Chapter 15, Problem 1.

Find the Laplace transform of:

(a) cosh *at* (b) sinh *at*

[*Hint:* $\cosh x = \frac{1}{2}\left(e^{x} + e^{-x}\right)$, $\sinh\ x = \frac{1}{2}\left(e^{x} - e^{-x}\right)$.]

Chapter 15, Solution 1.

(a) $$\cosh(at) = \frac{e^{at} + e^{-at}}{2}$$

$$L\left[\cosh(at)\right] = \frac{1}{2}\left[\frac{1}{s-a} + \frac{1}{s+a}\right] = \underline{\mathbf{\frac{s}{s^2 - a^2}}}$$

(b) $$\sinh(at) = \frac{e^{at} - e^{-at}}{2}$$

$$L\left[\sinh(at)\right] = \frac{1}{2}\left[\frac{1}{s-a} - \frac{1}{s+a}\right] = \underline{\mathbf{\frac{a}{s^2 - a^2}}}$$

Chapter 15, Problem 2.

Determine the Laplace transform of:

(a) $\cos(\omega t + \theta)$ (b) $\sin(\omega t + \theta)$

Chapter 15, Solution 2.

(a) $$f(t) = \cos(\omega t)\cos(\theta) - \sin(\omega t)\sin(\theta)$$

$$F(s) = \cos(\theta)\,L\left[\cos(\omega t)\right] - \sin(\theta)\,L\left[\sin(\omega t)\right]$$

$$F(s) = \underline{\mathbf{\frac{s\cos(\theta) - \omega\sin(\theta)}{s^2 + \omega^2}}}$$

(b) $$f(t) = \sin(\omega t)\cos(\theta) + \cos(\omega t)\sin(\theta)$$

$$F(s) = \sin(\theta)\,L\left[\cos(\omega t)\right] + \cos(\theta)\,L\left[\sin(\omega t)\right]$$

$$F(s) = \underline{\mathbf{\frac{s\sin(\theta) - \omega\cos(\theta)}{s^2 + \omega^2}}}$$

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Chapter 15, Problem 3.

Obtain the Laplace transform of each of the following functions:

(a) $e^{-2t}\cos 3tu(t)$ (b) $e^{-2t}\sin 4tu(t)$

(c) $e^{-3t}\cosh 2tu(t)$ (d) $e^{-4t}\sinh tu(t)$

(e) $te^{-t}\sin 2tu(t)$

Chapter 15, Solution 3.

(a) $$L\left[e^{-2t}\cos(3t)u(t)\right] = \underline{\mathbf{\frac{s+2}{(s+2)^2+9}}}$$

(b) $$L\left[e^{-2t}\sin(4t)u(t)\right] = \underline{\mathbf{\frac{4}{(s+2)^2+16}}}$$

(c) Since $L\left[\cosh(at)\right] = \frac{s}{s^2-a^2}$

$$L\left[e^{-3t}\cosh(2t)u(t)\right] = \underline{\mathbf{\frac{s+3}{(s+3)^2-4}}}$$

(d) Since $L\left[\sinh(at)\right] = \frac{a}{s^2-a^2}$

$$L\left[e^{-4t}\sinh(t)u(t)\right] = \underline{\mathbf{\frac{1}{(s+4)^2-1}}}$$

(e) $$L\left[e^{-t}\sin(2t)\right] = \frac{2}{(s+1)^2+4}$$

If $f(t) \longleftrightarrow F(s)$

$t\,f(t) \longleftrightarrow \frac{-d}{ds}F(s)$

Thus, $$L\left[te^{-t}\sin(2t)\right] = \frac{-d}{ds}\left[2\left((s+1)^2+4\right)^{-1}\right]$$

$$= \frac{2}{((s+1)^2+4)^2}\cdot 2(s+1)$$

$$L\left[te^{-t}\sin(2t)\right] = \underline{\mathbf{\frac{4(s+1)}{((s+1)^2+4)^2}}}$$

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Chapter 15, Problem 4.

Find the Laplace transforms of the following:

(a) $g(t) = 6\cos(4t - 1)$

(b) $f(t) = 2tu(t) + 5e^{-3(t-2)}u(t-2)$

Chapter 15, Solution 4.

(a) $$G(s) = 6\frac{s}{s^2 + 4^2}e^{-s} = \underline{\frac{6se^{-s}}{s^2 + 16}}$$

(b) $$F(s) = \underline{\frac{2}{s^2} + 5\frac{e^{-2s}}{s+3}}$$

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Chapter 15, Problem 5.

Find the Laplace transform of each of the following functions:

(a) $t^2 \cos(2t + 30°)u(t)$ (b) $3t^4 e^{-2t} u(t)$

(c) $2tu(t) - 4\frac{d}{dt}\delta(t)$ (d) $2e^{-(t-1)}u(t)$

(e) $5u(t/2)$ (f) $6e^{-t/3}u(t)$

(g) $\frac{d^n}{dt^n}\delta(t)$

Chapter 15, Solution 5.

(a)

$$\mathrm{L}\left[\cos(2t+30°)\right] = \frac{s\cos(30°) - 2\sin(30°)}{s^2+4}$$

$$\mathrm{L}\left[t^2\cos(2t+30°)\right] = \frac{d^2}{ds^2}\left[\frac{s\cos(30°)-1}{s^2+4}\right]$$

$$= \frac{d}{ds}\frac{d}{ds}\left[\left(\frac{\sqrt{3}}{2}s - 1\right)\left(s^2+4\right)^{-1}\right]$$

$$= \frac{d}{ds}\left[\frac{\sqrt{3}}{2}\left(s^2+4\right)^{-1} - 2s\left(\frac{\sqrt{3}}{2}s-1\right)\left(s^2+4\right)^{-2}\right]$$

$$= \frac{\frac{\sqrt{3}}{2}(-2s)}{\left(s^2+4\right)^2} - \frac{2\left(\frac{\sqrt{3}}{2}s-1\right)}{\left(s^2+4\right)^2} - \frac{2s\left(\frac{\sqrt{3}}{2}\right)}{\left(s^2+4\right)^2} + \frac{(8s^2)\left(\frac{\sqrt{3}}{2}s-1\right)}{\left(s^2+4\right)^3}$$

$$= \frac{-\sqrt{3}s - \sqrt{3}s + 2 - \sqrt{3}s}{\left(s^2+4\right)^2} + \frac{(8s^2)\left(\frac{\sqrt{3}}{2}s-1\right)}{\left(s^2+4\right)^3}$$

$$= \frac{(-3\sqrt{3}s+2)(s^2+4)}{\left(s^2+4\right)^3} + \frac{4\sqrt{3}s^3 - 8s^2}{\left(s^2+4\right)^3}$$

$$\mathrm{L}\left[t^2\cos(2t+30°)\right] = \underline{\frac{\mathbf{8 - 12\sqrt{3}s - 6s^2 + \sqrt{3}s^3}}{\left(\mathbf{s^2+4}\right)^3}}$$

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(b) $\mathcal{L}\left[3t^4 e^{-2t}\right] = 3 \cdot \dfrac{4!}{(s+2)^5} = \underline{\dfrac{72}{(s+2)^5}}$

(c) $\mathcal{L}\left[2t\,u(t) - 4\dfrac{d}{dt}\delta(t)\right] = \dfrac{2}{s^2} - 4(s \cdot 1 - 0) = \underline{\mathbf{\dfrac{2}{s^2} - 4s}}$

(d) $2e^{-(t-1)}\,u(t) = 2e^{-t}\,u(t)$

$\mathcal{L}\left[2e^{-(t-1)}\,u(t)\right] = \underline{\mathbf{\dfrac{2e}{s+1}}}$

(e) Using the scaling property,

$\mathcal{L}\left[5u(t/2)\right] = 5 \cdot \dfrac{1}{1/2} \cdot \dfrac{1}{s/(1/2)} = 5 \cdot 2 \cdot \dfrac{1}{2s} = \underline{\mathbf{\dfrac{5}{s}}}$

(f) $\mathcal{L}\left[6e^{-t/3}\,u(t)\right] = \dfrac{6}{s+1/3} = \underline{\mathbf{\dfrac{18}{3s+1}}}$

(g) Let $f(t) = \delta(t)$. Then, $F(s) = 1$.

$$\mathcal{L}\left[\frac{d^n}{dt^n}\delta(t)\right] = \mathcal{L}\left[\frac{d^n}{dt^n}f(t)\right] = s^n\,F(s) - s^{n-1}\,f(0) - s^{n-2}\,f'(0) - \cdots$$

$$\mathcal{L}\left[\frac{d^n}{dt^n}\delta(t)\right] = \mathcal{L}\left[\frac{d^n}{dt^n}f(t)\right] = s^n \cdot 1 - s^{n-1} \cdot 0 - s^{n-2} \cdot 0 - \cdots$$

$$\mathcal{L}\left[\frac{d^n}{dt^n}\delta(t)\right] = \underline{\mathbf{s^n}}$$

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Chapter 15, Problem 6.

Find $F(s)$ given that

$$f(t)=\begin{cases}2t, & 0<t<1\\ t, & 1<t<2\\ 0, & otherwise\end{cases}$$

Chapter 15, Solution 6.

$$F(s)=\int_0^\infty f(t)e^{-st}dt=\int_0^1 2te^{-st}dt+\int_1^2 2e^{-st}dt$$

$$2\frac{e^{-st}}{s^2}(-st-1)\Big|_0^1+2\frac{e^{-st}}{-s}\Big|_1^2=\underline{\frac{2}{s^2}(1-e^{-s}-se^{-2s})}$$

Chapter 15, Problem 7.

Find the Laplace transform of the following signals:

(a) $f(t)=(2t+4)u(t)$

(b) $g(t)=(4+3e^{-2t})u(t)$

(c) $h(t)=(6\sin(3t)+8\cos(3t))u(t)$

(d) $x(t)=(e^{-2t}\cosh(4t))u(t)$

Chapter 15, Solution 7.

(a) $F(s)=\underline{\frac{2}{s^2}+\frac{4}{s}}$

(b) $G(s)=\underline{\frac{4}{s}+\frac{3}{s+2}}$

(c) $\mathrm{H(s)}=6\frac{3}{s^2+9}+8\frac{s}{s^2+9}=\underline{\frac{8s+18}{s^2+9}}$

(d) From Problem 15.1,

$$L\{\cosh at\}=\frac{s}{s^2-a^2}$$

$$X(s)=\frac{s+2}{(s+2)^2-4^2}=\underline{\frac{s+2}{s^2+4s-12}}$$

$$\underline{(a)\ \frac{2}{s^2}+\frac{4}{s},(b)\frac{4}{s}+\frac{3}{s+2},(c)\frac{8s+18}{s^2+9},(d)\frac{s+2}{s^2+4s-12}}$$

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Chapter 15, Problem 8.

Find the Laplace transform $F(s)$, given that $f(t)$ is:

(a) $2tu(t-4)$

(b) $5\cos(t)\delta(t-2)$

(c) $e^{-t}u(t-t)$

(d) $\sin(2t)u(t-\tau)$

Chapter 15, Solution 8.

(a) 2t=2(t-4) + 8

f(t) = 2tu(t-4) = 2(t-4)u(t-4) + 8u(t-4)

$$F(s) = \frac{2}{s^2}e^{-4s} + \frac{8}{s}e^{-4s} = \underline{\left(\frac{2}{s^2} + \frac{8}{s}\right)e^{-4s}}$$

(b) $$F(s) = \int_0^\infty f(t)e^{-st}dt = \int_0^\infty 5\cos t\delta(t-2)e^{-st}dt = 5\cos te^{-st}\Big|_{t=2} = \underline{\mathbf{5cos(2)e^{-2s}}}$$

(c) $$e^{-t} = e^{-(t-\tau)}e^{-\tau}$$

$$f(t) = e^{-\tau}e^{-(t-\tau)}u(t-\tau)$$

$$F(s) = e^{-\tau}e^{-\tau s}\frac{1}{s+1} = \underline{\frac{e^{-\tau(s+1)}}{s+1}}$$

(d) $$\sin 2t = \sin[2(t-\tau)+2\tau] = \sin 2(t-\tau)\cos 2\tau + \cos 2(t-\tau)\sin 2\tau$$

$$f(t) = \cos 2\tau \sin 2(t-\tau)u(t-\tau) + \sin 2\tau \cos 2(t-\tau)u(t-\tau)$$

$$F(s) = \underline{\cos 2\tau e^{-\tau s}\frac{2}{s^2+4} + \sin 2\tau e^{-\tau s}\frac{s}{s^2+4}}$$

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Chapter 15, Problem 9.

Determine the Laplace transforms of these functions:

(a) $f(t) = (t-4)u(t-2)$

(b) $g(t) = 2e^{-4t}u(t-1)$

(c) $h(t) = 5\cos(2t-1)u(t)$

(d) $p(t) = 6[u(t-2) - u(t-4)]$

Chapter 15, Solution 9.

(a) $f(t) = (t-4)\,u(t-2) = (t-2)\,u(t-2) - 2\,u(t-2)$

$$F(s) = \underline{\frac{\mathbf{e^{-2s}}}{\mathbf{s^2}} - \frac{\mathbf{2\,e^{-2s}}}{\mathbf{s^2}}}$$

(b) $g(t) = 2\,e^{-4t}\,u(t-1) = 2\,e^{-4}\,e^{-4(t-1)}\,u(t-1)$

$$G(s) = \underline{\frac{\mathbf{2\,e^{-s}}}{\mathbf{e^4\,(s+4)}}}$$

(c) $h(t) = 5\cos(2t-1)\,u(t)$

$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$
$$\cos(2t-1) = \cos(2t)\cos(1) + \sin(2t)\sin(1)$$

$$h(t) = 5\cos(1)\cos(2t)\,u(t) + 5\sin(1)\sin(2t)\,u(t)$$

$$H(s) = 5\cos(1)\cdot\frac{s}{s^2+4} + 5\sin(1)\cdot\frac{2}{s^2+4}$$

$$H(s) = \underline{\frac{\mathbf{2.702\,s}}{\mathbf{s^2+4}} + \frac{\mathbf{8.415}}{\mathbf{s^2+4}}}$$

(d) $p(t) = 6u(t-2) - 6u(t-4)$

$$P(s) = \underline{\frac{\mathbf{6}}{\mathbf{s}}\mathbf{e^{-2s}} - \frac{\mathbf{6}}{\mathbf{s}}\mathbf{e^{-4s}}}$$

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Chapter 15, Problem 10.

In two different ways, find the Laplace transform of

$$g(t) = \frac{d}{dt}\left(te^{-t}\cos t\right)$$

Chapter 15, Solution 10.

(a) By taking the derivative in the time domain,

$$g(t) = (-t\,e^{-t} + e^{-t})\cos(t) - t\,e^{-t}\sin(t)$$

$$g(t) = e^{-t}\cos(t) - t\,e^{-t}\cos(t) - t\,e^{-t}\sin(t)$$

$$G(s) = \frac{s+1}{(s+1)^2+1} + \frac{d}{ds}\left[\frac{s+1}{(s+1)^2+1}\right] + \frac{d}{ds}\left[\frac{1}{(s+1)^2+1}\right]$$

$$G(s) = \frac{s+1}{s^2+2s+2} - \frac{s^2+2s}{(s^2+2s+2)^2} - \frac{2s+2}{(s^2+2s+2)^2} = \underline{\mathbf{\frac{s^2(s+2)}{(s^2+2s+2)^2}}}$$

(b) By applying the time differentiation property,

$$G(s) = sF(s) - f(0)$$

where $f(t) = t\,e^{-t}\cos(t)$, $f(0) = 0$

$$G(s) = (s)\cdot\frac{-d}{ds}\left[\frac{s+1}{(s+1)^2+1}\right] = \frac{(s)(s^2+2s)}{(s^2+2s+2)^2} = \underline{\mathbf{\frac{s^2(s+2)}{(s^2+2s+2)^2}}}$$

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Chapter 15, Problem 11.

