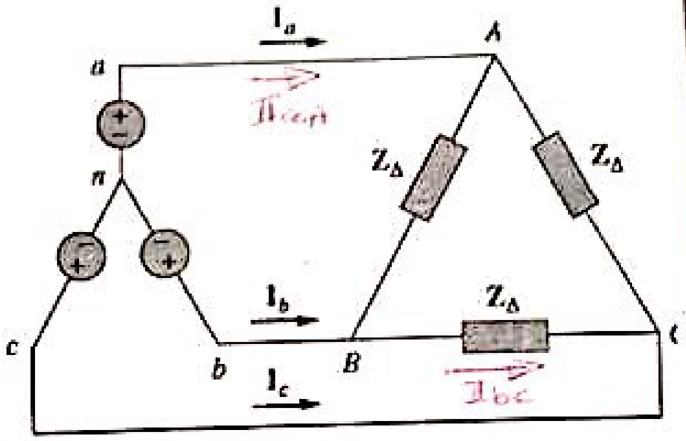
Name: KEY **ID#** **Section #** **Serial #**

Questions 1 (4 points)

The power absorbed by the load in the balanced three-phase Y-Δ system shown in the figure is 150 kVA at 0.85 PF lagging. If $V_{an} = 400\angle 0$, find:

a. the phase current I_{bc} (magnitude and phase).
b. the line current I_{aA} (magnitude and phase).
c. the per-phase impedance.



$$V_{AB} = \sqrt{3} * 400 \angle{+30} = 692.82 \angle{+30}$$

$$S_{1\phi} = 50 \angle \cos^{-1}(0.85) = 50 \angle 31.788 \text{ kVA}$$

(a)

$$S = V_{AB} I_{AB}^*$$

$$I_{AB}^* = S / V_{AB} = \frac{50*10^3 \angle 31.79}{692.8 \angle 30} = 72.2 \angle 1.79$$

$$I_{AB} = 72.2 \angle{-1.79}$$

$$I_{BC} = 72.2 \angle{-121.79}$$

(b)

$$I_{aA} = \sqrt{3}\, I_{AB} \angle{-30} = 125 \angle{-31.79}$$

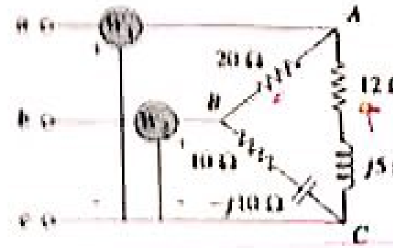
$$Z_\Delta = \frac{V_{AB}}{I_{AB}} = \frac{692.82 \angle{+30}}{72.2 \angle{-1.79}} = 9.6 \angle 31.79 = 8.16 + j5.06$$

2	1	1
I_{bc}	I_{aA}	Z_Δ
$72.2 \angle{-121.79}$	$125 \angle{-31.79}$	$9.6 \angle 31.79$

Question 2 (7 points)

In the figure below, two wattmeters are properly connected to the unbalanced load supplied by a balanced source such that $V_{an} = 100\angle 0°\ V_{rms}$

a. Determine the reading of each Wattmeter P1 & P2.
b. Determine the total real power of the load
c. Determine the total reactive power of the load
d. Calculate the total load power factor



