الجامعة الأردنية	0301102 تفاضل وتكامل 2	الإختبار الأول: 2017/3/16
اسم الطالب: ابراهيم يوسف شاكر الدلماني الرقم الجامعي: 0161663	20	مدرس المادة: .... وقت المحاضرة: 10—9

يتكون الامتحان من جزئين في ثلاث صفحات:

V

Part I

In questions 1 – 7, fill in the blanks to get correct statements (2 marks each)

[1] The partial fractions decomposition for $\frac{x^2+5}{x^7-25x^3}$ is (Don't evaluate the constants)

$\int\left(\frac{A_1}{x}+\frac{A_2}{x^2}+\frac{A_3}{x^3}+\frac{B_1}{x-\sqrt{5}}+\frac{B_2}{x+\sqrt{5}}+\frac{Cx+D}{x^2+5}\right)\cancel{dx}$

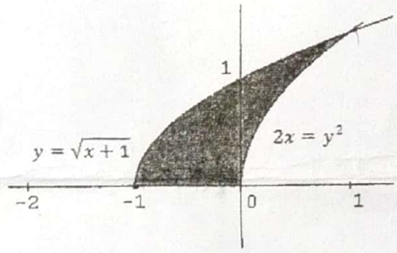
[2] The trigonometric substitution that solves the integral $\int\sqrt{x^2-4x}\,dx$ is

$x-2=2\sec\theta$

$dx=2\sec\theta\tan\theta\, d\theta$

[3] The integral that evaluates the area of the shaded region in the companion figure is

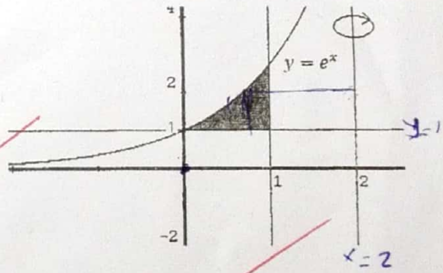
$\int_{-1}^{1}\left(\sqrt{x+1}-\sqrt{2x}\right)dx$



12

[4] The integral that represents the volume of the solid obtained by rotating the shaded region about $x=2$ using the method of cylindrical shells is

$\int_0^1\left(2\pi(e^x-1)(2-x)\right)dx$



[5] $\int\frac{x^5}{\sqrt{1-x^{12}}}\,dx=$ $\cancel{\frac{1}{6}\sin^{-1}(x)+c}=\frac{1}{6}\sin^{-1}(x^6)+C$

[6] Write down the improper integrals (but don't evaluate) using limits to solve

$\int_0^\infty\frac{e^x}{x-2}\,dx$ $\lim_{i\to2^-}\int_0^i\frac{e^x}{x-2}dx+\lim_{i\to2^+}\int_i^3\frac{e^x}{x-2}dx+\lim_{j\to\infty}\int_3^j\frac{e^x}{x-2}dv$

[7] $\int\sin(7x)\cos(4x)\,dx=$ $\cancel{\frac{1}{2}(}$ $\frac{1}{2}\left(\frac{-\cos 3x}{3}-\frac{\cos 11x}{11}\right)+c$

$\int\frac{1}{2}\sin(A-B)+(\sin(A+B)$ $\frac{1}{2}\int\sin 3x-\sin 11x$

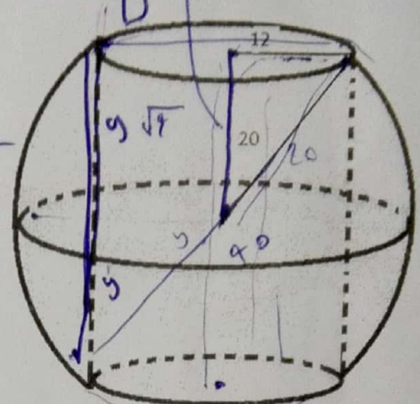
Part II

In questions 8 – 10, give a detailed answer (4 marks each)

$\sqrt{400-144}$

$= \sqrt{256}$

[8] A sphere of radius 20 is drilled by a cylinder of radius 12. Find the volume of the resulting solid using the method of slicing by finding appropriate cross sections.



