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Circuits1

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POWERUNIT

Fall2016

No. chapter 1

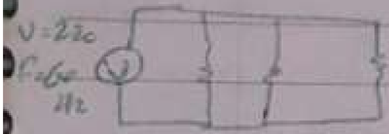
→ Electric circuit is a mathmatical model to approximate the behavior of an actual power system.

→ lumbed parameter system is the system where the effects are studied instantaneously and the signal propagates in it is larger than 10 times the dimension of the system

V 220 v
f 60 Hz

$$\lambda = \frac{c}{f} = \frac{3\times10^{8}\ m/s}{60\ Hz} = 5\times10^{6}\ m$$

The dimension of the system $\leq 5\times10^{5}$ m



Enternational system of units SI

length	m
mass	kg
time	s
current	A
thermo dynamic temp	k
luminimu	cd

frequency	f	Hz	s^{-1}
force	f	N	$\frac{m}{s^2}$
energy/work	w	J	N.m
electric charge	C	c	A.s
power	p	W	J
electric potential	V	v	J
electric resistance	R	Ω	V/A
electric conductance	G	S	$\frac{1}{\Omega}$
electric capasitance	C	f	$\frac{C}{V}$
magnatic flux	wb		V.s
Indectans	L	H	$\frac{wb}{A}$

⇒ Many of SI units has small or large values, then we use prefix to quantify them:-

atto	a	10^{-18}	→ as
femto	f	10^{-15}	→ fs
pico	p	10^{-12}	→ pF, pS, pm
nano	n	10^{-9}	→ ns, nm, nF
micro	μ	10^{-6}	→ μs, μm, μF, μH
milli	m	10^{-3}	→
centi	c	10^{-2}	→ cm
deci	d	10^{-1}	
deka	da	10	
hecto	h	10^{2}	
kilo	k	10^{3}	→ kg, kΩ
mega	M	10^{6}	
giga	G	10^{9}	
tera	T	10^{12}	

- ~~engineers~~

- engineers uses prefixes that has power divisable by 3
exventum

10^{9} ns ✓

10 000 000 ps ✗

0.01 ms ✗

900 ns ✓

1001 ns ✗

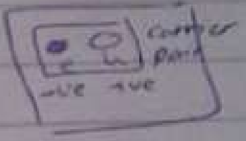
1.001 ms ✓

No

⇒ voltage and current $q = 1.602 \times 10^{-19}$ C

↓

electrical effects are attributed to the seperation of charges and motion of charges



h hole
e electron

⇒ voltage is the rate of change of energy per unit charge created by seperation

$$U = \frac{dw}{dq} \quad \frac{J}{C}$$

⇒ current is the rate of flow of charge per second

$$f = \cancel{dq} \frac{dq}{dt} \quad \frac{C}{s}$$

I is in A

$$v = \frac{dw}{dq}$$

$$i = \frac{dq}{dt}$$

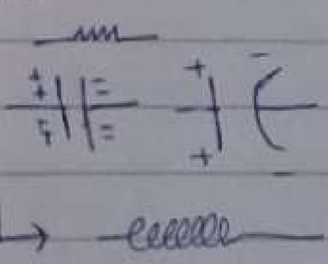
current elements

symbol circuit symbol

① Resistor R dissipate energy →
Capacitor C charges →
inductor L magnatic field →

passive elements

storage elements

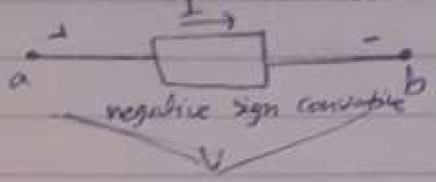


	symbol	circuit symbol
④ Sources voltage V	V	
⑤ source current	I	

V Dc supplied

$\tilde{V}$ Ac supplies

two terminal elements:

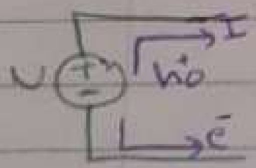


current flowing in the element is I voltage drop is voltage from terminal a to terminal b

→ Ideal Basic circuit Element

Ideal element is a non ~~[illegible]~~ realizable element (not practical) values don't change with time

Basic element can't be broken to other element

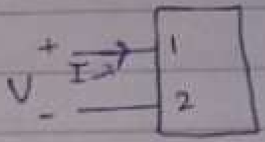


No.

power:-

$$p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt}$$

$$\boxed{\begin{array}{c} p = V \cdot I \\ W = V \cdot A \end{array}}$$



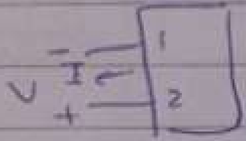
$p = VI$

$p = -VI$

$p > 0$ power is delivered to the circuit element

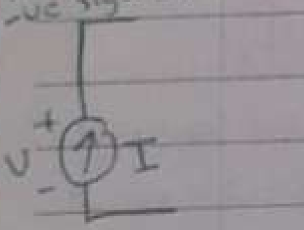
$p < 0$ power is extracted from the element

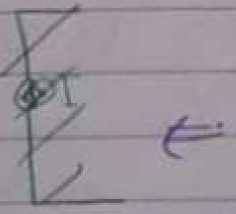
$p = -VI$



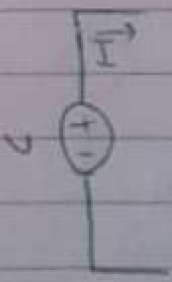
$p = VI$

not -ve sign convention





generating power to the circuit
"extracting"



$V, I > 0$
generating power to the circuit

$V, I > 0$
extracting power from the circuit

No.

Sources

- voltage
- Current
  - independent
    The voltage or current value doesn't depend on other current or voltage anywhere in the circuit
  - dependent
    the current or voltage value depend on the current or voltage in the circuit
    - current
      - voltage controlled
      - current controlled
    - voltage
      - current controlled
      - voltage controlled

$V_s = \mu \, \boxed{V_x}$

dependent voltage controlled voltage source

unit of μ is dimension less

$I_s = \alpha \, \boxed{V_x}$

dependent voltage controlled current source

$\alpha : \Omega$

$V_s = \rho \, \boxed{I_x}$

dependent current controlled voltage source

$\rho = \frac{V}{A} = S$

$I_s = \beta \, \boxed{I_x}$

dependent current-controlled current source

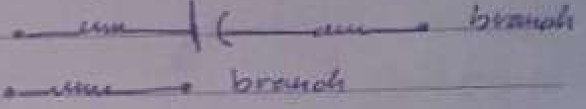
$y = f(x)$ ↑

β = dimension less

Scanned by CamScanner

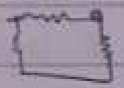
Circuit Analysis :-

- elements
- branch : two terminal device that may contain more than one element



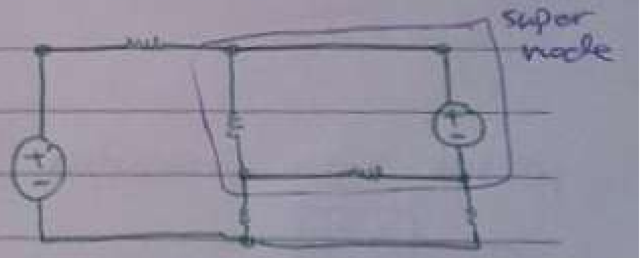
- Node : is a junction between two or more branches

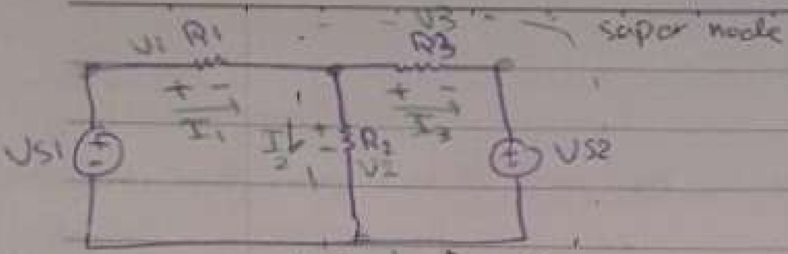
trivial between two branches



essential → three or more branches

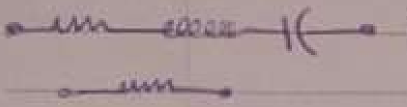
Super → Region that contains more than one node





- nodes < trivial / essential / super node

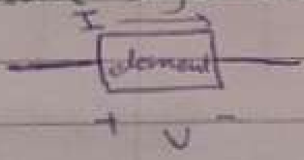
- branches


part of the circuit with two terminals

Loop :- closed path of branches
Mesh :- a loop with no loop inside

→ circuit Analysis : to find ~~the~~ branches current
find the nodal voltages

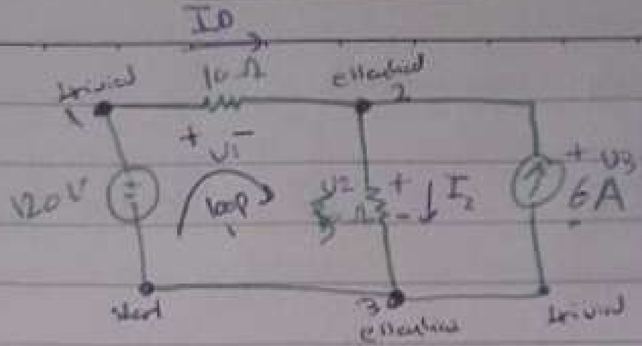
passive sign convention



known	unknown
Sources voltage	branches current
Sources current	nodals voltage

circuit laws
- ohm's law U = RI
- kirchoff's voltages law (KVL)
- kirchoff's current law (KCL)

No.



KVL

- works in loops
- $\sum_{j=1}^{k} \boxed{V_j} = 0$

is the voltage drop across an element

k # of elements in the loop

- find I_0
- check power generated and dissipiated for each element?

KCL

- works at nodes
- $\sum_{j=1}^{k} I_j = 0$

k = number of branches connected to the node

$\xrightarrow[I_1]{\text{in the node}} \bullet \xrightarrow[I_2]{\text{outer of the node}}$

$+ \qquad -$

KVL for loop #1

$-120 + V1 + V2 = 0$ CW
clock wise

$-V2 - V1 + 120 = 0$ CCW
counter clock wise

$\rightarrow -120 + 10 I_0 + 50 I_2 = 0 \; \ldots \; \boxed{1}$

KCL (using node 2)

$+I_0 - I_2 + 6 = 0 \; \ldots \; (2)$

$I_0 = -3A$ (current is in oppsite direction set on the diagram)

$I_2 = 3A$

dissipated power $\quad P = VI = IR.I = I^2R$

$-P_{R1} = I_0^2 R1$

$= (-3)^2 (10)$

40+450+360 $\overset{?}{=}$ 90 W ✓

$-P_{R2} = I_2^2 R2$

$= (3)^2 (50)$

$= 450$ W

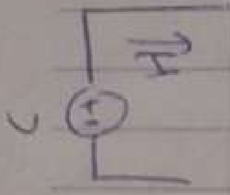
$P_{S1} = +VI$

$= 120 \times (-3)$

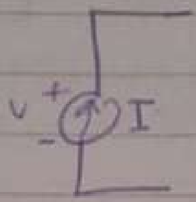
$= -360$ W

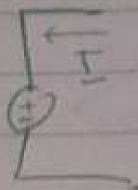
source VS1 is extracting power from the circuit

No.

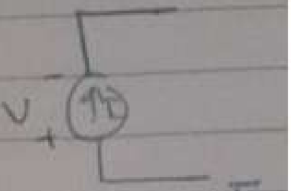


$p = vI$

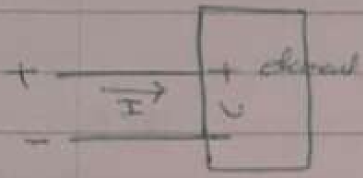




$p = -vI$



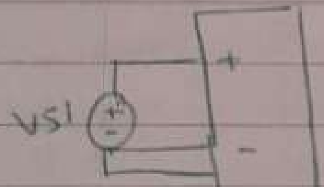
$p = -vI$



Passive sign convention

$p = +Iv$

$p < 0$



$p = +vI$

$= 120 \times -3 = -360$ W

$P_{s2} = +vI$

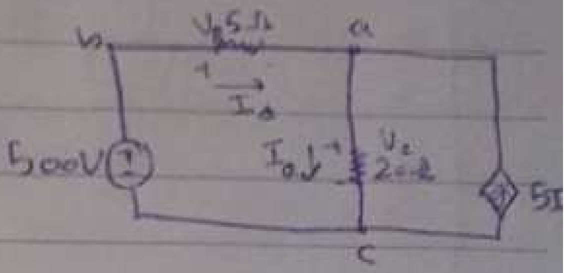
$= 130 \times 6$

$= 900$ W

$\frac{125 I_\Delta}{125} = \frac{500}{125}$ [illegible]

$I_\Delta = 4$ No. [illegible]

Analysis of a current containing Dependent sources



using kcl of node a:-

$$\sum_{j=1}^{3} I_j = 0$$

$$-I_o + 5I_\Delta + I_\Delta = 0$$

$$6I_\Delta = I_o \quad \ldots \boxed{1}$$

→ using kvL for loop 1:

$$\sum_{j=0}^{3} V_j = 0 \Rightarrow -500 + 5I_\Delta + 20I_o = 0$$

$$20I_o = 500 - 5I_\Delta \quad \ldots ②$$

$$I_o = 24\ A$$

$$V_o = 20I_o$$

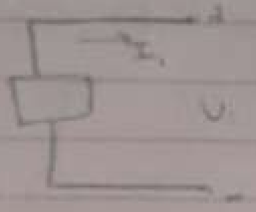
$$= 20 \times 24 = 480\ V$$

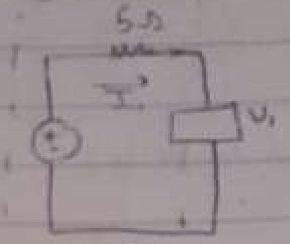
~~physiological relation~~	current	physiological relation
~~Barely feel it~~ ~~Barely feel~~ ↳	3-5 mA	Barely feel it
~~Extreem pain~~	35-50 mA	Extreem pain
~~muscles paralysis~~	50-70 mA	muscles paralysis
~~heart stop~~	50 V/mA	heart stop

No.

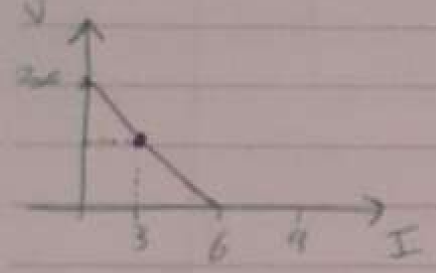
block box

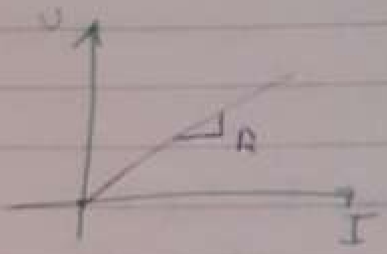
V_1	I_1
30	0
15	3
0	6





- ~~Constant~~ construct circuit model for the device shown.





$V = R^*I$

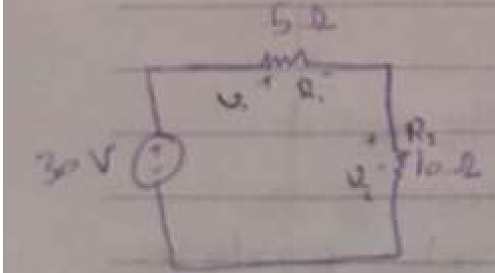
$V = IR + C$

$k = \frac{-I_0}{5} = \frac{-V}{A} \rightarrow 30 = \overset{0}{\cancel{-5}}I + C$

$C = 30$

$V_1 = -5I_1 + 30$

→ Using the circuit model predict the power that the device will deliver 10 Ω resistor?



KVL :-

$-30 + 5I_1 + 10I_1 = 0$

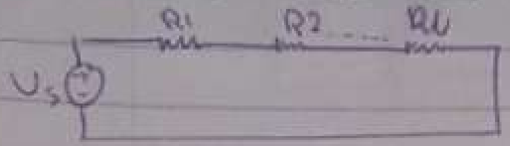
$-30 + 15I_1 = 0$

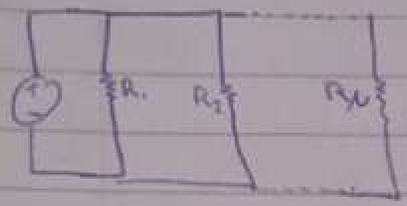
$I_1 = 2A$

$P_{R_1} = I^2R = 4*10 = 40 W$

* Simple Resister circuits :-

→ Resistors in devices



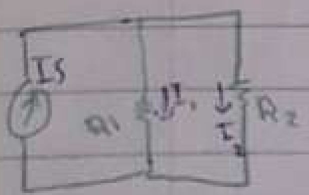


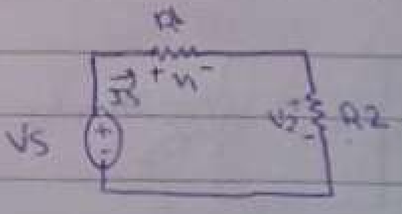
R
Resistor

G
Conductor

$$R = \frac{1}{G}$$

unit Ω ---- S





$$I_2 = \frac{R1}{R1+R2} IS$$

$$V_2 = Is\ R2$$

$$I_s = \frac{Vs}{Req}$$

$$V2 = Vs\ \frac{R2}{Req}$$

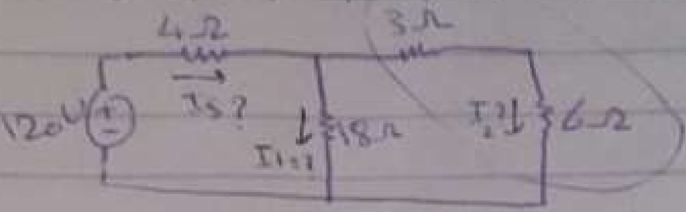
$$\boxed{V2 = Vs\ \frac{R2}{R1+R2}}$$

$$Is = I_1 = I_2$$

$$= \frac{V_1}{R_1} = \frac{V2}{R2}$$

Ex 3.1

Nelson



$R_{eq}? \rightarrow I_s = \frac{V_s}{R_{eq}}$

$3 + 6 = 9\,\Omega$

$9 \parallel 18 \rightarrow \frac{9 \times 18}{9+18} = 6\,\Omega$

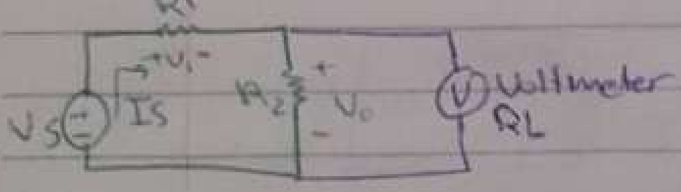
$4 + 6 = 10\,\Omega \rightarrow R_{eq}$

$I_s = \frac{V_s}{R_{eq}} = \frac{120}{10} = 12A$

$I_1 = \frac{9}{9+18} \times 12 = 4A$

$I_2 = \frac{18}{18+9} \times 12 = 8A$

Loading Effect



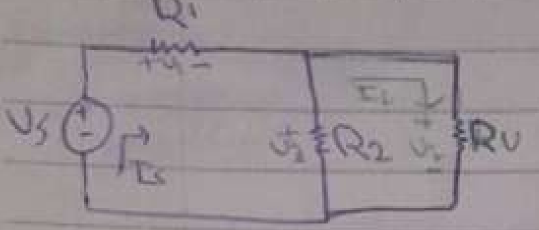
Theoritically : $V_o = \frac{R2}{R_1 + R_2} V_s$

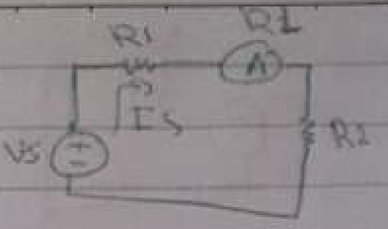
$I_2 = I_s$

Practically :-

$I_2 \neq I_s$

$I_2 = I_s - I_L$





Theoritically

$I_S = \frac{V_S}{R_1 + R_2}$

↓

RL as high as possible → ∞

practically

$I_s = \frac{V_s}{R_1 + R_2 + R_3}$

RL as small as possible → 0Ω

$V_o = I_2 R_2$

$V_o = I_L R_L$

$V_o = I_s R_{eq}$ ⊛

$$R_{eq} = \frac{R_2 R_L}{R_2 + R_L}$$

$$I_S = \frac{V_s}{R_1 + R_{eq}} = \frac{V_s}{R_1 + \frac{R_2 R_L}{R_2 + R_L}} = \frac{V_s}{\frac{R_1 R_2 + R_1 R_L}{R_2 + R_L} + R_2 R_L}$$

$$V_o = \overset{R_{eq}}{\left(\frac{R_2 R_L}{R_2 + R_L}\right)} \overset{⊛\ I_s}{\left(\frac{V_s}{\frac{R_1 R_2 + R_1 R_L + R_2 R_L}{R_2 + R_L}}\right)} = \frac{R_2 V_s}{\frac{1}{R_L}\left[R_1 R_2 + R_1 R_L + R_2 R_L\right]}$$

$$V_o = \frac{R_2 V_s}{R_1\left[1 + \frac{R_2}{R_L}\right] + R_2}$$

$$V_o = \frac{R_2}{R_1 + R_2} V_s$$

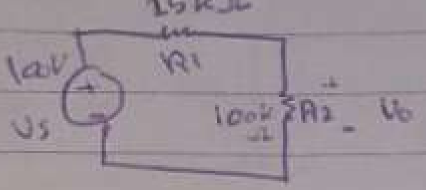
$$\frac{R_2}{R_L} \longrightarrow 0$$

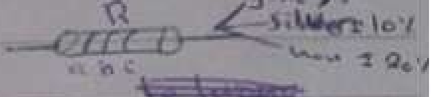
$R_L \longrightarrow \infty$ * it's an open circuit

1st oct
12th oct

No.

Ex 3.2
Q 84





~~tolerance~~
tolerance

Vo min
Vo max
when
R1 and R2 has tolearance values of ± 10% ?

black	0	10^0
brown	1	10^1
Red	2	10^2
orange	3	10^3
yellow	4	10^4
Green	5	10^5
blue	6	10^6
Violet	7	10^7
Gray	8	10^8
white	9	10^9

$V_o = \frac{R_2}{R_1+R_2} V_S$

$\pm \frac{10}{100} \times 25000 = \pm 2500\ \Omega$

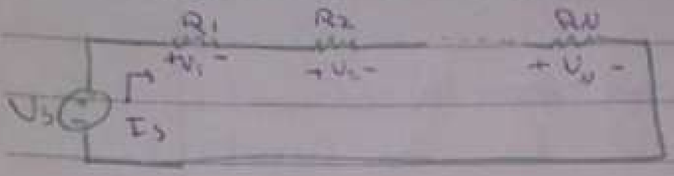
$25000 - 2500 \leq R^1 \leq 25000 + 2500$

$\pm \frac{10}{100} \times 100\ 000 = 10\ 000\ \Omega$

$100\ 000 - 10\ 000 \leq R_2 \leq 100000 + 10\ 000$

R1	R2	Vo	
R_1min	R_2min ✓	80V	
R_1 min	R_2 max	83.02V	Vo max
R_1 max	R_2 min	76.60 V	Vo min
R_1 max	R_2 max ✓	80V	

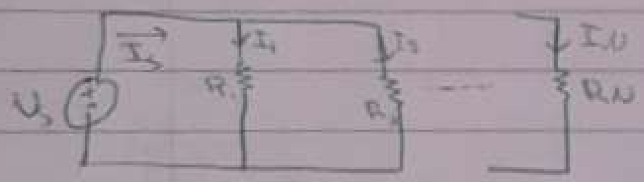
Generalization of current divider and voltage divider



$V_j = I_s . R_j$ $\quad j = 1, 2, \ldots, N$

$I_s = \frac{V_s}{R_{eq}}$

$\boxed{V_j} = V_s \frac{R_j}{R_{eq}}$



$I_j = \frac{V_s}{R_j}$

$V_s = I_s . R_{eq}$

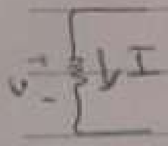
$\boxed{I_j} = I_s \frac{R_{eq}}{R_j}$

passive sign Convention

$p<0$ power is extracted from the element
power is supplied to the circuit

$p>0$ power is delivered to to the element

practical

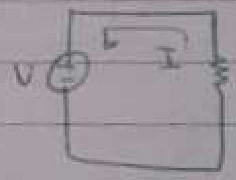


$p=IV$

$p=VI$

$p>0$

- power is delivered to the circuit
- supplied to the surface



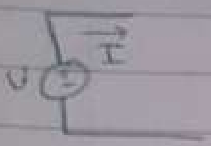
$p=-VI$

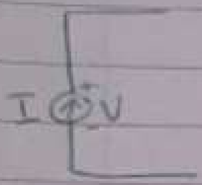
$p<0$

power is delivered to the source

No

idependent Source





$I_j = I_s \frac{R_{eq}}{R_j}$

$V_j = V_s \frac{R_j}{R_{eq}}$

dependent Source

V voltage depends

current Controlled / voltag Controled

$V = \_ I$ $\quad V = ..V$

current dependant

current / voltage

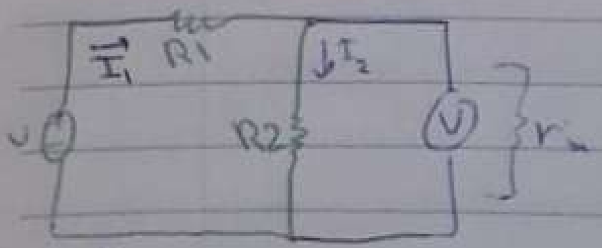
$I = ..V$ $\quad I = ..V$

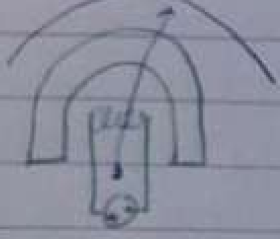
Voltmeter

- measur voltage
- parallel to the element
- very high resistor

Ameter

- measur current
- Connected ~~to~~ in Seires
- very low resistor



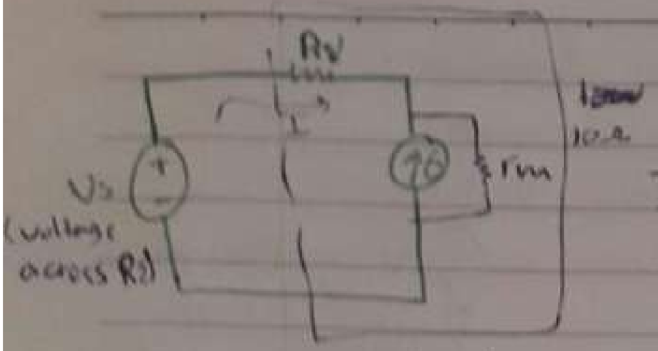


$V = I_2 R_2$

$I_2 = ??$

$I_2 = I_s \frac{r_{in}}{r_{in} + R_2}$

rin is the internl resistor of the meter



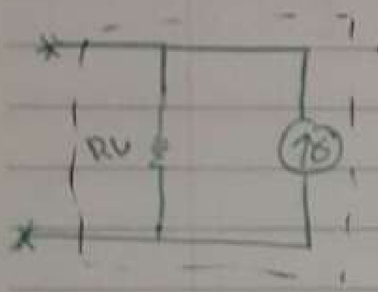
$$I = \frac{V_s}{r_m + R_V}$$

voltmeter

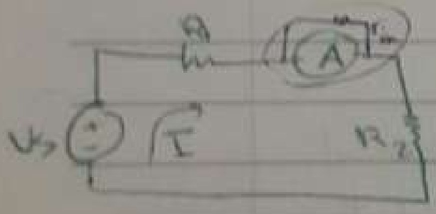
Rw very high

and connected in

Seires



Ameter



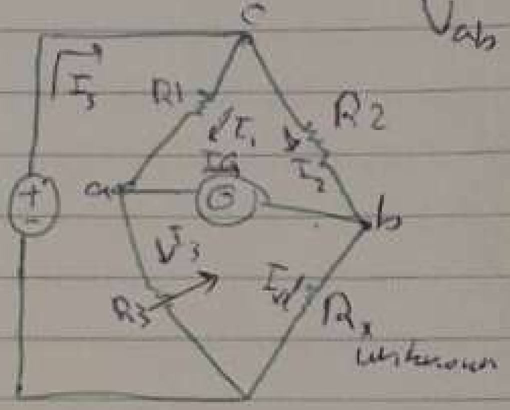
$$I = \frac{V_s}{R_1 + r_{in} + R_2}$$

* Measuring Resistances using wheat stone Bridge

$V_{ab} = V_a - V_b$

→ Adjust R3 until G reads 0 A.

$V_{ab} = 0\ V$



$I_1 = I_3 \quad , I_2 = I_x$

$V_a = V_b$

Then $V_{ad} = V_{bd} \Rightarrow I_3 R_3 = I_1 R_1$ G: Gelvano meter

$V_{ca} = V_{cb} \Rightarrow$ ~~$I_3 R_3 = I_x R_x$~~ $I_1 R_1 = I_2 R_2 \rightarrow I_2 = I_1 \frac{R1}{R2}$

$I_3 R_3 = I_2 R_x$

$\cancel{I_3} R_3 = \cancel{I_3} \frac{R_1}{R_2} R_x \Longrightarrow R_x = \frac{R_2}{R_1} R_3$

$\frac{R_2}{R_2} = 1 \Rightarrow$ This is not right (practical)

R3 range 0 - 10[illegible] Ω

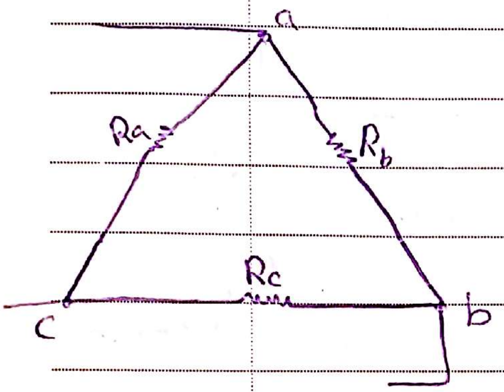
[illegible] measure $R_x = 1000\ \Omega$ (No We can't)

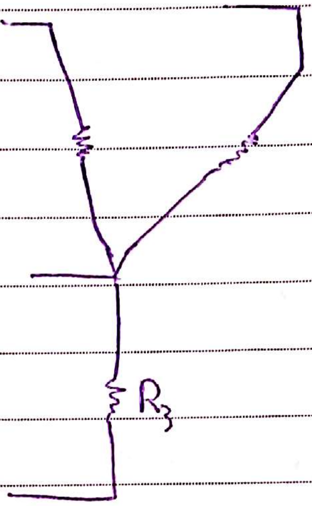
typical { value $\frac{R2}{R1} \approx 0.01$ to 1000

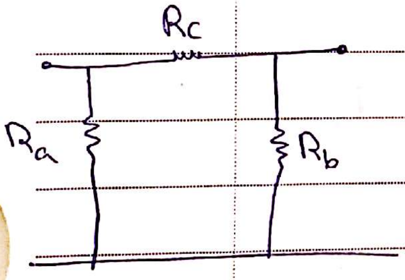
R3 11 k Ω

range of R_x that can be meased up to $\boxed{1M\ Hz}$

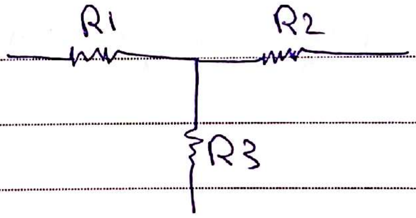
(Delta - wye) equivilant circuits
· π + T



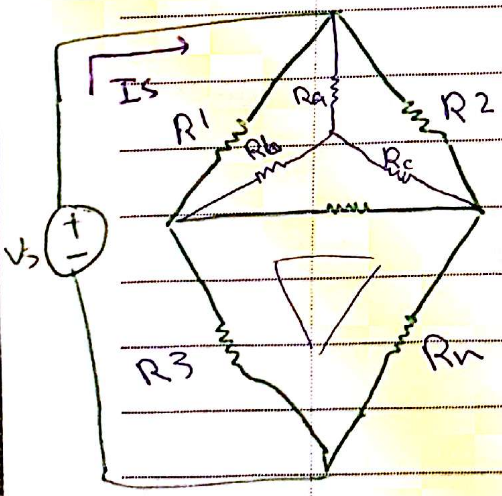




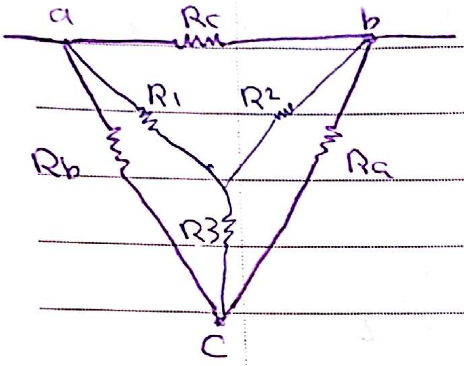
π

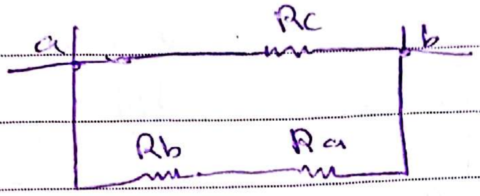


* * *



No.





in Δ-conf.

in Y-conf

Terminals a and b & equivalent Resistance in Δ Confgration should be equal to equivalent Resistence in Y-Confgration

$$\frac{R_c(R_b+R_a)}{R_c+R_b+R_a} = R_1+R_2 \quad \Longleftarrow \quad R_c \,//\, R_b+R_a = R_1+R_2$$

$$R_b \,//\, R_c+R_a = R_1+R_3$$

$$R_a \,//\, R_b+R_c = R_3+R_2$$

$$\frac{R_b(R_c+R_a)}{R_b+R_c+R_a} = R_1+R_3$$

$\Delta \rightarrow Y$

$$R_1 = \frac{R_b R_c}{R_a+R_b+R_c}$$

$$R_2 = \frac{R_c R_a}{R_a+R_b+R_c}$$

$$R_3 = \frac{R_b R_a}{R_a+R_b+R_c}$$

$Y \rightarrow \Delta$

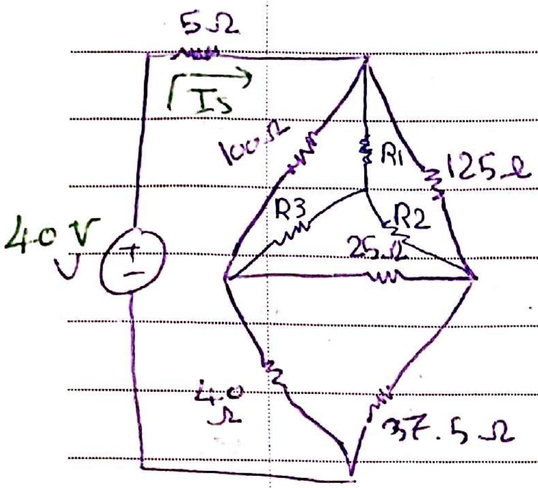
$$R_a = \frac{R_1R_2+R_2R_3+R_1R_3}{R_1}$$

$$R_b = \frac{R_1R_2+R_2R_3+R_1R_3}{R_2}$$

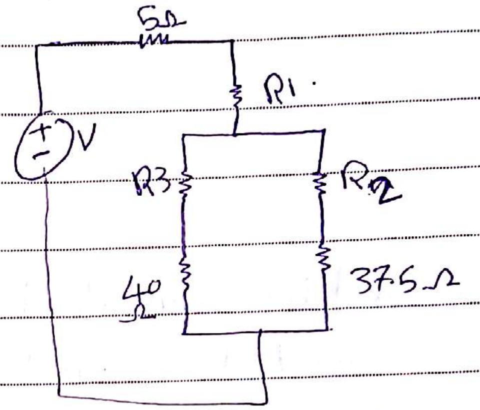
$$R_c = \frac{R_1R_2+R_2R_3+R_1R_3}{R_3}$$

Scanned by CamScanner

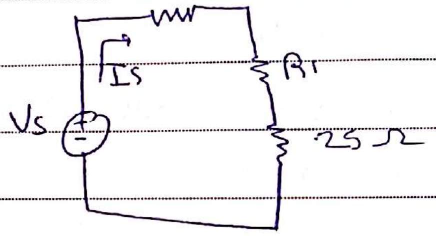
Find current and power supplied by the source.



