EM2

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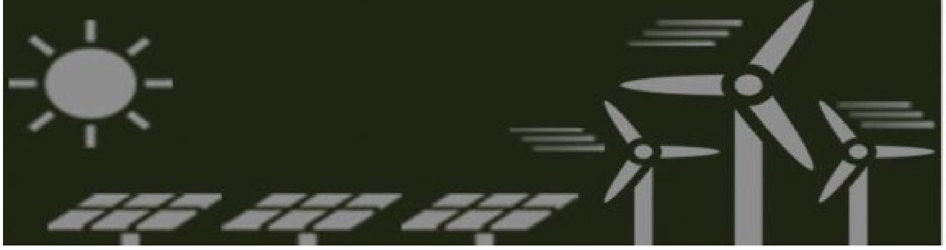

28/1/2018 No. EM2 Dr. Yanal Al faouri

* preview the 1^{st} three chapters

* differential elements :
$\vec{dL}$, $\vec{ds}$, dv

* Electrostatics $\bar{E}$
① coloumbs law
② Gauss law]→ charge is known
③ potential , $\bar{E} = -\nabla V$]→ potential is known
④ B.V.P]→ charge and potential is knaon at a certin location
⑤ methods of image

⇒ $D = \epsilon \bar{E}$ ⇒ $\int D \cdot \vec{ds} = Q$
⇒ $\bar{J} = \sigma \bar{E}$ ⇒ $\int_s \bar{J} \cdot \vec{ds} = I$
⇒ $w_e = \frac{1}{2} \epsilon E^2$ → Energy density

⇒ $R = \frac{V}{I} = \frac{-\int_L \bar{E} \cdot \vec{dL}}{\int_s \sigma \bar{E} \cdot \vec{ds}}$

⇒ $C = \frac{Q}{V} = \frac{\int_s \epsilon \bar{E} \cdot \vec{ds}}{-\int_L \bar{E} \cdot \vec{dL}}$

* Magnetic static
① Bio Savart's law
② Ampere's law ⇒ $\oint_L \bar{H} \cdot \vec{dL} = I_{enc}$]⇒ current is known
③ magnetic potential
- scalar V_m
- vector $\bar{A}$

*

No.

$\bar{B} = \nabla \times \bar{A}$

$B = \mu H$

$\bar{B}$: magnetic flux density

$\bar{A}$: magnetic potintial

V_m: Scalar magnetic potential

V: Scalar electric potential

④ B.V.P

⑤ Method of images

* For time varying field

$$I_{out} = I_{in}$$

$$\oint_s \bar{J} \cdot \bar{ds} = -\frac{d\,Q_{in}}{\partial t}$$

→ continuity equation, $Q = \int_v \rho_v \, dv$

$$= \frac{-d}{dt} \int_v \rho_v \, dv$$

⇒ Applying divergence theorem for $\oint_s \bar{J} \cdot \bar{ds} = \int_v \nabla \cdot \bar{J} \, dv$

$$\Rightarrow \int_v \nabla \cdot J \, dv = \frac{-d}{dt} \int_v \rho_v \, dv$$

$$\cancel{\int_v} \nabla \cdot \bar{J} \, \cancel{dv} = -\cancel{\int_v} \frac{d \rho_v}{dt} \, \cancel{dv}$$

$$\boxed{\nabla \cdot \bar{J} = -\frac{d \rho_v}{dt}}$$ → Continouity equation

* $\rho_v = \rho_{v_0} e^{-\frac{t}{T_r}}$] $\rho_v(t)$

T_r = Relaxation time

$= RC_{dc}$

- $e^{-1} = 1\,T_r = 0.368 = 36.8\,\%$
- $e^{-5} = 0.001 < 1\,\% = 5T_r$ → Steady state ; it means all charges are on the outside

No.

* 9.5 Maxwell's equation in time varying field

* $\nabla \cdot \bar{D} = \rho_v \rightarrow \oint_s \bar{D} \cdot \bar{ds} = \int_v \rho_v \, dv = Q$

* $\nabla \cdot \bar{B} = 0 \rightarrow \oint_s \bar{B} \cdot \bar{ds} = 0 = Q_m$

* $\nabla \times \bar{E} = \frac{-\partial \bar{B}}{dt} \rightarrow \oint_L \bar{E} \cdot \bar{dl} = \oint_s \nabla \times \bar{E} \, \bar{ds} = -\oint_s \frac{d\bar{B}}{dt} \cdot \bar{ds}$ (Faraday)

* $\nabla \times \bar{H} = J + \frac{d\bar{D}}{dt} \rightarrow \oint \bar{H} \cdot \bar{dl} = \oint \left(J + \frac{d\bar{D}}{dt}\right) \bar{ds}$

$J \rightarrow$ Conduction currents

Displacement current (I_d)

* Maxwells eq + Constitutive relation

$\nabla \cdot \epsilon \bar{E} = \rho_v$

$\nabla \cdot \bar{B} = 0$

$\nabla \times \bar{E} = -\mu \frac{d\bar{H}}{dt}$

$\nabla \times \bar{H} = \sigma \bar{E} + \epsilon \frac{d\bar{E}}{dt}$

$\nabla \cdot \bar{D} = \rho_v \quad ; \bar{D} = \epsilon \bar{E}$

$\nabla \cdot \bar{E} = \frac{\rho_v}{\epsilon}$, if ϵ is homogeneous

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EM 2 ...

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* Time varying potentials.

for static field:

$$V = \int_v \frac{\rho_v \, dv}{4\pi\epsilon R} \quad , \quad Q = \int_v \rho_v \, dv$$

$$\bar{A} = \int_v \frac{\mu \bar{J} \, dv}{4\pi R}$$

Recall :-

$\bar{B} = \nabla \times \bar{A}$, $\bar{B} = \mu \bar{H}$

Maxwell's eq.

$$\nabla \cdot \bar{D} = \rho_v$$

$$\nabla \cdot \bar{B} = 0$$

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\nabla \times \bar{E} = -\frac{\partial}{\partial t}(\nabla \times \bar{A})$$

$$\nabla \times \underbrace{\left(\bar{E} + \frac{\partial \bar{A}}{\partial t}\right)}_{} = 0$$

in general |||

$$\nabla \times (-\nabla V) = 0$$

No.

$$\bar{E} + \frac{\partial \bar{A}}{\partial t} = -\nabla V$$

$$\bar{E} = -\nabla V - \frac{\partial \bar{A}}{\partial t}$$

$$\bar{E} = -\nabla V \quad \text{in static}$$

$$\nabla \cdot \bar{D} = \rho \quad , \bar{D} = \epsilon \bar{E}$$

$$\nabla \cdot E = \frac{\rho_v}{\epsilon} \quad , \text{if } \epsilon \text{ is homogeneous}$$

$$\nabla \cdot \left(-\nabla V - \frac{\partial \bar{A}}{\partial t}\right) = \frac{\rho_v}{\epsilon}$$

$$-\nabla^2 V - \frac{\partial}{\partial t}(\nabla \cdot \bar{A}) = \frac{\rho_v}{\epsilon}$$

$$\nabla^2 V + \frac{\partial}{\partial t}(\nabla \cdot \bar{A}) = -\frac{\rho_v}{\epsilon} \quad \ldots (1)$$

poisson's equation for time varying fields.

from $\bar{B} = \nabla \times \bar{A}$

Take $\nabla \times$ for both sides

$$\nabla \times \nabla \times \bar{A} = \nabla \times \bar{B} = \nabla \times \mu \bar{H} = \mu(\nabla \times \bar{H})$$

$$\nabla \times \nabla \times \bar{A} = \mu \bar{J} + \mu \epsilon \frac{\partial \bar{E}}{\partial t}$$

but $\nabla \times \nabla \times \bar{A} = \nabla(\nabla \cdot \bar{A}) - \nabla^2 A$ → given

$$\boxed{\nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A} = \mu \bar{J} - \mu \epsilon \nabla\left(\frac{dv}{dt}\right) - \mu \epsilon \frac{\partial^2 \bar{A}}{\partial t^2}}$$

No.

we can write :

$$\nabla . \bar{A} = -\mu \epsilon \frac{\partial v}{\partial t}$$ ... sub in ①

$$\nabla^2 A = -\mu \bar{J} + \mu \epsilon \frac{\partial^2 A}{\partial t^2}$$

$$\boxed{\nabla^2 \bar{A} - \mu \epsilon \frac{\partial^2 \bar{A}}{\partial t^2} = -\mu \bar{J}}$$ ..... ⓐ in static $\nabla^2 \bar{A} = -\mu \bar{J}$

↳ vector poisson's equation

$$\boxed{\nabla^2 v - \mu \epsilon \frac{\partial^2 v}{\partial t^2} = \frac{-\rho_v}{\epsilon}}$$ ... ⓑ

↳ scalar poisson's equation.

(a . b) → wave equation

⇒ Solution for a, b :-

$$V = \int_v \frac{[\rho_v] \, dv}{4\pi \epsilon R}$$

$[\rho_v]$ = Retarted (lag) volume charge density (C/m^3)

$[\bar{J}]$ = Retarted volume current density (A/m^2)

$$\bar{A} = \int_v \frac{\mu [J] \, dv}{4\pi R}$$

in time varying

$\rho_v (x, y, z ; t)$

in Ch.13 Antenna

$[\rho_v] (x, y, z ; t')$.

$t' = t - \frac{R}{u}$ $\boxed{t > t'}$

CH.10 EM waves

No.

EM waves travelling in unbounded (unguided) media.

properties of EM waves :-

1) All function of space and time $\left(\bar{J}_d = \frac{\partial \bar{D}}{\partial t}\right)$

2) waves are travelling at high speed, max speed is $c = 3 \times 10^8$ m/s
speed of light in free space.

3) They rediate outward from a source in all directions.

*Types of media :-

1) lossy dielectric medium :-

$\epsilon = \epsilon_0 \epsilon_r$, $\mu = \mu_0 \mu_r$, $\sigma \neq 0$

2) lossless dielectric medium :-
$\epsilon = \epsilon_0 \epsilon_r$, $\mu = \mu_0 \mu_r$, $\sigma = 0$

3) Free space medium :-
$\epsilon = \epsilon_0$, $\mu = \mu_0$, $\sigma = 0$

4) perfect conducting medium
$\epsilon = \epsilon_0$, $\mu = \mu_0 \mu_r$, $\sigma \approx \infty$

No.

* wave equation :- (lossless medium)

$$\nabla^2 V - \mu\epsilon \frac{\partial^2 V}{\partial t^2} = \frac{-\rho_v}{\epsilon}$$

→ we choose the scalar poisson's equation

→ Assume the wave travelling in one direction only (assume the Z-dir)

$$\frac{\partial V}{\partial x} = 0 \quad , \quad \frac{\partial V}{\partial y} = 0$$

$$\frac{\partial^2 V}{\partial z^2} - \mu\epsilon \frac{\partial^2 V}{\partial t^2} = \frac{-\rho_v}{\epsilon}$$

→ for source free region ($\rho_v = 0$, $\bar{J} = 0$)

sub $V = E$, $V = -\int_l \bar{E} \cdot d\bar{l}$

$$\frac{\partial^2 \bar{E}}{\partial z^2} - \mu\epsilon \frac{\partial^2 E}{\partial t^2} = 0$$

$$\frac{\partial^2 E}{\partial t^2} - \frac{1}{\mu\epsilon} \frac{\partial^2 \bar{E}}{\partial z^2} = 0$$

$$\boxed{\frac{\partial^2 E}{\partial t^2} - u^2 \frac{\partial^2 \bar{E}}{\partial z^2} = 0} \quad , \quad u = \frac{1}{\sqrt{\mu\epsilon}} = \text{wave velocity m/s}$$

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* wave equation

$$\frac{\partial^2 E}{\partial t^2} - u^2 \frac{\partial^2 E}{\partial z^2} = 0$$

one - dimentional wave eq. in lossless media and source - free region.

→ solution

→ convert to phasor

$$\frac{\partial}{\partial t} \to j\omega$$

$$E(x,y,z;t) \to E_s(x,y,z)e^{j\omega t}$$

$$\frac{(j\omega)^2}{-u^2} E_s \cancel{e^{j\omega t}} - \frac{u^2}{-u^2} \frac{\partial^2 E_s}{\partial z^2} \cancel{e^{j\omega t}} = 0$$

$$\frac{\partial^2 E_s}{\partial z^2} + \frac{\omega^2}{u^2} E_s = 0$$

let $\frac{\omega}{u} = B$ ← rad/s, ← m/s

$B \equiv$ phase constant (rad/m)

$$u = \frac{1}{\sqrt{\mu \epsilon}}$$

$$\boxed{\frac{\partial^2 E_s}{\partial z^2} + B^2 E_s = 0}$$ Helmholtz's wave equation. "Phasor"

2^{nd} order homogeneous Diff eq.

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let $\frac{\partial}{dz} = m$

$m^2 E_s + \beta^2 E_s = 0$

$m^2 + \beta^2 = 0$

$m = \pm j\beta \rightarrow$ pure imagenary

$\frac{\partial E_s}{dz} = \pm j \beta E_s$

$\int \frac{dE_s}{E_s} = \int \pm j\beta \, dz$

$\ln E_s = \pm j\beta z$

$E_s = e^{\pm j\beta z}$

* Euler's identity

$E_s = \cos(\beta z) \pm j \sin(\beta z)$

$\bar{E} = Re\{E_s e^{j\omega t}\}$

$\bar{E} = Re\left\{ \underbrace{E_0^+ e^{j(\omega t - \beta z)}}_{\text{Forward wave } (+\hat{a}_z)} + \underbrace{E_0^- e^{j(\omega t + \beta z)}}_{\text{backward wave } (-\hat{a}_z)} \right\}$

choose the forward solution :

$\boxed{E = \underbrace{E_0^+}_{\text{Amplitude}} \underbrace{\cos(\omega t - \beta z)}_{\text{Phase}} \ V/m}$

$\omega t \Rightarrow$ Time تأثير ال

$\beta z \Rightarrow$ Space 〃 〃

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$\omega t - \beta z = \text{constant}$

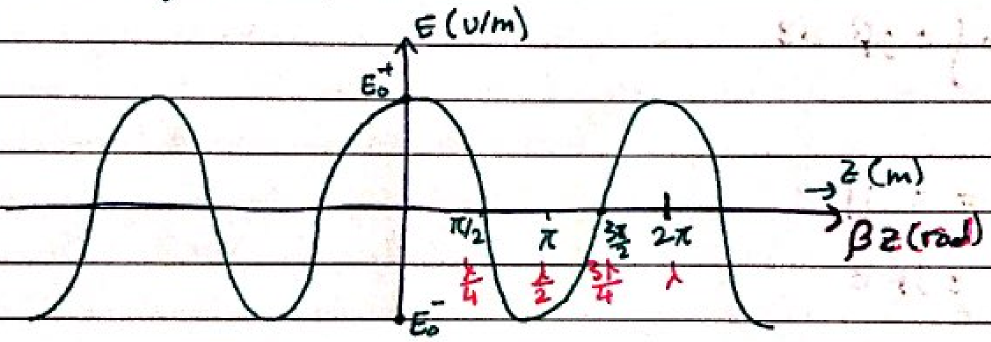
$\omega dt - \beta dz = 0 \Rightarrow \omega dt = \beta dz$

$$\frac{dz}{dt} = \frac{\omega}{\beta} = u$$

Sketch :-

at $t=0$ (wave as a function of space)

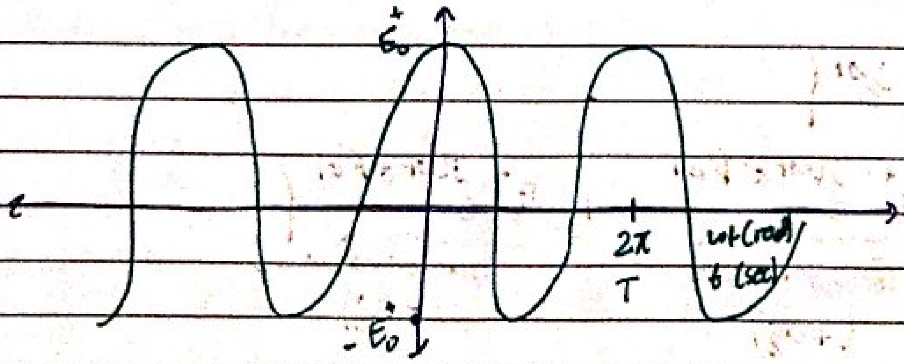
$\bar{E} = E_0^+ \cos(+\beta z)$



$\lambda \equiv$ wave length (m)

at $z=0$ (wave as a function of time)

$\bar{E} = E_0^+ \cos(\omega t)$



$T \equiv$ period , $T = \frac{1}{f}$ (s) $\Rightarrow u = \frac{\lambda}{T} = \lambda f$

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$\beta = \frac{2\pi}{\lambda} = \frac{rad}{m}$

$\omega = 2\pi f$

$u = \frac{\omega}{\beta} = \frac{2\pi f}{\frac{2\pi}{\lambda}} = \lambda f$

$$\boxed{u = \frac{1}{\sqrt{\mu\epsilon}} = \lambda f = \frac{\omega}{\beta}}$$

* properties of sinusoids :-

$\sin(-\varphi) = -\sin(\varphi)$

$\cos(-\varphi) = \cos(\varphi)$

$\sin(\varphi \mp \frac{\pi}{2}) = \mp \cos(\varphi)$

$\sin(\varphi \mp \pi) = -\sin\varphi$

$\cos(\varphi \pm \frac{\pi}{2}) = \mp \sin(\varphi)$

$\cos(\varphi \pm \pi) = -\cos(\varphi)$

* Ex. given $\bar{E} = 50 \cos(10^8 t + \beta x)\, \hat{a_y}$ V/m travels in free space. Find:

a) direction of propagation.

b) Calculate β, and time it takes to travel a distance of $\frac{\lambda}{2}$

c) sketch the wave at $t = 0, \frac{T}{4}, \frac{T}{2}$

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a) $\hat{a}_k = \boxed{-\hat{a}_x}$

b) i) $\beta = \frac{\omega}{u} = \frac{10^8}{3\times10^8} = \boxed{0.333 \text{ rad/m}}$

b ii) $\lambda \longrightarrow T$

$\frac{\lambda}{2} \longrightarrow \frac{T}{2}$

$\omega = 2\pi f = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{10^8}$

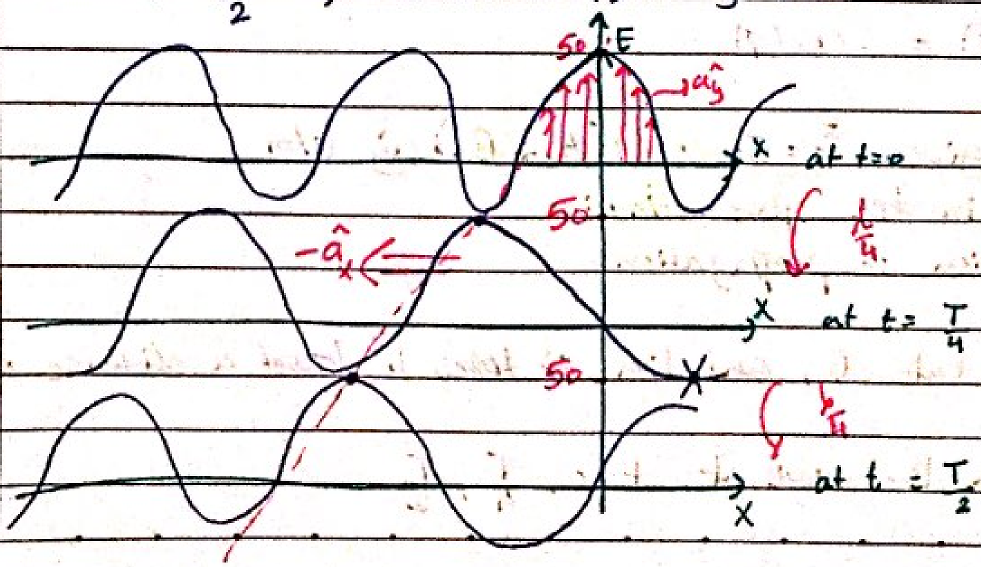
$\boxed{T = 62.84 \text{ ns}}$

to travel $\frac{\lambda}{2} \longrightarrow \boxed{t = \frac{T}{2} = 31.42 \text{ ns}}$

c) at $t=0$: $\bar{E} = 50\cos(\beta x)\,\hat{a}_y$ V/m

at $t = \frac{T}{4} \rightarrow \bar{E} = -50\sin(\beta x)\,\hat{a}_y$ V/m

at $t = \frac{T}{2} \rightarrow \bar{E} = -50\cos(\beta x)\,\hat{a}_y$ V/m



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* wave propagation in lossy dielectrics :-

$\epsilon = \epsilon_0 \epsilon_r \qquad \mu = \mu_0 \mu_r \qquad \sigma \neq 0$

→ lossy → imperfect conductor, imperfect dielectric
losses in power within the wave.

$$\rho = \frac{1}{\sigma}$$

from maxwell's equation in source free region ($\rho_v = 0$)

$$\nabla \cdot \bar{D} = 0 \rightarrow \nabla \cdot \vec{E} = 0$$
$$\nabla \cdot \bar{B} = 0 \rightarrow \nabla \cdot \vec{H} = 0$$
$$\nabla \times \bar{E}_s = -j\omega\mu \bar{H}_s$$
$$\nabla \times \bar{H}_s = \bar{J} + j\omega\epsilon \bar{E}_s$$
$$= (\sigma + j\omega\epsilon)\vec{E}_s$$

$$\nabla \times (\nabla \times \vec{E}_s) = \nabla \times (-j\omega\mu \vec{H}_s)$$
$$= -j\omega\mu \, \nabla \times \bar{H}_s$$

$$\nabla \times \nabla \times \bar{E} = \nabla(\cancelto{0}{\nabla \cdot \vec{E}_s}) - \nabla^2 \bar{E}_s \qquad \rightarrow \text{in general}$$

$$-\nabla^2 \vec{E}_s = -j\omega\mu((\sigma + j\omega\epsilon)\vec{E}_s)$$
$$\nabla^2 \vec{E}_s = j\omega\mu \bar{E}_s(\sigma + j\omega\epsilon) \qquad \therefore j\omega\mu(\sigma + j\omega\epsilon) = \text{constant} = \gamma^2$$
$$\nabla^2 \bar{E}_s - \gamma^2 \bar{E}_s = 0$$

$$\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$$

$$\underline{\underline{\gamma}} = \underline{\alpha} + j\underline{\beta}$$

β → phase constant (rad/m)

α → attenuation constant (Np/m or dB/m)

γ → propagation constant

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if $\sigma = 0$
$\gamma^2 = -\omega^2 \mu \epsilon$

$\gamma = j\omega\sqrt{\mu\epsilon} = \alpha + j\beta$
$\alpha = 0 \quad \beta = \omega\sqrt{\mu\epsilon}$ lossless

$\nabla^2 \bar{E}_s - \gamma^2 \bar{E}_s = 0$ → vector Helmholtz's wave equation

if $\nabla \times \nabla \times \bar{H}_s = (\sigma + j\omega\epsilon)\, \nabla \times \bar{E}_s$
$\nabla^2 \bar{H}_s - \gamma^2 \bar{H}_s = 0$

→ Solve for $\bar{E}_s$, Assume the wave travel in z-direction and the electric field is in the x-direction:

$E = E_{xs}(z)$?

$\frac{d^2E}{dx^2} = 0$, $\frac{d^2E}{dy^2} = 0$, $E(z)$

$\frac{d^2\bar{E}_{xs}}{dz^2} - \gamma^2 \bar{E}_{xs} = 0$

$\left(\frac{d^2}{dz^2} - \gamma^2\right) \bar{E}_{xs} = 0$

$(m^2 - \gamma^2)\, \vec{E}_{xs} = 0$

$m^2 - \gamma^2 = 0$
$m^2 = \gamma^2$
$m = \pm\sqrt{\gamma}$ two real solutions

Sol ⇒ $e^{\pm\gamma z}$ "without j"

$E_{xs}(z) = \underbrace{E_o^+ e^{-\gamma z}}_{\text{forward}} + \underbrace{E_o^- e^{\gamma z}}_{\text{backward}}$

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$E = R_e \{ E_{xs}(z) e^{j\omega t} \}$, instantaneous form (time domain)

- take only forward solution

$= R_e \{ E_o^+ e^{-(\alpha + j\beta)z} e^{j\omega t} \}$

$= R_e \{ E_o^+ e^{-\alpha z} e^{-j\beta z} e^{j\omega t} \}$

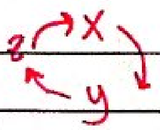
$= R_e \{ E_o^+ e^{-\alpha z} e^{j(\omega t - \beta z)} \}$

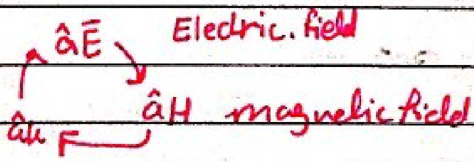
$$\boxed{\vec{E}(z;t) = E_o^+ e^{-\alpha z} \cos(\omega t - \beta z)\, \hat{a_x}}$$

for $\bar{H}$?

by symmetry

$$\boxed{\vec{H}(z,t) = H_o^+ e^{-\alpha z} \cos(\omega t - \beta z)\, \hat{a_y}}$$

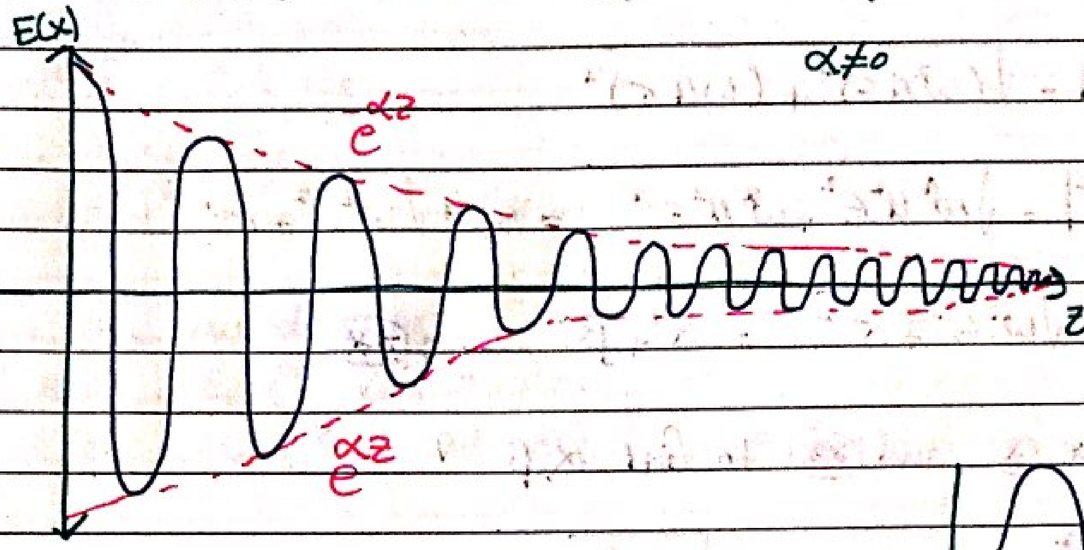


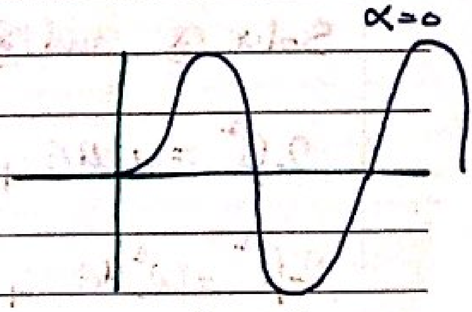


$\hat{a}H = \hat{a}k \times \hat{a}\bar{E}$

$= \hat{a}_z \times \hat{a}_x = +\hat{a}_y$

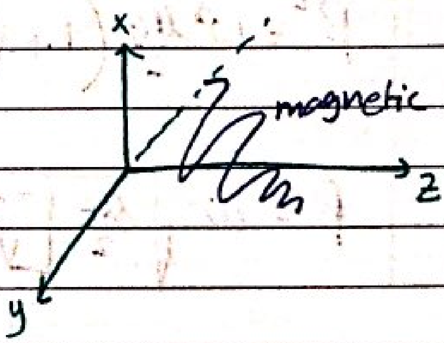
* Sketch : $\bar{E}(z,t)$ at $(t=0)$ → function of space





→ $\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon) \dashrightarrow ①$

$\gamma = \alpha + j\beta \longrightarrow ②$



$\gamma^2 = \alpha^2 + 2j\alpha\beta - \beta^2 \dashrightarrow ②$

take Real of eq① and eq②

$\alpha^2 - \beta^2 = -\omega^2\mu\epsilon \longrightarrow (*)$

magnitude of $|\gamma^2|$ from eq②

$$|\gamma^2| = \sqrt{(\alpha^2-\beta)^2 + (2\alpha\beta)^2}$$

$$= \sqrt{\alpha^4 - 2\alpha^2\beta^2 + \beta^4 + 4\alpha^2\beta^2}$$

$$= \sqrt{\alpha^4 + 2\alpha^2\beta^2 + \beta^4}$$

$$= \sqrt{(\alpha^2+\beta^2)^2} \Longrightarrow |\gamma^2| = \alpha^2 + \beta^2$$

magnitude of $|\gamma^2|$ from eq ①

$$|\gamma^2| = \sqrt{(\omega^2\mu\epsilon)^2 + (\omega\mu\sigma)^2}$$

$$|\gamma^2| = \sqrt{\omega^4\mu^2\epsilon^2 + \omega^2\mu^2\sigma^2} = \omega\mu\sqrt{\omega^2\epsilon^2+\sigma^2}$$

$$\omega\mu\sqrt{\omega^2\epsilon^2+\sigma^2} = \alpha^2+\beta^2 \dashrightarrow (**)$$

Solve (*) and (**) to find α, β ?

$$2\beta^2 = \omega^2\mu\epsilon + \omega\mu\sqrt{\omega^2\epsilon^2+\sigma^2}$$

$$2\beta^2 = \omega^2\mu\epsilon + \omega^2\mu\epsilon\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}$$

$$\beta^2 = \omega^2\frac{\mu\epsilon}{2}\left(1+\sqrt{\left(\frac{\sigma}{\omega\epsilon}\right)^2+1}\right)$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\mu\epsilon}\right]^2}+1\right]}$$

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right]}$$

No. 6/2/2018

* wave propagation in lossy dielectric

$$\nabla^2 \bar{E}_s - \gamma^2 E_s = 0$$

$$\nabla^2 \bar{H}_s - \gamma^2 \bar{H}_s = 0$$

sol. (if $\hat{a}_k = \hat{a}_z$)

$$\bar{E} = (E_o^+ e^{-\gamma z} + E_o^- e^{\gamma z})\, \hat{a}_x \quad V/m$$

Take the forward wave

$$\bar{E}(z,t) = E_o e^{-\alpha z} \cos(\omega t - \beta z)\, \hat{a}_x \quad V/m$$

$$\bar{H}(z,t) = H_o e^{-\alpha z} \cos(\omega t - \beta z)\, \hat{a}_y \quad A/m$$

* Intrinsic impedance (η)

$$\eta = \frac{E}{H} \left[\frac{V/m}{A/m} = \frac{V}{A} = \Omega \right]$$

in general

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} \rightarrow \text{from maxwell's equations}$$

$$\nabla \times \bar{E}_s = -j\omega\mu \bar{H}_s$$

$$\bar{E}_s = E_o e^{-\gamma z} \hat{a}_x$$

$$\bar{H}_s = H_o e^{-\gamma z} \hat{a}_y$$

$$\bar{H}_s = \frac{-1}{j\omega\mu} \nabla \times \bar{E}_s$$

$$= \frac{-1}{j\omega\mu} \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ d/dx & d/dy & d/dz \\ E_o e^{-\gamma z} & 0 & 0 \end{vmatrix} = \frac{-1}{j\omega\mu} \left[+ \left(-\gamma E_o e^{-\gamma z} \right) \hat{a}_y \right] = H_o e^{-\gamma z} \hat{a}_y$$

$$H_o = \frac{\gamma E_o}{j\omega\mu}$$

$$\frac{E_o}{H_o} = \eta = \frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma + j\omega\epsilon)}} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

No.

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = |\eta| \angle \theta_\eta$$

$$\eta^2 = \frac{j\omega\mu}{\sigma + j\omega\epsilon} + \frac{\sigma - j\omega\epsilon}{\sigma - j\omega\epsilon}$$

$$|\eta| = \frac{\sqrt{\mu/\epsilon}}{\left[1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/4}}$$

$$\tan(2\theta_\eta) = \frac{\sigma}{\omega\epsilon}$$, $Z = x + jy = r\angle\theta$

$\theta = 2\theta_\eta$ → loss angle

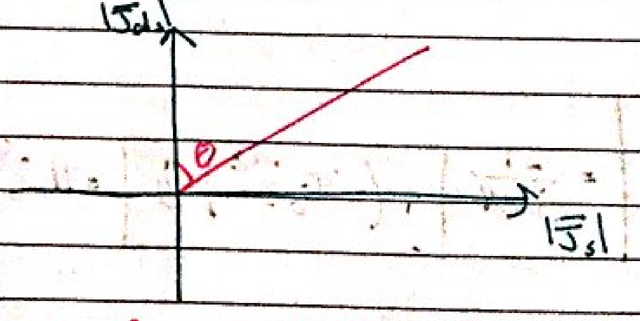
$\sqrt{Z} = \sqrt{r}\angle\theta/2$

$$\boxed{\tan\theta = \frac{\sigma}{\omega\epsilon}}$$ → loss tangent

$$\tan\theta = \frac{\sigma}{\omega\epsilon} = \frac{|\bar{J}_s|}{|\bar{J}_{d_s}|} = \frac{|\sigma \bar{E}_s|}{|j\omega\epsilon\bar{E}_s|} = \frac{\sigma}{\omega\epsilon}$$

$$J_d = \frac{\partial \bar{D}}{\partial t}$$

$$\bar{J}_{d_s} = j\omega\bar{D}_s = j\omega\epsilon\bar{E}_s$$



$\theta = 90^\circ \Rightarrow$ perfect conductor $(\sigma = \infty)$
↳ $\theta_\eta = 45^\circ$

$\theta = 0^\circ \Rightarrow$ perfect dielectric media $(\sigma = 0)$ → ① lossless ② freespace
↳ $\theta_\eta = 0^\circ$

* lossy case vary between $90^\circ - 0^\circ$ (general case)

No.

from η :-

$\bar{E} = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\, \hat{a}_x$ V/m , $\eta = \frac{E_0}{H_0}$

$\bar{H} = \frac{E_0}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)\, \hat{a}_y$ A/m

$H_0 = \frac{E_0}{\eta}$ ← complex

$H_0 = \frac{E_0}{|\eta| \angle \theta_\eta}$

$\omega, \mu, \epsilon, \sigma \rightarrow \alpha, \beta, \eta$

E leads H by θ_η

V " I

∴ Inductive media

⟹ units :-

$\delta \longrightarrow$ /m

$\beta \rightarrow$ rad/m

$\alpha \rightarrow$ Np/m or dB/m

$1\,np = 10 \log_{10} e = 8.686$ dB

$\alpha = 0.5$ dB/m

$\lambda = \frac{2\pi}{\beta}$, $u = \frac{1}{\sqrt{\mu \epsilon}} = \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}} = \frac{c}{\sqrt{\mu_r \epsilon_r}}$

if non-magnetic $(\mu_r = 1) \rightarrow u = \frac{c}{\sqrt{\epsilon_r}}$

No.

$u = \frac{\omega}{\beta}$ for lossless or free space

$\beta = \omega\sqrt{\mu\epsilon} = \frac{\omega}{u}$

$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right)}$

* Complex permittivity (ϵ_c)

from maxwell's eq.

$\nabla \times \bar{H}_s = (\sigma + j\omega\epsilon)\bar{E}_s$

$= j\omega\left(\epsilon\left(1+\frac{\sigma}{j\omega\epsilon}\right)\right)\bar{E}_s$

$\boxed{\nabla \times \bar{H}_s = j\omega\,\epsilon_c\,\bar{E}_s}$, if source free ($\bar{J}=0$)

$\nabla \times \bar{H}_s = j\omega\epsilon\bar{E}_s$

$\epsilon_c = \epsilon' - j\epsilon'' = \epsilon\left(1+\frac{\sigma}{j\omega\epsilon}\right) = \cancel{\epsilon}\left(\epsilon - \frac{j\sigma}{\omega\cancel{\epsilon}}\right)$

$\boxed{\epsilon' = \epsilon = \epsilon_0\epsilon_r}$ (real part)

$\boxed{\epsilon'' = \frac{\sigma}{\omega}}$ (imaginary part)

* special cases :-

* wave propagation in lossless medium :-

$\epsilon = \epsilon_0\epsilon_r$, $\mu = \mu_0\mu_r$, $\sigma = 0$

$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right)}$

$\alpha = 0$

No.

$\beta = \omega\sqrt{\mu\epsilon} = \frac{\omega}{u}$

$u = \frac{1}{\sqrt{\mu\epsilon}}$, $\lambda = \frac{2\pi}{\beta}$

$\eta = \sqrt{\frac{j\omega\mu}{\sigma^2 + j\omega\epsilon}} \Rightarrow \eta = \sqrt{\frac{\mu}{\epsilon}}$, $\theta_\eta = 0^\circ$

$\bar{E} = E_0 \cos(\omega t - \beta z)\, \hat{a}_x$ V/m

$\bar{H} = \frac{E_0}{\eta} \cos(\omega t - \beta z)\, \hat{a}_y$ A/m

$\bar{E}$ and $\bar{H}$ are in phase.

* wave propagation in free space.

$\epsilon = \epsilon_0$, $\mu = \mu_0$, $\sigma = 0$ ↳ E, H → in phase

$\alpha = 0$

$\beta = \omega\sqrt{\mu_0\epsilon_0} = \frac{\omega}{c}$, $u = c$, $\lambda = \frac{2\pi}{\beta}$

$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = \sqrt{\frac{4\pi \times 10^{-7}}{\frac{10^{-9}}{36\pi}}} = 377\ \Omega$
$\simeq 120\pi\ \Omega$

No. 8/2/2018

$u = \frac{\omega}{\beta}$ for all mediums

$\beta = \frac{\omega}{u} = \omega\sqrt{\mu\epsilon}$ → lossless ($\sigma = 0$) → free space $\omega\sqrt{\mu_0 \epsilon_0}$

$\rightarrow \beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right)} \longrightarrow$ lossy

$u = \frac{1}{\sqrt{\mu\epsilon}}$ → lossless → free space

* in free space

$\bar{E} = E_0 \cos(\omega t - \beta z)\,\hat{a}_x$

$\bar{H} = \frac{E_0}{\eta_0}\cos(\omega t - \beta z)\,\hat{a}_y$

$\bar{E}$ and $\bar{H}$ are in phase

$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\,\Omega = 120\pi\,\Omega$ $= \frac{E_0}{H_0}$ ohm's law

$\hat{a}_E = \hat{a}_x$

$\hat{a}_H = \hat{a}_y$

$\hat{a}_k = \hat{a}_z$

$\hat{a}_k \perp \hat{a}_E$, $\perp \hat{a}_H$ } TEM

TEM = Transverese Electro magnetic
(normal)
(perpendicular)

No.

if $\hat{a_k} = \hat{a_z}$ $\hat{a_E} \equiv$ the polarization of the wave.

No $\bar{E}$ or $\bar{H}$ in z-dir

* it is possible that
$\bar{E} = (E_x, E_y, 0)$

$\bar{H} = (H_x, H_y, 0)$

if $\hat{a_k} = \hat{a_z}$

* wave propagation in good Conductors :-

$\epsilon = \epsilon_0 \epsilon_r$, $\mu = \mu_0$, $\sigma \equiv \infty$

non-magnitic meduim $\mu_r = 1$

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right)}$$

$\sigma = \infty$

$$\alpha = \cancel{\omega} \sqrt{\frac{\mu \cancel{\epsilon} \omega^2}{2}\left(\frac{\sigma}{\cancel{\omega} \cancel{\epsilon}}\right)}$$

$$\alpha = \sqrt{\frac{\omega \mu \sigma}{2}} = \beta$$

→ if $\omega = 2\pi f$

$$\alpha = \cancel{\beta} \sqrt{\pi f \mu \sigma} = \beta$$

Np/m rad/m

No.

$\lambda = \frac{2\pi}{\beta}$, $u = \frac{\omega}{\beta}$

$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$ if $\sigma \simeq \infty$

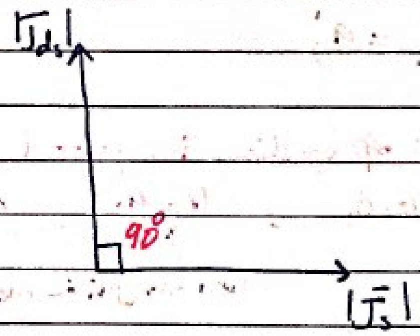
$\omega\epsilon \rightarrow 10^9 (10^{-12})$

$\eta = \sqrt{\frac{j\omega\mu}{\sigma}} = |\eta| \angle \theta_\eta$

$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$

$\theta_\eta = 45^\circ$

$\theta = 2\theta_\eta = 90^\circ$ $\Longrightarrow$



* when the angle $= 90^\circ$ then all current is a conduction current and no displacement current. (when perfect conductor)

$\bar{E} = E_0 e^{-\alpha z} \cos(\omega t - \beta z) \hat{a}_x$ v/m

$\bar{H} = \frac{E_0}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - 45^\circ) \hat{a}_y$ v/m

$\bar{E}$ lead $\bar{H}$ by 45°

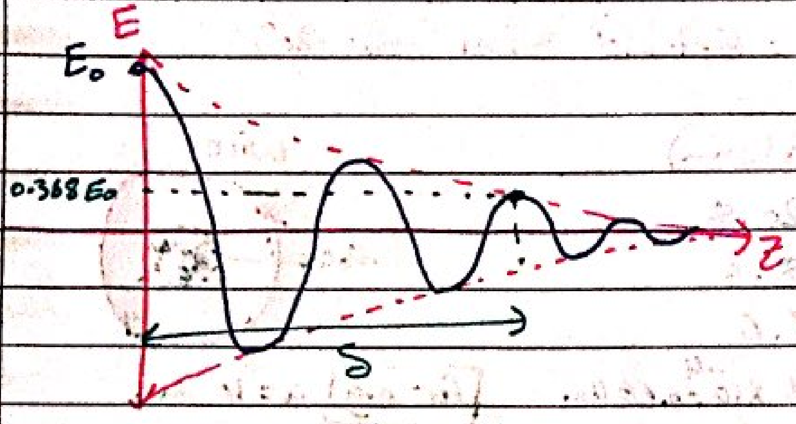
* biggest angle between H and E is 45°

$\theta = 45^\circ$ perfect conductor

$\theta = 0^\circ$ lossless, free space

$0^\circ < \theta < 45^\circ$ lossy

No.



→ when the magnitude of the wave drop to 36.8% of its max this effect is called the skin effect

$\delta \equiv$ penetration distance

$E_z = 0.368 E_0$ when $z = \delta$

, $z = \delta$

$e^{-\alpha\delta} = e^{-1}$

$\alpha\delta = 1 \longrightarrow \boxed{\delta = \frac{1}{\alpha}}$ m

$\Rightarrow \delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}}$ m

$1\,\delta \rightarrow e^{-1} \rightarrow 36.8\%$
$2\,\delta \rightarrow e^{-2} \rightarrow 14.\%$
$3\,\delta \rightarrow e^{-3} \rightarrow 5\%$
⋮
$5\,\delta \rightarrow e^{-5} \rightarrow$ less than 1% (steady state) (thick material)

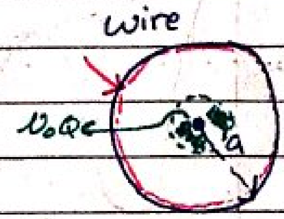
* if $\delta = 5$ it means the wave won't get through the conductor

$\sigma_{cu} = 5.7 * 10^7$ S/m at 20°C

No.

Ex. for Cu, $\sigma_{Cu} = 5.7\times10^{7}$ S/m at 20°c

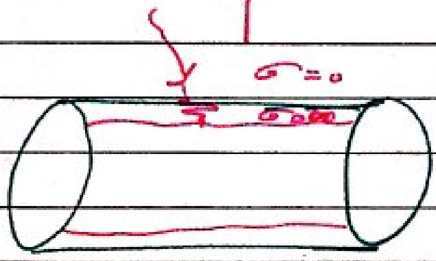
f (Hz)	δ (mm)
10	20.8
60	8.6
100	6.6
10^{10}	$6.6\times10^{-4} = 0.66\,\mu m$



$\boxed{\text{for e.g}}$ $a = 10$ mm

Diameter = 20 mm

So current will get ~~throug all~~ pass through the wire even in the centre.



* we are talking about a wave that propagate on a conductor from free space.