Find $F(s)$ if:

(a) $f(t) = 6e^{-t}\cosh 2t$ (b) $f(t) = 3te^{-2t}\sinh 4t$

(c) $f(t) = 8e^{-3t}\cosh tu(t-2)$

Chapter 15, Solution 11.

(a) Since $\mathcal{L}\left[\cosh(at)\right] = \dfrac{s}{s^2 - a^2}$

$$F(s) = \frac{6(s+1)}{(s+1)^2 - 4} = \underline{\mathbf{\frac{6(s+1)}{s^2+2s-3}}}$$

(b) Since $\mathcal{L}\left[\sinh(at)\right] = \dfrac{a}{s^2 - a^2}$

$$\mathcal{L}\left[3e^{-2t}\sinh(4t)\right] = \frac{(3)(4)}{(s+2)^2 - 16} = \frac{12}{s^2 + 4s - 12}$$

$$F(s) = \mathcal{L}\left[t \cdot 3e^{-2t}\sinh(4t)\right] = \frac{-d}{ds}\left[12(s^2 + 4s - 12)^{-1}\right]$$

$$F(s) = (12)(2s+4)(s^2 + 4s - 12)^{-2} = \underline{\mathbf{\frac{24(s+2)}{(s^2+4s-12)^2}}}$$

(c) $\cosh(t) = \dfrac{1}{2}\cdot(e^t + e^{-t})$

$$f(t) = 8e^{-3t}\cdot\frac{1}{2}\cdot(e^t + e^{-t})u(t-2)$$

$$= 4e^{-2t}u(t-2) + 4e^{-4t}u(t-2)$$

$$= 4e^{-4}e^{-2(t-2)}u(t-2) + 4e^{-8}e^{-4(t-2)}u(t-2)$$

$$\mathcal{L}\left[4e^{-4}e^{-2(t-2)}u(t-2)\right] = 4e^{-4}e^{-2s}\cdot\mathcal{L}\left[e^{-2}u(t)\right]$$

$$\mathcal{L}\left[4e^{-4}e^{-2(t-2)}u(t-2)\right] = \frac{4e^{-(2s+4)}}{s+2}$$

Similarly, $\mathcal{L}\left[4e^{-8}e^{-4(t-2)}u(t-2)\right] = \dfrac{4e^{-(2s+8)}}{s+4}$

Therefore,

$$F(s) = \frac{4e^{-(2s+4)}}{s+2} + \frac{4e^{-(2s+8)}}{s+4} = \underline{\mathbf{\frac{e^{-(2s+6)}\left[(4e^2 + 4e^{-2})s + (16e^2 + 8e^{-2})\right]}{s^2+6s+8}}}$$

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Chapter 15, Problem 12.

If $g(t) = e^{-2t}\cos 4t$ find $G(s)$.

Chapter 15, Solution 12.

$$G(s) = \frac{s+2}{(s+2)^2 + 4^2} = \underline{\frac{s+2}{s^2 + 4s + 20}}$$

Chapter 15, Problem 13.

Find the Laplace transform of the following functions:

(a) $t\cos t\,u(t)$ (b) $e^{-t}t\sin t\,u(t)$ (c) $\frac{\sin \beta t}{t}u(t)$

Chapter 15, Solution 13.

(a) $tf(t) \longleftrightarrow -\frac{d}{ds}F(s)$

If f(t) = cost, then $F(s) = \frac{s}{s^2+1}$ and $-\frac{d}{ds}F(s) = -\frac{(s^2+1)(1) - s(2s)}{(s^2+1)^2}$

$$\underline{\mathcal{L}(t\cos t) = \frac{s^2 - 1}{(s^2+1)^2}}$$

(b) Let $f(t) = e^{-t}\sin t$.

$$F(s) = \frac{1}{(s+1)^2 + 1} = \frac{1}{s^2 + 2s + 2}$$

$$\frac{dF}{ds} = \frac{(s^2 + 2s + 2)(0) - (1)(2s+2)}{(s^2 + 2s + 2)^2}$$

$$\underline{\mathcal{L}(e^{-t}t\sin t) = -\frac{dF}{ds} = \frac{2(s+1)}{(s^2 + 2s + 2)^2}}$$

(c) $\frac{f(t)}{t} \longleftrightarrow \int_s^\infty F(s)ds$

Let $f(t) = \sin \beta t$, then $F(s) = \frac{\beta}{s^2 + \beta^2}$

$$\mathcal{L}\left[\frac{\sin \beta t}{t}\right] = \int_s^\infty \frac{\beta}{s^2 + \beta^2}ds = \beta\frac{1}{\beta}\tan^{-1}\frac{s}{\beta}\Big|_s^\infty = \frac{\pi}{2} - \tan^{-1}\frac{s}{\beta} = \underline{\tan^{-1}\frac{\beta}{s}}$$

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Chapter 15, Problem 14.

Find the Laplace transform of the signal in Fig. 15.26.

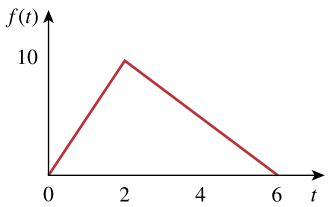

Figure 15.26

For Prob. 15.14.

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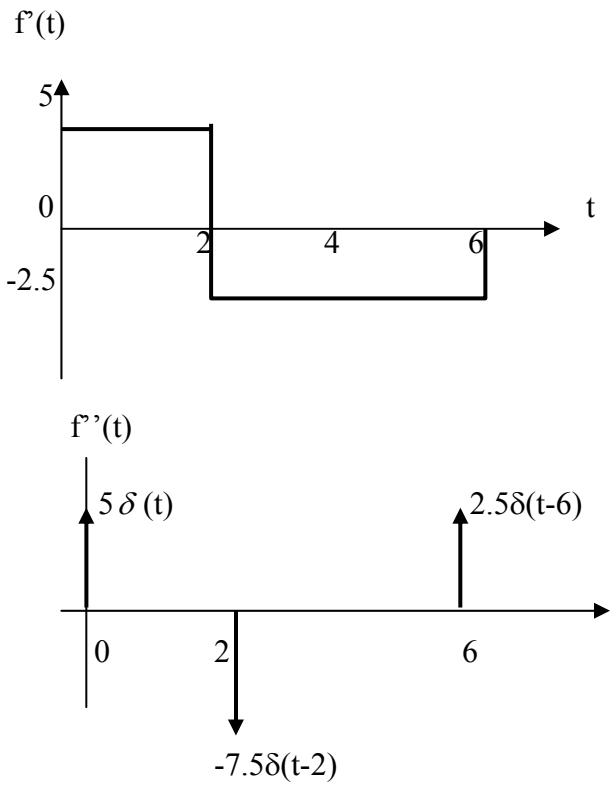
Chapter 15, Solution 14.

Taking the derivative of f(t) twice, we obtain the figures below.



$$f'' = 5\delta(t) - 7.5\delta(t-2) + 2.5\delta(t-6)$$

Taking the Laplace transform of each term,

$$s^2F(s) = 5 - 7.5e^{-2s} + 2.5e^{-6s} \text{ or } F(s) = \underline{\frac{5}{s} - 7.5\frac{e^{-2s}}{s^2} + 2.5\frac{e^{-6s}}{s^2}}$$

Please note that we can obtain the same answer by representing the function as,

$$f(t) = 5tu(t) - 7.5u(t-2) + 2.5u(t-6).$$

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Chapter 15, Problem 15.

Determine the Laplace transform of the function in Fig. 15.27.

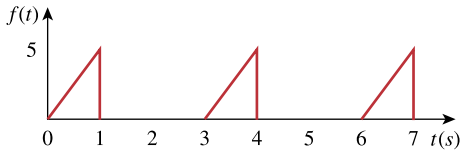


Figure 15.27

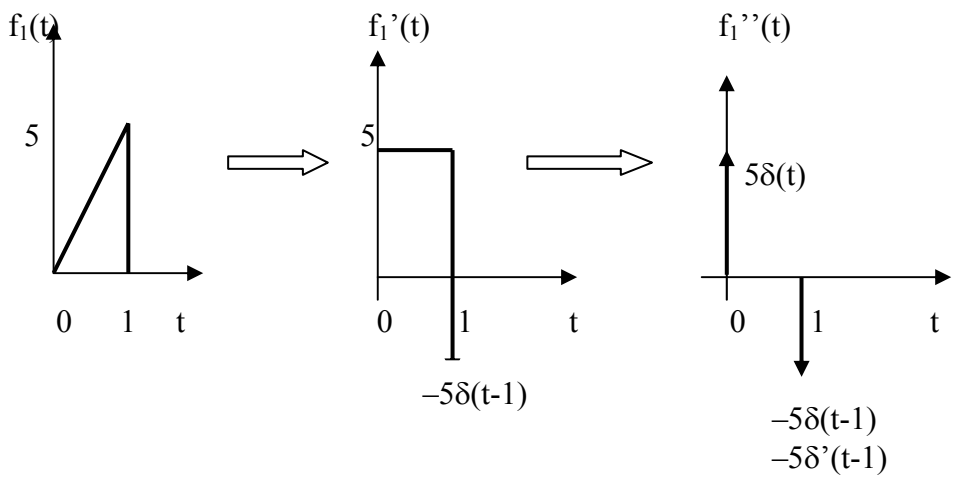
For Prob. 15.15.

Chapter 15, Solution 15.

This is a periodic function with T=3.

$$F(s) = \frac{F_1(s)}{1-e^{-3s}}$$

To get $F_1(s)$, we consider f(t) over one period.



$f_1'' = 5\delta(t) - 5\delta(t-1) - 5\delta'(t-1)$

Taking the Laplace transform of each term,

$$s^2F_1(s) = 5 - 5e^{-s} - 5se^{-s} \quad \text{or} \quad F_1(s) = 5(1 - e^{-s} - se^{-s})/s^2$$

Hence,

$$F(s) = \underline{5\frac{1-e^{-s}-se^{-s}}{s^2(1-e^{-3s})}}$$

Alternatively, we can obtain the same answer by noting that $f_1(t) = 5tu(t) - 5tu(t-1) - 5u(t-1)$.

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Chapter 15, Problem 16.

Obtain the Laplace transform of $f(t)$ in Fig. 15.28.

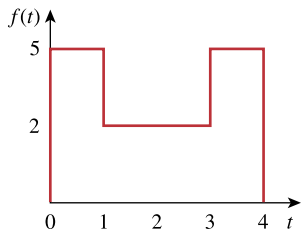

Figure 15.28
For Prob. 15.16.

Chapter 15, Solution 16.

$$f(t) = 5u(t) - 3u(t-1) + 3u(t-3) - 5u(t-4)$$

$$F(s) = \underline{\frac{1}{s}\left[5 - 3e^{-s} + 3e^{-3s} - 5e^{-4s}\right]}$$

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Chapter 15, Problem 17.

Find the Laplace transform of $f(t)$ shown in Fig. 15.29.

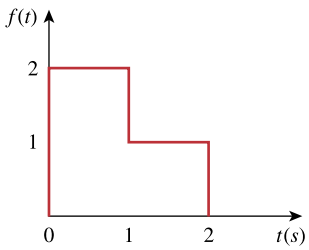
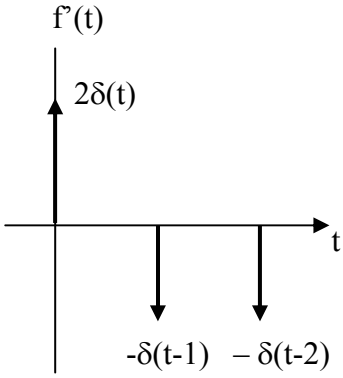


Figure 15.29
For Prob. 15.17.

Chapter 15, Solution 17.

Taking the derivative of f(t) gives f'(t) as shown below.



$$f'(t) = 2\delta(t) - \delta(t-1) - \delta(t-2)$$

Taking the Laplace transform of each term,

$$sF(s) = 2 - e^{-s} - e^{-2s} \quad \text{which leads to}$$

$$F(s) = \underline{\mathbf{[2 - e^{-s} - e^{-2s}]/s}}$$

We can also obtain the same answer noting that $f(t) = 2u(t) - u(t-1) - u(t-2)$.

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Chapter 15, Problem 18.

Obtain the Laplace transforms of the functions in Fig. 15.30.

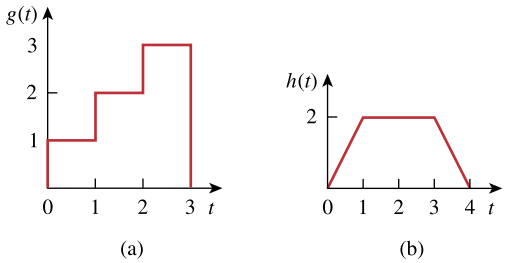

Figure 15.30

For Prob. 15.18.

Chapter 15, Solution 18.

(a)
$$g(t) = u(t) - u(t-1) + 2\left[u(t-1) - u(t-2)\right] + 3\left[u(t-2) - u(t-3)\right]$$
$$= u(t) + u(t-1) + u(t-2) - 3u(t-3)$$

$$G(s) = \underline{\mathbf{\frac{1}{s}(1 + e^{-s} + e^{-2s} - 3e^{-3s})}}$$

(b)
$$h(t) = 2t\left[u(t) - u(t-1)\right] + 2\left[u(t-1) - u(t-3)\right]$$
$$+ (8 - 2t)\left[u(t-3) - u(t-4)\right]$$

$$= 2t\,u(t) - 2(t-1)u(t-1) - 2u(t-1) + 2u(t-1) - 2u(t-3)$$
$$- 2(t-3)u(t-3) + 2u(t-3) + 2(t-4)u(t-4)$$

$$= 2t\,u(t) - 2(t-1)u(t-1) - 2(t-3)u(t-3) + 2(t-4)u(t-4)$$

$$H(s) = \frac{2}{s^2}(1 - e^{-s}) - \frac{2}{s^2}e^{-3s} + \frac{2}{s^2}e^{-4s} = \underline{\mathbf{\frac{2}{s^2}(1 - e^{-s} - e^{-3s} + e^{-4s})}}$$

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Chapter 15, Problem 19.

Calculate the Laplace transform of the train of unit impulses in Fig. 15.31.

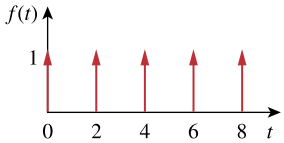

Figure 15.31

For Prob. 15.19.

Chapter 15, Solution 19.

Since $L[\delta(t)] = 1$ and $T = 2$, $F(s) = \underline{\mathbf{\dfrac{1}{1-e^{-2s}}}}$

Chapter 15, Problem 20.

The periodic function shown in Fig. 15.32 is defined over its period as

$$g(t)\begin{cases}\sin \pi t, & 0 < t < 1\\ 0, & 1 < t < 2\end{cases}$$

Find $G(s)$

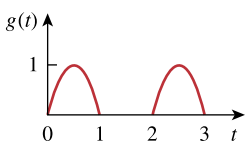

Figure 15.32

For Prob. 15.20.

Chapter 15, Solution 20.

Let $g_1(t) = \sin(\pi t), \quad 0 < t < 1$

$$= \sin(\pi t)[u(t) - u(t-1)]$$

$$= \sin(\pi t)u(t) - \sin(\pi t)u(t-1)$$

Note that $\sin(\pi(t-1)) = \sin(\pi t - \pi) = -\sin(\pi t)$.

