p in this case is called ((Exitation Current))



$\Rightarrow$ For practical transformer, There is losses:

losses = core losses + electrical losses.

$$= \left((P_h + P_e) + i_p^2 R_p \right)$$

$R_p \equiv$ Resistance of primary windings.

usually $P_e + P_h >> i_p^2 R_p$

$\therefore$ i_p has two functions:

1. To generate flux. $\Rightarrow i_m$.
2. To supply or account for core losses. $\Rightarrow i_c$

$\therefore i_p = i_m + i_c$ → Loss Component.

i_m → Magnetization Component.

* Objective:

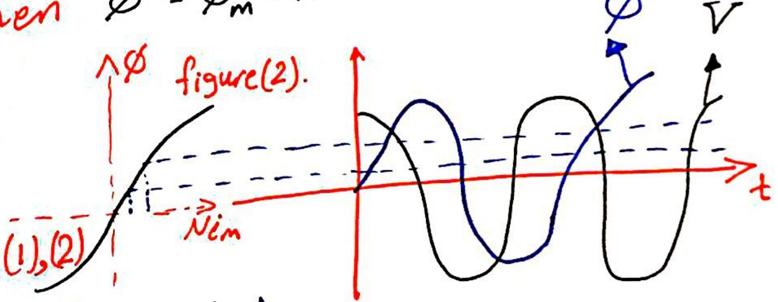
To find the wave shape of i_p.

1) find waveshape of i_m.
2) find wave shape of i_c.
3) Sum of the two waves.

$v = N\frac{d\phi}{dt} \Rightarrow \therefore \phi = \frac{1}{N}\int v\,dt$.

if $v = V_m \cos\omega t$, Then $\phi = \phi_m \sin\omega t$.

$\Rightarrow$ since $\underline{N i_m = \mathcal{R}\phi}$



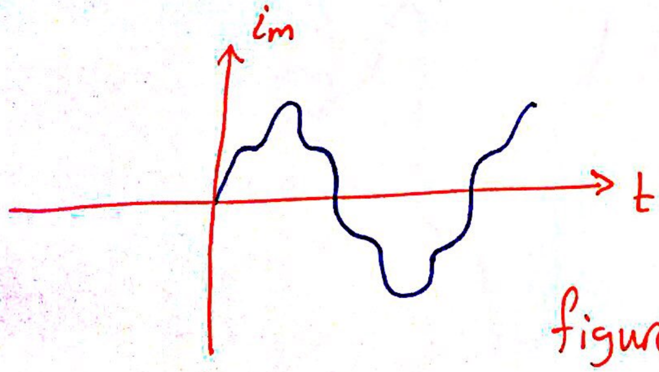
Then by using Figure(1),(2)

& the magnetization curve (ϕ Vs i_m)

figure (1).

$\Rightarrow$ one can graphically deduce the wave shape of i_m.

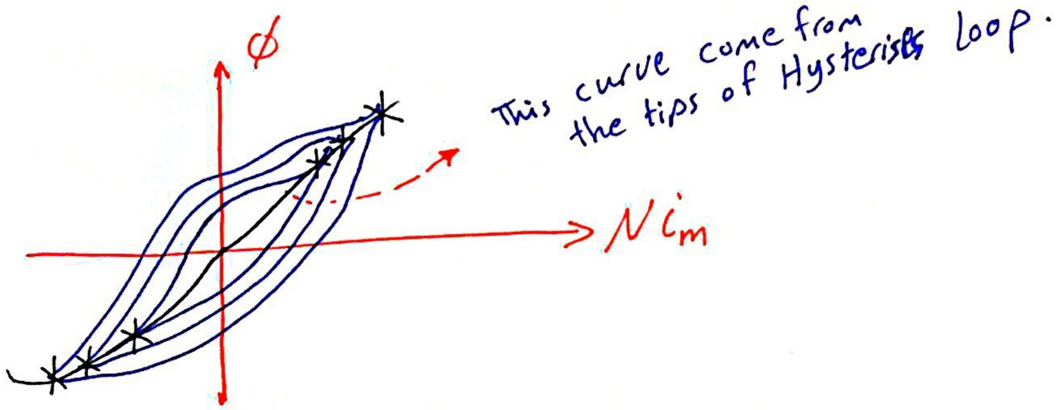
It can be found the shape will be as follows: $\Rightarrow$ figure (3)



$\Rightarrow i_m$ is Non-Pure sinusoidal

i.e <u>distorted</u>.

figure (3).

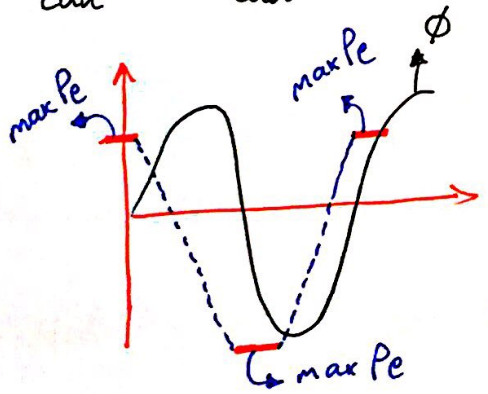


i_c: To represent eddy + hys. Losses.

$$i_{edd} \propto e_{edd} \propto \frac{d\phi}{dt}$$

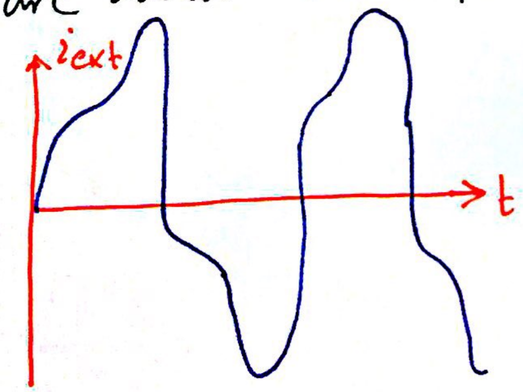
Since $P_{edd} \propto i_{edd}^2$

$\therefore P_e$ is maximum when $\phi = 0$



** P_{hys} is a non-linear Complex function of Applied voltage.

When i_m & i_c are added the shape of i_{ext} will be as follow:



* i_{ext} is a periodic Non-sinusoidal current.

Comment:

By using Fourier Series, any periodic waveform can be written as sum of cos & sin terms:

$$i_{ext} = \underbrace{I_{m_1}\cos\omega t}_{\text{Fundemental Component.}} + \underbrace{I_{m_2}\cos 2\omega t + I_{m_3}\cos 3\omega t + I_{m_4}\cos 4\omega t + \cdots}_{\text{Harmonics.}}$$

⇒ If (+ve) & (−ve) cycles are Identical, Then the expression has only odd harmonics.

$$\therefore \; \boxed{i_{ext} = I_{m_1}\cos\omega t + I_{m_3}\cos 3\omega t + I_{m_5}\cos 5\omega t + \cdots}$$

※ Equivalent circuit of Real Transformer:

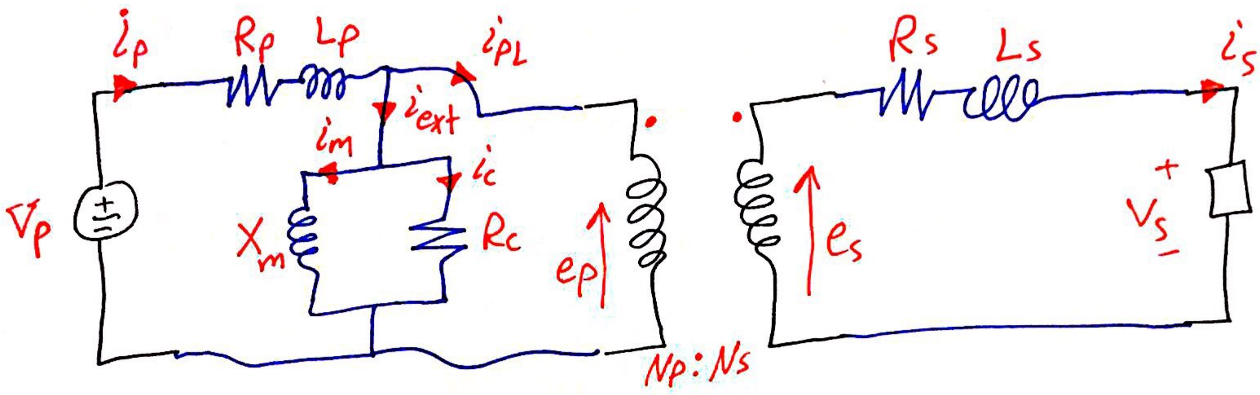
The equivalent cct should take the followings into account:

1] Electrical Losses in the primary & secondary windings. (I^2R).

2] Core Losses.

3] LeaKage flux. (i.e $N\frac{d\phi_L}{dt}$).

⟹



***

$R_p, R_s \equiv$ To represent electrical Losses in primary & Secondary.

$L_p, L_s \equiv$ To represent LeaKage flux in primary & secondary.

$R_c \equiv$ To represent Core Losses.

$X_m \equiv$ To represent generated flux by i_m.

$i_{PL} \equiv$ Load component of primary current.

$i_{ext} \equiv$ No-load. $\quad$ $\underline{i_p = i_{ext} + i_{PL}}$

$e_p, e_s \equiv$ Induced voltages in primary & secondary.

$V_p \equiv$ Primary applied voltage.

$V_s \equiv$ Terminal Secondary voltage.

$\Downarrow$

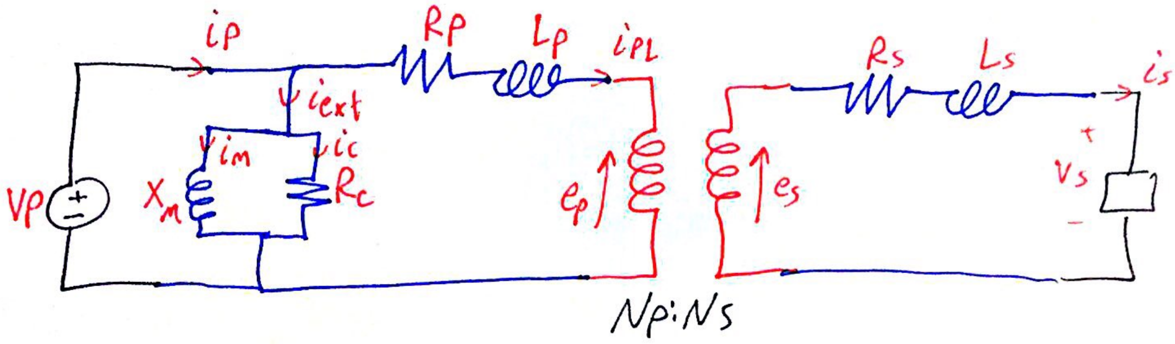
This circuit called

"Exact equivalent Circuit"

Scanned by CamScanner

⇒ Since $I_{ext} << i_{PL}$ & the voltage drop in R_p & L_p is very small, Then:

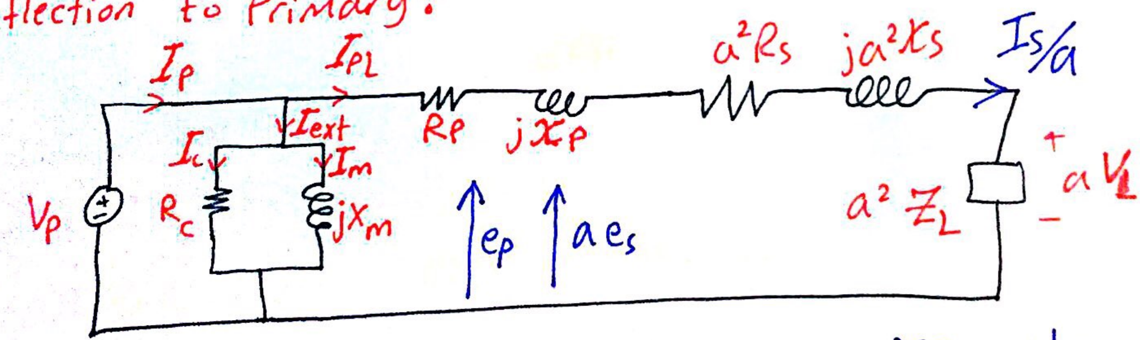
The parallel branch ($X_m // R_c$) can be shifted to the input.



$a = \frac{N_p}{N_s} : 1$

⇒ one can Reflect Primary to secondary or other way round.

*Reflection to Primary:



$X_p = \omega L_p$
$X_s = \omega L_s$

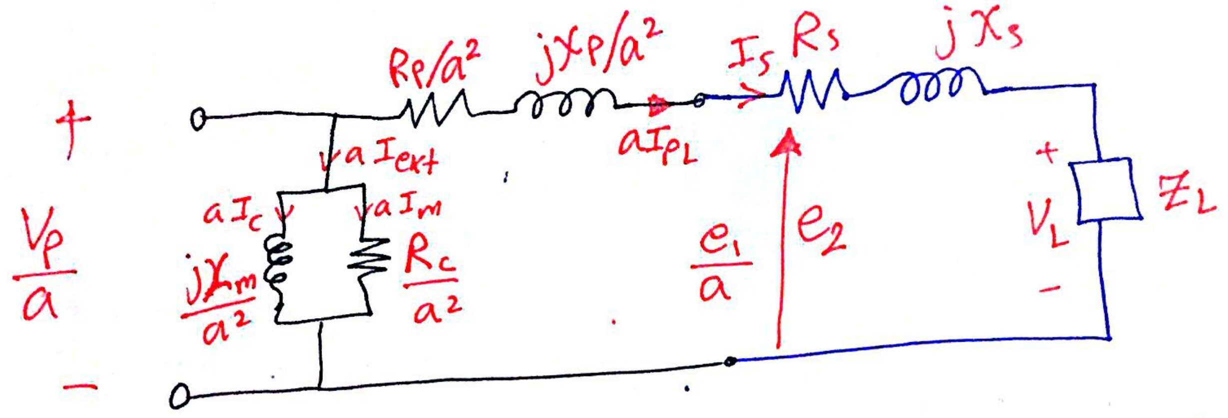
$$R_{eq(P)} = R_p + a^2 R_s$$
$$X_{eq(P)} = X_p + a^2 X_s$$

$$\boxed{I_{ext} = I_c + I_m}$$

R_{eq}, X_{eq} ≡ Equivalent Resistance & Reactance of the transformer. (series).

Scanned by CamScanner

*Now Reflection to Secondary:



$$\boxed{\begin{aligned} R_{eq(s)} &= R_P/a^2 + R_S \\ X_{eq(s)} &= \frac{X_P}{a^2} + X_S \end{aligned}}$$

*Notice that:

In Ω $R_{eq(P)} \neq R_{eq(s)}$

$X_{eq(P)} \neq X_{eq(s)}$

*But However as will be shown later in PU system R_{eq} & X_{eq} is the same irrespective of the reflection side.

** $\boxed{Z_{eq} = R_{eq} + jX_{eq}}$

* Evaluation of Transformer Parameter:

What?! ⇒ To measure R_{eq}, X_{eq}, X_m & R_c.

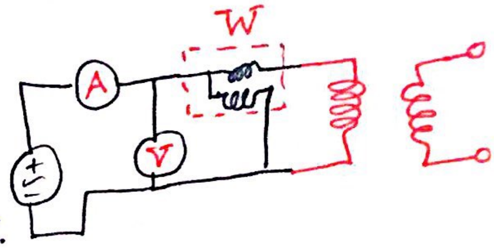
Why?! ⇒ Because these parameters are required in the process of Transformer analysis.

How?! ⇒ By Performing o/c & s/c Test on the Transformer.

Scanned by CamScanner

※ O/C Test:

1] o/c the secondary (i.e $Z_L = \infty$).
2] Apply Rated Voltage to the Primary & take the readings of input voltage, current & power.
(i.e A, V, W)



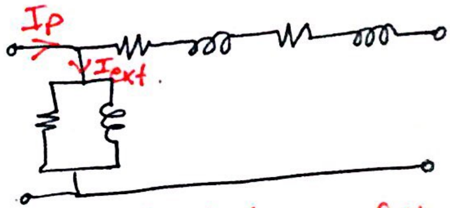
* Illustration:

e.g. Transformer Rateing (132/33) kV.

Note: As a Convention in the o/c test the low voltage side is made as primary because it is easier to measure its rating voltage. Hence, Readings are taken on LV side.

* Analysis:

This can be performed by using the equ. cct. of S → P.



$\therefore \frac{I_s}{a} = I_{PL} = 0$

$\therefore I_p = I_{ext} = I_c + I_m$

$\therefore$ In the o/c test R_c & X_m can be evaluated as follows:

$I_p \equiv$ Ammeter Reading.

$V_p \equiv$ Voltmeter Reading.

Wattmeter Reading $\equiv$ losses in the cct. $\equiv$ losses in R_c

** Calculations:

Wattmeter reading $P_{oc} = \frac{V_{oc}^2}{R_c} = \frac{(\text{voltmeter reading})^2}{R_c}$

$\therefore R_c = \frac{V_{oc}^2}{P_{oc}}$

Admittance $|Y| = \frac{\text{Ammeter reding}}{\text{Voltmeter reading}}$

$\Longrightarrow$

$$\Rightarrow \quad Y = \frac{1}{R_c} + \frac{1}{jX_m} \quad \Rightarrow |Y| = \sqrt{\left(\frac{1}{R_c}\right)^2 + \left(\frac{1}{X_m}\right)^2}$$

$$\therefore \quad \frac{1}{X_m} = \sqrt{|Y|^2 - \left(\frac{1}{R_c}\right)^2}$$

Another way to find X_m:

$Y = |Y| \angle -\theta$

$P_{oc} = V_{oc} I_{oc} \cos\theta$

$\Rightarrow \theta = \cos^{-1}\left[\frac{P_{oc}}{I_{oc} V_{oc}}\right]$

$\Rightarrow$ find Y then: $Y = \frac{1}{R_c} + \frac{1}{jX_m}$

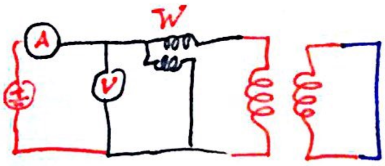
$\Rightarrow$ find X_m.

S/C Test:

1] S/c the terminal of the secondary.

2] Apply Rated current to the primary and take the Reading of W, V & A

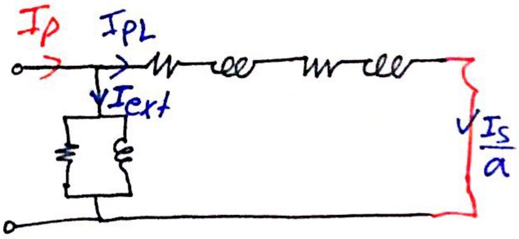
(i.e P_{sc}, V_{sc} & I_{sc})



As a Convention the HV is made primary.

* Analysis:

using equivalent circuit:



since $I_{PL} >> I_{ext}$ (i.e core losses are neglected)

$\therefore I_{PL} = I_p =$ Ammeter Reading. $\equiv I_{sc}$.

$\therefore Z_{eq} = R_{eq} + jX_{eq}$.

Wattmeter reading $P_{sc} = I_{sc}^2 R_{eq}$. $\quad \therefore R_{eq} = \frac{P_{sc}}{I_{sc}^2}$

$|Z_{eq}| = \frac{V_{sc}}{I_{sc}} = \sqrt{R_{eq}^2 + X_{eq}^2}$ $\quad \therefore X_{eq} = \sqrt{|Z_{eq}|^2 - R_{eq}^2}$

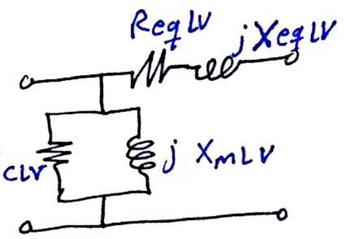
$\Rightarrow$ OR: $Z_{eq} = |Z_{eq}| \angle \theta \Rightarrow \theta = \cos^{-1}\left[\frac{P_{sc}}{V_{sc} I_{sc}}\right]$

ex.) 1380 VA, 230/115 V Transformer has been tested to detemine its equivalent cct. The results were as follows:

O/C Test	S/C Test
V = 230 Volt.	V = 13.2 Volt.
I = 0.45 A.	I = 6 A.
P = 30 W.	P = 20.1 W.

* Find the equivalent cct referred to Low voltage side?

Solution:



* Since in o/c test the applied voltage should be the rated one.

∴ In this example the applied voltage is that of HV.
∴ In the o/c Test the HV was primary.

* in s/c test, Rated current should be applied.

$I_{rated,HV} = 1380/230 = 6A$
$I_{rated,LV} = 1380/115 = 12A.$

$\Rightarrow$ from the given results it can be deduced that in the s/c test primary was HV.

* o/c test results:

Here R_{CHV} & X_{mHV} can be evaluated as follows:

$P_{oc} = \frac{V_{oc}^2}{R_c} \Rightarrow R_c = \frac{V_{oc}^2}{P_{oc}} = \frac{230^2}{30} \Rightarrow \boxed{R_{C_{HV}} = 1763.3\Omega}$

$|Y| = \sqrt{\left(\frac{1}{R_c}\right)^2 + \left(\frac{1}{X_m}\right)^2} \Rightarrow |Y| = \frac{I_{oc}}{V_{oc}} = \frac{0.45}{230} \Rightarrow \boxed{|Y| = 0.001957\,S}$

$\rightarrow$ solving: $\boxed{X_{m_{HV}} = 533.9\,\Omega}$

continue $\Longrightarrow$

Scanned by CamScanner

$\Rightarrow$ or use: $Y = |Y| \angle\theta$, $\theta = \cos^{-1}\left[\frac{P_{os}}{V_{oc} I_{oc}}\right]$

* S/C test results:

Here Req_{HV} & Xeq_{HV} are evaluated as follows:

$Zeq = Req + jXeq$

$P_{sc} = I_{sc}^2 * Req$

$\Rightarrow Req = P_{sc}/I_{sc}^2 = \frac{20.1}{36}$

$\Rightarrow \boxed{Req_{HV} = 0.558\,\Omega}$

$|Zeq| = \frac{V_{sc}}{I_{sc}} = \sqrt{Req^2 + Xeq^2} = \frac{13.2}{6} = 2.2$

$\hookrightarrow$ solving: $\boxed{Xeq_{HV} = 2.128\,\Omega}$

230 115

$N_{HV} = 230$, $N_{LV} = 115$

$a_1 = \frac{230}{115} : 1$

$a_1 = 2 : 1$

OR

$1 : \frac{115}{230} = 0.5 = a_2$

$R_{CLV} = \frac{R_{CHV}}{a_1^2}$ & $X_{mLV} = \frac{X_{mHV}}{a_1^2}$

$Req_{LV} = \frac{Req_{HV}}{a_1^2}$ & $Xeq_{LV} = \frac{Xeq_{HV}}{a_1^2}$

or if we use a_2 multiply by a_2^2.

as a final answer:

$\boxed{R_{CLV} = 440.8\,\Omega}$, $\boxed{X_{mLV} = 133.3\,\Omega}$, $\boxed{Req_{LV} = 0.139\,\Omega}$, $\boxed{Xeq_{LV} = 0.532\,\Omega}$

* Per Unit (PU) system:

What?! Definition: Here the basic quantities V, I, Z & S are not expressed by units of voltage, current, ohm, VA but they are expressed as a Fraction of a Reference or Base value.

$$\text{PU value} = \frac{\text{Actual Value}}{\text{Base OR Reference value.}}$$

PU it is dimensionless & Real +ve value.

