2&1 WEEK

Power 1 Notebook

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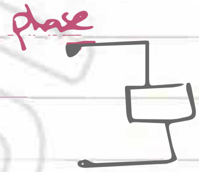
Monday
28/9

* Basic Concept (per unit revision) :-

in the solution of the P.S. problems the per phase ckt is used.



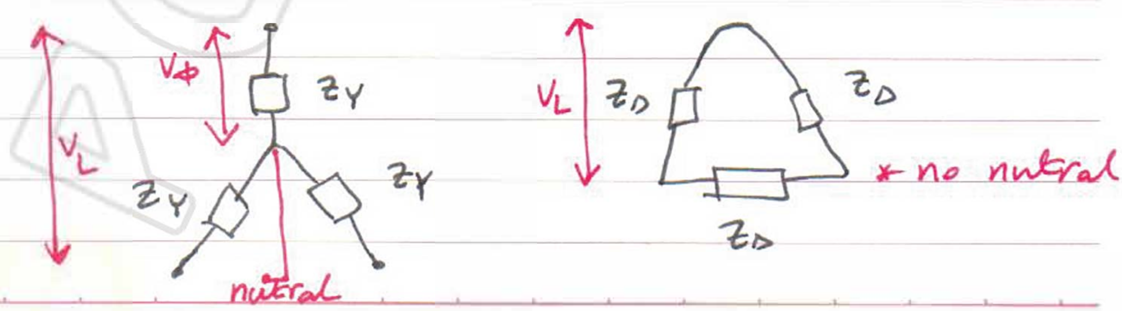
$$Z_b = \frac{V_b^2}{S_b}$$

$$\Delta Z_b = \frac{(V_b/\sqrt{3})^2}{S_{3\phi}/3}$$

* usually in the specification of power systems lines voltages and 3ϕ apparent power are given.

$$Z_b = \left(\frac{V_L^2}{S_{3\phi}}\right) = \frac{(V_\phi/\sqrt{3})^2}{S_{3\phi}/3}$$

→ this is the express of Z_b irrespective of the type of connection

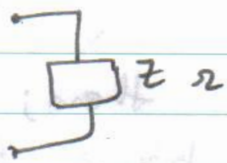


Power1

Wed 30/9

* The given per unit value of the impedence of the power system components (i.e : generator, transformer, lines & loads) are usually based on the ratings of such components ; however, to solve a given power system, then a common reference value should be used. Consequently all the given per unit values should be updated according to this common reference, as follows :-

* if one select a reference value say Z_{b1} Ω



$$Z_1 (pu) = \frac{Z}{Z_{b1}}$$

* if the base value is Z_{b2} Ω

$$\therefore Z_2 (pu) = \frac{Z}{Z_{b2}}$$

$$\frac{Z_2\ (pu)}{Z_1\ (pu)} = \frac{Z}{Z_{b2}} * \frac{Z_{b1}}{Z} = \frac{Z_{b1}}{Z_{b2}}$$

$$= \frac{(V_{b1})^2}{S_{b1}} \cdot \frac{S_{b2}}{(V_{b2})^2}$$

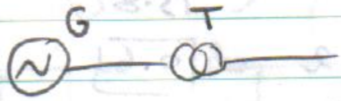
$$= \left(\frac{V_{b1}}{V_{b2}}\right)^2 * \frac{S_{b2}}{S_{b1}}$$

* if 1 represent old value
  2 represent new value

then;

$$Z_{new}\ (pu) = Z_{old}\ (pu) * \left(\frac{V_{b\ old}}{V_{b\ new}}\right)^2 \cdot \left(\frac{S_{b\ new}}{S_{b\ old}}\right)$$

* illustration:



13.8 kv
50 MVA
$x = 30\%$
$= 0.3$ pu

(13.8 / 132) kv
50 MVA
$x = 5\%$

→ equivalent reactance of the transformer $= 0.05$ pu

* irrespective of the reflection side

* Usually in power station G is a 3-phase synch. generator. When generated voltage $= 13.8$ kv
synch. reactance $= x = 0.3$ pu

* If this system is to be solved by using base value of (13.8 kv) and (100 MVA), then update the per unit values if necessary as follows:

→

G : $X_{new} = \cancel{X_{old}}^{0.3} * \left(\frac{13.8}{13.8}\right)^2 * \frac{100}{50}$

$= 0.3 * 1 * 2 = \boxed{0.6}$

T : $X_{new} = 0.05 * \left(\frac{13.8}{13.8}\right)^2 * \left(\frac{100}{50}\right) = \boxed{0.1}$

* Power system Representation :-

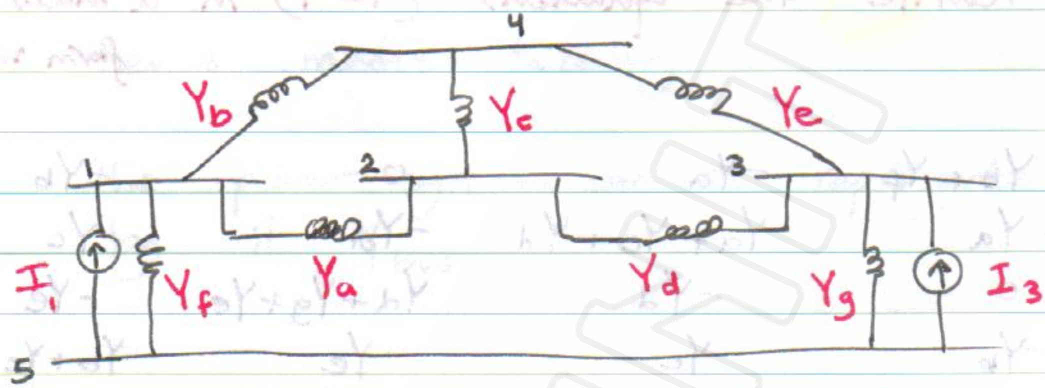
There are two basic approaches :

1) mathematical rep.
2) graphical rep.

[1] mathematical :

this is based on the concept of the node equations, as follows :-

$\rightarrow$



* Apply KCL @ each ~~node~~ node :

(node 1) : $(V_1 - V_2) Y_a + (V_1 - V_4) Y_b + V_1 Y_f = I_1$

$$V_1 (Y_a + Y_b + Y_f) - V_2 Y_a - Y_b V_4 = I_1 \quad \text{---} 1$$

* by similar approach at other nodes
it can be found :-

(node 2) : $-V Y_a + V_2 (Y_a + Y_c + Y_d)$

$$-V_3 Y_d - V_4 Y_c = 0 \quad \text{---} 2$$

(node 3) : $-V_2 Y_d + (Y_d + Y_G + Y_e) V_3$

$$- V_4 Y_e = I_3 \quad \text{----} 3$$

(node 4) : $-V_1 Y_b - V_2 Y_c - V_3 Y_e + V_4 (Y_b + Y_c + Y_e)$

$$= 0 \quad 4$$

* rewrite the equations (1–4) in a matrix form :-

$$\begin{bmatrix} Y_a+Y_b+Y_f & -Y_a & 0 & -Y_b \\ -Y_a & Y_a+Y_c+Y_d & -Y_d & -Y_c \\ 0 & -Y_d & Y_d+Y_g+Y_e & -Y_e \\ -Y_b & -Y_c & -Y_e & Y_b+Y_e+Y_c \end{bmatrix} \quad (*)$$

$$\begin{bmatrix} * \end{bmatrix} * \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} I_0 \\ 0 \\ I_3 \\ 0 \end{bmatrix}$$

(5) can be written :

$$[Y][V] = [I]$$

- $[Y]$: this is called bus admittance matrix written Y_{bus}
- $[V]$: nodal or bus voltages
- $[I]$: the current source entering nodes or sometimes called the current injected to the nodes

where :

i) its element $Y_{ii} = \Sigma$ of admittances conected directly to the (i^{th}) bus

ii) its element $Y_{ij} = -1 *$ equivalent admittance between the (i^{th}) and (j^{th}) buses

* hence mathematically:

the power system can be represented by its Y_{bus}

note : $[Y_{bus}]^{-1} \triangleq Z_{bus}$

where Z_{bus} is called Bus impedence matrix $= \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1n} \\ \vdots & & & \\ Z_{n1} & Z_{n2} & \cdots & Z_{nn} \end{bmatrix}$

Note : to find Z_{bus} :

i) find Y_{bus}

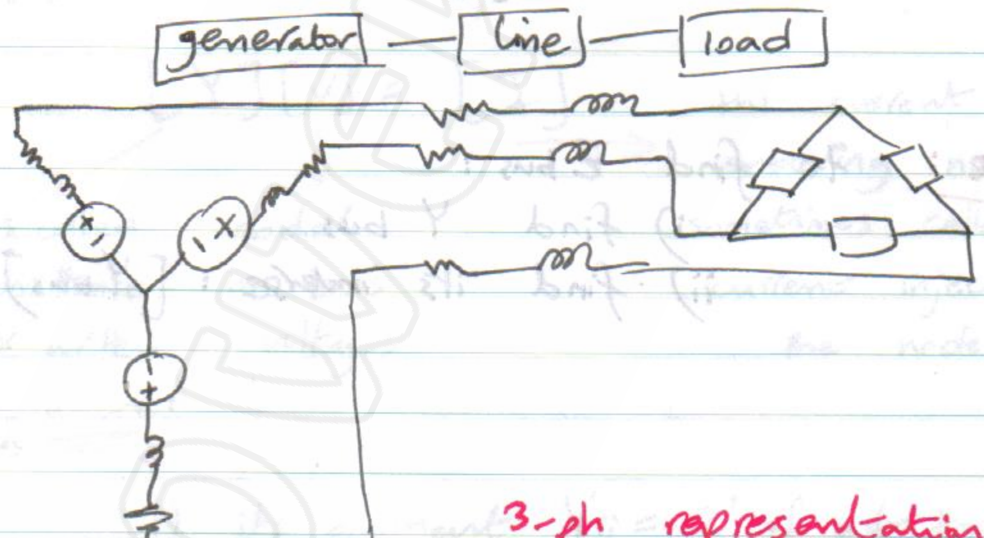
ii) find its inverse : $[Y_{bus}]^{-1}$

* given the Y_{bus} matrix, find the corresponding equ. network:

* [2] Graphical representation :-

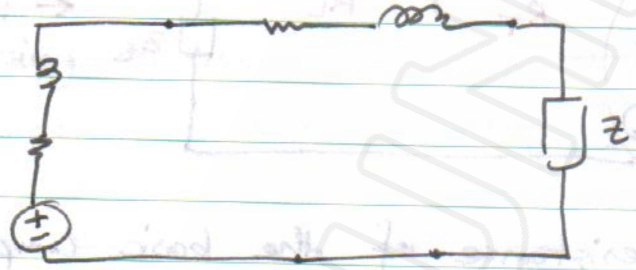
this is represented by the ~~so~~ called one-line or single-line diagram, defined and deduced as follows:-

* Consider the following sys. :-



3-ph representation

* if the system is balanced, then the perphase representation can be used



→ this is also called impedence diagram.

* if the resistance of the elements are neglected then it is called reactance diagram

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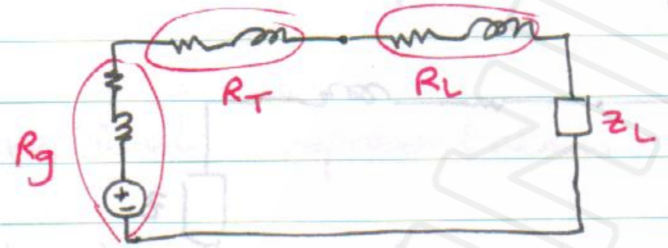
* gen — Tran — line — load

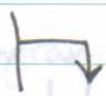
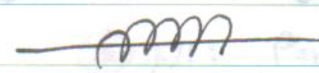


← impedence diagram

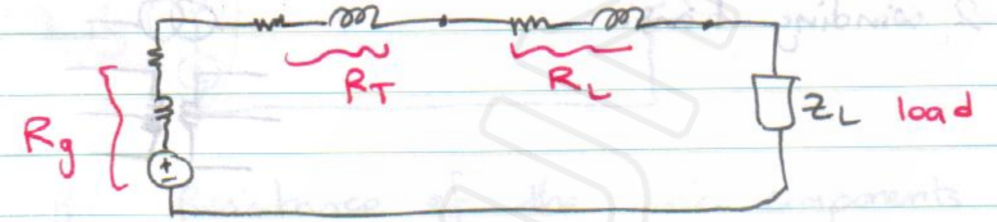
* if the Resistance of the basic components are neglected, then the diagram is called "reactance diagram".

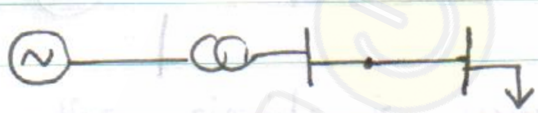
note: this circuit is usually used in power system analysis

now if one is only interested to see how the components of a given power sys. are interconnected, then these components are represented by the standard symbol instead of its equivalent ckt, the resultant diagram called single-line or one-line diagram

* Component	Symbol
rotating machinary	 ~ : generator M : motor
2 winding trans.	
Transmission line	
Busbar (node)	
load	
ckt breaker	 or 
isolator	
Current transformer	

hence, for our system, the single line diagram as follows :

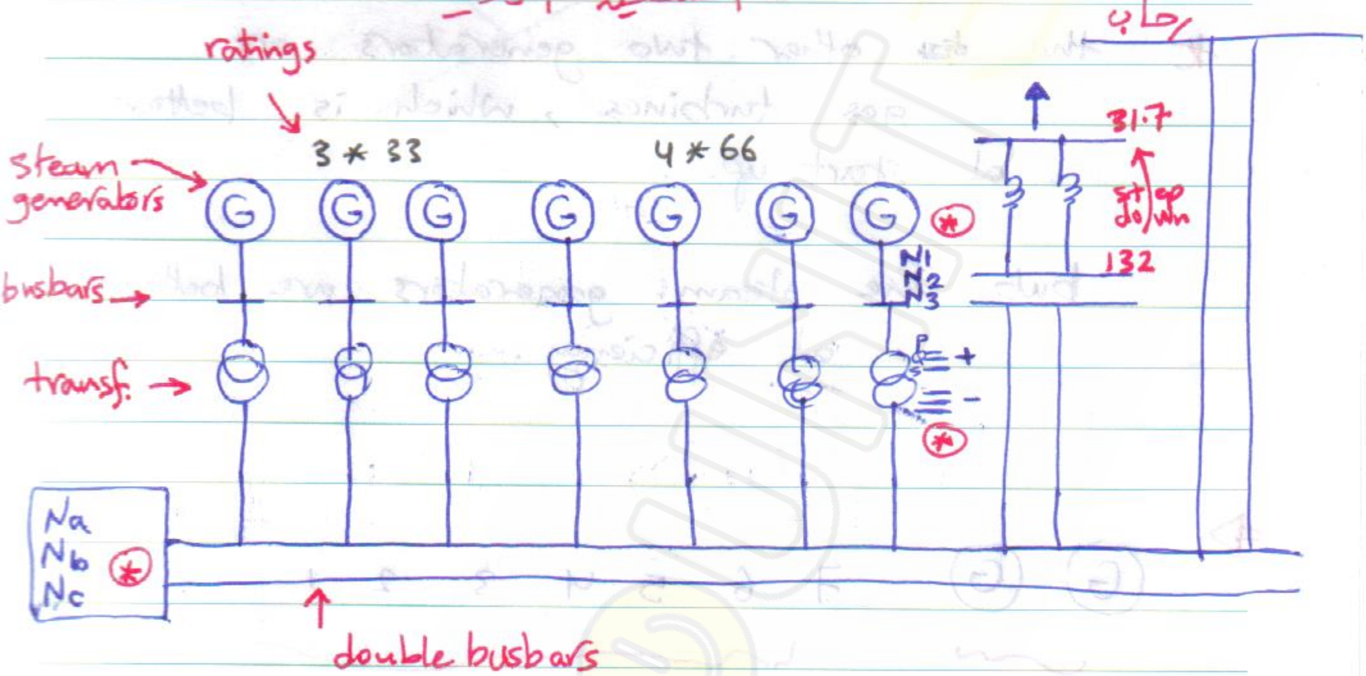




* given data on the single line diagram depend on the required analysis or Application

* As a practical example, consider the Jordanian electric single-line-diagram

* الحسية الحرارية



* the data on the diagram are the results of a given load flow analysis.

* load flow : is to calculate voltages at busbars and power flow in the system for a given load conditions.

* Na : voltage in kV
Nb : voltage in per unit
Nc : voltage in angle

* N1 : Power (MW)
N2 : reactive Q (MVAR)
N3 : apparent S (MVA)

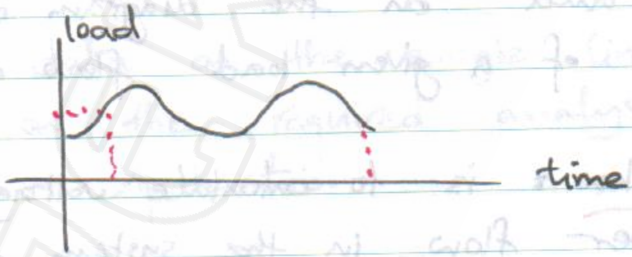
* when we add + & − values it will give the losses

* the other two generators are gas turbines, which is better at start up.

but the steam generators are better at efficiency.

⇒ G G 7 6 5 4 3 2 1

gas steam.

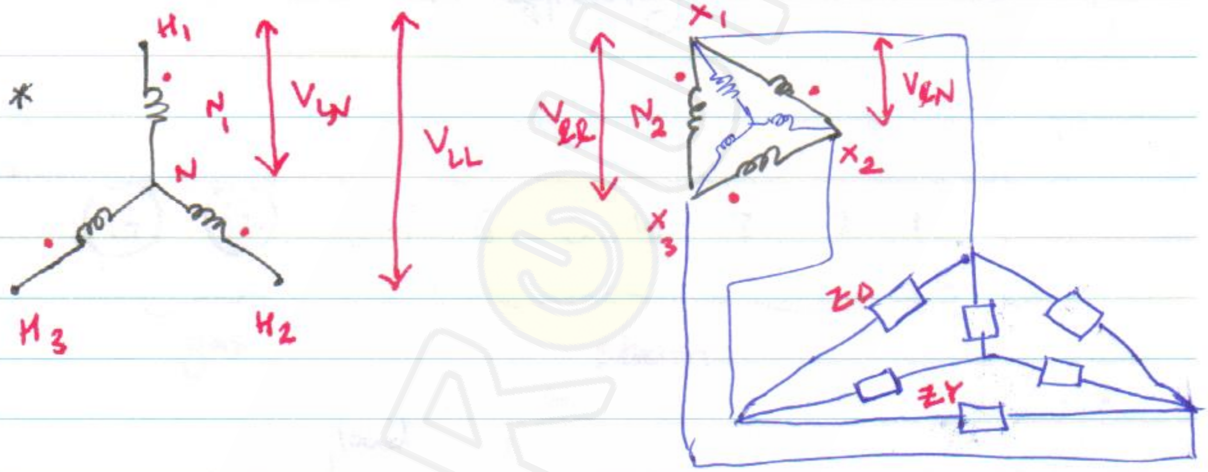


* $\Sigma P_{in} = \Sigma P_{out}$

Ex: Draw the single-line diagram of a given subsystem ?

* Consider a Y-Δ Transformation (Yd):

N_1 and N_2 are the number of turns at the HV and LV winding, which have voltage in phase.

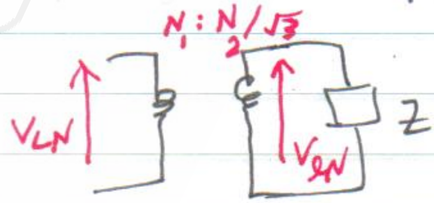


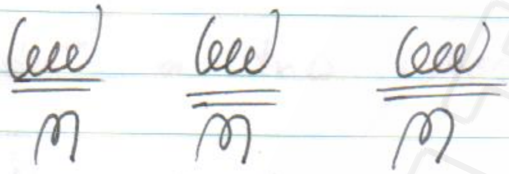
* from figure * :

$$\frac{|V_{LN}|}{|V_{ll}|} = \frac{N_1}{N_2} \quad ---- (1)$$

$$\frac{V_{LN}}{\sqrt{3}\, V_{lN}} = \frac{N_1}{N_2} \;,\quad \frac{|V_{LN}|}{|V_{ln}|} = \frac{N_1}{N_2/\sqrt{3}} \quad ---- (2)$$

from 2: deduce the equ. per-phase ckt:

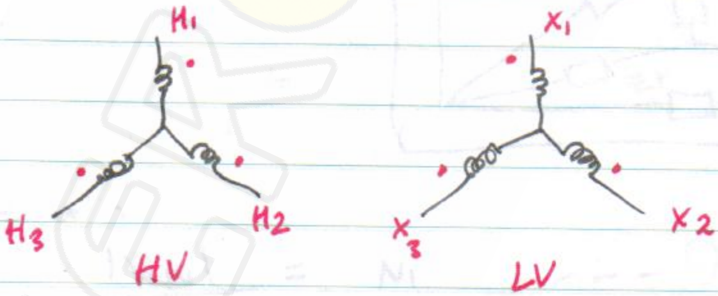




* Illustration:

consider a Y-Y connection among the rep. is that the letters H_1, H_2, H_3 used for high voltage side

and X_1, X_2, X_3 for low voltage.



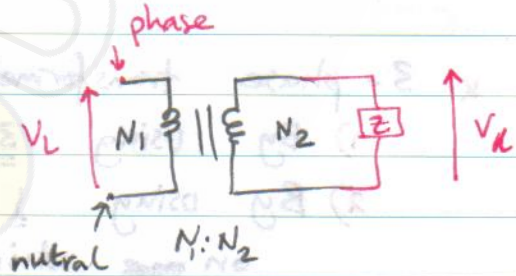
sometimes the letters A, B, C used for HV and a, b, c used for LV

* Concept of parameters Refliction in 3 phase Tx

It was found that the Refliction in the single phase Tx is as follows :-

→ N_1 , N_2 are the actual # at turns ratio.

a: b the turns ratio



$a = \frac{N_1}{N_2} : 1$

* It was found that :-

1) to reflect Z to (a) side * it by a^2
2) " " V to (a) side * it by a
3) " " I to (a) side ÷ it by a

* 3 - phase transformer :-

The objective is to find its equivalent per phase ckt for any type or connection

* Revision and Basic concept :-

* 3 - phase transformer can be constructed by :
  1) By using $\underline{\underline{3}}$ single phase Tx.
  2) By using $\underline{\underline{1}}$ single ~~phase~~ core on ~~one~~ which there are 3 pairs of windings.

* The winding can be connected as:
  Y-Y , Y-Δ , Δ-Y , Δ-Δ

* in the Y-Y or Δ-Δ the phase voltages of the HV and LV side one in phase. However in the Y-Δ or Δ-Y there is a phase shift between the phase voltages

* The winding or the core which are linked by the same flux linkage there induced voltages in phase.
(useful in graphical rep.)

* Consequently in ghraphical ~~core~~ core schematic rep. such coils are drown parallel to each other with a dot is located at one end of each coil or the coil at one phase are drown parallel to each other.

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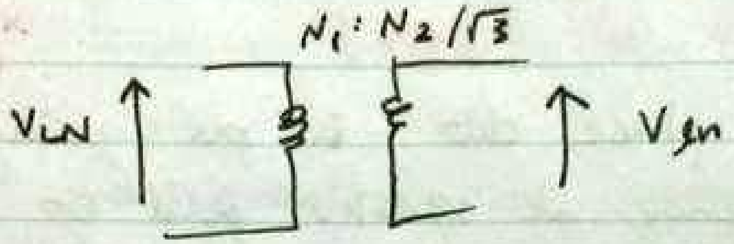
HV ← $\dfrac{V_{LN}}{V_{ln}} = \dfrac{N_1}{N_2/\sqrt{3}}$ → effective turns ratio
LV ←



(fig1)

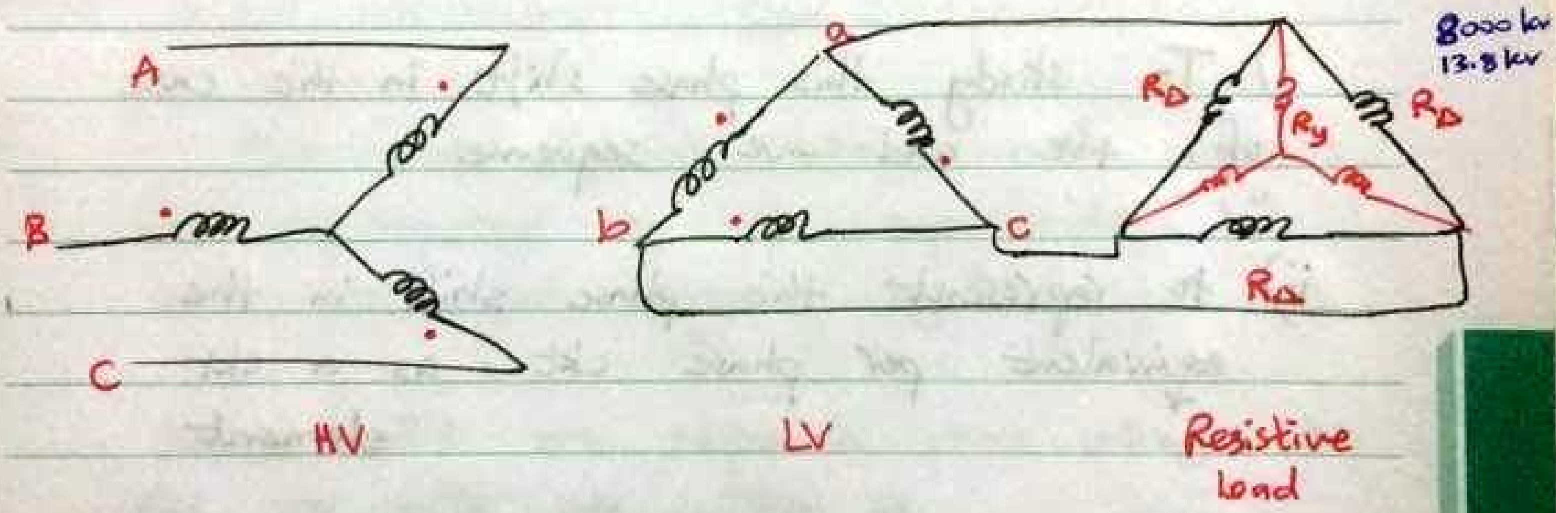
for all type of connections ; although the effective turns ratio is different , but it is also equal to line voltage ratio, hence (fig 1) can be used to reflect low voltage impedence (Z_L) to the high voltage impedence (Z_H) or the other ways round:

$$Z_H = \left(\frac{V_{LL}}{V_{ll}}\right)^2 Z_L$$

or

$$Z_L = \left(\frac{V_{ll}}{V_{LL}}\right)^2 Z_H$$

* ~~e.g~~ e.g.: A balanced Δ - connected resistive load of 8000 kW is supplied by a low voltage Δ connected side of a Y-Δ transformer rated at 10^4 kVA, 138 / 13.8 kV.
find the load resistance in ohms seen between the phase and neutral on the HV side assume rated voltage is supplied to primary.



$\Rightarrow$ $R_\Delta = ?$, $P = V^2/R$

$$\rightarrow R_\Delta = \frac{V^2}{P} = \frac{(13.8 \times 10^3)^2}{\frac{8000 \times 10^3}{3}} = 71.415\ \Omega$$

$$R_Y \triangleq \frac{R_\Delta}{3} = \frac{71.415}{3} = 23.805\ \Omega$$

* Requirement: $R_H = \left(\frac{V_{LL}}{V_{\ell\ell}}\right)^2 * R_L \leftarrow R_Y$

$$R_H = \left(\frac{138}{13.8}\right)^2 * 23.805 = 2380.5\ \Omega$$

* Phase Shift in 3-phase transformer :-

As stated before Y-Δ or Δ-Y introduce phase shift between the corresponding voltages on the HV and LV sides.

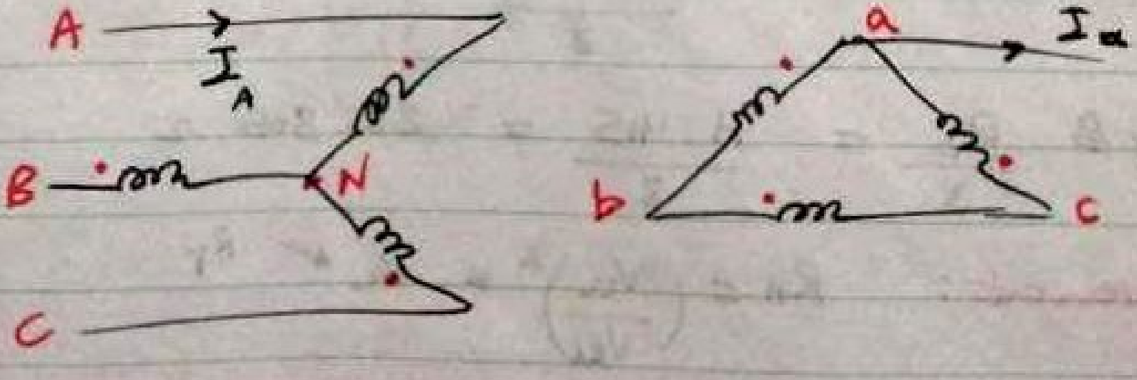
* Objectives :

i) To study this phase shift in the case of +ve and -ve sequence

ii) to represent this phase shift in the equivalent per phase ckt. as a ckt element

* Procedure :

consider a Yd 3 phase transformer



Now:

V_{AN} is in phase with V_{ab}
V_{BN} - - - - - V_{bc}
V_{CN} - - - - - V_{ca}

let: N_1 and N_2 are the number of turns of the HV and LV winding of the corresponding inphase winding.

