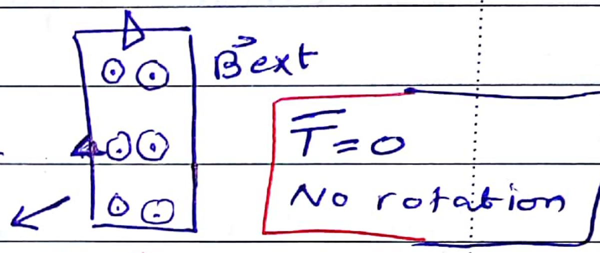
# Magnatic Tourque, and magnatic moment.

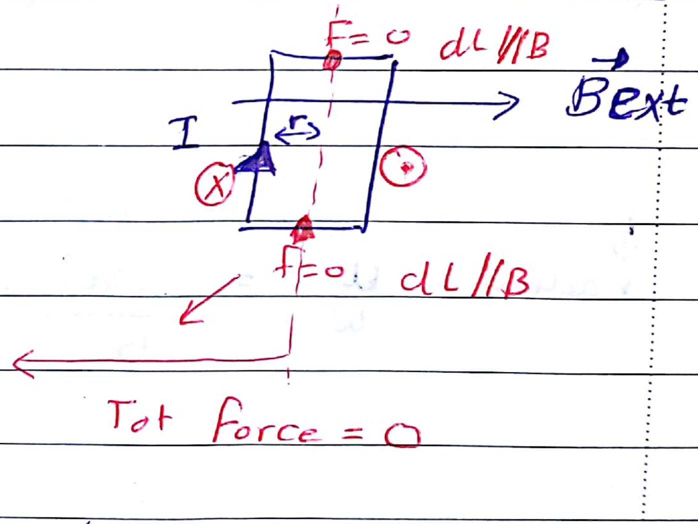
in CH4 $\bar{p} = Q\bar{d}$

↳ dipole moment

$$\text{Tourque} = \vec{r} \times \vec{F} \quad (N.m)$$




$$F_m = \oint I \, dL \times \bar{B}_{ext}$$




$$\bar{m} = I \, \vec{S} \, \hat{a_n}$$

Area

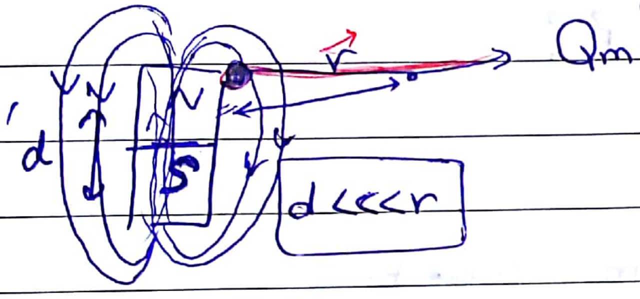
$$I \, (wL) \, \hat{a_n} \quad (A.m^2)$$

$$T = \vec{m} \times \vec{B}$$

$$A.m^2 \times \frac{wb}{m^2} = \boxed{A.wb} = \boxed{A^2 H}$$

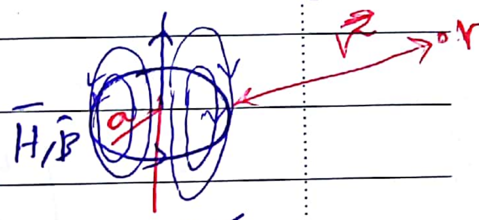
## magnatic Dipole

Small bar magnet




$\equiv$

Small loop carrying curre




$$\vec{m} = Q_m \vec{d}$$

$$\boxed{a <<< r}$$

$$\vec{m} = I s \, \hat{a_n}$$

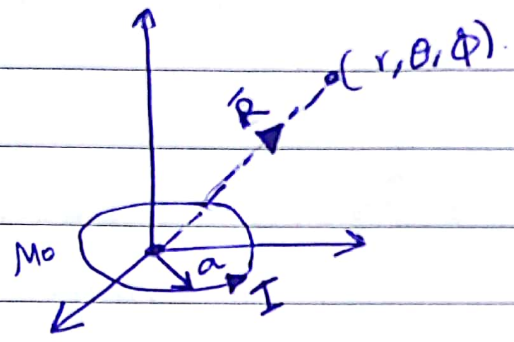
$$\vec{m} = I \pi (a^2) \, \hat{a_n}$$

$$Q_m \vec{d} = I s$$

$$\boxed{Q_m = \frac{I s}{d}}$$

North Pole:

* Magnatic Dipole
(small loop carrying current




$\bar{R} \ggg a$

$\vec{A}$?
$\vec{B}$?

$$\vec{A} = \int_L \frac{\mu_0 I\, dl}{4\pi R}$$ for line current.

$$\vec{A} = \frac{\mu_0 I\pi a^2 \;\cancel{\sin\phi}\; \hat{a\phi}}{4\pi R^2} \quad \sin\theta$$

$\longrightarrow$ $m = I\pi a^2$

$\Downarrow$

$$\vec{A} = \frac{\mu_0 \cdot \bar{m} \sin\theta}{4\pi R^2} = \frac{\mu_0 \bar{m} \times \hat{ar}}{4\pi R^2}$$

$m\, \hat{an} \times \hat{ar}$

$a\hat{z} \times a\hat{r} = \sin\theta\, a\hat{\phi}$

$a_z = \cos\theta\, a\hat{r} - \sin\theta\, a\hat{\theta} + 0\, a\hat{\phi}$

$(a\hat{z} \times a\hat{r}) = 0 + \sin\theta\, a\hat{\phi}$

from the Matrix.

$$\bar{B} = \nabla \times \vec{A}$$

$$\vec{B} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{a_r} & r\hat{a_\theta} & r\sin\theta\, \hat{a_\phi} \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ \cancel{A_r} & \cancel{rA_\theta} & r\sin\theta\, A_\phi \end{vmatrix}$$

$$A_\phi = \frac{\mu_0 I \pi a^2 \sin\theta}{4\pi r^2}$$

$$\vec{B} = \frac{1}{r^2 \cancel{\sin\theta}} \hat{a_r} \left( \frac{\mu_0 I \pi a^2}{2\pi r} 2\cancel{\sin\theta}\cos\theta \right)$$

$$- \cancel{r}\,\hat{a_\theta} \left( \frac{\mu_0 I \pi a^2 \sin\theta}{4\pi} \frac{(-1)}{(r^2)} \right)$$

$$\vec{B} = \frac{\mu_0 \overset{m}{I\pi a^2}}{4\pi r^3} \left( 2\cos\theta\, \hat{a_r} + \sin\theta\, \hat{a_\theta} \right)$$

$$\vec{B} = \frac{\mu_0 m}{4\pi r^3} \left( 2\cos\theta\, \hat{a_r} + \sin\theta\, \hat{a_\theta} \right) T$$

scalar

Ex. small loop of current $(L_1)$ $\bar{m} = 5\hat{a_z}$ A.m² located at the origin, while another small loop $(L_2)$ with $m_2 = 7\hat{a_y}$ A.m² located at $(4, -3, 10)$. Determine the torque on $(L_2)$?

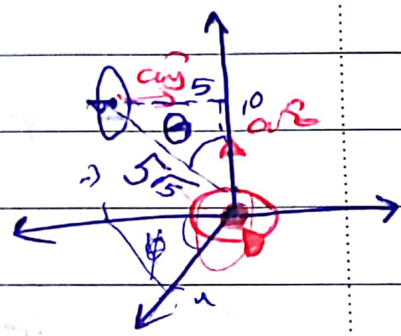
$$T_2 = \vec{m}_2 \times \vec{B}_1$$

$$B_1 = \frac{\mu_0 m_1}{4\pi r_1^3} \left(2\cos\theta\, \hat{a_r} + \sin\theta\, \hat{a_\theta}\right).$$

scalar

$$r_1 = (4,-3,10) - (4,-3,10)$$

$$r_1 = \sqrt{(4)^2 + (3)^2 + (10)^2} = 5\sqrt{5}\ m$$



$$\sin\theta = \frac{5}{5\sqrt{5}} = \frac{1}{\sqrt{5}}$$

$$\cos\theta = \frac{10}{5\sqrt{5}} = \frac{2}{\sqrt{5}}$$

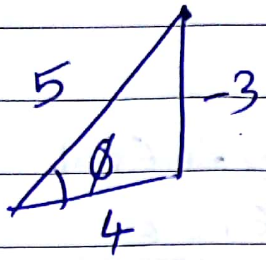
$$\vec{B}_1 = \frac{10^{-7}}{625}\left(4\hat{a_r} + \hat{a_\theta}\right) T$$

$m_2 = 3\hat{a_y}$ ⇝ Convert to spherical.

$$\bar{m}_2 = 3\left((\sin\theta \sin\phi)\, \hat{a_r} + \cos\theta \sin\phi\, \hat{a_\theta} + \cos\phi\, \hat{a_\phi}\right)$$

$$\tan\phi = \frac{y}{x} \qquad \phi = \tan^{-1}\frac{y}{x} = \tan^{-1}\left(\frac{-3}{4}\right)$$

N O T E B O O K




$\sin\phi = -3/5$

$\cos\phi = 4/5$

$$T_2 = -0.384\,\hat{a}_r + 1.536\,\hat{a}_\theta + 0.9015\,\hat{a}_\phi \quad nN.m$$

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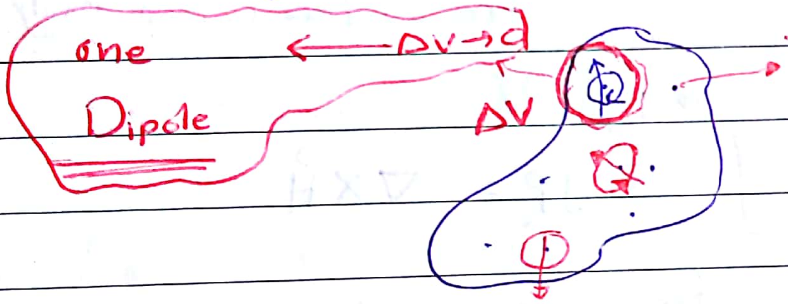
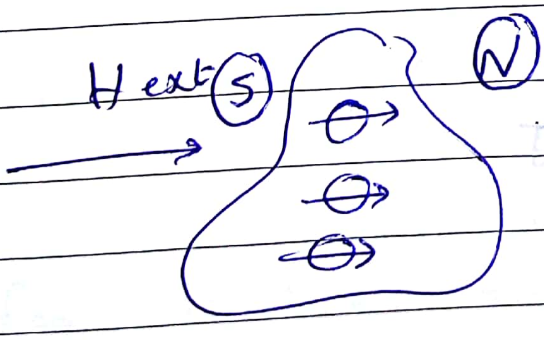
## Magnetization in Materials.




