

Components of power systems

centrally Controlled (Generation) → Create Power
(fuel sources)

Control Room

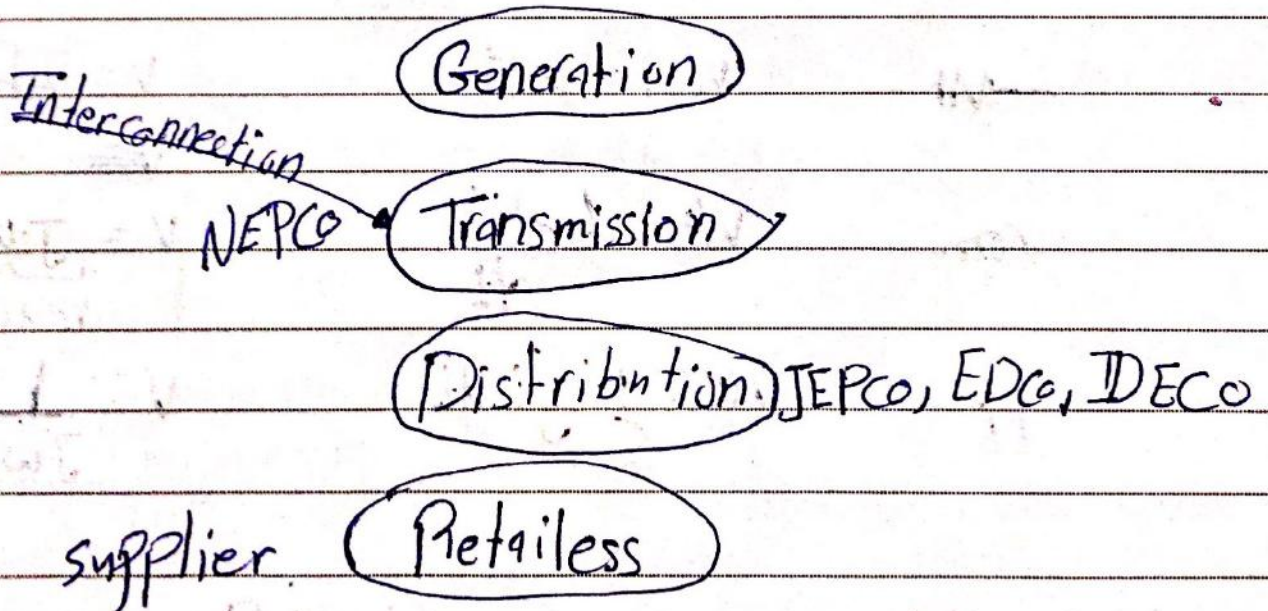
Distributed Network
Transmission & Distribution

LV ($\leq 1\text{kV}$)

Transmission $> 33\text{kV}$
Distribution $\leq 33\text{kV}$

Distributed Loads
Not Controlled
Customer Consumer
Prosumer.

Natural Monopolies
vertical



→ Review of Phasor

$$v(t) = V_{max} \cos(\omega t + \theta_v)$$

Simplify the analysis ^{fixed}

$$V_{rms} = \sqrt{\frac{1}{T} \int v^2(t) dt} = \frac{V_{max}}{\sqrt{2}}$$

$$v(t) = \sqrt{2} V_{rms} \cos(\omega t + \theta_v)$$

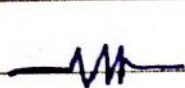
rms cosine-reference $j(\omega t + \theta_v)$?

$$v(t) = \sqrt{2} V_{rms} \operatorname{Re} \left\{ e^{j(\omega t + \theta_v)} \right\} \text{ enter}$$

$$V = V_{rms} \angle \theta_v$$

time

Phasor



$$v(t) = i(t)R$$

$$V = IR$$



$$v(t) = L \frac{di(t)}{dt}$$

$$V = \underbrace{j\omega L}_\text{reactance} I$$



$$v(t) = \frac{1}{C} \int i dt$$

$$V = \frac{1}{j\omega C} I$$

Complex Power = $S = P + jQ$

resistance

heating

inductor / capacitor

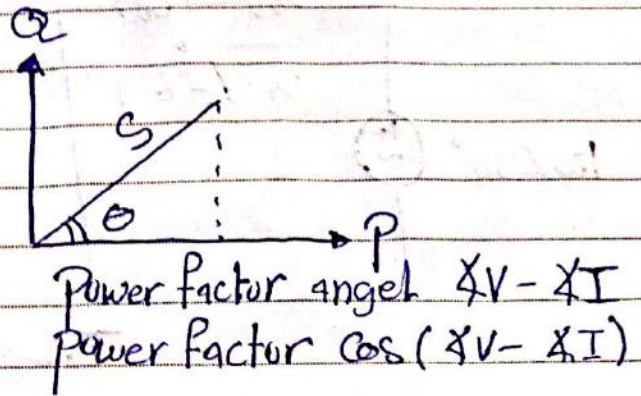
(" motor ")

Consumption Injection

$$S = VI^*$$

$$S = V(t)I(t)$$

$$\text{Power} = P_{avg}$$

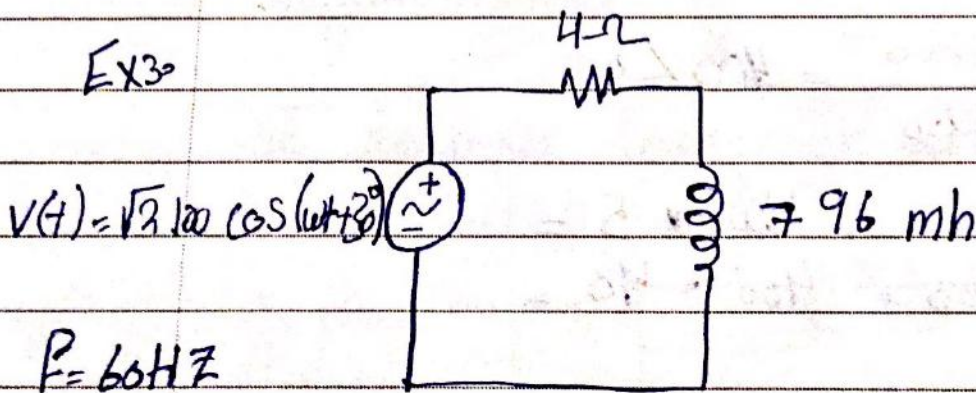


$$X_e = j\omega L \quad \text{let } \angle I = 0$$

$$V = j\omega L I \Rightarrow \angle V = 90^\circ$$



EX3

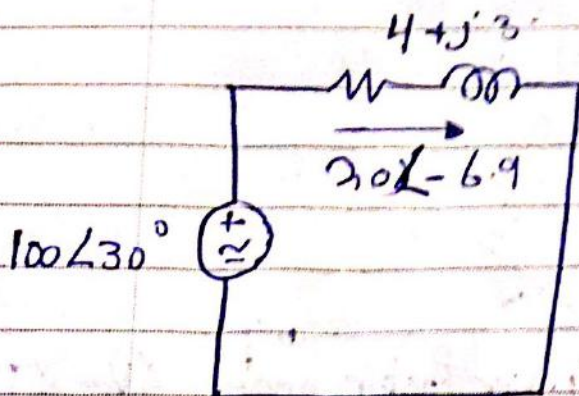


$$X_L = j\omega L = j(2\pi f)L = j3$$

$$Z = R + j\omega L = 4 + j3 = 5 \angle 36.9^\circ$$

$$\underline{I} = \frac{100 \angle 30^\circ}{5 \angle 36.9^\circ} = 20 \angle -6.9^\circ \text{ (A)} = 20\sqrt{2} \cos(\omega t - 6.9^\circ)$$

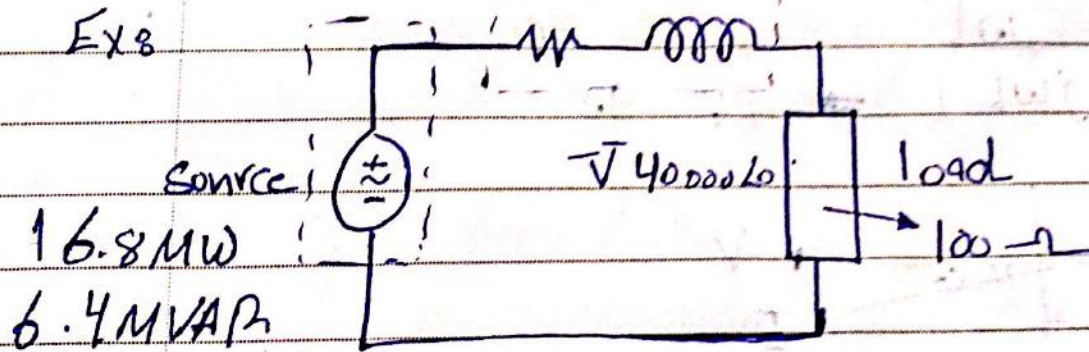
no. 26/9/2016



$$P_P = (20)^2 \times 4 = 1600 \text{ W}$$

$$P_L = (20)^2 \times 3 = 1200 \text{ Var}$$

Ex 8



$$I = \frac{40000}{100} = 400 \angle 0$$

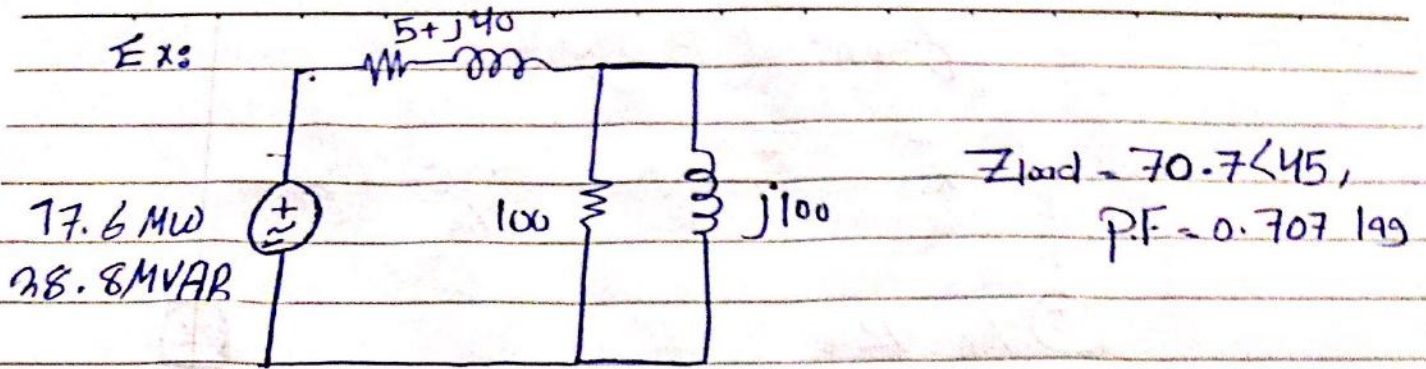
$$\text{Power losses} = 400^2 \times 5 =$$

$$\text{② losses} = 400^2 \times 40 =$$

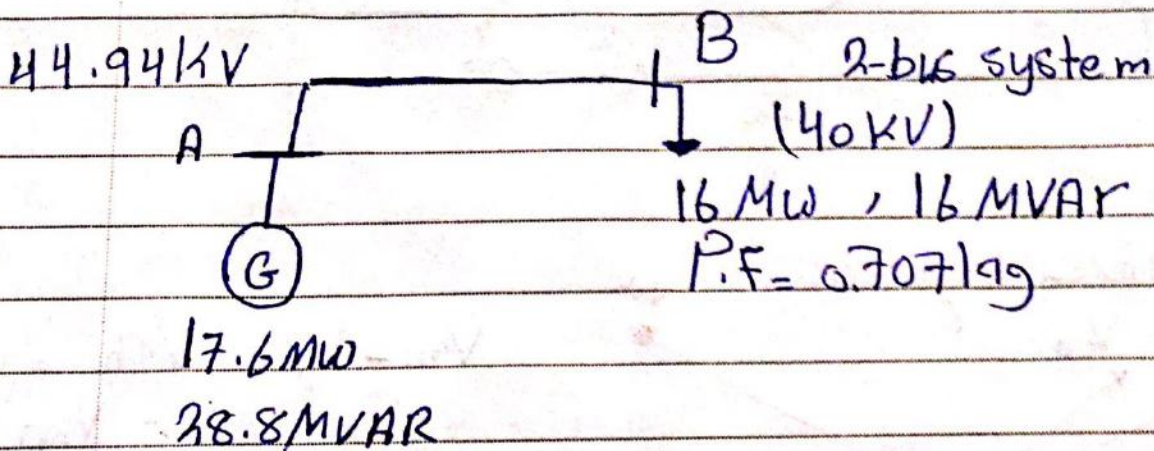
$$-V_s + I(5 + j40) + 40,000 \angle 0 = 0$$

$$V_s = 44.9 \angle 20.8 \text{ kV}$$

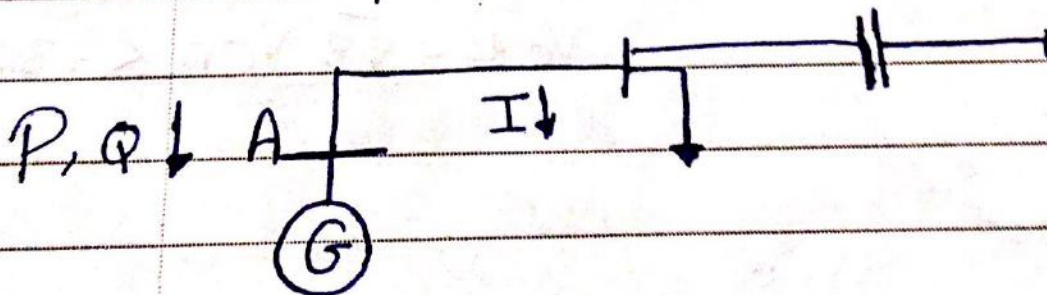
$$S = V_s I^* = (16.8 + j6.4) \text{ MVA}$$



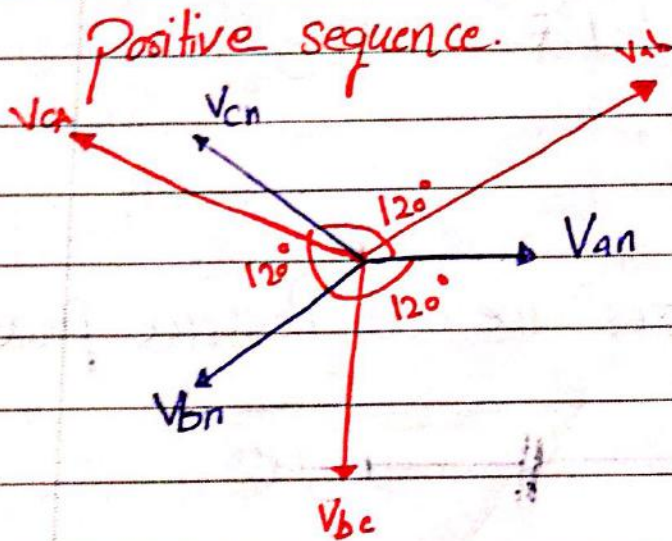
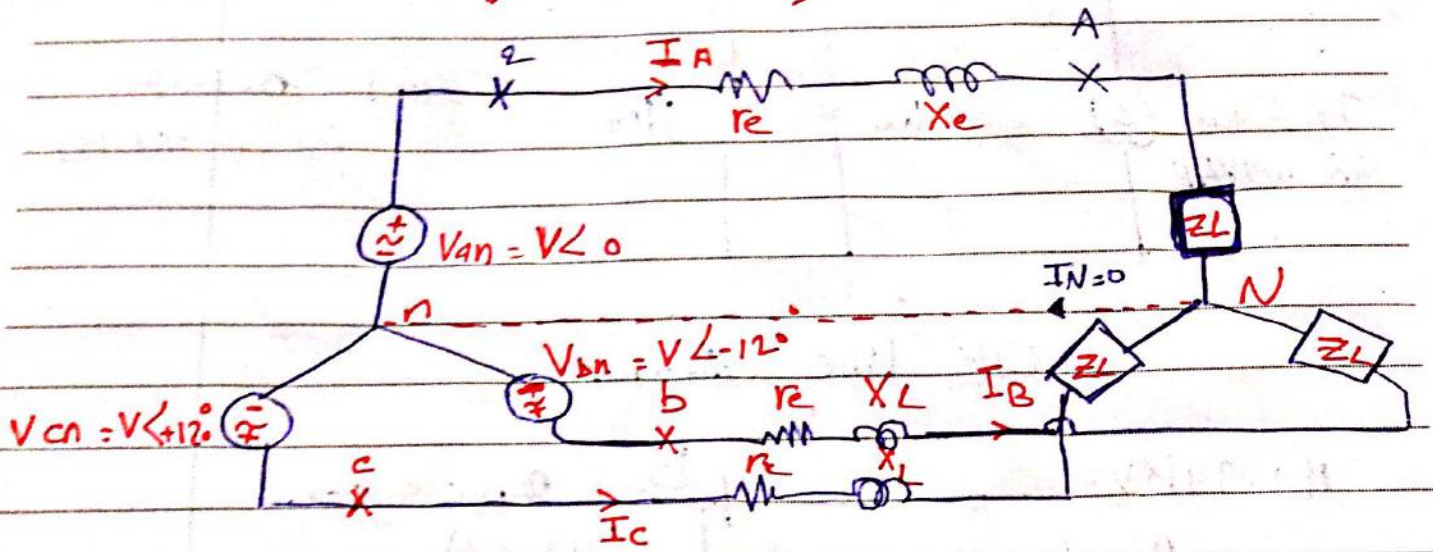
One line diagram



To decrease losses we add Reactive Power Compensation



Balanced 3 ϕ System



$$V_{an} - V_{bn} - V_{ab} = 0$$

$$V_{ab} = V_{an} - V_{bn}$$

$$V_{ab} = \sqrt{3} V_{an} \angle +30^\circ$$

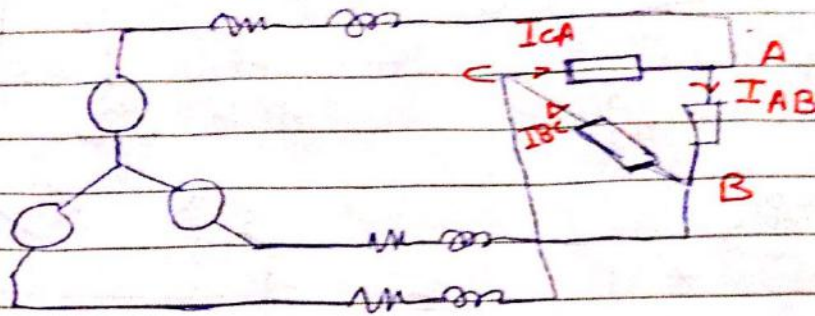
$$V_{bc} = \sqrt{3} V_{bn} \angle +30^\circ$$

$$V_{ca} = \sqrt{3} V_{cn} \angle +30^\circ$$

$$I_A = \frac{V_{an}}{Z_L}$$

$$I_B = \frac{V_{bn}}{Z_L}$$

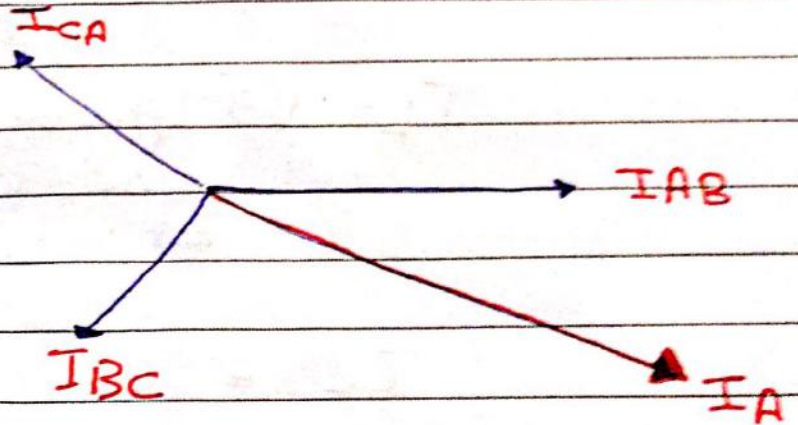
$$I_C = \frac{V_{cn}}{Z_L}$$



$$I_{AB} = \frac{V_{AB}}{Z_L}$$

$$I_{BC} = \frac{V_{BC}}{Z_L}$$

$$I_{CA} = \frac{V_{CA}}{Z_L}$$



$$I_A = I_{AB} - I_{CA}$$

$$= I_{AB} - I_{AB} \angle 120^\circ = \sqrt{3} I_{AB} \angle -30^\circ$$

$$S = V_{AB} I_{AB}^* + V_{BC} I_{BC}^* + V_{CA} I_{CA}^*$$

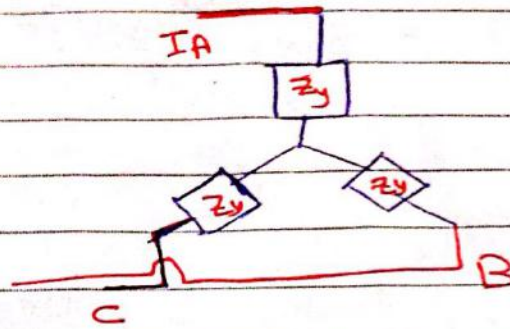
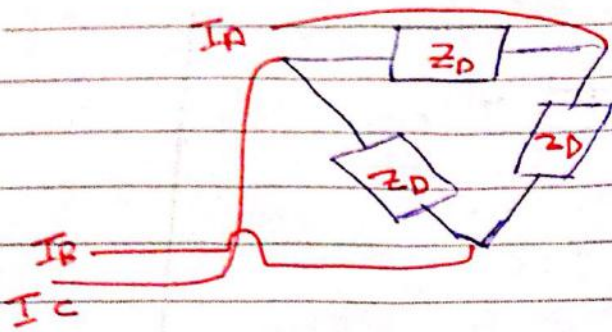
$$S_{3\phi} = \sqrt{3} V_L I_L$$

$$P_{3\phi} = 3 P_{1\phi}$$

$$= 3 V_{ph} I_{ph} \cos(\theta) \quad \checkmark \text{ PF}$$

$$Y\text{-load} \Rightarrow 3 \frac{V_L - V}{\sqrt{3}} I_L \cos(\theta)$$

$$P_{3\phi} = \sqrt{3} V_{L-L} I_L \cos \theta$$



$$I_L = \sqrt{3} I_{ph} \angle -30^\circ$$

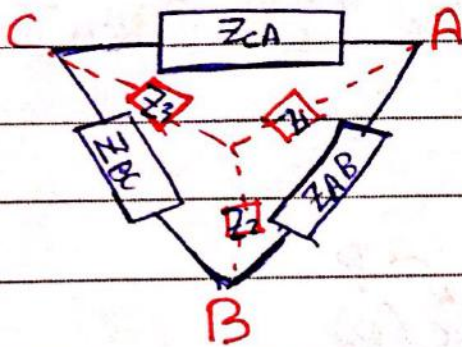
$$= \sqrt{3} \frac{V_{AB} \angle -30^\circ}{Z_\Delta}$$

$$I_L = \frac{V_{AN}}{Z_Y} = \frac{I_{ph}}{\sqrt{3}} = \frac{V_{AB} \angle -30^\circ}{\sqrt{3} Z_Y}$$

$$I_L = I_L$$

$$\frac{\sqrt{3} V_{AB} \angle -30^\circ}{Z_\Delta} = \frac{V_{AB} \angle -30^\circ}{\sqrt{3} Z_Y}$$

$$Z_Y = \frac{Z_\Delta}{3}$$



$$Z_1 = \frac{Z_{AB} Z_{CA}}{Z_{AB} + Z_{BC} + Z_{CA}}$$

$$Z_2 = \frac{Z_{AB} Z_{BC}}{Z_{AB} + Z_{BC} + Z_{CA}}$$

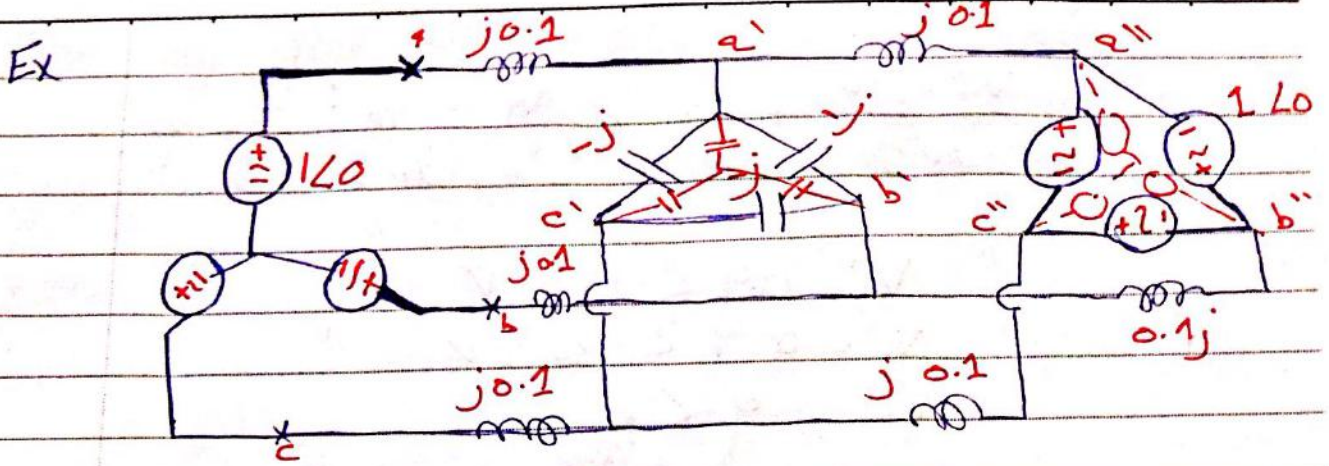
$$Z_3 = \frac{Z_{BC} Z_{CA}}{Z_{AB} + Z_{BC} + Z_{CA}}$$

$$Z_{AB} + Z_{BC} + Z_{CA}$$

$$Z_{AB} + Z_{BC} + Z_{CA}$$

$$Z_{AB} + Z_{BC} + Z_{CA}$$

« unbalanced »
load



- Q ① V_a
 ② total power supplied by each generator.

Solution: load ($\Delta \rightarrow Y$), $Z_Y = \frac{Z_\Delta}{3} = \frac{-j}{3}$

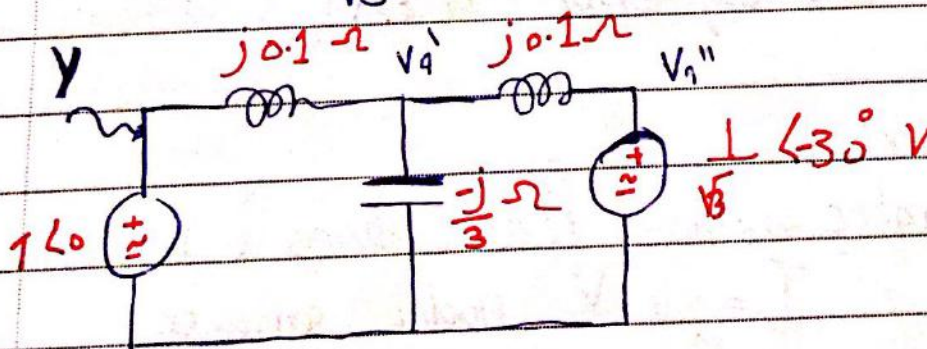
Source ($\Delta \rightarrow Y$)

$$V_L = \sqrt{3} V_P \angle +30^\circ$$

$$V_P = \frac{V_L}{\sqrt{3}} \angle -30^\circ$$

$$= \frac{1}{\sqrt{3}} \angle -30^\circ$$

* No resistive load to take real power.



$$\frac{V_a' - 1\angle 0}{j0.1} + \frac{V_a'}{-j/3} + \frac{V_c' - \frac{1}{\sqrt{3}}\angle -30^\circ}{j0.1} = 0$$

$$V_a' = 0.9 \angle -10.9^\circ \text{ V}$$

$$V_b' = 0.9 \angle -130.9^\circ \text{ V}$$

$$V_c' = 0.9 \angle 109.1^\circ$$

$$S_y \text{ generator} = 3 V_a I_a^*$$

$$= 3 \cdot 1\angle 0 \left(\frac{1\angle 0 - 0.9\angle -10.9^\circ}{j0.1} \right)$$

$$= 5.1 + j3.5 \text{ VA} \rightarrow \text{deliver } 3.5 \text{ VAR}$$

L. generat, deliver 5.1 W

$$S_\Delta \text{ generator} = 3 \frac{1}{\sqrt{3}} \angle -30^\circ * \left(\frac{V_a'' - V_a'}{j0.1} \right)$$

$$= -5.1 - j4.7 \text{ VA}$$

Capacitor bank \rightarrow injection Power (Q)

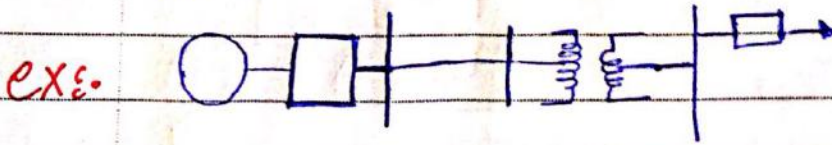
H.W \Rightarrow No capacitor Bank.

Note: Power world.

Systematic method for Solving CRT

$$I = Y V \text{ nodal analysis}$$

* Single line diagram « one line diagram »
 Relative interconnections of transformers, generators,
 line, loads, circuit breakers.

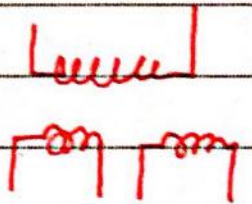


Generator/motor

power transformer
Two winding



three winding transformer
 Primary
 Secondary
 tertiary



- Delta Connection

- Wye Connection


- Wye connection neutral grounded loads

Solidly grounded

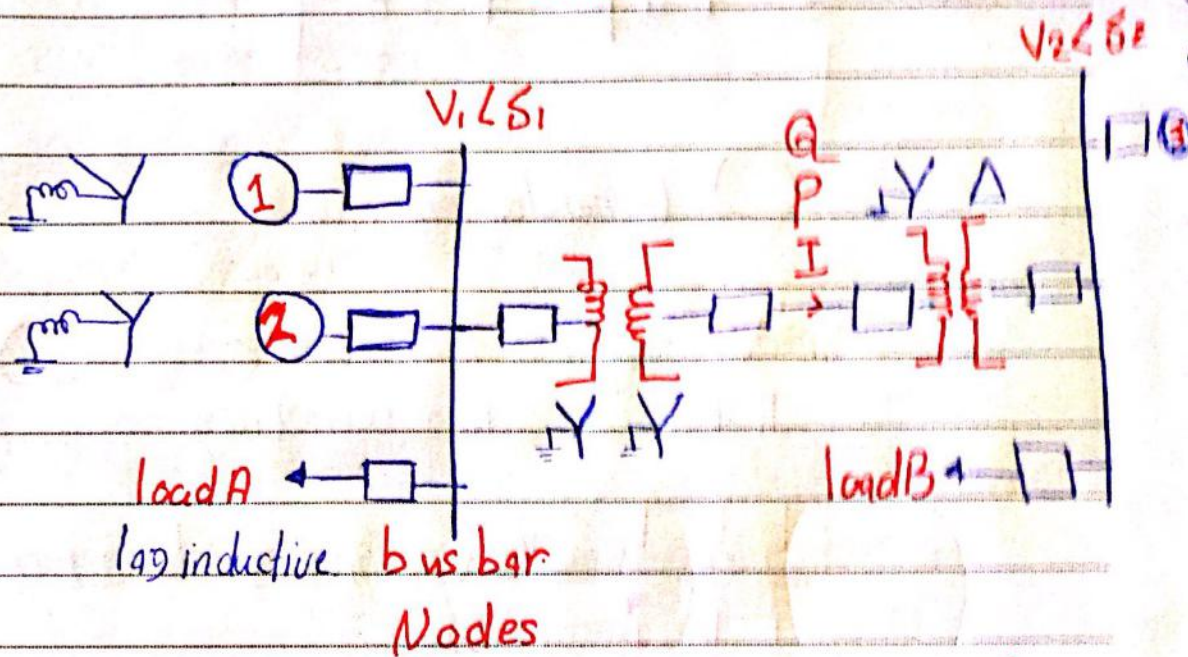
- Circuit breaker

- Voltage transformer

Fuse 

Current Transformer 

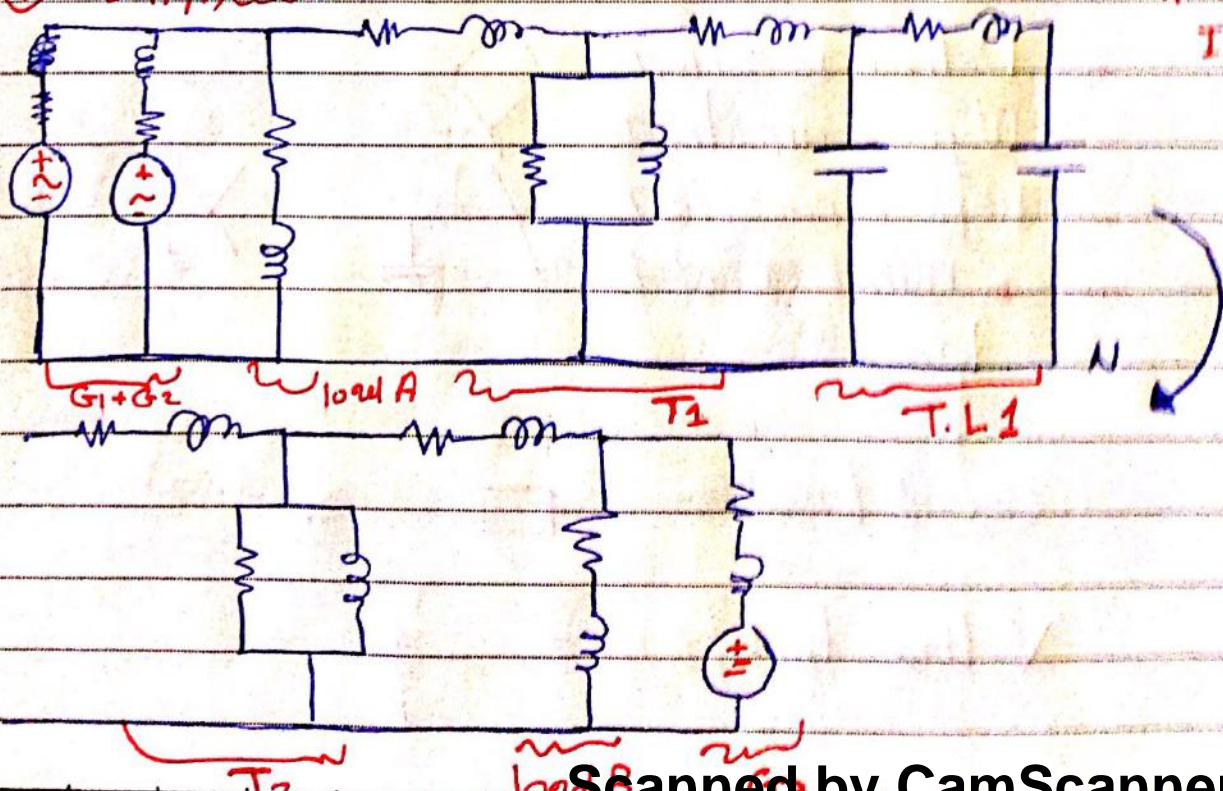
Ex.



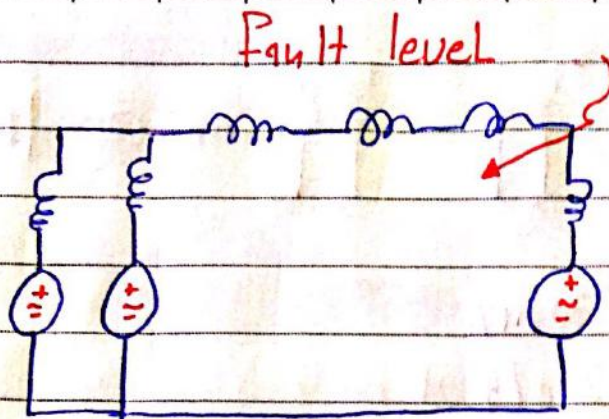
Reactance Diagram / Impedance :-
 ① Balanced

Note:

 if Balanced $T=0$.



No. 3/10/2016



Lo fault voltage
Li loads (gener)

The per unit systems %

- * Manufacture specify impedances of machine/transformers
- * $V^- \leq V \leq V^+$

statutory limit

$$0.94 \text{ pu} \leq V \leq 1.1 \text{ pu}, LV$$

$$\text{p.u Quantity} = \frac{\text{actual}}{\text{Base}}$$

V I S V

جاءت V I S في الترتيب ←

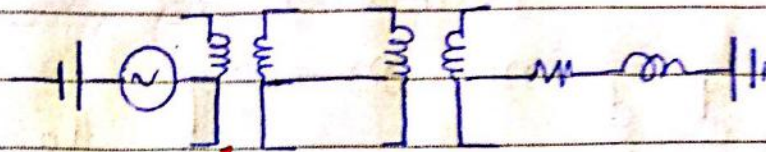
1 ϕ system

- Pick 1 ϕ VA for entire system
- Voltages are related by transformer turns ratio
- impedance base.

$$Z_{\text{base}} = \frac{V_{\text{base}}^2}{S_{\text{base}}}$$

$$I_{\text{base}} = \frac{V_{\text{base}}}{Z_{\text{base}}}$$

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30 kVA

240/480V

$X_{eq} = 0.1 \text{ pu}$

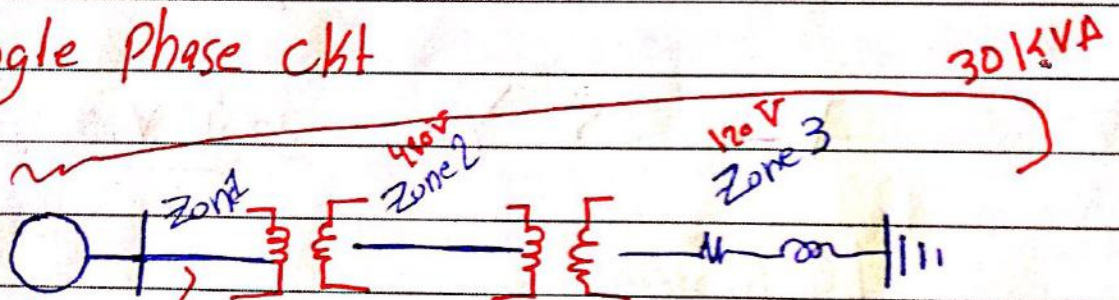
$\frac{(V_{base, old})^2}{S_{old}}$

$$Z_{p.u. new} = \frac{Z_{actual}}{Z_{base, new}} = Z_{p.u.} \times \frac{Z_{base, old}}{Z_{base, new}}$$

$\frac{(V_{base, new})^2}{S_{new}}$

$$Z_{p.u. new} = Z_{p.u. old} \times \left(\frac{V_{base, old}}{V_{base, new}} \right)^2 \times \left(\frac{S_{new}}{S_{old}} \right)$$

Ex: Single Phase ckt



320/20

30 kVA

240/480

$X_{eq} = 0.1 \text{ pu}$

20 kVA

480/115V

$X_{eq} = 0.1 \text{ pu}$

$Z_{load} = 0.9 + j0.2$

base 30 kVA

240

Q: Find load current P.u. Ampere etc

No. 5/10/2016

Ignore magnetization current of Transformers.
Solution :-

$$V_{base, 2} = 240 \times \left(\frac{480}{240} \right) = 480 \bar{V}$$

$$V_{base, 3} = 480 \times \left(\frac{115}{460} \right) = 120 \bar{V}$$

$$X_{T_1, p.u.} = 0.1 \times \left(\frac{30}{30} \right) \times \left(\frac{240}{240} \right)^2 = 0.1$$

$$X_{T_2, p.u.} = 0.1 \times \left(\frac{30}{20} \right) \times \left(\frac{115}{120} \right)^2 = 0.1378 \text{ p.u.}$$

$$= 0.1 \times \left(\frac{30}{20} \right) \times \left(\frac{460}{480} \right)^2 = 0.1378 \text{ p.u.}$$

$$X_{L, p.u.} = X(\text{actual})$$

$$X(\text{base})$$

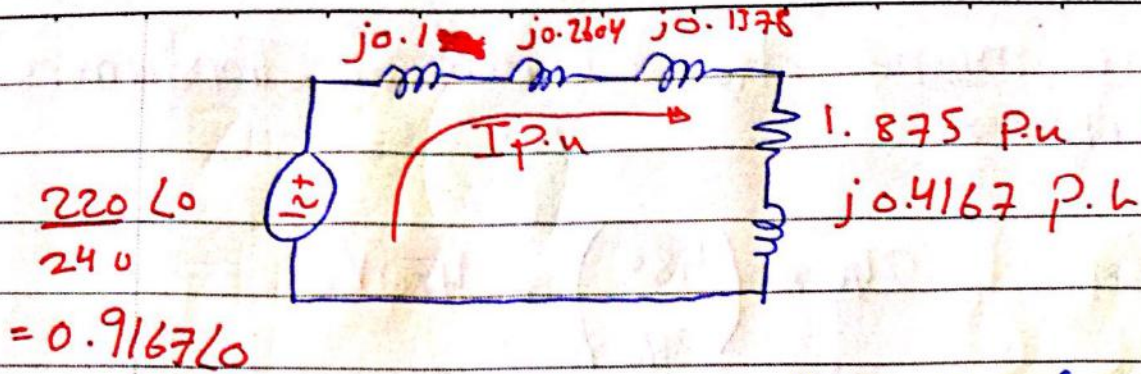
$$Z_{base} = X_{base} = \frac{V_{base}^2}{S_{base}} = \frac{(480)^2}{30 \text{ kVA}} = 7.68 \Omega$$

$$X_{line, p.u.} = 0.2604 \text{ p.u.}$$

$$Z_{load, p.u.} = \frac{0.9 + j0.2}{0.48} = 1.875 + j0.417 \text{ p.u.}$$

$$Z_{base, zone 3} = \frac{(120)^2}{30 \text{ kVA}}$$

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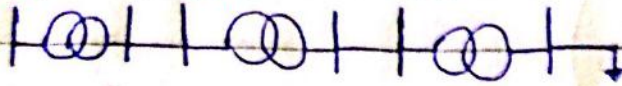


$$I_{p.u.} = 0.4395 \angle -26^\circ \text{ P.u.}$$

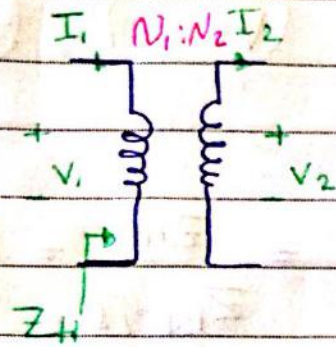
$$I_{base(3)} = \frac{S_{base(3)}}{V_{base(3)}} = \frac{30 \text{ KVA}}{120} = 250 \text{ A}$$

$$I_{actual} = (0.4395 \angle -26^\circ) \times 250 = 109.9 \angle -26.01^\circ$$

132KV 3KV 33KV 11KV 11KV 415



Ideal Transformer (Zero losses in winding copper losses $\mu \rightarrow \infty$, No core losses)



$$\frac{V_1}{V_2} = \frac{N_1}{N_2}, N_1 I_1 = N_2 I_2 \rightarrow \frac{I_1}{I_2} = \frac{N_2}{N_1}$$

$$Z_H = Z_L \left(\frac{N_1}{N_2} \right)^2$$

$\rightarrow V_1 \text{ p.u.} = V_2 \text{ p.u.} \quad ??$

$$V_1 \text{ p.u.} = \frac{V_1}{V_{1, \text{base}}} = \frac{V_2}{V_{\text{base}, 2}} = V_2 \text{ p.u.}$$

