EM 2

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No.

Review:

Electrostatic: $\epsilon\ (F/m)$

$q\ (C)$

$\rho_L\ (C/m)$

$\rho_s\ (C/m^2)$

$\rho_v\ (C/m^3)$

$\vec{E} \xleftrightarrow{\epsilon} \vec{D} \quad V \quad \vec{F}_e$

V/m $\quad$ C/m² $\quad$ V $\quad$ N

$$\vec{E} = \iiint_V \frac{\rho_v\, dv}{4\pi\epsilon R^2}\,\vec{a}_R \quad V/m$$

$$\vec{D} = \epsilon\vec{E}$$

$$V = \iiint_V \frac{\rho_v\, dv}{4\pi\epsilon R} = -\int_L \vec{E}\cdot d\vec{l}$$

$$\vec{F}_e = q\vec{E} = \frac{q_1 q_2}{4\pi\epsilon R^2}\,\vec{a}_R$$

$\nabla\cdot\vec{D} = \rho_v$

$\vec{E} = -\nabla V$

$\boxed{\nabla\times\vec{E} = 0}$ KVL (ckt)

* Capacitance:

$$C = \frac{q}{V} \quad q \to \iiint_V \rho_v\, dv \quad V \to -\int_L \vec{E}\cdot d\vec{l}$$

$$W_e = \frac{1}{2}CV^2 \quad J$$

$\epsilon = \epsilon_0\,\epsilon_r$

$\epsilon_0 = \frac{10^{-9}}{36\pi}$ F/m

ϵ_r: most of the time > 1

for vacume $\epsilon_r = 1$

Teflon 2.3

glass 1.26 – 1.5

Magnetostatic: $\mu\ (H/m)$

$i\ (A)$

$K\ (A/m)$

$J\ (A/m^2)$

$\vec{B} \leftarrow \vec{H} \to V_m$ A-turn, $\vec{A}$ wb/m

$\vec{B}$: T, wb/m² $\quad \vec{H}$: A/m

$$\oint \vec{H}\cdot d\vec{l} = I = \int \vec{K}\cdot d\vec{l} = \iint \vec{J}\cdot d\vec{s}$$

$\boxed{\nabla\times\vec{H} = \vec{J}}$

$$\vec{B} = \iiint_V \mu\,\frac{\vec{J}\,dv}{4\pi R^2}\times\vec{a}_R$$

$\boxed{\vec{B} = \nabla\times\vec{A}}$

$$\vec{A} = \mu\iiint_V \frac{\vec{J}\,dv}{4\pi R}$$

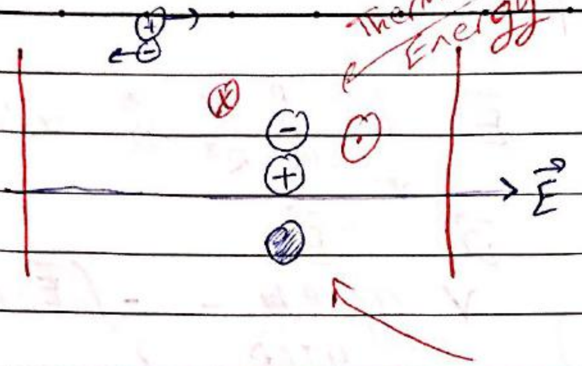
$\oiint \vec{B}\cdot d\vec{s} = $ Zero

$\Rightarrow \boxed{\nabla\cdot\vec{B} = 0}$

conductivety:

σ ($\mho$/m)

No.



$\Rightarrow$ R

$$R = \iiint \frac{d\ell}{\sigma\, dA}$$

✓ $\nabla \cdot \vec{D} = \rho_v$	$\nabla \cdot \vec{B} = 0$ ✓	? $\nabla \cdot \vec{J} = 0$ (KCL) at a point
? $\vec{E} = -\nabla V$	$\vec{B} = \nabla \times \vec{A}$	
? $\nabla \times \vec{E} = 0$	$\nabla \times \vec{A} = \vec{J}$?	(σ) $\mho$/m
(KVL) → at a point	magnetostatic	
electrostatic (ϵ) F/m	(μ) H/m	

σ for free space = Zero (vacume)

Time Varying Field:

general Current (GKCL)

Electric Energy — Magnetic Energy

Farday's law (GKVL)

* مصدر إشارة الناقص بالقانون
مصدر أ الناتج يعاكس المنتج

$$\text{Emf} = \oint \vec{E} \cdot d\vec{l} = 0 - \frac{\partial}{\partial t} \iint_s \vec{B} \cdot d\vec{s}$$

GKVL "faraday's Law"

$\psi_M = \vec{B} \cdot \vec{A}$

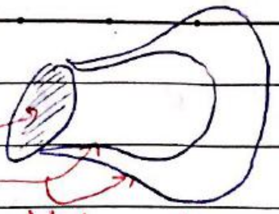
$\oint \vec{E} \cdot d\vec{l} =$

- → const. Area, Magnetic flux density ↗ (communication system & Transformer Relation)
- → Area ↗, const Magnetic flux density (Motor & generator) "Translation Relation"
- → Area ↗, magnetic flux density ↗ (General Case)

No.

* Case(i): s is fixed & B changes:

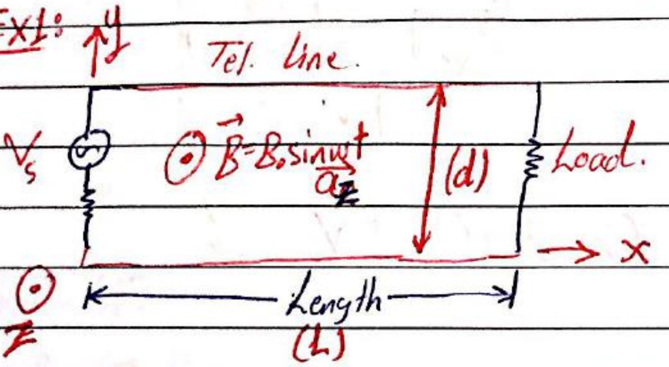
$$emf = \oint \vec{E}\cdot\vec{dl} = -0 - \iint_s \frac{\partial \vec{B}}{\partial t}\cdot\vec{ds}$$



(open surface)

* s could be all or part of the surface.

Ex1: Tel. line.



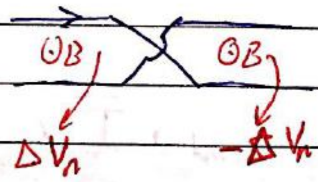
always put the coordinate system befor start to solve.

Need: V_{noise} ??

$$V_n = -\int_{y=0}^{d}\int_{x=0}^{\ell} B_o\, \omega \cos\omega t\ \vec{a_z}\cdot dxdy\ \vec{a_z} = \boxed{-B_o\, \omega\, d\, \ell \cos\omega t}\ \text{volt.}$$

** To reduce V_n:

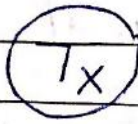
$\Rightarrow$

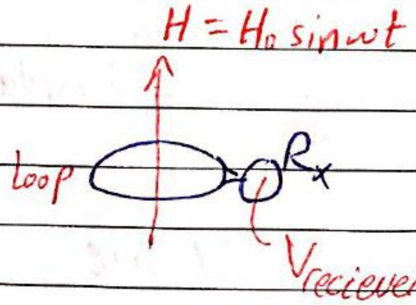


$\Rightarrow \sum_{n=1}^{N} \Delta V_n$ → 0 (if N even) / → $\pm \Delta V_n$ (if N odd)

Ex2: Radio Rx:

$$\Rightarrow V_r = -\iint \frac{\partial \vec{B}}{\partial t}\cdot\vec{ds}$$

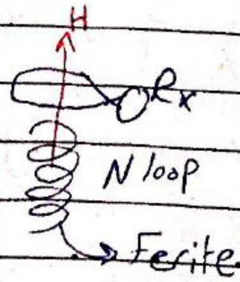




$$= \boxed{-\mu_o H_o\, \omega\ \text{area}\ \cos\omega t}\ \text{volt.}$$

** Need to maximize V_r:

$\Rightarrow$ one loop → N loop and use Ferite material with $\sigma << 1$ & $\mu_r >> 1$



$$\Rightarrow V_r = -N\mu H_o\, \omega\ \text{Area} \cos\omega t \quad \mu V$$

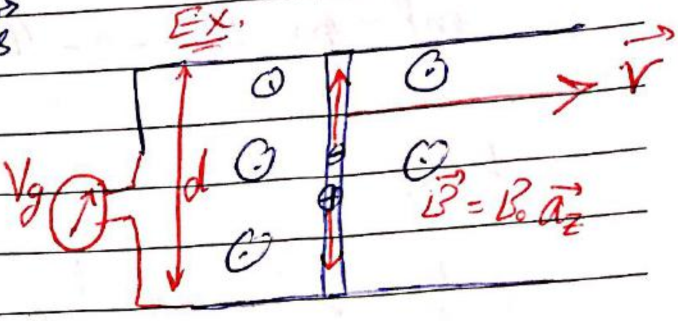
$$\Rightarrow \boxed{V_{r\,(rms)} = \frac{5}{\sqrt{2}}\ \mu V}$$

→ doesn't include (-) & $\cos\omega t$.

No.

* Case(ii): B is fixed & s is changing:

$$emf = \oint \vec{E}\cdot\vec{dl} = -0 - \frac{\partial}{\partial t}\iint \vec{B}\cdot\vec{ds}$$



$$\vec{F}_m = q\,\vec{V}\times\vec{B} \Rightarrow \vec{E} = \frac{\vec{F}}{q} = \vec{V}\times\vec{B}$$

$$\Rightarrow \boxed{\oint \vec{E}\cdot\vec{dl} = \oint \vec{V}\times\vec{B}\cdot\vec{dl}}$$

$$\Rightarrow \text{in } \underline{Ex.} \Rightarrow \oint \vec{E}\cdot\vec{dl} = \oint \vec{V}\times\vec{B}\cdot\vec{dl} = \pm\, \underline{V B_0 d}$$

* Case(iii): $\vec{B}$ & $\vec{S}$ are changing:

$$emf = \oint \vec{E}\cdot\vec{dl} = -0 - \iint_s \frac{\partial\vec{B}}{dt}\cdot\vec{ds} + \oint \vec{V}\times\vec{B}\cdot\vec{dl}$$

* General Current:

Electric Energy → magnetic energy

Current → Conduction Current. i_{cond}, $\vec{J}_{cond}$

→ Convection Current. $i_{conv.}$, $\vec{J}_{conv.}$

→ Displacement Current. i_{disp}, $\vec{J}_{disp}$

$$i \equiv \frac{\text{Charge}}{\text{Time}} = \frac{q}{t} = \frac{VC}{t} = \frac{\epsilon A \vec{E} d}{t\, d} = \frac{DA}{t}$$

$$\Rightarrow \vec{J} = \frac{i}{A} = \frac{\vec{D}}{t} \Rightarrow \vec{J}_{disp} = \frac{d\vec{D}}{\partial t}$$

$$mmf = \oint \vec{H}\cdot\vec{dl} = \boxed{\iint_{s'} \vec{J}_{\substack{cond\\+conv}}\cdot\vec{ds'} + \frac{\partial}{\partial t}\iint_{s''} \vec{D}\cdot\vec{ds''}}$$

* Here could $\vec{D}$ or s change But the changing in s hasn't big effect so we assum the changing is always could be in $\vec{D}$.

$$= \iint_{s'} J_{\substack{cond\\+conv}}\cdot\vec{ds'} + \iint_{s''} \frac{\partial\vec{D}}{\partial t}\cdot\vec{ds''}$$

take 1000 $\quad \simeq 10^{-6}$

No. $\mu_r \mu_o = \underline{\underline{10^{-3}}}$

magnetic energy → $\boxed{\mu}$ → Electric Energy. $\quad 10^{8} \quad 10^{-8}$

Electric Energy → $\boxed{\epsilon}$ → magnetic energy.

→ $\epsilon_r \epsilon_o \simeq 10^{-11}$, $= 10^{-11}$

for air = 1

* Maxwell's Equations in integral form:

① $emf = \oint \vec{E} \cdot \vec{dl} = -0 - \frac{\partial}{\partial t} \iint \vec{B} \cdot \vec{ds}$

② $mmf = \oint \vec{H} \cdot \vec{dl} = \iint \vec{J} \cdot \vec{ds} + \iint \frac{\partial \vec{D}}{\partial t} \cdot \vec{ds}$

③ $\vec{D} = \epsilon \vec{E}$, $\vec{B} = \mu \vec{H}$, $\vec{J} = \sigma \vec{E}$

④ $\oiint \vec{D} \cdot \vec{ds} = \iiint \rho_v \, dv$, $\oiint \vec{B} \cdot \vec{ds} =$ Zero

* Divergence Theorem: $\oiint_S \vec{D} \cdot \vec{ds} = \iiint_V \nabla \cdot \vec{D} \, dv$ **

* StoK's Theorem: $\oint \vec{H} \cdot \vec{dl} = \iint_S \nabla \times \vec{H} \cdot \vec{ds}$ **

⇒ * Maxwell's Equations in differential form:

$\oint \vec{E} \cdot \vec{dl} = \iint_S \nabla \times \vec{E} \cdot \vec{ds} = -0 - \frac{\partial}{\partial t} \iint_S \vec{B} \cdot \vec{ds}$ $\quad l \to \Delta l \to 0$, $s \to \Delta s \to 0$, $s' \to \Delta s' \to 0$

$\Rightarrow \nabla \times \vec{E} \cdot \vec{\Delta s} \simeq -0 - \frac{\partial}{\partial t} (\vec{B} \cdot \vec{\Delta s}) = -\frac{\partial \vec{B}}{\partial t} \cdot \vec{\Delta s} \Rightarrow \boxed{\nabla \times \vec{E} = 0 - \frac{\partial \vec{B}}{\partial t}}$ ---- ①

⇒ As the same: $\boxed{\nabla \times \vec{E} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$ ---- ②

$\boxed{\vec{D} = \epsilon \vec{E}, \vec{B} = \mu \vec{H}, \vec{J} = \sigma \vec{E}}$ ---- ③ $\qquad \boxed{\nabla \cdot \vec{D} = \rho_v, \ \nabla \cdot \vec{B} = 0}$ ---- ④

for: $\left[\nabla \times \vec{H} = \vec{J} + \partial \vec{D}/\partial t = \frac{\sigma}{\epsilon} \vec{D} + \frac{\partial \vec{D}}{\partial t} \right]$ take the divergence to it.