$\bar{M} = 0$

$$\vec{M} = \frac{\lim\limits_{\Delta V = 0} \sum\limits_{k=1}^{n} \bar{m}_k}{\Delta V}$$




$\vec{M} \neq 0$

Usually $\mu$ is Tensor non-isotropic

for some Magnatic materials

$$\vec{M} = \chi_m \vec{H} \quad A/m^2$$

A/m

$$\chi_m = (\mu_r - 1)\,\vec{H})$$

$$\mu_r = \frac{\mu}{\mu_0}$$

$\chi_e = \epsilon_r - 1$

$\epsilon_r = \frac{\epsilon_1}{\epsilon_0}$

$\epsilon = \epsilon_0 \epsilon_r$

$\rho_{ps} = \bar{P} \cdot \hat{a}_n$

A/m ← $\bar{K}_b$ = bound surface current density

$\rho_{q}v = -\nabla \cdot \bar{P}$

A/m² ← $\bar{J}_b$ = Volume = =

free surface current density

$$\bar{K}_b = \boxed{\bar{M} \times \hat{a}_n} \rightarrow K_f = \nabla$$

$$\bar{J}_b = \boxed{\nabla \times \bar{M}} \rightarrow J_f = \nabla \times \vec{H}$$

$J$

$$\boxed{\chi_m \bar{J}}$$

free volume current density

$$J = \nabla \times \vec{H}$$

$$J_b = \nabla \times M$$

$$= \nabla (\mu_r - 1)\,\vec{H}$$

$$= (\mu_r - 1)\,\nabla \times \vec{H}$$

$$J_b = (\mu_r - 1)\,J$$

$$\boxed{J_b = \chi_m \bar{J}}$$

$$\vec{B} = \mu \vec{H} = \mu_o \mu_r \vec{H}$$ only for linear Materials.

* Classification of Magnatic Materials :-

Conductors

1. Diamagnatic $\Rightarrow$ $\mu_r \leq 1$
2. paramagnatic $\Rightarrow$ $\mu_r > 1$
3. Ferromagnatic $\Rightarrow$ $\mu_r >>> 1$

Linear materials $\boxed{\bar{B} = \mu \bar{H}}$

$\rightarrow$ (Iron), (Nickle) (Cobalt) (Ferrite) (Nd)

↳ Copper $\boxed{\epsilon = \epsilon_o}$ $\boxed{\mu = \mu_o}$ $\boxed{\sigma = \infty}$

↳ Air $\boxed{\epsilon = \epsilon_o}$ $\boxed{\mu = \mu_o}$ $\boxed{\sigma = 0}$

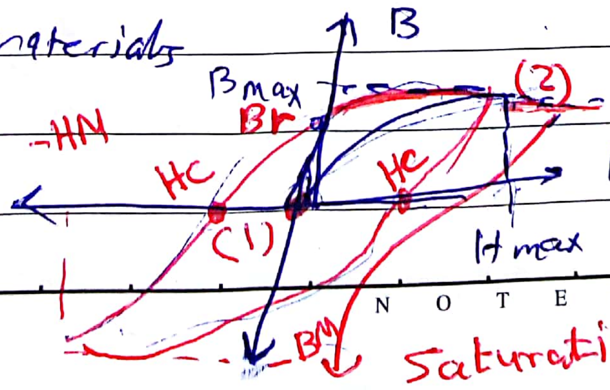
Non-linear
$B = \mu H$
at pointform.

for linear materials


B/H curve for linear material

for non-linear materials

(2) S N



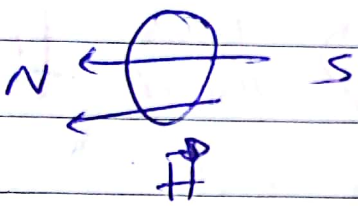

saturation.

(1) $H = 0 \rightarrow \bar{M} = 0$, $B = 0$

$\vec{H}_{ext}$

Br = residual magnatic flux density (T)

Br occre at ($\vec{H}=0$)




Hc = Corrrecieve

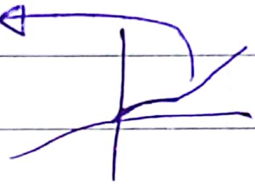
(Demagnitized Magnatic field intensity)

$\boxed{B=0}$

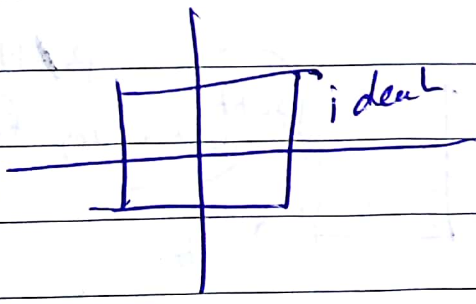
(Hard materials)

# Hysteresis loop

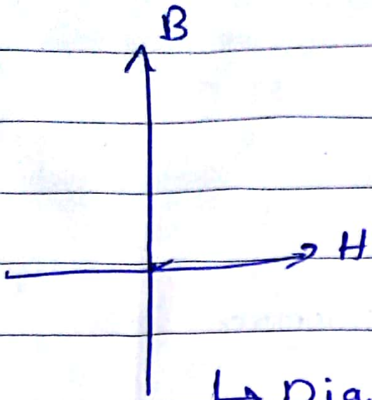
↓ (Lag)



# → initial path.



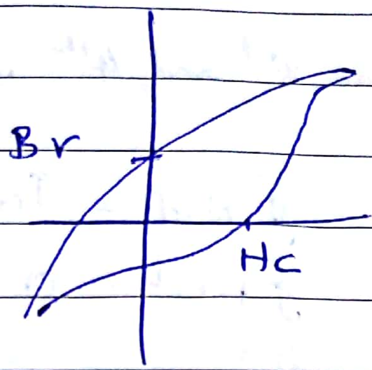
ideal materials.




↳ Dia magnatic $\mu_r \leq 1$

↳ paramagnatic $\mu_r \geq 1$

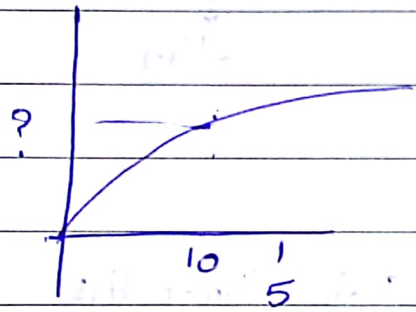
$\bar{B} = \mu \bar{H}$




Ferromagnatic

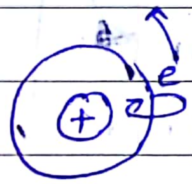
$\mu_r >>> 1$

Tc :




$\mu_r = \frac{?}{10}$

orbitalmotion

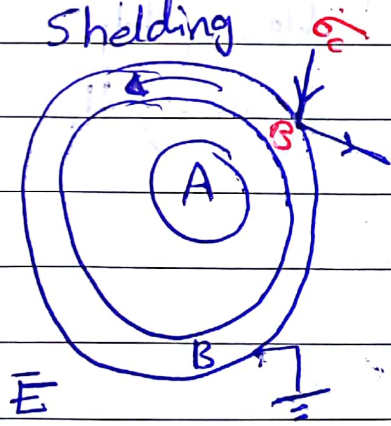


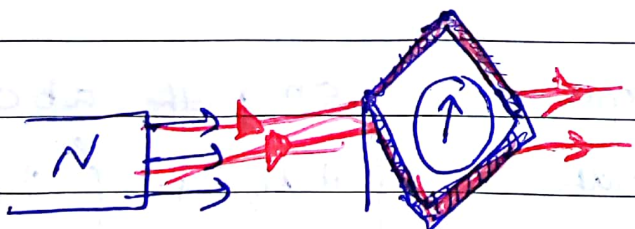

$I \rightarrow B$ orbital

spinmotion

$I \rightarrow B$ spin

Shelding




$\mu_r >>> 1$

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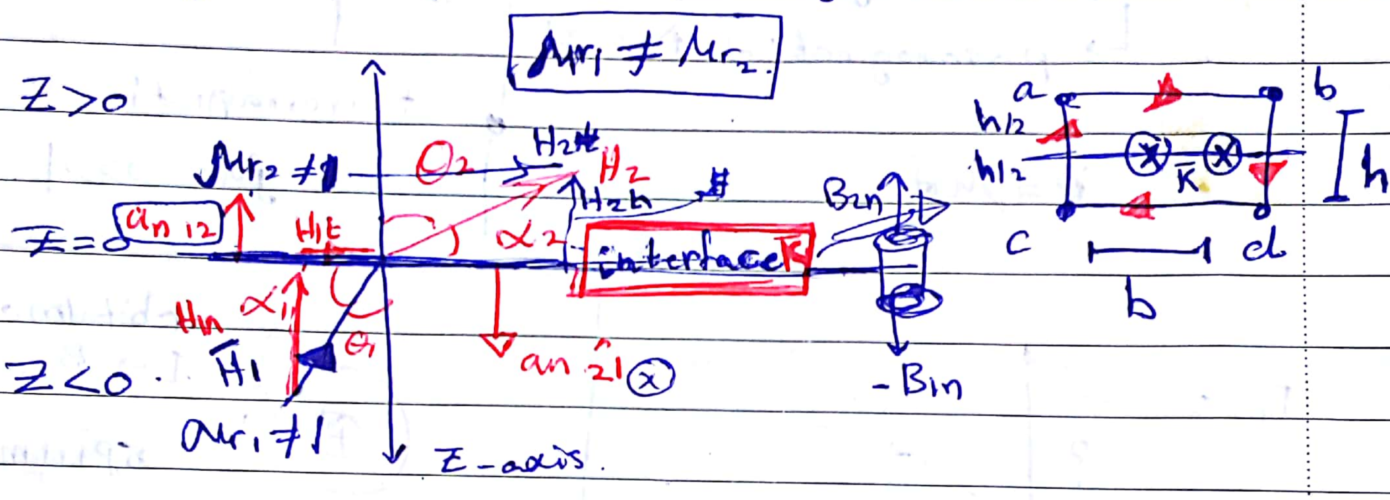
## Magnetic Boundary Conditions:-