So, $g_1(t) = \sin(\pi t)u(t) + \sin(\pi(t-1))u(t-1)$

$$G_1(s) = \frac{\pi}{s^2 + \pi^2}(1 + e^{-s})$$

$$G(s) = \frac{G_1(s)}{1 - e^{-2s}} = \underline{\mathbf{\frac{\pi(1+e^{-s})}{(s^2+\pi^2)(1-e^{-2s})}}}$$

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Chapter 15, Problem 21.

Obtain the Laplace transform of the periodic waveform in Fig. 15.33.

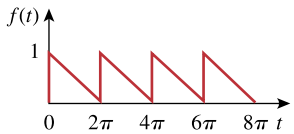

Figure 15.33

For Prob. 15.21.

Chapter 15, Solution 21.

$$T = 2\pi$$

Let $$f_1(t) = \left(1 - \frac{t}{2\pi}\right)\left[u(t) - u(t - 2\pi)\right]$$

$$f_1(t) = u(t) - \frac{t}{2\pi}u(t) + \frac{1}{2\pi}(t - 2\pi)u(t - 2\pi)$$

$$F_1(s) = \frac{1}{s} - \frac{1}{2\pi s^2} + \frac{e^{-2\pi s}}{2\pi s^2} = \frac{2\pi s + \left[-1 + e^{-2\pi s}\right]}{2\pi s^2}$$

$$F(s) = \frac{F_1(s)}{1 - e^{-Ts}} = \underline{\frac{2\pi s - 1 + e^{-2\pi s}}{2\pi s^2 (1 - e^{-2\pi s})}}$$

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Chapter 15, Problem 22.

Find the Laplace transforms of the functions in Fig. 15.34.

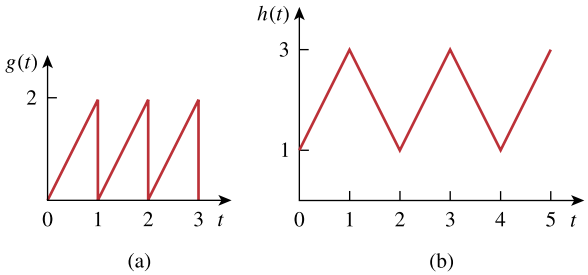

Figure 15.34

For Prob. 15.22.

Chapter 15, Solution 22.

(a) Let $g_1(t) = 2t, \quad 0 < t < 1$

$$= 2t\left[u(t) - u(t-1)\right]$$

$$= 2t\,u(t) - 2(t-1)u(t-1) + 2u(t-1)$$

$$G_1(s) = \frac{2}{s^2} - \frac{2e^{-s}}{s^2} + \frac{2}{s}e^{-s}$$

$$G(s) = \frac{G_1(s)}{1-e^{-sT}}, \quad T = 1$$

$$G(s) = \underline{\frac{\mathbf{2(1-e^{-s}+se^{-s})}}{\mathbf{s^2(1-e^{-s})}}}$$

(b) Let $h = h_0 + u(t)$, where h_0 is the periodic triangular wave.

Let h_1 be h_0 within its first period, i.e.

$$h_1(t) = \begin{cases} 2t & 0 < t < 1 \\ 4-2t & 1 < t < 2 \end{cases}$$

$$h_1(t) = 2t\,u(t) - 2t\,u(t-1) + 4u(t-1) - 2t\,u(t-1) - 2(t-2)u(t-2)$$

$$h_1(t) = 2t\,u(t) - 4(t-1)u(t-1) - 2(t-2)u(t-2)$$

$$H_1(s) = \frac{2}{s^2} - \frac{4}{s^2}e^{-s} - \frac{2e^{-2s}}{s^2} = \frac{2}{s^2}(1-e^{-s})^2$$

$$H_0(s) = \frac{2}{s^2}\frac{(1-e^{-s})^2}{(1-e^{-2s})}$$

$$H(s) = \underline{\mathbf{\frac{1}{s} + \frac{2}{s^2}\frac{(1-e^{-s})^2}{(1-e^{-2s})}}}$$

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Chapter 15, Problem 23.

Determine the Laplace transforms of the periodic functions in Fig. 15.35.

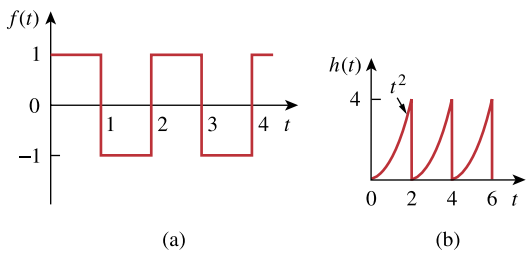

Figure 15.35

For Prob. 15.23.

Chapter 15, Solution 23.

(a) Let $f_1(t) = \begin{cases} 1 & 0 < t < 1 \\ -1 & 1 < t < 2 \end{cases}$

$$f_1(t) = [u(t) - u(t-1)] - [u(t-1) - u(t-2)]$$
$$f_1(t) = u(t) - 2u(t-1) + u(t-2)$$

$$F_1(s) = \frac{1}{s}(1 - 2e^{-s} + e^{-2s}) = \frac{1}{s}(1 - e^{-s})^2$$

$$F(s) = \frac{F_1(s)}{(1 - e^{-sT})}, \quad T = 2$$

$$F(s) = \underline{\frac{\mathbf{(1 - e^{-s})^2}}{\mathbf{s(1 - e^{-2s})}}}$$

(b) Let $h_1(t) = t^2[u(t) - u(t-2)] = t^2 u(t) - t^2 u(t-2)$

$$h_1(t) = t^2 u(t) - (t-2)^2 u(t-2) - 4(t-2)u(t-2) - 4u(t-2)$$

$$H_1(s) = \frac{2}{s^3}(1 - e^{-2s}) - \frac{4}{s^2}e^{-2s} - \frac{4}{s}e^{-2s}$$

$$H(s) = \frac{H_1(s)}{(1 - e^{-Ts})}, \quad T = 2$$

$$H(s) = \underline{\frac{\mathbf{2(1 - e^{-2s}) - 4s\,e^{-2s}(s + s^2)}}{\mathbf{s^3(1 - e^{-2s})}}}$$

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Chapter 15, Problem 24.

Given that

$$F(s) = \frac{s^2 + 10s + 6}{s(s+1)^2(s+3)}$$

Evaluate $f(0)$ and $f(\infty)$ if they exist.

Chapter 15, Solution 24.

$$f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{s^2 + 10s + 6}{(s+1)^2(s+2)} = \lim_{s\to\infty} \frac{1/s + 10/s^2 + 6/s^3}{(1+1/s)(1+2/s)} = \frac{0}{1} = \underline{0}$$

$$f(\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{s^2 + 10s + 6}{(s+1)^2(s+2)} = \frac{6}{(1)(2)} = \underline{3}$$

Chapter 15, Problem 25.

Let

$$F(s) = \frac{5(s+1)}{(s+2)(s+3)}$$

(a) Use the initial and final value theorems to find $f(0)$ and $f(\infty)$.

(b) Verify your answer in part (a) by finding $f(t)$, using partial fractions.

Chapter 15, Solution 25.

(a) $$f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{5s(s+1)}{(s+2)(s+3)} = \lim_{s\to\infty} \frac{5(1+1/s)}{(1+2/s)(1+3/s)} = \underline{5}$$

$$f(\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{5s(s+1)}{(s+2)(s+3)} = \underline{0}$$

(b) $$F(s) = \frac{5(s+1)}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

$$A = \frac{5(-1)}{1} = -5, \qquad B = \frac{5(-2)}{-1} = 10$$

$$F(s) = \frac{-5}{s+2} + \frac{10}{s+3} \qquad \longrightarrow \qquad f(t) = -5e^{-2t} + 10e^{-3t}$$

f(0) = -5 + 10 = **5**

f(∞)= -0 + 0 = **0.**

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Chapter 15, Problem 26.

Determine the initial and final values of $f(t)$, if they exist, given that:

(a) $F(s) = \dfrac{s^2 + 3}{s^3 + 4s^2 + 6}$

(b) $F(s) = \dfrac{s^2 - 2s + 1}{(s-2)(s^2 + 2s + 4)}$

Chapter 15, Solution 26.

(a) $$f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{s^3 + 3s}{s^3 + 4s^2 + 6} = \underline{\mathbf{1}}$$

Two poles are not in the left-half plane.
$f(\infty)$ **does not exist**

(b) $$f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{s^3 - 2s^2 + s}{(s-2)(s^2 + 2s + 4)}$$

$$= \lim_{s\to\infty} \frac{1 - \frac{2}{s} + \frac{1}{s^2}}{\left(1 - \frac{2}{s}\right)\left(1 + \frac{2}{s} + \frac{4}{s^2}\right)} = \underline{\mathbf{1}}$$

One pole is not in the left-half plane.
$f(\infty)$ **does not exist**

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Chapter 15, Problem 27.

Determine the inverse Laplace transform of each of the following functions:

(a) $F(s) = \frac{1}{s} + \frac{2}{s+1}$

(b) $G(s) = \frac{3s+1}{s+4}$

(c) $H(s) = \frac{4}{(s+1)(s+3)}$

(d) $J(s) = \frac{12}{(s+2)^2(s+4)}$

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Chapter 15, Solution 27.

(a) $f(t) = u(t) + 2e^{-t}u(t)$

(b) $G(s) = \dfrac{3(s+4)-11}{s+4} = 3 - \dfrac{11}{s+4}$

$g(t) = \underline{3\delta(t) - 11e^{-4t}u(t)}$

(c) $H(s) = \dfrac{4}{(s+1)(s+3)} = \dfrac{A}{s+1} + \dfrac{B}{s+3}$

$A = 2, \qquad B = -2$

$H(s) = \dfrac{2}{s+1} - \dfrac{2}{s+3}$

$h(t) = \underline{\mathbf{2e^{-t} - 2e^{-3t}}}\ \underline{\mathbf{u(t)}}$

(d) $J(s) = \dfrac{12}{(s+2)^2(s+4)} = \dfrac{A}{s+2} + \dfrac{B}{(s+2)^2} + \dfrac{C}{s+4}$

$B = \dfrac{12}{2} = 6, \quad C = \dfrac{12}{(-2)^2} = 3$

$12 = A(s+2)(s+4) + B(s+4) + C(s+2)^2$

Equating coefficients :

$s^2: \quad 0 = A + C \longrightarrow A = -C = -3$

$s^1: \quad 0 = 6A + B + 4C = 2A + B \longrightarrow B = -2A = 6$

$s^0: \quad 12 = 8A + 4B + 4C = -24 + 24 + 12 = 12$

$J(s) = \dfrac{-3}{s+2} + \dfrac{6}{(s+2)^2} + \dfrac{3}{s+4}$

$j(t) = \underline{\mathbf{3e^{-4t} - 3e^{-2t} + 6te^{-2t}}}\ \underline{\mathbf{u(t)}}$

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Chapter 15, Problem 28.

Find the inverse Laplace transform of the following functions:

(a) $F(s) = \dfrac{20(s+2)}{s(s^2+6s+25)}$

(b) $P(s) = \dfrac{6s^2+36s+20}{(s+1)(s+2)(s+3)}$

Chapter 15, Solution 28.

(a) $F(s) = \dfrac{20(s+2)}{s(s^2+6s+25)} = \dfrac{A}{s} + \dfrac{Bs+C}{s^2+6s+25}$

$20(s+2) = A(s^2+6s+25s) + Bs^2 + Cs$

Equating components,

s^2 : 0 = A + B or B= - A

s: 20 = 6A + C

constant: 40 – 25 A or A = 8/5, B = -8/5, C= 20 – 6A= 52/5

$$F(s) = \frac{8}{5s} + \frac{-\frac{8}{5}s + \frac{52}{5}}{(s+3)^2+4^2} = \frac{8}{5s} + \frac{-\frac{8}{5}(s+3) + \frac{24}{5} + \frac{52}{5}}{(s+3)^2+4^2}$$

$$\underline{f(t) = \frac{8}{5}u(t) - \frac{8}{5}e^{-3t}\cos 4t + \frac{19}{5}e^{-3t}\sin 4t}$$

(b) $P(s) = \dfrac{6s^2+36s+20}{(s+1)(s+2)(s+3)} = \dfrac{A}{s+1} + \dfrac{B}{s+2} + \dfrac{C}{s+3}$

$$A = \frac{6-36+20}{(-1+2)(-1+3)} = -5$$

$$B = \frac{24-72+20}{(-1)(1)} = 28$$

$$C = \frac{54-108+20}{(-2)(-1)} = -17$$

$$P(s) = \frac{-5}{s+1} + \frac{28}{s+2} - \frac{17}{s+3}$$

$$p(t) = \underline{(-5e^{-t} + 28e^{-2t} - 17e^{-3t})u(t)}$$

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Chapter 15, Problem 29.

Find the inverse Laplace transform of:

$$V(s) = \frac{2s + 26}{s\left(s^2 + 4s + 13\right)}$$

Chapter 15, Solution 29.

$$V(s) = \frac{2}{s} + \frac{As + B}{(s+2)^2 + 3^2};\ 2s^2 + 8s + 26 + As^2 + Bs = 2s + 26 \rightarrow A = -2 \text{ and } B = -6$$

$$V(s) = \frac{2}{s} - \frac{2(s+2)}{(s+2)^2 + 3^2} - \frac{2}{3}\frac{3}{(s+2)^2 + 3^2}$$

$$v(t) = \underline{(2 - 2e^{-2t}\cos 3t - \frac{2}{3}e^{-2t}\sin 3t)u(t),\ \ t \geq 0}$$

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Chapter 15, Problem 30.

Find the inverse Laplace transform of:

(a) $F_1(s) = \dfrac{6s^2 + 8s + 3}{s(s^2 + 2s + 5)}$

(b) $F_2(s) = \dfrac{s^2 + 5s + 6}{(s+1)^2(s+4)}$

(c) $F_3(s) = \dfrac{10}{(s+1)(s^2 + 4s + 8)}$

Chapter 15, Solution 30.

(a) $$F_1(s) = \frac{6s^2 + 8s + 3}{s(s^2 + 2s + 5)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 2s + 5}$$

$$6s^2 + 8s + 3 = A(s^2 + 2s + 5) + Bs^2 + Cs$$

We equate coefficients.

s^2 : 6 = A + B

s: 8= 2A + C

constant: 3=5A or A=3/5

B=6-A = 27/5, C=8-2A = 34/5

$$F_1(s) = \frac{3/5}{s} + \frac{27s/5 + 34/5}{s^2 + 2s + 5} = \frac{3/5}{s} + \frac{27(s+1)/5 + 7/5}{(s+1)^2 + 2^2}$$

$$f_1(t) = \underline{\left[\frac{3}{5} + \frac{27}{5}e^{-t}\cos 2t + \frac{7}{10}e^{-t}\sin 2t\right]u(t)}$$

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(b) $F_2(s) = \dfrac{s^2+5s+6}{(s+1)^2(s+4)} = \dfrac{A}{s+1} + \dfrac{B}{(s+1)^2} + \dfrac{C}{s+4}$

$s^2+5s+6 = A(s+1)(s+4) + B(s+4) + C(s+1)^2$

Equating coefficients,

s^2 : 1=A+C
s: 5=5A+B+2C
constant: 6=4A+4B+C
Solving these gives
A=7/9, B= 2/3, C=2/9

$$F_2(s) = \frac{7/9}{s+1} + \frac{2/3}{(s+1)^2} + \frac{2/9}{s+4}$$

$$\underline{f_2(t) = \left[\frac{7}{9}e^{-t} + \frac{2}{3}te^{-t} + \frac{2}{9}e^{-4t}\right]u(t)}$$

(c) $F_3(s) = \dfrac{10}{(s+1)(s^2+4s+8)} = \dfrac{A}{s+1} + \dfrac{Bs+C}{s^2+4s+8}$

$10 = A(s^2+4s+8) + B(s^2+s) + C(s+1)$

s^2 : 0 = A + B or B = -A
s: 0=4A+ B + C
constant: 10=8A+C
Solving these yields
A=2, B= -2, C= -6

$$F_3(s) = \frac{2}{s+1} + \frac{-2s-6}{s^2+4s+8} = \frac{2}{s+1} - \frac{2(s+1)}{(s+1)^2+2^2} - \frac{4}{(s+1)^2+2^2}$$

$f_3(t)$ = **<u>$(2e^{-t} - 2e^{-t}\cos(2t) - 2e^{-t}\sin(2t))u(t)$</u>**.