$\Rightarrow^{part(1)} \ 0 = \nabla \cdot \vec{J} + \frac{\partial}{\partial t} \rho_v \Rightarrow \nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}$ ⇒ GKCL at a point.

$\Updownarrow$

This called: $\boxed{\oiint \vec{J} \cdot \vec{ds} = -\frac{\partial}{\partial t} \iiint \rho_v \, dv}$

"Continuoty Relation"

part (2):

$$\Rightarrow \nabla\cdot\left[\nabla\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}\right] \Rightarrow 0 = \nabla\cdot\frac{\sigma\vec{D}}{\epsilon} + \nabla\cdot\frac{\partial\vec{D}}{\partial t}$$

$$\Rightarrow 0 = \overbrace{\frac{\sigma}{\epsilon}}^{\frac{1}{\tau}}\rho_v + \frac{\partial\rho_v}{dt} \Rightarrow \frac{\partial\rho_v}{\partial t} = \frac{-\sigma}{\epsilon}\rho_v \Rightarrow \ln|\rho_v| = \frac{-t}{\tau} + \ln\rho_o$$

$$\Rightarrow \boxed{\rho_v = \rho_o\, e^{-t/\tau}}$$ ⇒ "Relaxation Relation"

*some numbers:

$\tau = \frac{\epsilon}{\sigma} \simeq \frac{10^{-11}}{\sigma}$ sec

σ (v/m)	τ (sec)	10τ
10^{7}	10^{-18}	10^{-17}
10^{-6}	10^{-5}	10^{-4}
10^{-19}	10^{8}	10^{9}

at $t = 10\tau$: $\rho_v = \rho_o e^{-10} \simeq 0$.

* Note:

if $\tau \ll 1$ ⇒ "good Conducting".

$\tau \gg 1$ ⇒ "good Dielectric".

**حيث أن خصائص المواد تتغير تبعاً للمرجع الزمني.

* Time Harmonic Sources & Fields:

$\rho, \vec{J}, \vec{E}, \vec{D}, \vec{H}, \vec{B}$ $\left(\begin{matrix} x, y, z \\ r, \phi, z \\ r, \theta, \phi \end{matrix}, t\right)$ → $\sin\omega t$ or $\cos\omega t$ → $e^{\pm j\omega t}$

↳ most of time we take $e^{+j\omega t}$

⇒ Now Re-writing for Maxwell's Eqs. (Max. Eqs. in Harmonic fields):

① $\boxed{\nabla\times\vec{E} = -0 - j\omega\vec{B}} = -0 - j\omega\mu\vec{H}$

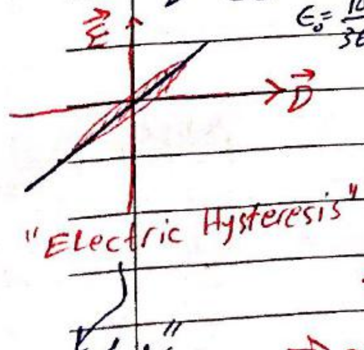
② $\boxed{\nabla\times\vec{H} = \vec{J} + j\omega\vec{D}} = \vec{E}(\sigma + j\omega\epsilon)$

③ $\vec{B} = \mu\vec{H}$, $\vec{D} = \epsilon\vec{E}$, $\vec{J} = \sigma\vec{E}$

④ $\nabla\cdot\vec{D} = \rho_v$, $\nabla\cdot\vec{B} = 0$

* Magnetic & Electric Hysteresis:

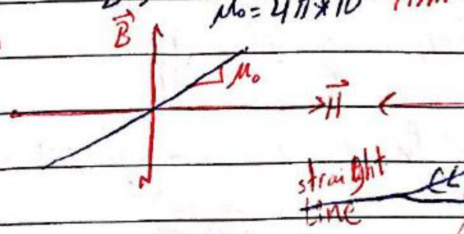
$\vec{B} = \epsilon\vec{E}$ $\qquad$ $\epsilon_o = \frac{10^{-9}}{36\pi}$ F/m

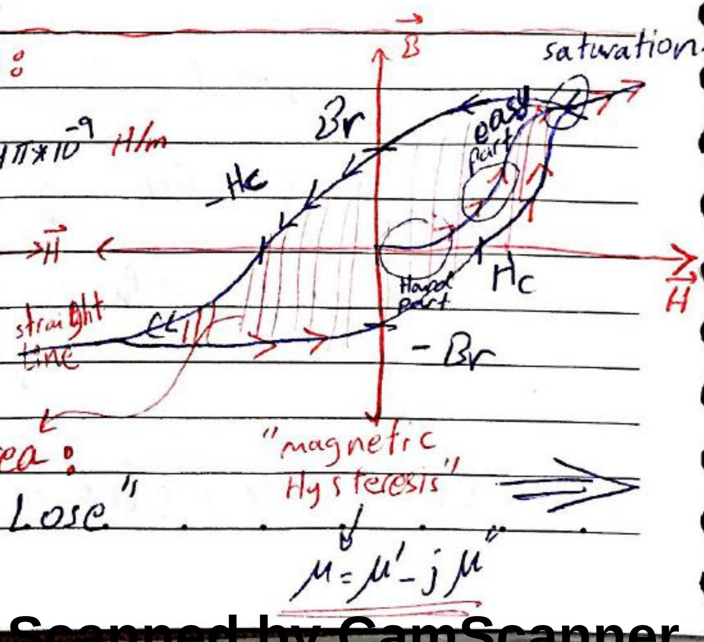


"Electric Hysteresis"

$\epsilon = \epsilon' - j\epsilon''$

$\vec{B} = \mu\vec{H}$ $\qquad$ $\mu_o = 4\pi\times10^{-9}$ H/m





The red shaded area: ⇒ called: "Hysteresis Lose".

"magnetic Hysteresis" ⇒

$\mu = \mu' - j\mu''$

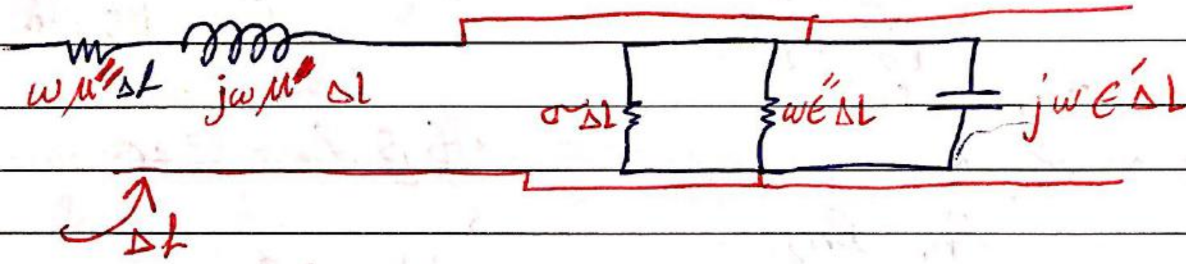
$$\Rightarrow \mu \equiv \underset{\text{for store}}{\mu'} - j\,\underset{\text{for lose}}{\mu''}$$

No.

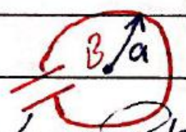
⇒ Maxwell's Equations:

$$\nabla \times \vec{E} = -0 - \{ j\omega\mu' + \omega\mu'' \}\vec{H}$$
$$\nabla \times \vec{H} = (\sigma + \omega\epsilon'' + j\omega\epsilon')\vec{E}$$

* Thevenin CKt representation:



* Example:



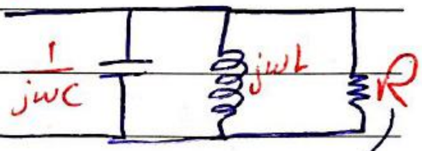
Capacitor

$(C) \equiv \frac{\epsilon A}{d}$

↳ loop represent a block of the Inductor.

$(L) \equiv \frac{\psi_{mL}}{I} = \frac{BA}{I}$ — B at the centre (Derived in EM(I)), $A = \pi a^2$

equivelant cKt:



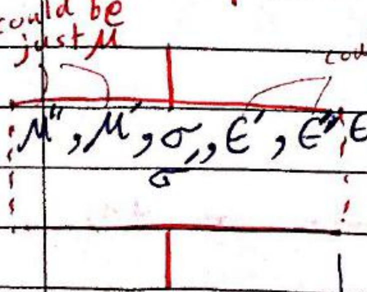
radiation resistance.

fres !!

$f_{res} = \frac{\omega}{2\pi}$, where $\omega = \frac{1}{\sqrt{LC}}$

* Ex: parallel plate capacitor:

$C = \frac{\epsilon' A}{d}$



Assume No μ here.

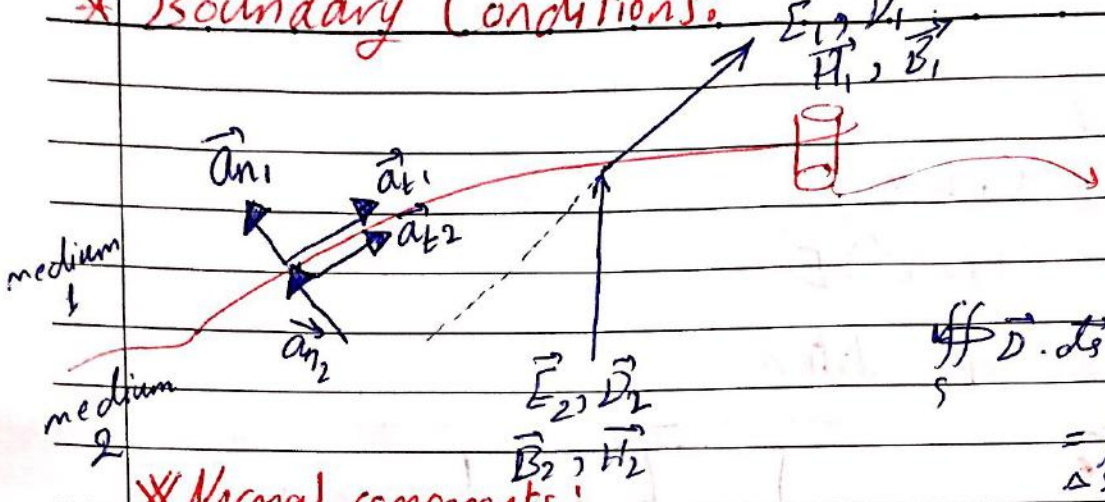
$\frac{\sigma A}{L} = G_{o.L}$

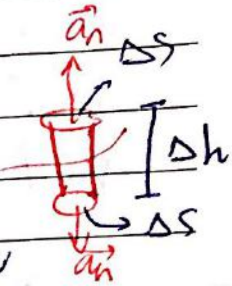
$G_{h.L} = \frac{\sigma' A}{L}$

ohmic loss.

hysterysis loss.

* Boundary Conditions:





$$\oint\!\!\oint_S \vec{D}\cdot\vec{ds} = \iiint \rho_v\, dv$$

$$= \iint_{\Delta S} \rho_s \int_{\Delta h} \rho_v\, dh \quad \text{--- (1)}$$

※ Normal components:

$$\oint\!\!\oint_S \vec{B}\cdot\vec{ds} = \text{Zero} \quad \text{--- (2)}$$

$B_{n_1}\,\Delta S \simeq B_{n_2}\,\Delta S$

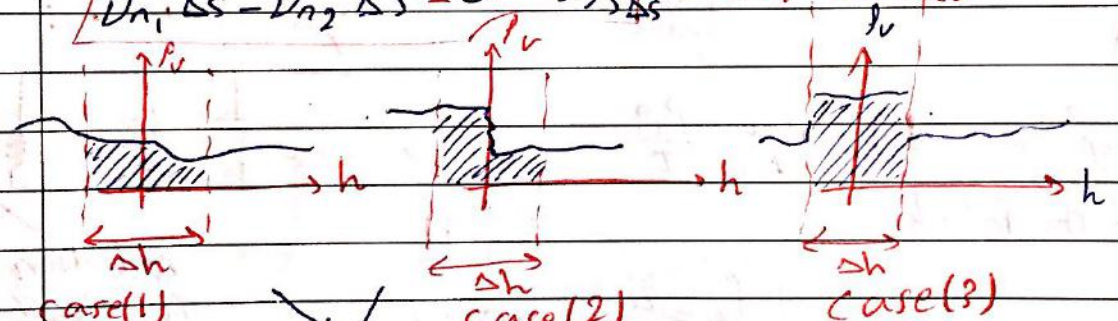
$\Delta h \to \phi$

$\Delta S \to 0 \Rightarrow \boxed{B_{n_1} = B_{n_2}}$

from (1):

$D_{n_1}\Delta S - D_{n_2}\Delta S = \rightarrow 0 \rightarrow$ if case (1) or (2)

$\rightarrow \rho_s \Delta S \rightarrow$ if case (3).



$$\lim_{\Delta h \to 0} \int_{\Delta h} \rho_v\, dh = \text{Zero}$$

$$\lim_{\Delta h \to 0} \int_{\Delta h} \rho_v\, dh = \rho_s$$

$\Delta S \to 0 \Rightarrow \boxed{D_{n_1} - D_{n_2} = 0 \text{ or } \rho_s}$

※ Tangential Components:

Δh, ΔL

$$\oint_{\Delta h \to 0} \vec{E}\cdot\vec{dl} = -0 \iint \frac{\partial \vec{B}}{\partial t}\cdot\vec{ds} \Rightarrow \boxed{E_{t_1} = E_{t_2}} \longleftrightarrow B_{n_1} = B_{n_2}$$

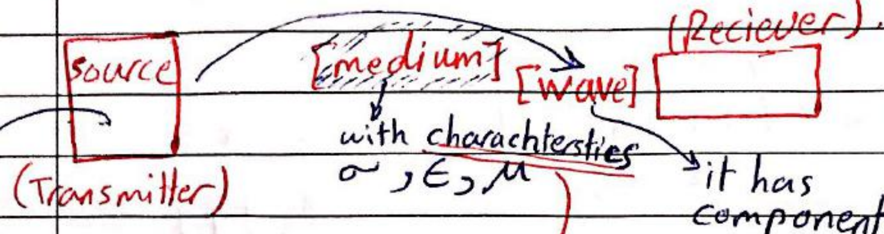
$$\rightarrow H_{t_1} - H_{t_2} = 0 \text{ or } K \Rightarrow \text{with } \vec{a}_n \times \vec{H} = \vec{K}$$

* * *

No.