Scanned by CamScanner

⇒ Actual value : could be Complex.

∴ PU value & Actual value : have the same value of angle.

Why?! Advantages:

1) As will be shown, it simplifies the process of solving system with many transformers.

Ex. R_{eq}, X_{eq}, R_c, X_m have the same value irrespective of the reflection side.

2) The Parameters of generators & transformers (i.e R, X) have the same range of values.

3) It simplifies the solution of 3-ph system, where $\sqrt{3}$ does not appear.

How?! Procedure:

since there are 4-quantities V, I, Z & Power, Then one may select the Base values for any Two quantities, Then Calculate the Base values for other two quantities.

* Usually, The base values of V & S are selected (i.e V_b & S_b)

$$\therefore \quad I_b = \frac{S_b}{V_b} \qquad Z_b = \frac{V_b}{I_b} = \frac{V_b}{S_b/V_b} = \frac{V_b^2}{S_b}$$

* Usually, Selected base values are the Rating of a component in the system.

Scanned by CamScanner

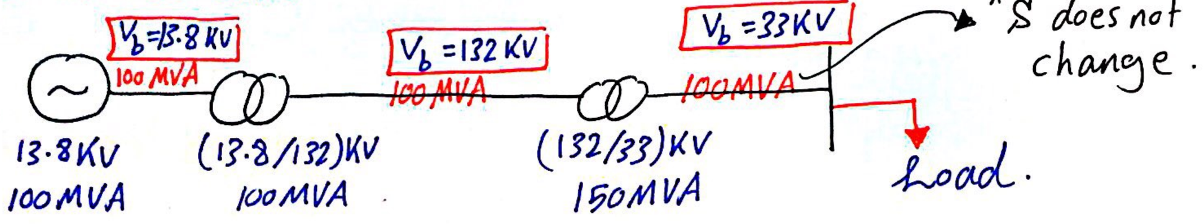
*** Note:**

since a transformer has the same S for its HV & LV sides; Then S_b does Not change one move from one side to other.

However, the voltage changes when one move from one side to other side.

This change should be according to voltage or Turns Ratio.

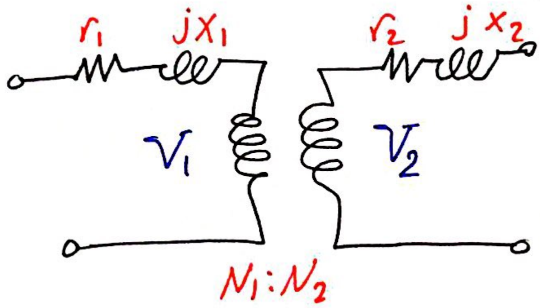
*** Illustration:**



Let $S_b = 100\,MVA$.

$V_b = 13.8\,KV$.

*** $Z_{eq}(PU)$ For Transformer:**



$a = \frac{N_1}{N_2} : 1$

$\Rightarrow$ reflect to a-side:

$R_{eq} = r_1 + a^2 r_2$

$X_{eq} = X_1 + a^2 X_2$

* let Base voltages be V_1, V_2.

where $\frac{V_1}{V_2} = a$

* let Base Power $= S$.

$\therefore\ Z_{1b} = V_1^2/S$

$Z_{2b} = V_2^2/S$

$\therefore\ R_{eq(PU)} = \frac{R_{eq_1}}{Z_{1b}} = \frac{R_{eq_1} S}{V_1^2}$

$\Longrightarrow$

$$\therefore \boxed{R_{eq_1}(PU) = (r_1 + a^2 r_2)\frac{S}{V_1^2}} \quad ---(1)$$

$$\boxed{X_{eq_1}(PU) = (X_1 + a^2 X_2)\frac{S}{V_1^2}} \quad ---(2)$$

* Reflect to (1)-side :

$$R_{eq_2} = \frac{r_1}{a^2} + r_2$$

$$X_{eq_2} = \frac{X_1}{a^2} + X_2$$

$$\Rightarrow \therefore R_{eq_2}(PU) = \frac{R_{eq_2}}{Z_{b_2}}$$

$$\boxed{R_{eq_2}(PU) = \left(\frac{r_1}{a^2} + r_2\right)\frac{S}{V_2^2}} \quad ---(3)$$

since $\frac{V_1}{V_2} = a$, $\therefore \boxed{V_2 = \frac{V_1}{a}}$ ---(4)

$\Rightarrow$ Substitute (4) into (3):

$$R_{eq_2}(PU) = \left(\frac{r_1}{a^2} + r_2\right)\frac{S a^2}{V_1^2}$$

$$\therefore \boxed{R_{eq_2}(PU) = (r_1 + a^2 r_2)\frac{S}{V_1^2}} \quad ---(5)$$

* similarly:

$$\boxed{X_{eq_2}(PU) = (X_1 + a^2 X_2)\frac{S}{V_1^2}} \quad (6)$$

*** Conclusion:

By Comparing (1) with (5) $\Rightarrow$ $R_{eq_1}(PU) = R_{eq_2}(PU)$

By Comparing (2) with (6) $\Rightarrow$ $X_{eq_1}(PU) = X_{eq_2}(PU)$

$\therefore$ $R_{eq}(PU)$ & $X_{eq}(PU)$ are the same irrespective of the side reflection.

Scanned by CamScanner

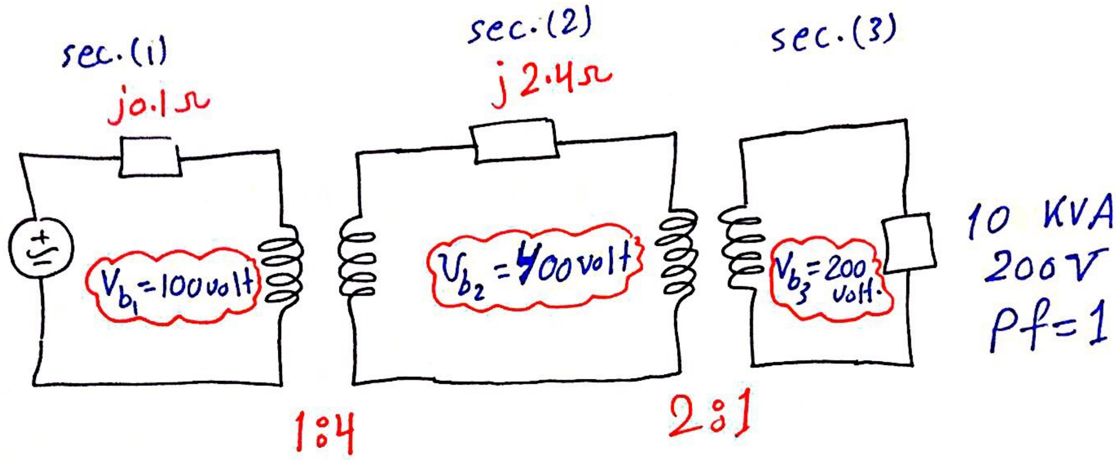


figure (1).

ex. Find the PU equivalent cct of the cct shown in figure (1). Take the rating of the load as Base values: $\therefore\ V_b = 200$ Volt.

$S_b = 10$ KVA.

$\Rightarrow S_b = 10$ KVA for all sections.

Next, Calculate Z_b for each section $Z_b = V_b^2 / S_b$

$$Z_{b_1} = \frac{(100)^2}{10 * 10^3}$$

$$Z_{b_2} = \frac{(400)^2}{10 * 10^3} \Rightarrow$$

$$Z_{b_3} = \frac{(200)^2}{10 * 10^3}$$

$Z_{b_1} = 1\Omega$

$Z_{b_2} = 16\,\Omega$

$Z_{b_3} = 4\,\Omega$

Now the PU values:

$j0.1\Omega \Rightarrow \frac{j0.1}{1} = \boxed{j0.1}$ PU.

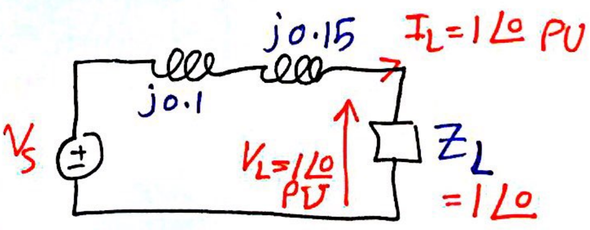
$j2.4\Omega \Rightarrow \frac{j2.4}{16} = \boxed{j0.15}$ PU.

why!?

$Z_L = \frac{V^2}{S} = 4\Omega$ $(Z_{b_3} = Z_L) \Rightarrow \boxed{Z_{L(PU)} = 1\text{PU}}$

Because its rating were selected as Base value.

* PU equ. cct:



$\Rightarrow$ for I_L:

$$I_L = \frac{10^4}{200} \angle 0 \ A \quad \Rightarrow I_{Lb} = \frac{10^4}{200}$$

$$I_L(PU) = \frac{I_L}{I_{Lb}} = \boxed{1 \angle 0}$$

for V_s:

$$V_s(PU) = 1\angle 0 \,(j0.1 + j0.15) + 1\angle 0$$

$$\Rightarrow \boxed{V_s(PU) = 1.0308 \angle 0.24^\circ}$$

Comment:

Given an impedance, Say $Z\,\Omega$ if the base value is

$$Z_{b_{old}} = \frac{V_{old}^2}{S_{old}} \quad \therefore Z(PU) = \frac{Z}{Z_{b_{old}}} = \frac{Z\, S_{old}}{V_{old}^2} \quad \ldots \boxed{1}$$

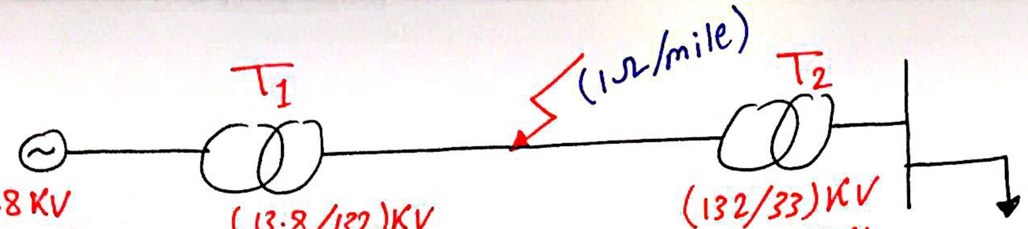
If the base value of Z is changed, say $Z_{b_{new}} = \frac{V_{new}^2}{S_{new}}$

$$\Rightarrow Z_{new}(PU) = \frac{Z\, S_{new}}{V_{new}^2} \quad \ldots \boxed{2}$$

$$\Rightarrow \frac{\boxed{2}}{\boxed{1}} \Rightarrow \frac{Z_{new}(PU)}{Z_{old}(PU)} = \frac{S_{new}}{S_{old}} * \frac{(V_{old})^2}{(V_{new})^2}$$

$$\therefore Z_{new}(PU) = Z_{old}(PU) * \left(\frac{V_{old}}{V_{new}}\right)^2 * \left(\frac{S_{new}}{S_{old}}\right)$$

Scanned by CamScanner



13.8 KV
100 MVA
$X = 0.01\,PU$
$= 1\%$

(13.8/132) KV
200 MVA
$X = 0.02\,PU$
$= 2\%$

(132/33) KV
200 MVA
$X = 0.03\,PU$
$= 3\%$

for example: let the base value be:

$$V_b = 13.8\,KV \quad \& \quad S_b = 100\,MVA.$$

for T_1:
$$X_{new} = X_{old} * \left(\frac{V_{old}}{V_{new}}\right)^2 * \left(\frac{S_{new}}{S_{old}}\right)$$
$$= 0.02\,(1)^2 * \left(\frac{100}{200}\right) \Rightarrow \boxed{X_{new} = 0.01}$$

---

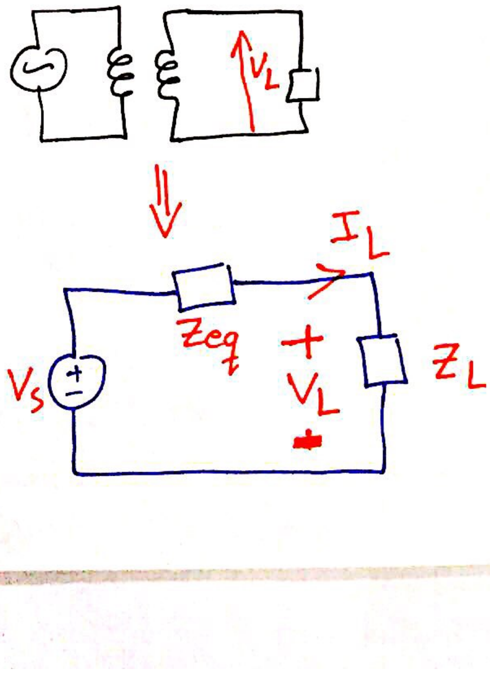
end of first material.

* Voltage Regulation:

This is a measure to Indicate the amount of voltage drop at the load due to the equivalent Impedance of the Transformer.

**

$$\Rightarrow V_L = V_s - I_L Z_{eq}$$
$$= V_s - I_L (R_{eq} + j X_{eq})$$



* Voltage Regulation:

$$\boxed{V_R\,\% \triangleq \frac{\underbrace{|V_L(NL)| - |V_L(FL)|}_{\text{magnitude}}}{|V_L(FL)|} * 100\%}$$

$\Rightarrow$ $V_L(NL)$ means $I_L = 0$

$\therefore$ $\boxed{V_L(NL) = V_S}$

* Sometimes V_L is called V_R.
↳ Receiving.

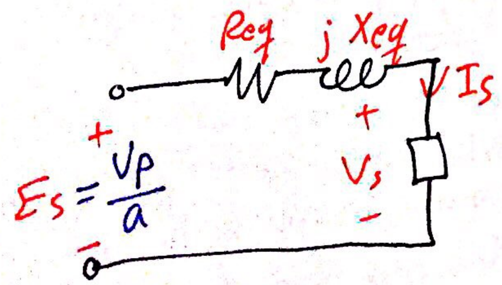
* Objective:

Is to see the magnitude of V_R for various loading conditions (i.e Inductive, Resistive, Capacitive) By Keeping $|\overset{V_s}{V_L}|$ & $|\overset{I_s}{I_L}|$ constants.
The pf angle is changed in order to find required E_S.

* Procedure:

By using the concept of phasor diagram. taking V_S as Refererence.

* Voltage regulation: Here usually the equivalent cct Reflected to secondary is used.



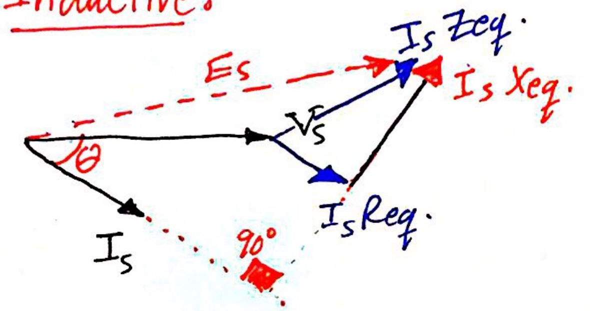
* Note: $V_S(NL) = E_S = \frac{V_P}{a}$

$$a = \frac{N_P}{N_S}$$

$$E_S = I_S\underbrace{(R_{eq} + jX_{eq})}_{\Rightarrow Z_{eq}} + V_S$$

Scanned by CamScanner

* Inductive:

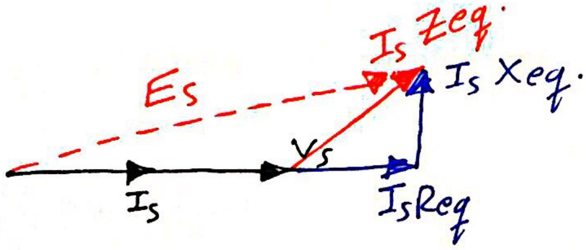


$\therefore |E_s| > |V_s|$

so we can reach that:

VR is +ve.

* Resistive:

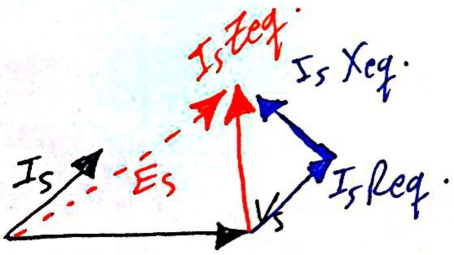


$|E_s| > |V_s|$ $\therefore$ VR is +ve.

But:

$|E_s|_{resistive} < |E_s|_{inductive}$.

* Capacitive:



$|E_s| < |V_s|$

$\therefore$ VR is -ve.

* Definition:

$\Rightarrow$ Ferranti Effect:

it is the increase in voltage occurring at the receiving end of along Transmission Line above the voltage at the sending end and it is more pronounced as how much the longer the line & the higher the voltage applied, also more pronounced in underground cables.

ex. A 10 KVA, 60 Hz, (4800/240) V transformer is tested by CONVENTIONAL procedure with the following results:

O/C : 240 V, 1.5 A, 60 W.

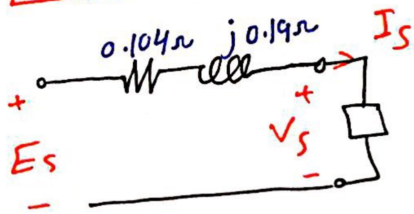
S/C : 180 V, rated A, 180 W.

1] Find the VR of this step-down Transformer, at unity pf at full load? (full Load → it is the quantity of current = $\frac{\text{apparent power}}{\text{voltage}}$.)

since step-down ⇒ secondary is LV.

By using calculations it can be found: $\boxed{R_{eq_{LV}} = 0.104\,\Omega}$

$\boxed{X_{eq_{LV}} = 0.19\,\Omega}$



$\therefore \boxed{V_s = 240\angle 0^\circ}$ volt.

$I_s = \frac{10*10^3}{240}\angle 0^\circ$

$\therefore E_s = I_s(0.104 + j0.19) + 240\angle 0$

$\Rightarrow \boxed{E_s = 244.17 + j7.92}$ volt. $\Rightarrow |E_s| = 244.3$ volt.

$\therefore VR = \frac{244.3 - 240}{240} * 100\% \Rightarrow VR = 1.8\%$

* Efficiency: (η)

$$\eta \triangleq \frac{\text{output}}{\text{input}} = \frac{\text{output}}{\text{output} + \text{Losses}}$$

Electrical Losses

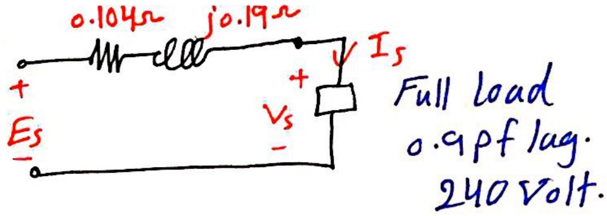
Core Losses.

continue. ⇒

Scanned by CamScanner

* for the last ex. :

2] Evaluate η of the transformer at full Load & $Pf = 0.9$ Lagging ?



$$\text{output} = |V_s||I_s|\cos\theta$$
$$= 240 * \left(\frac{10*10^3}{240}\right) * 0.9$$
$$= \boxed{9\,KW}$$

Losses: Electrical $= P_{sc} = 180W$. & Core $= P_{oc} = 60W$.

This happen just at full load.

$\therefore$ By substitution : $\eta = 97.4\%$.

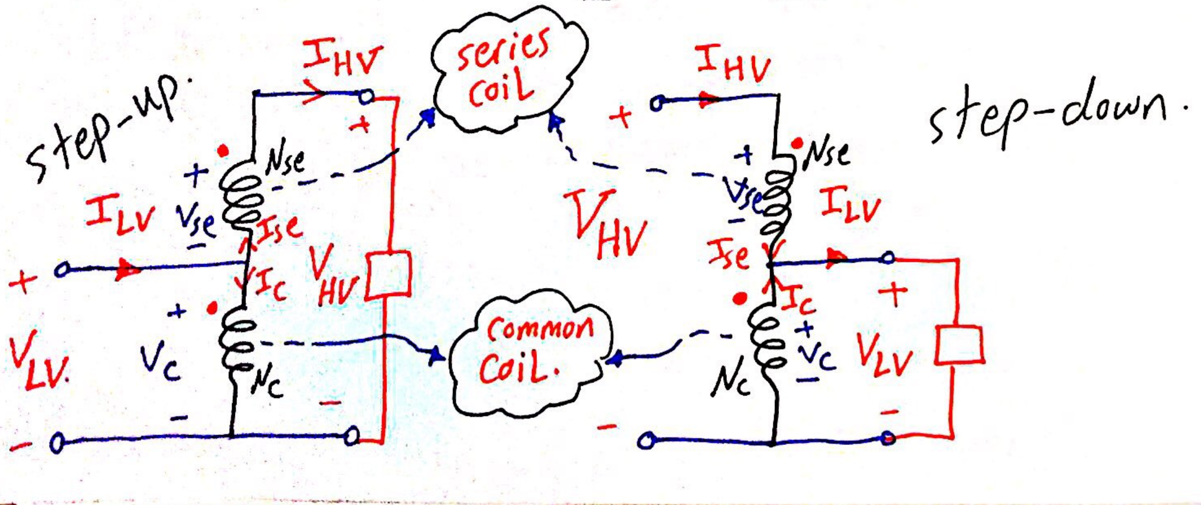
** Auto transformer:

* This is basically used when the voltage ratio is around 1.