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45^\circ, \quad \delta = \frac{1}{\sqrt{\pi f \mu \sigma}}, \quad f = \frac{\omega}{2\pi}$$

$$\delta = \frac{2}{\omega\mu}$$

$$\eta = \frac{\sqrt{2}}{\sigma\delta}\angle 45^\circ$$

$$\eta = \frac{\sqrt{2}}{\sigma\delta}e^{j\pi/4}\ \Omega$$

$$\eta = \frac{1}{\sigma\delta}(1+j)\ \Omega$$

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No. 11/2/2018

* Skin depth (δ) in (m)

$$\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

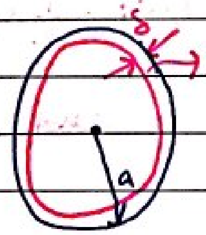
$$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ \; \Omega$$

$$= \frac{1}{\sigma\delta}(1+j) \; \Omega$$

Ac Resistance (R_{ac})

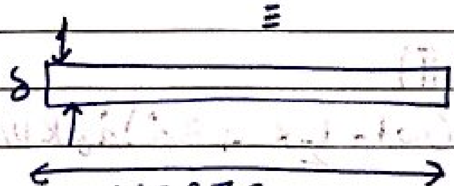
define surface resistance as (R_s) in Ω

$$R_s = R_e(\eta) = \frac{1}{\sigma\delta}$$



as $f\uparrow$ $\delta\downarrow$

$\equiv$



$w = 2\pi a$

$A_c = \delta w$

$= 2\pi a \delta$

$$R_{dc} = \frac{L}{\sigma A} = \frac{L}{\sigma\pi a^2}$$ at $f=0$ or very low freq

$$R_{ac} = \frac{L}{\sigma A} = \frac{L}{2\pi\sigma a\delta}$$

No. ....

$$\frac{R_{ac}}{R_{dc}} = \frac{\dfrac{l}{2\pi\sigma a\delta}}{\dfrac{l}{\sigma\pi a^2}} = \frac{a}{2\delta} \quad \underline{\underline{\text{ratio}}}$$

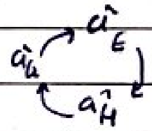
$\frac{R_{ac}}{R_{dc}} = 1$ for Dc or low freq

$\frac{R_{ac}}{R_{dc}} \ggg 1$ at high freq

* Ex:- A lossy ($\alpha \neq 0$) dielectric has $\eta = 200\angle 30^\circ\ \Omega$ at a certain freq.

2d surface ↲ if the $\boxed{\text{plane wave}}$ propagation through the dielectric has

$\bar{H} = 10\, e^{-\alpha x} \cos(\omega t - \frac{1}{2}x)\, \hat{a}_y$ A/m



Find:- $\bar{E}$, α, δ and the polarization.

i) $\bar{E} = \eta \bar{H}$ $\qquad \hat{a}_E = \hat{a}_H \times \hat{a}_k$

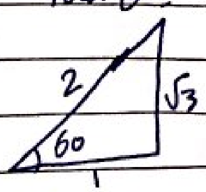
$= 200\, e^{j30^\circ} (\bar{H})$ $\qquad = \hat{a}_y \times \hat{a}_x$

$= 2\, e^{-\alpha x} \cos(\omega t - \frac{1}{2}x + 30^\circ)\, \hat{a}_z$ KV/m $\qquad = -\hat{a}_z$

ii) $\alpha = \omega \sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right)}$

$\beta = \omega \sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right)} = \frac{1}{2}$

$\tan\theta = \tan 2\theta_\eta = \tan 60^\circ = \frac{\sigma}{\omega\epsilon} = \sqrt{3}$



No.

$$\frac{\alpha}{\beta} = \frac{1}{\sqrt{3}}$$

$$\alpha = \frac{1}{\sqrt{3}}\beta = \frac{1}{\sqrt{3}} * \frac{1}{2} = \frac{1}{2\sqrt{3}} \ Np/m$$

iii) $\delta = \frac{1}{\alpha} = 2\sqrt{3} \ m$

iv) polarization $\Rightarrow -\hat{a}_z$ "direction of the electric field"

* Ex: In a lossless media $\eta = 60\pi \ \Omega$, $\mu_r = 1$

$\sigma = 0 \Rightarrow \alpha = 0$

$\bar{H} = -0.1 \cos(\omega t - z)\hat{a}_x + 0.5 \sin(\omega t - z)\hat{a}_y \ A/m$

$\rightarrow$ same time, same space

find: ϵ_r, ω, $\bar{E}$

$\beta = 1$

$\hat{a}_k = +\hat{a}_z$

$$\beta = \omega\sqrt{\mu_0 \epsilon_0 \epsilon_r}$$

$$\beta = \frac{\omega}{c}\sqrt{\epsilon_r} = 1$$

i) $\eta = \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_r}} = \frac{\eta_0}{\sqrt{\epsilon_r}} \Rightarrow 60\pi = \frac{120\pi}{\sqrt{\epsilon_r}} \Rightarrow \sqrt{\epsilon_r} = 2 \Rightarrow \boxed{\epsilon_r = 4}$

$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$

ii) $\omega = \frac{c}{2} = 1.5 \times 10^8 \ rad/s$

iii) $\bar{E} = \eta \bar{H}$

$\hat{a}_{E_1} = (-\hat{a}_x) \times (\hat{a}_z) = \hat{a}_y$

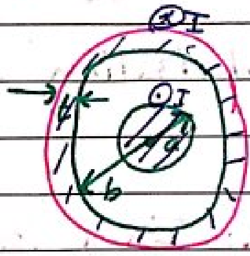
$\hat{a}_{E_2} = \hat{a}_x$

No.

$\nabla \times \bar{H} = (\sigma + j\omega\epsilon)\bar{E}_s$

$\bar{E} = \eta \bar{H}$

* Ex:- for a copper coaxial cable $a = 2mm$, $b = 6mm$, $t = 1mm$
$l = 2m$ $\sigma = 5.7\times10^7$ s/m



Find: R_{dc} and R_{ac} | $f = 100$ MHz

i) $R_{dc} = R_i + R_o$ (seires)

$R_i = \frac{l}{\sigma A} = \frac{2}{5.7\times10^7 * (\pi * 2*10^{-3})^2} = 2.744\, m\Omega$

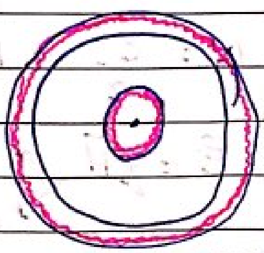
$R_o = \frac{l}{\sigma\pi((b+t)^2 - b^2)} = 0.8429\, m\Omega$

$R_{dc} = R_i + R_o = 3.587\, m\Omega$

ii) $R_{ac} =$

$R_s = \frac{1}{\sigma\delta}$

$\delta = \frac{1}{\sqrt{\pi f \mu_o \sigma_{cu}}} = 6.6\,\mu m << t$



$R_i = \frac{l}{\sigma(2\pi a\delta)} = 0.41\,\Omega$
$\underbrace{\quad}_{Area}$

up to 3GHz

$R_o = 0.1384\,\Omega$, $\frac{l}{\underbrace{\sigma(2\pi b)}_{\text{in the book}}}$

No.

$R_{ac} = 0.5484\,\Omega$

$\frac{R_{ac}}{R_{dc}} \cong 150$ time

13/2/2018

* The power and the Poynting vector.

→ we know that a wave has $\bar{E}$ and $\bar{H}$ fields.

$\nabla \times \bar{E} = -\frac{dB}{dt} \Rightarrow ①$ $\qquad \nabla \times \bar{H} = J + \frac{\partial \bar{D}}{\partial t} \Rightarrow ②$

$\bar{E}$ and $\bar{H}$ are together

$\bar{P} = \bar{E} \times \bar{H}^*$

→ From maxwell's equation

$\nabla \times \bar{E} = -\mu \frac{d\bar{H}}{\partial t} \rightarrow ①$ $\qquad \nabla \times \bar{H} = \sigma \bar{E} + \epsilon \frac{d\bar{E}}{\partial t} \rightarrow ②$

Take $\bar{E} \cdot$ eq② / you can take $\bar{H} \cdot$ eq①

$\bar{E} \cdot (\nabla \times \bar{H}) = \sigma \bar{E}^2 + \bar{E} \epsilon \frac{d\bar{E}}{\partial t}$

in general → $\nabla \cdot (\bar{A} \times \bar{B}) = \bar{B} \cdot (\nabla \times \bar{A}) - \bar{A} \cdot (\nabla \times \bar{B})$ → given

$\bar{E} \cdot (\nabla \times \bar{H}) = \nabla \cdot (\bar{H} \times \bar{E}) + \bar{H} \cdot (\nabla \times \bar{E}) = \sigma E^2 + \bar{E} \epsilon \frac{d\bar{E}}{dt} \ldots ③$

$\bar{H} \cdot (\nabla \times \bar{E}) = \bar{H} \cdot \left(-\mu \frac{d\bar{H}}{\partial t}\right) = -\frac{\mu}{2} \frac{d}{dt}(\bar{H} \cdot \bar{H})$

$= -\frac{\mu}{2} \frac{d}{dt} H^2$ $\qquad \left| H\frac{dH}{dt} + H\frac{dH}{dt} \right.$

$2H\frac{dH}{dt}$

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No.

Re arrange eq ③

$$-\nabla\cdot(\bar{E}\times\bar{H}) - \frac{\mu}{2}\frac{d\bar{H}^2}{dt} = \sigma\bar{E}^2 + \frac{\epsilon}{2}\frac{d\bar{E}^2}{dt}$$

$$\nabla\cdot(\bar{E}\times\bar{H}) = -\frac{\epsilon}{2}\frac{d\bar{E}^2}{dt} - \frac{\mu}{2}\frac{d\bar{H}^2}{dt} - \sigma\bar{E}^2$$

take volume integral to all sides

$$\int_v \nabla\cdot(\bar{E}\times\bar{H})\,\partial v = \int_v\left(-\frac{\epsilon}{2}\frac{\partial\bar{E}^2}{\partial t} - \frac{\mu}{2}\frac{d\bar{H}^2}{dt}\right)dv - \int_v \sigma\bar{E}^2\,\partial v$$

→ Apply divergence theorem

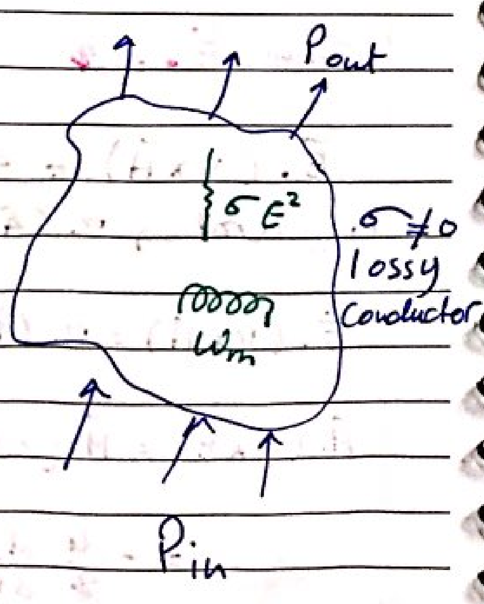
$$\oint_s \bar{E}\times\bar{H}\cdot\bar{ds} = -\frac{\partial}{\partial t}\int_v\left(\underbrace{\tfrac{1}{2}\epsilon\bar{E}^2 dv}_{w_E} + \tfrac{1}{2}\mu\bar{H}^2 dv\right) - \int_v \sigma\bar{E}^2 dv$$

↓ poynting theorem

$[w]$ $\qquad [\frac{J}{s} = w]$ $\qquad [\frac{1}{\Omega\cdot m}\cdot\frac{V^2}{m^2}\cdot m^3]$ → w

P_{loss} = decreased rate of change in magnetic and electric fields → heat

P_{loss} → $P_{out} - P_{in}$



No.

* Poynting vector

$\bar{P} = \bar{E} \times \bar{H} \rightarrow \frac{W}{m^2}$ "not power in a circuit"

(Time varriant) $\rightarrow E(t) \times H(t) \rightarrow p(t)$ instantaneous

define :-

Time average poynting vector: (W/m^2)

$S_{avg} = \bar{P}_{avg} = \frac{1}{T} \int_t \bar{P}\, dt \rightarrow$ (Time invariant)

$$= \frac{1}{2} \mathrm{Re}\{E_s \times H_s^*\}$$

* The Total time average power (W)

$\underline{\underline{\text{power}}} \rightarrow$ scalar

$$\boxed{P_{ave} = \int_s \bar{P}_{ave} \cdot \bar{ds}} \quad (W)$$

$\bar{P}_{ave}$: $\bar{S}_{ave}$ (W/m^2), $\bar{ds}$: m^2

$\bar{P}_{ave}$, $P_{ave} \neq |\bar{P}_{ave}|$

* assume:

$\bar{E}(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\hat{a}_x$ V/m

$\bar{H}(z,t) = \frac{E_0}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)\hat{a}_y$ A/m

$\bar{P} = \bar{E} \times \bar{H}$

$= \frac{E_0^2}{|\eta|} e^{-2\alpha z} \cos(\omega t - \beta z) \cos(\omega t - \beta z - \theta_\eta)\hat{a}_z$

No.

$$= \frac{E_o^2}{2|\eta|} e^{-2\alpha z} \left[\cos(\theta_\eta) + \cos(2\omega t - 2\beta z - \theta_\eta)\right] \hat{a_z} \quad w/m^2$$

$$\bar{P}_{ave} = \frac{1}{T} \int_0^t (\bar{E} \times \bar{H})\, dt$$

$$= \frac{E_o^2}{2|\eta|} e^{-2\alpha z} \cdot \cos(\theta_\eta)\, \hat{a_z} \quad w/m^2$$

to find the total power you need a plane with $\bar{ds}$ along $\hat{a_z}$ or part in $(\hat{a_z})$

15/2/2018

* EX:- In a nonmagnetic medium $\bar{E} = 4\sin(2\pi \times 10^7 t - 0.8x)\, \hat{a_z}$ v/m

Find:

a) ϵ_r, η

b) the time-average power carried by the wave.

c) the total power crossing 100 cm^2 of plane $2x+y=5$

a) $\beta = 0.8$ / $\omega = 2\pi \times 10^7$

$$\beta = \frac{\omega}{c}\sqrt{\epsilon_r} = \frac{2\pi \times 10^7}{3\times 10^8}\sqrt{\epsilon_r}$$

$\epsilon_r = 14.59 > 1$ $\therefore$ lossless medium

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \frac{\eta_o}{\sqrt{\epsilon_r}} = \frac{120\pi}{\sqrt{14.59}} = 10\pi^2\, \Omega = 98.7\, \Omega$$

* $\bar{P}_{ave}$ ~~waves~~ direction and the poynting vector are always with the wave's direction.

No.

(b) ~~(a)~~ $\bar{P}_{ave} = \frac{E_o^2}{2\eta} e^{-2\alpha z} \cos\theta_\eta \, \hat{a}_k$

$\bar{P}_{ave} = \frac{16}{20\pi^2} \hat{a}_x = 81 \, \hat{a}_x \; mW/m^2$

$\bar{H} = \frac{E_o}{\eta} \sin(\qquad) \hat{a}_H \; A/m$

$$\boxed{\begin{aligned} \hat{a}_H &= \hat{a}_k \times \hat{a}_E \\ &= \hat{a}_x \times \hat{a}_z \\ &= -\hat{a}_y \end{aligned}}$$

(c) $P_{ave} = \int_s \bar{P}_{ave} \cdot ds$ *

$= \int \bar{P}_{ave} \cdot \hat{a}_n \cdot ds \qquad \Rightarrow \bar{ds} = ds \cdot \hat{a}_n$

$= \bar{P}_{ave} \cdot \hat{a}_n (S) \qquad \Rightarrow \hat{a}_n = \frac{\nabla f}{|\nabla f|} = \frac{2\hat{a}_x + \hat{a}_y}{\sqrt{5}}$

$= 81 \cancel{\hat{a}_x} \times 10^{-3} \times 100 \times 10^{-4} \times \frac{2}{\sqrt{5}} \cancel{\hat{a}_x}$

$= 724.5 \, \mu w$

* polarization of electromagnitic waves :-

In general, polarization is the trace drawn by the $\bar{E}$-field as it propagates in the medium.

* types of EM polarization :-

1) linear polarization.
2) Circular ″ .
3) Elliptical ″ .

No.

* uniform plane wave (upw)

→ linearly polarized wave (LPW)

for a upw has:

$\bar{E} = E_1 e^{j(wt - ky)} \hat{a_x}$ V/m

$\bar{E} = [E_1 \cos(wt - ky) + j E_1 \sin(wt - ky)] \hat{a_x}$ V/m

* take the imaginary part.

$\bar{E} = \vec{E_1} \sin(wt - ky) \hat{a_x}$ V/m

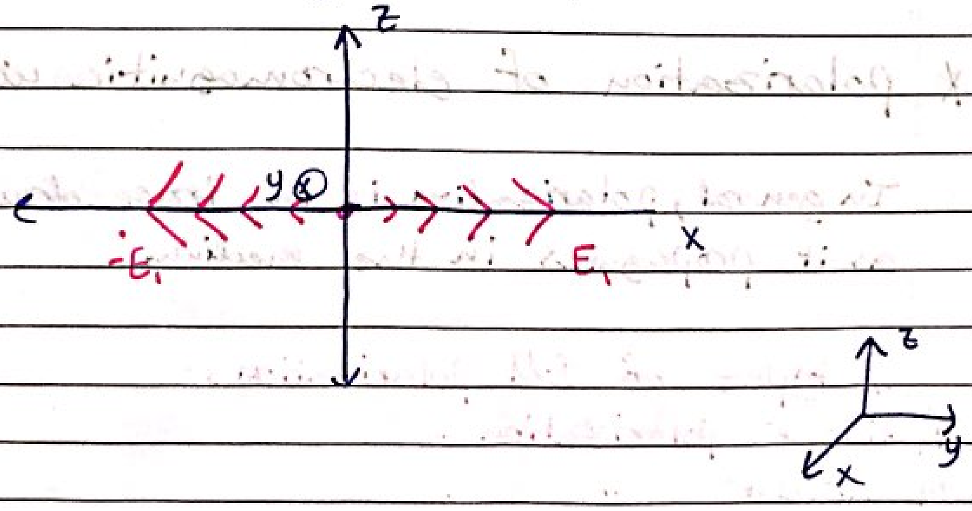
* Fix the space (let $y=0$)

$\bar{E} = E_1 \sin wt \ \hat{a_x}$ V/m → HLPW

Horizantally

* time is charging at $wt = 0$ → $\bar{E} = 0 \ \hat{a_x}$

at $wt = \frac{\pi}{2}$ → $\bar{E} = E_1 \hat{a_x}$

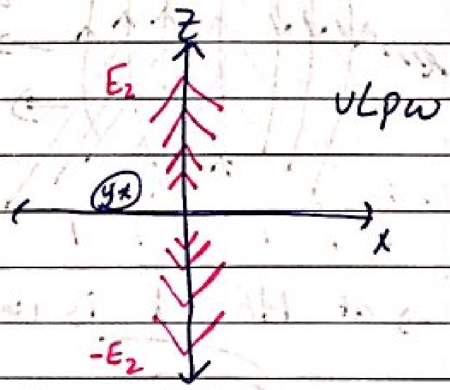


No.

For $\bar{E} = E_2 e^{j(\omega t - ky)} \hat{a_z}$ V/m

$\bar{E} = E_2 \sin(\omega t - ky) \hat{a_z}$ V/m
at $y = 0$

$\bar{E} = E_2 \sin \omega t \ \hat{a_z}$
at $\omega t = 0 \longrightarrow \bar{E} = 0 \ \hat{a_z}$
at $\omega t = \frac{\pi}{2} \longrightarrow \bar{E} = E_2 \hat{a_z}$

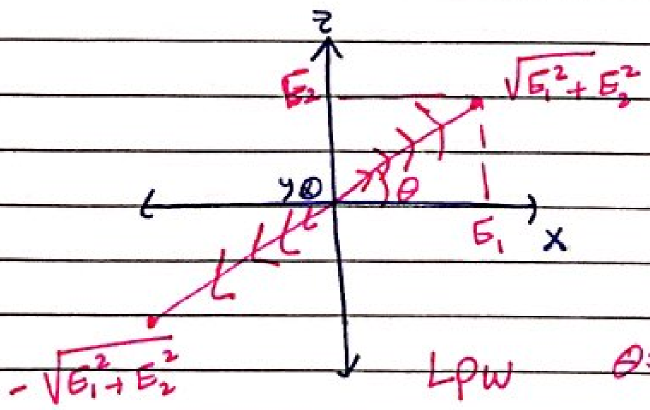


* For
$\bar{E} = E_1 e^{j(\omega t - ky)} \hat{a_x} + E_2 e^{j(\omega t - ky)} \hat{a_z}$ V/m

$\rightarrow$ Im, $y = 0$

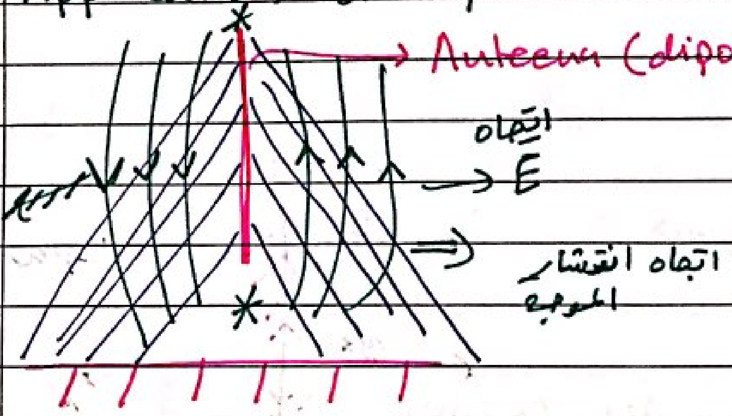
$\bar{E} = E_1 \sin \omega t \ \hat{a_x} + E_2 \sin \omega t \ \hat{a_z}$
at $\omega t = 0 \longrightarrow \bar{E} = 0$
at $\omega t = \frac{\pi}{2} \longrightarrow \bar{E} = E_1 \hat{a_x} + E_2 \hat{a_z}$

$|\bar{E}| = \sqrt{E_1^2 + E_2^2}$

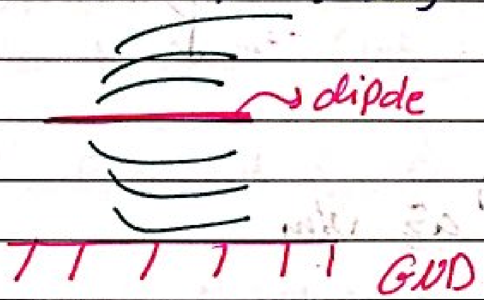


$\theta = \tan^{-1} \frac{E_2}{E_1}$

Applications of Lpw at LF (low-freq), and MF



→ at HF λ is very short they use HLpw



2) circulary polarized wave (cpw)

For a u.p.w has

$$\bar{E} = E_1 e^{j(\omega t - kz)} \hat{a_x} + E_1 e^{j(\omega t - kz \pm \frac{\pi}{2})} \hat{a_y} \quad V/m$$

for TEM mode $\Rightarrow k = \beta$

→ taking the imagenary part :

$$\bar{E} = E_1 \sin(\omega t - kz) \hat{a_x} + E_1 \sin(\omega t - kz \pm \frac{\pi}{2}) \hat{a_y} \quad V/m$$

$$\bar{E} = E_1 \sin(\omega t - kz) \hat{a_x} \pm E_1 \cos(\omega t - kz) \hat{a_y} \quad V/m$$

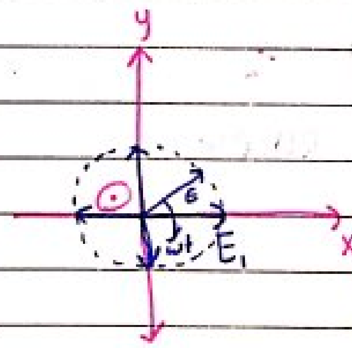
at $z = 0$

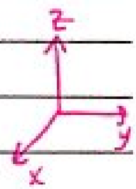
$$\bar{E} = E_1 \sin \omega t \, \hat{a_x} \; (\pm) \; E_1 \cos \omega t \, \hat{a_y} \quad V/m$$

→ if taking $+\frac{\pi}{2}$ phase shift

at $\omega t = 0 \longrightarrow \bar{E} = E_1 \hat{a_y}$

at $\omega t = \frac{\pi}{2} \longrightarrow \bar{E} = E_1 \hat{a_x}$





* الإبهام باتجاه الموجة
* الأصابع باتجاه ال rotation

CW - CPW

LHCPW

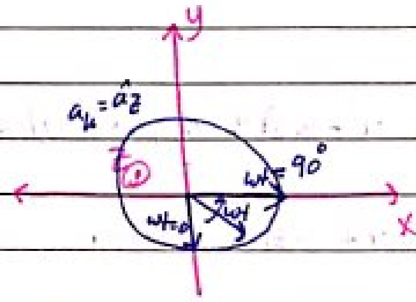
Since $E_x = E_y = E_1$

and $\phi = 90°$

taking $-\frac{\pi}{2}$

at $\omega t = 0 \rightarrow \bar{E} = -E_1 \hat{a}_y$ v/m

at $\omega t = \frac{\pi}{2} \rightarrow \bar{E} = E_1 \hat{a}_x$

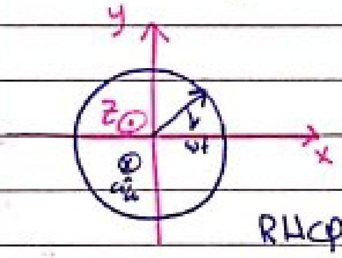


RHCPW

CCW-CPW

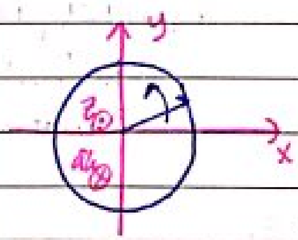
* if $\hat{a}_k = -\hat{a}_z$

if $\phi = \frac{\pi}{2}$ ...



RHCPW

if $\phi = -\frac{\pi}{2}$



LHCPW $\Rightarrow$ sense of rotation

No.

3) Elliptically polarized wave (Epw)

For a upw has

$\bar{E} = E_1 e^{j(\omega t - kz)} \hat{a}_x + E_2 e^{j(\omega t - kz \pm \psi)} \hat{a}_y$ v/m

Note:-

→ if $E_1 = 0$ or $E_2 = 0$ or $\psi = n\pi$, $n = 0, 1, 2 \ldots$
$E_1 \neq E_2$

it will be lpw

→ if $E_1 = E_2$, $\psi = (2n-1)\frac{\pi}{2}$, $n = 1, 2 \ldots$

it will be cpw

→ if $E_1 \neq E_2$, $\psi = (2n-1)\frac{\pi}{2}$, $n = 1, 2, \ldots$

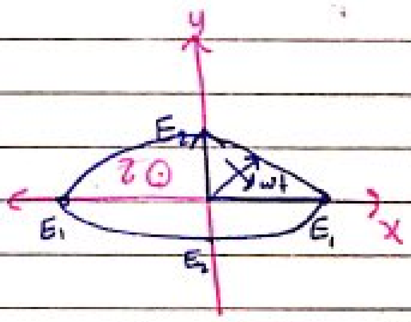
it will be an Epw

at $z = 0$, take $\psi = +\frac{\pi}{2}$

$\bar{E} = E_1 \sin\omega t \, \hat{a}_x \pm E_2 \cos(\omega t) \, \hat{a}_y$ v/m

at $\omega t = 0 \rightarrow \bar{E} = E_2 \hat{a}_y$
at $\omega t = \frac{\pi}{2} \rightarrow \bar{E} = E_1 \hat{a}_x$

if $E_1 > E_2$



CW - Epw
LHEpw

AR = $\frac{E_1}{E_2}$

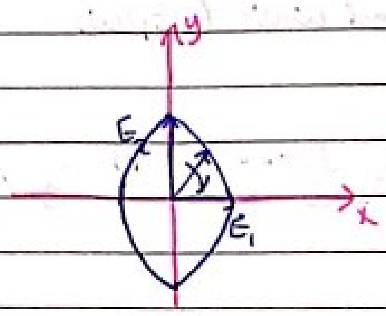
$\tau = 0°$

No.

if $E_2 > E_1$

LHEPW

$AR = \frac{E_2}{E_1}$



$\tau = 90°$

* define Axial Ratio (AR)

$AR = \frac{\text{major axis}}{\text{minor axis}}$

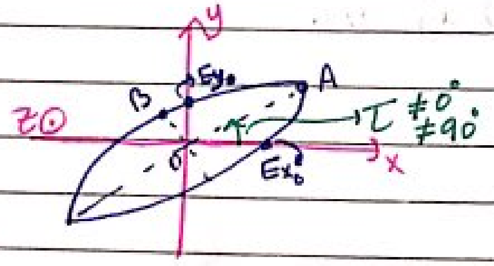
$1 \le AR < \infty$

CPW   LPW

* Tilt angle (τ)

τ is the angle between the major axis and the horizontal axis

if $\psi \neq (2n-1)\frac{\pi}{2}$



$AR = \frac{OA}{OB}$

$\tau \neq 0°$
$\neq 90°$

No.

$$\tau = \frac{1}{2}\tan^{-1}\left[\frac{2E_{x_0}E_{y_0}\cos\psi}{E_{x_0}^2 - E_{y_0}^2}\right]$$

from the horizontal axis

$$AR = \frac{OA}{OB}$$

$$OA = \left[\frac{1}{2}\left\{E_{x_0}^2 + E_{y_0}^2 \oplus \left[E_{x_0}^4 + E_{y_0}^4 - 2E_{x_0}^2E_{y_0}^2\cos(2\psi)\right]^{1/2}\right\}\right]^{1/2}$$

$$OB = \ldots\ldots \quad \ominus$$

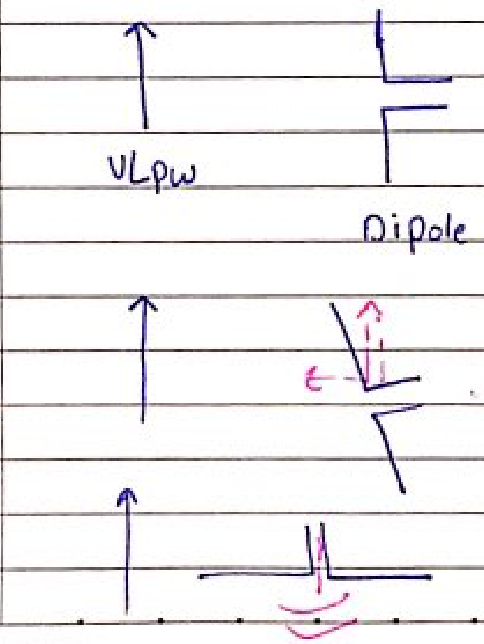
$\Downarrow$

if not 90° phase shift (in the exam there will be only 90° phase shift)

Application of polarization :-

1) Frequency reuse

2) for better reception of the signal



will lose some of the signal

will not recive any signal

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* Ex :- The $\bar{E}$-field for a upw in free space is

$$\bar{E}_s = (3\hat{a_x} + j4\,\hat{a_y})\, e^{-j0.5\pi z} \quad v/m$$

Find: a) frequency and $\bar{H}$

b) its polarization and AR and τ if any

c) Find $\bar{P}_{ave}$ and the total power crossing a $2*2\ m^2$ plane along xy plane

* τ does not exist if Lpw

a) $\beta = 0.5\pi$ rad/m

$= \frac{\omega}{c}$

$\omega = \beta c = 2\pi f$

$f = \frac{\beta c}{2\pi} \Rightarrow f = 75\ MHz$

$\bar{H} = \frac{\bar{E}}{\eta}$

$\eta = \eta_o = 120\pi$

$\hat{a_H} = \hat{a_k} \times \hat{a_E}$

$a_{H_1} = \hat{a_z} \times \hat{a_x} = \hat{a_y}$

$\hat{a_{H_2}} = \hat{a_z} \times \hat{a_y} = -\hat{a_x}$

$$\bar{H} = \frac{-j4\,\hat{a_x} + 3\hat{a_y}}{120\pi}\, e^{-j0.5\pi z} \quad A/m$$

b) $E_s = 3e^{-j0.5\pi z}\hat{a}_x + 4e^{-j(0.5\pi z - \frac{\pi}{2})}\hat{a}_y$

$E = \cancel{Re}\,Im\{E_s e^{j\omega t}\}$

$E = 3\sin(\omega t - 0.5\pi z)\hat{a}_x + 4\cos(\omega t - 0.5\pi z)$

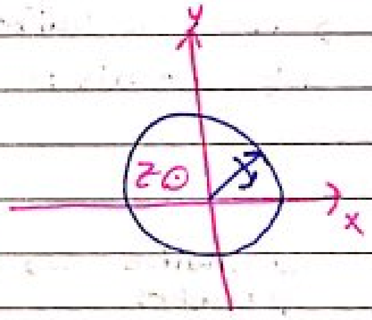
at $z = 0$

$E = 3\sin\omega t\,\hat{a}_x + 4\cos\omega t\,\hat{a}_y$

$\omega t \Rightarrow 0 \longrightarrow \bar{E} = 4\hat{a}_y$

$\omega t = \frac{\pi}{2} \longrightarrow \bar{E} = 3\hat{a}_x$

$AR = \frac{4}{3}$, $\tau = 90^\circ$



LHEPW

c) $\bar{P}_{ave} = \frac{1}{T}\int_0^T \underbrace{\bar{P}}_{\bar{E}\times\bar{H}}\,dt$

$= \frac{1}{2}Re\{\bar{E}_s \times \bar{H}_s^*\}$

$= \frac{E_0^2}{2\eta_0}\cancel{e^{-2\alpha z}}\cancel{\cos(\theta_\eta)}$

$E_0 = \sqrt{9+16} = \sqrt{25}$

$\bar{P}_{ave} = \frac{25}{240\pi}\hat{a}_z \quad W/m^2$

$= \frac{5}{48\pi}\hat{a}_z \quad W/m^2$

$P_{ave} = \int_s \bar{P}_{ave}\cdot d\bar{s} \Rightarrow d\bar{s} = dx\,dy\,\hat{a}_z$

No.

$P_{ave} = |\bar{P}_{ave}| (S)$

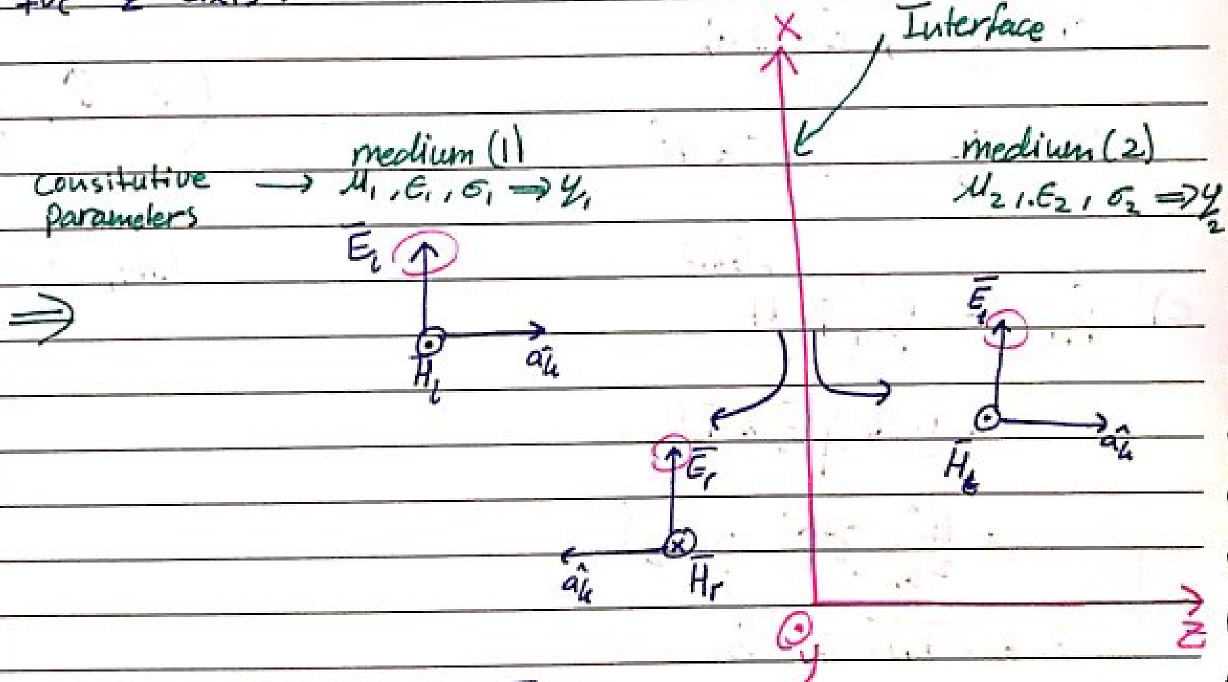
$= \frac{5}{48\pi} (4)$

$= 0.417\ W$

سقوط عامودي

* Reflection of a plane wave at normal Incidence :-

→ assume a uniform plane wave (upw) travels along the +ve z-axis :-



* polarization is the $\bar{E}$ direction because as we can see it didn't change along traveling in different mediums.

Expression for the fields :

For the incident fields :

$\bar{E}_{is} = E_{i0}\, e^{-\gamma z}\, a_x \quad , \quad \gamma = \alpha + j\beta$

No.

$$\bar{H}_{is} = \frac{E_{io}}{\eta_1} e^{-\gamma z} \hat{a}_y \quad A/m$$

* The reflected fields :

$$\bar{E}_{rs} = E_{ro}\, e^{\gamma z}\, \hat{a}_x \quad V/m$$

$$\bar{H}_{rs} = \frac{E_{ro}}{\eta_1} e^{\gamma z} (-\hat{a}_y) \quad A/m$$

* The transmitted fields :

$$\bar{E}_t = E_{to}\, e^{-\gamma_2 z}\, \hat{a}_x$$

$$\bar{H}_t = \frac{E_{to}}{\eta_2} e^{-\gamma_2 z} \hat{a}_y$$

Apply the boundry Condition :-

In medium (1) :-

$\bar{E}_1 = \bar{E}_i + \bar{E}_r$, $\bar{E}_2 = \bar{E}_t$

$\bar{H}_1 = \bar{H}_i + \bar{H}_r$, $\bar{H}_2 = \bar{H}_t$

$\bar{E}, \bar{H}$ must be Continuous at the interface

$z=0$

$E_i(0) + E_r(0) = E_t(0)$ ... ①

$a_k = \hat{a}_z$

$$\frac{E_i(0)}{\eta_1} - \frac{E_r(0)}{\eta_1} = \frac{E_t(0)}{\eta_2} \quad \text{...} ②$$

$E_{io} + E_{ro} = E_{to}$ ...①

$\frac{1}{\eta_1}(E_{io} - E_{ro}) = \frac{1}{\eta_2} E_{to}$ ...②

E_{ro} ? , E_{to} ?

$E_i = 1 \ v/m$

Solve (1) and (2) to find $\bar{E}_{ro}$ and $\bar{E}_{to}$

$\frac{E_{ro}}{E_{io}} = \boxed{\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \Gamma}$ ← Reflection Coefficient

$E_{ro} = \Gamma E_{io}$

$\frac{E_{to}}{E_{io}} = \frac{2\eta_2}{\eta_1 + \eta_2} = \tau$ ← Transmission Coefficient

$E_{to} = \tau E_{io}$

* Notes:-

$0 \leq |\Gamma| \leq 1$, $\Gamma + \tau = 1$

↑
total transmission (No reflection)

$\tau = 1 - |\Gamma|$

$\tau = 1 + \cancel{\tau}\Gamma$

No.

* special case :-

if medium (1) is a perfect dielectric and Medium (2) is a perfect conductor :

$\sigma_1 = 0$, $\sigma_2 \approx \infty$

$\Downarrow$ lossless

$\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}}$

$\Downarrow$ Perfect conductor

$\eta_2 = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$

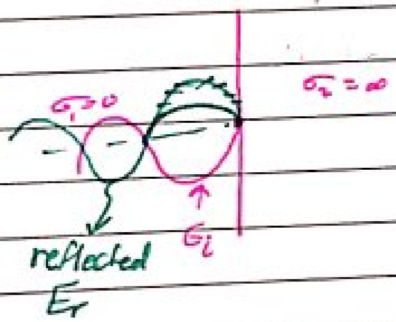
$= \sqrt{\frac{\mu\omega}{\sigma}} \angle 45$

$\eta_2 = 0$

$\Gamma = -1$

$\Gamma = 1\angle 180^\circ$

$E_r = -E_i$



$E_1 = E_i + E_r$

$E_i - E_r = 0$

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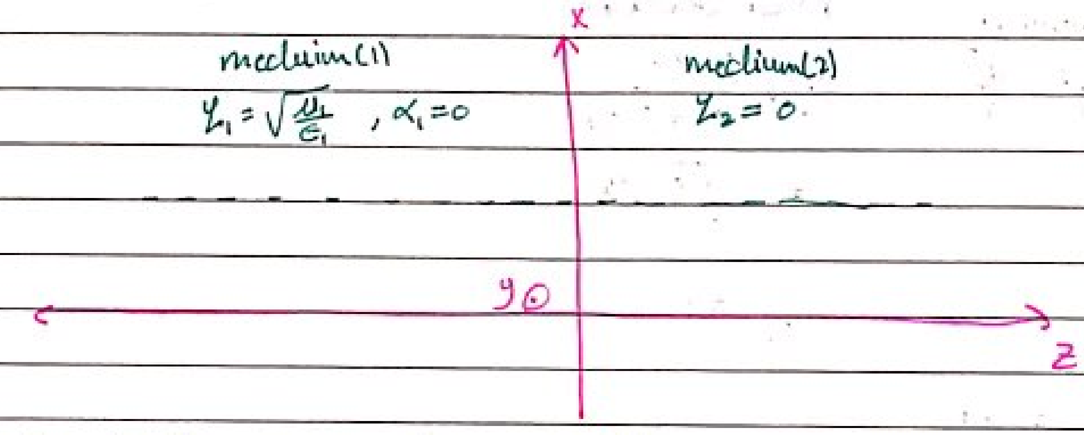
Special case :-

if medium(1) is a perfect dielectric $\sigma_1 = 0$

and medium (2) is a perfect conductor $\sigma_2 = \infty$

Assume $\hat{a_k} = +\hat{a}_z$



$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -1 = \frac{E_r}{E_i} = 1\angle 180^\circ$$

$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1} = 0$$

$$\tau = 1 + \Gamma = 0$$

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma_{\infty}}} \rightarrow 0$$

$$\bar{E}_{is} = E_{io}\, e^{-\gamma_1 z}\, \hat{a_x} \quad , \quad \gamma_1 = \alpha_1 + j\beta_1 \quad *$$

$$= E_{io}\, e^{-j\beta_1 z}\, \hat{a_x}$$

$$\bar{E}_{rs} = E_{ro}\, e^{+j\beta z}\, \hat{a_x} \quad , \quad E_{ro} = \Gamma E_{io}$$

$$= - E_{io}\, e^{+j\beta_1 z}\, \hat{a_x}$$

No.

$$\bar{E}_{1s} = \bar{E}_i + \bar{E}_r$$

$$= E_{i0}\left(e^{-j\beta_1 z} - e^{j\beta_1 z}\right)\hat{a_x}$$

$$= -E_{i0}\left(e^{j\beta_1 z} - e^{-j\beta_1 z}\right)\hat{a_x}$$

$$\bar{E}_{1s} = -2j \sin\beta_1 z \ \hat{a_x}$$

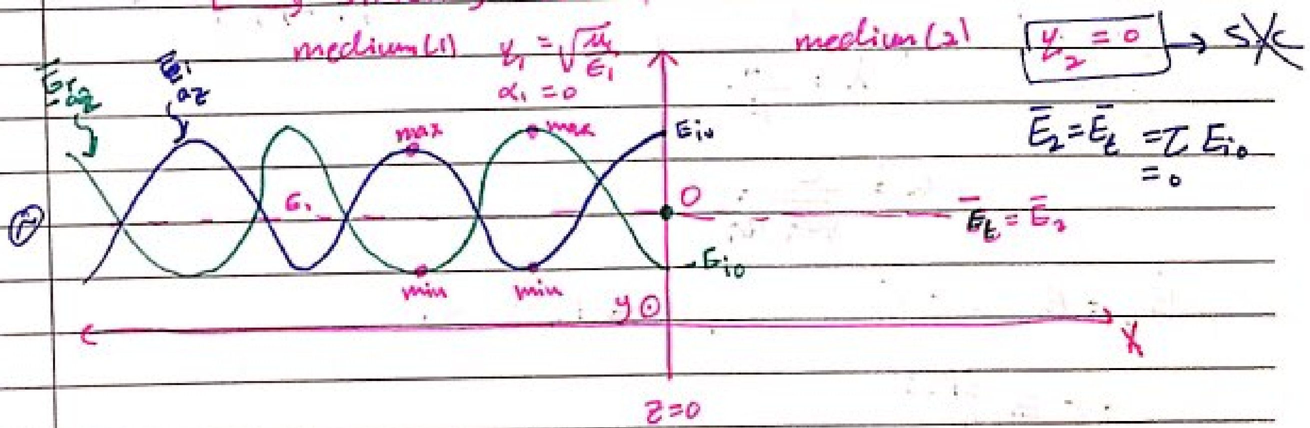
$$\sin\beta_1 z = \frac{e^{j\beta_1 z} - e^{-j\beta z}}{2j}$$

$$\rightarrow \bar{E}_1 = R_e\left\{\bar{E}_{1s}\, e^{j\omega t}\right\}$$

$$= R_e\left\{-E_{i0} \cdot 2j \sin\beta_1 z\,(\cos\omega t + j\sin\omega t)\right\}\hat{a_x}$$

$$\boxed{\bar{E}_1 = 2E_{i0}\sin\beta_1 z \sin\omega t \ \hat{a_x}}$$ → called a standing wave → summation of incident and reflected fields.

waves → propagating waves
→ standing waves



(2) if medium (1) is lossless with η_1 and medium (2) is also lossless with (η_2)

$\epsilon_1 \neq \epsilon_2$, $\mu_1 \neq \mu_2$, $\eta = \sqrt{\frac{\mu}{\epsilon}} \angle 0°$

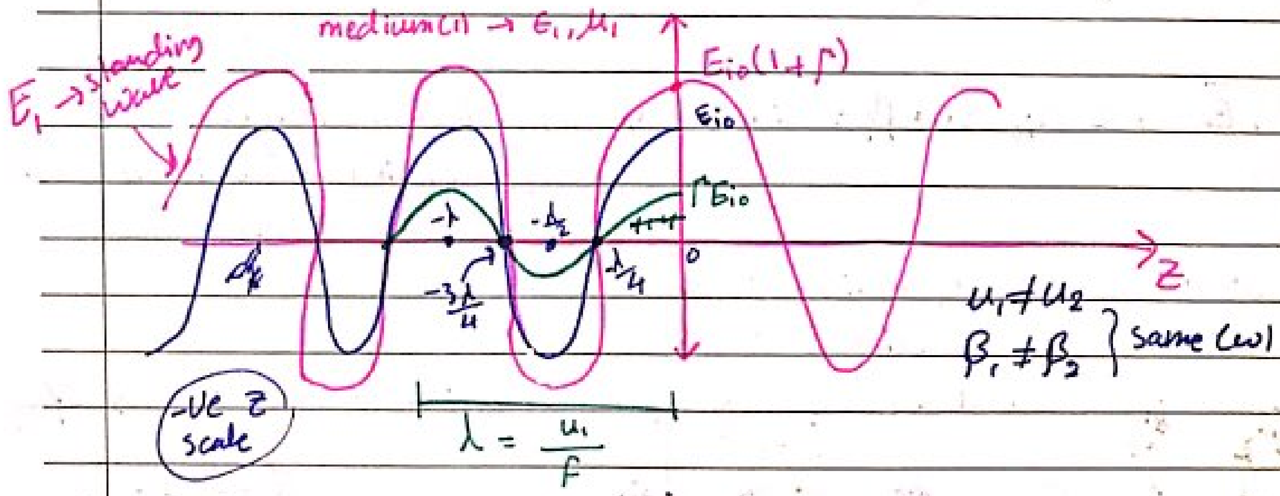
a) if $\eta_2 > \eta_1$ $\longrightarrow \Gamma$ is positive
$0 < \Gamma < 1$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\bar{E}_i = E_{io} e^{-j\beta_1 z} \hat{a}_x \quad , \alpha_1 = 0$$

$$\bar{E}_r = \Gamma E_{io} e^{+j\beta_1 z} \hat{a}_x$$

$$\bar{E}_{1s} = E_{io} \left(e^{-j\beta_1 z} + \Gamma e^{j\beta_1 z} \right) \hat{a}_x$$



$$u = \frac{c}{\sqrt{\epsilon_{r_1} \mu_{r_1}}}$$

$$\bar{E}_2 = \bar{E}_t = \tau \bar{E}_i \qquad , \tau = 1 + \Gamma > 1$$

$$= \tau E_{io} e^{-j\beta_2 z}$$

$$E_i(0) + E_r(0) = E_t(0) \implies (1+\Gamma) E_{io} = \tau E_{io}$$

No.

$$\bar{H}_1 = \frac{\bar{E}_1}{\eta_1} \quad , \bar{H}_2 = \frac{\bar{E}_2}{\eta_2}$$

→ E_{max} occurs when $\cos \beta_1 z$ is max

$-\beta_1 z = n\pi \quad , n = 0, 1, 2, \ldots$

↳ only because we draw it in the -ve z scale.

$$z_{max} = \frac{-n\pi}{\beta_1} = \frac{-n\pi}{\frac{2\pi}{\lambda_1}} = \frac{-n\lambda_1}{2}$$

$$-\beta_1 z_{min} = (2n+1)\frac{\pi}{2} \quad , n = 0, 1, 2, \ldots$$

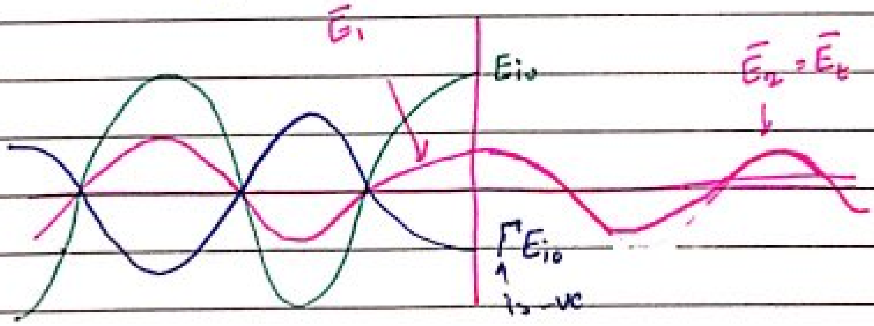
$$z_{min} = -\frac{(2n+1)\frac{\pi}{2}}{\beta_1} = -\frac{(2n+1)\lambda_1}{4}$$

b) if $\eta_2 < \eta_1$ → $\bar{E}_1 = \bar{E}_{i0} \sin\beta_1 z \sin\omega t \, \hat{a}_x$

(Γ is -ve)

$$z_{max} = \frac{-(2n+1)\lambda_1}{4}$$

$$z_{min} = \frac{-n\lambda_1}{2}$$



Standing wave ratio (SWR) or (S)

$$S = \frac{E_{max}}{E_{min}} = \frac{H_{max}}{H_{min}}$$ → magnitude

No. 25/2/2018

Standing wave Ratio (SWR) or (s)

$$S = \frac{E_{max}}{E_{min}} = \frac{H_{max}}{H_{min}}$$

$$S = \frac{|\bar{E}_i| + |\bar{E}_r|}{|\bar{E}_i| - |\bar{E}_r|} , \quad |\bar{E}_r| = |\Gamma| |\bar{E}_i|$$

$$S = \frac{|E_i|}{|E_i|} \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$\boxed{S = \frac{1+|\Gamma|}{1-|\Gamma|}}$$

$|\Gamma| = \frac{S-1}{S+1}$ → only magnitude

$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = |\Gamma| \angle \theta_\Gamma$ $\quad \eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$ lossy

$$0 \leq |\Gamma| \leq 1$$

total transmission ($\tau = 1$)

total Reflection ($\tau = 0$)

$$1 \leq S < \infty$$

→ $S(dB) = 20 \log_{10}(S)$

No.