* Wave Equation & Waves:

wave: that means used to transmitt. (energy / event / information) from one point to another.



*معظم الموجات تحتاج وسط بين T و R باستثناء بعض الموجات مثل electromagnetic wave

also it has components but, we just care for (freq) (ω or f)

* Source free medium.
$\Rightarrow (\nabla\cdot\vec{B}=0 \ \& \ \nabla\cdot D=0)$

** Medium: is homogenous & isotropic.
$\hookrightarrow (\sigma, \epsilon, \mu)$ are constants

* In the medium: Max. Equ. for EM fields.

$$\nabla\times\vec{E} = -j\omega\mu\vec{H} \longrightarrow \equiv Z \ \Omega/m$$ (without the minus sign).

$$\nabla\times\vec{H} = (\sigma + j\omega\epsilon)\vec{E} \longrightarrow \equiv Y \ \mho/m.$$

$$\Rightarrow \nabla\times\nabla\times\vec{E} = -Z\,\nabla\times\vec{H} = -ZY\vec{E}$$

$$\gamma^2 \leftarrow (j\omega\mu)(\sigma+j\omega\epsilon)$$

$j=\sqrt{-1}=e^{j\frac{\pi}{2}}$, $\sqrt{j} = e^{j\frac{\pi}{4}} = \angle\pi/4$

$$\Rightarrow \gamma = \sqrt{ZY} = (\omega\mu)^{\frac{1}{2}}(\sigma^2+\omega^2\epsilon^2)^{\frac{1}{4}} \angle\left(\frac{\pi}{4}+\frac{1}{2}\tan^{-1}\frac{\omega\epsilon}{\sigma}\right)$$

$$\Rightarrow \boxed{\gamma = \alpha + j\beta}$$

γ: Medium charachterstic & source freq.
α: Attenuation Constant $\left(\frac{\text{neper or dB}}{m}\right)$
β: used for phase shift $\left(\frac{\text{rad or degree}}{m}\right)$

$$\boxed{\begin{matrix}\alpha\\ \beta\end{matrix} = \left[(\omega\mu)\sqrt{\sigma^2+\omega^2\epsilon^2}\right]^{1/4} \begin{matrix}\cos\\ \sin\end{matrix}\left[\frac{\pi}{4}+\frac{1}{2}\tan^{-1}\frac{\omega\epsilon}{\sigma}\right]}$$

* $\sqrt{\sigma+j\omega\epsilon} = (\sigma^2+\omega^2\epsilon^2)^{\frac{1}{4}}\angle\frac{1}{2}\tan^{-1}\frac{\omega\epsilon}{\sigma}$

Continue.

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$$\Rightarrow \nabla\times\nabla\times\vec{E} = -\gamma^2\vec{E}$$

$$\nabla(\nabla\cdot\vec{E}) - \nabla^2\vec{E} = -\gamma^2\vec{E}$$

$\Rightarrow \vec{E} = \dfrac{\vec{D}}{\epsilon}$, $\nabla\cdot\vec{D} = 0$

$\dfrac{1}{\epsilon}\nabla(\nabla\cdot\vec{D}) = \text{Zero}$

$$\Rightarrow \nabla^2\vec{E} - \gamma^2\vec{E} = 0$$

$$\Rightarrow \left[\nabla^2\begin{pmatrix}E_x\\E_y\\E_z\end{pmatrix}(x,y,z) - \gamma^2\begin{pmatrix}E_x\\E_y\\E_z\end{pmatrix}(x,y,z) = 0\right] \quad * \cdot e^{j\omega t} \quad \boxed{1}$$

* remember:

$\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial}{\partial z^2}$ * 2^{nd} order homogenous partial differential equation.

* As the same way:

$\Rightarrow$ we derive that: $\nabla^2\vec{H}_{(x,y,z)} - \gamma^2\vec{H}_{(x,y,z)} = 0$ $\boxed{2}$

$\boxed{1}$ & $\boxed{2}$ are called:

"electromagnetic wave equation"

* Assuming we have one $\vec{E}$ in the direction of x and changing just in Z:

$E_{x,\cancel{y},\cancel{z}}(\cancel{x},\cancel{y},z)$ $\xrightarrow{\text{equation } \boxed{1}}$ $\dfrac{\partial^2 E_x}{\partial z^2} - \gamma^2 E_x = 0$

solve for it:

$E_x(z) = \underset{\text{constant}}{E^+} e^{-\gamma z} + E^- e^{\gamma z} \Rightarrow E_x(z,t) = E^+ e^{j\omega t - \gamma z} + E^- e^{-j\omega t + \gamma z}$

where $\gamma = \alpha + j\beta$:

$$E_x(z,t) = \left[E^+ e^{-\alpha z}\right] e^{j(\omega t - \beta z)} + \left[E^- e^{+\alpha z}\right] e^{j(\omega t + \beta z)} \quad V/m.$$

$\Rightarrow$ we could find $\vec{H}$ from: $\nabla\times\vec{E} = -j\omega\mu\vec{H}$

let $\vec{H}$ be: $H_y(z,t) = H^+ e^{j(\omega t - \beta z)} + H^- e^{j(\omega t + \beta z)}$

* Assume that the medium is Lossless:
(considering only the first term)

⇒ This assumption: $\alpha = 0$ ⇒ in general ($\alpha l = 0$)

and let $\beta = K$ ⇓ (No attenuation)

→ wave Number.

⇒ $E_x(Z,t) = E^+ e^{j(\omega t - KZ)}$, $H_y(Z,t) = H^+ e^{j(\omega t - KZ)}$

⇒ $\boxed{H^+ = \frac{E^+}{\eta}}$ where $\eta \equiv \sqrt{\frac{j\omega\mu}{j\omega\epsilon}} = 120\pi\,\Omega = 377\,\Omega$

⇒ "intrensic impedance" of the medium

just for free space.

$\eta = \frac{120\pi}{\sqrt{\epsilon_r}}$ ⇒ practically for lossless medium.

Now: $E_x = E^+ e^{j(\omega t - KZ)}$ V/m.

$H_y = \frac{E^+}{\eta} e^{j(\omega t - KZ)}$ A/m.

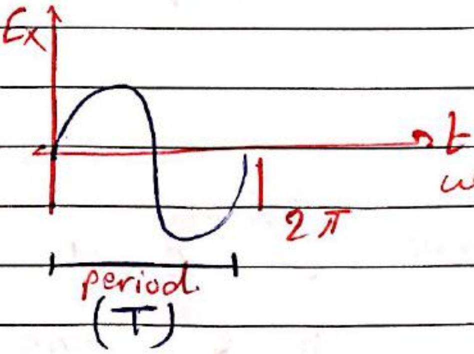
⇒ * physical expressions:

$E_x(Z,t) = E^+ \sin(\omega t - KZ)$

for $Z=0$
$E_x(0,t) = E^+ \sin\omega t$

$2\pi f = \omega$, $T = 2\pi$

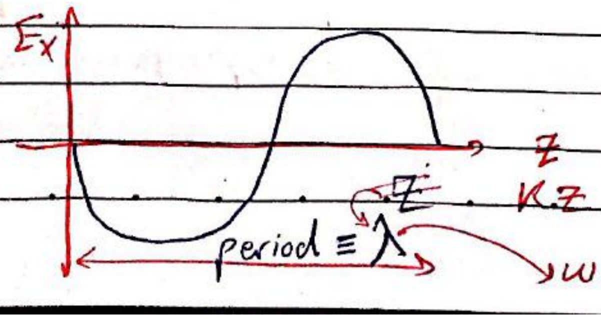
$f T = 1$ $f = \frac{1}{T}$



$t=0$, Z

$E_x(0,t) = -E^+ \sin KZ$

$KZ = 2\pi$
⇒ $K\lambda = 2\pi$



wave length.

$K = \omega\sqrt{\mu\epsilon}$, $K\lambda = 2\pi \Rightarrow \omega\sqrt{\mu\epsilon}\,\lambda = 2\pi$

No.

$\Rightarrow \beta = K = [\omega\mu]^{\frac{1}{2}}\,[\sigma^2 + \omega^2\epsilon^2]^{\frac{1}{4}} \sin\left[\frac{\pi}{4} + \frac{1}{2}\tan^{-1}\frac{\omega\epsilon}{\sigma}\right]$

$\boxed{\sigma = 0}$

$= \omega\sqrt{\mu\epsilon}$

$\eta = \sqrt{\frac{\mu}{\epsilon}}$

source ω — $\epsilon\mu$

f

$\underbrace{K}_{\lambda} = \omega\sqrt{\mu\epsilon}$

$\begin{matrix} E_x \\ H_y \end{matrix}(z,t) = \begin{matrix} E^+ \\ \frac{E^+}{\eta} \end{matrix} e^{j(\omega t - Kz)} = \begin{matrix} E^+ \\ \frac{E^+}{\eta} \end{matrix} \angle \psi(z,t)$

$\boxed{\psi(z,t) \equiv \text{phase} = \omega t - Kz}$

$\Rightarrow \psi(z,t) = \text{Constant} = \omega t - Kz$

(differentiate)! $0 = \omega\,dt - K\,dz$

$\Rightarrow \frac{dz}{dt} = \frac{\omega}{K} \equiv v_p$ (phase velocity)

$v_p = \frac{\omega}{\omega\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu\epsilon}}$ $\boxed{v_p = \frac{1}{\sqrt{\mu\epsilon}}}$

for $\mu_0\,\epsilon_0$:

$v_p = 3 \times 10^8$ m/s

if $\epsilon = \epsilon_0\epsilon_r$

$v_p = \frac{3 \times 10^8}{\sqrt{\epsilon_r}}$

in general:

$-\infty < v_p < \infty$

if v_p was mathematical velocity

source f — Medium σ, (ϵ, μ) $\rightarrow v_p = \frac{1}{\sqrt{\mu\epsilon}}$

homo & isopotric.

source free.

$\vec{E}$ & $\vec{H}$

$\eta = \sqrt{\frac{\mu}{\epsilon}}$

v_p mathematical

$\hookrightarrow$ one freq.

event (group of freq)

$\hookrightarrow$ physical velocity.

* group of freq: $\Rightarrow$ two freq.

f_1, f_2

$\omega_1 \quad \omega_2 \qquad \omega-\Delta\omega \qquad \omega+\Delta\omega \quad \omega$

$\qquad\qquad \beta-\Delta\beta \qquad \beta+\Delta\beta \quad \beta$

$$E_x(z,t) = \sin\left[(\omega-\Delta\omega)t - (\beta-\Delta\beta)z\right] + \sin\left[(\omega+\Delta\omega)t - (\beta+\Delta\beta)z\right]$$

$$= \cancel{\sin(\omega t-\beta z)\cos(\Delta\omega t-\Delta\beta z) - \cos(\omega t-\beta z)\sin(\Delta\omega t-\Delta\beta z)}$$

$$\cancel{+\sin(\omega t-\beta z)\cos(\Delta\omega t-\Delta\beta z) + \cos(\omega t-\beta z)\sin(\Delta\omega t-\Delta\beta z)}$$

$$= 2\sin(\omega t-\beta z)\cos(\Delta\omega t-\Delta\beta z)$$

$\Rightarrow v_p = \omega/\beta \;,\; v_g = \frac{d\omega}{d\beta}$ = group velocity;

$-\infty \le v_p \le \infty \qquad 0 < v_g \le 3\times 10^8 \quad \Rightarrow$ physical velocity.