* Construction:

It consists of 2 coils, electrically connected, with Polarity aiding.

→ connect +ve with -ve.



⇒ **Objective:** is to find voltage & current relationships at transformer terminals.

*** Characteristics:**

$$I_c N_c = I_{se} N_{se} \quad \cdots\cdots \boxed{1}$$

$$\frac{V_c}{V_{se}} = \frac{N_c}{N_{se}} \quad \cdots\cdots \boxed{2}$$

⇒ Consider Step-up Transformer:

$$\underline{V_{HV} = V_{se} + V_c} \quad \cdots\cdots \boxed{3}$$

$\Rightarrow V_{HV} = V_{se} + V_{LV}$ ⇒ from $\boxed{2}$: $V_{se} = V_c \dfrac{N_{se}}{N_c}$

$$\therefore V_{HV} = V_c \frac{N_{se}}{N_c} + V_{LV} = V_{LV}\left(\frac{N_{se}+N_c}{N_c}\right)$$

$$\therefore \frac{V_{HV}}{V_{LV}} = \frac{N_{se}+N_c}{N_c}$$ voltage relationship.

Now for the current:

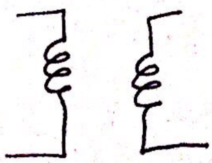
$I_{LV} = I_c + I_{se} = I_c + I_{HV}$ ⇒ from $\boxed{1}$: $I_{LV} = I_{se}\dfrac{N_{se}}{N_c} + I_{HV}$

$$\therefore I_{LV} = I_{HV}\left(\frac{N_{se}+N_c}{N_c}\right) \qquad \therefore \frac{I_{LV}}{I_{HV}} = \frac{N_{se}+N_c}{N_c}$$

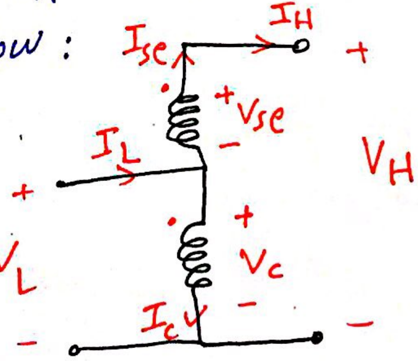
current relationship.

※ Apparent Power Advantage:

if the conventional transformer converted to Autotransformer, Then its apparent power will increase as shown below:



⇒



⇒

Scanned by CamScanner

$\therefore S_i = V_L I_L \quad \ldots\ldots \boxed{1}$ $\quad S_i \equiv$ input apparent power.

$S_o = V_H I_H \quad \ldots\ldots \boxed{2}$

It was shown before that:

$$\frac{V_H}{V_L} = \frac{I_L}{I_H} = \frac{N_c + N_{se}}{N_c} \quad \ldots\ldots \boxed{3}$$

$\Rightarrow$ find the relationship between S_i & S_o:

$\therefore$ By substituting $\boxed{3}$ into $\boxed{2}$:

$$S_o = \left(V_L \frac{N_c + N_{se}}{N_c}\right) * \left(I_L \frac{N_c}{N_c + N_{se}}\right) = V_L I_L = S_i$$

$$\therefore \quad S_o = S_i = S_{io}$$

* However the apparent power of transformer winding:

$$S_W = \underset{\substack{\downarrow \\ V_c = V_L}}{V_c} I_c = V_L (I_L - I_H) = V_L \left(I_L - I_L \frac{N_c}{N_c + N_{se}}\right)$$

$$= \underbrace{V_L I_L}_{S_{io}} \left(\frac{N_{se}}{N_c + N_{se}}\right)$$

$$\Rightarrow \quad \frac{S_{io}}{S_W} = \frac{N_c + N_{se}}{N_{se}}$$

$\Rightarrow$ Apparent Power Advantage.

* it can be shown that if a conv. Trans. converted to $\rightarrow$ Autotrans.

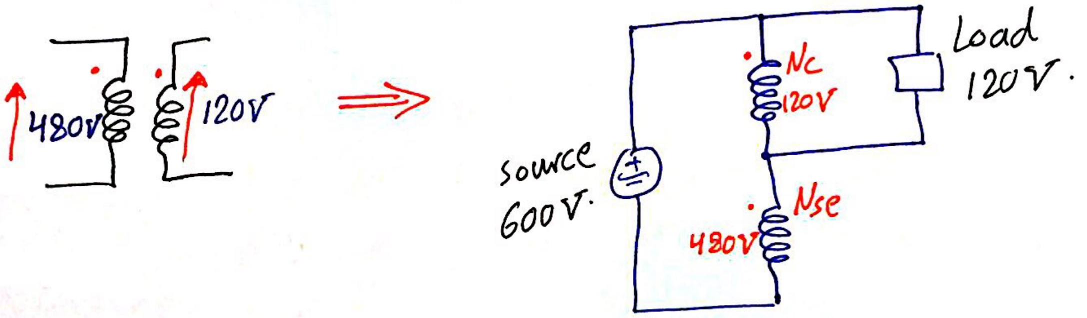
Then: its $Z_{eq(PU)}$ will be reduced by a factor $= \dfrac{N_c + N_{se}}{N_{se}}$

in Autotransformer divide the $Z_{eq(PU)}$ by this factor.

Scanned by CamScanner

ex.) A 5000 VA, (480/120) V. conventional Transformer is to be used to supply power from 600V source to 120V Load. Consider the transformer to be ideal, and assume that all insulation can handle 600 V.

1] Sketch the transformer connections that will do the required job?



2] Find the KVA rating of the produced AutoTransformer?

$$S_{io} = S_w \left(\frac{N_c + N_{se}}{N_{se}} \right)$$ we have: $N_c \propto V_c$, $N_{se} = V_{se}$

$\Rightarrow S_w = 5000\,VA.$

$$\therefore S_{io} = 5000 \left(\frac{120 + 480}{480} \right) \Rightarrow \boxed{S_{io} = 6250\,VA}$$

3] Find the max. primary & secondary currents of the Autotrans.?

$$I_{P_{max}} = \frac{6250}{600} \quad \& \quad I_{S_{max}} = \frac{6250}{120} \Rightarrow \boxed{\begin{array}{l} I_{P_{max}} = 10.4\,Amp. \\ I_{S_{max}} = 52.1\,Amp. \end{array}}$$

4] If $Z_{pu}(conv.) = 0.01 + j0.08$ find $Z_{pu}(Auto)$?

$$\therefore Z_{pu}(Auto) = (0.01 + j0.08) \Big/ \underbrace{\left(\frac{6250}{5000} \right)}_{1.25}$$

$$\Rightarrow Z_{pu}(Auto) = 0.008 + j0.064$$

*Note: See the Advantages & Disadvantages in the Book.

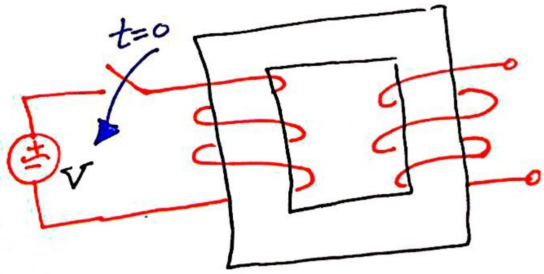
Scanned by CamScanner

※ Current Inrush:

This is a transient condition which may occur when a transformer is energized (i.e primary connected to source).

* Let $v = V_m \sin(\omega t + \theta)$

$\theta \equiv$ phase angle of applied voltage.



* Let $\theta = 0° \therefore v = V_m \sin\omega t$

⇒ find the value of ϕ over a half-cycle:

$$\omega = 2\pi f = \frac{2\pi}{T} \quad \therefore \quad T = \frac{2\pi}{\omega} \Rightarrow \text{half-cycle} = \frac{\pi}{\omega}$$

$$\Rightarrow \phi = \frac{1}{N}\int_0^{\pi/\omega} v\,dt = \frac{1}{N}\int_0^{\pi/\omega} V_m \sin\omega t\,dt$$

$$= \frac{-V_m}{\omega N}\left[\cos\omega t\right]_0^{\pi/\omega} \Longrightarrow \phi = 2\frac{V_m}{\omega N}$$

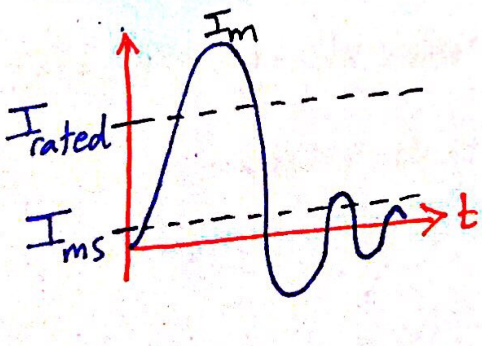
$\frac{V_m}{\omega N} \rightarrow \phi_m$

$\phi_m \equiv$ Normal max. value of flux.

$\therefore$ in this case $\boxed{\phi = 2\phi_m}$

$\therefore$ Under this condition, the primary will draw a very high current $\gg I_{rated}$.

This will occure if $v = 0$ at $t = 0$.



Scanned by CamScanner

※ Three-Phase Transformer:

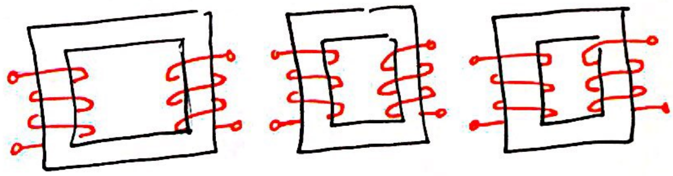
since electrical energy is generated, transmitted and ditributed in 3-ph AC voltage, Then 3-ph transformers are required for step-up & step-down.

*Note: see the advantages & disadvantages ⟹ of 3-ph supply over 3-ph & single phase

* Construction:

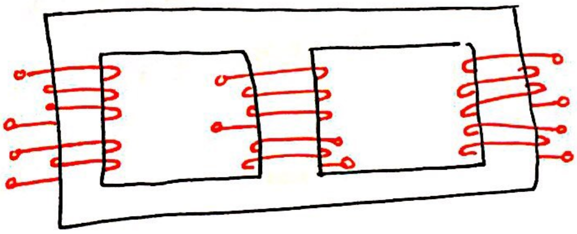
It can be one of two:

1] Using 3 1-ph Transformers.



This called:
"3-ph Transformer Bank."

2] A single core with 3 pairs of windings.



* The only advantage of first type on second type:
each unit in the bank could be replaced individually in the event of trouble. (saving money).

*Note: What is the advantages & disadvantages of these types?

*From Connection Point of view there are 4-types:

1] Y-Y 2] Y-Δ 3] Δ-Y 4] Δ-Δ

*Objective:

1] Find voltage ratio. 2] phase angle shift. 3] Advantages & Disadvantages of each type.

Scanned by CamScanner

* The advantage of using single 3-ph is:

Lighter, smaller, cheaper and slightly more efficient.

* Advantages of Autotransformer:

1] Gives more apparent power than the conventional as shown in the relation $\frac{S_{io}}{S_w} = \frac{N_{se}+N_c}{N_{se}}$

2] It is very advantageous to build transformers between two nearly equal voltages as auto transformers.

* Disadvantages of Autotransformer:

1] There is a direct physical connection between the primary & the secondary circuits so the electrical isolation of the two sides is Lost.

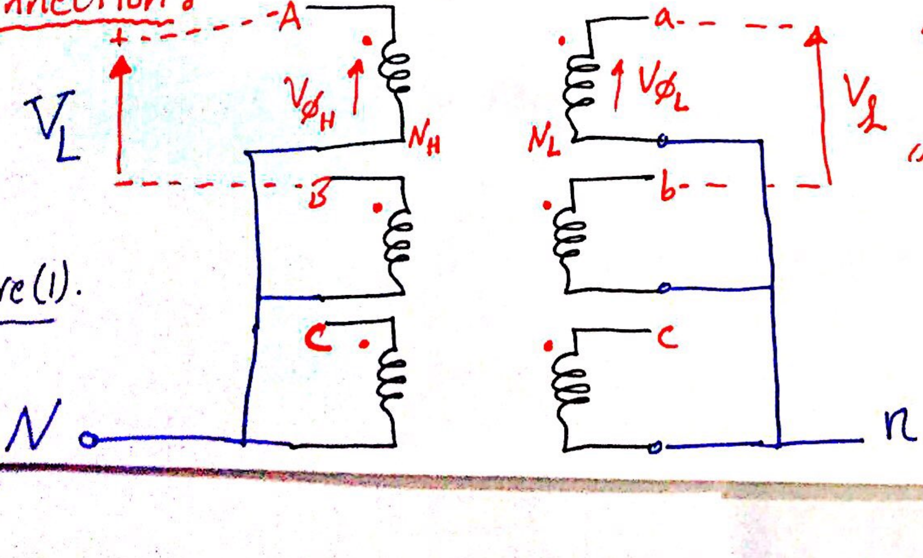
2] The effective PU impedance for Autotransformer is smaller by a factor equal to the reciprocal of the power advantage of auto transformer connection $\left(\frac{N_{se}+N_c}{N_{se}}\right)$.

* Introduction:

HV is represented by Capital Letters (e.g. A, B, C, Y, D).

LV is represented by small letters (e.g a, b, c, y, d)

Y-Y connection:



figure(1).

This called: "Wiring" or "Connection" Diagram ⟹

$V_L, V_\ell \equiv$ Line voltage of HV & LV sides.

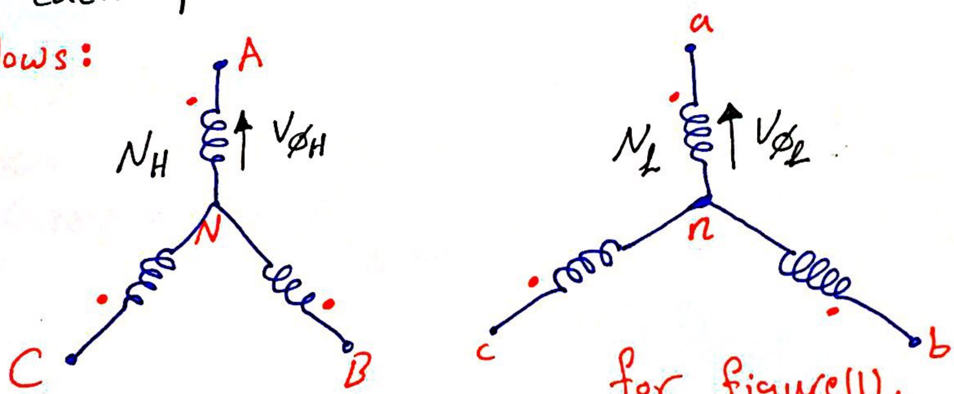
$V_{\phi H}, V_{\phi L} \equiv$ phase voltage of HV & LV sides.

$N_H, N_L \equiv$ Number of Turns of HV & LV sides.

$$\frac{|V_L|}{|V_\ell|} = \frac{\sqrt{3}\,|V_{\phi H}|}{\sqrt{3}\,|V_{\phi L}|} = \frac{|V_{\phi H}|}{|V_{\phi L}|} = a$$

$a \equiv$ Ratio of Phase voltages.

* Wiring Diagram is represented by a symbole where the windings of each phase are drawn Parallel to each other as follows:



for figure(1).

⇒⇒ Connection Designation:

1) First write the letter symbole of HV.

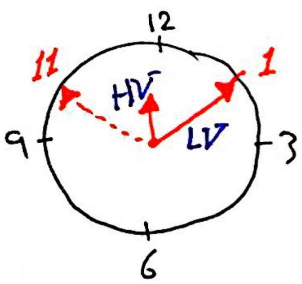
2) Next write the letter symbole of LV.

3) Indicate the phase of the low voltage with respect to High voltage.

↳ By using the clock arms:

* a) Hour arm always pointing at the 12:00 position to represent HV.

b) Minutes arm is used to represent LV.



Scanned by CamScanner

ex. Find the Connection Designation for figure (1) in page 49 ?

YNyn 0 → zero since No shift between the voltages.

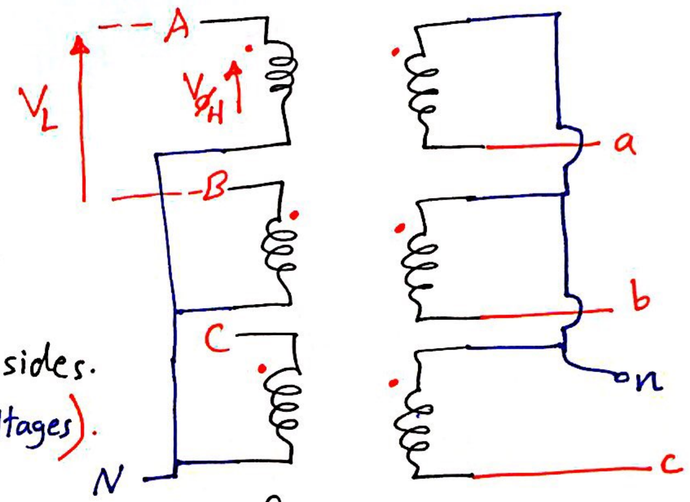
(we write it since the neutral is accessable.)

** In figure (2) just reverse the connection of the Low voltage side

In this case :

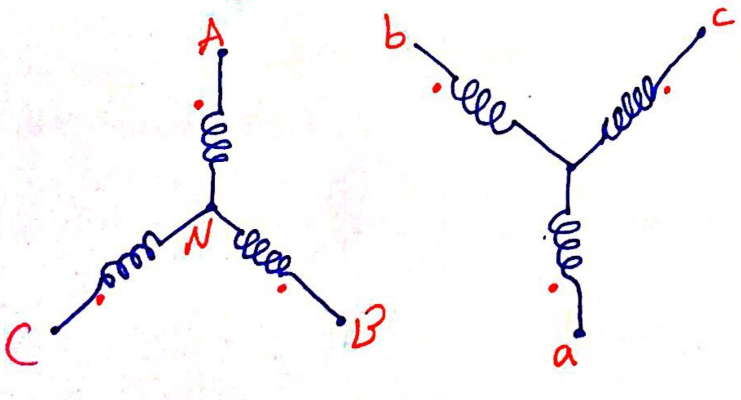
there is a phase shift of 180° between HV & LV sides.

(i.e Between Corresponding voltages).



figure(2).

for figure (2):



⇒⇒ Connection Designation:

YNyn 6

→ six since the phase shift is 180°.

---

* By reversing the connections:

There are only 4 possible cases for the phase shifts:

0° , 180° , −30° , +30°

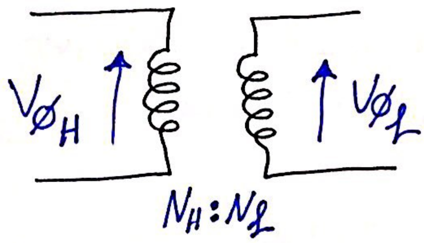
0° → 12:00, 180° → 6:00, −30° → 1:00, +30° → 11:00

Scanned by CamScanner

0°, 180°, −30°, +30°

group1, group2, group3, group4

* The Per-Phase cct of the Y-Y connection will be as follows:

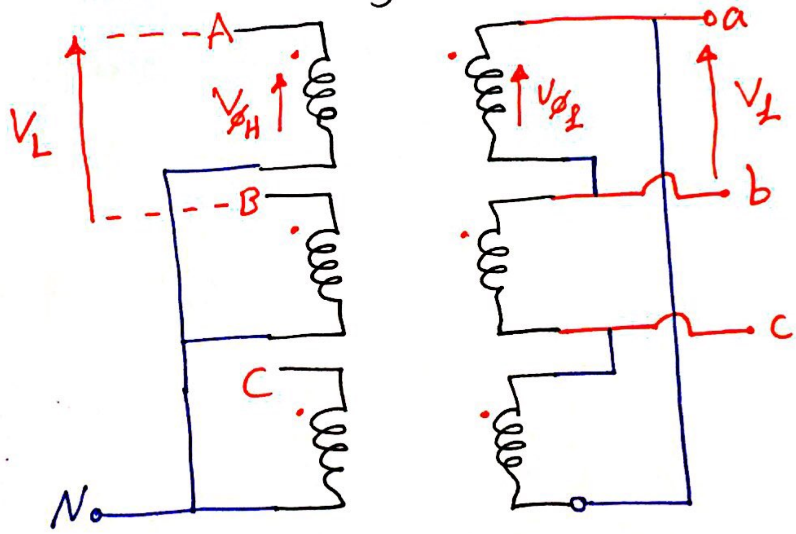


$$a = \frac{V_{\phi H}}{V_{\phi L}} = \frac{N_H}{N_L}$$

Y-Δ Connection:

$$\frac{|V_L|}{|V_L|} = \frac{\sqrt{3}\,|V_{\phi H}|}{|V_{\phi L}|} = \sqrt{3}\,\frac{|V_{\phi H}|}{|V_{\phi L}|} = \sqrt{3}\,a$$

figure (3).



* By similar approach, it can be found for:

Δ-Y: $\dfrac{|V_L|}{|V_L|} = \dfrac{a}{\sqrt{3}}$

Δ-Δ: $\dfrac{|V_L|}{|V_L|} = a$

*** if he ask to derive a relation for any type: DRAW the figure and put the variables on it then derive the needed relation.

Scanned by CamScanner

* in Y-Δ connection, there is a phase shift of 30° between corresponding voltages.

⇒ As a Convention:

* The terminal markings are made in such away that HV Leads LV by 30° in the case of +ve phase sequance.

* The terminal markings are made in such away that HV lags LV by 30° in the case of −ve phase sequance.

** Illustration: consider figure (3).

⇒ assume +ve phase sequance.