Use 3rd and 4th Maxwell's equation.

$$\oint H \cdot dl = I_{enc}$$
$$\oint B \cdot ds = 0$$

at interface




$$\vec{B_1} = \mu_0 \mu_{r1} \vec{H_1}$$
$$\vec{M_1} = (\mu_r - 1)\vec{H_1}$$

$$\begin{cases} H_1 = \vec{H}_{1n} + \vec{H}_{1t} \\ H_{1n} = (H_1 \cdot \hat{a}_n)\,\hat{a}_{n12} \\ H_{1t} = \vec{H_1} - \vec{H}_{1n} \end{cases}$$

or
$$\begin{cases} \vec{H}_{1t} = H_1 \times \hat{a}_{n12} \\ H_{1n} = H_1 - H_{1t} \end{cases}$$

* Apply $\oint H \cdot dl = I_{enc}$ on path abcd

$$\int_a^b H \cdot dl_1 + \int_b^c H \cdot dl_2 + \int_c^d H \cdot dl_3 + \int_d^a H \cdot dl_4 = Kb$$

$$H_{2t}(b) - \cancel{H_{2n}\left(\frac{h}{2}\right)} - \cancel{H_{1n}\left(\frac{h}{2}\right)} - H_{1t}(b)$$
$$\cancel{H_{1n}\left(\frac{h}{2}\right)} + \cancel{H_{2n}\left(\frac{h}{2}\right)} = K(b)$$

$$\boxed{H_{2t} - H_{1t} = K}$$ if $k=0$ $$\boxed{H_{2t} = H_{1t}}$$

Magni $|H_{2t}|$ only.

only if $k=0$

$$(\bar{H}_2 - \bar{H}_1) \times a_{n21} = \bar{K}$$

$$\boxed{(H_1 - H_2) \times a_{n12} = \bar{K}}$$

$$\boxed{\bar{H}_{2t} = \bar{H}_{1t}}$$

Apply

$$\oint B \cdot ds = 0.$$

$$B_{2n}\Delta s - B_{1n}\Delta s = 0$$

$$\boxed{\bar{B}_{2n} = \bar{B}_{1n}} \rightarrow \text{always}$$

$B = \mu H$ ( linear material.

$$\cancel{\mu_0 \mu_{r1} H_{1n}} = \cancel{\mu_0}\, \mu_{r2} H_{2n}$$

$$H_{2n} = \frac{\mu_{r1}}{\mu_{r2}} \bar{H}_{1n}$$

$$H_2 = H_{2t} + H_{2n}$$

$$B_2 = \mu_0 \mu_r \bar{H}_2$$

$$\bar{M}_2 = (\mu_{r2} - 1)\bar{H}_2$$

if constant.

$$K_b = \bar{M} \times a_{n12} \qquad J_{b2} = \nabla \times \bar{M} = 0$$

if $\bar{K}=0$ $\quad \bar{H}_{1t}=\bar{H}_{2t}$

$$H_1 \sin\theta_1 = H_2 \sin\theta_2 \longrightarrow (1)$$

$$\bar{B}_{1n} = \bar{B}_{2n}$$

$$\mu_{r_1} H_{1n} = \mu_{r2} H_{2n}$$

$$\mu_{r_1} H_1 \cos\theta_1 = \mu_{r_2} H_2 \cos\theta_2 \longrightarrow (2)$$

(1) $\div$ (2)

$$\frac{\tan\theta_1}{\mu_{r_1}} = \frac{\tan\theta_2}{\mu_{r_2}}$$

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_{r_1}}{\mu_{r_2}}}$$ if $\bar{K}=0$

* Example:

Given $\vec{H} = (-2, 6, 4)$ A/m, in Region $y-x-2 \leq 0$ where $\mu_1 = 5\mu_0$

Calculate.

(a) $\bar{M}_1$ and $\bar{B}_1$

(b) $H_2$ and $B_2$ for $y-x-2 \geq 0$ if $\mu_2 = 2\mu_0$.

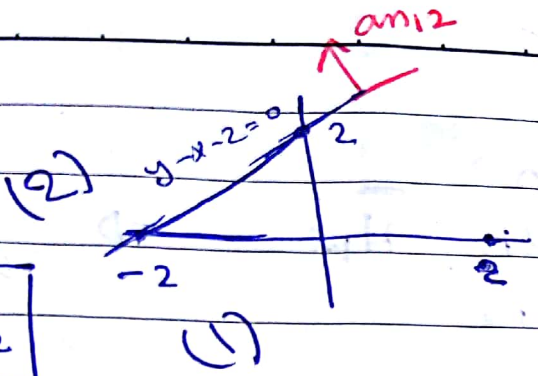
(a)

(1) $\bar{M}_1 = (\mu_{r_1} - 1) H_1 = 4\bar{H}_1 = (-8, 24, 16)$ A/m

(2) $\bar{B}_1 = 5\mu_0 H_1 = (-10, 30, 20)\mu_0$ T

Subject Date No.

(b) $\hat{a}_{n12}$




$y - x - 2 = 0$

$\boxed{\begin{matrix} x=0 \\ y=2 \end{matrix}}$ $\boxed{\begin{matrix} y=0 \\ x=-2 \end{matrix}}$

$a_{n12} = \dfrac{\nabla f}{\|\nabla f\|}$ , $f = y - x - 2$

$= \dfrac{(-1, 1, 0)}{\sqrt{2}}$ $\boxed{\hat{a}_n = \left\langle \dfrac{-1, 1, 0}{\sqrt{2}} \right\rangle}$

$\bar{H}_{1n} = (\bar{H}_1 \cdot \hat{a}_{n12}) \cdot a_{n12}$

$\left((-2, 6, 4) \cdot \left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)\right) \cdot \left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$

$\left(\frac{2}{\sqrt{2}} + \frac{6}{\sqrt{2}}, 0\right)\left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$\boxed{H_{1n} = (-4, 4, 0) \quad A/m}$

$H_{1t} = H_1 - H_{1n}$

$\boxed{H_{1t} = (2, 2, 4) \quad A/m}$

$k=0$

$\boxed{H_{1t} = H_{2t} = (2, 2, 4) \quad A/m}$

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$$\bar{B}_{1n} = \bar{B}_{2n}$$

$$H_{2n} = \frac{\mu_{r1}}{\mu_{r2}} \bar{H}_{1n} \Rightarrow \frac{5}{2} \bar{H}_{1n}$$

$$\boxed{\bar{H}_{2n} = (-10, 10, 0)\ A/m}$$

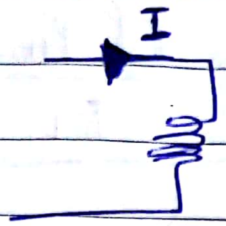
$$\bar{H}_2 = \bar{H}_{2t} + \bar{H}_{2n}.$$

$$\boxed{H_2 = (-8, 12, 4)\ A/m}$$

$$B_2 = 2\mu_0 H_2 = (-16, 24, 8)\mu_0\ T$$

$$\boxed{\bar{M}_2 = \bar{H}_2}$$

* inductors :-




$$I \xrightarrow{\alpha} \vec{B} \xrightarrow{\alpha} \varphi \xrightarrow{\alpha} \lambda$$

$\lambda = N\varphi$ (wb)

flux linkage.