$W_1 : V_{ac} \cdot I_a$

$W_2 : V_{bc} \cdot I_b$

$V_{ab} = 100\sqrt{3}\angle 30° = 173.2\angle 30°$

$V_{bc} = 173.2\angle -90°$

$V_{ca} = 173.2\angle 150°$

$V_{ac} = 173.2\angle -30°$

$I_{AB} = \dfrac{173.2\angle 30}{20} = 8.66\angle 30$

$I_{BC} = \dfrac{173.2\angle -90}{10 - j10} = 12.24\angle -45$

$I_{CA} = \dfrac{173.2\angle 150}{12 + j15} = 8.02\angle 98.6$

$I_{aA} = I_{AB} - I_{CA} = 16.8\angle -21.1\,A$

$I_{bB} = I_{BC} - I_{AB} = 13.0\angle -84.4\,A$

$P_1 = V_{ac} \cdot I_a \cdot \cos(\theta_V - \theta_I)$

$= 173.2 \cdot 16.8 \cdot \cos(-30 + 21$

$= 2880.73\,W$

$P_2 = V_{bc} \cdot I_b \cdot \cos(\theta_V - \theta_I)$

$= 173.2 \cdot 13.0 \cos(-90 + 84$

$= 2242.7\,W$

$P_{total} = P_1 + P_2 = 5123.43$

$Q_{total} = \sqrt{3}(P_2 - P_1) = -1105.1$

$\theta = \tan^{-1}\left(\dfrac{Q_{total}}{P_{total}}\right)$

$= -12.17$

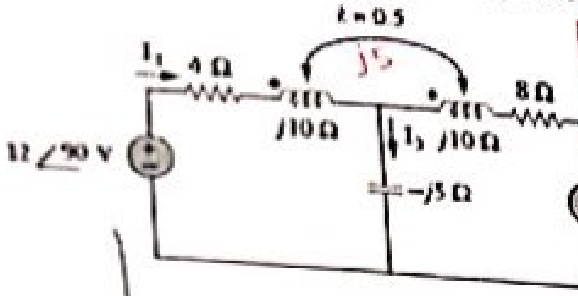
$PF = \cos\theta = 0.9775$

P_1	P_2	P_{total}	Q_{total}	PF
2880.73	2242.7	5123.43	-1105.1	0.9775

2 2 1 1 1

Questions 3 (4 points)

The circuit shown below depicts a circuit with two mutually coupled coils. Find the value of the mutual inductance M, and write down the two loop equations that govern the circuit in their simplest form. If $I_{1(RMS)} = 5\angle 30°$ A and $I_{2(RMS)} = 8\angle 50°$ A, then find the total energy stored in the two coils and their mutual inductance at t = 0.1 sec if w = 50 rad/sec.



$L_1 = 0.2$ H
$L_2 = 0.2$ H
$M = 0.1$ H

1

$$\boxed{M = k\sqrt{L_1 L_2} = 0.1}$$

$$\boxed{j\omega M = j5}$$

$$-12\angle 90 + 4I_1 + j10I_1 + (-j5)(I_1 - I_2) + j5I_2 = 0$$

$$20\angle 0 - j5(I_2 - I_1) + j10I_2 + 8I_2 + j5I_1 = 0$$

$$i_1(t) = \sqrt{2} \cdot 5\cos(\omega t + 30)$$

$$i_1(0.1) = \sqrt{2} \cdot 5 \cdot \cos\left(5 \cdot \frac{180}{\pi} + 30\right)$$

$$= \boxed{5.13}$$

$$i_2(t) = \sqrt{2} \cdot 8\cos(\omega t + 50)$$

$$i_2(1) = \sqrt{2} \cdot 8\cos\left(5 \cdot \frac{180}{\pi} + 50\right)$$

$$= \boxed{10.37}$$

$$W(0.1) = \frac{1}{2}L_1 i_1^2(0.1) + \frac{1}{2}L_2 i_2^2 + M i_1(0.1)\, i_2(0.1)$$

$$= \frac{1}{2} \cdot 0.2 \cdot (5.13)^2 + \frac{1}{2} \cdot 0.2 \cdot (10.37)^2 + 0.1 \cdot 5.13 \cdot 10.37$$

$$= \boxed{18.7\ J}$$

Equations	M	W_{total}
1) $(j5 + 4)I_1 + j10I_2 = 12\angle 90$ (1) 2) $j10I_1 + (8 + j5)I_2 = 20\angle 180$ (1)	j5 0.1 (1)	18.70 J (1)