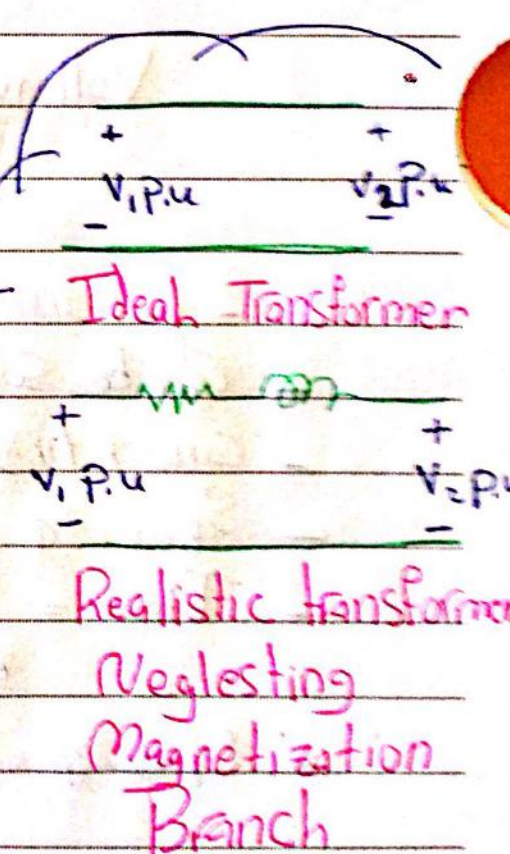
$$V_{\text{base}, 1} = V_{\text{base}, 2} * \left(\frac{N_1}{N_2} \right)$$

$$I_1 \text{ p.u.} = \frac{I_2 \text{ p.u.}}{I_1} = \left(\frac{N_2}{N_1} \right) I_2$$

$$I_{\text{base}, 1} = \left(\frac{N_2}{N_1} \right) I_{\text{base}, 2}$$

$I_{\text{base}, 1} = \frac{S_{\text{base}}}{V_{\text{base}, 1}}$
 $= \frac{S_{\text{base}}}{V_{\text{base}, 2} * \left(\frac{N_1}{N_2} \right)}$
 $= \frac{N_2}{N_1} I_{\text{base}, 2}$

$$I_1 \text{ p.u.} = \frac{I_2}{I_{\text{base}, 2}} = I_2 \text{ p.u.}$$



Ideal Transformer

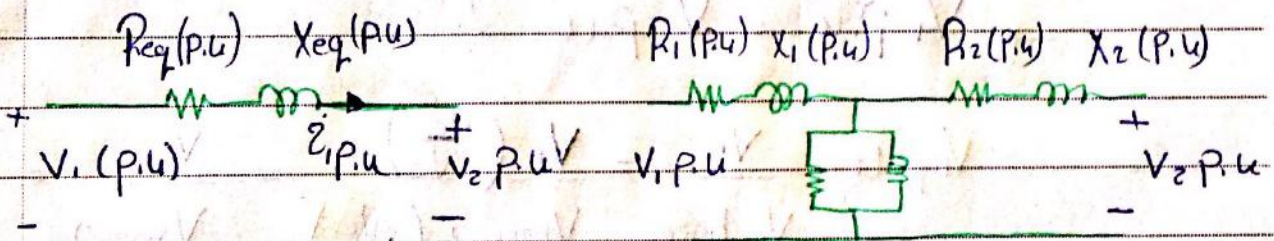
Realistic transformer
Neglecting Magnetization Branch

$$Z_H = Z_L \left(\frac{N_1}{N_2} \right)^2$$

$$Z_{H \text{ p.u.}} = Z_{L \text{ p.u.}} ??$$

$$Z_{L \text{ p.u.}} = \frac{Z_L}{Z_{\text{base}2}} \quad \frac{(V_{\text{base}2})^2}{S_{\text{base}}} = \frac{\left(\frac{N_2}{N_1} V_{\text{base}1} \right)^2}{S_{\text{base}}}$$

$$Z_{L \text{ p.u.}} = \frac{Z_H \left(\frac{N_2}{N_1} \right)^2}{\left(\frac{N_2}{N_1} \right)^2 \frac{V_{\text{base}1}^2}{S_{\text{base}}}} = \frac{Z_H}{Z_{\text{base}1}} = Z_{H \text{ p.u.}}$$



$$\text{Voltage regulation \%} = \frac{|V_2 \text{NL}| - |V_2 \text{FL}|}{|V_2 \text{FL}|} \times 100\%$$

Perunit in Balance Three Phase Circuits

- Can be solved in perunit on a perphase basis.
- Converting Δ load to equivalent Y impedances.
- $S_{\text{base}3\phi}$, V_{LL} «select»
- $I_{\text{base}} = \frac{S_{\text{base}3\phi}}{\sqrt{3} V_{LL, \text{base}}}$

$$S_{\text{base}3\phi} = P_{\text{base}3\phi} = Q_{\text{base}3\phi}$$

$$Z_{base} = \frac{V_{base, L}^2}{S_{base, 1\phi}} = \left(\frac{V_{base, L-L}}{\sqrt{3}} \right)^2$$

$$S_{base, 1\phi} \quad S_{base, 3\phi} / 3$$

$$Z_{base} = \frac{V_{base, L-L}^2}{S_{base, 3\phi}}$$

$$S_{base, 3\phi}$$

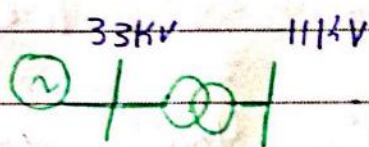
$$R_{base} = X_{base} = Z_{base} = \frac{1}{Y_{base}}$$

$$Z_{P.U. (new)} = Z_{P.U. (old)} * \left(\frac{V_{LL, old}}{V_{LL, new}} \right)^2 * \left(\frac{S_{new}}{S_{old}} \right)$$

Ex:

T₁ 10 MVA 33/44KV, X_{P.U.} = 18%

T₂ 25 MVA 33/11KV, X_{P.U.} = 10%



Q: high voltage drop ??
100 MVA

$$X_{T1, P.U.} = 10\% * \left(\frac{100}{10} \right) = 1 P.U. \quad \checkmark$$

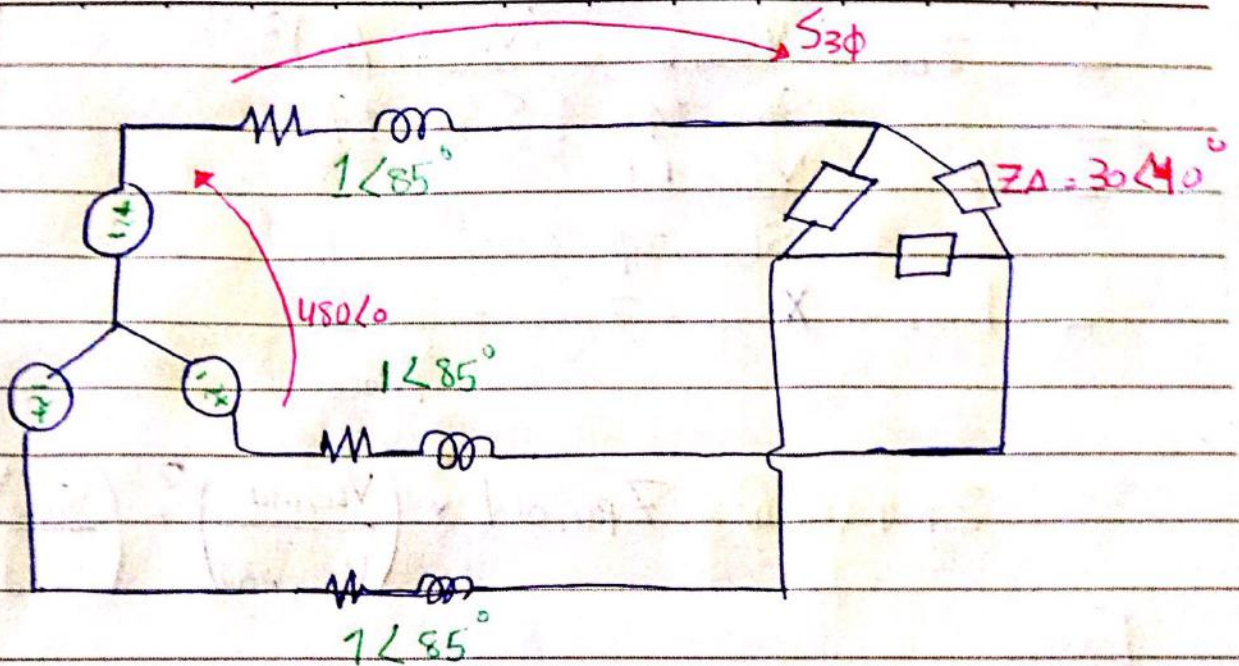
$$X_{T2, P.U.} = 10\% * \left(\frac{100}{25} \right) = 0.4 P.U.$$

Q A balanced Y connected Voltage Source

$E_{ab} = 480 \angle 0^\circ V$ is applied to a balanced Δ load,
 $Z_{\Delta} = 30 \angle 40^\circ \Omega$

The line impedance between source & load is $Z_{line} = 1 \angle 85^\circ \Omega$
For each phase calculate p.u & actual current in phase a using $S_{1\phi} = 10 KVA$, $V_{base, LL} = 480 V$.

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* Solution 8° $S_{3\phi} = 10 \times 3 = 30 \text{ KVA}$

$$Z_y = \frac{Z_\Delta}{3} = \frac{30 \angle 40^\circ}{3} = 10 \angle 40^\circ$$

$$Z_{base} = \frac{(V_{LL})^2}{S_{3\phi}} = \frac{(480)^2}{30} = 23.04 \Omega$$

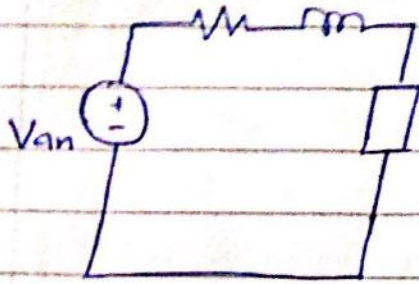
$$Z_{y \text{ p.u.}} = \frac{10 \angle 40^\circ}{23.04} = \frac{10 \cos 40^\circ + j 10 \sin 40^\circ}{23.04}$$

$$Z_{y \text{ p.u.}} = 0.434 \angle 40^\circ$$

$$Z_{line \text{ p.u.}} = \frac{1 \angle 85^\circ}{23.04} = 0.0434 \angle 85^\circ$$

$$V_{an} = \frac{480}{\sqrt{3}} \angle -30^\circ$$

$$V_{base} = \frac{480}{\sqrt{3}} = V_{an \text{ p.u.}} = 1 \angle -30^\circ$$



$$I_{p.u.} = \frac{1 \angle -30^\circ}{0.0434 \angle 85^\circ + 0.434 \angle 40^\circ}$$

$$= 2.147 \angle -73.78$$

$$I_{base} = \frac{S_{3\phi}}{\sqrt{3} V_{LL}} = \frac{30}{\sqrt{3} 480}$$

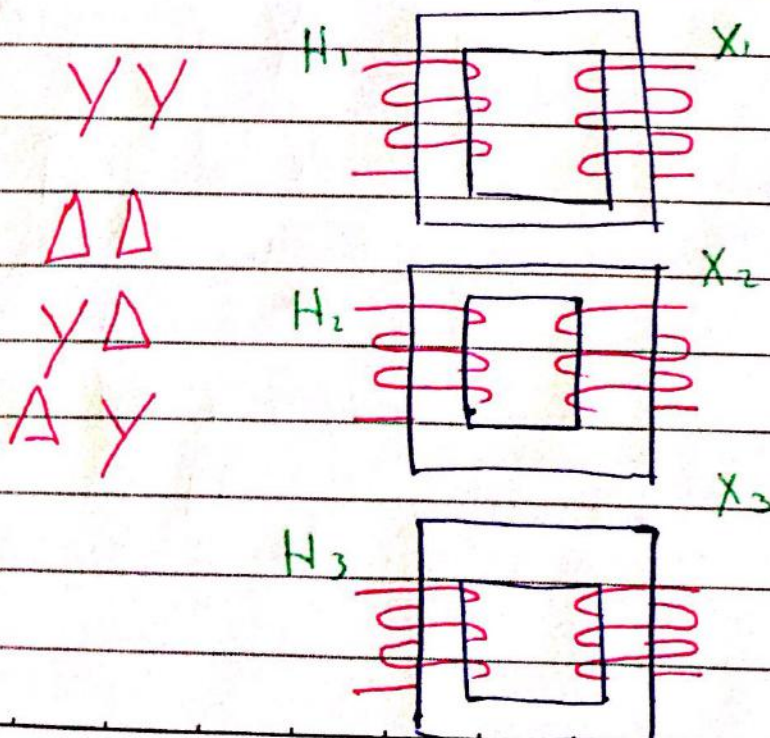
$$= 12.03 \text{ A}$$

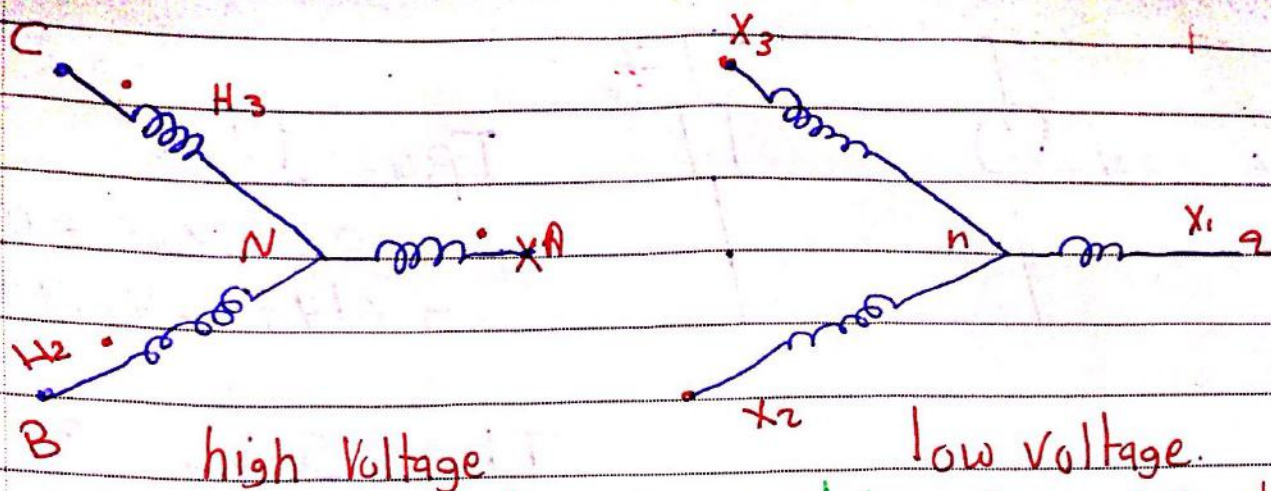
$$I_{actual} = 2.147 \angle -73.78 + 12.03$$

$$= 25.83 \angle -73.78 \text{ A}$$

Three Phase Transformer Connections

- three identical single phase two-winding transformer « more reliable, replace one unit in the case of failure »
- 3 ϕ Transformer are usually connected to share some core. « cheaper, less iron, less space »





high Voltage

low Voltage.

schematic representation → Not phasor

H → high side $\bar{a} \bar{b} \bar{c}$

X → Low side $\bar{a} \bar{b} \bar{c}$

AN // an

Windings of primary & secondary which are drawn in parallel directions are for the same single phase transformer "parallel winding have the same flux"

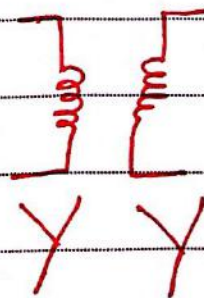
VAN is in phase with Van

Voltage from terminal H_1, H_2, H_3 to neutral leads voltages to neutral from X_1, X_2, X_3
"American Standard"

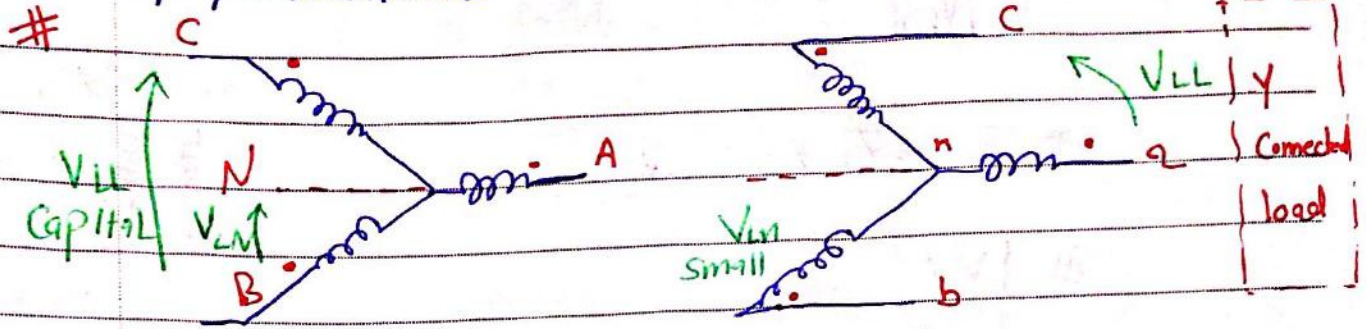
ΔY } → high leads Low $\underline{30^\circ}$
 $Y \Delta$ }

$\Delta \Delta, Y Y$ → In phase

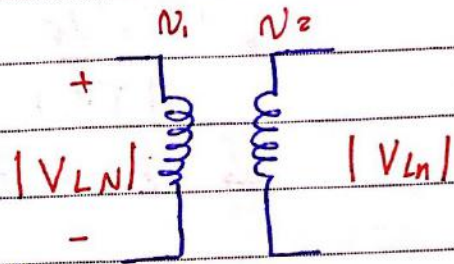
single line diagram:



Y-Y Balanced



$$\frac{|V_{LN}|}{|V_{Ln}|} = \frac{N_1}{N_2}$$



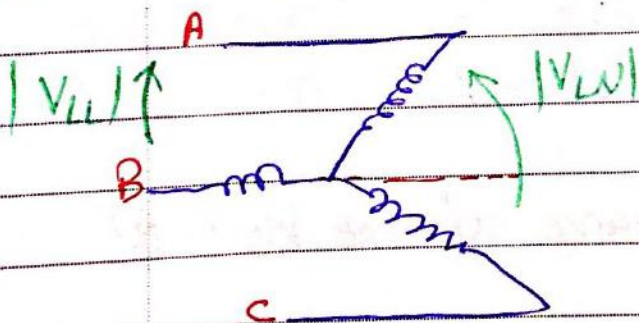
$$Z_{N \text{ per phase}} = Z_L * \left(\frac{V_{LN}}{V_{Ln}} \right)^2$$

$$Z_L * \left(\frac{V_{LL}/\sqrt{3}}{V_{LL}/\sqrt{3}} \right)^2$$

$$Z_{H \text{ per phase}} = Z_{L \text{ per phase}} * \left(\frac{V_{LL}}{V_{LL}} \right)^2$$

→ Capital
→ Small

Y-Δ



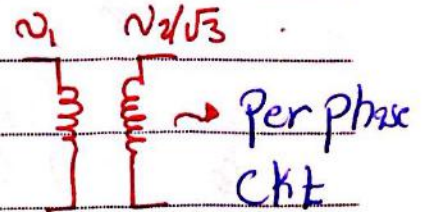
⇒ let assume

$$\frac{|V_{LN}|}{|V_{LL}|} = \frac{N_1}{N_2}$$

System) دوسو ۱۲ #
to per phase

$$\frac{V_{LN}}{\sqrt{3}|V_{2n}|} = \frac{N_1}{N_2} \Rightarrow \frac{|V_{LN}|}{|V_{2n}|} = \sqrt{3} \frac{N_1}{N_2} \quad \text{effective turn ratio}$$

$$\# Z_H = Z_L * \left(\sqrt{3} \frac{N_1}{N_2} \right)^2$$



$$\# \frac{|V_{LN}|}{|V_{22}|} = \frac{N_1}{N_2}$$

$$\hookrightarrow \frac{|V_{LL}|/\sqrt{3}}{|V_{22}|} = \frac{N_1}{N_2} \rightarrow \frac{|V_{LL}|}{|V_{22}|} = \sqrt{3} \frac{N_1}{N_2}$$

$$\hookrightarrow Z_H = Z_L \left(\frac{V_{LL}}{V_{22}} \right)^2$$

low \rightarrow high \rightarrow line to line \rightarrow phase to phase

Q Three Single Phase transformer peak rated 25 MVA, 38.1/3.81 kVA, are connected Y Δ with Balanced load 0.6 Ω (per phase) Y connected \ll choose a base of 75 MVA, 66 kV for HV side :-

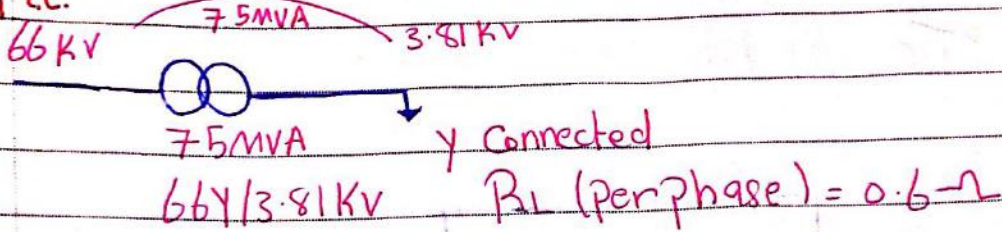
\hookrightarrow Rating 3 ϕ transformer

75 MVA

38.1 * $\sqrt{3}$ / 3.81 kV

Q.A Determine p.u resistance of the load @ high side ??

Solution 22.

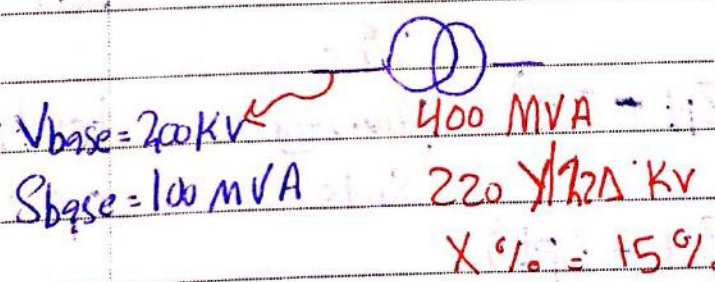


Note 3: $70 \text{ kV} \leftarrow 66 \text{ kV}$ (بالقوة الجهد)
 $\frac{70 \times 3.81}{66}$ (بالقوة الجهد)

~~$\frac{66 \times 3.81}{66}$~~ (بالقوة الجهد)

- R_L (seen from high side) = $0.6 \left(\frac{66}{3.81} \right)^2 = 180$
- $Z_{base} \text{ (HV)} = \frac{(V_{LL})^2}{S} = \frac{(66)^2}{75} = 58.1 \Omega$
- $R_{L.p.u} = \frac{180}{58.1} = 3.1 \text{ p.u.}$

Q three phase transformer 400 MVA, 220Y/22Δ kV

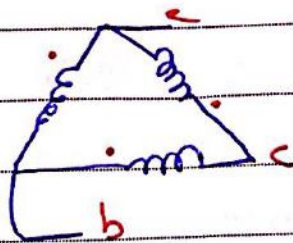
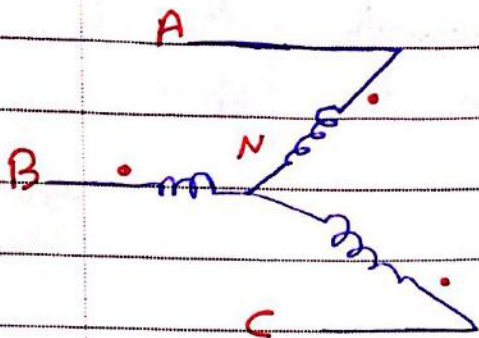


Find p.u transformer according to new base

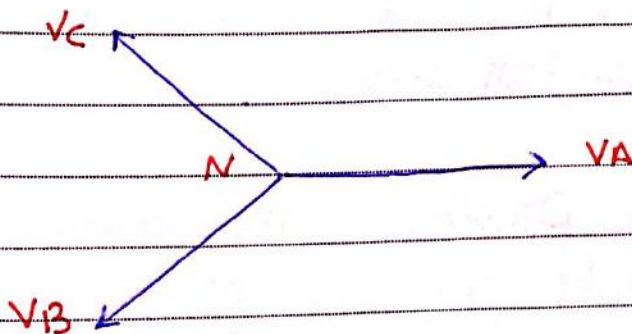
$$Z_{p.u} \text{ (new)} = Z_{p.u} \text{ old} \times \left(\frac{V_{old, LL}}{V_{new, LL}} \right)^2 \times \left(\frac{S_{new}}{S_{old}} \right)$$

$$X_{p.u} = 0.15 \times \left(\frac{220}{200} \right)^2 \times \left(\frac{100}{400} \right) = 4.5375\%$$

$Y\Delta$

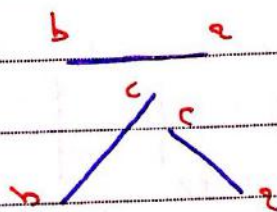


* draw the phasor diagram (the sequence)??

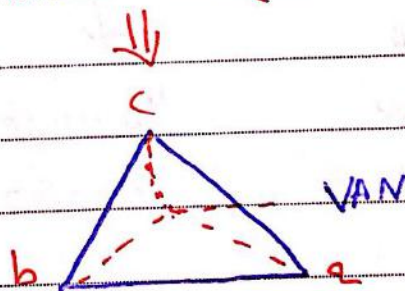


Y connection

AN // ab
BN // bc
CN // ca



دوتی فیلڈ
"dot"



$$V_A = V_{\phi} \angle +30^\circ$$

$$I_A = I_{\phi} \angle +30^\circ$$

$$S = \sqrt{3} V I^*$$

$$V_{\phi}^{(1)} * I_{\phi}^{(1)*}$$

$$V_A^{(1)} * I_A^{(1)*}$$

V_{AN} lead V_{ϕ} by 30°

(1) → turn ratio

Primary $\rightsquigarrow V_A^{(1)} I_A^{(1)}$
 Secondary $\rightsquigarrow V_A^{(2)} * I_A^{(2)*} = V_A^{(1)} \angle 30^\circ * I_A^{(1)} \angle 30^\circ *$

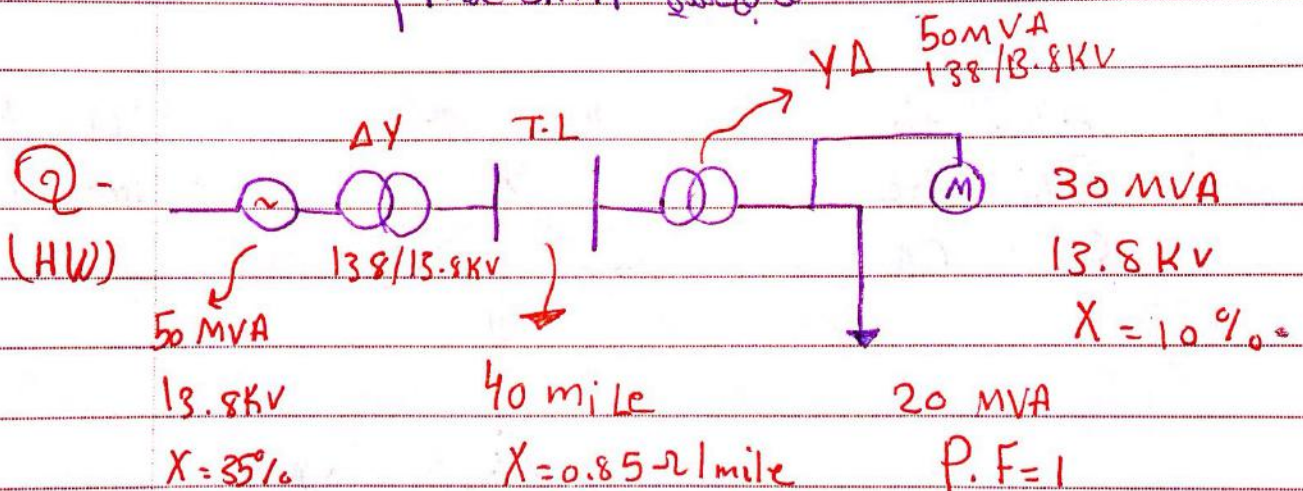
$P_{secondary} = V_A * I_A$

→ Power for two side are the same

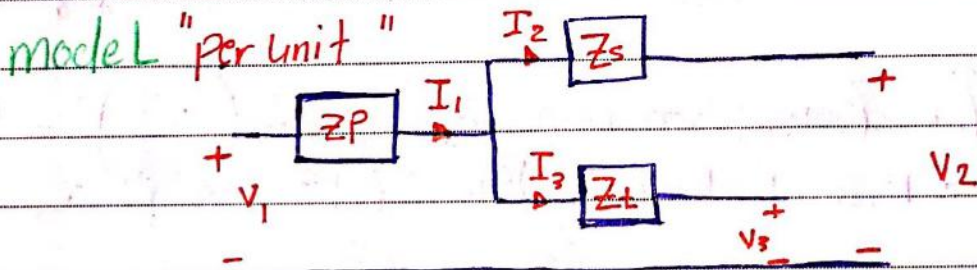
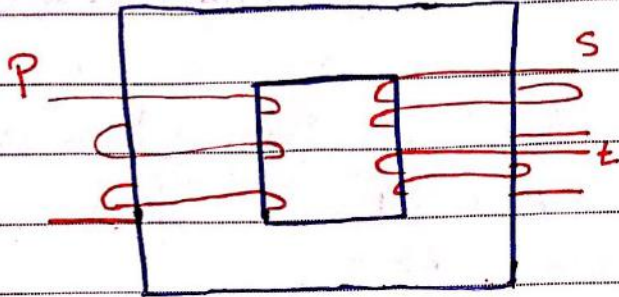
→ -ve sequence

V_{AN} lead $V_{an} - 3^\circ$
 Primary & secondary are in phase

→ Power & reactive Power
 Phase shift $\sin \theta$



Three winding transformer



→ $Z_{ps} = Z_p + Z_s$

→ $Z_{pt} = Z_p + Z_t$

→ $Z_{st} = Z_s + Z_t$

Z_{ps} → Secondary Short circuit

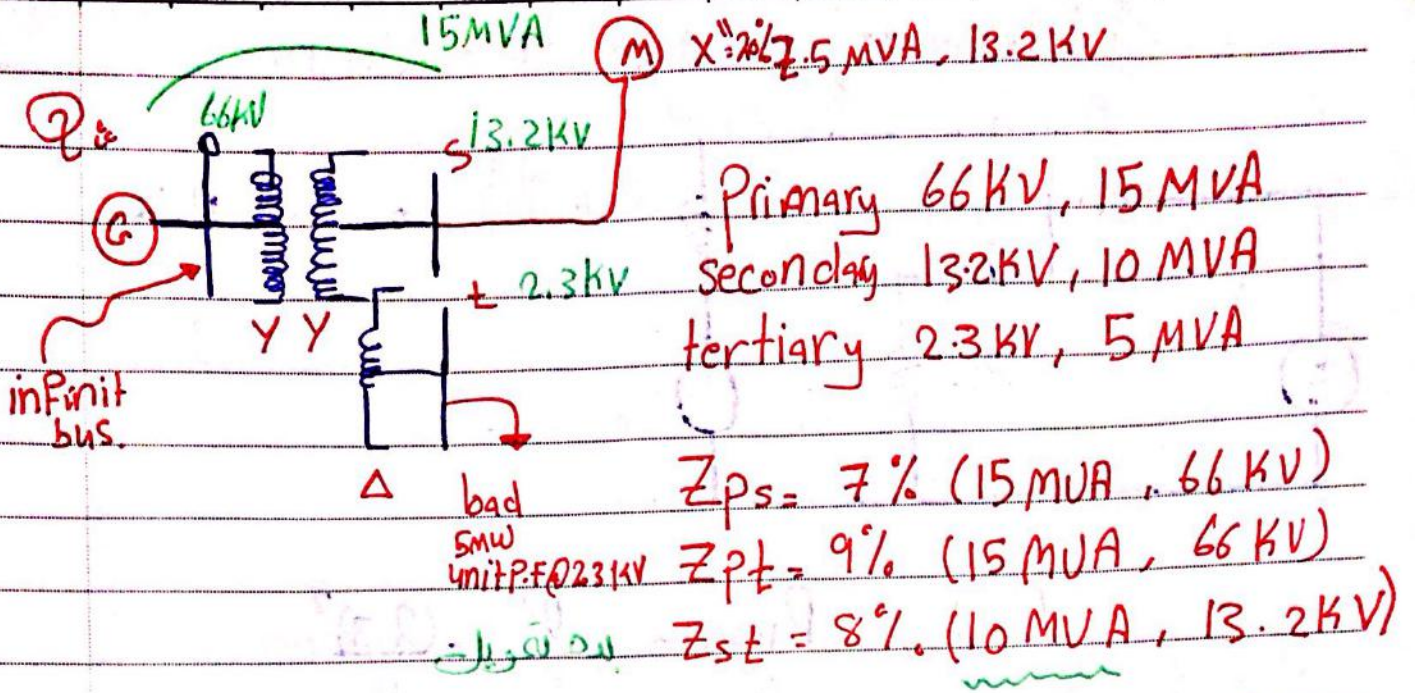
Seen @ Primary Side

tertiary open circuit

→ $Z_p = \frac{1}{2} (Z_{ps} + Z_{pt} + Z_{st})$

→ $Z_s = \frac{1}{2} (Z_{ps} + Z_{st} - Z_{pt})$

→ $Z_t = \frac{1}{2} (Z_{pt} + Z_{st} - Z_{ps})$



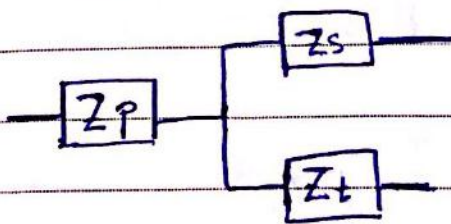
(A) Determine per unit impedances per-phase equivalent circuit "15MVA, 66kV @ primary side" ??

Sol 3: $Z_{st} = 8\% \times \left(\frac{15}{10}\right) = 12 \text{ p.u.}$

$Z_p = j0.02$

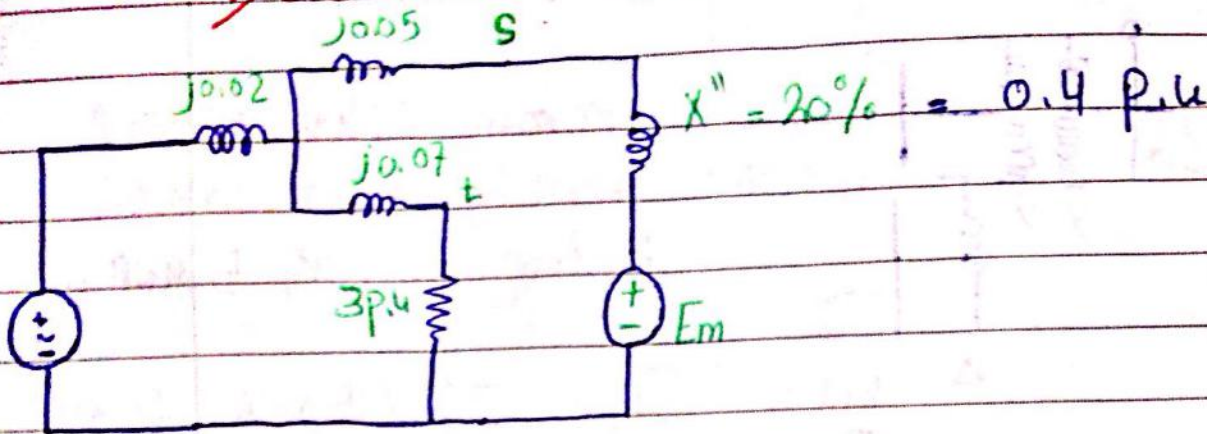
$Z_s = j0.05$

$Z_t = j0.07$



(B) Connect the model to infinite bus and connect load and connect motor

Determine :- 1) Reactance diagram

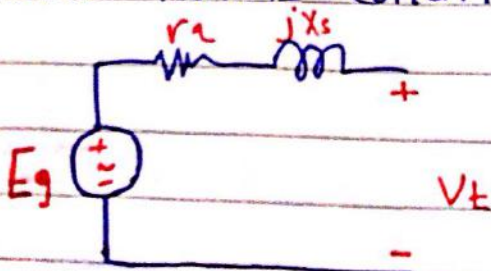


load $\Rightarrow R_{load} = \frac{V^2}{S} = \frac{(2.3)^2}{5}$

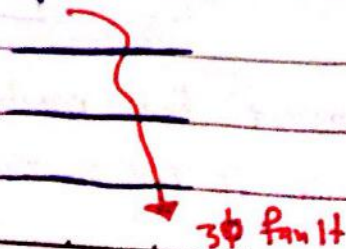
$R_{load, \text{per unit}} = \frac{R_{load}(\text{actual})}{Z_{base}} = \frac{(2.3)^2/5}{(2.3)^2/15} = \frac{15}{5} = 3 \text{ pu}$

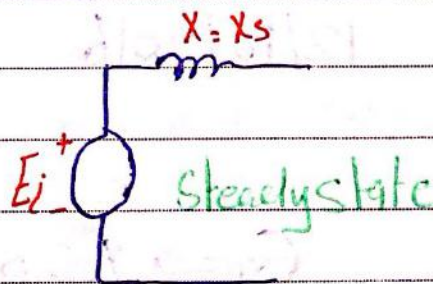
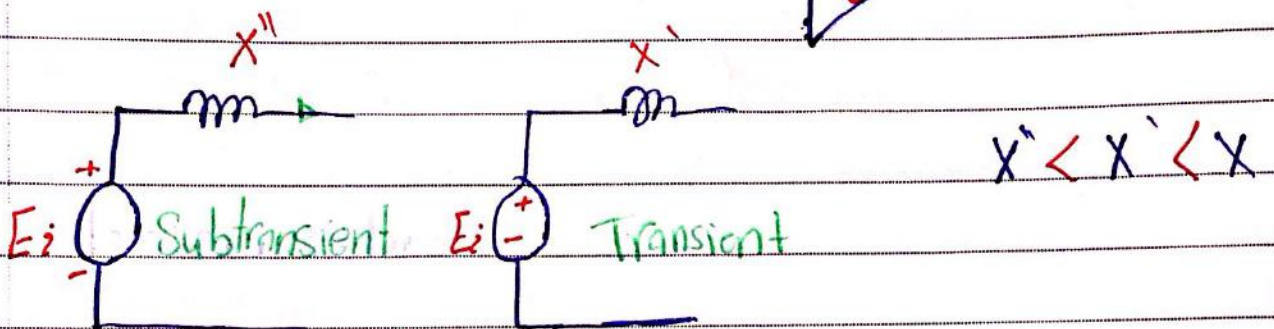
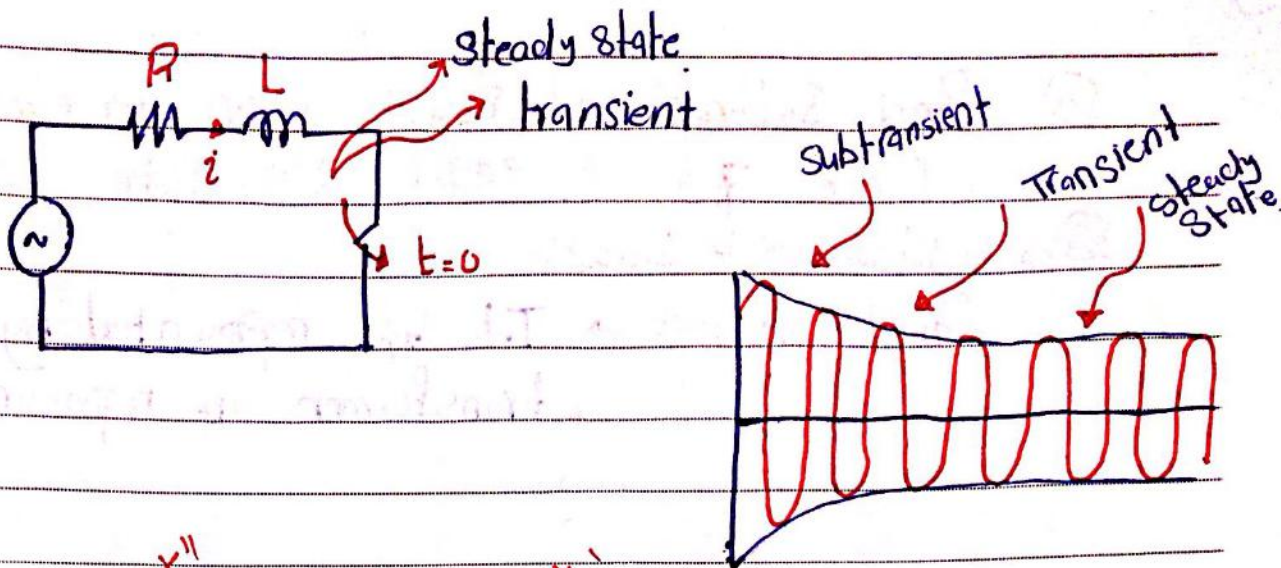
$X'' = 20\% \times \left(\frac{15}{7.5}\right) = 0.4 \text{ p.u.}$

Three phase Synchronous Generator under normal conditions

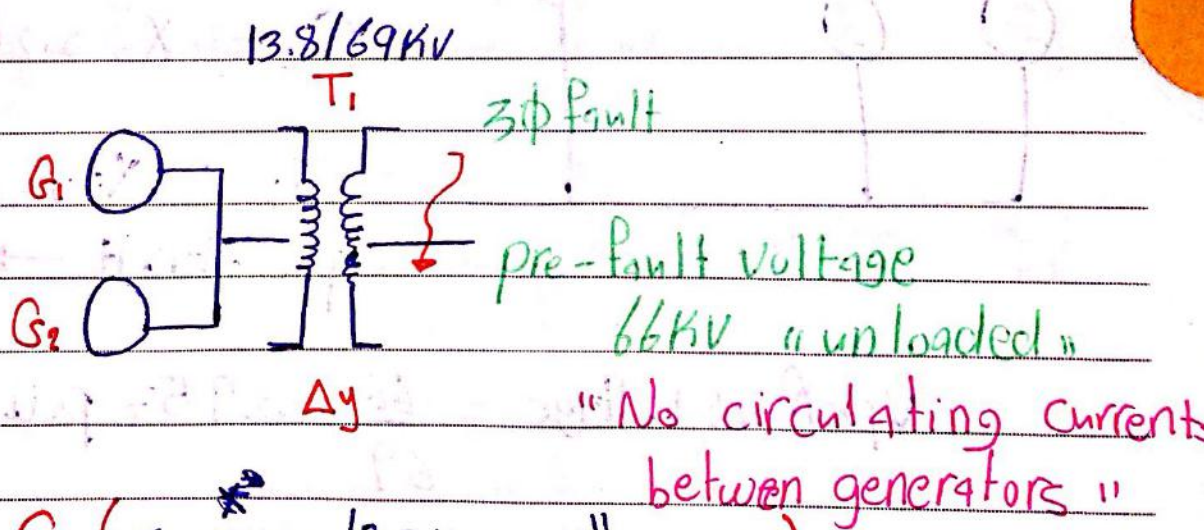


during fault





Q

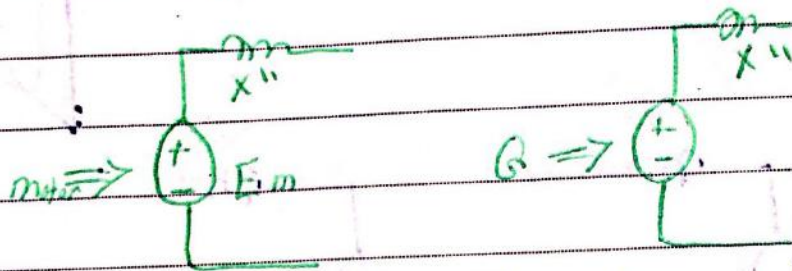


- G_1 (50 MVA, 13.8 KV, $X'' = 25\%$)
- G_2 (50 MVA, 13.8 KV, $X'' = 25\%$)
- T_1 75 MVA, 13.8 KV Δ / 69 KV Y, $X = 10\%$

Q Find subtransient fault current in each generator
 " Base 75 MVA, 69 kV @ HV side of T. "

~~Q Find subtransient fault~~

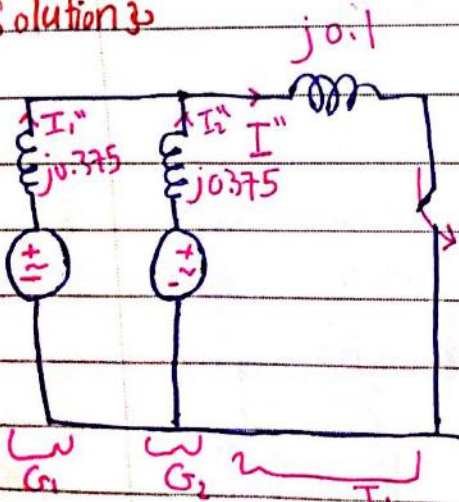
Notes: Fault analysis \rightarrow T.L are represented by their reactance
 \rightarrow transformer are represented by their leakage



load \Rightarrow ignored

12/10/2016

Solution 3



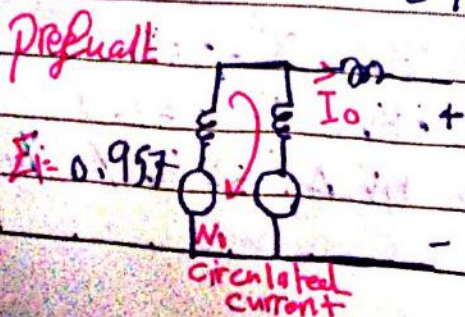
$$G_1 \Rightarrow X'' = 0.25 * \left(\frac{75}{50}\right) = 0.375 \text{ p.u.}$$