* Ex 3- Given a u.p.w in air as $\bar{E}_i = 40\cos(\omega t - \beta z)\,\hat{a}_x + 30\sin(\omega t - \beta z)\,\hat{a}_y$ v/m

a) Find $\bar{H}_i$

b) If the wave ~~encounts~~ encounters a perfectly conducting plane normal to the z-axis at $z=0$, find $\bar{E}_r$, $\bar{H}_r$

c) what are the total fields for $z \leq 0$

d) Calculate the time average poynting vector for $z \leq 0$, $z \geq 0$

a) $\bar{H}_i = \dfrac{E_i}{\eta}$

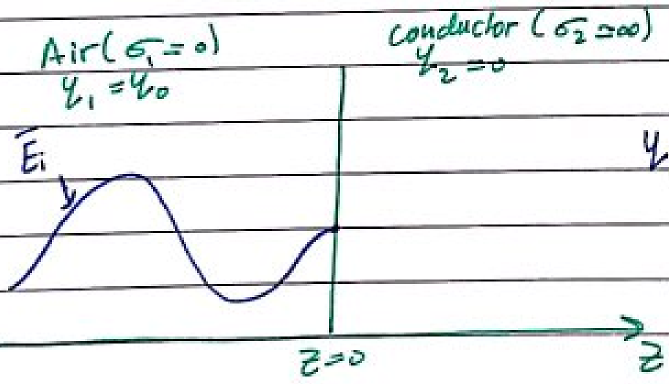
$\eta = \eta_0 = 120\pi$

$\bar{H}_i = \bar{H}_{1i} + \bar{H}_{2i}$

$\bar{H}_i = \dfrac{40}{120\pi}\cos(\omega t - \beta z)\,\hat{a}_y - \dfrac{30}{120\pi}\sin(\omega t - \beta z)\,\hat{a}_x$

$$\boxed{\begin{aligned}\hat{a}_{H_1} &= \hat{a}_k \times \hat{a}_{E_1} \\ &= \hat{a}_z \times a_x = \hat{a}_y \\ \hat{a}_{H_2} &= \hat{a}_k \times \hat{a}_{E_2} \\ &= \hat{a}_z \times \hat{a}_y = -\hat{a}_x\end{aligned}}$$

b)



$\eta_2 < \eta_1$

$\Gamma = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -1$

$\bar{E}_r = \Gamma \bar{E}_1 = -\bar{E}_i$

$\bar{E}_r = -40\cos(\omega t + \beta_1 z)\,\hat{a}_x - 30\sin(\omega t + \beta_1 z)\,\hat{a}_y$ v/m

$\frac{|E_{io}^2|}{2\eta_o} e^{-2\alpha z} \cos\theta_\eta$ ⇒ applied only when there's one medium

No.

$\bar{H}_r = \frac{\bar{E}_r}{\eta} = \frac{40}{120\pi}\cos(\omega t+\beta z)\hat{a}_y - \frac{30}{120\pi}\sin(\omega t+\beta z)\hat{a}_x$

$\boxed{\frac{\bar{H}_r}{\bar{H}_i} = -\Gamma} = -(-1) = 1 \Rightarrow \bar{H}_i = \bar{H}_r$

c) $\bar{E}_1$, $\bar{H}_1$?

$\bar{E}_1 = \bar{E}_i + \bar{E}_r$ ⇒ standing wave $E_1 = 2E_{io}\sin\beta z \sin\omega t\ \hat{a}_x + \ldots$
$2E_{io}\cos\beta z\cos\omega t\,\hat{a}_y$

$\bar{H}_1 = \frac{\bar{E}_1}{\eta_1}$

d) $\bar{P}_{ave} = \frac{1}{2}Re\{\bar{E}_s \times \bar{H}_s^*\} = \frac{|E_o|^2}{2\eta_o}\cancel{e^{-2\alpha z}}^{1}\cancel{\cos\theta_\eta}^{1}$, $E_{io} = E_i + E_r = E_i - E_i = 0$

$= \frac{1}{T}\int_0^T \bar{E}_s \times \bar{H}_s\, dt$

$\bar{P}_{ave} = 0 \rightarrow \bar{P}_{ave} = \frac{E_o^2}{2\eta_o}(\underbrace{1-|\Gamma|^2}_{0})\cancel{e^{-2\alpha}}\cos(\theta_\eta)$

→ for $z \geq 0$

$E_t = E_2 = \tau E_i = 0$

$\bar{H}_t = \frac{\bar{E}_t}{\eta_2} = 0$

$\bar{P}_{ave} = \frac{|\overset{0}{E_t}|^2}{2\eta_2}\underbrace{e^{-2\alpha_2}}_{\neq 0}\underbrace{\cos(\theta\eta_2)}_{\neq 0} = 0$

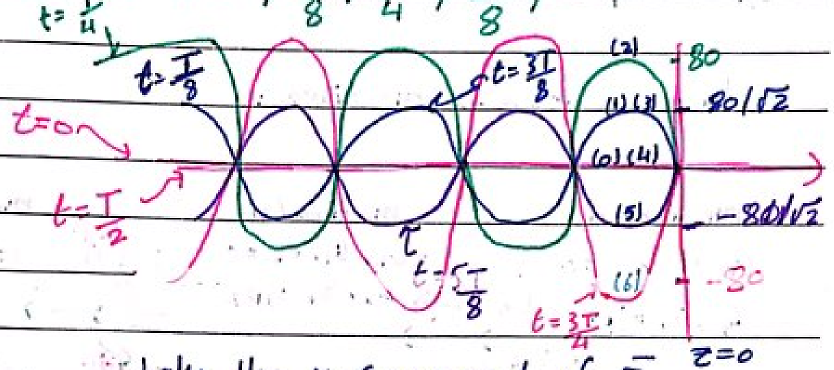
No.

E) sketch the standing wave for $t=0, \frac{T}{8}, \frac{T}{4}, \frac{3T}{8}, \ldots$

$\bar{E}_i$ ✓

$E_r = -\bar{E}_i$

$\Gamma = -1$



take the x-component of $\bar{E}_i$

$\bar{E}_{is} = 40 e^{-j\beta z} \rightarrow 40\cos(\omega t - \beta z)$

if $z = 0$

$40\cos(\omega t)$

$\bar{E}_{1s} = 2E_{io}\sin\beta_1 z \sin\omega t\, \hat{a}_x$

$= 80\sin\beta_1 z \sin\omega t\, \hat{a}_x$

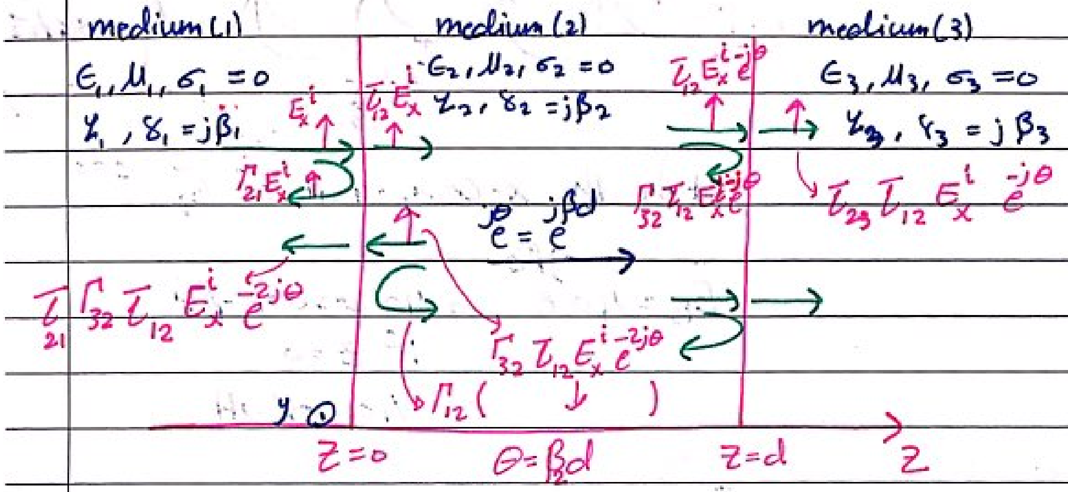
$(E_{1s} = E_{is} + E_{rs} = 40e^{-j\beta_1 z} - 40e^{+j\beta_1 z})$

$\rightarrow \omega t = \frac{2\pi}{\cancel{T}} \cdot \frac{\cancel{T}}{8}$

No. 27/12/2018

* Multiple Reflection of EM waves at normal Incidence:-

take 3 lossless mediums:-



*we can controll ϵ, μ, β only.

* To analyz this problem:-

1) Follow all the reflected and transmitted waves at $z=0, z=d$
2) Applying the B.C at $z=0, z=d$ after finding the fields in each region.
3) using the T.L theory (will be discussed in CH.11)

$$\Gamma_{21} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad \tau_{12} = \frac{2\eta_2}{\eta_1 + \eta_2}$$

$$\Gamma_{32} = \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2} \qquad \tau_{23} = \frac{2\eta_3}{\eta_2 + \eta_3}$$

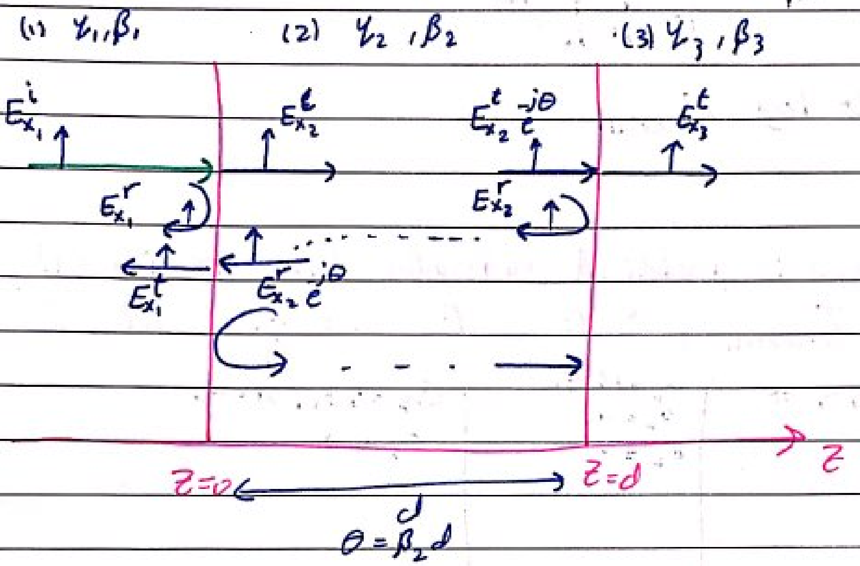
if d is given $\rightarrow \theta = \beta_2 d$

$$\Gamma_{overall} = \frac{E_{1x}^r}{E_{1x}^i} \qquad \tau_{overall} = \frac{E_{3x}^t}{E_{1x}^i}$$

No.

Shielding Effectiveness (S.E)

$$S.E = 20 \log (T_{overall}) \quad dB$$



⇒ Appling the B.c at $z=0$

$$E^i_{x_1} + E^r_{x_1} = E^t_{x_2} + E^r_{x_2} e^{-j\theta} \quad \ldots ①$$

انعكس تماماً موجة

$$(E^i_{x_1} - E^r_{x_1}) \cdot \frac{1}{\eta_1} = \frac{1}{\eta_2} (E^t_{x_2} - E^r_{x_2} e^{-j\theta}) \quad \ldots ②$$

B.c at $z=d$

$$E^t_{x_2} e^{-j\theta} + E^r_{x_2} = E^t_{x_3} \quad \ldots ③$$

$$\frac{1}{\eta_2} (E^t_{x_2} e^{-j\theta} - E^r_{x_2}) = \frac{E^t_{x_3}}{\eta_3} \quad \ldots ④$$

Solve for $E^r_{x_1}$, $E^r_{x_2}$

$E^t_{x_2}$, $E^t_{x_3}$

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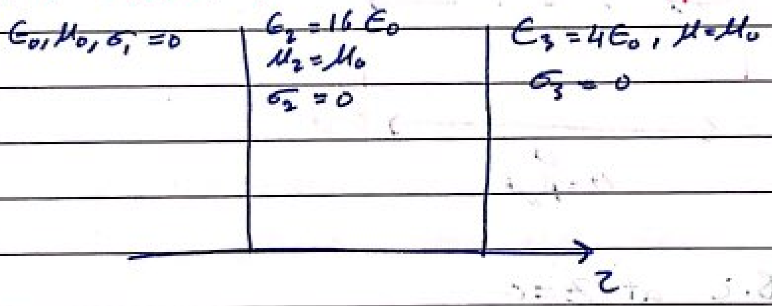
No.

$$\Gamma_{overall} = \frac{E_{x_1}^r}{E_{x_1}^i} = \frac{\Gamma_{21} + \Gamma_{32}\, e^{-2j\theta}}{1 + \Gamma_{21}\Gamma_{32}\, e^{-j2\theta}}$$

$$\tau_{overall} = \frac{E_{x_3}^t}{E_{x_1}^i} = \frac{\tau_{12}\,\tau_{23}\, e^{-j2\theta}}{1 + \Gamma_{21}\Gamma_{32}\, e^{-j2\theta}} , \qquad \theta = \beta_2 d$$

$$S.E = 20 \log \tau_{overall}$$

Ex: if a signal is incidient normally from region (1), find $\Gamma_{overall}$, $\tau_{overall}$, S.E for $d = \frac{\lambda}{4}$ and $d = \frac{\lambda}{2}$.



→ Three lossless and non-mag. mediums

$\eta_1 = \eta_0 = 120\pi\ \Omega$

$\eta_2 = \frac{\eta_0}{4} = 30\pi\ \Omega$

$\eta_3 = \frac{\eta_0}{2} = 60\pi\ \Omega$

$\Gamma_{21} = -0.6$

$\Gamma_{32} = \frac{1}{3}$

$\tau_{12} = 0.4$

$\tau_{23} = \frac{4}{3}$

$\tau = 1 + |\Gamma|$ → τ and Γ in this equation ~~are only~~ between ~~for~~ 2 mediums and not overall.

No.

$\theta = \beta_2 d$

for $d = \frac{\lambda}{4}$

$\theta = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$

$e^{-j2\theta} = e^{-j\pi} = \cos(\pi) - j\sin\pi$

$= -1$

$\Gamma_{overall} = -0.78$

$\tau_{overall} = -0.44$

$S.E = 20 \log \tau_{overall}$

if $d = \frac{\lambda}{2}$ $\Rightarrow \tau_{overall} = 0.67$ $\Gamma_{overall} = -0.33$

$\Rightarrow$ Find the characteristics for region (2) to have $\Gamma_{overall} = 0$ $\rightarrow$ at $d = \frac{\lambda}{4}$, $\mu = \mu_0$

$\Gamma_{overall} = 0 \Rightarrow \Gamma_{21} + \Gamma_{32} e^{-j\pi} = 0$

$\Gamma_{21} = \Gamma_{32}$

$\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2}$

solve for η_2

$\boxed{\eta_2 = \sqrt{\eta_3 \eta_1}} = 60\pi\sqrt{2}\ \Omega$

$\eta_2 = \frac{\eta_0}{\sqrt{\epsilon_{r_2}}} = 60\pi\sqrt{2} \Rightarrow \epsilon_{r_2} = 2$

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No. 1/3/2018

* Reflection of a plane wave at oblique Incidence :-

↳ it is more general than the normal incidence.

before $\bar{E}$ - Field was written in the form.

$$\bar{E}(z,t) = E_o \cos(\omega t - \beta z)\, \hat{a}_x$$

in general

$$\bar{E} = \bar{E}_o \cos(\bar{k}\cdot\bar{r} - \omega t) \quad , \quad \bar{E}_o = E_o \hat{a}_x$$

$$\cos(\theta) = \cos(-\theta)$$

$\bar{k} = k_x \hat{a}_x + k_y \hat{a}_y + k_z \hat{a}_z$ → phase constant vector

→ in lossless medium

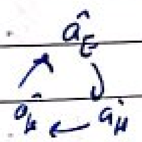
$k = \beta$

$k^2 = k_x^2 + k_y^2 + k_z^2 = \omega^2 \mu \epsilon = \beta^2$ (lossless)

$\bar{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$ → position vector

$$\bar{k}\cdot\bar{r} = k_x x + k_y y + k_z z$$

→ Maxwell's eq. can be written as follow:



$\nabla \cdot D = \rho_v$ $\qquad$ $\bar{k}\cdot\bar{E} = 0$

$\nabla \cdot \bar{B}_s = 0$ $\qquad$ $\bar{k}\cdot\bar{H} = 0$

$\nabla \times \bar{E}_s = -j\omega H_s$ $\qquad \Rightarrow \qquad$ $\bar{k}\times\bar{E} = \omega\mu\bar{H}$

$\nabla \times \bar{H}_s = j\omega\epsilon \bar{E}_s$, $(\sigma = 0)$ lossless $\qquad$ $k \times \bar{H} = -\omega\epsilon\bar{E}$

it is a TEM wave

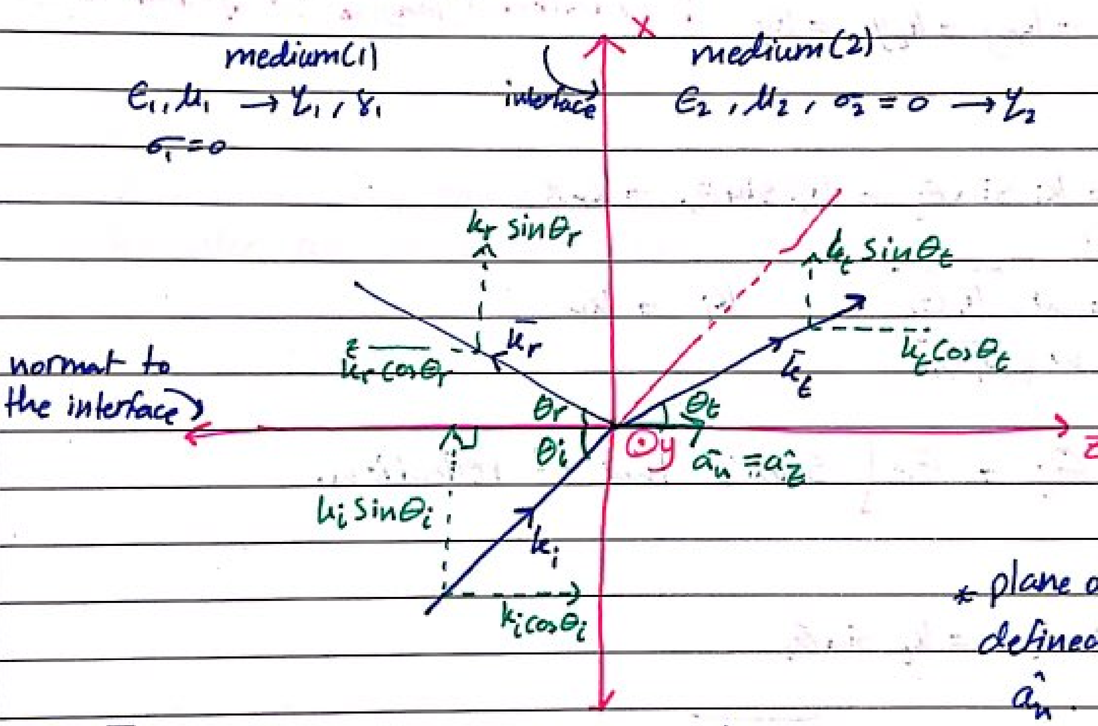
No.

$$\bar{H} = \frac{\bar{k} \times \bar{E}}{\omega\mu} \quad , \bar{k} = k\,\hat{a}_k$$

$$\bar{H} = \frac{k}{\omega\mu}(\hat{a}_k \times \bar{E}) = \frac{\omega\sqrt{\mu\epsilon}}{\omega\mu}(\hat{a}_k \times \bar{E}) = \sqrt{\frac{\epsilon}{\mu}}(\hat{a}_k \times \bar{E})$$

$$\boxed{\bar{H} = \frac{\hat{a}_k \times \bar{E}}{\eta}}$$

$\Rightarrow$ $\bar{E} = E_o \cos(k_x x + k_y y + k_z z - \omega t)$



* plane of incidience defined by $\bar{k}$ and $\hat{a}_n$
$\hat{a}_n \rightarrow \hat{a}_z$; $\bar{k} \rightarrow \hat{a}_x, \hat{a}_z$

$\bar{E}_i = \bar{E}_{io} \cos(k_{ix} x + k_{iy} y + k_{iz} z - \omega_i t)$

$\bar{E}_r = \bar{E}_{ro} \cos(k_{rx} x + k_{ry} y + k_{rz} z - \omega_r t)$

$\bar{E}_t = \bar{E}_{to} \cos(k_{tx} x + k_{ty} y + k_{tz} z - \omega_t t)$

* plane of interface is x-y plane

No.

$\Rightarrow$ Apply B.C at $z=0$

$\bar{E}_i(0) + \bar{E}_r(0) = \bar{E}_t(0)$

To satisfy the B.C

1) $\omega_i = \omega_r = \omega_t \longrightarrow$ freq. is unchanged

2) $k_{ix} = k_{rx} = k_{tx} = k_x$

3) $k_{iy} = k_{ry} = k_{ty} = k_y$

phase matching Condition (tangent component is const)

$k_{ix} = \cancel{k_i} \sin\theta_i = \cancel{k_r} \sin\theta_r = k_{rx}$

$k_i = \omega\sqrt{\mu_1 \epsilon_1}$ $\qquad k_i = k_r$

$k_r = \omega\sqrt{\mu_1 \epsilon_1}$

$\boxed{\theta_i = \theta_r}$ Snell's law of reflection.

$k_i \sin\theta_i = k_t \sin\theta_t$

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{k_t}{k_i} = \frac{\omega\sqrt{\mu_2\epsilon_2}}{\omega\sqrt{\mu_1\epsilon_1}} = \frac{u_1}{u_2}$$

$$\boxed{\frac{\sin\theta_i}{\sin\theta_t} = \frac{u_1}{u_2}}$$ Snell's law of refraction.

No.

in other way

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{n_2}{n_1} = \frac{c\sqrt{\mu_2\epsilon_2}}{c\sqrt{\mu_1\epsilon_1}}$$

$n \equiv$ refrative index $= \frac{c}{u}$

$n_1 = \frac{c}{u_1}$, $n_2 = \frac{c}{n_2}$

* Special Cases :-

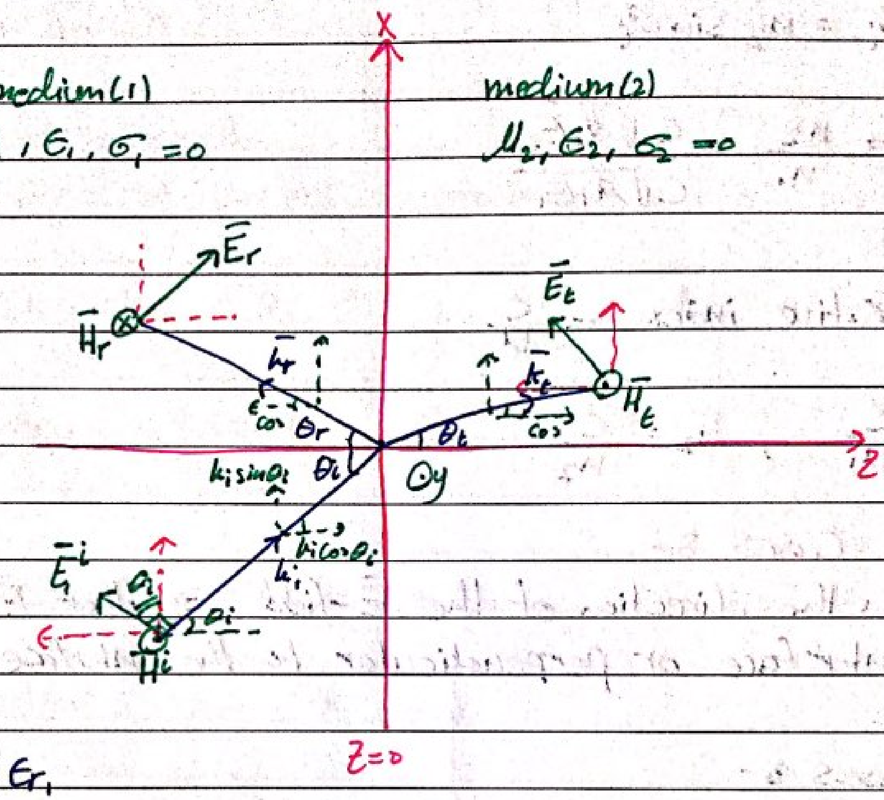
Based on the direction of the $\bar{E}$-field $\Rightarrow$ either $\bar{E}$ is parallel to the interface or perpendicular to the interface

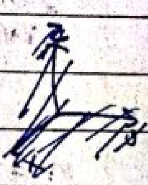
Two cases :-

1) parallel polarization
2) perpendicular polarization

No. 4/3/2018

A) parallel polarization (the perpendicular component will be reflected only)
↳ the $\bar{E}$-field is parallel to the plane of incident





if $\epsilon_{r_2} > \epsilon_{r_1}$

$\theta_t < \theta_i$

$n_1 \sin\theta_i = n_2 \sin\theta_t$

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{n_2}{n_1} = \sqrt{\frac{\mu_2 \epsilon_2}{\mu_1 \epsilon_1}}$$

* The incident fields :-

$$E_s^i = E_{i0}\, e^{-j\beta_1 (x\sin\theta_i + z\cos\theta_i)} \cdot (\cos\theta_i\, \hat{a}_x - \sin\theta_i\, \hat{a}_z)$$

$\beta = \omega\sqrt{\mu_1 \epsilon_1}$

$E_s = E_0\, e^{-j(\bar{k}\cdot\bar{r})}$

$\bar{k}\cdot\bar{r} = x\hat{a}_x + \cancel{y\hat{a}_y} + z\hat{a}_z$

$$H_{is} = \frac{E_{is}}{\eta_1} = \frac{E_{i0}}{\eta_1}\, e^{-j\beta_1 (x\sin\theta_i + z\cos\theta_i)}\ \hat{a}_y \ \ A/m$$

No.

The reflected field :-

$$\bar{E}_{rs} = E_{ro} \left(\cos\theta_r \hat{a}_x + \sin\theta_r \hat{a}_z \right) e^{-j\beta_1 (x\sin\theta_r - z\cos\theta_r)} \; v/m \qquad \theta_i = \theta_r$$

$$\bar{H}_{rs} = -\frac{E_{ro}}{\eta_1} e^{-j\beta_1 (x\sin\theta_r - z\cos\theta_r)} \hat{a}_y \; A/m$$

The Transmitted field :-

$$\bar{E}_{ts} = E_{to} \left(\cos\theta_t \hat{a}_x - \sin\theta_t \hat{a}_z \right) e^{-j\beta_2 (x\sin\theta_t + z\cos\theta_t)} \; v/m$$

$$\bar{H}_{ts} = \frac{\bar{E}_{ts}}{\eta_2} = \frac{E_{to}}{\eta_2} e^{-j\beta_2 (\quad)} \hat{a}_y \; A/m$$

* Apply the B.c at $z=0$

Tangent component must be cont.

$$\bar{E}_{is}(0) + \bar{E}_{rs}(0) = \bar{E}_{ts}(0) \Rightarrow \bar{E}_{1s}(0) = \bar{E}_{2s}(0)$$

$$E_{io}(\cos\theta_i) + E_{ro}\cos\theta_r = E_{to}\cos\theta_t$$

$$(E_{io} + E_{ro})\cos\theta_i = E_{to}\cos\theta_t \quad \ldots (1)$$

$$\boxed{e^{-j\beta_1 (x\sin\theta_i)} = e^{-j\beta_2 (x\sin\theta_t)} \Rightarrow \beta_1 \sin\theta_i = \beta_2 \sin\theta_t \quad \text{* Snell's law of reflection}}$$

$$\bar{H}_{is}(0) + \bar{H}_{rs}(0) = \bar{H}_{ts}(0)$$

$$\frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1} = \frac{E_{to}}{\eta_2}$$

$$\frac{1}{\eta_1}(E_{io} - E_{ro}) = \frac{E_{to}}{\eta_2} \quad \ldots (2)$$

Solve (1) and (2) to find E_{ro} and E_{to}

No.

$$\frac{E_{ro}}{E_{io}} = \Gamma_{\parallel} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$\frac{E_{to}}{E_{io}} = \tau_{\parallel} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$\boxed{1 + \Gamma_{\parallel} = \tau_{\parallel} \frac{\cos\theta_t}{\cos\theta_i}}$$

$$E_{ro} = \Gamma_{\parallel} E_{io} \quad , \quad E_{to} = \tau_{\parallel} E_{io}$$

$\Rightarrow$ Define the Brewster angle (polarizing angle)

$\theta_B = \theta_i$

which the angle that make $\Gamma_{\parallel} = 0$ (no reflection)

$$\Gamma_{\parallel} = 0 \Rightarrow \eta_2 \cos\theta_t = \eta_1 \cos\theta_B$$

$$\boxed{\theta_{B_{\parallel}} = \cos^{-1}\left(\frac{\eta_2}{\eta_1} \cos\theta_t\right)}$$

$$\cos\theta_B = \frac{\eta_2}{\eta_1} \cos\theta_t \quad , \quad \eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} \quad , \quad \eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}}$$

$$\cos^2\theta_B = \frac{\mu_2 \epsilon_1}{\mu_1 \epsilon_2} \cos\theta_t^2$$

$$1 - \sin^2\theta_{B_{\parallel}} = \frac{\mu_2 \epsilon_1}{\mu_1 \epsilon_2}\left(1 - \sin\theta_t^2\right) \quad \text{---} \ \circledast$$

No.

$$\mu_1 \sin\theta_{B_{\parallel}} = \mu_2 \sin\theta_t \rightarrow \sin^2\theta_t = \frac{\mu_2\epsilon_2}{\mu_1\epsilon_1}\sin^2\theta_{B_{\parallel}}$$

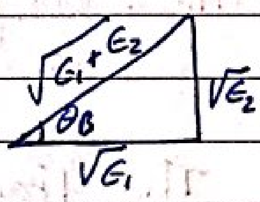
$$\boxed{\sin^2\theta_B = \frac{1-\frac{\mu_2\epsilon_1}{\mu_1\epsilon_2}}{1-\left(\frac{\epsilon_1}{\epsilon_2}\right)^2}}$$ general

if both mediums are non-magnetic $\mu_1 = \mu_2 = \mu_0$

$$\sin^2\theta_{B_{\parallel}} = \frac{1}{1+\frac{\epsilon_1}{\epsilon_2}} = \frac{\epsilon_2}{\epsilon_1+\epsilon_2}$$

$$\sin\theta_{B_{\parallel}} = \sqrt{\frac{\epsilon_2}{\epsilon_1+\epsilon_2}} \quad , \quad \theta_{B_{\parallel}} = \tan^{-1}\left(\sqrt{\frac{\epsilon_2}{\epsilon_1}}\right)$$

$$= \tan^{-1}\left(\frac{n_1}{n_2}\right)$$

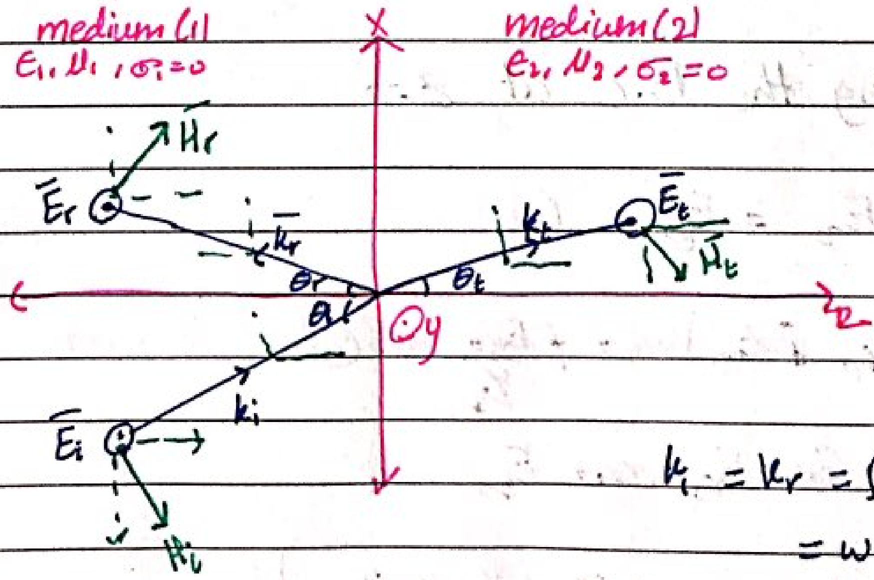


6/3/2018

B) perpendicular polarization:-

The $\bar{E}$ is perpendicular to the plane of incidence

(The parallel component of $\bar{H}$ will be reflected)



$k_1 = k_r = \beta_1 \leftarrow$ lossless

$= \omega\sqrt{\mu\epsilon}$

No.

The incident fields:-

$$\bar{E}_{is} = E_{io}\, e^{-j\beta_1 (x\sin\theta_i + z\cos\theta_i)}\, \hat{a}_y \quad V/m$$

$$\bar{H}_{is} = \frac{E_{io}}{\eta_1}\left(-\cos\theta_i\, \hat{a}_x + \sin\theta_i\, \hat{a}_z\right) e^{-j\beta_1 (x\sin\theta_i + z\cos\theta_i)} \quad A/m$$

The reflected fields:-

$$\bar{E}_{rs} = \bar{E}_{ro}\, e^{-j\beta_1 (x\sin\theta_i - z\cos\theta_i)}\, \hat{a}_y \quad V/m$$

$$\bar{H}_{rs} = \frac{E_{ro}}{\eta_1}\left(x\sin\theta_i\, \hat{a}_x + \sin\theta_i\, \hat{a}_z\right) e^{-j\beta_1 (\quad)} \quad A/m$$

The Transmitted field:-

$$\bar{E}_{ts} = E_{to}\, e^{-j\beta_2 (x\sin\theta_t + z\cos\theta_t)}\, \hat{a}_y$$

$$\bar{H}_{ts} = \frac{E_{to}}{\eta_2}\left(-\cos\theta_t\, \hat{a}_x + \sin\theta_t\, \hat{a}_z\right) e^{-j\beta (\quad)} \quad A/m$$

$\Rightarrow$ Appling the B.C at $z=0$

$$E_{io} + E_{ro} = E_{to} \quad \text{....} (1)$$

$$\pm\frac{1}{\eta_1}(+E_{i_o} \mp E_{r_o}) = \pm\frac{E_{to}}{\eta_2} \quad - (2)$$

solve for E_{r_o} and E_{to}

$$\boxed{\frac{E_{ro}}{E_{io}} = \Gamma_\perp = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i - \eta_1\cos\theta_t}}$$

No.

$E_{r0} = \Gamma_{\perp} E_{i0}$

$$\boxed{\frac{E_{t0}}{E_{i0}} = \tau_{\perp} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}}$$

$\boxed{1 + \Gamma_{\perp} = \tau_{\perp}}$, $\boxed{E_{t0} = \tau_{\perp} E_{i0}}$

$\Gamma_{\parallel}, \tau_{\parallel} \Rightarrow$ Fresnell's equations for parallel

$\Gamma_{\perp}, \tau_{\perp} \Rightarrow$ Fresnell's equations for perpendicular.

For No reflection $\Rightarrow \Gamma_{\perp} = 0$ at $\theta_i = \theta_{B\perp}$

$\eta_2 \cos\theta_{B\perp} = \eta_2 \cos\theta_t$ $\quad (n_1 \sin\theta_i = n_2 \sin\theta_t)$

$\eta_2^2 (1 - \sin^2\theta_{B\perp}) = \eta_1^2 (1 - \sin^2\theta_t)$, $\sin^2\theta_t = \left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_{B\perp}$

$$\boxed{\sin^2\theta_{B\perp} = \frac{1 - \frac{\mu_1 \epsilon_2}{\mu_2 \epsilon_1}}{1 - \left(\frac{\mu_1}{\mu_2}\right)^2}}$$

For $\mu_1 = \mu_2 = \mu_0$

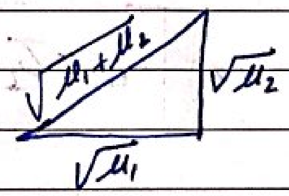
$\theta_{B\perp}$ doesn't exist for non-magnetic mediums.

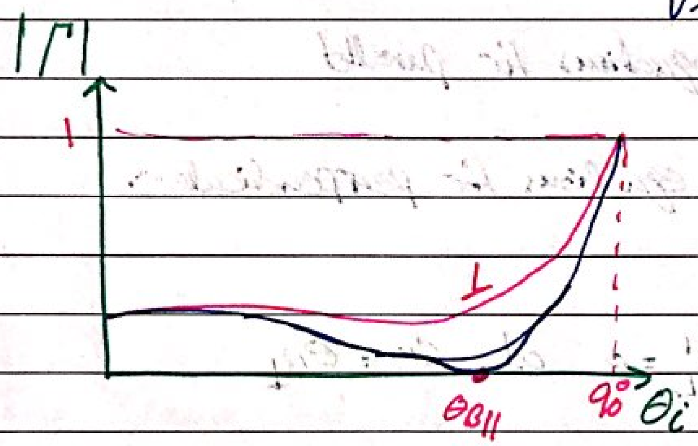
No.

for $\epsilon_1 = \epsilon_2 = \epsilon_0$

$$\sin^2\theta_{B\perp} = \frac{1}{1+\frac{\mu_1}{\mu_2}} = \frac{\mu_2}{\mu_1+\mu_2}$$

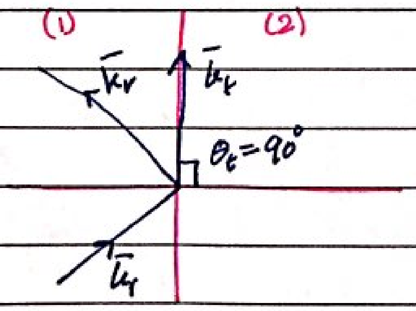
$$\boxed{\tan\theta_{B\perp} = \sqrt{\frac{\mu_2}{\mu_1}}}$$





* Critical angle (total internal Reflection)

$\theta_i = \theta_c$ at $\theta_t = 90^\circ$ $\qquad$ $\theta_i \geq \theta_c \rightarrow$ No Transmission

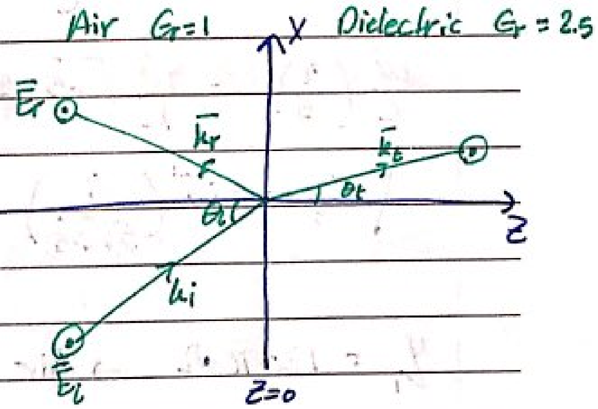


No. 8/3/2018

Ex8 A u.p.w in air with $\bar{E} = 8\cos(\omega t - 4x - 3z)\, \hat{a}_y$ V/m is incident on dielectric slab at $z = 0$, with $\mu_r = 1$, $\epsilon_r = 2.5$, $\sigma = 0$

Find:- $\hat{a}_n = \hat{a}_z$

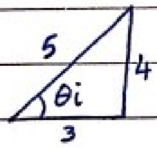
1) The type of polarization
2) The angle of incidence
3) The reflected $\bar{E}$-field
4) The transmitted $\bar{H}$-field



1) it is a perpendicular polarization

2) $\bar{k} = k_x \hat{a}_x + k_z \hat{a}_z$
$= 4\hat{a}_x + 3\hat{a}_z$

$|\bar{k}| = k = 5$



$\tan\theta_i = \dfrac{k_x}{k_z} = \dfrac{4}{3}$

* $k_r \cos\theta_r = \bar{k}_r \cdot -\hat{a}_z$

$\theta = 53.13°$

$\Rightarrow -k_i \cdot -\hat{a}_n = |k_i|\,(1)\cos\theta_i$

$(-4, 0, -3) \cdot -\hat{a}_z = 5\cos\theta_i$

$\dfrac{3}{5} = \cos\theta_i \quad \Rightarrow \theta = \cos^{-1}\dfrac{5}{3}$

3) $E_r = E_{r0}\cos(\omega t - 4x + 3z)$

$E_{r0} = \Gamma_\perp E_{i0}$

$\Gamma_\perp = \dfrac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$

θ_t? $\quad k_i \sin\theta_i = k_t \sin\theta_t$

No.