⇒



⇓



$$R_1 = \frac{125 \times 100}{100 + 25 + 125} = 50\,\Omega$$

$$R2 = \frac{125 \times 25}{100+25+125} = 12.5\,\Omega$$

$$R3 = \frac{100 \times 25}{100+25+125} = 10\,\Omega$$

$50\,\Omega \,//\, 50\,\Omega = 25\,\Omega$

$R_{eq\ total} = 5 + 50 + 25 = \cancel{4}80\,\Omega$

$$I_s = \frac{V_s}{R_{eq\ total}} = \frac{40}{80} = 0.5\text{ A}$$

-ve sign convention

$P = -VI$

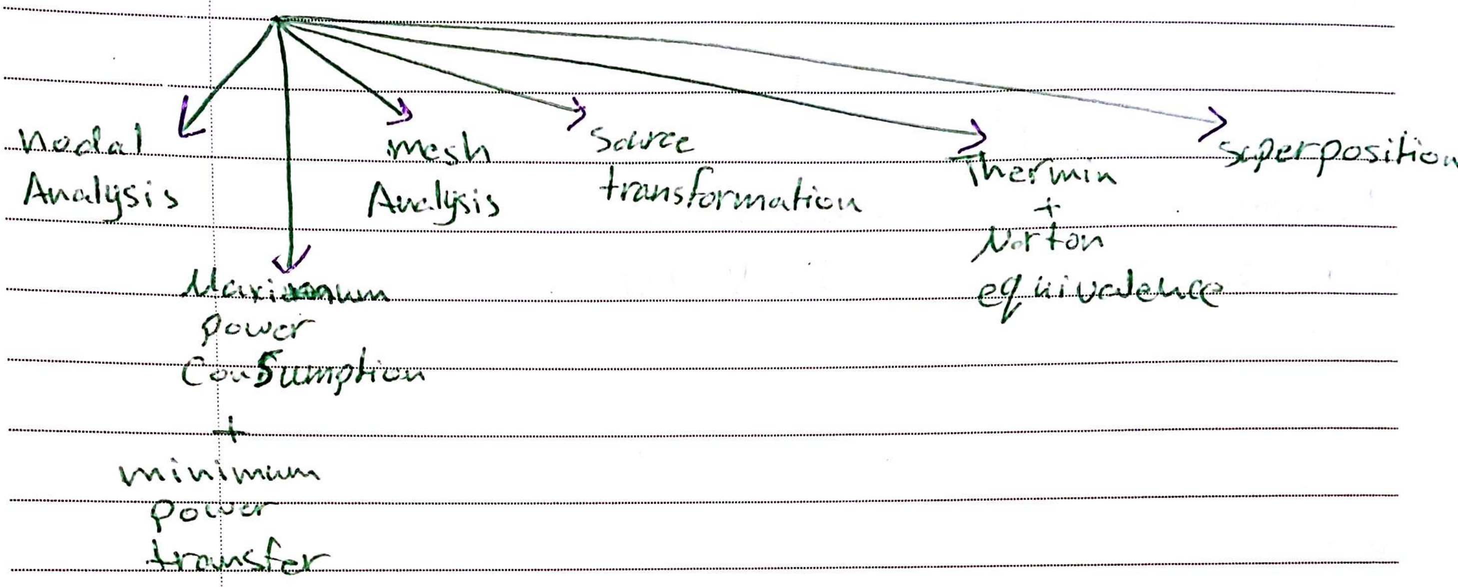
$= -40 \times 0.5$

$= -20$ W power is extracted from the element

No.

Circuit Analysis

finding nodes' voltage and branches' current.



Nodal Analysis :-

① Define the nodal voltages as independent variables.

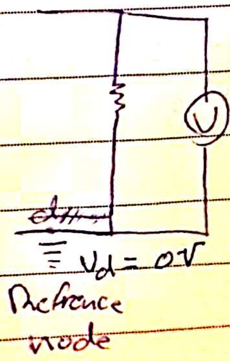
② Determine the number of equations to write.

~~③~~ * n# of nodes

* m# of independent or dependent voltage sources

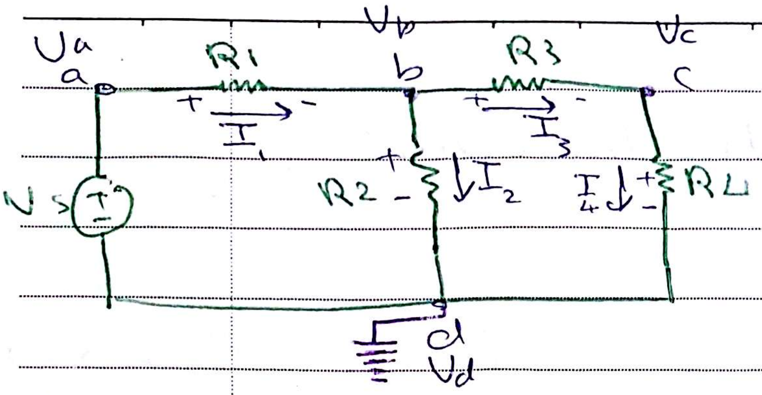
* Define the refernce node as the node with maximum number of elements connected to it.

\# of nodal equations = n − m − 1



③ use KCL at each node $\sum_{j=1}^{n} I_j = 0$

④ current in each branch is specified using node's voltage



knowns	unknown
R1, R2, R3, R4	~~Va~~, Vb, Vc, ~~Vd~~ 0V
Vs	Vs

n = 4

m = 1

~~#~~ eq → 4 - 1 - 1 = 2

mode b :

$I_1 - I_2 - I_3 = 0$

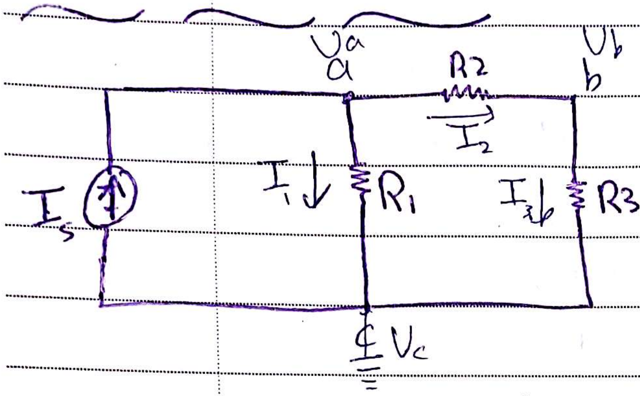
$$\frac{\overset{V_s}{\cancel{V_a}} - V_b}{R_1} - \frac{V_b - \overset{0}{\cancel{V_d}}}{R_2} - \frac{V_b - V_c}{R_3} = 0$$

$I_3 - I_4 = 0$

$$\frac{V_b - V_c}{R_3} - \frac{V_c - \overset{0}{\cancel{V_d}}}{R_4} = 0$$

n_e : # of essintial nodes

# of equation = $n_e - 1$



Sol:-

	Knowns	unknowns
$V_c = 0V$	R1, R2, R3	$V_a, V_b, \underset{0}{\underset{\downarrow}{V_c}}$
$V_a = ?$		
$V_b = ?$		

node a :

$$+I_s - I_1 - I_2 = 0$$

$$I_s - \frac{V_a - V_c}{R_1} - \frac{V_a - V_b}{R_2} = 0$$

$$I_s = \frac{V_a - \overset{0}{\cancel{V_c}}}{R_1} + \frac{V_a - V_b}{R_2}$$

$$I_s = V_a\left[\frac{1}{R_1} + \frac{1}{R_2}\right] - V_b\left[\frac{1}{R2}\right] \text{ ---- } ①$$

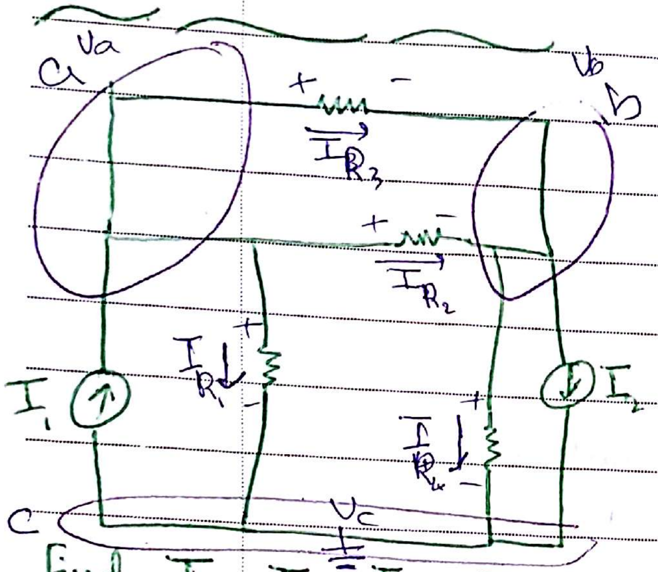
node b :

$$I_2 - I_3 = 0$$

$$\frac{V_a - V_b}{R_2} - \frac{V_b - \overset{0}{\cancel{V_c}}}{R_3} = 0$$

No.

$$V_a\left[\frac{1}{R_2}\right] - V_b\left[\frac{1}{R_2}+\frac{1}{R_3}\right] = 0 \;\ldots\ldots ②$$



$I_1 = 10\ \mu A$

$I_2 = 50\ mA$

R1 = 1 kΩ

R2 = 2 kΩ

R3 = 10 kΩ

R4 = 2 kΩ

find $I_{R_1}, I_{R_2}, I_{R_3}, I_{R_4}$, using nodal voltage method ?

$n = 3$ $\quad n_e = 3$ $\quad$ unknowns $\quad$ ~~Knowns~~

$m = 0$ $\quad 3 - 1 = 2$ $\quad V_a, V_b, V_c$

$3 - 0 - 1 = 2$

node a :-

$$I_1 - I_{R_1} - I_{R_2} - I_{R_3} = 0$$

$$I_1 - \frac{V_a - \cancel{V_c}^{\,0}}{R_1} - \frac{V_a - V_b}{R_2} - \frac{V_a - V_b}{R_3} = 0$$

$$I_1 = V_a\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right] - V_b\left[\frac{1}{R_2}+\frac{1}{R_3}\right] \ldots\ldots ①$$

node b :-

$$I_{R_3} + I_{R_2} - I_2 + I_{R_4} = 0$$

$$\frac{V_a - V_b}{R_3} + \frac{V_a - V_b}{R_2} - I_2 - \frac{V_b - \cancel{V_c}}{R_4} = 0$$

$$I_2 = V_a\left[\frac{1}{R_3} + \frac{1}{R_2}\right] - V_b\left[\frac{1}{R_3} + \frac{1}{R_2} + \frac{1}{R_4}\right] \quad \ldots ②$$

$I_{R_1} = -13.57$ mA current I_{R_1} has to be reversed ~~assum~~. as assumed

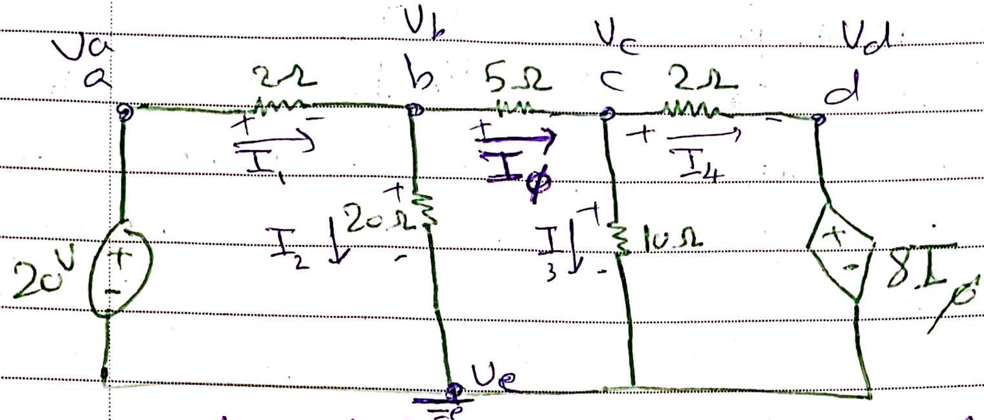
$I_{R_2} = 19.65$ mA

$I_{R_4} = -26.43$ mA

Cramer's Rule :-

$$\begin{aligned} a_1 x_1 + b_1 x_2 &= u \\ a_2 x_1 + b_2 x_2 &= v \end{aligned} \implies \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - b_1 a_2$$

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} u \\ v \end{bmatrix} \qquad x_2 = \frac{\begin{vmatrix} a_1 & u \\ a_2 & v \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$



Find power dissipated in 5Ω resistor using nodal voltage method

$n=5$

$m=2$

$5-2-1=2$

$n_e=3$

$n_e-1=2$

unknowns

$\underset{20V}{\underset{=}{V_a}}, b_b, V_c, \underset{8I_\phi}{\underset{=}{V_d}}, \underset{0V}{\underset{=}{V_e}}$

$$I_\phi = \frac{V_b - V_e}{5}$$

node b :

$$I_1 - I_2 - I_\phi = 0$$

$$\frac{\overset{20}{\cancel{V_a}} - V_b}{2} - \frac{V_b - \overset{0}{\cancel{V_e}}}{20} - \frac{V_b - V_c}{5} = 0$$

$$10 = V_b\left[\frac{1}{2} + \frac{1}{5} + \frac{1}{20}\right] - V_c\left[\frac{1}{5}\right] \ldots ①$$

node c :

$$I_\phi - I_3 - I_4 = 0$$

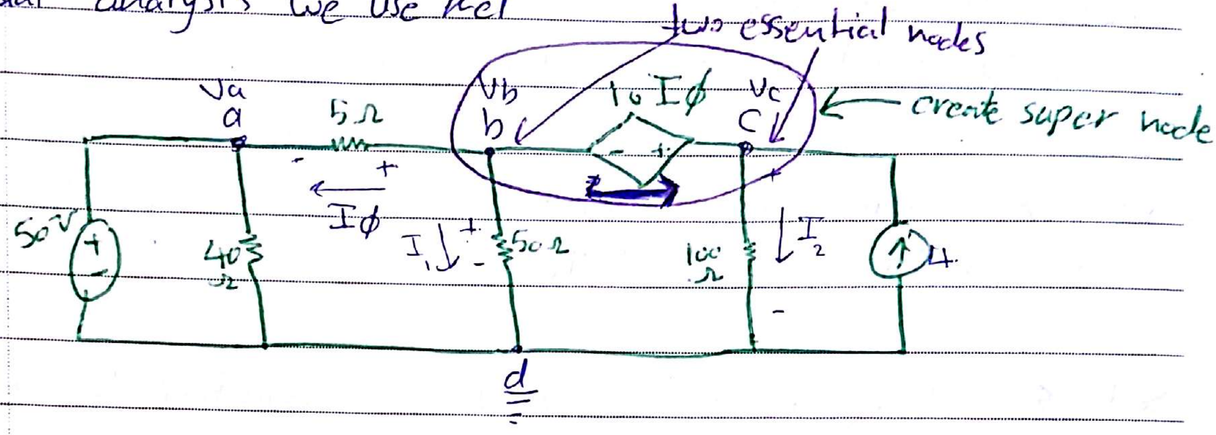
$$\frac{V_b - V_c}{5} - \frac{V_c - \overset{0}{\cancel{V_e}}}{10} - \frac{\overset{0}{\cancel{V_c}} - V_d}{2} = 0$$

$$V_d = 8I_\phi = 8\left[\frac{V_b - V_c}{5}\right] = \frac{8}{5}V_b - \frac{8}{5}V_c$$

$$\boxed{\begin{array}{l} V_b = 16\,V \\ V_c = 10\,V \\ P = 7.2\,W \end{array}}$$

* Special cases for nodal voltage method

* when there is a voltage source between two essential nodes
* In nodal analysis we use kcl



two essential nodes

create super node

$n = 4$ $n_e = 4$

$m = 2$ $n_e - 1 = 3$?

$4 - 2 - 1 = 1$?

kcl node b :

$-I_\phi - I_1 - I = 0$

$-\frac{V_b - V_a}{5} - \frac{V_b - \cancel{V_d}^{0}}{50} - I = 0$

$V_c - V_b = 10 I_\phi$

$I_\phi = \frac{V_b - \cancel{V_a}^{50}}{5}$

kcl node c :

$+I - I_2 + 4 = 0$

$I = I_2 - 4$

$I = \frac{V_c}{100} - 4$

⇒ using super node

kcl

$-I_\phi - I_1 - I_2 + 4 = 0$

$V_c - V_b = 10 I_\phi$

$V_c - V_b = 10 \frac{V_b - \cancel{V_a}^{50}}{5}$ ... ②

$-\frac{V_b - \cancel{V_a}^{50}}{5} - \frac{V_b - \cancel{V_d}}{50} - \frac{V_c - \cancel{V_d}^{0}}{100} + 4 = 0$ ... ①

$V_b = 60\ V$
$V_c = 80\ V$
$I_\phi = 2\ A$
$I = -3.2\ A$

No.

1.7.2 Electricity Bill

V

- Power Consumption W
- Electricity utility charge based on $\underline{\text{energy}}$
  $= P \cdot \text{time}$

→ power consumed in certain time amount (energy)
$200\,W \rightarrow 3hrs \xrightarrow{\text{Consumption}} 600\,Wh$

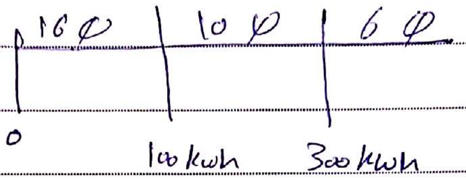
→ Base charge is a fee for being connected to the grid.
→ The peak hour (usually more money, they charge you).
→ The amount of consumed energy : they have ranges and your price goes up.

They charge you based on

[illegible]

Ex 1.9
Q 19

Consumes 700kwh / in a month
Base rate ~~12~~ $ 12.00
1st 100 kwh 16 ¢/kwh
next 200 kwh 10 ¢/kwh
over 300 kwh 6 ¢/kwh



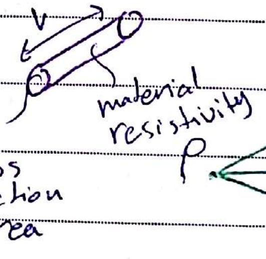
100 x 0.16 = $ 16
200 x 0.1 = $ 20
400 x 0.06 = $ 24

Base $ 12
$ 72

2.2 ohm's law

ρ material resist.

- Conductors
  - Cu, Au, Ag, AL
- insulaters
  - mica, teflon {for capacitors}
- semi Conductors
  - Gi Si C



ρ
- semi Conductors
- inculaters mica, teflon capacitors
- conductors ~~silver, gold~~ Cu, Au, Ag, AL

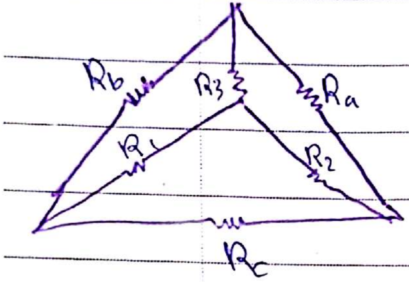
$$R = \frac{\rho \cdot L}{A}$$

by applying dective field it becomes a Conductor.

No.

2.7 Wye - Delta Transformation

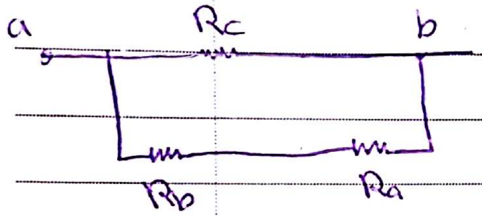
Network balanced →

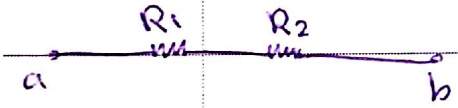


$R_1 = R_2 = R_3 = R_Y$

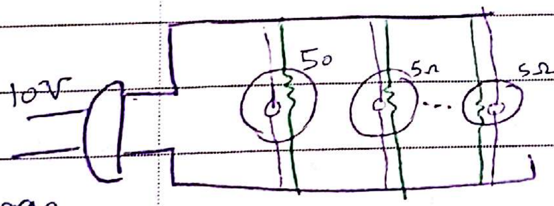
$R_a = R_b = R_c = R_\Delta$

$R_Y = \frac{R_\Delta}{3} \Rightarrow R_\Delta = 3R_Y$





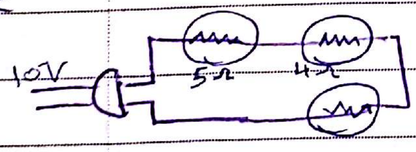
2.8.1 lighting system.



lamp could be defective

open Short

Voltage supply
DC
AC

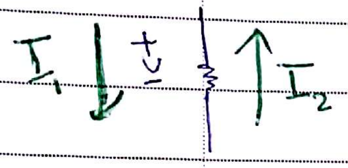


Mesh Analysis

① The independent Variables are the Mesh currents

② Use KVL $\sum_{j=1}^{n} V_j = 0$

③ Assign voltage for each branch then state the current flowing through it.

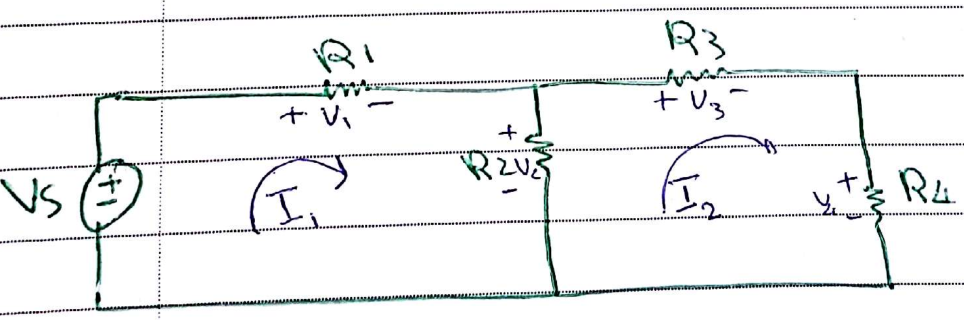


$R(I_1 - I_2) = V$

④ # of equations = # of meshes (no current sources in the mesh)

m

= n - m (for circuit with current sources)



KVL mesh #1 :

$-V_s + V_1 + V_2 = 0$

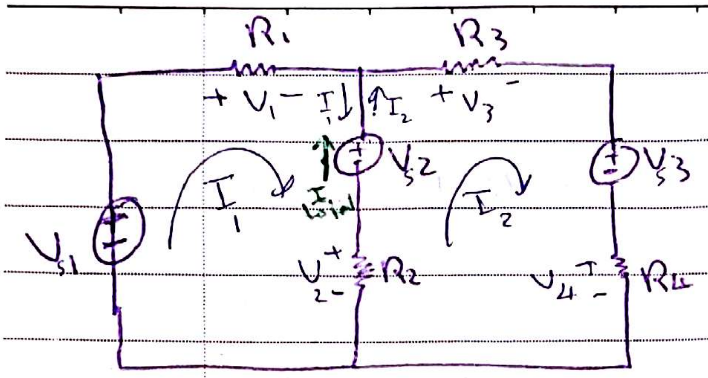
$-V_s + I_1R_1 + R_2(I_1 - I_2) = 0$

KVL mesh #2 :

$-V_2 + V_3 + V_4 = 0$

$-R_2(I_1 - I_2) + I_2R_3 + I_2R_4 = 0$

No.



$V_1 = 10V$

$V_2 = 9V$

$V_3 = 1V$

$R_1 = 5\Omega$

$R_2 = 10\Omega$

$R_3 = 5\Omega$

$R_4 = 5\Omega$

$V_2 = R_2(I_1 - I_2)$

kVL mesh ~~#~~1:

$-V_{s1} + V_1 + V_{s2} + V_2 = 0$

$-V_{s1} + R_1 I_1 + V_{s2} + R_2(I_1 - I_2) = 0$ ... ①

kVL mesh ~~#~~2:

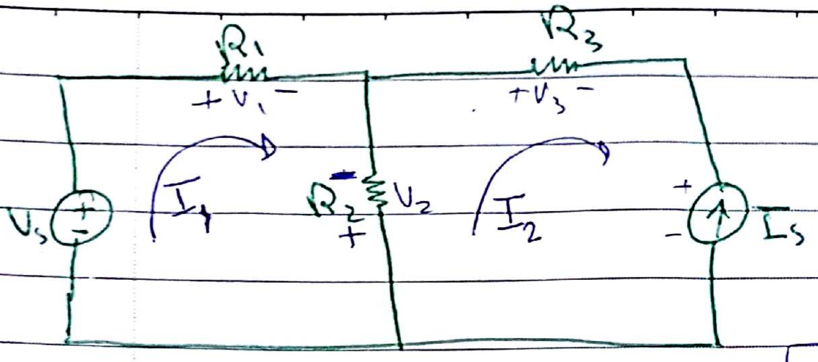
$-V_2 \underset{-V_{s2}}{\downarrow} + V_3 \underset{+V_{s3}}{\downarrow} + V_4 = 0$

$-R_2(I_1 - I_2) \underset{-V_{s2}}{\downarrow} + I_2 R_3 \underset{+V_{s3}}{\downarrow} + I_2 R_4 = 0$

$I_1 = \frac{1}{2}$ A

$I_2 = 0.65$ A

No.



$n = 2$

$m = 1$

$\#eq = 2 - 1 = 1$

$$\boxed{I_2 = -I_s}$$

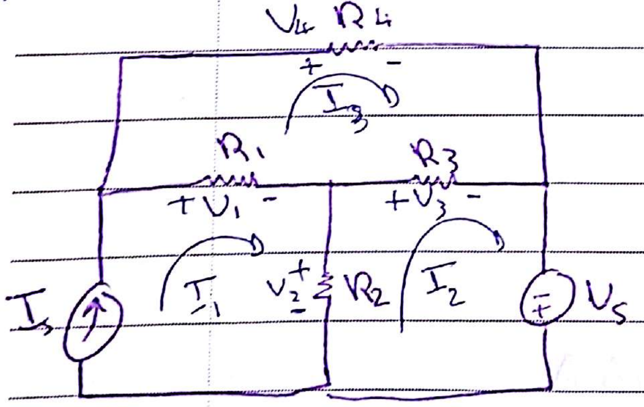
$$V_2 = R_2(I_2 - I_1)$$

mesh #1

KVL

$$-V_s + V_1 - V_2 = 0$$

$$-V_s + I_1R_1 - R_2(\underset{\substack{\downarrow \\ -I_s}}{I_2} - I_1) = 0$$



$R_1 = 3\Omega$

$R_2 = 8\Omega$

$R_3 = 6\Omega$

$R_4 = 4\Omega$

$V_s = 6V$

$I_s = 0.5A$

$$\boxed{I_3 = I_1}$$

$I_2 = ?$

$I_3 = ?$

No.

$n=3$

$m=1$

$3-1=2$

$V_1=R_1(I_1-I_3)$

$V_2=R_2(I_1-I_2)$

$V_3=R_3(I_2-I_3)$

KVL mesh #2

$-V_2+V_3-V_s=0$

$-R_2(I_1-I_2)+R_3(I_2-I_3)-V_s=0$ ... ①

KVL mesh #3

$+V_4-V_3-V_1=0$

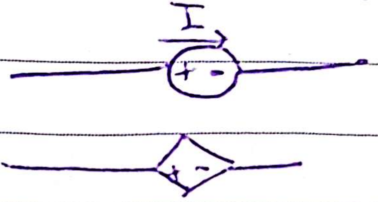
$R_4I_3-R_3(I_2-I_3)-R_1(I_1-I_3)$

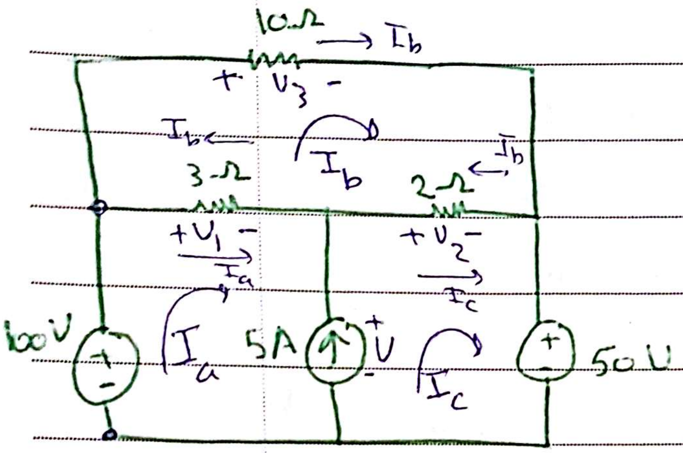
$I_2=0.95\ \text{A}$

$I_3=0.55\ \text{A}$

V_s is supplying Power

The Concept of super~~node~~ Mesh.





Super mesh:-

$-100 + V_1 + V_2 + 50 = 0$

$-100 + 3(I_a - I_b) + 2(I_c - I_b) + 50 = 0$

$V_1 = 3(I_a - I_b)$

$V_2 = 2(I_c - I_b)$

$V_3 = 10 I_b$

$5 = I_c - I_a$

mesh #3 :-

$+V_3 - V_2 - V_1 = 0$

$10 I_b - 2(I_c - I_b) - 3(I_a - I_b) = 0$

$I_a = 1.75\ A$

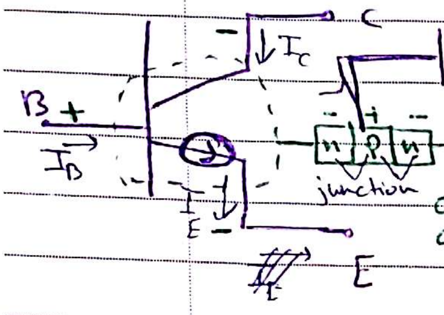
$I_b = 1.25\ A$

$I_c = 6.75\ A$

No. Applications

Transistor

*



$V_{BE} > 0.7$ V

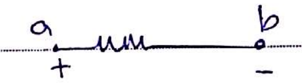
off ✱ both junctions off

active BC junction off

Sat. BE junction off

– kcl $I_B + I_C - I_E = 0$

$$I_E = I_B + I_C$$



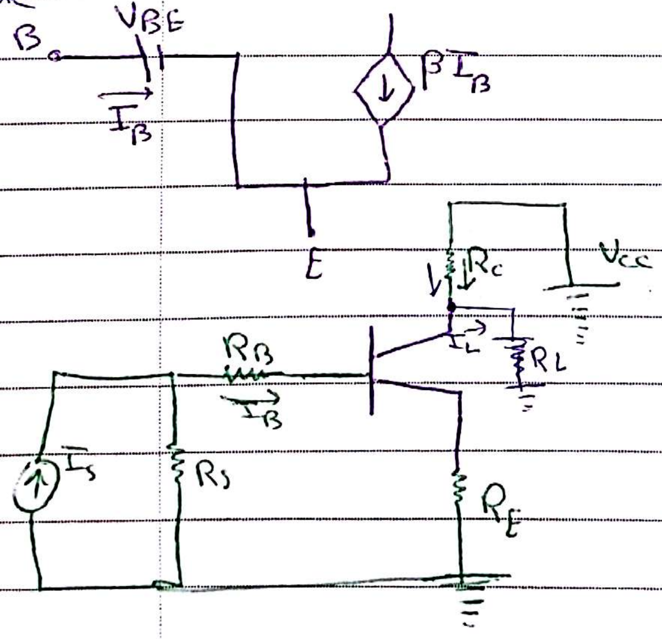
$V_{ab} > 0$

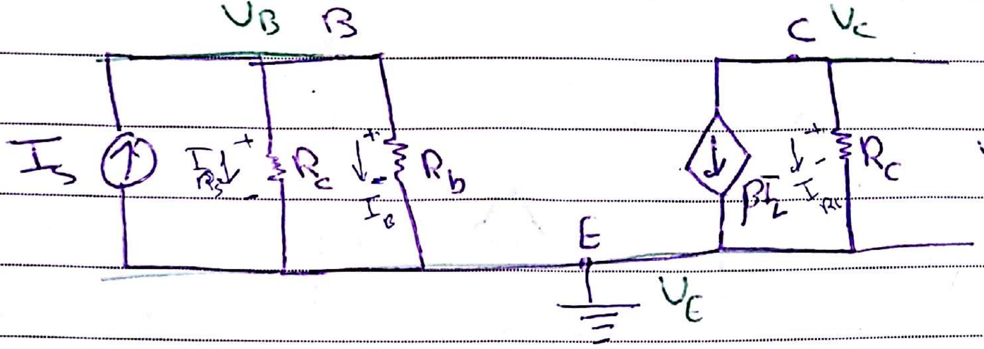
– $I_C = \beta I_B$

↘ 150, 100, 50

✱: BE junction on
BC junction on] Saturation

~~Example~~





Voltage gain

unknowns

V_B , V_E , V_c

→ $n=3$ $\quad$ $n_e = 2$

$m=0$ $\quad$ $2-1=1$

$3-0-1=2$

→ kcL node B :-

$$+I_s - I_{RS} - I_B = 0$$

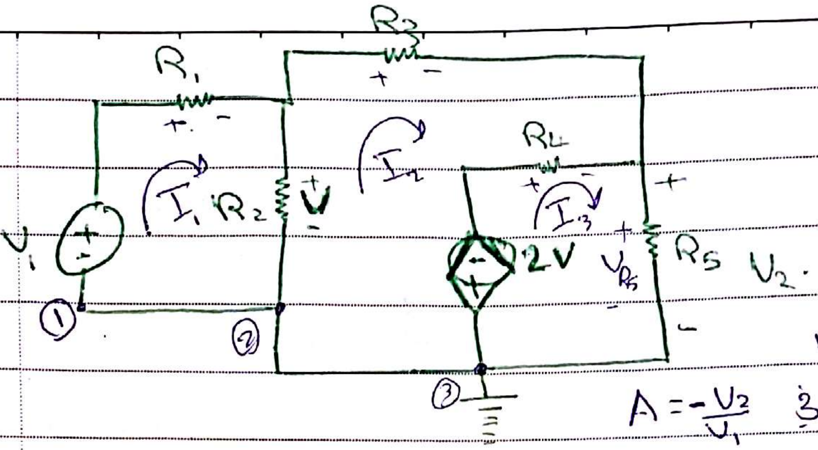
$$I_s - \frac{V_B - \overset{0}{\cancel{V_E}}}{R_s} - \frac{V_B - \overset{0}{\cancel{V_E}}}{R_b} = 0$$

→ kcl node c

$$-\beta I_L - I_{R_c} = 0$$

$$-\beta \frac{U_b}{R_b} - \frac{V_c}{R_c} = 0$$

No.



$$\boxed{V_{R5} = V_2}$$

اذا كان التيار عكس V_2

نضع اشارة سالب في $A = -\frac{V_2}{V_1}$

$R_i = 1\,\Omega$ $R_2 = 0.5\,\Omega$ $R_3 = 0.25\,\Omega$ $R_4 = 0.25\,\Omega$ $R_5 = 0.25\,\Omega$

$A = \frac{V_2}{V_1}$ using mesh Analysis

① $-V_1 + V_{R_1} + V = 0$ $\rightarrow V = R_2(I_1 - I_2)$

$-V_1 + R_1 I_1 + R_2(I_1 - I_2) = 0$

② $-V + V_3 - V_4 - 2V = 0$

$-R_2(I_1 - I_2) + R_3 I_2 - R_4(I_3 - I_2) - 2R_2(I_1 - I_2) = 0$

③ $+2V + V_4 + V_5 = 0$

$2R_2(I_1 - I_2) + R_4(I_3 - I_2) + R_5 I_3 = 0$

$$\begin{bmatrix} R_1 + R_2 & -R_2 & 0 \\ -3R_2 & 3R_2 + R_3 + R_4 & -R_4 \\ 2R_2 & -2R_2 - R_4 & R_4 + R_5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \\ 0 \end{bmatrix}$$

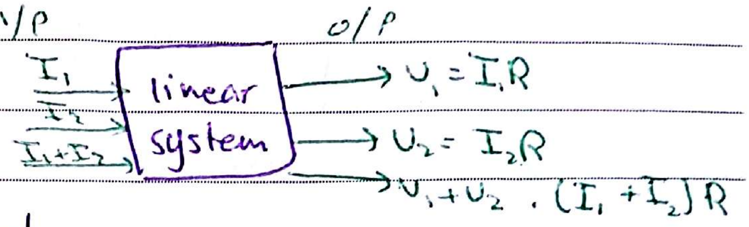
$I_1 = 0.88\,V_1$ $I_2 = 0.64\,V_1$ $I_3 = -0.16\,V_1 \rightarrow$ current is reversed as assume

$A = \frac{V_2}{V_1} = \frac{R_5 I_3}{V_1} = \frac{-R_5(0.16\,\cancel{V_1})}{\cancel{V_1}} = -0.04$

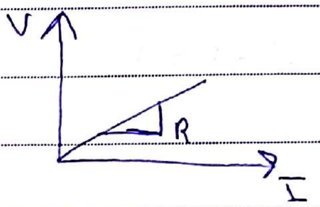
The priciple of super position.