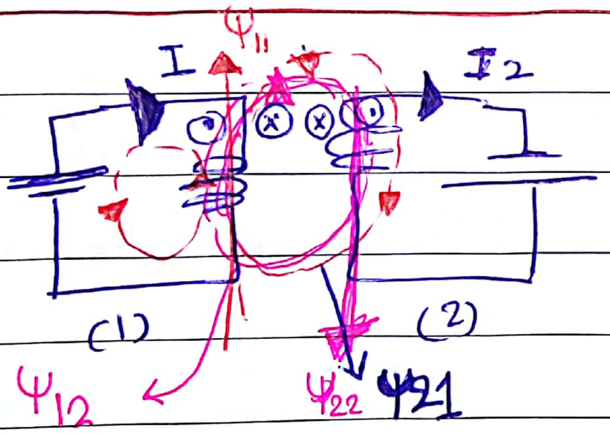
$\lambda \propto I$

① $\boxed{L = \frac{\lambda}{I}}$ $\quad \frac{wb}{A} = \frac{H\,A}{A} = (H)$

inductance

(H)

## self and mutual inductance




$\varphi_{11}$ ← current number.

skt number

$\varphi_{11}$, $\varphi_{22}$
self fluxes (wb)

$\varphi_{21}$, $\varphi_{12}$
mutual fluxes (wb)

self inductance $\boxed{L_{11} = \frac{\lambda_{11}}{I_1} = \frac{N_1 \Psi_{11}}{I_1}}$ (H)

$$L_{22} = \frac{\lambda_{22}}{I_2} = \frac{N_2 \Psi_{22}}{I_2} \quad (H)$$

Mutual ~~indu~~ inductance M

$\overline{M}$ A/m

(1) $M_{12} = \frac{\lambda_{12}}{I_2} = \frac{N_1 \Psi_{12}}{I_2}$ (H)

(2) $M_{21} = \frac{\lambda_{21}}{I_1} = \frac{N_2 \Psi_{21}}{I_1}$ (H)

total flux on circuit 1

$$\Psi_1 = \Psi_{11} \pm \Psi_{12}$$

$$\Psi_2 = \Psi_{22} \pm \Psi_{21}$$

(+) → in same direction

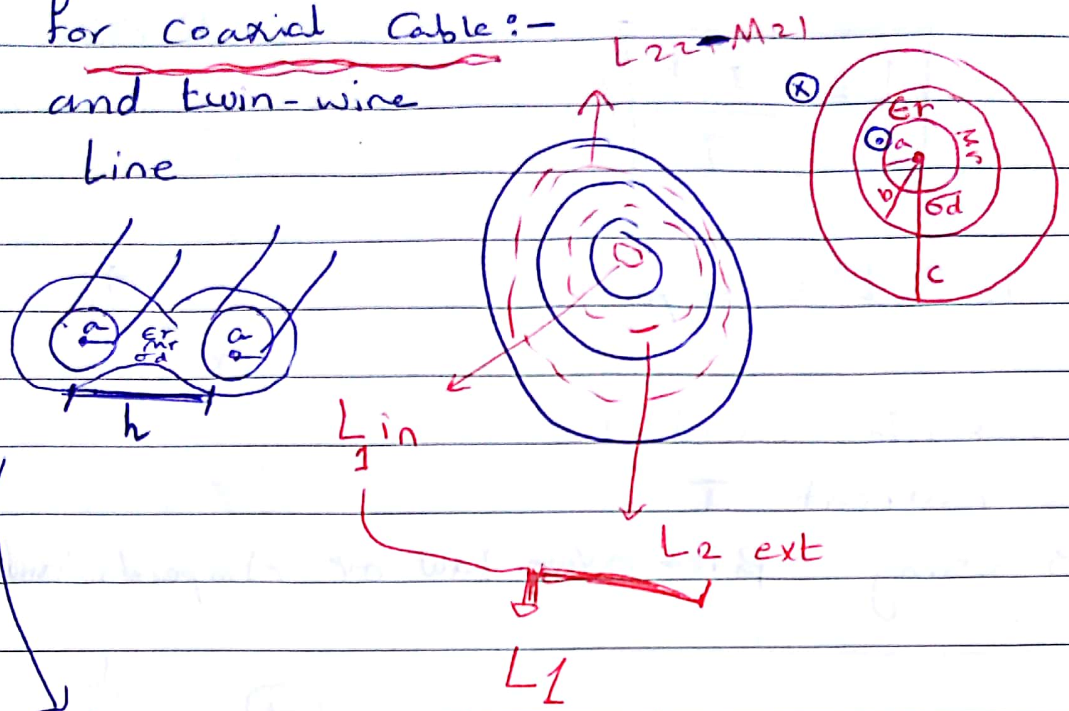
(−) → opposite direction

total inductance of ckt 1

$$L_1 = L_{11} \pm M_{12}$$

$$L_2 = L_{22} \pm M_{21}$$

For coaxial Cable :-
and twin-wire Line




$$\boxed{\begin{aligned} L_{ext} * C &= \mu\epsilon \\ RC &= \frac{\epsilon}{\sigma} \end{aligned}}$$

Valid for these two cases just

** Magnatic energy (Wm) (joul)

$$W_m = \frac{1}{2} \int_V \vec{H} \cdot \vec{B} \, dv \qquad \vec{B} = \mu_0 \vec{H}$$

$$\frac{1}{2} \mu \int_V H^2 \, dv = \frac{1}{2} \int_V \frac{B^2}{\mu} \, dv$$

$$= \boxed{\frac{1}{2} L I^2}$$

$$\boxed{L = \frac{2W_m}{I^2}} \qquad \boxed{L = \frac{N\psi}{I}}$$

## procedure to find L or M

1. choose a suitable coordinates
2. assume current $I$
3. Find $\vec{B}$ using Bio-Savart's law or Ampere's law.

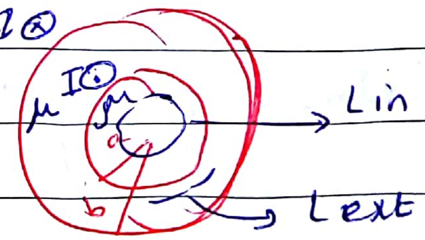
4. $\psi = \oint_S B \cdot ds$

$$L = \frac{N\phi}{I}$$

4. $W_m = \frac{1}{2}\int \frac{B^2}{\mu} dV$

$$L = \frac{2W_m}{I^2}$$

**Ex** Determine the self inductance of Coaxial cable of radii (a, b) (Total inductance of inner conductor)




from ch.7 (Ampere's law)

$$\vec{B} = \begin{cases} \frac{\mu I \rho}{2\pi a^2}\, \hat{a}_\phi & 0 \le \rho \le a \\ \frac{\mu I}{2\pi\rho}\, \hat{a}_\phi & a \le \rho \le b \end{cases}$$

N O T E B O O K

Lin ?

Using $\oint$ $L = \frac{2W_m}{I^2}$

$$W_{m\ int} = \frac{1}{2}\int_V \frac{B^2}{\mu}\,dv = \frac{1}{2}\int_0^L\int_0^{2\pi}\int_0^a \frac{\mu^{\cancel{2}} I^2 \rho^2}{4\pi^2 a^4 \cancel{\mu}}\,\rho\,d\rho\,d\phi\,dz$$

$$= \frac{\mu I^2 \cancel{a^4}}{8\ \cancel{8}\pi^{\cancel{2}} \cancel{a^4}\ \cancel{4}2} \frac{(\cancel{2\pi}) L}{} = \frac{\mu I^2 L}{16\pi} \text{ (joul)}$$

$$\boxed{L_{int} = \frac{2 w_{min}}{I^2} = \frac{\mu L}{8\pi}\ H \qquad \frac{L_{in}}{L}\ (\text{per unit length}) = \frac{\mu}{8\pi}\ (H/m)}$$

Lext

$$\psi = \oint_s B.ds = \int_0^L\int_a^b \frac{\mu I}{2\pi\rho}\,d\rho\,dz$$

$$\psi = \frac{\mu I}{2\pi} \ln\left(\frac{b}{a}\right) L \text{ (wb)}$$

$$\lambda = N\psi = \psi$$

$$L = \frac{1\psi}{I} = \quad L_{ext} = \frac{\mu L}{2\pi}\ln\left(\frac{b}{a}\right) H$$

$\frac{L_{ext}}{l}$ (per unit length) $= \frac{\mu}{2\pi} \ln\left(\frac{b}{a}\right)$ H/m

$L/l$ (per unit length $= \frac{\mu}{2\pi}\left(\frac{1}{4} + \ln\left(\frac{b}{a}\right)\right)$ H/m

* Magnetic circuit

$$\boxed{L_{ext} * C = \frac{\mu}{\epsilon}}$$

| Electrical circuit. | Magnatic circuit |
| --- | --- |
| $V_{emf} \Rightarrow V$ | $V_{mmF} \Rightarrow F$ |
| $I$ | $\psi$ |
| $\sigma$ | $\mu$ |
| $R = \frac{L}{\sigma A}$ $(\Omega)$ | $R_m = \frac{L_e}{\mu A_e}$ $(H^{-1})$ magnatic Resistance (Reluctance) |
| $G = \frac{1}{R}$ (s) | $P = \frac{1}{R_m}$ (permeance H) |
| ohm's law $\boxed{V = IR}$ | oh'ms low $\Rightarrow$ $\boxed{F = \psi R_m}$ |

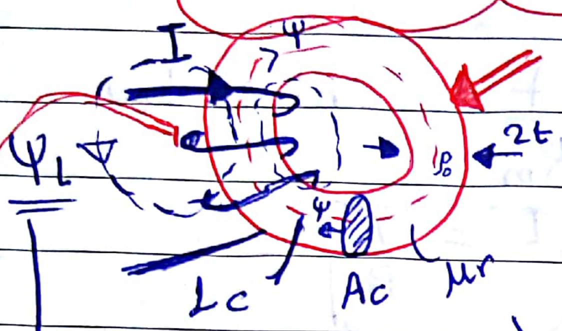
N O T E B O O K

$[L_c, \mu_r, A_c]$

* Magnatic structure → ① magnatic Core
② Excitation Coil
$(I, N)$

$F = NI \quad (A.t)$

mmF




magnatic core.

→ Gapless Circuit.

→ leakage flux.

$\sim 5\% \; \psi$

Excitation Coil

$L_c = 2\pi \rho_0$ (averge length of the Coil).