$k_t = \omega\sqrt{\mu_2 \epsilon_2} = \frac{\omega}{c}\sqrt{\epsilon_{r_2}}$

$k_i = \omega\sqrt{\mu_1 \epsilon_1} = \frac{\omega}{c}$

$\frac{\omega}{c}\left(\frac{4}{5}\right) = \frac{\omega}{c}\sqrt{2.5}\ \sin\theta_t$

$$\boxed{\theta_t = \sin^{-1}\left(\frac{4}{5\sqrt{2.5}}\right) = 30.39^\circ}$$

$\eta_1 = 120\pi\,\Omega \rightarrow$ Air

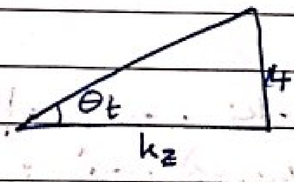
$\eta_2 = \frac{\eta_0}{\sqrt{\epsilon_r}} = \frac{120\pi}{\sqrt{2.5}} = 238.4\,\Omega$

$\Gamma_\perp = -0.389$

$E_{r_0} = -3.112$ V/m

4) $\bar{H}_t$? $\Rightarrow \bar{H}_t = \frac{\hat{a}_k \times \bar{E}_t}{\eta_2}$

$\bar{E}_t = \tau_\perp E_{i0} \cos(\omega t - 4x - k_{z_2} z)$



$\tan\theta_t = \frac{4}{k_z}$

$k_{z_2} = 6.819$ rad/m

$k = \sqrt{k_x^2 + k_z^2} = 7.906$ rad/m

$\tau_\perp = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} =$

or $1 + \Gamma_\perp = \tau_\perp = 0.611$

No.

$E_{to} = 4.888 \text{ v/m}$

$$\overline{H}_t = \frac{4\hat{a_x} + 6.819\hat{a_z}}{7.906(238.4)} \times 4.888 \cos(\omega t - 4x - 6.819z)\hat{a_y}$$

$$= \left(-17.69\hat{a_x} + 10.37\hat{a_z}\right)\cos(\omega t - 4x - 6.819z) \text{ mA/m}$$

$$\Rightarrow k = \frac{\omega}{c}\sqrt{\epsilon_r} \Rightarrow \omega = \frac{k_1 c}{\sqrt{\epsilon_r}} = \frac{5(3\times10^8)}{1} = 1.5\times10^8 \text{ rad/s}$$

No. Chapter 11 "Transmission lines"

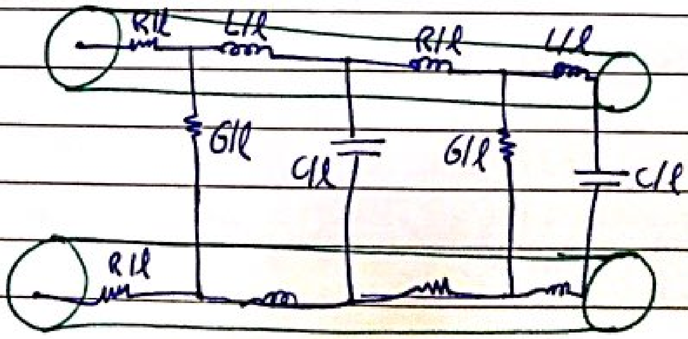
⇒ Two or more Conductors

examples of two - Conductors T.L
→ Coaxial Cable ⓞ $f < 3\,GHz$
→ Twin - wire line
$f < 300\,MHz$

→ planar T.L
- → strip - line
- → micro strip - line
- → slot line

* T.L parameters :-

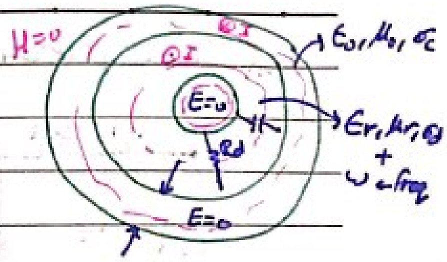
R/ℓ (Ω/m)
L/ℓ (H/m)
C/ℓ (F/m)
G/ℓ (S/m)



types of elements :-
→ Lumped elements.
→ Distriputed elements.

No.

For a Coaxial Cable



$R/\ell \longrightarrow$ AC resistance

$$R/\ell = \frac{1}{2\pi \delta \sigma_c}\left(\frac{1}{a} + \frac{1}{b}\right) \ \Omega/m$$

$$L/\ell = \frac{\mu}{2\pi} \ln\left(\frac{b}{a}\right) \longrightarrow \text{external inductance}$$

$$L_{in} \to 0 \quad \text{as} \quad \omega \to \infty$$

$$C/\ell = \frac{2\pi \epsilon}{\ln b/a} \ F/m$$

$$G/\ell = \frac{2\pi \sigma_d}{\ln \frac{b}{a}} \ S/m \longrightarrow G = \frac{1}{R_d} \neq \frac{1}{R}$$

$$L_{ext} \ C = \mu \epsilon$$

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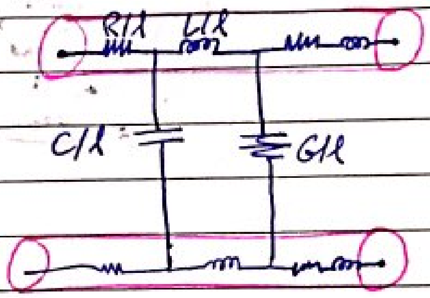
* T.L parameters



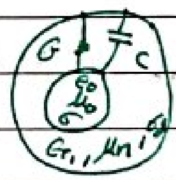
For coaxial cable :-

$$R_{ac}/l = \frac{1}{2\pi \sigma_c \delta}\left(\frac{1}{a}+\frac{1}{b}\right) \ \Omega/m$$

$$C/l = \frac{2\pi\epsilon}{\ln \frac{b}{a}} \ F/m$$

$$G/l = \frac{2\pi\sigma_d}{\ln\frac{b}{a}} \ S/m$$

$$L/l = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right) \ H/m$$

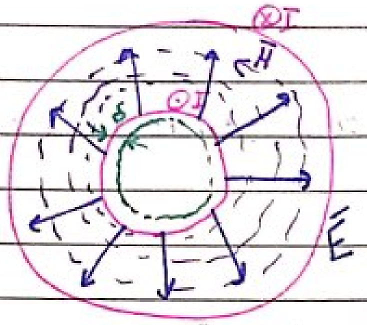


$$L_{ext} C = \mu\epsilon$$

$$R_d C = \frac{\epsilon}{d} \ \text{(or)} \ \frac{C}{G} = \frac{\epsilon}{\sigma}$$

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma_c}}$$

No.



$\bar{P} = \vec{E} \times \bar{H}$ in $\hat{a_k}$ direction

$\hat{a_k} \perp \vec{E} \Rightarrow$ TEM wave
$\quad \perp \bar{H}$

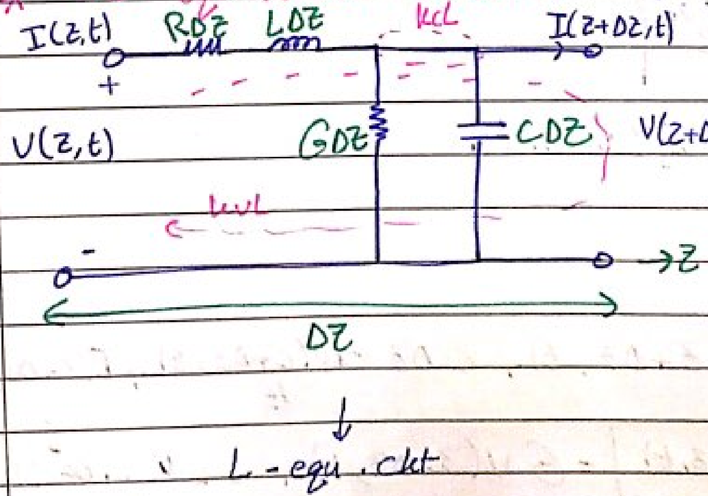
T.L equation :- (Telegrapher equation)

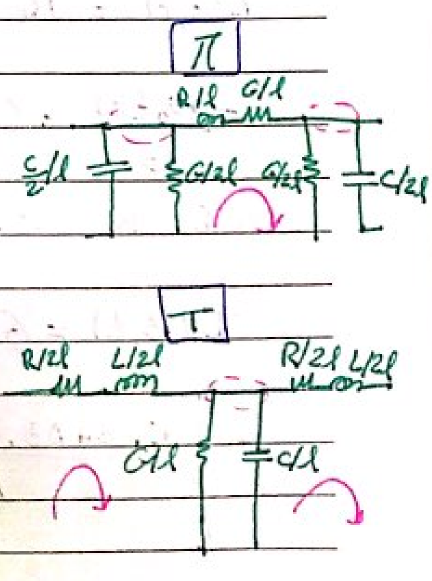
Since an $\bar{E}$ and $\bar{H}$ fields are related to voltage and current according to :

$$V = -\int_{l} \bar{E} \cdot \bar{dL} \quad , \quad I = \oint_{l} \bar{H} \cdot \bar{dL}$$

So, we will use circuit theory to analyze the T.L

* T.L equivelant circuit :- (π and T)





No.

write the kvL eq :-

$$-v(z,t) + I(z,t)R\Delta z + L\Delta z \frac{dI(z,t)}{dt} + v(z+\Delta z,t) = 0$$

$$-\boxed{\frac{v(z+\Delta z,t) - v(z,t)}{\Delta z}} = I(z,t)R + L\frac{dI(z,t)}{dt}$$

take $\lim_{\Delta z \to 0}$

$$-\frac{dv(z,t)}{dz} = I(z,t)R + L\frac{dI(z,t)}{dt}$$

Convert to phasor

$$v(z,t) = Re\{v_s(z)e^{j\omega t}\}$$

$$I(z,t) = Re\{I_s(z)e^{j\omega t}\}$$

$$-\frac{dv_s(z)}{dz} = (R+j\omega L)I_s(z) \quad \ldots (1)$$

write the kvL eq.

$$I(z,t) = G\Delta z\ v(z+\Delta z,t) + C\Delta z \frac{dv(z+\Delta z,t)}{dt} + I(z+\Delta z,t)$$

$$-\boxed{\frac{I(z+\Delta z,t) - I(z,t)}{\Delta z}} = G\,v(z+\Delta z,t) + C\frac{dv(z+\Delta z,t)}{dt}$$

No.

$$\lim_{\Delta z \to 0} \Rightarrow \frac{-dI(z,t)}{dz} = G\,v(z,t) + C\frac{d\,v(z,t)}{dt}$$

Convert to phasor

$$\frac{-dI_s(z)}{dz} = (G+j\omega C)\,V_s(z) \;\cdots ②$$

$$-\frac{dV_s(z)}{dz} = (R+j\omega L)\,I_s(z) \;\cdots ①$$

solve for $V_s(z)$ and $I_s(z)$

derive eq.(1) w.r.t. (z)

$$\frac{-d^2 V_s(z)}{\partial z^2} = (R+j\omega L)\frac{d\,I_s(z)}{dz} \;\cdots ③$$

sub (2) in the (3)

$$\frac{d^2 V_s(z)}{\partial z^2} = \underbrace{(R+j\omega L)(G+j\omega C)}_{\text{Constant}}\,V_s(z)$$

$$\boxed{\frac{\partial^2 V_s(z)}{\partial z^2} - \gamma^2 V_s(z) = 0} \Rightarrow \text{voltage wave equation}$$

$$\gamma^2 = (R+j\omega L)(G+j\omega C)$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$
$$= \alpha + j\beta$$

$$\boxed{\frac{\partial^2 I_s(z)}{\partial z^2} - \gamma^2 I_s(z) = 0} \Rightarrow \text{current wave equation}$$

No.

Solution (same as CH.10)

$$V_s(z) = \boxed{V_0^+ e^{-\gamma z}} + V_0^- e^{\gamma z}$$

$$I_s(z) = \boxed{I_0^+ e^{-\gamma z}} + \underbrace{I_0^- e^{\gamma z}}$$

forward wave — backward wave

* characteristic Impedance (Z_0) in (Ω)

$$Z_0 = \frac{V_0^+}{I_0^+} = \frac{-V_0^-}{I_0^-} = \frac{R+j\omega L}{\gamma} = \frac{\gamma}{G+j\omega C}$$

$$\text{or } Z_0 = \frac{R+j\omega L}{\sqrt{(R+j\omega L)(G+j\omega C)}}$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \quad (\Omega)$$

$$= R_0 + j X_0$$

resistance — reactance

$$V(z,t) = Re\left\{ V_s(z) e^{j\omega t} \right\}$$

$$= V_0^+ \cos(\omega t - \gamma z) + V_0^- \cos(\omega t + \gamma z)$$

$\hat{a}_k = +\hat{a}_z$ — $\hat{a}_k = -\hat{a}_z$

No.

* Characteristic Admittance (Y_o)

$$Y_o = \frac{1}{Z_o} \, (\mho) \text{ or } (S)$$

* wave length (λ)

$$\lambda = \frac{2\pi}{\beta}$$

* Velocity $\Rightarrow u = \frac{\omega}{\beta} = \lambda f$

* α in Np/m $\Rightarrow$ 1 Np = 8.688 dB

13/3/2017

special cases :-

A) lossless line :- $\sigma_c \sim \infty$, $\sigma_d = 0$

$R = 0$, $G = 0$ ($R_d = \infty$)

$$\gamma = j\omega\sqrt{LC} = j\beta$$

$$\boxed{\begin{array}{l} \alpha = 0 \\ \beta = \omega\sqrt{LC} \end{array}} \qquad \beta = \omega\sqrt{\mu\epsilon} \quad \boxed{\mu\epsilon = L_{ext} C}$$

$$Z_o = \sqrt{\frac{L}{C}} = R_o$$

$$R_o = \sqrt{\frac{L}{C}} , \; X_o = 0$$

pure real

No.

$$\lambda = \frac{2\pi}{\beta}$$

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} \stackrel{eq}{\equiv} \frac{1}{\sqrt{\mu\epsilon}}$$

$$V_s(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

standing wave

B) Distortionless line :-

$$\frac{R}{L} = \frac{G}{C}$$

$$\gamma = \sqrt{R\left(1 + \frac{j\omega L}{R}\right) G\left(1 + \frac{j\omega C}{G}\right)}$$

$$\gamma = \sqrt{RG}\left(1 + \frac{j\omega L}{R}\right)$$

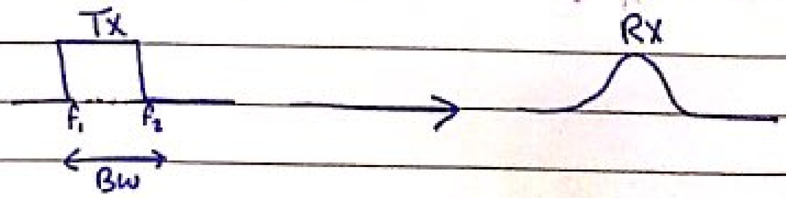
$$\gamma = \sqrt{RG} + j\omega\sqrt{\frac{L\cancel{R}GL}{R^2}}$$

$$= \sqrt{RG} + j\omega\sqrt{\frac{C}{G} \cdot GL}$$

$$= \sqrt{RG} + j\omega\sqrt{LC}$$

$$\alpha = \sqrt{RG} \quad , \beta = \omega\sqrt{LC}$$

α is not depending on f $\longrightarrow$ distortionless



if line is Distortion

No.

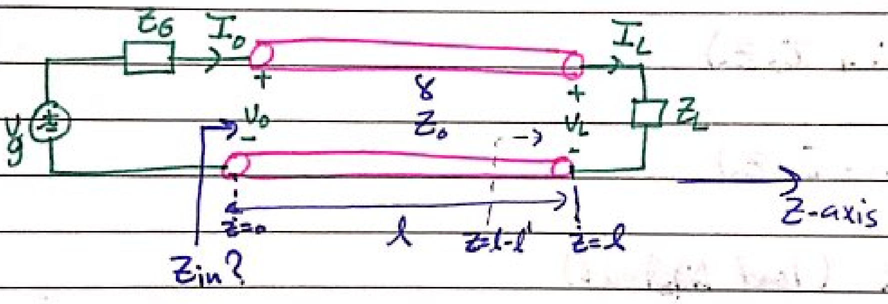
$$Z_0 = \sqrt{\frac{R(1+\cancel{j\omega \frac{L}{R}})}{G(1+\cancel{j\omega \frac{C}{G}})}} = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}$$

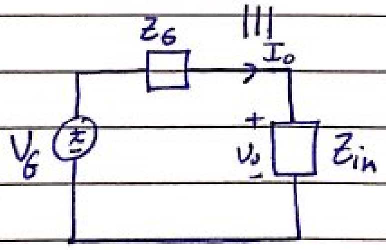
$$R_0 = \sqrt{\frac{L}{C}}, \quad X_0 = 0$$

$$V_s(z) = V_0^+ e^{-\alpha z} e^{-j\beta z} + V_0^- e^{\alpha z} e^{j\beta z}$$

$$\lambda = \frac{2\pi}{\beta}, \quad u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

* Input impedance, SWR, power





$$V_0 = \frac{V_G Z_{in}}{Z_{in} + Z_G}$$

$$I_0 = \frac{V_0}{Z_{in}} = \frac{V_G}{Z_{in} + Z_G}$$

The voltage and current on the line are :-

$$\boxed{V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}}$$

$$I_s(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

$$Z_0 = \frac{V_0^+}{I_0^+} = \frac{-V_0^-}{I_0^-}$$

No.

$$\boxed{I_s(z) = \frac{V_o^+}{Z_o} e^{-\gamma z} - \frac{V_o^-}{Z_o} e^{\gamma z}}$$

Solve for V_o^+ and V_o^-

→ at $z = 0$ (Generator end)

$V_s(0) = V_o$, $I_s(0) = I_o$

$V_o = V_o^+ + V_o^-$ ...①

$I_o = \frac{V_o^+ - V_o^-}{Z_o}$ ...②

$V_o^+ = \frac{1}{2}(V_o + I_o Z_o)$

$V_o^- = \frac{1}{2}(V_o - I_o Z_o)$

→ at $z = l$ (load end)

$V_s(l) = V_L$; $I_s(l) = I_L$

$V_L = V_o^+ e^{-\gamma l} + V_o^- e^{\gamma l}$ ...ⓐ

$I_L = \frac{1}{Z_o}\left(V_o^+ e^{-\gamma l} - V_o^- e^{\gamma l}\right)$ ...ⓑ

$V_o^+ = \left(V_L - V_o^- e^{\gamma l}\right) e^{\gamma l}$

$V_o^+ = \frac{1}{2}(V_L + I_L Z_o) e^{\gamma l}$

$V_o^- = \frac{1}{2}(V_L - I_L Z_o) e^{-\gamma l}$

No.

Input impedance (Z_{in})

$$Z_{in}\Big|_{z=0} = \frac{V_s(z)}{I_s(z)}\Big|_{z=0} = \frac{V_o^+ + V_o^-}{\frac{V_o^+ - V_o^-}{Z_o}}$$

$$\boxed{Z_{in} = Z_o \frac{V_o^+ + V_o^-}{V_o^+ - V_o^-}} = \frac{V_o}{I_o}$$

* Z_{in} at $z \neq 0$ (for any point)

Recall that :-

$$\frac{e^{\gamma l} + e^{-\gamma l}}{2} = \cosh \gamma l$$

$$\frac{e^{\gamma l} - e^{-\gamma l}}{2} = \sinh \gamma l$$

$$\frac{e^{\gamma l} - e^{-\gamma l}}{e^{\gamma l} + e^{-\gamma l}} = \tanh \gamma l$$

$\Rightarrow$ $$Z_{in} = Z_o \frac{V_o^+ + V_o^-}{V_o^+ - V_o^-}$$

sub. V_o^+ and V_o^- at $z = l$

$$Z_{in} = \frac{Z_o (V_L + I_L Z_o) e^{\gamma l} + (V_L - I_L Z_o) e^{-\gamma l}}{(V_L + I_L Z_o) e^{\gamma l} - (V_L - I_L Z_o) e^{-\gamma l}}$$

divide by I_L $\left(\frac{V_L}{I_L} = Z_L \right)$

No.

$$Z_{in} = Z_0 \frac{(Z_L + Z_0)e^{\gamma l} + (Z_L - Z_0)e^{-\gamma l}}{(Z_L + Z_0)e^{\gamma l} - (Z_L - Z_0)e^{-\gamma l}}$$

take Z_L and Z_0 as a common factor and divide by $(e^{\gamma l} + e^{-\gamma l})$

$$\boxed{Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}} \rightarrow \text{for lossy line}$$

iF lossless $\gamma = j\beta$

$$\boxed{\tan \gamma (j\beta l) = j \tan \beta l}$$

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$$

$\beta l \rightarrow \frac{2\pi}{\lambda} l$

$* \tanh \gamma l = \tanh(\alpha + j\beta) l \Rightarrow$ given in the exam

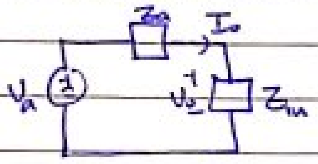
Scanned with CamScanner

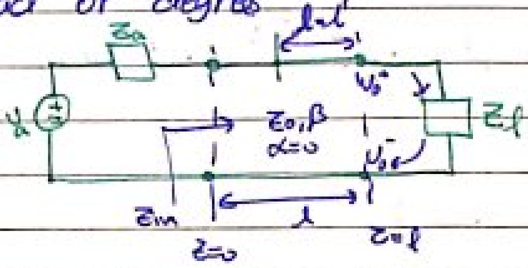
No. 15/3/2018

$$Z_{in} = Z_0 \frac{Z_\ell + Z_0 \tanh \gamma\ell}{Z_0 + Z_\ell \tanh \gamma\ell} \quad \text{(lossy)}$$

$$Z_{in} = Z_0 \frac{Z_\ell + jZ_0 \tan\beta\ell}{Z_0 + jZ_\ell \tan\beta\ell} \quad \text{(lossless)}$$

$\beta\ell$ = Electrical length in rad or degrees





Reflection coefficient

$$\Gamma = \frac{V_0^- e^{j\beta z}}{V_0^+ e^{-j\beta z}} = \frac{V_0^-}{V_0^+} e^{2j\beta z} \Rightarrow \text{anywhere in the T.L}$$

$$V_s(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

$$\Gamma = \frac{V_0^- e^{j\beta\ell}}{V_0^+ e^{-j\beta\ell}} = \frac{\frac{1}{2}\left((V_L - I_L Z_0) e^{-j\beta\ell}\right) e^{j\beta\ell}}{\frac{1}{2}(V_L - I_L Z_0) e^{j\beta\ell} e^{-j\beta\ell}} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

For $z = \ell - \ell'$

$$\Gamma = \frac{V_0^-}{V_0^+} e^{2j\beta(\ell - \ell')} = \frac{V_0^-}{V_0^+} e^{2j\beta\ell} e^{-2j\beta\ell'}$$

$$\Gamma = \Gamma_L e^{-j2\beta\ell'}$$

No.

Current reflection Coefficient

$$I_s(z) = I_o^+ e^{-j\beta z} + I_o^- e^{j\beta z}$$

$$\Gamma = \frac{I_o^-}{I_o^+} e^{2j\beta z} \qquad I_o^- = \frac{-V_o^-}{Z_o}$$

$$\Gamma = \frac{-V_o^-}{V_o^+} e^{2j\beta z} = -\Gamma \text{ voltage} \qquad I_o^+ = \frac{V_o^+}{Z_o}$$

Standing wave ratio & SWR or S

$$S = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}}$$

$$\frac{V_o^+ + V_o^-}{V_o^+ - V_o^-} \text{ or } \frac{V_o^+ e^{-j\beta z} + V_o^- e^{j\beta z}}{V_o^+ e^{-j\beta z} - V_o^- e^{j\beta z}}$$

divide by V_o^+

$$S = \frac{1+|\Gamma|}{1-|\Gamma|} \Rightarrow |\Gamma| = \frac{S-1}{S+1}$$

$$Z_{in_{max}} = \frac{V_{max}}{I_{min}}$$

$$\frac{V_{max}}{I_{min}}, \frac{V_{min}}{I_{min}} = S Z_o$$

$$Z_{in_{min}} = \frac{V_{min}}{I_{max}}$$

$$\frac{V_{min}}{I_{max}} * \frac{I_{min}}{I_{min}} = \frac{Z_o}{S}$$

No.

$0 \leq |\Gamma| \leq 1$ $\qquad\qquad$ $1 \leq S \leq \infty$

no reflection (matched load) $\Rightarrow Z_L = Z_0$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

power

$P_{avg} = \frac{1}{2} Re\{V_s(z)\, I_s^*(z)\}$ [w] $\qquad$ scalar

at the load $z = l$

$V_s(l) = V_0^+ e^{-j\beta l} + V_0^- e^{j\beta l}$

$I_s(l) = \frac{1}{Z_0}\{V_0^+ e^{-j\beta l} - V_0^- e^{-j\beta l}\}$

$P_{avg} = \frac{1}{2} Re\{V_0^+ (e^{-j\beta l} + |\Gamma| e^{j\beta l}) \frac{V_0^+}{Z_0}(e^{j\beta l} - |\Gamma| e^{-j\beta l})\}$

$= \frac{1}{2} Re\{\frac{V_0^{+2}}{Z_0}(1 - |\Gamma| e^{-j2\beta l} + |\Gamma| e^{j2\beta l} - |\Gamma|^2)\}$

$P_{avg} = \frac{(V_0^+)^2}{2Z_0}(1 - |\Gamma|^2)$ w

$P_t = P_i - P_r$

$= \frac{(V_0^+)^2}{2Z_0} - \frac{(|\Gamma| V_0^+)^2}{2Z_0} = \frac{(V_0^+)^2}{2Z_0} - \frac{(V_0^-)^2}{2Z_0}$

if $V_0^- = 0 \rightarrow \Gamma = 0 \rightarrow P_r = 0$

$P_t = P_i \rightarrow$ M.P.T

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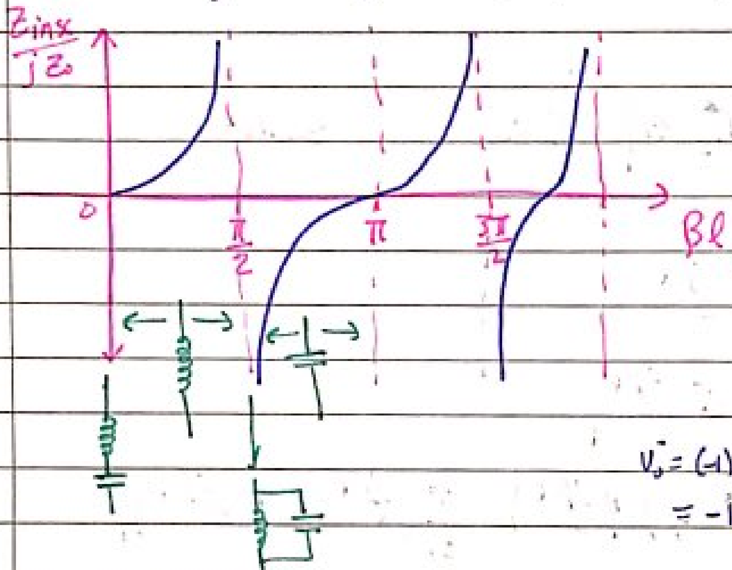
No.

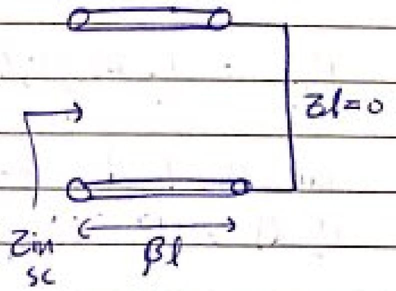
Special case from the load 8 (lossless T.L)

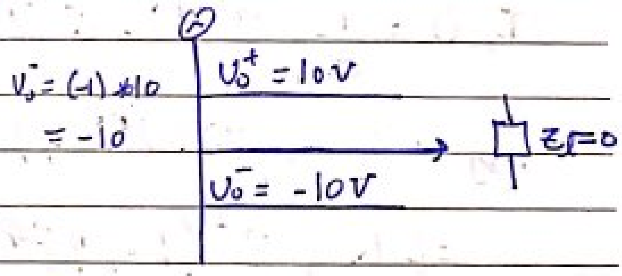
① Short circuit load ($Z_\ell = 0$)

$$Z_{in} = Z_0 \frac{Z_\ell + jZ_0 \tan\beta\ell}{Z_0 + jZ_L \tan\beta\ell}$$

$Z_{in\,sc} = j Z_0 \tan\beta\ell \longrightarrow$ pure imaginary







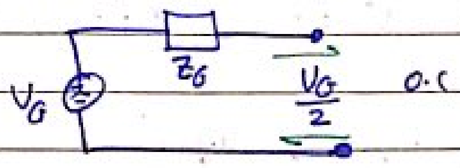
$$\Gamma = \frac{Z_\ell - Z_0}{Z_\ell + Z_0} = -1 \qquad S \rightarrow \infty$$

② open circuit load ($Z_\ell = \infty$)

divide Z_{in} by Z_ℓ

$$Z_{in\,oc} = Z_0 \left(\frac{1}{j\tan\beta\ell} \right) = -jZ_0 \cot\beta\ell \rightarrow \text{pure imaginary}$$

$\Gamma = 1 = 1\angle 0^\circ \qquad S \rightarrow \infty$



$$Z_{insc} * Z_{in\,oc} = Z_0^2$$

No. Chapter 11 "Transmission lines"

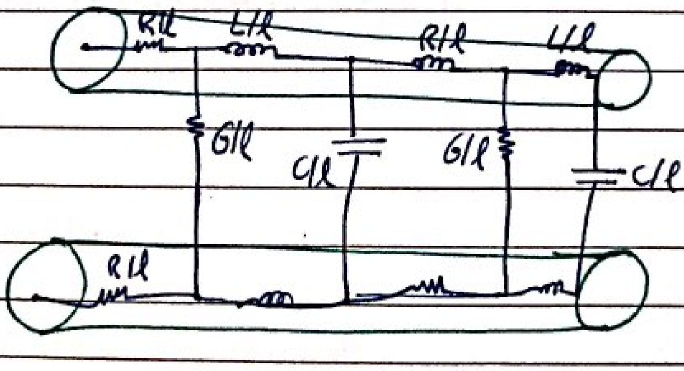
⇒ Two or more Conductors

examples of two- Conductors T.L
→ Coaxial Cable ⓞ $f < 3GHz$
→ Twin - wire line
$f < 300 MHz$

→ planar T.L
- → strip-line
- → micro strip-line
- → slot line

* T.L parameters :-

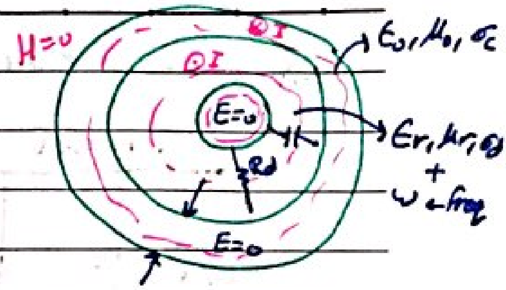
R/ℓ (Ω/m)
L/ℓ (H/m)
C/ℓ (F/m)
G/ℓ (S/m)



types of elements :-
→ Lumped elements.
→ Distriputed elements.

No.

for a Coaxial Cable



$R/l \longrightarrow$ AC resistance

$$R/l = \frac{1}{2\pi \delta \sigma_c}\left(\frac{1}{a}+\frac{1}{b}\right) \ \Omega/m$$

$$L/l = \frac{\mu}{2\pi} \ln\left(\frac{b}{a}\right) \rightarrow \text{external inductance}$$

$$L_{in} \rightarrow 0 \quad \text{as} \quad \omega \rightarrow \infty$$

$$C/l = \frac{2\pi\epsilon}{\ln b/a} \ F/m$$

$$G/l = \frac{2\pi\sigma_d}{\ln \frac{b}{a}} \ S/m \longrightarrow G = \frac{1}{R_d} \neq \frac{1}{R}$$

$$L_{ext} \cdot C = \mu\epsilon$$

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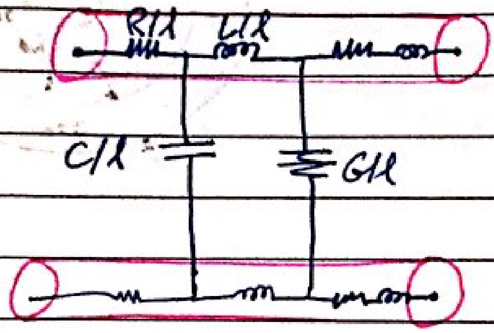
* T.L parameters



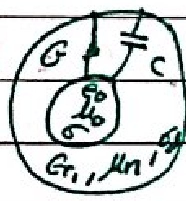
For coaxial cable :-

$$R_{ac}/l = \frac{1}{2\pi\sigma_c \delta}\left(\frac{1}{a}+\frac{1}{b}\right)\ \Omega/m$$

$$C/l = \frac{2\pi\epsilon}{\ln\frac{b}{a}}\ F/m$$

$$G/l = \frac{2\pi\sigma_d}{\ln\frac{b}{a}}\ S/m$$

$$L/l = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right)\ H/m$$

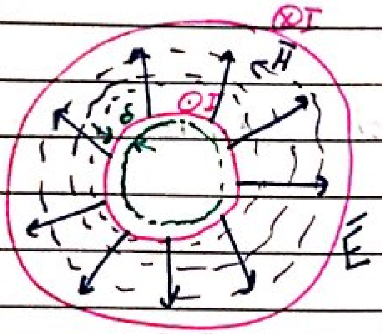


$$L_{ext}C = \mu\epsilon$$

$$R_d C = \frac{\epsilon}{d} \text{ (or) } \frac{C}{G} = \frac{\epsilon}{\sigma}$$

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma_c}}$$

No.



$\bar{P} = \bar{E} \times \bar{H}$ in $\hat{a}_k$ direction

$\hat{a}_k \perp \bar{E} \perp \bar{H} \Rightarrow$ TEM wave

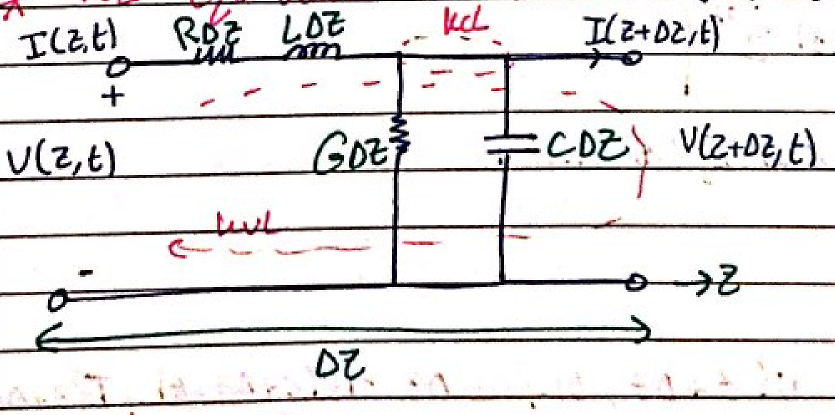
T.L equation :- (Telegrapher equation)

Since an $\bar{E}$ and $\bar{H}$ fields are related to voltage and current according to :

$$V = -\int_{l} \bar{E} \cdot \bar{dl} \quad , \quad I = \oint_{l} \bar{H} \cdot \bar{dl}$$

So, we will use circuit theory to analyze the T.L

* T.L equivelant circuit :- (π and T)



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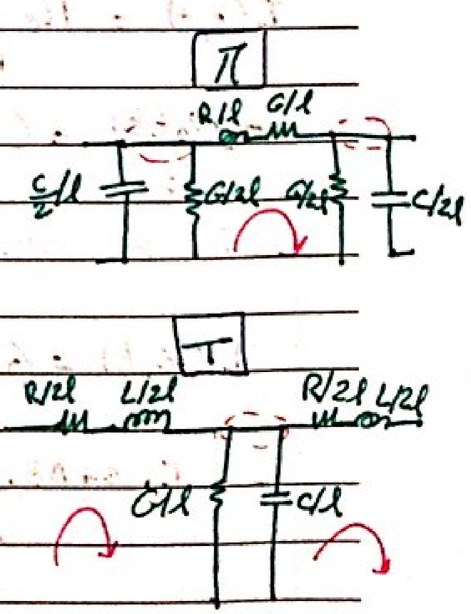
L-equ.ckt



No.

write the kvL eq :-

$$-v(z,t) + I(z,t) R \Delta z + L \Delta z \frac{dI(z,t)}{dt} + v(z+\Delta z,t) = 0$$

$$-\boxed{\frac{v(z+\Delta z,t) - v(z,t)}{\Delta z}} = I(z,t) R + L \frac{dI(z,t)}{dt}$$

take $\lim_{\Delta z \to 0}$

$$\frac{-dv(z,t)}{dz} = I(z,t) R + L \frac{dI(z,t)}{dt}$$

Convert to phasor

$$v(z,t) = Re\{v_s(z) e^{j\omega t}\}$$

$$I(z,t) = Re\{I_s(z) e^{j\omega t}\}$$

$$\frac{-dv_s(z)}{dz} = (R + j\omega L) I_s(z) \quad \ldots 1$$

write the kvL eq.

$$I(z,t) = G \Delta z \; v(z+\Delta z,t) + C \Delta z \frac{dv(z+\Delta z,t)}{dt} + I(z+\Delta z,t)$$

$$-\boxed{\frac{I(z+\Delta z,t) - I(z,t)}{\Delta z}} = G\, v(z+\Delta z,t) + C \frac{dv(z+\cancel{\Delta z},t)}{dt}$$

No.

$$\lim_{\Delta z \to 0} \Rightarrow -\frac{dI(z,t)}{dz} = G\,v(z,t) + C\frac{d\,v(z,t)}{dt}$$

Convert to phasor

$$-\frac{dI_s(z)}{dz} = (G+j\omega C)\,V_s(z) \quad \dots ②$$

$$-\frac{dV_s(z)}{dz} = (R+j\omega L)\,I_s(z) \quad \dots ①$$

solve for $V_s(z)$ and $I_s(z)$

derive eq.(1) w.r.t. (z)

$$-\frac{d^2V_s(z)}{\partial z^2} = (R+j\omega L)\frac{dI_s(z)}{dz} \quad ③$$

sub (2) in the (3)

$$\frac{d^2V_s(z)}{\partial z^2} = \underbrace{(R+j\omega L)(G+j\omega C)}_{\text{Constant}}\;V_s(z)$$

$$\boxed{\frac{\partial^2 V_s(z)}{\partial z^2} - \gamma^2 V_s(z) = 0} \Rightarrow \text{voltage wave equation}$$

$$\gamma^2 = (R+j\omega L)(G+j\omega C)$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$
$$= \alpha + j\beta$$

$$\boxed{\frac{\partial^2 I_s(z)}{\partial z^2} - \gamma^2 I_s(z) = 0} \Rightarrow \text{current wave equation}$$

No.

Solution (same as CH.10)

$$V_s(z) = \boxed{V_0^+ e^{-\gamma z}} + V_0^- e^{\gamma z}$$

$$I_s(z) = \boxed{I_0^+ e^{-\gamma z}} + I_0^- e^{\gamma z}$$

forward wave — backward wave

* Characteristic Impedance (Z_0) in (Ω)

$$Z_0 = \frac{V_0^+}{I_0^+} = \frac{-V_0^-}{I_0^-} = \frac{R+j\omega L}{\gamma} = \frac{\gamma}{G+j\omega C}$$

$$\text{or } Z_0 = \frac{R+j\omega L}{\sqrt{(R+j\omega L)(G+j\omega C)}}$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \quad (\Omega)$$

$$= \underset{\text{resistance}}{R_0} + j \underset{\text{reactance}}{X_0}$$

$$V(z,t) = Re\{V_s(z)\, e^{j\omega t}\}$$

$$= V_0^+ \cos(\omega t \underset{\hat{a}_k = +\hat{a}_z}{-} \gamma z) + V_0^- \cos(\omega t \underset{\hat{a}_k = -\hat{a}_z}{+} \gamma z)$$

No.

* Characteristic Admittance (Y_o)

$Y_o = \frac{1}{Z_o}$ ($\mho$) or (S)

* wave length (λ)

$\lambda = \frac{2\pi}{\beta}$

* Velocity $\Rightarrow u = \frac{\omega}{\beta} = \lambda f$

* α in Np/m $\Rightarrow$ 1 Np = 8.688 dB

13/3/2017

Special cases :-

A) lossless line :- $\sigma_c \simeq \infty$, $\sigma_d = 0$
$R = 0$, $G = 0$ ($R_d = \infty$)

$\gamma = j\omega\sqrt{LC} = j\beta$

$\boxed{\alpha = 0 \quad \beta = \omega\sqrt{LC}}$ $\beta = \omega\sqrt{\mu\epsilon}$ $\boxed{\mu\epsilon = L_{ext} C}$

$Z_o = \sqrt{\frac{L}{C}} = R_o$

$R_o = \sqrt{\frac{L}{C}}$, $X_o = 0$

pure real

$$\lambda = \frac{2\pi}{\beta}$$

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} \overset{eq}{\equiv} \frac{1}{\sqrt{\mu\epsilon}}$$

$$V_s(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

Standing wave

B) Distortionless line :-

$$\frac{R}{L} = \frac{G}{C}$$

$$\gamma = \sqrt{R(1+\frac{j\omega L}{R})\, G(1+\frac{j\omega C}{G})}$$

$$\gamma = \sqrt{RG}\,(1+\frac{j\omega L}{R})$$

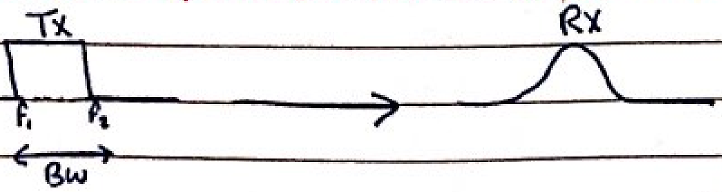
$$\gamma = \sqrt{RG} + j\omega\sqrt{\frac{L \cancel{R} G L}{R^2}}$$

$$= \sqrt{RG} + j\omega\sqrt{\frac{C}{G} \cdot GL}$$

$$= \sqrt{RG} + j\omega\sqrt{LC}$$

$$\alpha = \sqrt{RG} \quad , \beta = \omega\sqrt{LC}$$

α is not depending on f $\longrightarrow$ distortionless



iF line is Distortion

No.

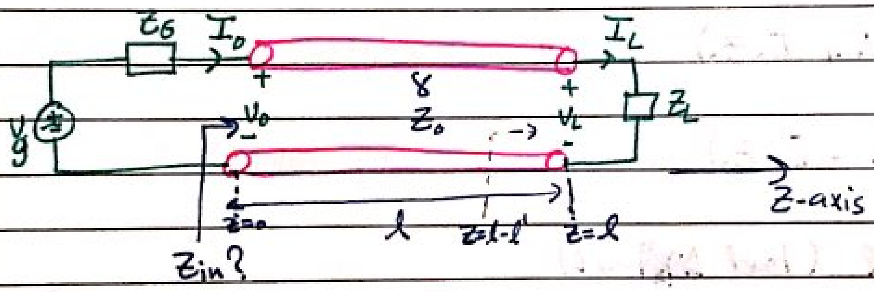
$$Z_0 = \sqrt{\frac{R(1+\cancel{j\omega \frac{L}{R}})}{G(1+\cancel{j\omega \frac{C}{G}})}} = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}$$

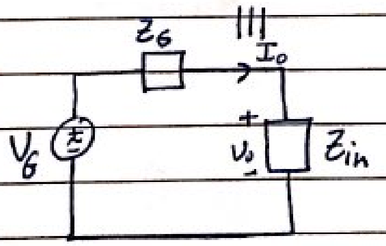
$$R_0 = \sqrt{\frac{L}{C}} \quad , \quad X_0 = 0$$

$$V_s(z) = V_0^+ e^{-\alpha z} e^{-j\beta z} + V_0^- e^{\alpha z} e^{j\beta z}$$

$$\lambda = \frac{2\pi}{\beta} \quad , \quad u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

* Input impedance, SWR, power





$$V_0 = \frac{V_G Z_{in}}{Z_{in} + Z_G}$$

$$I_0 = \frac{V_0}{Z_{in}} = \frac{V_G}{Z_{in} + Z_G}$$

The voltage and current on the line are :-

$$\boxed{V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}}$$

$$I_s(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

$$Z_0 = \frac{V_0^+}{I_0^+} = \frac{-V_0^-}{I_0^-}$$

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No.

$$\boxed{I_s(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z}}$$

Solve for V_0^+ and V_0^-

→ at $z = 0$ (Generator end)

$V_s(0) = V_0$, $I_s(0) = I_0$

$V_0 = V_0^+ + V_0^-$ ... ①

$I_0 = \frac{V_0^+ - V_0^-}{Z_0}$ ... ②

$V_0^+ = \frac{1}{2}(V_0 + I_0 Z_0)$

$V_0^- = \frac{1}{2}(V_0 - I_0 Z_0)$

→ at $z = l$ (load ~~side~~ end)

$V_s(l) = V_L$; $I_s(l) = I_L$

$V_L = V_0^+ e^{-\gamma l} + V_0^- e^{\gamma l}$ ... ⓐ

$I_L = \frac{1}{Z_0}\left(V_0^+ e^{-\gamma l} - V_0^- e^{\gamma l}\right)$ ... ⓑ

$V_0^+ = \left(V_L - V_0^- e^{\gamma l}\right) e^{\gamma l}$

$V_0^+ = \frac{1}{2}(V_L + I_L Z_0) e^{\gamma l}$

$V_0^- = \frac{1}{2}(V_L - I_L Z_0) e^{-\gamma l}$

No.