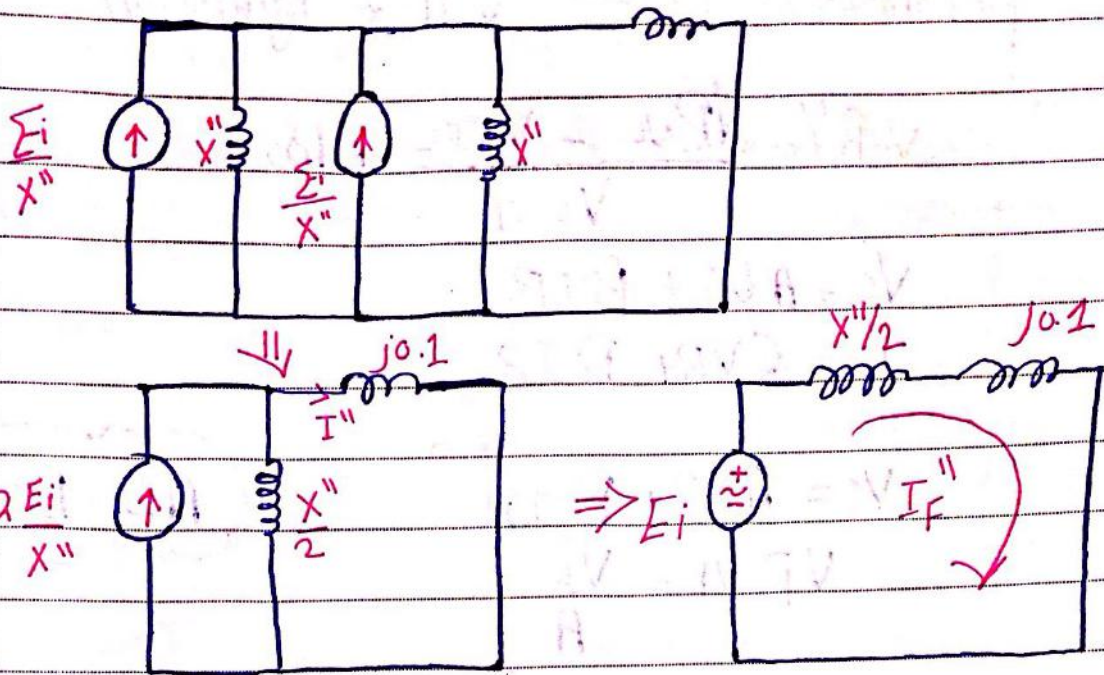
$$G_2 \Rightarrow X'' = 0.25 * \left(\frac{75}{50}\right) = 0.375 \text{ p.u.}$$

$$T_1 \Rightarrow X_{p.u} = 0.1 \text{ p.u.}$$

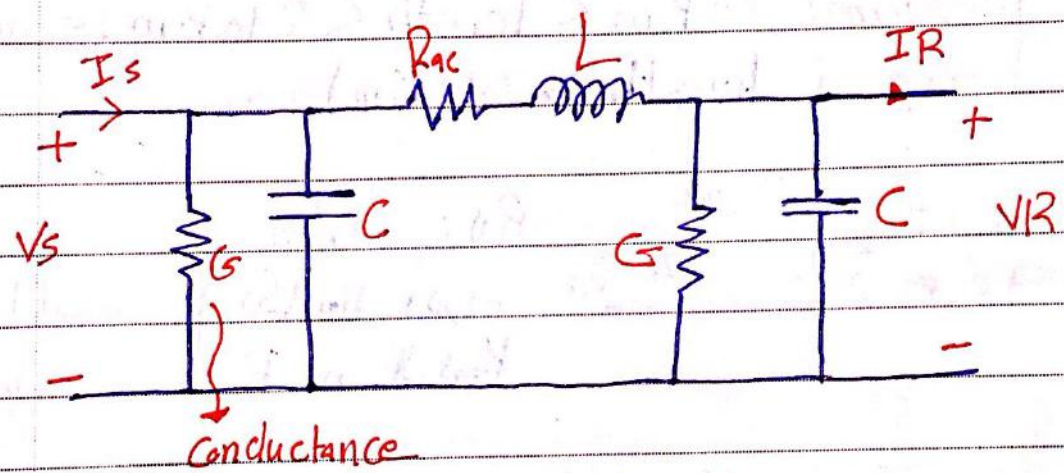
pre-fault \rightarrow unloaded

$$V_{\text{prefault voltage}} = \frac{66}{69} = 0.957 \text{ p.u.}$$

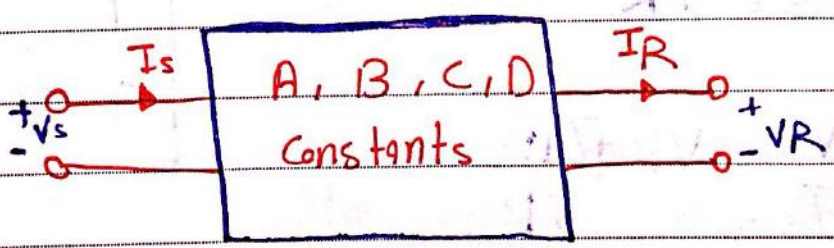




Transmission line modelling



Two Port Networks



$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

$$V_s = AV_R + BI_R$$

$$I_s = CV_R + DI_R$$

Performance (T.L.) \Rightarrow Voltage regulation

$$V.R\% = \frac{V_{R,NL} - V_{R,FL}}{V_{R,FL}} \times 100\%$$

$$V_S = A V_R + B I_R$$

$$I_S = C V_R + D I_R$$

$$V_S = A V_R + B I_R$$

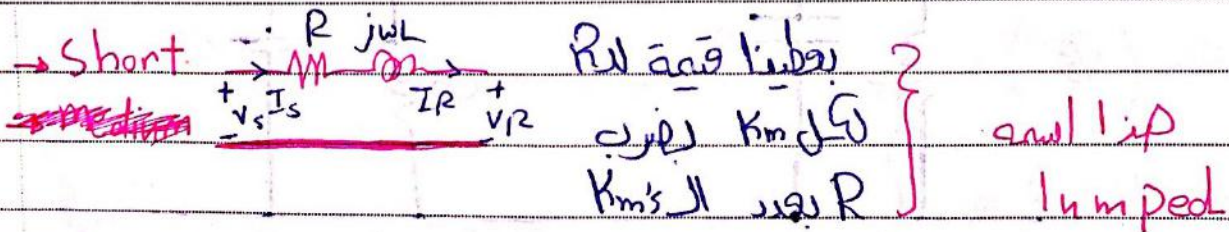
$$V_{R,NL} = \frac{V_S}{A}$$

$$AD - BC = 1$$

Short (length $< 80 \text{ km}$) \Rightarrow lumped Representation

medium (80 km \leq length \leq 240 km) \Rightarrow lumped

long (length $> 240 \text{ km}$) \Rightarrow



$$V_S = I_S Z + V_R$$

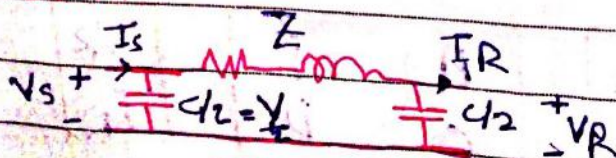
$$I_S = I_R$$

$$\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

$$V_S = V_R + I R$$

$$I_S = I_R$$

\rightarrow medium



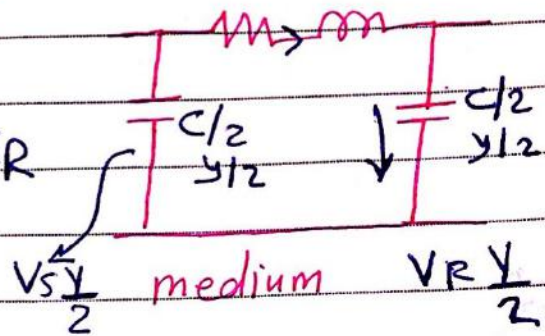
π model

$$V_s = V_R + Z(I_R + V_R \frac{Y}{2})$$

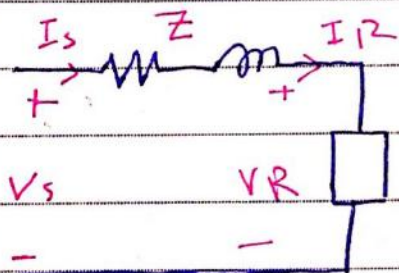
$$V_s = V_R(1 + YZ) + I_R Z$$

$$I_s = V_s \frac{Y}{2} + [I_R + V_R \frac{Y}{2}]$$

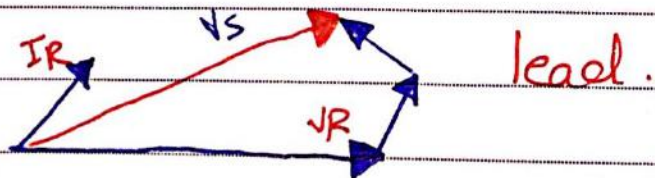
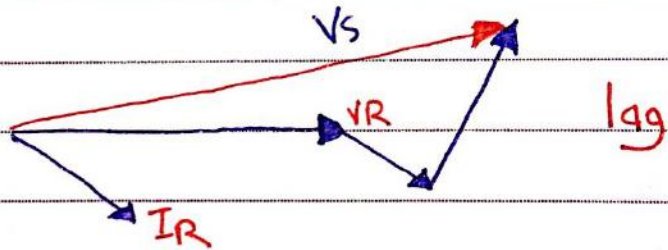
$$I_s = Y(1 + YZ) V_R + (1 + YZ) I_R$$



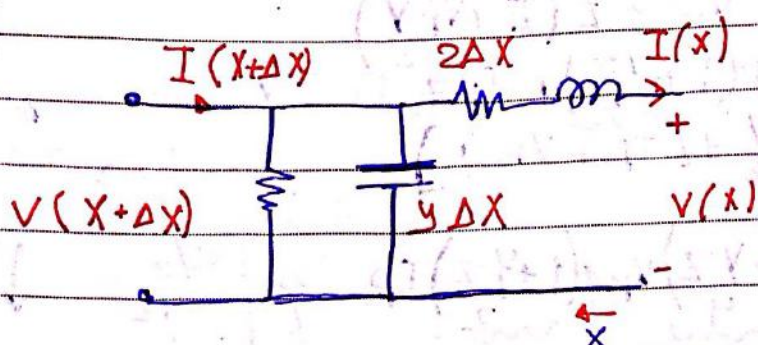
$\frac{1 + YZ}{2}$	Z
$\frac{1 + YZ}{4}$	$\frac{1 + YZ}{2}$



load
 ← unity P.F.
 ← lag P.F.
 ← lead P.F.



→ long Transmission line [cascaded π circuits



Δx (section length)

$R \frac{\Omega}{m} \rightarrow$ overall line $\frac{\Omega}{m} * \text{length}$

$L \frac{h}{m} \rightarrow$ short & medium

$C \frac{F}{m} \rightarrow$ lumped representation

$$V_s = A V_R + B I_R$$

$$I_s = C V_R + D I_R$$

$$V(x + \Delta x) = V(x) + I(x) Z \Delta x$$

$$\lim_{\Delta x \rightarrow 0} \frac{V(x + \Delta x) - V(x)}{\Delta x} = Z I(x)$$

$$\frac{\partial V(x)}{\partial x} = Z I(x) \dots \textcircled{1}$$

KCL \rightarrow

$$I(x+\Delta x) = I(x) + V(x+\Delta x) y \Delta x$$

$$\lim_{\Delta x \rightarrow 0} \frac{I(x+\Delta x) - I(x)}{\Delta x} = V(x+\Delta x) y$$

$$\frac{\partial I(x)}{\partial x} = V(x) y \dots (2)$$



$$\frac{\partial^2 V(x)}{\partial x^2} = -Z \frac{\partial I(x)}{\partial x}$$

$$\frac{\partial^2 V(x)}{\partial x^2} = -Z y V(x)$$

$$V(x) = A_1 e^{\gamma x} + A_2 e^{-\gamma x}$$

Propagation constant $\leftarrow \gamma = \sqrt{ZY} = \alpha + j\beta$

\rightarrow Phase constant
 \rightarrow attenuation constant

$$\frac{\partial V(x)}{\partial x} = A_1 \gamma e^{\gamma x} - A_2 \gamma e^{-\gamma x} = -Z I(x)$$

$$I(x) = \frac{A_1 e^{\gamma x} - A_2 e^{-\gamma x}}{Z/\gamma}$$

$$\frac{Z}{\gamma} = \frac{Z}{\sqrt{ZY}} = \sqrt{\frac{Z}{Y}} \rightsquigarrow \text{Characteristic impedance } Z_c (\Omega)$$

No. 17/10/2016

$$V(x) = A_1 e^{\delta x} + A_2 e^{-\delta x}$$
$$I(x) = \frac{A_1 e^{\delta x} - A_2 e^{-\delta x}}{Z_c}$$

initial condition $V(x=0)$ receiving end
 $I_R(x=0)$

$$V_R = A_1 + A_2$$

$$I_R = \frac{A_1 - A_2}{Z_c}$$

Solving two eqn

$$A_1 = \frac{V_R + Z_c I_R}{2}$$

$$A_2 = \frac{V_R - I_R Z_c}{2}$$

$$\rightarrow V(x) = \left(\frac{V_R + Z_c I_R}{2} \right) e^{\delta x} + \left(\frac{V_R - Z_c I_R}{2} \right) e^{-\delta x}$$

$$\rightarrow I(x) = \frac{\left(\frac{V_R + Z_c I_R}{2} \right) e^{\delta x} - \left(\frac{V_R - Z_c I_R}{2} \right) e^{-\delta x}}{Z_c}$$

$$V(x) = \left(\frac{e^{-\delta x} + e^{\delta x}}{2} \right) V_R + Z_c \left(\frac{e^{\delta x} - e^{-\delta x}}{2} \right) I_R$$

$\left(\frac{e^{-\delta x} + e^{\delta x}}{2} \right) \rightarrow \cosh \delta x$
 $\left(\frac{e^{\delta x} - e^{-\delta x}}{2} \right) \rightarrow \sinh \delta x$

$$I(x) = \frac{1}{Z_c} \left(\frac{e^{\delta x} - e^{-\delta x}}{2} \right) V_R + \left(\frac{e^{\delta x} + e^{-\delta x}}{2} \right) I_R$$

$\left(\frac{e^{\delta x} - e^{-\delta x}}{2} \right) \rightarrow \sinh \delta x$
 $\left(\frac{e^{\delta x} + e^{-\delta x}}{2} \right) \rightarrow \cosh \delta x$

$\rightarrow V(x) = \cosh \gamma x V_R + Z_c \sinh \gamma x I_R$
 $\rightarrow I(x) = \frac{\sinh \gamma x}{Z_c} V_R + \cosh \gamma x I_R$

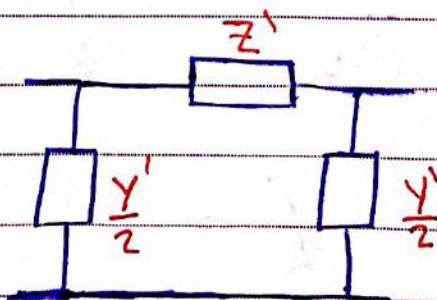
$\gamma = \alpha + j\beta$, if $x = l \rightarrow e^{\gamma l} = e^{(\alpha + j\beta)l} = e^{\alpha l} \angle \beta l$
 $e^{-\gamma l} = e^{-(\alpha + j\beta)l} = e^{-\alpha l} \angle -\beta l$

$\rightarrow \cosh((\alpha + j\beta)l) = \cosh(\alpha l) \cos(\beta l) + j \sinh(\alpha l) \sin(\beta l)$
 $\sinh((\alpha + j\beta)l) = \sinh(\alpha l) \cos(\beta l) + j \cosh(\alpha l) \sin(\beta l)$

$\gamma = \sqrt{ZY}$ YZ circuit
 use lumped
 circuit system
 ZY circuit

$e^{\gamma l} \rightarrow \gamma l = \sqrt{ZY} l = \sqrt{Zl Yl} = \sqrt{ZY}$

Equivalent π circuit model



π circuit
 $A = D = 1 + \frac{Y'Z'}{2}$
 $B = Z'$
 $C = Y' \left(1 + \frac{Y'Z'}{4} \right)$

$$A = \cosh \delta x$$

$$B = Z_c \sinh \delta x$$

$$C = \frac{\sinh \delta x}{Z_c}$$

$$D = \cosh \delta x$$

$x=L$

$$Z' = Z_c \sinh \delta L$$

$$Z' = \sqrt{\frac{Z}{Y}} \sinh \delta L \quad * \frac{ZL}{ZL}$$

$$Z' = ZL * \frac{\sinh \delta L}{\sqrt{ZY} L}$$

✓ $Z' = Z \frac{\sinh \delta L}{\delta L}$ → Correction Factor

$$\cosh \delta L = 1 + \frac{YZ'}{2}$$

$$\frac{Y'}{2} = \frac{\cosh \delta L - 1}{Z'}$$

$$\frac{Y'}{2} = \frac{\cosh \delta L - 1}{Z_c \frac{\sinh \delta L}{\delta L}} = \frac{\cosh \delta L - 1}{Z_c \sinh \delta L}$$

$$Z = \frac{Z_c}{\sqrt{ZY}}$$

$$= \sqrt{\frac{Z}{Y}}$$

$$= Z_c$$

$$\frac{Y'}{2} = \frac{\tanh(\delta L/2)}{Z_c} * \frac{YL/2}{YL/2}$$

✓ $\frac{Y'}{2} = \frac{Y'}{2} * \frac{\tanh(\delta L/2)}{\delta L/2}$ → Correction Factor

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$$A = 1 + \gamma' z' / 2$$

$$B = z'$$

$$C = \gamma' (1 + \gamma' z' / 4)$$

$$D = 1 + \gamma' z' / 2$$

$$\tanh^2 \gamma' = \frac{\cosh^2 \gamma' - 1}{\sinh^2 \gamma'}$$

→ lossless transmission line ($R=0, G=0$)

→ long transmission line → $A = \cosh \gamma L, B = Z_c \sinh \gamma L$
 " exact " $C = \frac{\sinh \gamma L}{Z_c}, D = \cosh \gamma L$

→ lossless $\gamma = \alpha + j\beta$ $\rightarrow \omega\sqrt{LC}$ " $X=L$ "

$$A = \cosh \gamma L = \frac{1}{2} (e^{\gamma L} + e^{-\gamma L})$$

$$\gamma = j\beta \rightarrow A = \frac{1}{2} (e^{j\beta L} + e^{-j\beta L}) = \cos \beta L$$

$$B = Z_c \sinh \gamma L = Z_c \frac{1}{2} (e^{\gamma L} - e^{-\gamma L})$$

$$B = Z_c \frac{1}{2} (e^{j\beta L} - e^{-j\beta L}) = j Z_c \sin \beta L$$

phase 90°

$$C = \frac{j}{Z_c} \sin \beta L, D = \cos \beta L$$

$\angle A = 0$
 $\angle B = 90^\circ$

$$V(x) = AV_R + BI_R$$

~~$$V_s = V(x=L) = \cos \beta L V_R + j Z_c \sin \beta L I_R$$~~

$$Z_c = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}} \quad \text{H/m}$$

real

is lossless

line

" surge impedance "

→ wave length: $\lambda = \frac{2\pi}{\beta} \rightsquigarrow \omega\sqrt{LC}$ Phase constant

$\lambda f = v \leftarrow$ velocity of propagation

$$\lambda f = \frac{1}{\sqrt{LC}}$$

LN $\leftarrow V_s = \cos\beta L V_R + j Z_c \sin\beta L I_R$

@ no load

$$V_s = \cos\beta L V_R + j Z_c \sin\beta L I_R$$

$$V_R = \frac{V_s}{\cos\beta L}$$

$$|V_R| > |V_s|$$

$$I_s = C V_R + D I_R$$

$I_s \neq 0$ because

← $I_s = \frac{j}{Z_c} \sin\beta L V_R$
(value) L, β

we have

Capacitor

No. 19/10/2016

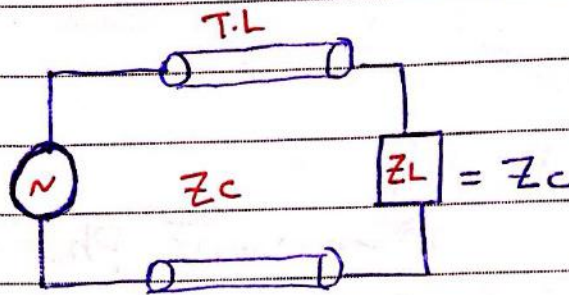
@ short circuit ($V_R = 0$)

$$V_S = A V_R + B I_R$$

$$I_S = C V_R + D I_R$$

$$|I_S| = \cos \beta L I_R$$

→ Surge impedance loading (SIL)



measure of line capacity \leftarrow $SIL = \frac{|V_{L, rated}|^2}{Z_c} = \frac{|V_{L, rated}|^2}{Z_c}$

→ Prove voltage, current for a lossless line with surge impedance Z_c constant?

$$V(x) = \cos \beta x V_R + j Z_c \sin \beta x I_R$$

$$I_R = \frac{V_R}{Z_c}$$

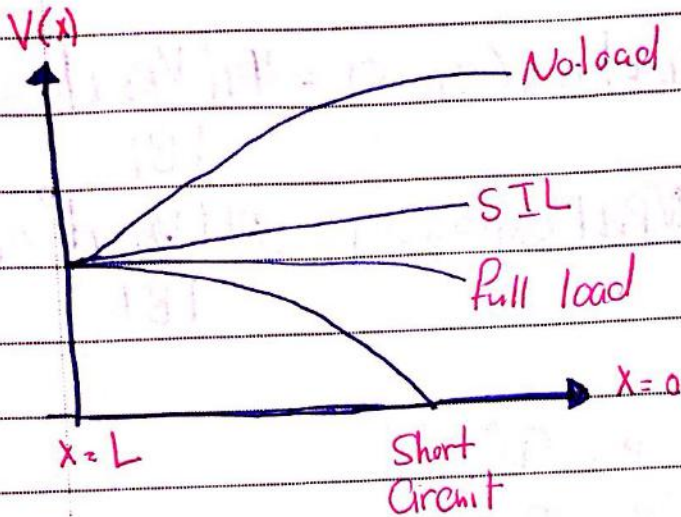
$$V(x) = \cos \beta x V_R + j Z_c \sin \beta x \frac{V_R}{Z_c}$$

$$= (\cos \beta x + j \sin \beta x) V_R$$

$$\Rightarrow V(x) = V_R e^{j \beta x}$$

$$|V(x)| = |V_R|$$

No. 19/10/2016



- * $A = |A| \angle \theta_A$
- * $B = |B| \angle \theta_B$
- * $V_S = |V_S| \angle S$ ← power angle

∴ $V_R = |V_R| \angle 0$

$V_S = A V_R + B I_R$

$I_R = \frac{V_S - A V_R}{B}$

$I_R = \frac{|V_S| \angle S - \theta_B - |A| |V_R| \angle \theta_A - \theta_B}{|B|}$

1φ

$P_R + j Q_R = \frac{|V_S| |V_R| \angle \theta_B - S - |A| |V_R|^2 \angle \theta_B - \theta_A}{|B|}$

for 3φ $\frac{V_{SL-L}}{\sqrt{3}} \times 3$

*3

→ $S_{P,3\phi} = \frac{|V_{SL-L}| |V_{RL-L}| \angle \theta_B - S - |A| |V_{RL-L}|^2 \angle \theta_B - \theta_A}{|B|}$

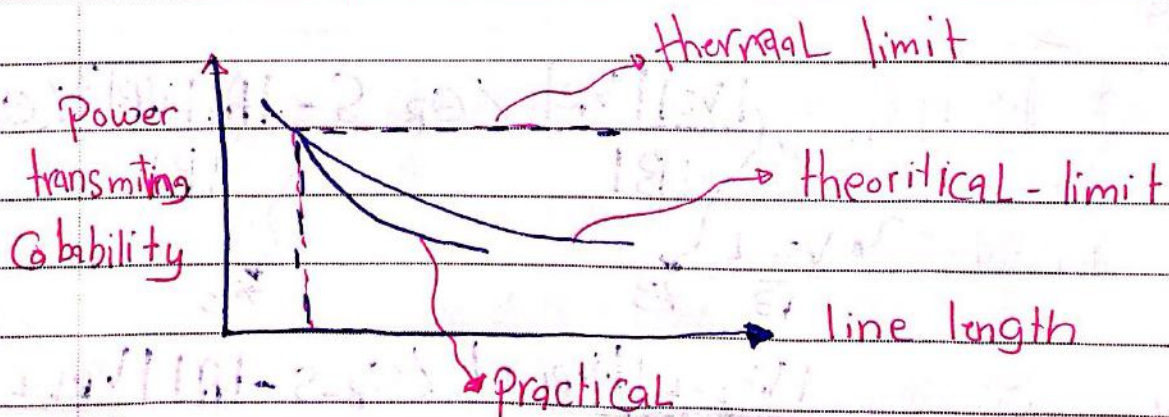
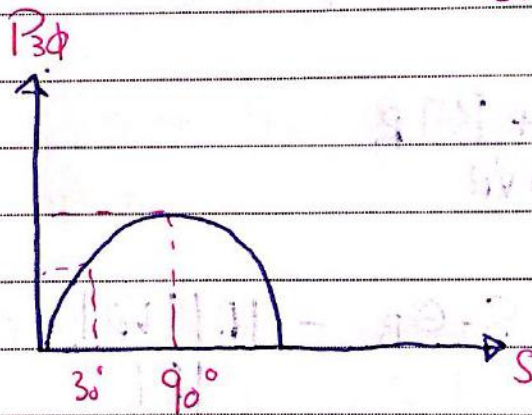
$$\rightarrow P_{R,3\phi} = \frac{|V_{S-L}| |V_{R-L}| \cos(\theta_B - \theta_S) - |A| |V_{R-L}|^2 \cos(\theta_B - \theta_A)}{|B|}$$

$$\rightarrow Q_{R,3\phi} = \frac{|V_{S-L}| |V_{R-L}| \sin(\theta_B - \theta_S) - |A| |V_{R-L}|^2 \sin(\theta_B - \theta_A)}{|B|}$$

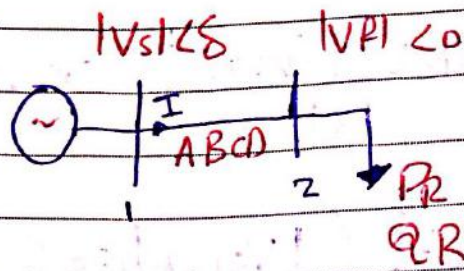
For loss less $\left(\begin{matrix} \theta_B = 90^\circ \\ \theta_A = 0 \end{matrix} \right)$

$$P_{R,3\phi} = \frac{|V_{S-L}| |V_{R-L}| \sin S}{|B|}$$

$Z \sin \beta L$



No. 24/10/2016



short line & losses

$$A = 1 = D, \quad B = Z = jX = |X| \angle 90^\circ$$

$$P_{R,3\phi} = \frac{|V_{S-L-L}| |V_{R-L-L}| \cos(\theta_B - \delta)}{|B|} - \frac{|A| |V_{R-L-L}|^2 \cos(\theta_B - \delta)}{|B|}$$

$$Q_{R,3\phi} = \frac{|V_{S-L-L}| |V_{R-L-L}| \sin(\theta_B - \delta)}{|B|} - \frac{|A| |V_{R-L-L}|^2 \sin(\theta_B - \delta)}{|B|}$$

$$\theta_B = 90^\circ \quad \theta_A = 0$$

$$Q_{P,3\phi} = \frac{|V_{SL-L}| |V_{RL-L}| \cos \delta - |V_{RL-L}|^2}{|X|}$$

$$= \frac{|V_{RL-L}|}{X} (|V_{SL-L}| \cos \delta - |V_{RL-L}|)$$

δ very small

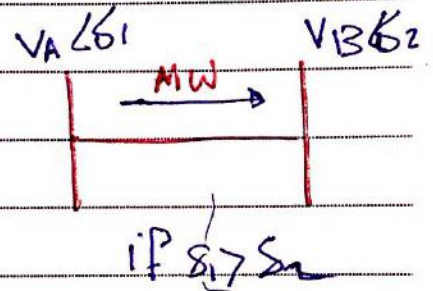
$$Q_{P,3\phi} = \frac{|V_{RL-L}|}{|X|} (|V_{SL-L}| - |V_{RL-L}|)$$

$$P_{R,3\phi} = \frac{|V_{SL-L}| |V_{RL-L}| \sin \delta}{|X|}$$

δ very small

$$P_{R,3\phi} = \frac{|V_{SL-L}| |V_{RL-L}| \delta}{|X|}$$

$Q \propto \Delta V$



$$|V_A| > |V_B|$$

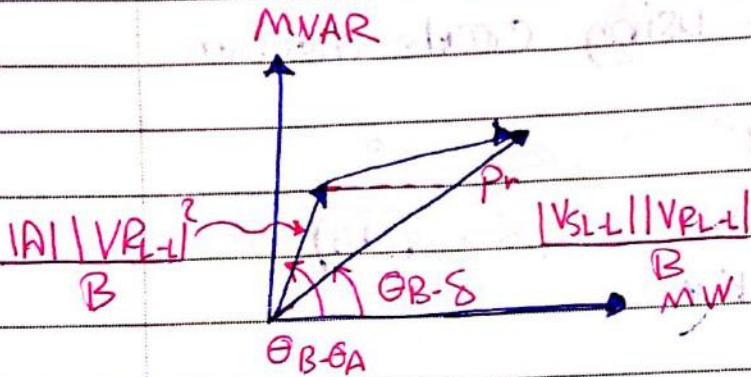
* Transfer real power

Real power flow \rightarrow angle
 reactive power flow \rightarrow magnitude

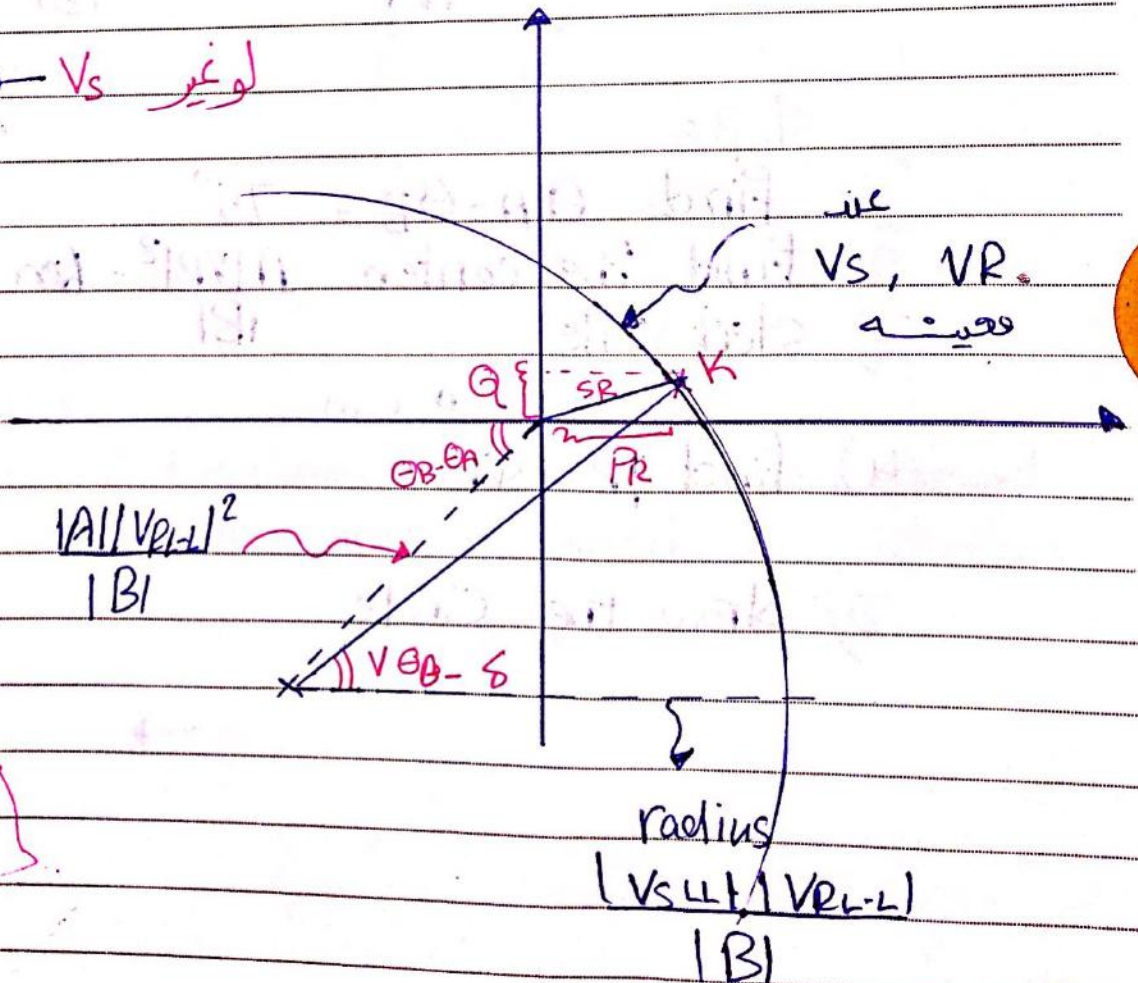
* Power of flow
 magnitude \rightarrow
 Voltage

→ Receiving end Power Circle diagram

$$S_3\phi = \frac{|V_{SL-L}| |V_{RL-L}| \angle \theta_B - \delta - |A| |V_{RL-L}|^2 \angle \theta_B - \theta_A}{|B|}$$



لو غير V_s ← لزيادة (radius)



→ if $\theta_B - \delta = 0 \rightarrow$ max power Transfer @ negative

Ex: A long T.L $A = 0.88 \angle -2^\circ$, $B = 120 \angle 77^\circ$,

load @ receiving end: 1.70 MW, 0.88 lag @ 300 kV

Find V_s , δ using circle diagram

$$\theta_B = 77^\circ, \theta_A = 2^\circ$$

$$P_R = 1.70 \text{ MW} \quad Q_R = \sqrt{\left(\frac{P_R}{p}\right)^2 - P_R^2}$$

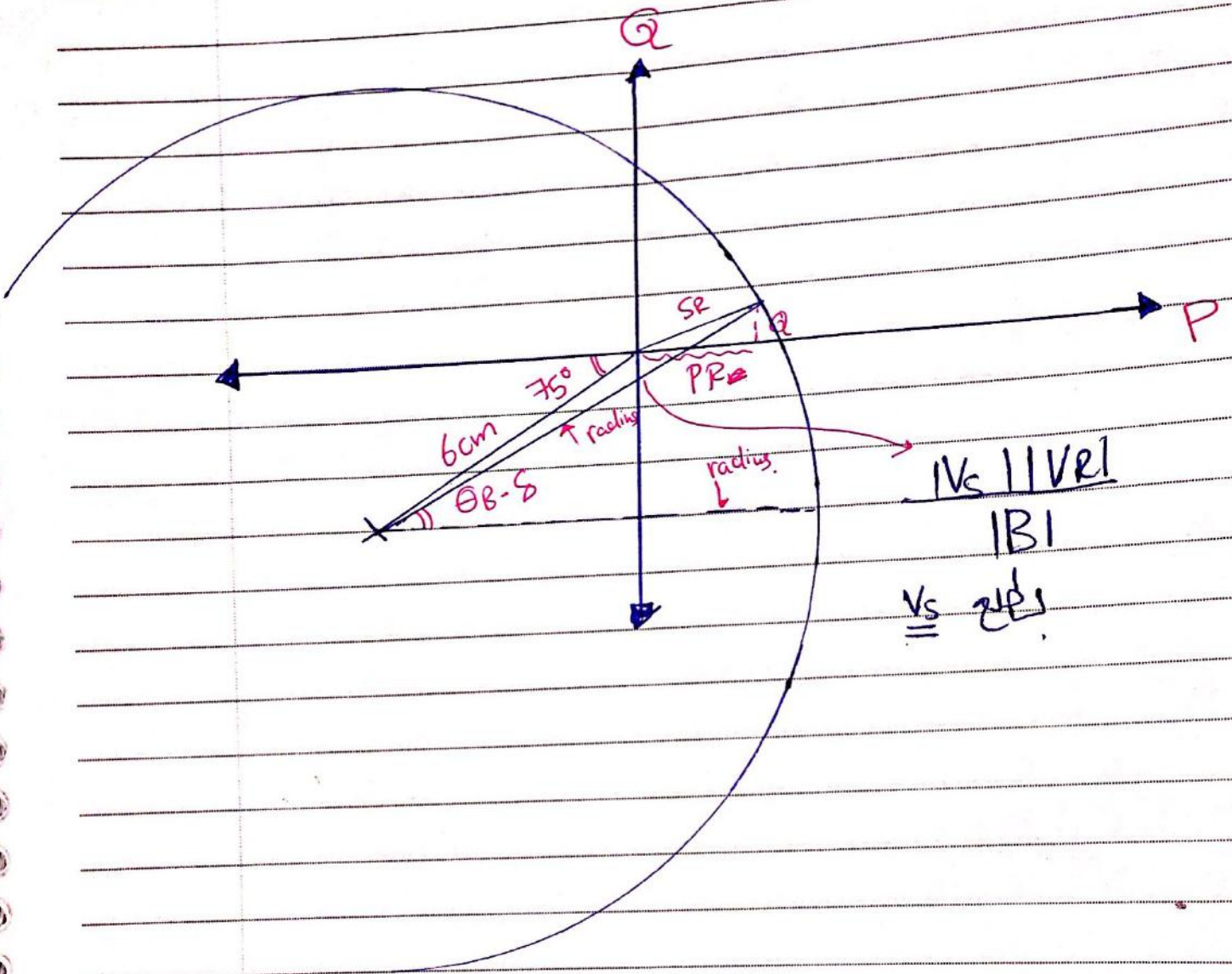
$$|V_R| = 300 \text{ kV}$$

$$\text{Center} = \frac{|A||V_R|^2}{|B|} = \frac{0.88 \times (300 \text{ kV})^2}{120} = 600$$

Steps:

- 1) Find $\theta_A - \theta_B = 75^\circ$
- 2) Find the center $\frac{|A||V_R|^2}{|B|} = 600$
- 3) select scale
- 4) Find P_R , Q_R
1.7 cm 0.9 cm
- 5) draw the circle

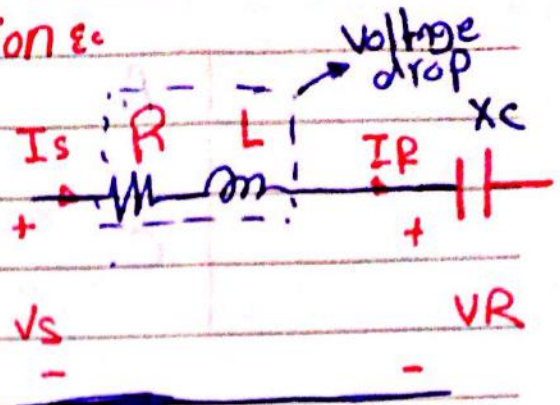




Note if the capability prevent me to exceed certain VR we increase V_s by capacitor Bank

* Reactive power Compensation etc

1) Series Compensation
 " heavy load "

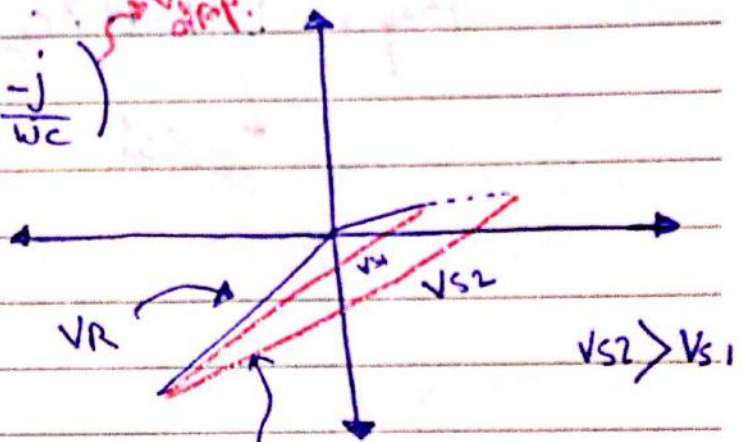


$X_c = \Delta$ Compensation Factor

Impedance X_L

$V_R = V_s - I_R (R + j\omega L - \frac{j}{\omega C})$

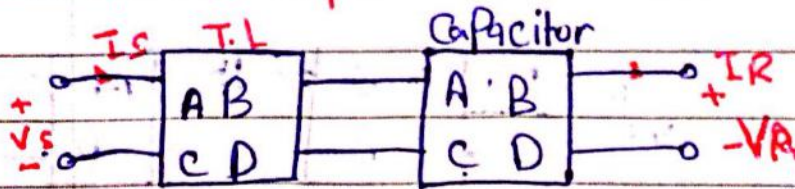
Adv \Rightarrow Increase maximum power that could be transmitted



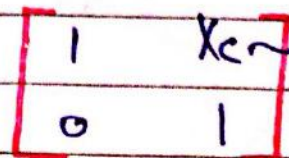
$P = \frac{|V_s| |V_R| \sin \delta}{|B|}$ lossless T.L

$Z \approx X$

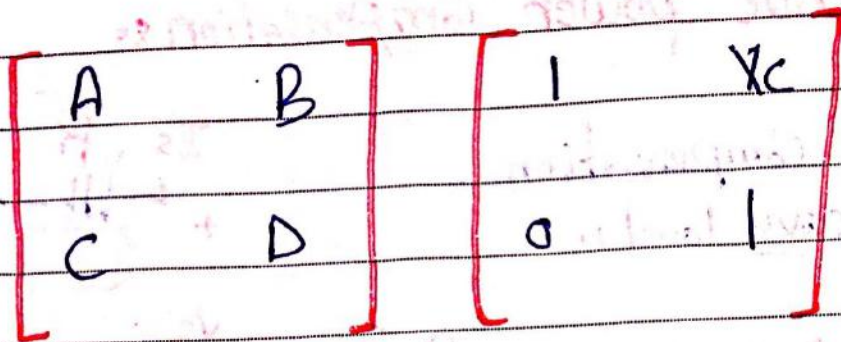
\Rightarrow ABCD capacitor



$I_s = I_R$
 $V_s = I_R X_c + V_R$



$|X_c| \angle -90^\circ$



$A_{eq} = A$

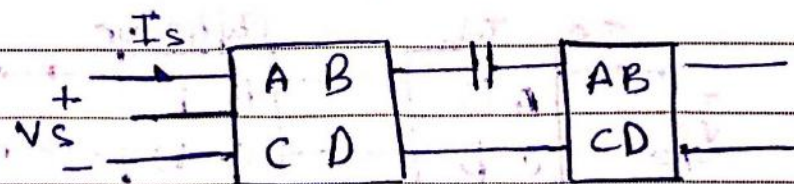
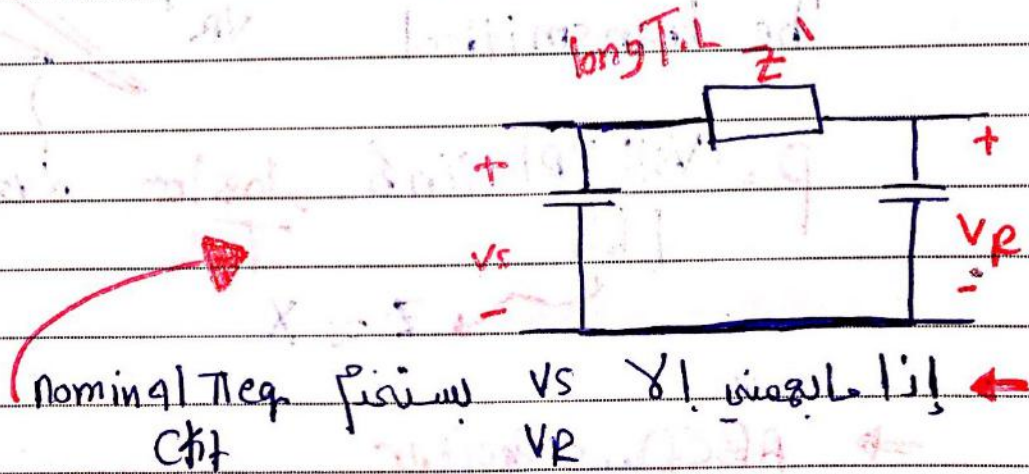
$B_{eq} = B + AX_c$

$\approx \angle 90^\circ$

$\approx \angle -90^\circ$

$B_{eq} \rightarrow$ max power transfer \uparrow

\rightarrow voltage drop \downarrow



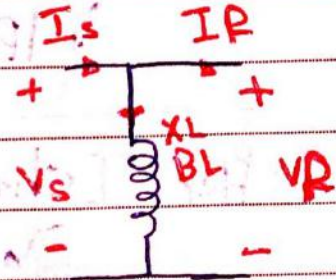
operating conditions are interest \ll model ABCD For each position \rightarrow

2) Shunt inductor & light load

@ No load → Voltage ↑

$$V_S = A V_R + B I_P$$

$A < 1$



Adv ⇒ increase A

Susceptance $B_L = \frac{1}{j\omega L}$

A B C D constant

$$V_S = V_R$$

$$I_S = I_P + \frac{V_R}{X_L}$$

→ Facts devices
→ STATCOM

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{X_L} & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_P \end{bmatrix}$$

$|B_L| \angle -90^\circ$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j|B_L| & 1 \end{bmatrix}$$

T.L

Inductor

$$A_{eq} = A + B * -j|B_L|$$

$$A_{eq} = A + B|B_L| \angle -90^\circ$$

1) Shunt reactor

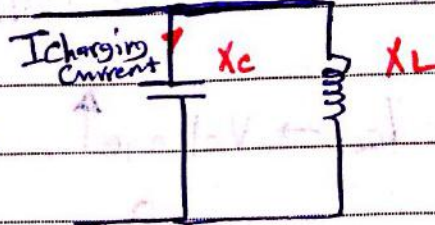
limiting Voltage

2) decreases Charging Current

before

$I_{charging}$

$\rightarrow |Bc| V_{LN}$

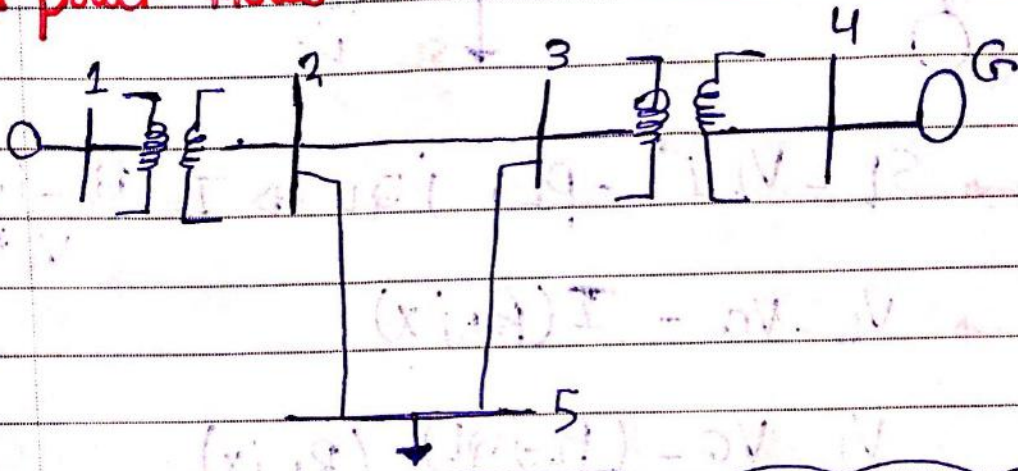


after

$I_{charging}$

$\rightarrow |Bc - BL| V$

→ Power Flows



→ objective "what to find"

- 1) V, S
- 2) P, Q throughout T.L.F
Transformers
- 3) Voltage & flows must be within limits
- 4) adverse conditions
 - loss of lines
 - loss of Generation

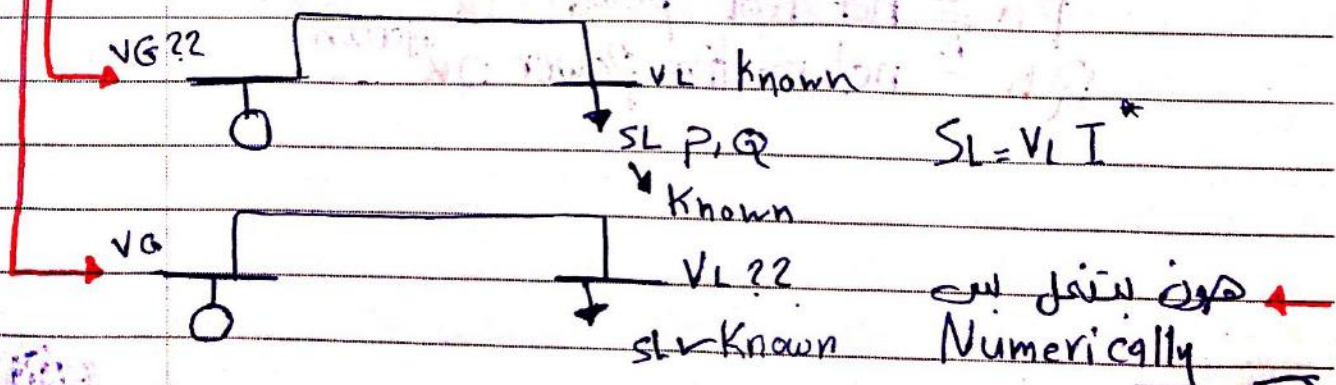
کے لیے پاور ویج
Transformer matching
Generator کے لیے پاور

* Note

Circuits ⇒ loads as constant impedance.