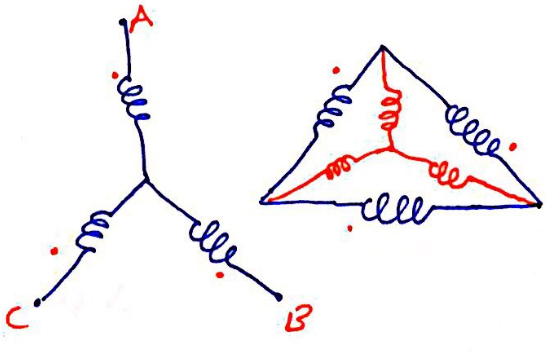
Let $\angle V_{AN} = \text{Zero}$. $\therefore \angle V_{AB} = +30°$

But V_{AN} & V_{ab} are in phase ⇒ $\angle V_{ab} = \angle V_{AN} = 0°$

$\therefore$ V_{AB} (i.e HV) Leads V_{ab} (i.e LV) by 30°.

* Find the Per-phase cct for Y-Δ:

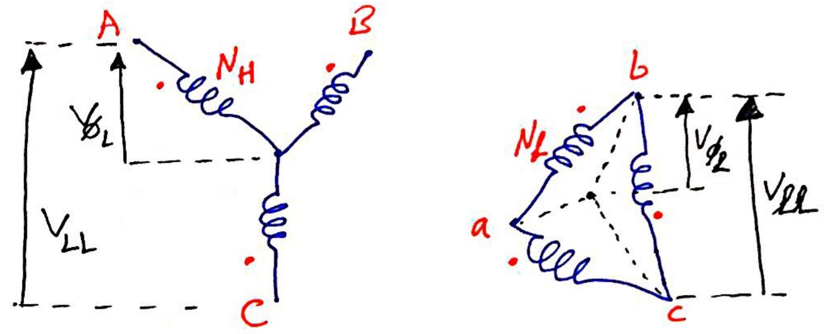
discussed in the Next page.



⇒

※ Effective Turns Ratio:

consider a Y–Δ Transformer:

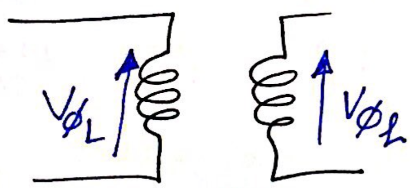


$$\frac{V_{\phi L}}{V_{LL}} = \frac{N_H}{N_L} \quad \ldots \boxed{1}$$

$$\frac{V_{\phi L}}{\sqrt{3}\, V_{\phi_L}} = \frac{N_H}{N_L} \quad \ldots \boxed{2}$$

$$\therefore \frac{V_{\phi L}}{V_{\phi_L}} = \frac{N_H}{N_L/\sqrt{3}} \quad \ldots \boxed{3}$$

$\therefore \boxed{3}$ can be used to find per phase equivalent cct:



$N_H : N_L/\sqrt{3}$

$a_{eff} \leftarrow \left(\frac{N_H}{N_L/\sqrt{3}}\right) : 1$

$$\therefore a_{eff} = \frac{N_H}{N_L/\sqrt{3}}$$

Effective Turns Ratio.

$\Rightarrow \boxed{1}$ can be written as follows:

$$\frac{V_{LL}/\sqrt{3}}{V_{LL}} = \frac{N_H}{N_L} \Rightarrow \therefore \frac{V_{LL}}{V_{LL}} = \frac{N_H}{N_L/\sqrt{3}} \equiv a_{eff} \equiv \text{Ratio of Line Voltage.}$$

** Conclusion:

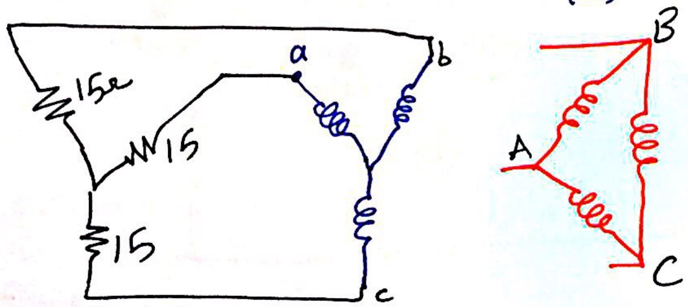
Irrespective of the type of 3-ph Transformer windings

$$a_{eff} \triangleq \text{Ratio of Line Voltages.}$$

ex. A (132/33) KV Transformer is D-Y connected. a balanced Y-connected resistive Load of 15Ω is connected to LV side.

Find how this load is seen from HV side?

from connection designation: 132/33 → D (Δ) ; 33 → y (Y)



$\therefore$ Load seen from HV side:

$15 * a_{eff}^2 = 15 * \left(\frac{132}{33}\right)^2$

$= 240\Omega$

(33 : 132) KV

$1 : \frac{132}{33} = a_{eff}.$

*Note : if the load was (Δ) with 45Ω:

first we convert $\Delta \longrightarrow Y \Rightarrow \frac{45}{3} = 15\Omega$

Then load seen from HV side: $= 15 * \left(\frac{132}{33}\right)^2 = 240\Omega$

$\Rightarrow$ We convert from $\Delta \longrightarrow Y$:

since we deal with single-per-phase circuit.

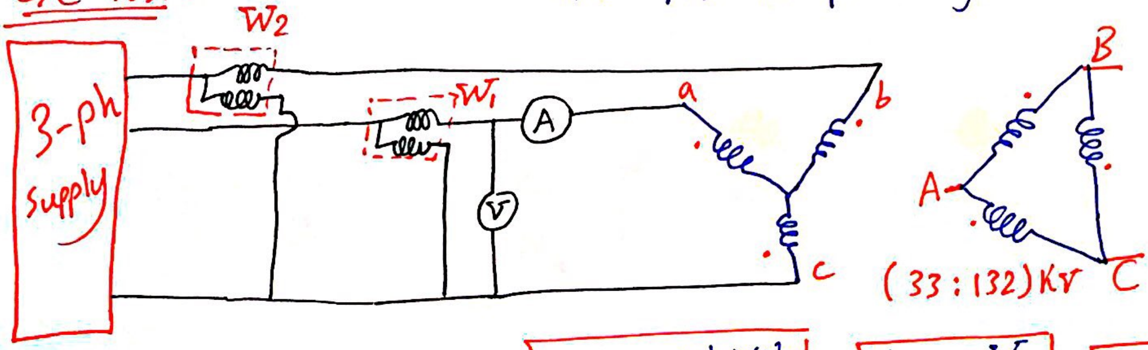
Scanned by CamScanner

※Application of O/C & S/C Tests To 3-ph Transformer:

In O/C Rated voltage is applied.
In S/C Rated current is applied.

Conventional Test: → O/C LV is primary. → S/C HV is primary.

O/C Test:

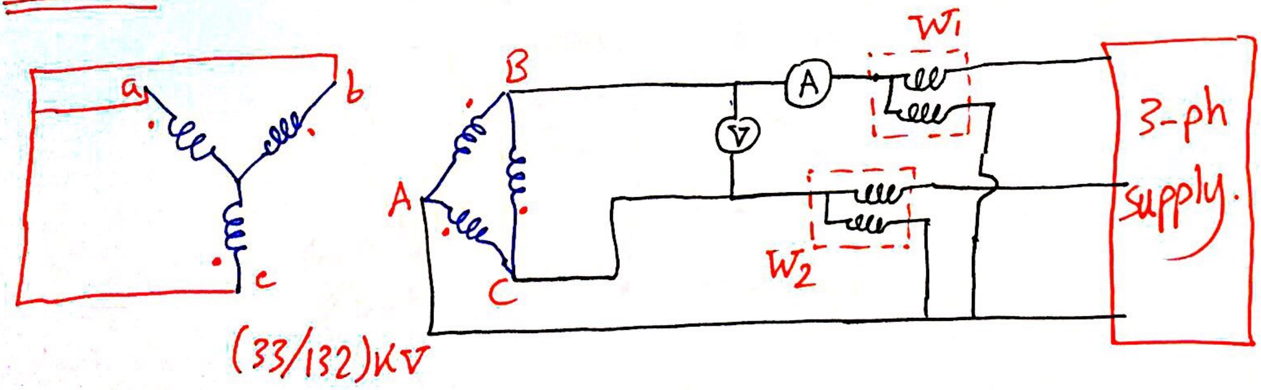


$$\underline{P_{3p} = W_1 + W_2}$$

$$\boxed{P_{oc} = \frac{W_1 + W_2}{3}}, \quad \boxed{V_{oc} = \frac{V}{\sqrt{3}}}, \quad \boxed{I_{oc} = A}$$

→ in this figure: Applied voltage $= V = 33$ K Volt.

S/C Test:



for example: 100KVA, (33/132) KV

$|S| = \sqrt{3}|V||I| \Rightarrow 100 * 10^3 = \sqrt{3} * 132 * 10^3 * |I|_{rated}$

$$\boxed{I_{sc} = |I|_{rated} = A}, \quad \boxed{V_{sc} = V/\sqrt{3}}, \quad \boxed{P_{sc} = \frac{W_1 + W_2}{3}}$$

Scanned by CamScanner

$\Rightarrow \therefore R_{eq}$ & X_{eq} can be calculated as before.

※ PU system in 3-ph Transformer:

Let $S_{3\phi} \equiv$ Apparent power of 3-ph (Base).

$V_{LL} \equiv$ Lin voltage.

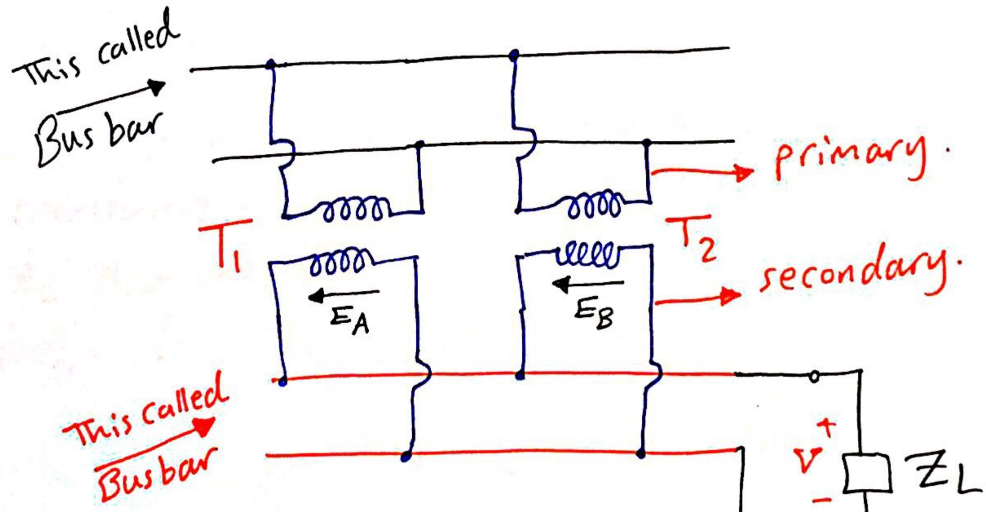
$\therefore$ For per-phase circuit: $\boxed{S_{1\phi} = \frac{S_{3\phi}}{3}}$, $\boxed{V_{1\phi} = \frac{V_{LL}}{\sqrt{3}}}$

Now for Z_b:

$$Z_b = \frac{(V_\phi)^2}{S_\phi} = \frac{(V_{LL}/\sqrt{3})^2}{S_{3\phi}/3} \Rightarrow \boxed{Z_b = \frac{V_{LL}^2}{S_{3\phi}}}$$

Base Impedance of per-phase circuit.

※ Parallel Operation of Transformers:



*** ⇒ Condition of Parallel operation:

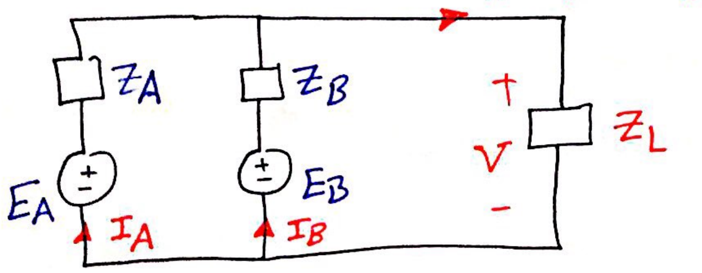
1) Same voltage ratio & turns ratio.
2) Same percentage impedance & $\frac{X}{R}$ ratio.
3) Identical Position of Tap Changer.
4) Same KVA ratings.
5) Same vector group.
6) same frequancy rating.
7) same Polarity.
8) same phase sequence.

Scanned by CamScanner

* Equivalent circuit:

For Two-Transformers in parallel.

$I_A + I_B = I_L$



KVL: $E_A = I_A Z_A + V = I_A Z_A + (I_A + I_B) Z_L \quad \cdots\cdots \boxed{1}$

$E_B = I_B Z_B + V = I_B Z_B + (I_A + I_B) Z_L \quad \cdots\cdots \boxed{2}$

⇒ By using $\boxed{1}$ & $\boxed{2}$ you can find expressions for I_A, I_B & V.

By rearrangment & substitution it can be found that:

$$\boxed{V = \frac{\frac{E_A}{Z_A} + \frac{E_B}{Z_B}}{\frac{1}{Z_A} + \frac{1}{Z_B} + \frac{1}{Z_L}}} \quad \cdots\cdots \boxed{1^*}$$

Advantages of $\boxed{1^*}$ is that it can be modified for any number of parallel transformers. For example for a 3rd Transformer with E_C & Z_C Then one add to:

The numerator $\frac{E_C}{Z_C}$ & The dominator $\frac{1}{Z_C}$

* when V is found one can find the current in any transformer:

For e.g. $I_A = \frac{E_A - V}{Z_A}$

* For the special case where $E_A = E_B$, Then:

$I_A Z_A = I_B Z_B$ since ⇒ $I_L = I_A + I_B$

$\therefore I_A = I_L \left(\frac{Z_B}{Z_A + Z_B} \right)$ $\qquad \therefore I_B = I \left(\frac{Z_A}{Z_A + Z_B} \right)$ ⇒

Scanned by CamScanner

$$(I_A * V) = (I * V)\left(\frac{Z_B}{Z_A+Z_B}\right), \quad (I_B * V) = I * V\left(\frac{Z_A}{Z_A+Z_B}\right)$$

apparent power of Load.

"sharing of apparent power" between Transformers.

ex. Two Parallel Transformers have the following Data:

	T_A	T_B
Rated current (A)	200	600
Z_{eq} (PU)	$0.02+j0.05$	$0.025+j0.06$
No load emf (V)	245	240

Calculate the terminal voltage when they are connected to $Z_L = 0.25+j0.1\ \Omega$?

Solution:

$Z_A = (0.02+j0.05)\frac{245}{200}$

$\Rightarrow \boxed{Z_A = 0.0245+j0.0613}\ \Omega$

$Z_B = (0.025+j0.06)\frac{240}{600}$

$\Rightarrow \boxed{Z_B = 0.01+j0.024}\ \Omega$

By substitution:

using $E_A = 245\angle 0^\circ$ & $E_B = 240\angle 0^\circ$

it can be found: $\boxed{V = 230\angle -3^\circ}$ volt.

End of CH2.

* * * *.

Scanned by CamScanner

CHAPTER (3):

※ Electrical Machines:

Machines → DC machines. / AC machines.

Machines → Generators OR Motors

Generators → input (mechanical), output (electrical)

Motors → input (electrical), output (mechanical).

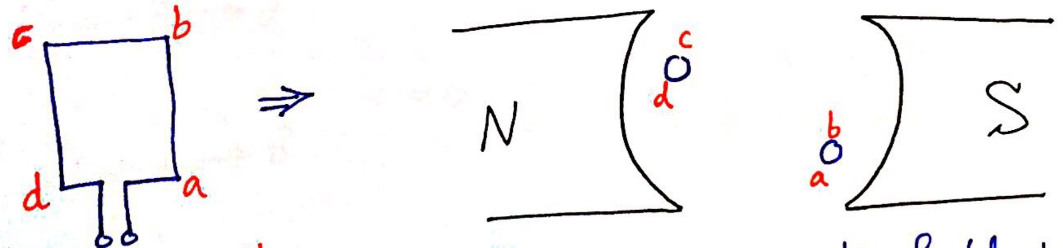
* Fundemental of machines:

in machines there are 3 requirements:

1. Magnetic Field. (Electro magnet).
2. Conductors.
3. Relative motion between $\boxed{1}$ & $\boxed{2}$.

* Voltage Induced in a single turn coil:

Consider the following structure:



single turn coil.

In this case: magnetic field is the "STATOR", and the coil is Rotating "ROTOR"

$$e \triangleq (V * B) \cdot \ell$$

$V \equiv$ linear velocity of the conductor.

$e \equiv$ Induced voltage in a conductor

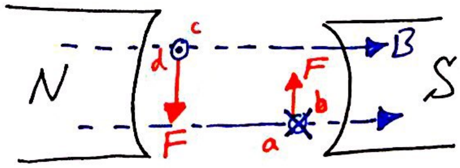
$B \equiv$ flux density.

$\ell \equiv$ Active length of the conductor

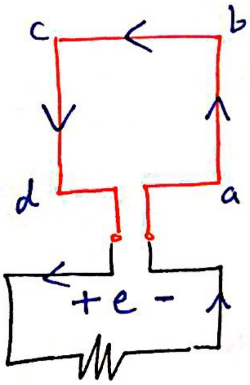
Scanned by CamScanner

*To find the direction of the induced voltage one may use RHR.

Fore finger ≡ Direction of magnetic Field.
Thumb ≡ Direction of Applied Force.
Middle finger ≡ Direction of Induced or Generated voltage.



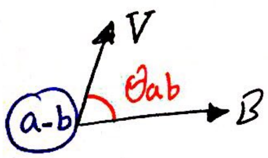
$F \equiv$ applied Force.



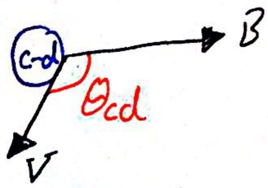
*If a load is connected to the coil, then a current will flow in the same direction of Induced voltage.

*Magnitude of e:

$\Rightarrow e_{ab} = vBl \sin\theta_{ab}$



$\Rightarrow e_{cd} = vBl \sin\theta_{cd}$



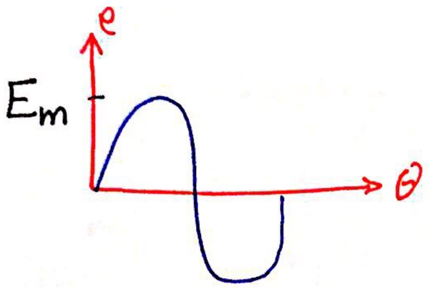
*However, No induced voltage in bc & da.

$\theta_{ab}+\theta_{cd}=180°$

$\therefore e = e_{ab} + e_{bc} + e_{cd} + e_{da} = vBl \sin\theta_{ab} + vBl \sin\theta_{cd}$

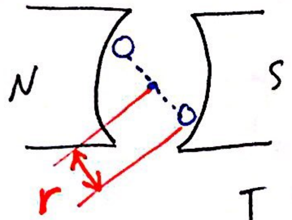
But: $\theta_{cd} = 180 - \theta_{ab} \Rightarrow e = vBl(\sin\theta_{ab} + \underbrace{\sin(180-\theta_{ab})}_{=\sin\theta_{ab}})$

$$\Rightarrow \boxed{e = 2vBl \sin\theta_{ab}}$$



$$e = \underbrace{2vBl}_{E_m} \sin\theta_{ab}$$

$$E_m = 2vBl.$$

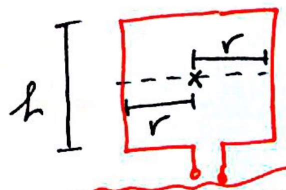


$\Rightarrow$ $v = \omega r$

$$E_m = 2vBl = 2\omega r B l$$

$$= A B \omega$$

remember: $\phi_m = AB$

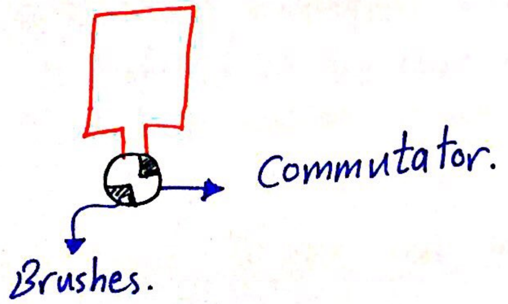


$$\boxed{Area = 2rl}$$

Plane Area of the coil.

$$\boxed{E_m = \phi_m \omega}$$

$\rightarrow$ this is for single turn coil, if there is more than one turn $\Rightarrow$ multiply by number of turns.

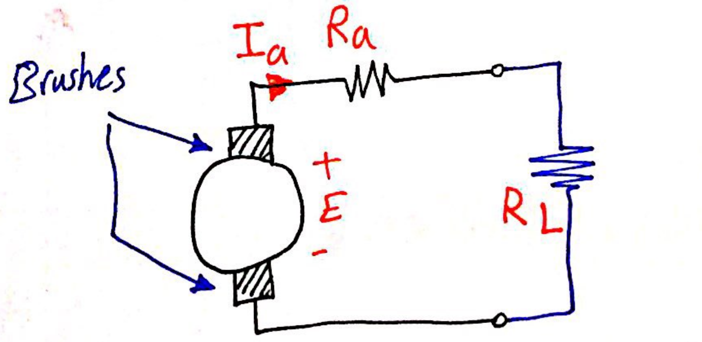


* Comment : The internal AC induced voltage is converted to a DC voltage by means of commutator segments (connected to rotating coil) is contact with Fixed brushes, This called "Commutation".