→ depends on the Concept of circuit linearity.

Voltage sources ⎤
Current sources ⎦ inputs



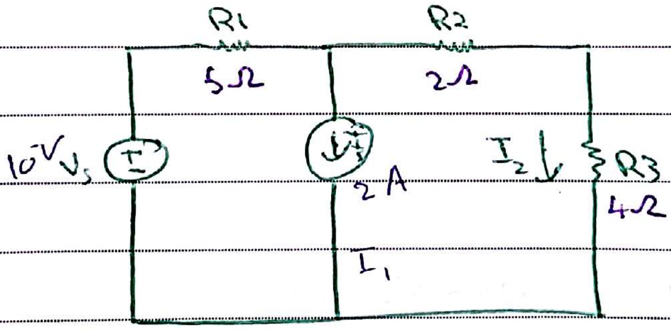
Voltage drop across any branch ⎤
Current in any branch ⎦ output



→ zeroing of

Voltage source → short circuit it.

Current source → open circuit it.

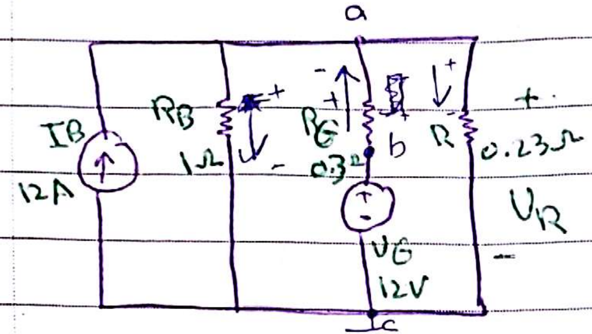


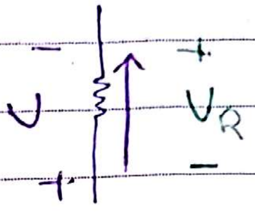
$$I_{2_{Vs}} = \frac{Vs}{R1+R2+R3} = 0.909 \text{ A}$$

if we removed Vs

↳ $I_{2_{Is}} = \frac{R1}{R1+R2+R3} I_s = \frac{5}{5+2+4} . 2 = -0.909$ A

$I_2 = 0$ A





$V_R = -U$

V_R? superposition method.

$3-1-1=1$ $n_c=2$

$2-1=1$

* * *

$V_b = 12$ V

$V_c = 0$ V

$V_a = ??$

~~KCL~~

node a:-

$-I_{RB} + I_{RG} - I_R = 0$

$$-\frac{V_a - V_c}{R_B} + \frac{V_b - V_a}{I_{RG}} - \frac{V_a - V_c}{R} = 0$$

$V_R = 4.61$ V

if we removed V_G

$I_R = I_B \cdot \frac{Req}{R}$ $\quad Req = \frac{1}{\frac{1}{1} + \frac{1}{0.3} + \frac{1}{0.23}}$

$= 12 \times \frac{Req}{0.23}$

$V_{R_I} = 1.38$ V $\quad V_{R_{I_s}} + V_{R_{V_s}} = 4.61 + 1.38 = 5.99$ V

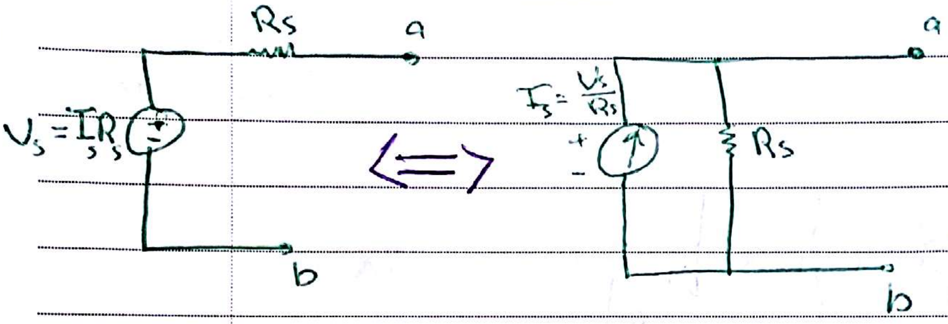
*

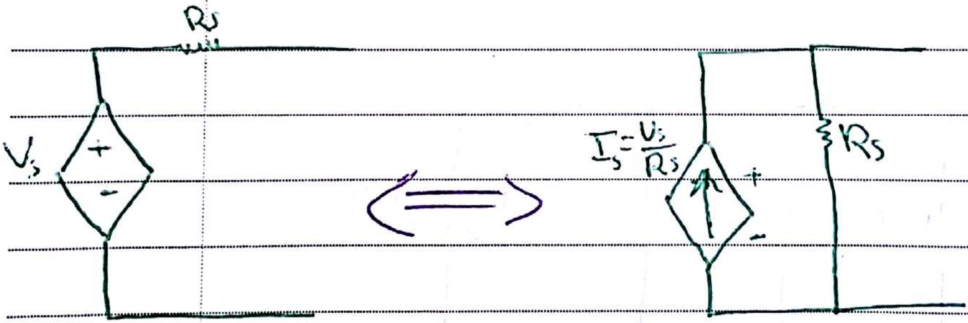
No.

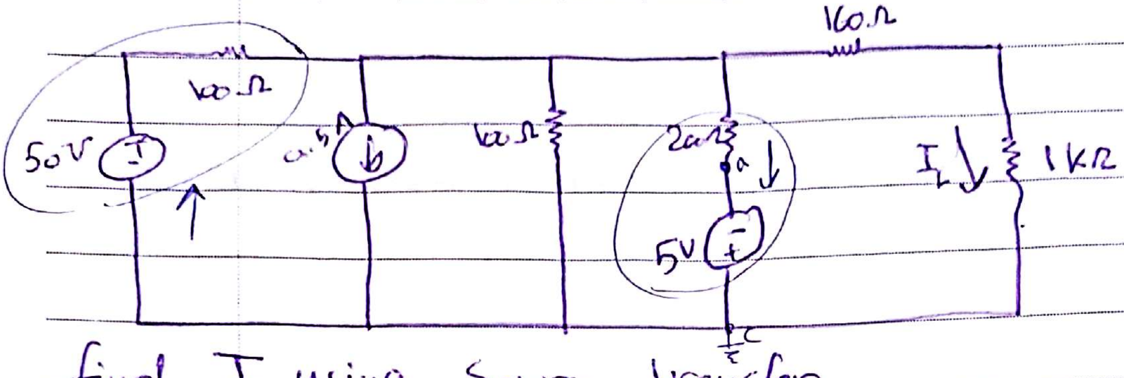
Source Transformation.

Voltage + Series resister ⟹ Current + parallel resistince

dep. indep.







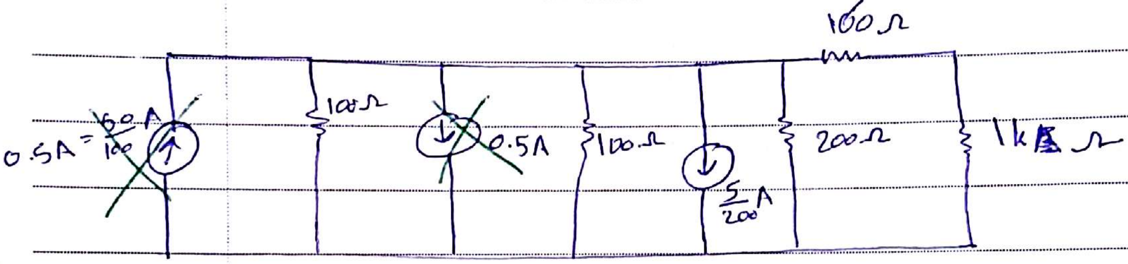
find I_L using Source transfer

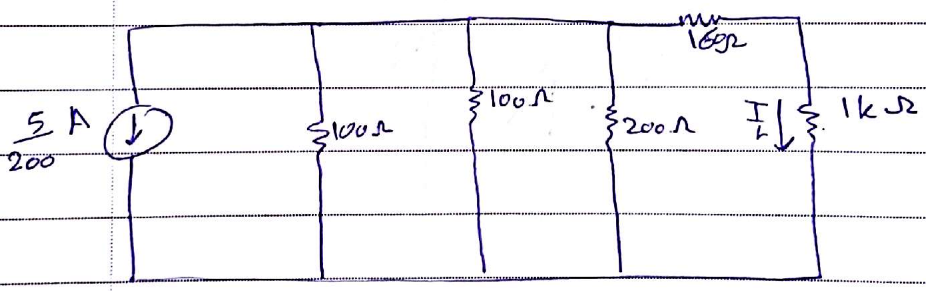
$V_{ca} = +5V$

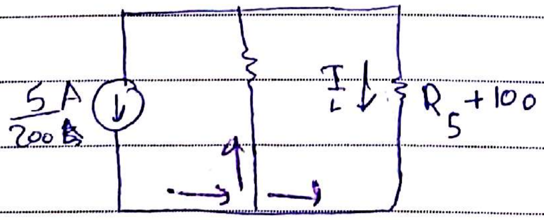
$V_c - V_a = 5$

$V_a = -5V$

Sol :-





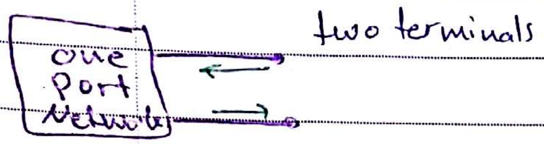


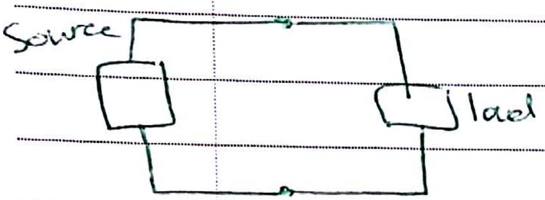
$$I_L = \frac{-40}{1000+160+40} \cdot \frac{5}{200} = -0.83 mA$$

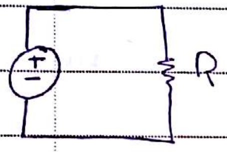
*

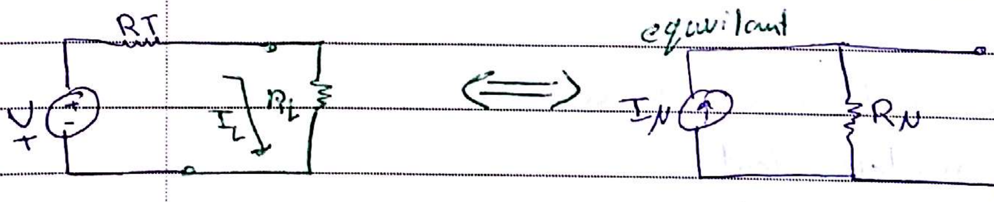
No.

One port Network. → Thevenin and Norton equivalence









$R_N = R_T$

$I_N = \frac{V_T}{R_T} - R_N$

Purpose

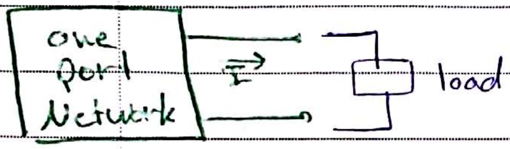
→ simply analysis for loud

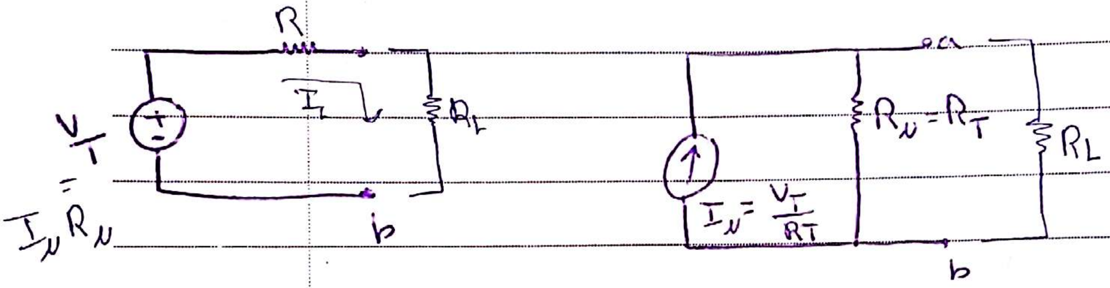
$I_L = \frac{V_I}{R_T + R_L}$

→ maximum power transfor

$R_T = R_L$

Norton and Thevenin equivelnt.





purpose of Thevenin + Norton :-

① Simplify the Circuit analysis when load is Connected

② Maximize the power transformed to the load.

we use Thevenin + Norton when :-

① all Sources independent

② Mixeture of dependent and independent sources

-③ dependent Sources only.

No.

Ⓘ only independent sources

Thevenin equivelant

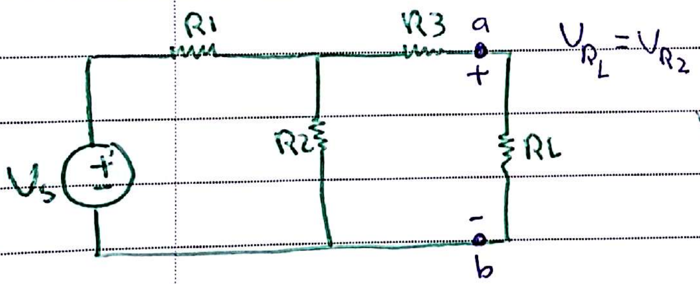
① remove load

② zero all sources

- → open circuit for current source
- → short " " voltage "

③ find R_{Th}

④ Find Voc (open circuit (oc)) voltage across the terminal where the load exist.



$V_{R_L} = V_{R_2}$

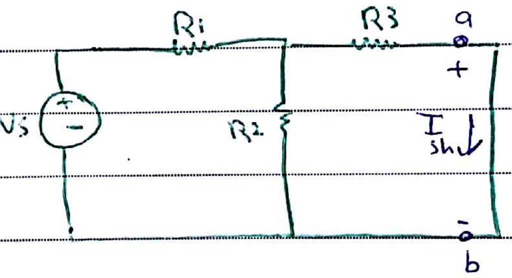
Norton eq.

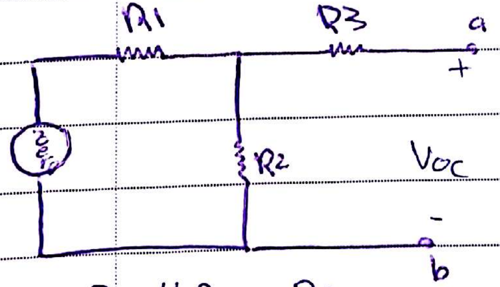
① replace the load with short circuit.

② zero all sources find R_{Th} when load is away of circuit.

③ find short circuit current

$I_{sh} = I_N$





3-1-1 = 2

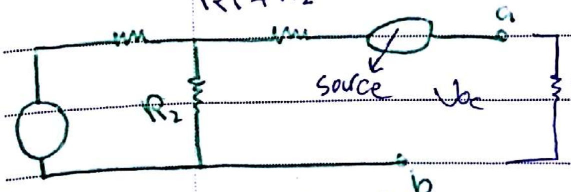
① Nodal method

② mesh method

③ Trans.

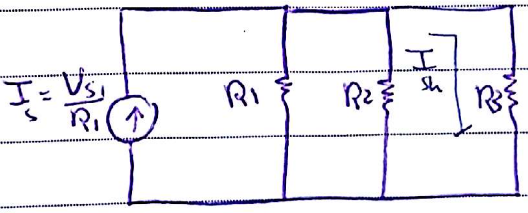
$R_{Th} = R_1 // R_2 + R_3$

$V_{oc} = \frac{R_2}{R_1+R_2} V_{S_1}$



هنا $V_{R_1} \neq V_{R_2}$ بسبب وجود
ال source لذا نعتبرها Mesh
أو loop ونكمل الحل

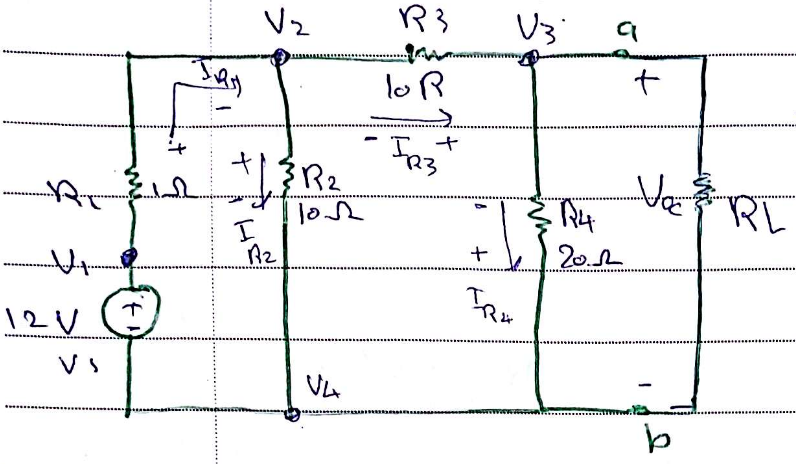
$-V_{oc} + V_{R_3} + V_{R_2} = 0$



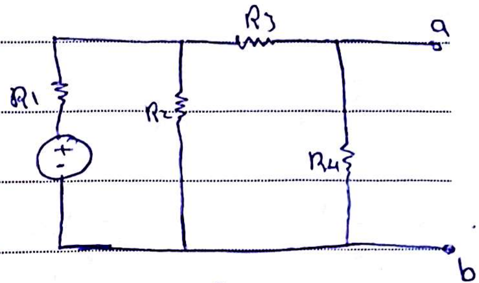
$\text{current division} = \frac{R_{eq}}{R_3} I_s$

④ توالي وتوازي

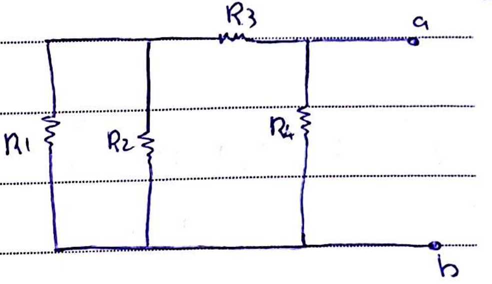
Ex :



⇒ ①



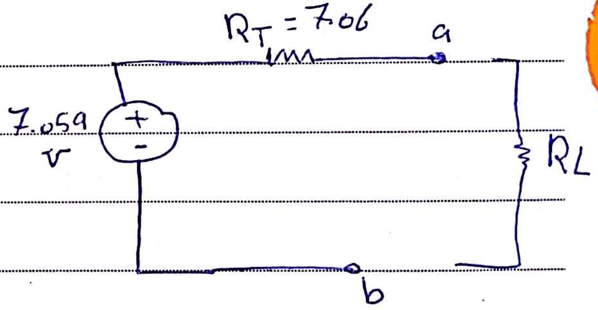
②



③ Thevinin across ab

$R_{Th} = (R_1 // R_2 + R_3) // R_4$

at last it will become like this.



Voc ?!

$$\boxed{V_{oc} = V_{R_4}}$$

(using nodal voltage methods) :

n = 4

m = 1

4 - 1 - 1 = 2

$V_1 = 12V$

node 2 :

$+I_{R_1} - I_{R_2} - I_{R_3} = 0$

$$\frac{\overset{12}{V_1} - V_2}{R_1} - \frac{V_2 - \overset{0}{\cancel{V_4}}}{R_2} - \frac{V_2 - V_3}{R_3} = 0 \sim ⓘ$$

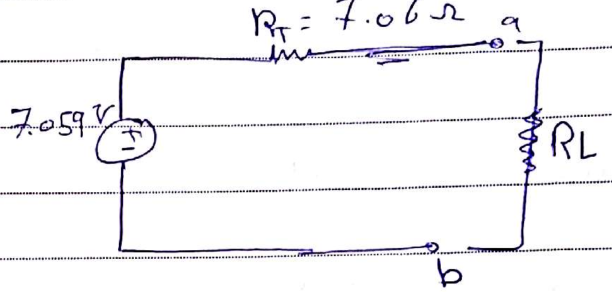
node 3 :

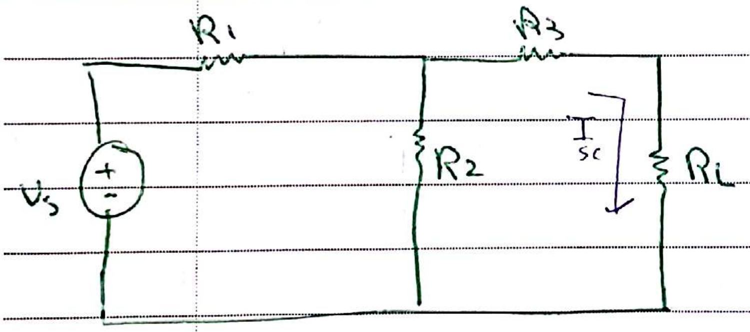
kcl

$+I_{R3} - I_{R4} = 0$

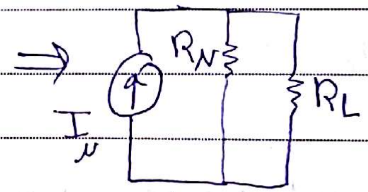
$$\frac{V_2 - V_3}{R_3} - \frac{V_3 - \overset{0}{\cancel{V_4}}}{R_4} = 0$$

$$\boxed{\begin{aligned} V_2 &= 10.59\ v \\ V_3 &= 7.059\ V \end{aligned}} \Longrightarrow$$



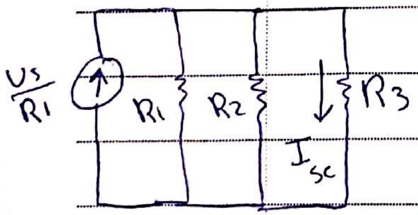


$\Longrightarrow$



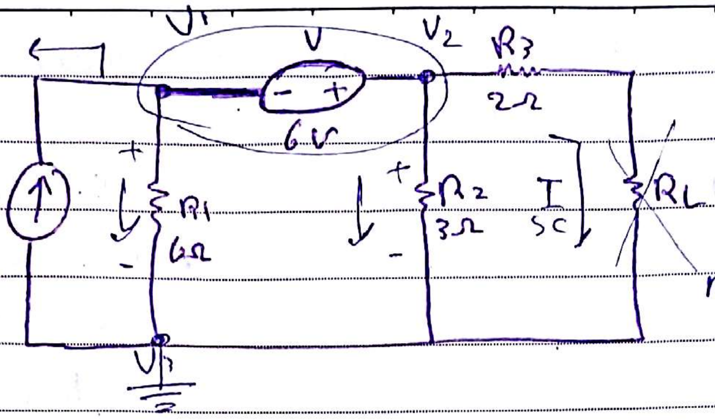
I_N , R_N ?!

$R_N = (R_1 // R_2) + R_3$



$$I_N = I_{sc} = \frac{R_{eq}}{R_3} \cdot I_s$$

making an open circuit in the first step



removing the load in the first step

Find I_N?

R_N?

$R_N = (R_1 // R_2) + R_3$

kcl supernode:-

$+I - I_{R_1} - I_{R_2} - I_{sc} = 0$

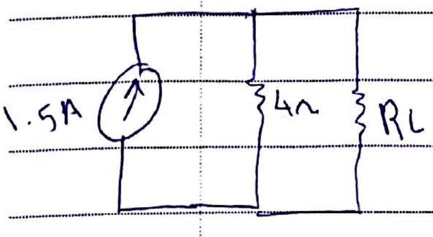
$2 - \frac{V1}{R_1} - \frac{V2}{R_2} - \frac{V2}{R_3} = 0$

$\boxed{V_2 - V_1 = 6}$

$V_2 = 3V$

$V_1 = -3V$

$I_{R_1} = 2.5A$, $I_{sc} = 1.5A$, $R_N = 4\Omega$



Ⅱ Combined dependent and Independent sources

- Regular way to find V_{oc}, I_{sc}
- find R_{Th} using

$$R_{Th} = \frac{V_{oc}}{I_{sc}}$$

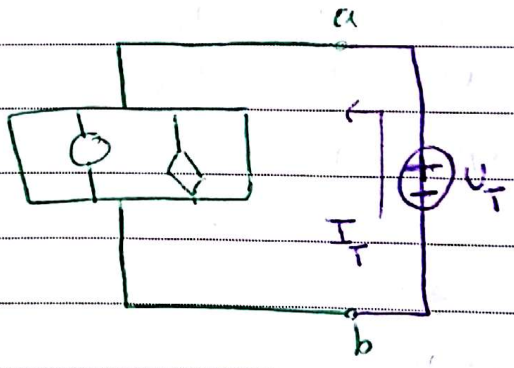
alternative way

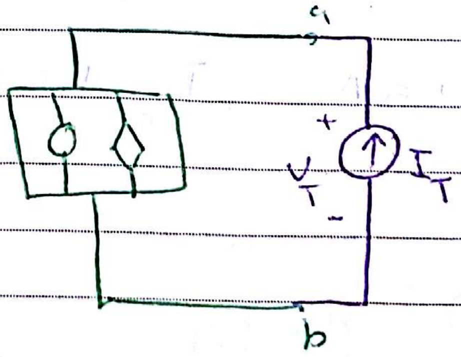
① Include a voltage test source V_T which supplies the test current I_T

② Zeroing all independent sources.

③ $R_{Th} = \frac{V_T}{I_T}$

④ find V_{oc} or I_{sc} using Regular way.

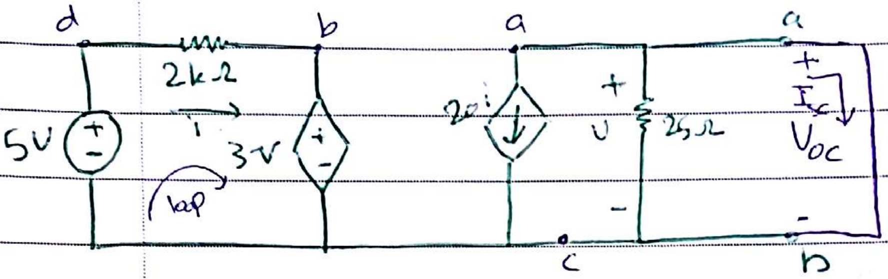




(III) only dependent sources.

⇒ we can find only R_{Th} using alterative scheme.

Example :



i)

$-V_{oc} + V = 0$

$\boxed{V_{oc} = V}$

kcl node c :-

$+20i + \frac{V}{25} = 0$

$20i = \frac{-V}{25}$

$\boxed{i = \frac{-V}{500}}$

kvl in the loop :-

$-5 + 2000i + 3V = 0$

$\boxed{V = -5V}$

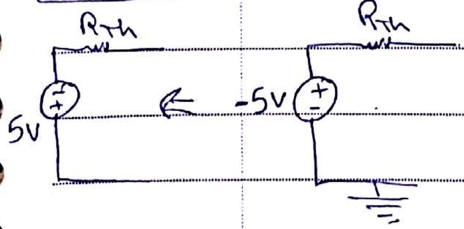
ii) $I_{sc} = -20i$ ⇒ node C

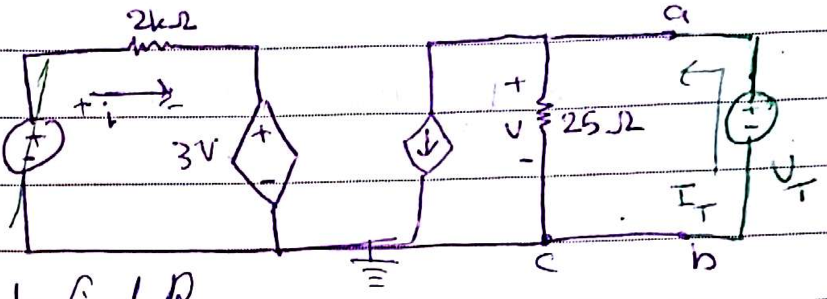
$-5 + 2000i = 0$

⋮

$I_{sc} = -50mA$

$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{-5}{-50m} = 100\Omega$





alternative way to find R_{Th}

$$\boxed{V_T = V}$$

kcl at c :

$$+20i + \frac{V}{25} - I_T = 0$$

$$I_T = 20 + \frac{V}{25} \quad \ldots \text{[I]}$$

$$2000i + 3\cancel{V_s} = 0$$

$$2000i + 3V_T = 0$$

$$i = \frac{-3}{2000} V_T$$

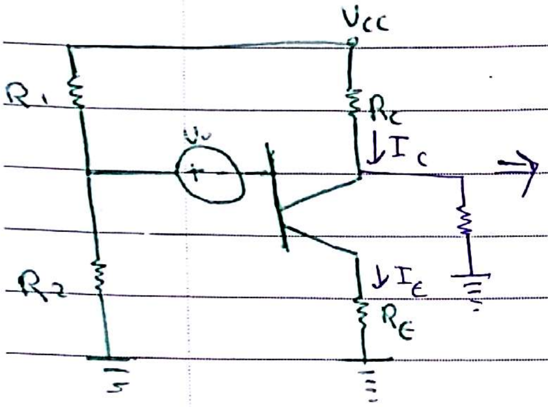
going back to [I]

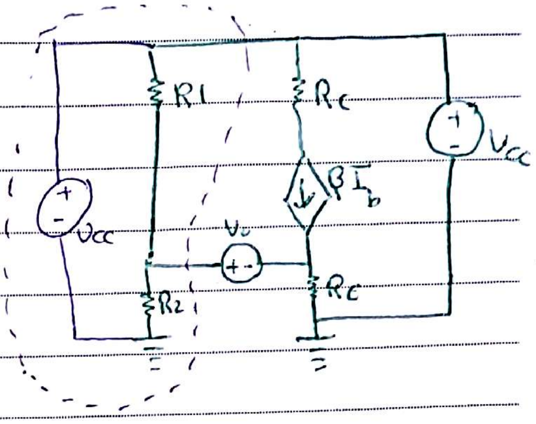
$$I_T = 20i + \frac{V_T}{25}$$

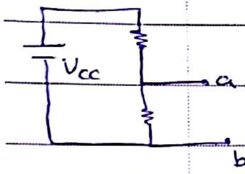
$$I_T = 20\left(\frac{-3}{2000} V_T\right) + \frac{V_T}{25}$$

$$\frac{V_T}{I_T} = 100\,\Omega$$

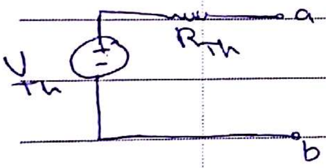
No.





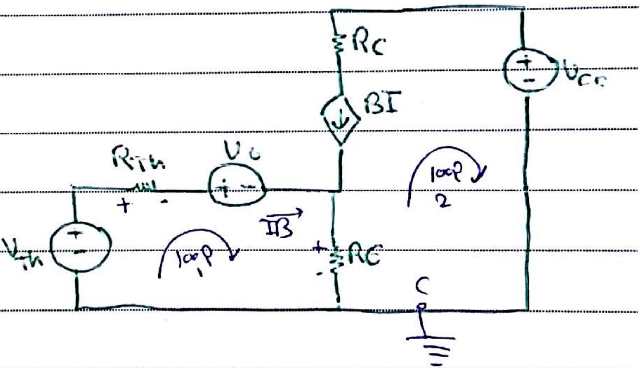


$\Downarrow$



Find I_B ?!

$\Downarrow$



$$V_{Th} = \frac{R_2}{R_1+R_2} V_{cc}$$

$$R_{Th} = R_1 // R_2 = \frac{R_1R_2}{R_1+R_2}$$

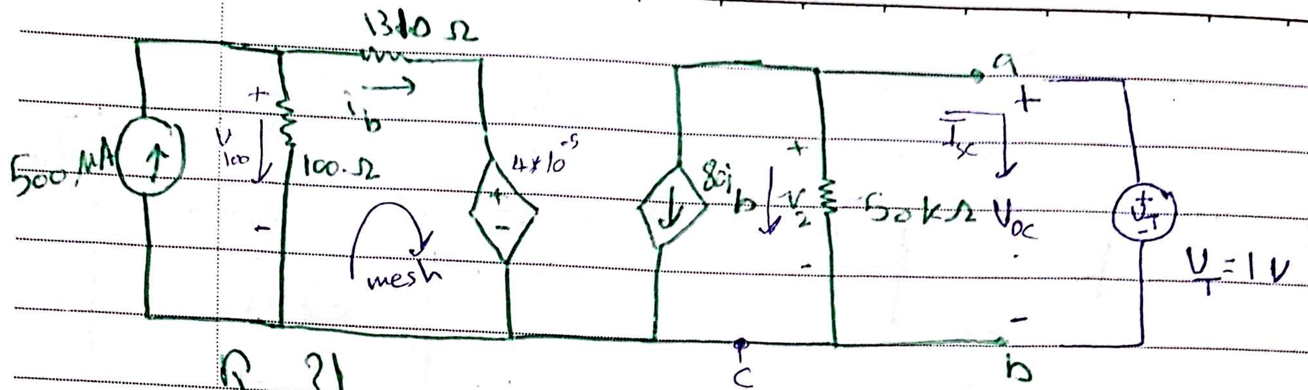
loop ①:-

$$-\overset{\checkmark}{V}_{Th} + \overset{\checkmark}{R}_{Th}\,\overset{?}{I}_B + \overset{\checkmark}{V}_o + \overset{?}{I}_E\,\overset{\checkmark}{R}_E = 0$$

KCL at node c:-

$$+I_B + \beta I_B - I_E = 0$$

$$(1+\beta)I_B = I_E \quad \Rightarrow \quad I_B = \frac{\frac{V_{cc}R_2}{R_1+R_2} - V_o}{\frac{R_1R_2 + R_E(1+\beta)}{R_1+R_2}}$$



R_{Th} ?!!

withaut V_T

with V_T

kcl at node C :-

$$80 i_b + \frac{V_2}{50000} = 0$$

$$0.5\times10^{-3} - \frac{V}{100} - i_b = 0$$

$$-V + 1310\, ib + 4\times10^{-5} V_2 = 0$$

$$V_2 = -50000 \times 80\, i_b$$

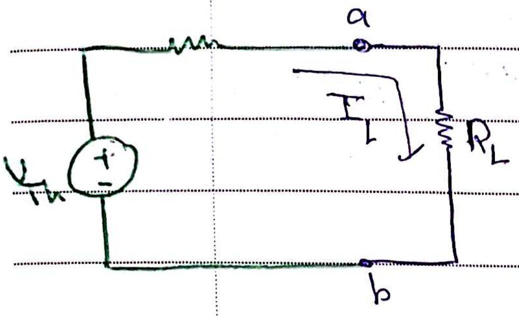
$$V_2 = -160\ V$$

$$V_{oc} = -160 V$$

$$I_{sc} = -80\, ib \qquad i_b = \frac{100}{100+1310}\ 0.5\times10^{-3} = 35.461\ mA$$

$$R_{Th} = 56.4 k\Omega$$

Maximum power Transfer



$$I_L = \frac{V_{Th}}{R_L+R_{Th}}$$

→ power dissipated in R_L

$$P = I_L^2 R_L$$

$$= \left(\frac{V_{Th}}{R_L+R_{Th}}\right)^2 R_L$$

to maximize P_L

$$\frac{dP_L}{dR_L} = \frac{d}{dR_L}\left[R_L V_T^2 (R_L+R_{Th})^{-2}\right]$$

$$= V_T^2 . \left[R_L(-2)(R_L+R_{Th})^{-3} + (R_L+R_{Th})^{-2} . 1\right]$$

$$= V_T^2 \left[\frac{-2R_L}{(R_L+R_{Th})^3} + \frac{1}{(R_L+R_{Th})^2}\right] = 0$$

$$\frac{dP_L}{dR_L} = V_T^2 \left[\frac{-2R_L + (R_L+R_{Th})}{(R_L+R_{Th})^3} \right] = 0$$

$$V_T \neq 0$$

$$(R_L + R_{Th}) \neq 0$$

$$-2R_L + (R_L + R_{Th}) = 0$$
$$2R_L = R_L + R_{Th}$$
$$R_L = R_{Th}$$

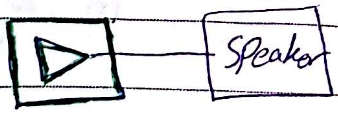
Determine the increase in power delivered to a loudspeaker when matching it's resistance to an amplifier equivclant resistor?