* Convention :-

The +ve sequence voltages and currents are represented by the superscript "1" and the -ve sequence by "2"

hence, the +ve sequence phase voltages on the HV side are written as :

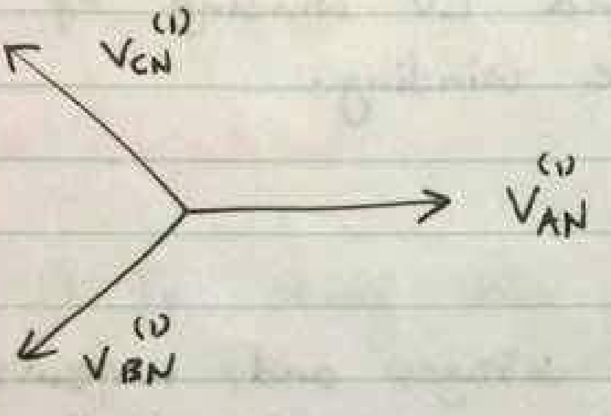
$$V_{AN}^{(1)} , V_{BN}^{(1)} , V_{CN}^{(1)}$$

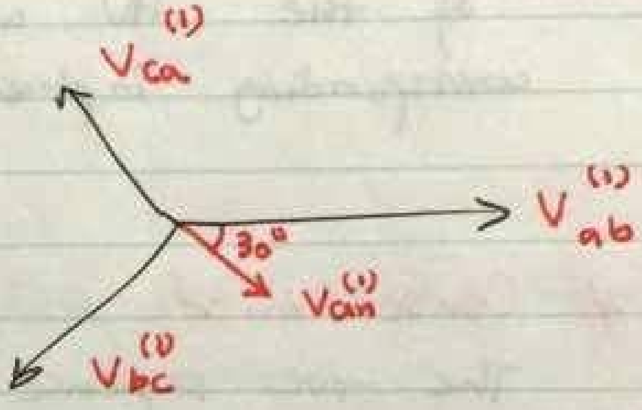
→ sometimes this is simplified to :

$$V_{A}^{(1)} , V_{B}^{(1)} , V_{C}^{(1)}$$

* Let us solve the problem by using the concept of phasor diagram:

+ve sequence:

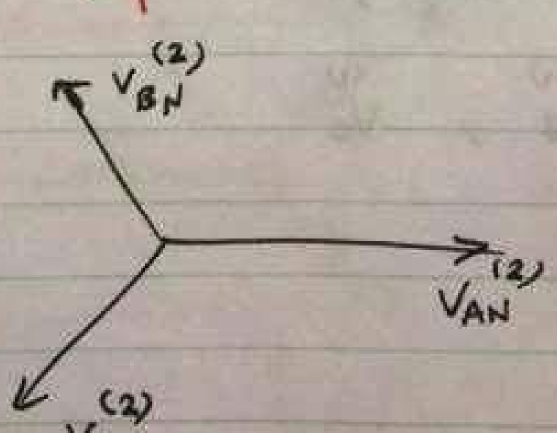


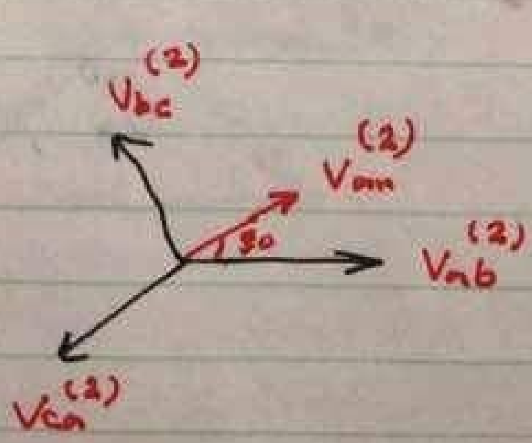


* $$V_{AN}^{(1)} = \frac{N_1}{N_2/\sqrt{3}} V_{an}^{(1)} * 1\angle 30^\circ$$

$$V_{AN}^{(1)} = \frac{N_1}{N_2/\sqrt{3}} V_{an}^{(1)} \angle 30 \quad \text{------} (1)$$

-ve sequence:-





$$* \quad V_{AN}^{(2)} = \frac{N_1}{N_2/\sqrt{3}} V_{an}^{(2)} * 1\angle{-30}$$

$$V_{AN}^{(2)} = \frac{N_1}{N_2/\sqrt{3}} V_{an}^{(2)} \angle{-30} \quad ---- \ (2)$$

* Comments :-

i) in the +ve phase sequence HV quantities (i.e: voltages and currents) lead the corresponding LV V quantities by 30°, where in the -ve phase sequence, the HV quantities lags LV by 30°.

ii) since it was found that :

$$\frac{|V_{LL}|}{|V_{ll}|} = \frac{N_1}{N_2/\sqrt{3}}$$

∴ (1) and (2) can be written :-

$$\frac{V_{AN}^{(1)}}{|V_{LL}|} = \frac{V_{an}^{(1)}}{|V_{ll}|} \angle 30 \quad ---- \ (3)$$

$$\frac{V_{AN}^{(2)}}{|V_{LL}|} = \frac{V_{an}^{(2)}}{|V_{ll}|} \angle{-30} \quad ---- \ (4)$$

* if $|V_{LL}|$ and $|V_{ll}|$ are the rated voltages and used as base values, then :

3 → $$V_{AN}^{(1)} (pu) = V_{an}^{(1)} (pu) \angle 30 \quad \text{----} (5)$$

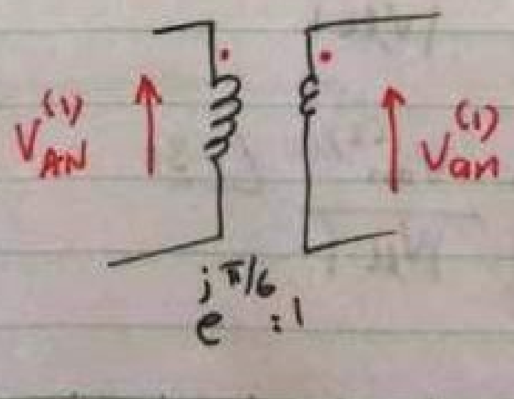
4 → $$V_{AN}^{(2)} (pu) = V_{an}^{(2)} (pu) \angle -30 \quad \text{----} (6)$$

hence, in (pu) the (HV) and (LV) quantities have the same magnitude

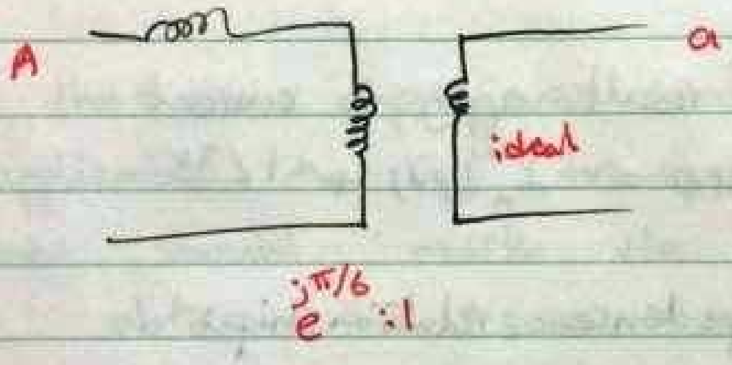
iii) in normal operation, one use +ve sequence:

∴ 5 → $$\frac{V_{AN}^{(1)} (pu)}{V_{an}^{(1)} (pu)} = 1\angle 30 = \frac{e^{j\pi/6}}{1} \quad \text{----} (7)$$

graphically as a ckt diagram or element (7) can be represented as:



∴ phase shift in Y–Δ or Δ–Y is represented by ideal transformer with a complex turns ratio



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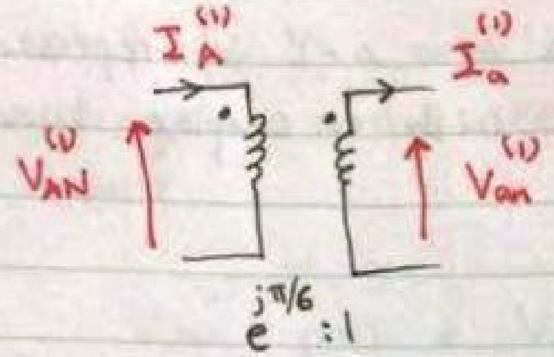


* Comments :-

i) The same result apply to current

$$I_A^{(1)}(pu) = I_a^{(1)}(pu) + 1\angle 30$$

ii) hence, Impedence relationship:

$$\frac{V_{AN}^{(1)}}{I_A^{(1)}} = \frac{V_{an}^{(1)} * 1\angle 30}{I_a^{(1)} * 1\angle 30}$$

$$Z_{HV}\ (pu) = Z_{LV}\ (pu)$$

iii) power relationship:

$$V_{AN}^{(1)} * (I_A^{(1)})^* = (V_{an}^{(1)} \angle 30) * (I_a^{(1)} \angle 30)$$

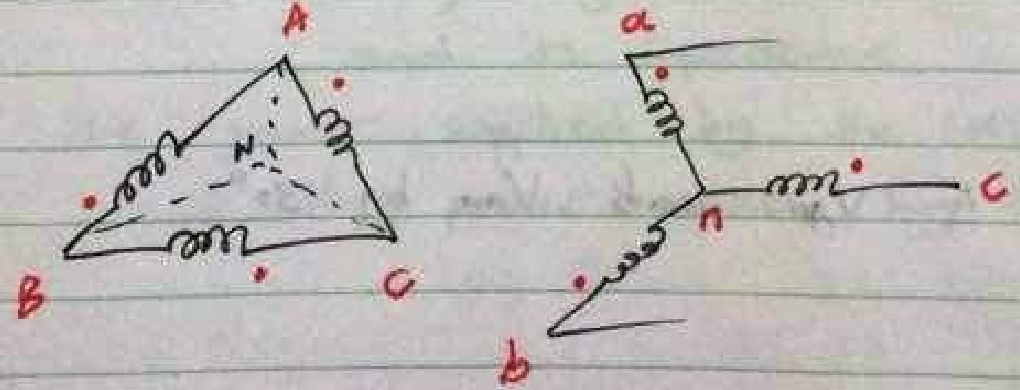
$$V_{AN}^{(1)} (I_A^{(1)})^* = V_{an}^{(1)} I_a^{(1)*}$$

→ If one is only interested in power calculations then there is no need to represent Y-D or D-Y by an Ideal transformer.

iv) in general as far as analysis concern, then one may not include the ideal transformer in the (pu) perphase ckt. But should take into account the phase shift of 30°, when moving from HV ⇄ LV

v) the same previous results of Yd are applicable to Dy transformer, However one should mark the terminals to obtain the same results

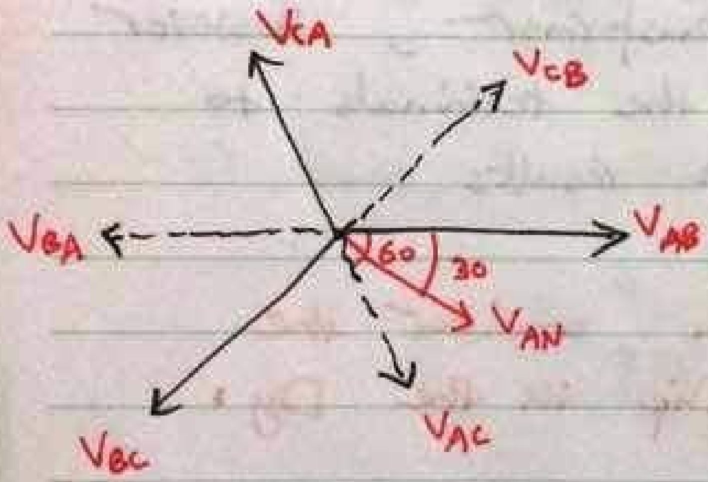
* As an illustration, consider the voltage relationship in the Dy:

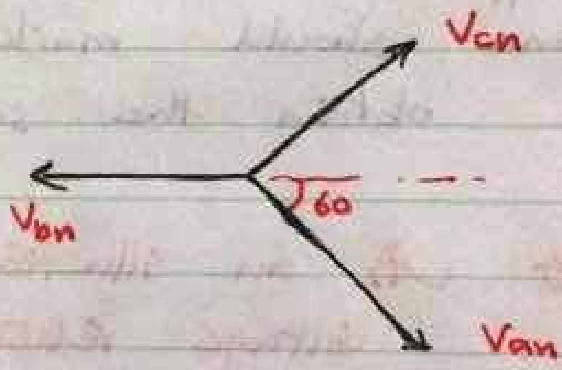


* from the schematic diagram :

V_{AC} is in phase with V_{an}
V_{CB} — — — — V_{cn}
V_{BA} — — — — V_{bn}

* by using the power diagram for +ve phase sequence





HV side LV side

$\therefore$ V_{AN} leads V_{an} by $30°$

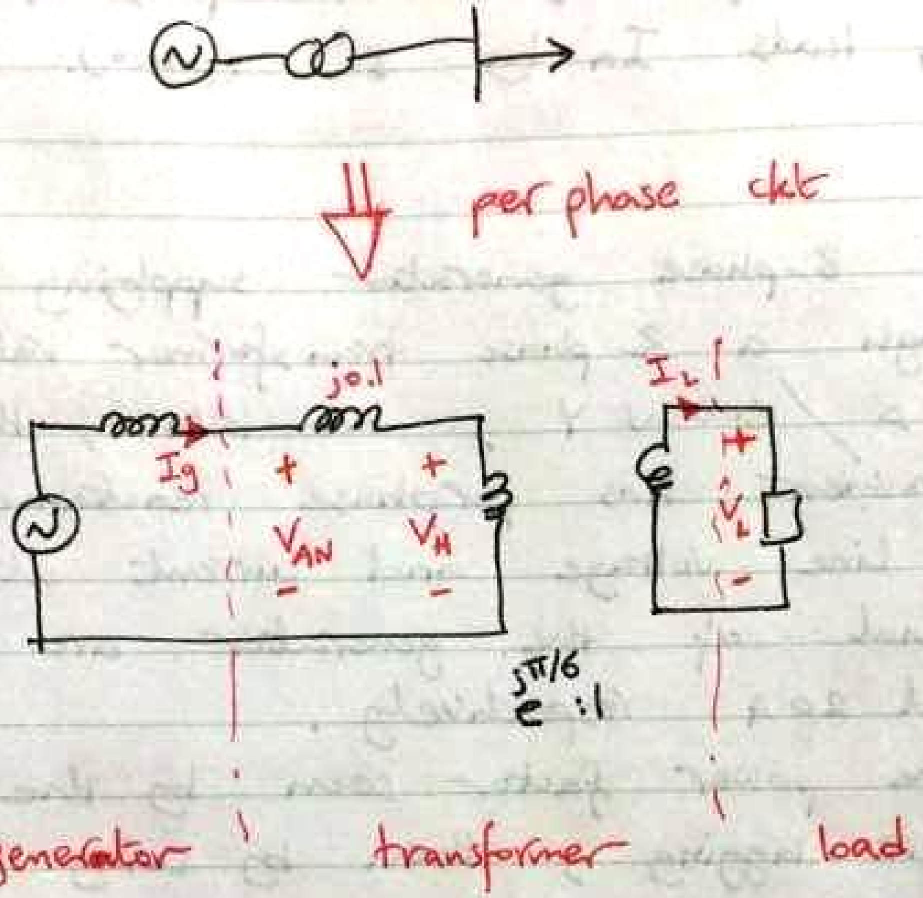
* Ex: Show for Dy in +ve phase sequence I_A leads I_a by $30°$ (prove).

* e.g: A 3-phase generator supplying a load through a 3 phase transformer rated at 12 kV Δ / 600 V Y, 600 kVA, the transformer has perphase reactance of 10%. The line voltage and current at the terminal of the generator are 11.9 kV and 20 A respectively.
If the power factor seen by the G is 0.8 lagging, then by using +ve phase sequence, find:

i) the current, voltage at the load and its impedence
ii) the power supplied by the (HV) side and the power taken by the load

→D

Sol:



* let the base values be (12 kV) and (600 kVA)

$\therefore \quad 600 * 10^3 = \sqrt{3} * 12 * 10^3 * I_b$

$\rightarrow \quad I_b = 28.87 \text{ A}$

$\rightarrow \quad |I_g| = \dfrac{20}{28.87} = 0.69 \text{ pu}$

$\rightarrow \quad |V_{AN}| = \dfrac{11.9}{12} = 0.99 \text{ pu}$

$\therefore \quad V_{AN} = 0.99 \angle 0^\circ$

$I_g = 0.69 \angle \cos^{-1}(0.8) = 0.69 \angle -36.87^\circ$

$$V_H = 0.99\angle 0 - j0.1 * 0.69\angle -36.87$$

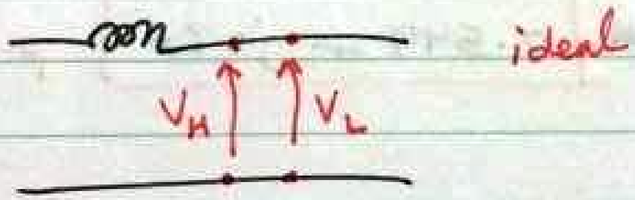
$$V_H = 0.952\angle -3.6^\circ$$

$$\therefore V_L = 0.952\angle -3.6^\circ - 30^\circ$$

$$= 0.952\angle -33.6$$

$$I_L = 0.69\angle -36.87^\circ - 30^\circ$$

$$= 0.69\angle -66.87$$



$$Z = \frac{V_L}{I_L} = \frac{0.952\angle -33.6}{0.69\angle -66.87} = 1.38\angle 33.27^\circ \text{ pu}$$

$$Z_b = \frac{(V_b)^2}{S_b} = \frac{(600/\sqrt{3})^2}{\frac{600 * 10^3}{3}}$$

$$Z(\Omega) = Z(pu) * Z_b$$

$$S_{gen} = V_{AN} * I_g^*$$
$$= 0.99 \angle 0 * 0.69 \angle 36.87$$
$$= 0.6831 \angle 36.87$$

$\boxed{S = 0.546 + j0.41}$ pu   in

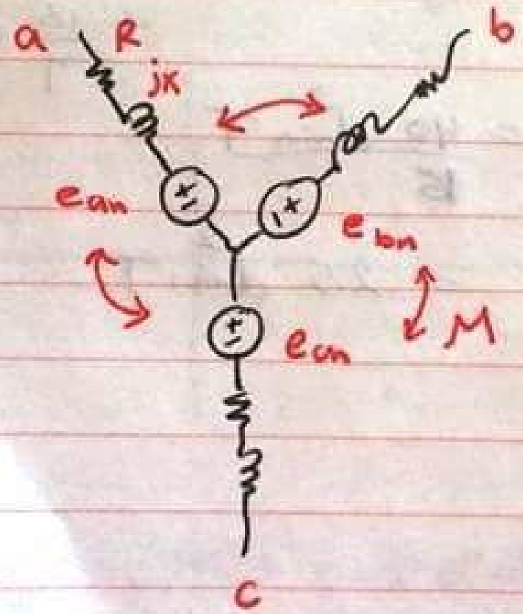
$$S_{load} = V_L * I_L^*$$
$$= 0.952 \angle -33.6 * 0.69 \angle 66.89$$

$= \cancel{0.4} \boxed{0.549 + j0.36}$ pu   out

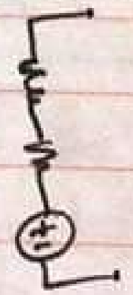
we have no resistors → real part ≈
but in the imaginary we have losses because of $\sim$

* 3-phase synch. generator :-

Revision: i) it contains of stator (which carry armature winding, in which the generator voltage is induced) + Rotor (field winding into which field current is applied)



* equivalent ckt in the steady state :



R = armature winding resistance

$jx = j(\omega L_s + \omega M)$

synch reactance — self inductance — mutual inductance

Monday
19/10

* Equivalent circuit of 3-phase generator under short circuit conditions:

* problem:
if a solution (s.c) is applied at the terminals of the generator, find its equivalent ckt ?

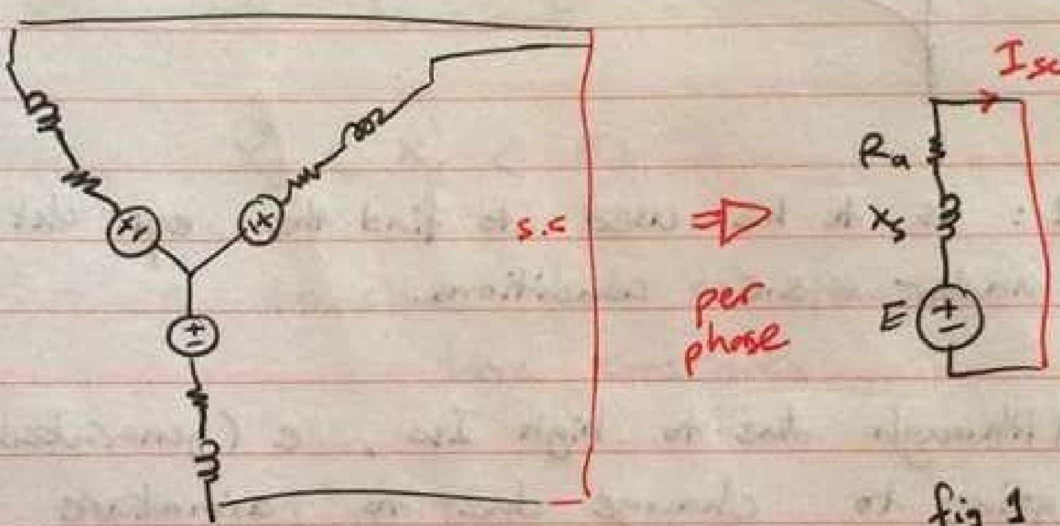
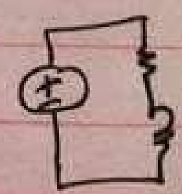


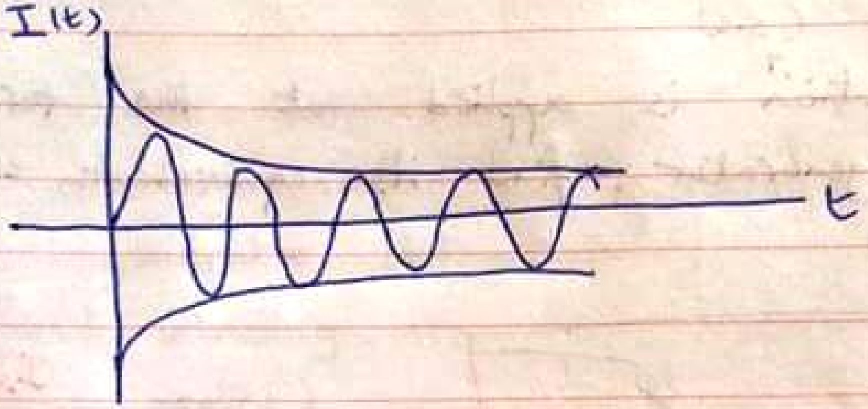
fig 1

* (fig 1) looks like an RL circuit to which a sinusoidal facing function (f.f) is suddenly applied to it.

* As found before, the response consist of
transient + forced
↓
dc response



* If the DC of transient response is neglected then the s.c current will have the following form:



(fig 2)

* fig2: is to be used to find the equ. ckt under s.c conditions.

* Although due to high I_{sc}, e (generated v) is going to change due to armature reaction; however, in the modeling it is assumed that e (i.e: no load generated voltge) remains constant and it is assumed that the reactance of the generator changes

* Consequently the magnitude of the current in fig 2 can be expressed as follows:-

$$\boxed{I(t) = \frac{E}{X_d} + E\left(\frac{1}{X'_d} - \frac{1}{X_d}\right) e^{-t/\tau'} + E\left(\frac{1}{X''_d} - \frac{1}{X'_d}\right) e^{-t/\tau''}} \quad (1)$$

* E = RMS value of the generated voltage.

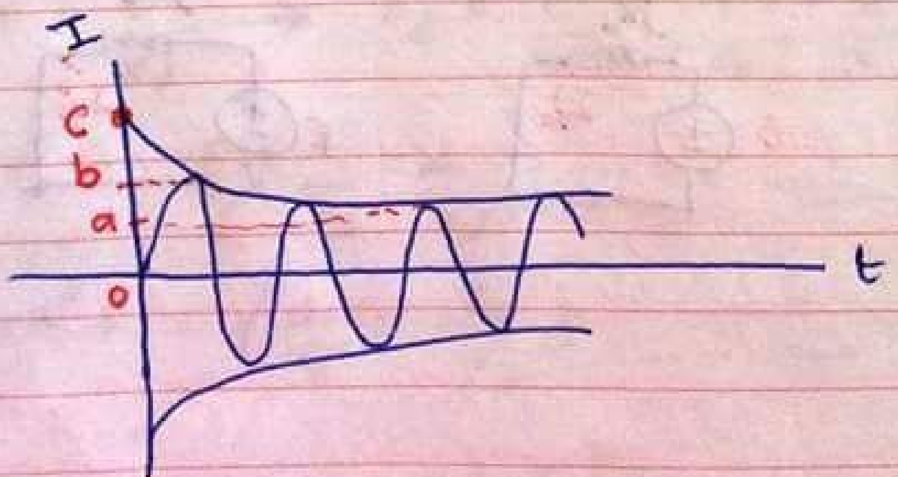
X_d : steady state reactance
X'_d : transient reactance
X''_d : sub- transient "

where: $X''_d < X'_d < X_d$

* τ', τ'' = transient and sub-transient time constants

where $\tau'' < \tau'$

* if (fig 2) is given then one can determine X_d, X'_d, X''_d as follows:



* oa → mag

(RMS) $\frac{oa}{\sqrt{2}} = \frac{E}{X_d}$ ------ (2)

X_d can be found from (2)

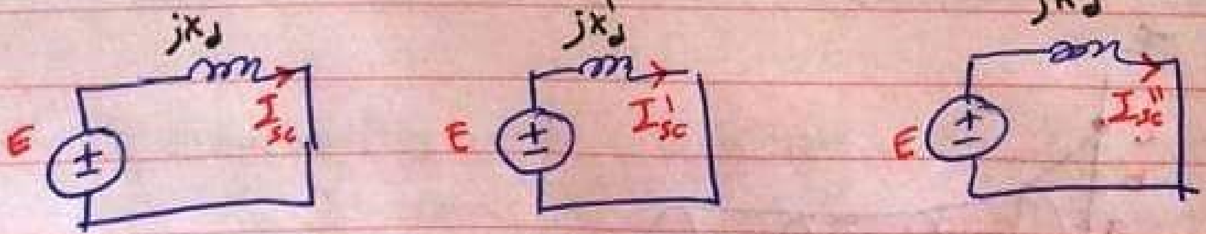
* $\frac{ob}{\sqrt{2}} = \frac{E}{X_d'}$ ------ (3) * neglect the 1st two cycles

X_d' can be found from 3

* $\frac{oc}{\sqrt{2}} = \frac{E}{X_d''}$ ------ (4)

X_d'' can be found from 4

* If X_d, X_d', X_d'' one can evaluate $I_{sc}, I_{sc}', I_{sc}''$ by using the corresponding equ. ckt as follows:



* **Ex:** a 300 MVA, 13.8 kV, 3Φ - Y connected 60 Hz generator is adjusted to rated voltage at open ckt. A balanced 3Φ force is applied to its terminals at $t = 0$.

The obtained symmetrical current $i(t)$ is:

$$i = 10^4 \left[\underset{\text{SS}}{1} + \underset{\text{trans.}}{e^{-t/\tau_1}} + \underset{\text{sub-trans.}}{6 e^{-t/\tau_2}} \right] \text{ A}$$

when $\tau_1 = 200$ ms

$\tau_2 = 15$ ms

→ find X_d, X_d', X_d'' ?

Sol: $E = \frac{13.8 \text{ kw}}{\sqrt{3}}$

by comparing the coeff. of 1 and 5, then:-

1) $10^4 = \frac{E}{X_d} \rightarrow X_d = \frac{E}{10^4} = \frac{13.8 * 10^3}{\sqrt{3} * 10^4} = \boxed{0.797}\ \Omega$

2) $10^4 = E\left(\frac{1}{X_d} - \frac{1}{X_d'}\right)$ ---- — $X_d' = \boxed{0.398}\ \Omega$

3) $6 \times 10^4 = E\left(\frac{1}{X_d'} - \frac{1}{X_d''}\right)$ ---- — $X_d'' = \boxed{0.0996}\ \Omega$

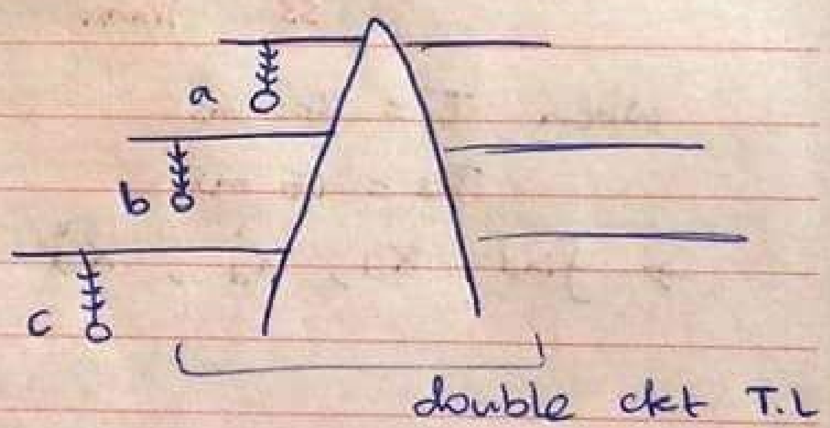
* to find the values in (pu) → calculate Z_b ....

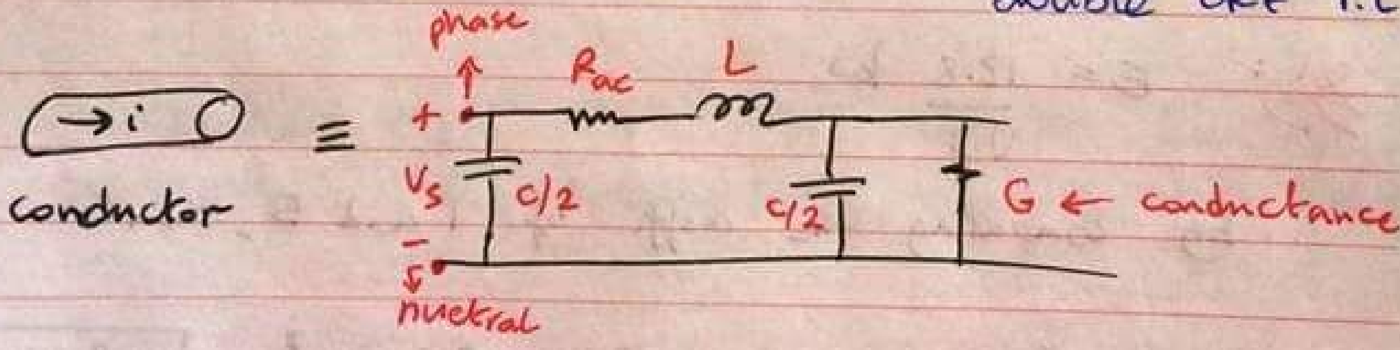
* Transmission line :

objective: to find the relationships for voltage, current, power.

introduction to its equivalent ckt.