$\rho_0 = a + t$

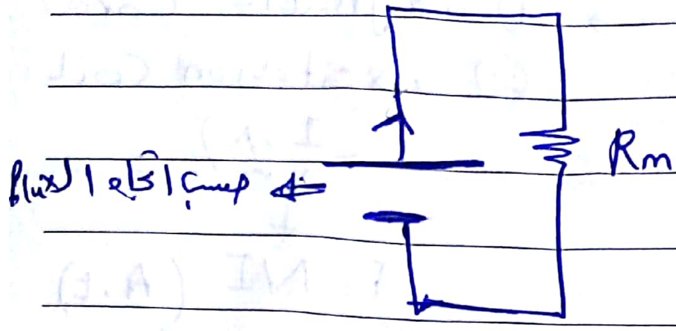
$\mu = \mu_0 \mu_r \qquad \mu_0 = 4\pi * 10^{-7}$

$A_c$ = (Cross sectional area of the core) ⇒ العدى بتاع ال flux

$= \pi t^2$

$R_m = \dfrac{L_c}{\mu_0 \mu_r A_c}$ (Reluctance) $(H)^{-1}$

→ Convert to magnatic circuit.

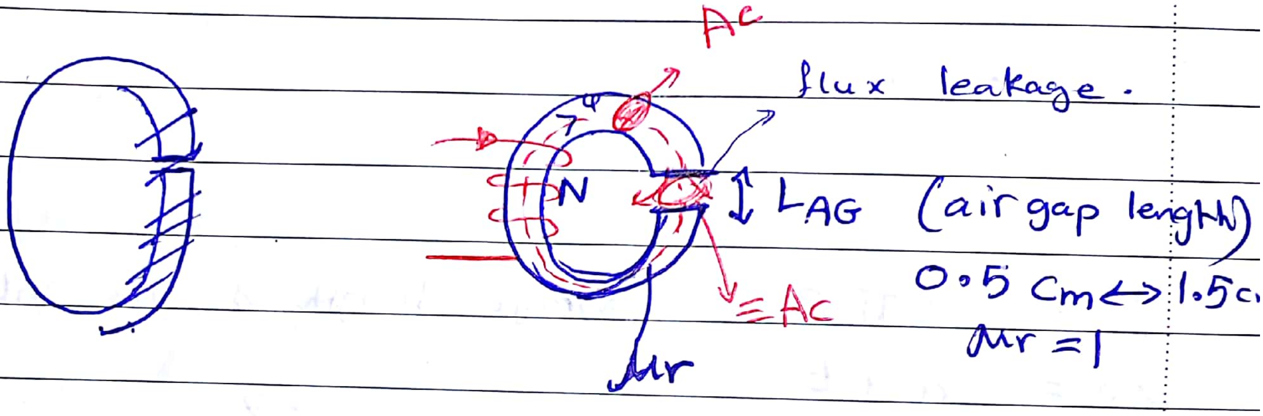



Series circuit

$$\Psi = \frac{F}{R_m} \quad \text{ohm's law}$$

$$\equiv \boxed{I = \frac{V}{R}}$$

Structure with air Gap.



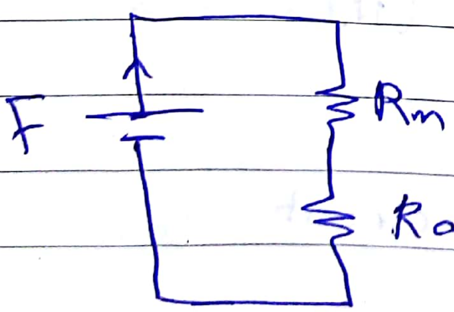

$l = l_{core} + l_{air\,Gap}$

$\downarrow$

$2\pi\rho_o$

$\underline{L_c} = L - L_{air\,gap}$

$R_{air\,gap} > R_m$




\# $F = NI$

\# $R_m = \dfrac{L_c}{\mu_o \mu_r A_c}$

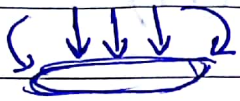
$R_{eq} = R_m + R_{air\,gap}$

\# $R_{air\,gap} = \dfrac{L_{air\,gap}}{\mu_o A_c}$

# $\Psi_{leackage} = 5\%$

↳ $\Psi = 95\% * \Psi \longrightarrow \left[ \Psi = \boxed{BA_c} \Rightarrow \text{Uniform} \right]$

air gap flux



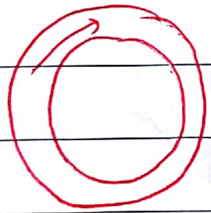
$$\frac{\text{Area app}}{\text{Area physical} \Rightarrow (A_c)} = \underset{\downarrow}{FF}$$

Frienging factor $\geq 1$

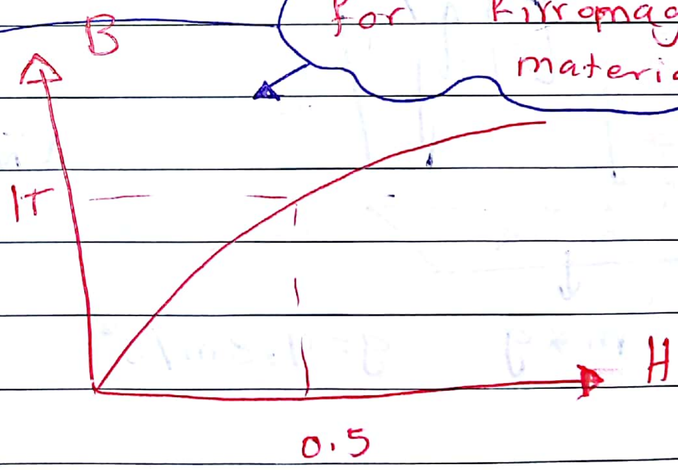
$$R_{AG} = \frac{L_{AG}}{\mu_o \underbrace{\mu_r}_{=1} * A_{capp}}$$

---

* $B = 1T$



for firromagnatic materials




$\mu_r = \frac{B}{H} = 2$

:

* $F = NI$
* $F = \Psi R_m$

$F = HL$ ← from ampere's law (kvl eq)

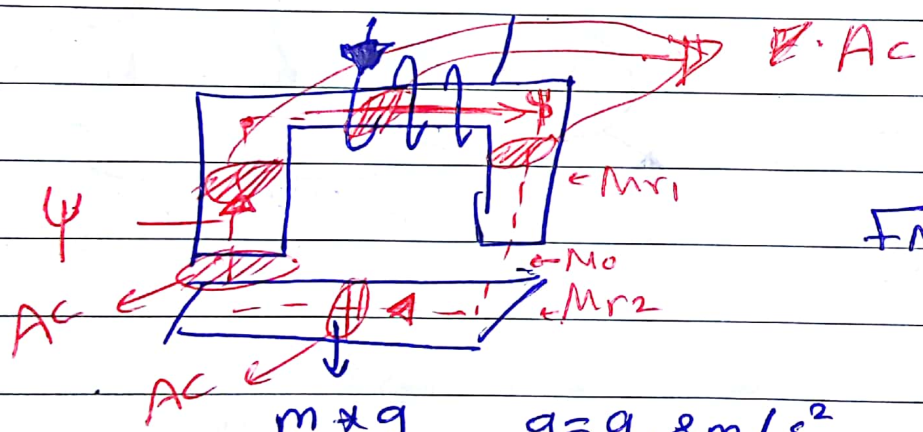
$\vec{F}_m(\text{force}) = \int_V J\,dV \times B_{ext}$

## * Force in the airgap

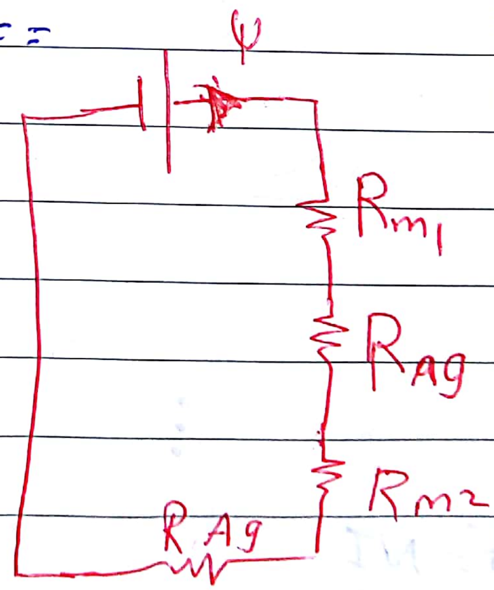
$F_{MT}$ ( Magnatic Tractive force) (N)

$$F_{MT} = \frac{B^2 * A_c}{2 \mu_0}$$ For one airgap

$$\varphi = \frac{F}{R_m} \qquad B = \frac{\varphi}{A_c}$$

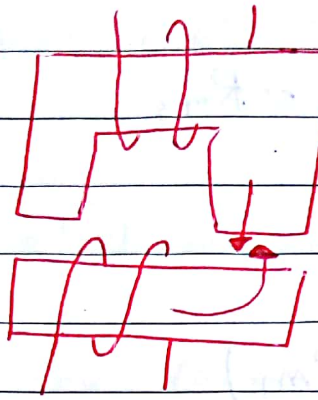



$F_{Mt} =$




$m * g \qquad g = 9,8 \, m/s^2$

$A_{AG}$ $A_{pp} = FF * A_c$

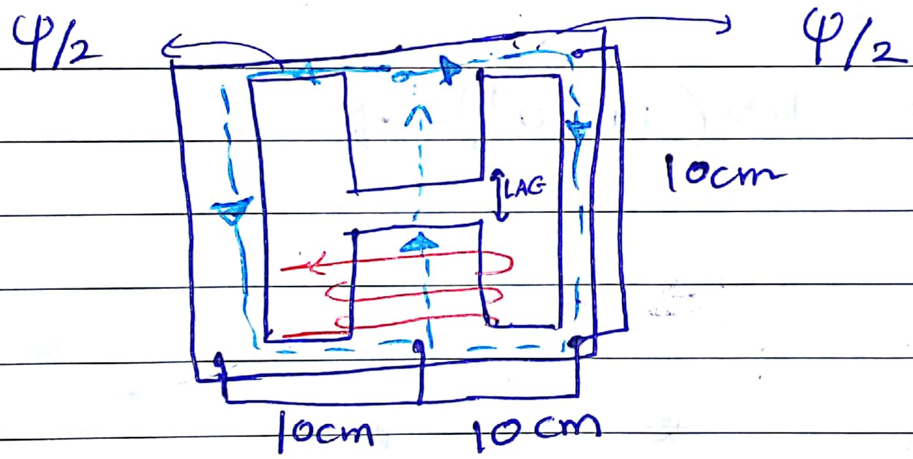


$$\text{pressure} = \frac{F_{MT}}{A_c} = \left[\frac{B^2}{2\mu_0}\ N/m^2\right]$$