Ex: $V = j\omega LI$ "Linear"

power sys ⇒ loads constant power.



نمبری طور پر
Numerically



→ $S_L = V_L I^* = P_L + jQ_L \Rightarrow I = \frac{P_L - jQ_L}{V_L^*}$

→ $V_L = V_G - I(R + jX)$

$V_L = V_G - \frac{(P_L - jQ_L)}{V_L^*} (R + jX)$

non-linear

initial value $V_L^{(0)} = 1 < 0$

IF $|V_L^{(i+1)} - V_L^{(i)}| \leq \epsilon$

stop

Tolerance

→ Formulation of power flow ω \rightarrow all J

→ For each bus k

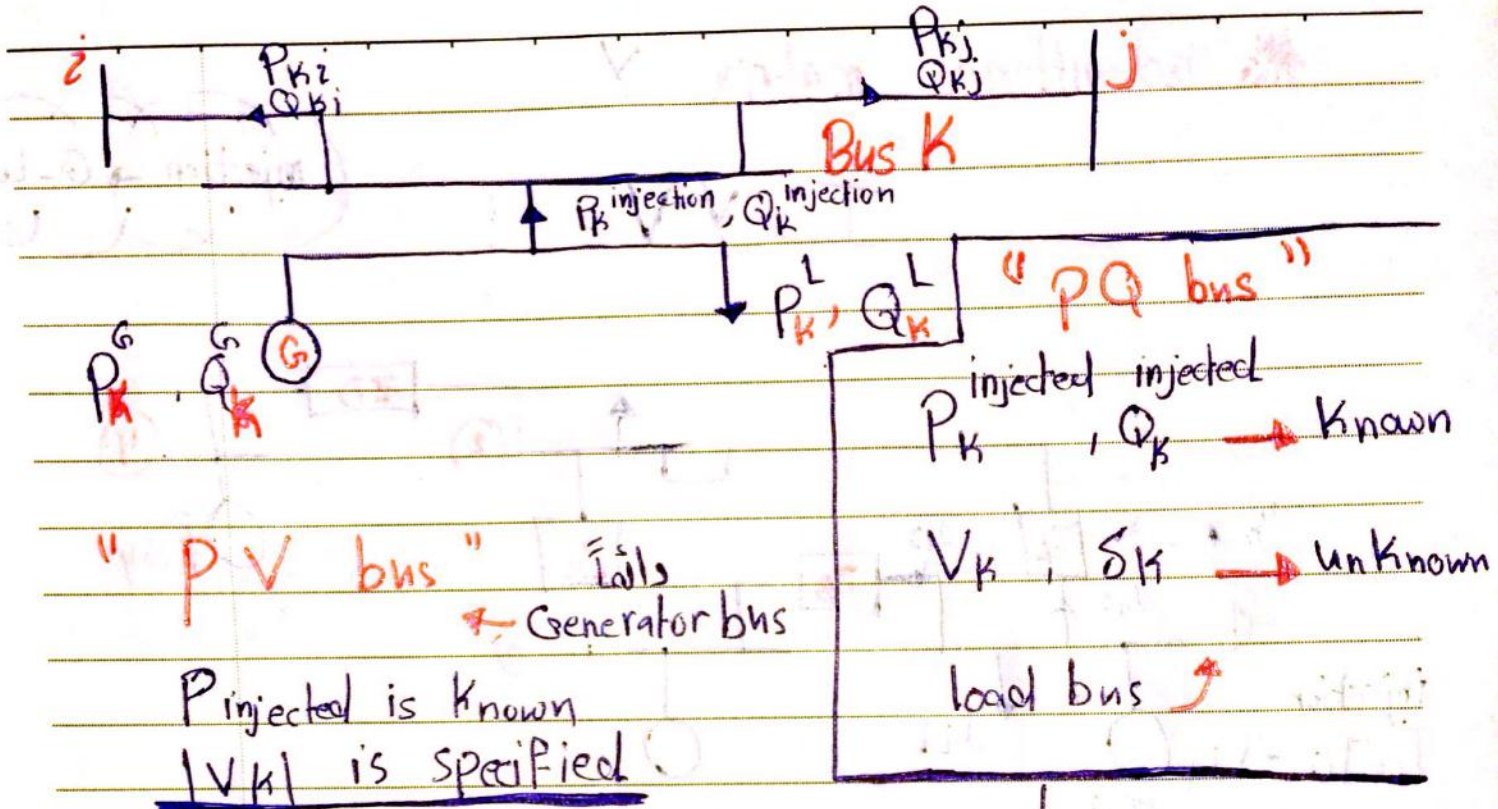
V_k

S_k

$P_k \triangleq$ net real power $P_k^{injection}$

$Q_k \triangleq$ net reactive power $Q_k^{injection}$

Steady state



δ_k
 Q_k "must have VAR sources at each PV bus"

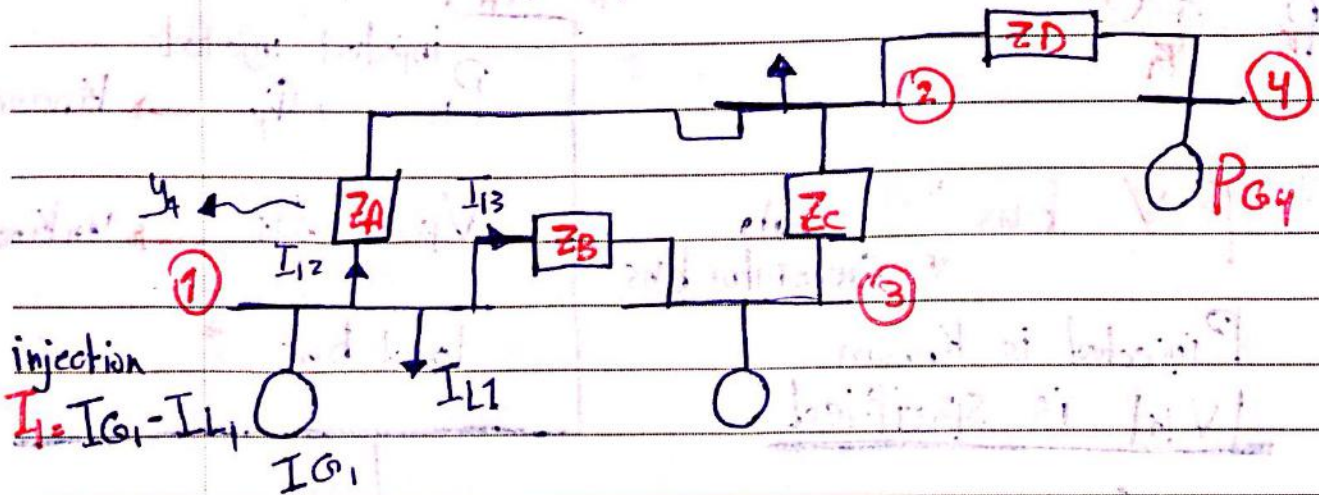
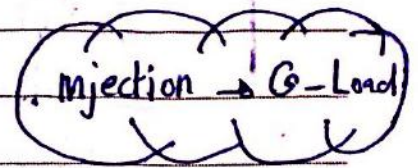
"Slack bus" \rightarrow Balanced system
 Load $\leftarrow G$ (بیلن G)
 losses

Equation: $P_{system}^{Generation} = P_{system}^{load} + P_{system}^{losses}$

P, Q unknown
 $|V_k|, \delta_k \Rightarrow 120 p.u$
 voltage "unknown"

* admittance matrix Y

$$I = YV$$



injection
 $I_1 = I_{G1} - I_{L1}$

$$I_1 = I_{12} + I_{13}$$

$$I_1 = I_{12} + I_{13}$$

$$= \frac{V_1 - V_2}{Z_A} + \frac{V_1 - V_3}{Z_B}$$

$$= (V_1 - V_2) Y_A + (V_1 - V_3) Y_B$$

$$I_1 = V_1 (Y_A + Y_B) - Y_A V_2 - Y_B V_3$$

$$I_2 = V_1 + V_2 + V_3 + V_4$$

$$I_3 =$$

$$I_4 =$$

$$\begin{bmatrix}
 Y_A + Y_B & -Y_A & -Y_B & 0 \\
 -Y_A & Y_A + Y_B + Y_D & -Y_C & -Y_D \\
 -Y_B & -Y_C & Y_B + Y_C & 0 \\
 0 & -Y_D & 0 & Y_D
 \end{bmatrix}$$

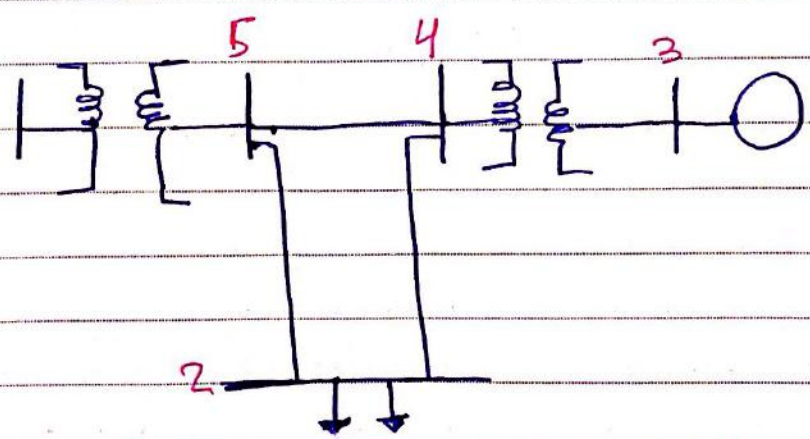
Symmetrical matrix

off diagonal

Bus 1110

most entries are zero "sparse matrix"

H.W



lines	R (p.u)	X (p.u)	B (p.u)
2-4	0.009	0.1	1.72
2-5	0.0045	0.05	0.88
4-5	0.00225	0.025	0.44

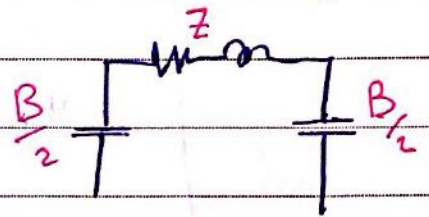
TX	R	X
1-5	0.0015	0.02
3-4	0.00075	0.01

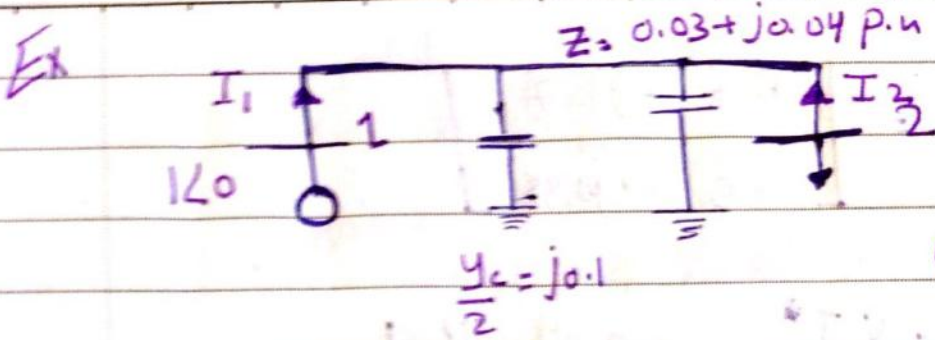
No. 2/11/2016

Q Calculate admittance matrix α element of second Row 11.

Note Medium Transmission line

Sol: $Y_{21} = Y_{23} = 0$
 $Y_{24} = 9.9572 \angle 95.143 \text{ p.u.}$
 $Y_{25} = 19.9145 \angle 95.143 \text{ p.u.}$
 $Y_{22} = 28.5847 \angle -84.624 \text{ p.u.}$





"shunt capacitor effect the diagonal of matrix"

- ① Find Y
- ② Find I , injected power @ bus 1, if

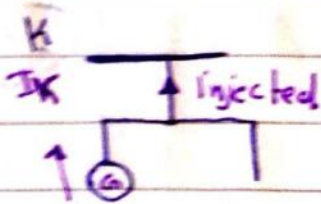
$V_1 = 120$
 $V_2 = 0.82 \angle -14$

Sol: $Y = \frac{1}{Z} = \frac{1}{0.03 + j0.04} = 12 - j16 \text{ p.u.}$

$$Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 12 - j16 + j0.1 & -(12 - j16) \\ (12 - j16) & 12 - j16 + j0.1 \end{bmatrix}$$

"12 - j15.9"

"12 - j15.9"



"load is Capacitive بتأثر على Y_{22} "

$I = YV$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

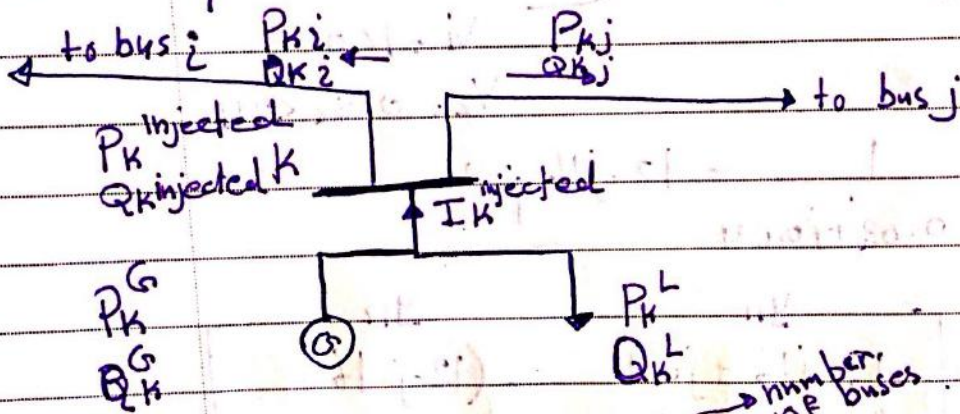
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 5.6 - j0.7 \\ -5.58 + j0.88 \end{bmatrix}$$

$$S_1 = V_1 I_1^* = 0.37 - j4.51$$

$$S_2 = V_2 I_2^*$$

* Power Flow Formulation

$$X = F(x)$$



$$P_k^{\text{injected}} = \sum_{i=1}^N P_{ki} = \sum_{i=1}^N P_k^G - P_k^L$$

$$Q_k^{\text{injected}} = \sum_{i=1}^N Q_{ki} = \sum_{i=1}^N Q_k^G - Q_k^L$$

$$S_k^{\text{injected}} = V_k I_k^{\text{injected}}$$

$$I_k^{\text{injected}} = \sum_{i=1}^N y_{ki} V_i$$

Power through Branches

$$I = YV$$

$$S_k^{\text{injected}} = V_k \left[\sum_{i=1}^N y_{ki} V_i \right]^*$$

$$S_k^{\text{injected}} = \sum_{i=1}^N y_{ki}^* V_k V_i^*$$

let $V_k = |V_k| \angle \theta_k$, $V_i = |V_i| \angle \theta_i$

$$y_{ki} = G_{ki} + jB_{ki}$$

injected

$$P_k + jQ_k = S_k^{\text{injected}} = \sum_{i=1}^N (G_{ki} - jB_{ki}) |V_k| \angle \theta_k |V_i| \angle -\theta_i$$

$$= \sum_{i=1}^N (G_{ki} - jB_{ki}) |V_k| |V_i| \angle \theta_k - \theta_i$$

$$\rightarrow P_k^G - P_k^L = P_k = \sum_{i=1}^N |V_k| |V_i| (G_{ki} \cos(\theta_k - \theta_i) + B_{ki} \sin(\theta_k - \theta_i))$$

$$\rightarrow Q_k^G - Q_k^L = Q_k = \sum_{i=1}^N |V_k| |V_i| (G_{ki} \sin(\theta_k - \theta_i) - B_{ki} \cos(\theta_k - \theta_i))$$

PQ bus $\rightarrow P_k, Q_k$, unknown $V_k, \theta_k \rightarrow$ load

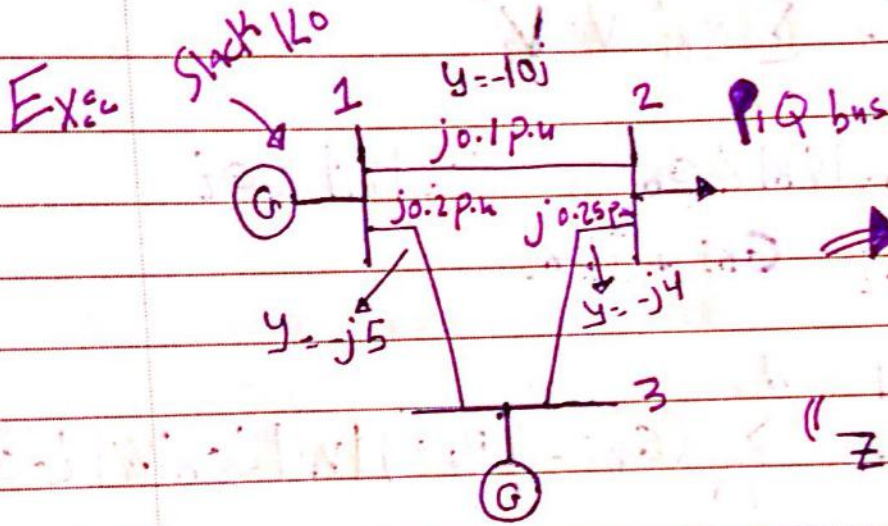
PV bus $\rightarrow P_k, |V_k|$, unknown $Q_k, \theta_k \rightarrow$ generator

slack bus $\rightarrow P_k$ is adjusted to keep the system balance, $|V_k|, \theta_k$

\rightarrow $1 < 0$ abaw ille

Equations & Variables

2 * N equation
4 * N variables



Write Power flow equations

("لا تلتزم + ج عرف انجا")

$$Y = \begin{bmatrix} -j15 & j10 & j5 \\ j10 & -14j & j4 \\ j5 & j4 & -j9 \end{bmatrix}$$

$$Q_i = |V_i| |V_j| \times (-15) \cos \theta_{11} + (-10) |V_i| |V_2| \cos \theta_{12} - 5 |V_i| |V_3| \cos \theta_{13}$$

$$P_1 = |V_1| |V_1| (-15) \sin(\theta_{11}) + |V_1| |V_2| (10) \sin(\theta_{12}) + |V_1| |V_3| \sin(\theta_{13}) (5)$$

$$\rightarrow P_1 = |V_1| |V_2| \times 10 \sin \theta_{12} + |V_1| |V_3| \times 5 \sin \theta_{13}$$

~~Q1 = 15 |V1|^2 - 10 |V1| |V2| cos theta 12 - 5 |V1| |V3| cos theta 13~~

$$\rightarrow Q_1 = 15 |V_1|^2 - 10 |V_1| |V_2| \cos \theta_{12} - 5 |V_1| |V_3| \cos \theta_{13}$$

Known implicit eqn

$$P_2 = 10 |V_2| |V_1| \sin \theta_{21} + 4 |V_2| |V_3| \sin \theta_{23}$$

$$Q_2 = 14 |V_2|^2 - 10 |V_2| |V_1| \cos \theta_{21} - 4 |V_2| |V_3| \cos \theta_{23}$$

$$P_3 = 5 |V_3| |V_1| \sin \theta_{31} + 4 |V_3| |V_2| \sin \theta_{32}$$

explicit
equ

$$P_1 = 10 |V_1| |V_2| \sin \theta_{12} + 5 |V_1| |V_3| \sin \theta_{13}$$

$$Q_1 = 15 |V_1|^2 - 10 |V_1| |V_2| \cos \theta_{12} - 5 |V_1| |V_3| \cos \theta_{13}$$

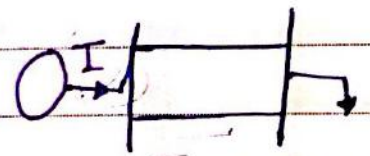
$$Q_3 = 9 |V_3|^2 - 5 |V_1| |V_3| \cos \theta_{31} - 4 |V_3| |V_2| \cos \theta_{32}$$

⇒ Power Flow Solution 3

* Solve for the unknown voltages through implicit equation "unknown left hand side"
P, Q

* obtain unknown real & reactive power
"explicit equations"

injected $\rightarrow I = YV$
 $S = VI^*$



⇒ Gauss seidel "simple iterative approach"

$$F(x) = 0$$

↳ Fixed point solution

$$X = F(x)$$

Solve $X - \sqrt{X} - 1 = 0$
 $X = \sqrt{X} + 1$

- initial Guess
- iterative
- stopping Criteria

$$X^{(0)} = 1 \Rightarrow X^{(1)} = 2 \Rightarrow X^{(2)} = \sqrt{2} + 1$$

9 iterations $\Rightarrow 2.617$

Iteration number \rightarrow

$$|X_i^{VH} - X_i^V| \leq \epsilon \rightarrow \text{tolerance}$$

$$X = F(x)$$

\rightarrow Current mismatch

$$\Delta I_i = 0$$

$$I_i - \sum_{j=1}^N y_{ij} V_j = 0$$

$$I_i - y_{ii} V_i - \sum_{\substack{j=1 \\ j \neq i}}^N y_{ij} V_j = 0$$

$$I_i = \frac{S_i^*}{V_i^*}$$

$$y_{ii} V_i = I_i - \sum_{\substack{j=1 \\ j \neq i}}^N y_{ij} V_j$$

$$V_i = \frac{1}{y_{ii}} \left[\frac{S_i^*}{V_i^*} - \sum_{\substack{j=1 \\ j \neq i}}^N y_{ij} V_j \right]$$

$$V_i = \frac{1}{y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{j=1 \\ j \neq i}}^N y_{ij} V_j \right]$$

from admittance matrix diameter

$y, V \Rightarrow \text{complex}$

$$V_i = \frac{1}{y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{j=1}^{i-1} y_{ij} V_j - \sum_{j=i+1}^N y_{ij} V_j \right]$$

iterative

Iteration \rightarrow (k+1)

$$V_i^{(k+1)} = \frac{1}{y_{ii}} \left[\frac{P_i - jQ_i}{V_i^{(k)*}} - \sum_{j=1}^{i-1} y_{ij} V_j^{(k+1)} - \sum_{j=i+1}^N y_{ij} V_j^{(k)} \right]$$

$$\max |\Delta V_i| = |V_i^{(k+1)} - V_i^{(k)}| \leq \epsilon$$

Convergence \rightarrow poor
heavy computation burden
 \Rightarrow more iterations

Ex:

5 bus (1 slack
4 PQ bus)

V_2, V_3, V_4, V_5 c. usv line

first

iteration

$$V_2 = F(V_1^{(0)}, V_2^{(0)}, V_3^{(0)}, V_4^{(0)}, V_5^{(0)})$$

$$V_3 = F(V_1^{(0)}, V_2^{(1)}, V_3^{(0)}, V_4^{(0)}, V_5^{(0)})$$

$$V_4 = F(V_1^{(0)}, V_2^{(1)}, V_3^{(1)}, V_4^{(0)}, V_5^{(0)})$$

\rightarrow Does not handle PV bus ?? "match"

The resulting $|V|$ will not coincide with its specified value

$$Q_i = \sum_{j \in N} |V_i||V_j| [G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}]$$

$$Q_i = Q_{Gi} - Q_{demand}$$

$$Q_{Gi} = Q_i + Q_{demand}$$

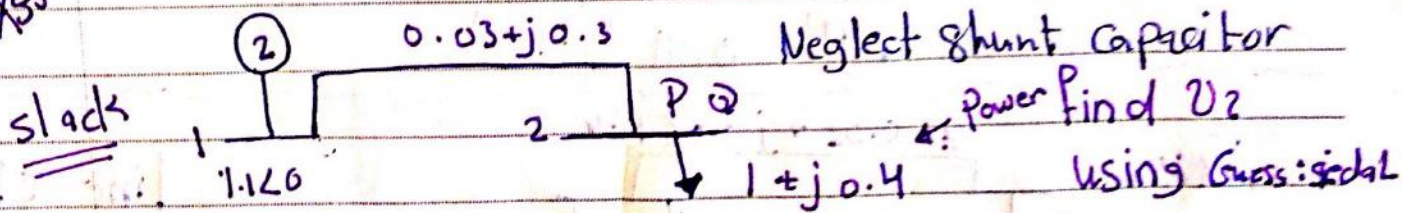
if $Q_{Gi} < \text{limit}$ (reactive resources)

use Q_{Gi} in the next iteration

if $Q_{Gi} > \text{limit} \Rightarrow Q_{Gi} = \text{limit}$

Bus type is changed from PV to PQ

Ex 30



$$Y \Rightarrow Y_{\text{Line}} = \frac{1}{0.03 + j0.3} = 0.33 - j3.3 \text{ p.u.}$$

$$Y = \begin{bmatrix} 0.33 - j3.3 & 0.33 + j3.3 \\ -0.33 + j3.3 & 0.33 - j3.3 \end{bmatrix}$$

$$PQ \text{ bus} \Rightarrow S_2 \Rightarrow P_2 = P_{G2} - P_{D2}$$

$$= 0 - 1$$

$$= -1 \text{ p.u}$$

$$Q_2 = Q_{G2} - Q_{D2}$$

$$= -0.4 \text{ p.u}$$

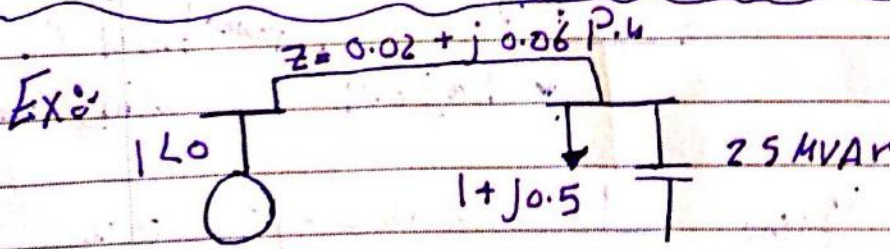
injected $S_2 = -1 - j0.4$

$$V_2 = \frac{1}{Y_{22}} \left(\frac{S_2^*}{V_2^{(0)}} - V_1 Y_{21} \right)$$

$$V_2^{(1)} = \frac{1}{0.33 - j3.3} \left[\frac{(-1 + j0.4)}{1 \angle 0} - (1 \angle 0) * (-0.33 + j3.3) \right]$$

$$V_2^{(0)} = 1 \angle 0 \text{ p.u}$$

$$V_2^{(1)} = 0.91 \angle -15^\circ$$



line charging current @ each end = 5 MVAR

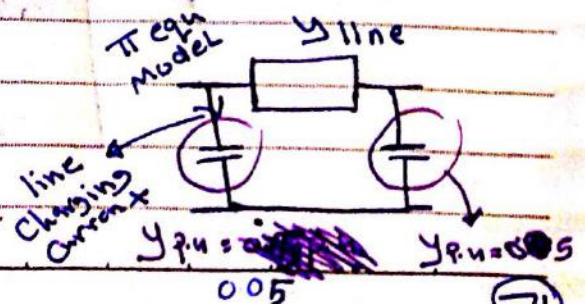
base of 100 MVA, Find V_2, P_1, Q_1

Answer:

$$V_2 = 0.9687 \angle -3.36$$

$$P_1 = 102.3 \text{ MW}$$

$$Q_1 = -23.9 \text{ MVAR}$$



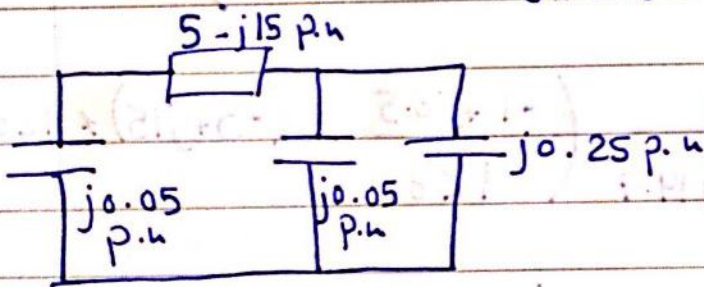
Solⁿ:

$$Y_{\text{line}} = \frac{1}{0.02 + j0.06} = 5 - j15 \text{ p.u.}$$

* shunt capacitance:

$$b \Rightarrow \frac{1 \text{ p.u.}}{V^2} Y_c = 5 \text{ MVA} \cdot 0.05 \text{ p.u.}$$

$$Y_{c \text{ p.u.}} = \frac{Y_{\text{actual}}}{Y_{c \text{ base}}} = 0.05 \text{ p.u.}$$



Note:
make sure
all the numbers
are in p.u.

For Capacitor $\rightarrow 25 \text{ mVAR} \rightarrow Y_{\text{p.u.}} = \frac{25}{100} = 0.25 \text{ p.u.}$

$$Y = \begin{bmatrix} 5 - j15 + j0.05 & -(5 - j15) \\ -(5 - j15) & 5 - j15 + j0.05 + j0.05 \end{bmatrix}$$

$$Y = \begin{bmatrix} 5 - j14.95 & -5 + j15 \\ -5 + j15 & 5 - j14.7 \end{bmatrix}$$

$$V_2 = \frac{1}{y_{22}} \left(\frac{S_2^*}{V_2^*} - y_{21} V_1 \right)$$

$$S_2 = 0 - (1 + j0.5) = -1 - j0.5$$

$$\rightarrow V_2^{(0)} = 1 \angle 0$$

$$\rightarrow V_1^{(0)} = 1 \angle 0$$

$$V_2 = \frac{1}{y_{22}} \left(\frac{S_2^*}{V_2^{*(0)}} - y_{21} V_1^{(0)} \right)$$

$$V_2 = \frac{1}{5 - j14.7} \left(\frac{-1 + j0.5}{1 \angle 0} - (-5 + j15) * 1 \angle 0 \right)$$

$$= 0.9671 - j0.0568 = 0.9687 \angle -3.36$$

$$\text{Capacitor} = |V_2|^2 \cdot y_c$$

$$= (0.9638)^2 * 0.25$$

$$= 0.232 \text{ p.u.}$$

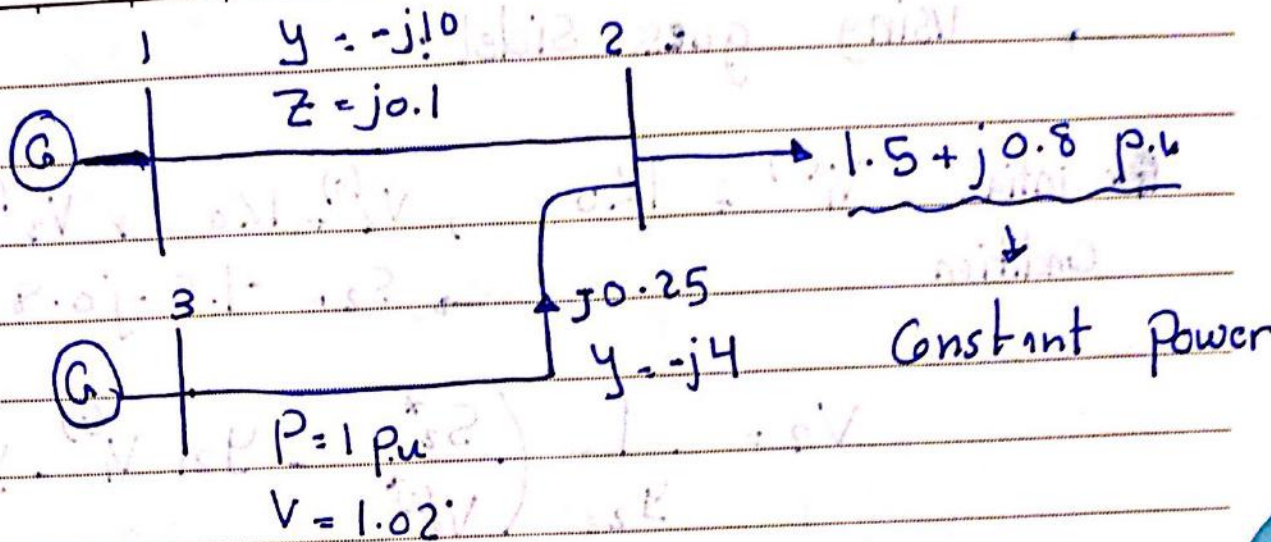
$$= 23.2 \text{ MVAR}$$

$$I = YV \rightarrow I_1 = y_{11} V_1 + y_{12} V_2$$

$$S_1 = V_1 I_1^*$$

يكون مصدر بالسؤال

Ex:



Find

- ① Y matrix
- ② perform 2 iterations of the gauss seidel

Sol:

only transmission lines do not include the load

$$Y = \begin{bmatrix} -j10 & j10 & 0 \\ j10 & -j14 & j4 \\ 0 & j4 & -j4 \end{bmatrix}$$

→ bus 2 (PQ bus) $\Rightarrow S_2 = 0 - (1.5 + j0.8) = -1.5 - j0.8$
 unknown: $|V_2| < \delta_2$

→ bus 3 (PV bus) $\Rightarrow P = 1 \text{ pu}$, $|V_3| = 1.02$ (Known)

bus 1 $\rightarrow K_0$ (slack) unknown δ_3, Q

No. 14/11/2016

→ using guess sidel ::

initial condition $V_1^{(0)} = 1 \angle 0$, $V_2^{(0)} = 1 \angle 0$, $V_3^{(0)} = 1.02 \angle 0$
→ $S_2 = -1.5 - j0.8$, $P_3 = 1$

$$V_2 = \frac{1}{y_{22}} \left(\frac{S_2^*}{V_2^{(0)*}} - y_{21} V_1^{(0)} - y_{23} V_3^{(0)} \right)$$

$$V_2 = 0.9514 - j0.10714 = 0.95744 \angle -6.2454$$

PV bus

$$V_3^{(1)} = \frac{1}{y_{33}} \left(\frac{S_3^*}{V_3^*} - y_{31} V_1^{(0)} - y_{32} V_2^{(1)} \right)$$

P Q??

to find Q

$$S_k = V_k I_k^* \\ = V_k \left[\sum_{i=1}^N y_{ki} V_i \right]^*$$

$$P_k + jQ_k = V_k \left[\sum_{i=1}^N y_{ki} V_i \right]^*$$

$$P_k + jQ_k = V_k^* \left[\sum_{i=1}^N y_{ki} V_i \right]$$

$$\rightarrow Q_k = -\text{Im} \left\{ V_k^* \left(\sum_{i=1}^N y_{ki} V_i \right) \right\}$$

$$\text{So, } Q_3 = -\text{Im} \left\{ \underbrace{(V_3^{(\omega)})^*}_{1.02 \angle 0} \cdot \left[\underbrace{y_{31} V_1^{(\omega)}}_{1 \angle 0} + \underbrace{y_{32} V_2^{(i)}}_{0.957 \angle -6.24} + y_{33} V_3^{(\omega)} \right] \right\}$$

$$Q = 0.32038 \text{ p.u.}$$

note: Sometimes in other questions, they give a range of Q:

$$Q_{\min} \leq Q \leq Q_{\max}$$

[So] if $Q_2 > Q_{\max} \rightarrow$ assume $Q_3 = Q_{\max}$

V_3
↓
(not constant any more)



$$V_3^{(1)} = 1.0395515 < 7.626 \text{ p.u}$$

↳ not equal to the given Voltage

given in the question

$$V_3^{(1)}, \text{ correct} = |V_3| \times \frac{V_3^{(u)}}{|V_3^{(u)}|}$$

$$\rightarrow V_3^{(1)} = 1.02 < 7.626$$

if $Q_3 > Q_{max}$ when given a range then this value 1.039 of the Voltage is accepted

Gauss Sidel

$$V_i^{(v+1)} = \frac{1}{y_{ii}} \left(\frac{S_i^*}{V_i^{(v)}} - \sum_{k=1}^{i-1} y_{ik} V_k^{(v+1)} - \sum_{k=i+1}^n y_{ik} V_k^{(v)} \right)$$

already known @ iteration k+1
last iteration

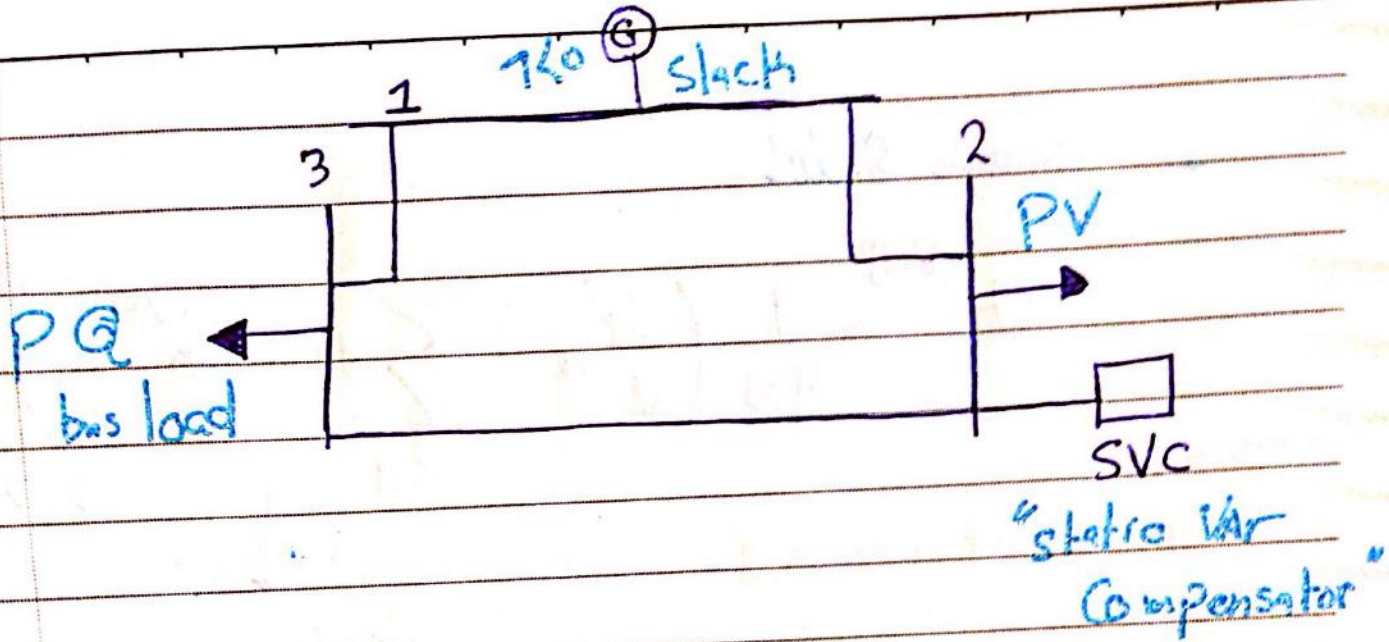
⇒ PV bus

$$Q_i^{v+1} = -\text{Im} \left\{ (V_i^v)^* \left[\sum_{k=1}^{i-1} y_{ik} V_k^{(v+1)} + \sum_{k=i}^n y_{ik} V_k^v \right] \right\}$$

Check Voltage

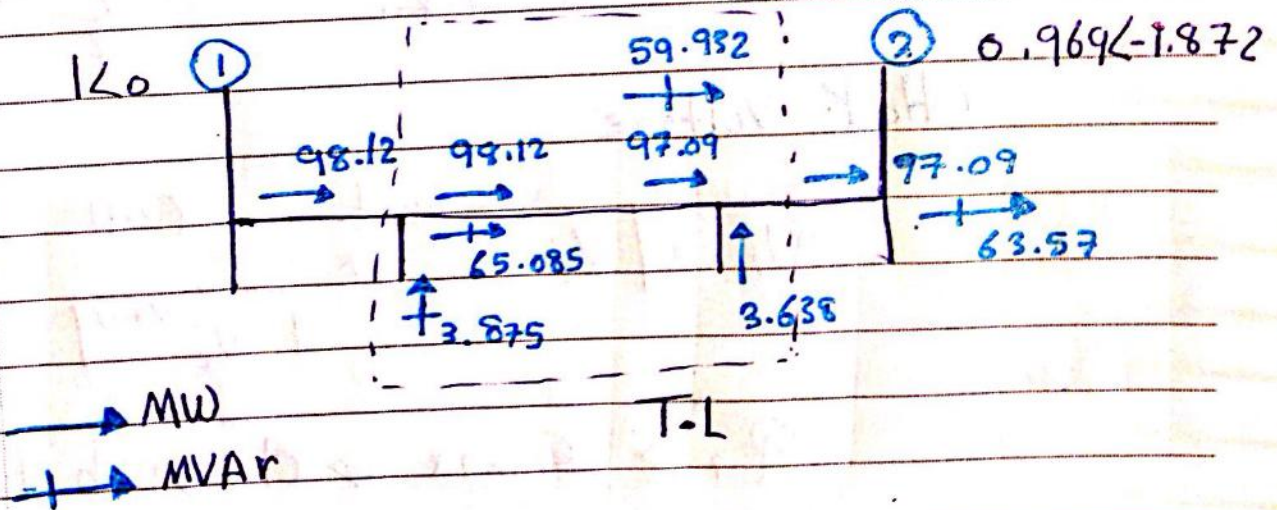
$$V_i^{v+1} = \left| \frac{V_i^{\text{specified}}}{V_i^{(v+1)}} \right| \cdot V_i^{(v+1)}$$

$Q_i^v > Q_{\max} \Rightarrow$ change bus to PQ bus
 $Q_i = Q_{\max}$

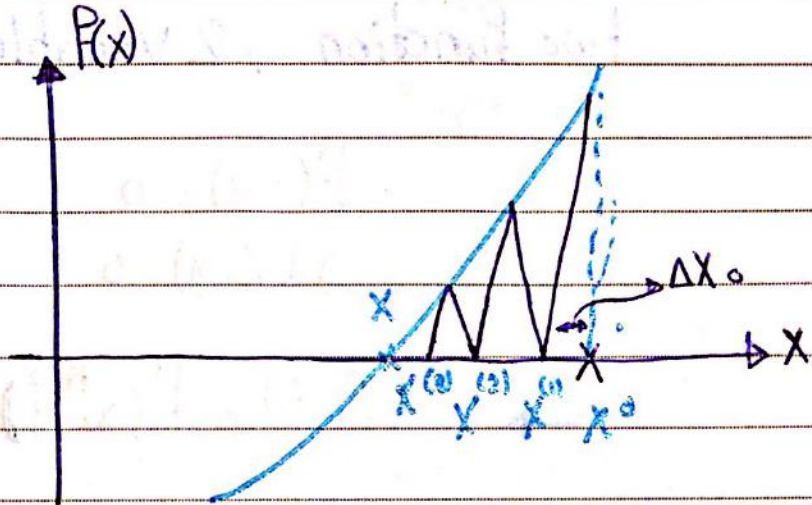


Voltage is controlled by G & SVC to maintain voltages of bus 1 & 2 close to unity, Find Q reactive restore @ bus 2?