Input impedance (Z_{in})

$$Z_{in}\Big|_{z=0} = \frac{V_s(z)}{I_s(z)}\Big|_{z=0} = \frac{V_o^+ + V_o^-}{\frac{V_o^+ - V_o^-}{Z_o}}$$

$$\boxed{Z_{in} = Z_o \frac{V_o^+ + V_o^-}{V_o^+ - V_o^-}} = \frac{V_o}{I_o}$$

$*$ Z_{in} at $z \neq 0$ (for any point)

Recall that :-

$$\frac{e^{\gamma l} + e^{-\gamma l}}{2} = \cosh \gamma l$$

$$\frac{e^{\gamma l} - e^{-\gamma l}}{2} = \sinh \gamma l$$

$$\frac{e^{\gamma l} - e^{-\gamma l}}{e^{\gamma l} + e^{-\gamma l}} = \tanh \gamma l$$

$\Rightarrow$ $$Z_{in} = Z_o \frac{V_o^+ + V_o^-}{V_o^+ - V_o^-}$$

sub. V_o^+ and V_o^- at $z = l$

$$Z_{in} = Z_o \frac{(V_L + I_L Z_o) e^{\gamma l} + (V_L - I_L Z_o) e^{-\gamma l}}{(V_L + I_L Z_o) e^{\gamma l} - (V_L - I_L Z_o) e^{-\gamma l}}$$

divide by I_L $\left(\frac{V_L}{I_L} = Z_L \right)$

No.

$$Z_{in} = Z_0 \frac{(Z_L + Z_0) e^{\gamma l} + (Z_L - Z_0) e^{-\gamma l}}{(Z_L + Z_0) e^{\gamma l} - (Z_L - Z_0) e^{-\gamma l}}$$

take Z_L and Z_0 as a common factor and divide by $(e^{\gamma l} + e^{-\gamma l})$

$$\boxed{Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}} \rightarrow \text{for lossy line}$$

if lossless $\gamma = j\beta$

$$\boxed{\tan \gamma (j\beta l) = j \tan \beta l}$$

$$Z_{in} = Z_0 \frac{Z_L + j Z_0 \tan \beta l}{Z_0 + j Z_L \tan \beta l} \qquad \beta l \rightarrow \frac{2\pi}{\lambda} l$$

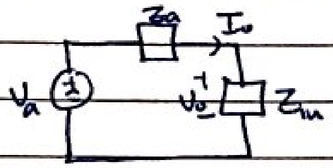
* $\tanh \gamma l = \tanh (\alpha + j\beta) l$ $\Rightarrow$ given in the exam

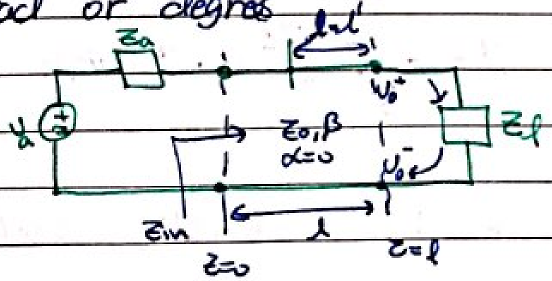
No. @15/3/2018

$$Z_{in} = Z_0 \frac{Z_l + Z_0 \tan\gamma l}{Z_0 + Z_l \tan\gamma l} \quad \text{(lossy)}$$

$$Z_{in} = Z_0 \frac{Z_l + jZ_0 \tan\beta l}{Z_0 + jZ_l \tan\beta l} \quad \text{(lossless)}$$

βl = Electrical length in rad or degrees





Reflection Coefficient

$$\Gamma = \frac{V_0^- e^{j\beta z}}{V_0^+ e^{-j\beta z}} = \frac{V_0^-}{V_0^+} e^{2j\beta z} \Rightarrow \text{anywhere in the T.L}$$

$$V_s(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

$$\Gamma = \frac{V_0^- e^{j\beta l}}{V_0^+ e^{-j\beta l}} = \frac{\frac{1}{2}\left((V_L - I_L Z_0) e^{-j\beta l}\right) e^{j\beta l}}{\frac{1}{2}(V_L - I_L Z_0) e^{j\beta z} e^{-j\beta l}} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

For $z = l - l'$

$$\Gamma = \frac{V_0^-}{V_0^+} e^{2j\beta(l-l')} = \frac{V_0^-}{V_0^+} e^{2j\beta l} e^{-2j\beta l'}$$

$$\Gamma = \Gamma_L e^{-j2\beta l'}$$

No.

Current reflection Coefficient

$$I_s(z) = I_o^+ e^{-j\beta z} + I_o^- e^{j\beta z}$$

$$\Gamma = \frac{I_o^-}{I_o^+} e^{2j\beta z} \qquad I_o^- = \frac{-V_o^-}{Z_o}$$

$$\Gamma = \frac{-V_o^-}{V_o^+} e^{2j\beta z} = -\Gamma \text{ voltage} \qquad I_o^+ = \frac{V_o^+}{Z_o}$$

Standing wave ratio & SWR or S

$$S = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}}$$

$$\frac{V_o^+ + V_o^-}{V_o^+ - V_o^-} \text{ or } \frac{V_o^+ e^{-j\beta z} + V_o^- e^{j\beta z}}{V_o^+ e^{-j\beta z} - V_o^- e^{j\beta z}}$$

divide by V_o^+

$$S = \frac{1+|\Gamma|}{1-|\Gamma|} \Rightarrow |\Gamma| = \frac{S-1}{S+1}$$

$$Z_{in_{max}} = \frac{V_{max}}{I_{min}}$$

$$\frac{V_{max}}{I_{min}}, \frac{V_{min}}{I_{min}} = S Z_o$$

$$Z_{in_{min}} = \frac{V_{min}}{I_{max}}$$

$$\frac{V_{min}}{I_{max}} * \frac{I_{min}}{I_{min}} = \frac{Z_o}{S}$$

No.

$0 \leq |\Gamma| \leq 1$ $\qquad\qquad\qquad$ $1 \leq S \leq \infty$

no reflection (matched load) $\Rightarrow Z_L = Z_0$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

power

$$P_{avg} = \frac{1}{2} Re\{V_s(z)\, I_s^*(z)\} \; [W]$$ scalar

at the load $z = l$

$$V_s(l) = V_0^+ e^{-j\beta l} + V_0^- e^{j\beta l}$$

$$I_s(l) = \frac{1}{Z_0}\{V_0^+ e^{-j\beta l} - V_0^- e^{-j\beta l}\}$$

$$P_{avg} = \frac{1}{2} Re\{V_0^+ (e^{-j\beta l} + |\Gamma| e^{j\beta l}) \frac{V_0^+}{Z_0}(e^{j\beta l} - |\Gamma| e^{-j\beta l})\}$$

$$= \frac{1}{2} Re\{\frac{V_0^{+2}}{Z_0}(1 - |\Gamma| e^{-j2\beta l} + |\Gamma| e^{j2\beta l} - |\Gamma|^2)\}$$

$$P_{avg} = \frac{(V_0^+)^2}{2Z_0}(1 - |\Gamma|^2) \quad W$$

$$P_t = P_i - P_r$$

$$= \frac{(V_0^+)^2}{2Z_0} - \frac{(|\Gamma| V_0^+)^2}{2Z_0} = \frac{(V_0^+)^2}{2Z_0} - \frac{(V_0^-)^2}{2Z_0}$$

if $V_0^- = 0 \rightarrow \Gamma = 0 \rightarrow P_r = 0$

$P_t = P_i \rightarrow$ M.P.T

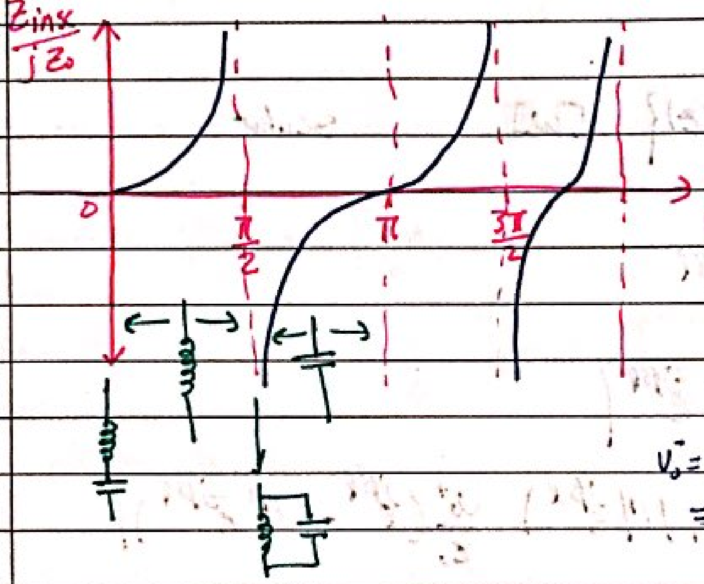
No.

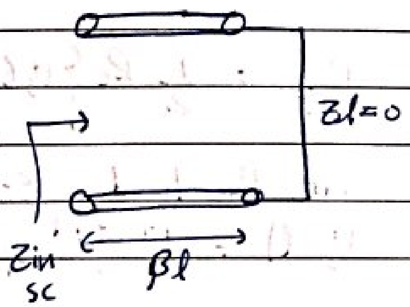
Special case from the load 8 (lossless T.L)

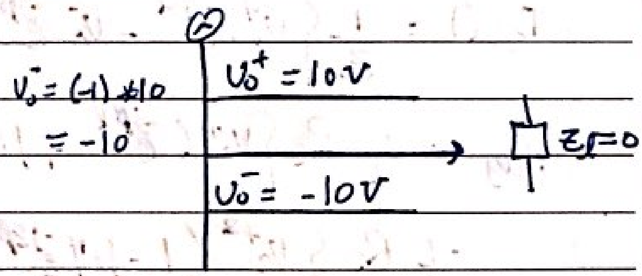
① Short circuit load ($Z_\ell = 0$)

$$Z_{in} = Z_0 \frac{Z_\ell + jZ_0 \tan\beta\ell}{Z_0 + jZ_L \tan\beta\ell}$$

$Z_{in\,sc} = j Z_0 \tan\beta\ell \longrightarrow$ pure imaginary







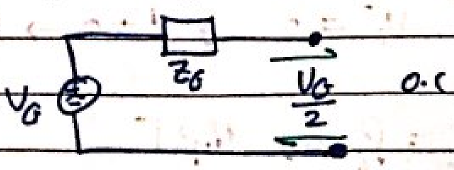
$$\Gamma = \frac{Z_\ell - Z_0}{Z_\ell + Z_0} = -1 \qquad S \to \infty$$

② open circuit load ($Z_\ell = \infty$)

divide Z_{in} by Z_L

$Z_{in\,oc} = Z_0 \left(\frac{1}{j\tan\beta\ell} \right) = -jZ_0 \cot\beta\ell \rightarrow$ pure imaginary

$\Gamma = 1 = 1 \angle 0^\circ \qquad S \to \infty$



$Z_{in\,sc} * Z_{in\,oc} = Z_0^2$

No. 18/3/2015

③ matched load ($Z_\ell = Z_0$)

$Z_{in} = Z_0$, $\Gamma = 0$, $S = 1$

$P_t = P_i = \frac{(V_0^+)^2}{2Z_0}$ M.P.T

* Smith chart :

Construction of smith chart

$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$ ← for lossless line $Z_0 = \sqrt{\frac{L}{C}}$

$\Gamma = \frac{z_L - 1}{z_L + 1}$

$Z_L = R + jX$

Normalize the impedance :

$z_L = \frac{Z_L}{Z_0} = \frac{R + jX}{Z_0} = r + jx$

$z_L = \frac{1 + \Gamma_L}{1 - \Gamma_L}$, $\Gamma_L = \Gamma_r + j\Gamma_i$

$z_L = \frac{(1 + \Gamma_r) + j\Gamma_i}{(1 - \Gamma_r) - j\Gamma_i} = r + jx$

multiply the conjugate and equating real and imagenary parts.

$r = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1 - \Gamma_r)^2 + \Gamma_i^2}$

$x = \frac{2\Gamma_i}{(1 - \Gamma_r)^2 + \Gamma_i^2}$

Rearranging these equations:

No.

$$r \Rightarrow \left[\Gamma_r - \frac{r}{1+r}\right]^2 + \Gamma_i^2 = \left(\frac{1}{1+r}\right)^2$$

$$x \Rightarrow (\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2$$

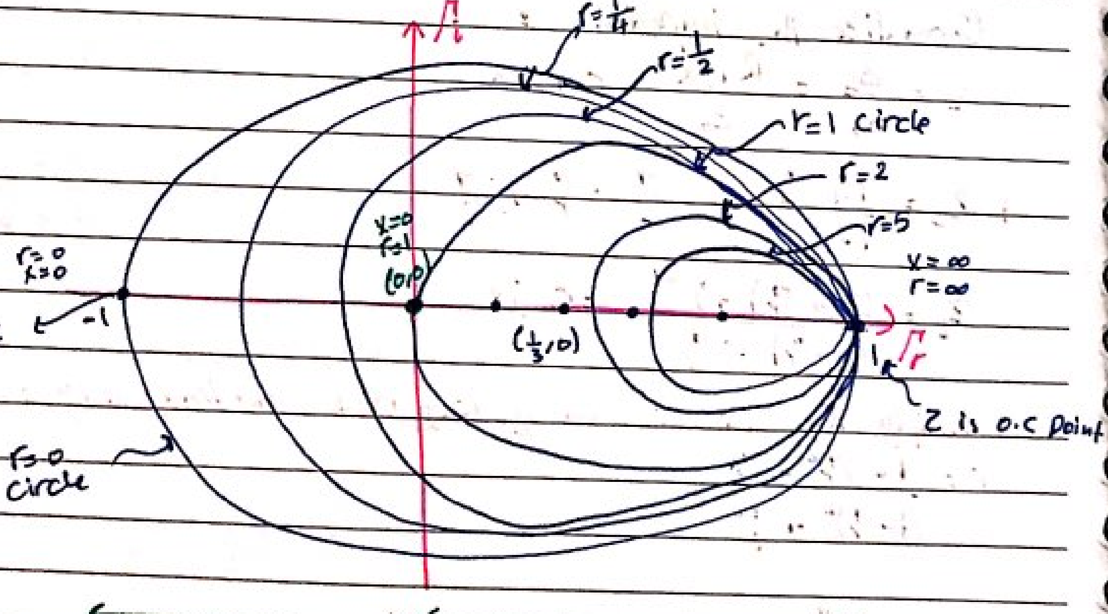
These are circle equations

r-circle : Center $= \left(\frac{r}{1+r}, 0\right)$ (Γ_r, Γ_i)

radius $= \frac{1}{1+r}$

x-circle : Center $= \left(1, \frac{1}{x}\right)$

radius $= \frac{1}{x}$



$r = 0 \pm jx$

$x = 0$

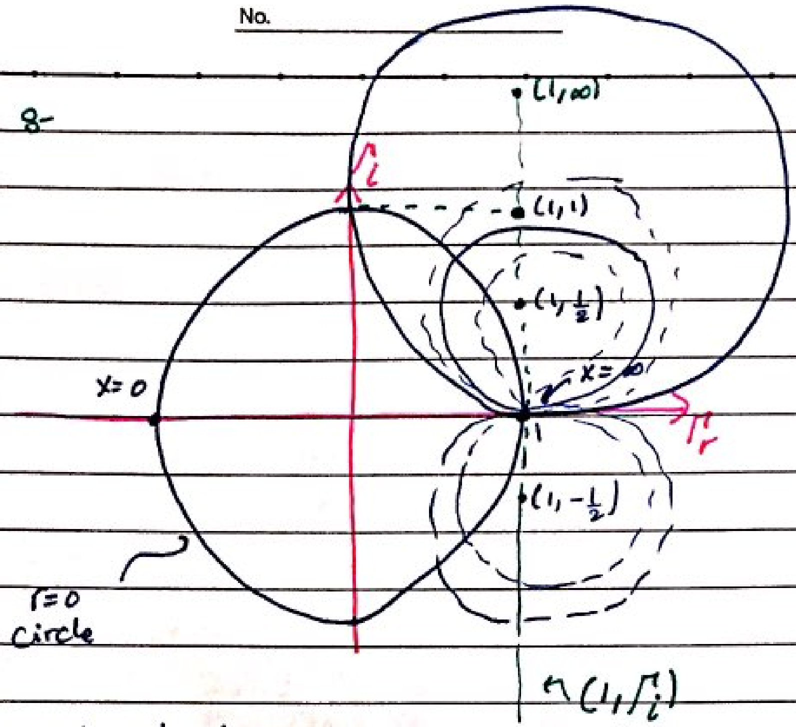
$x = \infty, r = \infty$

$0 \le r < 1$ $\quad$ $1 \le r \le \infty$

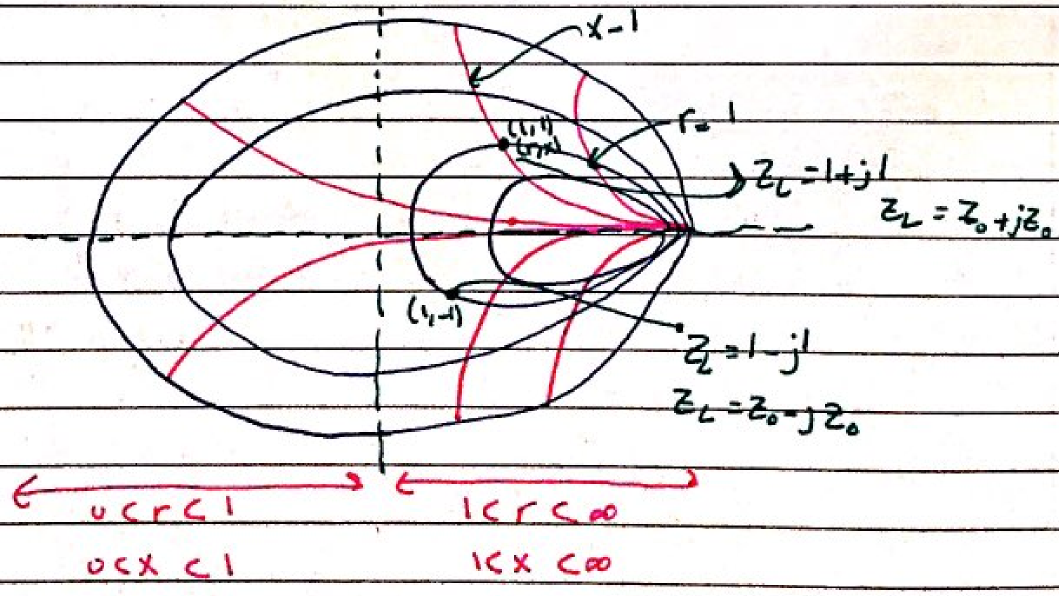
x if $Z_L = 1 \Rightarrow \boxed{Z_L = Z_0}$ matched load

No.

X-Circles :-



Complete Smith chart



* don't solve complex impedances using smith chart.

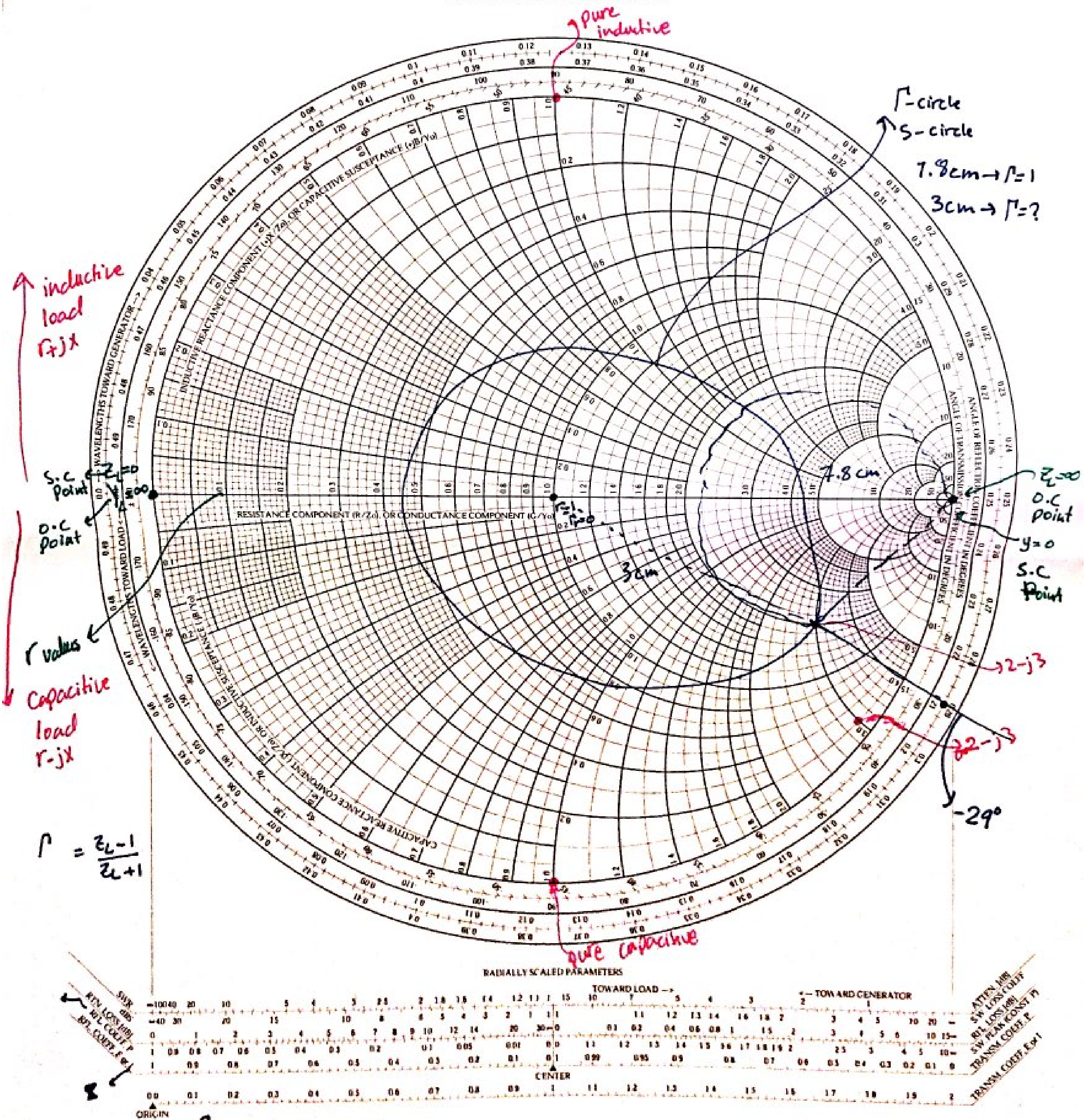
$z=0 \rightarrow y = \frac{1}{0} = \infty$

$Y_o = \frac{1}{Z_o}$

$Y_L = \frac{1}{Z_L}$

The Smith Chart

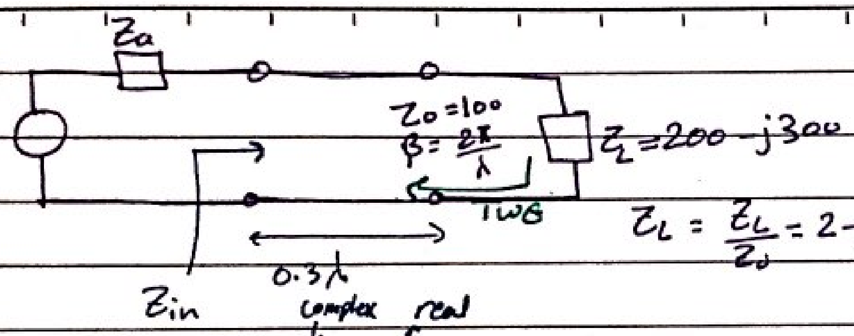
Microwaves101.com



$\Gamma = \frac{z_L - 1}{z_L + 1}$

Γ circle with $-\Gamma_r$ axis

20/3/2018



$\bar{Z}_L = \frac{Z_L}{Z_0} = 2 - j3$

complex real

$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma| \angle \theta_r$

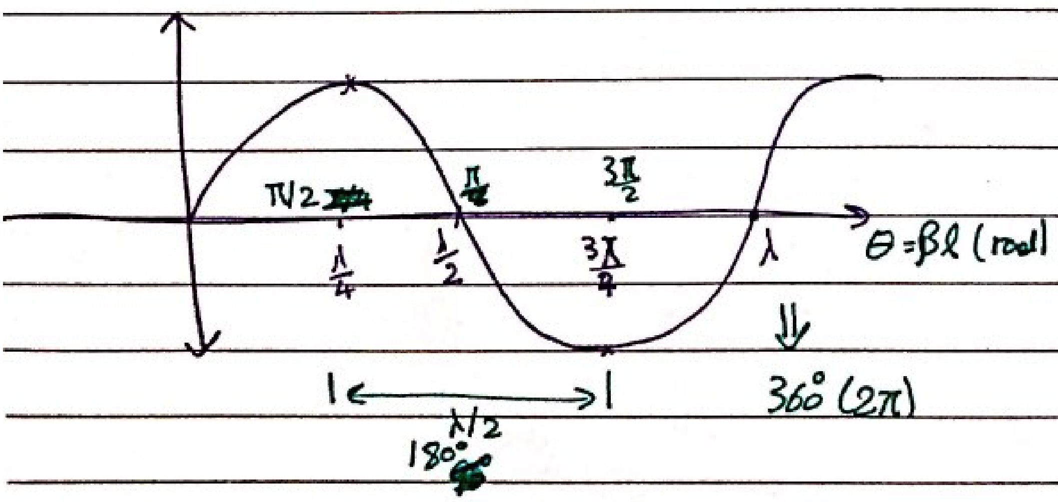
* go back to smith chart

$\Rightarrow Z_L = 200 - j300 \Rightarrow \bar{Z}_L = \frac{Z_L}{Z_0}$ $\qquad Z_0 = \frac{1}{Y_0}$

$Y_L = \frac{1}{200 - j300} \Rightarrow \frac{Y_L}{Y_0} = Y_L Z_0$

$Z_L = 360.5 \angle -56.3° \ \Omega$

$Y_L = \frac{1}{36.5 \angle -56.3} = 2.77 \times 10^{-3} \angle 56.3° \ S$



$2\beta l$

$360° \to 0.5\lambda$

$1\lambda = 720°$

θ_L (1) θ_r (2) WTL (3) WTG (4) ccw cw → Smith chart scales

The Smith Chart

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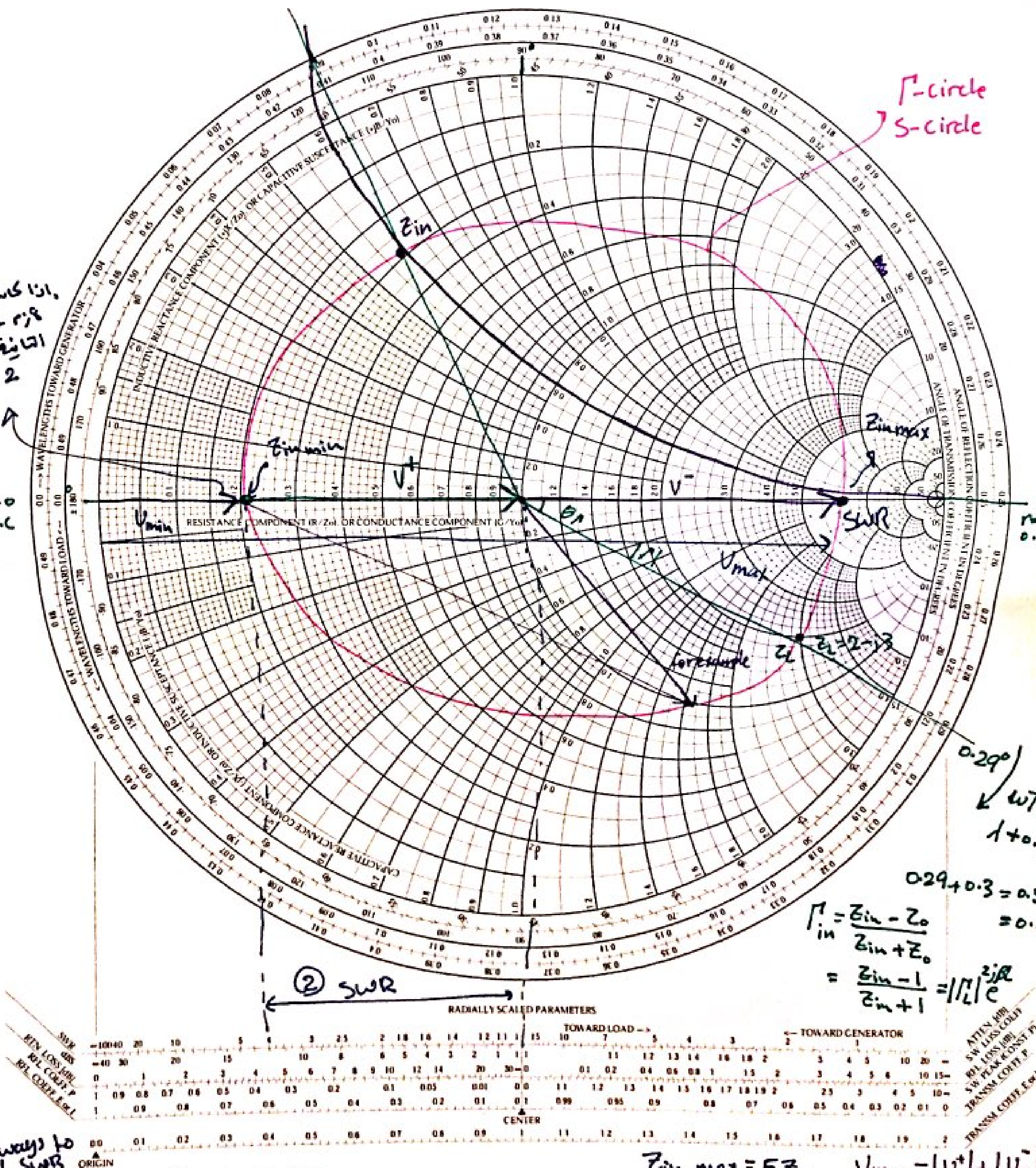


اذا كانت 0.2 جزء على المحور الثانية تكون 2 وهكذا

$\lambda = 0.5$ r=0 $\lambda=0$ S.C

0° r=∞ O.C

0.29° WTG $\lambda + 0.3\lambda$

$0.29 + 0.3 = 0.59 - 0.5 = 0.09$

$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{z_{in} - 1}{z_{in} + 1} = |\Gamma_L| e^{2j\beta l}$$

two ways to find SWR

① SWR ⇒ تقاطع ال Γ-circle مع S-circle ال +ve real axis

$Z_{in\,max} = 5Z_0$

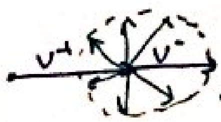
$Z_{in\,min} = \frac{Z_0}{5}$

$z_{in\,max} = 5$

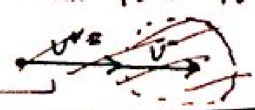
$z_{in\,min} = \frac{1}{5}$

$V_{max} = |V^+| + |V^-| = V^+(1 + |\Gamma|)$

$V_{min} = |V^+| - |V^-|$



cranck diagram

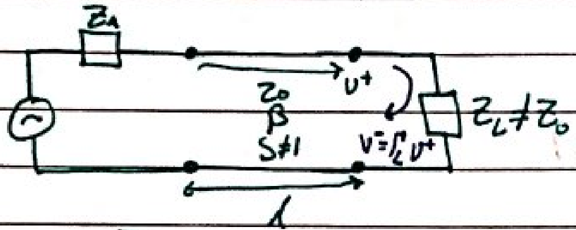


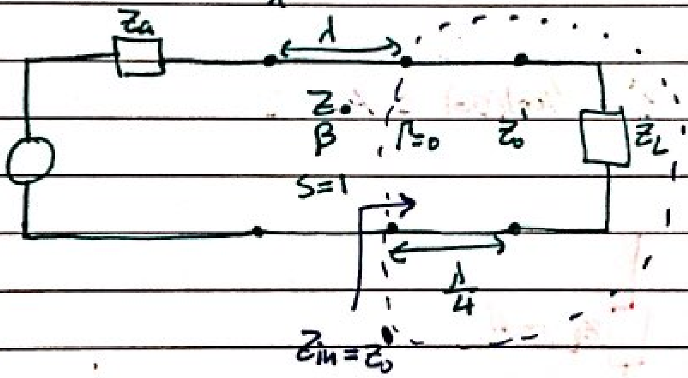
No. 22/3/2018

* Application of T.L

(How to achieve matching)

* $\frac{\lambda}{4}$ Transformer :





$$Z_{in} = Z_o' \frac{Z_L + jZ_o \tan\beta l}{Z_o + jZ_L \tan\beta l}$$

$$\beta l = \frac{2\pi}{\lambda} * \frac{\lambda}{4} = \frac{\pi}{2}$$

divide by $\tan\frac{\pi}{2} = \infty$

$$Z_{in} = Z_o' \frac{jZ_o'}{jZ_L}$$

$$Z_{in} = \frac{Z_o'^2}{Z_L} = \frac{Z_o' Z_o'}{Z_L}$$

$$\frac{Z_{in}}{Z_o'} = \frac{Z_o'}{Z_L}$$

$$z_{in} = \frac{1}{z_L} = y_L$$

Smith chart

Scanned with CamScanner

No.

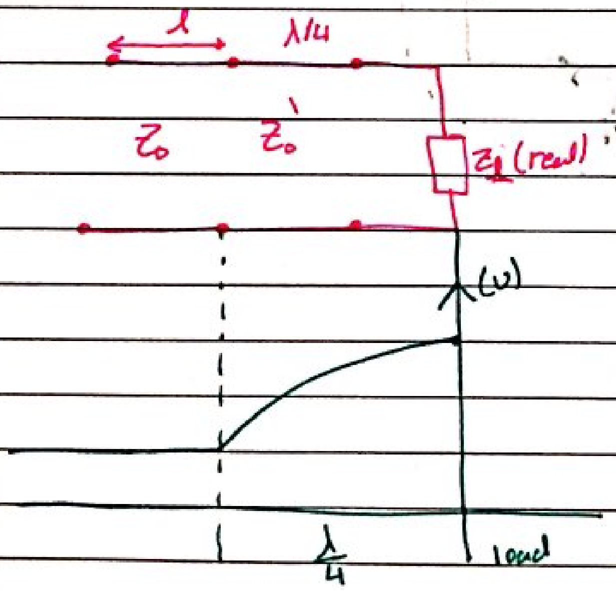
$Z_0' = \sqrt{Z_L \, Z_{in}}$ Z_0

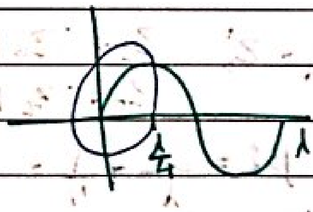
if $Z_L = 100\,\Omega$
$Z_0 = 50\,\Omega$

$\Gamma_L = \frac{100-50}{100+50} = \frac{1}{3} \neq 0$

$S = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \frac{\frac{4}{3}}{\frac{2}{3}} = 2 \neq 1$

add a $\frac{\lambda}{4}$ Tr. with $Z_0' = \sqrt{(100)(50)} \simeq 70.1\,\Omega$



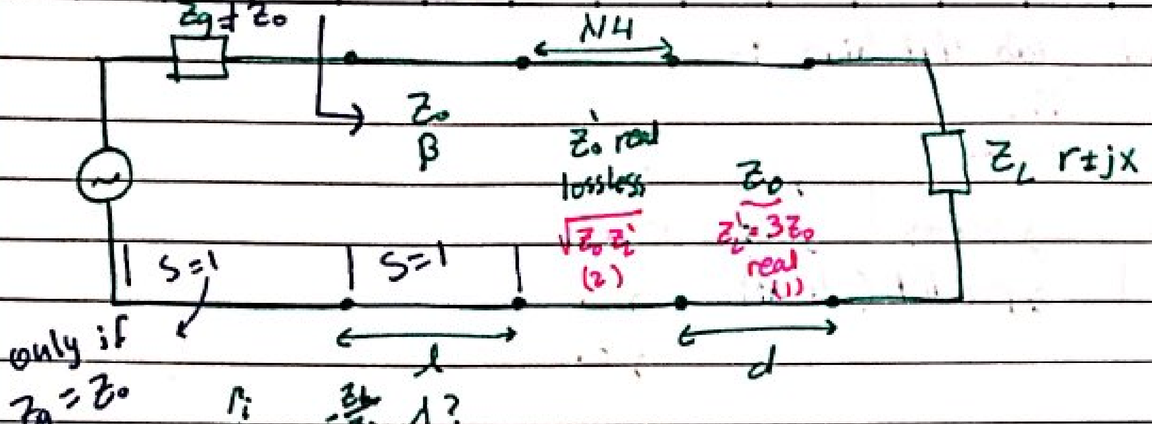


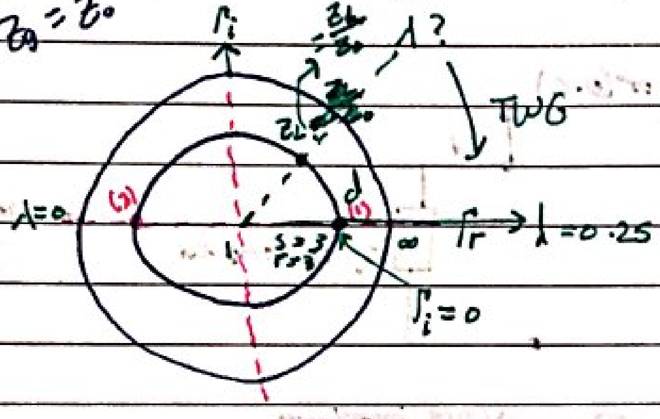
$\frac{V_{max}}{V_{min}} = 2$

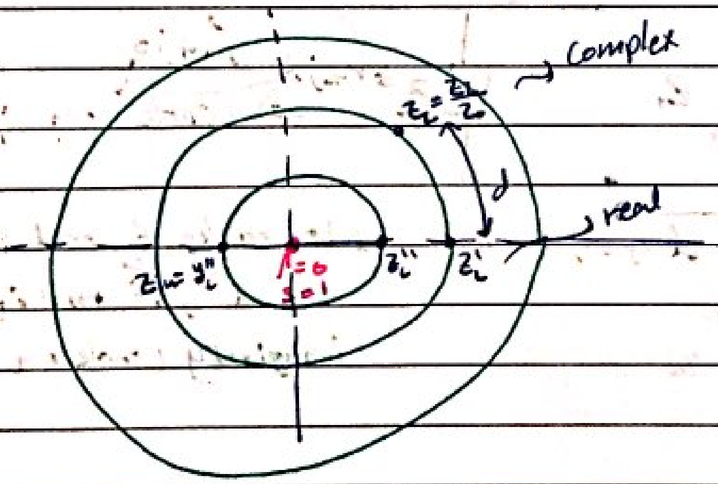
* $\frac{\lambda}{4}$ Transformer cause narrow Bandwidth

* if load is complex then move a distance (d) from the load to make the load (real) then add the $\frac{\lambda}{4}$ Tr.

$$V_0 = V_g \frac{Z_{in}}{Z_{in}+Z_g}$$







$Z_L' = z_L' Z_0 \ (\Omega)$

$z_L'' = \frac{Z_L'}{Z_0'}$

No.

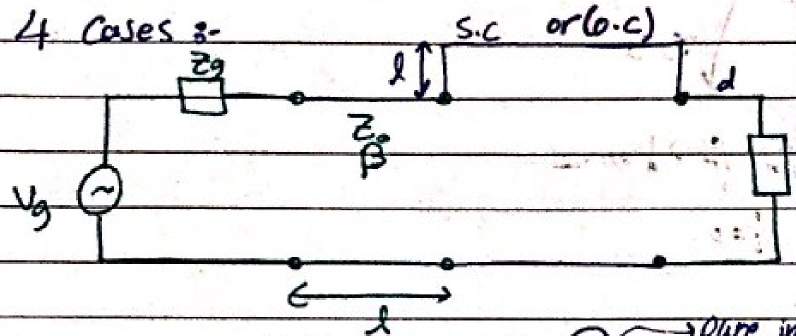
2) single stub Matching :-

T.L lossless

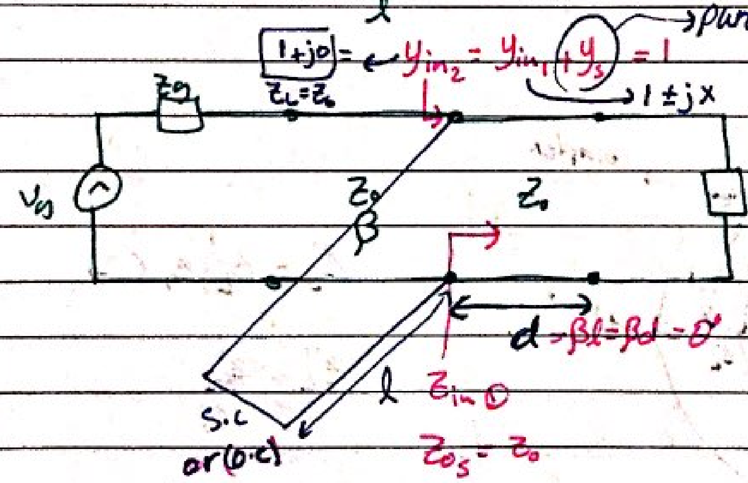
Stub → Seires → S.C ($\Gamma = 0$) / O.C ($\Gamma = \infty$)

Stub → parallel (shunt) → S.C / O.C

4 Cases :-



Z_L is Complex ⟹ work on Z



$Z_L = r \pm jX \xrightarrow{d} 1 + j0$, l

⟹ work on $Y = \frac{1}{Z}$

* to make Z_L real work on d, if making it imaginary work on l

* The best is shunt S.C (high Freq.)

O.C (low Freq.)

No. 27/3/2018

Single Stub Matching : (Tuner)

Example :- Match a load $Z_L = 100 + j80\,\Omega$ to a $50\,\Omega$ T.L using

A) Single Seires open-circuited stub : (follow the smith chart in next page)

$z_L = \frac{100 + j80}{50} = 2 + j1.6$

$z_1 = 1 - j1.33$
$z_2 = 1 + j1.33$

$d_1 = 0.328\lambda - 0.208\lambda = 0.12\lambda$
$d_2 = 0.463\lambda \Rightarrow (0.5\lambda - 0.208\lambda) + 0.172\lambda$

$z_{in} = 1 + j0 = z_1 + z_s$
$= 1 - j1.33 + j1.33 = 1 + j0$

$l_1 = 0.398\lambda$
$l_2 = 0.102\lambda$

* Sol ① shortest distance $\rightarrow d = 0.12\lambda$, $l = 0.398\lambda$
  Sol ② shortest length $\rightarrow d = 0.463\lambda$, $l = 0.102\lambda$

$l_1 + l_2 = 0.5\lambda$ (always)

$Z_s = z_s Z_0 = j(1.33)(50) = j66.5\,\Omega = j\omega L$
$\omega L = 66.5\,\Omega$

No.