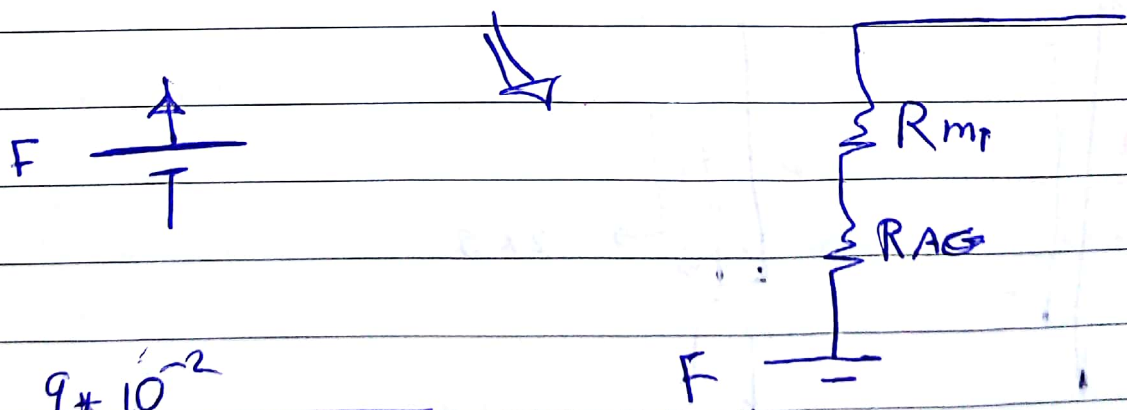
* * * *

# $\boxed{\text{Example}}$

Find $I$ to produce $B = 1.5\,T$ in the airgap if $\mu = 50\mu_0$, $L_{AG} = \cancel{10cm}$ 1cm, $N = 400$ turns and all branches have $A_c = 10cm^2$




$$R_{AG} = \frac{1*10^{-2}}{4*\pi*10^{-7} * 10*10^{-4}}$$




$$R_{m1} = \frac{9*10^{-2}}{50\,(4\pi*10^{-7})\,(10*10^{-4})}$$

$$R_{m2} = \frac{30 \times 10^{-2}}{50(4 * \pi * 10^{-7})(10 * 10^{-4})} = R_{m3}$$

$$R_{m(eq)} = (R_{m2} // R_{m3}) + (R_{mAG} + R_{m1})$$

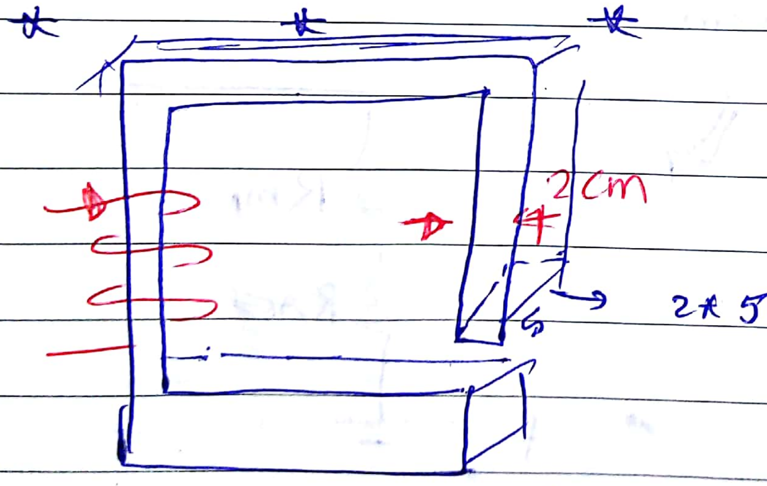
$$R_{eq} = \left( \frac{7.4 \times 10^{8}}{30\pi} * \cancel{[4]}^{-1} \right)$$

$$F = N\cancel{I} = \underline{\varphi} R_{meq}$$

$$\varphi = BA_c = 1.5(10 * 10^{-4}) \Rightarrow$$

$$I = \frac{\varphi R_m}{N}$$

$$\boxed{I = 44.16 A}$$



# CH(9): Maxwells Equation

9.6* / X

## Electromagnatic Fields :

Time - Varying fields.

Maxwells equation in static field

| | |
|---|---|
| $\nabla \cdot \bar{D} = \rho_v$ | $\oint_S D \cdot ds = \int_V \rho_v \, dv$ |
| $\nabla \times \vec{E} = 0$ | $\oint_L E \cdot dL = 0$ |
| $\nabla \times \vec{H} = J$ | $\oint_L H \cdot dL = \int_S J \cdot d\bar{s}$ |
| $\nabla \cdot \vec{B} = 0$ | $\oint_S B \cdot ds = 0$ |

No changes

Sources for Ac fields.

① Charge moving with acceleration

② Ac current flowing in awire

all fields and sources are function of space and time

$\vec{E}(x, y, z; t)$
$\bar{D}(x, y, z; t)$
$H(x, y, z; t)$
$B(x, y, z; t)$

fields

sources $\begin{cases} \vec{J} \rightarrow Ac & (x, y, z; t) \\ \rho_V \rightarrow a \neq 0 & (x, y, z; t) \end{cases}$

# Faraday's law :-

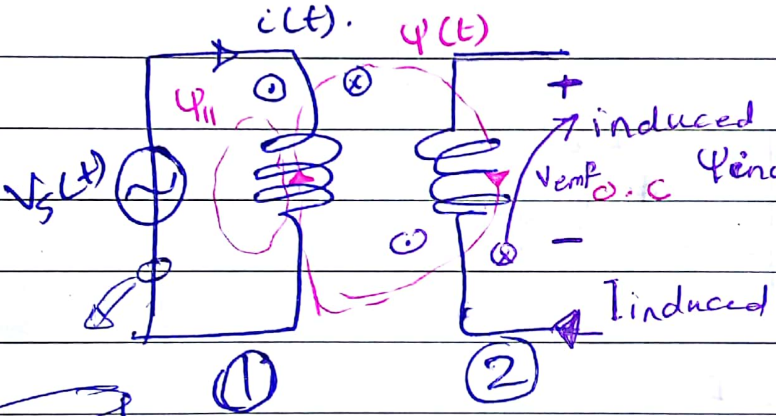
* Faraday's law :- (generator action).

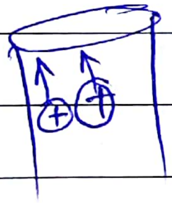
$$V_{emf} = \frac{-d\lambda}{dt} = \frac{-dN\varphi}{dt} = \frac{-Nd\varphi}{dt}$$

if $N=1$ turns

$$\boxed{V_{emf} = \frac{\ominus d\varphi}{dt}}$$

lenze's law

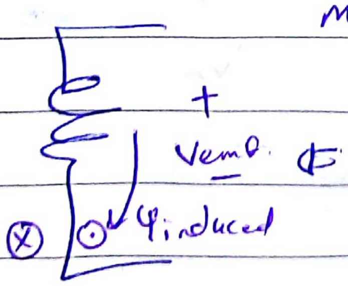


moving to (1)

$$\frac{d\varphi}{dt}\uparrow$$

moving far from 1 $\downarrow \frac{d\varphi}{dt}$ (−ve)




φinduced the same direction with φ(t).

$$\varphi = \int_S B \cdot ds$$

$$V_{emf} = -\frac{d}{dt}\int_S B \cdot ds$$

$\boxed{\text{from CH 4}}$

$$V_{emf} = -\oint_L E \cdot dL \qquad \boxed{\oint_L E \cdot dL = 0}$$

$$V_{emf} = \oint_L E \cdot dL = \frac{-d}{dt}\int_S B \cdot ds$$

2nd Maxwells equation in integral form (in time varying field)

applying stoke's theorem

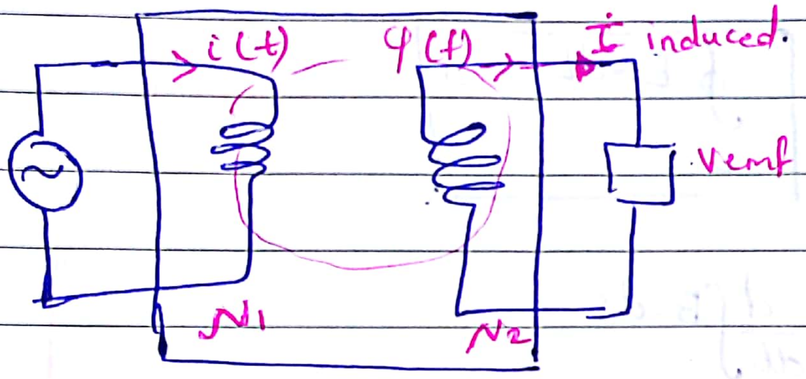
$$\int_S \nabla \times E \cdot \cancel{ds} = -\frac{d}{dt}\cancel{\int_S} \vec{B} \cdot \cancel{ds}$$

$$\boxed{\nabla \times E = -\frac{\partial B}{\partial t}}$$

[Second maxwell's eq in diff form]

3 cases of faradays law.