$\therefore$ The equivalent circuit will be as follows:

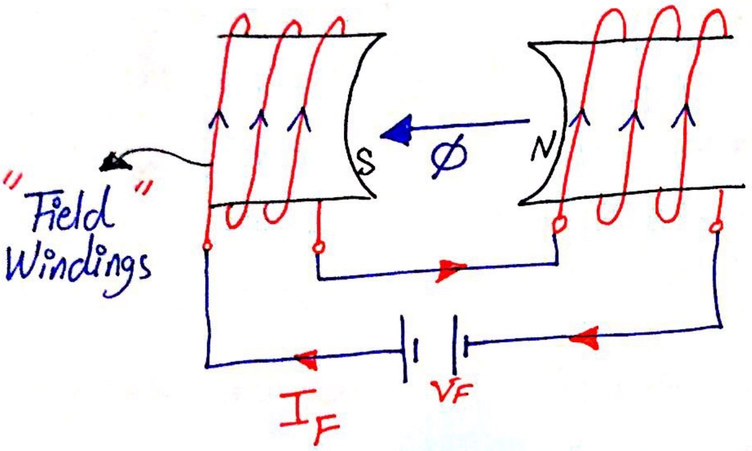
$R_a \equiv$ Internal Resistance of the Rotor windings called "Armature Windings".

$I_a \equiv$ Armature Current.

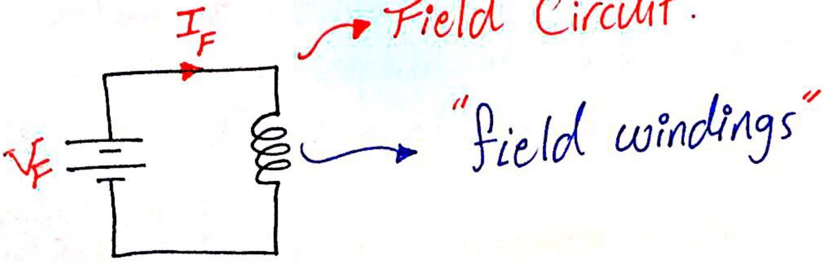


"Armature Circuit."

Scanned by CamScanner



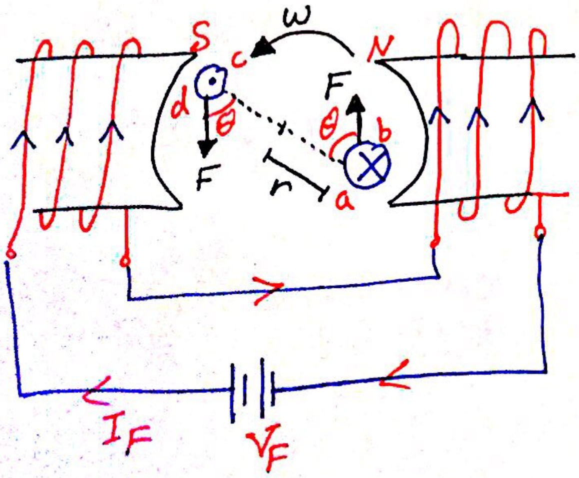
$I_F \equiv$ Field Current. (i.e current required to generate magnetic field.)

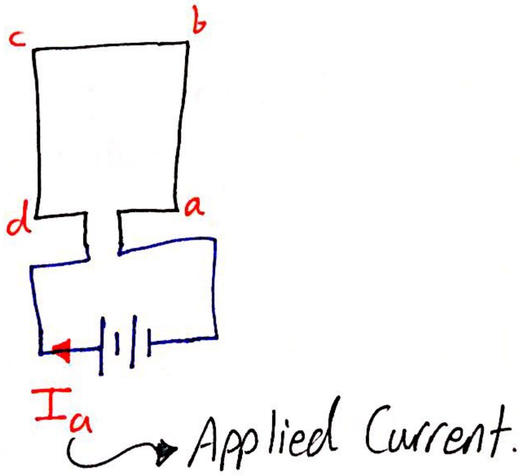


* Armature Windings: They are the windings in which a voltage is induced in the case of generator, OR the windings into which a current is applied in the case of Motor.

* Induced Torque in a single turn coil:

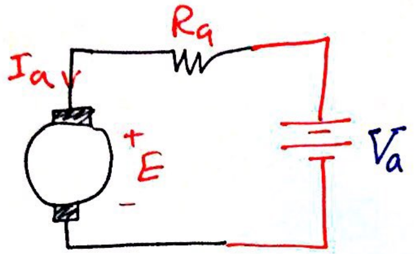
In this case A current is applied to Armature Windings.





⊙, ⊗ ≡ Direction of Applied Current.

$w \equiv$ Direction of Developed Angular Speed.



Consequently, Force F will be developed on the Armature conductors.

where: $F = (B * \ell)\, i_a$

* The direction of F: By means of LHR.

forefinger ≡ Field Direction.
middle finger ≡ Direction of Applied Current.
Thumb ≡ Direction of generated or developed Force.

* Only Torques are developed on the sides ab & cd.
(i.e No Torques on bc & da).

$\tau_{ab} = F * r\sin\theta = B I_a \ell r \sin\theta$
$\tau_{cd} = F * r\sin\theta = B I_a \ell r \sin\theta$

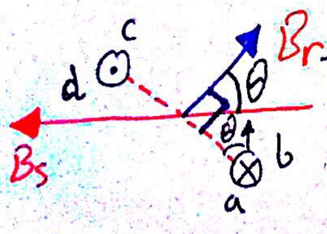
∴ Developed Torque:
$\tau = \tau_{ab} + \tau_{cd}$
∴ $\tau = 2 B I_a \ell r \sin\theta$

$\Rightarrow \boxed{\tau = A B I_a \sin\theta}$ .... $\boxed{I}$

$\Rightarrow \boxed{I}$ can be rewritten in another form:

$\tau = A B_s I_a \sin\theta$

B_s ≡ This is to idicate the flux density produced by field or stator windings.



determined by RHR.

⟹

$$\Rightarrow N I_a = \mathcal{R}\, \phi_R = \mathcal{R}\, B_R A_R$$

$G \equiv$ constant.

$$N I_a = \frac{l}{\mu A_R} B_R A_R \Rightarrow I_a = \frac{l}{N} \cdot \frac{B_R}{\mu}$$

$$I_a = \frac{G\, B_R}{\mu} \quad \ldots \boxed{2}$$

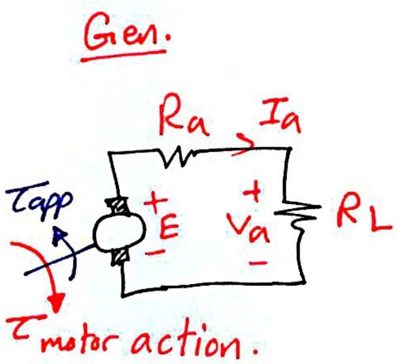
* Substitute $\boxed{2}$ in $\boxed{1}$:

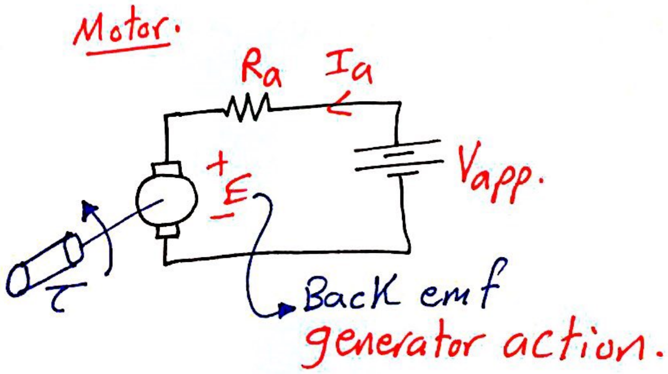
$$\tau = A B_s \frac{G\, B_R}{\mu} \sin\theta = K\, B_s B_R \sin\theta$$

$$\tau = K\,(B_s \times B_R)$$

$K \equiv$ constant.
↳ Related to the construction of the machine.

* In a generator action there is a motor action.
* In a motor action there is a generator action.

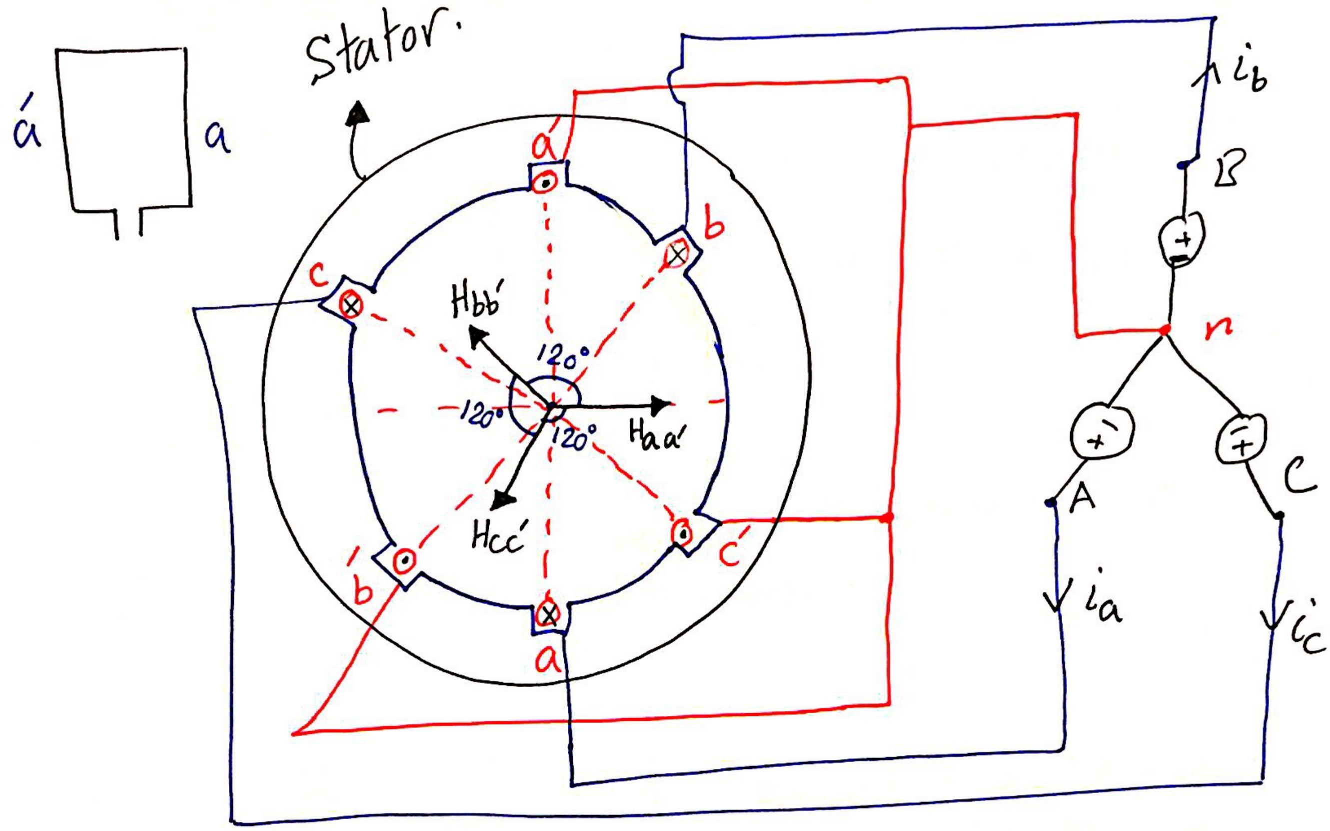




* Rotating magnetic field:

If a Balanced 3-ph voltage is applied to a 3-ph winding Located on the Stator, Then a Rotating magnetic field is produced within the stator, as follows:

⟹ ⟹



* Consequantly, Balanced 3-ph currents will flow to the terminals i_a, i_b, i_c

$\therefore$ $i_a = I_m \sin(\omega t)$

$i_b = I_m \sin(\omega t - 120^\circ)$

$i_c = I_m \sin(\omega t - 240^\circ)$

* Each current will generate its magnetic field whose:

1] magnitude $\propto$ current.

2] Direction can be found by RHR.

since $\vec{B} = \mu \vec{H}$

$\therefore$ $H_{aa'} = H_m \sin(\omega t) \angle 0^\circ$

$H_{bb'} = H_m \sin(\omega t - 120^\circ) \angle 120^\circ$

$H_{cc'} = H_m \sin(\omega t - 240^\circ) \angle 240^\circ$

$\therefore$ $B_{aa'} = B_m \sin(\omega t) \angle 0^\circ$

$B_{bb'} = B_m \sin(\omega t - 120^\circ) \angle 120^\circ$

$B_{cc'} = B_m \sin(\omega t - 240^\circ) \angle 240^\circ$

Scanned by CamScanner

∴ Resultant flux density:

$$B_R = B_{aa'} + B_{bb'} + B_{cc'}$$

$$\Rightarrow B_R = B_m \sin(\omega t)\angle 0^\circ + B_m \sin(\omega t - 120^\circ)\angle 120^\circ + B_m \sin(\omega t - 240^\circ)\angle 240^\circ$$

* By converting to cartesian form it can be found that:

$$\boxed{B_R = 1.5 B_m \left(\sin(\omega t) - j\cos(\omega t)\right)}$$

$$\therefore |B_R| = 1.5 B_m$$

Next investigate the direction of B_R as ωt changes.

ωt	B_R
0°	$-j1.5B_m = 1.5B_m\angle 90^\circ$
90°	$1.5B_m = 1.5B_m\angle 0^\circ$
180°	$j1.5B_m = 1.5B_m\angle 90^\circ$
270°	$-1.5B_m = 1.5B_m\angle 180^\circ$
360°	$-j1.5B_m = 1.5B_m\angle -90^\circ$

* hences in this case when the current make one electrical cycle, the Rotating magnetic field makes one mechanical Rotation.

Next consider the following case $\omega t = 90^\circ$

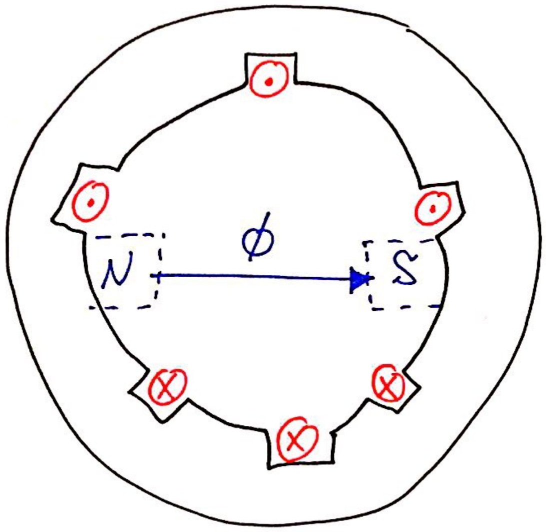
Then: $i_a = I_m \sin(\omega t) = +ve$

$i_b = I_m \sin(\omega t - 120) = -ve$

$i_c = I_m \sin(\omega t - 240) = -ve$

* let +ve current represented by ⊗
* let −ve current represented by ⊙

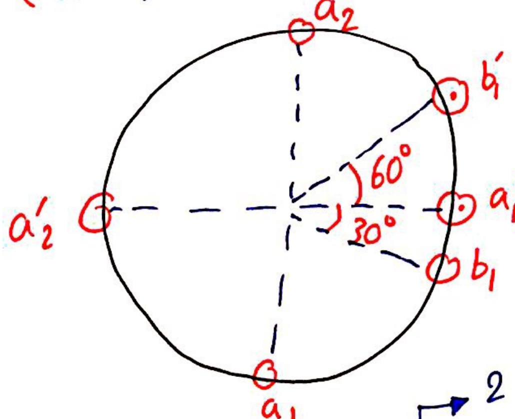
Scanned by CamScanner



* hence this type of stator windings represent 2-pole winding.

* The sequance of this winding is $a c' b a' c b'$
(i.e span between coil sides of each sides of each phase $(a \rightarrow a') = 180°$)
and the span between phases
(i.e $a \rightarrow b = 120°$)

⇒ However, the sequance is repeated twice $a c' b a' c b'$ $a c' b a' c b'$
(i.e span between coil sides $= 90°$ & span between phases $= 60°$)



see Text Book.

→ 2 seq. & 4 poles.

⇒ Then in the previous case electrical freq. $= 2 *$ Mechanical freq.

in general: $f_e = P f_m$

$f_e \equiv$ electrical freq. of applied current.
$f_m \equiv$ mechanical freq of rotating flux.
$P \equiv$ Number of pole-pairs of stator windings.

* End of Second Material *

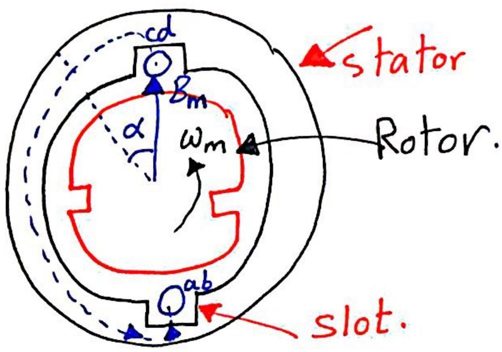
Scanned by CamScanner

* Voltage Induced in AC machines: (such as synchronous generator).

Here the field windings are located on the Rotor and the Armature windings are located on the Stator.

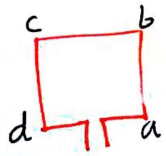
* The field current is supplied from External DC source to fieldwindings through sliprings & brushes.

* The Field Windings are designed in such away asinusoidal flux is generated.
  ⇒ This flux is going to cut or LinK Aramature Windings. , Consequently a voltage is induced, whose magnitude is derived as follows:

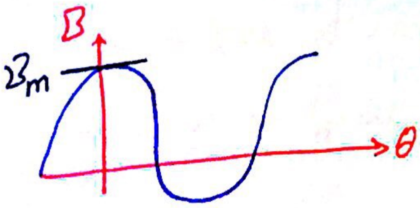


*voltage induced (i.e magnitude):

Consider a single turn coil Located in 2 slots.



⇒ flux generated can be expressed as:



θ ≡ Position at Stator.

→ As a Reference: Let B_m along +ve y-direction.

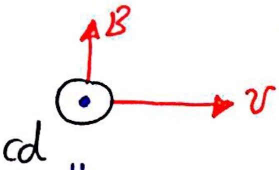
⇒ At a given Location or position on the stator with angle α, Then: $B = B_m \cos\alpha$

* If the Rotor is rotating at angular speed $\equiv \omega_m$
  ↳ m stands for mechanical.

$$\therefore \quad B = B_m \cos(\omega_m t - \alpha)$$

↳ This is for outward direction of flux.

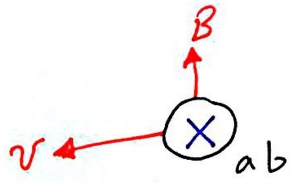
$\Rightarrow$ voltage induced can be found by using RHR.
(However this was applied when the field is stator).
, to apply it here, Then to simulate that the field is the stator, Then the observer sit on the Rotor and then look at Armature windings as follows:



$\Rightarrow \therefore e_{cd} = (B_m \cos(\omega_m t)) \ell v$ --- ①

$\Downarrow$

$\alpha = 0$



$\therefore e_{ab} = -B_m \cos(\omega_m t - 180°) \ell v$

$= (B_m \cos(\omega_m t)) \ell v$ --- ②

$\alpha = 180°$

since we deal with the outward direction, and actually it is inward direction.

As before there is No induced voltage in cb & da
$\therefore$ the Total induced voltage in the single turn coil:

① + ② $\Longrightarrow$ $\boxed{e = 2 B_m \ell v \cos(\omega_m t)}$

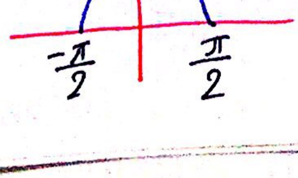
$\therefore$ Amplitude of Induced voltage is: $E_m = 2 B_m \ell v$ ($v = \omega_m r$)
$= 2 B_m \ell \omega_m r$

since $\omega_e = \frac{P}{2} \omega_m$ electrical $P \equiv$ Number of poles.

$\Rightarrow \therefore \omega_m = \frac{2 \omega_e}{P}$

$\Rightarrow \therefore E_m = \frac{4 B_m \ell \omega_e r}{P}$ .... ①

*Average value of flux under a pole:



$$B_{av} = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} B_m \cos\alpha \, d\alpha = \frac{1}{\pi} \left[B_m \sin\alpha \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \frac{2 B_m}{\pi}$$

$\Longrightarrow$

Scanned by CamScanner

* But surface area of a pole, $A_p = \frac{2\pi \ell r}{P}$

∴ flux per pole, $\phi = B_{av} * A_p = \frac{2B_m}{\pi} * \frac{2\pi \ell r}{P}$

$= \frac{4 B_m \ell r}{P} \quad \ldots \{2\}$

∴ Substitute {2} in {1}:

$\Rightarrow \quad E_m = \phi \, \omega_e$

** For N-turns coil:

Then, $E_m = N \phi \, \omega_e \quad \ldots \{3\}$

or in RMS:

$E_{rms} = \frac{E_m}{\sqrt{2}} = \frac{N\phi \, \omega_e}{\sqrt{2}} = \frac{N\phi \, (2\pi f)}{\sqrt{2}}$

$\Rightarrow \quad E_{rms} = 4.44 \, N \phi f \quad \ldots \{4\}$

* in deriving {3} & {4} the following were assumed:

1) Flux is Sinusoidal, which isn't the case.
This is taken into account by Introducing Pitch Factor (K_p).

2) That the turns are concentrated in a pair of slots.
However in practice, the coils are Distributed
This is taken into account by a Distribution Factor (K_d)

∴ The general expression for induced voltage:

$E_{rms} = 4.44 \, \underbrace{K_p K_d}_{K_\omega} \, N \phi f$

$K_\omega \equiv K_p K_d$ and called Winding Factor, defined as follows:

$K_p \triangleq \sin(P/2)$

$P \equiv$ coils Pitch in electrical degree.

$\Rightarrow$

$\Rightarrow$

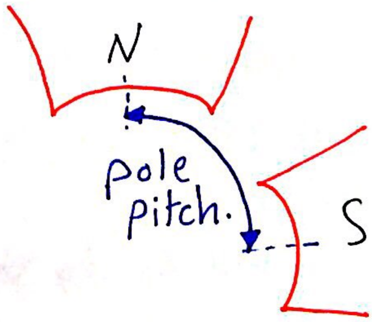
$$K_d = \frac{\sin\left(\frac{n\gamma}{2}\right)}{n\sin(\gamma/2)}$$

$\gamma \equiv$ Slot Pitch in electrical degree.