* π equivalent ckt of transmission line (T.L)





* $R_{dc} = \frac{\rho l}{A}$

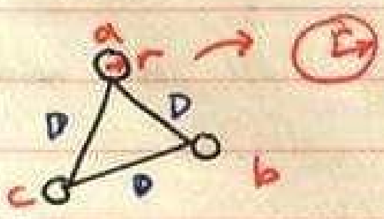
* $R_{ac} > R_{DC}$

* G is usually neglected

Wednesday
21/10

* Inductance:

This depends on the configuration of the transmission line (TL), for example if the phase conductors are equally spaced.



then it can be found that:

$$\boxed{L = 2 * 10^{-7} \ln\left(\frac{D}{D_s}\right)} \quad H/m$$

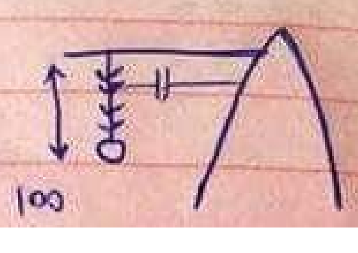
D_s it is called the geometrical mean radius and can be found from standards Tables

* Capacitance:

For the same equally spaced conductors, it can be found that capacitance to Nuetral C_n is:

$$\boxed{C_n = \frac{2\pi\epsilon}{\ln(D/r)}}$$

r: radius of conductor (F/m)

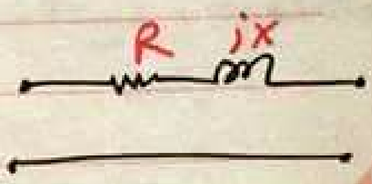


* Classifications of T.L :-

According to how one deal with the shunt capacitance, T.Ls are classified into :

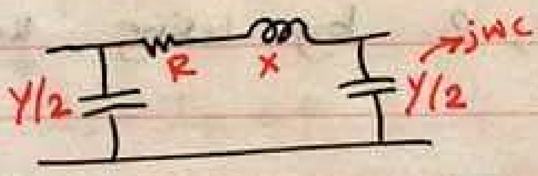
i) Short T.L < 80 km

→ here C is neglected.



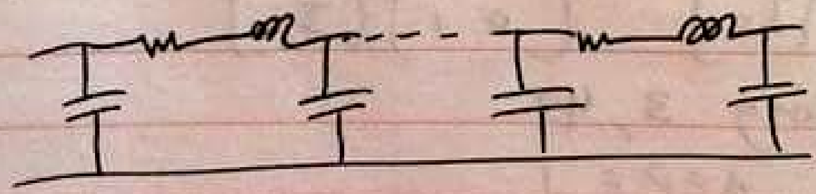
ii) medium line : $80 < l < 240$ km

→ here C is taken into account and the line is represented by one π-ckt (i.e: lumped parameter)



iii) long T.L : $l > 240$ km

→ here C is taken into account, and the line is represented by a set of π-ckts connected in cascade.



* distributed parameters

* Short T.L :

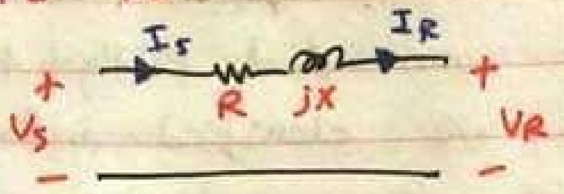


fig 1

$x = \omega L$
$= 2\pi f L$
$Z = R + j\omega L$

* from the equivalent circuit, fig 1

$$V_s = V_R + I_R Z \quad \text{-----} (1)$$
$$Z = R + j\omega L$$

$$I_s = I_R \quad \text{-----} (2)$$

→ rewrite 1, 2 by using two ports network parameters:

→

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \quad \text{-----} (3)$$

1 and 2 ↓

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \quad \text{-----} (4)$$

→ by equating 3, 4 :

$A = D = 1$

$B = Z$ (total series impedance of the line)

$C = 0$

⇒ V_s, V_R : sending and recieving end voltages respectively

* One of the tools used to measure the performance of T.L is called voltage regulation (VR)

→ where : $$VR\% = \frac{|V_{RNL}| - |V_{RFL}|}{|V_{RFL}|} * 100\%$$

⇒ NL : No load
FL : full load. (i.e : rated current)

$$\therefore \ |V_{RNL}| = |V_s|$$
$$\downarrow$$
$$I_s = I_R = 0$$

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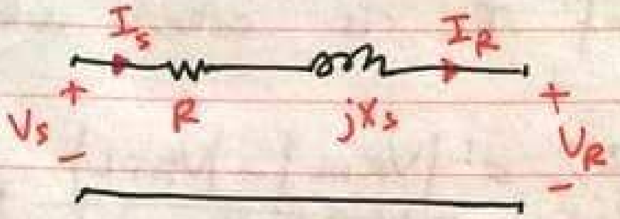
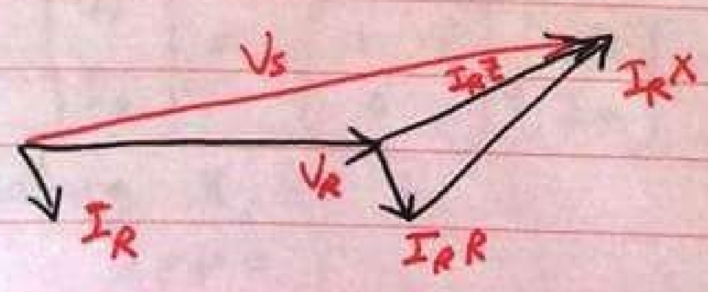
* Performance of the line under varuis types of loads :



* objective : to find relationship between required V_s for given V_R for inductive , resistive , capacitive loads .

* procedure : here it is assumed that $|V_R|$ and $|I_R|$ are constants and the p.f is changing . Hence, by using phasor diagram , it can be found that :

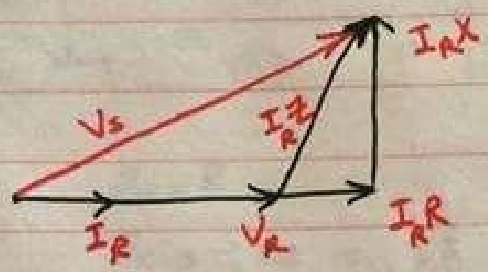
* lagging p.f :



$|V_{SL}| > |V_R|$

lagging

* Unity P.f : (Resistive)

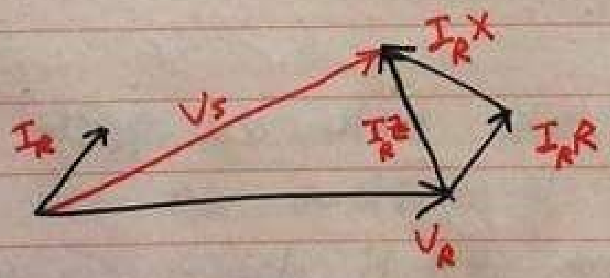


$$|V_{su}| > |V_R|$$

unity

note: $|V_{su}| < |V_{sL}|$

* leading P.f : (capacitive)



$$|V_{sc}| < |V_R|$$

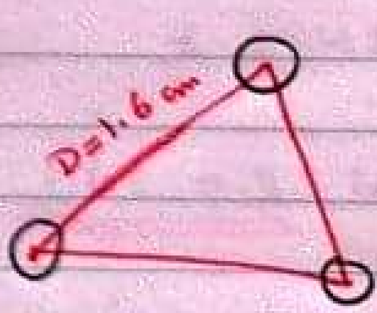
capacitive

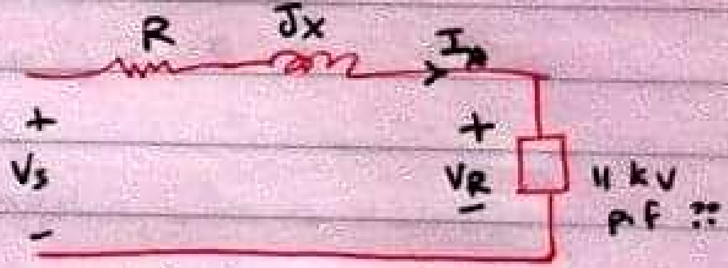
* see: p 750, table A3

26/10
Monday

* **Ex:** An 18 km, 60 Hz, single ckt, 3phase transmission line is componnd of parridge conductor equally spaced with 1.6 m between centers, the line delivers 2500 kw of 11 kv to a balanced load. Assume a wire temprature of 50°c find:-

1) the three phase series impedence of the line
2) the required sending-end voltages when the pf is 0.8 lag, unity, 0.9 leading





* $Z = R + Jx$
* from the standard table, is shown in the Appendix, it can be found that @ 50°:

$\Rightarrow R = 0.3792\ \Omega/m$

$= 0.3792 * \dfrac{18}{1.609} = 4.24\ \Omega$

* for equally spaced $\Rightarrow$ $\boxed{L = 2*10^{-7} \ln\left(\dfrac{D}{D_s}\right)\ H/m}$ *

$\rightsquigarrow$

* $X = 2\pi f L$, $f = 60$ Hz

→ from standard table : $D_s = 0.0217$ ft (foot)

→ 1 foot = 0.3048 m

→ $L = 2 * 10^{-7} \ln\left(\frac{1.6}{0.0217 * 0.3048}\right) = 10.98 * 10^{-7}$ H/m

$L = 10.98 * 10^{-7} * 10^{9} = 0.0197$ H

* $X = 2\pi f L = 7.45\ \Omega$

$Z = 4.24 + j7.45\ \Omega$

P.f = 0.8 lagging.

$V_R = \frac{11}{\sqrt{3}} * 10^{3} \angle 0$ V.

$I_R = \frac{2500 * 10^{3}}{\sqrt{3} * 11 * 10^{3} * 0.8} \angle -63.8 = 164.02 \angle -36.87$ A

$V_s = V_R + I_R Z = 7660.66 \angle 4.19°$

→

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Pf = 1 unity:

V_R = the same

$$I_R = \frac{2500 * 10^3}{\sqrt{3} * 11 * 10^3 * 1} \angle 0^\circ$$

$$V_s = V_R + I_R Z = 6975.9 \angle 8.4^\circ \text{ V}$$

Pf = 0.9 leading:

V_R = the same

$$I_R = \frac{2500 * 10^3}{\sqrt{3} * 11 * 10^3 * 0.9} \angle \cos^{-1}(0.9)$$

$$V_s = 6553.6 \angle 10.97^\circ \text{ V}$$

* p.f	$\lvert V_s \rvert$	V_R%
0.8 lag	7660.66	20.6
1	6975.9	9.84
0.9 lead	6553.6	3.19

* high unacceptable V.R. can be solved by using the concept of compensation as will be explained later.

* By KVL:

$$V_s = Z\left(I_R + V_R \frac{Y}{2}\right) + V_R$$

$$V_s = V_R \underbrace{\left(1+\frac{ZY}{2}\right)}_{A} + \underbrace{Z}_{B} I_R \quad \text{---} ①$$

* By KCL:

$$I_s = V_s \cdot \frac{Y}{2} + \left(I_R + V_R \cdot \frac{Y}{2}\right) \quad \text{---} ②$$

sub ① into ②:

$$I_s = \frac{Y}{2}\left(V_R\left(1+\frac{ZY}{2}\right) + Z I_R\right) + I_R + V_R \frac{Y}{2}$$

$$= V_R\left(\frac{Y}{2} + Z\frac{Y^2}{4} + \frac{Y}{2}\right) + I_R\left(\frac{ZY}{2} + 1\right)$$

$$I_s = V_R \underbrace{\left(Y + \frac{ZY^2}{4}\right)}_{C} + \underbrace{\left(1+\frac{ZY}{2}\right)}_{D} I_R$$

8. $A = D = 1 + \frac{ZY}{2}$

$B = Z$

NL $\rightarrow I_R = 0$

$C = Y + \frac{ZY^2}{4}$

$$VR = \frac{|VR_{NL}| - |VR_{FL}|}{|VR_{FL}|}$$

① $\Rightarrow VR_{NL} = V_S / A$

$$VR = \frac{\left|\frac{V_S}{A}\right| - VR_{FL}}{VR_{FL}}$$

* long transmission line:

let a line of length $= l$
it's series impedence $= z$ Ω/m

$\therefore$ shunt impedence $= y$ S/m
V_s, V_R = it's sending and Recieving voltages

* Consider a segment Δx of this line
@ a distance x from the recieving end

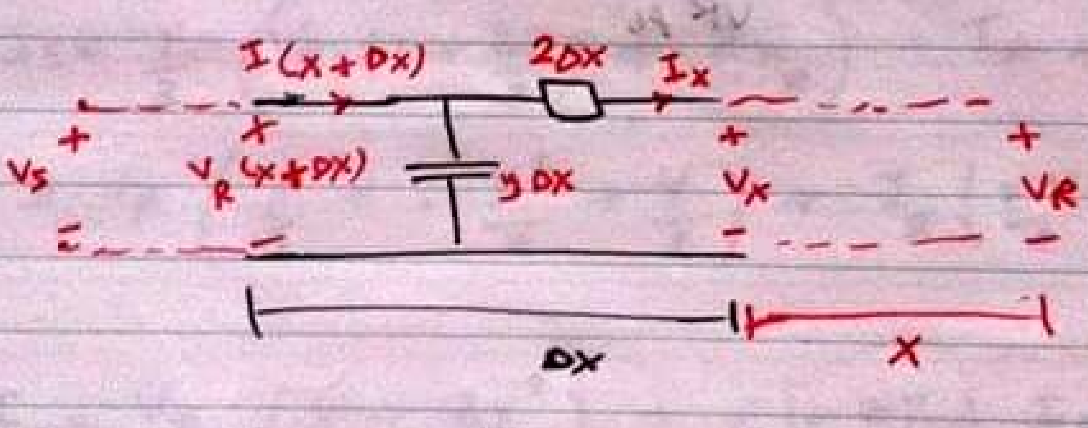

fig1

* objective: to use figure.1 in order to formulate necessary equations in order to evaluate $V(x)$ and $I(x)$

* Analysis:-

→ By KVL:

$$V(x+\Delta x) = Z\Delta x\, I(x) + V(x)$$

$$\therefore \frac{V(x+\Delta x) - V(x)}{\Delta(x)} = Z\, I(x)$$

* in the limit when $\Delta x \to 0$

$$\therefore \frac{dV}{dx} = Z\, I(x) \quad \text{—— } ①$$

→ By KCL:

$$I(x+\Delta x) = V(x+\Delta x)\, y\, \Delta x + I(x)$$

$$\frac{I(x+\Delta x) - I(x)}{\Delta x} = V(x+\Delta x)\, y$$

* in the limit $\Delta x \to 0$

$$\frac{dI}{dx} = V(x)\, y \quad \text{—— } ②$$

diff ① w.r.t x ⇒ $\frac{d^2V}{dx^2} = Z\frac{dI}{dx}$ —— ③

sub ② in ③ ⇒ $\frac{d^2V}{dx^2} = Z \cdot y \cdot V(x) = ZyV(x)$ —— ④

Similarly it can be found $\frac{d^2I}{dx^2} = ZY\, I(x)$ —— ⑤

$$\frac{dv}{dx} = z\, I(x) \quad \text{---} \; 1 \qquad\qquad \frac{d^2v}{dx^2} = zy\, V(x) \quad \text{---} \; 3$$

$$\frac{dI}{dx} = y\, V(x) \quad \text{---} \; 2 \qquad\qquad \frac{dI^2}{dx^2} = zy\, I(x) \quad \text{---} \; 4$$

* To simplify ~~let~~ $\gamma^2 = zy$

$$\gamma = \sqrt{zy}$$

γ is called ~~propag~~ propagation constant

$\therefore$ solve 1st equation 3:

$$\frac{d^2v}{dx^2} = \gamma^2 V(x) \quad \text{-----} \; \cancel{4} \; 5$$

* Solution of 5 is:

$$V(x) = A_1 e^{\gamma x} + A_2 e^{-\gamma x} \quad \text{---} \; 6$$

* A_1 and A_2 can be found from initial conditions.

$$V(0) = V_R = A_1 + A_2 \quad \text{----} \; 7$$

$$\frac{dv}{dx} = A_1 \gamma e^{\gamma x} - A_2 \gamma e^{-\gamma x} \quad \text{---} \; 8$$

* Substitute: ① into ②:

$$Z\,I(x) = A_1 \gamma e^{\gamma x} - A_2 \gamma e^{-\gamma x}$$

$$\therefore Z\,I(0) = Z I_R = A_1 \gamma - A_2 \gamma \text{ ---- } ⑨$$

Solve 7, 9: to find A_1, A_2:

$$\therefore \gamma * ⑦ + ⑨$$

$$\gamma V_R + Z I_R = 2\gamma A_1$$

$$\therefore A_1 = \frac{\gamma V_R + Z I_R}{2\gamma}$$

$$= \frac{1}{2}\left(V_R + \frac{Z}{\gamma} I_R\right) \text{ ---- } ⑩$$

$$= \frac{1}{2} \quad V_R + \sqrt{\frac{Z}{y}}\, IR$$

is called characterstic impedance: Z_c

$$= \frac{1}{2}\left(V_R + Z_c I_R\right)$$

* $A_2 = V_R - A_1$

$$= V_R - \frac{1}{2}\left(V_R + Z_c I_R\right)$$

$$\therefore A_2 = \frac{1}{2}\left(V_R - Z_c I_R\right) \text{ ---- } ⑪$$

* substitute: (10) and (11) into (6):

$$V(x) = \frac{1}{2}\left(V_R + Z_c I_R\right) e^{\gamma x}$$

$$+ \frac{1}{2}\left(V_R - Z_c I_R\right) e^{-\gamma x}$$

$$\therefore \quad V(x) = V_R \underbrace{\frac{1}{2}\left(e^{\gamma x} + e^{-\gamma x}\right)}_{\cosh(\gamma x)} + I_R Z \underbrace{\frac{1}{2}\left(e^{\gamma x} - e^{-\gamma x}\right)}_{\sinh(\gamma x)}$$

$$V_R \cosh(\gamma x) + I_R Z_c \sinh(\gamma x)$$

* By repeating the same procedure, it can be found that:

$$\boxed{I(x) = \frac{1}{Z_c} \sinh(\gamma x) V_R + \cosh(\gamma x) I_R}$$

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* when $x = l$ $\therefore$ $V(l) = V_s$ and
$I(l) = I_s$

$\therefore$ $V_s = V_R \cosh \gamma l + I_R Z_c \sinh \gamma l$

A B

$I_s = V_R \frac{1}{Z_c} \sinh \gamma l + I_R \cosh \gamma l$

C D

$\therefore$ $A = D = \cosh \gamma l$

$B = Z_c \sinh \gamma l$

$C = \frac{1}{Z_c} \sinh \gamma l$

* $VR\% = \frac{\left|\frac{V_s}{A}\right| - |V_R|}{|V_R|} * 100\%$

* e.g: A 60 Hz, 3Φ T.L. is 175 miles long.
it has total series impedence = $135 + j140\ \Omega$
and total shunt admittance = $930 * 10^{-6} \angle 90$ S
it delivers 40 MW at 220 kV and 0.9 p.f
lagging: a) find: voltage current and p.f
at the sending end
b) find V_R and efficiency of line.

$$V_S = AV_R + BI_R$$
$$I_S = CV_R + DI_R$$

$$Z_c = \sqrt{z/y}$$
$$= \sqrt{\frac{35 + j140}{930 * 10^{-6} \angle 90}} = 393.9 \angle -7^\circ$$

* $\gamma l = \sqrt{zy}$

$$= \sqrt{\underbrace{z * l}_{Z} * \underbrace{y * l}_{Y}}$$

$$= \sqrt{(35 + 140j)(930 * 10^{-6} \angle 90)}$$

$$= 0.3663 \angle 83^\circ$$

* $F = 5 e^{j30}$
$\sqrt{F} = \sqrt{5} e^{j30}$
$= (5)^{1/2} e^{j30(\frac{1}{2})}$
$= (5)^{1/2} e^{j15}$
$1 \angle 15$

$$e^{\gamma l} = e^{0.3663} \angle 83^\circ$$

$$= e^{0.0446 + j0.3636} = e^{0.0446} * e^{j0.3636}$$

$$= 1.0456 \angle 20^\circ$$

$$\therefore \ e^{-\gamma l} = \frac{1}{1.0456 \angle 20}$$

$$= 0.9563 \angle -20$$

$$A = D = \cosh \gamma l$$

$$\frac{1}{2}(e^{\gamma l} + e^{-\gamma l})$$

by substitution:-

$$A = D = 0.9407 \angle 1^\circ$$

$$B = Z_c \sinh \gamma l = Z_c \frac{1}{2}(e^{\gamma l} - e^{-\gamma l})$$

$$= 135.9 \angle 76^\circ$$

$$C = \frac{1}{Z_c} \sinh \gamma l = 875 * 10^{-6} \angle 90^\circ$$

$$\rightarrow \ V_R = \frac{220 * 10^3 \angle 0}{\sqrt{3}}$$

$$\rightarrow \ I_R = \frac{40 * 10^6}{\sqrt{3} * 220 * 10^3 * 0.9} \angle -\cos^{-1} 0.9 = \boxed{116.6 \angle 25.8^\circ \ A}$$

→ $V_s = AV_R + BI_R = 13.04 \angle 6.3^\circ$ Kv

→ $I_s = CV_R + DI_R = 119 \angle 33^\circ$

$P.f = \cos(6.3^\circ - 33^\circ)$

$= 0.89$ leading.

(b) VR ? η ?

→ $VR\% = \dfrac{\left|\dfrac{V_s}{A}\right| - |V_R|}{|V_R|} * 100\%$

$= \dfrac{\dfrac{130.4}{0.9407} - \dfrac{200}{\sqrt{3}}}{\dfrac{220}{\sqrt{3}}} * 100\% = 9.15\%$

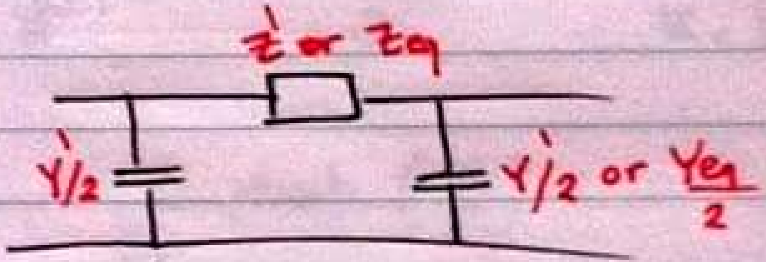
→ $\eta = \dfrac{P_{out}}{P_{in}} = \dfrac{40 * 10^6}{3V_s * I_s * P.f_s}$

$= \dfrac{40 * 10^6}{3 * 130.4 * 10^3 * 119 * 0.89}$

* Equivalent ckt of long T.L. :-

Since such lines are part of a given power system then it is required to represent it by an equ. ckt.

it is represented by a π— ckt as that of medium T.L as follows :-



* for medium line

→ $V_s = V_R \left(1 + \frac{ZY}{2}\right) + Z I_R$

However; for Long line:

$V_s = V_R \cosh \gamma L + Z_c \sinh \gamma L . I_R$

Power 1 Notebook

DR. DAIFALLAH DALBEEH

BY : SAUSAN ALMOHTASEB

* To use the same π- equ. :-

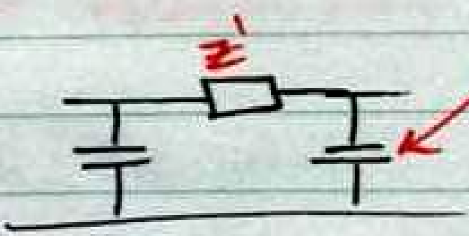
force: $Z_c \sinh \gamma L = Z$ ~ —— ①

$\cosh \gamma L = 1 + \frac{ZY}{2}$ ---- ②

* $Z' = Z_c \sinh \gamma L$

② → $\frac{Y}{2} = \frac{\cosh \gamma L - 1}{Z} = \frac{\cosh \gamma L - 1}{Z_c \sinh \gamma L}$

$= \frac{1}{Z_c} \tanh \left(\frac{\gamma L}{2} \right)$



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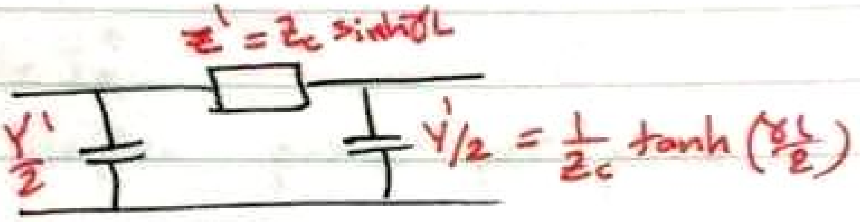
c) find the equivalent cct of the line in the previous e.g:



* by substitution it can be found that:

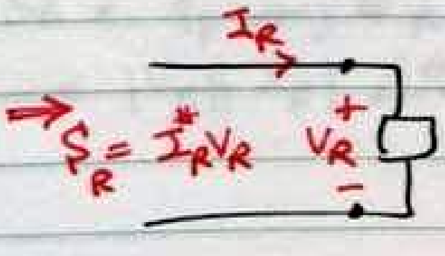
$$Z' = Z_c \sinh \gamma l = 135.8995 \angle 76^\circ$$

$$\frac{Y'}{2} = \frac{1}{Z_c}\left(\frac{e^{\gamma l/2} - e^{-\gamma l/2}}{e^{\gamma l/2} + e^{-\gamma l/2}}\right) = 4.64 * 10^{-4} \angle 90.98^\circ$$

* Power relationship for T.L :-

→ Although the power at any point along the line can be calculated if one knows the V, I, P.f

→ the objective here is to find an expression for the power ~~system~~ in terms of $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ parameters



* Procedure :

$$S_R = V_R I_R^* \quad \text{---} \; ①$$

S_R : complex power per phase

$$V_s \triangleq A V_R + B I_R$$

$$\therefore \; I_R = \frac{V_s - A V_R}{B} \quad \text{---} \; ②$$

<u>let:</u>

$$\left.\begin{aligned} V_s &= |V_s| \angle \delta \\ A &= |A| \angle \alpha \\ B &= |B| \angle \beta \\ V_R &= |V_R| \angle 0^\circ \end{aligned}\right\} \quad \text{---} \; ③$$

* Sub ③ into ① :

$$I_R = \frac{|V_s|}{|B|} \angle \delta - \beta - \frac{|A||V_R|}{|B|} \angle \alpha - \beta \quad \text{---} \; ④$$

* sub 4 into 1 :

$$S_R = \frac{|V_R||V_s|}{|B|} \angle \beta - \delta - \frac{|A||V_R|^2}{|B|} \angle \beta - \alpha \quad \text{---} \; ⑤$$

Since : $S_R = P_R + j Q_R$

$$= |V_R||I_R| \angle \theta \quad \text{---} \; ⑥$$

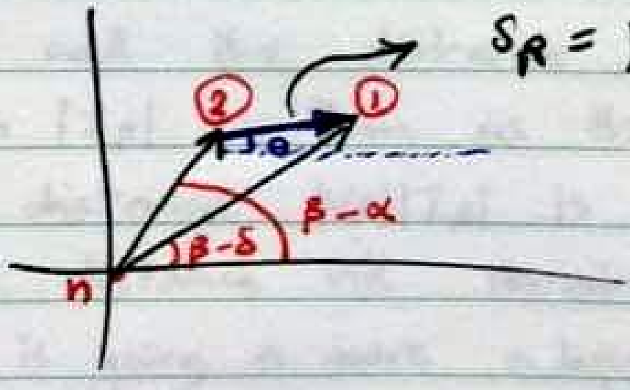
* hence, from 5 & 6 the following comments and conclusions can be made:

$$P_R = \frac{|V_R||V_s|}{|B|} \cos(\beta - \delta) - \frac{|A||V_R|^2}{|B|} \cos(\beta - \alpha) \quad (7)$$

$$Q_R = \frac{|V_R||V_s|}{|B|} \sin(\beta - \delta) - \frac{|A||V_R|^2}{|B|} \sin(\beta - \alpha) \quad (8)$$

⇒ S_R can be expressed graphically by using the concept of phasor diagram, where in (5) S_R is the resultant of two phases as follows:

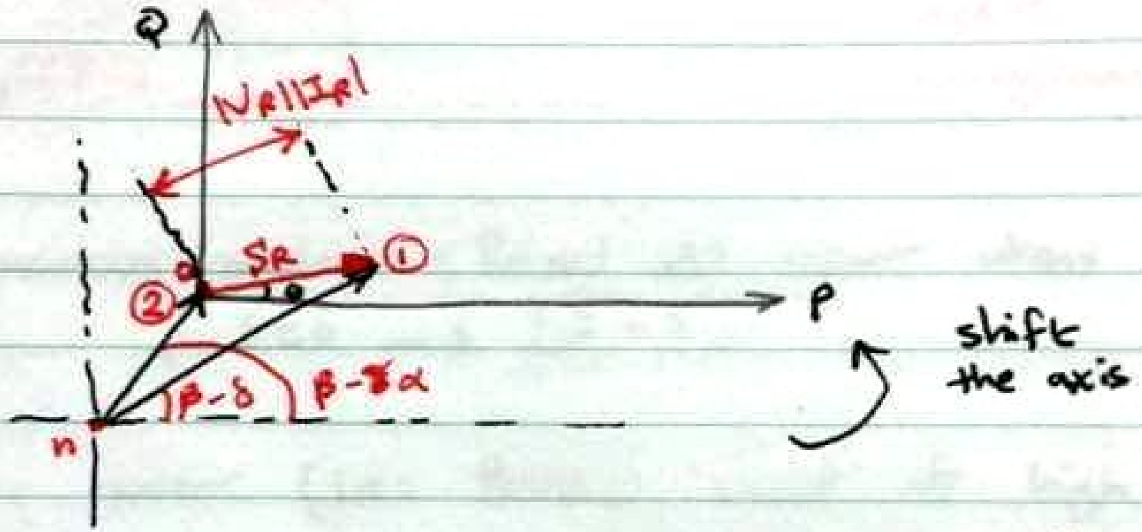
$$(5): S_R = \underbrace{\frac{|V_R||V_s|}{|B|} \angle \beta - \delta}_{\boxed{1}} - \underbrace{\frac{|A||V_R|^2}{|B|} \angle \beta - \alpha}_{\boxed{2}}$$



$S_R = |V_R||I_R| \angle \theta$

(resultant)

* if the axis are shifted from point (n) to point (0), then the resultant diagram is the power diagram

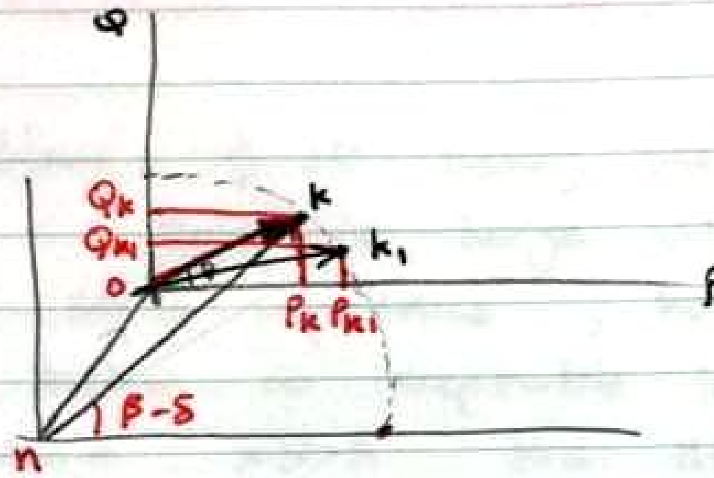


* find the locus of the power when the load changes keeping $|V_R|$ and $|V_s|$ constants

$|I_R|$ θ

* Since n and the distance nk doesn't depend on $|I_R|$, then as the load changes then the distance $|V_R||I_R|$ is going to change with the distance nk remains constant, hence k is going to move along a circle

note: here (δ) changes with load.



power circle diagram

(a) maximum power (P_{max}) will occur when
$\beta - \delta = 0 \longrightarrow \boxed{\beta = \delta}$

(b) max. power (i.e: P_{max}) occur at high value of leading current

* hence in practice $\frac{|V_S|}{|V_R|}$ is limited to 0.95 or higher, δ is limited to about 35°

* hence from (7): $P_{max} = \frac{|V_R||V_S|}{|B|} - \frac{|A||V_R|^2 \cos(\beta-\alpha)}{|B|}$ (9)

from (9):

$P_{max} \propto \frac{1}{|B|}$

for medium & long lines $\longrightarrow |B| \propto |Z|$

$\therefore$ Z affect power capability of T.L

7

* Reactive Compensation :

problem : 1) at heavy load (i.e. higher I_R) there is large voltage drop in the series impedence , hence $|V_R|$ may be below an acceptable $|B|$ level , and since $x \gg R$ then this drop is mainly due to inductive reactance.