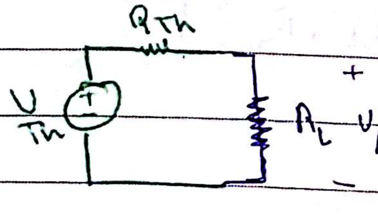
Assuming amp. is linear (amp. has resistor load)

$R_T = 8\ \Omega$

$R_{LU} = 16\ \Omega$ (load unmatched)

$R_{Lm} = 8\ \Omega$ (match resistance of the load)





$$V_L = \frac{R_L}{R_L + R_{Th}} \cdot V_{Th}$$

$$P_1 = \frac{V_L^2}{R_L}$$

$$P_L = \frac{\left(\frac{R_L}{R_{Th}+R_L}\right)^2 V_{Th}^2}{R_L}$$

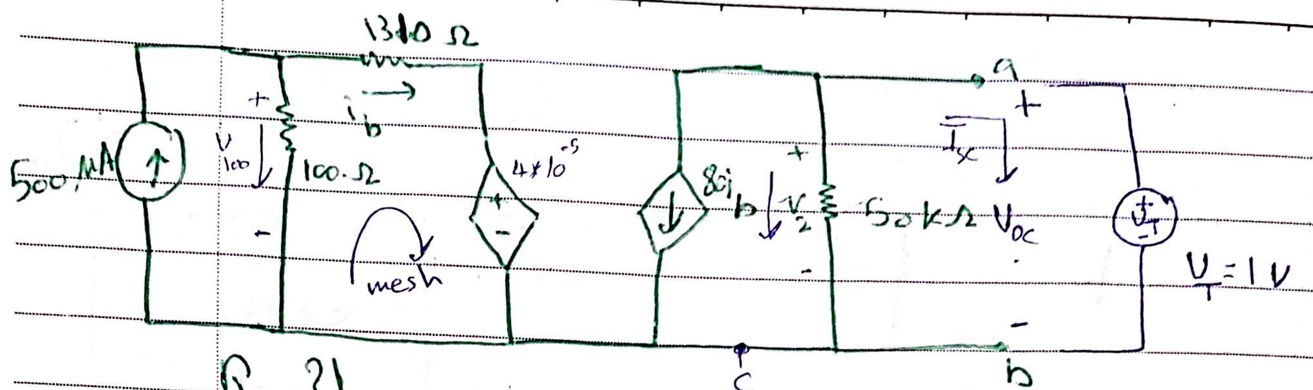
$$R_{LV} = \frac{\left(\frac{16}{8+16}\right)^2 V_{Th}^2}{16} = 0.0278 V_{Th}^2$$

$$P_{LU} = \frac{\left(\frac{8}{8+8}\right)^2 V_{Th}^2}{8}$$

$$= 0.03125 V_{Th}^2$$

Percentage of the increase $\frac{0.03125 V_{Th}^2 - 0.0278 V_{Th}^2}{0.0278 V_{Th}^2} = 12.5 \%$

No.



R_{Th} ?!!

without V_T

with V_T

kcl at node C :-

$$80 i_b + \frac{V_2}{50000} = 0$$

$$0.5\times10^{-3} - \frac{V}{100} - i_b = 0$$

$$-V + 1310\, i_b + 4\times10^{-5} V_2 = 0$$

$$V_2 = -50000 \times 80\, i_b$$

$$V_2 = -160\ V$$

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* * *

$I_{sc} = -80\, i_b$

$i_b = \frac{100}{100+1310}\ 0.5\times10^{-3} = 35.461\ mA$

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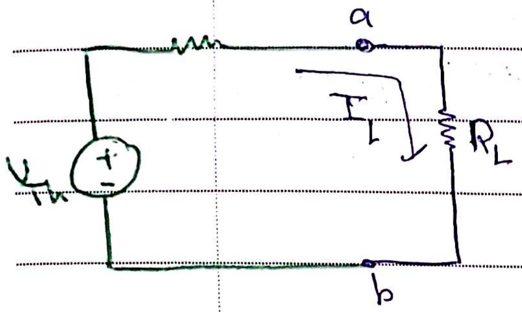
Maximum power Transfer.



$$I_L = \frac{V_{Th}}{R_L + R_{Th}}$$

→ power dissipated in RL

$$P = I_L^2 R_L$$

$$= \left(\frac{V_{Th}}{R_L + R_{Th}}\right)^2 R_L$$

to maximize P_L

$$\frac{dP_L}{dR_L} = \frac{d}{dR_L}\left[R_L V_T^2 (R_L + R_{Th})^{-2}\right]$$

$$= \frac{V^2}{T} \cdot \left[R_L(-2)(R_L + R_{Th})^{-3} + (R_L + R_{Th})^{-2} \cdot 1\right]$$

$$= \frac{V^2}{T}\left[\frac{-2R_L}{(R_L + R_{Th})^3} + \frac{1}{(R_L + R_{Th})^2}\right] = 0$$

$$\frac{dP_L}{dR_L} = V_T^2 \left[\frac{-2R_L + (R_L+R_{Th})}{(R_L+R_{Th})^3} \right] = 0$$

$$V_T \neq 0$$

$$(R_L + R_{Th}) \neq 0$$

$$-2R_L + (R_L + R_{Th}) = 0$$
$$2R_L = R_L + R_{Th}$$
$$R_L = R_{Th}$$

Determine the increase in power delivered to a loudspeaker when matching it's resistance to an amplifier equivclant resistor?

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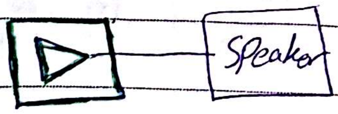
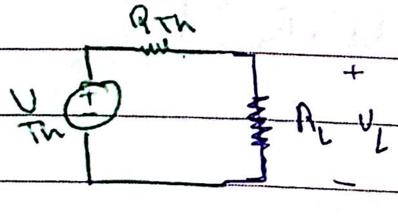




No.

$$V_L = \frac{R_L}{R_L + R_{Th}} \cdot V_{Th}$$

$$P_1 = \frac{V_L^2}{R_L}$$

$$P_L = \frac{\left(\frac{R_L}{R_{Th} + R_L}\right)^2 V_{Th}^2}{R_L}$$

$$R_{LV} = \frac{\left(\frac{16}{8+16}\right)^2 V_{Th}^2}{16} = 0.0278\, V_{Th}^2$$

$$P_{LM} = \frac{\left(\frac{8}{8+8}\right)^2 V_{Th}^2}{8}$$

$$= 0.03125\, V_{Th}^2$$

percentage of the increase $\dfrac{0.03125\, V_{Th}^2 - 0.0278\, V_{Th}^2}{0.0278\, V_{Th}^2} = 12.5\,\%$

No. chapter "6"

Ac Network Analysis

in Dc regime

① resistors

② Sources

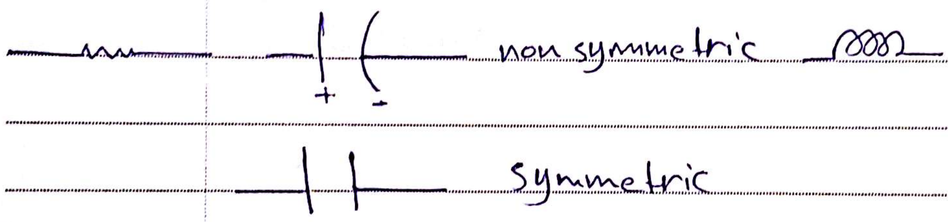
$\left.\begin{matrix}\text{Ac}\\ \text{Dc}\end{matrix}\right]$ regimes

when we have

R, c, L

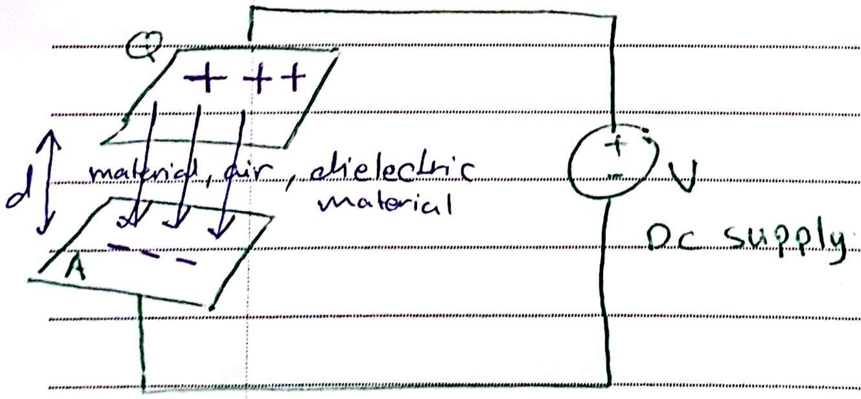
⇝ passive elements

R, c, L



non symmetric

symmetric



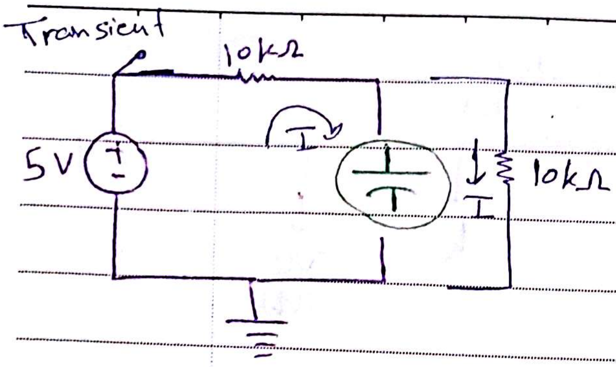
$Q = C V$

$C = \varepsilon \frac{A}{d}$ [F] farad

↓ permitivity of material

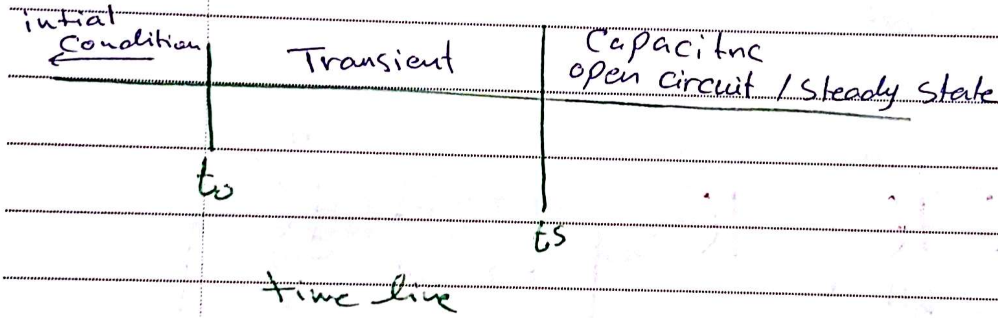
$i = \frac{dq}{dt} = 0 \, A$

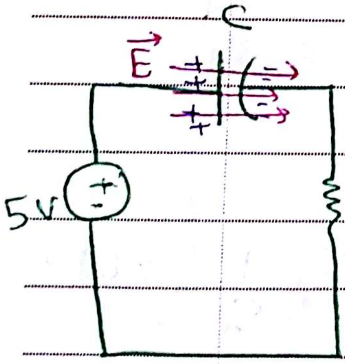
when Source is Dc, charge is Constant



$$I = \frac{5}{10k + 10k}$$

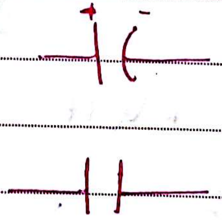
$$= 0.2 \text{ mA}$$





$Q = CV$

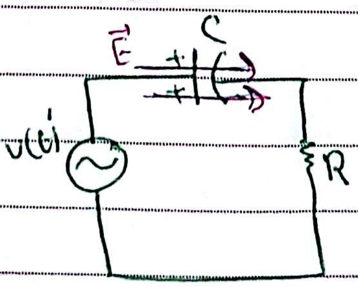
$$i = \frac{dq}{dt}$$



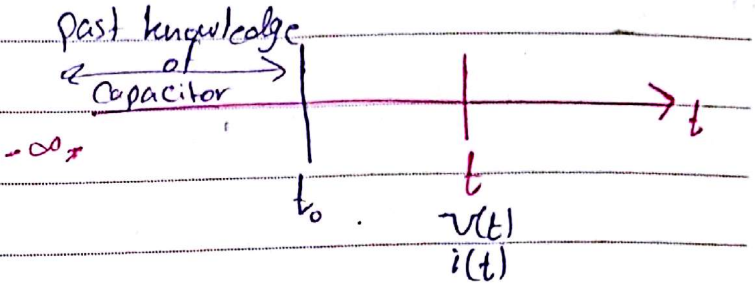
i.

$$i(t) = \frac{d\,(V(t))}{dt}$$

$$i(t) = C\,\frac{d\,V(t)}{dt}$$



$$V(t) = \frac{1}{C}\int_{-\infty}^{t} i(t')\,dt'$$



$$= v(t) = \frac{1}{C}\int_{-\infty}^{t} i(t)\, dt$$

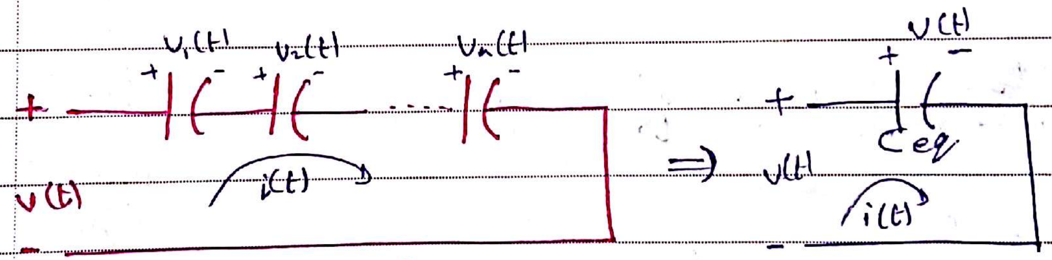
$$= \boxed{\frac{1}{C}\int_{-\infty}^{t} i(t')\, dt'} + \frac{1}{C}\int_{t_0}^{t} i(t')\, dt$$

initial condition

V_0

$$v(t) = V_0 + \frac{1}{C}\int_{t_0}^{t} i(t')\, dt$$

Ex:-



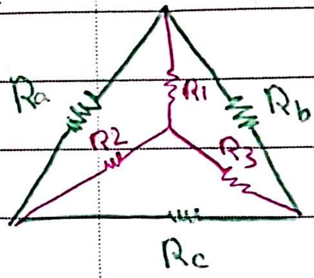
$$v(t) = v_1(t) + v_2(t) + \dots + V_n(t)$$

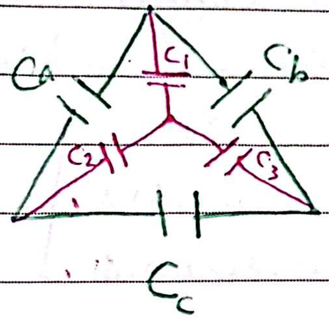
$$\frac{1}{C_{eq}}\int_{-\infty}^{t} i(t')\, dt' = \frac{1}{C_1}\int_{-\infty}^{t} i_1(t')\, dt' + \frac{1}{C_2}\int_{-\infty}^{t} i_2(t')\, dt' + \dots + \frac{1}{C_n}\int_{-\infty}^{t} i_2(t')\, dt'$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots \frac{1}{C_n}$$ ....Same to resistance in parallel.

No.

Ex:-





Δ→Y

$$R_1 = \frac{R_aR_b}{R_a+R_b+R_c}$$

$$C_1 = \frac{C_aC_b+C_aC_c+C_bC_c}{C_c}$$

Y→Δ

$$R_c = \frac{R_1R_2+R_1R_3+R_2R_3}{R_1}$$

* Calculate the charge Stored in an ultraCapacitor, how long it will take to discharge the capacitor at menimum current rate.

- capacitor 100F (±10% - 30%)
- Series resistence (DC) 15 mΩ ± 25%
  (AC) at 1k Hz 7 mΩ ± 25%

Voltage continuoes 2.5 V , peak 2.7 V

current rated 25A

$Q = CV$

$= 100 \times 2.5$

$= 250 C$

$\Delta t = \frac{250}{25} = 10$ sec.

No.

Calculate current flow through a capacitor

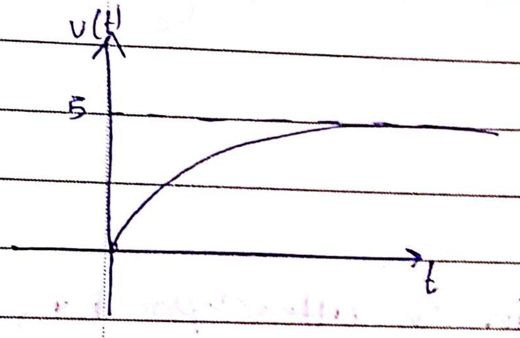
$v(t) = 5(1 - e^{-t/10^{-6}})\,V \qquad t \geq 0$ sec.

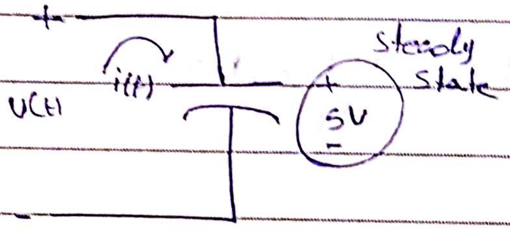
$C = 0.1\ \mu F$

اذا كان ثابت فلا يوجد تيار يسير عبره

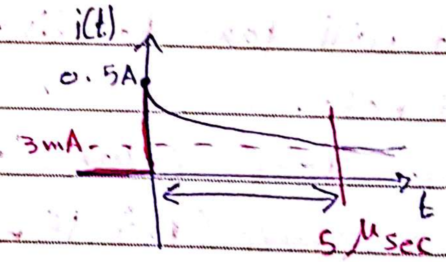
لكن هنا يوجد متغير فسيمر تيار عبره.

Sol:-





$$i(t) = C \frac{dv\,v(t)}{dt}$$



$$= 0.1 \times 10^{-6} \frac{d}{dt}\left[5 - 5e^{-t/10^{-6}}\right]$$

$$= 0.1 \times 10^{-6}\left[-5\left(\frac{-1}{10^{-6}}\right) e^{-t/10^{-6}}\right]$$

$$= 0.5\, e^{-t/10^{-6}} \qquad t \geq 0$$

$i(15) = 0.003$ A

$= 3$ mA

- Max charge stored in capacitor

$Q_{max} = C\,V_{max} \qquad = 0.1 \times 10^{-6} \times 5$

$= 0.5\ \mu C$

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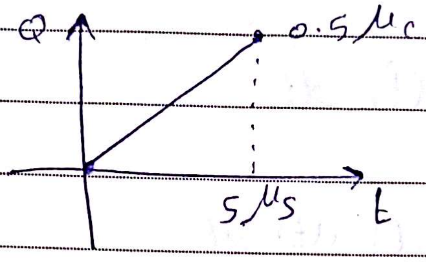
No.

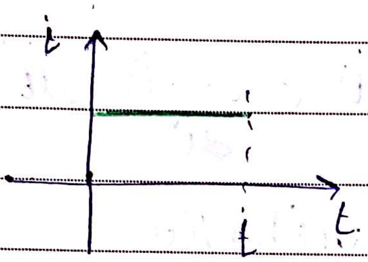
- within 5 μs assuming charge increased to max value, find current flowing in the capacitor

$$i(t) = \frac{dq}{dt}$$



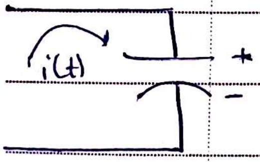
$$i(t) = \frac{0.5 \times 10^{-6}}{5 \times 10^{-6}}$$

$= 0.1 A$



Find voltage across the capacitor from the current flowing through it given that $V_c(t) = 2\ V$ at $t=0$

$C = 1000\ \mu f$



$$i_c(t) = \begin{cases} 0 & t<0 \quad sec \\ 10\ mA & 0 \le t \le 1 \\ 0 & t>1 \end{cases}$$

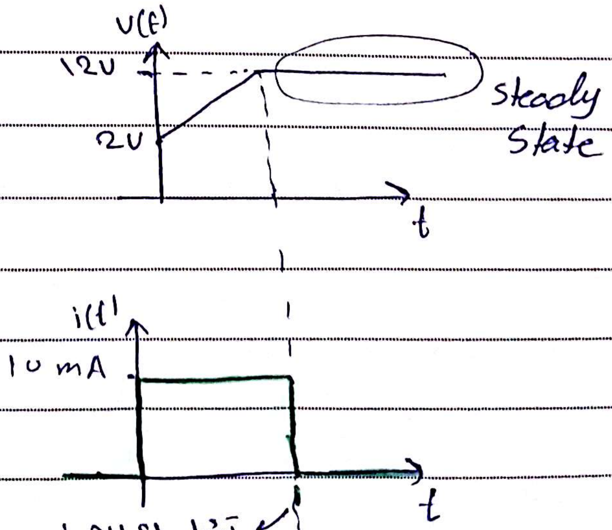
$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(t)\,dt + \cancel{V_0}^{2}$$

$$= \frac{1}{10^{-3}}\int_{0}^{t} 10^{-2}\,dt + 2$$

$$= 10^{3}\ 10^{-2} t + 2$$

$$= 10t + 1$$

$\uparrow$
1



تؤول إلا الصفر

لأن V ← Constant

لا يوجد Current

No.

$\rho = \frac{dw}{dt}$

$p = v(t) . i(t)$

$$W(t) = \int_{-\infty}^{t} p(t') \, dt'$$

$$= \int v(t') \, i(t') \, dt'$$

$$= \int v(t') \, C \frac{dv(t')}{\cancel{dt'}} \, \cancel{dt'}$$

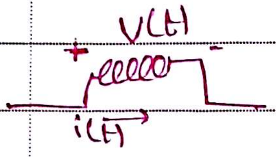
$$= C \int v(t') \, dv(t)$$

$$= \frac{1}{2} C V^2(t) + \text{Cons}$$

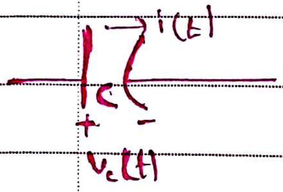
W_{max} = Steady State $W = \frac{1}{2} C v^2$

τ_c

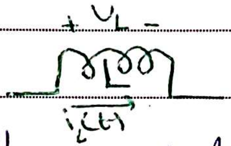
Inductors

L 

$v(t) = \frac{L\,di(t)}{dt}$

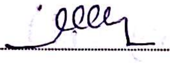
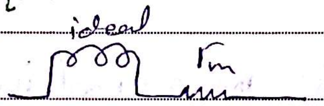
C 

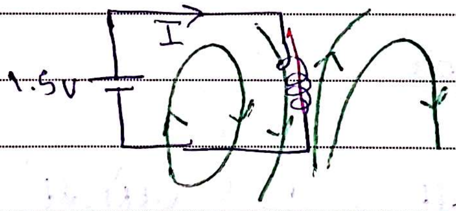
$i(t) = C\frac{dv(t)}{dt}$

- The ideal inductor 

$V(t) = L\frac{di}{dt}$

- No resistance for the Coil

practical coil  ⇒ 



faraday's law:-

$V_L = \frac{d\lambda}{dt}$

$v = \frac{d}{dt} N^2 \rho\, i$

$= \underbrace{N^2\rho}_{L}\frac{di}{dt}$

λ = magnatic flux

$\lambda = N Q$ N # of turns

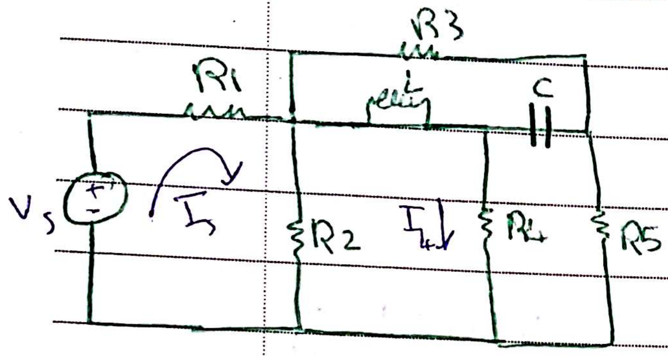
$\lambda = N \rho N\, i$

↑ pereance

DC analysis

C
open circuit

L
short circuit



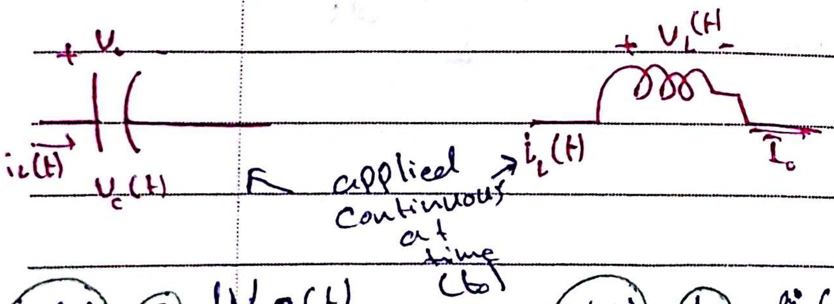
L ≡ inductance is measured in Henry (H).

$$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v(t')\,dt'$$

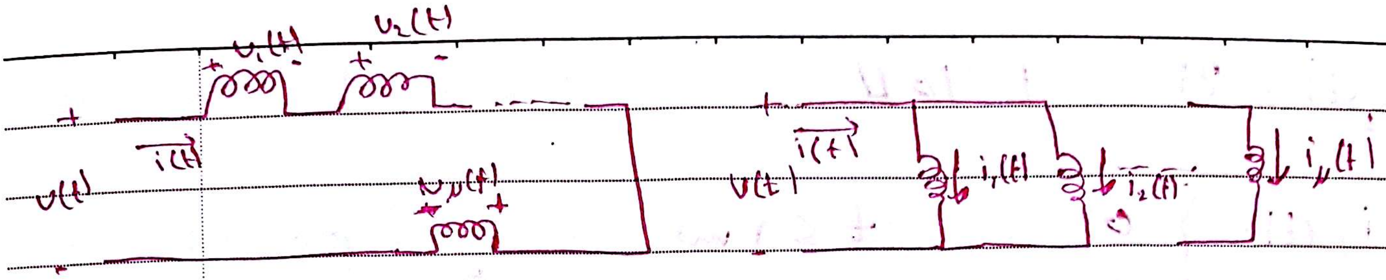
$$= \underbrace{\frac{1}{L}\int_{-\infty}^{t_0} v(t')\,dt'}_{\text{initial Condition}} + \frac{1}{L}\int_{t_0}^{t} v(t')\,dt'$$

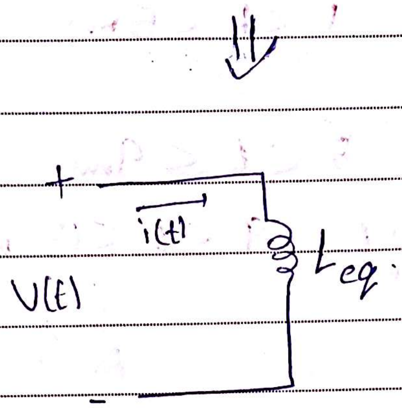
$$i_L(t) = I_0 + \frac{1}{L}\int_{t_0}^{t} v_L(t')\,dt'$$

duality concept



$$i_c(t) = C\frac{dV_c(t)}{dt} \qquad V_L(t) = L\frac{di_L(t)}{dt}$$

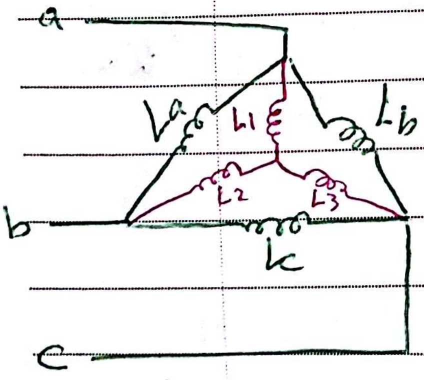




$$v(t) = L_{eq}\frac{d\cancel{i(t)}}{dt} = v_1(t)+v_2(t)+\dots+v_N(t)$$

$$L_1\frac{d\cancel{i(t)}}{dt} + L_2\frac{d\cancel{i(t)}}{\cancel{dt}} + \dots + L_N\frac{d\cancel{i(t)}}{\cancel{dt}}$$

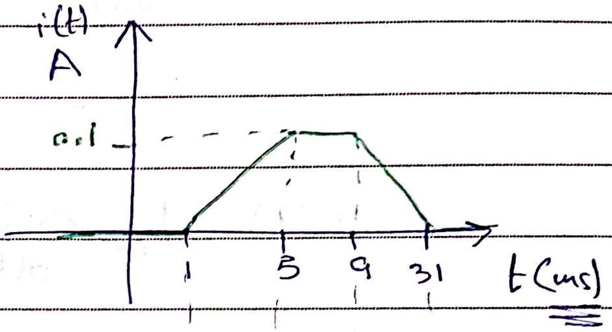
$$L_{eq} = L_1 + L_2 + \dots + L_{eq}$$



No.

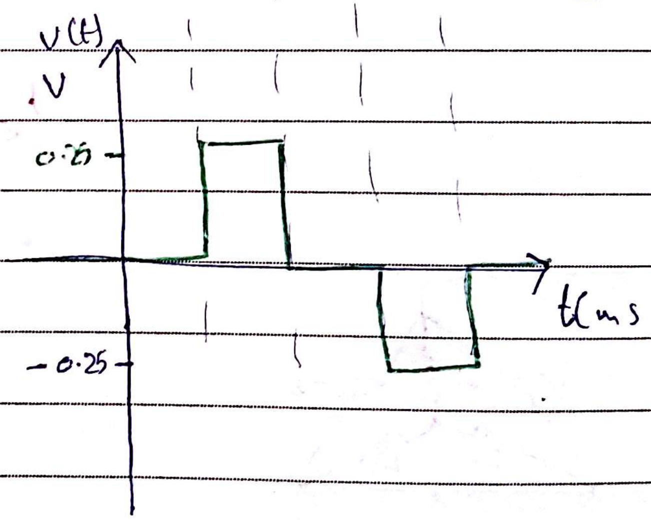
$v_L(t) = ?!$ $L = 10H$

$$i_L(t) = \begin{cases} 0 & t < 1ms \\ \frac{-0.1}{4} + \frac{0.1}{4}t & 1 \le t \le 5ms \\ 0.1 & 5 < t \le 9ms \\ 13 \times \frac{0.1}{4} - \frac{0.1}{4}t & 9 < t \le 13 ms \end{cases}$$



$$V_L(t) = L\frac{di(t)}{dt}$$

$$V_L(t) = \begin{cases} 0.V & t < 1ms \\ 0.25 & 1 \le t < 5ms \\ 0 & .5 \le t \le 9ms \\ -0.25 & 9 < t < 13ms \\ 0 & t > 13ms \end{cases}$$



$i_L(t)$? $L = 10\,mH$

$i_L(t)$ DC

$\}$ ↓

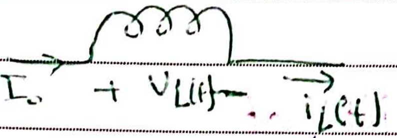
$V_L(t) = 0$

$$v_L(t) = \begin{cases} 0 & t < 0 \text{ sec} \\ -10\,mV & 0 \le t \le 1 \\ 0 & t > 1\,s \end{cases}$$

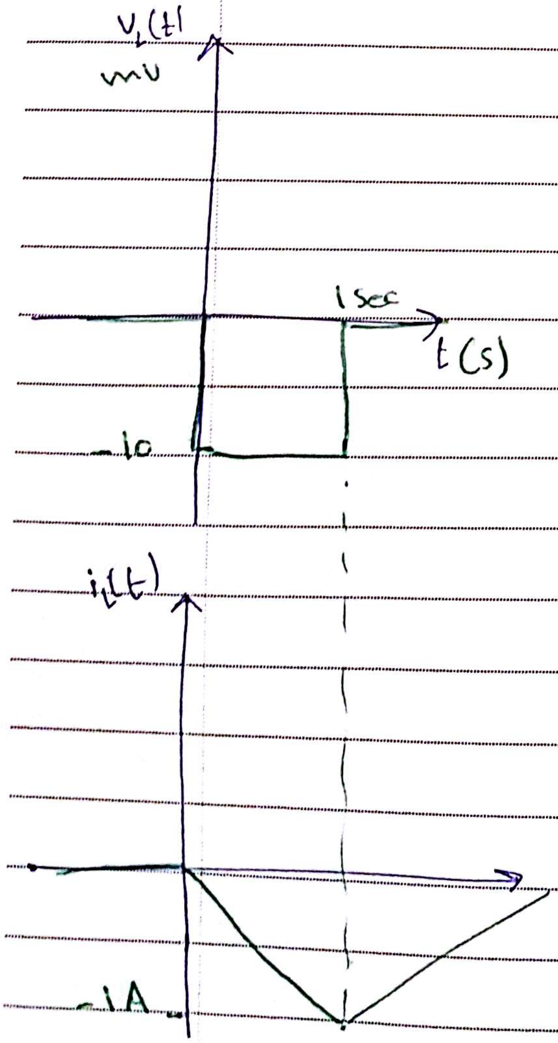
$I_0 = 0\,A$

~ since $V_L(t) = 0$ so we have a constant current.

Sol:-



$$i_L(t) = I_0 + \frac{1}{L}\int_{t_0}^{t} v_L(t')\,dt'$$



$$-\cancel{10m} \times \frac{1}{\cancel{10m}} = -1$$

No.

intial
Condition
or
Steady
State

$$W_C = \frac{1}{2} C V^2$$

$$W_L = \frac{1}{2} L I^2$$

$$W = \int p(t)\, dt$$

$$W_L(t) = i \int_{\cancel{\infty}\, t_0}^{t} i_L(t)\, di(t)$$

$$= \int v(t)\, i(t)\, dt$$

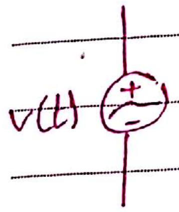
$$= \int L \frac{d\,i(t)}{\cancel{dt}}\, i(t)\, \cancel{dt}$$

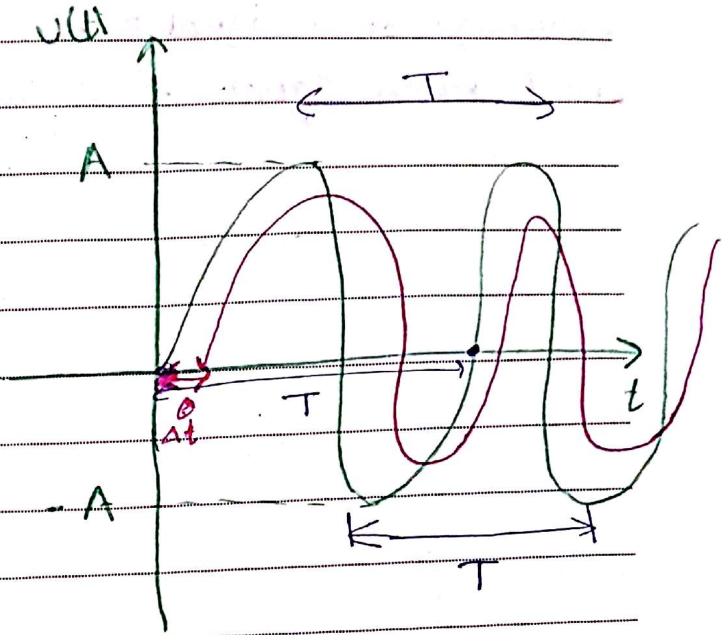
$$= \frac{1}{2} L I^2 + \text{Const.}$$

Scanned by CamScanner

No. Some notes about signals from ch 7.8.9

Sinsusoidal Sources





* $v(t) = A\sin(\omega t+\theta)$

$\omega = 2\pi f \to \frac{1}{T}$

* $\underset{\text{Peak}}{V_p} = A$

* $\underset{\text{peak to peak}}{V_{pp}} = 2A$

* θ measeard in degrees, in rad

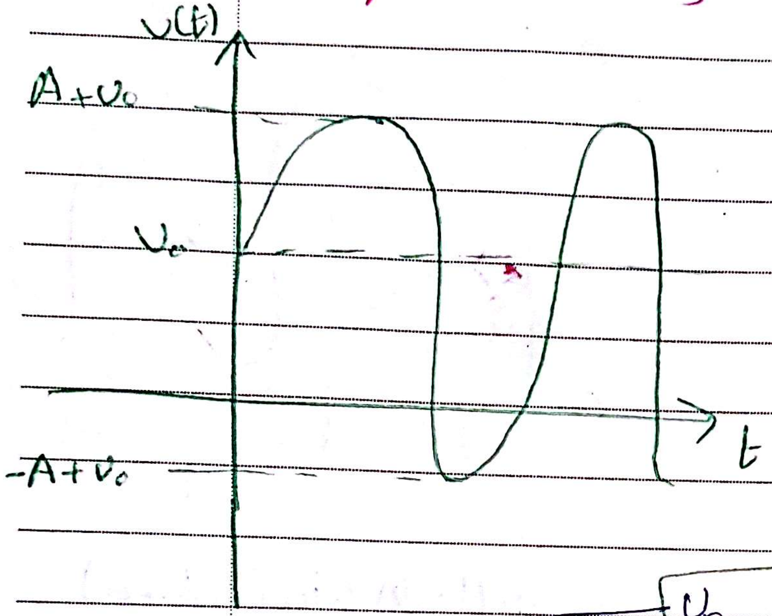
$\frac{\text{rad}}{2\pi}$ $\frac{\text{degree}}{360}$

θ θ

$$\Delta t = \frac{T\theta}{360}$$

$$\Delta t = \frac{T\theta}{2\pi}$$

- average of a signal ≡ DC Component of the Signal.
- root mean square of a signal



periodic signal
$v(t) = v(t-nT)$

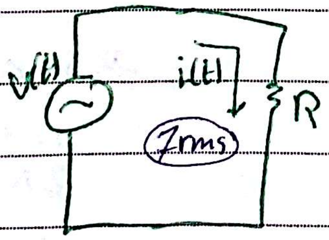
$v(t) = A\sin(\omega t+\theta) + V_0$ $\boxed{V_{rms} = 0.707 A}$

$\overline{v(t)} = V_0$

DC of the signal $= \overline{v(t)} = V_0 = \frac{1}{T}\int_T v(t)\,dt$

$$V_{rms} = \underbrace{\sqrt{\underbrace{\frac{1}{T}\int_T \underbrace{v^2(t)}_{s}\,dt}_{m}}}_{r}$$

*



$P_{DC} = I_{eff}^2 R$

power dissipated in the resistor for ac or dc supplies should be the same.