$v_p = \frac{\omega}{\beta}$ $\quad$ lossless: $\frac{\omega}{\omega\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu\epsilon}}$

$v_g = \frac{\partial\omega}{\partial\beta}$ $\quad$ $\frac{\partial\omega}{\sqrt{\mu\epsilon}\,\partial\omega} = \frac{1}{\sqrt{\mu\epsilon}}$

$v_p = \frac{1}{\sqrt{\mu\epsilon}} = v_g$

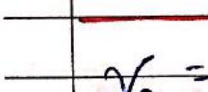
could equal in Numbers but can't be physical = mathmatical

medium char:

σ, ϵ, μ & $\omega \Rightarrow \beta = (\omega\mu)^{1/2}(\sigma^2+\omega^2\epsilon^2)^{1/4}\sin\left[\frac{\pi}{4}+\frac{1}{2}\tan^{-1}\left(\frac{\omega\epsilon}{\sigma}\right)\right]$

$\hookrightarrow$ Non linear function $f(\omega)$

ϵ, μ & $\omega \Rightarrow \beta = \omega\sqrt{\mu\epsilon}$



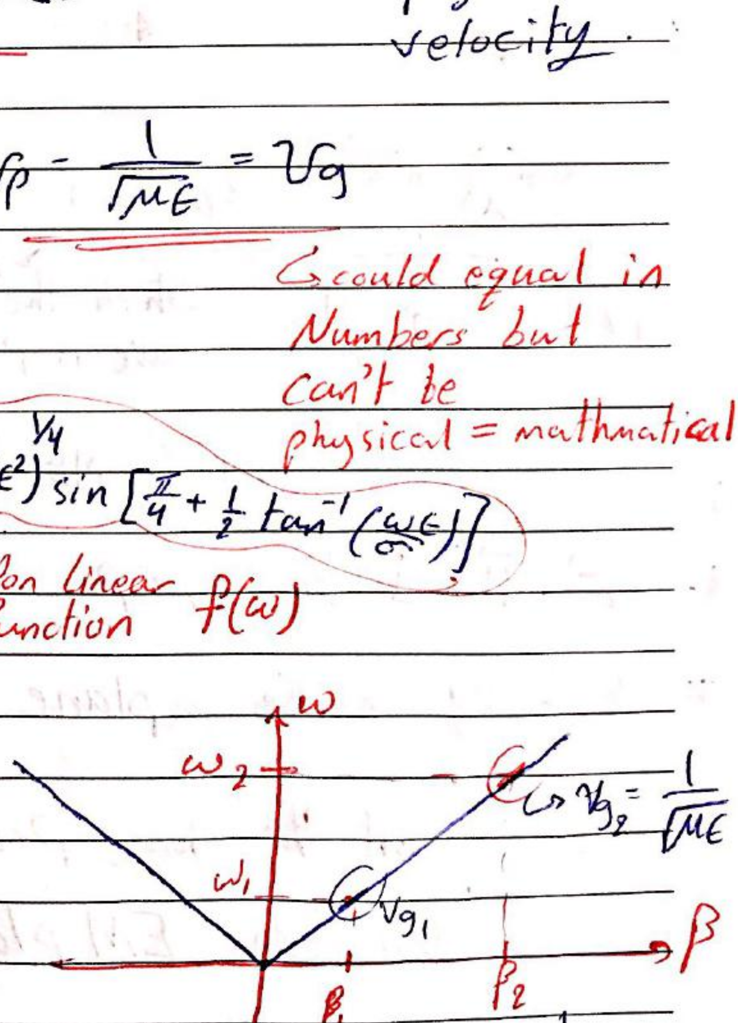
lossless

$\beta = \omega\sqrt{\mu\epsilon}$

$v_p = \frac{1}{\sqrt{\mu\epsilon}} = v_g$

$\hookrightarrow$ non distorting medium.

Replica (the same)

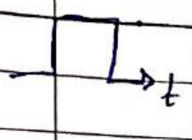


$v_p = \frac{1}{\sqrt{\mu\epsilon}}$

$v_{g_1} = v_{g_2}$

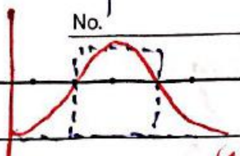
المساحة تحت المنحنى الأحمر تساوي
المساحة تحت المربع المنقط.

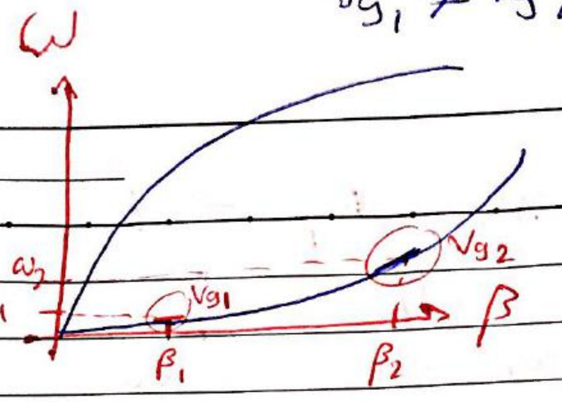
$v_{g_1} \neq v_{g_2}$



Lossless

$\beta = f(\omega)$ → distorting medium. (dispersive medium).





$v_{p_1} < v_{p_2}$

$E_x(Z,t) = E^+ \angle \omega t - KZ + E^- \angle \omega t + KZ \Rightarrow t \uparrow \ Z \downarrow$

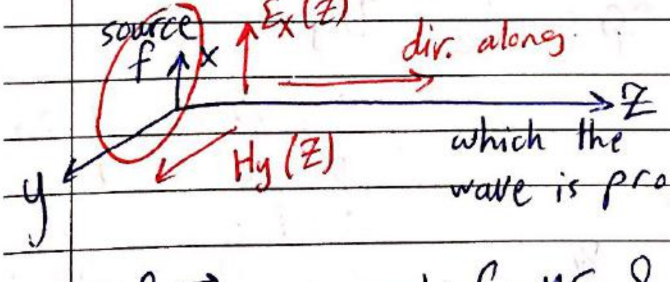
if we want to maintain this value since t always increasing so Z must be increasing.

$t \nearrow \ Z \nearrow$

$$E_x(Z,t) = \underline{E^+ e^{j(\omega t - KZ)}} + \underline{E^- e^{-j(\omega t + KZ)}} \quad V/m$$

Forward incident wave →

Backward reflected wave ←



$E_x = E^+ e^{j(\omega t - KZ)}$

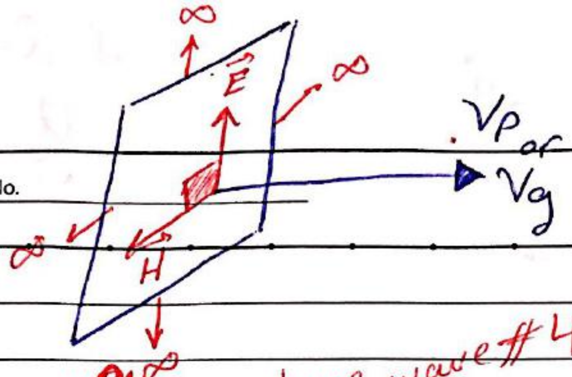
$H_y = \frac{E^+}{\eta} e^{j(\omega t - KZ)}$

* $\vec{E}$ & $\vec{H}$ ⟹ satisfy ME. & wave equation.
* $(\vec{E} \perp \vec{H}) \perp$ dir of prop.
* E_x & H_y make a plane (xy-plane) & they are constant in this plane.

↳ All the three points above: "Uniform EM plane wave" (UEMPW)

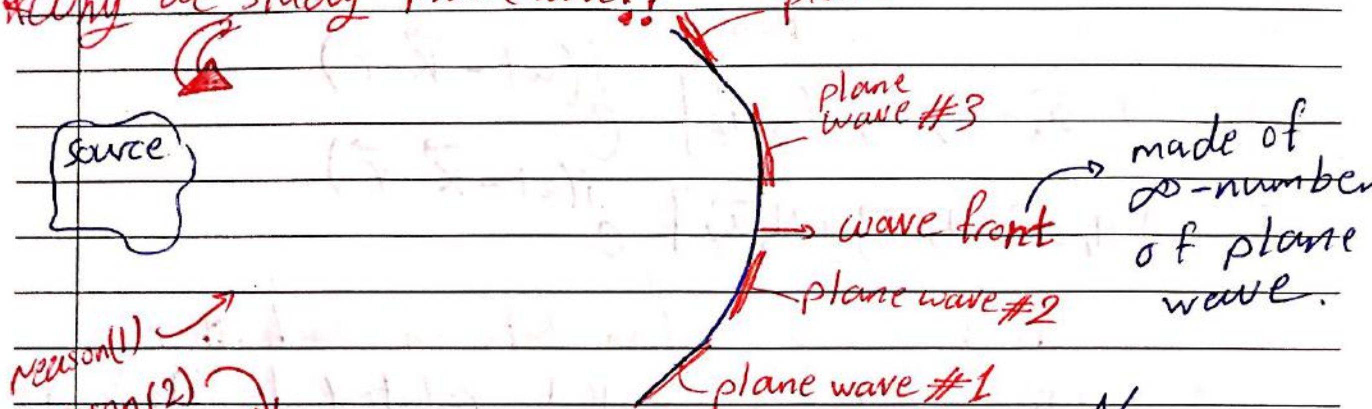
↳ so, Now we will take about (plane wave)

* plane Wave:

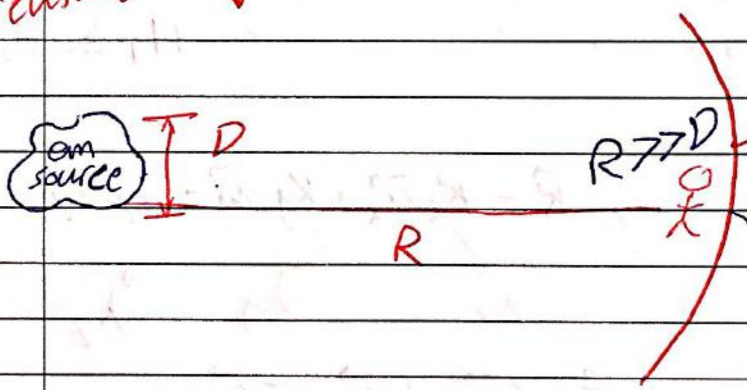


No.

* Why we study plane wave?!



$$\Rightarrow = \sum_{n=1}^{N} (\text{plane wave})_n$$



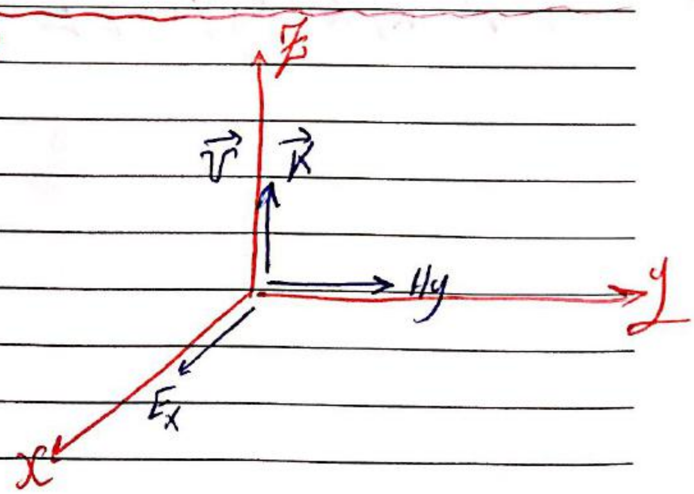
* locally to the R_x ⇒ wave front is plane wave

* General Plane Wave:

$\vec{K} = \omega\sqrt{\mu\epsilon} = \frac{\omega}{\vec{v}}$

Now:

$\Rightarrow E_x(z,t) = E^+ e^{j[\omega t - \vec{K} \cdot \vec{z}]}$



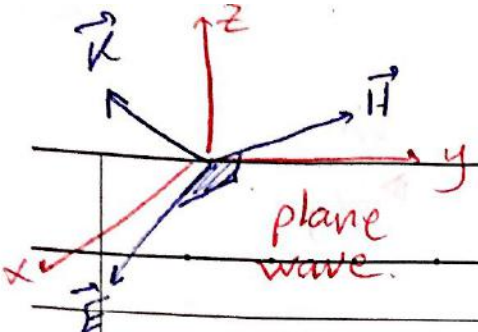
$\vec{K} = K\vec{a}_z$

$\vec{z} \Rightarrow \vec{r} = x\vec{a}_x + y\vec{a}_y + z\vec{a}_z$

it becomes: $(\vec{K} \cdot \vec{r})$.

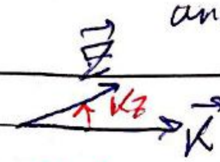
$\vec{K} \cdot \vec{r} = Kz$

as we saw it before



if $\vec{K}$ was: $\longrightarrow \vec{K}$
and $\vec{Z} \longrightarrow \vec{Z}$

$\vec{K} = K\vec{a_z}$



$\vec{K}$ will have three components.

* general wave equation:

$$\vec{E} = \left[E_1\vec{a_x} + E_2\vec{a_y} + E_3\vec{a_z}\right] e^{j(\omega t - \vec{K}\cdot\vec{r})}$$

&

$$\vec{H} = \left[H_1\vec{a_x} + H_2\vec{a_y} + H_3\vec{a_z}\right] e^{j(\omega t - \vec{K}\cdot\vec{r})}$$

* E_3 & H_3 has No relation between eachother.

if we talking about E_1, it will be related to H_2 & H_3
" " " " E_2, " " " " " H_1 & H_3
and so on.

$$\eta = \frac{\sqrt{E_1^2 + E_2^2 + E_3^2}}{\sqrt{H_1^2 + H_2^2 + H_3^2}}$$

$\vec{K} = K_x\vec{a_x} + K_y\vec{a_y} + K_z\vec{a_z}$

$\frac{2\pi}{\lambda}$ λ_x λ_y λ_z

$\vec{r} = x\vec{a_x} + y\vec{a_y} + z\vec{a_z}$

so $\vec{K}\cdot\vec{r}$:

$\vec{K}\cdot\vec{r} = K_x X + K_y Y + K_z Z$

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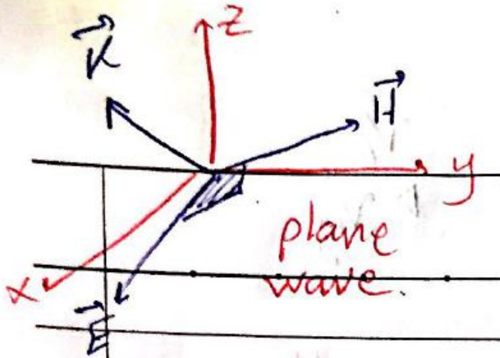
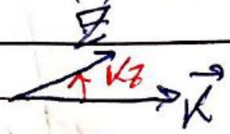


if $\vec{K}$ was: $\longrightarrow \vec{K}$
and $\vec{Z} \longrightarrow \vec{Z}$

$\vec{K} = K\vec{a_z}$



$\vec{K}$ will have three components.