Consequently, this problem can be solved by introducing series capacitor called : series reactive compensation

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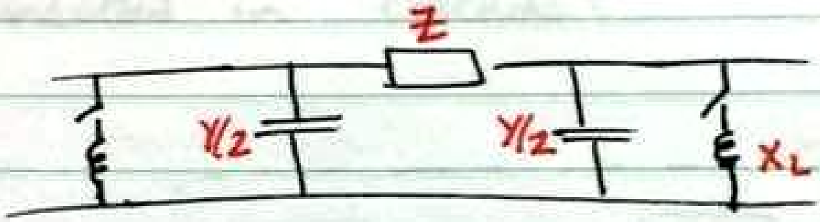
* At light load (no load), then inductive compensation is used to reduce the rise in recieving voltage.



* The compensation factor in this case is $\frac{B_L}{B_C}$; where. $B_L = \frac{1}{X_L}$ → reactance of compensation ($j\omega L$)

$B_C = j\omega C$

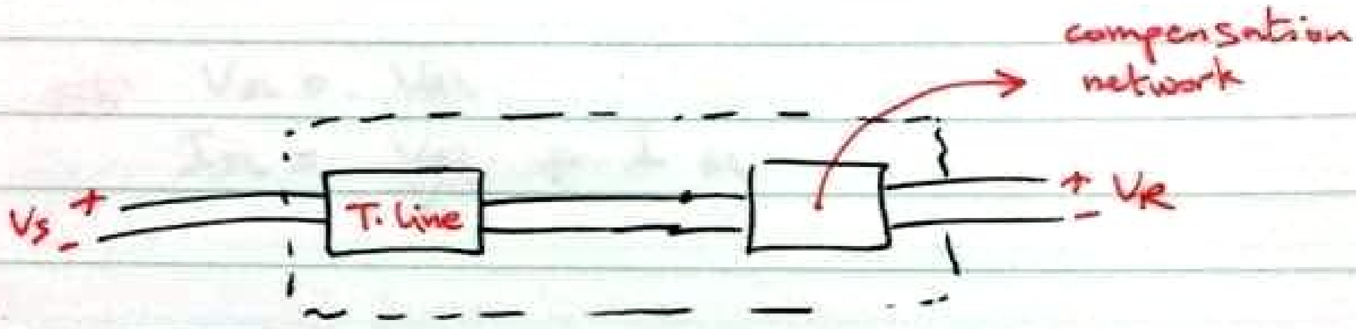
* ~~note~~ : in ~~this~~ the case of series or capacitive compensation the compensation factor = $\frac{X_C}{X_L}$

X_L → inductive reactance of the T.L

X_C → reactance of the added capacitor

* Analysis :

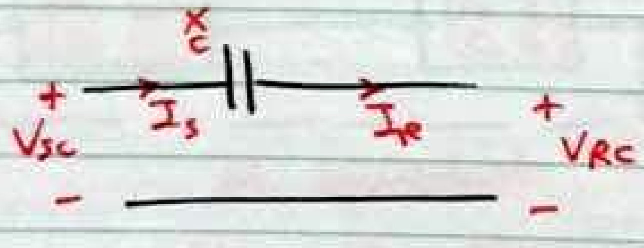
Hence after introducing compensation, there will be two 2-port networks connected in cascade :



equivalent network

* Parameters of compensation network :

* series or capacitor compensation :

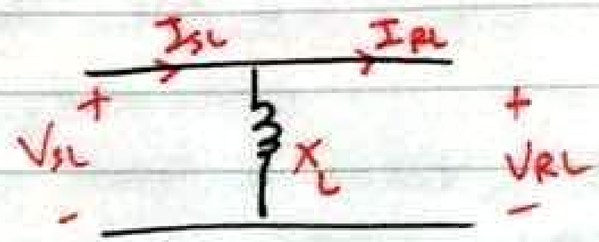


$\therefore$ $V_{sc} = V_{RC} + I_R X_c$

$I_{sc} = I_{RC}$

$$\therefore \begin{bmatrix} V_{sc} \\ I_{sc} \end{bmatrix} = \begin{bmatrix} \overset{A}{1} & \overset{B}{X_c} \\ \overset{C}{0} & \overset{D}{1} \end{bmatrix} \begin{bmatrix} V_{RC} \\ I_{RC} \end{bmatrix}$$

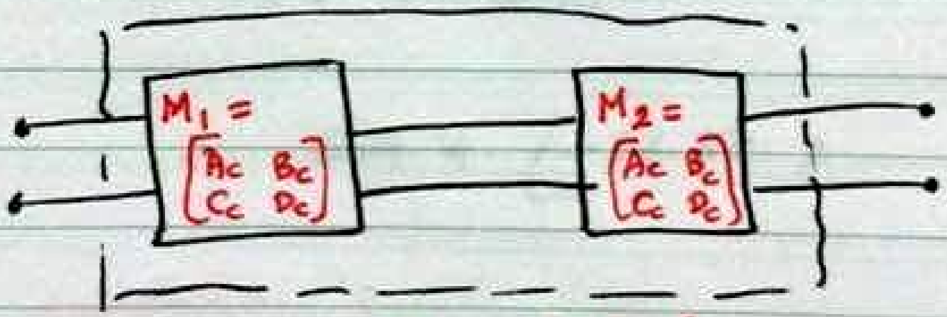
* Parallel or inductive compensation :



$\Rightarrow$ $V_{SL} = V_{RL}$

$$I_{SL} = \frac{V_{RL}}{X_L} + I_{RL}$$

$$\begin{bmatrix} V_{SL} \\ I_{SL} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ * \, 1/X_L & 1 \end{bmatrix} \begin{bmatrix} V_{RL} \\ I_{RL} \end{bmatrix}$$



M- equivalent

$M_{eq} = M_1 * M_2$

* e.g: 3- phase T.L. is 300 mile long and supply a load of 400 MVA with 0.8 pf lagging at 345 kv. The line has the following parameters:

$$A = D = 0.818 \angle 1.3^\circ$$
$$B = 172.2 \angle 84.2^\circ$$
$$C = 0.001933 \angle 90.4^\circ$$

a) find V_s and VR% ?

$$V_s = A V_R + B I_R$$

$$V_R = \frac{345 \angle 0^\circ}{\sqrt{3}} \text{ kv} = 199.2 \angle 0^\circ$$

$$I_R = \frac{400 * 10^6}{\sqrt{3} * 345 * 10^3} \angle -\cos^{-1} 0.8$$

$$= 669.4 \angle -36.87 \text{ A}$$

* By substitution:

$$V_s = 256.8 \angle 20.1^\circ \text{ kv}$$

$$VR\% = \frac{\frac{256.8}{0.818} - 199.2}{199.2} * 100\% = 57.6\%$$

b) If a series capacitor bank having reactance of 146.6 Ω is to be installed at the mid point of the line, and the $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ constants for each 150 mile portion are

$$A = D = 0.9534 \angle 0.3^\circ$$
$$B = 90.33 \angle 84.1^\circ$$
$$C = 0.00104 \angle 90.1^\circ$$

b.1: find the equivalent $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ constants for cascade connection:

$$\underbrace{\begin{bmatrix} 150 \\ \text{mile} \end{bmatrix}}_{\text{given}} * \begin{bmatrix} \text{capacitor} \end{bmatrix} * \underbrace{\begin{bmatrix} 150 \\ \text{mile} \end{bmatrix}}_{\text{given}}$$

$$\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -j146.6 \\ 0 & 1 \end{bmatrix}$$

* it can be found:

$$= \begin{bmatrix} 0.96 \angle 1.2^\circ & 42 \angle 64.8^\circ \\ 0.002 \angle 90 & 0.96 \angle 1.19^\circ \end{bmatrix}$$

A_{eq} B_{eq}

b.2: find V_s and VR%

$$V_s = A_{eq} V_R + B_{eq} I_R$$

by sub: $V_s = 216.7 \angle 4.5°$

$$VR = \frac{\frac{216.7}{A_{eq}} - 199.2}{199.2} * 100\% = \boxed{13.3\%}$$

* Comments on the equation of long line:

it was found that:

$$v(x) = \underbrace{\frac{V_R + I_R Z_c}{2} e^{\gamma x}}_{\text{incident wave}} + \underbrace{\frac{V_R - I_R Z_c}{2} e^{-\gamma x}}_{\text{reflected wave}}$$

$$I(x) = \frac{\frac{V_R}{Z_c} + I_R}{2} e^{\gamma x} - \frac{\frac{V_R}{Z_c} - I_R}{2} e^{-\gamma x}$$

* where: $\gamma = \alpha + j\beta$

* in this case no reflected wave

* surge impedence = $\sqrt{L/C}$ for lossless

* $\boxed{e^{\gamma x} = e^{\alpha} \cdot e^{j\beta}}$

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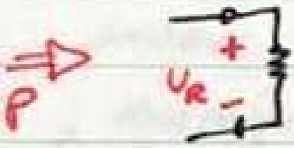
* Surge Impedence Loading (SIL):

This is the power transmitted by the line to a pure resistive load whose value equal to surge impedance.



$$R = \sqrt{L/C}$$

$$\begin{aligned} \rightarrow \quad P &= \sqrt{3}\, V_L I_L \cos\theta \\ &= \sqrt{3}\, V_L I_L \\ &= \sqrt{3}\, V_L \left(\frac{V_L}{\sqrt{3}\,\sqrt{L/C}} \right) = \frac{V_L^2}{\sqrt{L/C}} \end{aligned}$$

* The end of the voltage current for T.L.

* Fault Analysis:

* objective:

what? defintion: Any failure which causes the interruption of normal current or power flow is called Fault.

why? under fault, extreme high current flow, therefore power system should be protected against such high currents.

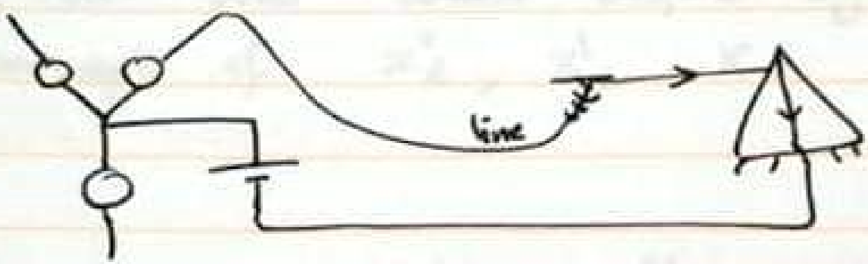
* Protection systems consist of the following components:

for analysis is used to calculate fault current in order to make setting for Relay & circuit breaker.

How ?

* Classifications of Faults :

here we are concerned with s/c fault



So, need to stop the current by circuit breaker.

balanced or symmetrical fault → per phase ckt

unbalanced or unsymmetrical fault →
1. line to ground fault
2. line to line fault
3. line to line to ground fault

* Balanced fault :-

* There are two cases :
  ① Unloaded generator suffer short circuit at it's terminal.
  → This was solved before by using the concept of x''_d , x'_d , x_d.

  ② Loaded generator suffer s.c ~~and~~ at a certain point & analyzed as follows:

* Consider the following sys :-

let the problem that a short circuit occur at the terminal of the load and ~~the~~ let the load by 3-ph synch. motor

That perphase equ. ckt will be as follows:

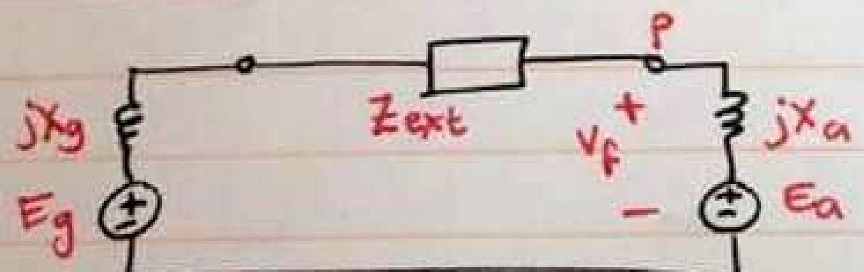
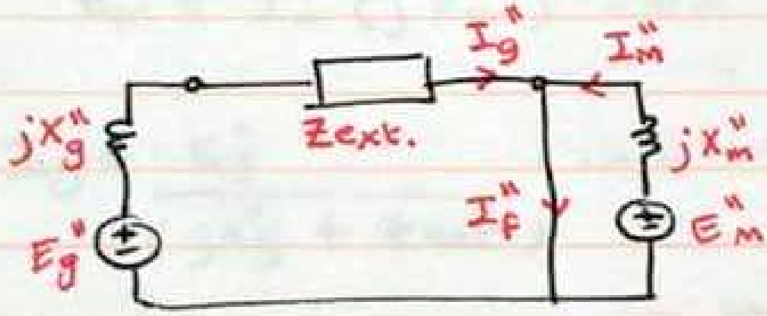


fig (1) : before fault.

$Z_{ext.}$: equ. impedance between the generator terminals and load.

P : is the location of the fault.
V_f : pre fault voltage.

* after fault:



sub-tran.

* fig (2)

* I_f'' : fault current at subtransient fault

$$I_f'' = I_g'' + I_m''$$

* objective to evaluate: I_g'' , I_m'' , I_f''

* Analysis :

Here it is assumed that: E_g'' & E_m'' has the same value immediatly before and immediatly after the fault.

→

* from fig (1):

$$E_g'' = I_L(jX_g'' + Z_{ext}) + V_f \quad --- \ ①$$
$$E_m'' = V_f - I_L jX_m'' \quad --- \ ②$$

* from fig (2):

$$E_g'' = I_g''(jX_g'' + Z_{ext}) \quad ---- \ ③$$
$$E_M'' = I_m''(jX_m'') \quad --- \ ④$$

3: $$I_g'' = \frac{E_g''}{(jX_g'' + Z_{ext})} \quad ---- \ ⑤$$

→ sub ① into ⑤:

⇒ $$I_g'' = I_L + \left(\frac{V_f}{(jX_g'' + Z_{ext})}\right) \quad ---- \ ⑦$$

4: $$I_m'' = \frac{E_m''}{jX_m''} \quad ---- \ ⑥$$

→ sub ② into ⑥:

⇒ $$I_m'' = \frac{V_f}{jX_m''} - I_L \quad ---- \ ⑧$$

∴ $$I_f'' = I_g'' + I_m'' \quad ---- \ ⑦+⑧$$

So: $$I_f'' = \frac{V_f}{jX_m''} - \cancel{I_L} + \cancel{I_L} + \frac{V_f}{jX_g'' + Z_{ext}} \quad ---- \ ⑨$$

* This method up to How it is called the internal voltages method.

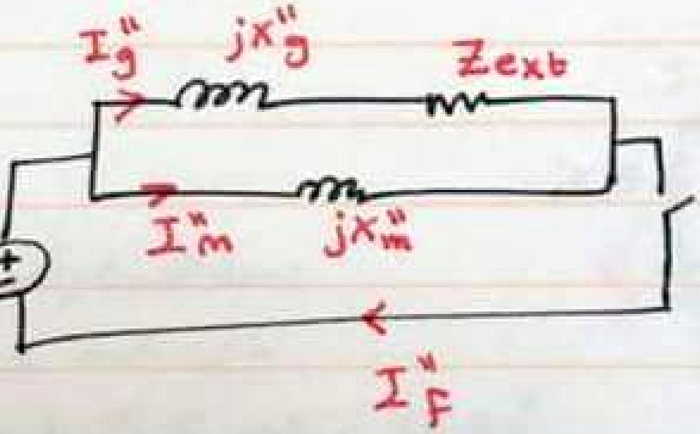
* comment :

(9) can be written as follows:

$$I''_f = V_f \left(\frac{1}{jX''_m} + \frac{1}{jX''_g + Z_{ext}} \right)$$

$$= \frac{V_f}{\underbrace{(jX''_g + Z_{ext}) \,//\, (jX''_m)}_{\text{parallel}}} \quad \text{---- (10)}$$

* so (10): can be represented by a thevenin equivalent as follows:



→ so, thevenin equ. is used to evaluate fault current (I''_f)

→ so, I''_g and I''_m can be found by using current division.

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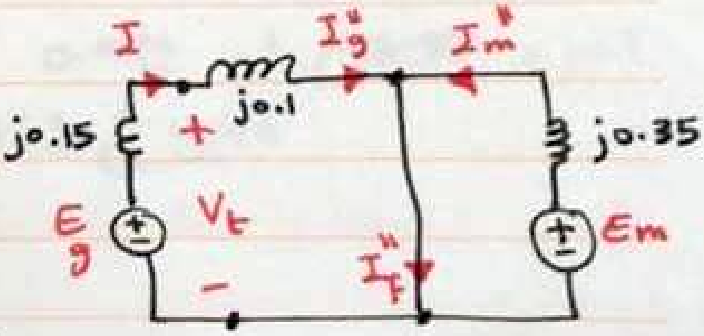
e.g: A generator is connected through a transformer to a synch. motor for the same base pu reactances of the components $X''_g = 0.15$, $X_T = 0.1$, $X''_m = 0.35$.
A 3-ph fault occur at the terminal of the motor, when the terminal voltage of the ~~motor~~ generator $= 0.9$ pu at the gen- current $= 1$ pu at 0.8 pf leading.

Find: the sub-transient current in pu in the fault gen. and motor:

⇒ sol:



* given information:

$V_t = 0.9 \angle 0$

$I = 1 \angle \cos^{-1} 0.8 = 36.87^\circ$

* using internal voltage method:

before: $E''_g = V_t + jX''_g I = 0.82 \angle 8.42$

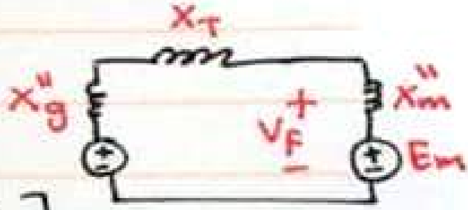
$E''_m = V_t - I(jX_T + jX''_m) = 1.22 \angle -17.1$

(after): $I_g'' = \dfrac{I_g''}{j0.15 + j0.1} = 8.28 \angle 81.58$

$$I_m'' = \frac{E_m''}{j0.35} = 3.49 \angle \overline{1}07.1$$

$$I_f = I_g'' + I_m'' = 6.6 \angle -94.78$$

* Using Thevenin:



$$I_f = V_f / \left[(jX_g'' + jX_T) \,//\, jX_m'' \right]$$

$$V_f = V_t - I X_{ext} = 0.9 \angle 0 - 1 \angle 36.87 * j0.1$$
$$= 0.963 \angle -4.76$$

* by substitution:

$$I_f'' = 6.6 \angle -94.76$$

* by current division:

$$I_g'' = I_f'' \left(\frac{jX_m''}{jX_m'' + jX_g'' + jX_T} \right) = 3.85 \angle -94.76$$

→ $I_m'' = I_f'' - I_g'' = 2.75 \angle -94.76$

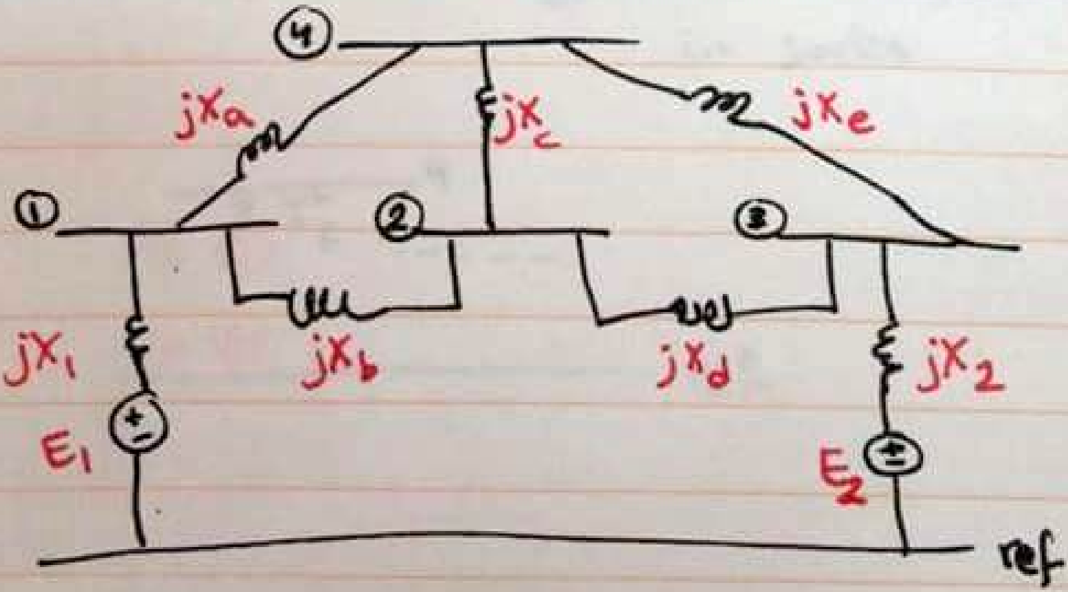
* The difference in the answer for I''_g and I''_m in the two methods is due to fact that Thev. method neglect I_L

$\therefore$ To take into account add I_L to I''_g and subtract it from I''_m

* Z-bus method:

this method is used to evaluate the fault current due to a balanced 3-ph fault for a general power system.

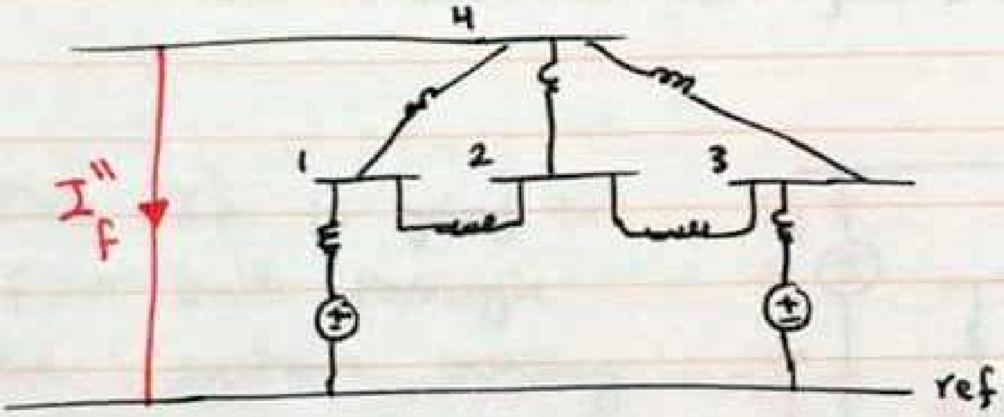
* procedure: Consider the following sys.:



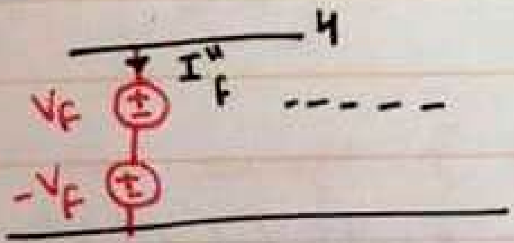
* problem :

for all given sub-transient parameters, evaluate the fault current (I_f'') and the internal current when a fault occur at any busbar.

* for e.g: let the fault at Bus 4 .



now, the s.c of the fault can be represented by 2 voltage sources (i.e: V_f , $-V_f$) in series.



* Since there are 4 sources then super position can be used to evaluate (I''_f)

ⓐ if E''_1, E''_f, V_f are taken together, then $I''_f = 0$, because these sources represent pre-fault condition.

ⓑ hence, I''_f is due only to the source ($-V_f$) and evaluated as follows:

* This I''_f is going to flow back through the system causing changes of voltages at the busbars = $\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \\ \Delta V_4 \end{bmatrix}$

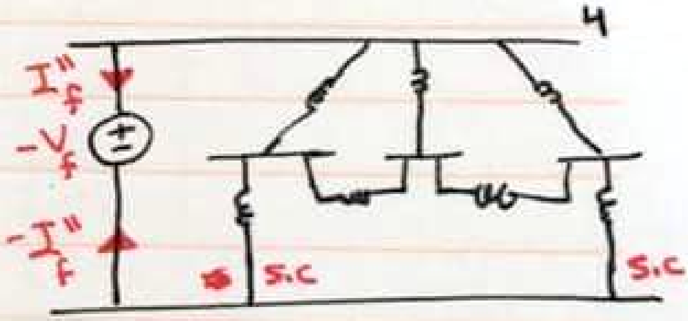


fig ①

* By using the concept of [Y] and [Z]:

$\therefore \quad [I] = [Y][V]$

or $\quad [Y]^{-1}[I] = [V] \qquad * \; Z \triangleq [Y]^{-1}$

or $\quad [V] = [Z][I] \quad$ ---①

* by applying ① to fig ①:

$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \\ \Delta V_4 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -I_f'' \end{bmatrix}$$

$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \\ -V_f \end{bmatrix} = -I_f'' \begin{bmatrix} Z_{14} \\ Z_{24} \\ Z_{34} \\ Z_{44} \end{bmatrix}$$

$\therefore \quad -V_f = -I_f'' Z_{44}$

$\therefore \quad I_f'' = V_f / Z_{44}$

* in general: $\quad I_{f,k}'' = \dfrac{V_{f,k}}{Z_{kk}}$

$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \\ \Delta V_4 \end{bmatrix} = - \begin{bmatrix} Z_{14} \, V_f / Z_{44} \\ Z_{24} \, V_f / Z_{44} \\ Z_{34} \, V_f / Z_{44} \\ Z_{44} \, V_f / Z_{44} \end{bmatrix}$$

due to the source ($-V_f$)

* total voltages at the busbars:

now the voltages due to the sources E_1'', E_2'', V_f and neglecting load currents, will be:

$$\begin{bmatrix} V_f \\ V_f \\ V_f \\ V_f \end{bmatrix}$$

→ total voltage at busbars:

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \underbrace{\begin{bmatrix} V_f \\ V_f \\ V_f \\ V_f \end{bmatrix}}_{\text{due to 3 sources}} + \underbrace{\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \\ \Delta V_4 \end{bmatrix}}_{\text{due to } (-V_f)}$$

* e.g.:

$$V_4 = V_f + \Delta V_4$$
$$= V_f - V_f$$
$$= 0$$

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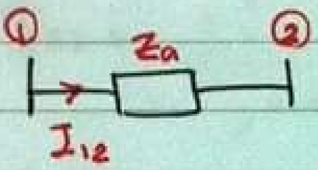
* $I_f'' = V_F / Z_{kk}$ ------ ①

$V_j = V_f - Z_{jk} I''_{fk}$ ------ ②

$$V_j = V_f - \frac{Z_{jk} V_f}{Z_{kk}}$$

* Having evaluated the voltages at the busbars then one can calculate internal fault current.



$$I_{12} = \frac{V_1 - V_2}{Z_a}$$

* eg: the bus-impedance matrix of a 4-bus system is:

$$Z = j \begin{bmatrix} 0.15 & 0.08 & 0.04 & 0.07 \\ 0.08 & 0.15 & 0.06 & 0.09 \\ 0.04 & 0.06 & 0.13 & 0.05 \\ 0.07 & 0.09 & 0.05 & 0.72 \end{bmatrix}$$

* The system has generators connected to buses ① and ② if pre-fault current is neglected evaluate (I''_f), for a 3 phase fault at bus ④

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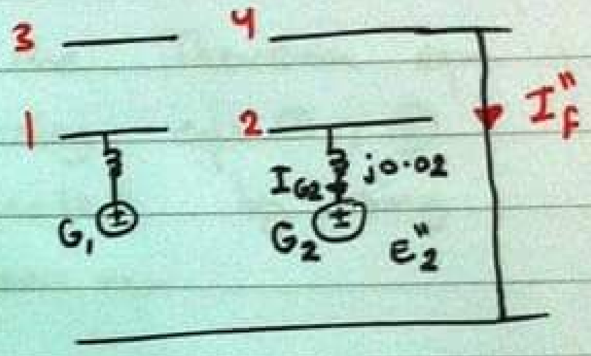
* Assume $V_f = 1\angle 0$ pu, find also the current from generator ② whose $X''_{G2} = j0.02$

* $I''_f = V_f / Z_{44}$
$= 1\angle 0 \,/\, j0.12$
$= 8.33 \angle -90°$



* $I_{G2} = \dfrac{E''_2 - V_2}{j0.02}$

→ $V_2 = V_f - Z_{24} I''_f$
$= 1\angle 0 - j0.09 * 8.53 \angle -90$
$= 0.251 \angle 0$

→ $E_2 = 1\angle 0$

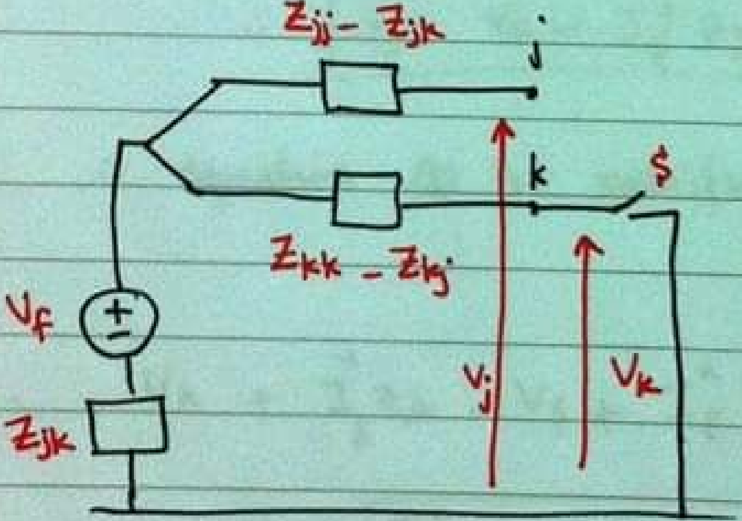
* by sub: $I''_{G2} = -3.75$
$= 3.75 \angle -90$

* Equivalent ckt of Z-bus matrix :

it is impossible to find a single equ. ckt. for the Z-bus method.