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Chapter 15, Problem 31.

Find $f(t)$ for each $F(s)$:

(a) $\dfrac{10s}{(s+1)(s+2)(s+3)}$

(b) $\dfrac{2s^2+4s+1}{(s+1)(s+2)^3}$

(c) $\dfrac{s+1}{(s+2)(s^2+2s+5)}$

Chapter 15, Solution 31.

(a) $$F(s) = \frac{10s}{(s+1)(s+2)(s+3)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$A = F(s)(s+1)\Big|_{s=-1} = \frac{-10}{2} = -5$$

$$B = F(s)(s+2)\Big|_{s=-2} = \frac{-20}{-1} = 20$$

$$C = F(s)(s+3)\Big|_{s=-3} = \frac{-30}{2} = -15$$

$$F(s) = \frac{-5}{s+1} + \frac{20}{s+2} - \frac{15}{s+3}$$

$$f(t) = \underline{(-5e^{-t} + 20e^{-2t} - 15e^{-3t})u(t)}$$

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(b) $$F(s) = \frac{2s^2+4s+1}{(s+1)(s+2)^3} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2} + \frac{D}{(s+2)^3}$$

$$A = F(s)(s+1)\Big|_{s=-1} = -1$$

$$D = F(s)(s+2)^3\Big|_{s=-2} = -1$$

$$2s^2+4s+1 = A(s+2)(s^2+4s+4) + B(s+1)(s^2+4s+4)$$
$$+ C(s+1)(s+2) + D(s+1)$$

Equating coefficients :

s^3: $\quad 0 = A + B \longrightarrow B = -A = 1$

s^2: $\quad 2 = 6A + 5B + C = A + C \longrightarrow C = 2 - A = 3$

s^1: $\quad 4 = 12A + 8B + 3C + D = 4A + 3C + D$

$\quad 4 = 6 + A + D \longrightarrow D = -2 - A = -1$

s^0: $\quad 1 = 8A + 4B + 2C + D = 4A + 2C + D = -4 + 6 - 1 = 1$

$$F(s) = \frac{-1}{s+1} + \frac{1}{s+2} + \frac{3}{(s+2)^2} - \frac{1}{(s+2)^3}$$

$$f(t) = -e^{-t} + e^{-2t} + 3te^{-2t} - \frac{t^2}{2}e^{-2t}$$

$$\underline{f(t) = (-e^{-t} + \left(1 + 3t - \frac{t^2}{2}\right)e^{-2t})u(t)}$$

(c) $$F(s) = \frac{s+1}{(s+2)(s^2+2s+5)} = \frac{A}{s+2} + \frac{Bs+C}{s^2+2s+5}$$

$$A = F(s)(s+2)\Big|_{s=-2} = \frac{-1}{5}$$

$$s+1 = A(s^2+2s+5) + B(s^2+2s) + C(s+2)$$

Equating coefficients :

s^2: $\quad 0 = A + B \longrightarrow B = -A = \frac{1}{5}$

s^1: $\quad 1 = 2A + 2B + C = 0 + C \longrightarrow C = 1$

s^0: $\quad 1 = 5A + 2C = -1 + 2 = 1$

$$F(s) = \frac{-1/5}{s+2} + \frac{1/5 \cdot s + 1}{(s+1)^2 + 2^2} = \frac{-1/5}{s+2} + \frac{1/5(s+1)}{(s+1)^2+2^2} + \frac{4/5}{(s+1)^2+2^2}$$

$$\underline{f(t) = (-0.2e^{-2t} + 0.2e^{-t}\cos(2t) + 0.4e^{-t}\sin(2t))u(t)}$$

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Chapter 15, Problem 32.

Determine the inverse Laplace transform of each of the following functions:

(a) $\dfrac{8(s+1)(s+3)}{s(s+2)(s+4)}$ (b) $\dfrac{s^2-2s+4}{(s+1)(s+2)^2}$ (c) $\dfrac{s^2+1}{(s+3)(s^2+4s+5)}$

Chapter 15, Solution 32.

(a) $$F(s)=\frac{8(s+1)(s+3)}{s(s+2)(s+4)}=\frac{A}{s}+\frac{B}{s+2}+\frac{C}{s+4}$$

$$A=F(s)s\big|_{s=0}=\frac{(8)(3)}{(2)(4)}=3$$

$$B=F(s)(s+2)\big|_{s=-2}=\frac{(8)(-1)}{(-4)}=2$$

$$C=F(s)(s+4)\big|_{s=-4}=\frac{(8)(-1)(-3)}{(-4)(-2)}=3$$

$$F(s)=\frac{3}{s}+\frac{2}{s+2}+\frac{3}{s+4}$$

$$f(t)=\underline{\mathbf{3u(t)+2e^{-2t}+3e^{-4t}}}$$

(b) $$F(s)=\frac{s^2-2s+4}{(s+1)(s+2)^2}=\frac{A}{s+1}+\frac{B}{s+2}+\frac{C}{(s+2)^2}$$

$$s^2-2s+4=A(s^2+4s+4)+B(s^2+3s+2)+C(s+1)$$

Equating coefficients :

s^2: $1=A+B \longrightarrow B=1-A$

s^1: $-2=4A+3B+C=3+A+C$

s^0: $4=4A+2B+C=-B-2 \longrightarrow B=-6$

$A=1-B=7$ $C=-5-A=-12$

$$F(s)=\frac{7}{s+1}-\frac{6}{s+2}-\frac{12}{(s+2)^2}$$

$$f(t)=\underline{\mathbf{7e^{-t}-6(1+2t)e^{-2t}}}$$

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(c) $$F(s) = \frac{s^2+1}{(s+3)(s^2+4s+5)} = \frac{A}{s+3} + \frac{Bs+C}{s^2+4s+5}$$

$$s^2+1 = A(s^2+4s+5) + B(s^2+3s) + C(s+3)$$

Equating coefficients :

s^2: $1 = A + B \longrightarrow B = 1 - A$

s^1: $0 = 4A + 3B + C = 3 + A + C \longrightarrow A + C = -3$

s^0: $1 = 5A + 3C = -9 + 2A \longrightarrow A = 5$

$B = 1 - A = -4$ $\qquad$ $C = -A - 3 = -8$

$$F(s) = \frac{5}{s+3} - \frac{4s+8}{(s+2)^2+1} = \frac{5}{s+3} - \frac{4(s+2)}{(s+2)^2+1}$$

$f(t) = \underline{\mathbf{5e^{-3t} - 4e^{-2t}\cos(t)}}$

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Chapter 15, Problem 33.

Calculate the inverse Laplace transform of:

(a) $\dfrac{6(s-1)}{s^4-1}$ (b) $\dfrac{se^{-\pi s}}{s^2+1}$ (c) $\dfrac{8}{s(s+1)^3}$

Chapter 15, Solution 33.

(a) $$F(s) = \frac{6(s-1)}{s^4-1} = \frac{6}{(s^2+1)(s+1)} = \frac{As+B}{s^2+1} + \frac{C}{s+1}$$

$$6 = A(s^2+s) + B(s+1) + C(s^2+1)$$

Equating coefficients :

s^2: $0 = A + C \longrightarrow A = -C$

s^1: $0 = A + B \longrightarrow B = -A = C$

s^0: $6 = B + C = 2B \longrightarrow B = 3$

$A = -3$, $B = 3$, $C = 3$

$$F(s) = \frac{3}{s+1} + \frac{-3s+3}{s^2+1} = \frac{3}{s+1} + \frac{-3s}{s^2+1} + \frac{3}{s^2+1}$$

$$f(t) = \underline{(3e^{-t} + 3\sin(t) - 3\cos(t))u(t)}$$

(b) $$F(s) = \frac{s\,e^{-\pi s}}{s^2+1}$$

$$f(t) = \underline{\mathbf{\cos(t-\pi)u(t-\pi)}}$$

(c) $$F(s) = \frac{8}{s(s+1)^3} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{(s+1)^2} + \frac{D}{(s+1)^3}$$

$A = 8$, $D = -8$

$$8 = A(s^3+3s^2+3s+1) + B(s^3+2s^2+s) + C(s^2+s) + Ds$$

Equating coefficients :

s^3: $0 = A + B \longrightarrow B = -A$

s^2: $0 = 3A + 2B + C = A + C \longrightarrow C = -A = B$

s^1: $0 = 3A + B + C + D = A + D \longrightarrow D = -A$

s^0: $A = 8,\quad B = -8,\quad C = -8,\quad D = -8$

$$F(s) = \frac{8}{s} - \frac{8}{s+1} - \frac{8}{(s+1)^2} - \frac{8}{(s+1)^3}$$

$$f(t) = \underline{\mathbf{8\left[1 - e^{-t} - te^{-t} - 0.5t^2e^{-t}\right]u(t)}}$$

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Chapter 15, Problem 34.

Find the time functions that have the following Laplace transforms:

(a) $F(s) = 10 + \dfrac{s^2+1}{s^2+4}$ (b) $G(s) = \dfrac{e^{-s}+4e^{-2s}}{s^2+6s+8}$ (c) $H(s) = \dfrac{(s+1)e^{-2s}}{s(s+3)(s+4)}$

Chapter 15, Solution 34.

(a) $$F(s) = 10 + \frac{s^2+4-3}{s^2+4} = 11 - \frac{3}{s^2+4}$$

$$f(t) = \underline{\mathbf{11\delta(t) - 1.5\sin(2t)}}$$

(b) $$G(s) = \frac{e^{-s}+4e^{-2s}}{(s+2)(s+4)}$$

Let $$\frac{1}{(s+2)(s+4)} = \frac{A}{s+2} + \frac{B}{s+4}$$

$A = 1/2$ $B = 1/2$

$$G(s) = \frac{e^{-s}}{2}\left(\frac{1}{s+2} + \frac{1}{s+4}\right) + 2e^{-2s}\left(\frac{1}{s+2} + \frac{1}{s+4}\right)$$

$$g(t) = \underline{\mathbf{0.5\left[e^{-2(t-1)} - e^{-4(t-1)}\right]u(t-1) + 2\left[e^{-2(t-2)} - e^{-4(t-2)}\right]u(t-2)}}$$

(c) Let $$\frac{s+1}{s(s+3)(s+4)} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+4}$$

$A = 1/12$, $B = 2/3$, $C = -3/4$

$$H(s) = \left(\frac{1}{12}\cdot\frac{1}{s} + \frac{2/3}{s+3} - \frac{3/4}{s+4}\right)e^{-2s}$$

$$h(t) = \underline{\mathbf{\left(\frac{1}{12} + \frac{2}{3}e^{-3(t-2)} - \frac{3}{4}e^{-4(t-2)}\right)u(t-2)}}$$

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Chapter 15, Problem 35.

Obtain $f(t)$ for the following transforms:

(a) $F(s) = \dfrac{(s+3)e^{-6s}}{(s+1)(s+2)}$ (b) $F(s) = \dfrac{4 - e^{-2s}}{s^2 + 5s + 4}$ (c) $F(s) = \dfrac{se^{-s}}{(s+3)(s^2+4)}$

Chapter 15, Solution 35.

(a) Let $G(s) = \dfrac{s+3}{(s+1)(s+2)} = \dfrac{A}{s+1} + \dfrac{B}{s+2}$

$A = 2, \qquad B = -1$

$G(s) = \dfrac{2}{s+1} - \dfrac{1}{s+2} \longrightarrow g(t) = 2e^{-t} - e^{-2t}$

$F(s) = e^{-6s}\,G(s) \longrightarrow f(t) = g(t-6)\,u(t-6)$

$f(t) = \underline{\mathbf{\left[2e^{-(t-6)} - e^{-2(t-6)}\right]u(t-6)}}$

(b) Let $G(s) = \dfrac{1}{(s+1)(s+4)} = \dfrac{A}{s+1} + \dfrac{B}{s+4}$

$A = 1/3, \qquad B = -1/3$

$G(s) = \dfrac{1}{3(s+1)} - \dfrac{1}{3(s+4)}$

$g(t) = \dfrac{1}{3}\left[e^{-t} - e^{-4t}\right]$

$F(s) = 4G(s) - e^{-2t}\,G(s)$

$f(t) = 4g(t)\,u(t) - g(t-2)\,u(t-2)$

$f(t) = \underline{\mathbf{\frac{4}{3}\left[e^{-t} - e^{-4t}\right]u(t) - \frac{1}{3}\left[e^{-(t-2)} - e^{-4(t-2)}\right]u(t-2)}}$

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(c) Let $G(s) = \dfrac{s}{(s+3)(s^2+4)} = \dfrac{A}{s+3} + \dfrac{Bs+C}{s^2+4}$

$A = -3/13$

$s = A(s^2+4) + B(s^2+3s) + C(s+3)$

Equating coefficients :

$s^2: \quad 0 = A + B \longrightarrow B = -A$

$s^1: \quad 1 = 3B + C$

$s^0: \quad 0 = 4A + 3C$

$A = -3/13, \quad B = 3/13, \quad C = 4/13$

$$13\,G(s) = \frac{-3}{s+3} + \frac{3s+4}{s^2+4}$$

$$13\,g(t) = -3e^{-3t} + 3\cos(2t) + 2\sin(2t)$$

$$F(s) = e^{-s}\,G(s)$$

$$f(t) = g(t-1)\,u(t-1)$$

$$f(t) = \underline{\mathbf{\frac{1}{13}\left[-3e^{-3(t-1)} + 3\cos(2(t-1)) + 2\sin(2(t-1))\right]u(t-1)}}$$

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Chapter 15, Problem 36.

Obtain the inverse Laplace transforms of the following functions:

(a) $X(s) = \dfrac{1}{s^2(s+2)(s+3)}$

(b) $Y(s) = \dfrac{1}{s(s+1)^2}$

(c) $Z(s) = \dfrac{1}{s(s+1)(s^2+6s+10)}$

Chapter 15, Solution 36.