$x^2+y^2=400$

$3(12)^2\pi h$

$V = V_{spher} - V_{cylinder}$

$\frac{20}{40} = \sqrt{256}$

$V = \frac{4}{3}\pi r^3 - \int_{y_1}^{y_2} \cancel{\pi r^2}\ L \cdot D\, dy$

$V = \frac{4}{3}\pi(20)^3 - \int_{-12}^{12} 2\sqrt{256}\ \sqrt{400-y^2}$

$x^2+y^2=400$

$x^2+y^2=400$

$-20 \quad -12 \quad 20$

$x^2+y^2=400$

$-20 \quad 20$

4

[9] Evaluate $\int \frac{dx}{x\sqrt{9x+1}}$

$y = \sqrt{9x+1}$

$\boxed{x = \frac{y^2-1}{9}}$

$y^2 = 9x+1$

$2y\,dy = 9\,dx$

$\int \frac{2y\,dy}{9\cdot\frac{(y^2-1)}{9}\cdot y}$

$= \int \frac{2\,dy}{y^2-1} = \int \frac{A}{y-1} + \frac{B}{y+1}$

$A = 1$

$B = -1$

$= \ln(y-1) - \ln(y+1) + C$

$= \ln(\sqrt{9x+1}-1) - \ln(\sqrt{9x+1}+1) + C$

[10] (a) Use integration by parts to show that

$$\int_1^e x^n(\ln x)^m\,dx = \frac{e^{n+1}}{n+1} - \frac{m}{n+1}\int_1^e x^n(\ln x)^{m-1}\,dx$$

$u = (\ln x)^m$ $\quad dv = x^n$

$du = \frac{m(\ln x)^{m-1}}{x}$ $\quad v = \frac{x^{n+1}}{n+1}$

$= (\ln x)^m \cdot \frac{x^{n+1}}{n+1}\Big|_1^e - \int_1^e \frac{m(\ln x)^{m-1}}{x} \cdot \frac{x^{n+1}}{n+1}\,dx$

$= (\ln e)^m \frac{e^{n+1}}{n+1} - (\ln 1)^m \cdot \frac{1^{n+1}}{n+1} - \frac{m}{n+1}\int_1^e x^n(\ln x)^{m-1}$

$= \frac{e^{n+1}}{n+1} - 0 - \frac{m}{n+1}\int_1^e x^n(\ln x)^{m-1}\,dx$

$= \frac{e^{n+1}}{n+1} - \frac{m}{n+1}\int_1^e x^n(\ln x)^{m-1}\,dx$

2

(b) Use the formula in (a) to evaluate $\int_1^e x^4(\ln x)^2\,dx$

$u = (\ln x)^2$ $\quad dv = x^4$

$du = \frac{2\ln x}{x}$ $\quad v = \frac{x^5}{5}$

$(\ln x)^2 \cdot \frac{x^5}{5}\Big|_1^e - \int_1^e \frac{2}{5}\ln x \cdot x^4\,dx$

$u = \frac{2}{5}\ln x$ $\quad dv = x^4$

$du = \frac{2}{5x}$ $\quad v = \frac{x^5}{5}$

$= (\ln x)^2 \cdot \frac{x^5}{5} - \frac{2}{25}\ln x\, x^5 + \int_1^e \frac{2}{25}x^4\,dx$

$\frac{(\ln x)^2}{5} \cdot x^5 - \frac{2}{25}\ln x \cdot x^5 + \frac{2}{125}$

$\to =$

$= \frac{e^5}{5} - \frac{2}{5}\int_1^e x^4(\ln x)$

$u = \ln x$ $\quad dv = x^4$

$du = \frac{1}{x}$ $\quad v = \frac{x^5}{5}$

$= \frac{e^5}{5} - \frac{2}{5}\left(\ln x \cdot \frac{x^5}{5} - \int_1^e \frac{x^4}{5}\,dx\right)$

$= \frac{e^5}{5} - \frac{2}{5}\left(\ln x \cdot \frac{x^5}{5} - \frac{x^5}{25}\Big|_1^e\right)$

$= \frac{e^5}{5} - \frac{2}{5}\left(\frac{e^5}{5} - \frac{e^5}{25} - \left(0 - \frac{1}{25}\right)\right)$

$= \frac{e^5}{5} - \frac{2}{5}\left(\frac{e^5}{5} - \frac{e^5}{25} + \frac{1}{25}\right)$

2

Scanned by CamScanner