$W_{dc} = T < P_{ac} >$

$= T \frac{1}{T} \int_T i^2(t) R \, dt$

$I_{eff}^2 T = \int i(t)^2 dt$

$= \int_T i^2(t) R \, dt$

$= \boxed{R \quad I_{eff}^2} \; T \rightarrow P_{dc}$ power

$$\boxed{I_{eff}^2 = \frac{1}{T} \int_T i^2(t) \, dt}$$

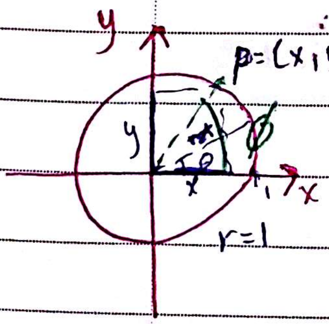
$I_{eff} = \sqrt{\frac{1}{T} \int_T i^2(t) \, dt}$

Ch.10

→ phasor Representation.

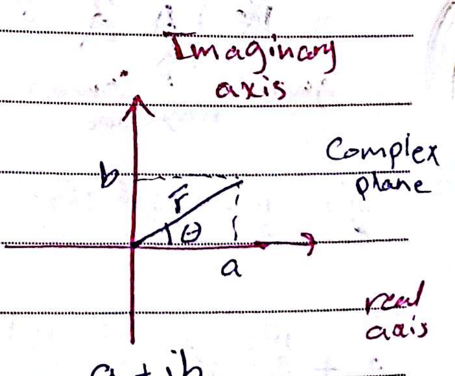
Euler identity

$v(t) = A \cos(\omega t + \theta)$



cartesian $\Longrightarrow$

Coordinates



$x = 2 \cos\phi$

$y = 1 \sin\phi$

$(1 \cos\phi, 2 \sin\phi)$

(x, y)

$p = (1, \phi)$

polar Coordinates

(cylindrical Coordinates)

$a + jb$

$j = \sqrt{-1}$

$|\vec{r}| < \theta$

$\sqrt{a^2 + b^2}$

$\theta = \tan^{-1} \frac{b}{a}$

Euler representation.

$$\vec{r} = |\vec{r}|\, e^{j\theta}$$

$$= |\vec{r}| \left\{ \cos\theta + j\sin\theta \right\}$$

$$v(t) = A\cos(\omega t + \theta)$$

$$v(t) = \underset{\text{real}}{Re} \left\{ \underbrace{A\, e^{j(\omega t+\theta)}} \right\} \longrightarrow \text{Euler representation}$$

$$= Re \left\{ A \left[\cos(\omega t+\theta) + j\sin(\omega t+\theta) \right] \right\}$$

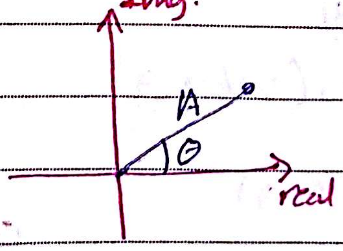
$$= Re \left\{ A\cos(\omega t+\theta) + jA\sin(\omega t+\theta) \right\}$$

$$= A\cos(\omega t+\theta)$$

$$v(t) = A\cos(\omega t+\theta)$$

$$= Re \left\{ A\, e^{j(\omega t+\theta)} \right\} \longrightarrow v(t) = A e^{j\theta}$$

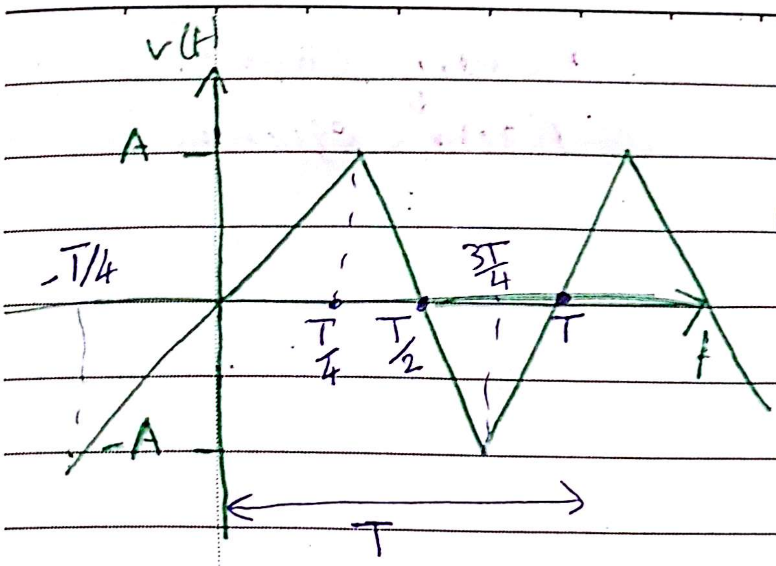
$$= \underset{\text{ignored}}{\cancel{Re}} \left\{ \underset{\uparrow\ \text{magnitude}}{A}\ \underset{\text{ignored}}{\cancel{e^{j\omega t}}}\ e^{j\theta} \right\}$$



$$e^{j\theta_1} \cdot e^{j\theta_2} = e^{j(\theta_1+\theta_2)}$$

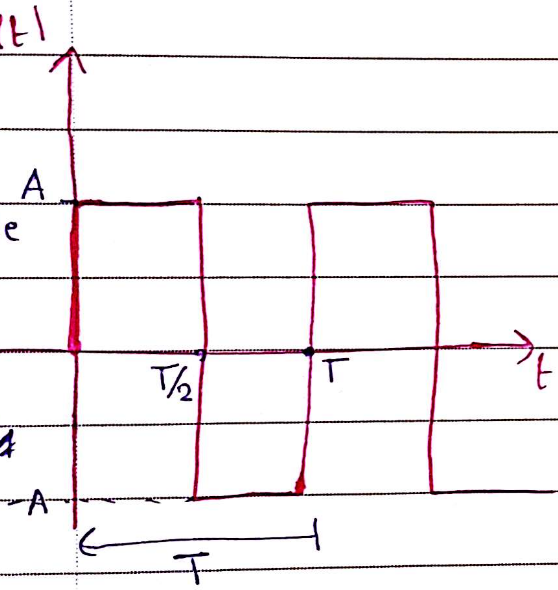
$$e^{j\theta_1} + e^{j\theta_2} \neq e^{j(\theta_1+\theta_2)}$$

Triangle wave



square wave

Duty cycle



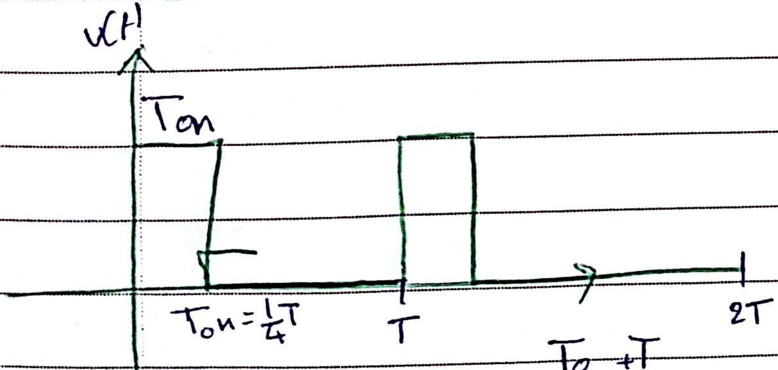
$D = \frac{T_{on}}{T\cancel{4}}$

⇓

$D = \frac{1}{2}$

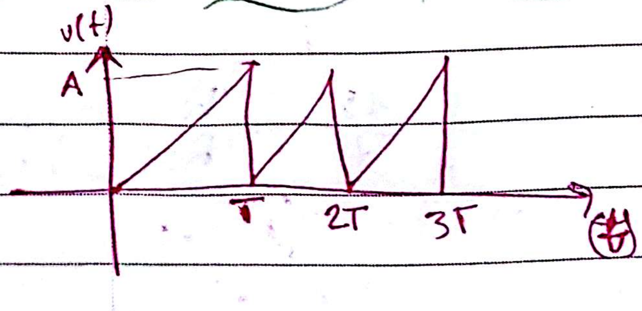
$= 50\%$

pulsed signal



$D = \frac{T_{on}}{T}$

saw tooth



No. CH. 7

differantial equations

RL , RC — first order

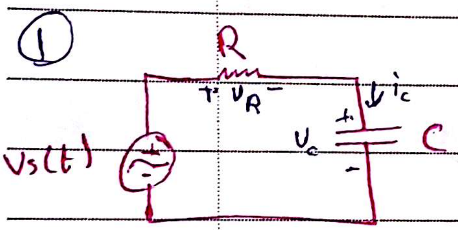
DC/switching → ch.7, ch.8

AC → ch.9, ch.10

Resistive circuit
↓
Algebric equations

RLC (2nd order) → parallel, Series

①



$V_s(t) = A\cos wt$

two ways : ① kcL
② kvL

kcl

$i_R - i_c = 0$

$$\frac{V_s - V_L}{R} - C\frac{dV_c}{dt} = 0$$

$$\frac{V_s}{R} - \frac{V_L}{R} - C\frac{dV_c}{dt} = 0$$

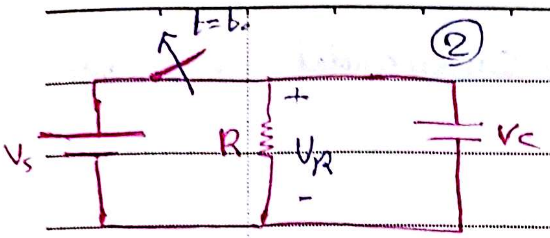
$$\frac{C\,dV_c}{dt} + \frac{V_c}{R} = \frac{V_s}{R}$$

$$\boxed{\frac{dV_c}{dt} + \frac{V_c}{RC} = \frac{V_s}{RC}}$$ step Response

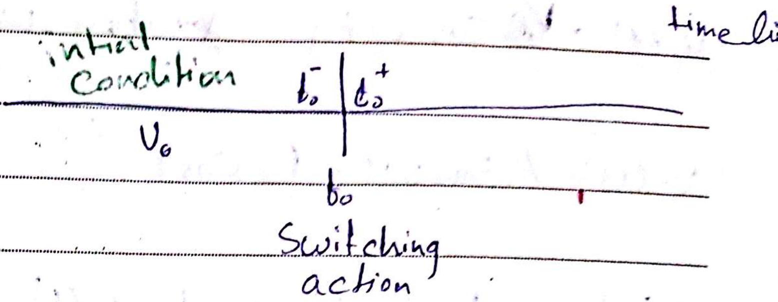
$$\boxed{\frac{dV_L}{dt} + \frac{V_c}{RC} = 0}$$ natural response

No.

②



⟹



→ switch stays in it's position for long time.

→ voltage continuous at switching action

→ Current is discont. 〃 〃 〃.

back to ①:-

$-V_S + V_R + V_C = 0$

$\frac{d}{dt}$ $-V_s(t) + Ri(t) + \frac{1}{C}\int_{-\infty}^{t} i(t')\,dt' = 0$

$V_0 + \frac{1}{C}\int_{t_0}^{t} i(t)'\,dt'$

$-\frac{d}{dt}V_s(t) + R\frac{di}{dt} + \frac{1}{C}i(t) = 0$

$R\frac{di}{dt} + \frac{1}{C}i(t) = \frac{d}{dt}V_s(t)$

$$\boxed{\frac{di(t)}{dt} + \frac{1}{RC}i(t) = \frac{1}{R}\frac{d}{dt}V_s(t)}$$

Time Constant

$\tau = RC$

$\boxed{V_s(t) = V \cos \omega t}$ *when we have a sinsusoidal source we use phasor.

$\boxed{V_c(t)} = \enclose{circle}{A} \sin \omega t - \enclose{circle}{B} \cos t$

$$\frac{d\, v_c(t)}{dt} + \frac{1}{Rc} V_c(t) = \frac{1}{Rc} V_s(t)$$

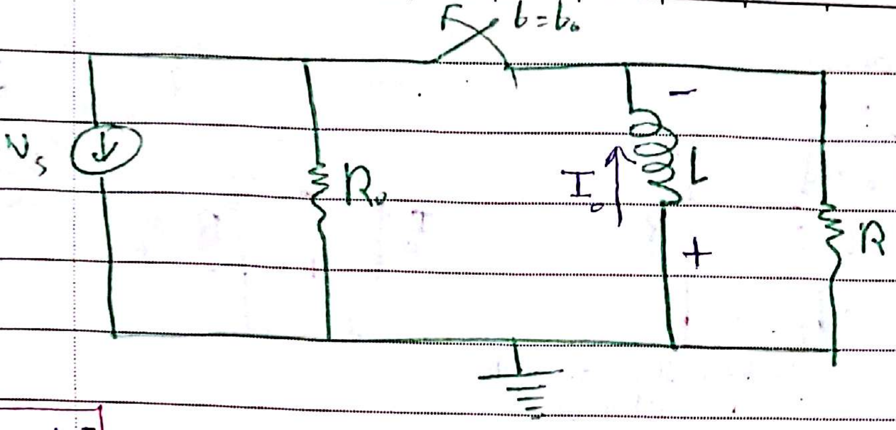
$$A\omega \cos \omega t - B\omega \sin \omega t + \frac{A}{Rc} \sin \omega t + \frac{B}{Rc} \cos \omega t = \frac{1}{Rc} V \cos \omega t$$

$$\left(\frac{A}{Rc} - B\omega\right) \sin \omega t + \left(A\omega + \frac{B}{Rc} - \frac{V}{Rc}\right) \cos \omega t = 0$$

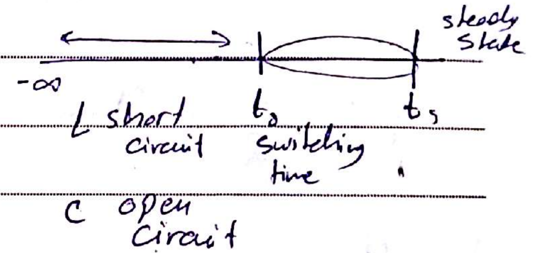
$$\frac{A}{Rc} - B\omega = 0 \quad , \quad A\omega + \frac{B}{Rc} - \frac{V}{Rc} = 0$$

$$A = \frac{V \omega Rc}{1 + \omega^2 (Rc)^2}$$

$$B = \frac{V}{1 + \omega^2 (Rc)^2}$$



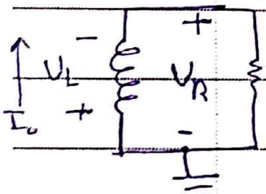
$\boxed{* \text{at } t = t_o^-}$



$I_L = I_o = I_s$

$I_o = I_s$

$\boxed{* \text{at } t = t_o}$



KVL :-

$+V_c + V_R = 0$

$$L \frac{di(t)}{dt} + Ri = 0$$

$$L \frac{di(t)}{dt} = -Ri$$

$$\frac{di(t)}{dt} = -\frac{R}{L} i(t)$$

$$\int_{i(t_o)}^{i(t)} \frac{di(t)}{i(t)} = \int_{t_o}^{t} -\frac{R}{L} dt \Rightarrow \frac{i(t)}{i(t_o)} = e^{-\frac{R}{L}t}$$

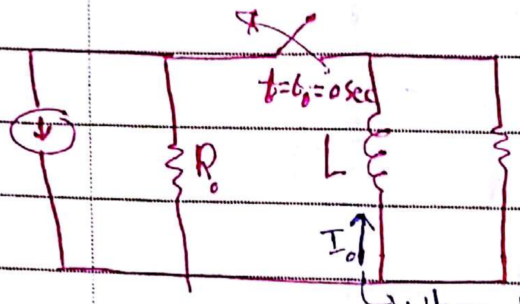
$$\ln i(t) \Big|_{i(t_o)}^{i(t)} = -\frac{R}{L}(t - \overset{0}{\cancel{t_o}})$$

$$\ln i(t) - \ln i(t_o) = -\frac{R}{L} t$$

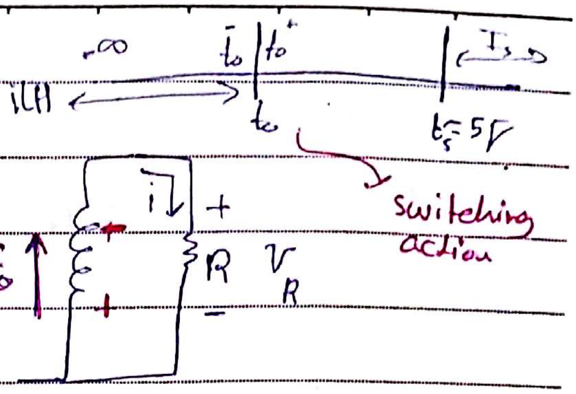
$$\ln \frac{i(t)}{i(t_o)} = -\frac{R}{L} t$$

$$i(t) = I_o e^{-\frac{R}{L}t} \quad t \geq 0$$

RL natural Response



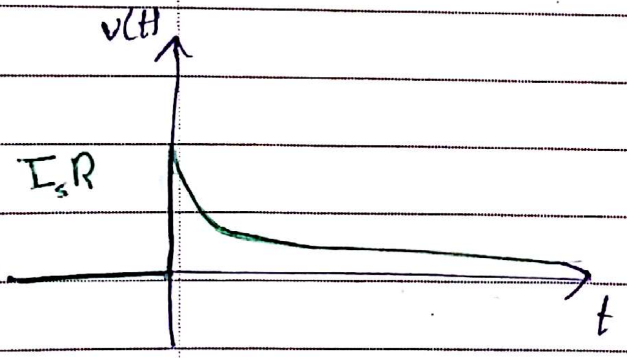
when the switch is closed

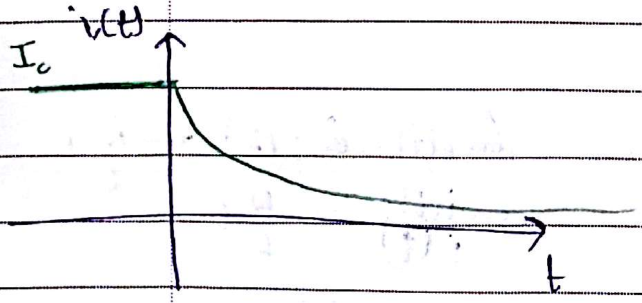


$$i(t) = I_s e^{-t/\tau} \qquad t \geq 0$$

$$\boxed{\tau = \frac{L}{R}}$$ Time constent (decay factor)

$$V_R = iR$$
$$= I_s R e^{-t/\tau} \qquad t > 0$$





t	$e^{-t/\tau}$
0	1
τ	$e^{-1} = 0.37$
2τ	$= 0.14$
3τ	$= 0.05$
4τ	$= 0.02$
5τ	$= 0.007$ ≈ 0.01
10τ	$4.\tau \times 10^{-\tau}$

Steady state time $t_s = 5\tau$ ⟵ 5τ

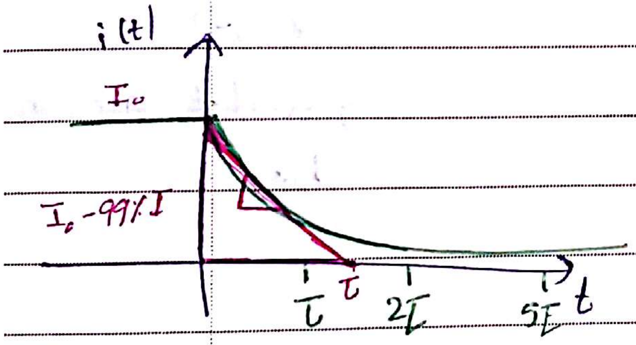
→ How to find τ graphically.

$i(t) = I_o e^{-t/\tau}$

$$\frac{di(t)}{dt}\Bigg|_{t=t_o} = I_o\left(-\frac{1}{\tau}\right)e^{-t/\tau}\Bigg|_{t=t_0}$$

$$= \frac{-I_o}{\tau}$$

we can find τ using the slope of the line



* * *

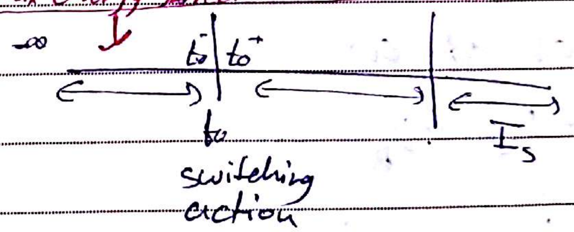
$p = v(t)\,i(t)$

$= R\,i^2(t)$

$= R I_s^2 e^{-2t/\tau}$ $t \cancel{>} 0$

$t > 0$

max energy stored in the inductor



$W = \int p\,dt$

energe dissipated in the resister

$$= \frac{-2}{\tau}\int_{t_o}^{t} R I_s^2 e^{-2t'/\tau}\,dt'$$

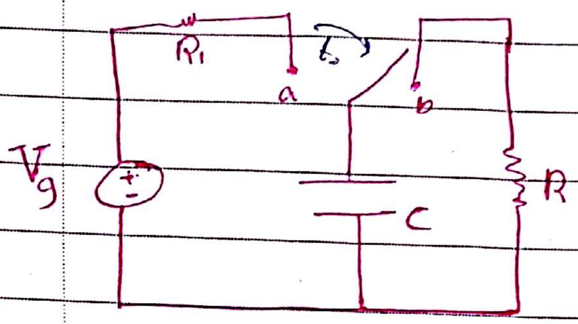
$$= \frac{-\tau}{2} R I_s^2 e^{\frac{-2t'}{\tau}}\Bigg|_0^t$$

$$\cancel{\frac{L}{R}} = \frac{\tau}{2}\cancel{R} I_s^2\left(1 - e^{-2t/\tau}\right) = \frac{1}{2} L I_s^2\left(1 - e^{-2t/\tau}\right)$$

$t \geq 0$ ✓

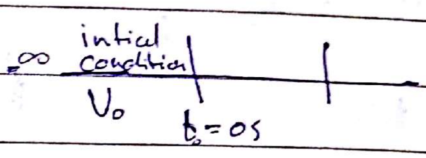
$t > 0$ ✗

Natural response for Rc

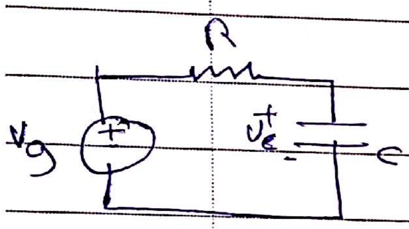


 Single pole single throw

→ Single pole double throw



step 1 :- switch stays at a for long time

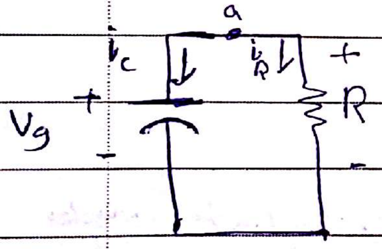


$t = t_0^- = 0^-$

$V_c(t) = V_g$

$t = 0^+$

→ switch goes to b.



kcl at a:-

$-i_c - \cancel{2} i_R = 0$

$i = c = -i_R$

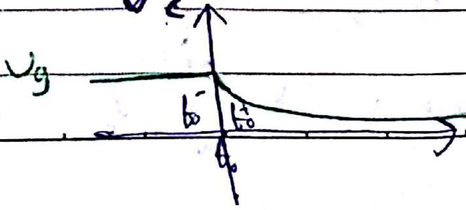
$c\frac{dv_c}{dt} = -\frac{\overset{v(t_0)}{\cancel{v_g}}}{R}$

$\frac{dv_c}{dt} = -\frac{v(t_0)}{Rc}$

$\int_{v(t_0)}^{v(t)} \frac{dv_c}{\cancel{dt}v_c} = \int_{t_0}^{t} -\frac{1}{R_c}dt$

$v_c(t) = \overset{V_0}{\cancel{V_g}}e^{-t/Rc}$

$\Rightarrow \boxed{T = Rc}$ Time Constant

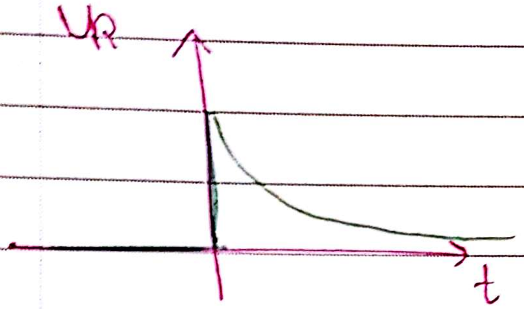


No.

$U_R = V_g e^{-t/\tau}$ $t>0$

$U_R(t=t_0^+) = V_g$

$V_c(t_0^+) = V_c(t_0^-) = V_c(t_0) = V_g$



$P = i^2 R = \frac{Vg^2}{R} e^{-2t/\tau}$ $t>0$

$= \frac{V^2}{R}$

$W_R = \int_{t_0}^{t} \frac{Vg^2}{R} e^{-2t'/\tau} dt' = \frac{\tau}{2} \frac{Vg^2}{R}\left[1-e^{-2t/\tau}\right]$

$= \frac{\cancel{R}C}{2} \frac{Vg^2}{\cancel{R}}\left[1-e^{-2t/\tau}\right]$

$W_R = \frac{1}{2} C \, Vg^2\left[1-e^{2t/\tau}\right]$ $t>0$

6/11/2016

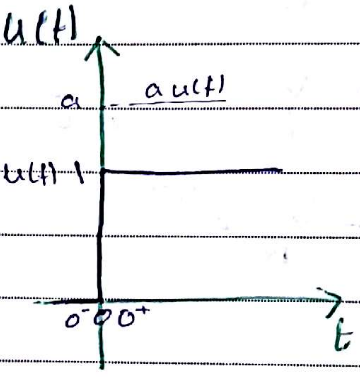
forced response for RL

Sinutial CH.9

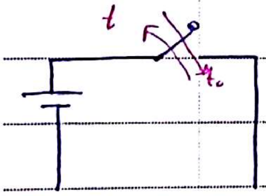
Step response

Step function

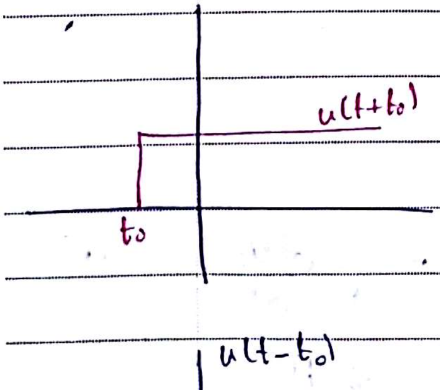
$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$



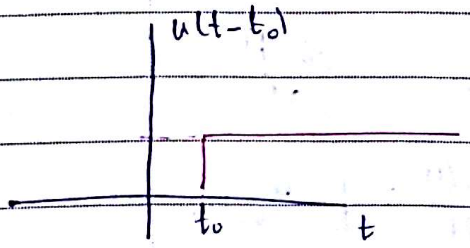
$t = t_0 = 0$



→ we call it step response because we apply DC surface.

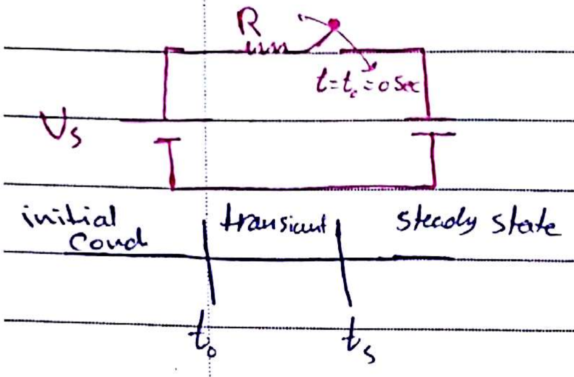


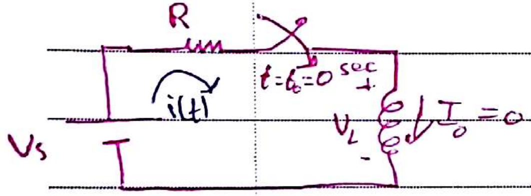
$$u(t+t_0) = \begin{cases} 0 & t < t_0 \\ 1 & t > t_0 \end{cases}$$



$$u(t-t_0) = \begin{cases} 0 & t < t_0 \\ 1 & t > t_0 \end{cases}$$

$\frac{du(t)}{dt} = \delta(t)$ delta function





KVL

$$-V_s + Ri + V_L = 0$$

$$-V_s + Ri + L\frac{di}{dt} = 0$$

$$L\frac{di}{dt} = V_s - Ri$$

$$L\frac{di}{dt} = -R\left(i - \frac{V_s}{R}\right)$$

$$\frac{di}{dt} = \frac{-R}{L}\left(i - \frac{V_s}{R}\right)$$

$$\int_{i(t_0)}^{i(t)} \frac{di'}{i' - \boxed{\frac{V_s}{R}}} = \int_{t_0}^{t} \frac{-R}{L}\, dt'$$

I_f

$\Rightarrow$

$$\int_{\boxed{I_0}}^{i(t)} \frac{dx}{x - \frac{V_s}{R}} = \int_{t_0}^{t} \frac{-R}{L}\, dy.$$

$I_{initial}$

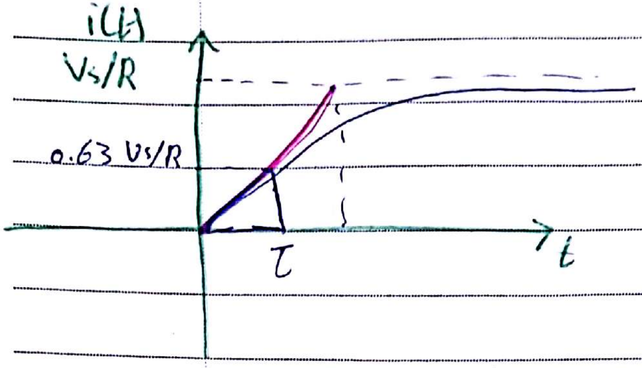
$$\frac{\ln\left(i(t) - \frac{V_s}{R}\right)}{I_0 - \frac{V_s}{R}} = \frac{-R}{L}(t - t_0)$$

$$i(t) - \frac{V_s}{R} = \left(I_0 - \frac{V_s}{R}\right) e^{-\frac{R}{L}(t-t_0)}$$

$$i(t) = \boxed{\frac{V_s}{R}} + \left(I_0 - \frac{V_s}{R}\right) e^{-\frac{R}{L}(t-t_0)}$$

$$I_f + (\cancel{I}_{initial} - I_f)\, e^{-(t-\cancel{t_0})/\tau}$$

No.