(a) $$X(s) = \frac{1}{s^2(s+2)(s+3)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+2} + \frac{D}{s+3}$$

$B = 1/6, \qquad C = 1/4, \qquad D = -1/9$

$$1 = A(s^3 + 5s^2 + 6s) + B(s^2 + 5s + 6) + C(s^3 + 3s^2) + D(s^3 + 2s^2)$$

Equating coefficients :

$s^3: \quad 0 = A + C + D$

$s^2: \quad 0 = 5A + B + 3C + 2D = 3A + B + C$

$s^1: \quad 0 = 6A + 5B$

$s^0: \quad 1 = 6B \longrightarrow B = 1/6$

$A = -5/6\,B = -5/36$

$$X(s) = \frac{-5/36}{s} + \frac{1/6}{s^2} + \frac{1/4}{s+2} - \frac{1/9}{s+3}$$

$$x(t) = \underline{\mathbf{\frac{-5}{36}u(t) + \frac{1}{6}t + \frac{1}{4}e^{-2t} - \frac{1}{9}e^{-3t}}}$$

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(b) $$Y(s) = \frac{1}{s(s+1)^2} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{(s+1)^2}$$

$A = 1, \qquad C = -1$

$$1 = A(s^2 + 2s + 1) + B(s^2 + s) + Cs$$

Equating coefficients :

$s^2: \quad 0 = A + B \longrightarrow B = -A$

$s^1: \quad 0 = 2A + B + C = A + C \longrightarrow C = -A$

$s^0: \quad 1 = A, \quad B = -1, \quad C = -1$

$$Y(s) = \frac{1}{s} - \frac{1}{s+1} - \frac{1}{(s+1)^2}$$

$$y(t) = \underline{\mathbf{u(t) - e^{-t} - t\,e^{-t}}}$$

(c) $$Z(s) = \frac{A}{s} + \frac{B}{s+1} + \frac{Cs + D}{s^2 + 6s + 10}$$

$A = 1/10, \qquad B = -1/5$

$$1 = A(s^3 + 7s^2 + 16s + 10) + B(s^3 + 6s^2 + 10s) + C(s^3 + s^2) + D(s^2 + s)$$

Equating coefficients :

$s^3: \quad 0 = A + B + C$

$s^2: \quad 0 = 7A + 6B + C + D = 6A + 5B + D$

$s^1: \quad 0 = 16A + 10B + D = 10A + 5B \longrightarrow B = -2A$

$s^0: \quad 1 = 10A \longrightarrow A = 1/10$

$A = 1/10, \qquad B = -2A = -1/5, \qquad C = A = 1/10, \qquad D = 4A = \frac{4}{10}$

$$10\,Z(s) = \frac{1}{s} - \frac{2}{s+1} + \frac{s+4}{s^2 + 6s + 10}$$

$$10\,Z(s) = \frac{1}{s} - \frac{2}{s+1} + \frac{s+3}{(s+3)^2 + 1} + \frac{1}{(s+3)^2 + 1}$$

$$z(t) = \underline{\mathbf{0.1\left[1 - 2e^{-t} + e^{-3t}\cos(t) + e^{-3t}\sin(t)\right]u(t)}}$$

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Chapter 15, Problem 37.

Find the inverse Laplace transform of:

(a) $H(s) = \dfrac{s+4}{s(s+2)}$

(b) $G(s) = \dfrac{s^2+4s+5}{(s+3)(s^2+2s+2)}$

(c) $F(s) = \dfrac{e^{-4s}}{s+2}$

(d) $D(s) = \dfrac{10s}{(s^2+1)(s^2+4)}$

Chapter 15, Solution 37.

(a) $H(s) = \dfrac{s+4}{s(s+2)} = \dfrac{A}{s} + \dfrac{B}{s+2}$

s+4 =A(s+2) + Bs
Equating coefficients,
s: 1 = A + B
constant: 4= 2 A $\longrightarrow$ A =2, B=1-A = -1

$$H(s) = \frac{2}{s} - \frac{1}{s+2}$$
$$h(t) = 2u(t) - e^{-2t}u(t) = \underline{(2-e^{-2t})u(t)}$$

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(b) $G(s) = \dfrac{A}{s+3} + \dfrac{Bs+C}{s^2+2s+2}$

$$s^2 + 4s + 5 = (Bs + C)(s+3) + A(s^2 + 2s + 2)$$

Equating coefficients,

s^2:	1= B + A	(1)
s:	4 = 3B + C + 2A	(2)
Constant:	5 =3C + 2A	(3)

Solving (1) to (3) gives

$$A = \frac{2}{5}, \quad B = \frac{3}{5}, \quad C = \frac{7}{5}$$

$$G(s) = \frac{0.4}{s+3} + \frac{0.6s+1.4}{s^2+2s+2} = \frac{0.4}{s+3} + \frac{0.6(s+1)+0.8}{(s+1)^2+1}$$

$$g(t) = \underline{0.4e^{-3t} + 0.6e^{-t}\cos t + 0.8e^{-t}\sin t}$$

(c) $f(t) = \underline{e^{-2(t-4)}u(t-4)}$

(d) $D(s) = \dfrac{10s}{(s^2+1)(s^2+4)} = \dfrac{As+B}{s^2+1} + \dfrac{Cs+D}{s^2+4}$

$$10s = (s^2+4)(As+B) + (s^2+1)(Cs+D)$$

Equating coefficients,

s^3 :	0 = A + C
s^2 :	0 = B + D
s:	10 = 4A + C
constant:	0 = 4B+D

Solving these leads to

A = -10/3, B = 0, C = -10/3, D = 0

$$D(s) == \frac{10s/3}{s^2+1} - \frac{10s/3}{s^2+4}$$

$$d(t) = \underline{\frac{10}{3}\cos t - \frac{10}{3}\cos 2t}$$

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Chapter 15, Problem 38.

Find $f(t)$ given that:

(a) $F(s) = \dfrac{s^2 + 4s}{s^2 + 10s + 26}$

(b) $F(s) = \dfrac{5s^2 + 7s + 29}{s(s^2 + 4s + 29)}$

Chapter 15, Solution 38.

(a)
$$F(s) = \frac{s^2 + 4s}{s^2 + 10s + 26} = \frac{s^2 + 10s + 26 - 6s - 26}{s^2 + 10s + 26}$$
$$F(s) = 1 - \frac{6s + 26}{s^2 + 10s + 26}$$
$$F(s) = 1 - \frac{6(s+5)}{(s+5)^2 + 1^2} + \frac{4}{(s+5)^2 + 1^2}$$

$$f(t) = \underline{\boldsymbol{\delta(t) - 6e^{-t}\cos(5t) + 4e^{-t}\sin(5t)}}$$

(b)
$$F(s) = \frac{5s^2 + 7s + 29}{s(s^2 + 4s + 29)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 4s + 29}$$

$$5s^2 + 7s + 29 = A(s^2 + 4s + 29) + Bs^2 + Cs$$

Equating coefficients :

s^0: $\quad 29 = 29A \longrightarrow A = 1$

s^1: $\quad 7 = 4A + C \longrightarrow C = 7 - 4A = 3$

s^2: $\quad 5 = A + B \longrightarrow B = 5 - A = 4$

$A = 1, \qquad B = 4, \qquad C = 3$

$$F(s) = \frac{1}{s} + \frac{4s + 3}{s^2 + 4s + 29} = \frac{1}{s} + \frac{4(s+2)}{(s+2)^2 + 5^2} - \frac{5}{(s+2)^2 + 5^2}$$

$$f(t) = \underline{\boldsymbol{u(t) + 4e^{-2t}\cos(5t) - e^{-2t}\sin(5t)}}$$

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Chapter 15, Problem 39.

*Determine $f(t)$ if:

(a) $F(s) = \dfrac{2s^3 + 4s^2 + 1}{(s^2 + 2s + 17)(s^2 + 4s + 20)}$

(b) $F(s) = \dfrac{s^2 + 4}{(s^2 + 9)(s^2 + 6s + 3)}$

* An asterisk indicates a challenging problem.

Chapter 15, Solution 39.

(a) $$F(s) = \frac{2s^3 + 4s^2 + 1}{(s^2 + 2s + 17)(s^2 + 4s + 20)} = \frac{As + B}{s^2 + 2s + 17} + \frac{Cs + D}{s^2 + 4s + 20}$$

$$s^3 + 4s^2 + 1 = A(s^3 + 4s^2 + 20s) + B(s^2 + 4s + 20)$$
$$+ C(s^3 + 2s^2 + 17s) + D(s^2 + 2s + 17)$$

Equating coefficients :

s^3: $2 = A + C$

s^2: $4 = 4A + B + 2C + D$

s^1: $0 = 20A + 4B + 17C + 2D$

s^0: $1 = 20B + 17D$

Solving these equations (Matlab works well with 4 unknowns),
$A = -1.6$, $B = -17.8$, $C = 3.6$, $D = 21$

$$F(s) = \frac{-1.6s - 17.8}{s^2 + 2s + 17} + \frac{3.6s + 21}{s^2 + 4s + 20}$$

$$F(s) = \frac{(-1.6)(s+1)}{(s+1)^2 + 4^2} + \frac{(-4.05)(4)}{(s+1)^2 + 4^2} + \frac{(3.6)(s+2)}{(s+2)^2 + 4^2} + \frac{(3.45)(4)}{(s+2)^2 + 4^2}$$

$$f(t) = \underline{\mathbf{-1.6e^{-t}\cos(4t) - 4.05e^{-t}\sin(4t) + 3.6e^{-2t}\cos(4t) + 3.45e^{-2t}\sin(4t)}}$$

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(b) $$F(s) = \frac{s^2+4}{(s^2+9)(s^2+6s+3)} = \frac{As+B}{s^2+9} + \frac{Cs+D}{s^2+6s+3}$$

$$s^2+4 = A(s^3+6s^2+3s) + B(s^2+6s+3) + C(s^3+9s) + D(s^2+9)$$

Equating coefficients :

s^3: $0 = A + C \longrightarrow C = -A$

s^2: $1 = 6A + B + D$

s^1: $0 = 3A + 6B + 9C = 6B + 6C \longrightarrow B = -C = A$

s^0: $4 = 3B + 9D$

Solving these equations,

$A = 1/12$, $B = 1/12$, $C = -1/12$, $D = 5/12$

$$12F(s) = \frac{s+1}{s^2+9} + \frac{-s+5}{s^2+6s+3}$$

$$s^2+6s+3 = 0 \longrightarrow \frac{-6 \pm \sqrt{36-12}}{2} = -0.551, -5.449$$

Let $$G(s) = \frac{-s+5}{s^2+6s+3} = \frac{E}{s+0.551} + \frac{F}{s+5.449}$$

$$E = \frac{-s+5}{s+5.449}\Big|_{s=-0.551} = 1.133$$

$$F = \frac{-s+5}{s+0.551}\Big|_{s=-5.449} = -2.133$$

$$G(s) = \frac{1.133}{s+0.551} - \frac{2.133}{s+5.449}$$

$$12F(s) = \frac{s}{s^2+3^2} + \frac{1}{3}\cdot\frac{3}{s^2+3^2} + \frac{1.133}{s+0.551} - \frac{2.133}{s+5.449}$$

$$f(t) = \underline{\mathbf{0.08333\cos(3t) + 0.02778\sin(3t) + 0.0944e^{-0.551t} - 0.1778e^{-5.449t}}}$$

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Chapter 15, Problem 40.

Show that

$$L^{-1}\left[\frac{4s^2+7s+13}{(s+2)(s^2+2s+5)}\right]=\left[\sqrt{2}e^{-t}\cos(2t+45°)+3e^{-2t}\right]u(t)$$

Chapter 15, Solution 40.

$$\text{Let } H(s)=\left[\frac{4s^2+7s+13}{(s+2)(s^2+2s+5)}\right]=\frac{A}{s+2}+\frac{Bs+C}{s^2+2s+5}$$

$$4s^2+7s+13=A(s^2+2s+5)+B(s^2+2s)+C(s+2)$$

Equating coefficients gives:

s^2 : $\quad 4=A+B$

s : $\quad 7=2A+2B+C \quad\longrightarrow\quad C=-1$

constant : $\quad 13=5A+2C \quad\longrightarrow\quad 5A=15$ or $A=3,\ B=1$

$$H(s)=\frac{3}{s+2}+\frac{s-1}{s^2+2s+5}=\frac{3}{s+2}+\frac{(s+1)-2}{(s+1)^2+2^2}$$

Hence,

$$h(t)=3e^{-2t}+e^{-t}\cos 2t-e^{-t}\sin 2t=3e^{-2t}+e^{-t}(A\cos\alpha\cos 2t-A\sin\alpha\sin 2t)$$

where $A\cos\alpha=1,\quad A\sin\alpha=1 \quad\longrightarrow\quad A=\sqrt{2},\quad \alpha=45^o$

Thus,

$$h(t)=\underline{\left[\sqrt{2}e^{-t}\cos(2t+45^o)+3e^{-2t}\right]u(t)}$$

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Chapter 15, Problem 41.

* Let $x(t)$ and $y(t)$ be as shown in Fig. 15.36. Find $z(t) = x(t) * y(t)$.

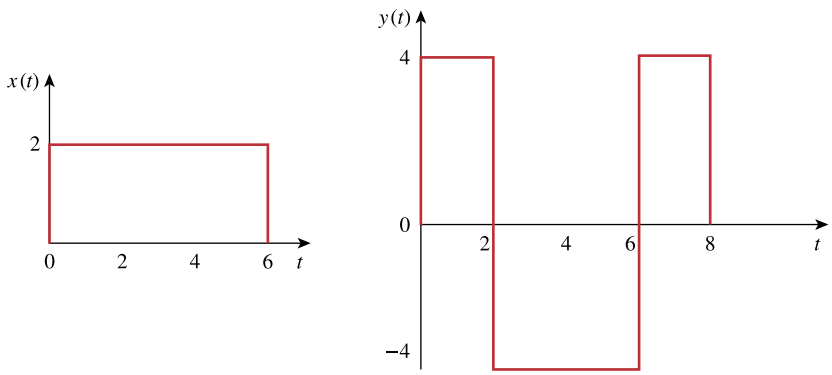

Figure 15.36
For Prob. 15.41.

* An asterisk indicates a challenging problem.

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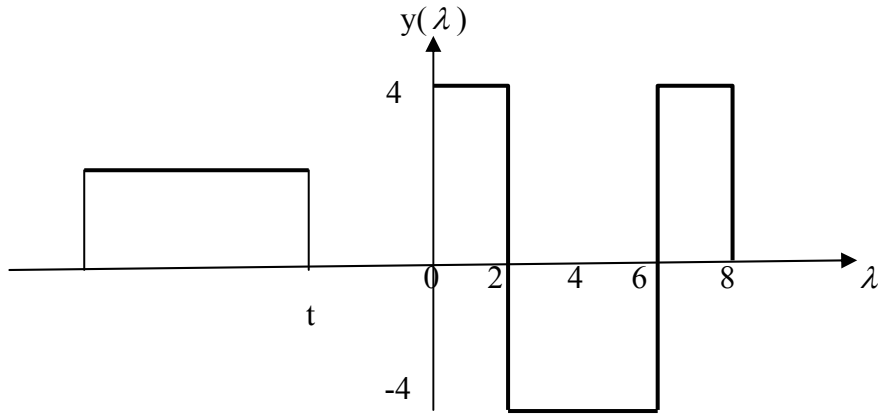
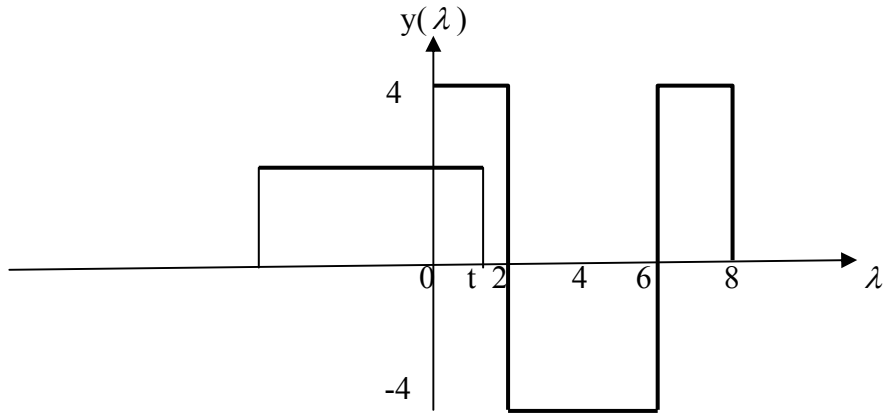
Chapter 15, Solution 41.

We fold x(t) and slide on y(t). For t<0, no overlapping as shown below. x(t) =0.