Bus 2 voltage is 0.969



Newton Raphson



Taylor series

$$P(x^*) = P(x^0) + P'(x) \Delta x^0 + \frac{P''(x)}{2!} (\Delta x^0)^2 + \frac{P'''(x)}{3!} (\Delta x^0)^3 + \dots$$

neglected

$$P(x^*) = 0$$

$$P(x^*) = P(x^0) + P'(x) \Delta x^0 = 0$$

$$\Delta x^0 = -(P'(x))^{-1} P(x^0)$$

$$x^1 = x^0 + \Delta x^0$$

Ex: $P(x) = x^2 - 5x + 4$, $P(x) = 0$, x ??

$$x^0 = 6$$

Sol: =

$$P'(x) = 2x - 5$$

$$P(x^0) = 10$$

$$P'(x^0) = 7$$

$$\Delta x^0 = - (P'(x^0))^{-1} P(x^0)$$

$$\Delta x^0 = -\frac{10}{7}, \quad x^1 = x^0 + \Delta x^0 = 6 - \frac{10}{7} = 4.57$$

Two function, 2 variables

$$F(x, y) = 0$$

$$g(x, y) = 0$$

$$\rightarrow F(x^*, y^*) = F(x^k, y^k) + \frac{\partial F(x^k, y^k)}{\partial x} \Delta x^k + \frac{\partial F(x^k, y^k)}{\partial y} \Delta y^k = 0$$

$$\rightarrow g(x^*, y^*) = g(x^k, y^k) + \frac{\partial g(x^k, y^k)}{\partial x} \Delta x^k + \frac{\partial g(x^k, y^k)}{\partial y} \Delta y^k = 0$$

$$\begin{bmatrix} \frac{\partial F(x^k, y^k)}{\partial x} & \frac{\partial F(x^k, y^k)}{\partial y} \\ \frac{\partial g(x^k, y^k)}{\partial x} & \frac{\partial g(x^k, y^k)}{\partial y} \end{bmatrix} \begin{bmatrix} \Delta x^k \\ \Delta y^k \end{bmatrix} = - \begin{bmatrix} F(x^k, y^k) \\ g(x^k, y^k) \end{bmatrix}$$

Jacobian
- J

$$\begin{bmatrix} \Delta x^k \\ \Delta y^k \end{bmatrix} = -J^{-1} \begin{bmatrix} P(x^k, y^k) \\ g_j(x^k, y^k) \end{bmatrix}$$

unknown

mismatch

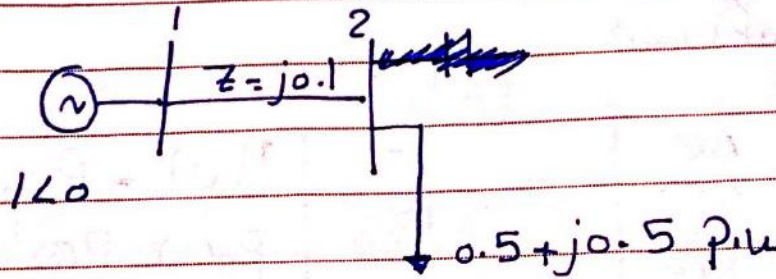
$$\begin{bmatrix} \Delta \theta_2 \\ \Delta \theta_3 \\ \vdots \\ |\dot{v}_2| \\ \vdots \\ |\dot{v}_3| \end{bmatrix} = -J^{-1} \begin{bmatrix} P_2(x) - P_{G_2} + P_{D_2} \\ \vdots \\ P_n(x) - P_{G_n} + P_{D_n} \\ \vdots \\ Q_n(x) - Q_{G_n} + Q_{D_n} \end{bmatrix}$$

$$P_{G_2} - P_{D_2} = P_2(x)$$

$$\begin{bmatrix} \frac{\partial P_2}{\partial \theta_2} \\ \vdots \\ \frac{\partial P_n}{\partial \theta_2} \end{bmatrix} \quad \dots \quad \begin{bmatrix} \frac{\partial P_2}{\partial \theta_n} \\ \vdots \\ \frac{\partial P_n}{\partial \theta_n} \end{bmatrix}$$

⇒ Newton Raphson

Two bus power flow



$V_2 \angle \theta_2, \theta_2 ??$

* $y = \frac{1}{j0.1} = -j10, Y = \begin{bmatrix} -j10 & j10 \\ j10 & -j10 \end{bmatrix}$

* unknown $|V_2|, \theta_2$ | Known $S_2 = 0.5 - j0.5$
 P_1, Q_1 | $= -0.5 - j0.5$

$V_1 = 1\angle 0$

$f(x) = 0$

power & reactive mismatch

$P_k = \sum_i |V_k| |V_i| [G_{ki} \cos \theta_{ki} + B_{ki} \sin \theta_{ki}]$

$Q_k = \sum_i |V_k| |V_i| [G_{ki} \sin \theta_{ki} - B_{ki} \cos \theta_{ki}]$

$$P_1 = |V_1| |V_2| * 10 * \sin(-\theta_2) = -10 |V_2| \sin \theta_2$$

explicit equation

$$Q_1 = 10 - 10 |V_2| \cos \theta_2$$

implicit equation

$$P_2 = 10 |V_2| \sin \theta_2$$

$$Q_2 = -10 |V_2| \cos \theta_2 + 10 |V_2|^2$$

$$\begin{cases} |V_1| = 1 \\ \theta_1 = 0 \\ \sin \theta_1 = 0 \\ \cos \theta_1 = 1 \end{cases}$$

دالفا بنبلست بن

$$S_2 = -0.5 - j0.5$$

$$P(\theta, V) = 10 |V_2| \sin \theta_2 = -0.5$$

$$P(\theta, V) = 10 |V_2| \sin \theta_2 + 0.5 = 0$$

Power & reactive mismatch

$$Q(\theta, V) = 10 |V_2|^2 - 10 |V_2| \cos \theta_2 + 0.5 = 0$$

→ Newton Raphson (unknown V_2, θ_2)

$$\begin{bmatrix} \frac{\partial P(\theta^k, V^k)}{\partial \theta} & \frac{\partial P(\theta^k, V^k)}{\partial V} \\ \frac{\partial Q(\theta^k, V^k)}{\partial \theta} & \frac{\partial Q(\theta^k, V^k)}{\partial V} \end{bmatrix} \begin{bmatrix} \Delta \theta^k \\ \Delta V^k \end{bmatrix} = - \begin{bmatrix} P(\theta^k, V^k) \\ Q(\theta^k, V^k) \end{bmatrix}$$

J

mismatch

$$J = \begin{bmatrix} 10V^k \cos \theta^k & 10 \sin \theta^k \\ 10V^k \sin \theta^k & 20V^k - 10 \cos \theta^k \end{bmatrix}$$

decoupled.
 → power is independent of V so
 $\frac{\partial P(\theta, V)}{\partial V} = 0$

1st iteration

Flat start $\theta^0 = 0, V^0 = 1 \rightarrow$ load bus

① $P(\theta^0, V^0) = 0.5$
 $Q(\theta^0, V^0) = 0.5$

② $J(\theta^0, V^0) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$

③ $\begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix} = -J^{-1} \begin{bmatrix} P \\ Q \end{bmatrix} = - \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$

④ $J^{-1} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$

⑤ $\begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix} = \begin{bmatrix} -0.05 \\ -0.05 \end{bmatrix}$ ← rad

$$\begin{bmatrix} \theta \\ V' \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -0.05 \\ -0.05 \end{bmatrix} = \begin{bmatrix} -0.05 \\ 0.95 \end{bmatrix}$$

$$\begin{bmatrix} \theta^3 \\ V^3 \end{bmatrix} = \begin{bmatrix} -0.05288 \\ 0.94575 \end{bmatrix}$$

عشان نطلع θ^3 iteration
 بنفي الـ (5) خطأ initial condition
 بنسب الـ initial condition

miss $\begin{cases} P(\theta, V) = 0.00012 \\ Q(\theta, V) = 0.00015 \end{cases}$
 match
 for 3rd iteration

$\rightarrow V', \theta'$

slack bus active & reactive power.

$$P_1 = 10 V_2 \sin \theta_2 = 0.4999 \text{ p.u.}$$

$$Q_1 = 10 - 10 V_2 \cos \theta_2 = 0.5557 \text{ p.u.}$$

general equation Slack $\rightarrow 1, \dots, N, P, Q$ bus

$$P_2(\theta_2, \theta_3, \dots, \theta_N, V_2, V_3, \dots, V_N) = 0$$

miss match

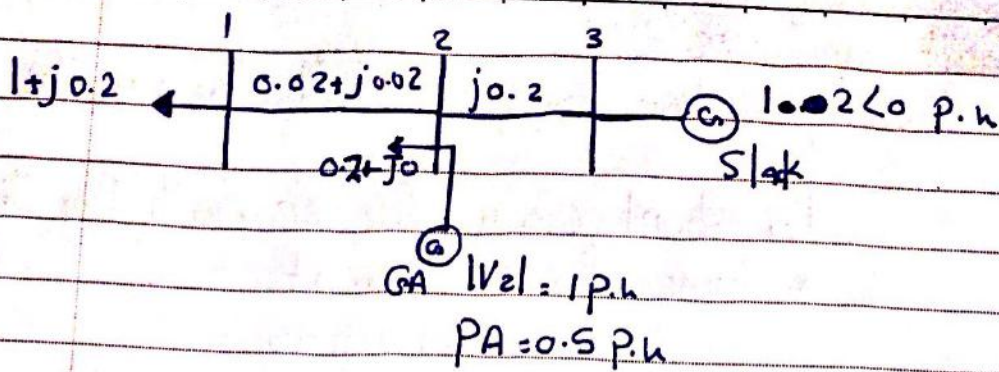
$$P_N(\theta_2, \theta_3, \dots, \theta_N, V_2, V_3, \dots, V_N) = 0$$

$$Q_2(\theta_2, \theta_3, \dots, \theta_N, V_2, V_3, \dots, V_N) = 0$$

$$Q_N(\theta_2, \theta_3, \dots, \theta_N, V_2, V_3, \dots, V_N) = 0$$

$$\mathbf{J} = \begin{array}{c|cc}
 \begin{array}{c} \frac{\partial P_2}{\partial \theta_2} \quad \frac{\partial P_2}{\partial \theta_3} \\ \vdots \\ \frac{\partial P_N}{\partial \theta_2} \quad \frac{\partial P_N}{\partial \theta_3} \\ \vdots \\ \frac{\partial Q_2}{\partial \theta_2} \\ \vdots \\ \frac{\partial Q_N}{\partial \theta_2} \end{array} & \begin{array}{c} J_{11} \\ \vdots \\ J_{21} \end{array} & \begin{array}{c} \frac{\partial P_2}{\partial v_2} \\ \vdots \\ \frac{\partial P_N}{\partial v_2} \\ \vdots \\ \frac{\partial Q_2}{\partial v_2} \\ \vdots \\ \frac{\partial Q_N}{\partial v_2} \end{array} \\
 \hline & & \begin{array}{c} J_{12} \\ \vdots \\ J_{22} \end{array} \\
 \hline & & \begin{array}{c} \frac{\partial P_2}{\partial v_n} \\ \vdots \\ \frac{\partial P_N}{\partial v_n} \\ \vdots \\ \frac{\partial Q_2}{\partial v_n} \\ \vdots \\ \frac{\partial Q_N}{\partial v_n} \end{array}
 \end{array}$$

$$\begin{array}{c} \Delta \theta_2 \\ \Delta \theta_3 \\ \vdots \\ \Delta \theta_n \\ \Delta v_2 \\ \Delta v_3 \\ \vdots \\ \Delta v_n \end{array} = \mathbf{J} \begin{array}{c} \vdots \\ \vdots \\ \vdots \end{array} \rightarrow \text{mismatch}$$



Unknown θ_1, V_1, θ_2
 implicit functions P_1, Q_1, P_2

$$J = \begin{bmatrix} \frac{\partial P_1}{\partial \theta_1} & \frac{\partial P_1}{\partial \theta_2} & \frac{\partial P_1}{\partial V_1} \\ \frac{\partial P_2}{\partial \theta_1} & \frac{\partial P_2}{\partial \theta_2} & \frac{\partial P_2}{\partial V_1} \\ \frac{\partial Q_1}{\partial \theta_1} & \frac{\partial Q_1}{\partial \theta_2} & \frac{\partial Q_1}{\partial V_1} \\ \frac{\partial P_2}{\partial \theta_1} & \frac{\partial P_2}{\partial \theta_2} & \frac{\partial P_2}{\partial V_1} \\ \frac{\partial Q_2}{\partial \theta_1} & \frac{\partial Q_2}{\partial \theta_2} & \frac{\partial Q_2}{\partial V_1} \end{bmatrix} \begin{bmatrix} \Delta \theta_1 \\ \Delta \theta_2 \\ \Delta V_1 \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ Q_1 \end{bmatrix}$$

⇒ Fault analysis

* Failure in the system. (Breakdown in the insulation)

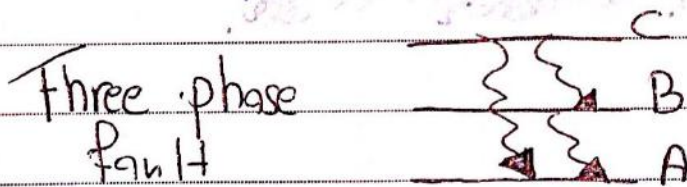
↳ short circuit (easier path for the current)

* large fault current

- ↳ dangerous
- ↳ destroy equipment
- ↳ make system unstable

* Why?

- lightning
- contacts between conductor
- Movement of conductor "wind, sag"
- Damage
- Snow & ice
- Ageing of insulating



Three phase

Fault

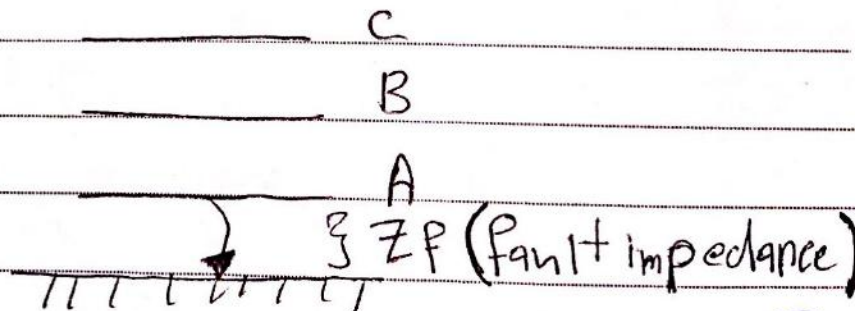
Balanced

3 ϕ "All phases are affected ⇒ Single phase in the same way"

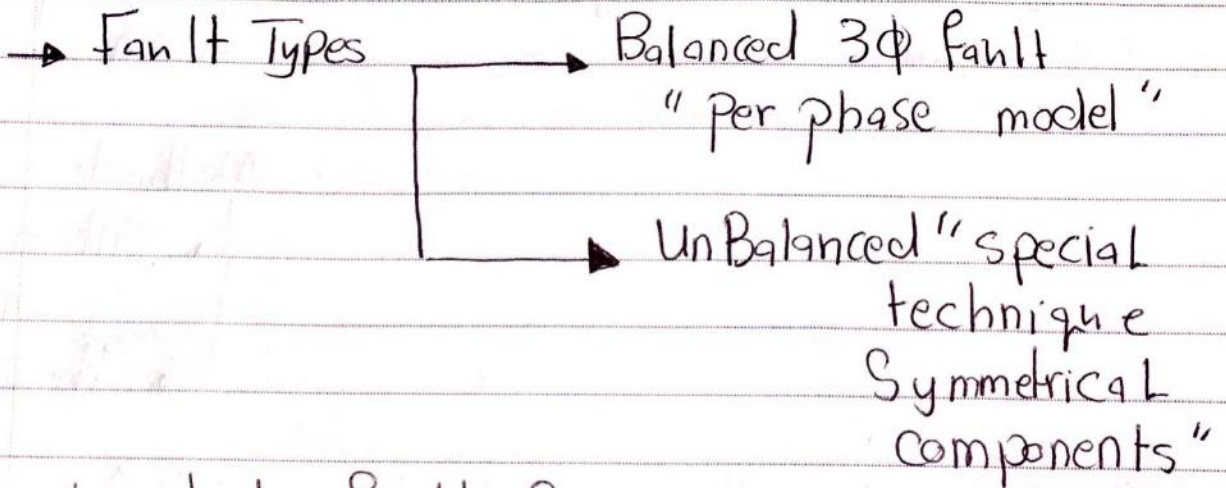
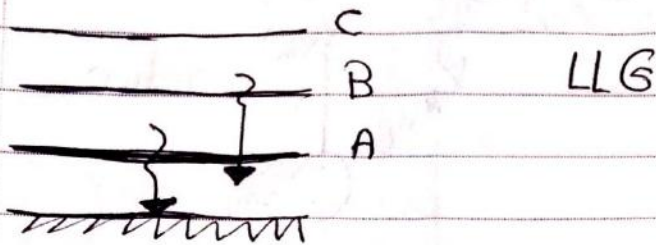
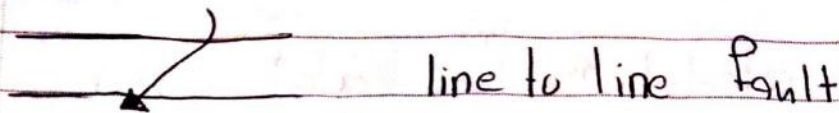
Single line

to ground

Fault 70%



Z_F (Fault impedance)



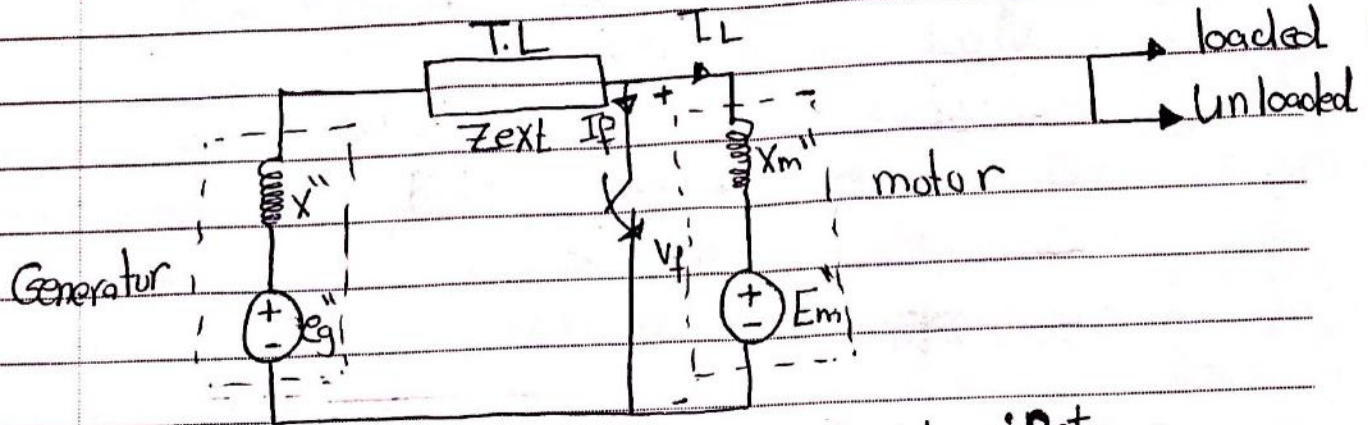
→ why study faults?

- rating circuit breakers
- Make sure current is large enough
- system stability "large disturbance"
- power quality "voltage drop"

Max fault current

و لڳو fault min fault current

⇒ Fault calculations for loaded system



لحالة Subtransient Current X'' و Current I_f و Voltage V_f ←

Methods

- ↳ Internal voltage method
- ↳ Thevenin

Internal voltage method

$V_f \triangleq$ pre fault voltage

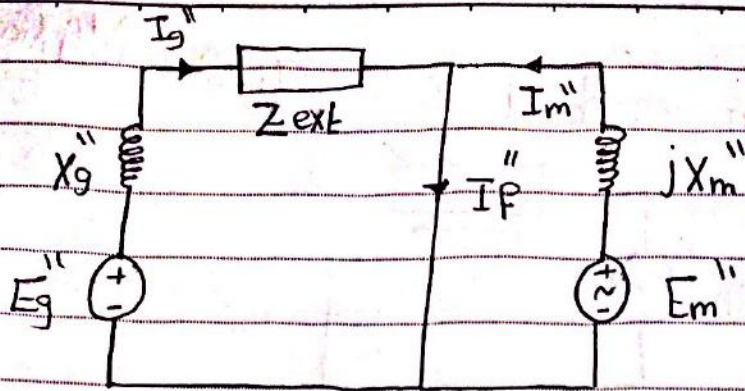
لحالة $V_f = K_0 V_{pin}$

$$E_g'' = V_f + I_L (Z_{ext} + jX_g'')$$

$$E_m'' = V_f - I_L (jX_m'')$$

switch open

- ① I_L, V_f
- ② E_g'', E_m''
- ③ $I_g'', I_m'' \Rightarrow I_f''$



$$\begin{aligned} I_p'' &= I_g'' + I_m'' \\ I_g'' &= \frac{E_g''}{Z_{ext} + jX_g''} = \frac{V_P + I_L (Z_{ext} + jX_g'')}{Z_{ext} + jX_g''} \end{aligned}$$

$$I_g'' = \frac{V_P}{Z_{ext} + jX_g''} + I_L$$

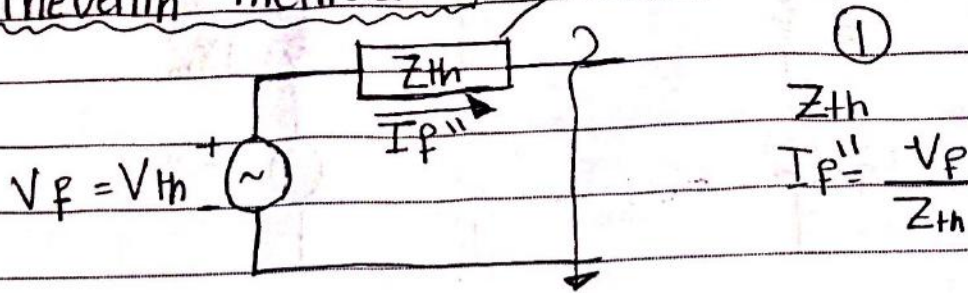
$$\begin{aligned} I_m'' &= \frac{E_m''}{jX_m''} \\ &= \frac{V_P - I_L (jX_m'')}{jX_m''} \end{aligned}$$

$$I_m'' = \frac{V_P}{jX_m''} - I_L$$

$$\begin{aligned} I_p'' &= I_g'' + I_m'' \\ &= \frac{V_P}{jX_g'' + Z_{ext}} + \frac{V_P}{jX_m''} = V_P \left(\frac{1}{jX_g'' + Z_{ext}} + \frac{1}{jX_m''} \right) \end{aligned}$$

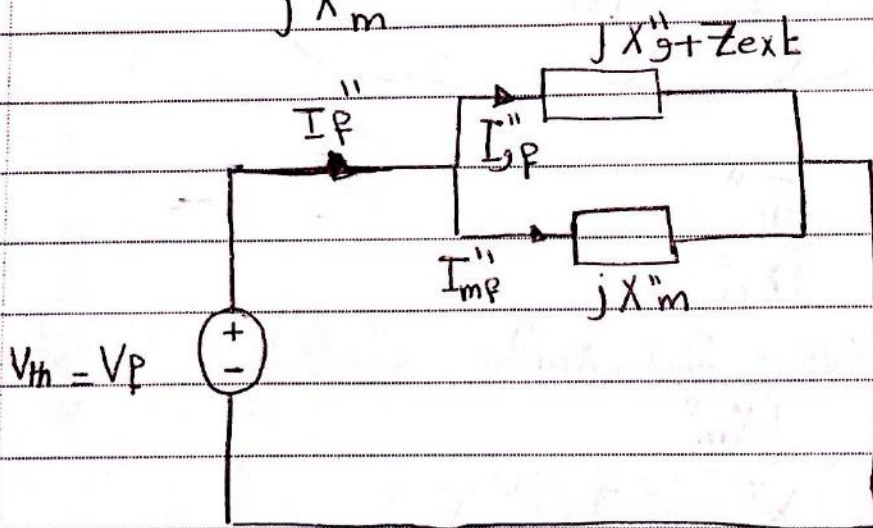
Thevenin

Thevenin method



$$I_g'' = \frac{V_p}{Z_{ext} + jX_g''} - I_L$$

$$I_m'' = \frac{V_p}{jX_m''} - I_L$$



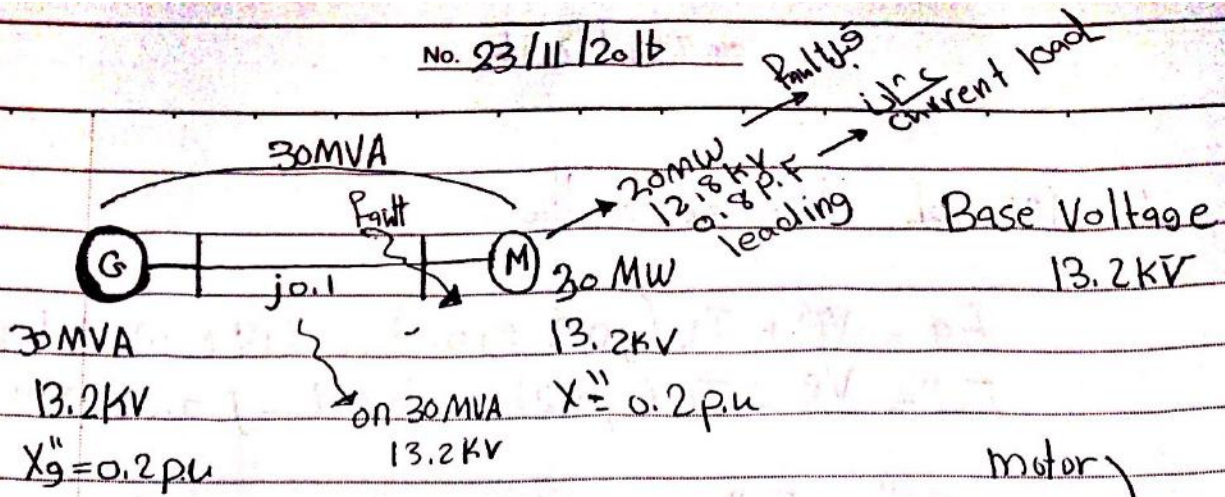
②
$$I_{gF}'' = \frac{V_p}{Z_{ext} + jX_g''}$$

$$I_{mF}'' = \frac{V_p}{jX_m''}$$

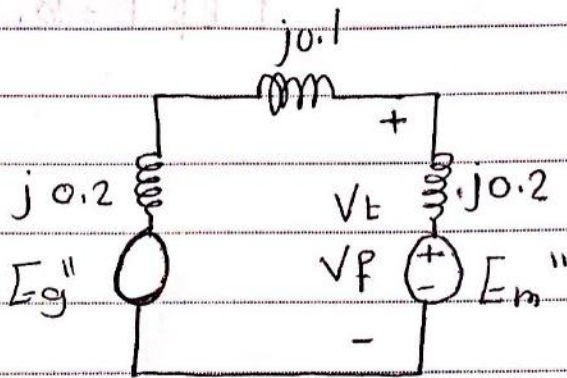
} correction ③

$$I_g'' = I_{gF}'' + I_L$$

$$I_m'' = I_{mF}'' - I_L$$



Q Find I_F'' , I_G'' , I_M'' subtransient fault current? 3ϕ fault



$$I_L = \frac{20 \text{ MW} / 0.8}{\sqrt{3} * 12.8} = 1128 \angle +36.9^\circ \text{ A}$$

$$I_{\text{Base}} = \frac{30 \text{ MVA}}{13.2 \text{ kV} * \sqrt{3}} = 1312 \text{ A}$$

$$I_L \text{ pu} = \frac{1128 \angle 36.9^\circ}{1312} = 0.86 \angle 36.9^\circ \text{ pu}$$

$$V_F = \frac{12.8}{13.2} = 0.97 \angle 0^\circ \text{ pu}$$

S

(i) internal Voltage method

$$E_g'' = V_F + I_L (j0.2 + j0.1) = 0.814 + j0.207 \text{ p.u.}$$

$$E_m'' = V_F - I_L (j0.2) = 1.074 - j0.138 \text{ p.u.}$$

$$I_g'' = \frac{E_g''}{j0.3}$$

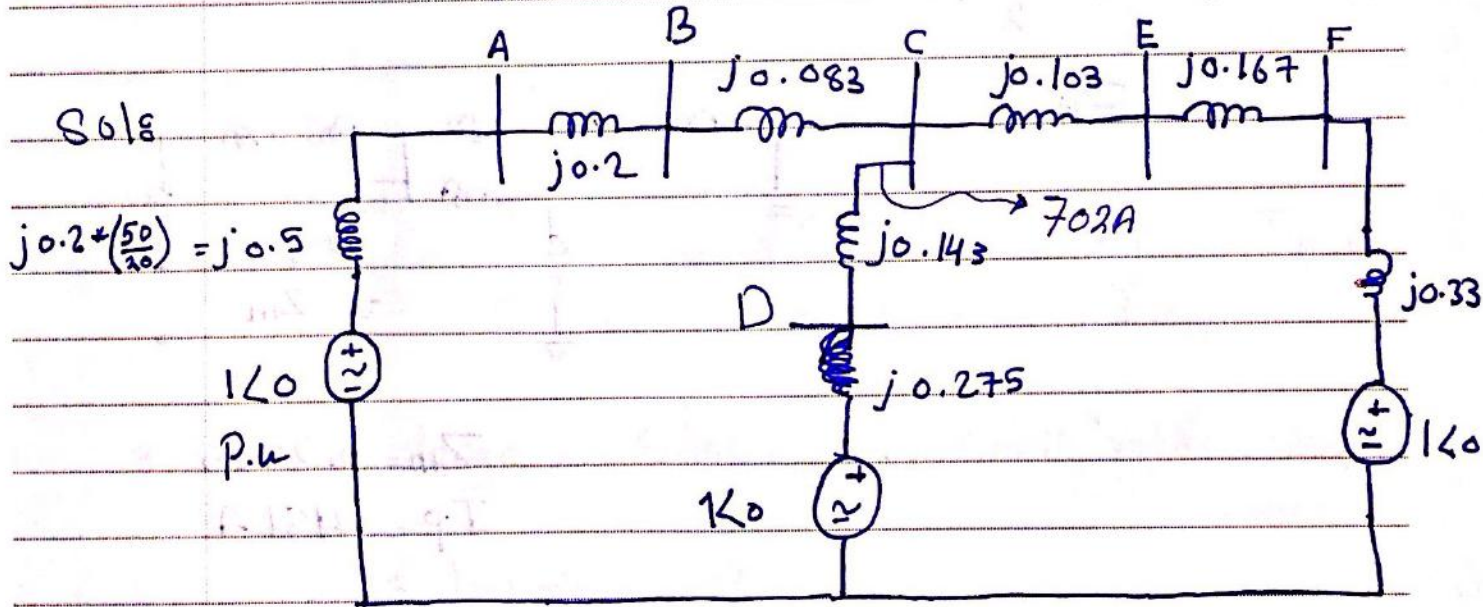
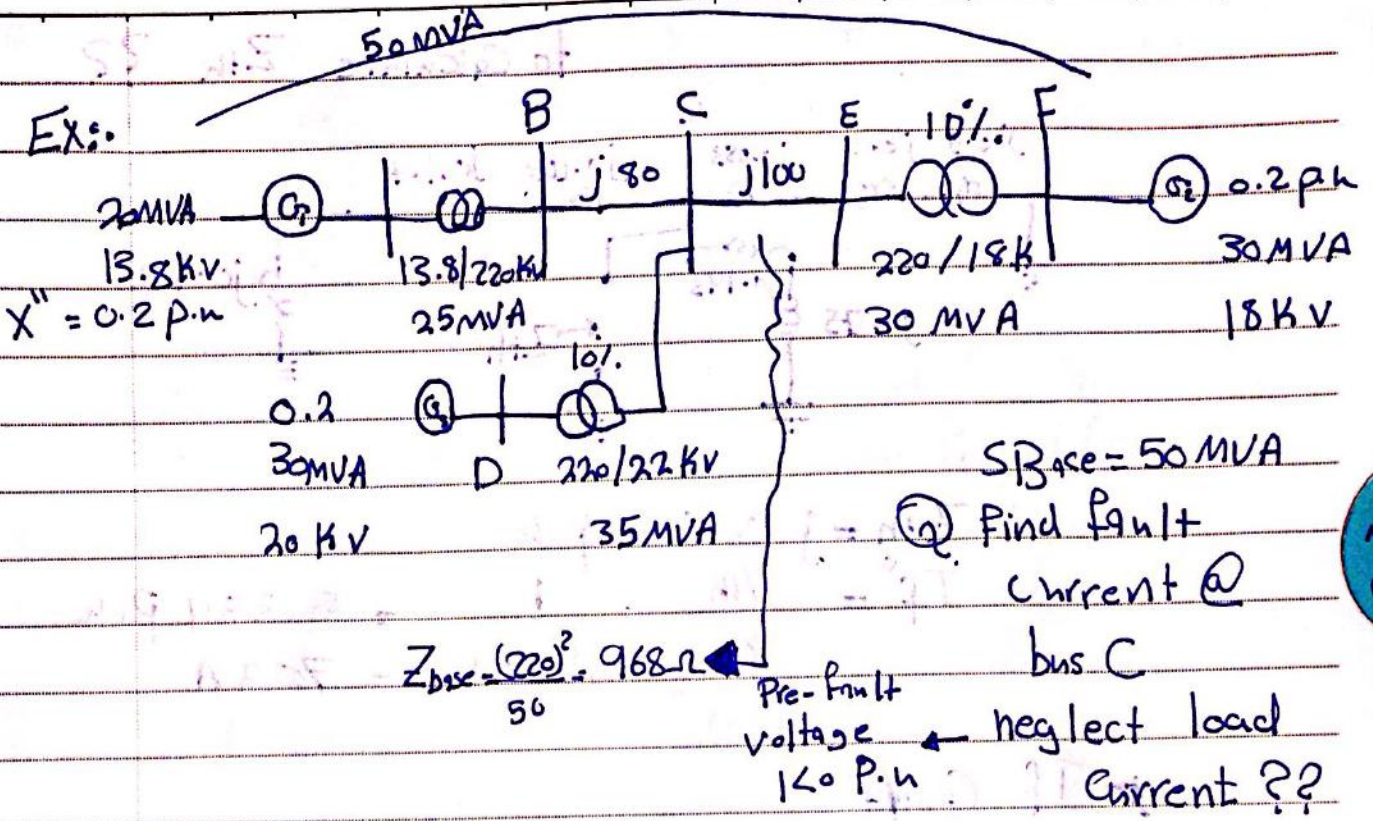
$$I_m'' = \frac{E_m''}{j0.2}$$

$$I_p'' = I_g'' + I_m''$$

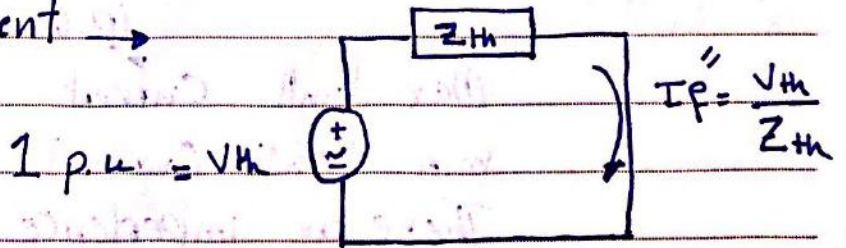
$$= -j8.08 \text{ p.u.}$$

$$= -j10.6 \text{ kA}$$

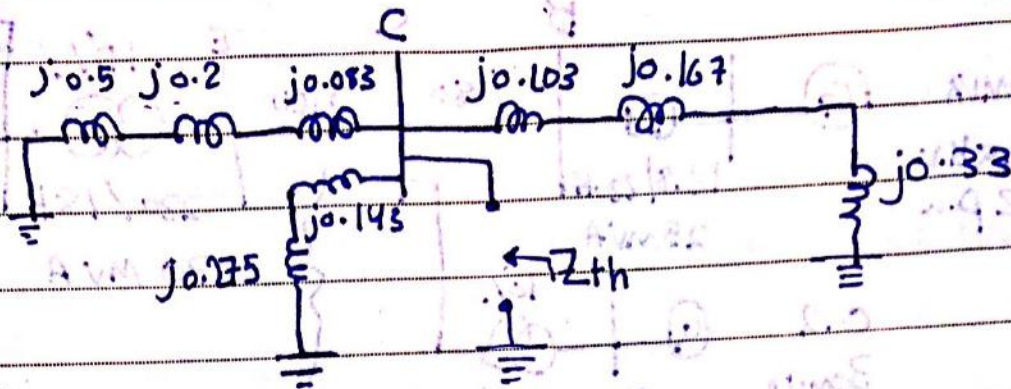
$$|I_p''| = 10.6 \text{ kA}$$



Thevenin equivalent



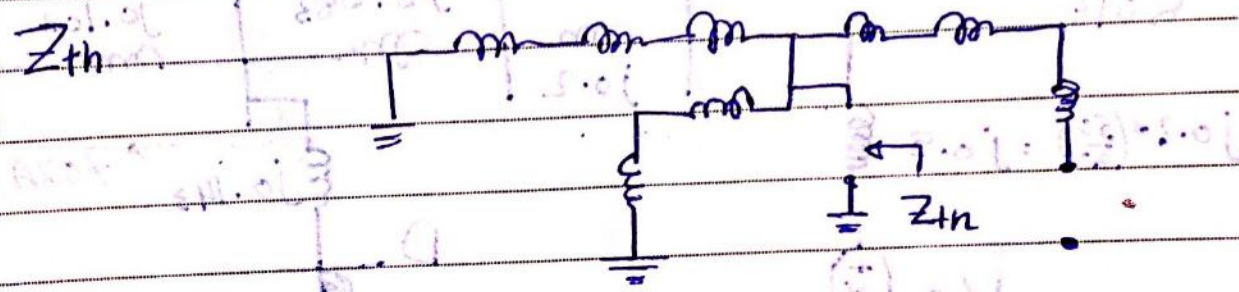
to calculate Z_{th} ??



$$Z_{th} = j0.187 \text{ p.u.}$$

$$I_p'' = \frac{V_{th}}{Z_{th}} = \frac{1}{j0.187} = 5.384 \text{ p.u.} = 702 \text{ A}$$

⇒ I_p G_2 off



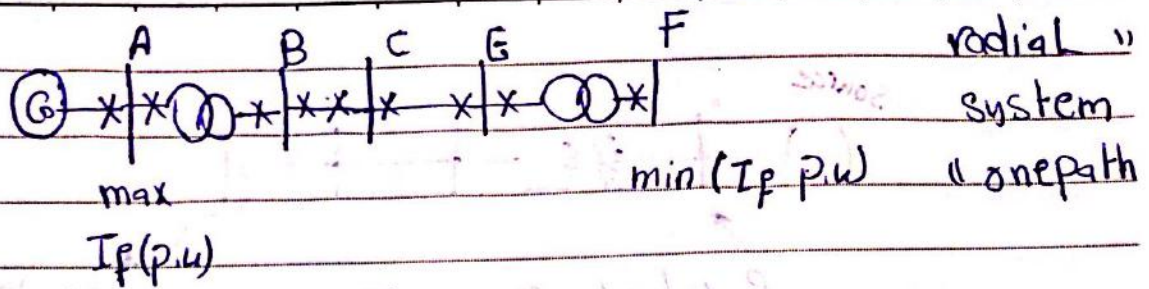
$$Z_{th} = 0.2725$$

$$I_p = 481 \text{ A}$$

⇒ I_p G_2, G_3 off

(p.u.)