$n \equiv$ Number of phase belts

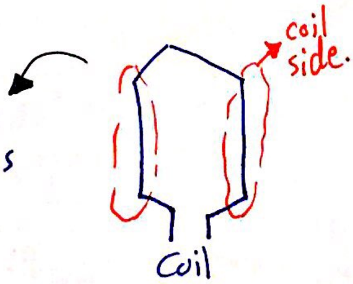
$\Rightarrow$ $n = \frac{\text{Number of slots}}{3 * P}$; P → number of poles.

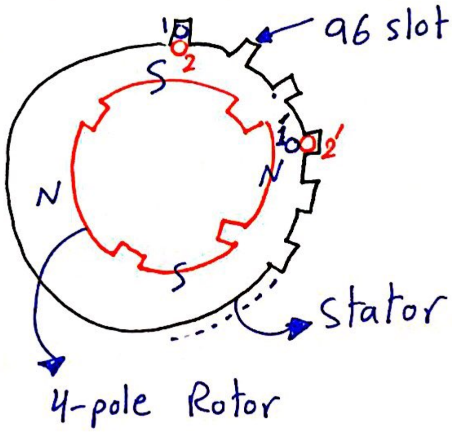
Pole Pitch $\equiv$ Distance between adjacent Poles.



ex. A 3-ph, 4-pole synchrounus generator has 96 slots. The slot contain a double Layer winding with 4 turns per coil. The coil pitch is $\frac{19}{24}$, Evaluate the following:

∴ Each coil has 2 coil sides.





⇒ Double Layer windings: means that each slot has 2 coil sides.

$\equiv$ equal to one coil.

→ side connected at the bottom of the slot & the other side connected at the top of the slot.

∴ Total Number of coils $\equiv$ Number of slots.

∴ Total Number of turns $\equiv$ Number of slots $*$ Number of turns per coil ⇒

⇒

⇒

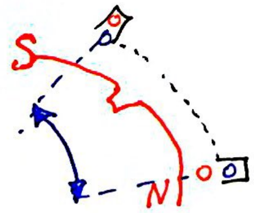
⇒ Now for the coil pith: it is the distance or span between coil sides of a given coil.

ex.
⇒ on the figure: span between (1 & 1') OR (2 & 2')

Note: The Span is measured in mechanical degree or in electrical degree.

*Important Definitions:

→ Full Pitch Coil: if a coil side is under the center of a S-pole, then its other side will be under the center of adjacent North pole. ⇒ This is equal to pole's pitch.



→ Fractional Pitch Coil: if the coil's pitch is < Full-pitch.

$$\therefore \boxed{\text{Pole's Pitch} = \frac{360^\circ}{P} \text{ mech.}^\circ}$$

P → Number of poles.

always:

$$\boxed{\text{Pole's pitch} = 180^\circ \text{ electrical}}$$

* In this example:

1) Coil's pitch $= 19/24$ means: $\frac{19}{24}$ * Full Pitch.

2) Pole's Pitch $= \frac{360^\circ}{4} = 90^\circ$ mechanical. OR Pole's Pitch $= 180^\circ$ electrical.

⇒ ∴ 1° mech $\equiv 2^\circ$ elect.

⟹

Continue.

Scanned by CamScanner

NOW Back To the Example:

1) Find slot & Coil Pitch in electrical degrees?

* slot pitch, $\gamma \equiv \frac{360^\circ}{96} = 3.75^\circ$ mech. $= 3.75*2 = 7.5^\circ$ elect.

* Coil pitch, $\rho = \frac{19}{24} *$ Pole's Pitch $= \frac{19}{24} * 90^\circ$ mech $= 71.25^\circ$ mech.

$= \frac{19}{24} * 90^\circ * 2 = 142.5^\circ$ elect.

2) Find Pitch, distribution and winding factors?

* $K_p = \sin\left(\frac{\rho}{2}\right) = \sin\left(\frac{142.5}{2}\right) \Rightarrow \boxed{K_p = 0.947}$

* $K_d = \frac{\sin(n\gamma/2)}{n \sin(\gamma/2)}$; n = Number of phase belts $= \frac{\text{No. of slots}}{3 * P} = \frac{96}{3*4} = 8$

$\Rightarrow \boxed{K_d = 0.956}$

* $K_w = K_p K_d \Rightarrow \boxed{K_w = 0.905}$

3) If flux per pole is 0.06 Wb and speed of Rotor is 1800 rpm, Find the magnitude of gen. voltage per phase?

$E_{rms} = 4.44\ K_w N \phi f$; $N \equiv$ Number of turns per phase.

$= \frac{96 * 4}{3} = 128$

Total Number of coils.

Number of Turns/coil.

Number of phases.

$f = f_e = \frac{P}{2} n\,(rps)$

$= \frac{4}{2} * \frac{1800}{60} = 60\ HZ$

$\Rightarrow E_{rms} = (4.44)(0.905)(128)(0.06)(60)$

$\Rightarrow \boxed{E_{rms} = 1851.59 \text{ volt}}$

* * *

Scanned by CamScanner

Three-phase Synch. Generator:

* Construction: explained before.

⇒ But Rotor could be of Two Types:

1] Cylindrical OR Non-salient (i.e 2-poles Rotor).

2] Non-Cylindrical OR Salient (i.e more than 2-poles).

$f = n\,p \longrightarrow$ No. of pair poles.

S.T ⇒ $n = 3000$ rpm
$f = 50$ HZ.

$\therefore\ 50 = \frac{3000}{60} * p \Rightarrow \underline{\underline{p = 1}}$

∴ Cylindrical Rotor is used.

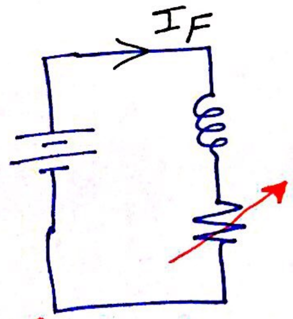
Hydro $n = (1000 - 1400)$ rpm.

∴ required poles > 2

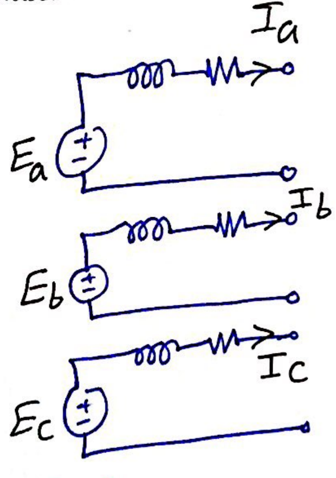
∴ Salient Rotor is used.

* Equivalent cct of gen.:

since the gen. has 3-phases:



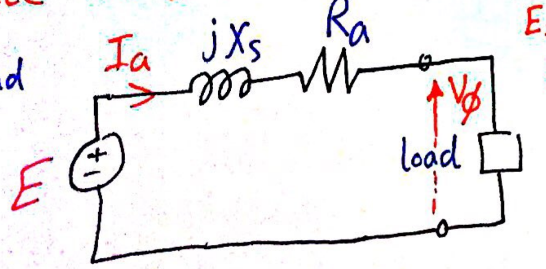
field cct.



Armature cct.

⇒ since the gen. is Balanced, Then one use per-phase cct:

At No-load
$V_\phi = E$



$E, V_\phi \equiv$ generated & Terminal phase voltages.

$X_s \equiv$ Synch. Reactance, whose components will be explained Later.

$R_a \equiv$ Armature winding Resistance.

$I_a \equiv$ Armature Current.

Scanned by CamScanner

*** Armature Reaction:**

The flux Produced by Armature current effect the main flux which is produced by Field windings, causing its reduction

since $E = K\phi\omega$, then E will be reduced.

$\therefore$ The effect of A.R is represented by a Reactance X.

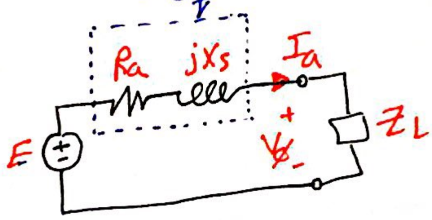
$\Rightarrow$ * since Armature winding has a self inductance, $L_a \Rightarrow X_a$

The sum of X & X_a is called "Synchronous Reactance", X_s

$$X_s = X + X_a$$

*** Phasor Diagram of Synch. Gen.:**

$\Rightarrow$ Here one uses the per-ph equ. cct.



- **objective:** To find voltage & current relationships under various loading condition.

This is the condition of generator operating alone.

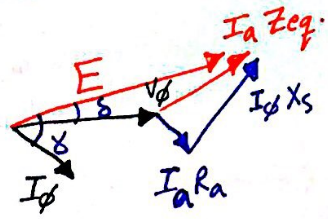
- **Procedure:** Keep $|V_\phi|$ & $|I_a|$ constants & vary Pf Angle of the load.

$$E = I_a Z_{eq} + V_\phi$$

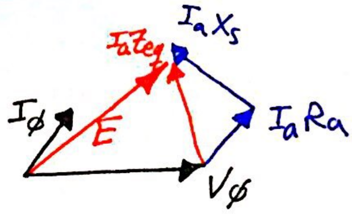
$\gamma \equiv$ angle between E & I_ϕ

$\delta \equiv$ angle between E & V_ϕ

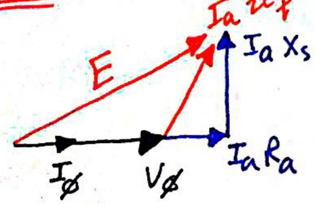
- **Inductive Load:**



- **Capacitive Load:**



- **Resistive Load:**



$\therefore |E_L| > |E_R| > |E_c|$

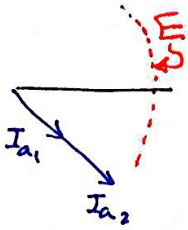
& $|E_c|$ couldbe $< |V_\phi|$

* Since $VR = \dfrac{|E| - |V_\phi|}{|V_\phi|}$ $\therefore$ VR for inductive load has high +ve magnitude than Resistive Load.

Scanned by CamScanner

* However, it can be deduced if $|E|$ is Kept constant Then as the Load changes $|I_a|$ is varied then $|V_\phi|_L < |V_\phi|_R < |V_\phi|_C$

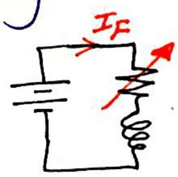
Note: see the corresponding phasor diagram in the Text book.



$\Rightarrow$ here in this case $|V_{\phi_2}| < |V_{\phi_1}|$

* **Comment:** As load increases (i.e $I_a\uparrow$) $\rightarrow$ Then voltage drop (i.e $I_a Z_{eq}\uparrow$) $\rightarrow$ then $V_\phi\downarrow$ ($V_\phi = E - I_a Z_{eq}$)

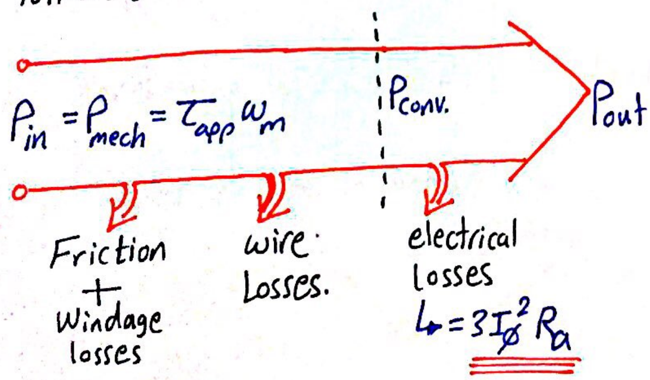
Hence to maintain V_ϕ at a constant value then one should $E\uparrow$ here one may $\uparrow\phi$ or $\uparrow\omega$ $\rightarrow$ But $\rightarrow$ since $E = K\phi\omega$ ω can NOT be increased in order to maintain f at specified value, Therefore one $\phi\uparrow$ by $I_F\uparrow$ by reducing the variable resistance in the field windings.



* Power & Torque Relationships in 3-ph Synch. Gen.:

This can be introduced by using power flow diagram as follows:

$P_{conv.} \equiv$ Amount of power converted to electrical.



$$P_{conv} = 3|E||I_\phi|\cos\delta = \sqrt{3}\,|E||I_\phi|\cos\delta$$

$$P_{out} = 3\,V_\phi I_\phi \cos\theta = \sqrt{3}\,V I_\phi \cos\theta$$

$$P_{conv} = \tau_{ind}\,\omega_m$$

$$\therefore \boxed{\tau_{ind} = \frac{P_{conv}}{\omega_m}}$$

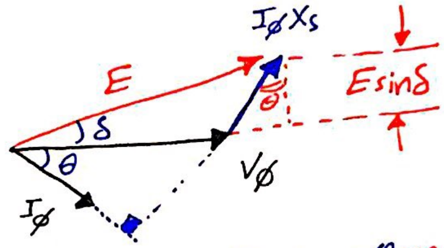
$\theta \equiv$ pf angle of load.

Scanned by CamScanner

⇒⇒ Approximate Expression for P & τ :

* Here R_a is neglected.

∴ The phasor diagram will be as follows:



∴ $P_{conv} = P_{out}$

$\Rightarrow E \sin\delta = I_\phi X_s \cos\theta \; \cdots \boxed{1}$

since $P_{out} = 3 V_\phi I_\phi \cos\theta \; \cdots\cdots \boxed{2}$

Sub. ① into ② :

$$P_{out} = \frac{3 V_\phi E \sin\delta}{X_s} \; \cdots\cdots \boxed{3}$$

since $P_{conv} = P_{out} = \tau_{ind}\,\omega_m$ ∴ $\tau_{ind} = P_{out}/\omega_m$

$$\Rightarrow \tau_{ind} = \frac{3 V_\phi E \sin\delta}{\omega_m X_s} \; \cdots \boxed{4}$$

***Comment:**

from $\boxed{3}$ it can be observed that $P_{out(max)}$ is when $\delta = 90°$.

$$\therefore \boxed{P_{out(max)} = \frac{3 V_\phi E}{X_s}}$$ ⇒ This called: Static Stability Limit.

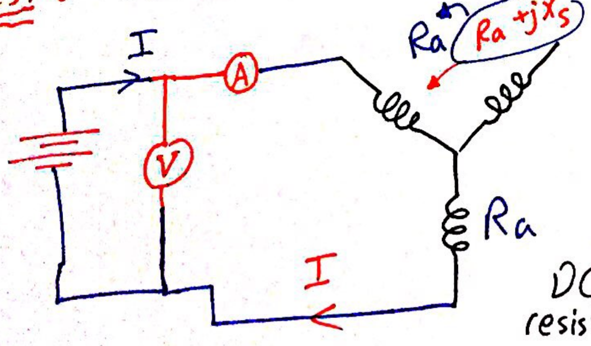
* in practice, δ never reach 90° usually it is in the Range of 20° → 30°

what?!
* Measurement of R_a & X_s :

why?! Because there values are required in the performance analysis of the generator.

How?! By Performing DC, O/C & S/C Tests on the generator.

* DC Test: This is used to measure R_a.



Armature winding usually Y-connected.

$$\therefore 2R_a = \frac{\text{voltmeter reading}}{\text{Ammeter reading}}$$

$$\therefore R_a = \frac{1}{2}\,\frac{V}{A}$$

DC resistance.

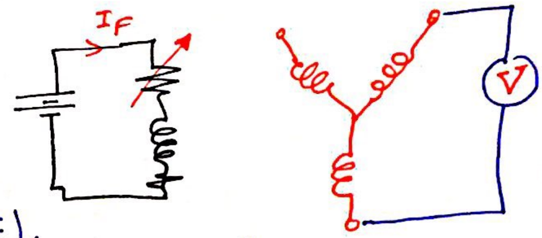
$R_{a,ac} = \text{factor} * R_{a,dc}$

Scanned by CamScanner

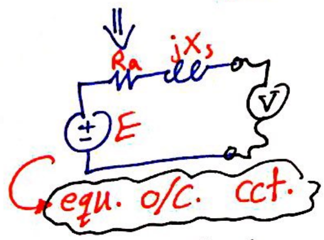
* O/C Test:

⇒ procedure: with terminals o/c, run the gen. at constant speed and then vary the field current.
Taken the readings of terminal voltage & field current.

I_F	I_1 .... I_n
V	V_1 .... V_n

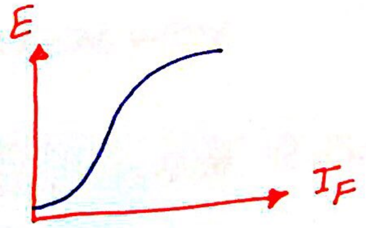


∴ In o/c V Reading ≡ Generated Voltage (E).



since $E = K\phi n$ ← constant (speed).

⇒ $E = K'\phi$ * But ϕ is generated by I_F.

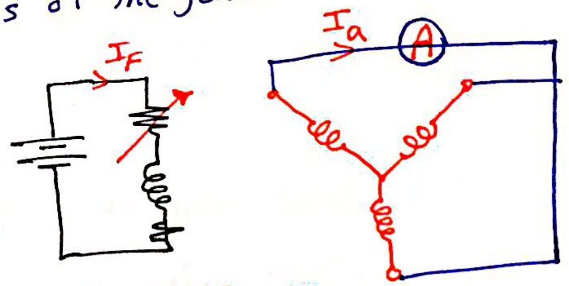


⇒ This is "o/c char. of the gen."
OR "Magnetization Curve."

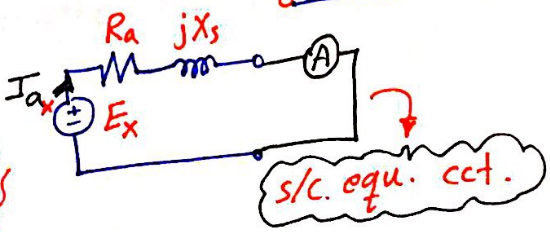
* S/C Test:

⇒ short circuit the terminals of the generator.

Then, vary I_F & take the corresponding Reading of I_a.

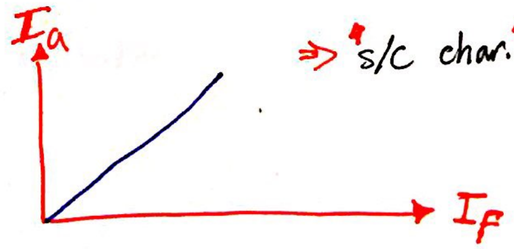


I_F	I_{F_1} .... I_{F_n}
I_a	I_{a_1} .... I_{a_n}

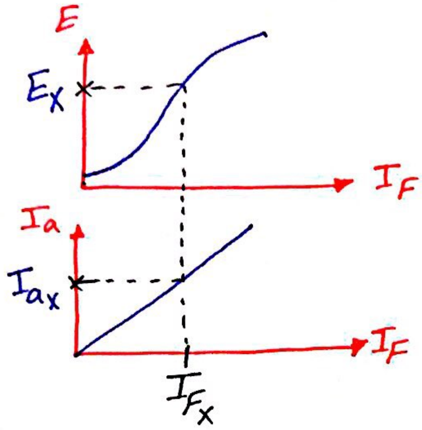


$$\Rightarrow I_a = \frac{E}{R_a + jX_s} \Rightarrow I_a = \frac{E}{Z_{eq}}$$

$$\Rightarrow \boxed{|I_a| = \frac{|E|}{|Z_{eq}|}}$$



⇒ "s/c char."



** For a given I_{Fx}, one would obtain from o/c & s/c characteristics the corresponding E & I_a ⇒ say E_x, I_{ax}

$$\therefore \frac{E_x}{I_{ax}} = |Z_{eq}|$$

⇒ since $Z_{eq} = R_a + jX_s$ ⇒ $\therefore |Z_{eq}| = \sqrt{R_a^2 + X_s^2}$

so $$X_s = \sqrt{|Z_{eq}|^2 - R_a^2}$$

* Name Plat of a generator contain the following:

1) Rated voltage 2) Rated current. 3) frequancy

* short circuit Ratio (SCR):

$$SCR \triangleq \frac{\text{The field current which produces Rated voltage at o/c.}}{\text{The field current which produces Rated armature current at s/c}}$$

$$\Rightarrow \boxed{SCR = \frac{I_{fv}}{I_{fI}}}$$

* It can be found that:

$$SCR = \frac{1}{X_s\,(PU)}$$

* Generator could be operated on its own called "Alone Generator".

OR * Generator could be working with other generators called "Parallel Generators".

what?! * Parallel Operation of 3-ph generator:

why?! ⇒ Objectives or Advantages:

1] More economical from operation Point of view.

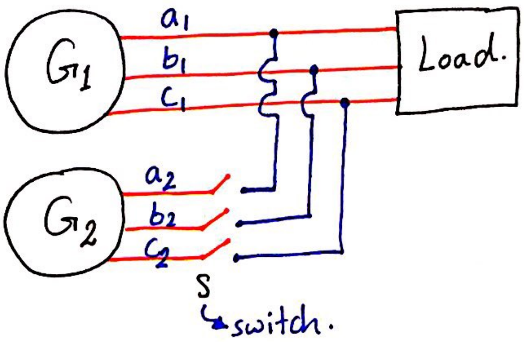
2] More Reliable.

Scanned by CamScanner

How ?! A generator could be connected in parallel with other generator OR Connected in parallel with a power system. (i.e National Grid).

* Conditions for Parallel Operation:

G_2 is to be connected with G_1 which is already suppling load.



* G_2 is called: "On coming Generator".

⇒ Condition of Parallel operation:
(i.e what should be done before closing S)

1] G_1 & G_2 should have the same voltage (i.e magnitude & phase angle).

The magnitude of G_2 Can be checked by means of VOLTMETER.

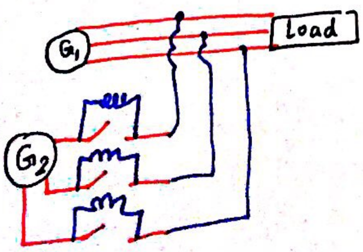
2] They should have the same sequence.

⇒ This could be checked by:

{a} using a small 3-ph induction motor ⇒ first connected to G_1 and then to G_2, observe direction of rotation.

→ if same direction, then same phase sequence.
→ otherwise, reverse the connection of 2-phases in G_2.

{b} Using 3-bulb method.



⇒ The brightness of the lamp depends on the phase shift between its terminals.

→ when 3-bulbs Light-up & dim-down simultanuously together, then G_1 & G_2 have the same phase sequence.