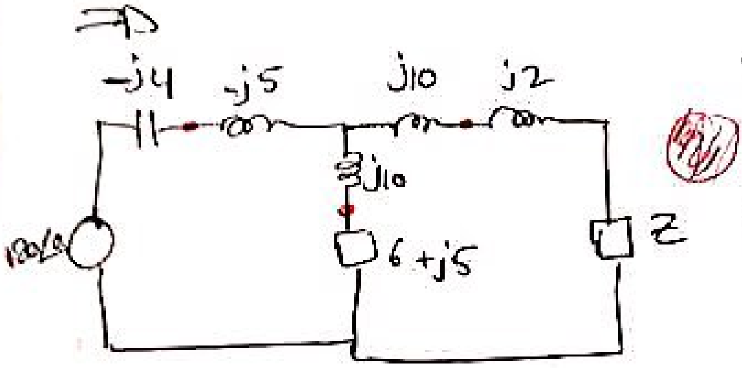
12.405

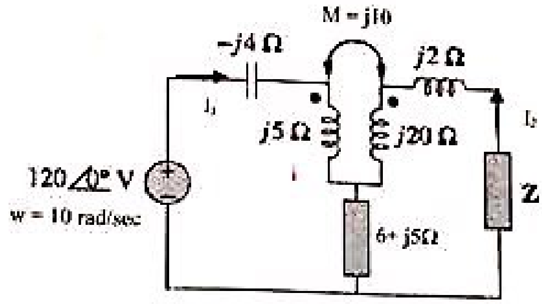
9.34

$I_3 = I_1 - I_2$

Question 4 (5 points)

For the circuit shown in the figure, replace the transformer by its T- equivalent model and then find the value of the load Z that maximize the power transferred to the load. What is this maximum power.





$Z_{Load} = Z_{Th}^{*}$

$Z_{Th} = -j9 // (6+j15) + j12$ ①

$= \dfrac{-j9 * (6+j15)}{6+j6} + j12$

$= 7.72 \angle -29 = 6.75 - j3.75$

$\boxed{Z_{Load} = 7.72 \angle +29 = 6.75 + j3.75}$

$P_{max} = \dfrac{V_{th}^2}{8R}$ → rms, or 4 ½

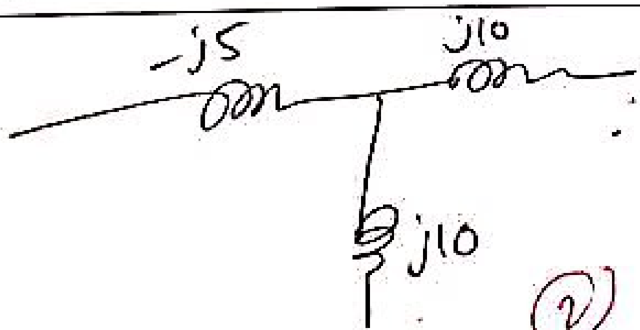
$V_{th} = \dfrac{6+j15}{6+j6} \times 120\angle 0$

$= 228.47 \angle 23.2$ ½

$R = Z \cos\theta$

$= 6.75\ \Omega$

$P_{max} = 1933.28$ oR 966.64

T-equivalent (draw and label)	Z_{Load}	P_{max}
 ②	$7.72\angle 29$ ½	1933.28 or 966.64 ½

both

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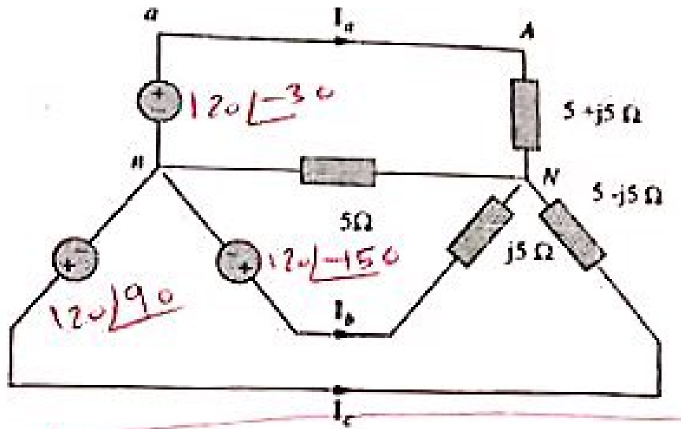
Questions 5 (5 points)

If the line voltage in the figure below is 208∠0 V(with abc sequence), determine the source currents, the current in the neutral line I_{nN}, and the load side neutral voltage V_N.