Max fault current at bus A
 min fault current at bus F because
 There are impedance before it



→ Short circuit level "Fault level"
* express fault in MVA [to cater for different voltage levels]

$$S_c^{MVA} = \sqrt{3} V_B I_f$$

Base (actual) (actual)

$$S_c^{MVA} = \sqrt{3} V_B I_f$$

$$S_c^{p.u} = \frac{S_c^{MVA}}{S_{Base}} = \frac{\sqrt{3} V_B I_f}{\sqrt{3} V_B I_B} = \frac{I_f}{I_{Base}} = I_{p.u}$$

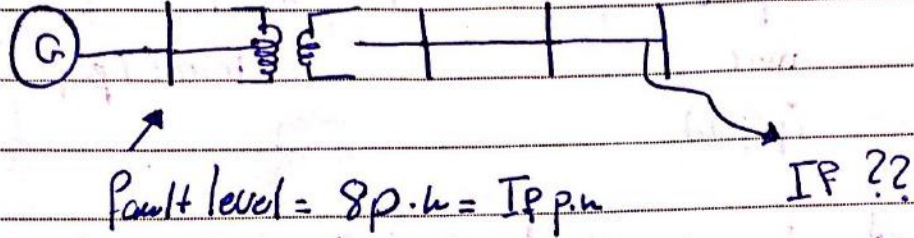
* S_c is large \Rightarrow Strong " Z_{th} Small " voltage drop Small "

* S_c is low \Rightarrow Weak

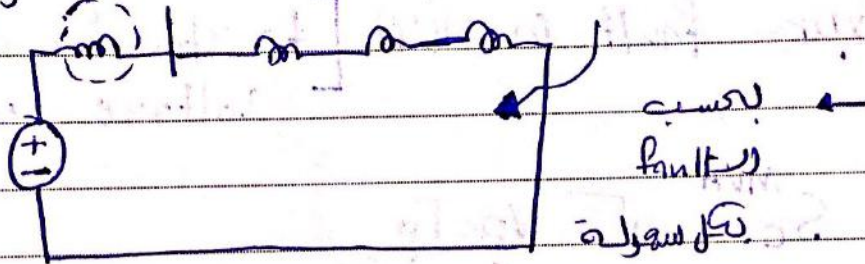
Finite bus
 $Z_{th} = 0$

No. 28/11/2016

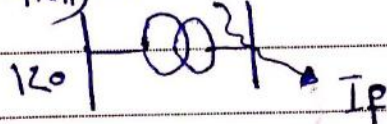
Source



$$\frac{1}{8} = j\omega L$$



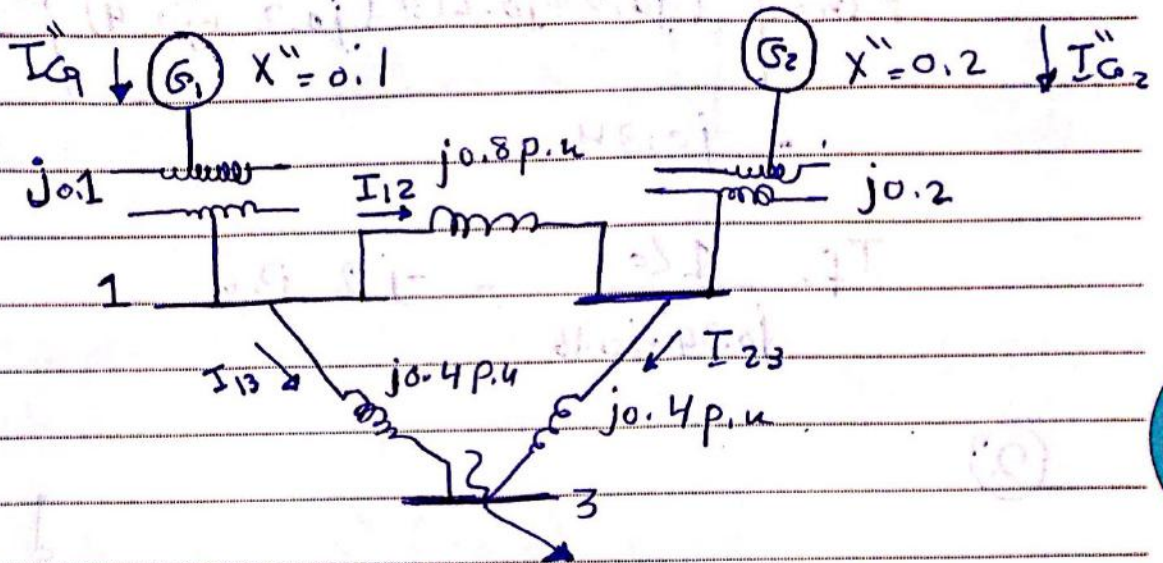
Strong $x = 10\%$



$I_{Fault} \rightarrow I_{Full load}$

$$I_{Fault} > 10 I_{Full load}$$

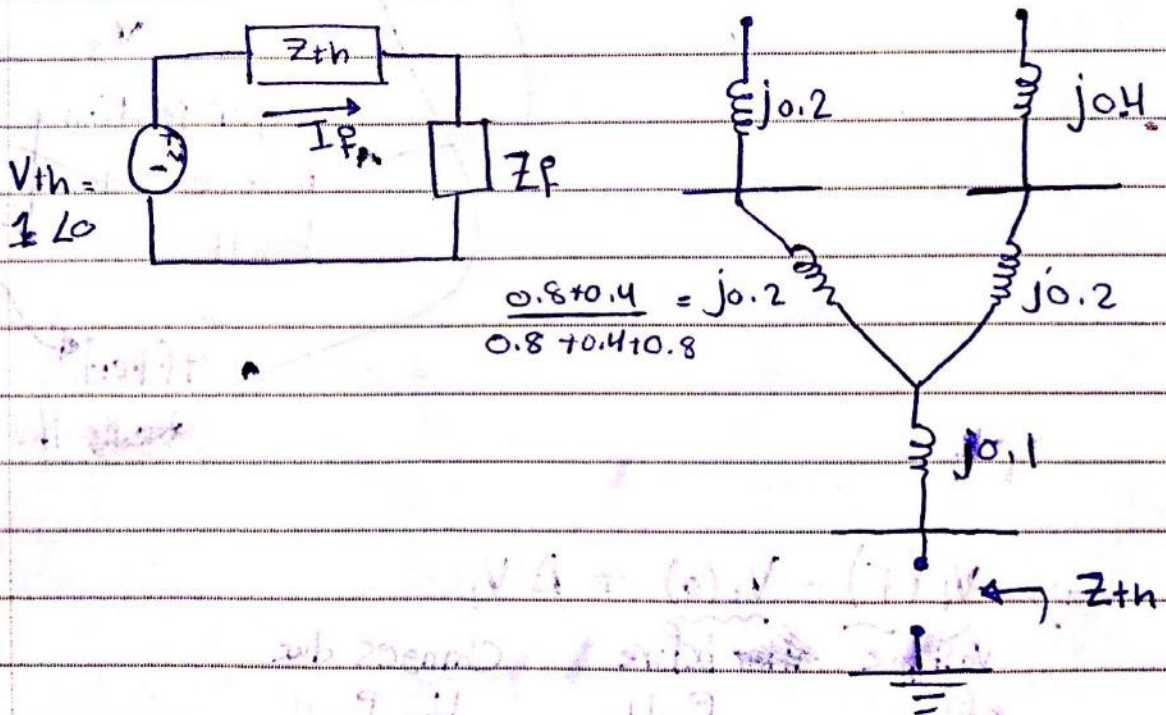
Ex 2



$Z_p = j0.16 \text{ p.u.}$

- Q ① I_{fault} "unloaded"
 ② changes in bus voltage $\Delta V_1, \Delta V_2, \Delta V_3$

Soln

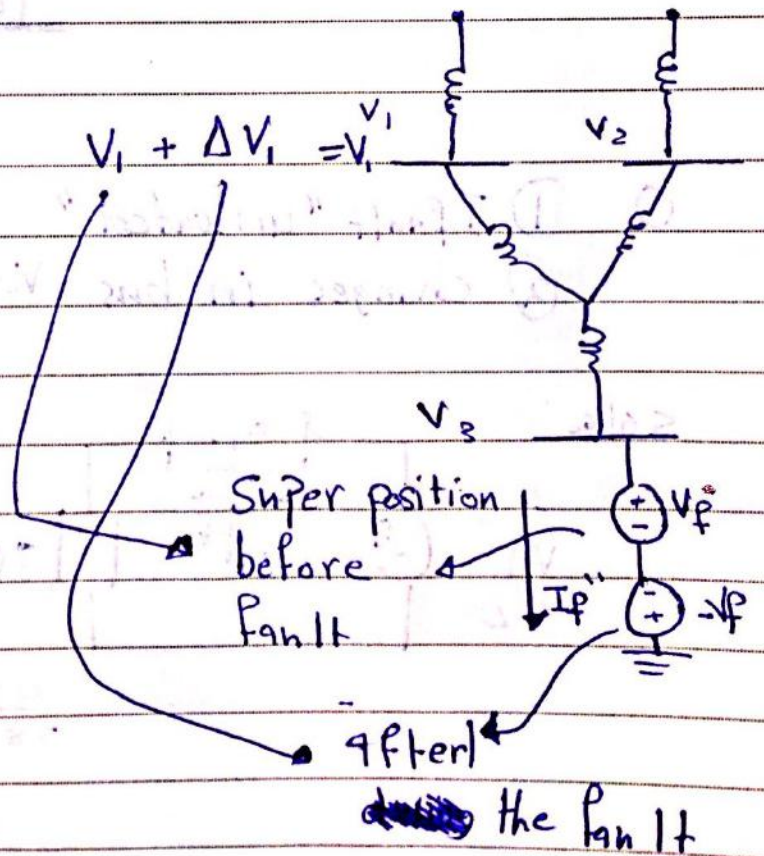


$$Z_{th} = \left[(j0.2 + j0.2) \parallel (j0.2 + j0.4) \right] + j0.1$$

$$= j0.34$$

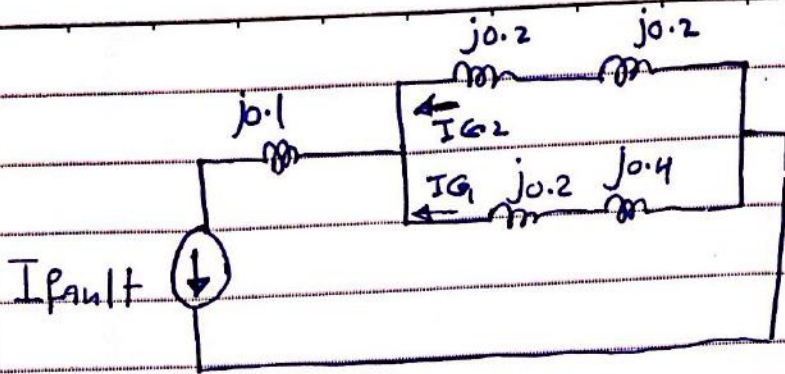
$$I_f = \frac{1 \angle 0}{j0.34 + j0.16} = -j2 \text{ p.u.}$$

(2)



$$\Rightarrow \underbrace{V_1(F)}_{\substack{\text{voltage} \\ \text{after} \\ \text{fault}}} = \underbrace{V_1(0)}_{\substack{\text{before} \\ \text{fault}}} + \Delta V_1$$

Changes due to the fault



Current	$I_{G1}'' = -j1.2 \text{ p.u.}$
division	$I_{G2}'' = -j0.8 \text{ p.u.}$

$$\Delta V_1 = -I_{G1}'' (j0.2) = -0.24 \text{ p.u.}$$

$$\Delta V_2 = -I_{G2}'' (j0.4) = -0.34 \text{ p.u.}$$

$$V_1 = 1 - 0.24 = 0.76 \text{ p.u.}$$

$$V_2 = 1 - 0.34 = 0.66 \text{ p.u.}$$

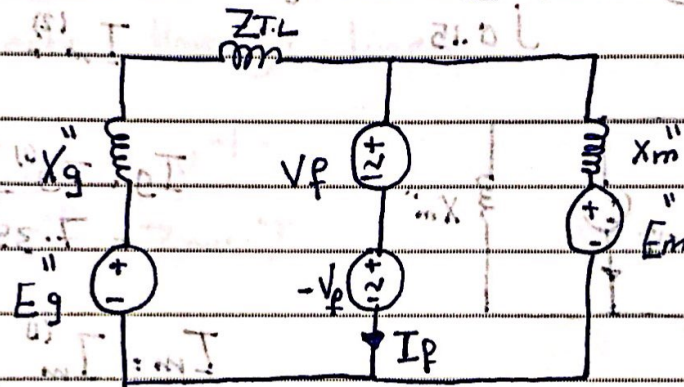
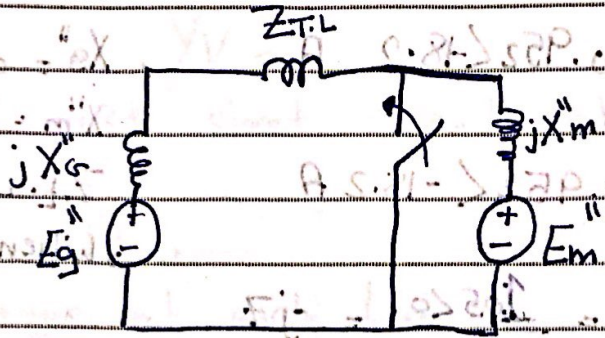
$$I_{12} = \frac{V_1 - V_2}{j0.8}$$

CH-9.1, 9.2, 9.3, 9.5, 9.4 guess
det.

CH-10.1, 10.2

No. 5/12/2016

⇒ fault analysis : super position

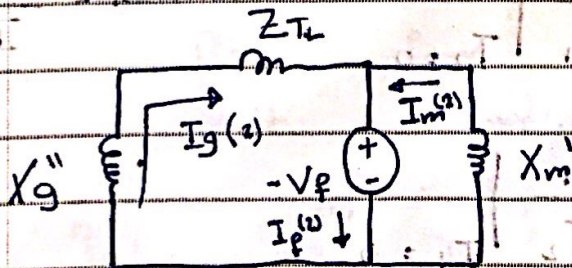


V_p pre fault voltage

* Super position

- ① V_p off, others on $\Rightarrow I_p^{(1)}$
- ② V_p on, others off $\Rightarrow -I_p^{(2)}$

$$I_p = I_p^{(1)} + I_p^{(2)}$$



$$I_g = I_g^{(1)} + I_g^{(2)}$$

before contribution
Fault in Fault

Ex. $V_f = 1.05 \angle 0^\circ$: $X_g'' = 0.15$ p.u. $X_m'' = 0.5$ p.u. $Z_{L-L} = 0$

$I_g^{(1)} = 0.952 \angle -18.2^\circ$ A

$X_g'' = 0.15$ p.u.

$X_m'' = 0.5$ p.u.

$I_m^{(1)} = -0.952 \angle -18.2^\circ$ A

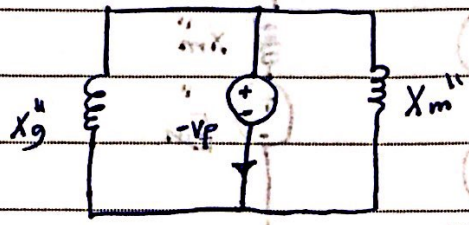
$Z_{L-L} = 0$

When fault @ motor side?

Solve

$I_g^{(2)} = \frac{1.05 \angle 0^\circ}{j0.15} = -j7$

$I_m^{(2)} = \frac{1.05 \angle 0^\circ}{j0.5} = -j2.1$



$I_g = I_g^{(1)} + I_g^{(2)}$
 $= 7.35 \angle -82.9^\circ$

$I_m = I_m^{(1)} + I_m^{(2)}$

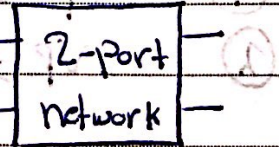
*

$YV = I$

$Z = Y^{-1}$

$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$



$Z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0}$

$Z_{21} = \frac{V_2}{I_1} \Big|_{I_2=0}$

$Z_{12} = \frac{V_1}{I_2} \Big|_{I_1=0}$

$I_1 = I_2 = I$

Extension to large system

$$YV = I$$

* I.F. a 3 ϕ short circuit at bus k \Rightarrow V_1

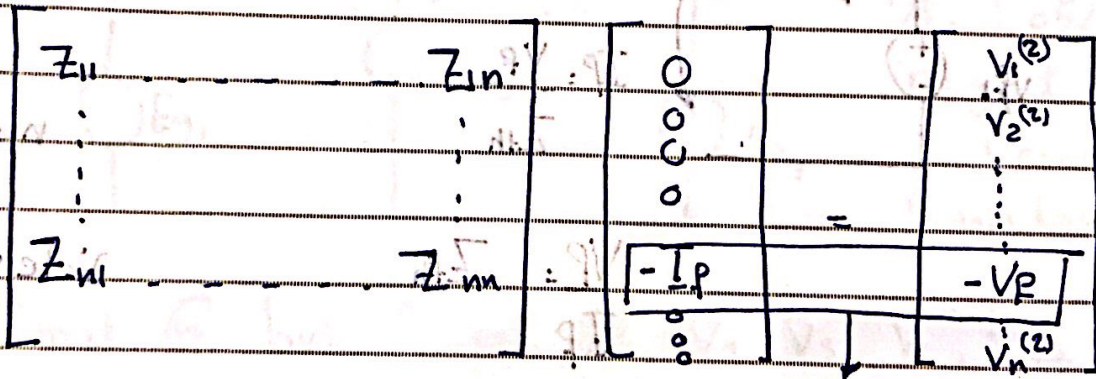
Fault Flow V_2

voltage at each bus V_3

currents through lines V_n

$$Z_{bus} = Y^{-1}$$

$$V^{(2)} = Z_{bus} I$$



bus k

$$V_i = V_i^{(1)} + V_i^{(2)}$$

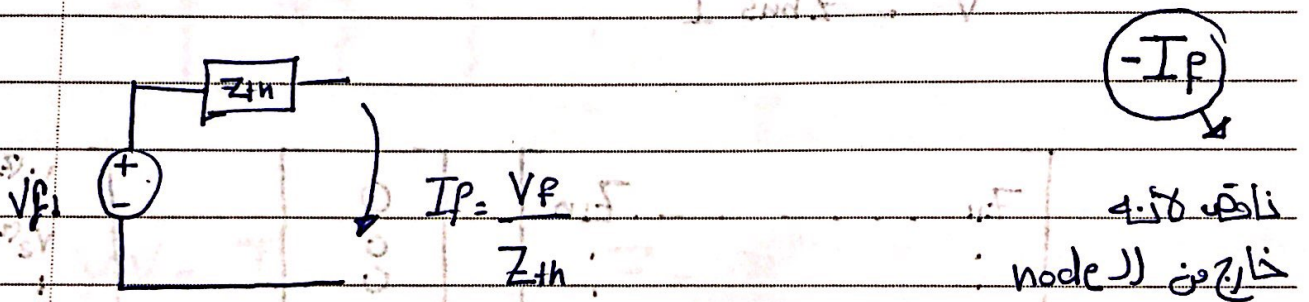
Pre fault Z matrix

$$V_i = V_i^{(1)} + \Delta V_i \rightarrow V_i^{(2)}$$

Pre fault Changes due to $-V_P$

Assume 4 bus, fault @ bus 2

Z_{11}	Z_{12}	Z_{13}	Z_{14}	⊖	ΔV_1
Z_{21}	Z_{22}	Z_{23}	Z_{24}	$-I_P$	V_P
Z_{31}	Z_{32}	Z_{33}	Z_{34}	0	ΔV_3
Z_{41}	Z_{42}	Z_{43}	Z_{44}	0	ΔV_4



$$- \frac{V_P}{I_P} = Z_{th}$$

Note: I_P → diameter

$$\Rightarrow Z_{22} (-I_P) = -V_P \Rightarrow Z_{22} = \frac{V_P}{I_P}$$

$$\Delta V_1 = Z_{12} (-I_P)$$

$$\Delta V_1 = -Z_{12} \left(\frac{V_P}{Z_{22}} \right) = - \frac{Z_{12}}{Z_{22}} V_P$$

$$V_1 = V_1^{(1)} + \frac{-Z_{12}}{Z_{22}} V_P$$

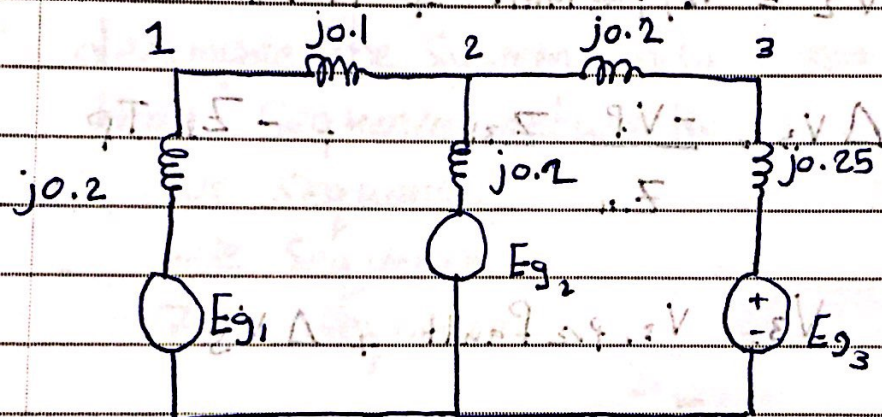
Fault a bus voltage ΔV ...

$$\Delta V_3 = \frac{-Z_{32} V_F}{Z_{22}}$$

Ymatrix ... Network ...

$Z_{11} \Rightarrow Z_{th}$ when the fault happen in bus 1

Exs



$V_F = 1.05 V_0$, neglect load current "unloaded"

Fault @ bus 1 $\Rightarrow I_F, V_2, V_3, V_1, T_{23}$

$$Y = j \begin{bmatrix} -15 & 10 & 0 \\ 10 & -20 & 5 \\ 0 & 5 & -9 \end{bmatrix}$$

$$Z_{11} \neq Y_{11}$$

$$Z = Y^{-1} = j \begin{bmatrix} 0.008 & 0.063 & 0.035 \\ 0.063 & 0.094 & 0.052 \\ 0.035 & 0.052 & 0.1409 \end{bmatrix}$$

$$I_{FA} = \frac{V_F}{Z_{11}} = \frac{1.05}{j0.0108} = -j9.6 \text{ p.u.}$$

$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix} \begin{bmatrix} -I_{FA} \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta V_2 = -\frac{V_F}{Z_{11}} Z_{21}$$

$$V_2 = V_{2, \text{pre fault}} + \Delta V_2$$

$$\Delta V_3 = -\frac{V_F}{Z_{11}} Z_{31} = -Z_{31} I_{FA}$$

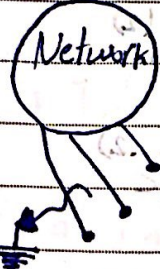
$$V_3 = V_{3, \text{pre fault}} + \Delta V_3$$

Ex page (100)

Z_{11}	Z_{12}	Z_{13}
Z_{21}	Z_{22}	Z_{23}
Z_{31}	Z_{32}	Z_{33}

V_1	V_2	V_3
I_1	I_2	I_3
S_1	S_2	S_3

⇒ Symmetrical Components of Asymmetrical faults



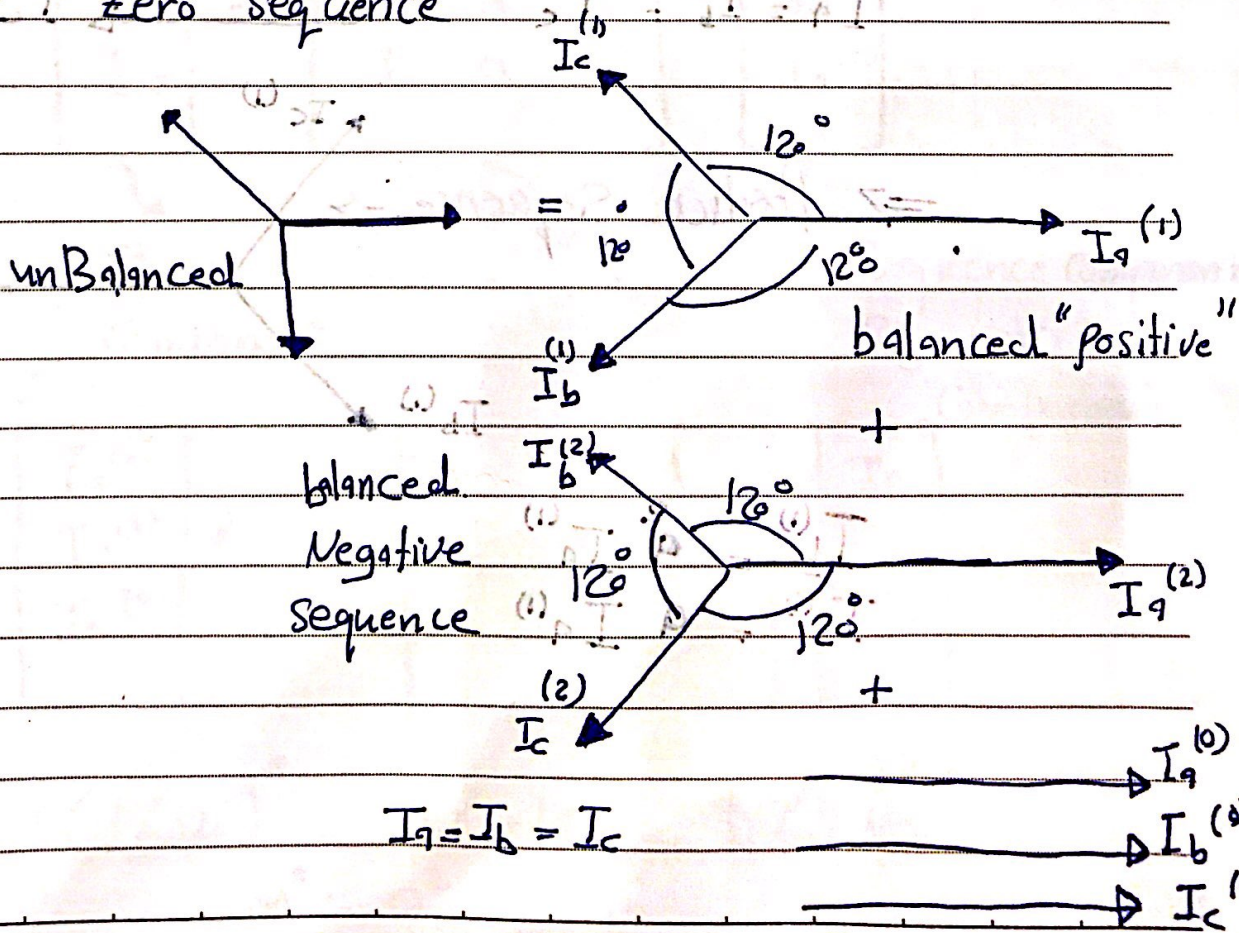
Unbalanced Faults ⇒ NOT Symmetrical System

* look what happens in each phase

↓
Symmetrical Components

decompose the system into three sequence networks

- (1) +ve sequence
- (2) -ve sequence
- (0) Zero sequence



$I_a = I_b = I_c$

$$I_a = I_a^{(0)} + I_a^{(1)} + I_a^{(2)}$$

$$I_b = I_b^{(0)} + I_b^{(1)} + I_b^{(2)}$$

$$I_c = I_c^{(0)} + I_c^{(1)} + I_c^{(2)}$$

I_a & I_c, I_b, I_a sequence

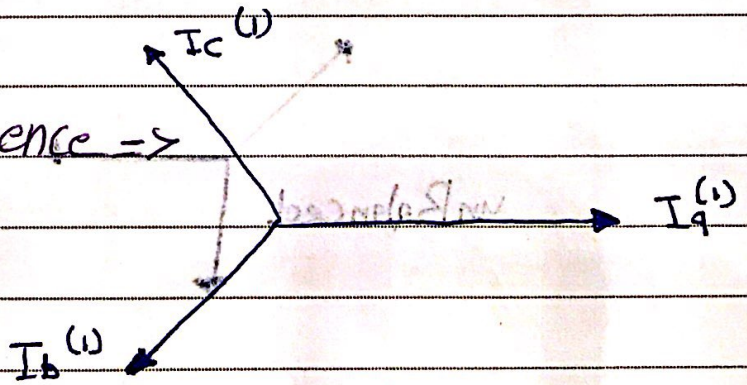
\Rightarrow a operator

$$\Rightarrow a = 1 \angle 120^\circ, a^2 = 1 \angle 240^\circ, a^3 = 1 \angle 0^\circ$$

\Rightarrow Zero Sequence \Rightarrow

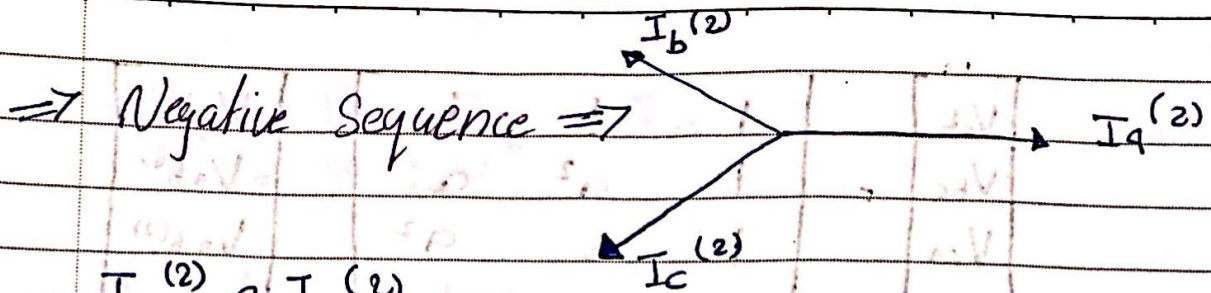
$$I_a^{(0)} = I_b^{(0)} = I_c^{(0)}$$

\Rightarrow Positive sequence \Rightarrow



$$I_b^{(1)} = a^2 I_a^{(1)}$$

$$I_c^{(1)} = a I_a^{(1)}$$



$$I_b^{(2)} = a I_a^{(2)}$$

$$I_c^{(2)} = a^2 I_a^{(2)}$$

$$I_a = I_a^{(0)} + I_a^{(1)} + I_a^{(2)}$$

$$I_b = I_a^{(0)} + a^2 I_a^{(1)} + a I_a^{(2)}$$

$$I_c = I_a^{(0)} + a I_a^{(1)} + a^2 I_a^{(2)}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix}$$

Phase

Component

A

Sequence Component

Symmetrical

Component

$$\begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$\angle 120^\circ = 1 \angle 120^\circ = 1 \angle 240^\circ$$

$$a^* = a^2$$

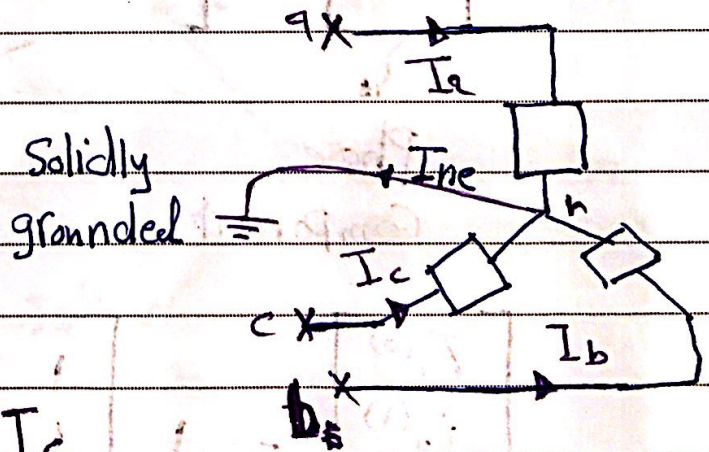
$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{ab}^{(0)} \\ V_{ab}^{(1)} \\ V_{ab}^{(2)} \end{bmatrix}$$

$$\begin{bmatrix} V_{ab}^{(0)} \\ V_{ab}^{(1)} \\ V_{ab}^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix}$$

$$V_{ab}^{(0)} = \frac{1}{3} (V_{ab} + V_{bc} + V_{ca}) = 0$$

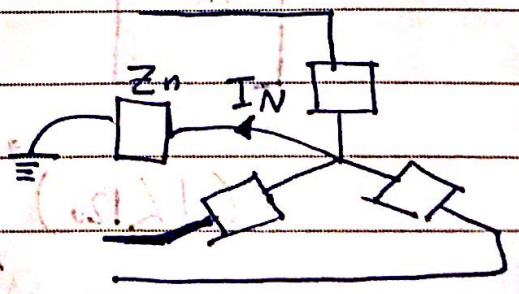
KVL "closed path"

IP unbalanced
 $I_{ne} > 0$



$$I_N = I_a + I_b + I_c$$

Earth is n



Zn

$$I_N = I_A + I_B + I_C = 3 I_A^{(0)}$$

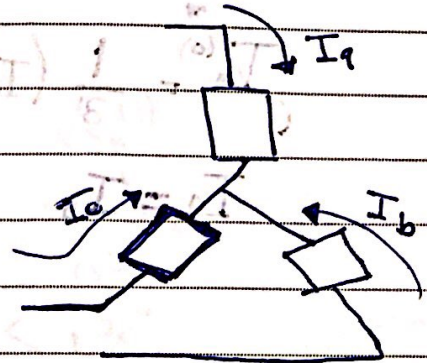
⇒ if not grounded

$$I_A^{(0)} = 0$$

No Zero Sequence currents

"line currents have

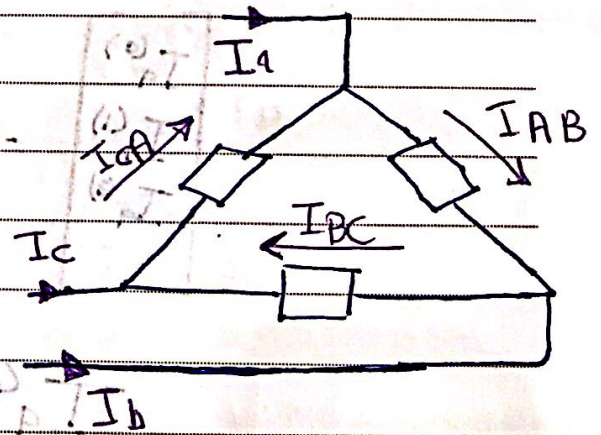
no Zero Sequence currents"



"line currents have no Zero Sequence Components"

$$I_A = I_A^{(0)} + I_A^{(1)} + I_A^{(2)}$$

$$I_{AB}^{(0)} = I_{BC}^{(0)} = I_{CA}^{(0)}$$

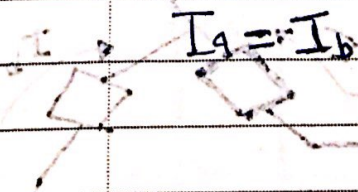


$$\begin{bmatrix} I_A^{(0)} \\ I_A^{(1)} \\ I_A^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}$$

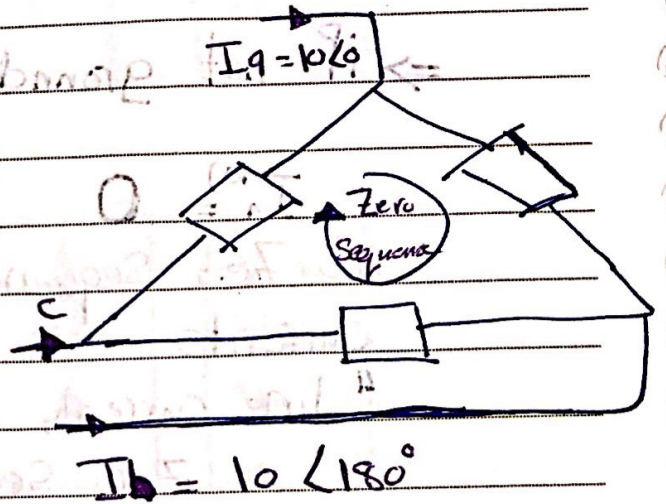
$$I_A^{(0)} = \frac{1}{3} (I_A + I_B + I_C)$$

Exo:

$$I_A^{(0)} = \frac{1}{3} (I_A + I_B + I_C)$$

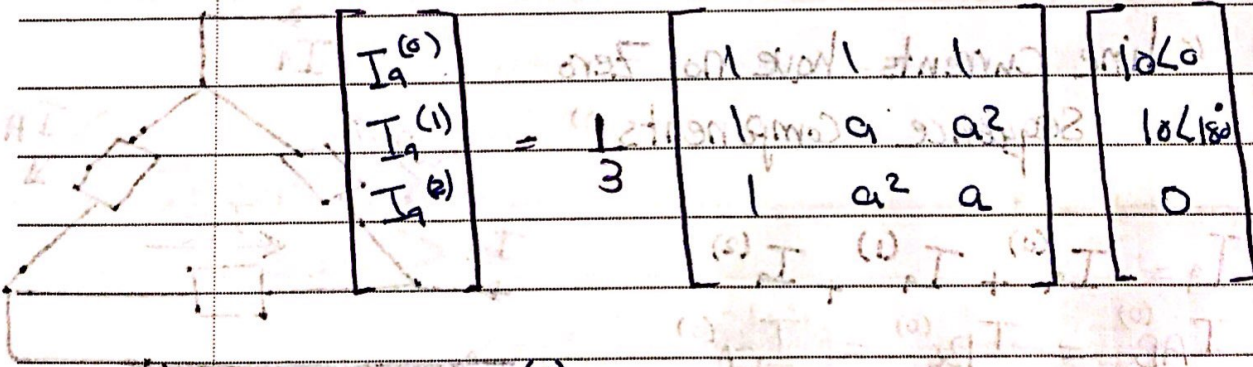


open circuit
"I_C = 0"



$$I_B = 10 \angle 180^\circ$$

Q find Symmetrical Components ??



$$I_A^{(0)} = 0$$

$$I_A^{(1)} = 5.78 \angle -30^\circ$$

$$I_A^{(2)} = 5.78 \angle 30^\circ$$

$$I_B^{(0)} = 0 = I_C^{(0)}$$

$$I_B^{(1)} = a^2 I_A^{(1)} = 0$$

$$I_B^{(2)} = a I_A^{(2)} = 5.78 \angle 90^\circ$$

$$I_C^{(1)} = a I_A^{(1)} = 5.78 \angle -90^\circ$$

$$I_C^{(2)} = a^2 I_A^{(2)} = 0$$

$$I_C = 0$$

$$I_C^{(1)} = 5.78 \angle -90^\circ$$

$$I_C^{(2)} = 0$$

$$I_C = 0$$

$$I_C = 0$$

⇒ Objective :- Relation ship between line currents & phase currents using Symmetrical components.

Y connected load

$$\begin{aligned} V_{ab}^{(1)} &= V_{an}^{(1)} - V_{bn}^{(1)} \\ &= V_{an}^{(1)} - a^2 V_{an}^{(1)} \\ &= V_{an}^{(1)} (1 - a^2) \\ &= \sqrt{3} V_{an}^{(1)} \angle +30^\circ \end{aligned}$$

← Magnitude of line current is $\sqrt{3}$ times per unit of phase current.

$$\begin{aligned} 1 - a^2 &= \sqrt{3} \angle 30^\circ \end{aligned}$$

$$\begin{aligned} V_{ab}^{(2)} &= V_{an}^{(2)} - V_{bn}^{(2)} \\ &= V_{an}^{(2)} - a V_{an}^{(2)} \\ &= V_{an}^{(2)} (1 - a) \\ &= V_{an}^{(2)} \sqrt{3} \angle -30^\circ \end{aligned}$$

Δ connected load :

$$I_1^{(1)} = \sqrt{3} I_{AB}^{(1)} \angle -30^\circ$$

$$I_1^{(2)} = \sqrt{3} I_{AB}^{(2)} \angle +30^\circ$$

$$S_{3\phi} = 3 V_a I_1^* \quad \text{"balanced"}$$

$$S_{3\phi} = V_a I_a^* + V_b I_b^* + V_c I_c^* \quad \text{"unbalanced"}$$

$$S_{3\phi} = 3 V_a^{(0)} I_1^{(0)} + 3 V_a^{(1)} I_1^{(1)} + 3 V_a^{(2)} I_1^{(2)}$$

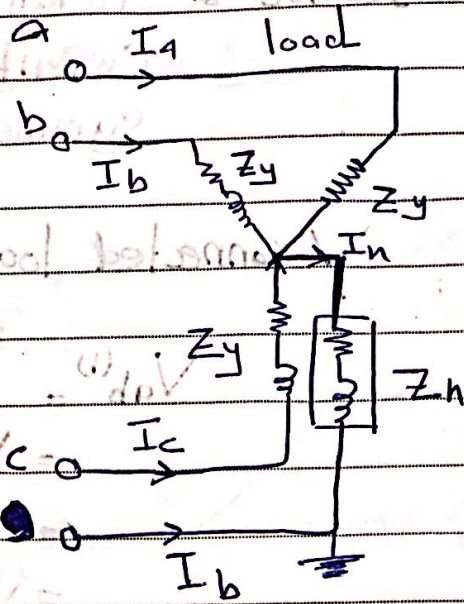
⇒ Sequence network

unbalanced

I_a

I_b

I_c



→ $V_{ag} = I_a Z_y + Z_n I_n$ (Coupling between phases)

→ $V_{ag} = I_a Z_y + Z_n (I_a + I_b + I_c)$

$V_{ag} = (Z_y + Z_n) I_a + Z_n I_b + Z_n I_c$

→ $V_{bg} = Z_n I_a + (Z_y + Z_n) I_b + Z_n I_c$ $g \triangleq$ ground

→ $V_{cg} = Z_n I_a + Z_n I_b + (Z_n + Z_y) I_c$ $e \triangleq$ earth

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \begin{bmatrix} Z_y + Z_n & Z_n & Z_n \\ Z_n & Z_y + Z_n & Z_n \\ Z_n & Z_n & Z_y + Z_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

→ $V_p = Z_p I_p$ $p \triangleq$ phase

$A^{-1} V_s = Z_p A I_s$

$V_s = A^{-1} Z_p A I_s$

$$\bar{A}^{-1} Z_p A = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} Z_y + Z_n & Z_n & Z_n \\ Z_n & Z_y + Z_n & Z_n \\ Z_n & Z_n & Z_y + Z_n \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$a^2 + a = -1$$

$$1 + a^2 + a = 0$$

$$= \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} Z_y + 3Z_n & Z_y & Z_y \\ Z_y + 3Z_n & a^2 Z_y & a Z_y \\ Z_y + 3Z_n & a Z_y & a^2 Z_y \end{bmatrix}$$

$$\bar{A}^{-1} Z_p A = \begin{bmatrix} Z_y + 3Z_n & 0 & 0 \\ 0 & Z_y & 0 \\ 0 & 0 & Z_y \end{bmatrix}$$

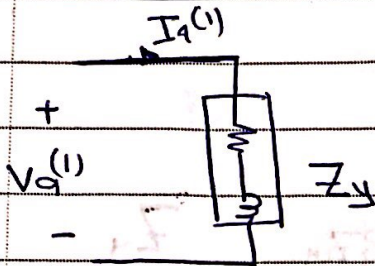
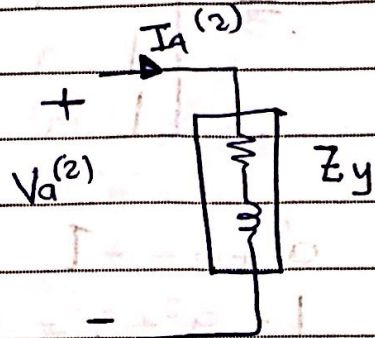
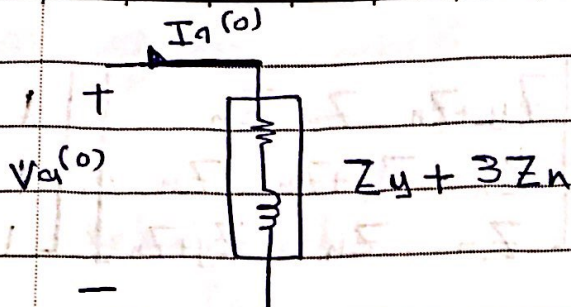
← decoupled in sequence network

$$\begin{bmatrix} V_a^{(0)} \\ V_a^{(1)} \\ V_a^{(2)} \end{bmatrix} = \begin{bmatrix} Z_y + 3Z_n & 0 & 0 \\ 0 & Z_y & 0 \\ 0 & 0 & Z_y \end{bmatrix} \begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix}$$

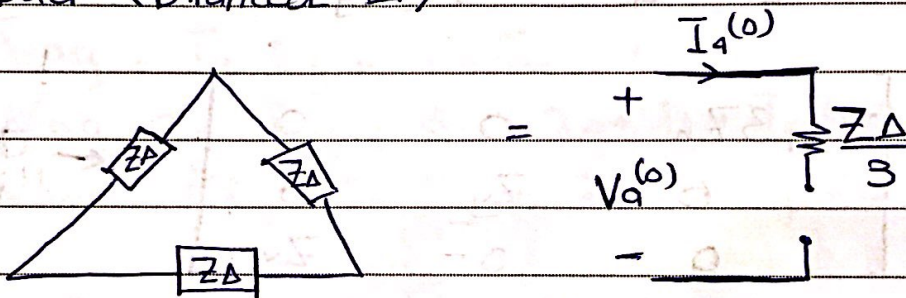
$$V_a^{(0)} = (Z_y + 3Z_n) I_a^{(0)}$$

$$V_a^{(1)} = Z_y I_a^{(1)}$$

$$V_a^{(2)} = Z_y I_a^{(2)}$$

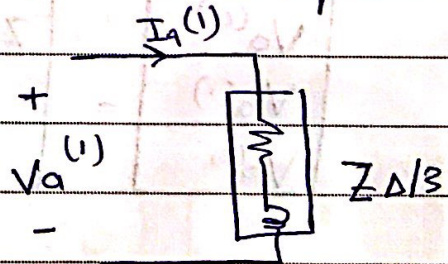


⇒ bad (balanced Δ)