B) Single seires short-circuited stub:

$Z_L = ✓$ (same as A)
$Z_1 = ✓$
$Z_2 = ✓$
$d_1 = ✓$
$d_2 = ✓$

$l_{1_{s.c}} = l_{1\,o.c} - 0.25\lambda$ → because l_1 is bigger than 0.25
$l_{2_{s.c}} = l_{2\,o.c} + 0.25\lambda$ → because l_2 is smaller than 0.25

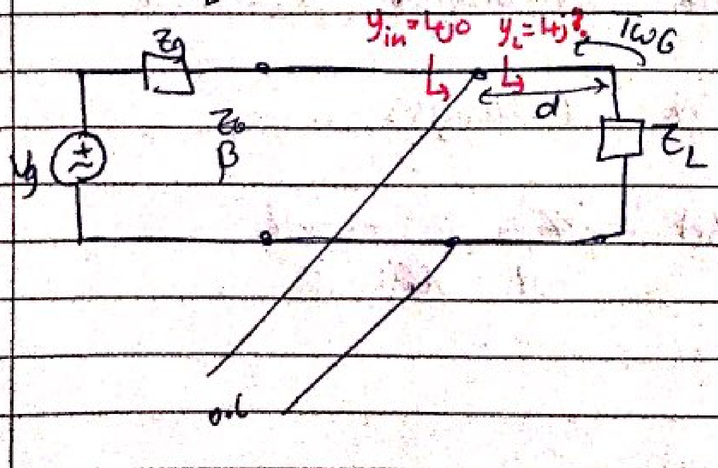
* (always diff. between o.c and s.c is $\frac{\lambda}{4}$)

$l_1 = 0.148\lambda$
$l_2 = 0.352\lambda$

c) Single shunt open-circuited stub:

$Z_L = 2 + j1.6$
$y_L = \frac{1}{Z_L} = 0.3 - j0.24 \Rightarrow y_1 = 1 + j1.33$, $y_2 = 1 - j1.33$

$d_1 = 0.214\lambda$
$d_2 = 0.371\lambda$
$l_1 = 0.352\lambda$
$l_2 = 0.148\lambda$
$= 0.5\lambda - 0.35\lambda$
$l_1 + l_2 = 0.5\lambda$

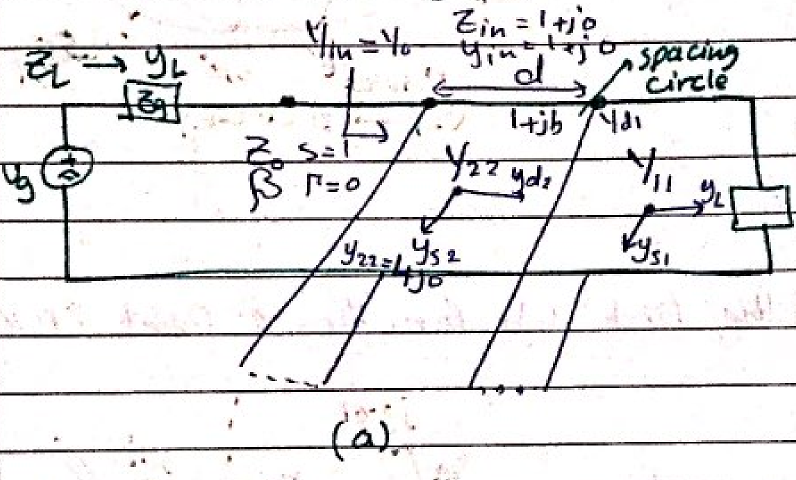


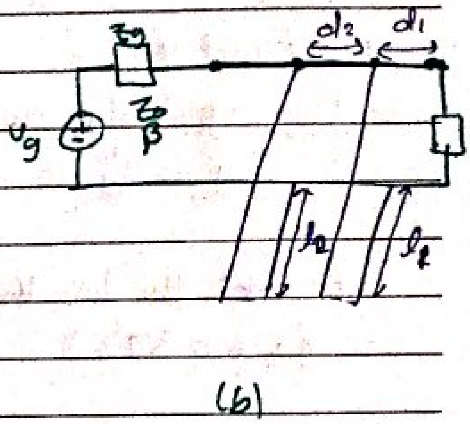
No.

The best solution
seires S.C
$d_1 = 0.12\lambda$
$l_2 = 0.148\lambda$

29/3/2017

Double stub metering : (shunt stubs)





Example 8 Match the load 100+j100 Ω to a 50 Ω T.L using shunt double stub short circuited stub if $d1 = 0.4\lambda$ (from the load) and $d_2 = \frac{3\lambda}{8}$ (from the first stub)

100+j100 + 2 stub = 50

① $z_L = \frac{100+j100}{50} = 2+j2$

② Draw S-circle and locate y_L $\quad y_L = 0.25 - j0.25$

③ if $d_1 \neq 0 \rightarrow$ move from y_L TWG a distance of d_1 and locate y_{d_1}. $(0.458\lambda + 0.4\lambda = 0.858\lambda)$

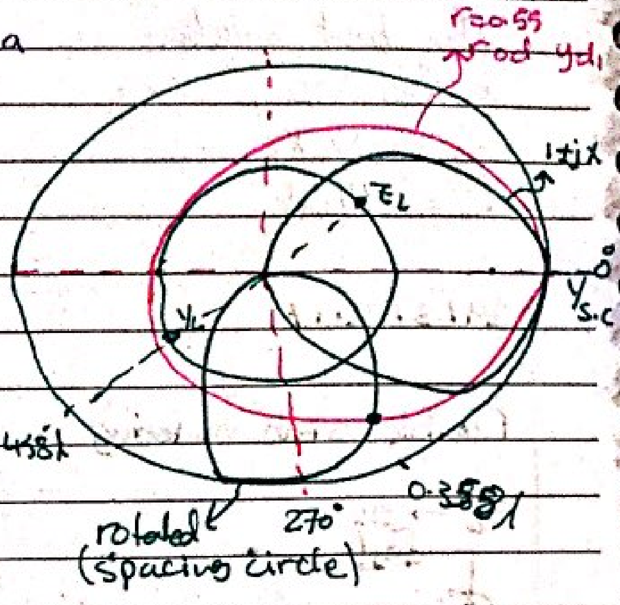
$0.858\lambda > 0.5\lambda \rightarrow (-0.5\lambda) \rightarrow 0.358\lambda$

$y_{d_1} = 0.55 - j1.08$ (if $d_1 = 0$ skip step 3)

No.

④ Rotate the $1 \pm jx$ circle TWL a distance d_2

$d_2 = \frac{3\lambda}{8} = 270^\circ$



⑤ locate the intersections between the R-circle of y_{d_1} and the spacing circle

$Y_{11} = 0.55 - j0.11 \quad Y_{11}' = 0.55 - j1.88$

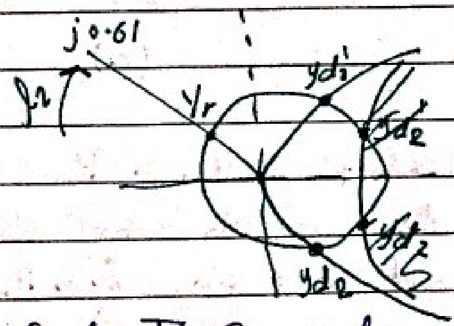
⑥ Find y_{s_1}

$y_{s_1} = y_{11} - y_{d_1} = j0.97$

$y_{s_1}' = y_{11}' - y_{d_1} = -j0.8$

⑦ Find the length of the first stub from the SC point (TWG)

$l_1 = 0.373\lambda$



⑧ Draw y_{11} or y_{11}' S-circle and locate the inter sections with the $1 \pm jx$ circle

⑨ Rotate y_{11} or y_{11}' a distance of d_2 TWG and locate y_{d_2}, y_{d_2}'

$y_{22} = 1 - j0.61 \qquad y_{22}' = 1 + j?$

⑩ Find y_{s_2} or y_{s_2}'

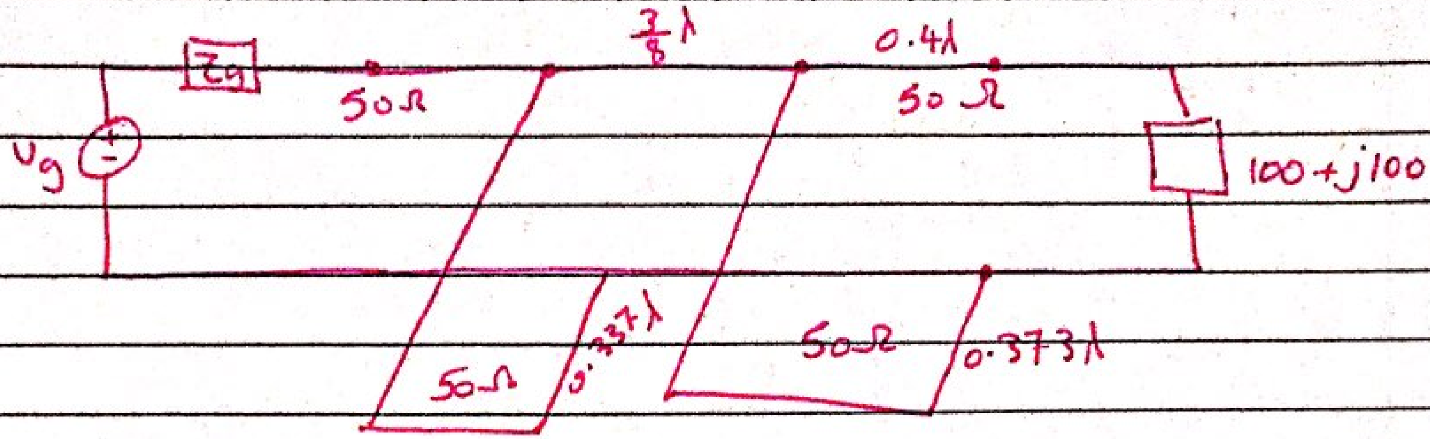
$y_{s_2} = y_{22} - y_{d_2} = j0.61$

$y_{s_2}' = y_{22}' - y_{d_2}'$

and Find l_2 and l_2'

$l_2 = 0.337\lambda$

No.



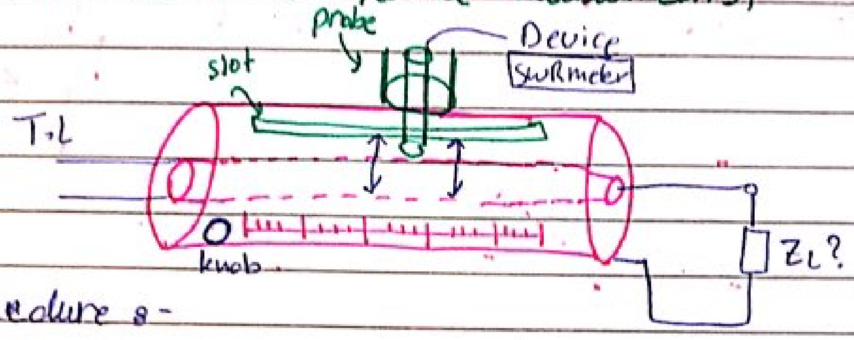
Example: Match a load of $60-j80\ \Omega$ using double shunt open stubs if the distance between the stubs is $\frac{\lambda}{8}$ and $Z_0 = 50\Omega$

Sol: $l_1 = 0.146\lambda$ $\quad$ $l_2 = 0.204$ $\rightarrow$ shortest solution

$l_1' = 0.482\lambda$ $\quad$ $l_2' = 0.35\lambda$

No. 1/4/2018

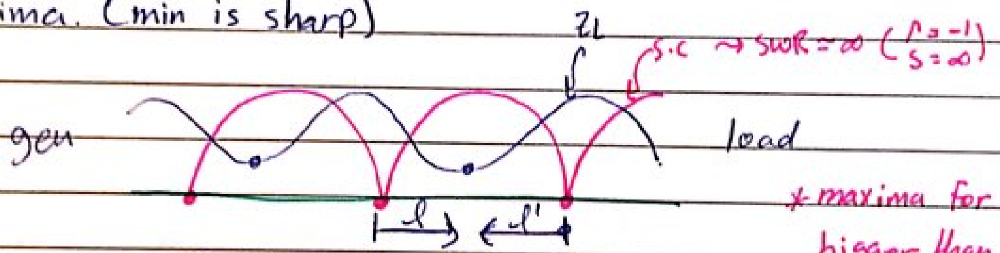
c) Slotted line :- (Impedance measurments)



procedure :-

1) with the load connected to the slotted line locate the voltage minima. (If two ~~eon~~ Negiboring minima)

2) ~~with~~ Replace the load with a S.C and again locate the minima. (min is sharp)



* maxima for S.C is bigger than Z_L

3) measure the distance from the short circuit minima to the load minima.

$l \Rightarrow$ TWL

$l' \Rightarrow$ TWG

4) use smith chart to locate the load

No.

Ex:- unknown load connected to a slotted air line has $S=2$ and minima are found at 11 cm, 19 cm on the scale when the load is connected and at 16 cm and 24 cm when S.C is connected. If $Z_0 = 50\Omega$

find: λ, f, Z_L

i) $\frac{\lambda}{2} = \underset{\text{or } 24-16}{19-11} = 8\,cm \longrightarrow \boxed{\lambda = 16\,cm}$ — difference between two minima equals $\frac{\lambda}{2}$

ii) $l = 16 - 11 = 5\,cm$ (TWL)

$l' = 24 - 19 = 3\,cm$ (TWG)

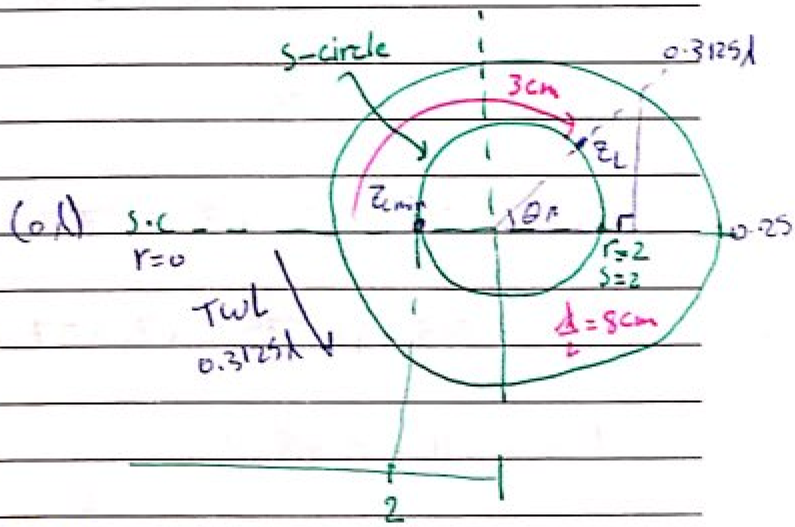
$l = \frac{5}{16}\lambda = 0.3125\lambda$

$f = \frac{u}{\lambda} = \frac{c}{\lambda} = \frac{3\times10^8}{16\times10^{-2}} = 1.875\ GHz$

$Z_L = z_L Z_0$

$= (1.4 + 0.75)50$

$= 70 + j37.5\ \Omega$



$S = \frac{1+|\Gamma|}{1-|\Gamma|}$, $|\Gamma| = \checkmark$

~~Z_L~~ $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma| \angle \theta_r$

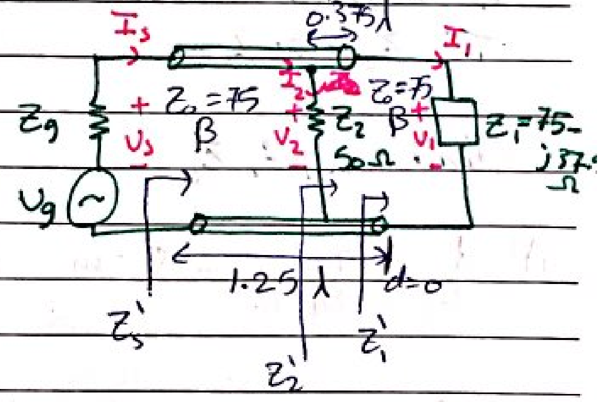
it's complex

θ_r ?

$\theta_r = \pi + 2\beta\, l_{min} = \pi + \frac{4\pi}{\lambda} l_{min}$ 5cm

No.

Ex:- A lossless T.L is connected to 30 V rms with a 75 Ω internal impedance and to a two loads Z_1 and Z_2, if $Z_0 = 75\Omega$ find the following:-



a) $z_1', y_1', y_2', z_2', z_s', y_s'$

b) $V_1, I_1, V_2, I_2, V_s, I_s$

c) Draw $|V(d)|$ and $|I(d)|$ Vs. d

a) $z_1 = 1 - j0.5 \rightarrow$ S-circle move 0.375λ TWG

$z_1' = 1.6 + j0.2 \rightarrow Z_1' = 120 + j15\ \Omega$

$y_1' = 0.61 - j0.75 \rightarrow Y_1' = 0.107 + j0.0053\ \Omega^{-1}$

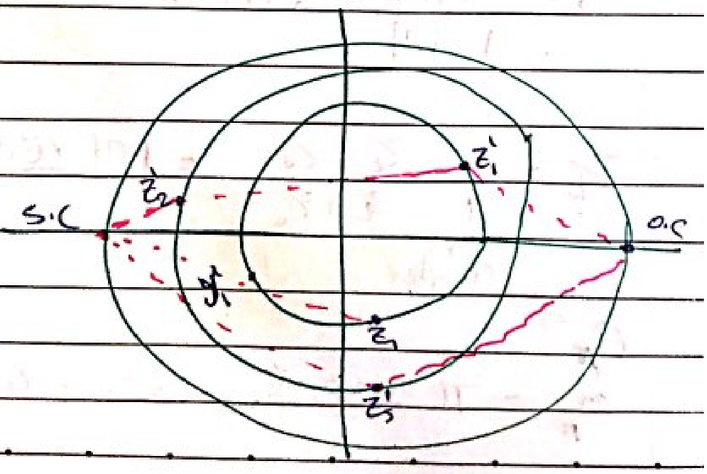
$Y_2' = \frac{1}{Z_2} + Y_1' = 0.0281 - j0.001\ \Omega^{-1}$

$y_2' = 2.11 - j0.075$

$z_2' = 0.5 + j0.03 \rightarrow Z_2' = 37.5 + j2.25\ \Omega$

$z_s' = 0.765 - j0.575 \rightarrow Z_s = 57.4 - j43.1\ \Omega$

$Y_s = 0.011 + j0.008\ \Omega^{-1}$



No.

b) $V_s = \frac{V_g Z_s'}{Z_s' + Z_g} = 15.5 \angle{-18.9^\circ}\ V_{rms}$

$I_s = \frac{V_g}{Z_s' + Z_g} = 0.216 \angle 18^\circ\ A_{rms}$

Drawing scale

$V \rightarrow \frac{15.5}{8cm} = 1.9375\ V/cm$

$I \rightarrow \frac{0.216}{8.2cm} = 0.026\ A/cm$

$V_2' = (5.1\ cm) \times 1.9375 \frac{V}{cm} = 9.9V\ rms$

$I_2 = 0.27\ A_{rms}$

$\rightarrow$ To find V_1', I_1'

$\frac{\overset{V_2 = V_1'}{9.9V}}{9.5cm} = 1.042\ V/cm$

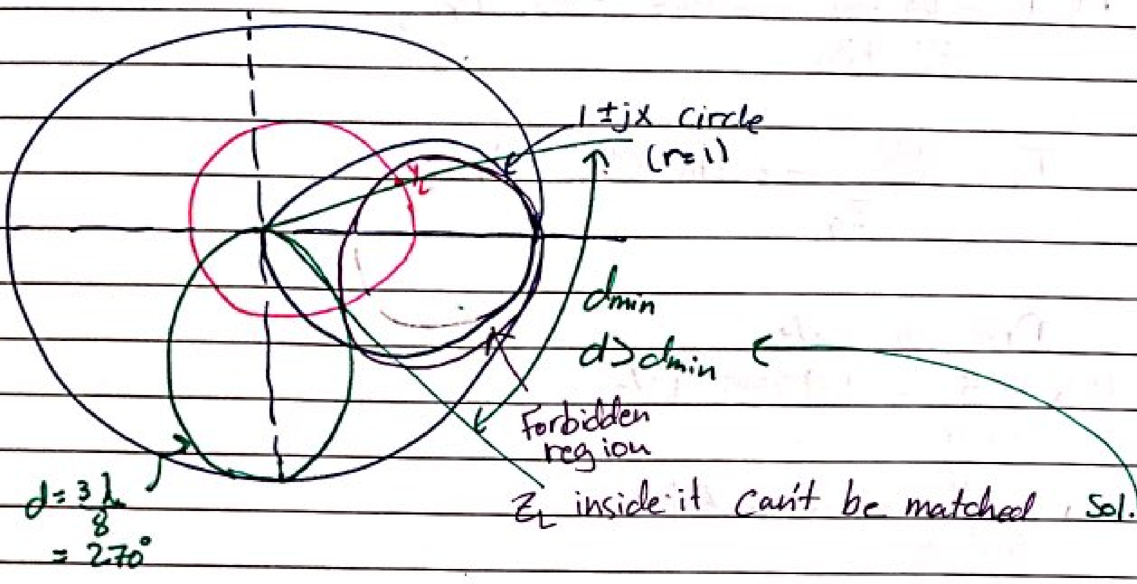
$I_1' = V_2' = \frac{V_1'}{|Z_2'|} = 0.082\ A$

$\frac{0.082A}{5.9cm} = 0.0139\ A/cm$

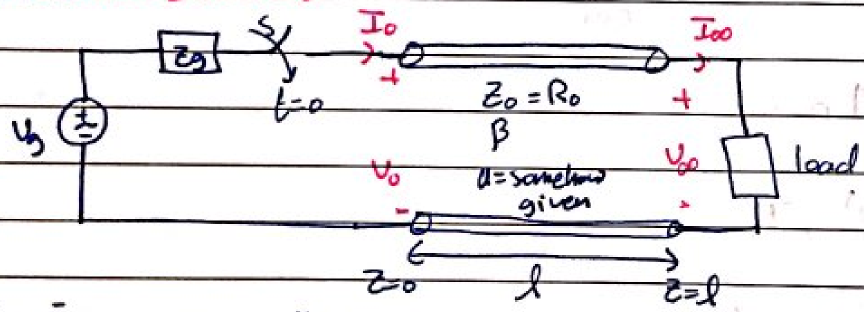
$V_1 = 8.3cm \times 1.042 \frac{V}{cm} = 8.65\ V_{rms}$

$I_1 = 0.1\ A_{rms}$

No. 3/4/2017



* Transients on T.L



at $t = 0^-$ → No signal on transmition line

at $t = 0^+$

$z = 0$ → generator end

$z = l$ → load end

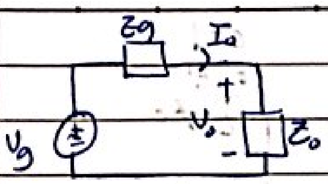
* IF $Z_L = R_L$ (real)

* at the generator end

$V(z,t) = V(0, 0^+)$

$\uparrow z \quad \uparrow t$

No.



$$V_0 = \frac{V_g Z_0}{Z_0 + Z_g}$$

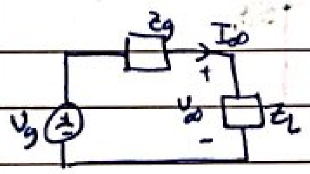
$$I_0 = \frac{V_0}{Z_0} = \frac{V_g}{Z_g + Z_0}$$

$$t_1 = \frac{l}{u} = \text{Transient time}$$

$\Rightarrow$ at the load end:

$V(l, t_1) = V_\infty$ $\quad$ $t_1 \Rightarrow$ time when signal approches the load

$I(l, t_1) = I_\infty$



$$V_\infty = \frac{V_g Z_L}{Z_L + Z_g}$$

$$I_\infty = \frac{V_\infty}{V_L} = \frac{V_g}{Z_L + Z_g}$$

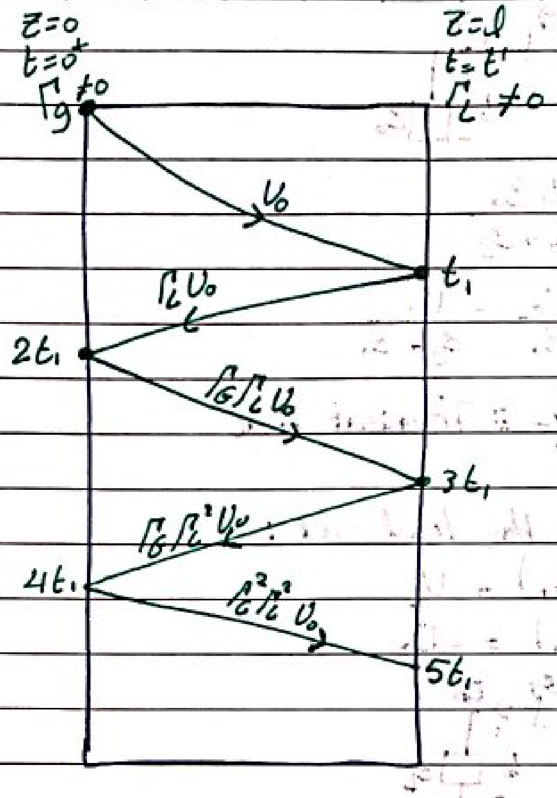
Since $Z_g \neq Z_0$, $Z_L \neq Z_0$

$$\Gamma_g = \frac{Z_g - Z_0}{Z_g + Z_0}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Scanned with CamScanner

No.



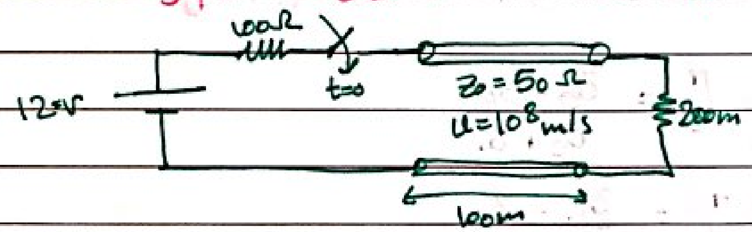
lattice diagram
Bounce diagram
Zigzag "
space-time "

gen. end

$$V(0,t) = V_0 + \Gamma_L V_0 + \dots$$

$$V(l,t) = V_0 + \Gamma_G \Gamma_L V_0 + \Gamma_G^2 \Gamma_L^2 V_0 + \dots$$

* Ex:- For The T.L shown:-
Calculate and sketch :-
a) The voltage at the load and the generator and for $0 < t < 6\,\mu s$
b) The current at the load and generator and for $0 < t < 6\,\mu s$



$$t_1 = \frac{l}{u} = \frac{10^2}{10^8} = 1\,\mu s$$

$$\Gamma_g = \frac{100 - 50}{100 + 50} = \frac{1}{3}$$

$$\Gamma_L = \frac{200 - 50}{200 + 50} = \frac{3}{5}$$

No.

$$V_0 = \frac{V_g Z_0}{Z_0 + Z_g} = \frac{(12)(50)}{150} = 4V$$

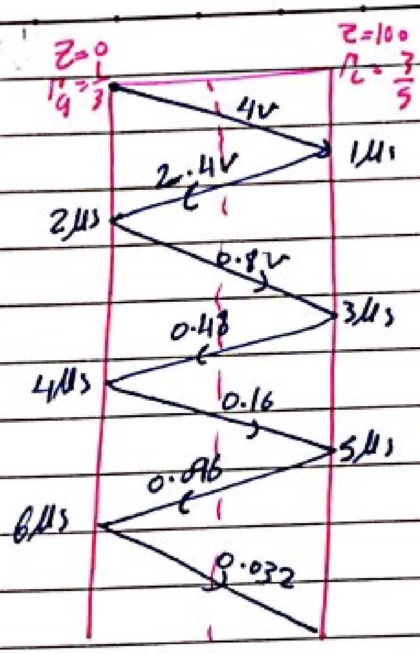
$$I = \frac{V_0}{Z_0} = \frac{4}{50} = 80mA$$

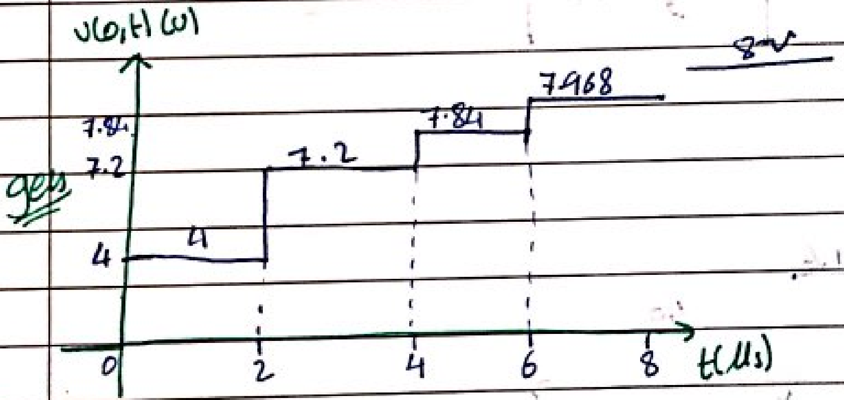
$$V_\infty = \frac{V_g Z_L}{Z_g + Z_L} = \frac{12(200)}{250} = 8V$$

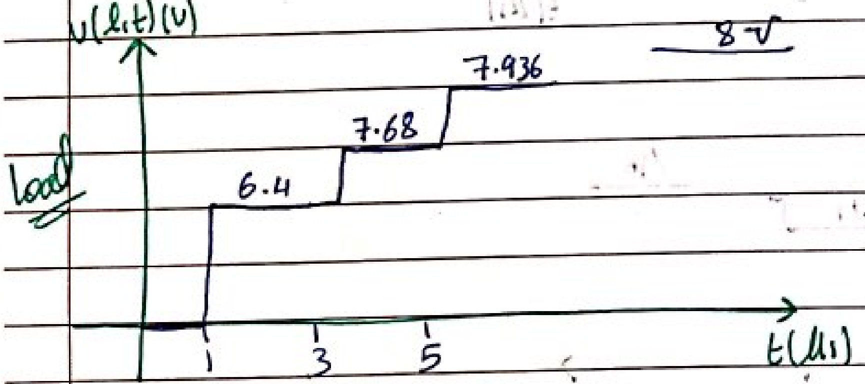
$$I_\infty = \frac{8}{200} = 40mA$$

$$\Gamma_{Gi} = \frac{-1}{3}$$

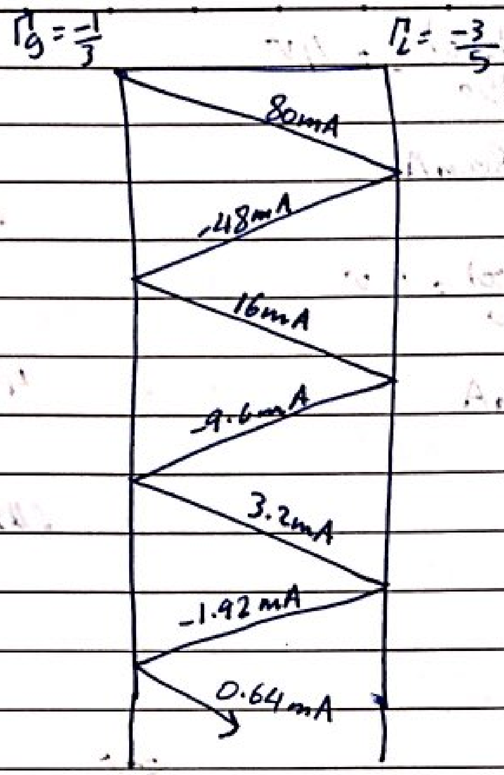
$$\Gamma_{Li} = \frac{-3}{5}$$

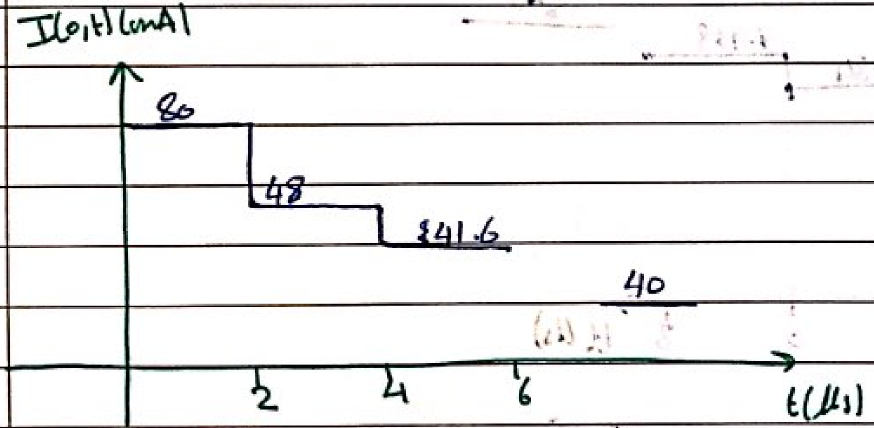


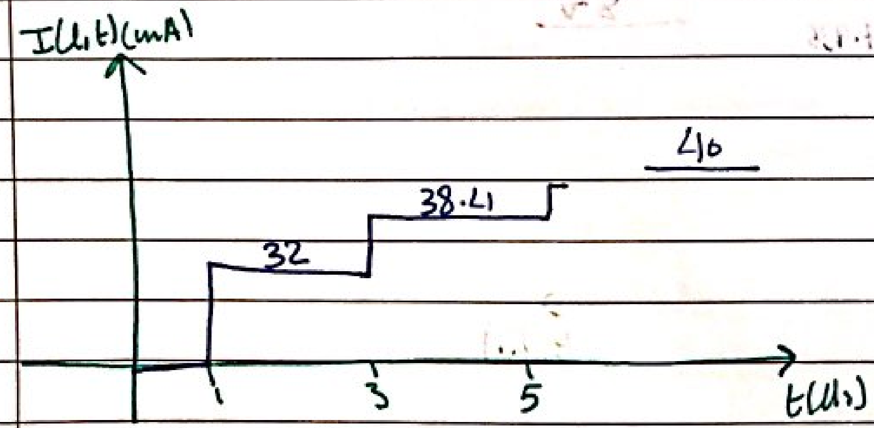




No.

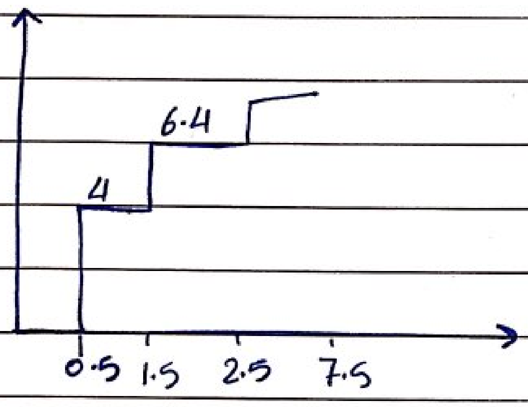






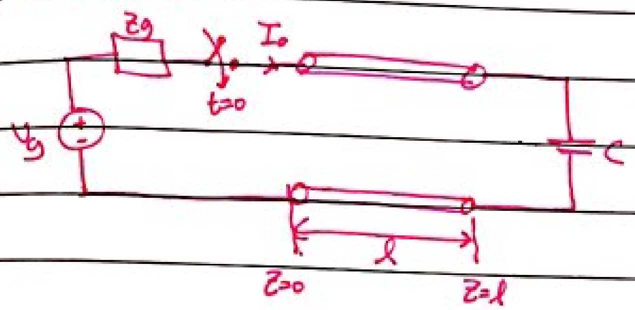
* if was asked to draw V(50,t) (V) go back to lattice diagram and draw a line in the middle

No.



5/4/2018

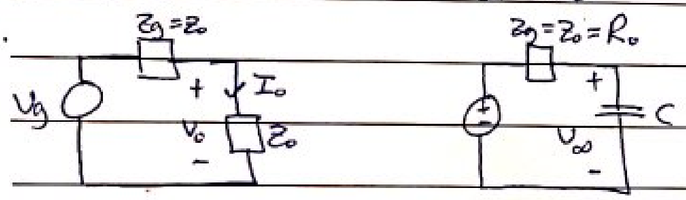
* Transients :-



Sketch $V(0,t)$, $I(0,t)$
$V(l,t)$, $I(l,t)$

$t_1 = \frac{l}{u}$

at $t=0^+$ at the



$V_0 = \frac{V_g}{2}$

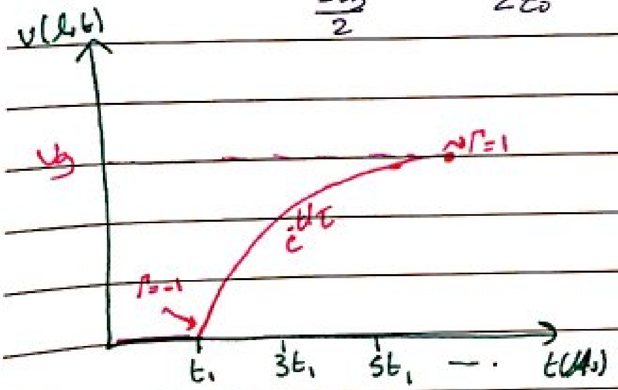
$I_0 = \frac{V_g}{2Z_0}$

C is empty (initial voltage is zero)

$\rightarrow$ S.C

$\Gamma = -1\ (V) \rightsquigarrow \frac{V_g}{2}, \frac{V_g}{2Z_0}$

$\Gamma = 1\ (i) \leftarrow \frac{-V_g}{2}, \frac{V_g}{2Z_0}$

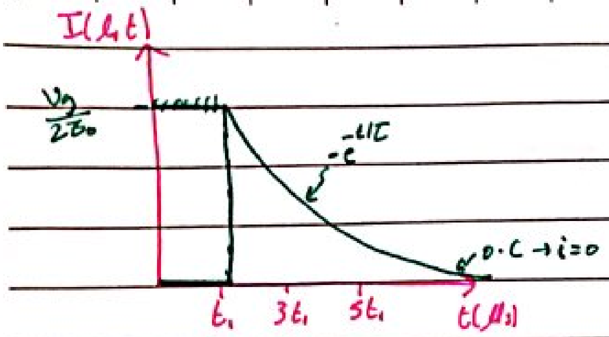


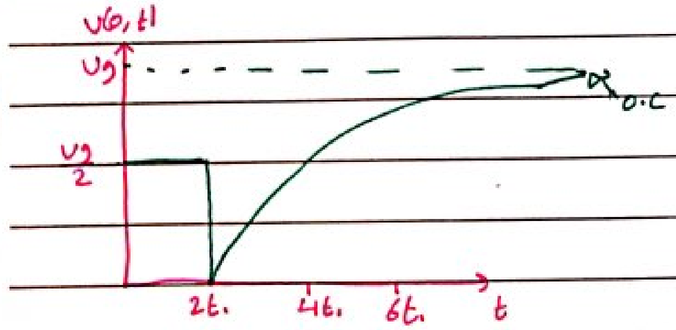
$V_c(t) = V_0 e^{-t/\tau}$

$\tau = RC$

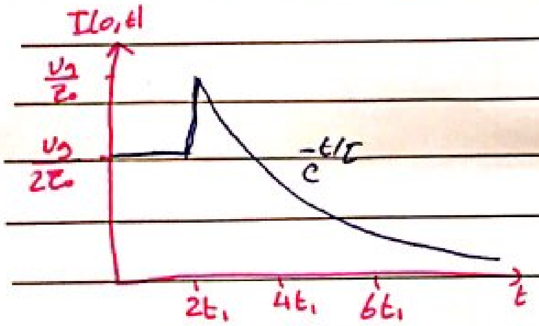
$= Z_g C = Z_0 C$

$Z_0 = R_0$

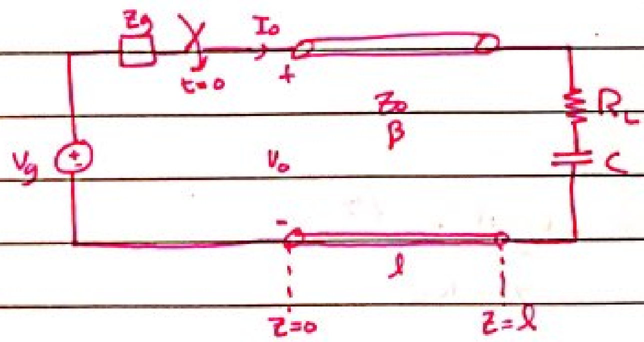




$$V_c(t) = V_\infty + (V_0 - V_\infty)\, e^{-t/\tau}$$



* For inductor ⇒ Voltage diagrams are the same as current diagrams for capacitor and vice versa

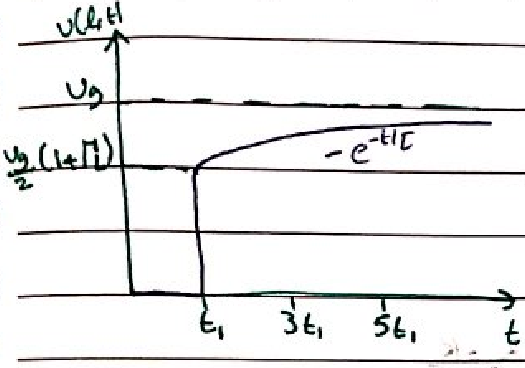


Sketch $V(0,t)$, $I(0,t)$

$V(l,t)$, $I(l,t)$

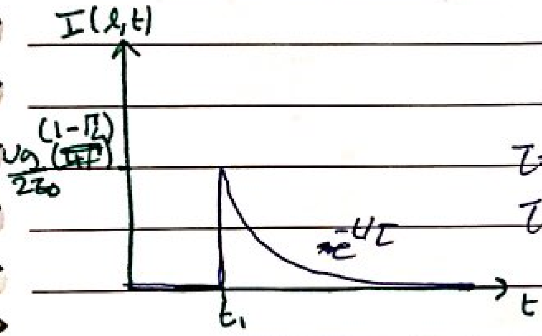
$t_1 = \frac{l}{u}$, $V_0 = \frac{V_g}{2}$

$I_0 = \frac{V_g}{2Z_0}$



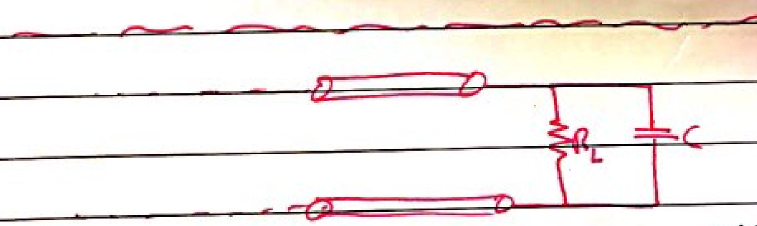
at $t = t_1^- \longrightarrow C$ is S.C

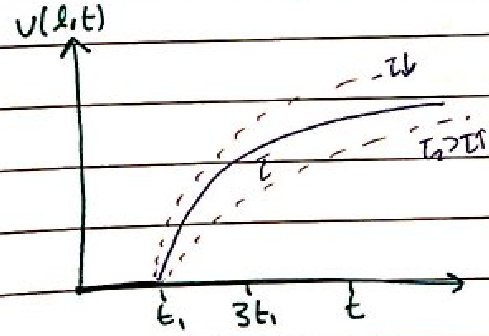
$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} \longrightarrow (-V)$$

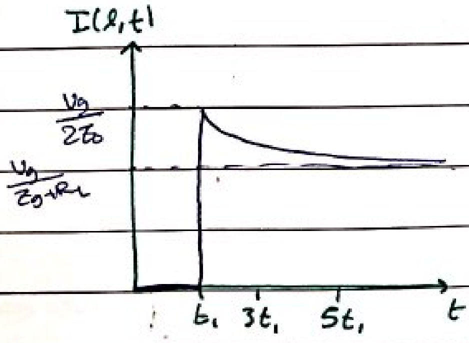


$\tau = R_{eq} C$

$\tau = (R_L + R_0) C$ ← Z_g

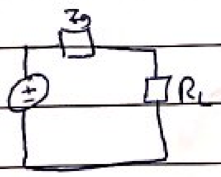






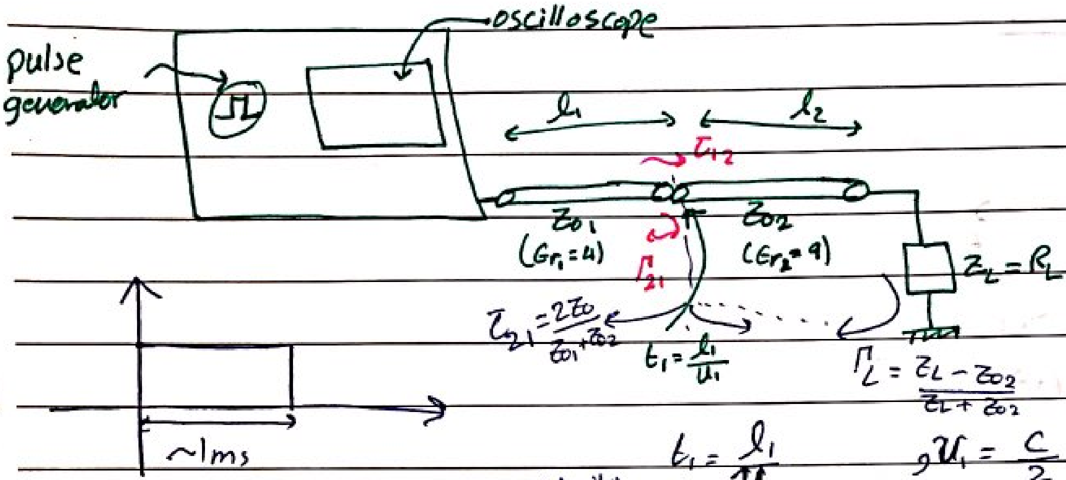
$\tau = R_{eq} C$

$$R_{eq} = \frac{R_L R_0}{R_L + R_0}$$



$$I = \frac{V_g}{Z_g + R_L}$$

TDR : Time Domain Reflectometer:

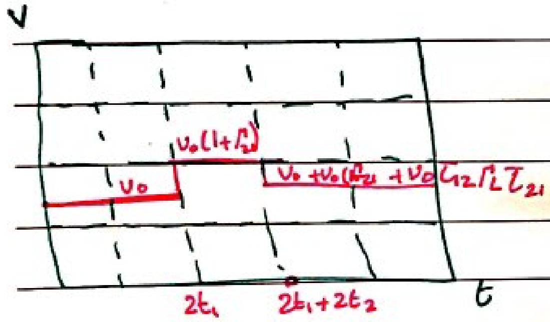


$t_1 = \dfrac{l_1}{u_1}$, $u_1 = \dfrac{c}{2}$, $u_2 = \dfrac{c}{3}$

Velocity ← u_1

$$\Gamma_{21} = \frac{Z_{02} - Z_{01}}{Z_{01} + Z_{02}}$$

$$\tau_{12} = \frac{2Z_{02}}{Z_{02}+Z_{01}}$$

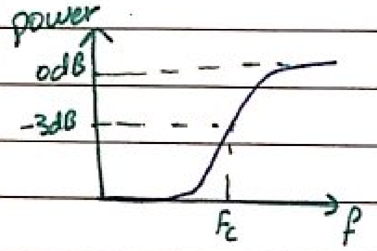


No. CH.12 :- Wave guides

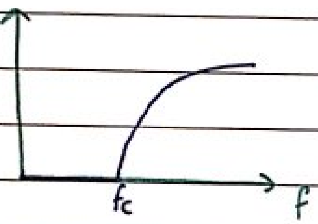
⇒ wave guides are used for (3-300 GHz)
↳ micro wave range

* it's a one conductor T.L, it's not a TEM mode, it's a HPF.