For 0 < t < 2, there is overlapping, as shown below.



$$z(t) = \int_0^t (2)(4)dt = 8t$$

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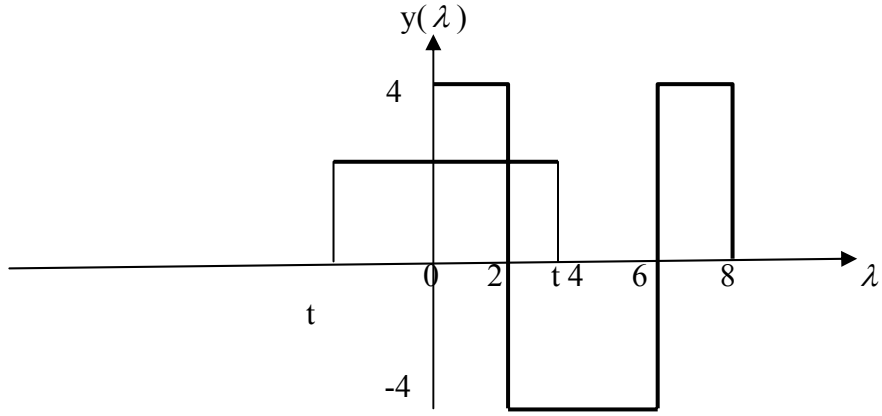
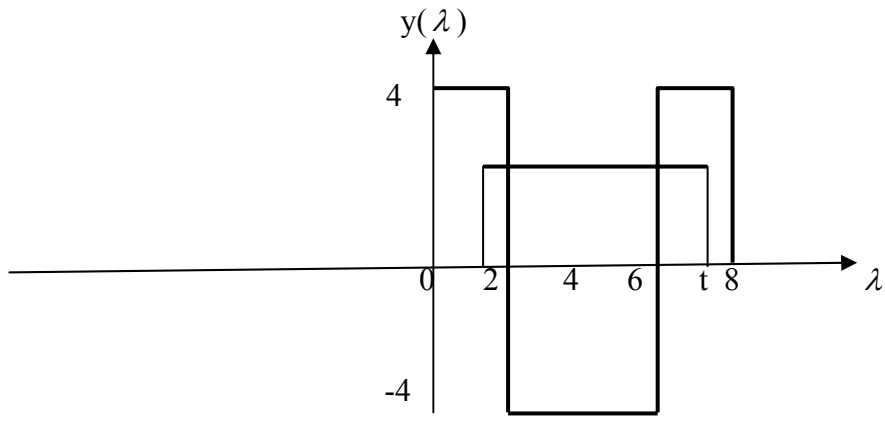
For 2 < t < 6, the two functions overlap, as shown below.



$$z(t) = \int_0^2 (2)(4)d\lambda + \int_0^t (2)(-4)d\lambda = 16 - 8t$$

For 6<t<8, they overlap as shown below.



$$z(t) = \int_{t-6}^{2} (2)(4)d\lambda + \int_2^6 (2)(-4)d\lambda + \int_6^t (2)(4)d\lambda = 8\lambda\Big|_{t-6}^{2} - 8\lambda\Big|_2^6 + 8\lambda\Big|_6^t = -16$$

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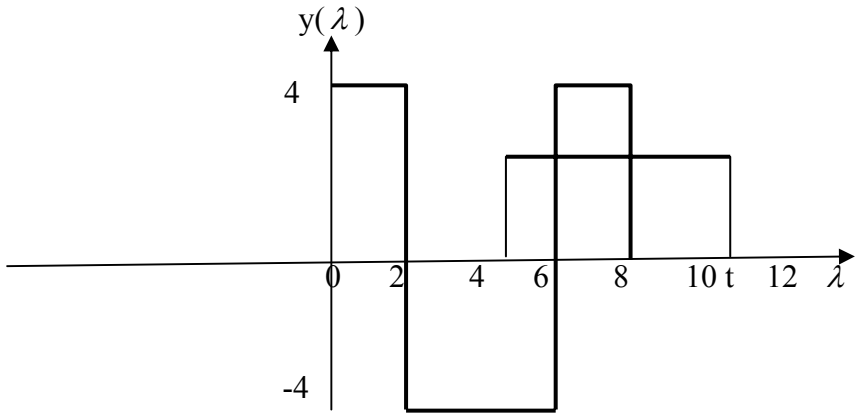
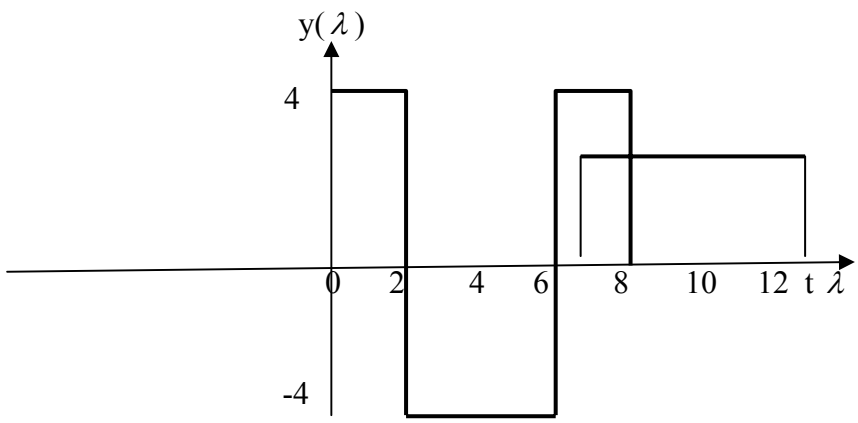
For 8< t <12, they overlap as shown below.



$$z(t) = \int_{t-6}^{6} (2)(-4)d\lambda + \int_{6}^{8} (2)(4)d\lambda = -8\lambda\Big|_{t-6}^{6} + 8\lambda\Big|_{6}^{8} = 8t - 80$$

For 12 < t < 14, they overlap as shown below.



$$z(t) = \int_{t-6}^{8} (2)(4)d\lambda = 8\lambda\Big|_{t-6}^{8} = 112 - 8t$$

Hence,

z(t) = **8t, 0<t<2**
16–8t, 2<t<6
–16, 6<t<8
8t–80, 8<t<12
112–8t, 12<t<14
0, otherwise.

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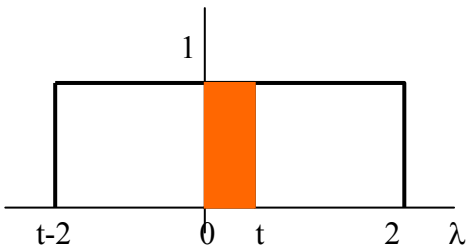
Chapter 15, Problem 42.

Suppose that $f(t) = u(t) - u(t-2)$. Determine $f(t) * f(t)$.

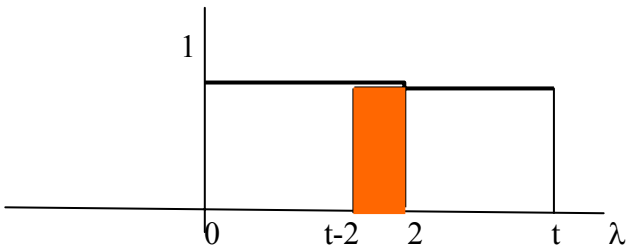
Chapter 15, Solution 42.

For 0<t<2, the signals overlap as shown below.



$$y(t) = f(t) * f(t) = \int_0^t (1)(1)d\lambda = t$$

For 2 < t< 4, they overlap as shown below.



$$y(t) = \int_{t-2}^{2} (1)(1)d\lambda = t\Big|_{t-2}^{2} = 4 - t$$

Thus,

$$y(t) = \begin{cases} t, & 0 < t < 2 \\ 4 - t, & 2 < t < 4 \\ 0, & \text{otherwise} \end{cases}$$

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Chapter 15, Problem 43.

Find $y(t) = x(t) * h(t)$ for each paired $x(t)$ and $h(t)$ in Fig. 15.37.

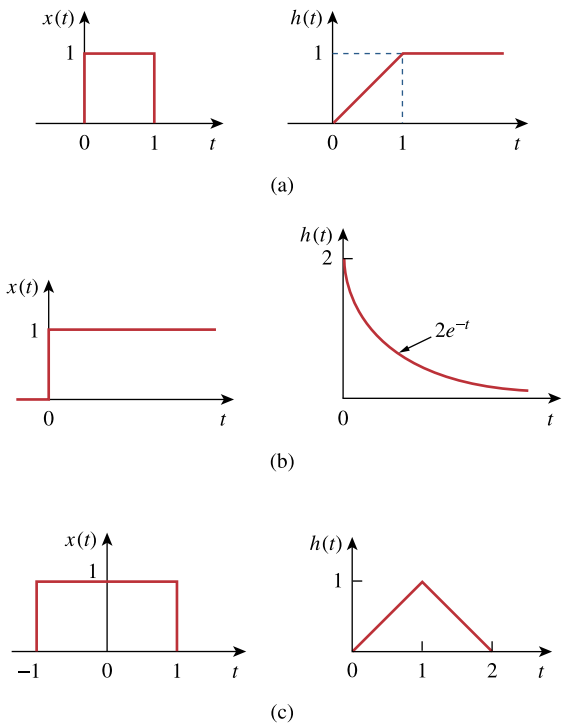

Figure 15.37

For Prob. 15.43.

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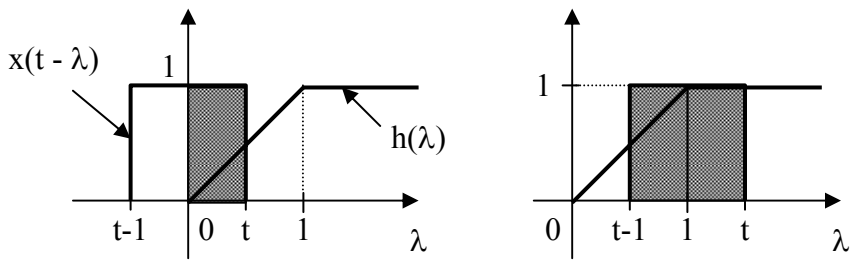
Chapter 15, Solution 43.

(a) For $0 < t < 1$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (a).

$$y(t) = x(t) * h(t) = \int_0^t (1)(\lambda)\, d\lambda = \frac{\lambda^2}{2}\Big|_0^t = \frac{t^2}{2}$$



(a) **(b)**

For $1 < t < 2$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (b).

$$y(t) = \int_{t-1}^{1} (1)(\lambda)\, d\lambda + \int_1^t (1)(1)\, d\lambda = \frac{\lambda^2}{2}\Big|_{t-1}^{1} + \lambda\Big|_1^t = \frac{-1}{2}t^2 + 2t - 1$$

For $t > 2$, there is a complete overlap so that

$$y(t) = \int_{t-1}^{t} (1)(1)\, d\lambda = \lambda\Big|_{t-1}^{t} = t - (t-1) = 1$$

Therefore,

$$y(t) = \begin{cases} \mathbf{t^2/2,} & \mathbf{0 < t < 1} \\ \mathbf{-(t^2/2) + 2t - 1,} & \mathbf{1 < t < 2} \\ \mathbf{1,} & \mathbf{t > 2} \\ \mathbf{0,} & \mathbf{otherwise} \end{cases}$$

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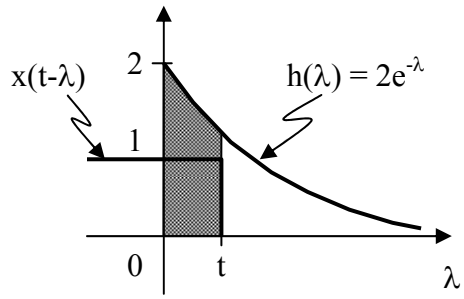
(b) For $t > 0$, the two functions overlap as shown in Fig. (c).

$$y(t) = x(t) * h(t) = \int_0^t (1)\, 2\, e^{-\lambda}\, d\lambda = -2\, e^{-\lambda}\Big|_0^t$$



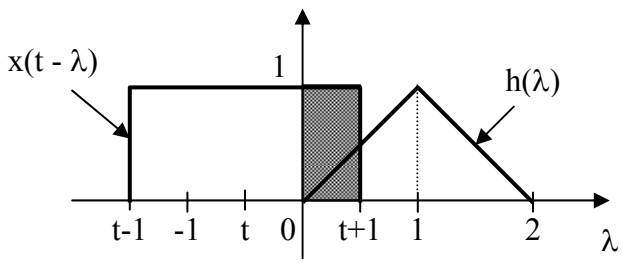
(c)

Therefore,

$$y(t) = \underline{\mathbf{2(1 - e^{-t}),\quad t > 0}}$$

(c) For $-1 < t < 0$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (d).

$$y(t) = x(t) * h(t) = \int_0^{t+1} (1)(\lambda)\, d\lambda = \frac{\lambda^2}{2}\Big|_0^{t+1} = \frac{1}{2}(t+1)^2$$

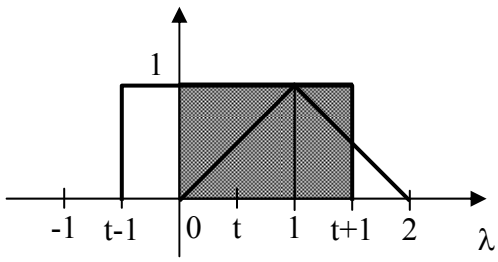


(d)

For $0 < t < 1$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (e).

$$y(t) = \int_0^1 (1)(\lambda)\, d\lambda + \int_1^{t+1} (1)(2-\lambda)\, d\lambda$$

$$y(t) = \frac{\lambda^2}{2}\Big|_0^1 + \left(2\lambda - \frac{\lambda^2}{2}\right)\Big|_1^{t+1} = \frac{-1}{2}t^2 + t + \frac{1}{2}$$



(e)

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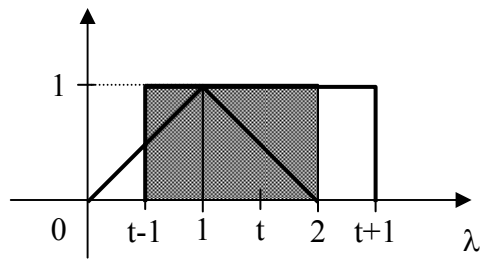
For $1 < t < 2$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (f).

$$y(t) = \int_{t-1}^{1} (1)(\lambda)\, d\lambda + \int_{1}^{2} (1)(2-\lambda)\, d\lambda$$

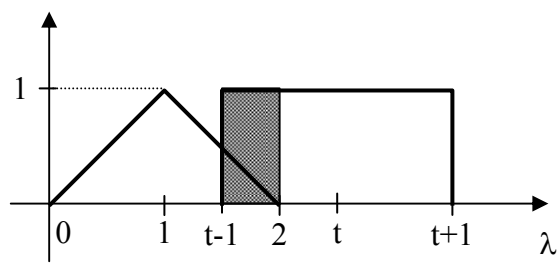
$$y(t) = \frac{\lambda^2}{2}\Big|_{t-1}^{1} + \left(2\lambda - \frac{\lambda^2}{2}\right)\Big|_{1}^{2} = \frac{-1}{2}t^2 + t + \frac{1}{2}$$



(f)

For $2 < t < 3$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (g).

$$y(t) = \int_{t-1}^{2} (1)(2-\lambda)\, d\lambda = \left(2\lambda - \frac{\lambda^2}{2}\right)\Big|_{t-1}^{2} = \frac{9}{2} - 3t + \frac{1}{2}t^2$$



(g)

Therefore,

$$y(t) = \begin{cases} \mathbf{(t^2/2) + t + 1/2,} & \mathbf{-1 < t < 0} \\ \mathbf{-(t^2/2) + t + 1/2,} & \mathbf{0 < t < 2} \\ \mathbf{(t^2/2) - 3t + 9/2,} & \mathbf{2 < t < 3} \\ \mathbf{0,} & \textbf{otherwise} \end{cases}$$

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Chapter 15, Problem 44.

Obtain the convolution of the pairs of signals in Fig. 15.38.