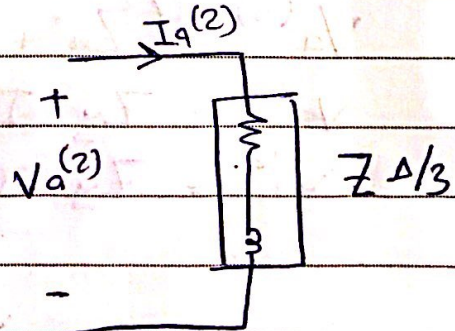


Zero sequence

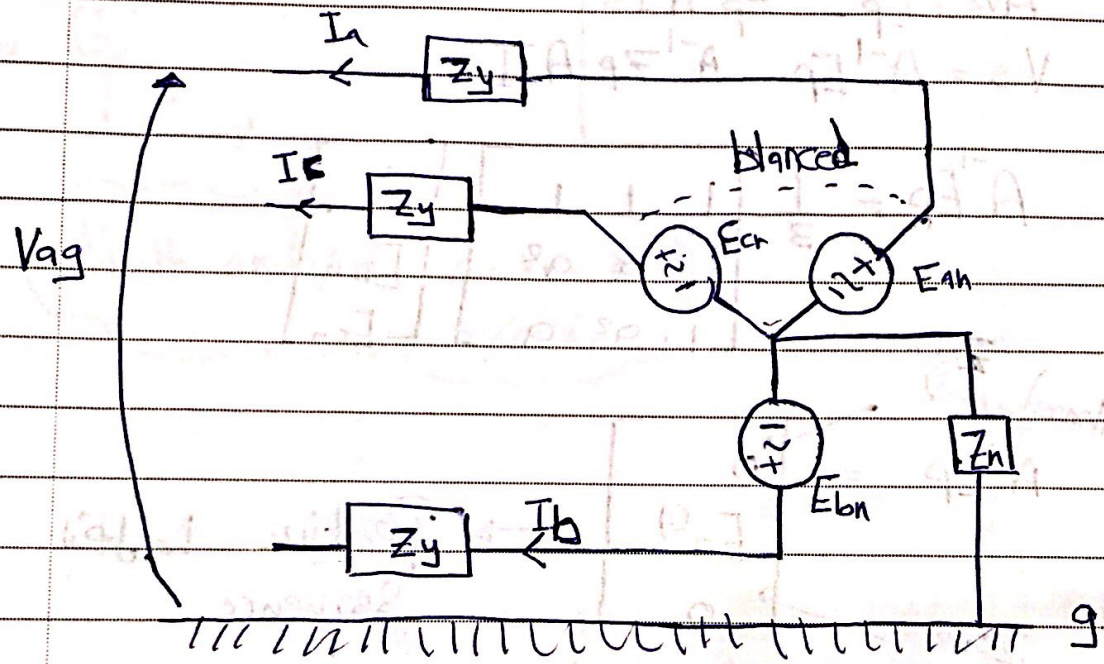
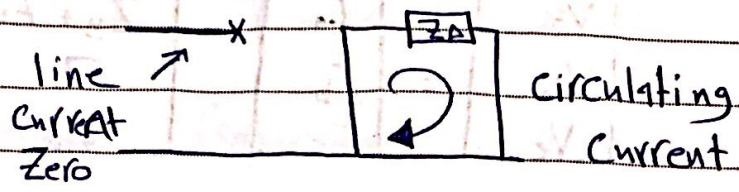
Positive



negative



Δ model

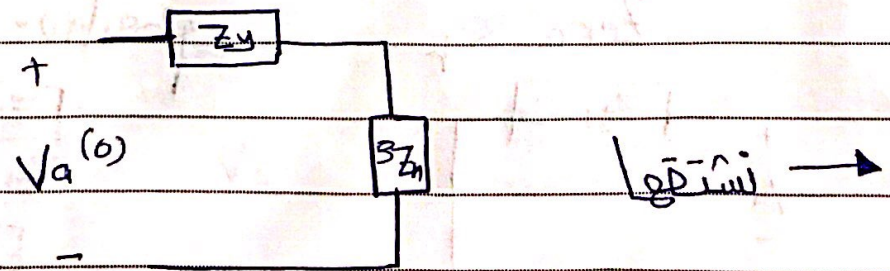


$$V_{ag} = E_{an} - I_a (Z_n + Z_y) - I_b Z_n - I_c Z_n$$

$$V_{bg} = E_{bn} - I_a Z_n - I_b (Z_y + Z_n) - I_c Z_n$$

$$V_{cg} = E_{cn} - I_a Z_n - I_b Z_n - I_c (Z_y + Z_n)$$

Zero sequence network



V_a		E_a	$Z_y + Z_n$	Z_n	Z_y	$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$
V_b	-	E_b	Z_n	$Z_y + Z_n$	Z_y	
V_c		E_c	Z_n	Z_n	$Z_y + Z_n$	

$$V_p = E_p - Z_p I_p$$

$$AV_s = E_p - Z_p A I_s$$

$$V_s = A^{-1} E_p - A^{-1} Z_p A I_s$$

$$A^{-1} E_p = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix}$$

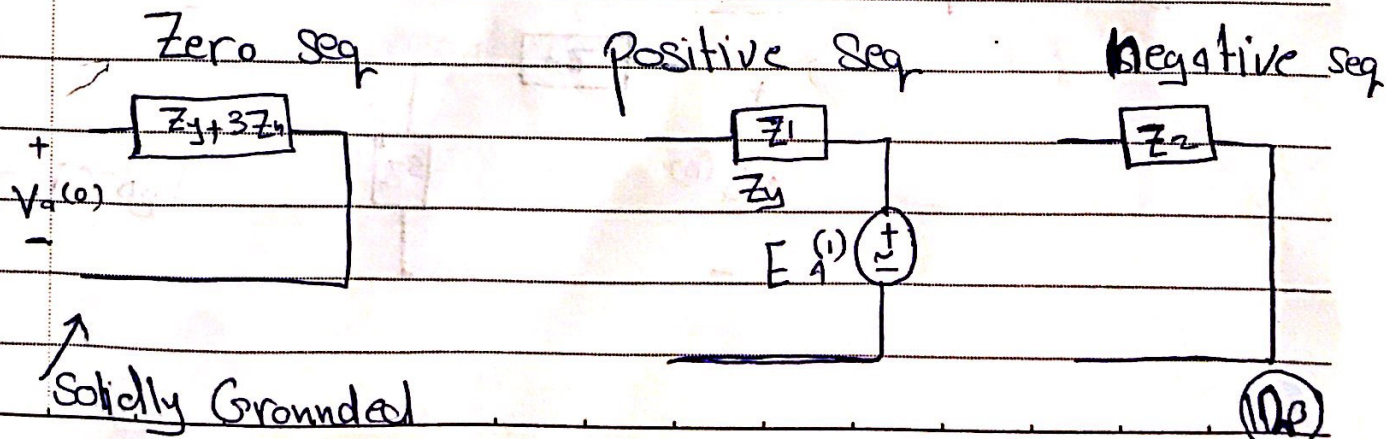
Balanced ^{Zero} _{field}

$$A^{-1} E_p = \begin{bmatrix} 0 \\ E_a^{(1)} \\ 0 \end{bmatrix}$$

→ Positive sequence

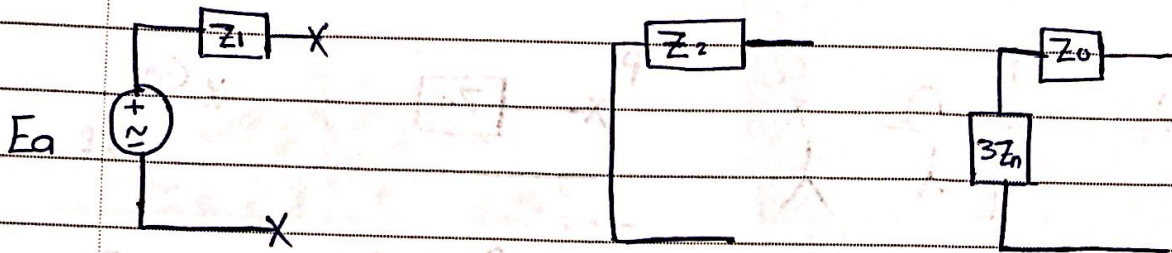
Zero _{Balanced field}

$$A^{-1} Z_p A = \begin{bmatrix} Z_y + Z_n & 0 & 0 \\ 0 & Z_y & 0 \\ 0 & 0 & Z_y \end{bmatrix}$$

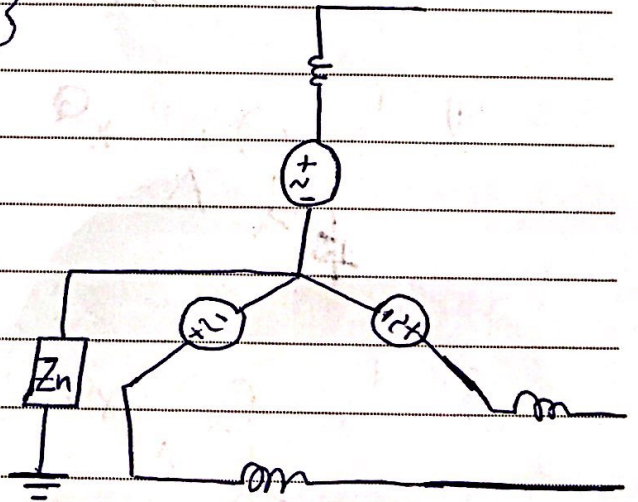


lines, cables, transformer's $Z_1 = Z_2$
 $Z_0 \approx 2-3.5 Z_1$

⇒ Sequence diagram Generators

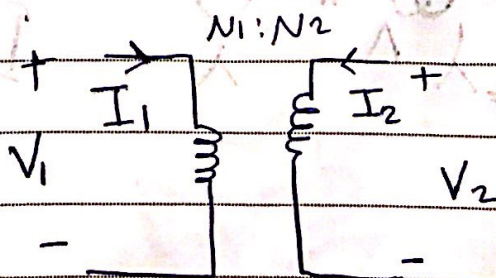


Fault analysis $X_1 = X_2 = X''$
 $X_0 \ll X_1$



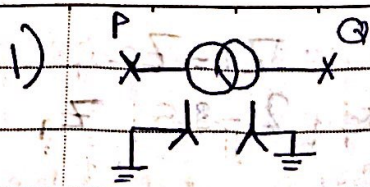
⇒ Sequence diagram transformer's

Ideal Transformer

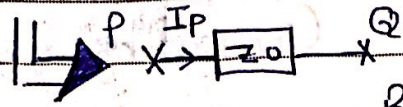


$N_1 \cdot I_1 = N_2 \cdot I_2$

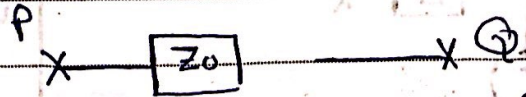
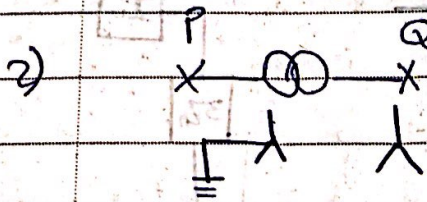
A current flow in the primary if a current flow in the secondary



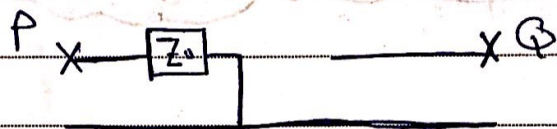
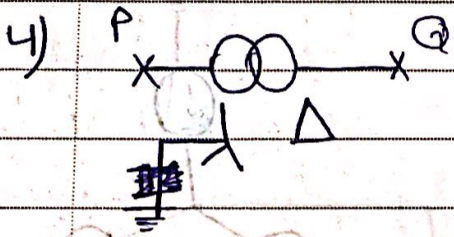
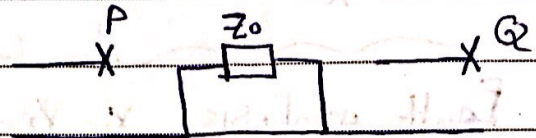
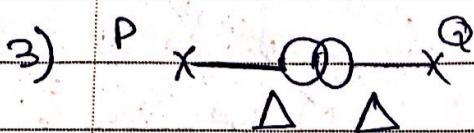
Z_1 positive
 Z_2 negative



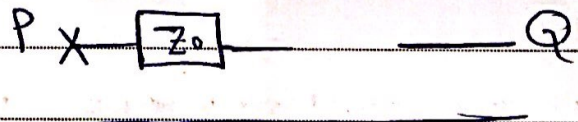
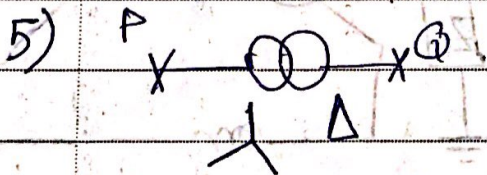
Reference



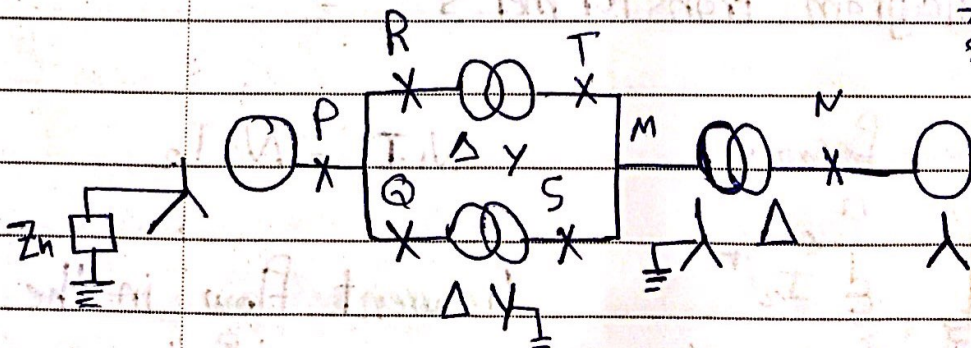
Ref



line \rightarrow Zero



Zero seq



Draw zero sequence network

⇒ Unsymmetrical Fault calculations "unbalanced"

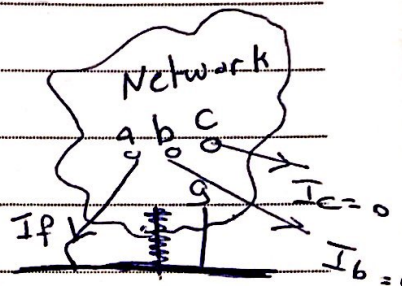
- Single line to Ground Faults
- line to line (LL) faults
- Double line to Ground Faults

* Single line to Ground Faults

$$I_a = I_f \neq 0$$

$$I_b = I_c = 0$$

$$\begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

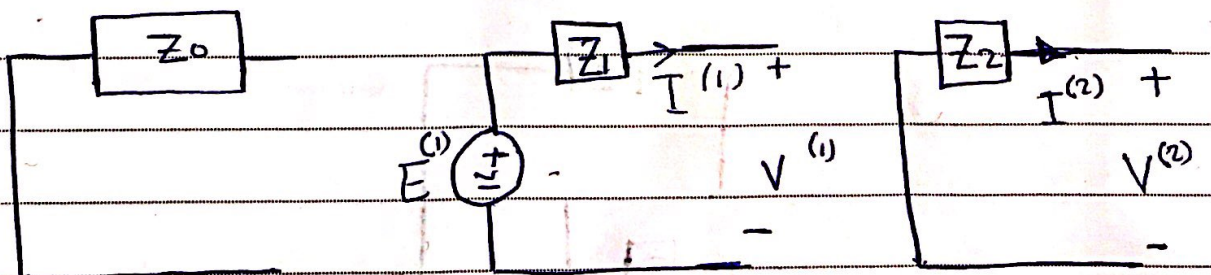


$$I^{(0)} = \frac{1}{3} I_a = \frac{1}{3} I_f$$

$$\frac{1}{3} I_f = I^{(0)} = I^{(1)} = I^{(2)}$$

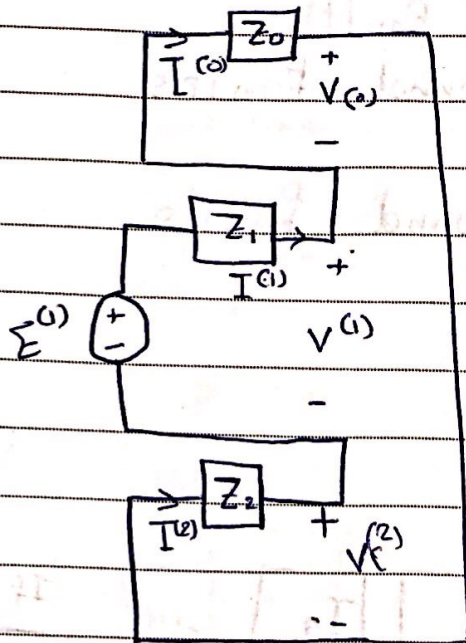
$$I^{(1)} = \frac{1}{3} I_a, \quad I^{(2)} = \frac{1}{3} I_a$$

↓ coupled Series



$Z_0, Z_1, Z_2 \Rightarrow$ thevenin impedences seen from the fault location

connect it Series



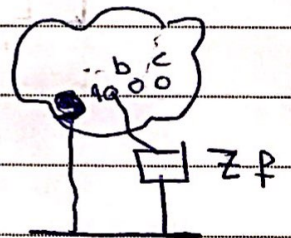
$$I^{(0)} = I^{(1)} = I^{(2)} = \frac{E_1}{Z_0 + Z_1 + Z_2}$$

$$I_f = 3I^{(0)} = \frac{3E_1}{Z_0 + Z_1 + Z_2}$$

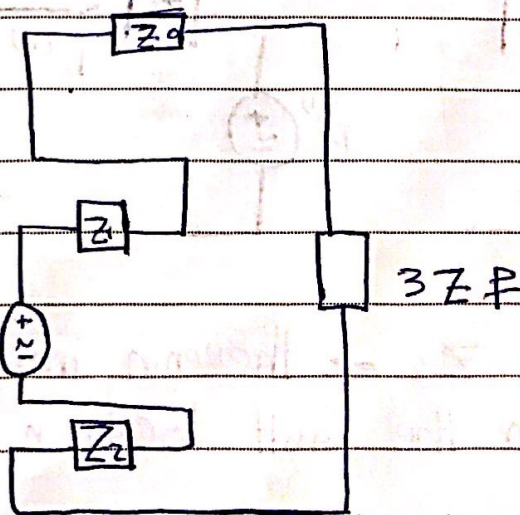


to Ground Fault

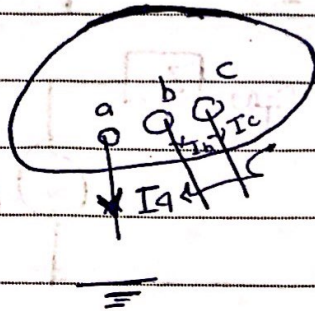
$$Z_f \neq 0$$



$$V_g = Z_f I_g = Z_f I_f = Z_f (3I^{(0)}) = 3Z_f I^{(0)}$$



"line to line Fault"



$$I_a = 0$$

$$I_b = I_f$$

$$I_c = -I_f$$

$$\begin{bmatrix} I^{(0)} \\ I^{(1)} \\ I^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

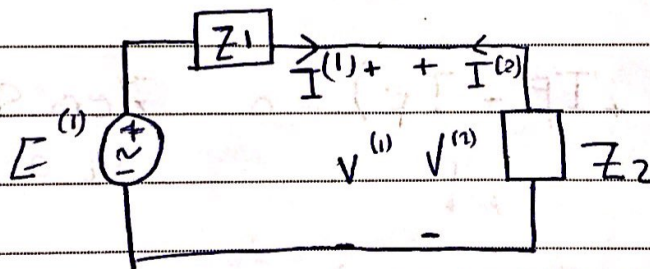
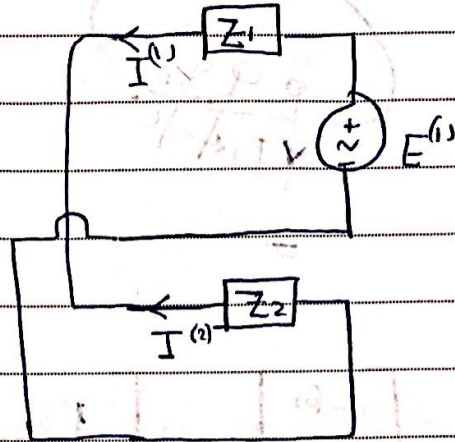
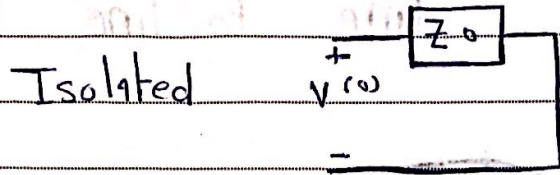
$$\Rightarrow I^{(0)} = \frac{1}{3} (0 + I_f - I_f) = 0 \quad \text{Zero Sequence} = 0$$

No access to ground

$$\Rightarrow I^{(1)} = \frac{1}{3} (a I_f - a^2 I_f)$$

$$I^{(2)} = \frac{1}{3} (a^2 I_f - a I_f)$$

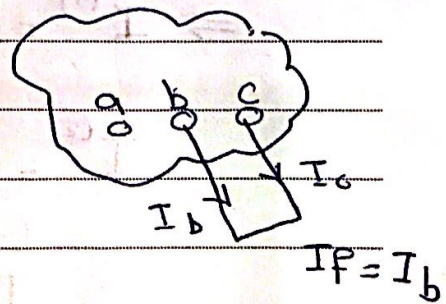
$$I^{(1)} = -I^{(2)}$$



$$I^{(1)} = \frac{E^{(1)}}{Z_1 + Z_2}$$

$$I^{(2)} = -I^{(1)}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I^0 \\ I^1 \\ I^2 \end{bmatrix}$$



$$I_b = a^2 I^{(1)} + a I^{(2)}$$

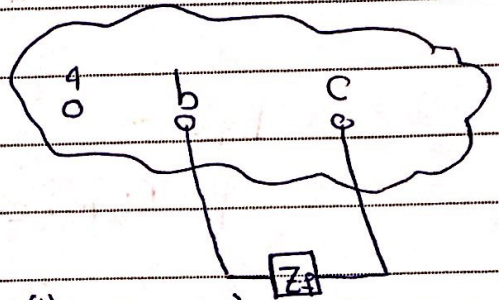
$$I_b = a^2 I^{(1)} - a I^{(1)} = I^{(1)} (a^2 - a)$$

$$I_b = -j\sqrt{3} I^{(1)}$$

$$|I_P| = \frac{\sqrt{3} E^{(1)}}{Z_1 + Z_2}$$

$Z_P \neq 0$

$$V_b - V_c = I_P Z_P$$



$$(V^{(0)} + a^2 V^{(1)} + a V^{(2)}) - (V^{(0)} + a V^{(1)} + a^2 V^{(2)}) = I_P Z_P$$

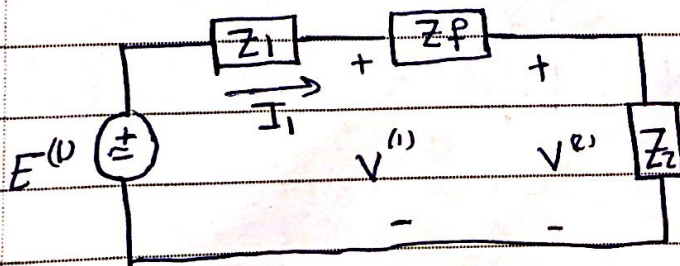
3 ZP و 3 ZP یعنی 3 لود
Ground 3 لود ای

$$I_P = I_b = I^{(2)} + a^2 I^{(1)} + a I^{(2)}$$

$$I_P = (a^2 - a) I^{(1)}$$

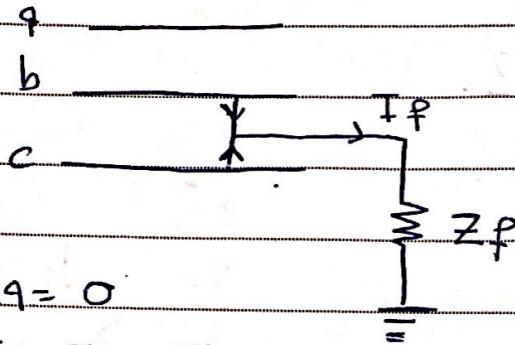
$$V^{(1)}(a^2 - a) - V^{(2)}(a^2 - a) = Z_P(a^2 - a) I^{(1)}$$

$$V^{(1)} - V^{(2)} = Z_P I^{(1)}$$



$$I_P = \frac{\sqrt{3} E^{(1)}}{Z_1 + Z_2 + Z_P}$$

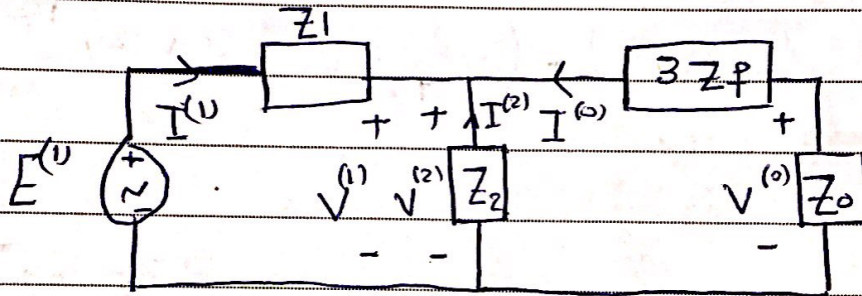
Double line to Grounds



$$I_a = 0$$

$$I_p = I_b + I_c$$

في طرف الأخرى
Zero في
Seq



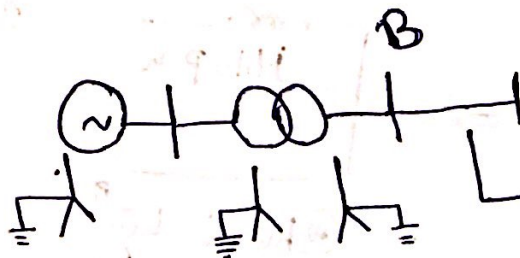
$$I^{(0)} + I^{(1)} + I^{(2)} = 0$$

$$I^{(1)} = \frac{E'}{Z_1 + [Z_2 \parallel (Z_0 + 3Z_f)]}$$

$$I^{(0)}, I^{(2)}$$

$$I_p = I_b + I_c$$

Ex 10.0



$Z_0 = j1 \text{ p.u.}$
 $Z_1 = j0.5 \text{ p.u.}$
 $Z_2 = j0.6 \text{ p.u.}$

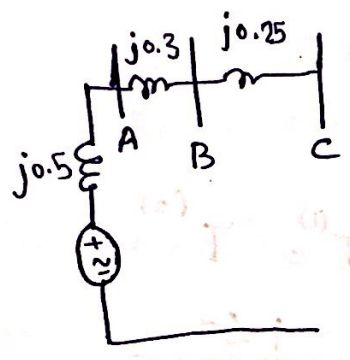
$Z_0 = j0.3 \text{ p.u.}$
 $Z_1 = j0.3 \text{ p.u.}$
 $Z_2 = j0.3 \text{ p.u.}$

line $\rightarrow Z_0 = j0.45 \text{ p.u.}$
 $Z_1 = j0.25 \text{ p.u.}$
 $Z_2 = j0.25 \text{ p.u.}$

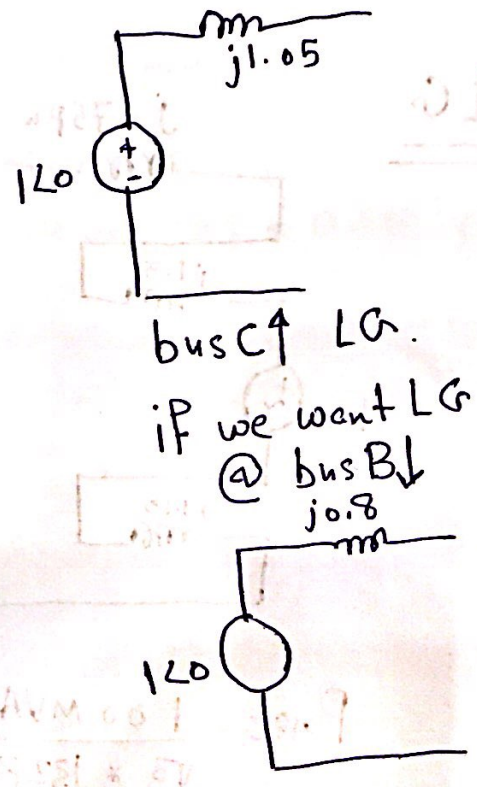
Q system is unloaded & operating @ normal voltage,
 100 MVA base : 1) LG Fault current @ bus C
 2) LL Fault current @ bus B

Solution 10.0

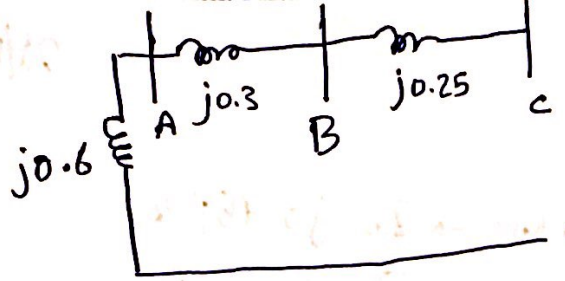
+ve sequence



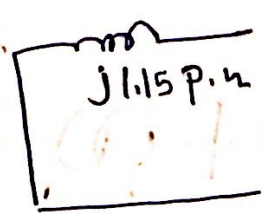
Therminin \Rightarrow



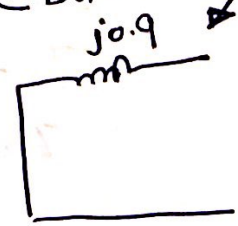
-ve sequence



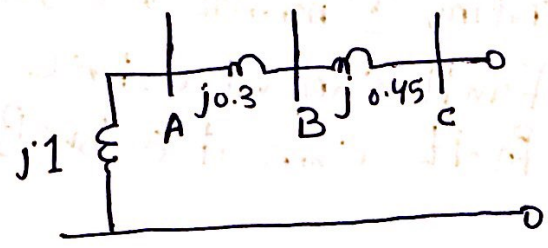
=>



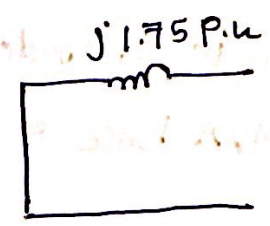
@ bus C LG
@ bus B LG



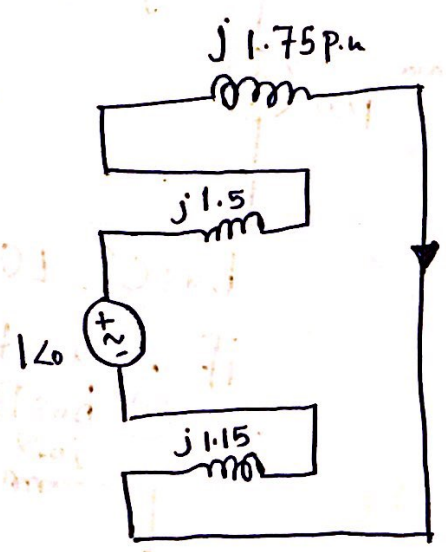
Zero Sequence



=>



LG



$$I^{(0)} = I^{(1)} = I^{(2)}$$

$$I^{(0)} = \frac{140}{Z_0 + Z_1 + Z_2}$$

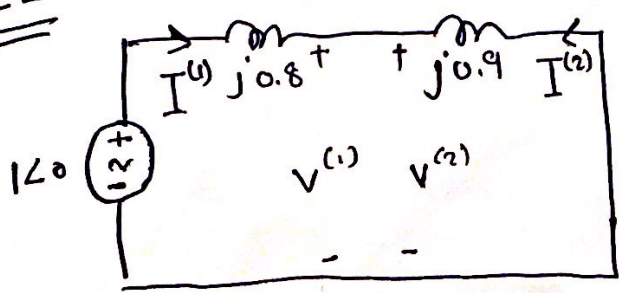
$$I^{(0)} = 0.253 \text{ p.u.}$$

$$I_F = 3 I^{(0)} = 0.759 \text{ p.u.}$$

$$\text{Base} = \frac{100 \text{ MVA}}{\sqrt{3} \times 132 \text{ kV}} = 437.4 \text{ A}$$

$$I_F = 437.4 \times 0.759 = 322.2 \text{ A}$$

LL



$$I_1 = -I_2$$

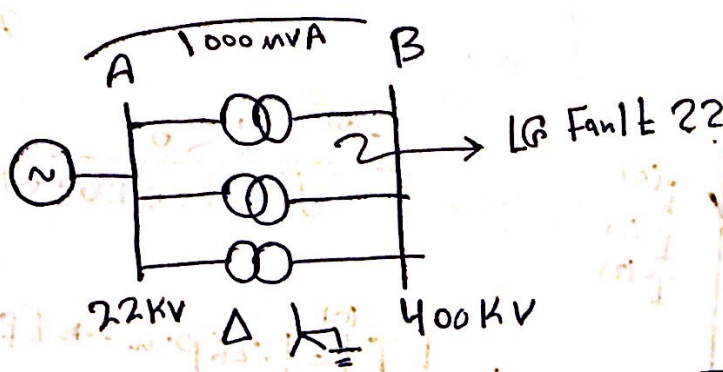
$$I_F = \left(\frac{120}{j0.8 + j0.9} \right) \sqrt{3}$$

$$I_b = -I_c$$

$$I_F = 445.6$$

Ex 280

21/12/2016



$$\frac{3E^{(1)}}{Z_1 + Z_2 + Z_0}$$

Generator → 825 MVA

$$X_1 = 0.14 \text{ p.u.}$$

$$X_2 = 0.13 \text{ p.u.}$$

$$X_0 = 0.15 \text{ p.u.}$$

Unloaded, $V_f = 1.20$

Transformers

300 MVA

$$X_1 = 0.14 \text{ p.u.}$$

$$X_2 = 0.14 \text{ p.u.}$$

$$X_0 = 0.14 \text{ p.u.}$$

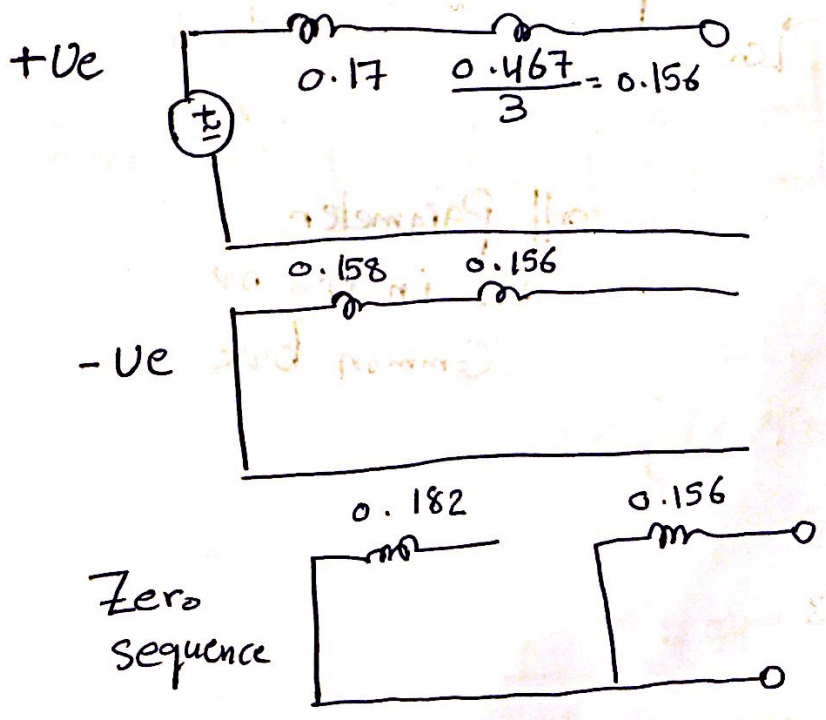
1000 MVA base

Solⁿ Generator ⇒ $X_1 = 0.14 * \left(\frac{1000}{825}\right) = 0.17 \text{ p.u.}$

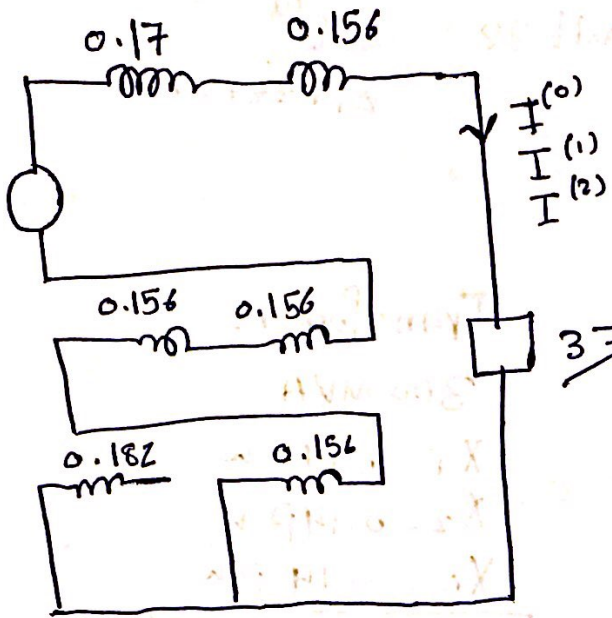
$$X_2 = 0.13 * \left(\frac{1000}{825}\right) = 0.158 \text{ p.u.}$$

$$X_0 = 0.15 * \left(\frac{1000}{825}\right) = 0.182 \text{ p.u.}$$

Transformer ⇒ $X_1 = X_2 = X_0 = 0.14 * \left(\frac{1000}{300}\right) = 0.467 \text{ p.u.}$



LG Fault

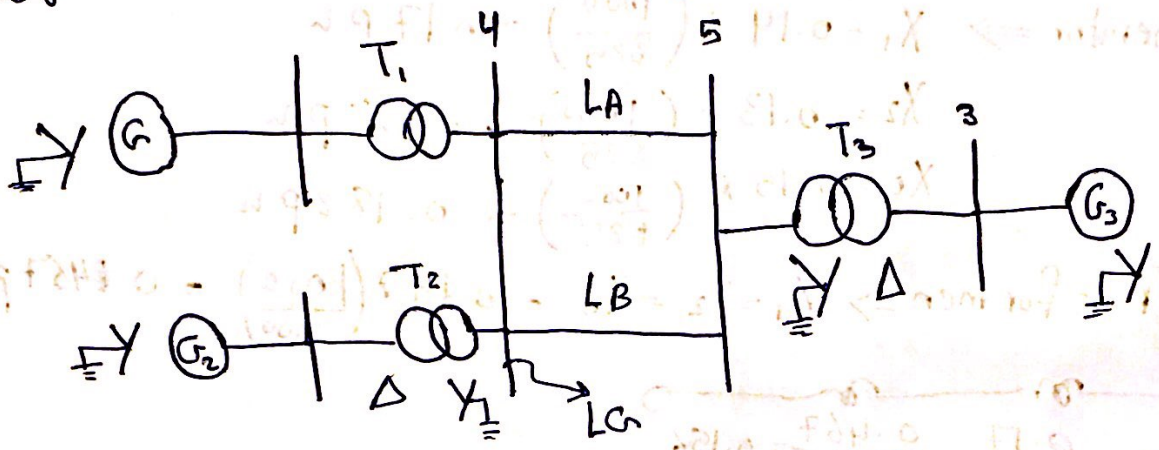


$$I^{(0)} = \frac{1 \angle 0}{j(0.17 + 0.156 + 0.156 + 0.156 + 0.156)}$$

$$|I^{(0)}| = 1.26 \text{ p.u.} \rightarrow I_F = 3I^{(0)} = 3.78 \text{ p.u.}$$

$$I_F = 3.78 \times \left(\frac{10000}{\sqrt{3} \times 400} \right) = 5.453 \text{ KA}$$

Ex 3.8

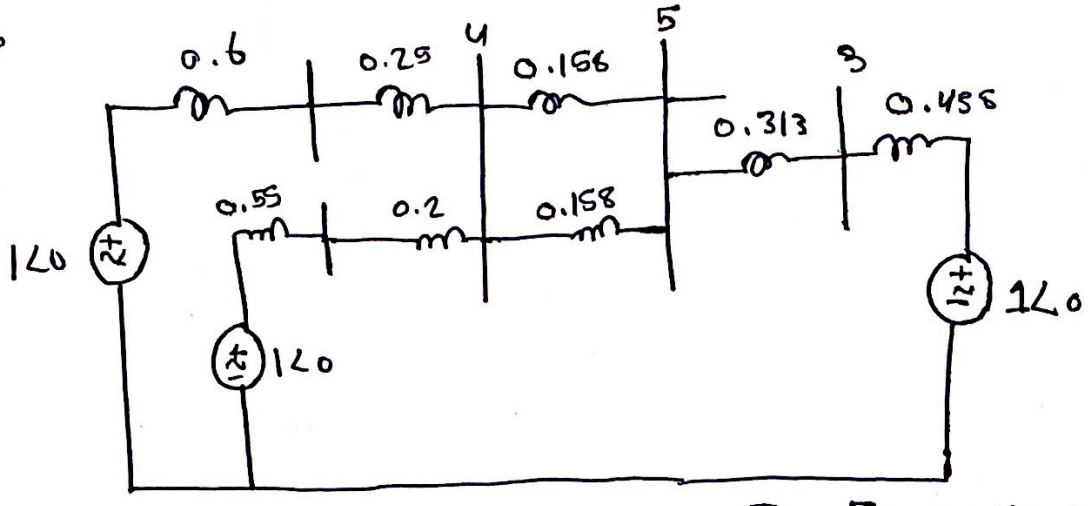


	X_1	X_2	X_0
G_1	0.6	0.5	0.3
G_2	0.55	0.45	0.25
G_3	0.438	0.375	0.25
T_1	0.25	0.25	0.25
T_2	0.2	0.2	0.2
T_3	0.313	0.313	0.313
LA	0.158	0.158	0.473
LB	0.158	0.158	0.473

all parameter are in p.u on common base.

Sol 80

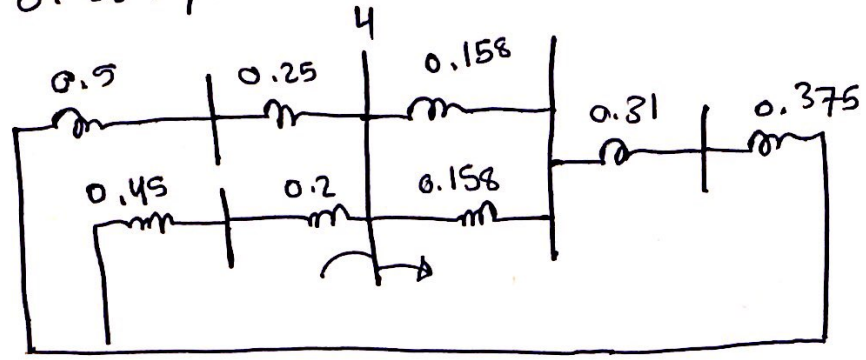
+ve



$$Z_1 = [(0.6 + 0.25) \parallel (0.55 + 0.2)] \parallel [(0.158 \parallel 0.158) + 0.313 + 0.455]$$

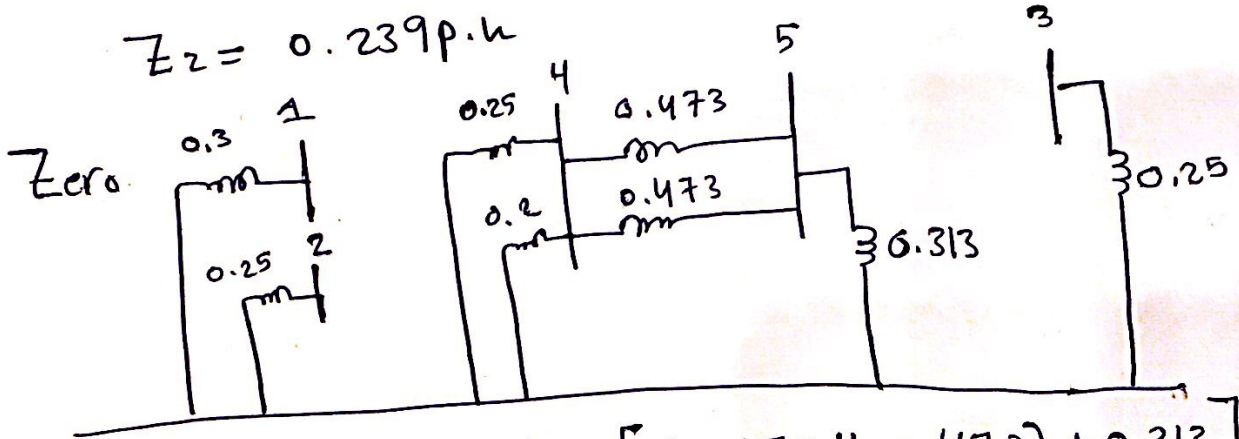
$$= 0.269 \text{ p.u.}$$

-ve



$$Z_2 = [(0.5 + 0.25) \parallel (0.45 + 0.2)] \parallel [(0.158 \parallel 0.158) + 0.31 + 0.375]$$

$$Z_2 = 0.239 \text{ p.u.}$$



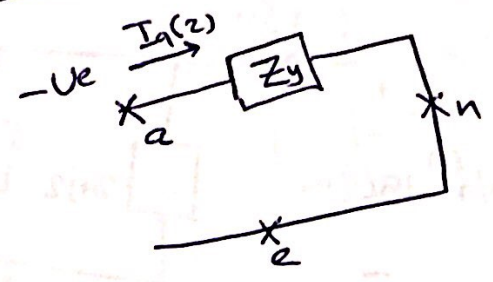
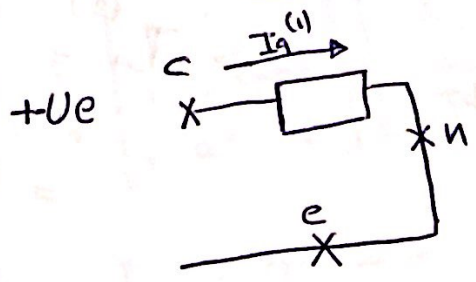
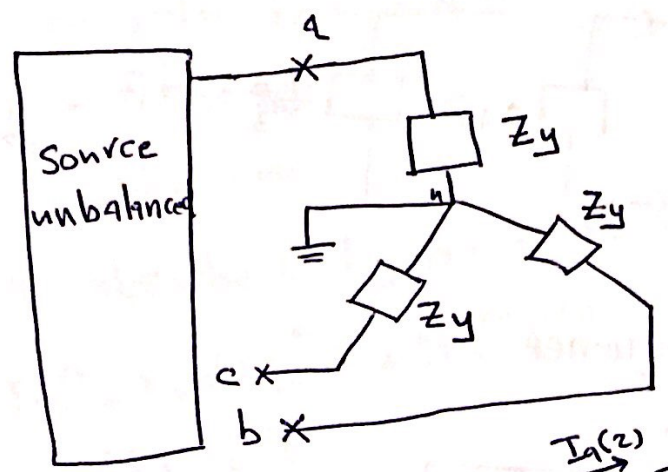
$$Z_0 = (0.2 \parallel 0.25) \parallel [(0.473 \parallel 0.473) + 0.313]$$

$$Z_0 = 0.092 \text{ p.u.}$$

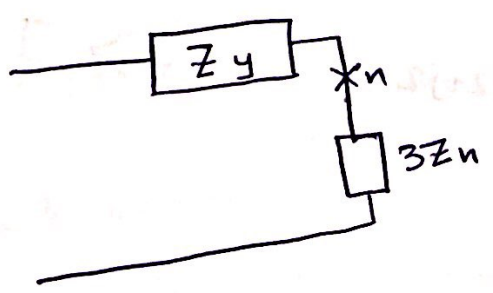
$$I_P = \frac{3 * 120}{Z_1 + Z_2 + Z_0} = 4.997 \text{ p.u.}$$

$$I^0 = I_1 = I_2$$

$V_a = V_{ac} = V_{ag}$



Zero seq



$$I_a^{(0)} + I_b^{(0)} + I_c^{(0)} = 3 I_a^{(0)}$$

$$\rightarrow S_{3\phi} = 3 V_a^{(0)} I_a^{(0)*} + 3 V_a^{(1)} I_a^{(1)*} + 3 V_a^{(2)} I_a^{(2)*}$$

Ex: A 3φ impedance load

- balanced Δ load ($Z_{\Delta} = 6 + j6$) in parallel
- Y-load ($Z_{Y\text{Load}} = 2 + j2$)

The Yload is grounded through a neutral impedance

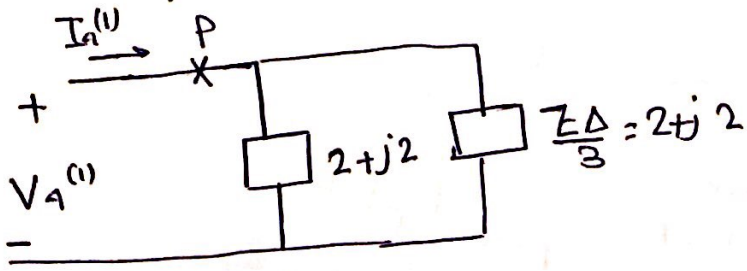
$Z_n = j1 \Omega$

* unbalanced source $V_a^{(0)} = 10 \angle 60^\circ$, $V_a^{(1)} = 100 \angle 0^\circ$, $V_a^{(2)} = 15 \angle 200^\circ$

* Determine the total complex power?