→ when 3-bulbs dim-down, then they are in-phase.

3] Frequancy of G_2 should be slightly higher than G_1

⟹ for stability reasons.

Scanned by CamScanner

* This process of Parallel operation is called:
"Synchronization."

ex. "Example 4-1 in Text Book".
200 KVA, 480 Volt, 50 HZ. Y-connected synch. Gen. with rated field current 5A was tested, and the following were taken:

$V_{T,oc}$ at rated I_F = 540 volt.
$I_{L,sc}$ at rated I_F = 300 A.
when a DC volt 10-V was applied a 25A was measured.

Find R_a & X_s ?

From the DC Test one can obtain R_a:

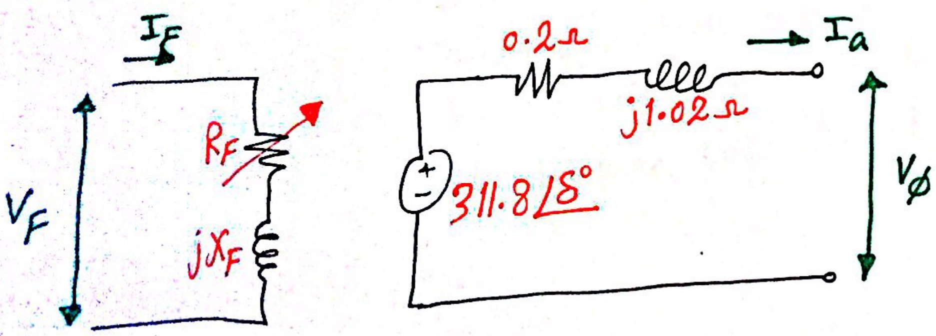
$$\rightarrow R_a = \frac{1}{2}\left(\frac{\text{Voltmeter}}{\text{Ammeter}}\right) = \frac{1}{2}\left(\frac{10}{25}\right) \Rightarrow \boxed{R_a = 0.2\,\Omega}$$

from the o/c Test one can obtain E_A:
since it is Y-connected: $E_A = \frac{V_{T,oc}}{\sqrt{3}} = \frac{540}{\sqrt{3}} \Rightarrow \boxed{E_A = 311.8 \text{ volt.}}$

from the s/c Test one can obtain I_a:
since it is Y-connected: $\boxed{I_a = I_{L,sc} = 300A}$

Now: $|Z_{eq}| = \frac{E_A}{I_a} = \frac{311.8}{300} = 1.04\,\Omega = \sqrt{R_a^2 + X_s^2}$

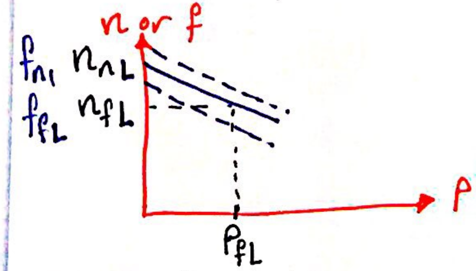
so $X_s = 1.021\,\Omega$



* Power-frequancy & voltage-Reactive power Relationships:

As the load (i.e P) on the gen ↑ the speed (i.e n) ↓ due to the rotating torque.

This decrease is non-linear, but by using a Governor Mechanizm the drop is made linear.



figure(1).

$$\therefore \boxed{\text{speed Droop} = \frac{n_{nL} - n_{fL}}{n_{fL}}}$$

No Load (n_{nL}), full load (n_{fL}).

since $f = np$.

∴ since drop is linear, then the output of the gen. can be epressed as:

$$P = S_p (f_{nL} - f_{fL})$$

where S_p unit is : MW/HZ.

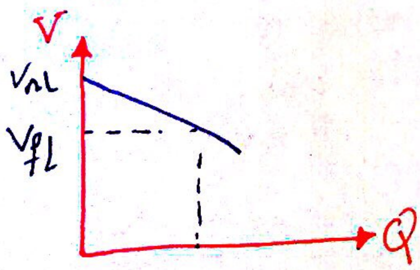
*By means of the Governor, the no-load set-point can be changed.

∴ charachteristics can be shifted up or down.

*Application of this characteristics: is that it can be used to find Load sharing between parallel generators as will be illustrated by an example.

*It was found before by using phasor diagram that as the inductive Load increases (i.e Reactive Power increases) then terminal voltage ↓.

By means of a voltage Regulator, this decreas is made Linear.



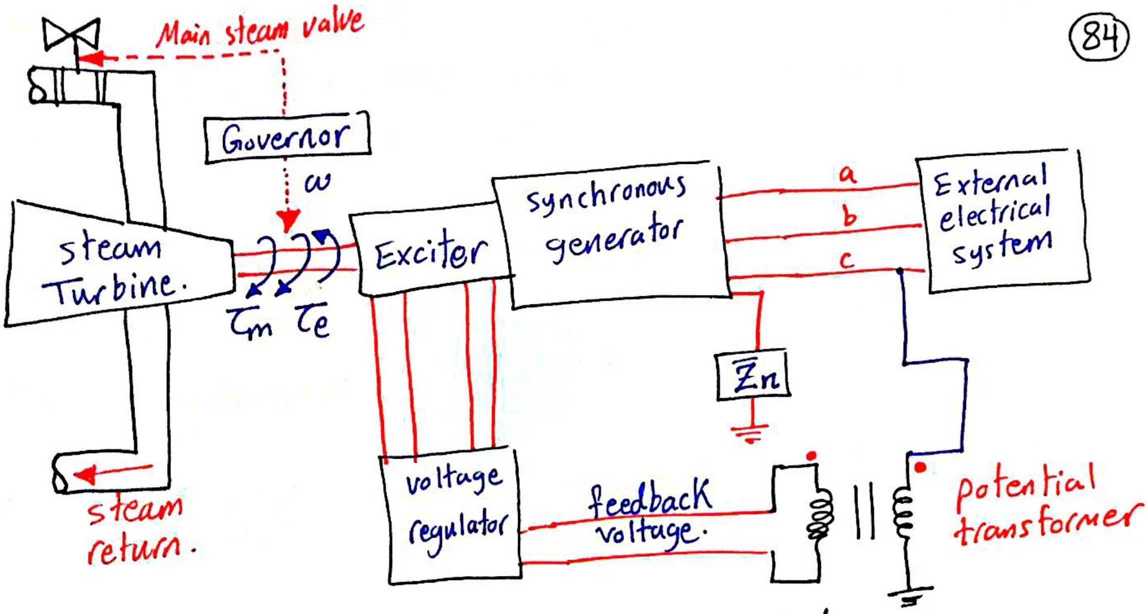
figure(2).

figure(1) & figure(2):

These 2 controls are Illustrated by the following Block diagram of Turbine-Generator-Exciter system.

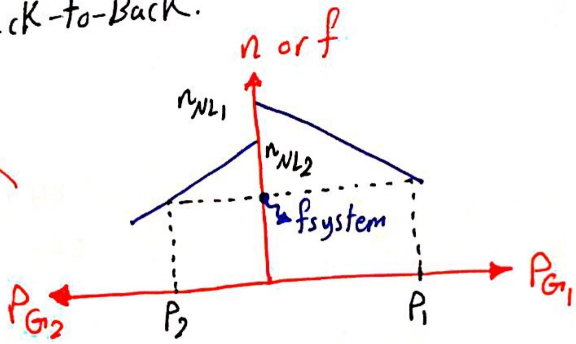
⇒

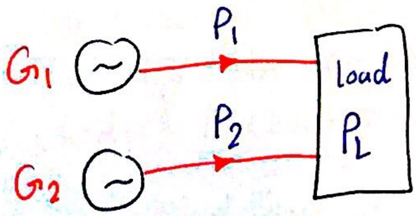
Scanned by CamScanner



* Power sharing between 2 generators can be solved by drawing their characteristics Back-to-Back.

This called: "Housing Diagram"



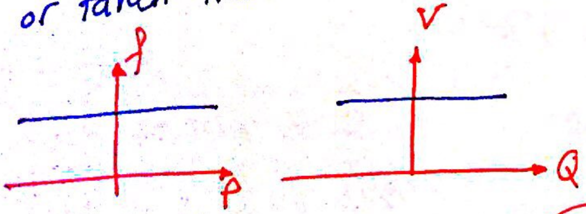


$$P_1 + P_2 = P_L$$

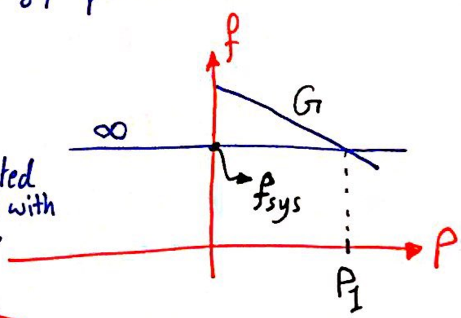
$$S_{P_1}(f_{NL_1} - f_{sys}) + S_{P_2}(f_{NL_2} - f_{sys}) = P_L$$

* Connecting a Generator to infinite system:

- Definition: ∞ system it is a large power system whose frequancy and voltage, does not change by the amount of power supplied to it or taken from it.



⇒ for G connected with parallel with ∞ system.



$$P_\infty = P_L - P_1$$

Scanned by CamScanner

ex.) A DC, o/c, S/C Tests as explained before, were performed on 25 MVA, 12.2 KV, 0.9 pf Lagging 3-ph, Y-connected generator with the following Results:

DC Test: V = 12 volt & I = 10 A.

O/C Test:

I_F (A)	320	365	380
V (KV)	15.4	17.5	18.3

S/C Test:

I_F (A)	320	365	380
I_a (A)	1040	1190	1240

Find the parameters of generator?

* Rated pf: This is the lowest acceptable pf of the gen. when it is operating at it rated apparent power.

we take any one from table.

$R_{dc} = \frac{1}{2}\left(\frac{12}{10}\right) \Rightarrow \boxed{R_{dc} = 0.6\Omega}$

$|Z| = \frac{V_{oc}}{I_{sc}} = \frac{(15.4/\sqrt{3}) * 10^3}{1040} = 8.55\Omega$

since we need phase Voltage.

$\Rightarrow X_s = \sqrt{|Z|^2 - R^2} \Rightarrow X_s = 8.53\Omega$

---

ex.) a 480 volt, 200 KW, 2 pole, 3 Ph, 50 HZ synch. gen. with its turbine has $n_{NL} = 3040$ rpm and $n_{FL} = 2975$ rpm, is in parallel with 480 V, 150 KW, 4 pole, 3-ph, 50 HZ synch. gen. with its turbine has $n_{NL} = 1500$ rpm & $n_{FL} = 1485$ rpm in order to supply a load of $P_L = 200$ KW at 0.85 pf Lagging.

1) Find Speed Droop (SD) for each gen?

∴ By substitution $\boxed{SD_{G_1} = 2.18\%}$, $\boxed{SD_{G_2} = 1.01\%}$

2) Find the operating frequancy of the system (i.e f_{sys}) and then load sharing?

$f = nP$, by using this:

	G_1	G_2
f_{nL}	50.67 HZ	50 HZ.
f_{fL}	49.58 HZ	49.5 HZ.

$SP_1 = \frac{200}{50.67 - 49.58} = 183.5$ KW/HZ.

$SP_2 = \frac{150}{50 - 49.5} = 300$ KW/HZ.

$\therefore 200 = \underbrace{183.5(50.67 - f_{sys})}_{P_{G_1}} + \underbrace{300(50 - f_{sys})}_{P_{G_2}}$

$\Rightarrow \boxed{f_{sys} = 49.84 \text{ HZ}}$

$P_{G_1} = 152.3$ KW. $P_{G_2} = 47.7$ KW.

Scanned by CamScanner

* DC Machines:

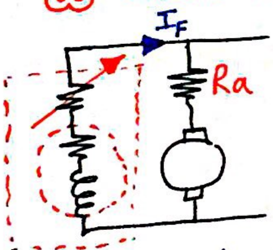
* Construction: It was explained during Fundementals of electrical machines ⇒ Field windings are located at the stator.
Armature windings are located at the Rotor.

* Classification:
According to the source of field current (I_F), DC Machines are classified into:

1] Separately Excited Machine.
I_F is supplied from External DC source.

2] Self-Excited DC Machine.
I_F depends on residual magnetizem in the poles of the machine and classified into:

(a) Shunt DC Machines.

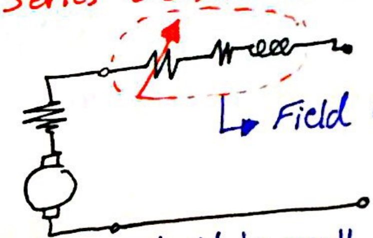


↳ Field cct.

field cct should be high for I_F to be small.

* Number of turns (N) in series is less than in shunt.

(b) Series DC Machines.

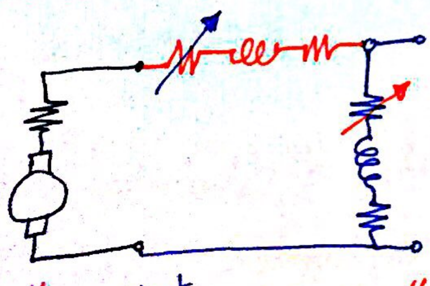


↳ Field cct.

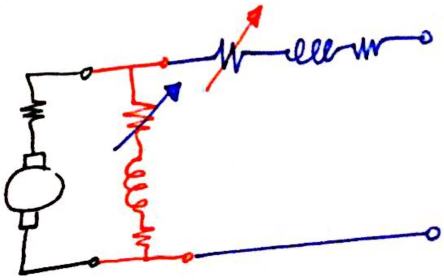
field cct should be small for voltage drop to be small

(c) Compound DC Machines.
It has 2 field windings (i.e series & shunt)

There are 2 possible connection:



" Long shunt Compound DC " Machines.



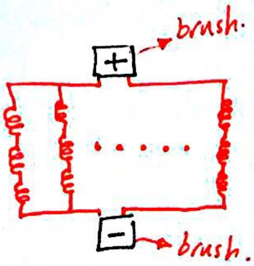
" short shunt compound " DC Machines.

Scanned by CamScanner

* If the series field aid shunt field the machines is called "Commulative Compound Machine".
* If the series field opposes shunt field the machines is called "Differential Compound Machine".

* If we need the machine Generator $\Rightarrow$ Connect R_L.
* If we need the machine Motor $\Rightarrow$ Connect a voltage source.

Introduction to the types of Armature Winding:



$\Rightarrow$ parallel paths. (Armature) winding

$a \equiv$ Number of parallel paths between +ve & −ve brushes.

$a \Rightarrow$ depends on the type of Armature windings.

$a = mP$ for Lap winding.

$a = 2m$ for Wave winding.

$m \equiv$ Multiplicity of Armature winding.

$m = 1$ for simplex.
$m = 2$ for Duplex.
$m = 3$ for Triplex.

$P \equiv$ Number of poles.

* Illustration:

consider a double Layer Armature winding with 36 slots, let the machine has 4 poles. If the machine (type of winding) is a simplex Lap winding.

$a = mp = 1 * 4 = \boxed{4}$

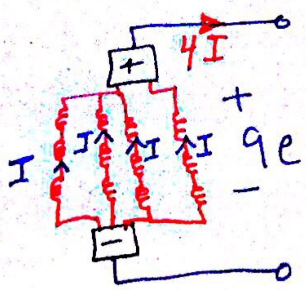
$\Rightarrow$ since it is a double layer so No. of coils = No. of slots.

No. of coils $= \boxed{36}$

$\Rightarrow$ No. of coils/path $= 36/4 = \boxed{9}$

if induced voltage per coil $= e$

$\therefore$ generated voltage (voltage between brushes) $= 9e$



if the rated current of coil $= I$

$\therefore$ Rated current of the machine $\equiv a * I = 4I$

$\Longrightarrow$

⇒ ∴ Rated power of the machine $\equiv 9e * 4I$

$= \boxed{36\,e\,I}$

* If the type of winding is a simplex wave winding.

⇒ $a = 2*m = 2*1 = \boxed{2}$

No. of coils / path $= 36/2 = \boxed{18}$

voltage between brushes $= 18e$

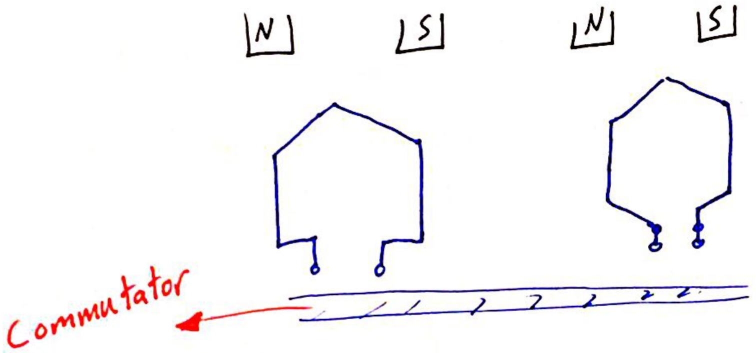
produced current $= I_a = 2I$

Rated power $= 18e * 2I = 36\,eI$

⇒ Conclusion :

* Lap winding used for Low voltage & High current application.

* Wave winding used for High voltage & Low current application.



see the Text Book for full figure.

* Voltage Induced in DC Generator (i.e voltage between brushes):

It was found before that the voltage induced in a single turn $e = 2B\ell v$ where a turn consists of 2 conductors.

∴ $e = 2B\ell\omega r$ ....① But Area per pole: $A_p = \frac{2\pi\ell r}{P}$ ....②

∴ flux per pole: $\phi = BA_p = \frac{B\,2\pi\ell r}{P}$ ∴ $2B\ell r = \frac{P\phi}{\pi}$ ....③

Substitute ③ into ①:

⇒ $e = \frac{P\phi\omega}{\pi}$ ....④ "Voltage induced in a single turn".

* If the total Number of Armature conductors $\equiv Z$.

∴ Total Number of Armature Turns $\equiv Z/2$

∴ Number of Turns per path $\equiv \frac{Z}{2a}$ ....⑤

∴ Voltage generated across brushes, E = Voltage per path = ④ * ⑤

Scanned by CamScanner

$\therefore E = \frac{P\phi\omega}{\pi} * \frac{Z}{2a}$ But $\omega = 2\pi n$ $n \equiv$ speed r.p.s.

$\Rightarrow \boxed{E = \frac{P\phi Z n}{a}}$ $K = \frac{PZ}{a}$

* For a given machine P, Z, a are constants so $\Rightarrow \underline{E = K\phi n}$

** For a generator, E is called "generated voltage".

** For a Motor, E is called "Back emf".

※ Induced Torque in DC Machine:

It was found before that the torque induced in a single turn

$\tau_{ind} = 2BlIr$, $I \equiv$ current passing through turn.

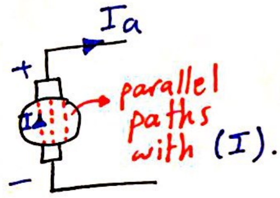
$\ldots$ (6)

* Sub. (3) into (6): $\tau_{turn.} = \frac{P\phi I}{\pi}$

$\therefore$ Total Torque produced, $\underline{T}$ $\Rightarrow T = \tau_{turn} *$ Total Number of Turns.

so, $T = \frac{P\phi I}{\pi} * \frac{Z}{2}$

* if the armature current is I_a, $\boxed{I = \frac{I_a}{a}}$



$\Rightarrow \boxed{T = \frac{P\phi Z I_a}{2\pi a}}$

* For a given machine P, Z, a constants: $\Rightarrow \underline{T = K'\phi I_a}$ $K' = \frac{PZ}{2\pi a}$

** For a DC Motor, T is called "Developed or Generated" Torque.

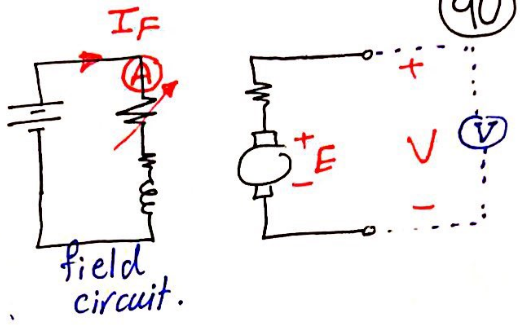
** For a DC generator, T is called "Opposing or Retarding Torque".

※ Characteristics of DC Generator:

* Objective: To find No-load & load char.

Separately Excited:

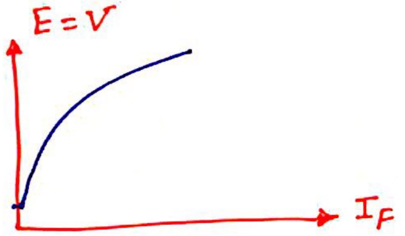
- **No-load:** $I_F \equiv$ field current. $E, V \equiv$ Generated & Terminal voltages.



*Procedure:
Run generator at constant speed then vary I_F, and take the reading of V.

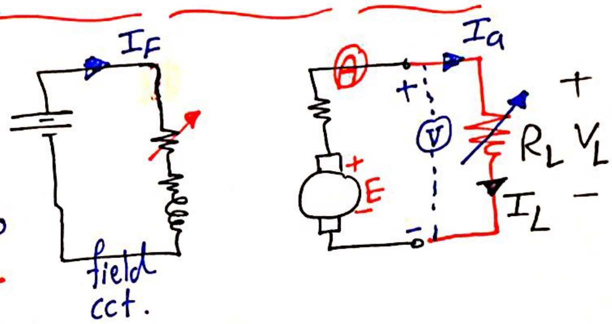
I_F	I_1 ...... I_n
V	V_1 ...... V_n

⟹ Then plot V vs. I_F since $E = K\phi n$.



⟹ This Called: Magnetization Curve. OR No-load char.

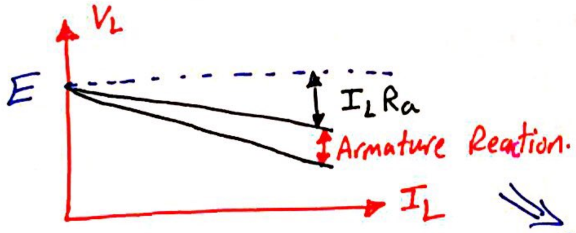
- **Load Characteristic:**



*procedure: Run generator at constant speed and for a given I_F, vary R_L, & record the readings of V_L & I_L.