$I_a + I_b + I_c = I_{Nn}$

$\Rightarrow$

$$\frac{V_a - V_N}{5+j5} + \frac{V_b - V_N}{j5} + \frac{V_c - V_N}{5-j5} = \frac{V_N}{5}$$

$\Rightarrow$

$$\frac{120\angle -30 - V_N}{5+j5} + \frac{120\angle -150 - V_N}{j5} + \frac{120\angle 90 - V_N}{5-j5} = \frac{V_N}{5}$$

$\Rightarrow$ Solving for V_N

$$\boxed{V_N = 57.15\angle 166.67}$$

$$\boxed{I_a = \frac{V_a - V_N}{5+j5} = 24.8\angle -69.6}$$

$$\boxed{I_b = \frac{V_b - V_N}{j5} = 17.54\angle 146.57}$$

$$\boxed{I_c = \frac{V_c - V_N}{5-j5} = 17.03\angle 107}$$

$$\boxed{I_{nN} = \frac{-V_N}{5} = 11.42\angle -13}$$

I_a	I_b	I_c	I_{nN}	V_N
24.8∠-69.6	17.5∠146.5	17.03∠107	11.42∠-13	57.15∠166.67

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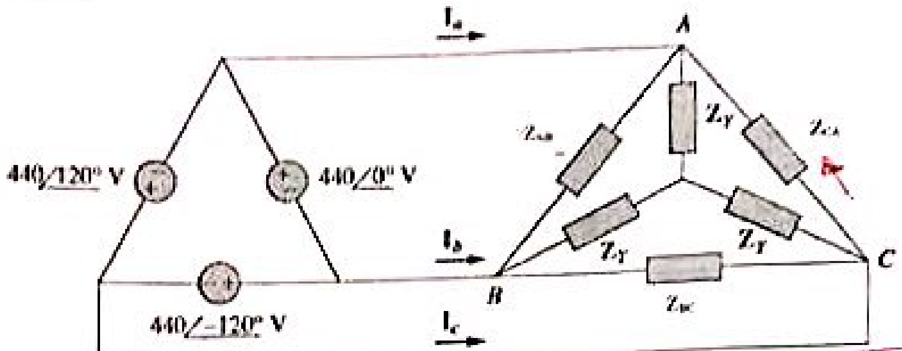
Questions 6 (5 points)

The circuit drawn shows a balanced 3 phase A source of 440 V line to line voltage suppling a balanced Y connected load of $Z_Y = 6+j8$ connected in parallel with **unbalanced** Delta load with $Z_{AB} = 6+j8$, $Z_{BC} = 8+j12$, and $Z_{CA} = 10+j6$. Find:

a) the phase current in phase A of the Y load.

b) The phase current in phase AB in the Delta load.

c) The total current in line A supplied by the source.



a) $I_{A(Y)} = \dfrac{V_{an}}{6+j8}$

$= \dfrac{\frac{440}{\sqrt{3}} \angle -30}{6+j8}$

$= \boxed{25.4 \angle -83.13}$

b) $I_{AB} = \dfrac{V_{AB}}{Z_{\Delta AB}} = \dfrac{440\angle 0}{6+j8}$

$= \boxed{44 \angle -53.13}$

c) $I_{BC} = \dfrac{V_{BC}}{Z_{BC}} = \dfrac{440\angle -120}{8+j12}$

$= 27.73 \angle -176.31$

$\boxed{I_{CA} = \dfrac{440\angle 120}{10+j6}} = 37.73\angle 89$

$\boxed{I_{A\Delta} = I_{AB} - I_{CA}} = 77.33 \angle -70.5$

$I_{A\,total} = I_{A\Delta} + I_{AY}$

$=$

$= 77.33\angle -70.56 + 25.4\angle -83.13$

$= 102.27 \angle -73.66$

$I_{A(phase\ Y)}$	$I_{AB(phase\ \Delta)}$	$I_{A(total)}$
$25.4\angle -83.13$	$44\angle -53.13$	$102.27\angle -73.66$