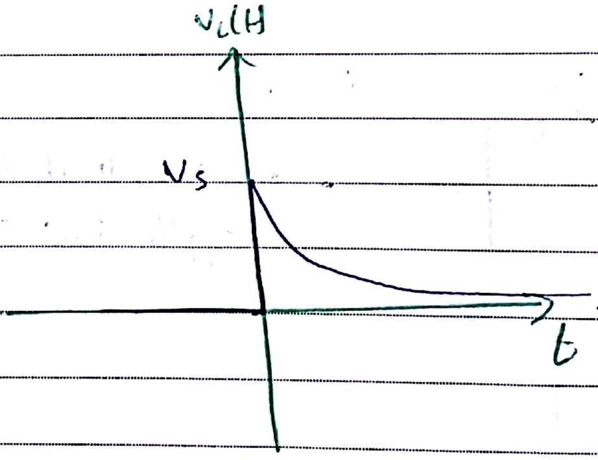
$$\left.\frac{di(t)}{dt}\right|_{t=0} = \frac{-V_s}{\cancel{R}} \cdot \left(\frac{-\cancel{R}}{L}\right)$$

$$= \frac{V_s}{L}$$

$t=\tau$

$$i(t) = \frac{V_s}{R} - \frac{V_s}{R} e^{-t/\tau}$$

Transiant



$$V_L(t) = L \frac{di(t)}{dt}$$

$$= \cancel{L}\left(\frac{-V_s}{\cancel{R}}\right)\left(\frac{-\cancel{R}}{\cancel{L}}\right) e^{-t/\tau}$$

$-\tau$

$$\boxed{V_L(t) = V_s e^{-t/\tau}}$$

$\rightarrow \frac{di(t)}{dt} + \frac{R}{L} i(t) = \frac{V_s}{L}$

$$\frac{di(t)}{dt} = -\frac{1}{R}\frac{dV_L}{dt} \quad \text{......} ①$$

$-V_s + Ri(t) + V_L = 0 \quad \Longrightarrow$

$$V_L = L \frac{di(t)}{dt}$$

$i(t) = \frac{1}{R} V_s - \frac{1}{R} V_L$

$$\frac{di(t)}{dt} = \frac{1}{L} V_L$$

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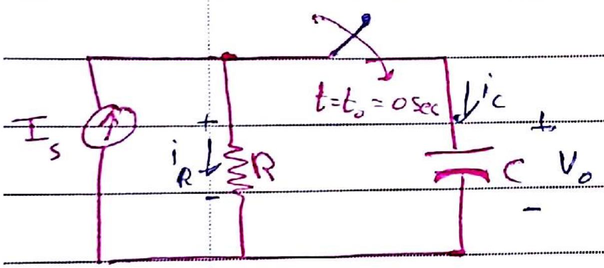
No.

$$\frac{1}{L} V_L = -\frac{1}{R} \frac{dV_L}{dt}$$

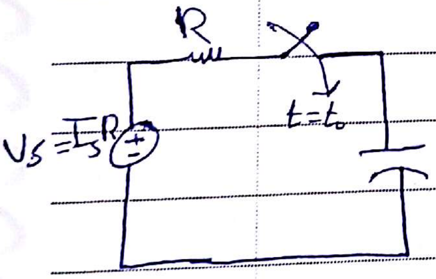
$$\int_{V_L(t_0)}^{v(t)} \frac{dV_L'}{V_L'} = \int_{t_0}^{t} -\frac{R}{L}\, dt'$$



$t = t_0^-$

$V_c(t) = V_0 = 0\,V$

$t = t_0^+$



$I_s - i_R - i_L = 0$

$$I_s - \frac{V_c(t)}{R} - C\frac{dv_c(t)}{dt} = 0$$

$$C\frac{dv_c(t)}{dt} = I_s - \frac{V_c}{R}$$

$$\frac{dv_c(t)}{dt} = -\left(\frac{V_c}{Rc} - \frac{I_s}{C}\right)$$

$$\frac{dv_c(t)}{dt} = -\frac{1}{Rc}\left(V_c - RI_s\right)$$

$$\int_{v(t=t_0)}^{v(t)} \frac{dv_c(t)}{dV_c - \underbrace{(RI_s)}_{I_f}} = \int_{t=t_0}^{t} -\frac{1}{Rc}\, dt'$$

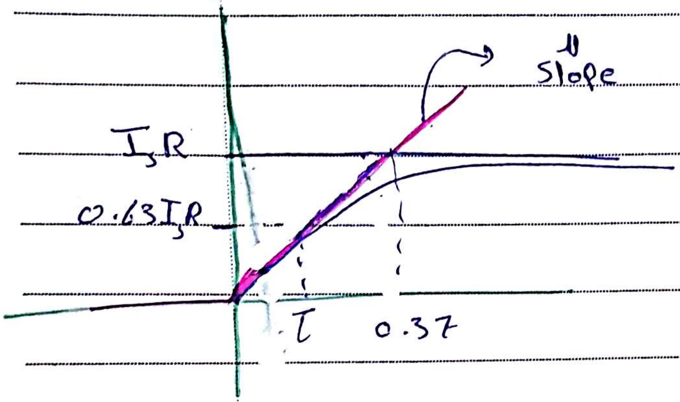
Initial $V_0 = 0$

$$\boxed{V_c(t) = V_f + (V_{in} - V_e)\, e^{-t/\tau}}$$ [illegible]

No.

$$v_c(t) = V_f + (V_{in} - V_f)\,e^{-t/\tau}$$

$$= I_s R - I_s R\, e^{-t/Rc} \qquad \boxed{t \geq 0}$$ مهم كتابتها

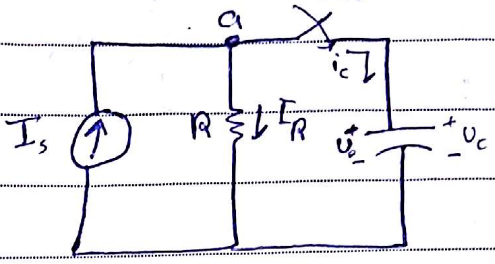


$$I_s R - I_s R\, e^{-1}$$

$$\left.\frac{d\,v_c(t)}{dt}\right|_{t=0} = -I_s R\left(\frac{-1}{Rc}\right) = +\frac{I_s}{c}$$

8/11/2016

→ The step response for Rc



kcl at point a

$I_s - i_R - i_c = 0$

$$I_s - \frac{V_c}{R} - c\frac{dV_c}{dt} = 0$$

$$i_c = c\frac{dv_c}{dt}$$

$$\frac{c\,dV_c}{dt} = I_s - \frac{V_c}{R}$$

$$\frac{dv_c}{dt} = \frac{I_s}{c} - \frac{V_c}{Rc}$$

$$\frac{dv_c}{dt} = -\frac{1}{Rc}\left[V_c - RI_s\right]$$

$$\int_{v_c(t_0)=v_0}^{v(t)} \frac{dv_c}{V_c - RI_s} = \int_{t_0=0}^{t} -\frac{1}{Rc}\,dt$$

$V_0 > t \geq 0^-$

$V_0 = 0\ V$

$V_f = I_s R$

⇒ The Voltage across the Capacitor is Continuous then $t \geq 0$

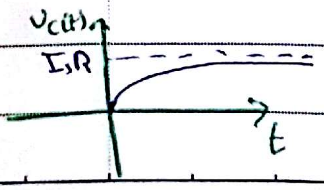
$\hookrightarrow v_c(t) = v_f + (v_0 - v_f)\,e^{-t/\tau}$ $\quad \tau = Rc$

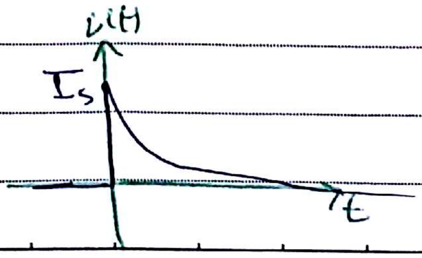
$V_c(t) = V_f + (V_0 - V_f)\,e^{-(t-t_0)/\tau}$ $\quad t_0 \neq 0$

$$i_c(t) = c\frac{d}{dt}\left[I_s R + (0 - I_s R)\,e^{-t/\tau}\right]$$

$$i_c(t) = c\left\{-(I_s R)\left(\frac{-1}{\tau}\right)e^{-t/\tau}\right\}$$

$i_c(t) = I_s e^{-t/\tau} \longrightarrow t > 0$





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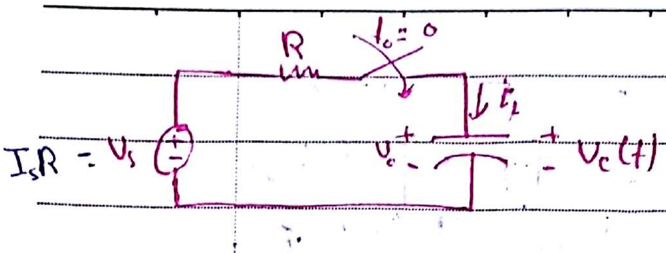


KVL :

$$\frac{d}{dt}\left[-V_s + R i_c + V_o + V_c(t) = 0\right]$$

$$-0 + R\frac{di_c}{dt} + 0 + \frac{dv_c(t)}{dt} = 0$$

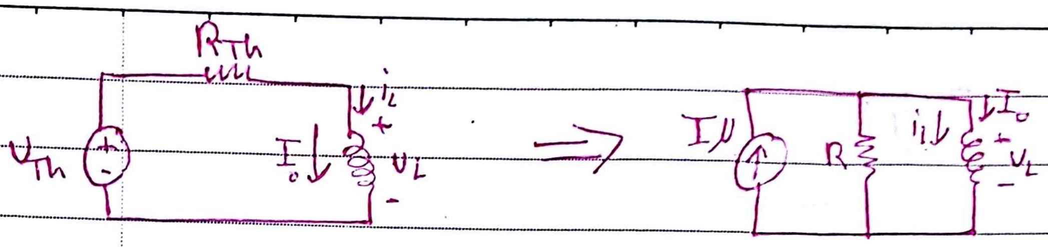
$$i_c = C\frac{dv_c}{dt}$$

$$\frac{dv_c}{dt} = \frac{1}{C} i_c \qquad i_c(t) = I_o e^{-t/\tau} \quad t > 0$$

$$R\frac{di_c}{dt} + \frac{1}{C} i_c = 0$$

$$\frac{di_c}{dt} = -\frac{1}{RC} i_c$$

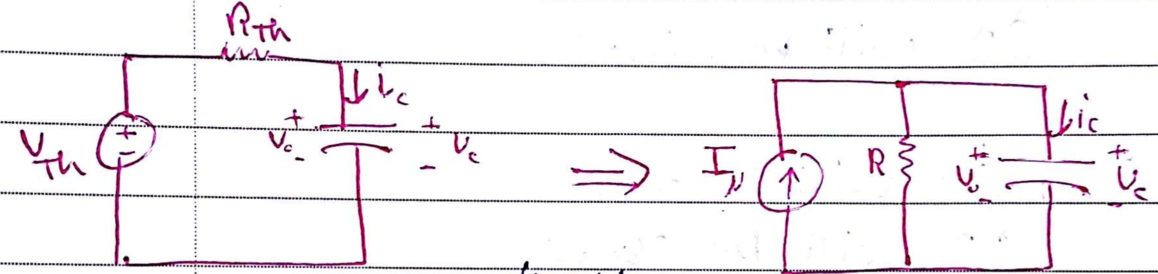
$$\tau \int_{i_c(t_o=0)}^{t} \frac{di_c}{i_c} = \int_{t_o=0}^{t} -\frac{1}{RC}\, dt$$



$i_L(t) = I_f + (I_0 - I_f)\, e^{-(t-t_0)/\tau}$ $t \geq t_0$

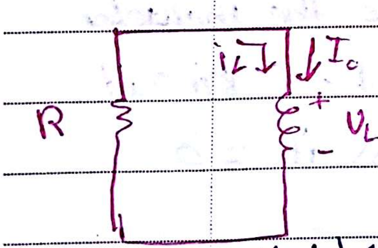
$v_L(t) = V_0\, e^{-(t-t_0)/\tau}$ $t > t_0$

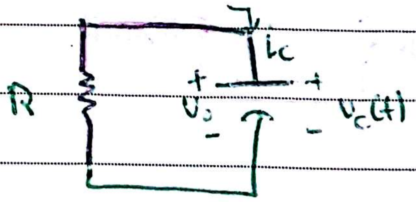
step response



$v_C(t) = V_f + (V_0 - V_f)\, e^{-(t-t_0)/\tau}$ $t \geq t_0$

$i_C(t) = I_0\, e^{-(t-t_0)/\tau}$ $t > t_0$



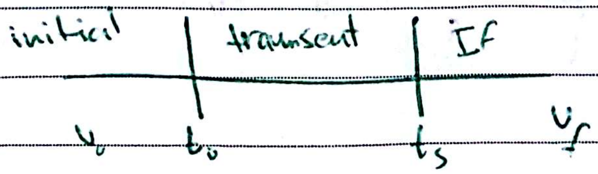


$i_L(t) = I_0\, e^{-(t-t_0)/\tau}$ $t \geq t_0$

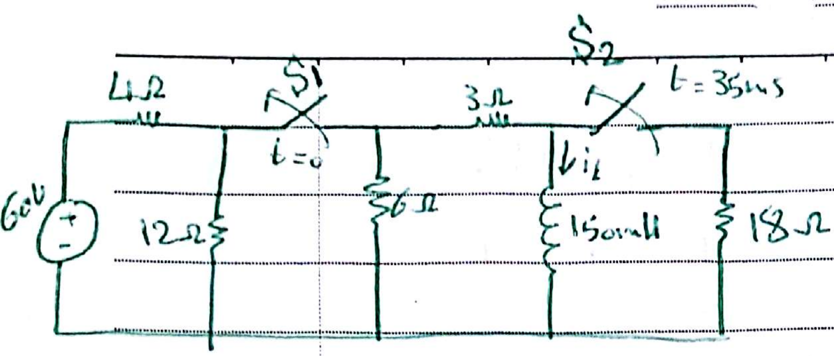
$v_L(t) = I_0 R\, e^{-(t-t_0)/\tau}$ $t > t_0$

$v_C(t) = V_0\, e^{-(t-t_0)/\tau}$ $t \geq t_0$

$i_C(t) = \frac{V_0}{R}\, e^{-(t-t_0)/\tau}$ $t > t_0$



Sequantial switching



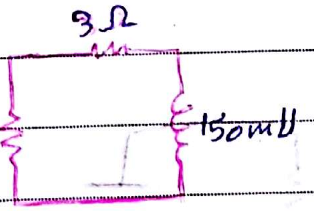
at t=0⁻ ⇒ the Switches will be closed

at t=0⁺ ⇒ $1 will open
$2 will be closed

at t=35ms⁻ ⇒ $1 will be open
$2 will be Closed

at t=35⁺ms ⇒ $1 will be open , $2 will open

↳

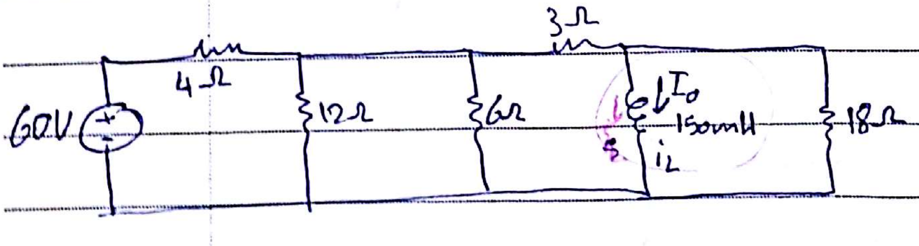


⇒ in Steady state the inductor will be like a short circuit so v across it = 0.

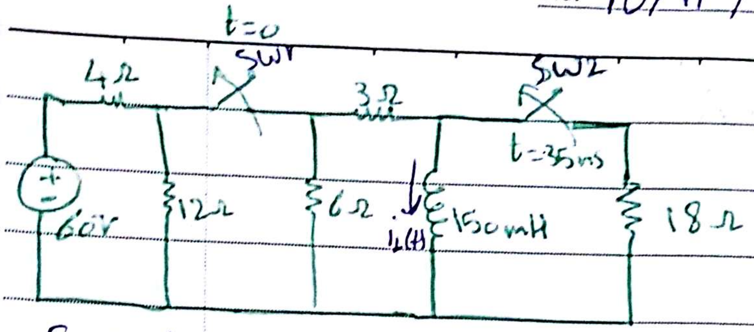
$$i_L(t) = I_o e^{-(t-\boxed{t_o})/\tau} \quad t \geq t_o$$

→ zero

⇒ at t=0⁻

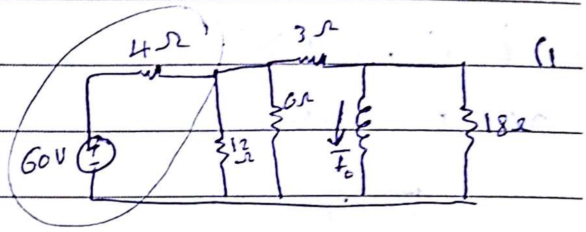


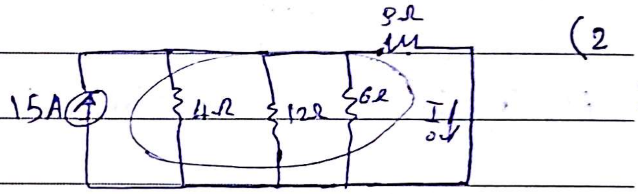
No. 10/11/2016



Find the $i_L(t)$ $0 \leq t \leq 35$ ms

$\boxed{i_L(t=0^-)} = I_0$

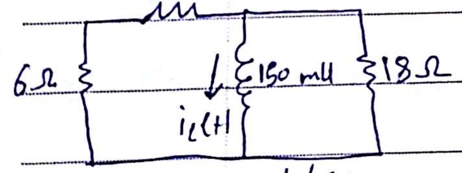




4 || 12 || 6 = 2Ω

$$I_0 = \frac{2}{2+3}(15) = 6A$$

$\boxed{\text{at } t=0^+}$



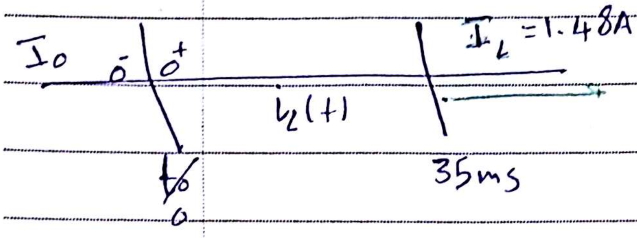
$i_L(t) = I_0 e^{-t/\tau}$ $\quad t \geq \overset{0\,sec}{\cancel{t_0}}$

$$\tau = \frac{L}{R} = \frac{150\times10^{-3}}{6} = 25\,mS$$

$R = (3+6) \,||\, 18 = 6\Omega$

$i_L(t) = 6e^{-t/25\times10^{-3}} = 6e^{-40t} \quad t \geq 0$

$i_L(35ms) = 6e^{-40(35\times10^{-3})} = 1.48A$



b) $i_L(t)$ for $t > 35ms$

at $t = 35^+ms$

$i_L(t) = I_0 e^{-t/\tau}$ $t \geq 35 ms$

$= 1.48 e^{-t/\tau}$

$\tau = \frac{L}{R} = \frac{150\times10^{-3}}{9} = 16.67 ms$

$i_L(t) = 1.48 e^{-t-\cancel{t_0}\,35ms/16.67\times10^{-3}} = 1.48e^{-60(t-35\times10^{-3})}$ $t \geq 35ms$

c) What is percentage of initial energy in the 150 mH inductor is dissipated in the 18Ω resistor?

$w = \int p\,dt$

$p = iv$

$= i^2R$

$= \frac{v^2}{R}$

$v_L = L\frac{di}{dt}$ $t \cancel{\geq 0}$, $t > 0$

$= 150\times10^{-3} \frac{d}{dt}\left[6e^{-40t}\right]$

$= -36e^{-40t}$ V

$0 < t < 35ms$

$V_{R=18\Omega} = V_L$

$= -36\, e^{-40t}$

$$W = \frac{\cancel{-40}}{-40}\int_0^{t=35ms} \frac{(-36e^{-40t})^2}{18}\, dt$$

$= 0.85\,J$

initial energy in inductor

$W_L = \frac{1}{2} L I_0^2$

$= \frac{1}{2} * 150 * 10^{-3} * 6^2$

$= 2.7\,J$

$\% W_{18} = \cancel{0.5}\ \frac{0.85}{2.7} \times 100\%$

$= 31.48\%$

d) percentage energy in 3Ω

For $I_{3\Omega}$ two cases to calculate energy dissipation?

① $0 < t < 35\,ms$

② $t > 35\,ms$

$V_{R3\Omega} = \frac{3}{3+6} \cdot V_L(t) = -12e^{-40t}$ $\quad 0 < t < 35\,ms$

$$W_{3\Omega} = \frac{-80}{-80}\int_0^{35m} \frac{(-12e^{-40t})^2}{3}\, dt = 0.5634\,J$$

$V_{3\Omega} = 3\, i_L$

$= 3\,(1.48\, e^{-60(t-0.035)t})$

$W = \int_{0.035}^{\infty} 6.57\, e^{-120(t-0.035)}\, dt$

$w_{3\Omega} = 0.055\ J$

$W_{3\Omega} = 0.055 + 0.5634$

$= 0.6182\ J$

$W\% = \frac{0.6182}{2.27} = 22.89\%$

e) in 6Ω

$W_{6\Omega} = 1236.48\ mJ$

$= 1.24\ J$

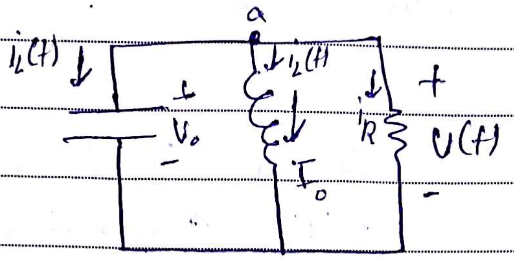
$W_{6\Omega}\ \% = \frac{1.236}{2.7} \times 100\% = 45.8\%$

$1236.48 + 618.24 + 8 + 5.27 = 2699.99\ mJ$

$2.7\ J$

No. "CH.8"

Natural response of RLC parallel



KCL at node a:-

$v_L = L\frac{di}{dt}$

$i_R + i_c + i_L = 0$

$$\left(\frac{v}{R} + C\frac{dv}{dt} + \frac{1}{L}\int v_L\,dt + I_0 = 0\right)\frac{d}{dt}$$

* for transient with DC supply

$v(t) = V_o e^{-t/\tau}$

$i(t) = I_o e^{-t/\tau}$

$v(t) = Ae^{st}$

$\frac{dv}{dt} = Ase^{st}$

$\frac{d^2v}{dt^2} = As^2et$

$$\frac{1}{R}\frac{dv}{dt} + C\frac{d^2v}{dt^2} + \frac{1}{L}v_L + 0 = 0$$

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = 0$$

$$As^2e^{st} + \frac{1}{RC}Ase^{st} + \frac{1}{LC}Ae^{st} = 0$$

$$\underset{\neq 0}{\boxed{Ae^{st}}}\left[s^2 + \frac{s}{RC} + \frac{1}{LC}\right] = 0$$

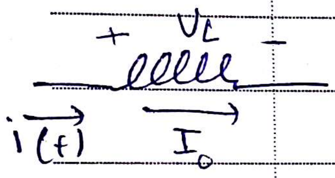
$$\underbrace{s^2 + \frac{s}{RC} + \frac{1}{LC}}_{\text{characicteristic equation}} = 0$$

$$S_1, S_2 = \frac{-1}{2RC} \mp \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$v_1(t) = A_1 e^{S_1 t}$

A_1, A_2 can be found using initial condition I_0, V_0

$v(t) = A_2 e^{S_2 t}$



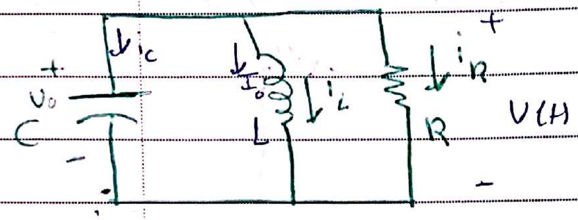
$i_L(t) = C\frac{dv(t)}{dt}$ $i_L(t_0^-) = i_L(t_0) = i_L(t_0^+) = I_0$

$= C\frac{d}{dt}\left(A_1 e^{s_1 t} + A_2 e^{s_2 t}\right)$

$= C A_1 S_1 \cancel{e^{S_1 t}} + C A_2 \cancel{S e^{S_2 t}}$

$= C A_1 S_1 + C A_2 S$

20/11/2016



$$\frac{d^2v}{dt^2} + \frac{1}{Rc}\frac{dv}{dt} + \frac{1}{Lc}v = 0$$

kcl :-

for $\underset{\text{initial condition}}{\Rightarrow}$ $i_L + i_c + i_R = 0$

$$S^2 + \frac{1}{Rc}S + \frac{1}{Lc} = 0$$ characteristic equation

$$v(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t}$$

$$S_1, S_2 = -\frac{1}{2Rc} \mp \sqrt{\frac{1}{(2Rc)^2} - \frac{1}{Lc}} = -\alpha \mp \sqrt{\alpha^2 - w_0^2}$$

$$= -\alpha \pm w_d$$

α ≡ Neper frequency ≡ rad/s $\alpha = \frac{1}{2Rc}$

damping factor

w_0 = resonance radian frequency ≡ rad/s

$$w_0 = \sqrt{\frac{1}{Lc}}$$

w_d = damping radian frequency ≡ rad/s

$$w_d = \sqrt{\alpha^2 - w_0^2}$$

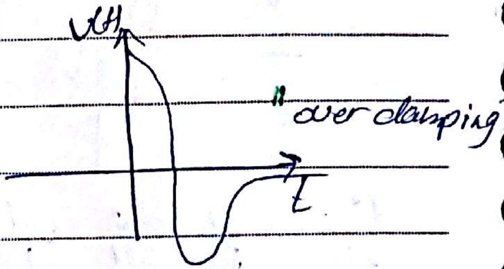
units of A_1, A_2 (v)

S_1, S_2 (sec^{-1})

No.

1) if $\alpha^2 > w_0^2$ (over damping)

we have two real, distic solution for the charateristic eq.

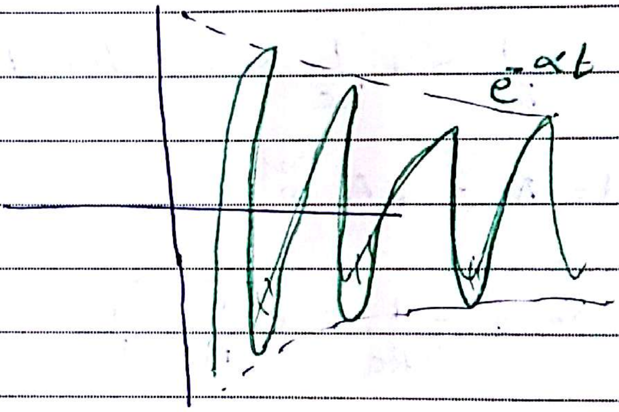


2) if $\alpha^2 < w_0^2$

$$\cancel{\sqrt{\alpha^2 - w_0^2} = \sqrt{-\alpha^2 - w_0^2}}$$

$$\cancel{= \sqrt{-1}\sqrt{\alpha^2 - w_0^2}}$$

$$\cancel{= j}$$



$$\sqrt{\alpha^2 - w_0^2} = \sqrt{-w_0^2 - \alpha^2}$$

$$= \sqrt{-1}\,\sqrt{w^2 - \alpha^2}$$

$$= j\,w_d$$

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$= A_1 \overset{s_1}{e^{(-\alpha + jw_d)t}} + A_2 \overset{s_2}{e^{(-\alpha - jw_d)t}}$$

$S_{1,2}$ = are complex conjugate solution

$$v(t) = A_1 e^{(-\alpha + w_d)t} + A_2 e^{-(\alpha - w_d)t}$$

3) $\alpha^2 = w_0^2$ → critical damping

↓

difficult to obtain in practice

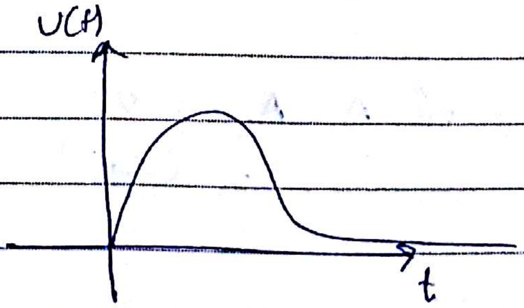
$S_{1,2} = -\alpha$

are real solution

$$v(t) = A_1 e^{-\alpha t} + A_2 e^{-\alpha t}$$

$$= (A_1 + A_2) e^{-\alpha t}$$

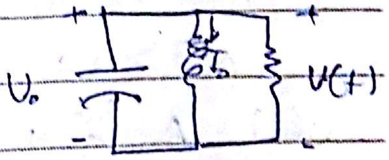
$$= \underline{A_0 e^{-\alpha t}} \times$$



$$v(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$$

$$\boxed{\begin{aligned} v(t) &= A e^{j\omega t} \\ &= A\cos\omega t + A j \sin\omega t \end{aligned}}$$

Euler



$\longrightarrow$ $\alpha^2 < \omega_0^2$ under damping

$$v(t) = A_1 e^{-\alpha t} e^{j\omega_d t} + A_2 e^{-\alpha t} e^{-j\omega_d t}$$ practic

$$= A_1 e^{-\alpha t}(\cos\omega_d t + j\sin\omega_d t) + A_2 e^{-\alpha t}(\cos\omega_d t - j\sin\omega_d t]$$

$$= e^{-\alpha t}\underbrace{(A_1 + A_2)}_{B_1}\cos\omega_d t + \underbrace{j\, e^{-\alpha t}(A_1 - A_2)}_{B_2}\sin\omega_d t$$

$$= B_1 e^{-\alpha t}\cos\omega_d t + B_2 e^{-\alpha t}\sin\omega_d t.$$

$$v(0) = \boxed{V_0 = B_1} \quad ①$$

$$i_L + i_L + i_R = 0$$

$$\frac{c\,dv_c}{dt} + I_0 + \frac{V_0}{R}$$

$$c\left[-B_1 e^{-\alpha t}\omega_d\, S\cancel{in}\,\omega_d t - B_1\cos\omega_d t\,\alpha e^{-\alpha t} + B_2 e^{-\alpha t}\omega_d\cos\omega_d t - B_2\sin\cancel{\omega_d t}\,\alpha e^{-\alpha t}\right]$$

$$c\left[B_2\omega_d - B_1\alpha\right] + I_0 + \frac{V_0}{R} = 0 \quad \ldots \quad ②$$

No.

→ $\alpha^2 = \omega_0^2$

$v(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$

$v(0) = V_0 = D_2$ — ①

V_c' $i_c + i_L + i_R = 0$

$$\frac{c\,dv_c(t)}{dt} + I_0 + \frac{V_0}{R} = 0$$

$$c\left[-D_1 \cancel{t\alpha e^{-\alpha t}} + D_1 e^{-\alpha t} + D_2(-\alpha) e^{-\alpha t}\right]$$

$$cD_1 - c\alpha D_2 + I_0 + \frac{V_0}{R} = 0 \quad \text{.... ②}$$

(Characteristic of under damping)

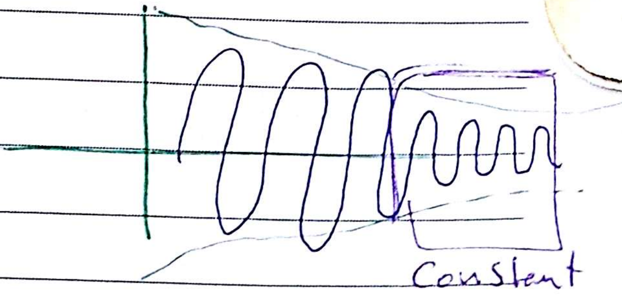
$P = \frac{V^2}{R}$

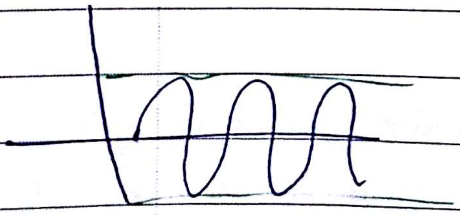
as dissipative losses ~~decreasing~~ decreases

$P_R \downarrow$ $R \uparrow$

$\alpha = \frac{1}{2Rc}$ $\omega_0 = \frac{1}{\sqrt{Lc}}$

$P \to 0$ $R \to \infty$ $\alpha \to 0$

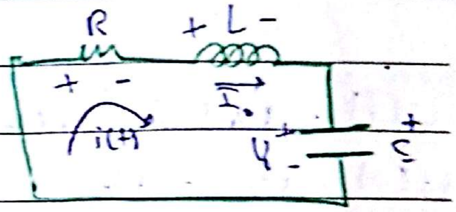




No. 22/11/2016

→ natural response for RLC series circuits

KVL:



$V_R + V_L + V_C = 0$

I_0R $v_R(0) + v_L(0) + v_c(0) = 0$

$\frac{d}{dt}$ $iR + L\frac{di}{dt} + \frac{1}{c}\int_0^t i(t')\,dt' + V_0 = 0$

$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{c}\,i(t) + 0 = 0$

$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}\,i(t) = 0$ ---①

$S^2 + \frac{R}{L}S + \frac{1}{LC} = 0$ characteristic equ.

$S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

→ in parallel RLC Circuit

$\alpha = \frac{1}{2RC}$, $\omega_0 = \frac{1}{\sqrt{LC}}$

→ in Seires RLC Circuit

$\alpha = \frac{R}{2L}$, $\omega_0 = \frac{1}{\sqrt{LC}}$

$S_{1,2} = -\frac{R}{2L} \mp \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

Reviews:-

→ $\alpha^2 > \omega_0^2$ over damped case

$i(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t}$

→ $\alpha^2 < \omega_0^2$ under damped

$i(t) = B_1 e^{\alpha t} \cos \omega_d t + B_2 e^{-\alpha t} \sin \omega_d t$

→ $\alpha^2 = \omega_0^2$ critical

$i(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$

$V_L(t)\Big|_{t=0} = L\frac{di_L(t)}{dt}\Big|_{t=0}$ = L[derive the proper equation for $i_L(t)$ depending on case of α^2 and ω_0^2 relationship]

$\alpha^2 > \omega_0^2$

$L \frac{d}{dt} i_L(t) = L A_1 S_1 e^{S_1 \cancel{t}\, t_0} + L A_2 S_2 e^{S_2 \cancel{t}\, t_0}$

$v_L(t)\Big|_{t=0} = L A_1 S_1 + L A_2 S_2$

$I_0 R + L A_1 S_1 + L A_2 S_2 + V_0 = 0 \dots$ ①

$I_0 = A_1 + A_2$ --- ②

$\Rightarrow$ To find constants B_1, B_2

$\alpha^2 < \omega_0^2$ underdamped case

$I_0 = B_1$ —①

$V_R + V_L + V_C = 0$

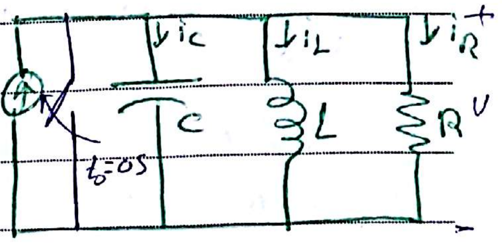
$R I_0 + \overset{L\omega_d B_2 - \alpha L B_1}{(\dots)} + V_0$

$V_L = L \frac{di}{dt}\Big|_{t=0}$

$$L\frac{di}{dt}\Big|_{t=t_0=0} = -L B_1 e^{-\alpha t} \cancel{\omega_d \sin \omega_d t} + B_1 L(-\alpha) e^{-\alpha t} \cos \omega_d t$$
$$+ L B_2 e^{-\alpha t} \omega_d \cos \omega_d t + L B_2 \cancel{(-\alpha) e^{-\alpha t}} \sin \omega_d t$$

No.

The step response & "forced response" for RLC parallel Circuits



$\boxed{t>0}$

$$I_s = i_L + i_c + i_R$$

$$\frac{d}{dt}\left[I_s = \frac{1}{L}\int_{-\infty}^{t} V_L(t)\,dt + C\frac{dv_c}{dt} + \frac{v_c}{R} \right]$$

$$\Downarrow$$

$$\frac{1}{L}\int_{0}^{t} v_c(t)\,dt + I_0$$

$$0 = \frac{v}{L} + C\frac{dv^2}{dt^2} + \frac{1}{R}\frac{dv}{dt}$$

$\alpha^2 > \omega_0^2$ $\quad$ $\alpha^2 < \omega_0^2$ $\quad$ $\alpha^2 = \omega_0^2$

KCL :-

$$I_s = i_L + C\frac{dv}{dt} + \frac{v}{R} \quad \ldots ⊛$$

$$V_L = V_c = V_R = V$$

$$V_L = L\frac{di}{dt}$$

$$\frac{dv}{dt} = L\frac{d^2 i}{dt^2}$$

back to ⊛

$$\boxed{I_s = i_L + CL\frac{d^2 i}{dt^2} + \frac{L}{R}\frac{di}{dt}}$$

To solve this equation $\left\langle \begin{array}{l} \text{direct} \\ \text{indirect} \end{array} \right.$

$t<0$

$V_o = 0$ (The switch is closed)

back to $t>0$

$\rightarrow \alpha^2 > \omega_0^2 \quad v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$\alpha^2 < \omega_0^2 \quad v(t) = B_1 e^{-\alpha t} \cos \omega_d t + B_2 e^{-\alpha t} \sin \omega_d t$

$\alpha^2 = \omega_0^2 \quad v(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$

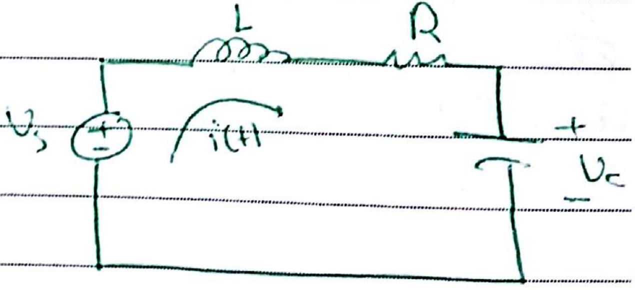
$i_L(t) = I_f + A_1' e^{s_1 t} + A_2' e^{s_2 t} \qquad \alpha^2 > \omega_0^2$

$i(t) = I_f + B_1' e^{-\alpha t} \cos \omega_d t + B_2' e^{-\alpha t} \sin \omega_d t \qquad \alpha^2 < \omega_0^2$

$i(t) = I_f + D_1' t e^{-\alpha t} + D_2' e^{-\alpha t} \qquad \alpha^2 = \omega_0^2$

The step response for RLC seires Circuit :-

kvl



$-V_s + V_L + V_R + V_c = 0 \quad --(*)$

$-V_s + L\frac{di}{dt} + iR + \frac{1}{C}\int_{-\infty}^{t} i(t')\,dt' = 0$

$0 + L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{1}{C} i(t) = 0$

$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

or

$i(t) = B_1 e^{-\alpha t} \cos \omega_d t + B_2 e^{-\alpha t} \sin \omega_d t$

or

$i(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$

~~$v_c(t)$~~

$v_L = L\frac{di}{dt}$, $i_c = C\frac{dv_c}{dt}$

$\boxed{\text{back to } \circledast}$

$-v_s + v_L + \overset{iR}{\underline{v_R}} + v_c = 0$

$-v_s + L\frac{di}{dt} + iR + v_c = 0$

$-v_s + L\frac{d}{dt}\left(C\frac{dv_c}{dt}\right) + C\frac{dv_c}{dt}R + v_c = 0$

$-v_s + LC\frac{d^2v_c}{dt^2} + RC\frac{dv_c}{dt} + v_c = 0$

$v_c(t) = v_f + A_1' e^{s_1 t} + A_2' e^{s_2 t}$

ⓞⓡ

$v_f + B_1' e^{-\alpha t}\cos w_d t + e^{-\alpha t}\sin w_d t$

ⓞⓡ

$v_f + D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$

No. 24/11/2016

$x(t) < \begin{matrix} i(t) \\ v(t) \end{matrix}$

8.7 General 2nd order RLC circuit analysis

① Determine the initial conditions

$x(0^-)$, $\frac{dx(0^+)}{dt}$

$i(0^-)$ $v(0^-)$ $\frac{di(0^-)}{dt}$, $\frac{dv(0^-)}{dt}$

② Determine the final value $x_{ss}(t)$

$x_{ss}(t)_{\to\infty} = x(\infty)$

③ turn off all independent sources and find the transient response $x_t(t)$ by applying kcl or kvl

④ solve the charateristic equation of 2nd order DE

$$a\frac{d^2v}{dt^2} + b\frac{dv}{dt} + cv = 0$$

$as^2 + bS + c = 0$ $S_{1,2} <$

⑤ find the proper form of the transiant response

$x(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ overdamped solution

$x(t) = B_1 e^{-\alpha t} \cos w_d t + B_2 e^{-\alpha t} \sin w_d t$

$x(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$

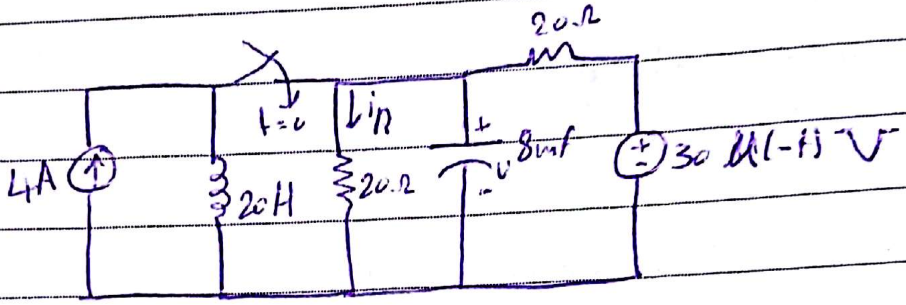
⑥ express the total response for x(t)

$$x(t) = x_t(t) + x_{ss}(t)$$

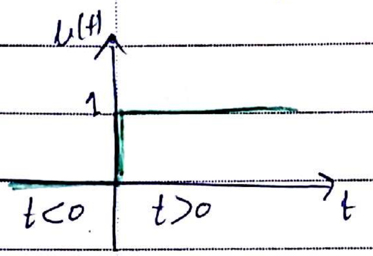
No.