* general wave equation:

$$\vec{E} = \left[E_1 \vec{a_x} + E_2 \vec{a_y} + E_3 \vec{a_z}\right] e^{j(\omega t - \vec{K}\cdot\vec{r})}$$

&

$$\vec{H} = \left[H_1 \vec{a_x} + H_2 \vec{a_y} + H_3 \vec{a_z}\right] e^{j(\omega t - \vec{K}\cdot\vec{r})}$$

* E_3 & H_3 has No relation between eachother.

if we talking about E_1, it will be related to H_2 & H_3
" " " " E_2, " " " " " H_1 & H_3
and so on.

$$\eta = \frac{\sqrt{E_1^2 + E_2^2 + E_3^2}}{\sqrt{H_1^2 + H_2^2 + H_3^2}}$$

$\frac{2\pi}{\lambda} \leftarrow \vec{K} = K_x \vec{a_x} + K_y \vec{a_y} + K_z \vec{a_z}$

λ_x λ_y λ_z

so $\vec{K}\cdot\vec{r}$:

$\vec{r} = x\vec{a_x} + y\vec{a_y} + z\vec{a_z}$

$$\vec{K}\cdot\vec{r} = K_x X + K_y Y + K_Z Z$$

↳ First 3 weeks.

4/10/2016

* Electromagnetic wave Polarization:

em wave $\longrightarrow \vec{E}(x,y,z,t) \rightarrow e^{j\omega t}$ $\Rightarrow \vec{E}$ at fixed space (location).
↳ $\vec{H}$

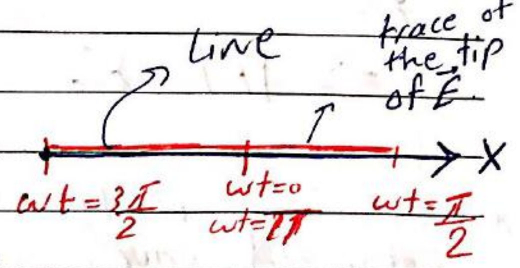
↳ $\vec{E}(\vec{r_0}, t)$ — trace of the tip of $\vec{E}$ as a function of time is the wave polarization.

No.

Ex:(1) em wave given by $\begin{matrix} E_x \\ H_y \end{matrix}(Z,t) = \begin{matrix} E^+ \\ \frac{E^+}{\eta} \end{matrix} e^{j(\omega t - ZK)}$

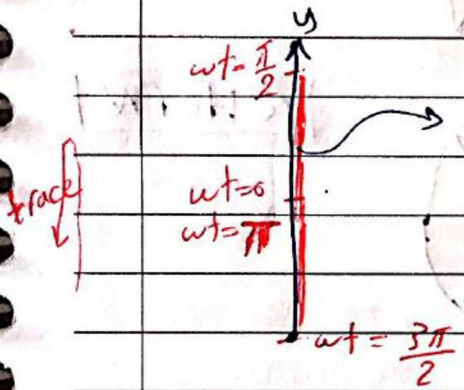
Need to identify the polarization of this wave ?

$E_x(Z,t)\Big|_{Z=0} = E^+ e^{j\omega t} \underset{\text{physical}}{=} E^+ \sin\omega t$



* wave is said to be Linearly polarized wave (LPW) $\Rightarrow$ along x.

Ex:(2) em wave $\begin{matrix} E_y \\ H_x \end{matrix} = \begin{matrix} E^+ \\ -\frac{E^+}{\eta} \end{matrix} e^{j(\omega t - Kz)}$ need pol. ?



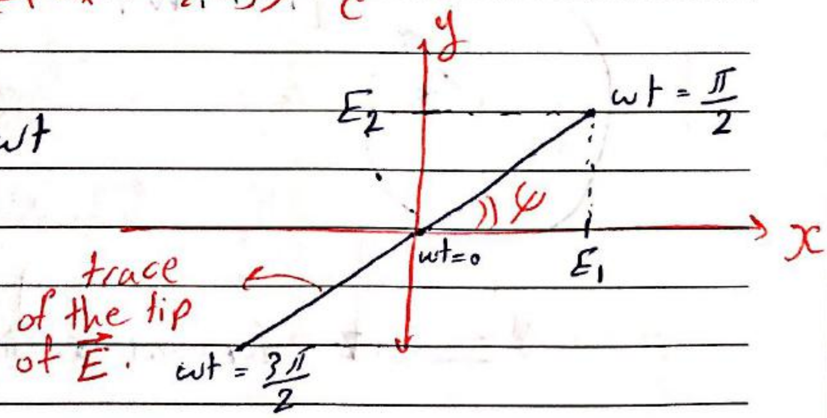
trace is a line

$\Rightarrow$ (LPW) along y.

Ex:(3) em wave $\vec{E} = (E_1 \vec{a}_x + E_2 \vec{a}_y)\, e^{j(\omega t - KZ)}$

Need pol. ?

$\vec{E}\Big|_{Z=0} = (E_1 \vec{a}_x + E_2 \vec{a}_y) \sin\omega t$



(LPW) along the line making an angle ψ w.r.t x-axis

Ex:(4) given an em wave with $\vec{E} = (E_1 \vec{a}_x + e^{+j\delta} E_2 \vec{a}_y)\, e^{j(\omega t - Kz)}$

Need to identify the wave polarization under assuming $E_1 = E_2 = E_0$?? $\Rightarrow\Rightarrow$

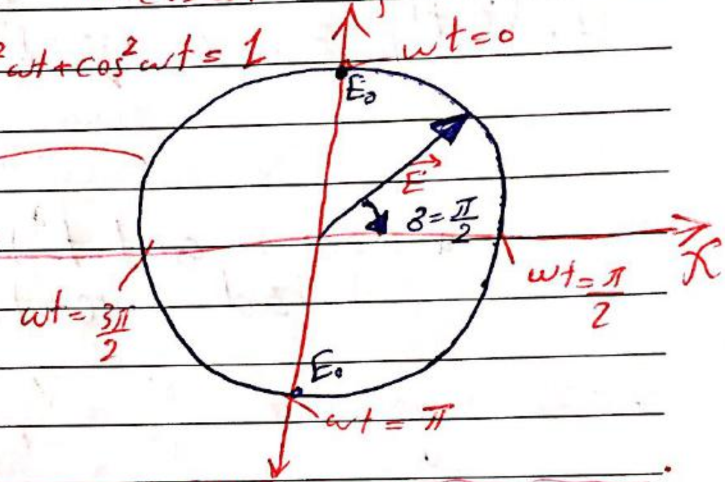
with $\delta = \pm \frac{\pi}{2}$

⇒ taking $\delta = \frac{+\pi}{2}$ (we always taking $\vec{E}$ and don't care to $\vec{H}$)

$$\vec{E}\Big|_{z=0} = E_0 \sin \omega t \; \vec{a_x} + E_0 \underbrace{\sin\left(\omega t + \frac{\pi}{2}\right)}_{\cos \omega t} \vec{a_y} \quad V/m.$$

$\Rightarrow |\vec{E}| = E_0 \Rightarrow$ since $\sin^2 \omega t + \cos^2 \omega t = 1$

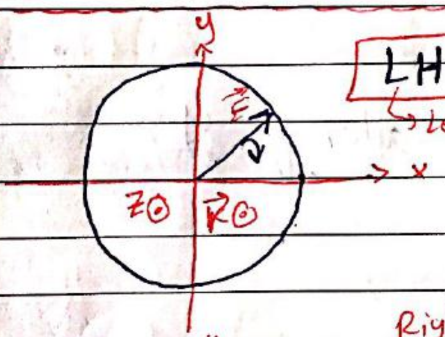
the trace is a circle



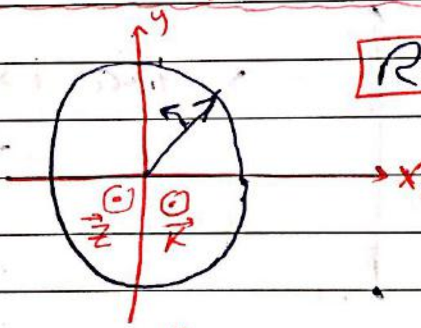
* wave is circularly polarized wave.

(CPW) ⇒ { clicking CW if $\delta = \frac{\pi}{2}$; clicking CCW if $\delta = \frac{-\pi}{2}$ }

Note:

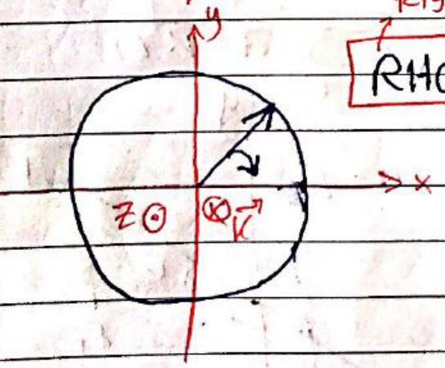


LHCPW → left handed.

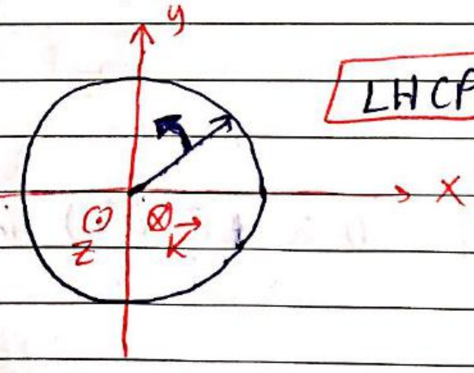


RHCPW

Right handed.



RHCPW



LHCPW

$e^{j(\omega t - KZ)} \longrightarrow +Z$ forward or incident.

$e^{j(\omega t + ZK)} \longrightarrow -Z$ backward or Reflected.

Ex: (5) em wave $\vec{E} = (E_1 \vec{a_x} + E_2 e^{j\delta} \vec{a_y})\, e^{j(\omega t - KZ)}$

(i) if ($E_1 = E_2$ or $E_1 \neq E_2$) & ($\delta = 0$ or $\delta = n\pi$) ⇒ (LPW)

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(ii) if $E_1 = E_2$ & $\delta = \pm(2n+1)\frac{\pi}{2}$

$\Rightarrow$ CPW

$\Rightarrow$ Otherwise (for both two cases) it is <u>Not</u> L or C PW

if $E_1 \neq E_2$ & $\delta \neq n\pi$

OR $E_1 = E_2$ & $\delta \neq (2n+1)\frac{\pi}{2}$

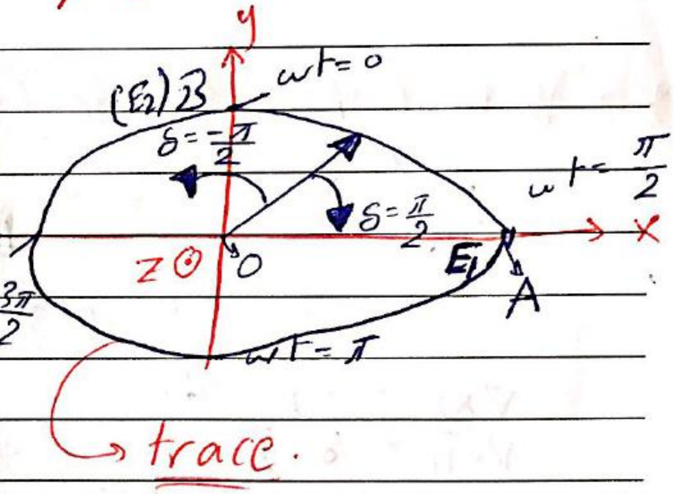
$E_1 \neq E_2$ & $\delta = (2n+1)\frac{\pi}{2}$

* given: $\vec{E} = \left(E_1 \vec{a}_x + E_2 \vec{a}_y e^{\pm j\frac{\pi}{2}}\right) e^{j(\omega t - KZ)}$

$\Rightarrow$ Assume $E_2 < E_1$, $\delta = \frac{\pi}{2}$

$\vec{E} = E_1 \sin\omega t \ \vec{a}_x + E_2 \cos\omega t \ \vec{a}_y$

at $Z=0$

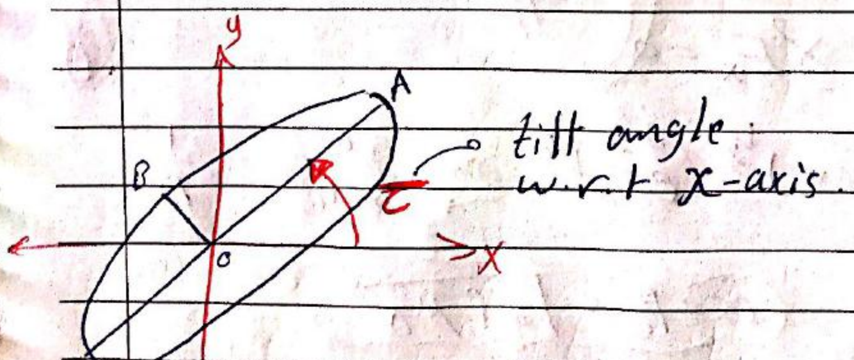


* wave is said to be Elliptically PW. (ELP) $\to$ LH or RH

$\Rightarrow$ with $\frac{\text{major axis}}{\text{minor axis}} = \frac{OA}{OB}$ = Axial Ratio $\equiv AR > 1$

also with tilt angle $(\tau) = 0^\circ$ (w.r.t x-axis)

$= 90^\circ$ (w.r.t y-axis)



***

AR $\to$ ~1 (CPW)

AR $\to$ (EPW)

AR $\to$ >>1 (LPW)

No. ⇐ بعض الفروق في الاستخدام لل linear PW & circular

** مابين Linear Horizontal و Linear vertical بنستخدم vertical لأن ال Horizontal يحتاج لتكلفة اقتصادية عالية جداً لأنه يحتاج أنه يكون على ارتفاع عالي عن الأرض حتى يستخدم بشكل فعّال بينما ال vertical يتم تثبيته بالأرض ونستفيد منه بشكل فعّال.