$\bar{E} \perp \bar{H}$ → $\perp \hat{a}_k$



HPF in ckt

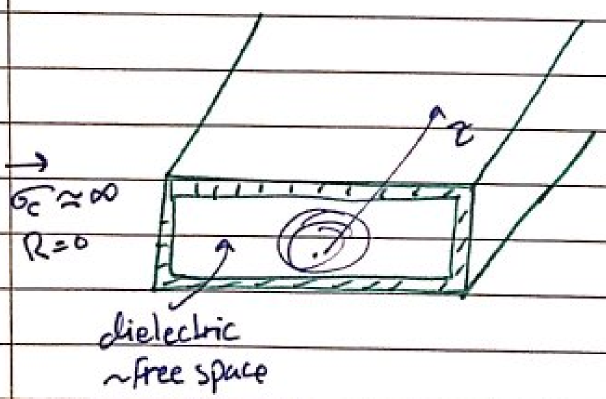


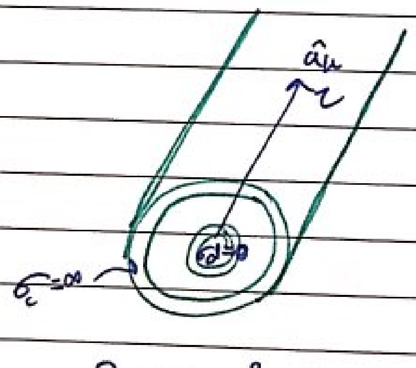
W.G

W.G.

R.W.G = Rectangular W.G

Circular W.G





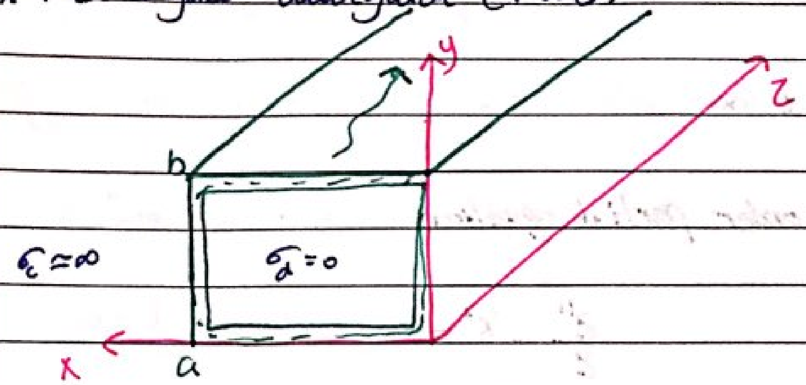
Bessel functions

→ $\sigma_d \approx 0$ → $G = 0$

$R_d = \infty$

No.

* Rectangular waveguide (RWG)



Assume the wave propagates along $+z$ from the wave equation:

$\nabla^2 \bar{E} + k^2 \bar{E} = 0$, in ch.10 $\nabla^2 E^2 - \gamma^2 \bar{E} = 0$

wave is function of 3-dimensions (x, y, z)

$\nabla^2 \bar{E}_{zs} + k^2 \bar{E} = 0$

$$\frac{\partial^2 E_{zs}}{\partial x^2} + \frac{\partial^2 E_{sz}}{\partial y^2} + \frac{\partial^2 E_{zs}}{\partial z^2} + k^2 \bar{E}_{zs} = 0$$

⇒ Seperation of variables

⇒ let $\bar{E}_{zs} = X(x)\, Y(y)\, Z(z)$ ← function of z, or $E_{zs} = XYZ$

$$X''YZ + XY''Z + XYZ'' + k^2 XYZ = 0$$

⇒ divide by XYZ

$$\frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} + k^2 = 0$$

let $k^2 = k_x^2 + k_y^2 + -\gamma^2$, $k_z^2 = -\gamma^2$

No.

to satisfy the B.C

$E \to 0$ as $z \to \pm\infty$

we have 3 second order partial equations

$\frac{X''}{X} + k_x^2 = 0$ $\qquad$ $\frac{Z''}{Z} - \gamma^2 = 0$

$\frac{Y''}{Y} + k_y^2 = 0$

$\Rightarrow X'' + k_x^2 X = 0$, $k_x > 0$

let $X' = m = \frac{d}{dx}$

$X'' = m^2$

$(m^2 + k_x^2) X = 0$

$m = \pm j k_x$ roots are imagenary

Solution $\to$ $\boxed{C_1 \cos k_x X + C_2 \sin k_x X}$ $X(x)$

or $C_1 e^{j k_x X} + C_2 e^{-j k_x X}$

$\Rightarrow Y'' + k_y^2 Y = 0$

$\hookrightarrow Y(y) = C_3 \cos k_y y + C_4 \sin k_y y$

$\Rightarrow Z'' - \gamma^2 Z = 0$

$m^2 - \gamma^2 = 0 \to m = \pm \gamma \to$ roots are real

Sol. either $Z(z) = C_5 \cosh \gamma z + C_6 \sinh \gamma z$

or $Z(z) = C_5 \underline{\underline{e^{-\gamma z}}} + C_6 e^{\gamma z}$ $\leftarrow$ wave in -ve dir

$C_6 = 0$ $\qquad$ $E \to \infty$ as $z \to \infty$

$E \to 0$ as $z \to \infty$

No.

$E_{zs} = (C_1 \cos k_x x + C_2 \sin k_x x)(C_3 \cos k_y y + C_4 \sin k_y y)\, C_5\, e^{-\gamma z}$

$C_1 C_3$

$E_{zs} = (A_1 \cos k_x x + A_2 \sin k_x x)\,(A_3 \cos k_y y + A_4 \sin k_y y)\, e^{-\gamma z}$

by following the same steps for H_{zs}

$\nabla^2 \bar{H}_{zs} + k^2 \bar{H}_{zs} = 0$

$\bar{H}_{zs} = (B_1 \cos k_x x + B_2 \sin k_x x)(B_3 \cos k_y y + B_4 \sin k_y y)\, e^{-\gamma z}$

for E_{xs}, E_{ys}
H_{xs}, H_{ys} ?

use maxwell's equation
for source free region ($\rho_v = 0$, $\bar{J} = 0$)

$\times\ \nabla \cdot \bar{D}_s = 0$

$\times\ \nabla \cdot \bar{B}_s = 0$

$\nabla \times \bar{E}_s = -j\omega\mu \bar{H}_s$

$\nabla \times \bar{H}_s = j\omega\epsilon \bar{E}_s$

$$\bar{H}_s = \frac{-1}{j\omega\mu} \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_{xs} & E_{ys} & E_{zs} \end{vmatrix} = \begin{bmatrix} H_{xs} \\ H_{ys} \\ H_{zs} \end{bmatrix}$$

$\rightarrow$ known

$$H_{xs} = \frac{-1}{j\omega\mu}\left(\frac{\partial E_{zs}}{\partial y} - \frac{\partial E_{ys}}{\partial z}\right) \quad \text{---1.a}$$

$$H_{ys} = \frac{-1}{j\omega\mu}\left(\frac{\partial E_{xs}}{\partial z} - \frac{\partial E_{zs}}{\partial x}\right) \quad \text{---1.b}$$

$$H_{zs} = \frac{-1}{j\omega\mu}\left(\frac{\partial E_{ys}}{\partial x} - \frac{\partial E_{xs}}{\partial y}\right) \quad \text{---1.c}$$

No.

$$\bar{E}_s = \frac{1}{j\omega\epsilon} \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_{xs} & H_{ys} & H_{zs} \end{vmatrix}$$

$$E_{xs} = \frac{1}{j\omega\epsilon}\left(\frac{\partial H_{zs}}{\partial y} - \frac{\partial H_{ys}}{\partial z}\right) \ldots 1.d \quad *$$

$$E_{ys} = \frac{1}{j\omega\epsilon}\left(\frac{\partial H_{xs}}{\partial z} - \frac{\partial H_{zs}}{\partial x}\right) \ldots 1.e$$

$$E_{zs} = \frac{1}{j\omega\epsilon}\left(\frac{\partial H_{ys}}{\partial x} - \frac{\partial H_{xs}}{\partial y}\right) \ldots 1.f$$

To find E_{xs} ?!

$(\quad)(B_3 \cos k_y y + B_4 \sin k_y y)e$

$E_{zs} \sim e^{-\gamma z}$

$E_{xs} \sim e^{-\gamma z}$

$\frac{\partial E_{zs}}{\partial z} \sim -\gamma e^{-\gamma z}$

$\frac{\partial E_z^2}{\partial z^2} \sim \gamma^2 e^{-\gamma z}$

going back to *

$$j\omega\epsilon E_{xs} = \frac{\partial H_{zs}}{\partial y} + \frac{1}{j\omega\mu}\left(\underbrace{\frac{\partial^2 E_{xs}}{\partial z^2}}_{\gamma^2 E_{xs}} - \underbrace{\frac{\partial^2 E_{zs}}{\partial x \partial z}}_{-\gamma \frac{\partial E_{zs}}{\partial x}}\right)$$

No.

$$\boxed{E_{xs} = \frac{-\gamma}{h^2}\frac{\partial E_{zs}}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial y}}$$

where $h^2 = \gamma^2 + \omega^2\mu\epsilon \rightsquigarrow h^2 = \gamma^2 + k^2$

$k^2 = \omega^2\mu\epsilon$

10/4/2018

$E_{zs} = (A_1 \cos k_x x + A_2 \sin k_x x)(A_3 \cos k_y y + A_4 \sin k_y y) e^{-\gamma z}$

$H_{zs} = (B_1 \quad + B_2 \quad)(B_3 \quad + B_4 \quad) e^{-\gamma z}$

Maxwell's eq.

$\nabla \times \bar{E}_s = -j\omega\mu\bar{H}_s$

$\nabla \times \bar{H}_s = j\omega\epsilon\bar{E}_s \,, (\bar{J} = 0)$ | 6-equations

$$E_{xs} = \frac{-\gamma}{h^2}\frac{\partial E_{zs}}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial y}$$

$h^2 = \gamma^2 + k^2 \quad , k = \sqrt{k_x^2 + k_y^2} = \omega\sqrt{\mu\epsilon}$

$$E_{ys} = \frac{-\gamma}{h^2}\frac{\partial E_{zs}}{\partial y} - \frac{j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial x}$$

$$H_{xs} = \frac{j\omega\epsilon}{h^2}\frac{\partial E_{zs}}{\partial y} - \frac{\gamma}{h^2}\frac{\partial H_{zs}}{\partial x}$$

$$H_{ys} = \frac{-j\omega\epsilon}{h^2}\frac{\partial E_{zs}}{\partial x} - \frac{\gamma}{h^2}\frac{\partial H_{zs}}{\partial y}$$

$\Rightarrow$ 6 Components :-

① $\hat{a}_k = +\hat{a}_z$

in TEM $\bar{E} \perp \bar{H} \Rightarrow \perp \hat{a}_k$

No.

For TEM $\rightarrow E_{zs} = 0 = H_{zs}$ $(h=0)$
All the Components = zero!!

W.G not supports the TEM mode.

2) if $E_{zs}=0$, $H_{zs} \neq 0$
TE Mode $(h \neq 0)$

$H_{zs}, E_{xs}, E_{ys}, H_{xs}, H_{ys} \Rightarrow$ 5 components

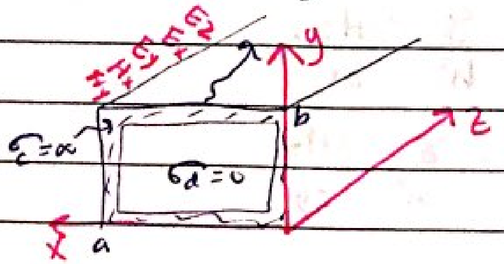
3) if $E_{zs} \neq 0$, $H_{zs}=0$
$(h \neq 0)$
TE mode
$E_{xs}, E_{ys}, E_{zs}, H_{xs}, H_{ys}$.

4) $E_{zs} \neq 0$, $H_{zs} \neq 0$
HE mode or (Hybrid mode)

$\rightarrow$ optical fiber.

$h^2 = \gamma^2 + k^2$
if $h^2 = 0$
$\therefore \gamma^2 = -k^2$
$\gamma = \pm jk$
$\gamma = \pm j\beta$
it is a TEM mode

* TM mode (Tranverse magnetic)
$(H_{zs}=0)$



B.C.S :-
$E_{zs}=0$, at $x=0$
$E_{zs}=0$, at $x=a$
$E_{zs}=0$, at $y=0$
$E_{zs}=0$, at $y=b$
} tangent to the W.G

$E_{zs} = (\underset{?}{A_1} \cos \underset{?}{k_x} x + \underset{?}{A_2} \sin \underset{?}{k_x} x)(\underset{?}{A_3} \cos k_y y + \underset{?}{A_4} \sin k_y y) e^{-\gamma z}$

$A_1 = 0 \quad , \quad A_3 = 0$

$E_{zs} = E_o \sin k_x x \sin k_y y \, e^{-\gamma z} \qquad , \quad E_o = A_2 A_4$

at $x = a \rightarrow E_{zs} = 0$

$E_o \neq 0 \Rightarrow$ to have a non-trivial solution.

$0 = E_o \sin k_x a \, \sin k_y y$

$\sin k_x a = 0 \qquad , a \neq 0$

$k_x a = m\pi$

$\boxed{k_x = \frac{m\pi}{a}}$ $, m = 1, 2, 3, \ldots$

$m \neq 0 \Rightarrow k_x = 0$ if $m = 0$ then no propagation in $\hat{a}_x$

$E_{zs} = 0 , \quad a_y y = b$

$\sin k_y b = 0$

$k_y = \frac{n\pi}{b} \quad , n = 1, 2, 3, \ldots$

$n \neq 0$

$\boxed{E_{zs} = E_o \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}}$ $a > b$

$n = 1, 2, 3, \ldots$

$m = 1, 2, 3, \ldots$

$\left.\begin{matrix} n \neq 0 \\ m \neq 0 \end{matrix}\right\}$ if $\left.\begin{matrix} n = 0 \\ m = 0 \end{matrix}\right\}$ TEM mode

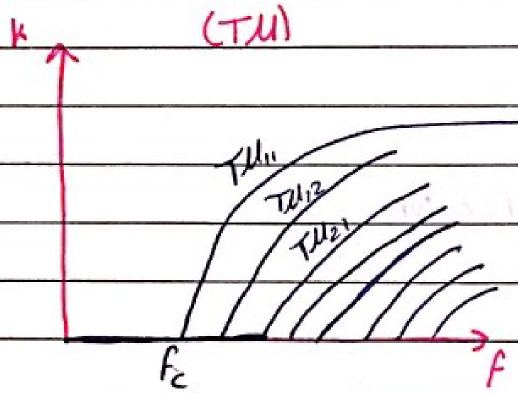
No.

n, m modes:

n=1 , m=1 $\Rightarrow TM_{11}$ (n, m)

$\boxed{TM_{nm} \text{ modes}}$

TM_{11} is the lowest mode in TM modes



$$E_{xs} = \frac{-\gamma}{h^2}\frac{\partial E_{zs}}{\partial x} - 0 \quad H_{zs}=0$$

$$= \frac{-\gamma E_0 m\pi}{h^2\, a}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-\gamma z}$$

$$E_{xs} = \frac{-\gamma}{h^2}\left(\frac{m\pi}{a}\right)E_0 \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-\gamma z}$$

$$E_{ys} = \frac{-\gamma}{h^2}\left(\frac{n\pi}{a}\right)E_0 \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-\gamma z}$$

$$E_{zs} = - - - -$$

$$H_{xs} = \frac{j\omega\epsilon}{h^2}\left(\frac{n\pi}{b}\right)E_0 \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{-\gamma z}$$

$$H_{ys} = \frac{j\omega\epsilon}{h^2}\left(\frac{n\pi}{a}\right)E_0 \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-\gamma z}$$

$$h^2 = \gamma^2 + k^2$$
$$= \gamma^2 + (k_x^2 + k_y^2 - \gamma^2)$$
$$h^2 = k_x^2 + k_y^2$$
$$h^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

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$$h = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$\gamma = \alpha + j\beta$$

$$\gamma^2 = \sqrt{h^2 - k^2}$$

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{a}\right)^2 + k^2}$$

Case 1 :- $h^2 = k^2$ $\qquad$ $h = \omega\sqrt{\mu\epsilon}$

$\gamma = \text{zero} \rightarrow$ cut off mode (no propagation or attenuation)

(ما في موجة)

$\rightarrow$ cut off freq f_c

$$\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = \omega^2\mu\epsilon$$

$$\omega_c = \frac{1}{\sqrt{\mu\epsilon}}\sqrt{\left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2}$$

$$f_c = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2} = \frac{u'}{2}\sqrt{\left(\frac{n}{b}\right)^2 + \left(\frac{m}{a}\right)^2}$$

$$u' = \frac{1}{\sqrt{\mu\epsilon}} = \text{TEM velocity}$$

$\rightarrow$ in chapter 10 $\rightarrow$ $a \rightarrow \infty$ $\quad$ $b \rightarrow \infty$ $\quad \rightarrow f_c = \text{zero}$

Case 2 :-

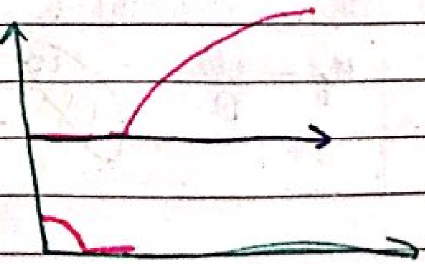
$f < f_c$

$k^2 < h^2$

$\omega^2\mu\epsilon < \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$ $\qquad$ (evanescent mode)

γ is real $\rightarrow$ α, $\beta = \text{zero}$ $\qquad$ vanishing mode

$$\alpha = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \mu\epsilon\omega^2}$$



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Case (3) $f > f_c \rightarrow$ propagation mode

$$\omega^2 \mu \epsilon > \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \Rightarrow k^2 > h^2$$

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \epsilon} = j\beta$$

is imagenary $\rightarrow \alpha = 0$ $\quad \sigma_d = 0$

$\sigma_c = \infty$

$$\gamma = j\sqrt{\omega^2 \mu \epsilon - \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]} = \cancel{j}\beta$$

$$\beta = \sqrt{\omega^2 \mu \epsilon - \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]}$$

$$\beta' = \omega\sqrt{\mu\epsilon}\Big|_{h=0} \rightarrow \text{TEM } \beta$$

we can write β :-

$$\boxed{\beta = \beta' \sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

$, f_c = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$ $\Rightarrow$ where the W.G starts to operate

$f \equiv$ operating freq.

cut-off wave length :-

$$\lambda_c = \frac{u'}{f_c} = \frac{u' \, 2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

phase velocity :- (u_p)

$$u_p = \frac{\omega}{\beta} = \frac{\omega}{\beta' \sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \rightarrow u_p' = \frac{1}{\sqrt{\mu\epsilon}}$$

No.

Intrinsic Impedance η :-

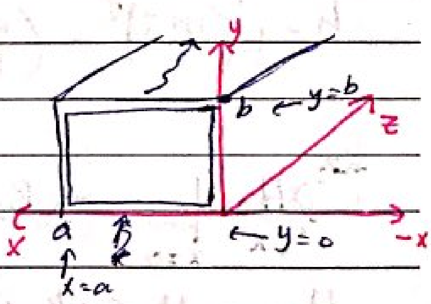
$$\eta_{TM} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \frac{\beta}{\omega\epsilon} = \frac{\beta'\sqrt{1-\left(\frac{f_c}{f}\right)^2}}{\omega\epsilon} \quad , \beta' = \omega\sqrt{\mu\epsilon}$$

$$\eta_{TM} = \frac{\sqrt{\mu}\sqrt{1-\left(\frac{f_c}{f}\right)^2}}{\sqrt{\epsilon}} \quad , \eta' = \sqrt{\frac{\mu}{\epsilon}}$$

$$\eta_{TM} = \eta'\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

* Tranverse Electric (TE) modes :-

$E_{zs} = 0 \quad , \bar{E} \perp \hat{a}_z$



$H_{zs} \neq 0$

Fields : $E_{xs}, E_{ys}, H_{xs}, H_{ys}, H_{zs}$

B.C.s :- $E_{ys} = 0 \quad , \quad x=0 \;\&\; x=a$

$E_{xs} = 0$ at $y=0 \;\&\; y=b$

$$E_{xs} = \frac{-\gamma}{h^2}\cancel{\frac{\partial E_{zs}}{\partial x}}^{\,0} - \frac{j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial y}\Bigg|_{\substack{y=0\\y=b}} = 0$$

$$\frac{\partial H_{zs}}{\partial y}\Bigg|_{y=0} = 0 \quad \text{--- (1)}$$

$$\frac{\partial H_{zs}}{\partial y}\Bigg|_{y=b} = 0 \quad \text{--- (2)}$$

$$\frac{\partial H_{zs}}{\partial x}\Bigg|_{\substack{x=0\\x=a}} \quad (3)\ (4)$$

$$H_{zs} = (B_1\cos k_x x + B_2 \sin k_x x)(B_3\cos k_y y + B_4 \sin k_y y)\underbrace{B_5 e^{-\gamma z}}_{\substack{\text{as } z\to\infty\\ H\to 0}}$$

$$\frac{\partial H_{zs}}{\partial y}\Bigg|_{y=0} = (\underset{\neq 0}{\quad})(-B_3 k_y \cancel{\sin k_y y} + B_4 k_y \cos k_y y)\underbrace{B_5 e^{-\gamma z}}_{\neq 0} = 0$$

$$\boxed{B_4 = 0}$$

No.

Apply $\left.\frac{\partial H_{zs}}{\partial x}\right|_{x=a}$, $\left.\frac{\partial H_{zs}}{\partial y}\right|_{y=b}$

to Find k_x and k_y

$k_x = \frac{m\pi}{a}$, $m = 0, 1, 2, \ldots$

$k_y = \frac{n\pi}{b}$, $n = 0, 1, 2, \ldots$

$(m,n) = (0,0)$ is not accepted since it will lead to ~~vanishing field~~ a TEM mode.

$h = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} = 0$ $\Rightarrow$ the lowest mode either $(m,n) = (0,1)$ or $(1,0)$

** $\left.\frac{\partial H_{zs}}{\partial x}\right|_{x=0}$ $\Rightarrow B_2 = 0$

$H_{zs} = H_o \cos k_x x \cos k_y y \, e^{-\gamma z}$

usually $a > b$

$\frac{1}{a} < \frac{1}{b}$

$m < n$

TE_{10} is the lowest TE mode

$f_{c_{10}} = \frac{u'}{2} \sqrt{\left(\frac{1}{a}\right)^2 + (0)}$ ← $n=0$

$f_{c_{10}} = \frac{u'}{2a} < f_{c_{01}} = \frac{u'}{2b}$

for $TM_{11} \rightarrow f_{c_{11}} = \frac{u'}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}$

So, TE_{10} is the dominant mode

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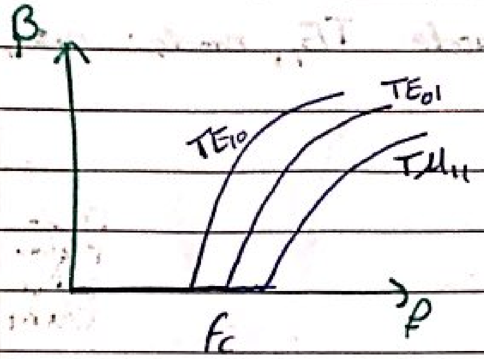


for TM and TE Modes

$(f_c, \beta, \lambda, U_p) \Rightarrow$ The same

α in evanescent mode $\Rightarrow$ The same

$$\eta_{TM} = \frac{E_x}{H_y} = \frac{-E_y}{H_x} = \frac{\beta}{\omega \epsilon} = \eta' \sqrt{1-\left(\frac{f_c}{f}\right)^2} \quad , \eta' = \sqrt{\frac{\mu}{\epsilon}}$$

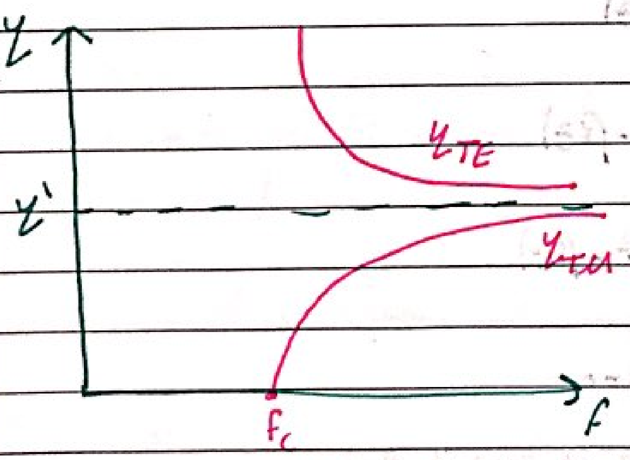
$$\eta_{TE} = \frac{E_x}{H_y} = \frac{-E_y}{H_x} = \frac{\omega \mu}{\beta}$$

$H_z = 0$

$$\eta_{TE} = \frac{\eta'}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$E_z = 0$

$$\eta_{TM} \cdot \eta_{TE} = \eta'^2$$



domainant mode (if a>b) is TE10

No.

* The field pattern for the dominant mode TE_{10} mode: $m=1, n=0$

$E_{zs} = 0$

~~TE_{00}~~ ~~TM_{00}~~ ~~TM_{10}~~ TM_{01}

$$H_{zs} = H_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$E_{xs} = \frac{j\omega\mu}{h^2}\left(\frac{n\pi}{b}\right) H_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z} = 0$$

$$E_{ys_{10}} = \frac{-j\omega\mu}{h^2}\left(\frac{m\pi}{a}\right) H_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{\cancel{n}\pi y}{b}\right) e^{-\gamma z}$$

$$H_{xs} = \frac{\gamma}{h^2}\left(\frac{m\pi}{a}\right) H_0 \sin(\quad) \cos(\quad) e^{-\gamma z}$$

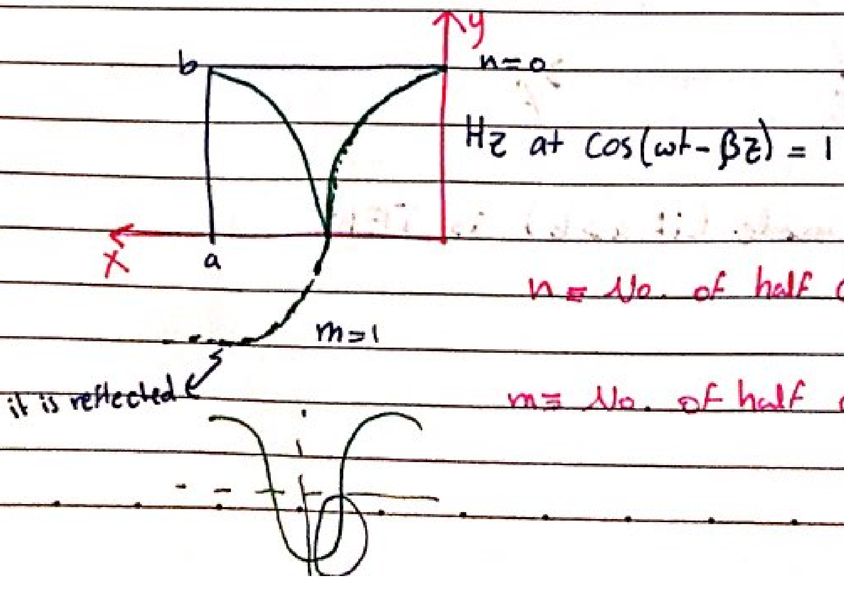
$$H_{ys} = \frac{\gamma}{h^2}\left(\frac{m\pi}{a}\right) H_0 \cos(\quad) \sin(0) = 0$$

in Time domain :- $H_z = Re\{H_s e^{j\omega t}\}$

$$H_z = H_0 \cos\left(\frac{\pi x}{a}\right) \cos(\omega t - \beta z)$$

$$E_y = \frac{\omega\mu a}{\pi} H_0 \sin\left(\frac{\pi x}{a}\right) \sin(\omega t - \beta z)$$

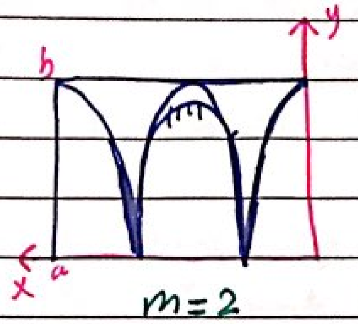
$$H_x = \frac{-\beta a}{\pi} H_0 \sin\left(\frac{\pi x}{a}\right) \sin(\omega t - \beta z)$$



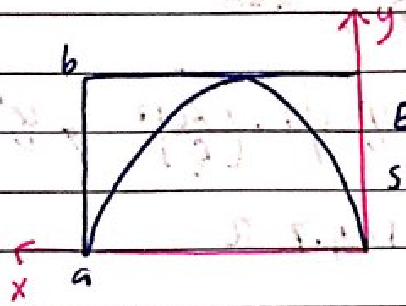
n = No. of half cycle variation along y-axis

m = No. of half cycle variation along x-axis

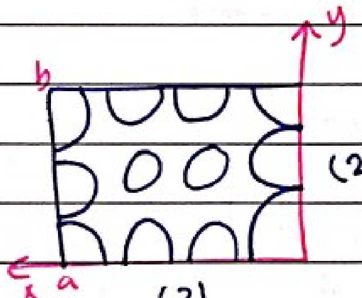
No.



$m=2$

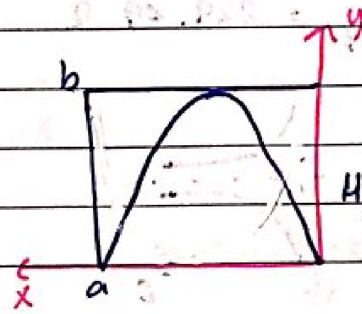


E_y
$\sin(\omega t-\beta z)=1$



(2)

(3)



$H_x \ \sin(\omega t-\beta z)=1$

Ex:- in a R.W.G for which $a=1.5\,cm$, $b=0.8\,cm$, $\sigma_d=0$, $\mu=\mu_0$, $\epsilon=4\epsilon_0$

$$H_x = 2\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{3\pi y}{b}\right)\sin(\pi\times 10^{11}t-\beta z)\ A/m$$

Determine:-

a) The mode of propagation

b) f_c, β, δ, γ

f) other field components for this mode

a) TE_{13} or TM_{13}

b) $f_c = \frac{u'}{2}\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{3}{b}\right)^2} = 28.57\ GHz$

$\beta = \beta'\sqrt{1-\left(\frac{f_c}{f}\right)^2}$, $\beta' = \omega\sqrt{\mu\epsilon} = \frac{2\omega}{c}$

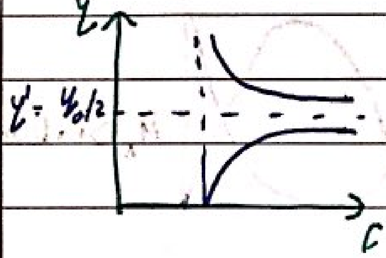
$\beta = 1718.81\ rad/m$

$\gamma = j\beta$, $\alpha = 0$

$$\eta_{TM} = \eta' \sqrt{1 - \left(\frac{f_c}{f}\right)^2} \rightarrow \eta' = \frac{\eta_0}{2}$$

$$= 154.7\ \Omega$$

$$\eta_{TE13} = 229.69\ \Omega$$



f) $H_z = 0$ for TM_{13}

$$H_y = \frac{-j\omega\epsilon}{h^2}\left(\frac{n\pi}{a}\right) E_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$H_x = \frac{-\omega\epsilon}{h^2}\left(\frac{n\pi}{b}\right) E_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

E_x

E_y

E_z

$$H_y = -\frac{2b}{3a} \cos\left(\frac{\pi x}{a}\right) \sin\left(\frac{3\pi y}{b}\right) e^{-\gamma z}$$

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* wave propagation in the R.W.G

All the fields contain sin, cos, $e^{-\gamma z}$

if lossless $\Rightarrow \alpha = 0$, $\gamma = j\beta$

So you can apply Euler's Identity

i.e for TE_{10} mode (H_z, E_y, H_x)

$\hookrightarrow m=1, n=0$

$$E_{y_s} = \frac{-j\omega\mu a}{\pi} \sin\left(\frac{\pi x}{a}\right) e^{-j\beta z}, \quad h^2 = \frac{\overset{m=1}{\pi^2}}{a^2} + \overset{n=0}{0}$$

$$= -\frac{j\omega\mu a}{\pi}\left(\frac{e^{j\frac{\pi x}{a}} - e^{-j\frac{\pi x}{a}}}{2j}\right) e^{-j\beta z} \qquad \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

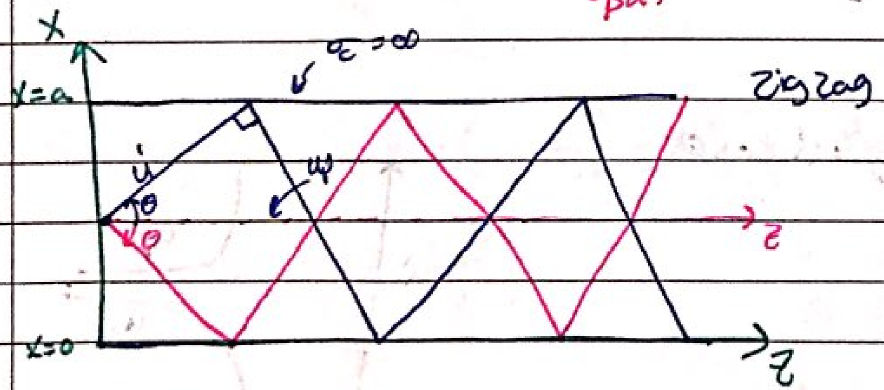
$$E_{y_s} = \frac{-\omega\mu a}{2\pi}\left(e^{-j\beta z} e^{j\pi\frac{x}{a}} - e^{-j\beta z} e^{-j\pi\frac{x}{a}}\right)$$

$$E_{y_s} = \frac{\omega\mu a}{2\pi}\left(e^{-j\beta\left(z + \frac{\pi x}{\beta a}\right)} - e^{-j\beta\left(z - \frac{\pi x}{\beta a}\right)}\right)$$

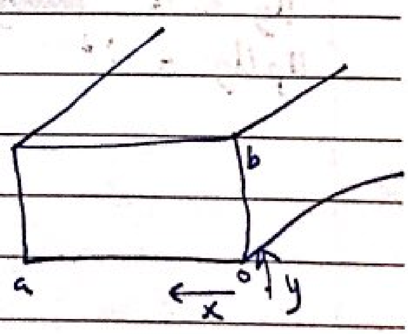
summation of 2-TEM waves

wave along +z, +x with a phase shift $\theta = \tan^{-1}\left(\frac{\pi}{\beta a}\right)$

with wave along +z, −x with a phase of $\theta = \tan^{-1}\left(\frac{-\pi}{\beta a}\right) = -\theta$



$$u' = \frac{1}{\sqrt{\mu\epsilon}}$$



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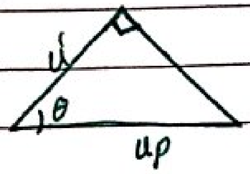


$\cos\theta = \frac{u'}{u_p} \Rightarrow u_p = \frac{u'}{\cos\theta}$

$u_p = \frac{u'}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \Rightarrow u_p = \frac{\omega}{\beta} = \frac{\overset{u'}{\omega}}{\beta'\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$

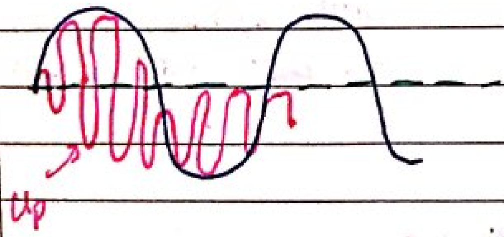
$\cos\theta = \sqrt{1-\left(\frac{f_c}{f}\right)^2}$, for propagation $f > f_c$

$u_p > u'$ always

*if the medium inside the wave guide is air $\Rightarrow u' = c$

$u_p > c$

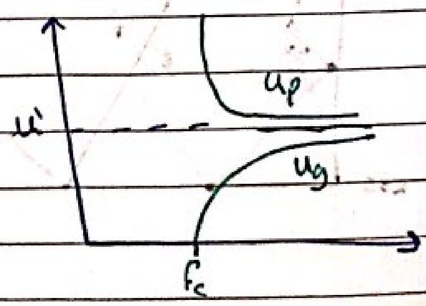
$\Rightarrow$ define $u_g \equiv$ group velocity (m/s) (speed of the envelope)



$\cos\theta = \frac{u_g}{u'} \Rightarrow u_g = u'\cos\theta$

$u_g < u'$ always

$u_p u_g = u'^2$



No.

u'

u_p

u_g = The speed of the wave inside the (w.G).

λ_g = wave guide wave length , $\lambda_{TE_{11}}$, $\lambda_c = \frac{u'}{f_c}$

$$\lambda_g = \frac{\lambda'}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\lambda_c = \frac{2}{\sqrt{\left(\frac{\pi}{a}\right)^2+\left(\frac{\pi}{b}\right)^2}}$$

$$\lambda_{g_{TM_{11}}} = \frac{\lambda'}{\sqrt{1-\left(\frac{f_{c_{TM_{11}}}}{f}\right)^2}}$$

* Power Transmission and Attenuation :-

⇒ **Power transmission :-** , $\bar{P} = \bar{E} \times \bar{H} \rightarrow w/m^2$

$$\bar{P} = \frac{1}{2} Re\{\bar{E}_s \times \bar{H}_s^*\} \quad w/m^2$$

E_x & E_y $\quad$ H_y & $-H_x$

$\hat{a}_z$ $\quad$ $\hat{a}_z$

$$P_{ave} = \frac{1}{2} Re\{E_{x_s} \times H_{y_s}^* - E_{y_s} \times H_{x_s}^*\}\hat{a}_z , \quad H = \frac{E}{\eta}$$

$$= \frac{1}{2} Re\left(\frac{E_{xs}^2}{\eta} - \frac{E_{ys}^2}{\eta}\right)$$

$$\eta_{TM} = \frac{E_x}{H_y} = \frac{-E_y}{H_x}$$

$\eta_{TE} =$

No.

$\bar{E}_s = (E_{xs}, E_{ys}, E_{zs})$ → one of them is Zero

$\bar{H}_s^* = (H_{xs}^*, H_{ys}^*, H_{zs}^*)$

$$\boxed{\bar{P}_{ave} = \frac{E_{xs}^2 + E_{ys}^2}{2\eta}\hat{a}_z \ \ w/m^2} = \bar{S}_{ave}$$

scalar

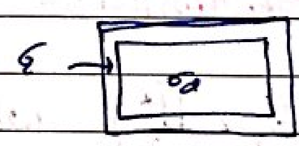
$$P_{ave} = \int_s \bar{P}_{ave} \cdot \bar{ds} \quad (w)$$

$$= \int_0^b \int_0^a \frac{E_{xs}^2 + E_{ys}^2}{2\eta} \hat{a}_z \cdot dx\, dy\, \hat{a}_z$$

⇒ Attenuation :-

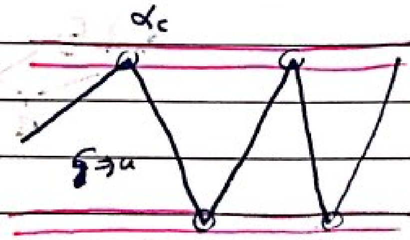
in general $\sigma_c \neq \infty$, $\sigma_d \neq 0$

$\Downarrow$ R $\quad$ $\Downarrow$ G



$R \neq 0$, $G \neq 0$ → not lossless

→ There is power loss



$\alpha = \alpha_c + \alpha_d$

$P_{ave} = P_0 e^{-2\alpha z} \hat{a}_z$

$$P_L = -\frac{\partial P_{ave}}{\partial z} = +2\alpha P_0 e^{-2\alpha z}$$

$$= 2\alpha P_{ave}$$

$$\alpha = \frac{P_L}{2P_{ave}}$$

$$= \alpha_c + \alpha_d$$

No.

derivation

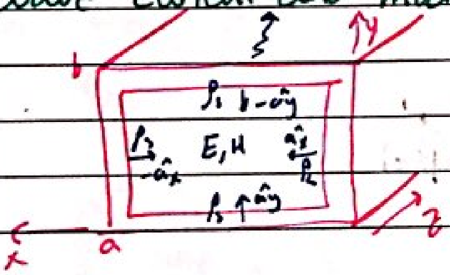
α_d, α_c $\alpha = 5\ Np/m$

$= \alpha_c + \alpha_d$

* α is always given in the exam and if it's not mentioned consider it lossless

22/4/2018

* wave current and mode Excitation :-



$\bar{k} = \bar{H} \times \hat{a_n}$ → outward unit normal vector

$\bar{k} = \hat{a_n} \times \bar{H}$ → inward normal unit vector

$\rho_s = \bar{D} \cdot \hat{a_n}$

$= \hat{a_n} \cdot \bar{D} \Rightarrow \boxed{\rho_s = \hat{a_n} \cdot \epsilon \bar{E}}$

for TE_{10}

H_z

E_y → $\hat{a_n} = \hat{a_y}$

H_x

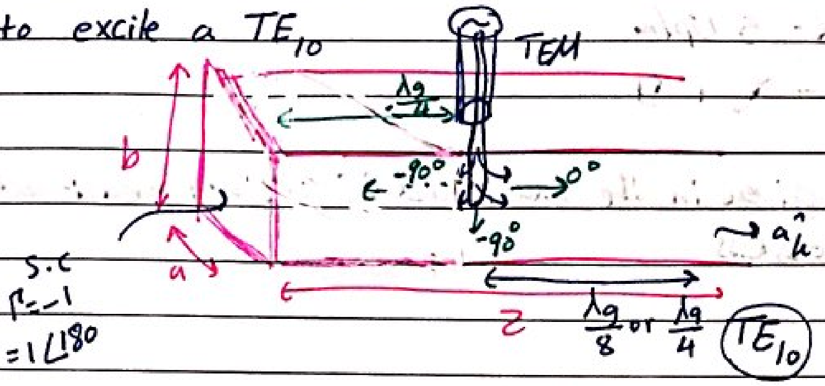
$\bar{k} = \hat{a_n} \times \bar{H}$

No.

How to excite a R.W.G ?

either to excite a TE or TM modes

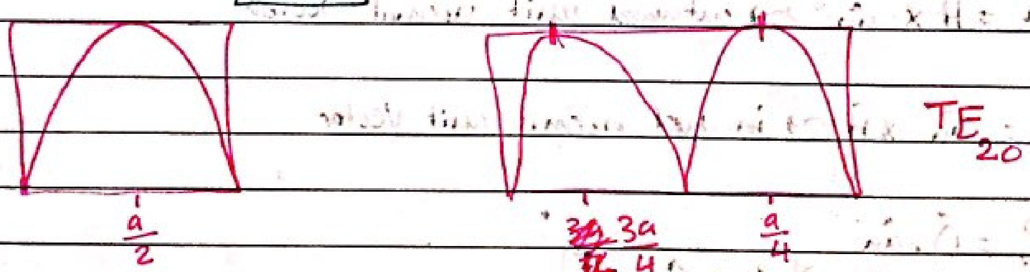
to excite a TE_{10}



$TE_{10} \rightarrow H_z, E_y, H_x$

$E_y(z,t) = (\quad) \sin\left(\frac{\pi x}{a}\right) \sin(\cancel{\omega t - \beta z})$

$\frac{\pi x}{a} = \frac{\pi}{2} \Rightarrow \boxed{x = \frac{a}{2}}$

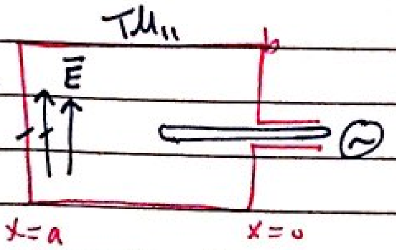


to excite a TM mode

$TM_{11} \Rightarrow E_x, E_y, E_z, H_x, H_y$

$$E_x = \frac{-j\beta}{h^2}\left(\frac{\pi}{a}\right) E_o \underbrace{\cos\left(\frac{\pi x}{a}\right)}_{\substack{\times\ x=0 \\ \times\ x=a}} \underbrace{\sin\left(\frac{\pi y}{b}\right)}_{y = \frac{b}{2}} e^{-\gamma z}$$

$\bar{E}$ is always in parallel with the narrower edge of the W.G



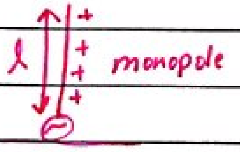
No.