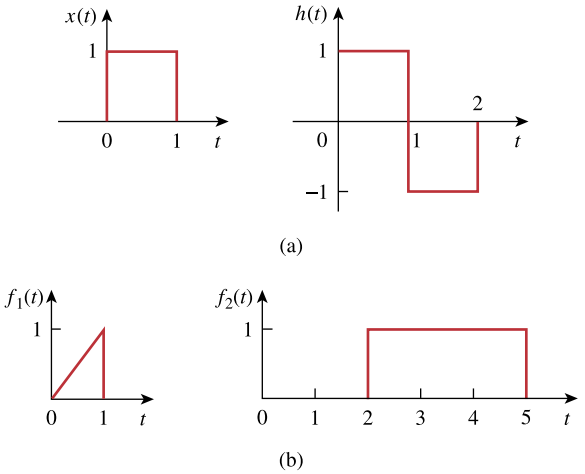

Figure 15.38

For Prob. 15.44.

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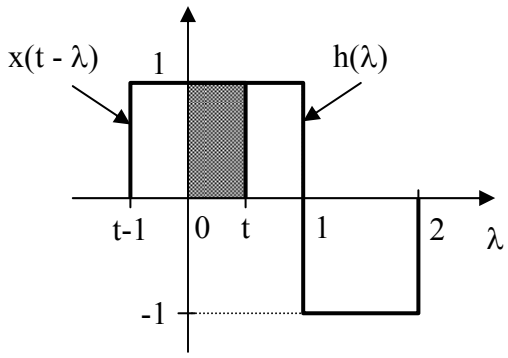
Chapter 15, Solution 44.

(a) For $0 < t < 1$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (a).

$$y(t) = x(t) * h(t) = \int_0^t (1)(1)\, d\lambda = t$$



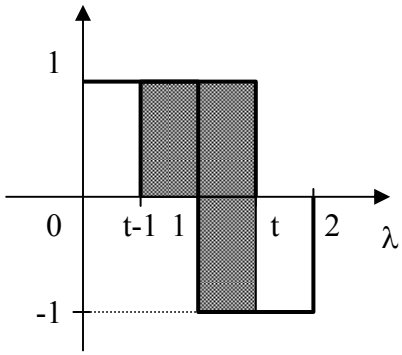
(a)

For $1 < t < 2$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (b).

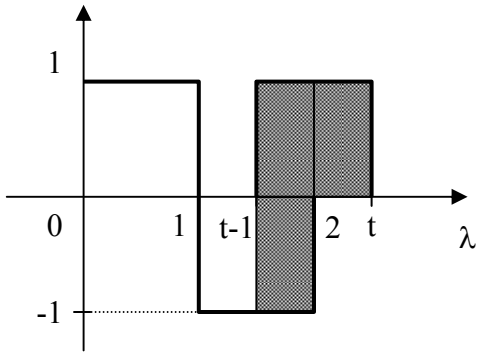
$$y(t) = \int_{t-1}^{1} (1)(1)\, d\lambda + \int_1^t (-1)(1)\, d\lambda = \lambda\Big|_{t-1}^{1} - \lambda\Big|_1^t = 3 - 2t$$

For $2 < t < 3$, $x(t-\lambda)$ and $h(\lambda)$ overlap as shown in Fig. (c).

$$y(t) = \int_{t-1}^{2} (1)(-1)\, d\lambda = -\lambda\Big|_{t-1}^{2} = t - 3$$



(b)



(c)

Therefore,

$$y(t) = \begin{cases} \mathbf{t,} & \mathbf{0 < t < 1} \\ \mathbf{3 - 2t,} & \mathbf{1 < t < 2} \\ \mathbf{t - 3,} & \mathbf{2 < t < 3} \\ \mathbf{0,} & \textbf{otherwise} \end{cases}$$

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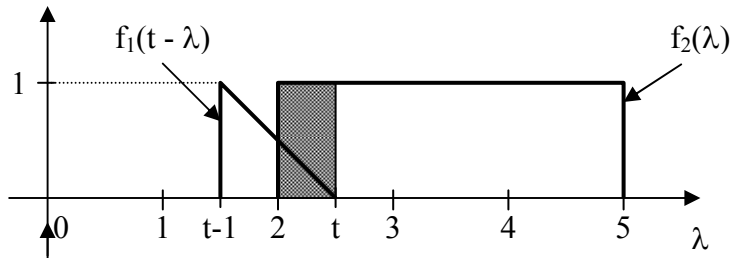
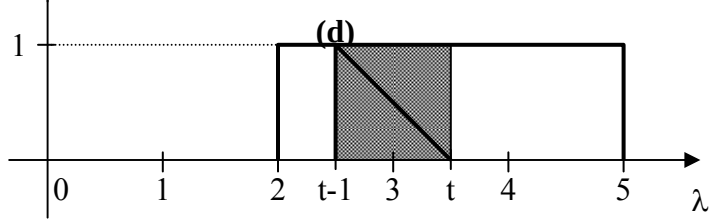
(b) For $t < 2$, there is no overlap. For $2 < t < 3$, $f_1(t-\lambda)$ and $f_2(\lambda)$ overlap, as shown in Fig. (d).

$$y(t) = f_1(t) * f_2(t) = \int_2^t (1)(t-\lambda)\, d\lambda$$



(d)



(e)

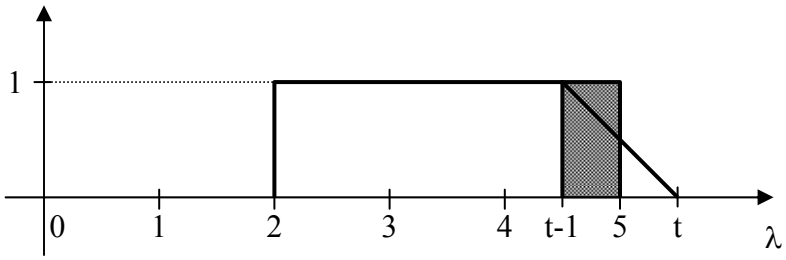
$$= \left(\lambda t - \frac{\lambda^2}{2}\right)\Big|_2^t = \frac{t^2}{2} - 2t + 2$$

For $3 < t < 5$, $f_1(t-\lambda)$ and $f_2(\lambda)$ overlap as shown in Fig. (e).

$$y(t) = \int_{t-1}^{t} (1)(t-\lambda)\, d\lambda = \left(\lambda t - \frac{\lambda^2}{2}\right)\Big|_{t-1}^{t} = \frac{1}{2}$$

For $5 < t < 6$, the functions overlap as shown in Fig. (f).

$$y(t) = \int_{t-1}^{5} (1)(t-\lambda)\, d\lambda = \left(\lambda t - \frac{\lambda^2}{2}\right)\Big|_{t-1}^{5} = \frac{-1}{2}t^2 + 5t - 12$$



(f)

Therefore, $$y(t) = \begin{cases} \mathbf{(t^2/2) - 2t + 2,} & \mathbf{2 < t < 3} \\ \mathbf{1/2,} & \mathbf{3 < t < 5} \\ \mathbf{-(t^2/2) + 5t - 12,} & \mathbf{5 < t < 6} \\ \mathbf{0,} & \mathbf{otherwise} \end{cases}$$

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Chapter 15, Problem 45.

Given $h(t) = 4e^{-2t}u(t)$ and $x(t) = \delta(t) - 2e^{-2t}u(t)$, find $y(t) = x(t) * h(t)$.

Chapter 15, Solution 45.

$$y(t) = h(t) * x(t) = \left[4e^{-2t}u(t)\right] * \left[\delta(t) - 2e^{-2t}u(t)\right]$$

$$= 4e^{-2t}u(t) * \delta(t) - 4e^{-2t}u(t) * 2e^{-2t}u(t) = 4e^{-2t}u(t) - 8e^{-2t}\int_0^t e^o d\lambda$$

$$= \underline{4e^{-2t}u(t) - 8te^{-2t}u(t)}$$

Chapter 15, Problem 46.

Given the following functions

$x(t) = 2\delta(t)$, $\quad y(t) = 4u(t)$, $\quad z(t) = e^{-2t}u(t)$,

evaluate the following convolution operations.

(a) $x(t) * y(t)$
(b) $x(t) * z(t)$
(c) $y(t) * z(t)$
(d) $y(t) * [y(t) + z(t)]$

Chapter 15, Solution 46.

(a) $x(t) * y(t) = 2\delta(t) * 4u(t) = \underline{8u(t)}$

(b) $x(t) * z(t) = 2\delta(t) * e^{-2t}u(t) = \underline{2e^{-2t}u(t)}$

(c) $y(t) * z(t) = 4u(t) * e^{-2t}u(t) = 4\int_0^t e^{-2\lambda} d\lambda = \left.\frac{4e^{-2\lambda}}{-2}\right|_0^t = \underline{2(1 - e^{-2t})}$

(d) $y(t) * [y(t) + z(t)] = 4u(t) * [4u(t) + e^{-2t}u(t)] = 4\int [4u(\lambda) + e^{-2\lambda}u(\lambda)] d\lambda$

$$= 4\int_0^t [4 + e^{-2\lambda}] d\lambda = 4[4t + \left.\frac{e^{-2\lambda}}{-2}]\right|_0^t = \underline{16t - 2e^{-2t} + 2}$$

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Chapter 15, Problem 47.

A system has the transfer function

$$H(s) = \frac{s}{(s+1)(s+2)}$$

(a) Find the impulse response of the system.
(b) Determine the output $y(t)$, given that the input is $x(t) = u(t)$

Chapter 15, Solution 47.

(a) $H(s) = \dfrac{s}{(s+1)(s+2)} = \dfrac{A}{s+1} + \dfrac{B}{s+2}$

s=A(s+2) + B(s+1)

We equate the coefficients.

s: 1= A+B
constant: 0 =2A +B

Solving these, A = -1, B= 2.

$$H(s) = \frac{-1}{s+1} + \frac{2}{s+2}$$

$$h(t) = \underline{(-e^{-t} + 2e^{-2t})u(t)}$$

(b) $H(s) = \dfrac{Y(s)}{X(s)} \longrightarrow Y(s) = H(s)X(s) = \dfrac{s}{(s+1)(s+2)}\dfrac{1}{s}$

$$Y(s) = \frac{1}{(s+1)(s+2)} = \frac{C}{s+1} + \frac{D}{s+2}$$

C=1 and D=-1 so that

$$Y(s) = \frac{1}{s+1} - \frac{1}{s+2}$$

$$y(t) = \underline{(e^{-t} - e^{-2t})u(t)}$$

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Chapter 15, Problem 48.

Find $f(t)$ using convolution given that:

(a) $F(s) = \dfrac{4}{\left(s^2 + 2s + 5\right)^2}$

(b) $F(s) = \dfrac{2s}{(s+1)\left(s^2 + 4\right)}$

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Chapter 15, Solution 48.

(a) Let $G(s) = \dfrac{2}{s^2 + 2s + 5} = \dfrac{2}{(s+1)^2 + 2^2}$

$$g(t) = e^{-t}\sin(2t)$$

$$F(s) = G(s)\,G(s)$$

$$f(t) = L^{-1}\left[\,G(s)\,G(s)\right] = \int_0^t g(\lambda)\,g(t-\lambda)\,d\lambda$$

$$f(t) = \int_0^t e^{-\lambda}\sin(2\lambda)\,e^{-(t-\lambda)}\sin(2(t-\lambda))\,d\lambda$$

$$\sin(A)\sin(B) = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$f(t) = \frac{1}{2}e^{-t}\int_0^t e^{-\lambda}\left[\cos(2t) - \cos(2(t-2\lambda))\right]d\lambda$$

$$f(t) = \frac{e^{-t}}{2}\cos(2t)\int_0^t e^{-2\lambda}\,d\lambda - \frac{e^{-t}}{2}\int_0^t e^{-2\lambda}\cos(2t-4\lambda)\,d\lambda$$

$$f(t) = \frac{e^{-t}}{2}\cos(2t)\cdot\frac{e^{-2\lambda}}{-2}\Big|_0^t - \frac{e^{-t}}{2}\int_0^t e^{-2\lambda}\left[\cos(2t)\cos(4\lambda) + \sin(2t)\sin(4\lambda)\right]d\lambda$$

$$f(t) = \frac{1}{4}e^{-t}\cos(2t)\,(-e^{-2t}+1) - \frac{e^{-t}}{2}\cos(2t)\int_0^t e^{-2\lambda}\cos(4\lambda)\,d\lambda$$

$$- \frac{e^{-t}}{2}\sin(2t)\int_0^t e^{-2\lambda}\sin(4\lambda)\,d\lambda$$

$$f(t) = \frac{1}{4}e^{-t}\cos(2t)\,(1-e^{-2t})$$

$$- \frac{e^{-t}}{2}\cos(2t)\left[\frac{e^{-2\lambda}}{4+16}\left(-2\cos(4\lambda) - 4\sin(4\lambda)\right)\right]\Bigg|_0^t$$

$$- \frac{e^{-t}}{2}\sin(2t)\left[\frac{e^{-2\lambda}}{4+16}\left(-2\sin(4\lambda) + 4\cos(4\lambda)\right)\right]\Bigg|_0^t$$

$$\mathbf{f(t) = \frac{e^{-t}}{2}\cos(2t) - \frac{e^{-3t}}{4}\cos(2t) - \frac{e^{-t}}{20}\cos(2t) + \frac{e^{-3t}}{20}\cos(2t)\cos(4t)}$$

$$\mathbf{+ \frac{e^{-3t}}{10}\cos(2t)\sin(4t) + \frac{e^{-t}}{10}\sin(2t)}$$

$$\mathbf{+ \frac{e^{-t}}{20}\sin(2t)\sin(4t) - \frac{e^{-t}}{10}\sin(2t)\cos(4t)}$$

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(b) Let $X(s) = \dfrac{2}{s+1}$, $\qquad Y(s) = \dfrac{s}{s+4}$

$x(t) = 2\,e^{-t}\,u(t)$, $\qquad y(t) = \cos(2t)\,u(t)$

$F(s) = X(s)\,Y(s)$

$$f(t) = L^{-1}\left[\,X(s)\,Y(s)\right] = \int_0^{\infty} y(\lambda)\,x(t-\lambda)\,d\lambda$$

$$f(t) = \int_0^{t} \cos(2\lambda)\cdot 2\,e^{-(t-\lambda)}\,d\lambda$$

$$f(t) = 2\,e^{-t}\cdot\frac{e^{\lambda}}{1+4}\left(\cos(2\lambda) + 2\sin(2\lambda)\right)\Big|_0^t$$

$$f(t) = \frac{2}{5}e^{-t}\left[\,e^{t}\left(\cos(2t) + 2\sin(2t) - 1\right)\right]$$

$$f(t) = \underline{\mathbf{\frac{2}{5}\cos(2t) + \frac{4}{5}\sin(2t) - \frac{2}{5}e^{-t}}}$$

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Chapter 15, Problem 49.

* Use the convolution integral to find:

(a) $t * e^{at}u(t)$

(b) $\cos(t) * \cos(t)u(t)$

* An asterisk indicates a challenging problem.

Chapter 15, Solution 49.

(a) $t*e^{\alpha t}u(t) =$

$$\int_0^t e^{a\lambda}(t-\lambda)d\lambda = t\frac{e^{a\lambda}}{a}\Bigg|_0^t - \frac{e^{a\lambda}}{a^2}(a\lambda-1)\Bigg|_0^t = \underline{\frac{t}{a}(e^{at}-1) - \frac{1}{a^2} - \frac{e^{at}}{a^2}(at-1)}$$

(b) $$\cos t * \cos t u(t) = \int_0^t \cos\lambda\cos(t-\lambda)d\lambda = \int_0^t \{\cos t\cos\lambda\cos\lambda + \sin t\sin\lambda\cos\lambda\}d\lambda$$

$$= \left[\cos t\int_0^t \frac{1}{2}[1+\cos 2\lambda]d\lambda + \sin t\int_0^t \cos\lambda d(-\cos\lambda)\right] = \left[\frac{1}{2}\cos t[\lambda + \frac{\sin 2\lambda}{2}]\Bigg|_0^t - \sin t\frac{\cos\lambda}{2}\Bigg|_0^t\right]$$

= **<u>0.5cos(t)(t+0.5sin(2t)) – 0.5sin(t)(cos(t) – 1)</u>**.