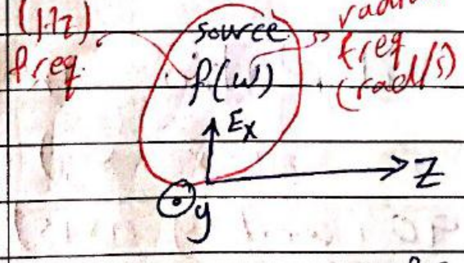
** مابين Linear و circular إذا كان الوسط الذي تنتقل فيه الموجة هو الغلاف الأيوني نستخدم circular لأن الغلاف الأيوني يعمل على حدوث التفاف بمقدار معين من الدرجات للموجة يعتمد على ترددها لذلك إذا كان Linear سيحدث ضياع للموجة بينما اذا كانت circular مهما حدث التفاف لا يؤثر.

** Wave Propagation in Good Conducting Medium:

$\sigma + \underbrace{j\omega\epsilon}_{\text{it will be neglected.}}$ ($j\omega\mu$) $\rightarrow \sigma >> \omega\epsilon \Rightarrow$ good conducting.

$$\nabla\times\vec{E} = -j\omega\mu\vec{H}$$
$$\nabla\times\vec{H} = \sigma\vec{E}$$

homo.
isotropic
source free ($\nabla\cdot\vec{D}=0$)
} $\sigma >> \omega\epsilon$



$$\nabla\times(\nabla\times\vec{E}) = -j\omega\mu\,\nabla\times\vec{H}$$
$$\nabla(\nabla\cdot\vec{E}) - \nabla^2\vec{E} = -j\omega\mu\sigma\vec{E}$$

$$\Rightarrow \frac{\partial^2 E_x}{\partial z^2} - \gamma^2 E_x = 0$$

$$\gamma = \sqrt{j\omega\mu\sigma} = (\omega\mu\sigma)^{1/2}\left(e^{j\frac{\pi}{2}}\right)^{1/2}$$ $\rightarrow \cos 45^\circ, \sin 45^\circ = \frac{1}{\sqrt{2}}$

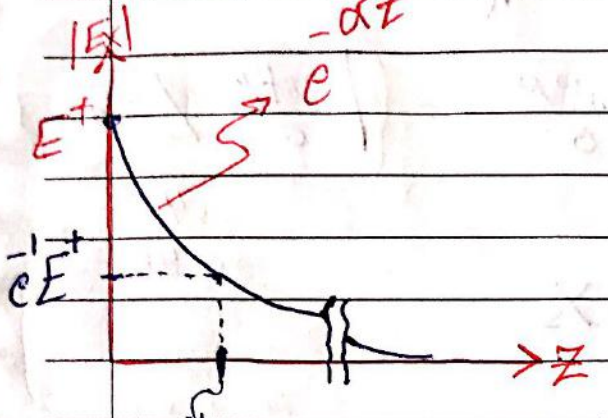
$$\therefore \boxed{\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} = \beta} \quad \text{or} \quad \alpha = \beta = \sqrt{\pi f \mu\sigma}$$

since $\omega = 2\pi f$

$$\Rightarrow E_x = \underbrace{\left(E^+ e^{-\alpha z}\right)e^{-j\beta z}}_{\text{forward}} + \underbrace{\left(E^- e^{\alpha z}\right)e^{+j\beta z}}_{\text{Backward}}$$

No.

⇒ as always taking Forward term:



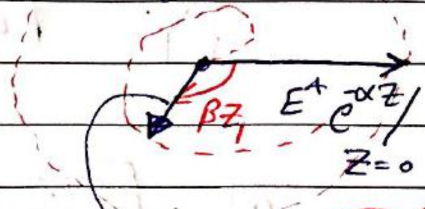
$z \equiv$ space constant

"skin depth" or "penetration depth"

⇒ we use the symbole δ for it

if $\alpha = 0$

$E_x|_{z=0} = E^+$



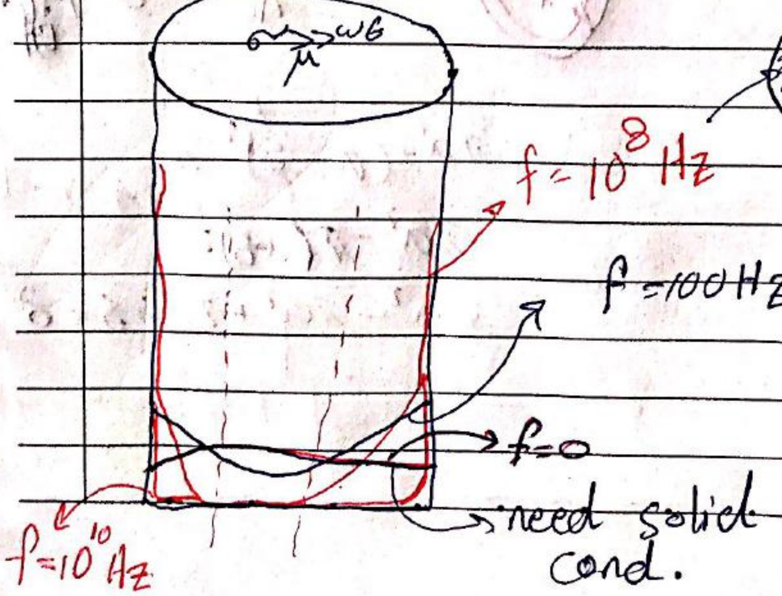
$$\boxed{E^+ e^{-\alpha z_1} \angle -\beta z_1}$$

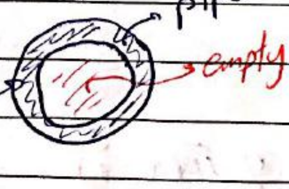
$$\boxed{\alpha = \sqrt{\pi f \mu \sigma}} \quad , \quad \boxed{\delta = \frac{1}{\alpha}} \quad ***$$

* if we assume $\pi\mu \simeq 10^{-6}$ & $\sigma \simeq 10^6$

↳ $\delta = \frac{1}{\sqrt{f}}$

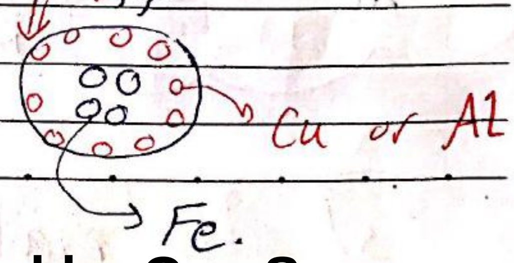
	f	δ
	→ 0 (DC)	→ ∞
power freq	100Hz	0.1m (10cm) (100mm)
(Radio freq) RF	10^8 Hz	0.1mm
M.wave	10^{10} Hz	$10\,\mu m$





pipe ⇒ mechanical is define.

f = 100Hz → نقوم بتقسيم المنطقة كما هي بالنقاط الحمراء.



$E_x(z,t) = E^+ e^{-\alpha z} e^{-j\beta z}$ ~~$E_x(z,t) = E^+ e^{-\alpha z} e^{-j\beta z}$~~

$\nabla \times \vec{E} = -j\omega\mu \vec{H} \implies H_y(z) = \frac{E^+}{\eta} e^{-\alpha z} e^{-j\beta z}$

$\implies \eta = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{2\pi f\mu}{\sigma}} \angle 45^\circ$

good cond.

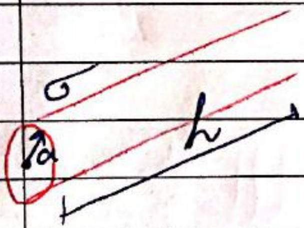
cos 45
sin 45

$\implies \eta = \sqrt{\frac{\pi f\mu}{\sigma}}(1+j) = R + jX$

$R = \sqrt{\frac{\pi f \mu}{\sigma}} = X$

assume:

$\frac{\pi\mu}{\sigma} \sim 10^{-12}$ ($\pi\mu \to 10^{-6}$, $\sigma \to 10^{6}$) $\implies R = 10^{-6}\sqrt{f}$

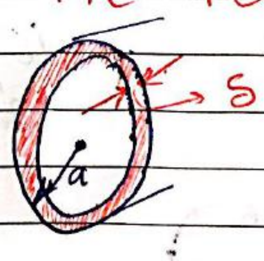


f	R
→ 0	→ 0
100 Hz	10 μΩ
10^8 Hz	10 mΩ
10^{10} Hz	0.1 Ω

* DC resistance:

$\boxed{R_{DC} = \frac{L}{\sigma \, \text{Area}}}$ → πa^2

* AC resistance:



$R_{AC} = \frac{L}{\sigma A_{eff.}}$

$\pi a^2 - (a-\delta)^2 \pi$ → very small

$\pi a^2 - \pi(a-\delta)^2$

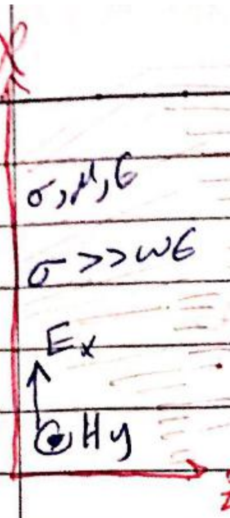
$\cancel{\pi a^2} - \cancel{\pi a^2} + 2\pi a \delta - \delta^2$ → very small

so $A_{eff} = 2\pi a \delta$

$R_{AC} = \frac{L}{\sigma \, 2\pi a \delta}$

First Material.

No.



$$E_x = E_0\, e^{-\alpha z}\, e^{-j\beta z}$$

$$H_y = \frac{E_0}{\eta_c}$$

$$J_x = J_0\, e^{-\alpha z}\, e^{-j\beta z}$$

$J_0 = \sigma E_0$

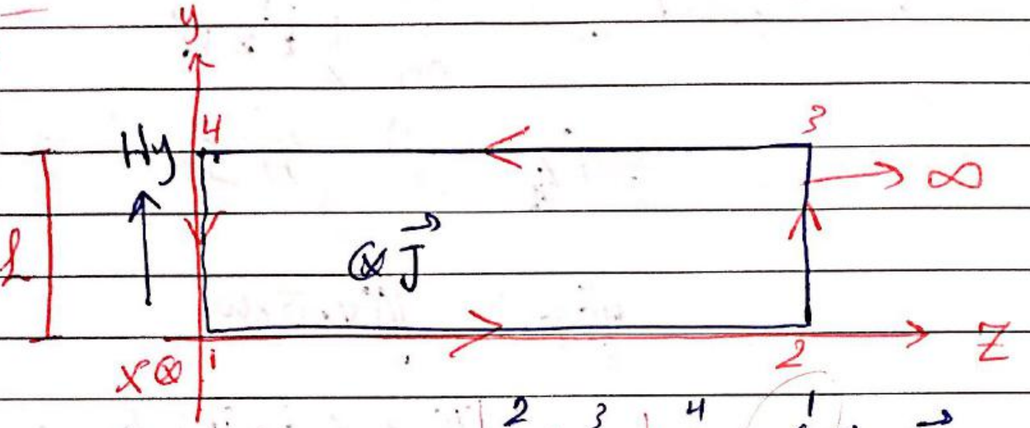
σ, μ, ϵ

$\sigma >> \omega\epsilon$

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} = \beta$$

$$\eta_c = R + jX$$

$$R = \sqrt{\frac{\omega\mu}{2\sigma}} = X$$



$$\oint_L \vec{H}\cdot\vec{dl} = \iint_S \vec{J}\cdot\vec{ds} = \int_1^2 + \int_2^3 + \int_3^4 + \int_4^1 \vec{H}\cdot\vec{dl}$$

at $z=\infty$ zero

$$\Rightarrow \int_4^1 \vec{H}\cdot\vec{dl} = \int_L^0 H_0\, dy = \int_0^L\int_{z=0}^{\infty} J_0 e^{-\gamma z}\, dy\, dz\ \vec{a}_x$$

$$= J_0 L \left.\frac{e^{-\gamma z}}{-\gamma}\right|_0^{\infty} = \frac{J_0 L}{\gamma} = H_0 L$$

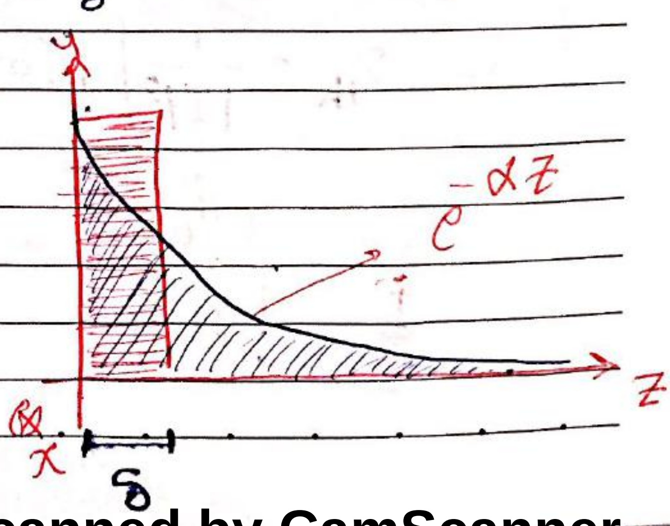
$$\Rightarrow \boxed{H_0 = \frac{J_0}{\gamma}} \quad ; \gamma = \alpha + j\beta \quad ; \alpha = \beta = \frac{1}{\delta}$$

$$\gamma = \frac{1+j}{\delta}$$

$$H_0 = \frac{J_0\,\delta}{1+j}$$

$$R_{ac} = \frac{L}{\sigma A_{eff}}$$

$2\pi a\delta$



Power flow through an area

No.