However, one may find the Thevenin eqn. for a pair of buses by using equ ①, ② so that (I_f'') can be evaluated by using Thev. equ.

* procedure :



k = faulty bus
j = healthy bus

↳

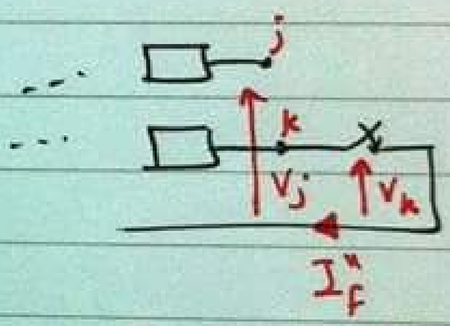
* **S open:** (pre-fault)

by applying KVL:

$$V_k = V_j = V_f$$

* **S closed:** (i.e: sc fault between k and ref)

the I''_f is going to flow
by KVL:



V_j: $-V_j + 0 + V_f - I''_f Z_{jk} = 0$

$$\therefore \boxed{V_j = V_f - I''_f Z_{jk}}$$

∴ equation ② is satisfied

V_k: $V_k + I''_f Z_{jk} - V_f + I''_f (Z_{kk} - Z_{kj}) = 0$

$$\therefore \boxed{I''_f = V_f / Z_{kk}}$$

∴ equation ① is satisfied

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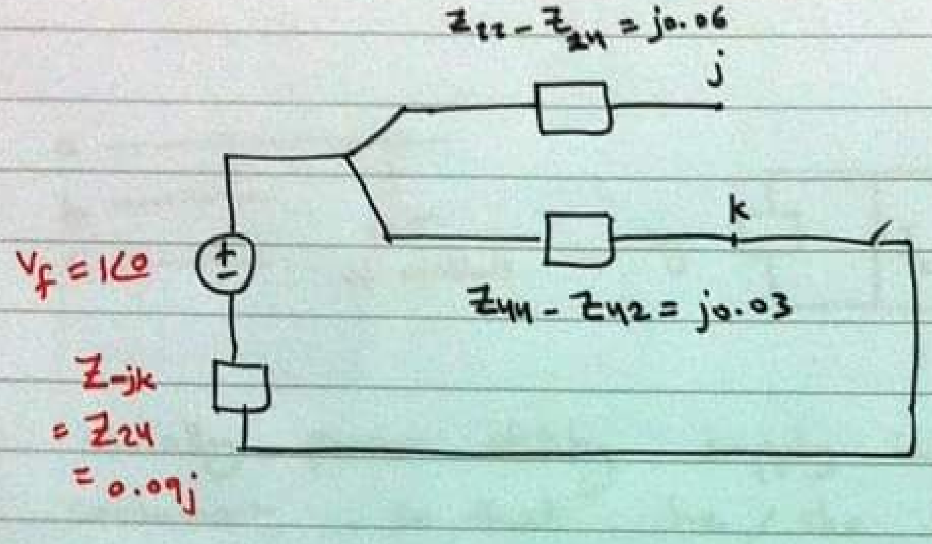
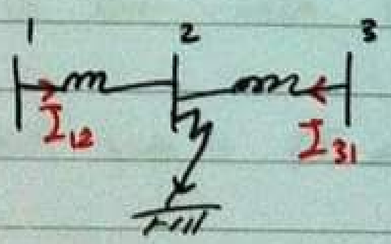
* solve the previous example by using Thev. eqn. :



* by sub: $I_f'' = 8.33 \angle -90$

$V_2 = 0.251 \angle 0$

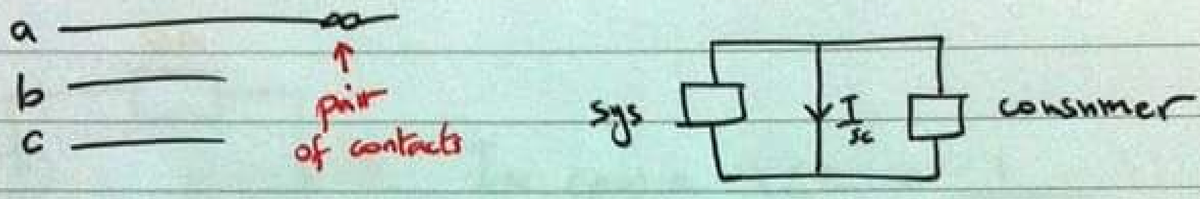
* See and study the example in the text book.
e.g (10.4)
page 396



by using Thev. equ.

* There are two steps for Thev. equ.

* Introduction for the selection of ckt breaker rating:



* usually power utility supply information to consumer, so that he / she can evaluate S.C current

* This information is called SC MVA and defined as follows:

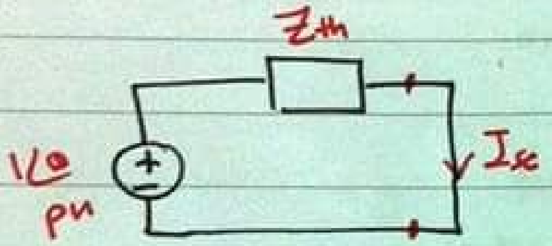
$$\text{SC MVA} \triangleq \sqrt{3} * (\text{nominal line voltage}_{(kV)}) * I_{sc\,(A)} * 10^{-3} \quad (1)$$

* if the MVA base is defined as:

$$\text{MVA base} = \sqrt{3} * (\text{base voltage})(\text{base current}) * 10^{-3} \quad (2)$$

$$\frac{(1)}{(2)} \Rightarrow \text{SC MVA (pu)} = I_{sc}\ (\text{pu})$$

* If the utility sys. looking from the consumer terminals is represented by its Thev. equ.
hence under sc :



$$I_{sc}\,(pu) = \frac{1\angle 0}{Z_{th}} = \frac{1}{Z_{th}}$$

* hence utility is supplying:
① expected sc
② Z_{th} of the sys.

* from current point of view, the most two important factors :

i) the max ~~✗~~ instantanuons current, which the breaker must with stand.
ii) the interrupting current (i.e. the current at which contact open)

→

i) in order to take into account the DC component then the sub-transient current (I'') is multiplied by a factor Σ (>1).
This factor depend on the type of breaker and its voltage.

* for e.g: for CB (circuit breaker) with rating > 5 kv, this factor $= 1.6$

* hence this is called: Momentary current =
$= \text{factor} * I''_F$

* this is current which the breaker should with stand for a short period 1, 2 cycles

ii) is defined by means of interrupting kVA =
$\sqrt{3}$ * bus voltage at which breaker is connected (kv) * interrupting current (A)

* this value depend on the speed of the breaker

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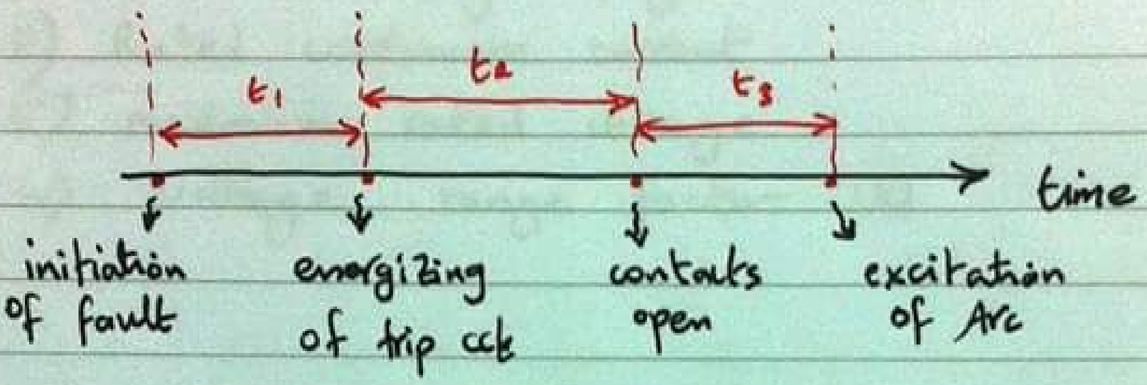
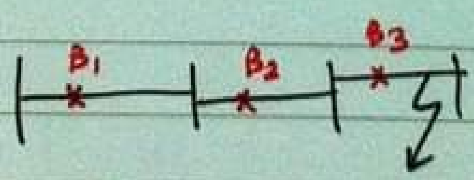
* Breakers with different speed are classified according to thier interrupting time, which is defined as follows:



→ $t_1 \equiv$ delay time of relay to take into account starting normal current or garding or coordination of protection.

→ $t_2 \equiv$ opening time

→ $t_3 \equiv$ Arc exciatation time



* $(t_2 + t_3)$ is called interrupting time.

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* Conclusion :

Among the major Ratings of C.B are :

i) nominal voltage (e.g : 132 kv)
ii) Rated continuous current
iii) maximum rated voltage
iv) voltage range factor (k)

* $k \triangleq \dfrac{\text{maximum rated voltage}}{\text{lower limit of voltage range}}$

→ k is selected in such away that the product of (sc current * operating voltage) is always constant

v) Rated sc current

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* e.g: A circuit breaker having nominal voltage rating of 34.5 kv continuous current rating of 1500 A , has k=1.65 rated maximum voltage is 38 kw and the rated SC current at this voltage is 22 kA

i) find the voltage below which rated sc current doesn't increase as operating voltage decreases and the value of that current.

$\Rightarrow$ $k = \frac{\text{max. voltage}}{\text{lower limit}}$ $\Rightarrow$ $V_{lower} = \frac{max}{K}$

$\frac{38}{1.65} = \boxed{23.03}$ kv

* SC current * operating voltage = constant

38 * 22 = 23.03 * I_{sc}

$\therefore I_{sc} = \frac{38 * 22}{23.03} = \boxed{36.3}$ kw

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* Unbalanced or Unsymmetrical faults :-

Here under unbalanced faults, current and voltages will be unbalanced.

Here the mathematical cocept of symmetrical components will be used and defined as :

* symmetrical components :-

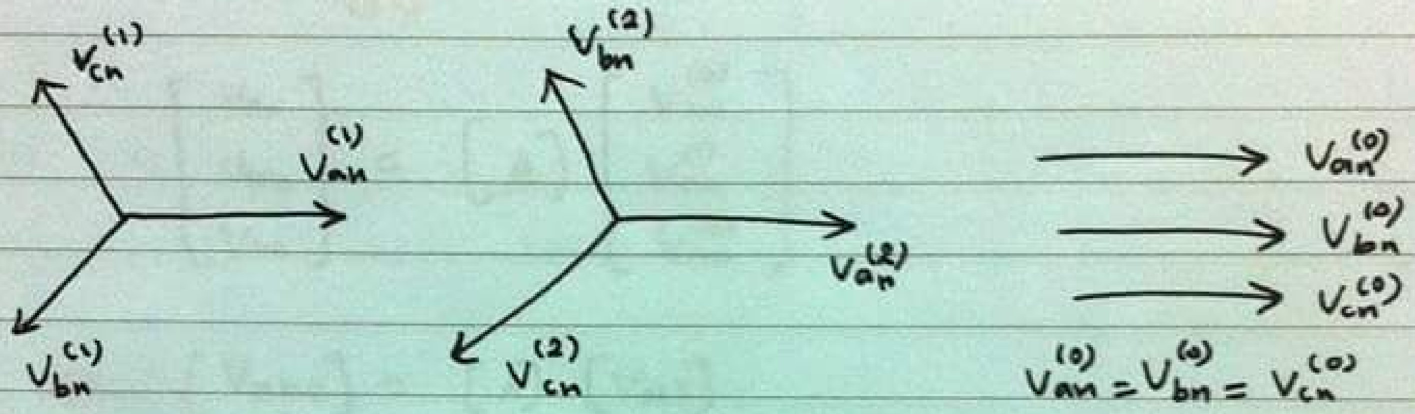
Here any unbalanced voltage or current can be expressed as the sum of 3 components called :

+ve sequence
-ve sequence
zero sequence

* Superscripts are used: (1) → +ve
(2) → -ve
(0) → zero

* for e.g: consider system of unbalanced voltages V_{an}, V_{bn}, V_{cn}

$$\therefore \quad V_{an} = V_{an}^{(0)} + V_{an}^{(1)} + V_{an}^{(2)}$$
$$V_{bn} = V_{bn}^{(0)} + V_{bn}^{(1)} + V_{bn}^{(2)}$$
$$V_{cn} = V_{cn}^{(0)} + V_{cn}^{(1)} + V_{an}^{(2)}$$



+ve　　　　-ve　　　　Zero

* By introducing the mathematical complex operator $a \triangleq 1\angle 120^\circ$
hence, by using (a) all the voltages V_{an}, V_{bn}, V_{cn} can be expressed in terms of the symmetrical components of phase (a), as:

$$\rightarrow \quad V_{an} = V_{an}^{(0)} + V_{an}^{(1)} + V_{an}^{(2)}$$
$$V_{bn} = V_{an}^{(0)} + a^2 V_{an}^{(1)} + a V_{an}^{(2)}$$
$$V_{cn} = V_{an}^{(0)} + a V_{an}^{(1)} + a^2 V_{an}^{(2)}$$

$\longrightarrow$

$$\Rightarrow \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}}_{[A]} \begin{bmatrix} V_{an}^{(0)} \\ V_{an}^{(1)} \\ V_{an}^{(2)} \end{bmatrix}$$

$$\therefore \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = [A] \begin{bmatrix} V_{an}^{(0)} \\ V_{an}^{(1)} \\ V_{an}^{(2)} \end{bmatrix}$$

$$\Rightarrow [V_{abc}] = [A]\,[V_{012}]$$

$$[V_{012}] = [A]^{-1}\,[V_{abc}]$$

* where: $[A]^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$

* <u>note:</u> the same expression can be applied to current:

$$[I_{abc}] = [A]\,[I_{012}]$$

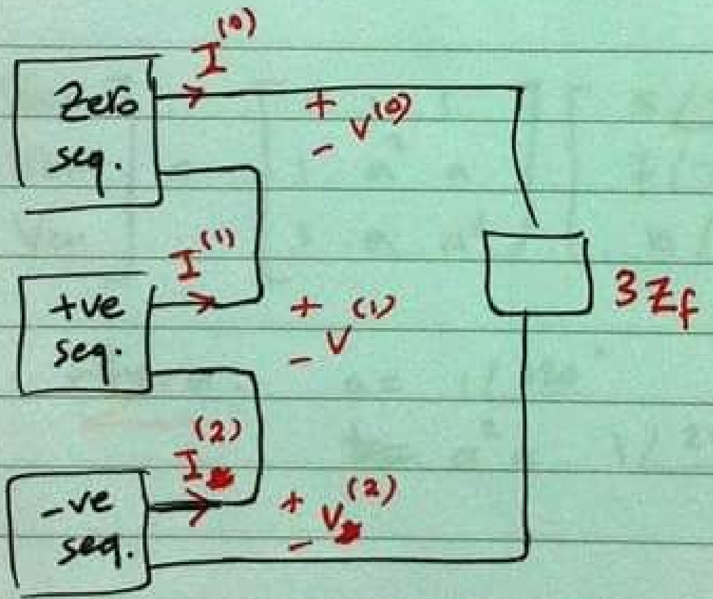
$$[I_{012}] = [A]^{-1}\,[I_{abc}]$$

* Procedure :

i) find the 0, +ve, -ve sequences ckt for each power system element
(i.e: generator, transf., line, load)

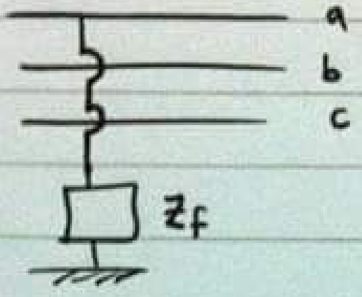
ii) the inter connection of these sequence ckts give sequence networks
(i.e: +ve, -ve, 0 sequence network)

*



iii) the interconnection between networks depends on the type of the fault (i.e: L-L
L-G
L-L-G)

* e.g.: (L - G)



as will be shown later

* e.g.: Given:

$V_{an}^{(0)} = 5\angle 30^\circ$; find $[V_{abc}]$?

$V_{an}^{(1)} = 7\angle -14^\circ$

$V_{an}^{(2)} = 10\angle 41^\circ$

$$\Rightarrow \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 5\angle +30 \\ 7\angle -14 \\ 10\angle 41 \end{bmatrix}$$

where: $a = 1\angle 120^\circ$

$a^2 = 1\angle 240^\circ$

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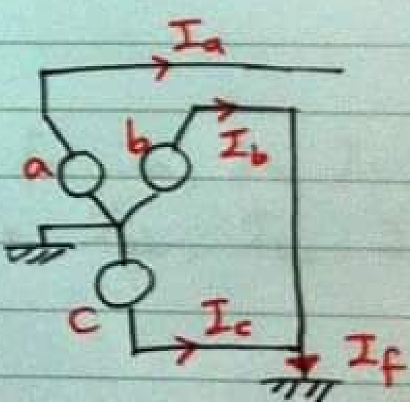
* e.g: When a 3phase generator has 1 terminal open cct and the other two terminals are shorted to ground, the symm. component of the currents as follows:

$$I_a^{(1)} = 600\angle -90$$
$$I_a^{(2)} = 250\angle 90$$
and
$$I_a^{(0)} = 350\angle 90$$

* find the corresponding phase current and fault current?

$$\Rightarrow \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I^{(0)} \\ I^{(1)} \\ I^{(2)} \end{bmatrix} \qquad \begin{cases} a = 1\angle 120 \\ a^2 = 1\angle 240 \end{cases}$$

* By substitution and multiplication, it can be found:



$$= \begin{bmatrix} 0 \\ 904.16\angle 44.5^\circ \\ 904.16\angle 33.5^\circ \end{bmatrix}$$

$$I_f = I_b + I_c = 1050.1\angle 90^\circ$$

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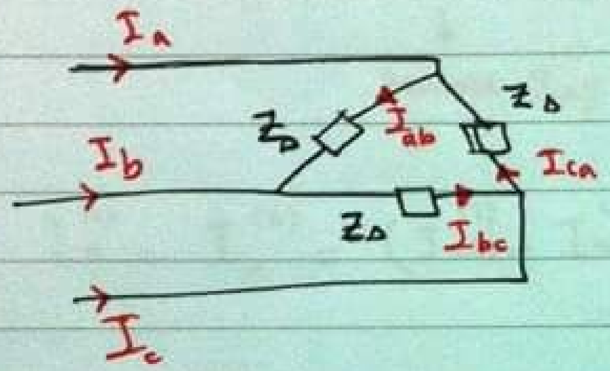
* Relationship between symmetrical components for currents and voltages of Y and Δ connections:

* consider a Δ connection:



* objective: to find relationship between:

$I_a^{(1)}$, $I_{ab}^{(1)}$

$I_a^{(2)}$, $I_{ab}^{(2)}$

note: (I_a) is taken as reference for I_a, I_b, I_c
and (I_{ab}) = = = = = I_{ab}, I_{bc}, I_{ca}

* Procedure: by KCL:

$$I_a = I_{ab} - I_{ca} \quad \text{---} \quad (1)$$
$$I_b = I_{bc} - I_{ab} \quad \text{---} \quad (2)$$
$$I_c = I_{ca} - I_{bc} \quad \text{---} \quad (3)$$

$$I_a^{(0)} \triangleq \frac{1}{3}(I_a + I_b + I_c) = 0$$ * by sub. 1,2,3

∴ linear currents don't have zero sequence components

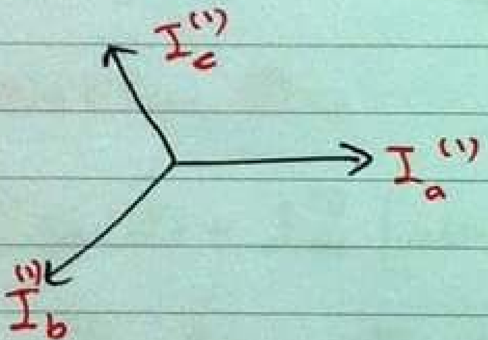
* rewrite ① in terms of its symmetrical components:

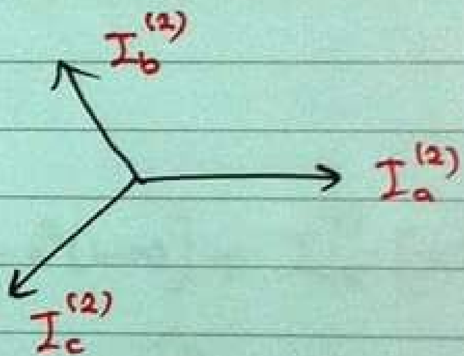
$$I_a^{(0)} + I_a^{(1)} + I_a^{(2)} = \left(I_{ab}^{(0)} + I_{ab}^{(1)} + I_{ab}^{(2)}\right) - \left(I_a^{(0)} + I_a^{(1)} + I_a^{(2)}\right)$$

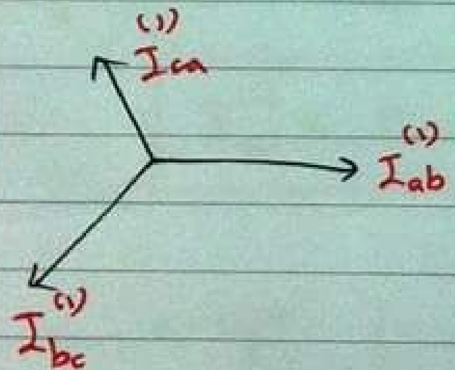
$$I_a^{(1)} + I_a^{(2)} = \underbrace{\cancel{\left(I_{ab}^{(0)} - I_{ca}^{(0)}\right)}}_{0} + \left(I_{ab}^{(1)} - I_{ca}^{(1)}\right) + \left(I_{ab}^{(2)} - I_{ca}^{(2)}\right)$$

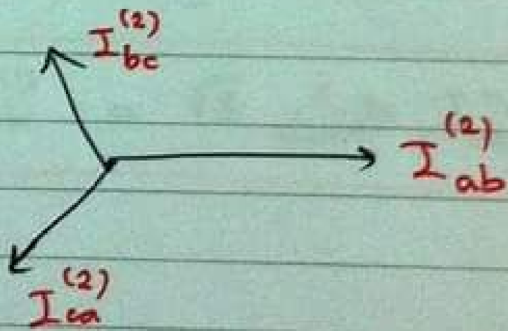
because $I_{ab}^{(0)} = I_{bc}^{(0)} = I_{ca}^{(0)}$

$$\therefore\ I_a^{(1)} + I_a^{(2)} = \left(I_{ab}^{(1)} - I_{ca}^{(1)}\right) + \left(I_{ab}^{(2)} - I_{ca}^{(2)}\right)$$









* If ① and ② are solved for $I_a^{(2)}$, it can be found that:

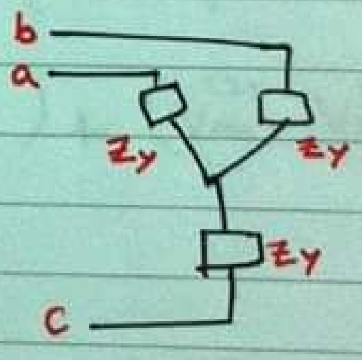
$$I_a^{(2)} = \sqrt{3}\,\angle 30\ I_{ab}^{(2)}$$

note: similar expression can be ~~found~~ obtained for $I_b^{(1)}$ & $I_b^{(2)}$ by replacing (I_{ab}) by (I_{bc})
$I_c^{(1)}$, $I_c^{(2)}$ by replacing (I_{ab}) by (I_{ca})

* symm- components of voltage:

* consider Y connection:

$V_{ab} = V_{an} - V_{bn}$ --- ①
$V_{bc} = V_{bn} - V_{cn}$ --- ②
$V_{ca} = V_{cn} - V_{an}$ --- ③



* let (V_{an}) reference for V_{an}, V_{bn}, V_{cn}
" (V_{ab}) " " V_{ab}, V_{bc}, V_{ca}

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* now;

$$V_{ab} \triangleq = \frac{1}{3}(V_{ab} + V_{bc} + V_{ca}) \quad \text{----} \; (4)$$

* sub 1, 2, 3 into (4) : $\boxed{V_{ab}^{(0)} = 0}$

Since $V_{ab}^{(0)} = V_{bc}^{(0)} = V_{ca}^{(0)} = 0$

∴ line voltages doesn't have zero seq. comp.

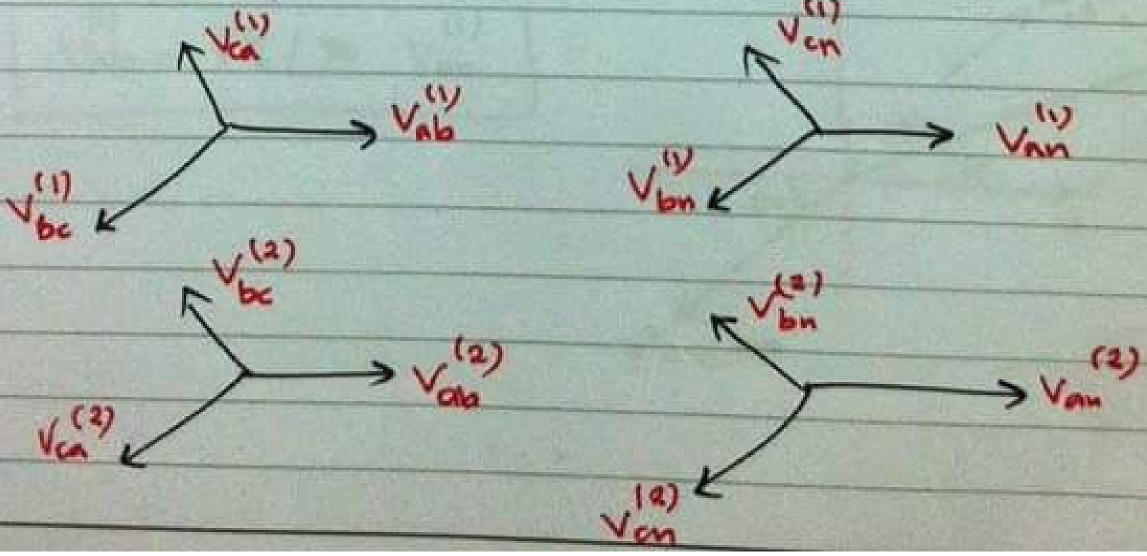
* rewrite (1) in terms of its symm. components:

$$\left(\cancel{V_{ab}^{(0)}}_{\,0} + V_{ab}^{(1)} + V_{ab}^{(2)}\right) = \left(V_{an}^{(0)} + V_{an}^{(1)} + V_{an}^{(2)}\right) - \left(V_{bn}^{(0)} + V_{bn}^{(1)} + V_{bn}^{(2)}\right)$$

$$\therefore \; V_{ab}^{(1)} + V_{ab}^{(2)} = \left(V_{an}^{(1)} - V_{bn}^{(1)}\right) + \left(V_{an}^{(2)} - V_{bn}^{(2)}\right) \quad \text{----} \; (5)$$

→ Since $\left(V_{an}^{(0)} - V_{bn}^{(0)}\right) = 0$

$$V_{ab}^{(1)} + V_{ab}^{(2)} = \left(V_{an}^{(1)} - a^2 V_{an}^{(1)}\right) + \left(V_{an}^{(2)} - a V_{an}^{(2)}\right)$$



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$$\Rightarrow \boxed{V_{ab}^{(1)} + V_{ab}^{(2)} = V_{an}^{(1)}(1-a^2) + V_{an}^{(2)}(1-a)} \quad \text{--- (6)}$$

by using (2) similar equation can be written for V_{bc}:

$$V_{bc}^{(1)} + V_{bc}^{(2)} = V_{bn}^{(1)}(1-a^2) + V_{bn}^{(2)}(1-a)$$

$$a^2 V_{ab}^{(1)} + a V_{ab}^{(2)} = a^2 V_{an}^{(1)}(1-a^2) + a V_{an}^{(2)}(1-a) \quad \text{---- (7)}$$

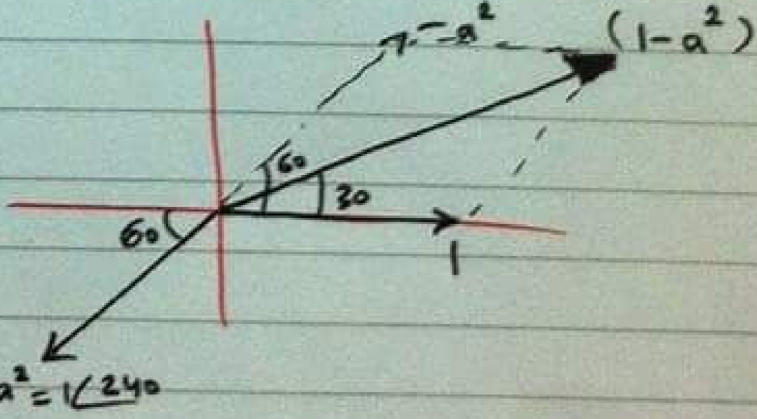
$\Rightarrow$ $a * (6) - (7)$:

$$V_{ab}^{(1)}(a-a^2) = V_{an}^{(1)}\left(a(1-a^2) - a^2(1-a^2)\right) + V_{an}^{(2)}\left(\underbrace{a(1-a) - a(1-a)}_{0}\right)$$

$$V_{ab}^{(1)}\cancel{(a-a^2)} = V_{an}^{(1)}(1-a^2)\cancel{(a-a^2)}$$

$$* \quad \boxed{V_{ab}^{(1)} = V_{an}^{(1)}(1-a^2)}$$

$$\therefore \boxed{V_{ab}^{(1)} = \sqrt{3}\angle 30 \; V_{an}^{(1)}}$$



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* if the same 2 equations are solved for $V_{ab}^{(2)}$, it can be found:

$$V_{ab}^{(2)} = \sqrt{3}\ \angle{-30}\ V_{an}^{(2)}$$

* note: similar equations can be obtained for V_{bc} and V_{ca} by:

replacing (V_{ab}) by (V_{bc}) and (V_{an}) by (V_{bn})

and " (V_{ab}) by (V_{ca}) and (V_{an}) by (V_{cn})

* Power in terms of symm- components :

* conclusions and comments :

⇒ symm- components

$I_a^{(1)} = \sqrt{3}\ I_{ab}^{(1)} \angle -30^\circ$ --- ①

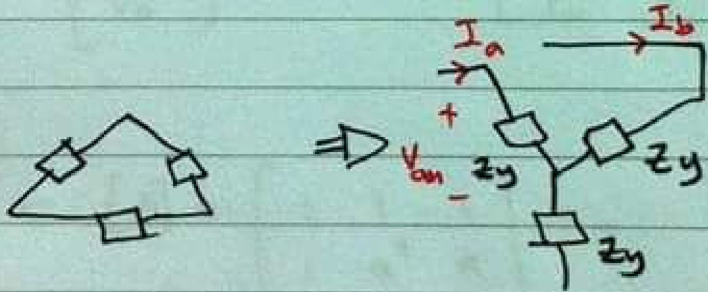
$I_a^{(2)} = \sqrt{3}\ I_{ab}^{(2)} \angle 30$ --- ②

$V_{ab}^{(1)} = \sqrt{3}\ V_{an}^{(1)} \angle 30$ --- ③

$V_{ab}^{(2)} = \sqrt{3}\ V_{an}^{(2)} \angle -30$ --- ④

$$Z_D = \frac{V_{ab}^{(1)}}{I_{ab}^{(1)}} = \frac{\sqrt{3}\ V_{an}^{(1)} \angle 30}{I_a^{(1)}/\sqrt{3}\ \angle 30}$$ by using 3, 1

$$Z_D = \frac{\cancel{\sqrt{3}}\,3\ V_{an}^{(1)} \angle 30}{\cancel{\sqrt{3}}\ I_a^{(1)} \angle 30} = \frac{3\ V_{an}^{(1)}}{I_a^{(1)}} = 3 Z_y$$



* power in terms of symm. components:

$$S \triangleq V_{an} I_a^* + V_{bn} I_b^* + V_{cn} I_c^*$$

$$= \begin{bmatrix} V_{an} & V_{bn} & V_{cn} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^*$$

$$\therefore \quad S = \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix}^T \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^* = \left([A] \begin{bmatrix} V_a^{(0)} \\ V_a^{(1)} \\ V_a^{(2)} \end{bmatrix} \right)^T * \left[[A] \begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} \right]^*$$

$$\rightarrow \quad S = \begin{bmatrix} V_a^{(0)} \\ V_a^{(1)} \\ V_a^{(2)} \end{bmatrix}^T [A]^T [A]^* \begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix}^* \text{ --- } (1)$$

$$* \quad [A]^T = [A] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}, \quad [A^*] = \begin{bmatrix} 1^* & 1^* & 1^* \\ 1^* & a^{2*} & a^* \\ 1^* & a^* & a^{2*} \end{bmatrix}$$

where:

$$\begin{cases} 1^* = 1 \\ a^* = 1\angle{-120} = 1\angle{240} = a^2 \\ a^{2*} = 1\angle{-240} = 1\angle{120} = a \end{cases}$$

$$[A]^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

⇒ Also it can be shown:

$$1 + a + a^2 = 0$$

$$[A]^T [A]^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \text{---} (2)$$

substitute (2) in (1):

$$S = \underbrace{\begin{bmatrix} V_a^{(0)} & V_a^{(1)} & V_a^{(2)} \end{bmatrix}}_{1\times3} \underbrace{\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}}_{3\times3} \underbrace{\begin{bmatrix} I_a^{(0)*} \\ I_a^{(1)*} \\ I_a^{(2)*} \end{bmatrix}}_{3\times1}$$

$$\Rightarrow \quad S = 3 V_a^{(0)} I_a^{(0)*} + 3 V_a^{(1)} I_a^{(1)*} + 3 V_a^{(2)} I_a^{(2)*}$$

* Ex: A balanced Y connected resistive load of 10Ω have the following voltages at its terminals:

$V_{ab} = 100\angle 0^\circ$, $V_{bc} = 80.8\angle -121.44^\circ$, $V_{ca} = 90\angle 130^\circ$ V

by assuming that their is no connection to the nuetral of the load. find:

1) line currents from the symm-comp. of the given lines voltages.