⟹ Then plot V_L vs. I_L.

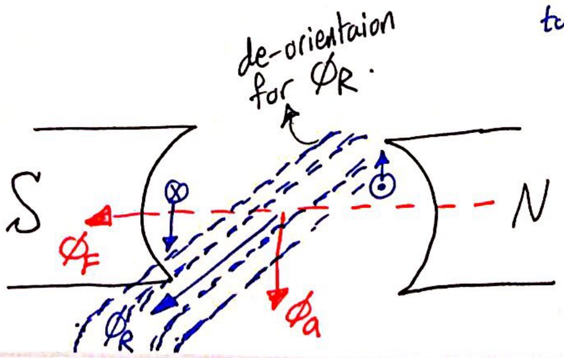
*when $I_L = 0$, (No-load); $V_L = E$.



* using KVL: $V_L = E - I_L R_a$

$R_a \equiv$ Equivalent Armature Resistance

⟹ so we have two types of drop.



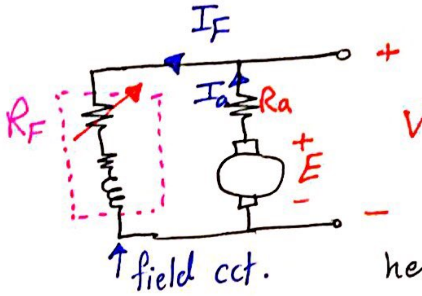
⟹ ϕ_a cause a de-orientation for the resultant flux.

Scanned by CamScanner

Self - excited:

- **shunt generator:**

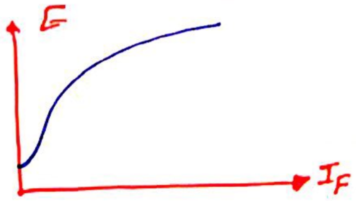
No load:



* Run gen. at constant speed.

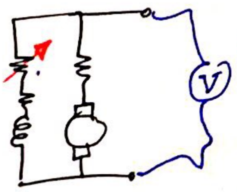
here $I_a = I_F$

since $E = K\phi n$.



⇒ This called: Build-up of voltage.
due to Residual Magnetism

$$I_a = I_F = \frac{E}{R_a + R_F}$$



→ if the voltmeter reads zero, we say voltage is failed to Build-up.

* Causes of failure to Build-up:

1] Lack of Residual Magnetism.

2] Field windings are connected in such a way to cancel Residual Magnetism.

3] open circuit in the field or Armature Windings.

Load:

$\Rightarrow V_L = E - I_a R_a$

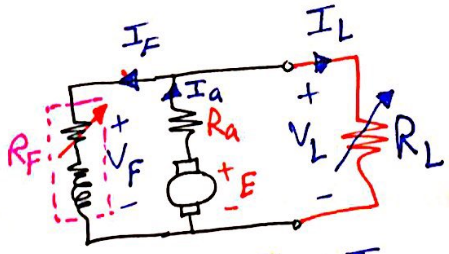
$\Rightarrow I_L\uparrow \rightarrow I_a\uparrow$

$\Downarrow$

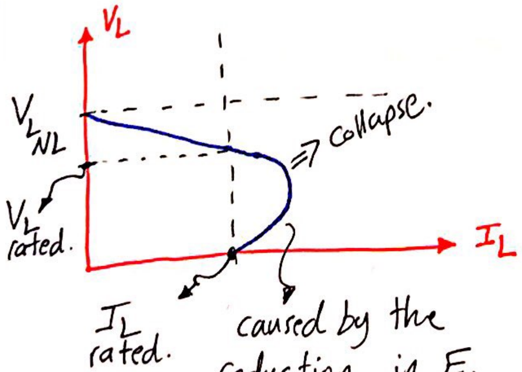
$I_a R_a \uparrow$

$\Downarrow$

$V_L \downarrow$



$I_a = I_L + I_F$



caused by the reduction in E. & $I_a R_a$.

also $V_L = V_F \Rightarrow V_L\downarrow \Rightarrow V_F\downarrow$

$$I_F = \frac{V_F}{R_F}$$

$V_F\downarrow \Rightarrow I_F\downarrow \Rightarrow \phi\downarrow \Rightarrow E\downarrow$

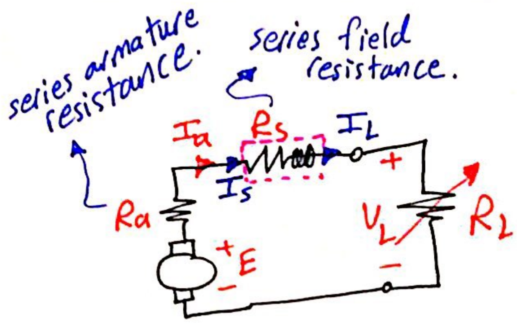
field cct Resistance.
"represent all of field cct"

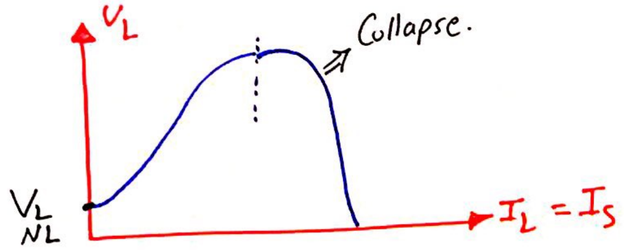
Scanned by CamScanner

• Series generator:

$V_L = E - I_L(R_a + R_s)$

$I_s = I_a = I_L$

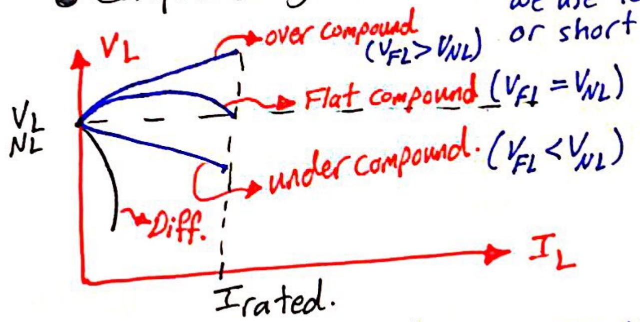


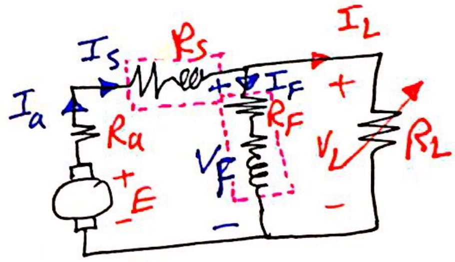


$I_L \uparrow \Rightarrow E = K\phi n \uparrow \Rightarrow I_L(R_a+R_s) \downarrow$

• Compound generator:

"No difference if we use long shunt or short shunt."





$R_F \equiv$ shunt field cct resistance.

$R_a \equiv$ Armature cct resistance.

$R_s \equiv$ Series Field resistance.

$V_L = V_F$

$I_a = I_s = I_F + I_L$

$E = I_a(R_a + R_s) + V_F$

$V_L = V_F = E - I_a(R_a+R_s)$

$I_L\uparrow \Rightarrow I_a\uparrow$

* Commulative:

$N_s I_s$ aid $N_F I_F$

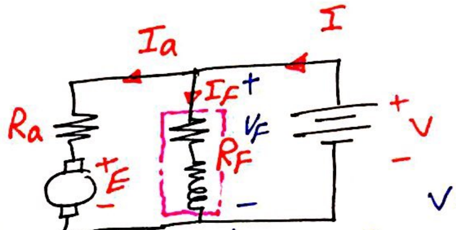
$N_s I_s$: mmf of series field

$N_F I_F$: mmf of shunt field.

* Depending on the amount of aiding there are 3 possible characteristic: see the above figure.

※ Characteristics of DC Motors:

● **Shunt Motor:**



$V, I \equiv$ Applied Voltage & Current.
$E \equiv$ Back EMF.

$V = V_F$.

* Objective: To find various relationships of voltage, current, power and Load characteristics.

* Procedure: $E = V - I_a R_a$ ····· ① ⇒ Multiply ① by I_a:

$$\Rightarrow \quad E I_a = V I_a - I_a^2 R_a \; \text{②} \quad ; (I_a = I - I_F) \cdots\cdots \text{③}$$

* sub. ③ into ②:

$$E I_a = \underbrace{V I}_{\text{input to motor.}} - \underbrace{V I_F}_{\text{field cct Losses.}} - \underbrace{I_a^2 R_a}_{\text{Armature cct Losses.}}$$

$$\Rightarrow \underbrace{E I_a}_{\text{Converted power } (P_{conv.})} = \underbrace{V I}_{\text{input.}} - \underbrace{(V I_F + I_a^2 R_a)}_{\text{Electrical Losses.}}$$

* **Note:**

It can be found that for ALL TYPES of motors that:

$$P_{conv.} = E I_a \quad ; \quad P_{conv} = \tau_{ind} \cdot \omega_m = E I_a \Rightarrow \tau_{ind} = \frac{E I_a}{\omega_m}$$

* We could be asked to prove $P_{conv.} = E I_a$ for any Type.

※ Speed Equation:

$$E = V - I_a R_a \Rightarrow K\phi\omega = V - I_a R_a \Rightarrow \therefore \omega = \frac{V - I_a R_a}{K\phi}$$

speed equation (SE).

This SE. shows the 3 methods of controlling the speed of shunt motor:

1) By varying applied voltage (V), (Armature Control).
2) By varying flux (ϕ), (Field Control).
3) By varying Armature cct. Resistance. (Rarely being used).

Scanned by CamScanner

* Field Control:

⇒ Vary R_F.

for ex. if $R_F \uparrow \Rightarrow I_F = \frac{V}{R_F} \downarrow \Rightarrow \phi \downarrow \Rightarrow E \downarrow$

since $I_a = \frac{V - E}{R_a} \Rightarrow E \downarrow \Rightarrow$ so $I_a \uparrow$ But $\tau_{ind} = K \phi I_a$ ($\downarrow$, $\phi \downarrow$, $I_a \uparrow$)

⇒ increasing in I_a > decreasing in ϕ

so $\tau_{ind} \uparrow \Rightarrow \omega \uparrow$ As $\phi \downarrow \Rightarrow \omega \uparrow$

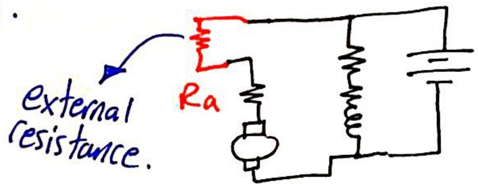
*** For shunt motor (Never ever open suddenly the shunt field cct)

since ⇒ $I_F = 0$ so $\phi = 0$ which lead to $\omega = \infty$.

* Armature Control:

⇒ vary the applied voltage.

* Varying Armature cct. resistance: (By adding an external resistance to R_a).



---

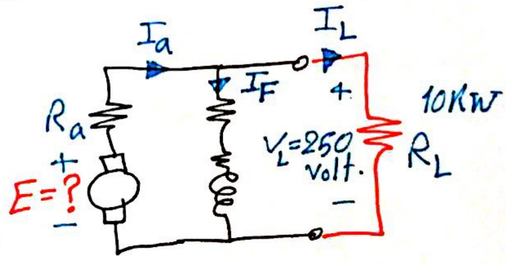
ex. A 10KW, 250V, DC shunt gen. is driven at 1000 rpm and given $R_a = 0.15\,\Omega$ & $I_F = 1.64\,A$. If Rotational losses = 540 W.

Find at Rated Conditions: 1] Generated Voltage. 2] Efficiency.

1] By KVL:

$E = I_a R_a + V_L$; $V_L = 250$ volt & $R_a = 0.15\,\Omega$

$\Rightarrow I_a = I_F + \underbrace{I_L}_{P_L / V_L} = 1.64 + \frac{10 * 10^3}{250} = 41.64\,A$



substitute: $\boxed{E = 256.25 \text{ volt.}}$

2] $\eta = \frac{P_{out}}{P_{out} + P_{losses}} * 100\%$, $P_{out} = P_L = 10 * 10^3\,W$. , P_{losses} = electrical + mechanical. (mechanical ≡ Rotational Losses.)

Electrical = Losses of Armature + losses of field $= I_a^2 R_a + V_F I_F = \boxed{670.08\,W.}$

substitute: $\eta = 89.2\%$

Scanned by CamScanner

ex. A 7.5 hP (hourse Power), 120 V DC series Motor has $R_a = 0.2\,\Omega$, $R_s = 0.16\,\Omega$. At Full Load the current input is 58A. at the rated speed of 1050 rpm. Given Core Losses = 200 W. , mechanical Losses = 240 W At full load. Assume mechanical losses is propotional with the cube of motor's speed. ($P_{mech.} \propto \omega^3$) & the Core losses are constant.

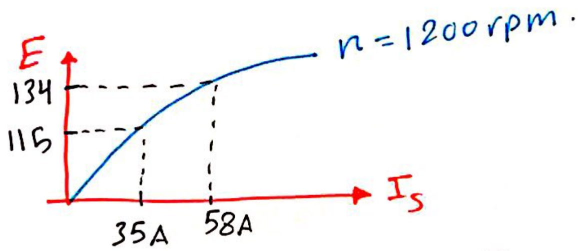
Also given from the magnetization curve which produced at $n = 1200$ rpm the followings:

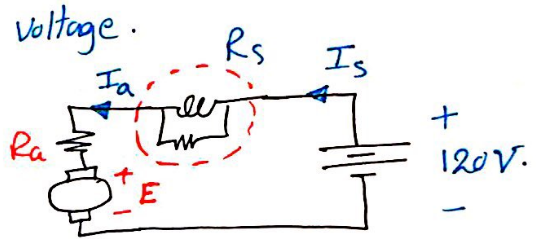
$I_F = 35A \rightarrow E = 115V$.

$I_F = 58A \rightarrow E = 134V$.

* Analysis for the Givens :

120 V in the example $\Longrightarrow$ it is a Terminal Voltage.





* Find the speed & efficiency of the motor if it is operating at Armature current 35A ?

Solution:

remember: E here is called Back emf.

$I_a = I_F$ $\Rightarrow$ $E = V - I_a(R_a + R_s)$

$R_a = 0.2\,\Omega$ for * $I_a = 35A \Rightarrow \boxed{E_1 = 107.4 \text{ volt}}$

$R_s = 0.16\,\Omega$ * $I_a = 58A \Rightarrow \boxed{E_2 = 99.1 \text{ volt.}}$

Since $E = K\phi n \Rightarrow \therefore \dfrac{E_1}{E_2} = \dfrac{\phi_1 n_1}{\phi_2 n_2}$ .... $\boxed{1}$

* Ratio $\dfrac{\phi_1}{\phi_2}$ can be evaluated from the given information as follows.

$E_1' = 115$ volt for $I_s = 35A$, $n = 1200$ rpm.

$E_2' = 134$ volt for $I_s = 58A$, $n = 1200$ rpm.

$\therefore \dfrac{E_1'}{E_2'} = \dfrac{115}{134} = \dfrac{\phi_1 \; 1200}{\phi_2 \; 1200}$ $\quad \therefore \dfrac{\phi_1}{\phi_2} = \dfrac{115}{134}$ .... $\boxed{2}$

Continue.

Scanned by CamScanner

⇒ ∴ Substituting ② into ①:

$$\frac{107.4}{99.1} = \frac{115}{134} \cdot \frac{n_1}{1050} \Rightarrow \boxed{n_1 = 1326\ rpm}. \#$$

Now for the efficiency:

$$\eta = \frac{P_{out}}{P_{in}} * 100\%$$

for the input: $P_{in} = (35)(120) \Rightarrow \boxed{P_{in} = 4200W}$

for the output: $P_{out} = P_{conv} - (P_{core} + P_{mech})$ ⇒ Always Put inmind the power flow Diagram.

$\Rightarrow P_{conv.} = E\, I_a = (107.4)(35)$

$\Rightarrow \boxed{P_{conv} = 3759\ W.}$

$\boxed{P_{core} = 200W}$ $P_{mech} = 240W$ @ $n = 1050$ rpm.

$P_{mech} = ?\ W$ @ $n = 1326$ rpm.

so $P_{mech} = 240 \left(\frac{1326}{1050}\right)^3 = \boxed{483.4\ W.}$

Now substituting: $\Rightarrow P_{out} = 3759 - (483.4 + 200) \Rightarrow \boxed{P_{out} = 3075.6W}$

so $\eta = P_{out}/P_{in} * 100\% \Rightarrow \boxed{\eta = 73.2\%}\ \#$

*** Always Draw the circuit before you start to solve.

---

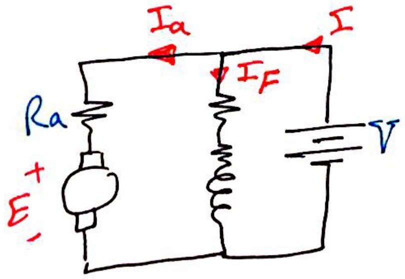
* Load Characteristic of DC Motors:

This give Relationship between two output of the Motor.

⇒ speed (n or ω) & Torque.

* Illustration:

For example, consider shunt Motor:



$E = V - I_a R_a$; $E = K\phi\omega$, $T = K'\phi I_a$

① ② ③

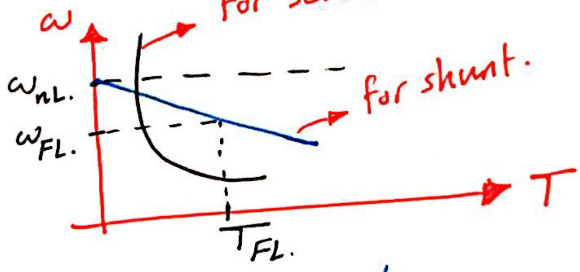
Continue.

⇒ Substitute ②,③ into ①:

$$K\phi\omega = V - \frac{T}{K'\phi} R_a \Rightarrow \therefore \boxed{\omega = \frac{V}{K\phi} - \frac{R_a}{KK'\phi^2} T}$$

constant   Constant.

V is constant ⇒ I is constant ⇒ ϕ is constant.
also I_f constant.



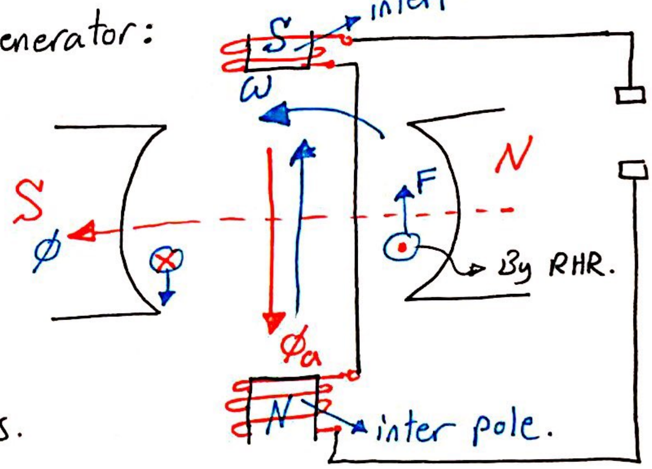
when T increased ⇒ ω decreased.

* speed Regulation:

$$SR = \frac{\omega_{nL} - \omega_{FL}}{\omega_{FL}}$$

※ Solution of Armature Reaction Problem:

- To illustrate Consider a DC Generator:



Armature Reaction: Intersection between ϕ_a & ϕ. where $\phi \propto I_a$

Solution → is to eliminate or Reduce ϕ_a.

By using for example: Interpoles.

⇒ These interpoles generate flux opposing ϕ_a.
hence they are connected in series with Armature windings.

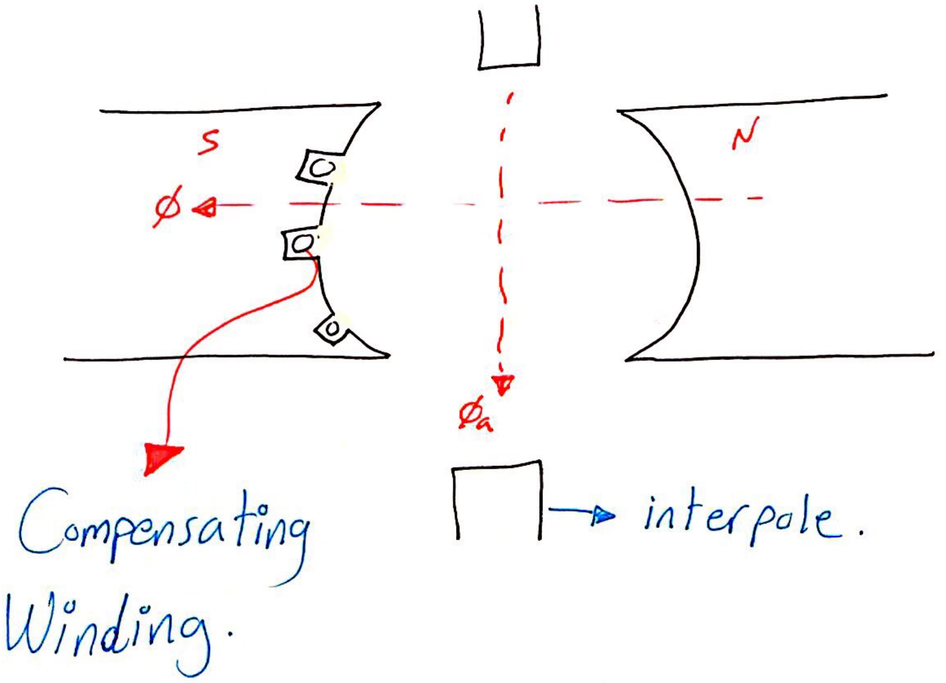
* polarty of interpole = Polarity of adjacent Main pole when moving in the direction of Rotation.

* Armature Resistance = R_a + interpole resistance.

- For DC Motor:
The same procedure for DC generator.

Scanned by CamScanner

⇒ * Some times it could be with Compensating winding.



* Here in this case, the total armature resistance would be as follows:

Armature Resistance = Ra + interpole Winding. + Compensating Winding.

* End of Material. *

Best of Luck.

Scanned by CamScanner