※ Poynting Vector:

CKt

$P_{dc} = VI$

$P_{avg} = \frac{1}{2} VI$

$\hat{P}_{\substack{ac \\ avg}} = \frac{1}{2} \underbrace{V}_{IZ} I^* = \frac{1}{2}|I|^2 Z_{th}$ W

EM

$\vec{S} = \underbrace{\vec{E}}_{V/m} \times \underbrace{\vec{H}^*}_{A/m} \Rightarrow W/m^2$

$\vec{a}_E \times \vec{a}_H$

$\vec{S}_{avg} = \frac{1}{2}\vec{E}\times\vec{H}$

$$\frac{1}{2}|I|^2 Z_{th} = \hat{P}_{avg} = \oiint_S \vec{S}_{avg}\cdot d\vec{s} = \frac{1}{2}\oiint \vec{E}\times\vec{H}^*\cdot d\vec{s}$$

* Remember: $\oiint_S \vec{D}\cdot d\vec{s} = \iiint_V \nabla\cdot\vec{D}\,dv$

$$\Rightarrow \hat{P} = \frac{1}{2}\iiint_V \nabla\cdot(\vec{E}\times\vec{H})\,dv$$

$$\nabla\times\vec{E} = -j\omega\mu\vec{H}$$

$$\nabla\times\vec{H} = (\sigma + j\omega\epsilon)\vec{E}$$

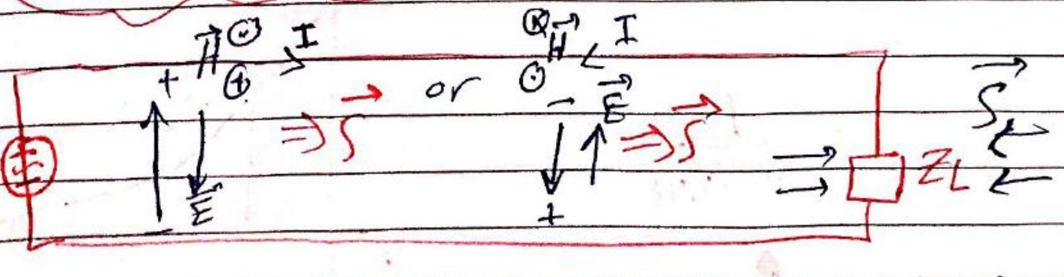
$$= \frac{1}{2}\iiint_V \left(\vec{H}^*\cdot\nabla\times\vec{E} - \vec{E}\cdot\underbrace{\nabla\times\vec{H}^*}_{(\sigma + j\omega\epsilon)\vec{E}^*}\right)dv$$

$$= -\frac{1}{2}\left[\iiint_V \sigma|\vec{E}|^2\,dv + j\omega\iiint_V\left(\mu|\vec{H}|^2 - \epsilon|\vec{E}|^2\right)dv\right]$$

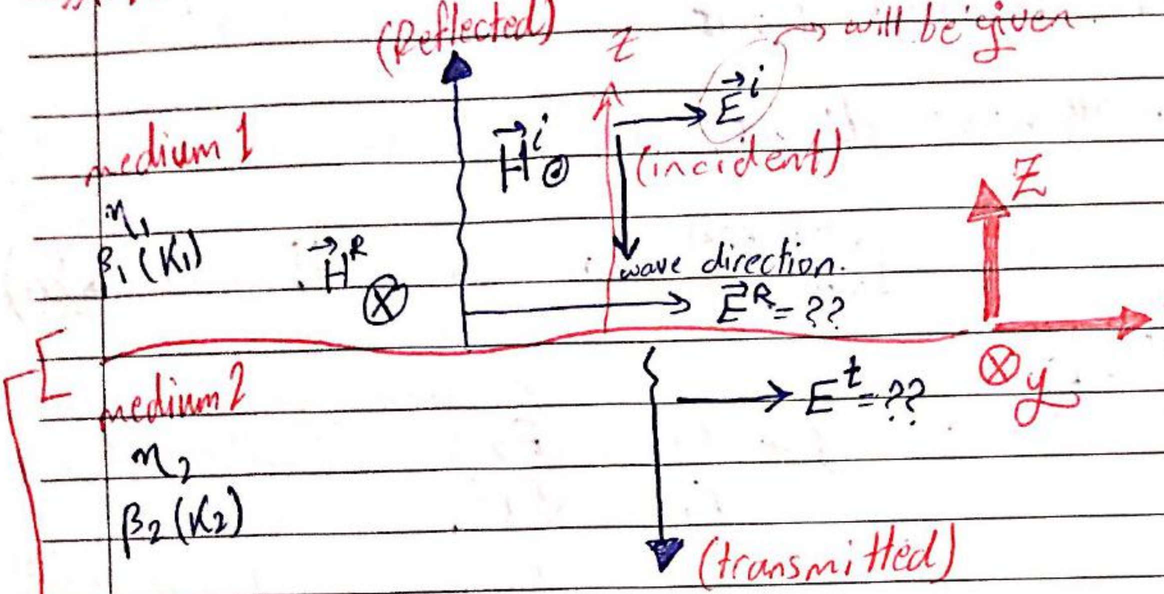
$$\Rightarrow -\oiint_S \vec{S}\cdot d\vec{s} = \frac{1}{2}\left[\iiint_V \sigma|\vec{E}|^2\,dv + j\omega\iiint_V\left(\mu|\vec{H}|^2 - \epsilon|\vec{E}|^2\right)dv\right] = \frac{|I|^2}{2}Z_{th}$$

σ, μ, ϵ

so $$Z_{th} = \frac{1}{|I|^2}\left[\iiint_V \sigma|\vec{E}|^2\,dv + j\omega\iiint_V\left(\mu|\vec{H}|^2 - \epsilon|\vec{E}|^2\right)dv\right]$$



* Normal Incidence of EM Wave :



source free
homo.
isotropic.

* Incident:

$\Rightarrow$ given: $E_x^i = E_1 \, e^{j(\omega t + K_1 Z)}$ — since it is in the (-ve) Z - direction (Be carful)

$$\Rightarrow H_y^i = \frac{-E_1}{\eta} e^{jK_1 Z}$$

* Transmitted: ??

$$E_x^t = E_2 \, e^{jK_2 Z} \quad , \quad H_y^t = \frac{-E_2}{\eta} e^{jK_2 Z}$$

* دائماً إذا كان المجال الكهربائي موازي للسطح $E^i // x_{axis}$ ⟸ بنفرض
المنعكس بنفس اتجاه E^i. E

* Reflected: ??

$$\vec{E}_x^r = E_1' \, e^{-jK_1 Z} \quad , \quad H_y^r = + \frac{E_1'}{\eta} e^{-jK_1 Z}$$

* Always Becarful $\Rightarrow$ for the (-ve) signs.

$\Rightarrow E_1', E_2$?? (using Boundary Conditions).

No.

$E_{tang.}$ are always Cont (D_{norm} is cont).

H_{tang} may have discont. (D_{norm} may have discont.)

in this case: η_1 real, η real (or non of them is $\to \infty$)

$$E_1 + E_1' = E_2 \Longrightarrow \boxed{1 + \frac{E_1'}{E_1} = \frac{E_2}{E_1}}$$

$\frac{E_2}{E_1}$ → Transmision Coeffecient (τ)

$\frac{E_1'}{E_1}$ → reflection Coeffecient (R) or (ρ)

$$1 + \rho = \tau \quad \text{--- (1)}$$

$$\frac{-E_1}{\eta_1} + \frac{E_1'}{\eta_1} = \frac{-E_2}{\eta_2} \Longrightarrow 1 - \rho = \frac{\eta_1}{\eta_2}\tau \quad \text{--- (2)}$$

(1) & (2):

$$\boxed{\rho = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}}, \quad \boxed{\tau = \frac{2\eta_2}{\eta_1 + \eta_2}}$$

* lossless media: $-1 \le \rho \le 1$, $0 \le \tau \le 2$

short cKt. ($\rho = -1$, $\tau = 0$) ; open cKt. ($\rho = 1$, $\tau = 2$)

$z \ge 0$:

$$E_x(z,t) = E_1\left(e^{+jk_1 z} + \rho\, e^{-jk_1 z}\right) e^{j\omega t} \quad V/m$$

$$H_y(z,t) = \frac{-E_1}{\eta_1}\left(e^{jk_1 z} - \rho\, e^{-jk_1 z}\right) e^{j\omega t} \quad A/m$$

$z \le 0$:

$$E_x(z,t) = E_1 \tau\, e^{+jk_2 z} e^{j\omega t} \quad V/m$$

$$H_y(z,t) = \frac{-E_1 \tau}{\eta_2} e^{jk_2 z} e^{j\omega t} \quad A/m.$$

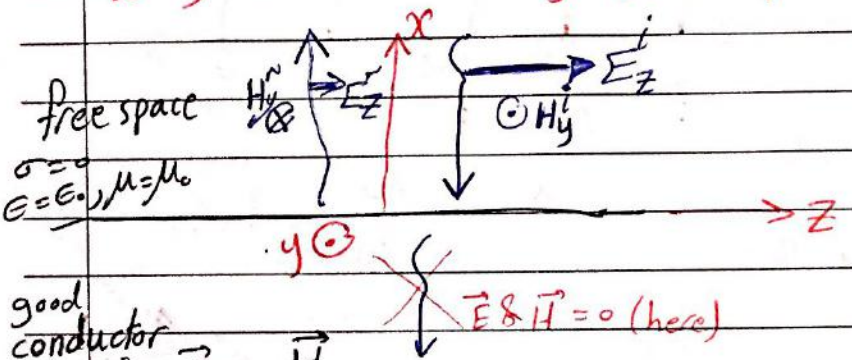
No.

⇒ find ρ_s & K:

$K = 0$ every where.

$$\rho_s \rightarrow \rho_{sf} = 0$$
$$\rho_s \rightarrow \rho_{sp} = 0$$

Ex. Normal incidence of em wave from free space to good conducting half space.



Determine $\vec{E}$ & $\vec{H}$ everywhere & define ρ_s & $\vec{K}$?

$$\rho_s \rightarrow \rho_{sf} = 0$$
$$\rho_s \rightarrow \rho_{sp} = 0$$

$\Rightarrow \vec{E} = 0 = \vec{H}$

*incident:

$$E_z^i = E_1 e^{jK_0 x}$$
$$H_y^i = \frac{E_1}{\eta_0} e^{jK_0 x}$$

*Reflected:

$$E_z^r = E_1 \rho \, e^{-jK_0 x}$$
$$H_y^r = -\frac{E_1 \rho}{\eta_0} e^{-jK_0 x}$$

* transmitted field:

$\vec{E} = 0$, $\vec{H} = 0$

⇒ Boundary Condition: E_{tang} is cont at $x = 0^+$

$$(E_z^i + E_z^r) = E_z^t$$

$E_1(1+\rho) = 0$

$\Rightarrow \rho = -1$

$$\rho = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{0 - \eta_0}{0 + \eta_0} = -1$$

⇒

No.

the remaining is the imaginary part.

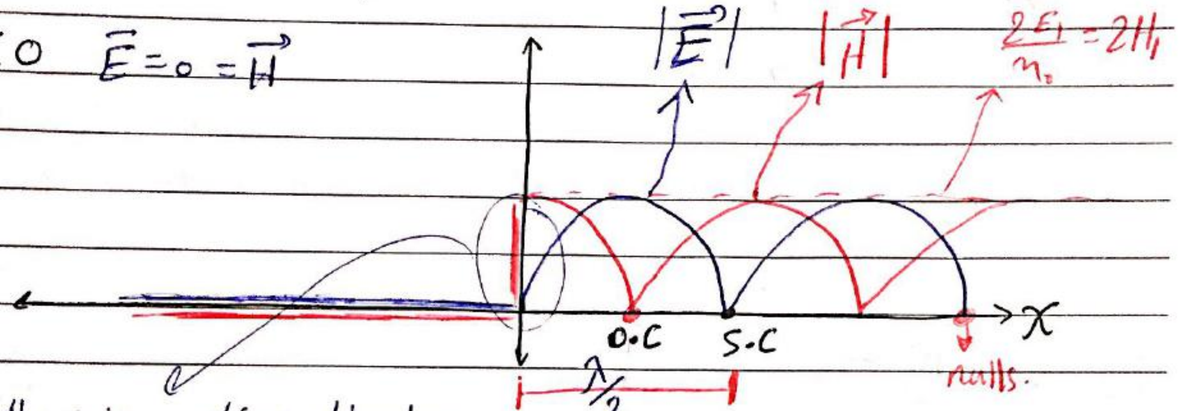
$\Rightarrow$

$x \geq 0$

$$E_z = E_1 \left[e^{jK_0 x} - e^{-jK_0 x} \right] = 2jE_1 \sin K_0 x$$

$$H_y = \frac{2E_1}{\eta_1} \cos K_0 x \rightarrow \text{real part.}$$

$X \leq 0 \quad \vec{E} = 0 = \vec{H}$



there is a discontinuity in $\vec{H}$ but No problem since the current caused this.

$$\Rightarrow K = \frac{2E_1}{\eta_1} \; A/m \; (\vec{a_z})$$

$\leftrightarrow \frac{\lambda}{4}$

$$a_n \times \vec{H} = \vec{K}$$
$$\vec{a_x} \times \vec{a_y} = \vec{a_z}$$

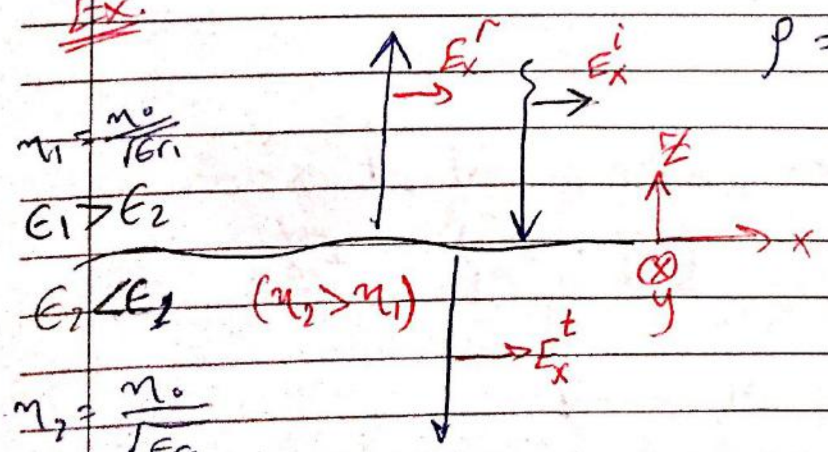
$\frac{2\pi}{\lambda}$

$K_0 X = n\pi$

$\Rightarrow X = n\frac{\lambda}{2}$

This type of waves Called: "Pure standing wave."