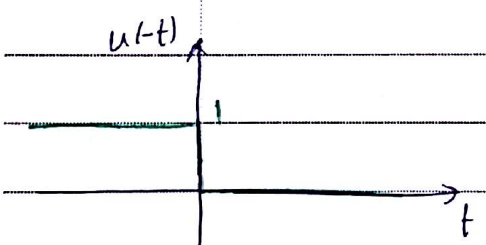
⑦ Find the Values of the Constants in the total Solution using $x(0)$ at $\frac{dx(0)}{dt}$

Ex 88
p. 335

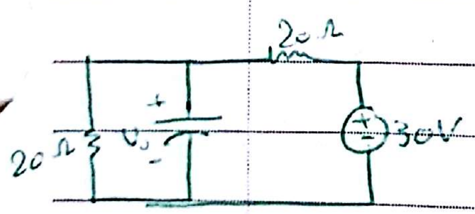


find $i(t)$, $i_R(t)$ $t > 0$

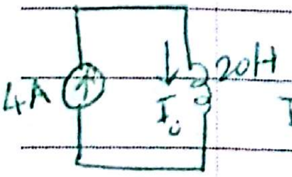




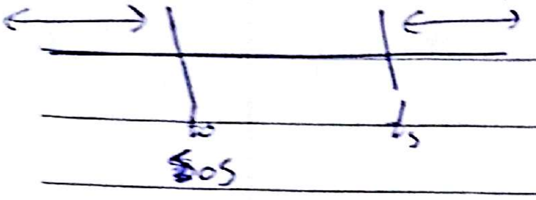
$t < 0$



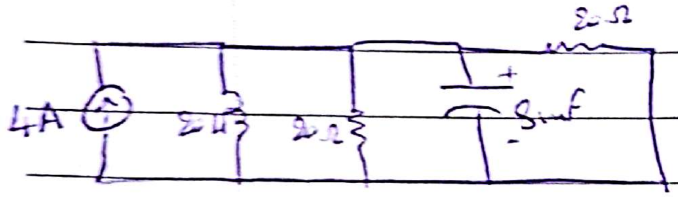
$$V_0 = \frac{20}{20+20}(30) = 15V$$

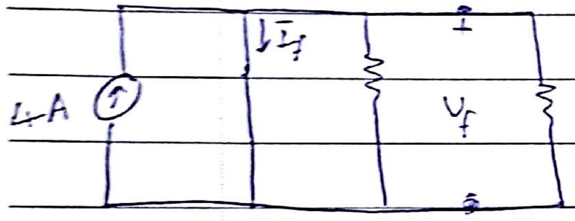


$I_0 = 4A$



$t \to \infty$

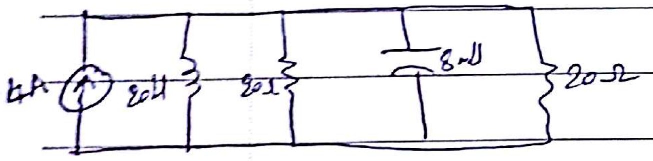


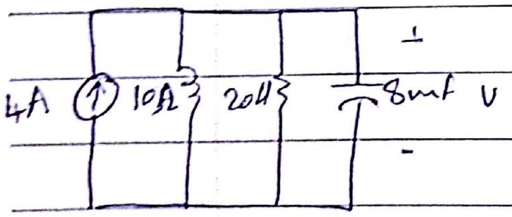


$I_f = 4A$

$V_f = 0\ V$

$t > 0$





$\alpha = \frac{1}{2RC} = \frac{1}{2\times10\times8\times10^{-3}} = 6.25\ rad/s$

$\sqrt{\underline{\underline{\alpha^2}} - w_0}$

bigger

So it's over damping

$w_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20\times8\times10^{-3}}} - 2.5\ rad/s$

$S = -\alpha I \sqrt{\alpha^2 - w_0^2}$

$S_{1,2} = -11.978\ s^{-1}, -0.5218\ s^{-1}$ $\quad v(t) = A_1 e^{-11.978t} + A_2 e^{-0.5218t}\ V$

$i_L(t) = \underset{4}{\cancel{I_s}} + A_1' e^{-11.978t} + A_2' e^{-0.5218t}$ A ... ⊛

$4 = 4 + A_1' + A_2'$

$A_1' + A_2' = 0$ .... ①

$v_L = L \frac{di(t)}{dt}$

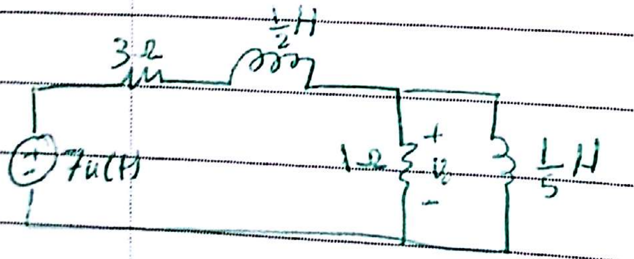
$v_L(t) = 20 \frac{d(⊛)}{dt}$

$A_1' = -0.0655$

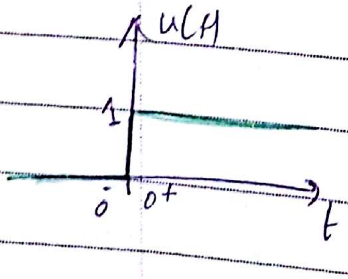
$A_2' = 0.0655$

$i_L(t) = 4 + 0.0655 \left[e^{-0.521t} - e^{-11.976t} \right]$ A

Ex 8.10
P. 339

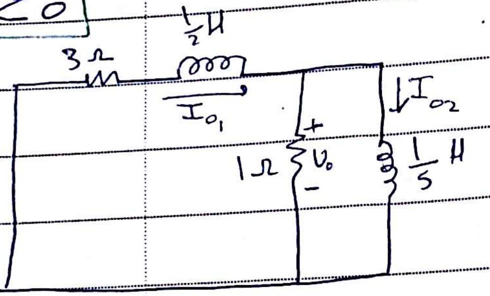


$v_o(t)$ $t > 0$



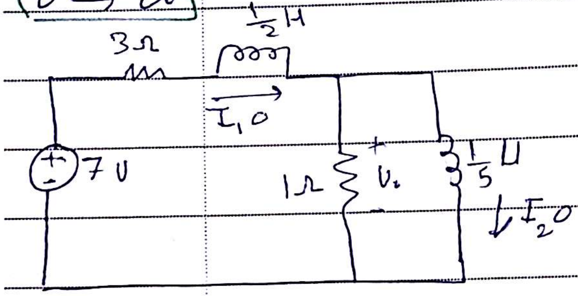
No.

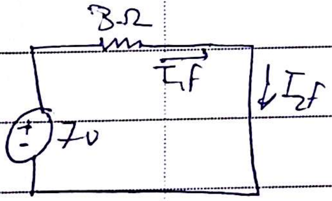
$\boxed{t<0}$



$I_{01} = I_{02} = 0A$

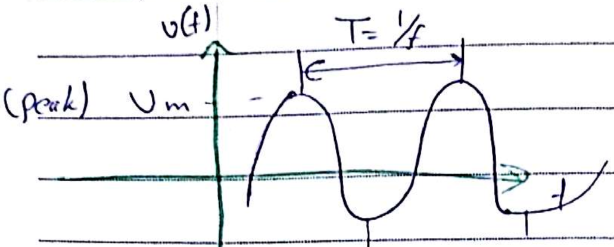
$\boxed{t\rightarrow\infty}$





$I_{2f} = I_{1f} = \frac{7}{3}A$

No. "CH. 9" 28/11/2016

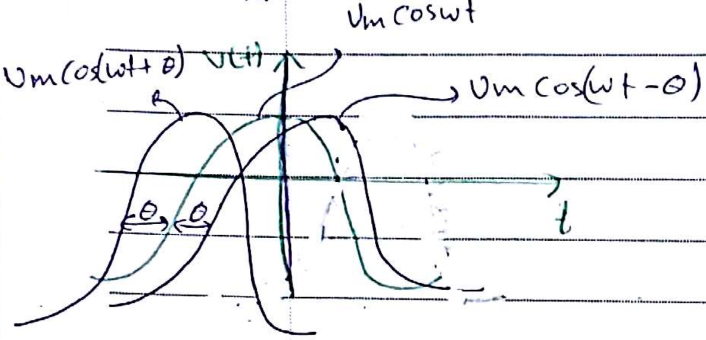


$$v(t) = V_m \cos(\overset{2\pi f}{\omega t} \pm \theta)$$

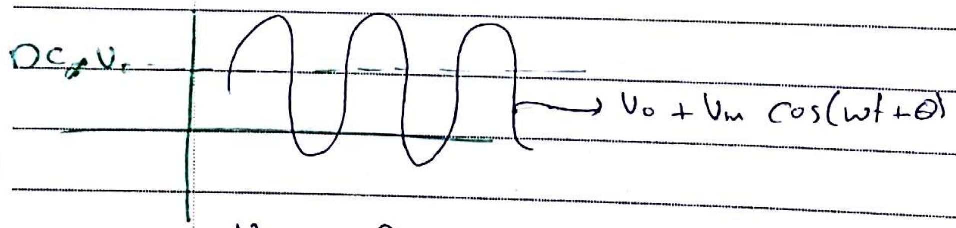
$+ \rightarrow$ advance

$- \rightarrow$ delay

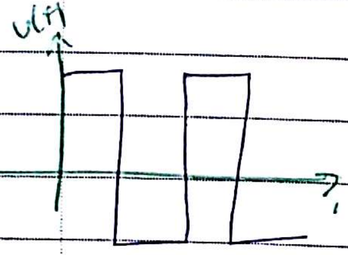
$i(t) = I_m$

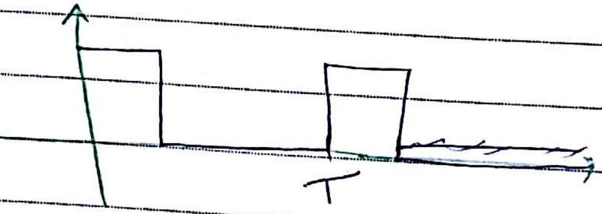


$$V_{rms} = \sqrt{\frac{1}{T}\int_T \underset{V_m\cos(\omega t+\theta)}{v^2(t)}\,dt} = 0.707\ V_m$$



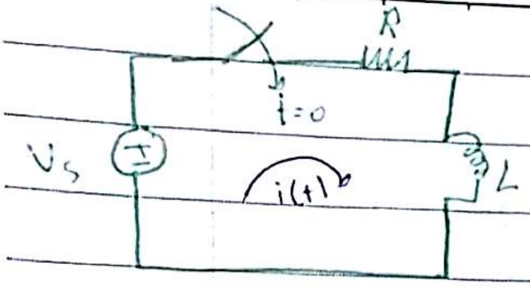
V_{mean} = DC component of a signal = $\frac{1}{T}\int_T v(t)\,dt$





$\overline{v(t)}$ = averag ?!

Vrms

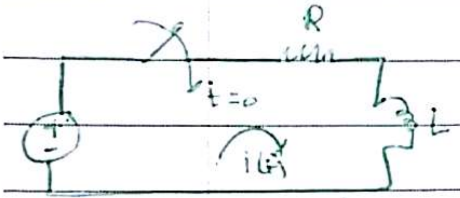


$$i(t) = I_f + (I_o - I_f)e^{-t/\tau}$$

$$\tau = \frac{L}{R}$$

$$I_f = \frac{V_s}{R}$$

$$I_o = 0A$$



$v_s(t) = V_m \cos(\omega t + \phi)$

$$-V_s(t) + i_R + L\frac{di}{dt} = 0$$

$$-V_m \cos(\omega t + \theta + i_R + L\frac{di}{dt} = 0$$

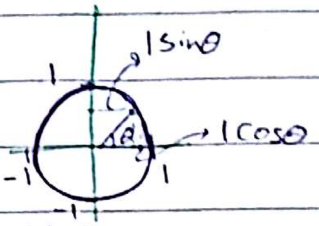
$$i(t) = \underbrace{\frac{-V_m}{\sqrt{R^2+\omega^2L^2}}\cos(\theta - B)e^{-\frac{R}{L}t}}_{\text{transient solution}} + \underbrace{\frac{V_m}{\sqrt{R^2+\omega^2L^2}}\cos(\omega t+\phi-B)}_{\text{steady state solution}}$$

$$\theta = \frac{\omega L}{R}$$

→ phasor is a complex number that carries the amplitude and phase information of a sinusoidal signal

$$v(t) = A\cos(\omega t + B)$$

Euler Representation



$v(t) = R_e\{A e^{j(\omega t+\theta)}\}$

$= R_e\{A\cos(\omega t+\theta) + jA\sin(\omega t+\theta)\} = R_e\{A e^{wt} e^{j\theta}\}$

$1e^{j\theta} = 1\cos\theta + j1\sin\theta$

$= A\cos(\omega t+\theta)$

$v_s(t) \rightarrow \vec{V}_s = A e^{j\theta} = A\underbrace{\cos\theta}_{a} + jA\sin\theta$

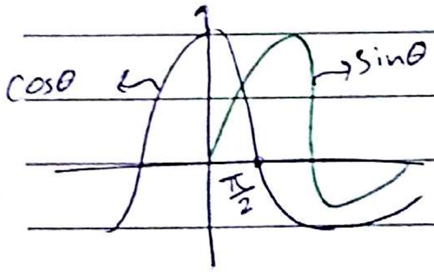
$a + jb$

phasor representation of the source voltage

$v(t) = 5\sin 2000t \quad V$

$= 5\cos(2000t - \frac{\pi}{2})$

$e^{j\theta_1} \cdot e^{j\theta_2} = e^{j(\theta_1+\theta_2)}$



$e^{j\theta_1} + e^{j\theta_2} \neq e^{j(\theta_1+\theta_2)}$

$\frac{e^{j\theta_1}}{e^{j\theta_2}} = e^{j(\theta_1-\theta_2)}$

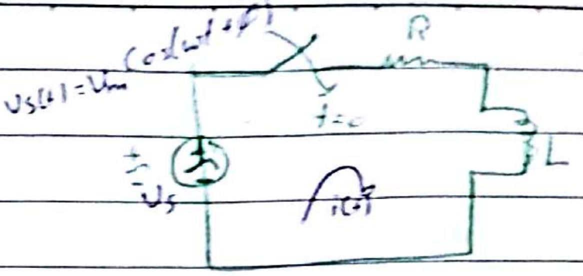
$\vec{V}_s = 5\angle -\frac{\pi}{2} \, V$

$= 5e^{-j\frac{\pi}{2}} \quad V$

$= 5\angle -90^\circ$

$$-V_s + Ri + L\frac{di}{dt} = 0$$

$$Ri + L\frac{di}{dt} = V_s$$



$$i(t) = I_m \cos(\omega t + \beta)$$

$$Re\{I_m R e^{j(\omega t+\beta)}\} + Re\{j\omega L I_m e^{j(\omega t-\beta)}\} = Re\{V_m e^{j(\omega t+\phi)}\}$$

$$\frac{Ld}{dt}\left[Re\left[I_m e^{j(\omega t+\beta)}\right]\right]$$

$$Re\{j I_m \omega L e^{j(\omega t+\beta)}\}$$

$$\boxed{R I_m^{j\beta} + j\omega L I_m e^{j\beta} = V_m} e^{j\phi}$$

$\frac{L}{R}$ ↓ $\frac{\omega}{R}$

L $j\omega L$

phasor equation for RL circuit

$$I_m e^{j\beta}[R + j\omega L] = V_m e^{j\phi}$$

$$\underbrace{v_1(t) + v_2(t)}_{\text{time domain}} \Longleftrightarrow \underbrace{\vec{V_1} + \vec{V_2}}_{\text{Frequency}}$$

$$I_m = \frac{V_m}{\underset{R}{\cancel{R}} + j\omega L} e^{j(\phi-\beta)}$$

No. 30/11/2016

Ex:- $y_1 = 20 \cos(wt - 3i)$

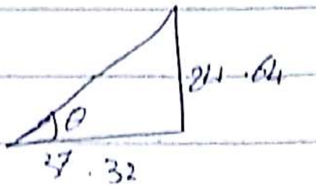
$y_2 = 40 \cos(wt + i_0)$

$y_1 + y_2 \Longleftrightarrow Y_1 + Y_2$

Sol:- $y_1 = 20 \cos wt (\cos 30^\circ + \sin wt (\sin 30^\circ)$

$y_2 = 40 \cos wt (\cos 60) - 40 \sin wt (\sin 60^\circ)$

$y_1 + y_2 = 37.32 \cos wt - 24.64 \sin wt$



→ phasors are complex quantities and characterized by frequency component (   ).

frequency ← $\vec{I} = 10 \angle 30^\circ$ mA $= 10 e^{j30^\circ} = 10 [\cos 30^\circ + j \sin 30^\circ]$

$w = 1000$ rad/sec $= 10 \underbrace{\cos 30^\circ}_{a} + j \underbrace{10 \sin 30^\circ}_{+ j b}$

T

$f = \frac{1}{T}$ $\qquad i(t) = 10 \cos(1000t + \underset{\frac{\pi}{6}}{\underline{30^\circ}})$ mA

$\omega = 2\pi f$

No.

$y_1(t) + y_2(t) \longleftrightarrow \vec{Y}_1 + \vec{Y}_2$

$\uparrow \; \uparrow$

must have same frequency

$17.32 - j10$

$y_1 = 20\cos(\omega t - 30^\circ) \rightarrow \vec{Y}_1 = 20 \angle -30^\circ = a + jb = 20\cos(-30) + j20\sin(-30)$

$y_2 = 40\cos(\omega t + 60^\circ) \rightarrow \vec{Y}_2' = 40 \angle 60^\circ = c + jd = 40\cos 60 + j40\sin 60$

$20 + j34.64$

$\vec{Y}_1 + \vec{Y}_2$

$\vec{Y}_1 + \vec{Y}_2 = 37.32 + j24.64 = \sqrt{37.32^2 + (-24.64)^2} = \angle \tan^{-1} \frac{24.64}{37.32}$

$= 44.72 \angle 33.43^\circ$

$y_1 + y_2 = 44.72\cos(\omega t + 33.43)$

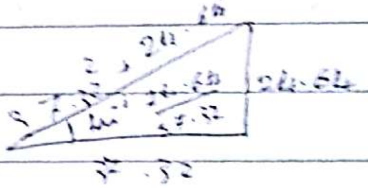
$\rightarrow y_1 = 20[\cos\omega t \cos(30^\circ) + \sin\omega t \sin 30^\circ]$

$\rightarrow y_2 = 40[\cos\omega t \cos(60^\circ) - \sin\omega t \sin 60^\circ]$

$y_1 + y_2 = 44.72 \frac{37.32}{44.72} \cos\omega t - 44.72 \frac{24.64}{44.72} \sin\omega t$

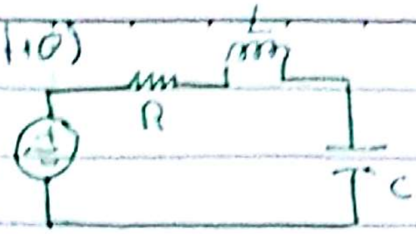
$= 44.72 \cos 33.43 \cos\omega t - \sin 33.43^\circ \sin\omega t$

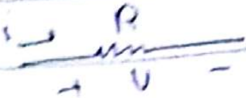
$= 44.72\cos(\omega t + 33.43^\circ)$



No.

$V_s = \boxed{V_m} \cos(\omega t + \theta)$





$V_s = \underline{V_m \angle +\theta}$

$i = I_m \cos(\omega t + \theta_i)$ A

$\bar{I} = I_m \angle \theta_i$ A

$t \rightarrow f$

$= I_m e^{j\theta_i}$

$\rightarrow V = iR$

$= R I_m \cos(\omega t + \theta_i)$

$= R I_m \cancel{Re} \left\{ e^{j(\omega t + \theta_i)} \right.$

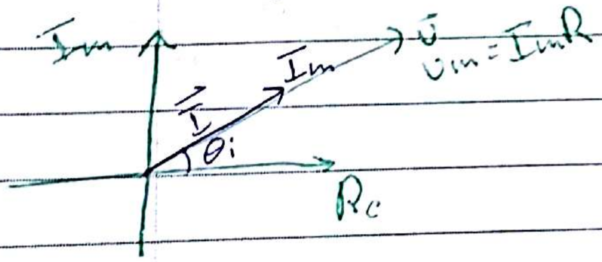
$\left. e^{j\omega t} e^{j\theta_i} \right\}$

$\vec{V} = R \underbrace{\boxed{I_m e^{j\theta_i}}}_{\bar{I}_m}$

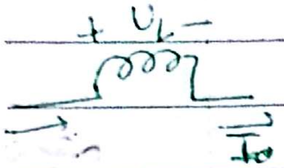
$\vec{V} = \underset{\overset{\uparrow}{\text{impedence}}}{R} \vec{I}_m$ ohm's law

$z = a + j\cancel{b}^{0}$

→ Impedence $(\bar{Z})$ is a complex number but not a phasor because no freq. component is attached to it.



No.



$i = I_m \cos(\omega t + \theta_i)$ A

$\vec{I} = I_m \angle \theta_i$ A

$V_L = L \frac{di}{dt}$

$= L \frac{d}{dt}\left[I_m \cos(\omega t + \theta_i) \right]$

$= L\, I_m \omega_m \sin(\omega t + \theta_i)$

$= -L\, I_m \omega_m \cos(\omega t + \theta_i - \frac{\pi}{2})$

$= -L\, I_m \omega_m \cancel{Re}\left\{ e^{j(\omega t + \theta_i - \frac{\pi}{2})} \cancel{e^{j\omega t}}\, e^{j\theta_i} \cdot \underbrace{\boxed{e^{-j\frac{\pi}{2}}}}_{-j} \right\}$

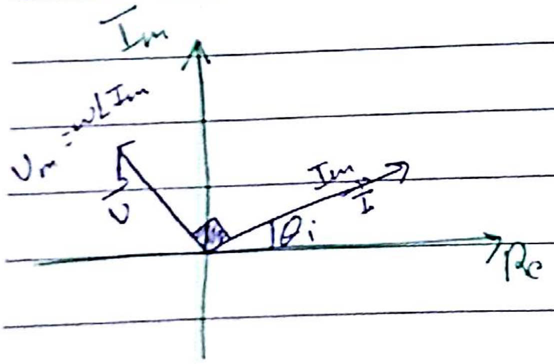
$\vec{V} = j \omega L \underbrace{\boxed{I_m e^{j\theta_i}}}_{I}$

$e^{j\frac{\pi}{2}} = \cos \frac{\pi}{2} + j \sin \frac{\pi}{2} = j$

$e^{-j\frac{\pi}{2}} = \cos \frac{\pi}{2} - j \sin \frac{\pi}{2} = -j$

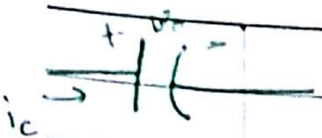
$e^{\pm j\pi} = \cos \pi \mp j \sin \pi = -1$

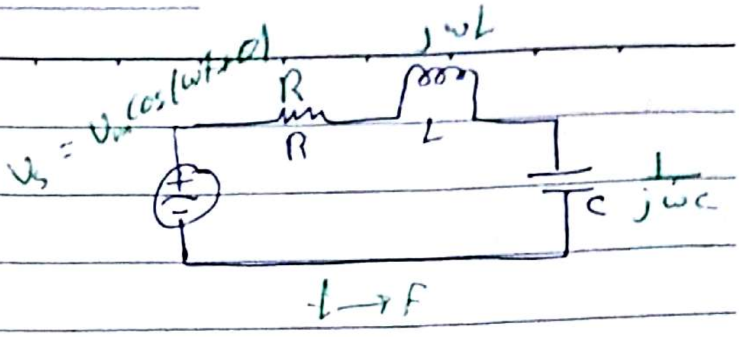
$\vec{V} = \underbrace{\boxed{j\omega L}}_{\vec{Z_L}} \vec{I}$



$v_L(t) = \omega L\, I_m \cos(\omega t + \theta_i \boxed{+ \frac{\pi}{2}})$

No.





$v_c = U_m\cos(wt+\theta_v)$

$i_c = \frac{C\,dv_c}{dt}$

$i_c = C\frac{d}{dt}\left[U_m\cos(wt+\theta_v)\right]$

$= C\,V_m(-w)\sin(wt+\theta_v)$

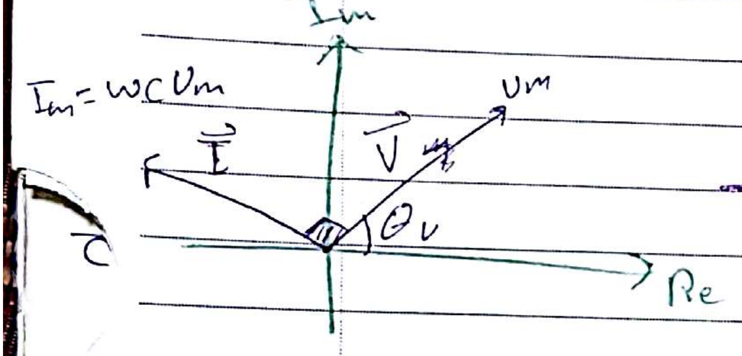
$= -C\,U_m w\cos(wt+\theta_v-\frac{\pi}{2})$

$= -C\,V_m w\,\mathrm{Re}\left\{ e^{j(wt+\theta_v-\frac{\pi}{2})} \right.$

$\left| e^{j\cancel{w}t}\, e^{j\theta_v}\, \underbrace{e^{-j\frac{\pi}{2}}}_{-j} \right.$

$\vec{I}_c = jCw\,U_m \underbrace{e^{j\theta_v}}_{\vec{U}}$

$$\boxed{\vec{U} = \underbrace{\frac{1}{jwc}}_{\vec{Z}}\vec{I}}$$



→ voltage is lagging the current

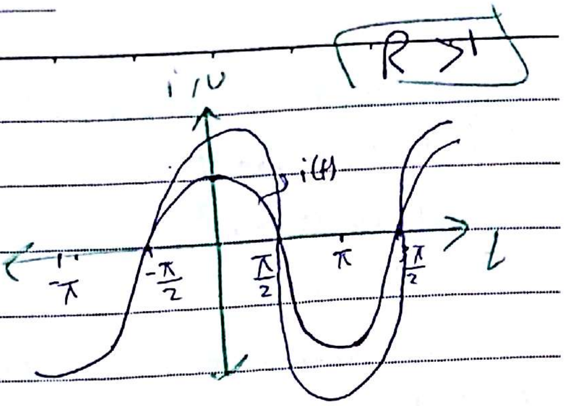
$i(t) = I_m\cos(wt+\cancel{\theta_i})$ 0

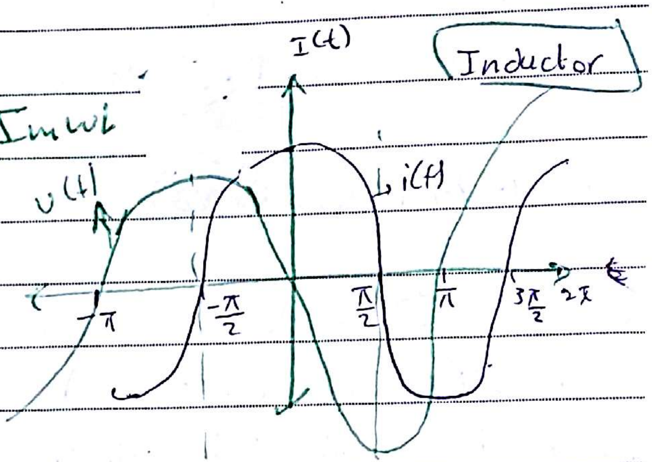
$U = iR$

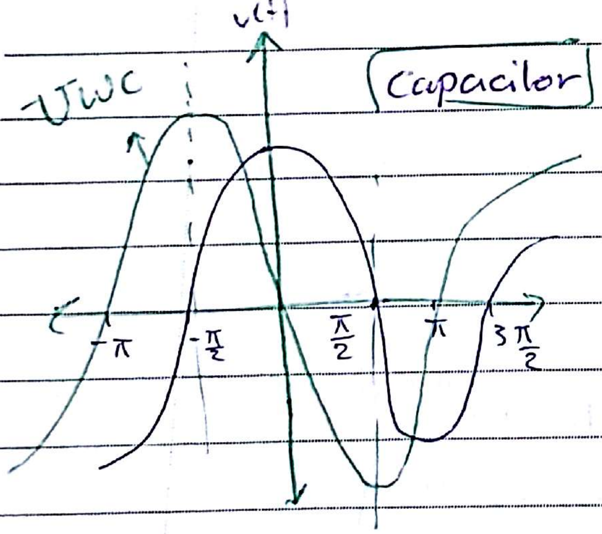
$\vec{U} = R\vec{I}$

$R>1$

No.


R >1
i, v
i(t)
-π
-π/2
π/2
π
3π/2
t

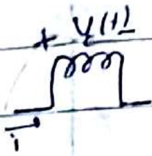

I(t)
Inductor
Imwl
v(t)
i(t)
-π
-π/2
π/2
π
3π/2
2π
t


v(t)
Capacilor
-VWC
-π
-π/2
π/2
π
3π/2

The current in a 20 mH inductor is 10 Cos (1000t + 30°) mA

ω = 10000 rad/s

→ inductive reactance



$X_L = 10000 \times 20 \times 10^{-3}$

$= 200 \Omega$

→ impedence of the inductor

$Z = jX_L = j200\Omega$ $\qquad Z = j\omega L$

$X_L = \omega L$

$\vec{V} = Z\vec{I} = j\,200\;10\angle 30°$

$= 200 \angle \pi/2 \; 10 \angle 30 = 2000 \angle 120° \text{ mV}$

$= 2 \angle 120° \text{ V}$

$i(t) = 10 \cos(10000t + 30°)$

$\vec{I} = 10 \angle 30° \text{ mA}$

$= 10(\cos 30° + j \sin 30°)$

→ find steady state voltage

$v(t) = 2\cos(10000t + 120°) \text{ V}$

kirchoff's voltage and current laws in frequency domain :-

$$\sum_{i=1}^{N} v_i(t) = 0 \implies \sum_{i=1}^{N} \vec{V}_i = 0$$

$$v_1(t) = V_{m_1} \cos(\omega t + \theta_1) \longrightarrow V_{m_1} \mathrm{Re}\{e^{j(\omega t + \theta_1)}\}$$

$$v_2(t) = V_{m_2} \cos(\omega t + \theta_2) \longrightarrow V_{m_2} \mathrm{Re}\{e^{j(\omega t + \theta_2)}\}$$

$$\vdots$$

$$V_N(t) = V_{m_N} \cos(\omega t + \theta_N) \longrightarrow V_{m_N} \mathrm{Re}\{e^{j(\omega t + \theta_N)}\}$$

$$\sum_{i=1}^{N} v_i(t) = V_1 + V_2 + V_3 + \cdots + V_N = 0$$

$V_{m_1} Re\{e^{j\omega t} e^{j\theta_1}\} + V_{m_2} Re\{e^{j\omega t} e^{j\theta_2}\} + \cdots + V_{m_N} Re\{e^{j\omega t} e^{j\theta_N}\} = 0$

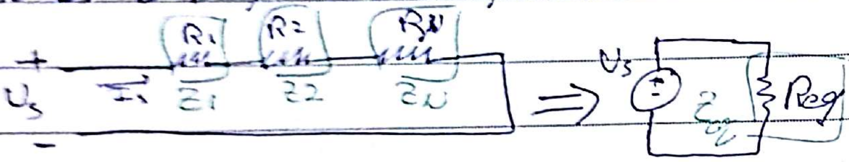
$V_{m_1} e^{j\theta_1} + V_{m_2} e^{j\theta_2} + \cdots + V_{m_N} e^{j\theta_N} = 0$

$\vec{V}_1 + \vec{V}_2 + \cdots + \vec{V}_N = 0$

kcl:-

$\sum_{j=1}^{N} i_j(t) = 0 \Longrightarrow \sum_{j=1}^{N} \vec{I}_j = 0$

series and parallel Impedance

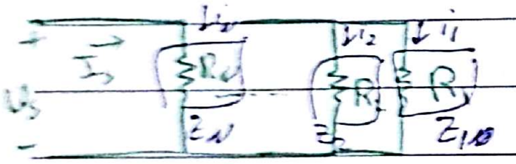


$= R_1 + R_2 + \cdots + R_N$

$Z_{eq} = Z_1 + Z_1 + \cdots + Z_N$

$V_s = I_s R_1 + I_s R_2 + \cdots + I_s R_N$

$I_s R_{eq}$



$Z_L = j\omega L$

$R_{eq} = \dfrac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}}$

$Z_C = \dfrac{1}{j\omega C}$

$Z_C = \dfrac{-j}{\omega C}$

$Z_{eq} = \dfrac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_N}}$

$z = a + j b$ → reactence X_L or X_c

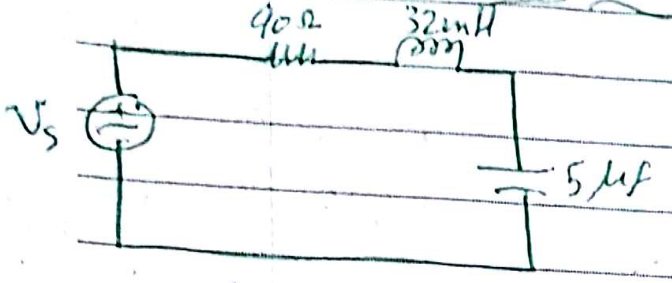
$a \rightarrow R$

$b \rightarrow C$, $\rightarrow L$

$z = a + j b$

$b \equiv L$ $\quad b > 0$

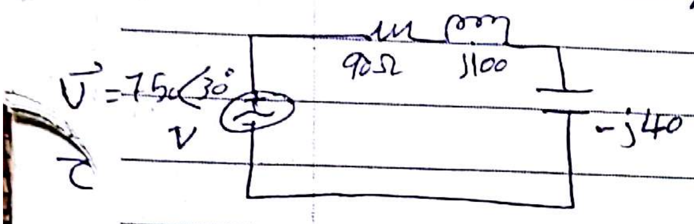
$b \equiv C$ $\quad b < 0$



$V_s(t) = 750 \cos(\underbrace{5000}_{\omega} t + 30°)\, V$

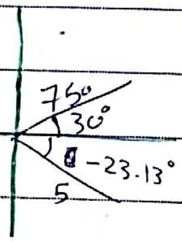
⇓

Find equivalent circuit in frequency domain



$Z_L = j\omega L = j\, 5000 \times 32 \times 10^{-3} = j\,160\ \Omega$

$Z_c = \frac{1}{j\omega c} = -j \frac{1}{5000 \times 5 \times 10^{-6}} = -j\,40\ \Omega$



→ calculate the steady state current?