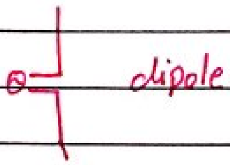
TE_{11} ← 5-fields, TM_{11} ← 5-fields

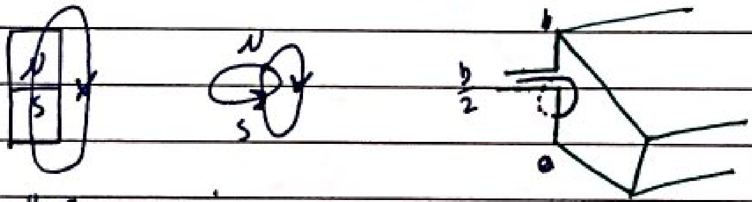
$f_{c_{11}}$ = same

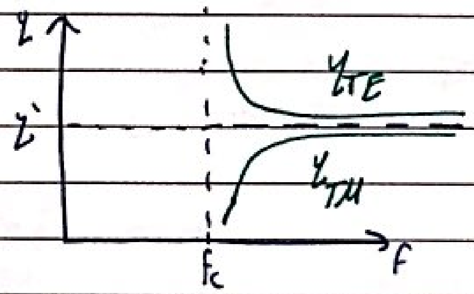
degenerate modes

TE_{10}, TE_{01} ⇝ non-degenerate mode



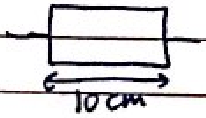






* wave guide Resonators :
(RLC)

at high freq $l \approx \lambda$



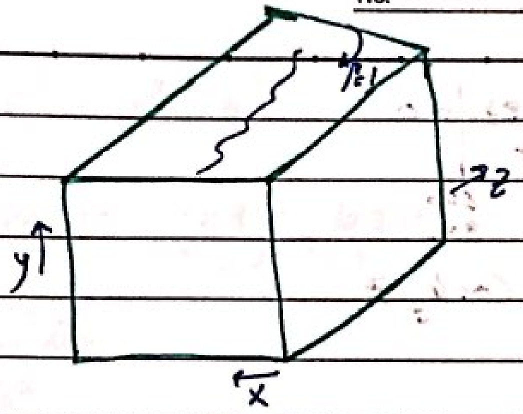
$\lambda = \frac{c}{f} = \frac{3\times10^8}{3\times10^{10}} = 1\,cm$ 30 GHz

$c = \lambda f$

$R = \frac{V}{I}$ ✗

$l < \frac{\lambda}{10}$

No.



All faces are S.C.s

A) TM for z

$E = X(x)\ Y(y)\ Z(z)$

$X(x) = C_1 \cos k_x x + C_2 \sin k_x x$

$Y(y) = C_3 \cos k_y y + C_4 \sin k_y y$

$Z(z) = C_5 \cos k_z z + C_6 \sin k_6 6$ ← Resonator

$E_z = 0$, $x=0$, $x=a$; $y=0$, $y=b$ $\Big\} \rightarrow C_1 = 0$, $C_3 = 0$; $k_x = \frac{m\pi}{a}$, $k_y = \frac{n\pi}{b}$

$E_x = 0$, at $z=0$, $z=c$
$E_y = 0$

TM

$E_x \Rightarrow j\omega\epsilon E_{xs} = \frac{1}{j\omega\mu}\left(\frac{\partial^2 E_{xs}}{\partial z^2} - \frac{\partial^2 E_{zs}}{\partial z \partial x}\right) + \cancel{\frac{\partial H_{zs}}{\partial y}}^{\,0}$

$E_y \Rightarrow j\omega\epsilon E_{ys} = \frac{-1}{j\omega\mu}\left(\frac{\partial^2 E_{zs}}{\partial y \partial z} - \frac{\partial^2 E_{ys}}{\partial z^2}\right) + \cancel{\frac{\partial H_{zs}}{\partial x}}^{\,0}$

Apply $E_y = 0$ at $z=0$, $z=c$
$E_x = 0$ at $z=0$, $z=c$

Since $E_x = 0 \rightarrow$ must be $\left.\dfrac{\partial E_{xs}}{\partial z}\right|_{\substack{z=0 \\ z=c}} = 0$

$Z'(z) = -C_5 k_z \sin k_z z + C_6 k_z \cos k_z z$

at $z=0 \Rightarrow C_6 = 0$

at $z=c \Rightarrow \quad -C_5 k_z \sin k_z c = 0$

$C_5 \neq 0$

$k_z \neq 0$

$\sin k_z c = 0$

$k_z c = p\pi \quad , p = 0, 1, 2, \dots$

$\boxed{k_z = \dfrac{p\pi}{c}} \quad p = 0, 1, 2, \dots$

24/4/2018

W.G. Resonent

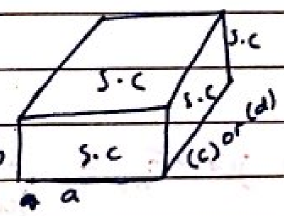
TM to (z)

$k_x = \dfrac{m\pi}{a} \quad , k_y = \dfrac{n\pi}{b} \quad , k_z = \dfrac{p\pi}{c}$ or $\dfrac{p\pi}{d}$

$E_z = X(x)\, Y(y)\, Z(z)$ $\quad e^{-\delta z}$ T.L

$= E_0 \sin\left(\dfrac{m\pi x}{a}\right) \sin\left(\dfrac{n\pi y}{b}\right) \cos\left(\dfrac{p\pi z}{c}\right)$

$C_2 C_4 C_5$



$m = 1, 2, 3, \dots$

$n = 1, 2, 3, \dots$

$p = 0, 1, 2, \dots$

No.

$k^2 = k_x^2 + k_y^2 + k_z^2$

$k^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{c}\right)^2$

$= \omega^2 \mu \epsilon$

$= \beta^2$

$\beta = \omega\sqrt{\mu\epsilon} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{c}\right)^2}$

$f_r = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2} \Rightarrow$ resonant freq.

lowest mode is TM_{110} in all TM modes

$\lambda_r = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}}$

$\boxed{\text{for TE to } (z)}$

$E_z = 0 \quad , H_z \neq 0$

$H_z = X(x) \cdot Y(y) \cdot Z(z)$

$= (B_1 \cos k_x x + B_2 \sin k_x x)(B_3 \cos k_y y + B_4 \sin k_y y)(B_5 \cos k_z z + B_6 \sin k_z z)$

$H_{zs} = 0$, at $z = 0$, $z = c$

$H_{xs} = 0$ at $x = 0$, $x = a$

$\hookrightarrow \frac{\partial H_{zs}}{\partial x} = 0$

$\Rightarrow$

$B_2 = 0$, $B_4 = 0$, $B_5 = 0$

$H_0 = B_1 B_3 B_6$

$H_{ys} = 0$ at $y = 0$, $y = b$

$\hookrightarrow \frac{\partial H_{zs}}{\partial y} = 0$

$H_{zs} = H_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \sin\left(\frac{p\pi z}{c}\right)$

$a > b$

$m = 0, 1, 2, \ldots$

$n = 0, 1, 2, \ldots$

$p = 1, 2, 3 \ldots$

lowest mode is

TE_{101} $(a > b)$

TE_{011} $(b > a)$

No.

* Quality factor :-

$$Q = \omega \left(\frac{W_e + W_m}{P_L} \right) \quad , \quad W_e = \frac{1}{2} \int_v \epsilon E^2 \, \partial v \leftarrow E \text{ is rms}$$

$\omega = 2\pi f_r$

$$W_m = \frac{1}{2} \int_v \mu H^2 \, \partial v$$

$$W_e = \frac{1}{4} \int_v \epsilon E^2 \, \partial v \leftarrow E \text{ is peak}$$

$P_{ave} = P_0 e^{-2\alpha z}$

$$P_L = 2\alpha (P_{ave}) = \frac{-\partial P_{ave}}{\partial z}$$

α: given

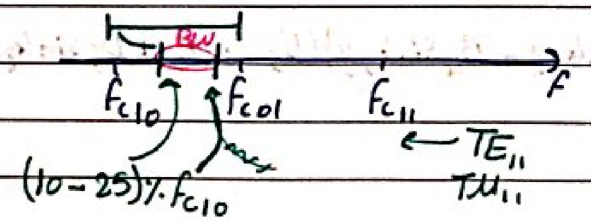
$$P_{ave} = \frac{1}{2} \cdot Re\{ E_s \times \bar{H}_s^* \}$$

$$P_{ave} = P_0 = \int_s P_{ave} \cdot ds$$

$$\Rightarrow Q_{101} = 2\pi f_{r101} \left(\frac{W_e + W_m}{P_L} \right) \leftarrow \alpha_{101}$$

* Safe guard:-

To ensure working on TE_{10} for example

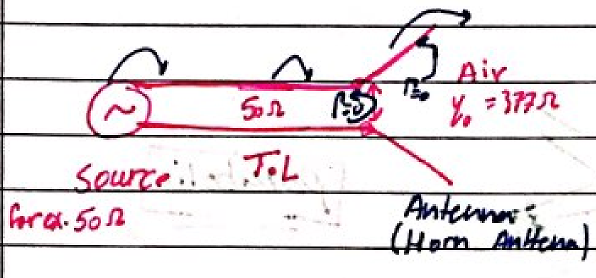


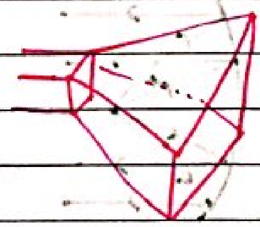
14.5
15.5

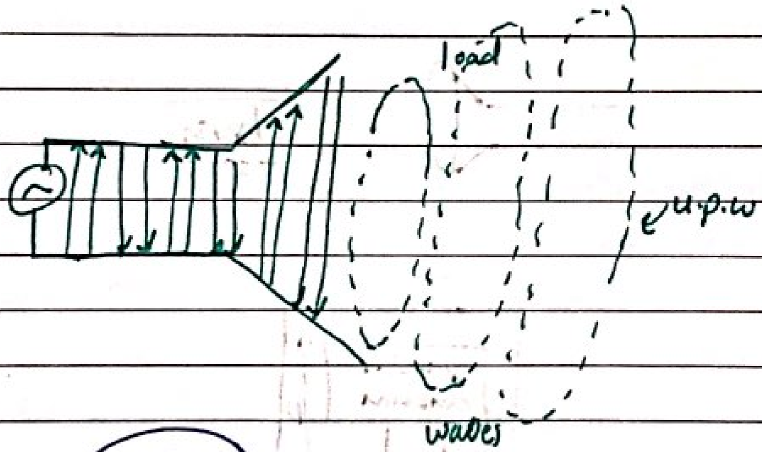
Chapter 13 Antennas:-

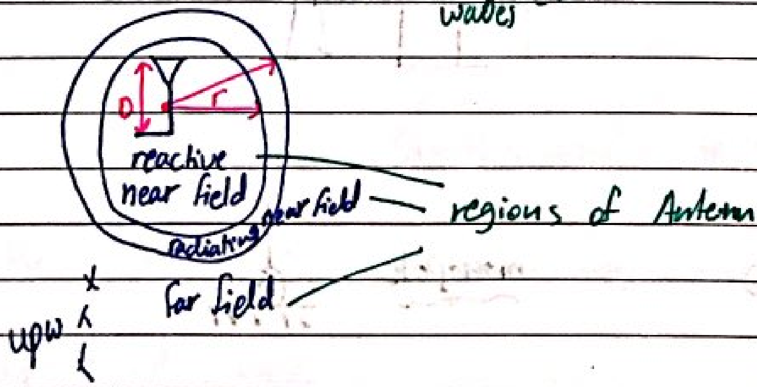
we use them for:-

1. Efficient radiation
2. Matching to the media





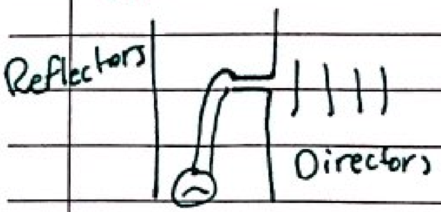




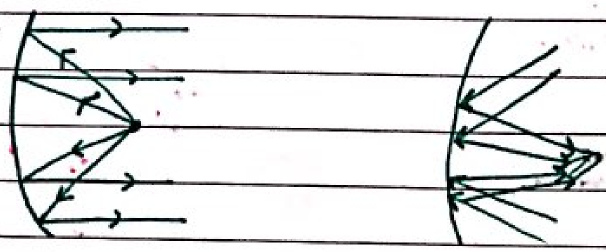
* D is the biggest diameter

No.

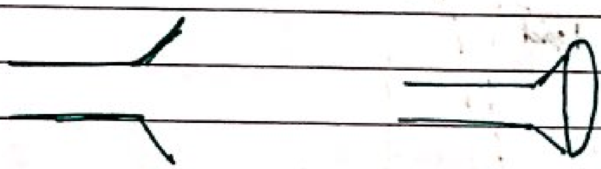
types of Antennas :-



Yagi-uda

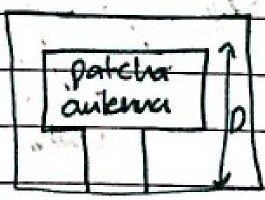


parabolic



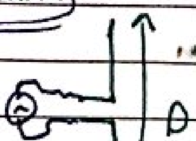
horn

planar antenna
- → planner T.L
- → TEM

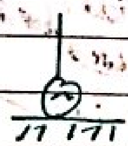


wire antenna

Dipole

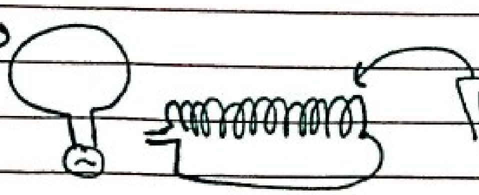


monopole



$\frac{\lambda}{2}$, $\frac{\lambda}{4}$ hertzian

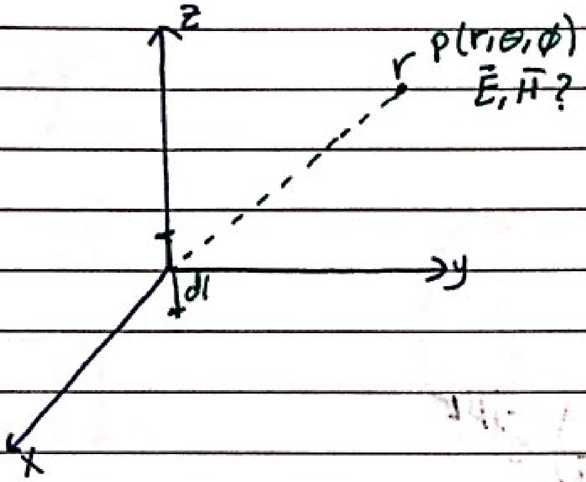
loop antenna



helix antenna

No. 26/4/2018

* Hertzian Dipole (infinitesmal dipole)



log
retarted current

$$\bar{A} = \int_v \frac{[J]\, dv}{4\pi R}$$

$$t' = t - \frac{r}{u}$$

retarted time

$$\bar{A} = \frac{\mu [I]\, dl}{4\pi R}\, \hat{a}_z$$

$I = I_o \cos\omega t$ → uniform in space

$$\Rightarrow A_{z} = \frac{\mu I_o \cos\omega t'\, dl}{4\pi r} \quad , \omega t' = \omega\left(t - \frac{r}{u}\right)$$

β

$$A_z = \frac{\mu I_o \{\cos(\omega t - \frac{\omega r}{u})\}\, dl}{4\pi r} \rightarrow e^{j(\omega t - \frac{\omega r}{u})}$$

$e^{j\omega t}$

$$\boxed{A_{zs} = \frac{\mu I_o\, e^{-j\beta r}\, dl}{4\pi r}} \Rightarrow \text{cart. cyl.}$$

convert to sph (matrix will be given)

$A_{rs} = A_{zs} \cos\theta$

$A_{\theta s} = -A_{zs} \sin\theta$

$A_{\phi s} = 0$

Scanned with CamScanner

Find $\bar{B}_s = \nabla \times \bar{A}_s$

$\bar{B}_s = \mu \bar{H}_s$

$H_{r_s} = 0$

$H_{\theta_s} = 0$

$$H_{\phi_s} = \frac{I_o\, dL}{4\pi} \sin\theta \left(\frac{j\beta}{r} - \frac{1}{r^2} \right) e^{-j\beta r}$$

To Find $\bar{E}$ $\rightarrow$ use Maxwell's eq.

$$\nabla \times \bar{H} = \cancelto{0}{\bar{J}_s} + j\omega\epsilon \bar{E}_s \longrightarrow \bar{E}_s = \frac{1}{j\omega\epsilon} \nabla \times \bar{H}_s$$

$$E_{r_s} = \frac{\eta\, I_o\, dL}{2\pi} \cos\theta \left(\frac{1}{r^2} - \frac{j}{\beta r^2} \right) e^{-j\beta r}$$

$E_{\theta_s} =$

$E_{\phi_s} = 0$

$$E_{\theta_s} = \frac{\eta\, I_o\, dL}{4\pi} \sin\theta \left(\frac{j\beta}{r} + \frac{1}{r^2} - \frac{j}{\beta r^3} \right) e^{-j\beta r}$$

$\eta = \sqrt{\frac{\mu}{\epsilon}}$

far field $\left(\frac{1}{r}\right)$ — inductive field $\left(\frac{1}{r^2}\right)$ — electrostatic field $\left(\frac{1}{r^3}\right)$

at Far-field

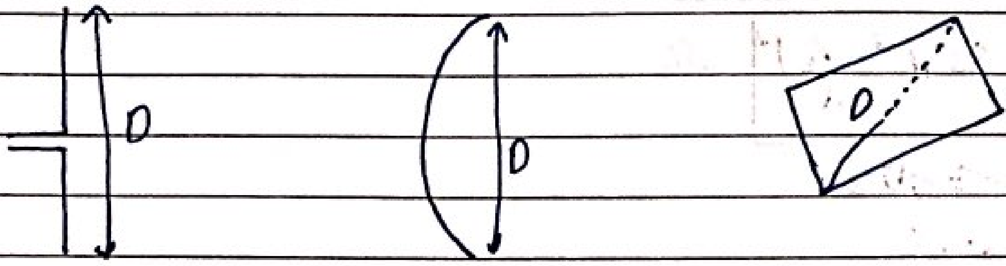
$$H_{\phi_s} = j \frac{I_o\, dL\, \beta \sin\theta}{4\pi r} e^{-j\beta r} \quad A/m$$

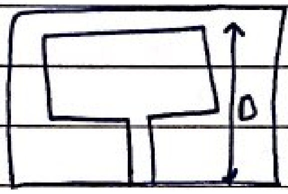
$$E_{\theta_s} = \eta\, H_{\phi_s}$$

No.

when far-field region begins

$$r = \frac{2D^2}{\lambda}$$ D: the biggest diameter of the antenna





$E_{r_s} = E_{\phi_s}$
$= H_{r_s} = H_{\theta_s} = 0$

power $\Rightarrow$ $\bar{P}_{ave} = \frac{1}{2} Re\{\bar{E}_s \times \bar{H}_s^*\}$ w/m^2

$= \frac{1}{2} Re\{\eta H_{\phi_s}^2 \hat{a}_r\}$ w/m^2

$P_{ave} = \int_s \bar{P}_{ave} \cdot \bar{d}_s$ (w) $= P_{rad}$

$$= \int_0^{2\pi}\int_0^{\pi} \frac{\frac{1}{2}\eta I_o^2 \beta^2 dL^2 \sin^2\theta}{16\pi^2 \cancel{r^2}} \cdot \cancel{r^2} \sin\theta \, d\theta \, d\phi$$

$$P_{rad} = \frac{I_o^2 \pi \eta}{3}\left(\frac{dL}{\lambda}\right)^2 \; w$$

$$= \frac{1}{2} I^2 R_{rad}$$

$$\boxed{R_{rad} = \frac{2P_{rad}}{I^2}}$$

if free space

$\eta = \eta_0 = 120\pi$

$\boxed{P_{rad} = 40\pi^2 \left(\frac{dl}{\pi}\right)^2 I_0^2}$ W

$\boxed{R_{rad} = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2}$ Ω

~~For $dl = \lambda/20$~~

29/4/2018

* Hertzian Dipole

Assume : $I = I_0$

$\bar{A}$

$\bar{B} = \nabla \times \bar{A}$

$\bar{H} = \frac{\bar{B}}{\mu}$

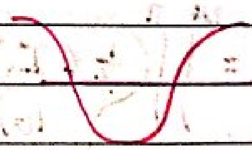
$\bar{E} \rightarrow \nabla \times \bar{H}_s = j\omega\epsilon \bar{E}_s \quad (\bar{J} = 0)$

$P_{rad} = \int_s \frac{1}{2} Re\{\bar{E}_s \times \bar{H}_s^*\} \cdot \bar{ds}$

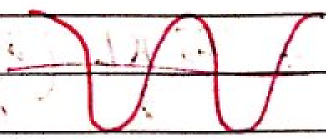
$= I_0^2 \, R_{rad}$

$R_{rad} = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2 \Omega$

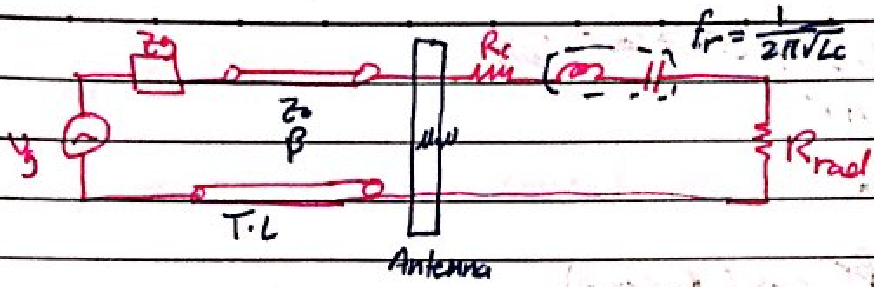
in free space

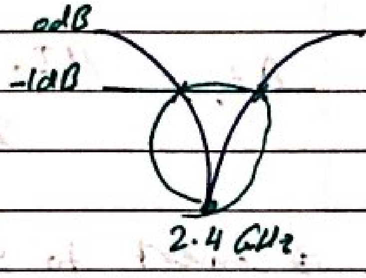
1 resonance  -10 dB

for $dl = \frac{\lambda}{20}$

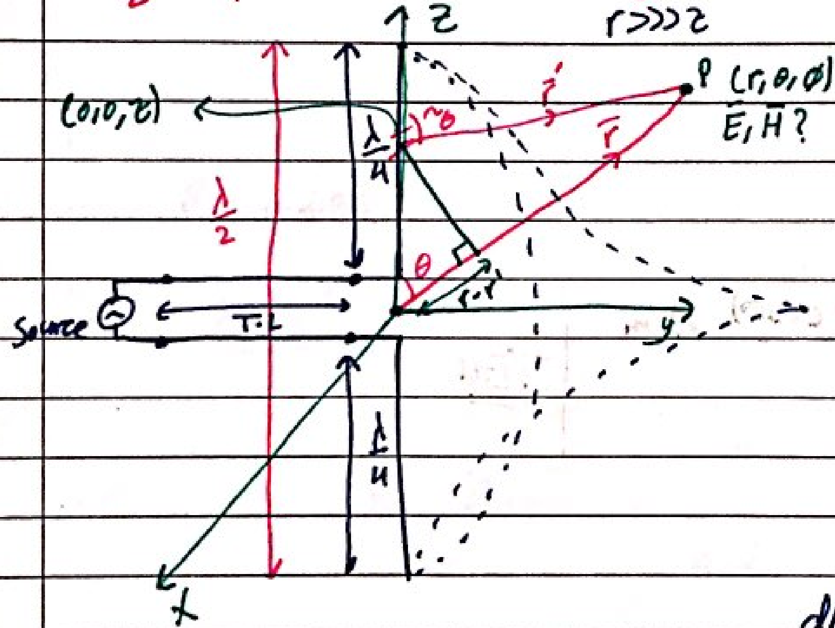
2 resonanse  -6dB

$R_{rad} = 2\ \Omega$





$\ast$ $\frac{\lambda}{2}$ dipole : half-wave length dipole antenna :-



$$I = I_o \cos(\omega t - \beta z)$$

$$I_s = I_o \cos \beta z$$

$$\cos\theta = \frac{r - \bar{r}}{z} \quad , t' = t - \frac{r}{u}$$

$$\omega t' = \omega t - \frac{\omega r}{u}$$

$$= \omega t - \beta r$$

$$r - r' = z \cos\theta$$

$$dA_{z_s} = \frac{\mu [I] \, dL}{4\pi R}$$

$$dA_{z_s} = \frac{\mu I_o \cos \beta z \, dz}{4\pi r'} e^{-j\beta r'}$$ → relative time

r' in Amplitude

$$\boxed{r' \simeq r}$$

in phase

$$r - r' = z \cos\theta$$

$$\boxed{r' = r - z \cos\theta}$$

No.

$$\partial A_{z_s} = \frac{\mu I_o \cos\beta z \, e^{-j\beta(r - z\cos\theta)} \partial z}{4\pi r}$$

$\Rightarrow$ do the integration

$$A_{z_s} = \frac{\mu I_o e^{-j\beta r} \cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r \beta \sin^2\theta} \rightarrow \text{given}$$

$\Rightarrow$ convert to sperical.

$$\bar{B}_s = \nabla \times \bar{A}_s$$

$$\bar{H}_s = \frac{\bar{B}_s}{\mu}$$

at far field

$$H_{\phi_s} = \frac{j I_o e^{-j\beta r} \cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r \sin\theta} \text{ A/m}$$

$$E_{\theta_s} = \eta H_{\phi_s} \text{ V/m}$$

TEM

$$\hat{a}_k = \hat{a}_r$$

$$\bar{P} = \bar{E} \times \bar{H} \text{ in W/m}^2 \, \hat{a}_r$$

$$\bar{P}_{ave} = \frac{1}{2} \text{Re}\{\bar{E}_s \times \bar{H}_s^*\}$$

$$P_{rad} = \int_s \bar{P}_{ave} \cdot \bar{d}_s = \int_0^{2\pi}\int_0^{\pi} \frac{1}{2} \text{Re}\{\eta H_{\phi_s}^2\} \hat{a}_r \cdot r^2 \sin\theta \, d\theta \, d\phi$$

$$\boxed{P_{rad} = 36.56 \, I_o^2} = \frac{1}{2} I_o^2 R_{ad}$$

Scanned with CamScanner

No.

$\boxed{R_{rad} = 73\,\Omega}$

easy match with a Coaxial Cable has $Z_0 = 75 + j42.5\,\Omega$

for a $\frac{\lambda}{2}$ dipole:-

$Z_{in} = R_{in} + jX_{in}$

$X_{in} = 42.5\,\Omega$

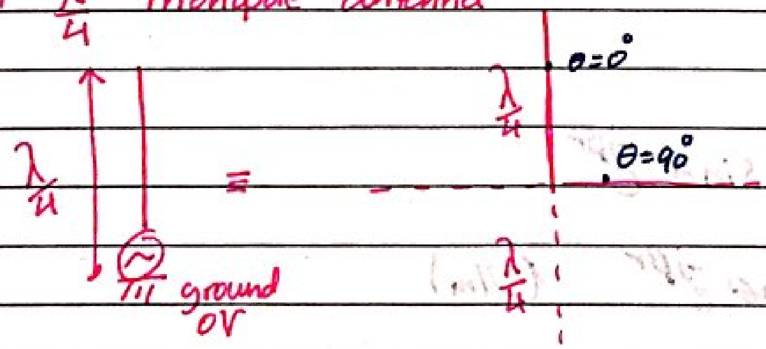
$\boxed{Z_{in} = 73 + j42.5\,\Omega} \rightarrow \varepsilon = \infty \sim R_s = 0$

if $\ell \nearrow$

For $\ell = 0.485\lambda$

$\boxed{Z_{in} = 73\,\Omega}$

for $\frac{\lambda}{4}$ monopole antenna



E_{θ_s} & H_{ϕ_s} → The Same

$0 \leq \theta \leq \frac{\pi}{2}$

$P_{rad} = 18.25\, I_0^2\; W$

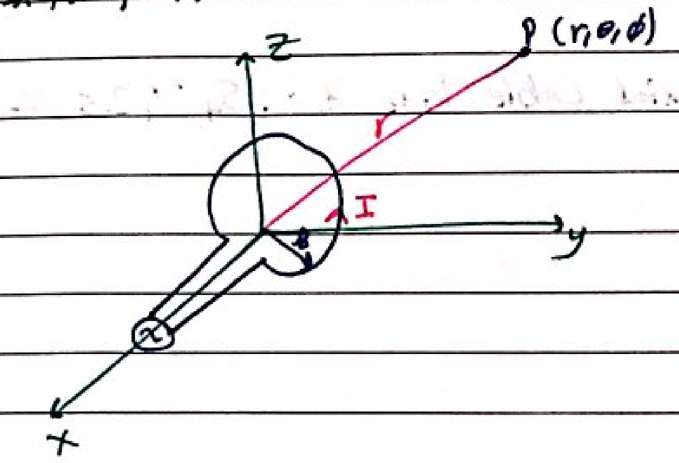
$R_{rad} = 36.5\,\Omega$

$Z_{in} = 36.5 + j21.25\,\Omega$

for $\ell = 0.2425\lambda \Rightarrow Z_{in} = 36.5\,\Omega$

No.

small
* Loop Antenna



$r \gg\gg \rho_0$

m ≡ magnetic moment
$= I S \hat{a}_n \quad A \cdot m^2$

$S = \pi \rho_0^2$

for N – loops

$S = N(\pi \rho_0^2)$

Assume $\Rightarrow I_s = I_0$

$$A_{\phi s} = \frac{\mu I_0 S}{4\pi r^2} \sin\theta \, (1 + j\beta r) \, e^{-j\beta r}$$

if $\beta = 0 \Rightarrow \frac{\mu \overbrace{I_0 S}^{m}}{4\pi r^2} \sin\theta \rightarrow$ same as D.C

$$\bar{B} = \nabla \times \bar{A}_s$$

$$\bar{H}_s = \frac{\bar{B}s}{\mu} \quad , \quad \nabla \times \bar{H}_s = j\omega\epsilon \bar{E}_s$$

At far field

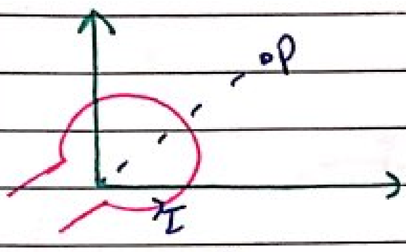
$$E_{\phi s} = \frac{\omega \mu I_0 S \beta}{4\pi r} \sin\theta \, e^{-j\beta r}$$

$$= \frac{\eta \pi I_0 S}{r \lambda^2} \sin\theta \, e^{-j\beta r} \quad (V/m)$$

$$H_{\theta s} = \frac{-E_{\phi s}}{\eta} \quad (A/m)$$

No. 3/5/2018

loop antenna :-



At far fields

$E_{\phi_s} =$

$E_{\theta_s} =$

$\bar{P} = \bar{E} \times \bar{H}$

$$\bar{P}_{ave} = \frac{1}{2} R_e \left\{ \bar{E}_{\phi_s} \times \bar{H}^*_{\theta_s} \right\}$$

$$\bar{P}_{ave} = \frac{1}{2} R_e \left\{ \frac{\bar{E}_{\phi_s}}{\eta} \hat{a}_r \right\} \ w/m^2$$

$$P_{rad} = \int_s \bar{P}_{ave} \cdot \bar{d}_s \quad , ds = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$$

$$P_{rad} = \frac{320 \pi^4 S^2 I_o^2}{2 \lambda^4} \ w \quad , \eta_o = 120\pi$$

$$= \frac{1}{2} I_o^2 R_{rad}$$

$$R_{rad} = \frac{320 \pi^4 S^2}{\lambda^4} \ \Omega$$

$S = \pi \rho_o^2$ for $N=1$

$S = N\pi \rho_o^2$ for N turns

No.

* Antenna characterstics :-

1) Radiation patterns :- 2D / 3D

- Amplitude pattern $\rightarrow E$
- power pattern $\rightarrow E^2$

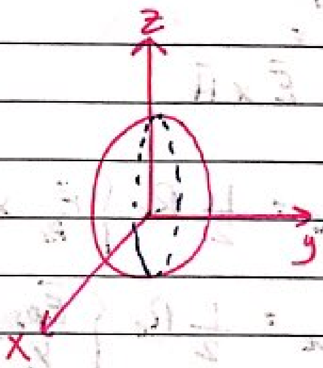
Can be divided into :-

a. $\bar{E}$-plane (vertical)

$\bar{E}$ vs. θ at a fixed ϕ

at $\phi = 0° \rightarrow$ x-z plane

at $\phi = 90° \rightarrow$ y-z plane



b. H-plane (horizantal)

$\bar{E}$ vs. ϕ at a fixed θ

at $\theta = \frac{\pi}{2} \rightarrow$ x-y plane

For Hertzian dipole :-

$$E_{\theta_s} = \eta H_{\phi_s} = \frac{j\eta I_o \beta dl}{4\pi r} \sin\theta \, e^{-j\beta r} \ \text{V/m}$$

phase

$\rightarrow$ Normalized $E_s = \dfrac{E_s}{|E_s|}$

$|E_{\theta_s}| = |\sin\theta| = \pm \sin\theta$

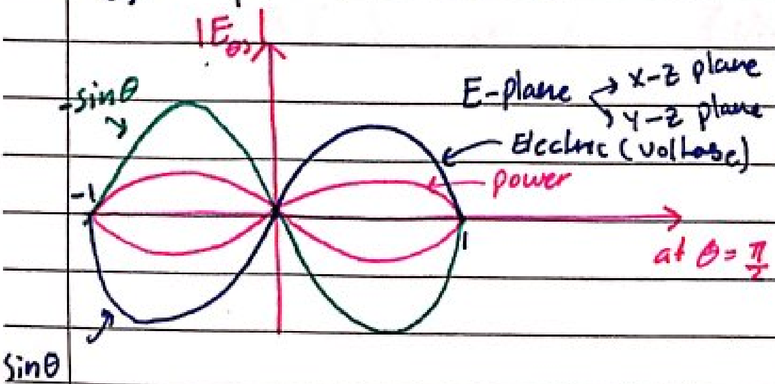
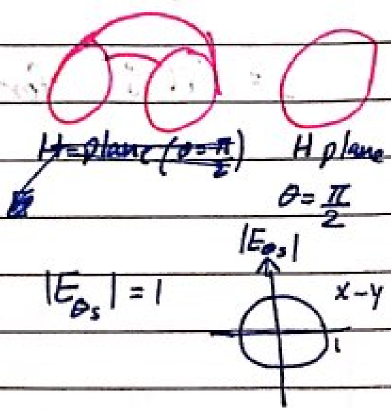


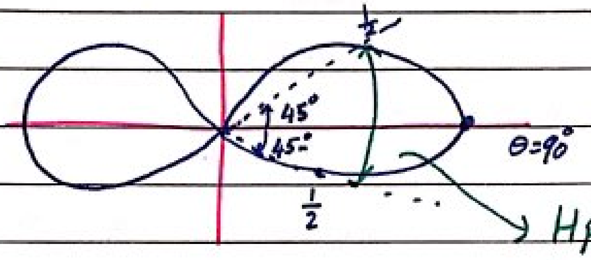
Figure of 8



No.

$|\bar{P}_{ave}| = \frac{1}{2}$ half power point

$\bar{E} = \frac{1}{\sqrt{2}}$



→ $HPBW = 2\theta_h$
half power beam width $= 90°$

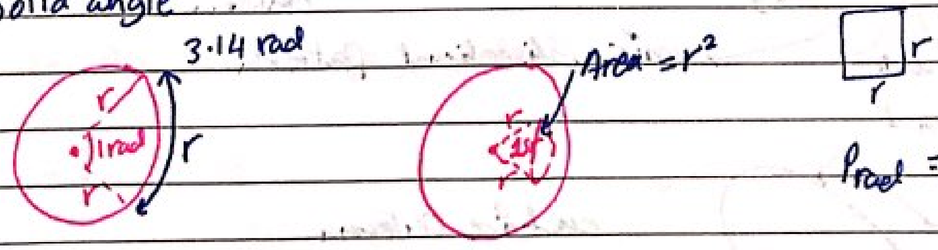
Zero in the pattern (Null)
$\theta_n = 0, \pi$

FNBW : First Null Beam width
$= 180° \rightarrow (\pi - 0)$

2) Radiation Intinsity :-

$U(\theta,\phi) = r^2 |\bar{P}_{ave}|$ $\quad \bar{P}_{ave} = \frac{1}{2} Re\{\bar{E}_s \times \bar{H}_s^*\}$
$= W/sr$
unit

$sr \equiv$ steradian $\Rightarrow 1sr = 4\pi$ rad
⇓
Solid angle



$P_{rad} = \int_s \bar{P}_{ave} \cdot d_s$

circle
1D

sphere
3D

No.

$u(\theta,\phi) = r^2 |\bar{P}_{ave}|$

$$\boxed{P_{rad} = \oint |\bar{P}_{ave}| \cdot ds}$$

$$= \int_0^{2\pi}\int_0^{\pi} |\bar{P}_{ave}|\, r^2 \sin\theta \, d\theta \, d\phi$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} u(\theta,\phi) \sin\theta \cdot d\theta \, d\phi$$

$$\boxed{P_{rad} = \oint u(\theta,\phi)\, d\Omega} = 4\pi u(\theta,\phi)$$

$$= 4\pi U_{ave}$$

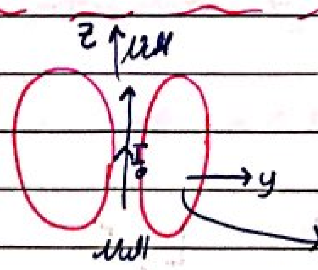
$$= 4\pi U_0$$

$d\Omega = \sin\theta \, d\theta \, d\phi$

$U_0 \equiv$ radiation intensity of an isotropic antenna

$$\Omega = \int_0^{2\pi}\int_0^{\pi} \sin\theta \, d\theta \, d\phi$$

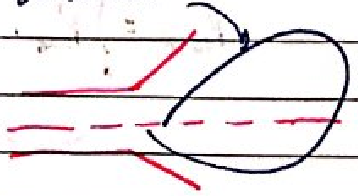
$$\boxed{\Omega = 4\pi} \text{ sr}$$



Broad side direction : max pattern is 90° with the Antenna

omni-directional pattern

directional



end-fire Antenna

No.

3) Directive gain (GD) $G_d(\theta,\phi)$
Directivity $D(\theta,\phi)$

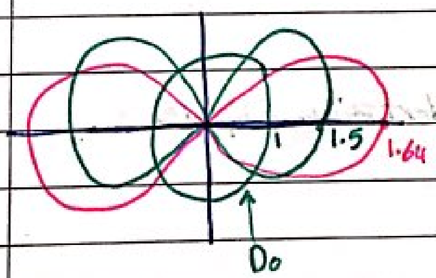
$$D(\theta,\phi) = \frac{u(\theta,\phi)}{u_{ave}}$$

$$u_{ave} = \frac{P_{rad}}{4\pi}, \quad P_{rad} = 4\pi\, u_{ave} = 4\pi\, u_0$$

$$D(dB) = 10\log(D)$$

$$D_0 \equiv D_{max}\Big|_{\sin\theta_{max}=1} = No.$$

ex: $D = 1.5\sin^2\theta \Rightarrow D_0 = 1.5$ (Hertzian dipole)
$D_0 = 1$ (isotropic antenna)



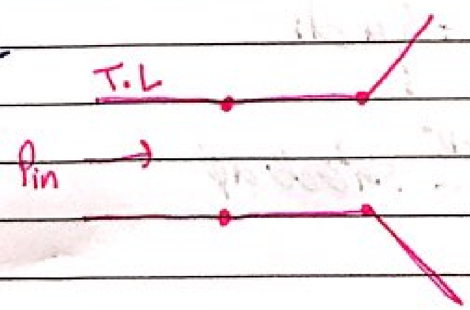
for $\frac{\lambda}{2}$ dipole

$$D = 1.64\,\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

$D_0 = 1.64$ $D\uparrow$ HpBw↓

4) power gain $G_p(\theta,\phi)$
gain → G
↳ take the losses into consideration

$\sigma_c \simeq \infty \longrightarrow R_L$



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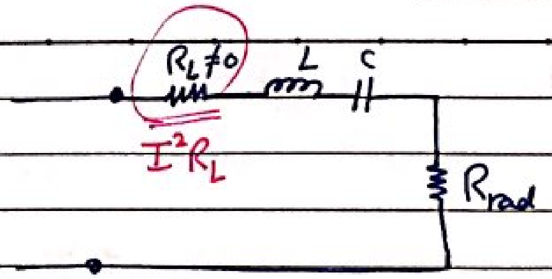

$P_{in} = P_L + P_{rad} = \frac{1}{2} |I_{in}|^2 (R_L + R_{rad})$

$G = \frac{u(\theta,\phi) * 4\pi}{P_{in}} = \eta_r D$

η_r : radiation Efficiency

$\eta_r : \frac{G}{D} = \frac{\frac{4\pi\, u(\theta,\phi)}{P_{in}}}{\frac{4\pi\, u(\theta,\phi)}{P_{rad}}} = \frac{P_{rad}}{P_{in}} = \frac{R_{rad}}{R_{rad} + R_L}$ if $\sigma = \infty$

$G = D$

$\eta_r = 100\%$

EX:- show that $D = 1.5 \sin^2\theta$ for Hertzian Dipole

$D = \frac{4\pi\, u(\theta,\phi)}{P_{rad}}$

$u(\theta,\phi) = r^2 |\bar{P}_{ave}| = f^2(\theta)$

$|\bar{P}_{ave}| = \frac{\frac{1}{2}\eta I_0^2 \beta^2 dl^2}{16\pi^2 r^2} \sin^2\theta = \frac{1}{2} Re\left\{\frac{E^2}{\eta}\right\}$

$P_{rad} = \oint_s |\bar{P}_{ave}|\, ds = \oint_s u\, ds$

$= \int_0^{2\pi}\int_0^{\pi} \frac{\eta I_0^2 \beta^2 dl^2}{32\pi^2} \sin^3\theta\, d\theta\, d\phi$

No.

$$P_{rad} = \frac{\eta I_o^2 \beta^2 dl^2}{16\pi^2}(2\pi)$$

$\int_0^{\pi} \sin^3\theta \, d\theta = \frac{4}{3}$ memorize

$D = 4\pi$ ______ $\sin^2\theta$ $= 1.5$

Ex: Determine E at a distance of 10 km from an antenna having $D = 5\,dB$ and $P_{rad} = 20\,kw$, $\eta_o = 120\pi\ \Omega$

$$D|_{dB} = 10 \log_{10} D$$

$$D = 10^{\frac{5}{10}} = 3.162$$

$$|\bar{P}_{ave}| = \frac{P_{rad}}{4\pi r^2} * D = \frac{|E_s|^2}{2\eta_o} , \quad |E_s| = 0.1948\ V/m$$

$$* P_{rad} = 4\pi \underbrace{r^2 P_{ave}}_{U_{ave}}$$

Ex:- $u(\theta,\phi) = \begin{cases} 2\sin\theta \sin^3\phi , & 0 \le \theta \le \pi \\ & 0 \le \phi \le \pi \\ 0 , & \text{else where} \end{cases}$

Find D?

$$D_o = \frac{U_{max}}{U_{ave}} , \quad D(\theta,\phi) = \frac{U(\theta,\phi)}{U_{ave} \rightarrow \frac{P_{rad}}{4\pi}}$$

$$D_o = \frac{2}{u_{ave}} = \frac{2}{\frac{1}{4\pi} \cdot 2 \cdot \frac{\pi}{2} \cdot \frac{4}{3}} = 6$$

$$U_{ave} = \frac{1}{4\pi} \underbrace{\int u \, d\Omega}_{P_{rad}} = \frac{1}{4\pi} \int_0^{\pi}\int_0^{\pi} 2 \sin^2\theta \sin^3\phi \, d\theta \, d\phi$$

($\frac{\pi}{2}$, $\frac{4}{3}$)

No.

$\therefore HPBW \to \theta_h$

at $\theta = \theta_h \Rightarrow U = 0.5$

~~2 sin θ_h~~ for $U = 2\sin^2\theta_h$

$\therefore 0.5 = 2\sin^2\theta_h$

$\theta_h = \sin^{-1}\sqrt{\frac{0.5}{2}}$

$HPBW = 2\theta_h$

at $\theta = \theta_n \xRightarrow{\text{null}} U = 0$

$2\sin^2\theta_n = 0$

Ex:- A magnetic field strength of $5\mu A/m$ is required at a point on $\theta = \frac{\pi}{2}$ 2 km from an antenna in air $\sigma_c \approx \infty$. how much power must the antenna transmit if it's :-

A) Hertzian dipole
B) $\frac{\lambda}{2}$ dipole
C) $\frac{\lambda}{4}$ dipole

} H_ϕ

d) a 10-turn loop antenna of radius $\rho = \frac{\lambda}{20}$ } H_θ

A) $|H_{\phi s}| = \frac{I_o \overset{\frac{2\pi}{\lambda}}{\beta}\, dL \sin\theta^{1}}{4\pi r} = 5\times10^{-6}$

$I_o = 0.5\ A$

$P_{rad} = 40\pi^2\left(\frac{dl}{\lambda}\right)^2 I_o^2 = 40\pi^2\left(\frac{\frac{\lambda}{25}}{\lambda}\right)^2 I_o^2 = 158\ mW$

No.

b) $|H_{\phi s}| = \dfrac{I_0 \cos((\pi/2)\cos\theta)}{2\pi r \sin\theta}$

$I_0 = 20\pi \ mA$

$R_{rad} = 73\,\Omega \Rightarrow P_{rad} = \frac{1}{2} I_0^2 R_{rad} = 144 \ mW$

$\hookrightarrow P_{rad} = 36.5 I_0^2$

c) $P_{rad} = 72 \ mW = 18.25 I_0^2$

d) $|H_{\theta s}| = \dfrac{\pi I_0 S}{r\lambda^2} \cancel{\sin\theta}^{1}$

$S = N\pi \rho_0^2 \quad , \rho_0 = \dfrac{\lambda}{20}$

$I_0 = 40.53 \ mA$

$R_{rad} = 192.3 \ \Omega \quad , P_{rad} = 158 \ mW$