1- ~~static~~ time varying $\vec{B}$-field and static loop (Transformer electromotive force)




only B is varying with time.

$$\boxed{V_{emf} = \oint E \cdot dL = -\int_S \frac{\partial B}{\partial t} \cdot ds}$$

$$\nabla \times \vec{E} = -\frac{\partial B}{dt}$$

* * * * * *

2. static B-field and moving loop → $\vec{u}$ (veloc

(motional EMF) ((DC generator))

$$V_{emf} = \oint_L E \cdot dL = \cancel{-\frac{d}{dt}\int B \cdot ds}$$ ($B$ is DC $\frac{dB}{dt} = 0$)

$$F_m = Q\,\vec{u} \times \vec{B}$$

$$\frac{F_m}{Q} = \vec{u} \times \vec{B}$$

like $E = \frac{F}{Q}$

$$E_m = \vec{u} \times \vec{B}$$

NOTEBOO

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$$\boxed{V_{emf} = \oint_L E.dL = \oint_L \vec{u} \times \vec{B} \cdot dl}$$ integral

(lenz's law will be considered in integral limits. (integrate according to induced current direction)).

$$\cancel{\int_S} \nabla \times E \cancel{.ds} = \cancel{\int_S} \nabla \times (\vec{u} \times \vec{B} \cancel{.ds}$$

$$\boxed{\nabla \times E = \nabla \times (\vec{u} \times \vec{B})}$$ diff

(3) Time varying magnatic field and moving loop. (motional + transformer Emf).

$$\boxed{V_{emf} = \oint_L E.dl = \int_S -\frac{dB}{dt} \cdot ds + \oint_L \vec{u} \times \vec{B} \cdot dl}$$

$$\boxed{\nabla \times E = -\frac{dB}{dt} + \nabla \times (\vec{u} \times \vec{B})}$$

* * * * *

* **Displacement current** :- (AC in Dielectric).

(Id) $J_d$ = displacement current Density

$$I_d = \oint_S J_d \cdot ds$$

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Amperés law

$$\oint_L H.dL = I_{enc}$$

$$\boxed{\nabla \times H = J}$$

for DC

$$\nabla \cdot \nabla \times H = \nabla \cdot J$$

$$\Rightarrow \text{Zero} = \nabla \cdot J \quad \text{but in CH 5}$$

$$\nabla \cdot J = -\frac{\partial \rho_v}{\partial t} \neq 0$$

with displacement current

$$\nabla \times H = J + J_d \Rightarrow \text{Zero in DC}$$

$$\cancel{\nabla \cdot (\nabla \times H)} = \nabla J + \nabla \cdot J_d$$

$$\nabla \cdot \bar{J}_d = -\nabla \cdot \bar{J}$$

$$\boxed{\nabla \cdot J_d = \frac{\partial \rho_v}{\partial t}}$$

$$\boxed{\nabla \cdot \bar{D} = \rho_v}$$

$$\cancel{\nabla} \cdot J_d = \frac{\partial}{\partial t} \cancel{\nabla} \cdot \vec{D}$$

$$\boxed{J_d = \frac{\partial \vec{D}}{\partial t}}$$

$$\oint H \cdot dL = \int_S J \cdot ds + \int_S J_d \cdot ds$$

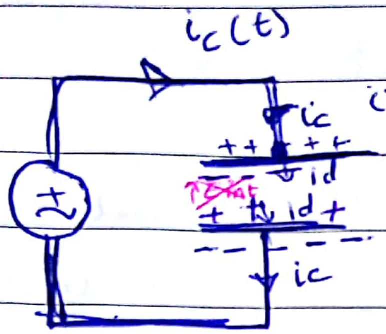
$$\boxed{\oint H \cdot d = \int_S \left(\bar{J} + \frac{\partial \bar{D}}{dt}\right) \cdot ds}$$ 3rd maxwell eq in integral form.

$$\int_S \nabla \times H \, \cancel{ds} = \int_S \bar{J} + \frac{\partial \vec{D}}{\partial t} \cancel{ds}$$

$$\boxed{\nabla \times H = J + \frac{\partial \bar{D}}{dt}}$$ 3rd maxwell eq in diff. form.

maxwells eq in time varying

| | |
|---|---|
| $\oint_S D \cdot ds = \int_V \rho_v \, dv$ | $\nabla \cdot \vec{D} = \rho_v$ |
| $\oint_S B \cdot ds = 0$ | $\nabla \cdot \vec{B} = 0$ |
| $\oint_L E \cdot dl = \int_S \frac{-\partial B}{dt} ds$ | $\nabla \times \vec{E} = -\frac{\partial B}{\partial t}$ |
| $\oint H \cdot dl = \int_S \left(\bar{J} + \frac{\partial D}{dt}\right) ds$ | $\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{dt}$ |




$i_c \equiv \vec{E}(t) = \oplus$

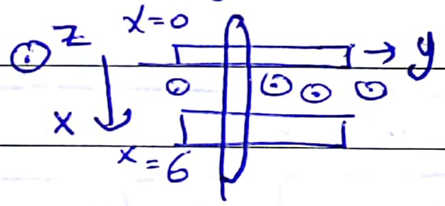
$\downarrow \vec{E}(t) \Rightarrow$ AC.

$$\frac{\partial \epsilon E(t)}{dt} = \frac{\partial D}{dt} = J_d$$

Displacement current $\Leftarrow$ $I_d = \int_S J_d \cdot ds$

*** Ex :** on faradays law :-

a conducting bar can slides freely over two conducting rails




Find emf if

(a) the bar is stationed at y = 8 cm and $\vec{B} = 4 \cos 10^6 t \, \hat{a_z}$ mT

(b) The bar slide at $\vec{u} = 20 \hat{a_y}$ m/s and $\vec{B} = 4 \hat{a_z}$ mT

(c) The bar slides at $\vec{u} = 20 \hat{a_y}$ m/s and $\vec{B} = 4 \cos(10^6 t - y) \hat{a_z}$ mT

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(a) $V_{emf} = \oint E \cdot dL = \int_S \frac{-dB}{dt} \cdot ds$

$$V_{emf} = \int_0^{0.08}\int_0^{0.06} 4 * 10^6 * 10^{-3} \sin 10^6 t \; \cancel{a_z \cdot a_z}^{=1} ) \, dx\, dy$$

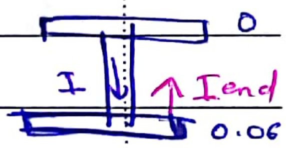
$$\boxed{V_{emf} = 19.2 \sin 10^6 t \; V}$$

* * * * *

(b)

$$V_{emf} = \oint_L E \cdot dL = \oint_L \vec{u} \times \vec{B} \cdot dL$$

$$V_{emf} = \int_{0.06}^{0} (20\, \cancel{a_y} \times 4 * 10^{-3}\, \cancel{a_z}) \cdot dx\, \cancel{a_x}^{=1}$$




$$\boxed{= -4.8\, mV}$$

(c) $V_{emf} = -\int_S \frac{dB}{dt} \cdot ds + \oint_L \vec{u} \times \vec{B} \cdot dL$

الحركة مستمرة ← (y)
لم يتوقف

$$= \int_0^{y}\int_0^{0.06} 4 * 10^6 * 10^{-3} \sin(10^6 t - y) \, dx\, dy$$

$$+ \int_{0.06}^{0} (20\, a_y \times 4 * 10^{-3} (\cos 10^6 t - y)\; a_z) \cdot dx\, a_x$$

9.7

# Time Harmonic fields.

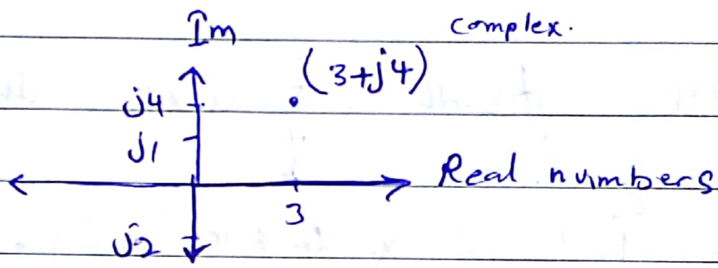
periodic
↓
sinusoidal ($\sin\theta$, $\cos\theta$).

Eulers identity:

$$e^{j\theta} = \cos\theta + j\sin\theta$$
$$e^{-j\theta} = \cos\theta - j\sin\theta$$

$j \equiv$ imaginary operator $= \sqrt{-1}$

complex.




$$\boxed{\begin{aligned}\cos\theta &= \frac{e^{j\theta} + e^{-j\theta}}{2} \\ \sin\theta &= \frac{e^{j\theta} - e^{-j\theta}}{2j}\end{aligned}}$$

$\ominus$ $\oplus$ ← $Z = x + jy$ (Rectangular form).

$Z = r\angle\theta$ (polar form) $\quad r = \sqrt{x^2+y^2}$

$\times \div$ $\quad \theta = \tan^{-1}\left(\frac{\text{Imaginary}}{\text{real}}\right)$

$Z = re^{j\theta}$ → [exponential form]

$$\sqrt{z} = \sqrt{r}\angle\theta/2$$

* * * * *

$$e^{\theta} = j\sinh\theta + \cosh\theta$$

for Example

$\vec{E}(x,y,z,t) \Rightarrow$ a time varying Electric field.

$$E(x,y,z) = Re\{\underbrace{E(x,y,z)}_{\text{Space factor}} \underbrace{e^{j\omega t}}_{\text{time factor}}\}$$

This only right if the source is sinusoidal.

$$E(x,y,z,t) = Re\{E_s e^{j\omega t}\}$$

$E_s$ (the phasor form) $= E(x,y,z)$

Time factor is exponential

$$\frac{d}{dt}(e^{j\omega t}) = j\omega e^{j\omega t}$$

$$\int e^{j\omega t}\,dt = \frac{1}{j\omega} e^{j\omega t}$$

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the first Maxwell's equ

$$\oint \bar{D} \cdot ds = \int \rho_v dv$$

$$\nabla \cdot \bar{D} = \rho_v$$

↓

$$\nabla \cdot \bar{D}(x,y,z,t) = \rho_v(x,y,z,t)$$

$$\nabla \cdot \bar{D}(x,y,z) \cancel{e^{j\omega t}} = \rho_{v_s} e^{j\omega t}$$

$$\boxed{\nabla \cdot \bar{D}_s = \rho_{v_s}}$$

1'st maxwells eq in phasor ~~domain~~ form.