2) the supplied power by using symm-comp. of voltage and current.

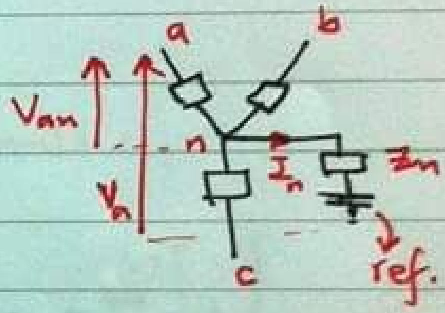
→

* sequence ckt of power system analysis :-

* objective : to find the equivalent zero, +ve, -ve sequence ccts of Generator, transformer, line, load.

* consider a 3ph Y connected load:

V_{an} = phase voltage
V_a = voltage between line terminal and ref.



* By KVL:

$$V_a = V_{an} + I_n Z_n \quad \text{--- (1)}$$
$$V_b = V_{bn} + I_n Z_n \quad \text{--- (2)}$$
$$V_c = V_{cn} + I_n Z_n \quad \text{--- (3)}$$
$$I_n = I_a + I_b + I_c$$
$$= (I_a^{(0)} + I_a^{(1)} + I_a^{(2)}) + (I_b^{(0)} + I_b^{(1)} + I_b^{(2)}) + (I_c^{(0)} + I_c^{(1)} + I_c^{(2)})$$
$$= (I_a^{(0)} + I_b^{(0)} + I_c^{(0)}) + \underbrace{\cancel{(I_a^{(1)} + I_b^{(1)} + I_c^{(1)})}}_{0} + \underbrace{\cancel{(I_a^{(2)} + I_b^{(2)} + I_c^{(2)})}}_{0}$$

$I_a^{(0)} = I_b^{(0)} = I_c^{(0)}$

(balanced)

$$\Rightarrow \boxed{I_n = 3 I_a^{(0)}} \quad \text{--- (4)}$$

* Rewrite 1, 2, 3:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} + 3Z_n I_a^{(0)}$$

$$= \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} + 3Z I_a^{(0)} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{---- } (5)$$

* rewrite in terms of symm ~~—~~ components:

$$[A] \begin{bmatrix} V_a^{(0)} \\ V_a^{(1)} \\ V_a^{(2)} \end{bmatrix} = [A] \begin{bmatrix} V_{an}^{(0)} \\ V_{an}^{(1)} \\ V_{an}^{(2)} \end{bmatrix} + 3Z_n I_a^{(0)} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{--- } (6)$$

$\Rightarrow$ ~~[illegible]~~ (6) $*$ $[A]^{-1}$:

$$\begin{bmatrix} V_{a}^{(0)} \\ V_a^{(1)} \\ V_a^{(2)} \end{bmatrix} = \begin{bmatrix} V_{an}^{(0)} \\ V_{an}^{(1)} \\ V_{an}^{(2)} \end{bmatrix} + 3Z_n I_a^{(0)} [A]^{-1} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{--- } (7)$$

note:
$V_{an} = Z_y I_a$
$V_{bn} = Z_y I_b$
$V_{cn} = Z_y I_c$

$$\Rightarrow \quad [A]\begin{bmatrix} V_{an}^{(0)} \\ V_{an}^{(1)} \\ V_{an}^{(2)} \end{bmatrix} = Z_y\,[A]\begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix}$$

$$\therefore \quad \begin{bmatrix} V_{an}^{(0)} \\ V_{an}^{(1)} \\ V_{an}^{(2)} \end{bmatrix} = Z_y\begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} \quad --- \; 8$$

sub 8 into 7 :

$$\begin{bmatrix} V_a^{(0)} \\ V_a^{(1)} \\ V_a^{(2)} \end{bmatrix} = Z_y\begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} + 3Z_n I_a^{(0)}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad --- \; 9$$

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$$* \quad \begin{bmatrix} V_a^{(0)} \\ V_a^{(1)} \\ V_a^{(2)} \end{bmatrix} = Z_Y \begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} + 3 I_a^{(0)} Z_n \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

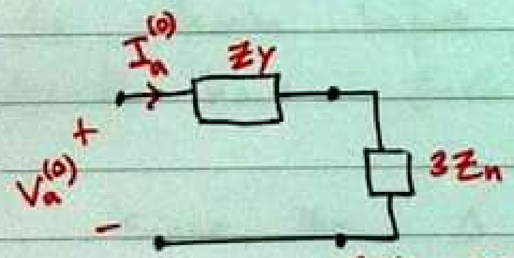
$$V_a^{(0)} = Z_Y I_a^{(0)} + 3 I_a^{(0)} Z_n \quad \text{---} ①$$

$$V_a^{(1)} = Z_Y I_a^{(1)} + 0 \quad \text{----} ②$$

$$V_a^{(2)} = Z_Y I_a^{(2)} \quad \text{----} ③$$

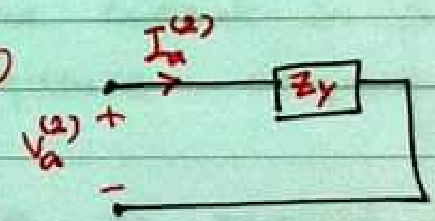
* 1, 2, 3 represent 3 decoupled equations.
Hence, these equations can be used to deduce the equivalent sequence ckt

①



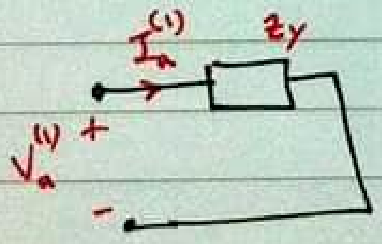
zero- seq.

③



-ve sequence

②



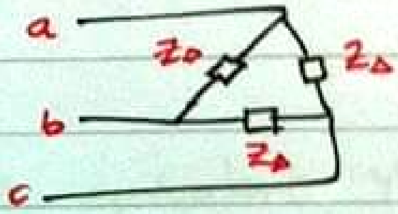
+ve sequence

$\therefore$

$$Z_0 = Z_Y + 3Z_n$$
$$Z_1 = Z_Y$$
$$Z_2 = Z_Y$$

* Consider the Δ_connected load:

$$V_{ab} = I_{ab} Z_\Delta$$
$$V_{bc} = I_{bc} Z_\Delta$$
$$V_{ca} = I_{ca} Z_\Delta$$



$\therefore \; (V_{ab} + V_{bc} + V_{ca}) = (Z_\Delta \; I_{ab} + I_{bc} + I_{ca})$

0

$$3 V_{ab}^{(0)} = Z_\Delta (3 I_{ab}^{(0)})$$

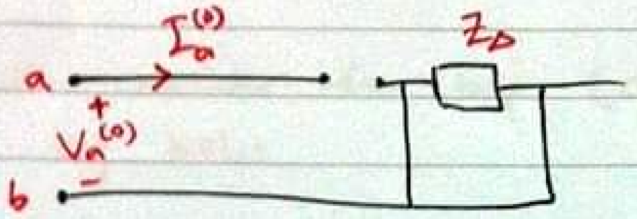
$$\therefore \quad 0 = 3 V_{ab}^{(0)} = Z_\Delta (3 I_{ab}^{(0)})$$

$$\therefore \quad V_{ab}^{(0)} = I_{ab}^{(0)} = \boxed{0}$$

➡ As shown before, line current in Δ connection doesn't have zero sequence component.

Since the Δ has no neutral return earth path

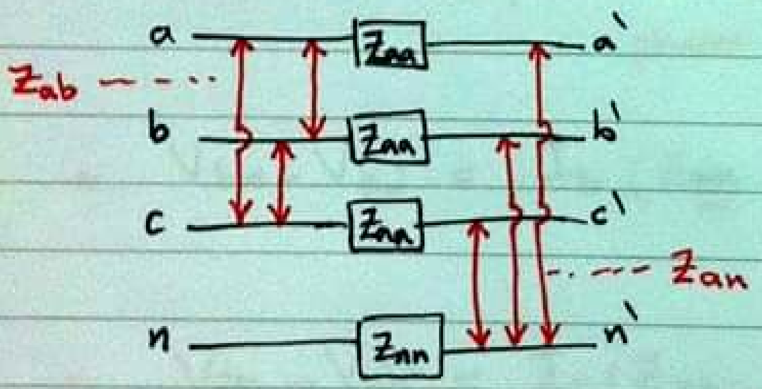
3



Hence, in converting Δ to its equivalent Y, then there is no connection between nuetral and ref.

* Sequence ccts of T.L :

* consider the following section of a symmetrical T.L :



* Z_{aa} = self impedence of each phase conductor
* Z_{nn} = self impedence of ~~each~~ neutral
* Z_{ab} = mutual impedence between phases
* Z_{an} = mutual impedence between the ~~two~~ neutral and each phase

* consider the loop $(a\,a'\,n'\,n\,a)$
by KVL:

$$V_{an} = I_a Z_{an} + I_b Z_{ab} + I_c Z_{ab} + I_n Z_{an} + V_{a'n'}$$
$$- \left(I_n Z_{nn} + I_a Z_{an} + I_b Z_{nn} + I_c Z_{an} \right)$$

$$\therefore \quad V_{an} - V_{a'n'} = I_a (Z_{aa} - Z_{an}) + I_b (Z_{ab} - Z_{an})$$
$$+ I_c (Z_{ab} - Z_{an}) + I_n (Z_{an} - Z_{nn})$$

$$\therefore \quad V_{an} - V_{a'n'} = I_a (Z_{aa} - Z_{an}) + (I_b + I_c)(Z_{ab} - Z_{an})$$
$$+ I_n (Z_{an} - Z_{nn}) \quad \text{---} \; (1)$$

* Similar equations can be written for phases b, c as follows:

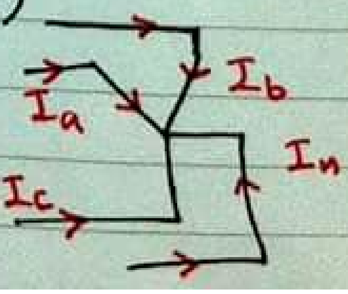
* $$V_{bn} - V_{b'n'} = I_b (Z_{aa} - Z_{an}) + (I_a + I_c)(Z_{ab} - Z_{an})$$
$$+ I_n (Z_{an} - Z_{nn}) \quad \text{---} \; (2)$$

* $$V_{cn} - V_{c'n'} = I_c (Z_{aa} - Z_{an}) + (I_a + I_b)(Z_{ab} - Z_{an})$$
$$+ I_n (Z_{an} - Z_{nn}) \quad \text{---} \; (3)$$

but: by KCL:

$$I_n = -(I_a + I_b + I_c)$$

(4)



* sub 4 into 1, 2, 3 : and rearrange :

$$V_{an} - V_{a'n'} = I_a \underbrace{(Z_{aa} - 2Z_{an} + Z_{nn})}_{Z_s}$$
$$+ I_b \underbrace{(Z_{ab} - 2Z_{an} + Z_{nn})}_{Z_m}$$
$$+ I_c (Z_{ab} - 2Z_{an} + Z_{nn}) \quad \text{----} \; (5)$$

$$V_{bn} - V_{b'n'} = I_a (Z_{ab} - 2Z_{an} + Z_{nn})$$
$$+ I_b (Z_{an} - 2Z_{an} + Z_{nn})$$
$$+ I_c (Z_{ab} - 2Z_{an} + Z_{nn}) \quad \text{----} \; (6)$$

$$V_{cn} - V_{c'n'} = I_a (Z_{ab} - 2Z_{an} + Z_{nn}) +$$
$$+ I_b (Z_{ab} - 2Z_{an} + Z_{nn})$$
$$+ I_c (Z_{aa} - 2Z_{an} + Z_{nn}) \quad \text{----} \; (7)$$

let:
$$Z_s = Z_{an} - 2Z_{an} + Z_{nn}$$
$$Z_m = Z_{ab} - 2Z_{an} + Z_{nn}$$

let:
$$V_{aa'} = V_{an} - V_{a'n'}$$
$$V_{bb'} = V_{bn} - V_{b'n'}$$
$$V_{cc'} = V_{cn} - V_{c'n'}$$

* 5, 6, 7 can be written as:

$$\begin{bmatrix} V_{aa'} \\ V_{bb'} \\ V_{cc'} \end{bmatrix} = \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \text{-----} ⑧$$

* express ⑧ in terms of symm. components:

$$[A] \begin{bmatrix} V^{(0)}_{aa'} \\ V^{(1)}_{aa'} \\ V^{(2)}_{aa'} \end{bmatrix} = \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} [A] \begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix} \text{----} ⑨$$

* multiply ⑨ by $[A]^{-1}$:

$$\begin{bmatrix} V^{(0)}_{aa'} \\ V^{(1)}_{aa'} \\ V^{(2)}_{aa'} \end{bmatrix} = [A]^{-1} \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} [A] \begin{bmatrix} I_a^{(0)} \\ I_a^{(1)} \\ I_a^{(2)} \end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} Z_s + 2Z_m & 0 & 0 \\ 0 & Z_s - Z_m & 0 \\ 0 & 0 & Z_s - Z_m \end{bmatrix}$$

$$V_{aa'}^{(0)} = I_a^{(0)}(Z_s + 2Z_m) = I_a^{(0)}\underbrace{(Z_{aa} + 2Z_{ab} - 6Z_{an} + 3Z_{nn})}_{Z_0}$$

$$V_{aa'}^{(1)} = I_a^{(1)}\underbrace{(Z_{aa} - Z_{ab})}_{Z_1}$$

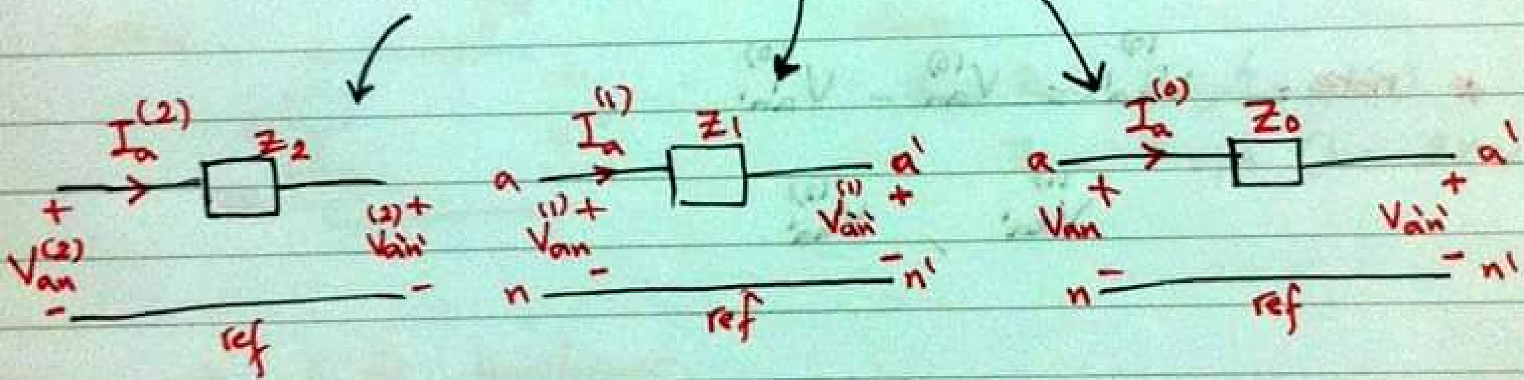
$$V_{aa'}^{(2)} = I_a^{(2)}\underbrace{(Z_{aa} - Z_{ab})}_{Z_2}$$

$$\Downarrow$$

$$V_{aa'}^{(0)} = I_a^{(0)} Z_0$$
$$V_{aa'}^{(1)} = I_a^{(1)} Z_1$$
$$V_{aa'}^{(2)} = I_a^{(2)} Z_2$$

* where: $Z_1 = Z_2$

$Z_{aa} = R + j\omega L$

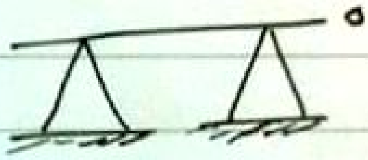


+ve sequence　　-ve sequence　　Zero-seq cct

* Comment :

usualy T.L have over head conductor (2 or more) which are ~~neglected~~ grounded at regular points

there conductors with the earth provide the so called:



(effective neutral return port)

* note: $V_{aa'}^{(0)} = V_{an}^{(0)} - V_{an'}^{(0)}$

$V_{aa'}^{(1)}$, $V_{aa'}^{(2)}$

* note: by applying some analysis and procedure one can find sequence. ccts of gen. + transf.

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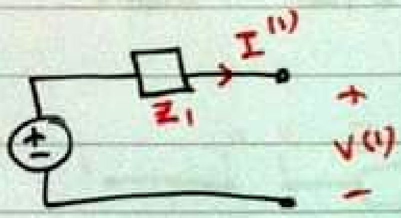
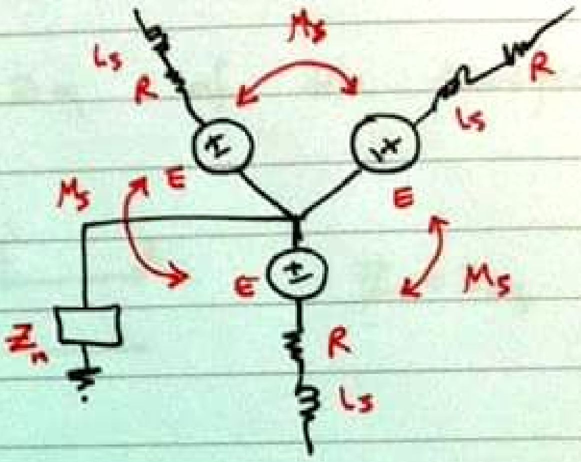
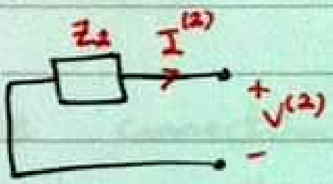
* Sequence ccts of 3 phase generator:

(+ve)

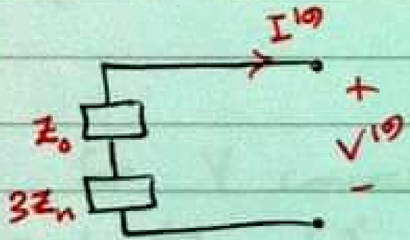




(-ve)



(zero)



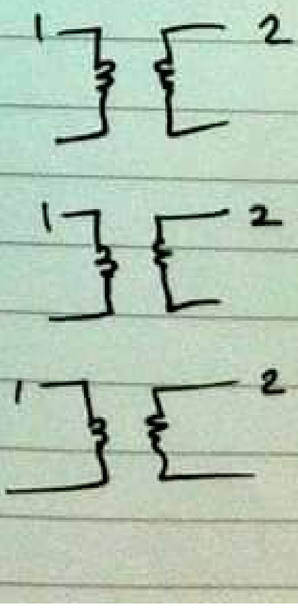
where:

$$Z_1 = Z_2 = R + j\omega(L_s + M_s)$$

$$Z_0 = R + j\omega(L_s - 2M_s)$$

* 3-ph transformer:

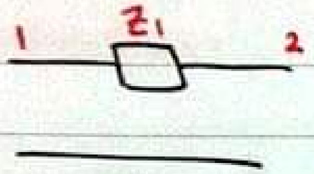
each side (1 and 2) can be connected as Δ or Y hence as shown before there are 4 types of connections YY, ΔΔ, ΔY, YΔ



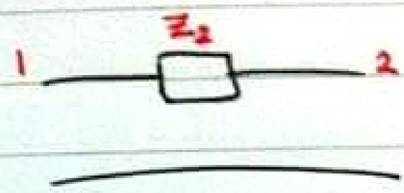
→

1°

+ve :

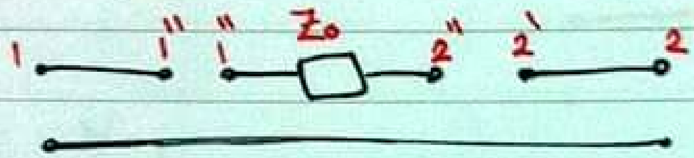


−ve :



where : $Z_1 = Z_2 = Z_{eq}$ of the transformer

Zero :



$Z_0 = Z_{eq}$

* the connection between (1' and 1'') and (2' and 2'') depends on the type of connection for the given side :

i) for Y connection, then connect (1', 1'') and /OR (2', 2'') by an impedance $= 3Z_n$ where Z_n = earthing impedance of the neutral.

ii) for Δ connection, then short cct 1'' or 2'' to ~~the~~ the ref.

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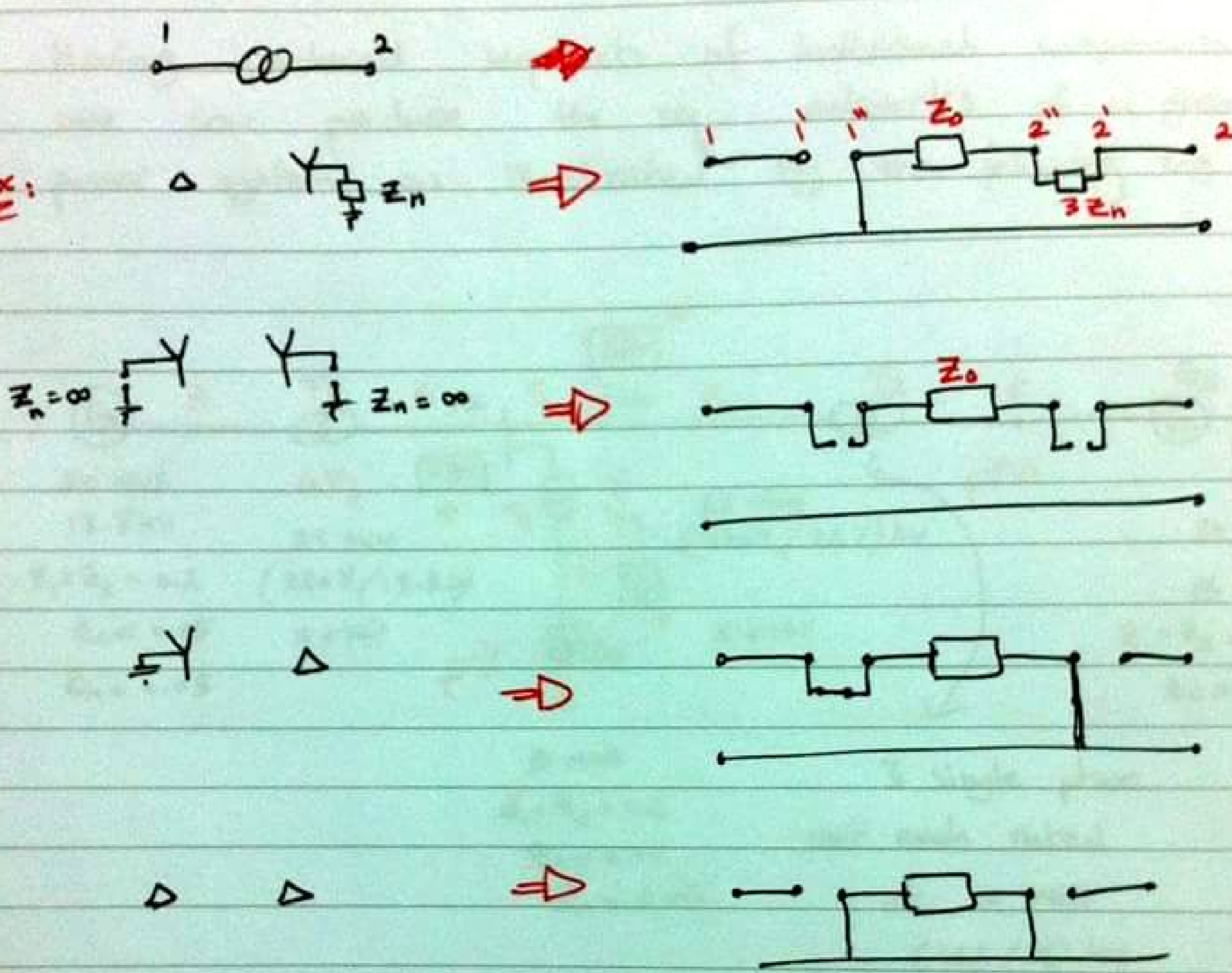
* illustration:

ex:



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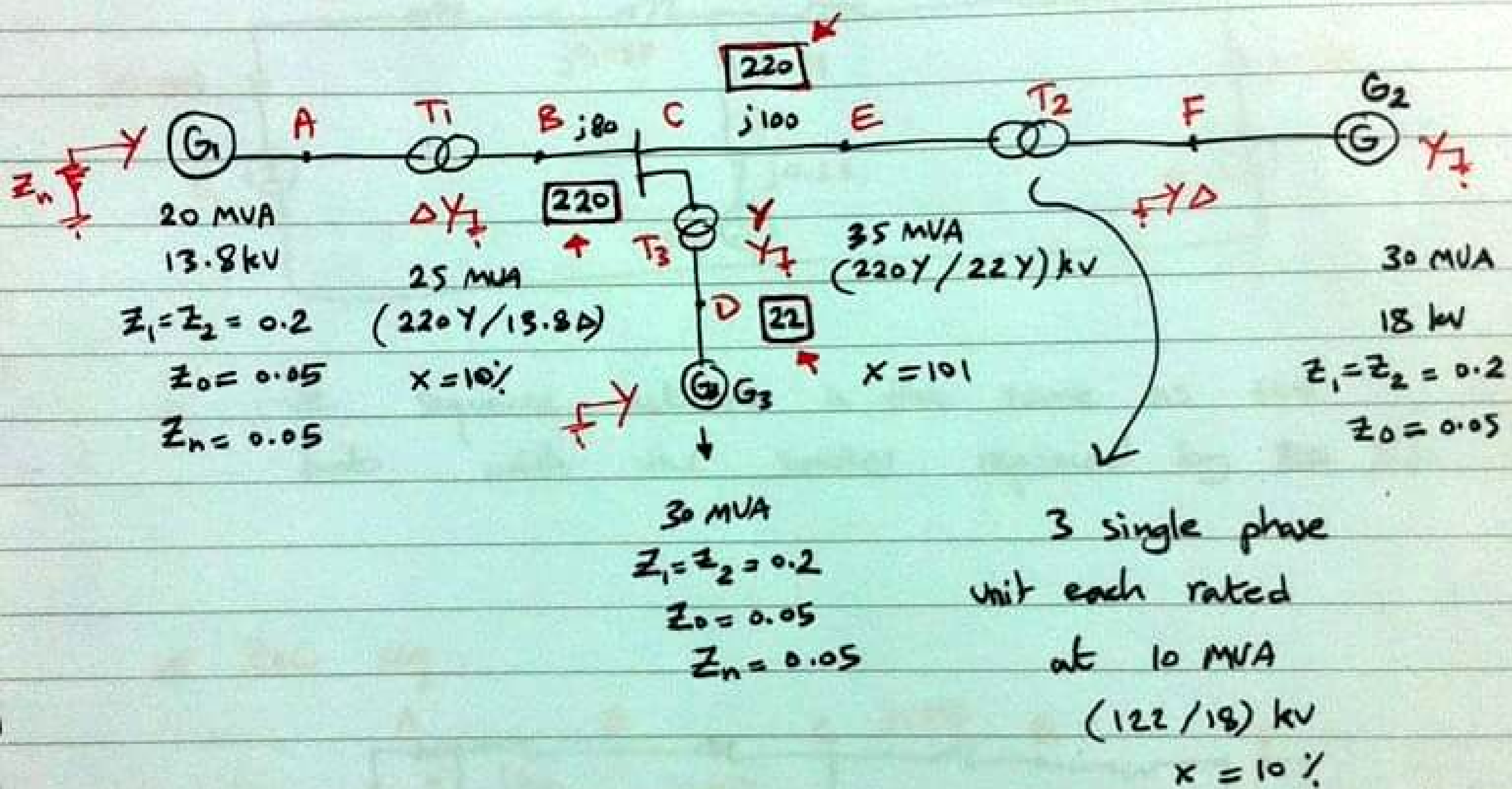
* Sequence networks:

Having produced seq. ccts. of individual components, one can produce the seq. networks of a given power system as illustrated by the following e.g:



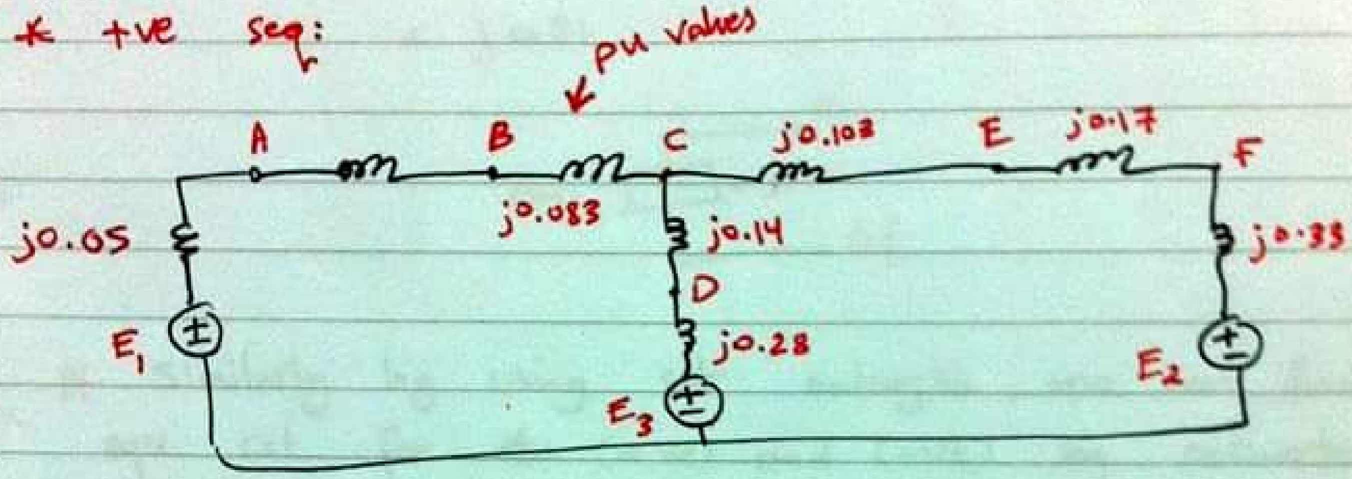
* for the given power system draw the per unit +ve, -ve, 0 seq networks by using phase values of (50 MVA) and (13.8 kV), also the zero seq impedence of T.L are for BC, $Z_0 = 210\ \Omega$

CE, $Z_0 = 250\ \Omega$

* put base voltages

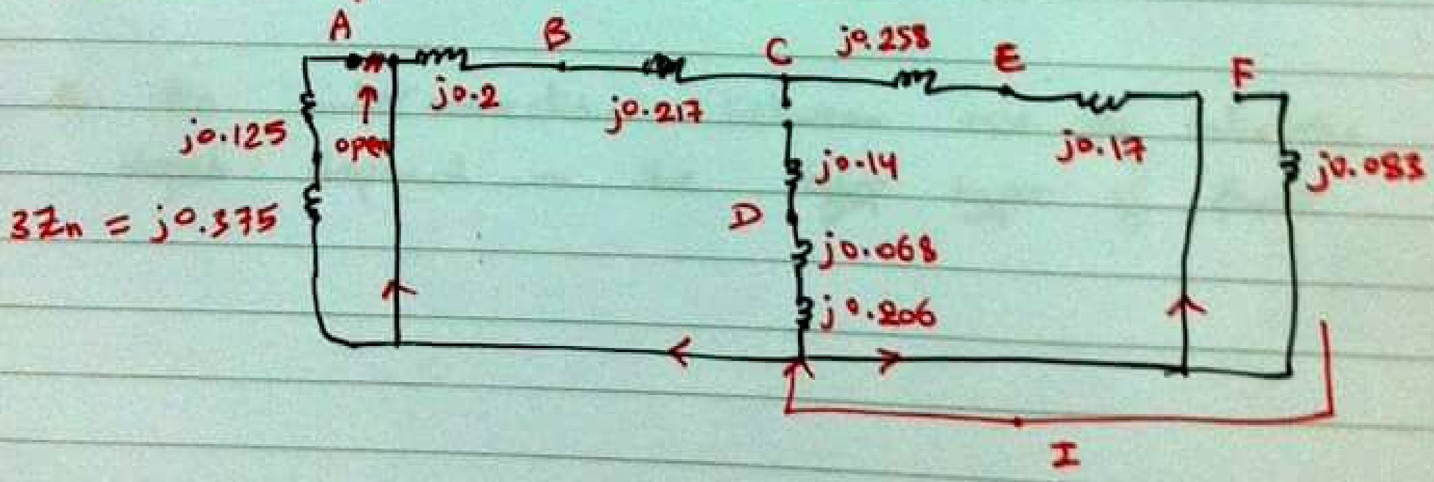
* note:
The pu values of each component are based on the rating of each component.

* +ve seq:



* -ve sequence network is the same as +ve but with the sources replaced by S.c

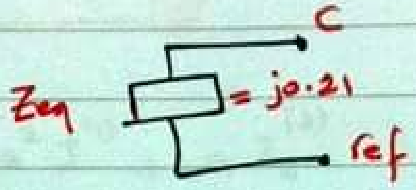
* Zero seq.:



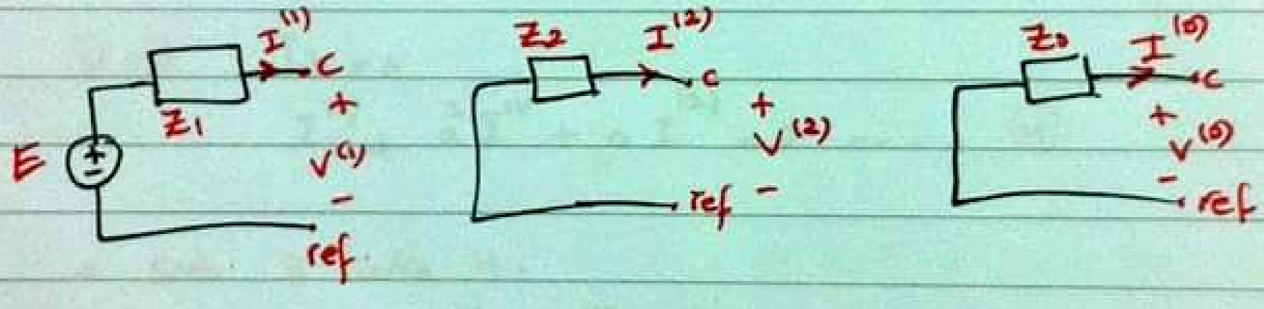
* Comments:

i) for example: find Z_{eqn} between C and the ref

$$Z_{eq} = (j0.217 + j0.2) \,//\, (j0.258 + j0.17)$$
$$= j0.21$$



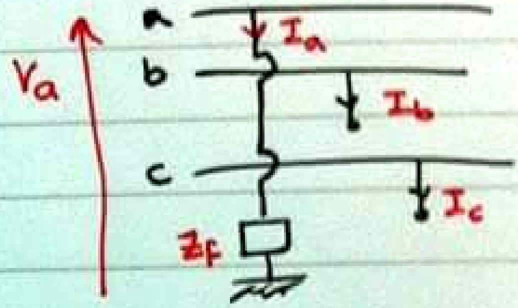
* similarly by using cct anlaysis, one can find equ. cct. for ~~the~~ (+ve) and (−ve) seq. networks



* the interconnection between tve seq. networks ~~deped~~ depend on the ~~type~~ type of fault

* Single line - to - ground fault

fig (1)



* from fig (1):

$I_b = I_c = 0$ ------ (1)

$V_a = I_a Z$ ------ (2)

Z_f = fault imped.

(1) $\Rightarrow$ $I_a^{(0)} + a^2 I^{(1)} + a I_2^{(2)}$

$= I_a^{(0)} + a I^{(1)} + a^2 I^{(2)}$

$\therefore \; I^{(1)} \cancel{(a^2 - a)} = I^{(2)} \cancel{(-a + a^2)}$

$\therefore \; I_b^{(1)} = I^{(2)}$ ------ (3)

(1) $\Rightarrow$ $I_b = 0$

$I^{(0)} + a^2 I^{(1)} + a I^{(2)} = 0$ ---- (4)

* Sub 3 into 4:

$I^{(0)} + I^{(1)} \underbrace{(a^2 + a)}_{\to -1} = 0$, since $(1 + a = 0)$

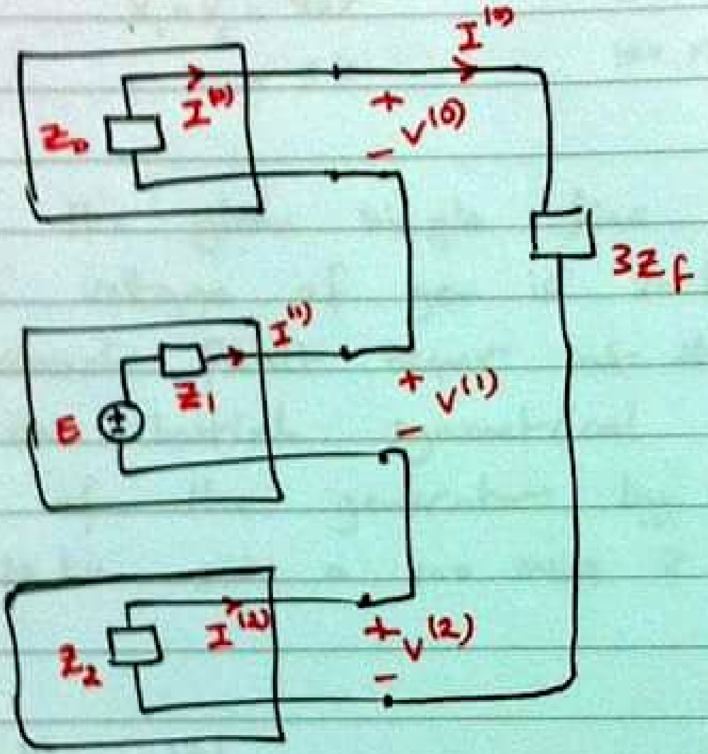
$\therefore \; I^{(0)} - I^{(1)} = 0$

$\boxed{I^{(0)} = I^{(1)} = I^{(2)}}$ --- (5)

(2) $\Rightarrow$ $V^{(0)} + V^{(1)} + V^{(2)} = Z_f \left(I^{(0)} + I^{(1)} + I^{(2)} \right)$

$\boxed{V^{(0)} + V^{(1)} + V^{(2)} = 3 Z_f I^{(0)}}$ --- (6)

* Sequence networks should be connected in such away that ⑤ and ⑥ are satisfied:



$$\therefore \quad I^{(0)} = I^{(1)} = I^{(2)} = \frac{E}{Z_0 + Z_1 + Z_2 + Z_f}$$

$$V^{(0)} = -I^{(0)} Z_0$$
$$V^{(1)} = E - I^{(1)} Z_1$$
$$V^{(2)} = -I^{(2)} Z_2$$

* Having found $[I_{012}]$ and $[V_{a12}]$ one can find

$[I_{abc}]$ and $[V_{abc}]$

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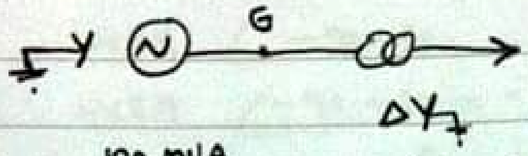
*** e.g.:**



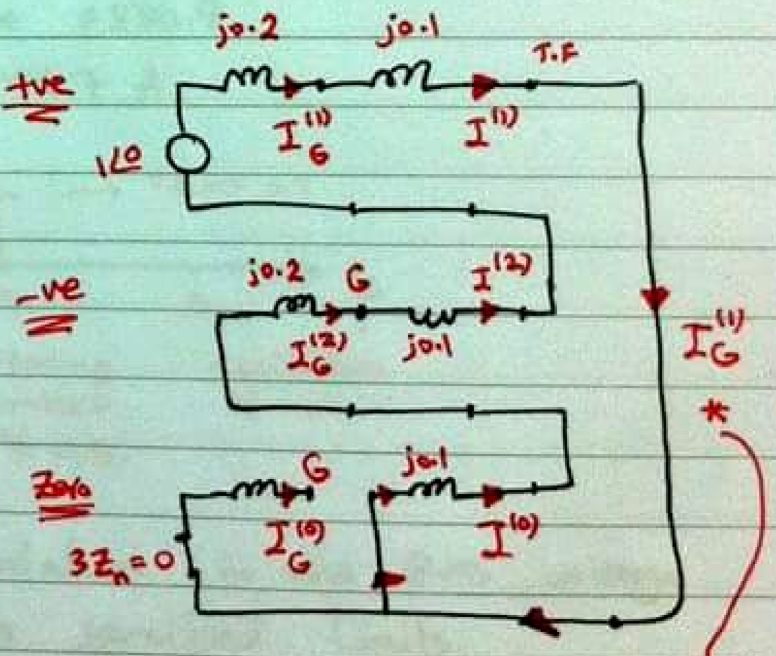
100 MVA
20 kv
$X_1 = X_2 = 20\%$
$X_0 = 5\%$

(20 Δ / 230 Y) kv
$X = 10\%$
100 MVA

* for the given single line diagram, when the terminal voltage of gen is 20 kv, a single line to ground fault occur at the high voltage side, → Find the initial symmetrical current and the phases of the generator by using phase values of 20 kv and a 100 MVA ?

* $I^{(0)} = I^{(1)} = I^{(2)} = ?$

$$= \frac{1\angle 0}{j0.2 + j0.1 + j0.2 + j0.1 + j0.1}$$



* for the transformer:

$$I_{abc} = [A][I_{012}]$$

$$= \begin{bmatrix} -4.287j \\ 0 \\ 0 \end{bmatrix}$$

↳

note here
$Z_f = 0$
i.e.: S/C
direct to grnd

$I_G^{(1)} = 1.429 \angle -90-30$

$*\; I_G^{(2)} = 1.429 \angle -90+30 = -60$

$I_G^{(0)} = 0$ because of the open circuit

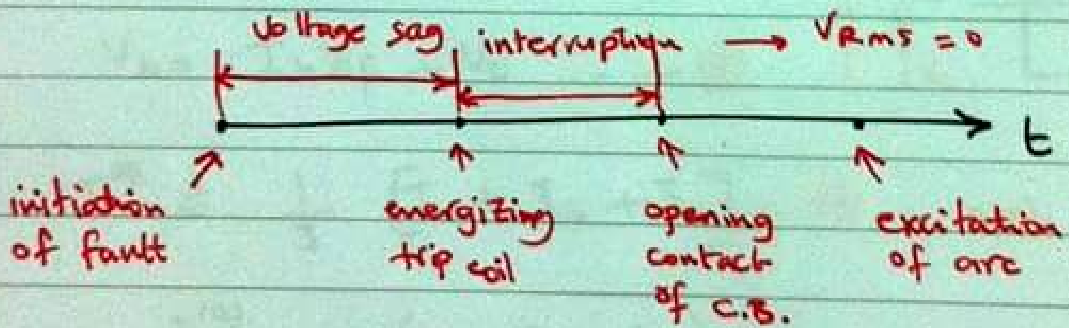
$$[I_{abc}]_G = [A]\,[I_{012}]_G = \begin{bmatrix} 2.475 \angle -90 \\ 2.475 \angle 90 \\ 0 \end{bmatrix}$$

* Comments:

$$I_{base} = I_{rated} = \frac{100 * 10^6}{\sqrt{3} * 20 * 10^3} = \boxed{2886.8}\ A$$

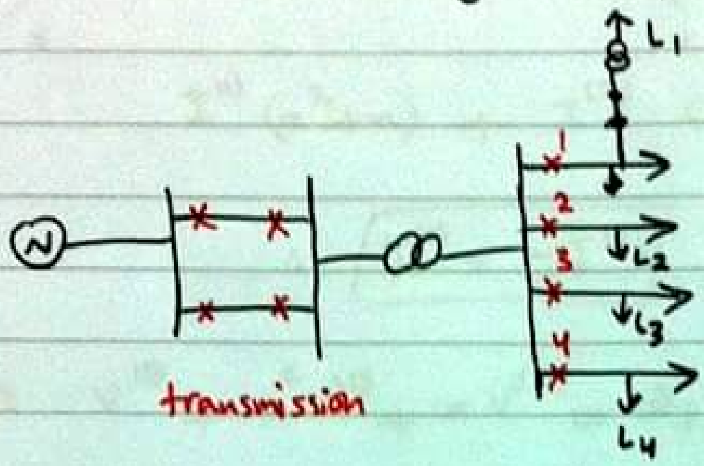
$\therefore\ I_{fault} = 2.475 * 2886.8$

$= 7144.7\ A$



voltage sag: large variation in the RMS voltage outside its specified limits.

* in power system quality there are the problems of voltage sag and interruption which occur during SC faults.

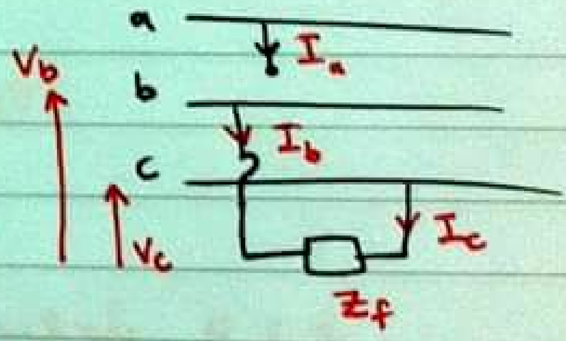


features of radial system

* line-to-line fault :

$I_a = 0$ —— ①

$I_b = -I_c$ —— ②



$V_b = I_b Z_f + V_c$ —— ③

$I^{(0)} \triangleq \frac{1}{3}\left[I_a + I_b + I_c\right]$ —— ④

sub 1,2 into ④

⇒ $I^{(0)} = \frac{1}{3}\left(0 + I_b - I_b\right)$

$I^{(0)} = 0$ —— ⑤

since, $V^{(0)} \triangleq -I^{(0)} Z_0$

$\therefore \boxed{V^{(0)} = 0}$

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* Zero seq. network is isolated:

② ⇒ $\cancel{I^{(0)}} + a^2 I^{(1)} + a I^{(2)} = -(\cancel{I^{(0)}} + a I^{(1)} + a^2 I^{(2)})$

$$I^{(1)}(a^2+a) + I^{(2)}(a+a^2) = 0$$

$$\therefore \boxed{I^{(1)} = -I^{(2)}} \quad --- ⑥$$

③ ⇒ $V^{(0)} + a^2 V^{(1)} + a V^{(2)} = Z_f(I^{(0)} + a^2 I^{(1)} + a I^{(2)}) + (V^{(0)} + a V^{(1)} + a^2 V^{(2)})$

$$V^{(1)}(a^2-a) - V^{(2)}(a^2-a) = Z_f\,(a^2 I^{(1)} - a I^{(1)})$$
$$= Z_f\, I^{(1)}\,(a^2-a)$$

$$\boxed{V^{(1)} - V^{(2)} = Z_f\, I^{(1)}} \quad --- ⑦$$

* connect seq. networks to satisfy 5, 6, 7

* second.

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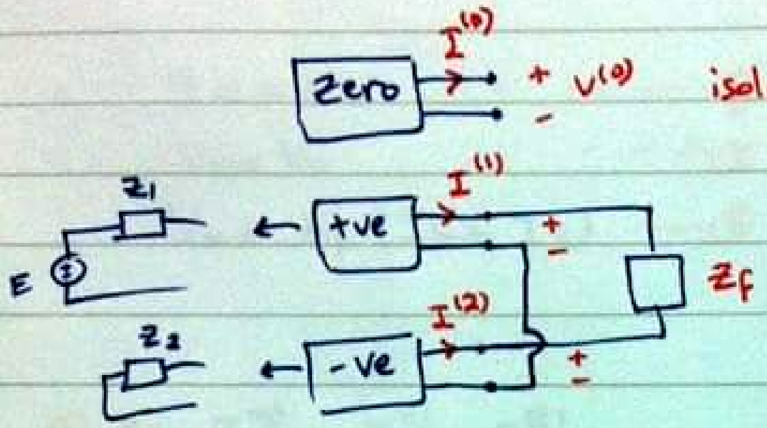
Power 1 Notebook

DR. DAIFALLAH DALBEEH

BY : SAUSAN ALMOHTASEB

DATE Monday 14/12

L-L fault:



isolated $\rightarrow I^{(0)} = V^{(0)} = 0$

Ex: solve the previous example by assuming L-L fault:

$$I^{(1)} = -I^{(2)} = \frac{E}{Z_1 + Z_2 + Z_f}$$

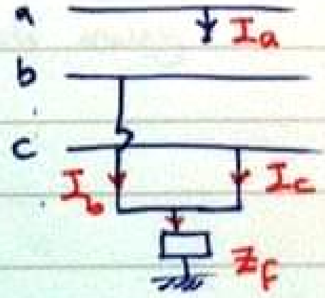
$$V_1 = E - I^{(1)} Z_1$$
$$V_2 = -I^{(2)} Z_2$$

* Line - Line - ground fault:

$\therefore \quad I_a = 0$ ---- ①

$V_b = V_c$ ---- ②

$V_b = Z_f\,(I_b + I_c)$ ---- ③



① ⇒ $I^{(0)} + I^{(1)} + I^{(2)} = 0$

$I^{(0)} = -(I^{(1)} + I^{(2)})$ ---- ④

② ⇒ $V^{(0)} + a^2V^{(1)} + aV^{(2)} = V^{(0)} + aV^{(1)} + a^2V^{(2)}$

$V^{(1)}(a^2 - a) = V^{(2)}(a^2 - a)$

$\therefore \quad V^{(1)} = V^{(2)}$ ---- ⑤

③ ⇒ $V^{(0)} + a^2V^{(1)} + aV^{(2)} = Z_f\,(I^{(0)} + a^2I^{(1)} + aI^{(2)} + I^{(0)} + aI^{(1)} + a^2I^{(2)}$

$\rightsquigarrow \quad V^{(0)} + V^{(1)}(a^2 + a) = Z_f\left(2I^{(0)} + I^{(1)}(a^2 + a) + I^{(2)}(a^2 + a)\right)$

$\therefore \quad (V^{(0)} - V^{(1)}) = Z_f\,(2I^{(0)} - I^{(1)} - I^{(2)})$

since $(a^2 + a = -1)$

$\therefore \quad = Z_f\left(2I^{(0)} - (I^{(1)} + I^{(2)})\right)$

$= 3Z_f\,I^{(0)}$

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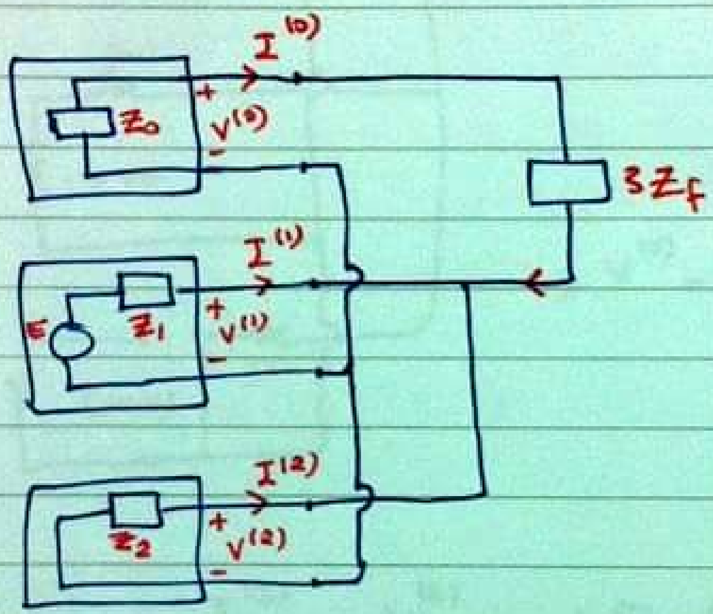
$$\therefore \boxed{V^{(0)} - V^{(1)} = 3Z_f\, I^{(0)}} \quad \text{---}\ 6$$

* connect the sequence ckts in such away to satisfy 3, 5, 6 :



→ $$I^{(1)} = \frac{E}{Z_1 + ((Z_0 + 3Z_f) \parallel Z_2)}$$

→ $$I^{(0)} = -I^{(1)} \frac{Z_0}{Z_2 + (Z_0 + 3Z_f)}$$

→ $$I^{(0)} = -(I_1 + I_2)$$

→ $V^{(0)} = -I^{(0)} Z_0$

→ $V^{(1)} = E - I^{(1)} Z_1$

→ $V^{(2)} = -I^{(2)} Z_2$

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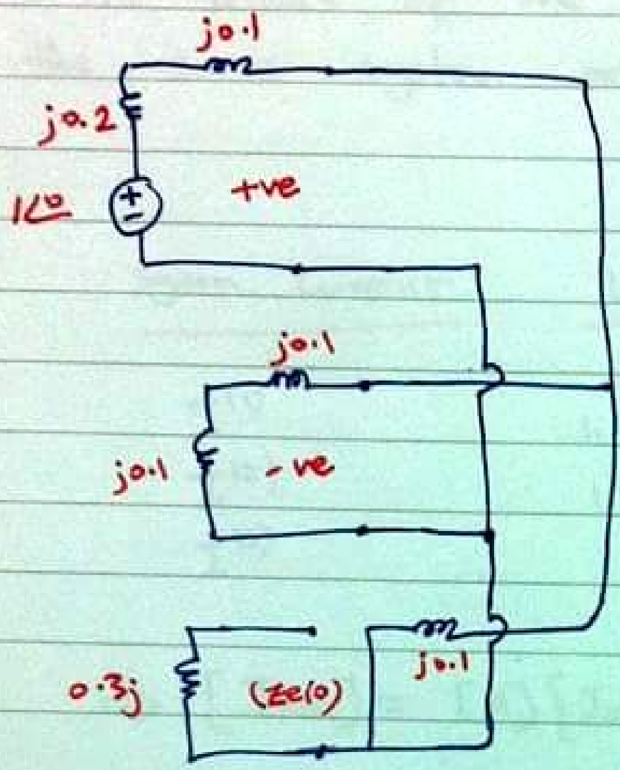
* **e.g:** Solve the previous e.g for L-L-G fault:



* by sub:

$\rightarrow I^{(1)} = \dfrac{1\angle 0}{j0.3 + (j0.3 \,//\, j0.1)}$

$= 2.67 \angle -90$

$\rightarrow V^{(0)} = V^{(2)} = V^{(1)} = E - I_1 Z_1$

$= 1\angle 0 - (2.67\angle 90)(j0.03)$

$= 0.2$

$\rightarrow \therefore I^{(2)} = \dfrac{-V^{(2)}}{Z_2} = 0.67 \angle 90$

$\rightarrow I^{(0)} = \dfrac{-V^{(0)}}{Z^{(0)}} = j2 = 2\angle 90$

* for the Generator:

$I_g^{(1)} = 2.67 \angle -90 - 30 = -120$

$I_g^{(2)} = 0.67 \angle 90 + 30 = 120$

$I_g^{(0)} = 0$

$\Rightarrow [I_{abc}] = [A][I_{012}]$ ----

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* Summary :

The results of the three types of faults for the same system are as follows:

* Symm. current	L-G	L-L	L-L-G
$I^{(1)}$	$1.429\angle{-120}$	$1.67\angle{-120}$	$2.67\angle{-120}$
$I^{(2)}$	$1.429\angle{-60}$	$1.67\angle{-60}$	$0.67\angle{120}$
$I^{(0)}$	0	0	0

$\Rightarrow [I_{abc}] = [A]\{I_{012}\}$ for each.

* Having performed fault analysis, which value of fault current is used to set the setting of protection system component. (search)

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* Load flow or power flow:

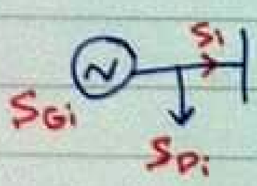
what ? : <u>defenition</u>: load flow is the conclusions of the voltages of the busbars at a given power system for a given load condition.

* having found voltages, one can find other quantities:
  1) current in the lines
  2) power flow in the power sys. components
  3) losses in the system.

why? <u>objective</u>: load flow analysis is used in the planning, design, and operation of power sys.

How? Mathematatical formulation of the load flow problem:

* consider a given busbar ~~sys~~ say the i^{th} busbar



S_{Gi} = complex generated power at the i^{th} busbar

S_{Di} = demand power at the i^{th} busbar (i.e. load)

S_i = complex power entering the i^{th} busbar

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* let: V_i = voltage at the i^{th} busbar

$\therefore \quad V_i = |V_i| \angle \delta_i$ ---- (1)

* by using the concept of bus admittance matrix

$$[I] = [Y][V]$$

$$I_i = \sum_{j=1}^{N} Y_{ij} V_j \quad \text{----- (2)}$$

I_i = current entering the busbar
N = No. of busbars

* let: $\quad Y_{ij} = |Y_{ij}| \angle \theta_{ij}$ ---- (3)

Now; $\quad S_i \triangleq V_i I_i^*$ ---- (4)

* sub 1, 2, 3 into 4:

$$S_i = |V_i| \angle \delta_i \left(\sum_{j=1}^{N} Y_{ij} \angle \theta_{ij} * |V_j| \angle \delta_i \right)^*$$

$$= |V_i| \angle \delta_i \sum_{j=1}^{N} |Y_{ij}||V_j| \angle (-\delta_j - \theta_{ij})$$

$$= \sum_{j=1}^{N} |V_i||V_j||Y_{ij}| \angle (\delta_i - \delta_j - \theta_{ij})$$

since: $\quad S_i = P_i + Q_i$

$$\therefore \quad P_i = \sum_{j=1}^{N} |V_i||V_j||Y_{ij}| \cos(\delta_i - \delta_j - \theta_{ij}) \quad \text{---- (5)}$$

$$Q_i = \sum_{j=1}^{N} |V_i||V_j||Y_{ij}| \sin(\delta_i - \delta_j - \theta_{ij}) \quad \text{---- (6)}$$

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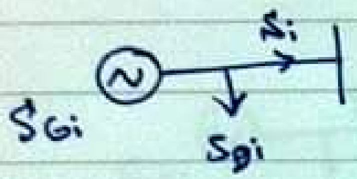
* Comments:

1) equations 5, 6 are called (power flow equations)
2) if the voltages are known then (P_i) and (Q_i) can be calculated, i.e: $P_{i,cal}$
$Q_{i,cal}$

eg:



* if, P_{Gi} = scheduled gen. power
P_{Di} = " load "

$$\begin{cases} P_{sch} = P_{Gi} - P_{Di} \\ Q_{sch} = Q_{Gi} - Q_{Di} \end{cases}$$

* Hence in the process of load flow solution
$P_{cal} \neq P_{sch}$ and $Q_{cal} \neq Q_{sch}$

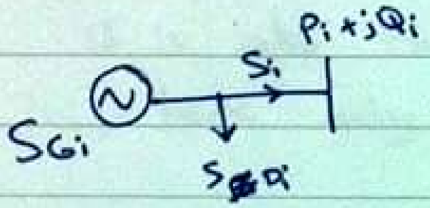
* one may say that there is a power mismach:
$\Delta P_i = P_{sch,i} - P_{cal,i}$
$\Delta Q_i = Q_{sch,i} - Q_{cal,i}$

⇒ Therefore a solution is obtained when ΔP_i and $\Delta Q_i = 0$, Hence one may say that a power balance is obtained. Consequently there are 2 functions to be satisfied.