$\vec{I} = \frac{\vec{V}}{Z_{eq}} = \frac{750∠30°}{150∠53.13°} = 5∠-23.13°\ A$

$Z_{eq} = Z_R + Z_L + Z_c = 90 + j160 - j40$

$= 90 + j120 = 150∠53.13°\ \Omega$

No.

Admittance Y unit is Siemens (S)

$$Y = \frac{1}{Z}$$

$$Z = \overset{\text{resistance}}{R} + j\,X \leftarrow \text{reactance}$$

$$Y = \underset{\text{conductance}}{G} + j\underset{\text{susceptance}}{B}$$

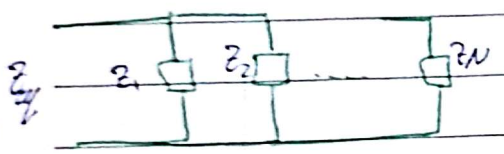
$$G \neq \frac{1}{R} \qquad B \neq \frac{1}{X}$$

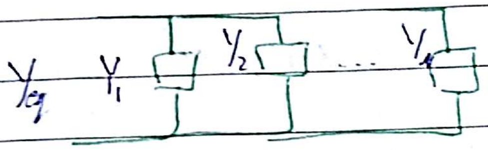
$$\frac{1}{R+jX} \cdot \frac{R-jX}{R-jX}$$

$$\frac{R-jX}{R^2+X^2}$$

$$\frac{R}{R^2+X^2} - j\,\frac{X}{R_2+X^2}$$

$$G + jB$$

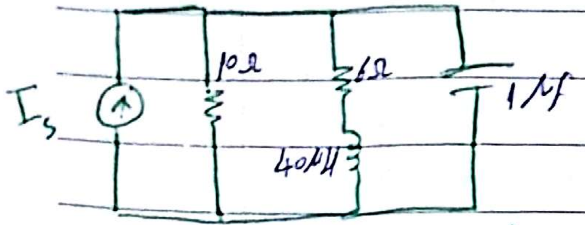




$$Z_{eq} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \dots + \frac{1}{Z_N}}$$

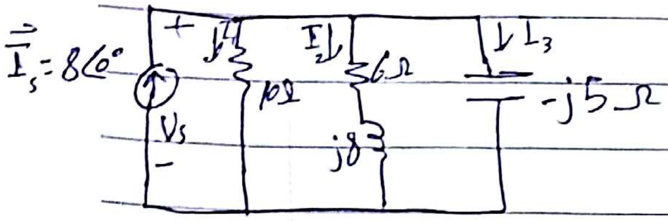
$$Y_{eq} = Y_1 + Y_2 + \dots + Y_N$$

	symbol	impedence	reactence	Admittance	Susecptance
Resistor	R	$Z_R = R$	0	$Y = \frac{1}{R}$	0
inductor	L	$Z = j\omega L$	ωL	$Y = \frac{1}{j\omega L}$	$\frac{-1}{\omega L}$
capaciter	C	$Z = \frac{1}{j\omega C}$ $\Downarrow$ $\frac{-j}{\omega C}$	$\frac{-1}{\omega C}$	$Y = j\omega C$	ωC



$i_s(t) = 8 \cos(200\,000 t)$

$\omega = 200\,000$ rad/sec



$Z_L = j\omega L = j\, 200\,000 \times 40 \times 10^{-6} = j8\,\Omega$

$Z_c = \frac{1}{j\omega C} = \frac{1}{j200\,000 \times 1 \times 10^{-6}} = -j5\,\Omega$

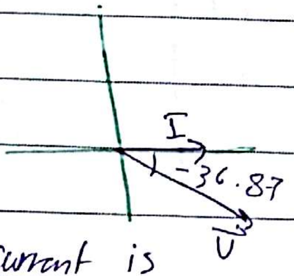
$Y_1 = \frac{1}{10}\,S$, $Y_2 = \frac{1}{6+j8} = \frac{1}{\sqrt{6^2+8^2}\angle \tan^{-1}\frac{8}{6}}$

$Y_3 = \frac{1}{-j5} = 0.2j\,S$

$Y_{eq} = Y_1 + Y_2 + Y_3$ = ~~0.06 - j0.08 S~~

$= 0.16 + j\,0.12$

$0.2 \angle 36.87°\,S$



Current is leading

$\vec{V} = Z I$

$Z = \frac{1}{Y} = \frac{1}{0.2\angle 36.870} = 5\angle -36.87°\,\Omega$

$V = 5\angle -36.87° \cdot 8\angle 0° = 40\angle -36.87°$

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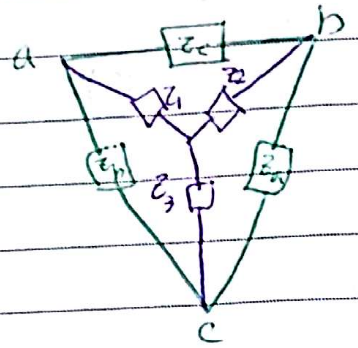
$v(t) = 40\cos(200\,000t - 36.87°)$

$\vec{\vec{I}}_1 = \frac{40\angle -36.87°}{10} = 4\angle -36.87°\,A$

Delta-to-wye conversion and vise versa



Δ-Y

$$Z_1 = \frac{Z_c Z_b}{Z_a + Z_b + Z_c}$$

$$Z_2 = \frac{Z_c Z_a}{Z_a + Z_b + Z_c}$$

$$Z_3 = \frac{Z_b Z_a}{Z_a + Z_b + Z_c}$$

Y-Δ

$$Z_a = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_1}$$

$$Z_b = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_2}$$

$$Z_c = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_3}$$

Time domain		frequency domain
$\frac{dv(t)}{dt}$	$\longleftrightarrow$	$jW\,\vec{V}$
$\int v(t)\,dt$	$\longleftrightarrow$	$\frac{V}{jW}$

No.

$$v(t) = V_m \cos(\omega t + \phi) \Longrightarrow \vec{V} = V_m \angle \phi$$

$$\frac{dv(t)}{dt} = \underbrace{V_m(t) \sin(\omega t + \phi)} = R_e \left\{ V_m e^{j(\omega t + \phi)} \right\}$$

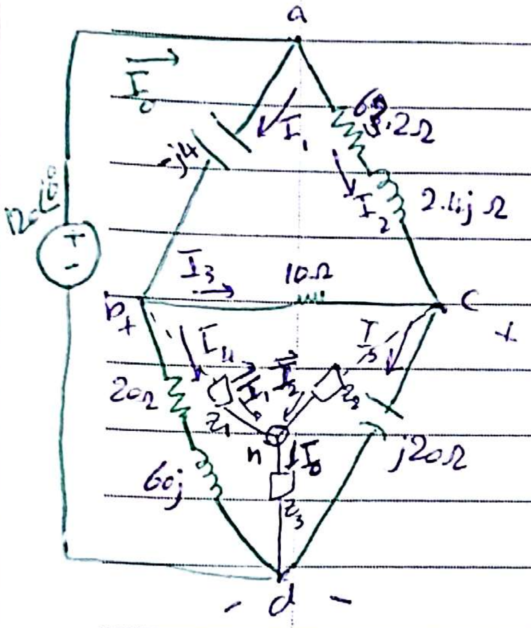
$$= R_e \left\{ V_m e^{j\omega t} e^{j\phi} \right\}$$

$$-V_m \omega \cos(\omega t + \beta - \frac{\pi}{2})$$

$$-V_m \omega \phi^{j\omega t} * e^{j\theta} e^{-j\frac{\pi}{2}}$$

$$j\omega (V_m e^{j\omega t} e^{j\phi})$$

Example :-



$\vec{I}_0, \vec{I}_1, \vec{I}_2, \vec{I}_3, \vec{I}_4, \vec{I}_5$
$\vec{V}_1$ and $\vec{V}_2$

$$Z_1 = \frac{(10)(20 + j60)}{10 + 20 + j60 - j20}$$

$$= \frac{200 + j600}{30 + j4} \cdot \frac{30 - j40}{30 - j40}$$ method ①

$$\frac{|\vec{Z}_1| \angle \theta_1}{|\vec{Z}_2| \angle \theta_2} = \frac{\vec{Z}_1}{\vec{Z}_2} \angle \theta_1 - \theta_2$$ method ②

$$= 12 + j4 \ \Omega$$

No.

$$Z_2 = \frac{(10)(-j20)}{30+j40} = -3.2 - j2.4\ \Omega$$

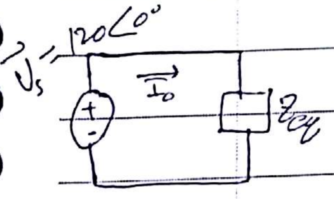
$$Z_3 = \frac{(-j20)(20+j60)}{30+j40} = 8 - j24\ \Omega$$

$$Z_{abn} = -j4 + 12 + j4 = 12\ \Omega$$

$$Z_{acn} = 63.2 + j2.4 - 3.2 - j2.4 = 60\ \Omega$$

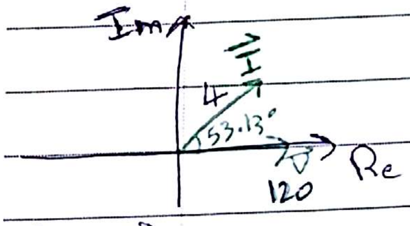
$$Z_{abn} \parallel Z_{acn} = \frac{12 \times 60}{12 + 60} = 10\ \Omega$$

$$Z_{eq} = 10 + 8 - j24 = 18 - j24\ \Omega$$



$$\vec{I}_o = \frac{\vec{V}}{Z_{eq}} = \frac{120\angle 0°}{18 - j24} = \frac{120\angle 0°}{30\angle -53.13°} = 4\angle 53.13°\ A$$

$$= 2.4 + j3.2\ A$$



$$\vec{V}_{nd} = \vec{I}_o ; Z_3$$

$$= 4\angle 53.13° (8 - j24)$$

$$= 4\angle 53.13° \quad 11\angle\theta$$

$$= 96 - j32\ V$$

KVL

$$-\vec{V}_s + \vec{V}_{an} + \vec{V}_{nd} = 0$$

$$\vec{V}_{an} = \vec{V}_s - \vec{V}_{nd}$$

$$= 120\angle 0° - 96 - j32 = 24 + j32\ V$$

No.

$$\vec{I}_1 = \frac{\vec{V}_{an}}{Z_1+(-j4)} = \frac{24+j32}{12} = 2+j2.67\,A$$

$$\vec{I}_2 = \frac{\vec{V}_{an}}{63.2+j2.4+Z2} = \frac{24+j32}{60} = 0.4+j0.53\,A$$

kVL

$$-\vec{V}_s + \vec{V}_{ab} + \overset{v_1}{\cancel{\vec{V}_{bd}}} = 0$$

$$\vec{V}_1 = \vec{V}_s - \vec{V}_{ab}$$

$$= 120\angle 0^\circ - (\vec{I}_1 \cdot (j4)) = 109.32 + j8\,V$$

kVL

$$-\vec{V}_3 + \vec{V}_{ac} + \overset{v_2}{\cancel{V_{cd}}} = 0$$

$$\vec{V}_2 = \vec{V}_3 - \vec{V}_{ac} \quad = 120\angle 0^\circ - (\vec{I}_2 \cdot (63.2+j24))$$

$$= 96 - j34.46\,V$$

kcl:

$$\vec{I}_1 = \vec{I}_3 + \vec{I}_4$$

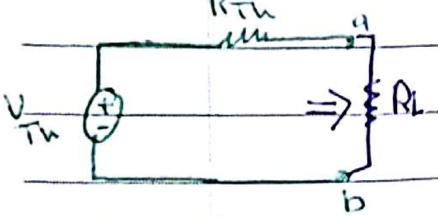
$$\vec{I}_3 = \vec{I}_1 - \boxed{\vec{I}_4} \longrightarrow \vec{I}_4 = \frac{v_1}{20+j60}$$

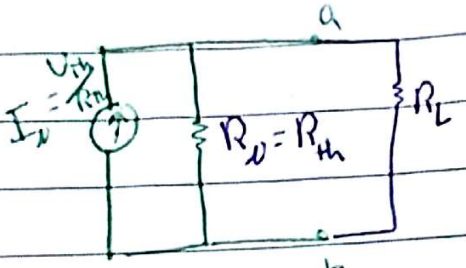
$$= 0.667 - j1.6\,A$$

$$= 1.33 + j4.27\,A$$

$$\vec{I}_5 = \frac{\vec{V}_2}{-j20} = 1.72 + j4.8\,A$$

No.





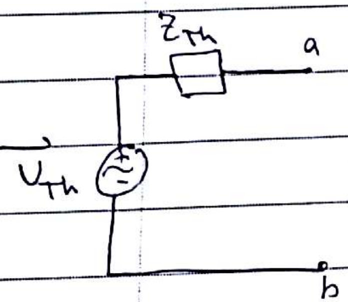
$V_{oc} = V_{Th}$

$I_{sc} = I_N$

- independent sources only
- independent and dependent
- dependent sources only

$R_{Th} = \frac{V_{Th}}{I_N} = \frac{V_{oc}}{I_{sc}}$

V_{Th} doesn't exist only R_{Th}



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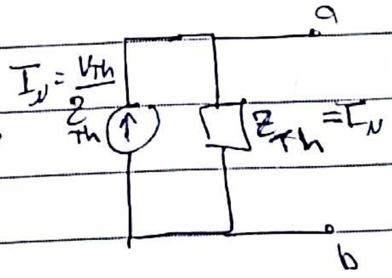
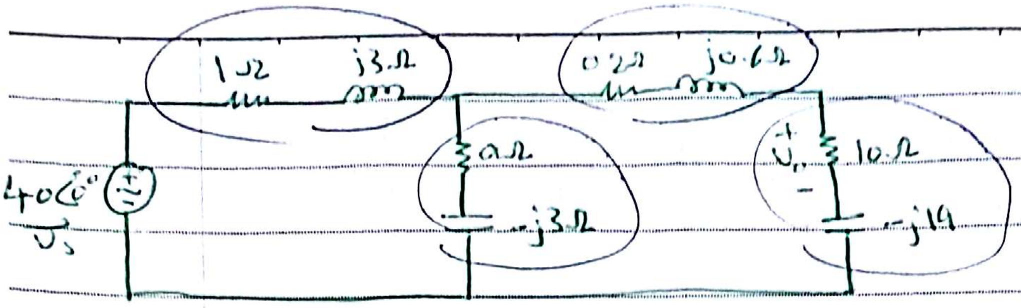
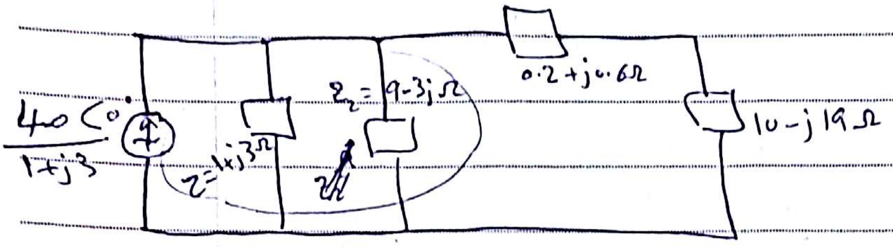
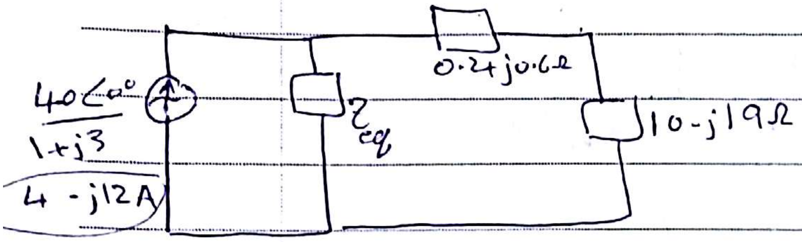
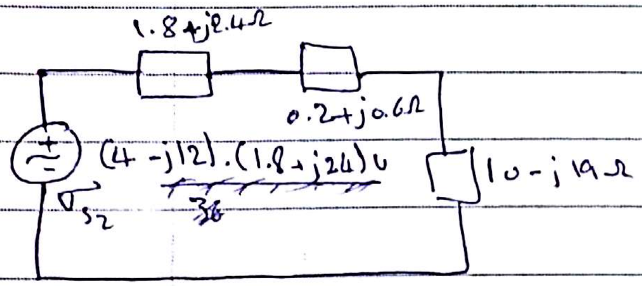






$$Z_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(1+j3)(9-3j)}{1+j3+9-j3} = 1.8 + j2.4\,\Omega$$





$$V_{s_2} = 36 - j\,12\ V$$

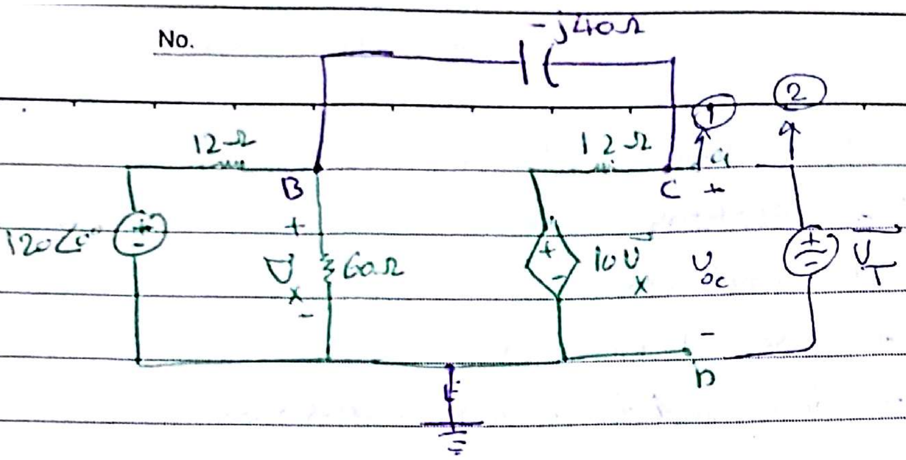
$$Z_{eq_2} = 1.8 + j2.4 + 0.2 + j0.6$$

$$= 2 + j3\,\Omega$$

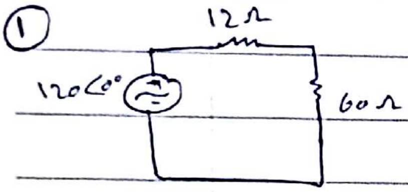
$$V_o' = \frac{Z_4}{Z_4 + Z_{eq2}} \cdot V_s \qquad V_o = \frac{10}{10+(-j19)} \cdot V_o'$$

$$= \frac{10 - j14}{10 - j19 + 2 + j3} \cdot (36 - j12) = 36.12 - j18.84\ V$$

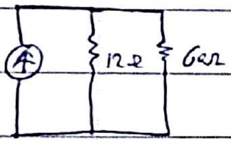
No.



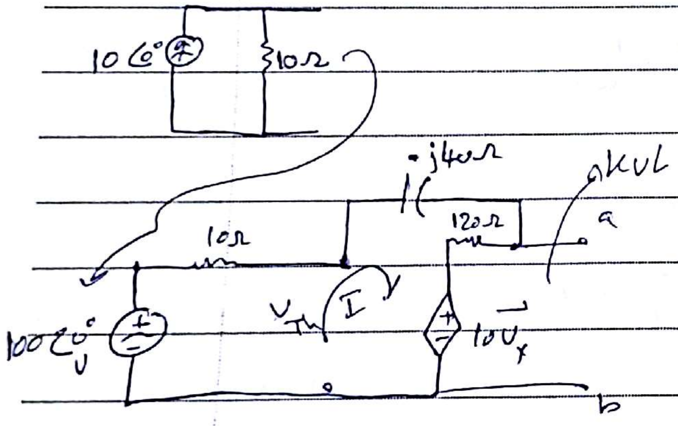
①



$=10\angle 0^\circ = \frac{120\angle 0^\circ}{12}$



$\Rightarrow \frac{60\times 12}{60+12} = \frac{720}{72} = 10\Omega$



$-10\vec{V}_x - 120\vec{I} + \vec{U}_{Th} = 0$

$-V_s + 10\vec{I} - j40(\vec{I}) + 120\vec{I} + 10\vec{V}_x = 0$

$-V_s + 10\vec{I} - 40j\vec{I} + 120\vec{I} + 10[100-10\vec{I}] = 0$

$\vec{U}_{Th} = 10\vec{V}_x + 120\vec{I}$

$\vec{I} = \frac{900}{-30+j40}$ $A = -10.8 - j14.4A$

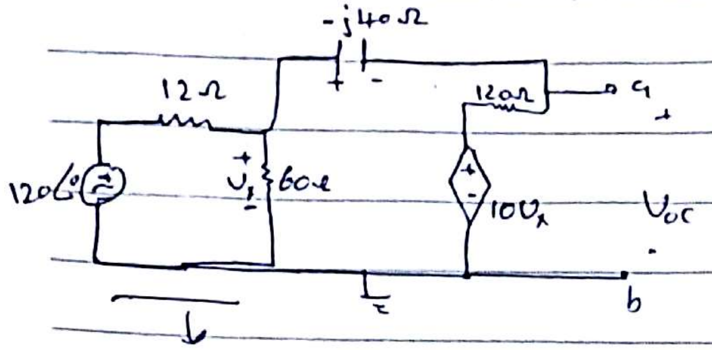
$\Rightarrow$

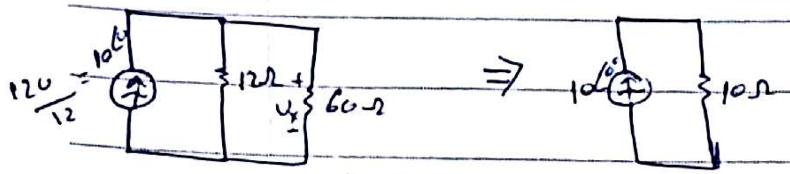
$\vec{U}_{Th} = 10[100-10\vec{I}] + 120\vec{I}$

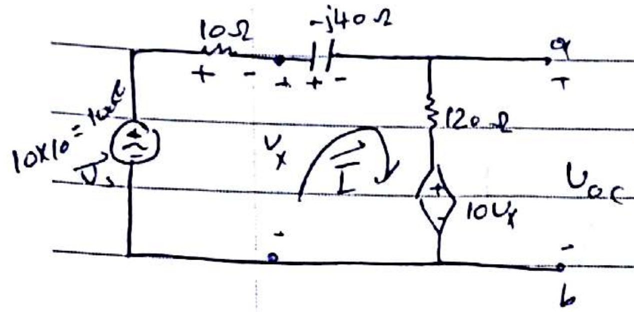
$= 18\angle -126.87^\circ A$

$= 1000 - 100\vec{I} + 120\vec{I}$

$V_{Th} = \cancel{V_x}\, 1000 + 20\vec{I}$







KVL: $-10V_x - 120\vec{I} + V_{oc} = 0$

$V_{oc} = 10V_x + 120\vec{I}$

but $V_x = 100\angle 0 - 10\vec{I}$

so $V_{oc} = 10(100 - 10\vec{I}) + 120\vec{I}$

$= 1000 + 20\vec{I}$ .. ①

KVL: $-100 + 10\vec{I} - j40\vec{I} + 120\vec{I} + 10(100 - 10\vec{I}) = 0$

$-100 + 130\vec{I} - j40\vec{I} + 1000 - 100\vec{I} = 0$

$\vec{I}(30 - j40) = -900$

$\vec{I} = -10.8 - j14.4$ A ... ②

Substituting ② in ①

$V_{oc} = V_{Th} = 1000 + 20(-10.8 - j14.4)$

$= 1000 - 216 - j288$

$\dot{V}_{Th} = 784 - j288$ V

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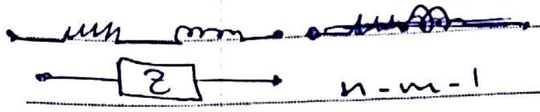
Nodal voltage method :-

① # of equations

# of nodes : n

# of essential nodes : n_e

# of inde. and dep. voltage sources : m



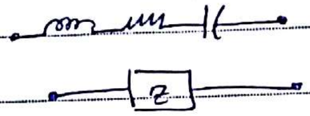
n-m-1

$n_e - 1$

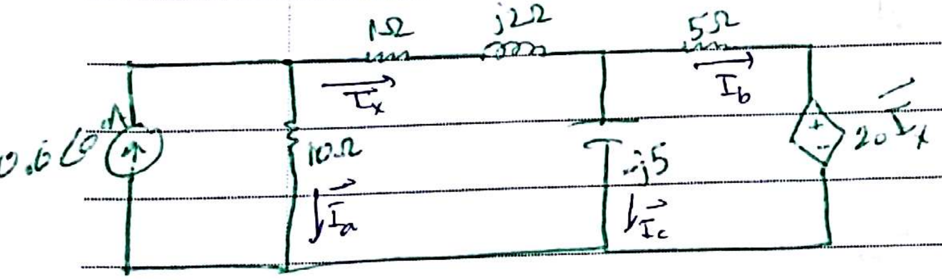
② every branch has ~~inde~~ impedence

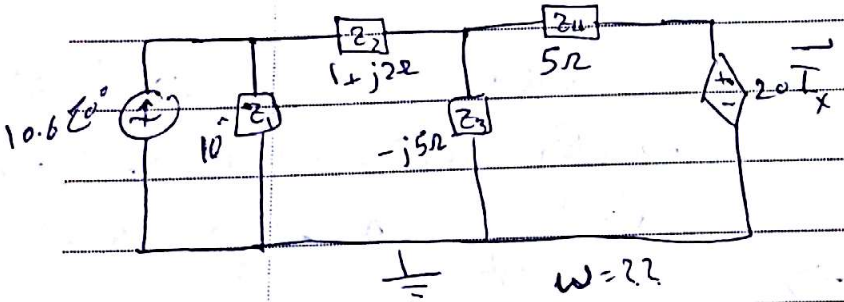
Z = a+jb

R C



$Z = R + j\omega L + \frac{1}{j\omega C}$





$n_e = 3$ m=4 n=1

$n_e - 1 = 3 - 1 = 2$

n - m - 1 = 4 - 1 - 1 = 2

ω = ??

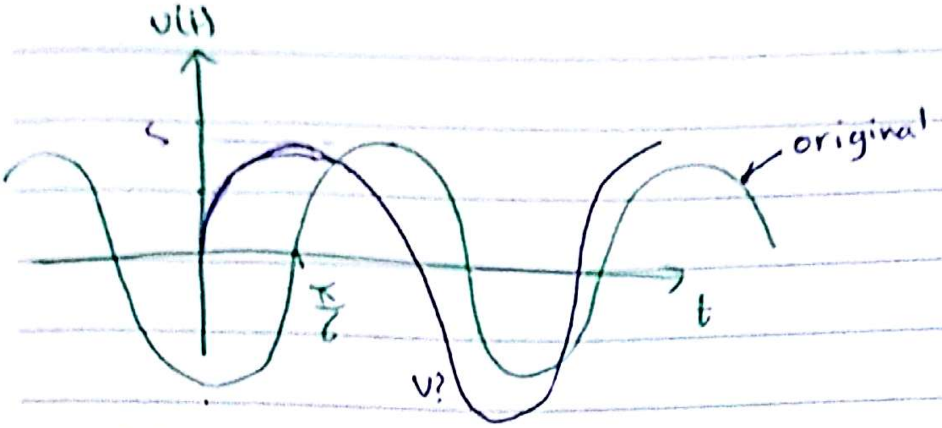
$+10.6 - \frac{\vec{V_a}}{Z_1} - \frac{\vec{V_a} - \vec{V_b}}{Z_2} = 0$

$10.6 - \frac{\vec{V_a}}{10} - \frac{\vec{V_a} - \vec{V_b}}{1+j2} = 0$

$10.6 + 0.1\vec{V_a} = \frac{V_a - V_b}{1+j2} = 0$

$\frac{10.6(1+j2) - 0.1\vec{V_a}(1+j2) - \vec{V_a} + \vec{V_b}}{(1+j2)} = 0$

$25j\vec{V_a} + (3 - 24j)V_b = 0$ .. ①



$\vec{V}$??.

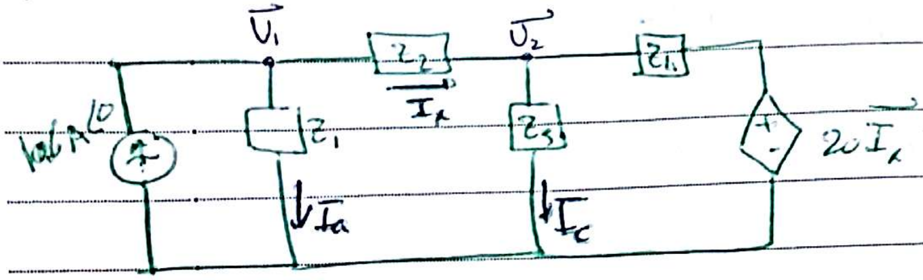
$v(t) = 5 \sin(\omega t + \phi)$

$= 5 \sin(\omega t + -\frac{\pi}{6})$

$= 5 \sin(\omega t - \frac{\pi}{6} - \frac{\pi}{2})$

$= 5 \cos(\omega t - \frac{4\pi}{6})$

~ ~ ~ ~ ~



$Z_1 = 10\ \Omega$ $Z_2 = 1+j2\ \Omega$ $Z_3 = -j5\ \Omega$ $Z_4 = 5\ \Omega$

$n_e = 3$

$n_e - 1 = 3 - 1 = 2$

$n = 4$

$m = 1$

$n - m - 1 = 4 - 1 - 1 = 2$

① $10.6 - \frac{\vec{V_1} - \vec{V_2}}{1+j2} - \frac{\vec{V_1}}{10} = 0$

② $$\frac{\vec{V}_1 - \vec{V}_2}{1+j2} - \frac{\vec{V}_2}{-j5} - \frac{\vec{V}_2 - 20\vec{I}_x}{5} = 0$$

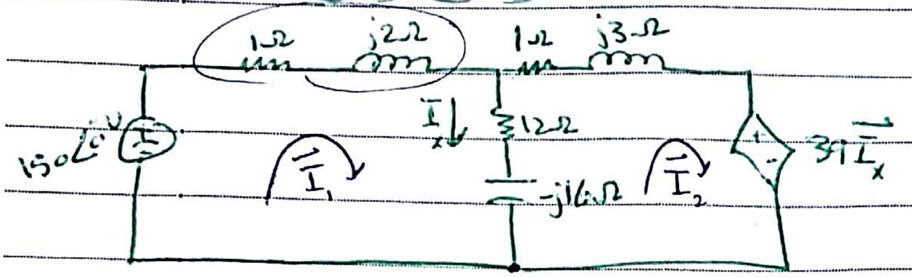
$\vec{V}_2 = 68 - j26\ \vec{V}$

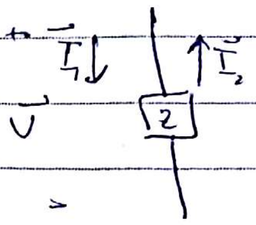
$\vec{V}_1 = 68.40 - j16.8\ V$

$\vec{I}_x = 3.76 + j1.68\ A$

$\vec{I}_b = -1.44 - j11.92\ A$

$\vec{I}_c = 5.2 + j13.6\ A$





$Z = a + jb$

$\vec{V} = (\vec{I}_1 - \vec{I}_2)Z$

$\rightarrow -150 + (1+j2)\vec{I}_1 + (12-j16)(\vec{I}_1 - \vec{I}_2) = 0$

$-(\vec{I}_1 - \vec{I}_2)(12-j16) + \vec{I}_2(1+j3 + 39\vec{I}_x = 0$

$\vec{I}_x = \vec{I}_1 - \vec{I}_2$

$\vec{I}_x = -2 + 6j\ A$

$\vec{I}_1 = -26. - 52j\ A$

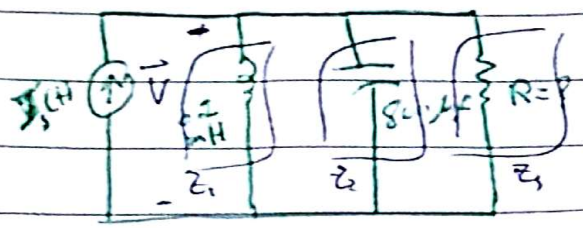
$\vec{I}_2 = -24 - j58\ A$

$\vec{V}_1 = (1+j2)\vec{I}_1 = 78.\ j104\ V$

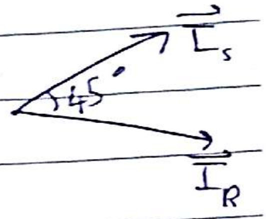
$\vec{V}_2 = (12+j16)\vec{I}_x = 72 + j104\ V$

$\vec{V}_3 = (1+j3)\vec{I}_2 = 150 - j130\ V$

Example:-



use phasor diagram to find R?!
which cause the current through the ~~resistor~~ resistor to lag the source current by 45°   w = 5k rd/sec



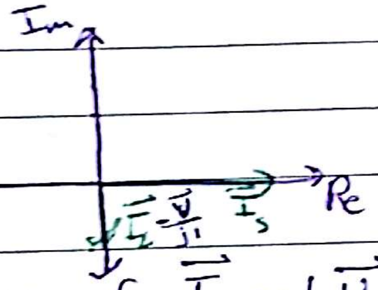
Sol:- $Z_1 = j\omega l$

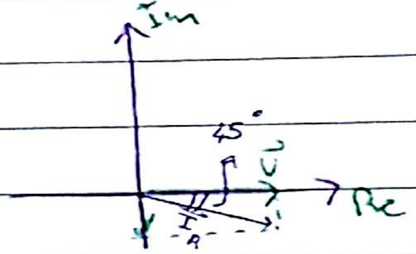
$= j\,5000 \times 0.2 \times 10^{-3}$

$= j1\,\Omega$

$Z_2 = \dfrac{1}{j\omega c}$

$= -j\dfrac{1}{5000 \times 800 \times 10^{-6}} = -j4\,\Omega$





phase of $\vec{I}_o$ and $\vec{V}$ are the same

$$\vec{I}_c = \frac{\vec{V}}{Z_c} = \frac{\vec{V}}{-j4} = j\frac{\vec{V}}{4}$$

$$I_s = \vec{I}_c + \vec{I}_L + \vec{I}_R \implies \vec{I}_R = \vec{I}_s - (\vec{I}_L + \vec{I}_c)$$

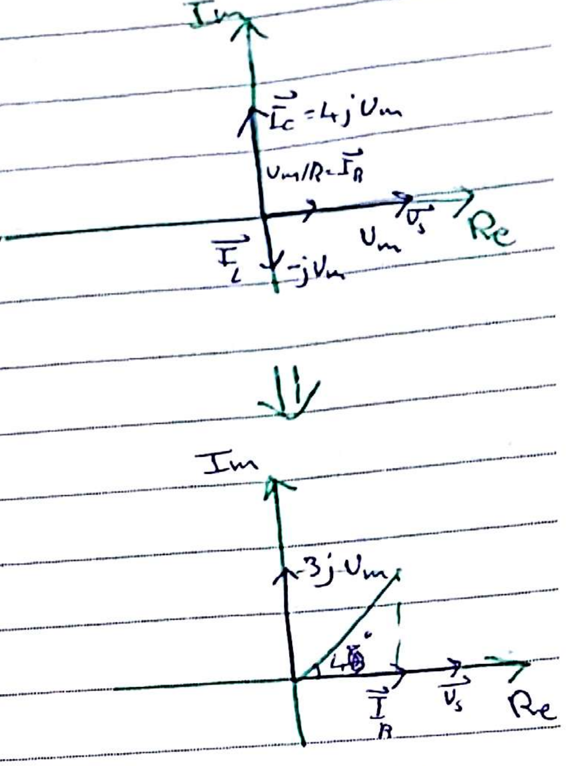
$$\vec{I}_L = \frac{\vec{V}_s}{Z_L} = \frac{V_m\angle 0^\circ}{j\omega L} = \frac{V_m}{j5\times 10^3\times 0.2\times 10^{-3}} = \frac{V_m}{j} = -jV_m$$

$$\vec{I}_c = \frac{\vec{V}_s}{Z_c} = \frac{V_m\angle 0^\circ}{\frac{1}{j\omega c}} = j\omega c\, V_m\angle 0^\circ$$

$$= j\,5\times 10^3\times 800\times 10^{-6}\, V_m$$

$$= j4\, V_m \text{ A}$$

$$\vec{I}_R = \frac{\vec{V}_s}{R} = \frac{V_m\angle 0}{R} = \frac{V_m}{R}$$



$$\tan 45^\circ = \frac{3V_m}{\frac{V_m}{R}}$$

$$1 = 3R$$

$$R = \frac{1}{3}$$