↓

same as ch7

Second Maxwell's equation.

$$\oint E \cdot dl = -\int_S \frac{d\bar{B}}{dt} ds$$

$$\nabla \times E = -\frac{d\bar{B}}{dt}$$

$$\nabla \times \bar{E}_s e^{j\omega t} = -\frac{d}{dt}\left(\bar{B}_s e^{j\omega t}\right)$$

$$\nabla \times E_s \cancel{e^{j\omega t}} = -j\omega \cancel{e^{j\omega t}} \bar{B}_s$$

$$\boxed{\nabla \times \bar{E}_s = -j\omega \bar{B}_s}$$

2nd Maxwell's eq in phasor form

$$\bar{E}\underline{(x,y,z,t)} = Re\left\{\bar{E}_s e^{j\omega t}\right\}$$

↳ instantaneous form

$$\oint_L E_s e^{j\omega t} . dl = -\int \frac{d}{dt}(\bar{B}_s e^{j\omega t})\, ds$$

$$\oint_L \bar{E}_s \cancel{e^{j\omega t}} . dl = -j\omega \int \bar{B}_s \cancel{e^{j\omega t}} . ds$$

$$\boxed{\oint \bar{E}_s . dl = -j\omega \int_S \bar{B}_s . ds}$$

Maxwell's equation in Phasor form.

* Diff form — integral form.

1. $\nabla . \bar{D}_s = \rho_{V_s}$ — $\oint_S D_s = \int_V \rho_{V_s}\, dV$

2. $\nabla \times \vec{E} = -j\omega \bar{B}_s$ — $\oint_L \bar{E}_s . dl = -j\omega \int_S B_s . ds$

   $= \nabla \times \bar{E} = -j\omega \mu \bar{H}_s$ — $= -j\omega \int_S \mu \bar{H}_s . ds$

3. $\nabla \times \bar{H}_s = \bar{J}_s + j\omega \bar{D}_s$ — $\oint_L \bar{H}_s . dL = \int_S (\bar{J}_s + j\omega \bar{D}_s)\, ds$

   $\nabla \times \bar{H}_s = \sigma \bar{E}_s + j\omega \epsilon \bar{E}_s$ — $\oint \bar{H}_s . dL = \int_S (\sigma + j\omega\epsilon) \bar{E}_s . ds$

4. $\oint \nabla . \bar{B}_s ds = 0$ — $\oint \bar{B}_s . ds = 0$

Ex Given $\bar{A} = 10\cos(10^8 t + 10x + 60^\circ)\hat{a}_z$

$$\bar{B}_s = \frac{20}{j}\hat{a}_x + 10e^{j2\pi x/3}\hat{a}_y$$

↳ Expres $\bar{A}$ in phasor form.
↳ and $\bar{B}$ in instantaneous form.

$$\bar{A} = Re\{\bar{A}_s e^{j\omega t}\}$$

$$Re\{10 e^{j(10^8 t - 10x + 60^\circ)}\hat{a}_z\}.$$

$$= Re\{10\, e^{j10^8 t} * e^{j(-10x+60^\circ)}\hat{a}_z\}$$

↓ $w = 10^8$ rad/s

$$\boxed{\bar{A}_s = 10e^{j(-10x+60^\circ)}\hat{a}_z}$$

$$\cancel{A_s = 10e^{j\omega t} - 10x + 60^\circ\, \hat{a}_z}$$

$$\boxed{\bar{A}_s = 10e^{j(-10x+60^\circ)}}$$

$$\bar{B}_s = \frac{20}{j}\hat{a}_x + 10e^{j2\pi x/3}$$

$$\bar{B} = Re\{\bar{B}_s e^{j\omega t}\}$$

$$\bar{B} = Re\{(-j20\hat{a}_x + 10e^{j2\pi y/3}\hat{a}_y)(e^{j\omega t})\}$$

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$j = e^{j\pi/2} = 1\angle 90^\circ$

$\bar{B} = Re\left\{\left[\left(-20\, e^{j(\omega t + \frac{\pi}{2})}\, \hat{a}_x + 10\, e^{j(\omega t + 2\pi x/3)}\, \hat{a}_y\right)\right]\right\}$

20 sin

$\bar{B} = -20 \cos(\omega t + \frac{\pi}{2})\, \hat{a}_x + 10 \cos\left(\omega t + \frac{2\pi x}{3}\right) \hat{a}_y$ T

$$\boxed{\bar{B} = 20 \sin(\omega t + \frac{\pi}{2})\, \hat{a}_y + 10 \cos(\omega t + \frac{2\pi x}{3})\, \hat{a}_y}$$

$x_y$
$(t, j)$

$\mu = \mu_o$ $\epsilon = \epsilon_o$ $\sigma = 0$

* Given the fields in free space and source free Region as $\vec{E} = \frac{50}{\rho} \cos(10^6 t + \beta z)\, \hat{a}_\phi$ V/m

$\vec{H} = \frac{H_o}{\rho} \cos(10^6 t + \beta z)\, \hat{a}_\rho$ A/m

$\rho_v = 0$
$\bar{J} = 0$

find $H_o$, $\beta$ after converting the field to phasor form)

* ① $\bar{E}_s = \frac{50}{\rho}\, e^{j\beta z}\, \hat{a}_\phi$ ② $\bar{H}_s = \frac{H_o}{\rho}\, e^{j\beta z}\, \hat{a}_\rho$

$\omega = 10^6$

( Maxwell's eq in integral form ~~can't~~ must ~~be~~ give the Region information)

↓ diff form

$\nabla \cdot \bar{D}_s = \rho_{vs} = 0$.

$\nabla \cdot \bar{B}_s = 0$.

curl
$$\begin{cases} \nabla \times \bar{E}_s = -j\omega \mu \bar{H}_s \\ \nabla \times \bar{H}_s = 0 + j\omega \epsilon \bar{E}_s \end{cases}$$

$\nabla \times \bar{E}_s = -j\omega \mu \bar{H}_s$.

$$\bar{H}_s = \frac{-1}{j\omega\mu} \nabla \times \bar{E}_s = \frac{j}{\omega\mu} \nabla \times \bar{E}_s$$

$$\bar{H}_s = \frac{j}{\omega\mu}\left(\frac{1}{\rho}\begin{vmatrix} a_\rho & \rho a_\phi & a_z \\ \partial/\partial\rho & \partial/\partial\phi & \partial/\partial z \\ 0 & \rho 50 e^{j\beta z} & 0 \end{vmatrix}\right)$$

$$\cancel{e^{j\beta z}} * H_0 \frac{\cancel{a_\rho}}{\cancel{\rho}} = \frac{j}{\omega\mu}\left(\frac{1}{\cancel{\rho}}\left(-50 j\beta \cancel{e^{j\beta z}} \cancel{a_\rho}\right)\right)$$

$$H_0 = \frac{50\beta}{\omega\mu} \Rightarrow (1)$$

$$\bar{E}_s = \frac{1}{j\omega\epsilon} \nabla \times \bar{H}_s$$

$$\frac{50}{\rho} e^{j\beta z} a_\phi = \frac{1}{j\omega\epsilon} \left| \frac{1}{\rho} \right| \begin{vmatrix} a_\rho & \rho a_\phi & a_z \\ \partial/\partial\rho & \partial/\partial\phi & \partial/\partial z \\ \frac{H_o e^{j\beta z}}{\rho} & 0 & 0 \end{vmatrix}$$

$$\frac{50}{\cancel{\rho}} \cancel{e^{j\beta z}} \cancel{a_\phi} = \frac{1}{j\omega\epsilon \cancel{\rho}} \left( -\rho \cancel{\hat{a}_\phi} \left( -j \frac{H_o \beta}{\cancel{\rho}} \cancel{e^{j\beta z}} \right) \right)$$

$$\boxed{50 = \frac{H_o \beta}{\omega\epsilon}} \rightarrow 2$$

from equ 1 sub in 2

$$\cancel{50} = \frac{\cancel{50} \beta^2}{\omega^2 \mu\epsilon}$$

$$\beta^2 = \omega^2 \mu\epsilon \qquad \beta = \pm\sqrt{\omega^2 \mu\epsilon}$$

$$\beta = \pm 3.33 \times 10^{-3}$$

*

$$H_o = \pm 0.1326$$

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$H_o = 0.1326$ A/m $\quad \beta = 3.33 * 10^{-3}$ $\quad \boxed{\beta = \pm \frac{\omega}{\omega}}$

$H_o = -0.1326$ A/m $\quad \beta = -3.33 * 10^{-2}$

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$\frac{1}{\sqrt{\mu_o \epsilon_o}} = 3 * 10^{8}$ m/s $= c$ = speed of light in free space

$\eta = \frac{E}{H}$ $\quad \eta = \frac{50/\beta}{\cancel{15\beta/\beta}}$ $\quad \eta = \frac{50}{H_o}$.

air impedence

$\eta_o = \sqrt{\frac{\mu_o}{\epsilon_o}}$

$\eta_o = \sqrt{\frac{\mu}{\epsilon}}$

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* محمد الشمري ... *

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