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Chapter 15, Problem 50.

Use the Laplace transform to solve the differential equation

$$\frac{d^2v(t)}{dt^2} + 2\frac{dv(t)}{dt} + 10v(t) = 3\cos 2t$$

subject to $v(0) = 1, dv(0)/dt = -2$.

Chapter 15, Solution 50.

Take the Laplace transform of each term.

$$\left[s^2 V(s) - s\,v(0) - v'(0)\right] + 2\left[s\,V(s) - v(0)\right] + 10\,V(s) = \frac{3s}{s^2+4}$$

$$s^2 V(s) - s + 2 + 2s\,V(s) - 2 + 10\,V(s) = \frac{3s}{s^2+4}$$

$$(s^2 + 2s + 10)\,V(s) = s + \frac{3s}{s^2+4} = \frac{s^3+7s}{s^2+4}$$

$$V(s) = \frac{s^3+7s}{(s^2+4)(s^2+2s+10)} = \frac{As+B}{s^2+4} + \frac{Cs+D}{s^2+2s+10}$$

$$s^3 + 7s = A(s^3 + 2s^2 + 10s) + B(s^2 + 2s + 10) + C(s^3 + 4s) + D(s^2 + 4)$$

Equating coefficients :

s^3: $1 = A + C \longrightarrow C = 1 - A$

s^2: $0 = 2A + B + D$

s^1: $7 = 10A + 2B + 4C = 6A + 2B + 4$

s^0: $0 = 10B + 4D \longrightarrow D = -2.5B$

Solving these equations yields

$$A = \frac{9}{26}, \qquad B = \frac{12}{26}, \qquad C = \frac{17}{26}, \qquad D = \frac{-30}{26}$$

$$V(s) = \frac{1}{26}\left[\frac{9s+12}{s^2+4} + \frac{17s-30}{s^2+2s+10}\right]$$

$$V(s) = \frac{1}{26}\left[\frac{9s}{s^2+4} + 6\cdot\frac{2}{s^2+4} + 17\cdot\frac{s+1}{(s+1)^2+3^2} - \frac{47}{(s+1)^2+3^2}\right]$$

$$v(t) = \underline{\mathbf{\frac{9}{26}\cos(2t) + \frac{6}{26}\sin(2t) + \frac{17}{26}e^{-t}\cos(3t) - \frac{47}{78}e^{-t}\sin(3t)}}$$

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Chapter 15, Problem 51.

Given that $v(0) = 2$ and $dv(0)/dt = 4$, solve

$$\frac{d^2v}{dt^2} + 5\frac{dv}{dt} + 6v = 10e^{-t}u(t)$$

Chapter 15, Solution 51.

Taking the Laplace transform of the differential equation yields

$$\left[s^2V(s) - sv(0) - v'(0)\right] + 5\left[sV(s) - v(0)\right] + 6V(s) = \frac{10}{s+1}$$

or $\left(s^2 + 5s + 6\right)V(s) - 2s - 4 - 10 = \dfrac{10}{s+1} \quad \longrightarrow \quad V(s) = \dfrac{2s^2 + 16s + 24}{(s+1)(s+2)(s+3)}$

Let $V(s) = \dfrac{A}{s+1} + \dfrac{B}{s+2} + \dfrac{C}{s+3}, \quad A = 5, \quad B = 0, \quad C = -3$

Hence,

$$v(t) = \underline{(5e^{-t} - 3e^{-3t})u(t)}$$

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Chapter 15, Problem 52.

Use the Laplace transform to find $i(t)$ for $t > 0$ if

$$\frac{d^2 i}{dt^2} + 3\frac{di}{dt} + 2i + \delta(t) = 0,$$

$$i(0) = 0, \qquad i'(0) = 3$$

Chapter 15, Solution 52.

Take the Laplace transform of each term.

$$\left[s^2 I(s) - s\,i(0) - i'(0)\right] + 3\left[s\,I(s) - i(0)\right] + 2\,I(s) + 1 = 0$$

$$(s^2 + 3s + 2)\,I(s) - s - 3 - 3 + 1 = 0$$

$$I(s) = \frac{s+5}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$A = 4, \qquad B = -3$$

$$I(s) = \frac{4}{s+1} - \frac{3}{s+2}$$

$$i(t) = \underline{\mathbf{(4e^{-t} - 3e^{-2t})u(t)}}$$

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Chapter 15, Problem 53.

* Use Laplace transforms to solve for $x(t)$ in

$$x(t) = \cos t + \int_0^t e^{\lambda - t} x(\lambda)\, d\lambda$$

* An asterisk indicates a challenging problem.

Chapter 15, Solution 53.

Transform each term.

We begin by noting that the integral term can be rewritten as,

$\int_0^t x(\lambda) e^{-(t-\lambda)} d\lambda$ which is convolution and can be written as $e^{-t} * x(t)$.

Now, transforming each term produces,

$$X(s) = \frac{s}{s^2+1} + \frac{1}{s+1} X(s) \rightarrow \left(\frac{s+1-1}{s+1}\right) X(s) = \frac{s}{s^2+1}$$

$$X(s) = \frac{s+1}{s^2+1} = \frac{s}{s^2+1} + \frac{1}{s^2+1}$$

x(t) = **<u>cos(t) + sin(t)</u>**.

If partial fraction expansion is used we obtain,

x(t) = **<u>1.4141cos(t–45˚)</u>**.

This is the same answer and can be proven by using trigonometric identities.

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Chapter 15, Problem 54.

Using the Laplace transform, solve the following differential equation for

$$\frac{d^2 i}{dt^2} + 4\frac{di}{dt} + 5i = 2e^{-2t}$$

Subject to $i(0) = 0, i'(0) = 2$.

Chapter 15, Solution 54.

Taking the Laplace transform of each term gives

$$\left[s^2 I(s) - si(0) - i'(0)\right] + 4\left[sI(s) - i(0)\right] + 5I(s) = \frac{2}{s+2}$$

$$\left[s^2 I(s) - 0 - 2\right] + 4\left[sI(s) - 0\right] + 5I(s) = \frac{2}{s+2}$$

$$I(s)(s^2 + 4s + 5) = \frac{2}{s+2} + 2 = \frac{2s+6}{s+2}$$

$$I(s) = \frac{2s+6}{(s+2)(s^2+4s+5)} = \frac{A}{s+2} + \frac{Bs+C}{s^2+4s+5}$$

$$2s + 6 = A(s^2 + 4s + 5) + B(s^2 + 2s) + C(s+2)$$

We equate the coefficients.

s^2 : 0 = A+ B
s: 2= 4A + 2B + C
constant: 6 = 5A + 2C

Solving these gives
A = 2, B= -2, C = -2

$$I(s) = \frac{2}{s+2} - \frac{2s+2}{s^2+4s+5} = \frac{2}{s+2} - \frac{2(s+2)}{(s+2)^2+1} + \frac{2}{(s+2)^2+1}$$

Taking the inverse Laplace transform leads to:

$$i(t) = \left(2e^{-2t} - 2e^{-2t}\cos t + 2e^{-2t}\sin t\right)u(t) = \underline{2e^{-2t}(1 - \cos t + \sin t)u(t)}$$

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Chapter 15, Problem 55.

Solve for $y(t)$ in the following differential equation if the initial conditions are zero.

$$\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + 8\frac{dy}{dt}e^{-t}\cos 2t$$

Chapter 15, Solution 55.

Take the Laplace transform of each term.

$$\left[s^3\,Y(s) - s^2\,y(0) - s\,y'(0) - y''(0)\right] + 6\left[s^2\,Y(s) - s\,y(0) - y'(0)\right]$$
$$+ 8\left[s\,Y(s) - y(0)\right] = \frac{s+1}{(s+1)^2 + 2^2}$$

Setting the initial conditions to zero gives

$$(s^3 + 6s^2 + 8s)\,Y(s) = \frac{s+1}{s^2 + 2s + 5}$$

$$Y(s) = \frac{(s+1)}{s(s+2)(s+4)(s^2+2s+5)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+4} + \frac{Ds+E}{s^2+2s+5}$$

$$A = \frac{1}{40}, \qquad B = \frac{1}{20}, \qquad C = \frac{-3}{104}, \qquad D = \frac{-3}{65}, \qquad E = \frac{-7}{65}$$

$$Y(s) = \frac{1}{40}\cdot\frac{1}{s} + \frac{1}{20}\cdot\frac{1}{s+2} - \frac{3}{104}\cdot\frac{1}{s+4} - \frac{1}{65}\cdot\frac{3s+7}{(s+1)^2+2^2}$$

$$Y(s) = \frac{1}{40}\cdot\frac{1}{s} + \frac{1}{20}\cdot\frac{1}{s+2} - \frac{3}{104}\cdot\frac{1}{s+4} - \frac{1}{65}\cdot\frac{3(s+1)}{(s+1)^2+2^2} - \frac{1}{65}\cdot\frac{4}{(s+1)^2+2^2}$$

$$y(t) = \underline{\mathbf{\frac{1}{40}u(t) + \frac{1}{20}e^{-2t} - \frac{3}{104}e^{-4t} - \frac{3}{65}e^{-t}\cos(2t) - \frac{2}{65}e^{-t}\sin(2t)}}$$

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Chapter 15, Problem 56.

Solve for $v(t)$ in the integrodifferential equation

$$4\frac{dv}{dt}+12\int_{-\infty}^{t} v\,dt=0$$

Given that $v(0)=2$.

Chapter 15, Solution 56.

Taking the Laplace transform of each term we get:

$$4\left[\,s\,V(s)-v(0)\right]+\frac{12}{s}V(s)=0$$

$$\left[\,4s+\frac{12}{s}\right]V(s)=8$$

$$V(s)=\frac{8s}{4s^2+12}=\frac{2s}{s^2+3}$$

$$v(t)=\underline{\mathbf{2\cos\left(\sqrt{3}t\right)}}$$

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Chapter 15, Problem 57.

Solve the following integrodifferential equation using the Laplace transform method:

$$\frac{dy(t)}{dt} + 9\int_0^t y(\tau)d\tau = \cos 2t, \qquad y(0) = 1$$

Chapter 15, Solution 57.

Take the Laplace transform of each term.

$$\left[s\,Y(s) - y(0)\right] + \frac{9}{s}Y(s) = \frac{s}{s^2+4}$$

$$\left(\frac{s^2+9}{s}\right)Y(s) = 1 + \frac{s}{s^2+4} = \frac{s^2+s+4}{s^2+4}$$

$$Y(s) = \frac{s^3+s^2+4s}{(s^2+4)(s^2+9)} = \frac{As+B}{s^2+4} + \frac{Cs+D}{s^2+9}$$

$$s^3+s^2+4s = A(s^3+9s) + B(s^2+9) + C(s^3+4s) + D(s^2+4)$$

Equating coefficients :

s^0: $\quad 0 = 9B + 4D$

s^1: $\quad 4 = 9A + 4C$

s^2: $\quad 1 = B + D$

s^3: $\quad 1 = A + C$

Solving these equations gives

$$A = 0, \qquad B = -4/5, \qquad C = 1, \qquad D = 9/5$$

$$Y(s) = \frac{-4/5}{s^2+4} + \frac{s+9/5}{s^2+9} = \frac{-4/5}{s^2+4} + \frac{s}{s^2+9} + \frac{9/5}{s^2+9}$$

$$y(t) = \underline{\mathbf{-0.4\sin(2t) + \cos(3t) + 0.6\sin(3t)}}$$

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Chapter 15, Problem 58.

Given that

$$\frac{dv}{dt} + 2v + 5\int_0^t v(\lambda)d\lambda = 4u(t)$$

with $v(0) = -1$, determine $v(t)$ for $t > 0$.

Chapter 15, Solution 58.

We take the Laplace transform of each term.

$$[sV(s) - v(0)] + 2V(s) + \frac{5}{s}V(s) = \frac{4}{s}$$

$$[sV(s) + 1] + 2V(s) + \frac{5}{s}V(s) = \frac{4}{s} \quad \longrightarrow \quad V(s) = \frac{4-s}{s^2 + 2s + 5}$$

$$V(s) = \frac{-(s+1)+5}{(s+1)^2 + 2^2} = \frac{-(s+1)}{(s+1)^2 + 2^2} + 5/2\,\frac{2}{(s+1)^2 + 2^2}$$

$$v(t) = \underline{(-e^{-t}\cos 2t + 2.5e^{-t}\sin 2t)u(t)}$$

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Chapter 15, Problem 59.

Solve the integrodifferential equation

$$\frac{dy}{dt} + 4y + 3\int_0^t y\,dt = 6e^{-2t}, \qquad y(0) = -1$$

Chapter 15, Solution 59.

Take the Laplace transform of each term of the integrodifferential equation.

$$\left[s\,Y(s) - y(0)\right] + 4\,Y(s) + \frac{3}{s}Y(s) = \frac{6}{s+2}$$

$$(s^2 + 4s + 3)\,Y(s) = s\left(\frac{6}{s+2} - 1\right)$$

$$Y(s) = \frac{s(4-s)}{(s+2)(s^2+4s+3)} = \frac{(4-s)s}{(s+1)(s+2)(s+3)}$$

$$Y(s) = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$$

A = –2.5, B = 12, C = -10.5

$$Y(s) = \frac{-2.5}{s+1} + \frac{12}{s+2} - \frac{10.5}{s+3}$$

$$y(t) = \underline{\mathbf{-2.5e^{-t} + 12e^{-2t} - 10.5e^{-3t}}}$$

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Chapter 15, Problem 60.

Solve the following integrodifferential equation

$$2\frac{dx}{dt} + 5x + 3\int_0^t x\,dt + 4 = \sin 4t, \qquad x(0) = 1$$

Chapter 15, Solution 60.

Take the Laplace transform of each term of the integrodifferential equation.

$$2\left[s\,X(s) - x(0)\right] + 5\,X(s) + \frac{3}{s}X(s) + \frac{4}{s} = \frac{4}{s^2+16}$$

$$(2s^2 + 5s + 3)\,X(s) = 2s - 4 + \frac{4s}{s^2+16} = \frac{2s^3 - 4s^2 + 36s - 64}{s^2+16}$$

$$X(s) = \frac{2s^3 - 4s^2 + 36s - 64}{(2s^2+5s+3)(s^2+16)} = \frac{s^3 - 2s^2 + 18s - 32}{(s+1)(s+1.5)(s^2+16)}$$

$$X(s) = \frac{A}{s+1} + \frac{B}{s+1.5} + \frac{Cs+D}{s^2+16}$$

$$A = (s+1)\,X(s)\Big|_{s=-1} = -6.235$$

$$B = (s+1.5)\,X(s)\Big|_{s=-1.5} = 7.329$$

When $s = 0$,

$$\frac{-32}{(1.5)(16)} = A + \frac{B}{1.5} + \frac{D}{16} \quad \longrightarrow \quad D = 0.2579$$

$$s^3 - 2s^2 + 18s - 32 = A(s^3 + 1.5s^2 + 16s + 24) + B(s^3 + s^2 + 16s + 16)$$
$$+ C(s^3 + 2.5s^2 + 1.5s) + D(s^2 + 2.5s + 1.5)$$

Equating coefficients of the s^3 terms,

$$1 = A + B + C \quad \longrightarrow \quad C = -0.0935$$

$$X(s) = \frac{-6.235}{s+1} + \frac{7.329}{s+1.5} + \frac{-0.0935s + 0.2579}{s^2+16}$$

$$x(t) = \underline{\mathbf{-6.235\,e^{-t} + 7.329\,e^{-1.5t} - 0.0935\cos(4t) + 0.0645\sin(4t)}}$$

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