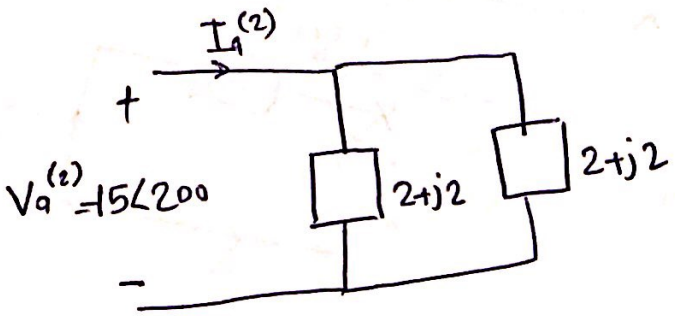
Sol: +ve sequence

(2)
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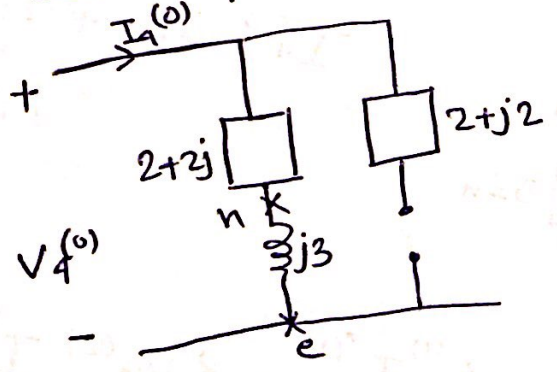
$$\Rightarrow I_A^{(1)} = \frac{100 \angle 0}{(2+j2) \parallel (2+j2)} = 70.71 \angle -45^\circ \text{ A}$$

-ve sequence



$$\Rightarrow I_A^{(2)} = \frac{15 \angle 200^\circ}{1+j1} = 10.61 \angle 155^\circ \text{ A}$$

Zero sequence.



$$\Rightarrow I_A^{(0)} = \frac{10 \angle 60^\circ}{2+j2+j3} = 1.84 \angle -0.26^\circ$$

$$S_{3\phi} = 3 V_A^{(1)} I_A^{(1)*} + 3 V_A^{(2)} I_A^{(2)*} + 3 V_A^{(0)} I_A^{(0)*}$$

$$|S| = 21.7 \angle 45^\circ \text{ KVA}$$

$$\Rightarrow V_{ab} = V_a - V_b$$

$$V_{ab} = V_{ab}^{(1)} + V_{ab}^{(2)} + V_{ab}^{(0)}$$

$$= \sqrt{3} V_{an}^{(1)} \angle 30^\circ + \sqrt{3} V_{an}^{(2)} \angle -30^\circ + 0$$

closed path

$$\Rightarrow \text{Voltage on neutral} \Rightarrow I_A^{(0)} \times j3$$

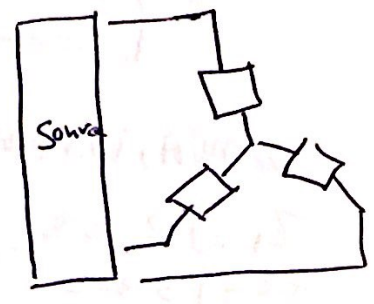
Ex: 3 ϕ unbalanced Voltage are applied to a star balanced load \rightarrow no zero seq

$V_{ab} = 400 \angle 0$, $V_{bc} = 415 \angle -135$

$I_a = 10 \angle -30^\circ$, $I_b = 16 \angle -100$

\Rightarrow Determine power absorbed by the load.?

Sol: $S_{3\phi} = 3V_a^{(0)} I_a^{(0)*} + 3V_a^{(1)} I_a^{(1)*} + 3V_a^{(2)} I_a^{(2)*}$



$$\begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$I_a + I_b + I_c = 0$ \rightarrow $\begin{matrix} \text{نول صفر} \\ \text{موجود بالازمنه} \end{matrix}$

$I_a^{(0)} = 0$

$I_c = -(I_a + I_b)$

$V_{ab} + V_{bc} + V_{ca} = 0$

$V_{ca} = -(V_{bc} + V_{ab})$

$$\begin{bmatrix} V_{ab}^{(0)} \\ V_{ab}^{(1)} \\ V_{ab}^{(2)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix}$$

$V_a^{(1)} = \frac{V_{ab}^{(1)}}{\sqrt{3}} \angle -30^\circ$

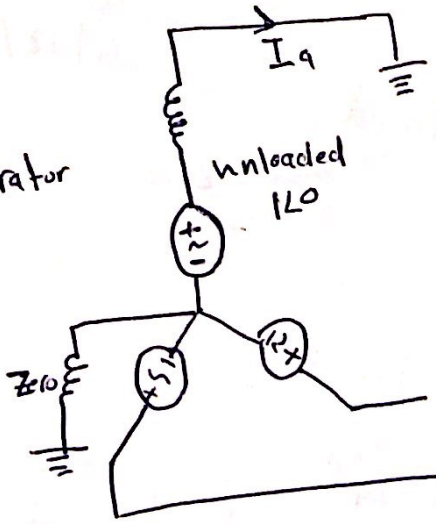
$V_a^{(2)} = \frac{V_{ab}^{(2)}}{\sqrt{3}} \angle +30^\circ$

$V_a^{(0)} = \text{Zero} \rightarrow$ $\begin{matrix} \text{نول صفر} \\ \text{ungrounded.} \end{matrix}$

Exer

26/12/2016

Generator



Fault
LG

$$V_A = 0$$

$$V_B = 8.071 \angle -102.25^\circ \text{ kV}$$

$$V_C = 8.071 \angle 102.25^\circ \text{ kV}$$

$$I_A, I_B, I_C ?$$

$$V_{AB}, V_{BC}, V_{CA} ?$$

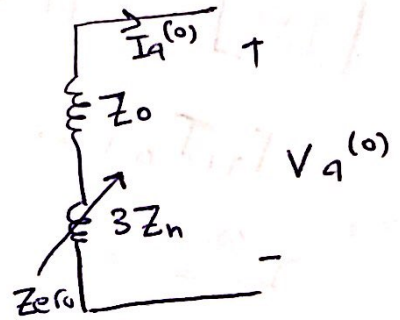
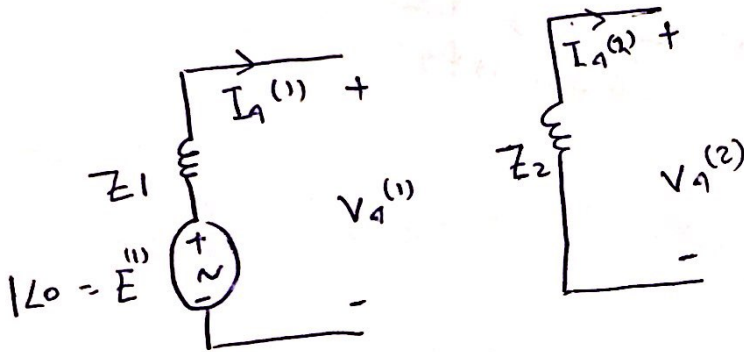
20 MVA, 13.8 kV

$$Z_1 = j 2.38 \Omega$$

$$Z_2 = j 3.3 \Omega$$

$$Z_3 = j 0.95 \Omega$$

Soln



$$\begin{bmatrix} V_A^{(0)} \\ V_B^{(0)} \\ V_C^{(0)} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} \Rightarrow$$

$$Z_1 |_{p.u} = \frac{j 2.38}{9.52} = j 0.25 \text{ p.u}$$

$$Z_2 |_{p.u} = \frac{j 3.3}{9.52} = j 0.35 \text{ p.u}$$

$$Z_0 |_{p.u} = \frac{j 0.95}{9.52} = j 0.1 \text{ p.u}$$

$$Z_{base} = \frac{V^2}{S}$$

$$= \frac{(13.8)^2}{20}$$

$$= 9.52$$

$$I_A^{(1)} = \frac{120 - V_A^{(1)}}{Z_1}, \quad I_A^{(0)} = \frac{-V_A^{(0)}}{Z_0 + 3Z_n}$$

$$I_A^{(2)} = \frac{-V_A^{(2)}}{Z_2}$$

$$V_A |_{p.u} = 0, \quad V_B = \frac{8.071}{13.8/\sqrt{3}} \angle -102.3^\circ \text{ p.u}$$

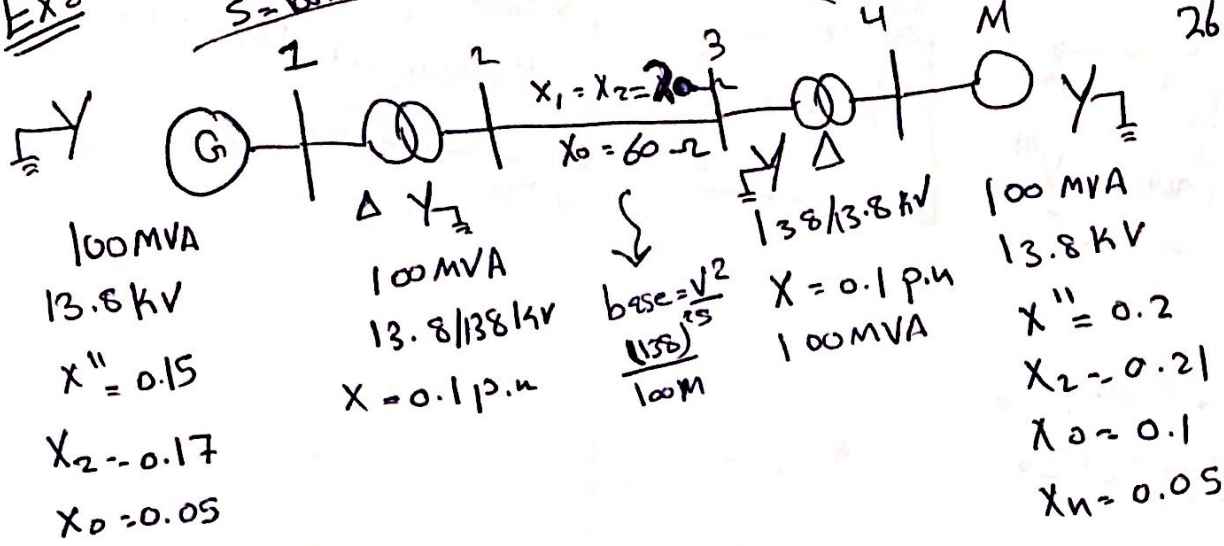
$$V_C = \frac{8.071}{13.8/\sqrt{3}} \angle 102.3^\circ \text{ p.u}$$

(137)

Ex 20

$S = 100 \text{ MVA}$

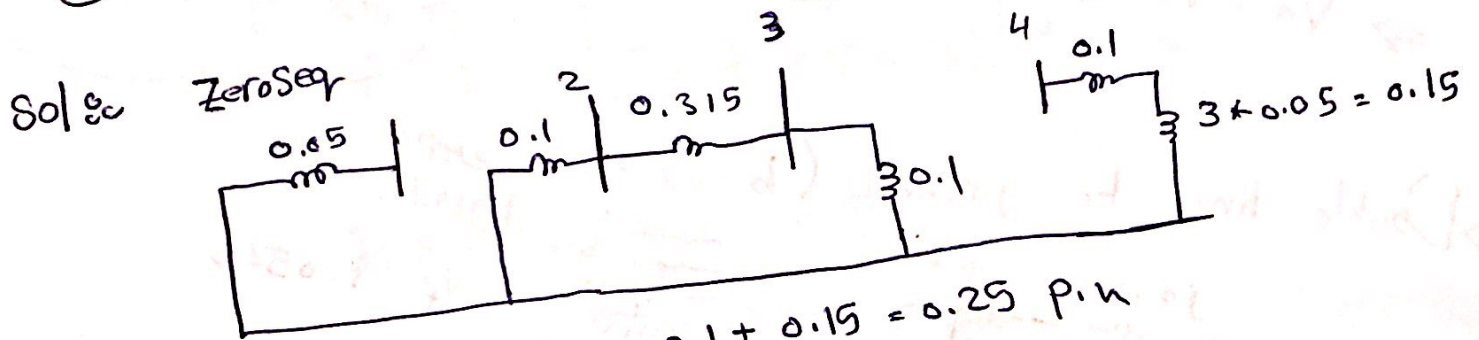
(5)
26/12/2016



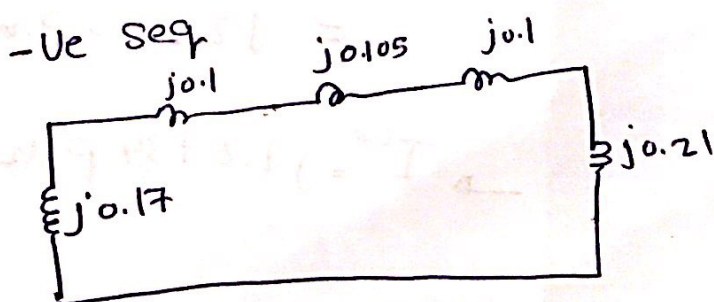
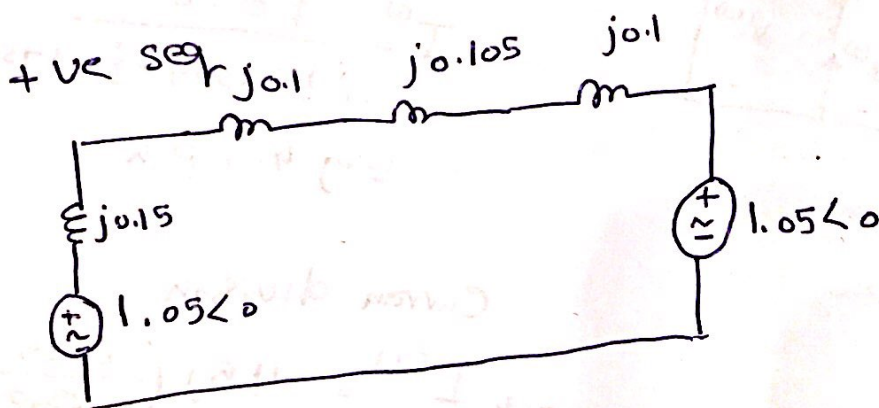
→ Pre fault voltages $1.05 \angle 0 \text{ p.u.}$

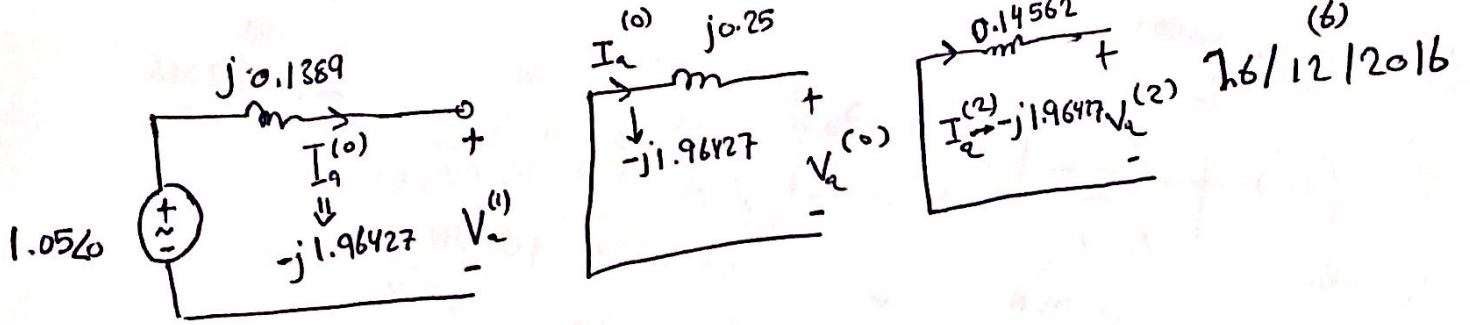
→ Neglect pre fault current phase-shift transformer

① LG @ bus 4 $\Rightarrow I_F, V_a, V_b, V_c$
@ bus 4



Thevenin $\Rightarrow Z_0 = 0.1 + 0.15 = 0.25 \text{ p.u.}$





$$I_{LG} = \frac{3 + 1.05}{Z_1 + Z_2 + Z_0} =$$

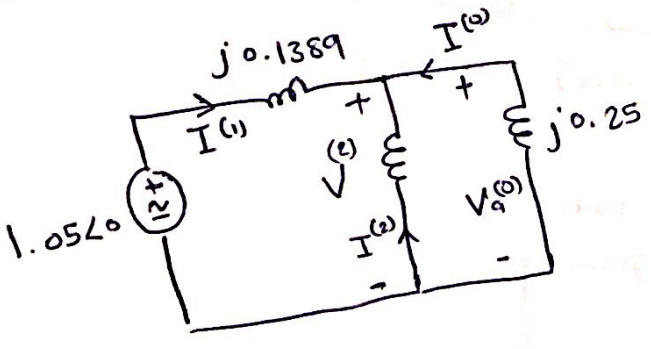
$$I_a^{(0)} = I_a^{(1)} = I_a^{(2)} = -j 1.96427$$

$$3I_a^{(0)} = I_F = -j 5.8928 \text{ p.u.}$$

$$\Rightarrow V_a = V_a^{(0)} + V_a^{(1)} + V_a^{(2)}$$

Final answer:
 $V_a = 0$
 $V_b = 1.179 \angle 231.3^\circ$
 $V_c = 1.179 \angle 128.7^\circ$

\Rightarrow Double line to ground (b-c) \rightarrow



دوبل لائن
 Parallel.
 $V_F = 1.05 \angle 0$

$$\rightarrow I^{(0)} = \frac{1.05 \angle 0}{j [0.1389 + (0.14562 \parallel 0.25)]} = -j 4.5 \text{ p.u.}$$

Current division

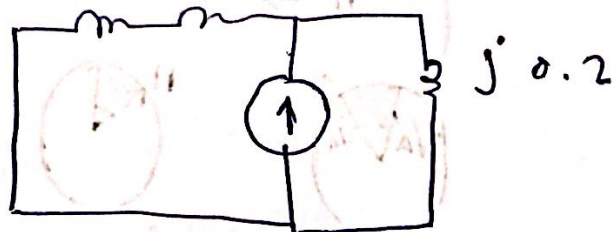
$$\rightarrow I^{(2)} = 4.5j \left(\frac{0.25}{0.25 + 0.14562} \right) = j 2.873 \text{ p.u.}$$

$$\rightarrow I^{(0)} = j 1.6734 \text{ p.u.}$$

$I_a = 0$

$I_b = 6.8983 \angle 158.66 \text{ P.u}$

$I_c = 6.8983 \angle 21.34 \text{ P.u}$

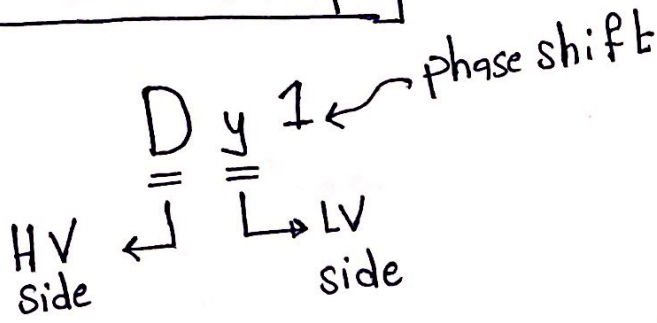


$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 2.123 \angle 90^\circ \\ 20.91 \angle 153^\circ \\ 20.91 \angle 26.83^\circ \end{bmatrix} \text{ KA}$$

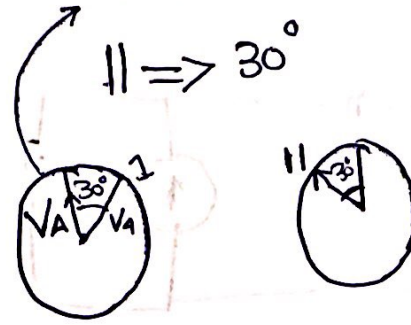
لو بونا نعرف كم لا Motor نخد في ال Fault
 زي قبل بنطبع I_m

Vector Group

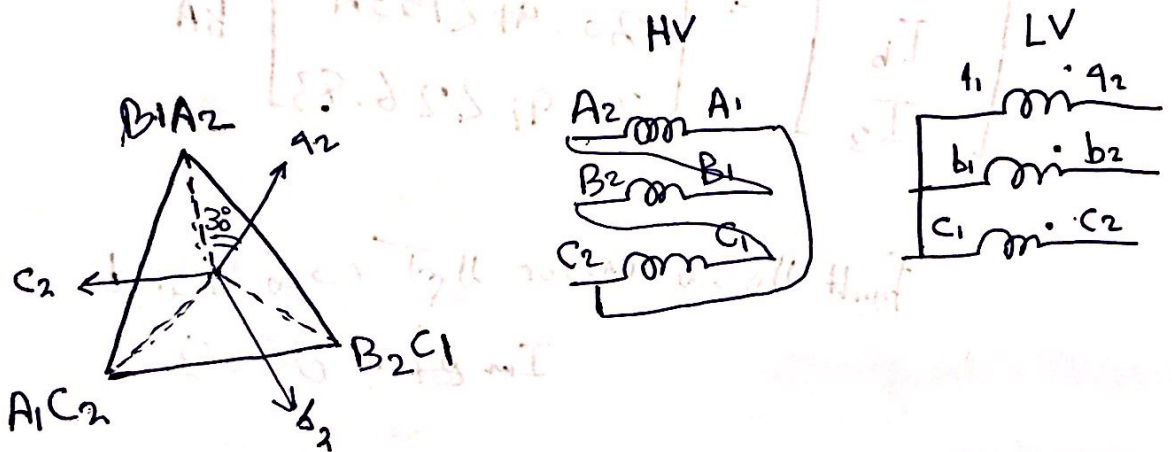
28/12/2016



0 \Rightarrow 0° LV phasor in phase with HV
 1 \Rightarrow 30° HV leads LV by 30°
 11 \Rightarrow 30°



Dy1 \rightarrow Vector diagram
 \rightarrow Winding diagram



$$V_{A2A1} \parallel V_{a2}V_{a1}$$

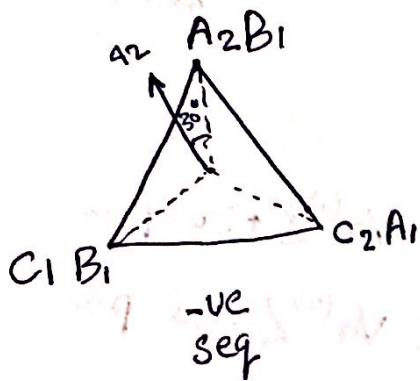
$$V_{B2B1} \parallel V_{b2}V_{b1}$$

Step's:

- 1) Draw Primary side.
- 2) Draw phasor.
- 3) لفظاً HV و LV فوازين لوصف جانب اعرف ال winding

* $V_{A1}^{(1)} = V_{a1}^{(1)} \angle +30^\circ \text{ p.u.}, I_{A1}^{(1)} = I_{a1}^{(1)} \angle 30^\circ \text{ p.u.}$

* $V_{A1}^{(2)} = V_{a1}^{(2)} \angle -30^\circ \text{ p.u.}, I_{A1}^{(2)} = I_{a1}^{(2)} \angle -30^\circ \text{ p.u.}$

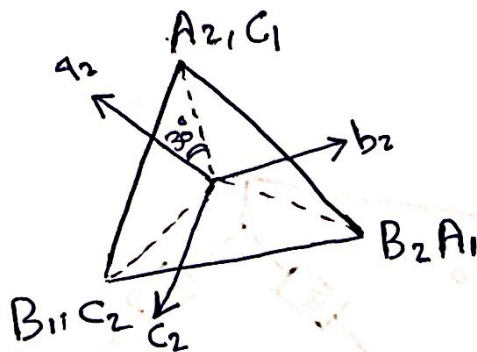


$$V_{A_2A_1} \parallel V_{A_2A_1}$$

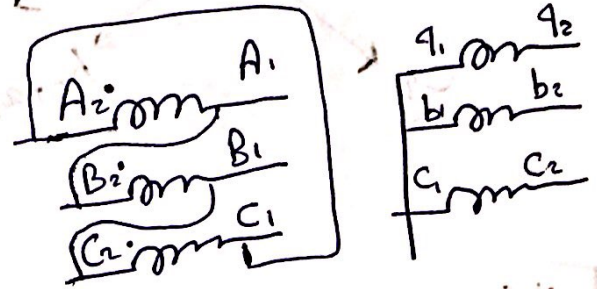
$$V_{A_1}^{(2)} = V_{A_1}^{(2)} \angle -30^\circ \text{ p.u.}$$

Dy11

→ Vector diagram
→ winding



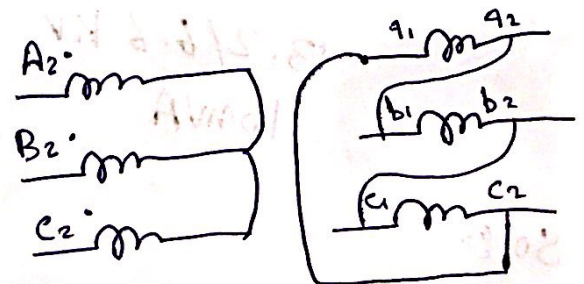
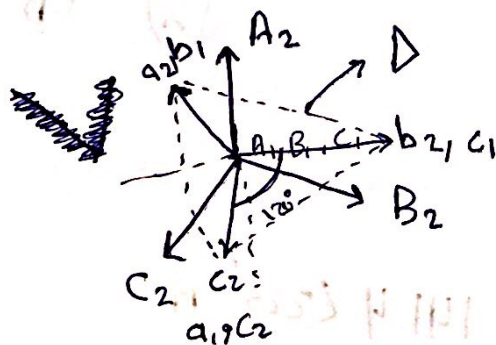
$$V_{A_2A_1} \parallel V_{A_2A_1}$$



$$* V_{A_1}^{(1)} = V_{A_1}^{(1)} \angle -30^\circ \text{ p.u.}$$

$$* V_{A_1}^{(2)} = V_{A_1}^{(2)} \angle +30^\circ \text{ p.u.}$$

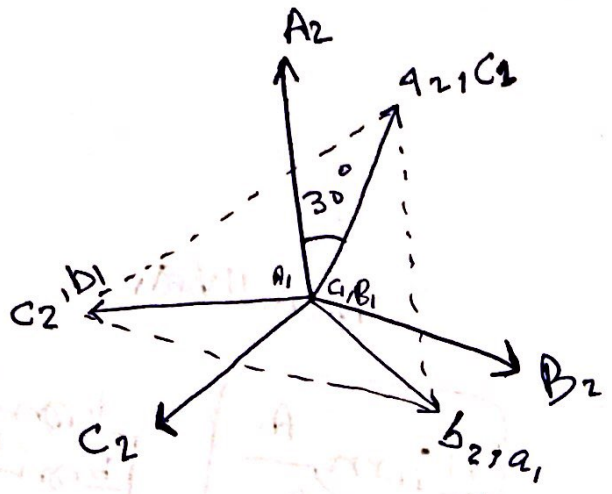
YΔ11
→ Vector diagram
→ winding



$$* V_{A_1}^{(1)} = V_{A_1}^{(1)} \angle -30^\circ \text{ p.u.}$$

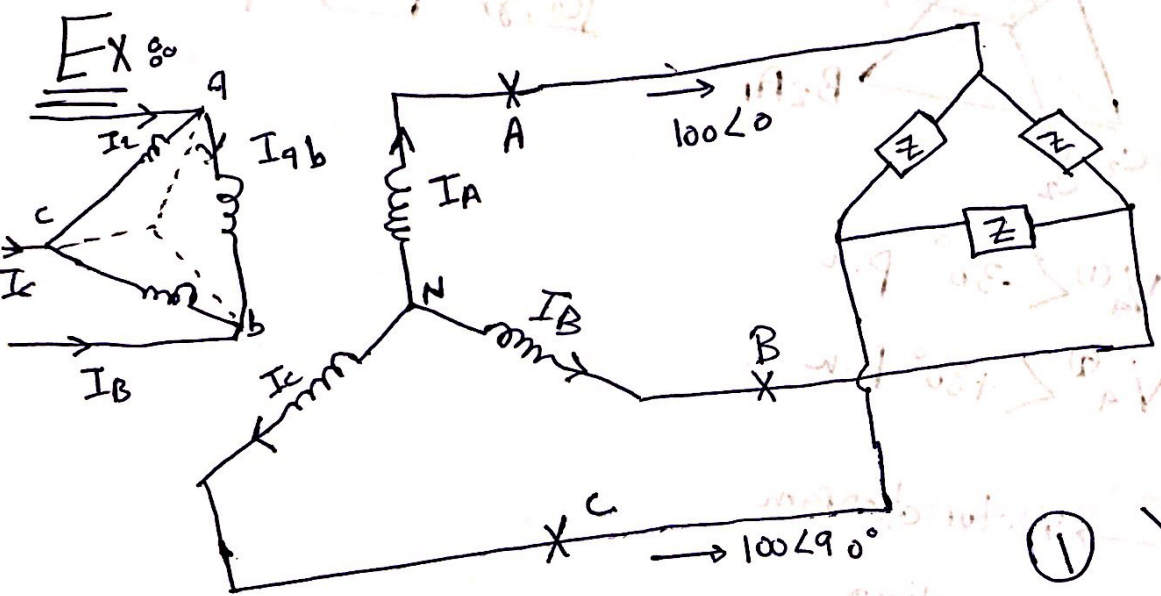
$$* V_{A_1}^{(2)} = V_{A_1}^{(2)} \angle +30^\circ \text{ p.u.}$$

YΔ 1
 → vector diagram
 → winding



$$V_A^{(1)} = V_a^{(1)} \angle +30^\circ \text{ p.u.}$$

$$V_A^{(2)} = V_a^{(2)} \angle -30^\circ \text{ p.u.}$$



13.2/6.6 KV
 10MVA

- ① YΔ 1 ← vector group
- ② IB ??

Solⁿ

$$I_A + I_B + I_C = 0$$

$$I_B = -(I_A + I_C) = 141.4 \angle 225^\circ \text{ A}$$

③ I_{ab} ??

$$I_{ab} = I_A * \left(\frac{66/\sqrt{3}}{13.2} \right) = 289 \angle 0^\circ \text{ A}$$

④ $I_{ca} ??$

$$I_{ca} = 100 \angle 90^\circ * \left(\frac{66/\sqrt{3}}{13.2} \right) = 289 \angle 90^\circ \text{ A}$$

⑤ $I_A = I_{ab} - I_{ca} = 408.71 \angle -45^\circ \text{ A}$

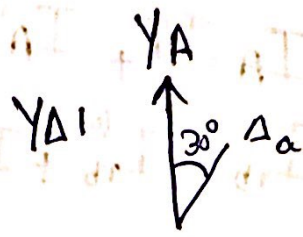
HV

$$I_A^{(0)} = 0$$

$$I_A^{(1)} = 111.5 \angle -15^\circ$$

$$I_A^{(2)} = 29.9 \angle 105^\circ$$

From matrix



LV

$$I_a^{(1)} = I_A^{(1)} \angle -30^\circ \text{ p.u.}$$

$$I_a^{(2)} = I_A^{(2)} \angle +30^\circ \text{ p.u.}$$

$$I_a^{(0)} = 0 \leftarrow \Delta$$

$$I_a^{(1)} = \frac{111.5 \angle -15^\circ}{\frac{10\text{M}}{\sqrt{3} * 66}} \angle -30^\circ = 1.274 \angle -45^\circ \text{ p.u.}$$

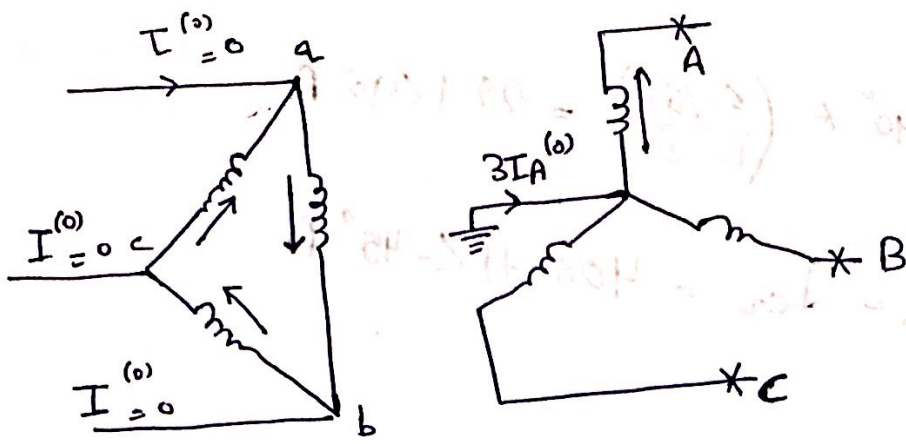
$$I_a^{(2)} = \frac{29.9 \angle 105^\circ}{\frac{10\text{M}}{\sqrt{3} * 66}} \angle +30^\circ = 0.342 \angle 135^\circ \text{ p.u.}$$

$$I_a = I_a^{(0)} + I_a^{(1)} + I_a^{(2)} = 0.932 \angle -45^\circ \text{ p.u.}$$

$$I_a = 0.932 \angle -45^\circ * \frac{10\text{M}}{\sqrt{3} * 13.2\text{K}} = 408.71 \angle -45^\circ$$

Ex 200

28/12/2016



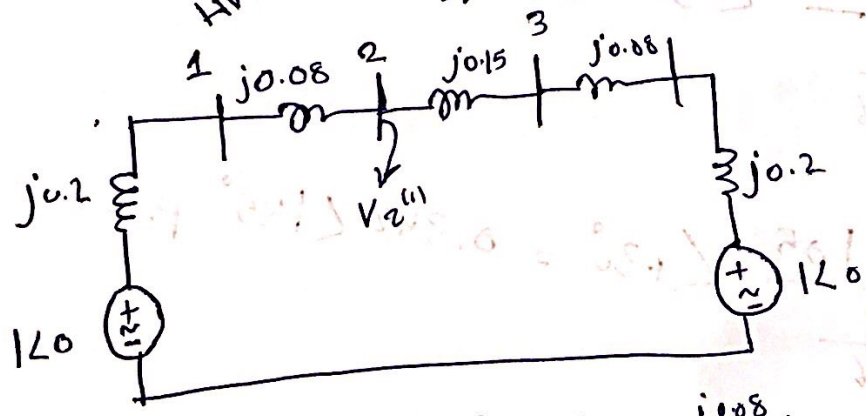
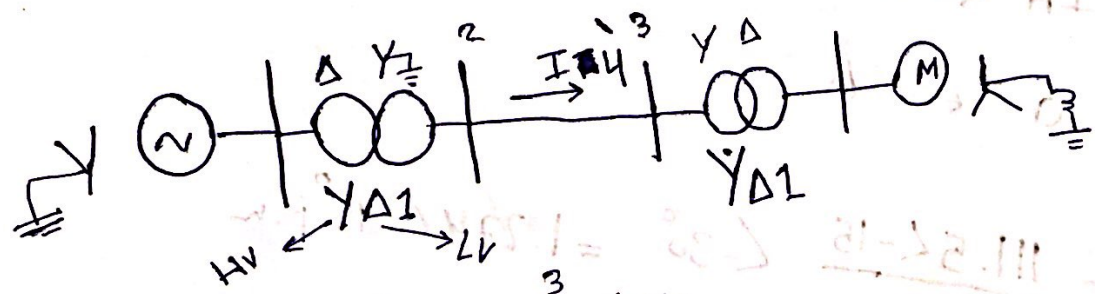
$$I_A = I_A^{(0)} + I_A^{(1)} + I_A^{(2)}$$

$$I_{ab} = I_{ab}^{(0)} + I_{ab}^{(1)} + I_{ab}^{(2)}$$

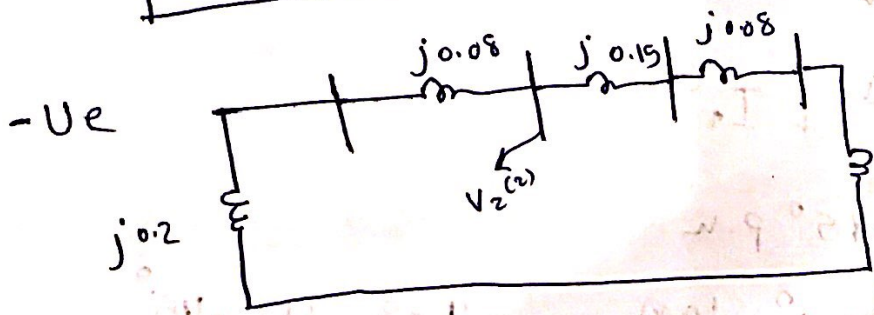
$$I_{ab}^{(0)} = I_A^{(0)} \times (\text{Turns ratio})$$

magnitude of I_f
مقدار I_f

Ex 200



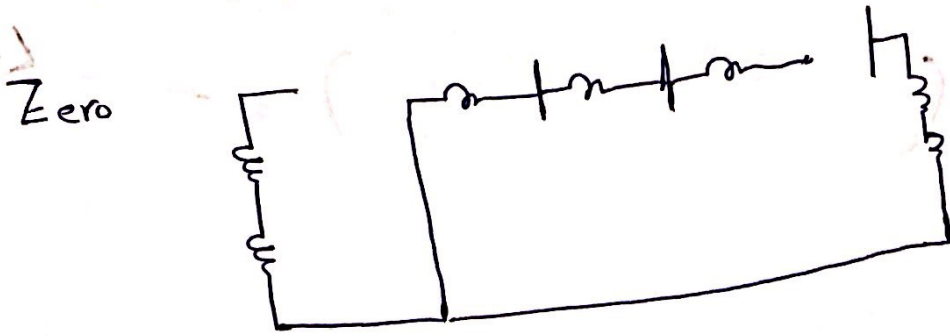
$$Z_1 = 0.1437 \text{ p.u.}$$



$$Z_2 = 0.1437 \text{ p.u.}$$

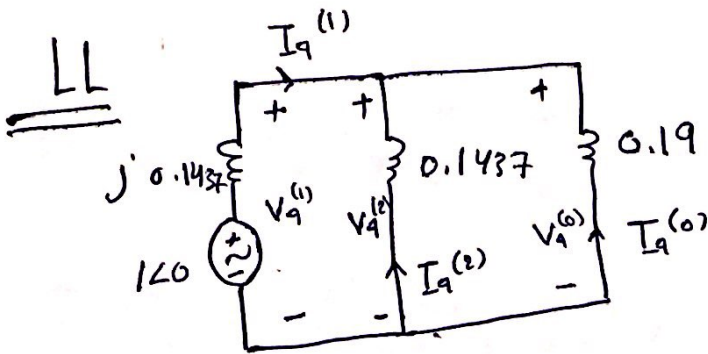
Q LL-G @ bus 4, unloaed system
 $b \rightarrow c \rightarrow E$

28/12/2016



$$Z_0 = 0.19 \text{ p.u.}$$

Zero seq current = Zero



$$I_a^{(1)} = -j4.3$$

$$I_a^{(2)} = j2.5$$

$$I_a^{(0)} = j1.9$$

$$I_a = \text{Zero}$$

$$I_b = 6.726 \angle 154.6^\circ$$

$$I_c = 6.726 \angle 25^\circ$$

$$\underline{V_4} \Rightarrow V_a^{(1)} = V_a^{(2)} = V_a^{(0)}$$

$$V_a^{(1)} = 120 - I_a^{(1)} \times j0.1437 = 0.3628 \text{ p.u.}$$

$$V_a^{(2)} = 0$$

$$V_a^{(0)} = 0$$

$$V_{4a} = 3 \times 0.3628 \text{ p.u.} = 3 V_a^{(1)}$$

$$V_{4b} = 0$$

$$V_{4c} = 0$$

③ I_{44} (HV side Tx)

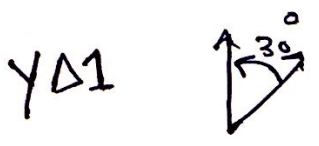
$$I_{44} \text{ (LV side Tx)} \Rightarrow I_{44}^{(1)} = -j4.4$$

$$I_{44}^{(2)} = -j1.2$$

$$I_4^{(1)} = -j4.4 * \left(\frac{j0.2}{j0.2 + j0.08 + j0.15 + j0.08 + j0.2} \right) = -j1.249 \angle +30^\circ$$

$$I_4^{(2)} = -j1.2 * \left(\right) = \text{---} \angle -30^\circ$$

$$I_4^{(0)} = 0$$



- $I_{4a}' \checkmark$
- $I_{4b}' \checkmark$
- $I_{4c}' \checkmark$

V_2 ??
 ↓
 vector group

$$V_{2a}^{(1)} = \angle 30^\circ - I_4^{(1)} (j0.2 + j0.08)$$

$$V_{2a}^{(2)} = \text{[scribble]}$$

⇒ Balanced fault fault @ bus n

1) Balanced fault =

$$I_n^{(1)} = \frac{V_F}{Z_{nn}^{(1)}} \leftarrow \text{Thevenin}$$

2) LG

$$I_n^{(2)} = I_n^{(0)} = I_n^{(1)} = \frac{V_F}{Z_{nn}^{(1)} + Z_{nn}^{(1)} + Z_{nn}^{(2)}}$$

line to line fault

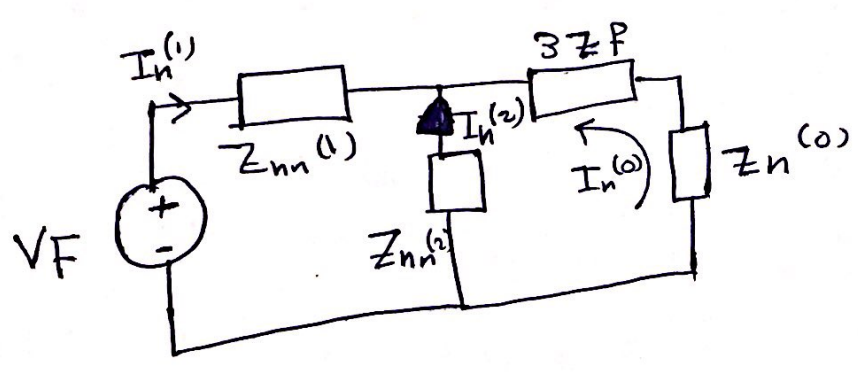
$$I_n^{(1)} = -I_n^{(2)} = \frac{V_F}{Z_{nn}^{(1)} + Z_{nn}^{(2)} + Z_F}$$

LLG

$$I_n^{(1)} = \frac{V_F}{Z_{nn}^{(1)} + (Z_{nn}^{(2)} \parallel Z_{nn}^{(0)} + 3Z_F)}$$

$$I_n^{(2)} = -I_n^{(1)} * \left[\frac{Z_{nn}^{(0)} + 3Z_F}{Z_{nn}^{(0)} + 3Z_F + Z_{nn}^{(2)}} \right]$$

$$I_n^{(0)} = - (I_n^{(1)} + I_n^{(2)})$$



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