$g_i' = P_{sch} - P_{cal} = (P_{Gi} - P_{Di}) - P_{cal} = 0$
$g_i'' = Q_{sch} - Q_{cal} = (Q_{Gi} - Q_{Di}) - Q_{cal} = 0$

→ In the 2 equations of the power flow, there are 4 unknowns: P_i, Q_i, $|V_i|$, δ_i

* Hence to overcome this problem, one has to specify values for 2 unknows and calculate values for other 2.



→ therefore, in practice 3 types of busbars are specified as follows:

i) load bus: this is a non-generator bus, Hence $P_{Gi} = Q_{Gi} = 0$ ∴ (unknowns → $|V_i|$, δ_i)
since load on the bus can be estimated by load forcast or historical data or measurement. This is usually for P_D, hence by assuming certain power factor for eg: 0.85, then Q_D can be found: $Q_D = P_D \tan\theta$
θ = P.F (angle)

∴ $Q_{i,sch} = 0 - Q_{Di}$, ∴ $P_{i,sch} = 0 - P_{Di}$

hence this bus is also called PQ - bus

∴ at this bus ΔP_i and ΔQ_i are to be satisfied.

ii) $\boxed{\text{Voltage controlled bus}}$: this is usally has a generator by means of its prime mover, one can control (P_i), and by means of its excitation → one can control $|V_i|$

∴ at this bus bar (P_i) and $|V_i|$ are specified here (Q_i) will be calculated when the load flow is complete ∴ the unknowns here is δ_i

it is also called $\boxed{\text{PV- bus}}$

→ here only $\Delta P_i = 0$ to be satisfied

iii) $\boxed{\text{reference or slack busbar}}$: as a convention busbar #1 is taken as a slack, here δ_1 is ~~satisfied~~ specified, and taken as the convention $= 0°$.

Here (P_1) and (Q_1) can't be satisfied in advance, as will be explained later.

→ Therefore, no need to satisfy $(\Delta P = 0)$ and $(\Delta Q = 0)$, at each busbar: $P_i \triangleq P_{Gi} - P_{Di}$

for the total number of busbars $= N$

there are N equations like ①

∴ Summating these equations:

$$\sum_{i=1}^{N} P_i = \sum_{i=1}^{N} P_{Gi} - \sum_{i=1}^{N} P_{Di} = P_{losses}$$

total generation ↲ ↳ total demand or load

→

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$$\Rightarrow \sum_{i=1}^{N} P_i = \sum_{i=1}^{N} P_{Gi} - \sum_{i=1}^{N} P_{Di} = P_{losses}$$

$$\Rightarrow P_{losses} = \sum 3|I|^2 R$$

this should be supplied the <u>slack bus</u>

* Similarly: $\sum Q_i = \sum Q_{Gi} - \sum Q_{Di} = Q_{losses}$ (reactive power loss)

* for the slack bus: $S_1 = P_{loss} + jQ_{loss}$

* <u>Conclusion</u>:

The unknown in the load flow problem are called state or dependent variables. Hence the number of state variables determine the number of equations to be solve as illustrated by table (9.1)

study.

→ $i=1$

→ $i=2, \dots, N_g+1$

→ $i=N_{g+2}, \dots, N$

equations $= 2N - N_g - 2$

$N = 10$ ⟶ $20 - 3 - 2 = 15$ equations

$N_g = 3$

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* Gauss - seidel method :

This method is based on the power flow equations as follows :

$$S_i = V_i I_i^*$$

$$S_i^* = V_i^* I_i$$

to simplify , let the system has 4 busbars with bus 1 is taken as a slack .

* bus 2 → $S_2^* = V_2^* I_2$

$$P_2 - jQ_2 = V_2^* \left(\sum_{j=1}^{4} Y_{ij} V_j\right)$$

$$= V_2^* \left(Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4\right)$$

$$\therefore V_2 = \frac{1}{Y_{22}} \left(\frac{P_2 - jQ_2}{V_2^*} - \left(Y_{21}V_1 + Y_{23}V_3 + Y_{24}V_4\right)\right) \text{----} (1)$$

* Similarly one can write equations for V_3 and V_4 as:

$$V_3 = \frac{1}{Y_{33}} \left(\frac{P_3 - jQ_3}{V_3^*} - \left(Y_{31}V_1 + Y_{32}V_2 + Y_{34}V_4\right)\right) \text{---} (2)$$

$$V_4 = \frac{1}{Y_{44}} \left(\frac{P_4 - jQ_4}{V_4^*} - \left(Y_{41}V_1 + Y_{42}V_2 + Y_{43}V_3\right)\right) \text{---} (3)$$

→ here it is assumed that 2, 3, 4 are PQ buses

* Procedure :

i) on the RHS of 2, 3, 4 one substitute the assumed solution and specified values

ii) initially one assume solutions for the unknowns $V_2^{(0)}, V_3^{(0)}, V_4^{(0)}$ usually $V_2^{(0)} = V_3^{(0)} = V_4^{(0)} = 1\angle 0$

this is called: flat start.

iii) always use most recent values

\# of iteration

$$\Rightarrow V_2^{(1)} = \frac{1}{Y_{22}}\left(\frac{P_2 - jQ_2}{V_2^{(0)*}} - \left(Y_{21}V_1 + Y_{23}V_3^{(0)} + Y_{24}V_4^{(0)}\right)\right)$$

specified $V_1 = 1\angle 0$

$$V_3^{(1)} = \frac{1}{Y_{33}}\left(\frac{P_3 - jQ_3}{V_3^{(0)*}} - \left(Y_{31}V_1 + Y_{32}V_2^{(1)} + Y_{34}V_4^{(0)}\right)\right)$$

$$V_4^{(1)} = \frac{1}{Y_{44}}\left(\frac{P_4 - jQ_4}{V_4^{(0)*}} - \left(Y_{41}V_1 + Y_{42}V_2^{(1)} + Y_{43}V_3^{(1)}\right)\right)$$

$\therefore$ 1st iteration is completed ✓

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* Check that for all buses $\left| V_i^{(k)} - V_i^{(k-1)} \right| \leq \epsilon$
where ϵ is certain specified tolerance, e.g: ($\epsilon = 10^{-6}$)

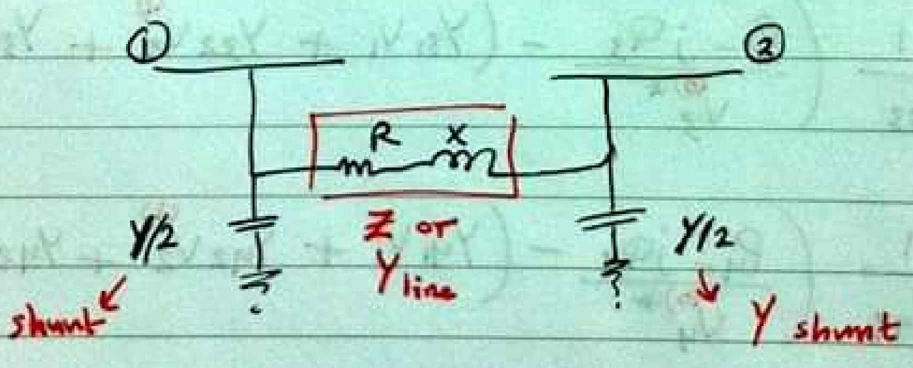
→ if Yes → solution is obtained
No → go to next iteration

* The general defining equation is :

$$V_i^{(k)} = \frac{1}{Y_{ii}} \left(\frac{P_i - jQ_i}{V_i^{(k-1)*}} - \sum_{j=1}^{i-1} Y_{ij} V_j^{(k)} - \sum_{j=i+1}^{N} Y_{ij} V_j^{(k-1)} \right)$$

(k: # of iteration)

* e.g 9.2 .(p 337):



↝

→ $L_{12} \rightarrow \quad Z = 0.01008 + j\,0.05040 = 0.05 \angle 78.69$

→ $Y_{line} = G + j\beta = \frac{1}{Z} = \frac{1}{0.05 \angle 78.69}$

$= 3.815629 - j\,19.078144$

* total charging MVAR is related to shunt capacitors of the line :

→ $VAR\ total = 3 V_p I_p = 3 V_p (V_p W_c)$

$\uparrow \nearrow$ phase quantity $= 3 V_p^2 . W_c = 3 V_p^2 Y_{shunt}$

$\therefore Y_{sh} = \frac{VAR\ (total)}{3 V_p^2} = \frac{10.25 * 10^6}{3 \left(\frac{230 * 10^3}{\sqrt{3}} \right)^2} = \frac{10.25}{(230)^2} \quad (S)$

→ $Y_{sh\,(b)} = \frac{1}{Z_b} = \frac{S_b}{V_b^2} = \frac{100}{(230)^2}$

→ $Y_{sh\,(pu)} = \frac{10.25}{(230)^2} \div \frac{100}{(230)^2} = \frac{10.25}{100} = 0.1025$

⇒ $\frac{Y_{sh}}{2} = \frac{0.1025}{2} = \boxed{0.05125}$

* Here it is assumed that each load has a power factor : pf = 0.85 lagging

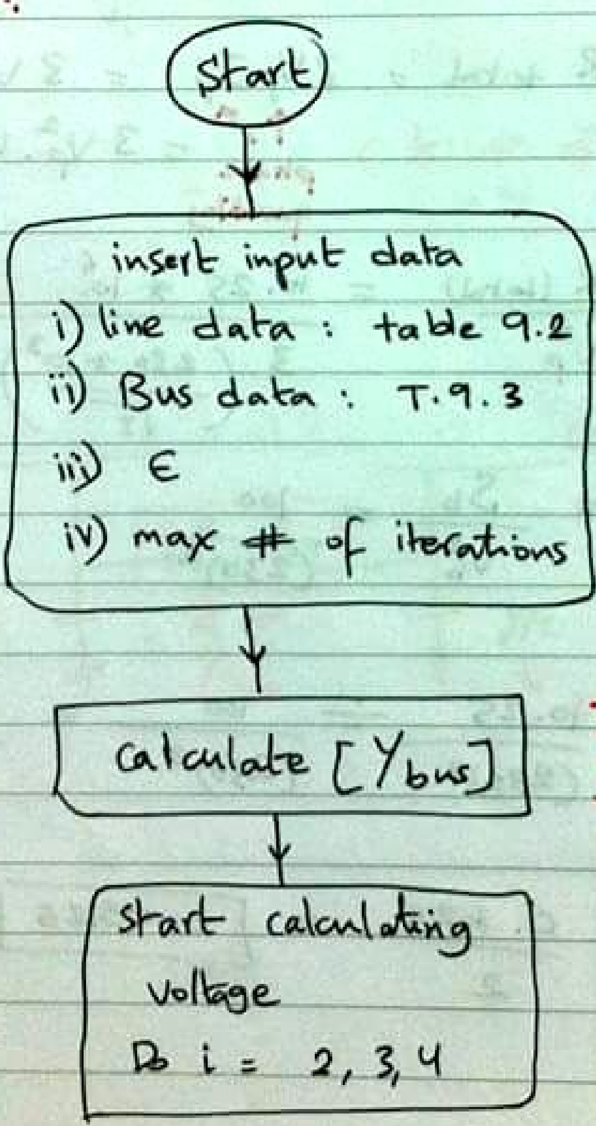
→ Bus 1

$P_L = 50$

$Q_L = P_L \tan\theta = 50 \tan(\cos^{-1}(0.85))$

$= 30.99$

* flow chart :



→ $Y_{bus} = 4 * 4$

→ e.g.: $Y_{11} = Y_{L12} + Y_{L13} + \frac{Y_{sh}}{2}(L_{12}) + \frac{Y_{sh}}{2}(L_{13})$

$= 8.985 - j44.83$

T.9.4:

→ $Y_{12} = -(Y_{L12})$

$= -3.81 + j19.07$

$$* \quad V_2^{(1)} = \frac{1}{Y_{22}}\left(\frac{P_{2,sch} - Q_{2,sch}}{V_2^{(0)*}} - \left(Y_{21}V_1 + \underbrace{Y_{23}}_{\substack{=0 \\ \text{(no line)}}}V_3^{(0)} + Y_{24}V_4^{(0)}\right)\right)$$

(page 339)

→ $P_{2,sch} = P_{2\,gen} - P_{2,load} = -P_{2,load} = -1.7$

→ $Q_{2,sch} = Q_{2,gen} - Q_{2,load} = -Q_{2,load} = 1.0535$

→ $V_2^{(0)} = 1\angle 0$

→ V_1 (slack bus) $= 1\angle 0$

→ $V_3^{(0)}$ (flat start) $= 1\angle 0$

→ $V_4 = \underbrace{|V_4|}_{\text{specified}} \underbrace{\angle 0^\circ}_{\text{flat start}} = 1.02\angle 0$

* by substitution:

$$V_2^{(1)} = 0.983564 - j0.032316$$

in order to accelerate the process of convergence, then the calculated value of $V_2^{(1)}$ has to be modified as follows:

$$\Rightarrow \overset{\text{\# of iteration}\rightarrow}{} V_{i,acc}^{(k)} = V_{i,acc}^{(k-1)} + \alpha\left(V_i^{k} - V_{i,acc}^{(k-1)}\right) \text{ ----- (9.21) book}$$

$$1 < \alpha < 2$$

usually: $\alpha = 1.6$

$$V_{2,acc}^{(1)} = V_{2,acc}^{(0)} + 1.6\left(V_2^{(1)} - V_{2,acc}^{(0)}\right)$$

calculated

(page: 340):

$$= 1\angle 0 + 1.6\left(V_2^{(1)} - V_{2,acc}^{(0)}\right)$$

calculated

$$= 1\angle 0 + 1.6\left[(0.98\ldots - j0.03\ldots) - 1\angle 0\right]$$

$$V_{2,acc}^{(1)} = 0.97\ldots - j\,0.05\ldots$$

* Next, apply the same procedure for bus 3 to find $V_3^{(1)}$ (use $V_{2,acc}^{(1)}$), then find $V_{3,acc}^{(1)}$

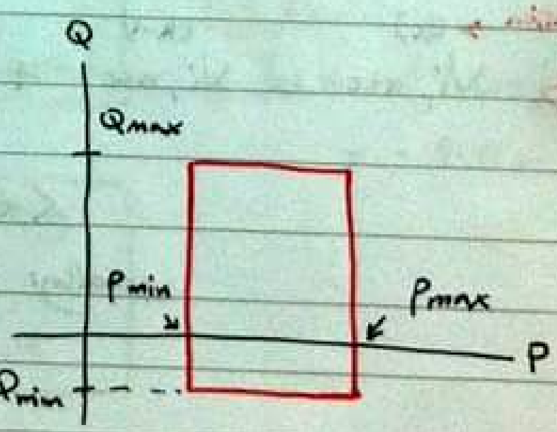
* Next evaluate $V_4^{(1)}$

$\Rightarrow$ the 1st thing evaluate $Q_4^{(1)}$ why ??

* for any 3-phase synch. generator, it has the following characteristics:

constraints $\Rightarrow$ $P_{min} \le P \le P_{max}$

$Q_{min} \le Q \le Q_{max}$



→ But for <u>PV- bus</u> : |P| is specified , and one can specify a value within its limit

→ How? $S_i = V_i I_i^*$

$S_i^* = V_i^* I_i$

$\therefore \quad P_i - jQ_i = V_i^* I_i$

$\therefore \quad Q_i = - * I_m [V_i^* I_i]$

↳ imaginary

$\therefore \quad Q_i = - * I_m \left[V_i^* \sum_{j=1}^{N} Y_{ij} V_j \right]$

* in our example : $i=4$, $\underset{\text{busbars}}{N=4}$, $k=1$

→ $Q_4^{(1)} = - * I_m \left\{ V_4^{(0)*} \left[Y_{41} V_1 + Y_{42} V_{2,acc}^{(1)} + Y_{43} V_{3,acc}^{(1)} + Y_{44} V_4^{(0)} \right] \right\}$

$= 1.654151$

* check that : $Q_{4_{min}} < Q_4^{(1)} < Q_{4\,max}$

→ if $\boxed{Yes}$: go and calculate $V_4^{(1)}$ by using the same equation for <u>PQ- buses</u> , using $Q_{4,sch}^{(1)}$

$\Rightarrow V_4^{(1)} = |V_4^{(1)}| \angle V_4^{(1)}$

* Next multiply $V_4^{(1)}$ by the factor: $\dfrac{|V_4|\ \text{specified}}{|V_4^{(1)}|}$

→ $V_4^{(1)}\ (\text{modified}) = |V_4^{(1)}| \angle V_4^{(1)} * \dfrac{|V_4|\ \text{specified}}{|V_4^{(1)}|}$

→ $\boxed{V_4^{(1)}\ (\text{modified}) = |V_4|_{\text{specified}} \angle V_4^{(1)}}$

→ if $\boxed{\text{No}}$: then set Q_4 at the violated limit,
i.e: $Q_4 = Q_{4\,max}$, if $Q_4^{(1)} > Q_{4\,max}$
$= Q_{4\,min}$ $Q_4^{(1)} < Q_{4\,min}$

→ and since now P_4 and Q_4 are specified then convert busbar 4 from PV- bus to PQ bus and calculate $V_4^{(1)}$, using the same equation of PQ- buses

→ next, check that $\left(|V_i^{(k)} - V_i^{(k-1)}| \leq \epsilon \right)$ for All buses

ϵ ← certain specified tolerance

if Yes : solution is obtained
if No : go to next iteration

* to find $V_2^{(2)}, V_3^{(2)}, V_4^{(2)}$ → 1st check Q_4 if within limits → Reinstate bus 4 in PV bus
keep $|V_4| = |V_4|$ specif.

DATE Monday 28/12

* Newton Raphson method, for power-flow solution:

This method is based on the taylor's series expansion of the function with 2 or more variables

* procedure:

i) mathematical concept
ii) its application to load flow

* Mathematical concept:

let the function: $h_1(x_1, x_2, u) = b_1$ ——— ①

and let the function: $h_2(x_1, x_2, u) = b_2$ ——— ②

x_1, x_2 = variables or unknowns to be found

u = is called independent control variable

b_1 = constant represent specified value of h_1

b_2 = " " " " " h_2

→ Since x_1 and x_2 will be evaluated by iterative techniques, then the following function: (g_1) and (g_2) are introduced as follows:

$g_1(x_1, x_2, u) = h_1(x_1, x_2, u) - b_1 = 0$ ——— ③

$g_2(x_1, x_2, u) = h_2(x_1, x_2, u) - b_2 = 0$ ——— ④

→ g_1, g_2 represent mismatch or difference between calculated and specified values.

2

→ let $x_1^{(0)}$, and $x_2^{(0)}$ be the initial estimates of x_1 and x_2.

→ if the actual solution is x_1^* and x_2^*, then a correction has to be ~~found~~ made to $x_1^{(0)}$ and $x_2^{(0)}$ to get the required answer; Hence:

$$g_1(x_1^*, x_2^*, u) = g_1(x_1^{(0)} + \Delta x_1^{(0)}, x_2^{(0)} + \Delta x_2^{(0)}, u) \text{ ---- } (5)$$

$$g_2(x_1^*, x_2^*, u) = g_2(x_1^{(0)} + \Delta x_1^{(0)}, x_2^{(0)} + \Delta x_2^{(0)}, u) \text{ ---- } (6)$$

→ the objective now is to evaluate $\Delta x_1^{(0)}$ and $\Delta x_2^{(0)}$

→ this can be achieved by the taylor's expansion about the assumed solution as follows:

$$\therefore g_1(x_1^*, x_2^*, u) = g_1(x_1^{(0)}, x_2^{(0)}, u) + \Delta x_1^{(0)} \frac{dg_1}{dx_1}\Big|^{(0)} + \Delta x_2^{(0)} \frac{dg_2}{dx_2}\Big|^{(0)} + \dots = 0 \quad (7)$$

$$g_2(x_1^*, x_2^*, u) = g_2(x_1^{(0)}, x_2^{(0)}, u) + \Delta x_1^{(0)} \frac{dg_2}{dx_1}\Big|^{(0)} + \Delta x_2^{(0)} \frac{dg_2}{dx_2}\Big|^{(0)} + \dots = 0 \quad (8)$$

→ the partial derivative of g_2 is evaluated at $x_1^{(0)}$ and $x_2^{(0)}$.

the same thing for other partial derivative

~~the~~

⇒ Here in ⑦ and ⑧ the higher partial derivative are neglected.

→ rewrite 7, 8 in a matrix form as:

$$\underbrace{\begin{bmatrix} \frac{dg_1}{dx_1} & \frac{dg_1}{dx_2} \\ \frac{dg_2}{dx_1} & \frac{dg_2}{dx_2} \end{bmatrix}^{(0)}}_{\text{called: Jacobian matrix : } J^{(0)}} \begin{bmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \end{bmatrix} = \begin{bmatrix} -g_1(x_1^{(0)}, x_2^{(0)}, u) \\ -g_2(x_1^{(0)}, x_2^{(0)}, u) \end{bmatrix} \quad \text{----- ⑨}$$

$$\therefore \quad J^{(0)} \begin{bmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \end{bmatrix} = \begin{bmatrix} -g_1(x_1^{(0)}, x_2^{(0)}, u) = b_1 - h_1(x_1^{(0)}, x_2^{(0)}, u) = \Delta g_1^{(0)} \\ -g_2(x_1^{(0)}, x_2^{(0)}, u) = b_2 - h_2(x_1^{(0)}, x_2^{(0)}, u) = \Delta g_2^{(0)} \end{bmatrix} \quad \text{---- ⑩}$$

$$\therefore \quad \begin{bmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \end{bmatrix} = \left[J^{(0)} \right]^{-1} \begin{bmatrix} \Delta g_1^{(0)} \\ \Delta g_2^{(0)} \end{bmatrix} \quad \text{---- ⑪}$$

i) since the RHS of ⑪ is known, then $\Delta x_1^{(0)}$ and $\Delta x_2^{(0)}$ can be calculated

ii) find new values: $x_1^{(1)} = x_1^{(0)} + \Delta x_1^{(0)}$

$x_2^{(1)} = x_2^{(0)} + \Delta x_2^{(0)}$

iii) repeat the same procedure to find new correction $\Delta x_1^{(1)}$ and $\Delta x_2^{(1)}$

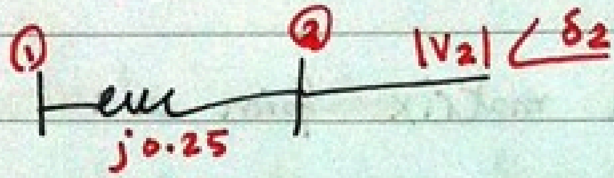
iv) the process terminates when $|\Delta x_1^{(k)}|$ and $|\Delta x_2^{(k)}| \leq \epsilon$

where: ϵ is ~~certain~~ certain sepecified tolerance

4

see:

* Ex (9.4), page (344):



$$P_i = \sum_{j=1}^{N} |V_i|\,|V_j|\,|Y_{ij}| \cos(\delta_i - \delta_j - Q_{ij})$$

$$Q_i = \text{---} \quad \sin(\;=\;)$$

here: $N=2$, $i=2$

* Application of mathematical concept to load flow problem:

The starting point is the complex power ($S_i = P_i + jQ_i$) which is entering the i^{th} bus

$$\rightarrow P_i = \sum_{j=1}^{N} \left(|V_i|\,|V_j|\,|Y_{ij}| \cos(\delta_i - \delta_j - \theta_{ij}) \right)$$

$$|V_i|^2 \underbrace{|Y_{ii}| \cos\theta_{ii}}_{G_{ii}} + \sum_{\substack{j=1 \\ j \neq i}} (\text{as})$$

note: $Y_{ij} = |Y_{ij}| \angle \theta_{ij}$

$$= \underbrace{|Y_{ij}| \cos\theta_{ij}}_{G_{ij}} + j \underbrace{|Y_{ij}| \sin\theta_{ij}}_{B_{ij}}$$

$$\rightarrow Q_i = \sum_{j=1}^{N} \text{---} \sin(\delta_i - \delta_j - \theta_{ij}) = |V_i|^2 B_{ij} + \sum_{\substack{j=1 \\ j \neq i}}^{N} 1$$

$\therefore$ ① and ② can be used to find the partial derivatives of P_i and Q_i with respect to $|V_i|$ and δ_i

$$\Delta P_i = P_{i,sch} - P_{i,cal} \quad ----- (3)$$

$$\Delta Q_i = Q_{i,sch} - Q_{i,cal} \quad ----- (4)$$

* assume the system has 4 buses, whose bus #1 is slack, and assume that the remaining 3 are PQ or load buses.

* the state variables are: $|V_2|, |V_3|, |V_4|, \delta_2, \delta_3, \delta_4$

* the mismatch equations will be:

$$\Delta P_i = \frac{\partial P_i}{\partial \delta_2} \cdot \Delta \delta_2 + \frac{\partial P_i}{\partial \delta_3} \cdot \Delta \delta_3 + \frac{\partial P_i}{\partial \delta_4} \cdot \Delta \delta_4 + |V_2| \frac{\partial P_i}{\partial |V_2|} \cdot \frac{\Delta V_2}{|V_2|}$$

$$+ |V_3| \frac{\partial P_i}{\partial |V_3|} \cdot \frac{\Delta V_3}{|V_3|} + |V_4| \frac{\partial P_i}{\partial |V_4|} \frac{\Delta V_4}{|V_4|} \quad --- (5)$$

* note: $\frac{\Delta |V_i|}{|V_i|}$ is introduced for mathematical simplification as will be shown later.

ΔQ_i is the same as (5) but with P_i replaced by Q_i

$$\Delta Q_i = \frac{\partial Q_i}{\partial \delta_2} \Delta \delta_2 + ----- + |V_4| \frac{\partial Q_i}{\partial |V_4|} \cdot \frac{\Delta V_4}{|V_4|} \quad --- (6)$$

* rewrite 5, 6 in a matrix form as follows:

eqn (9.45) in the book

$$
\left[\begin{array}{ccc|ccc}
\frac{\partial P_2}{\partial \delta_2} & \cdots & \frac{\partial P_2}{\partial \delta_4} & |V_2|\frac{\partial P_2}{\partial |V_2|} & \cdots & |V_4|\frac{\partial P_2}{\partial |V_4|} \\
\vdots & J_{11} & \vdots & \vdots & J_{12} & \vdots \\
\frac{\partial P_4}{\partial \delta_2} & \cdots & \frac{\partial P_4}{\partial \delta_4} & |V_2|\frac{\partial P_4}{\partial |V_2|} & \cdots & |V_4|\frac{\partial P_4}{\partial |V_4|} \\
\hline
\frac{\partial Q_2}{\partial \delta_2} & \cdots & \frac{\partial Q_2}{\partial \delta_4} & |V_2|\frac{\partial Q_2}{\partial |V_2|} & \cdots & |V_4|\frac{\partial Q_2}{\partial |V_4|} \\
\vdots & J_{21} & \vdots & \vdots & J_{22} & \vdots \\
\frac{\partial Q_4}{\partial \delta_2} & \cdots & \frac{\partial Q_4}{\partial \delta_4} & |V_2|\frac{\partial Q_4}{\partial |V_2|} & \cdots & |V_4|\frac{\partial Q_4}{\partial |V_4|}
\end{array}\right]
\left[\begin{array}{c}
\Delta\delta_2 \\ \Delta\delta_3 \\ \Delta\delta_4 \\ \hline \frac{\Delta|V_2|}{|V_2|} \\ \frac{\Delta|V_3|}{|V_3|} \\ \frac{\Delta|V_4|}{|V_4|}
\end{array}\right]
=
\left[\begin{array}{c}
\Delta P_2 \\ \Delta P_3 \\ \Delta P_4 \\ \hline \Delta Q_2 \\ \Delta Q_3 \\ \Delta Q_4
\end{array}\right] \quad \cdots (7)
$$

* (7) is used for load flow solution by using Newton Raphson method

1) The partial derivatives of the Jacobian, can be derived from the equations (1) and (2) (i.e: P_{cal}, Q_{cal})
   see equations: 9.52, 9.53, 9.55, 9.56, 9.58, 9.60

2) By assuming initial values for state variables, using flat start: $V_2^{(0)} = V_3^{(0)} = V_4^{(0)} = 1\angle 0°$, then:
   (a) the elements of the Jacobian matrix can be evaluated
   (b) evaluate P_{cal}, Q_{cal}, then find: $\Delta P_i = P_{i,sch} - P_{i,cal}$
   $\Delta Q_i = Q_{i,sch} - Q_{i,cal}$

   where: $P_{sch} = P_G - P_D$
   $Q_{sch} = Q_G - Q_D$ $\quad \boxed{\text{for } i = 2, 3, 4}$

3) evaluate the state variables:

$$\begin{bmatrix} \Delta\delta_2 \\ \vdots \\ \frac{\Delta|V_4|}{|V_4|} \end{bmatrix} = [J]^{-1} \begin{bmatrix} \Delta P_2 \\ \vdots \\ \Delta Q_2 \end{bmatrix}$$

4) calculate new values:

$$\delta_i^{(1)} = \delta_i^{(0)} + \Delta\delta_i^{(0)}$$

$$|V_i|^{(1)} = |V_i|^{(0)} + \Delta|V_i|^{(0)} = V_i^{(0)}\left(1 + \frac{\Delta|V_i|^{(0)}}{|V_i|^{(0)}}\right)$$

5) Repeat the process to calculate new values for the state variables

6) the process terminates when $|V_i^{(k)} - V_i^{(k-1)}| \leq \epsilon$ for all i
oR $|\Delta P_i|$ and $|\Delta Q_i| \leqslant \epsilon$ for all i

$i = 2, 3, 4$

* **Comments :**

i) in this example (i.e all buses except the slack) the order of the Jacobian is (6×6)

ii) if bus # 4 is PV (i.e $|V_4|$ is specified) $\therefore$ ~~$|\Delta V_4| = 0$~~

$|\Delta V_4| = 0$

$\therefore$ (a) the 6^{th} column of the [J] will be eliminated.

(b) Q_4 can not be calculated untill load flow is complete.

$\therefore$ 6^{th} row will be eliminated

$\therefore$ the order of the Jacobian will be : (5×5)

* in general the order of Jacobian $= (2N - N_g - 2)$

# of busbars

# of PV buses

in this example : $N = 4$

$N_g = 1$

$\therefore$ order $= (2(4) - 1 - 2) = 5$

* see: e.g : (9.5) page 353