Ex.



$$\rho = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} > 0$$

$Z \geq 0$:

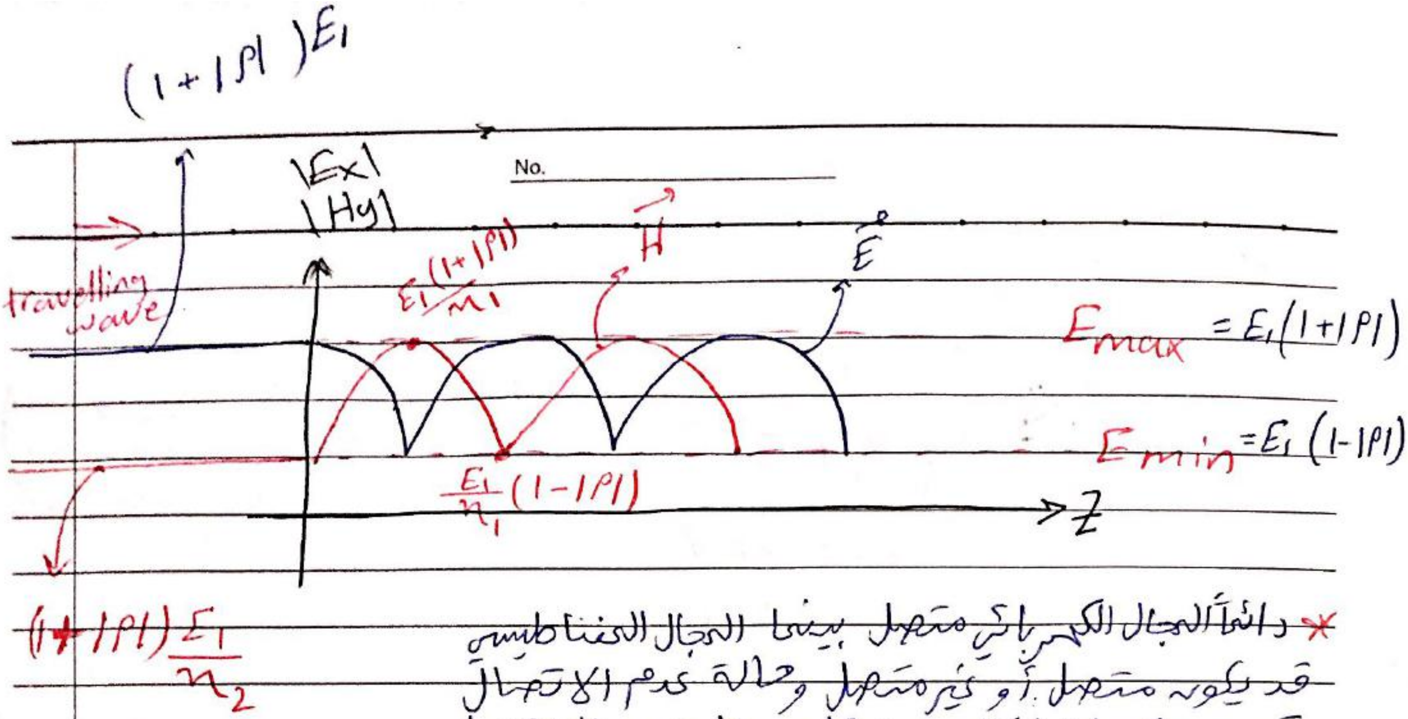
$$E_x = E_1 \left[e^{jK_1 Z} + \rho e^{-jK_1 Z} \right]$$

$Z \leq 0$

$$E_x = E_1 \tau e^{jK_2 Z}$$

in general:

$$0 \leq \rho \leq 1$$
$$0 \leq \tau \leq 2$$



* دائماً المجال الكهربائي متصل بينما المجال المغناطيسي قد يكون متصل أو غير متصل وحالة عدم الاتصال تحدث إذا كان عندنا وسط جيد التوصيل.

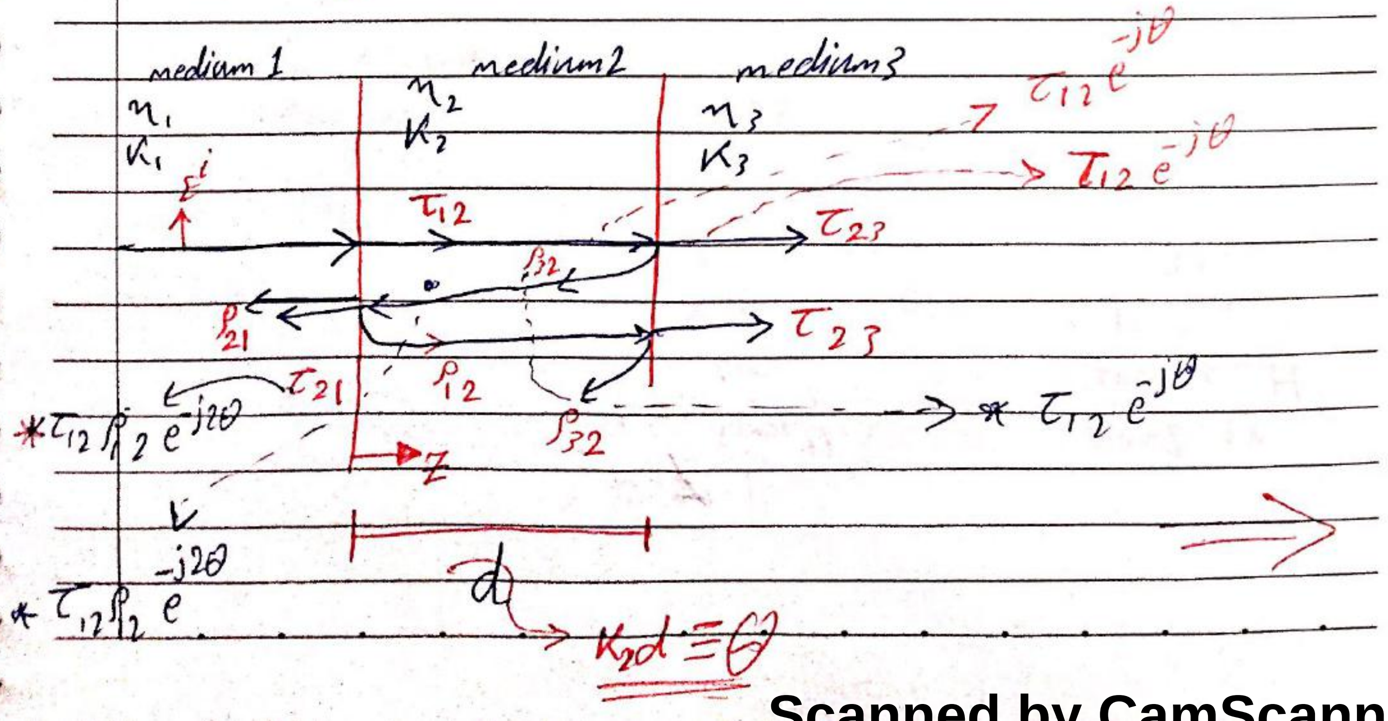
This type of waves called: "Standing Wave".

* Standing Wave Ratio ≡ SWR.

$$\Rightarrow \quad SWR = \left|\frac{E_{max}}{E_{min}}\right| = \left|\frac{H_{max}}{H_{min}}\right| = \frac{1+|\rho|}{1-|\rho|}$$

$$* \quad 1 \leq SWR \leq \infty \quad *$$

* Multi-layer Case: (3-Layers case).



No.

$\Rightarrow$ $z < 0$

reflected E^r $(=1)$

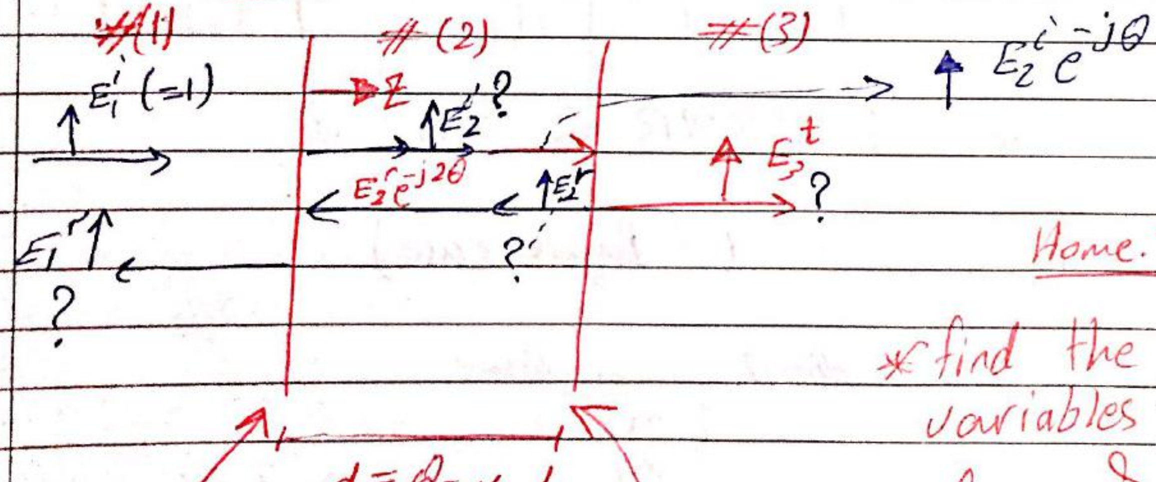
incident $E^i\left(\rho_{21} + \tau_{21}\,\tau_{12}\,\rho_{32}\,e^{-j2\theta} + \cdots e^{-j4\theta} + \cdots e^{-6j\theta} + \cdots\right)$

$\rho_{\text{over all}} = \dfrac{\text{Reflected}}{\text{incident}}$

$z > d$:

Transmitted $E^i\left(\tau_{12}\,\tau_{23}\,e^{-j\theta} + \text{—}\,e^{-j3\theta} - - + e^{-j5\theta} - -\right)$

$\tau_{\text{over all}}$. (find a suitable series to even & odd terms)



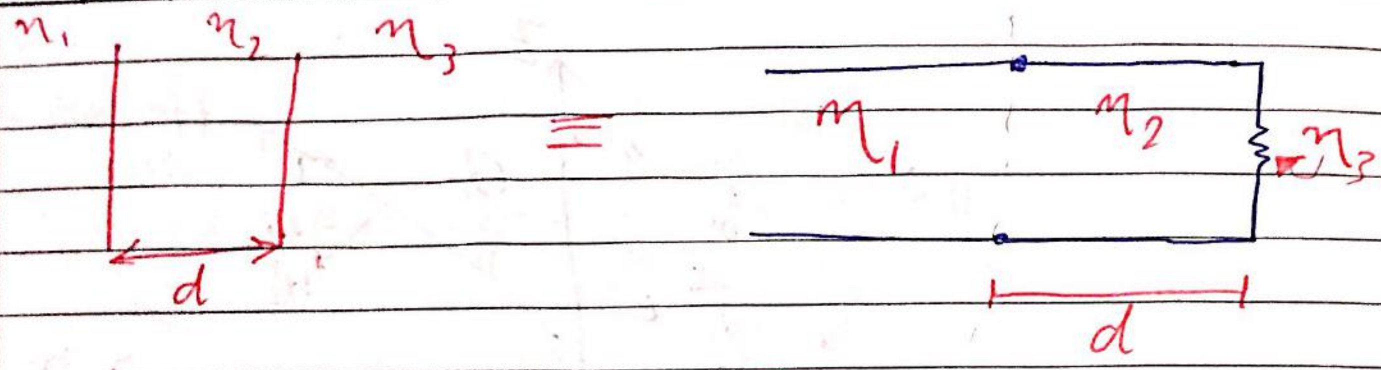
Home. Work.

* find the four variables & ρ_{overall} & $\tau_{\text{over all}}$?

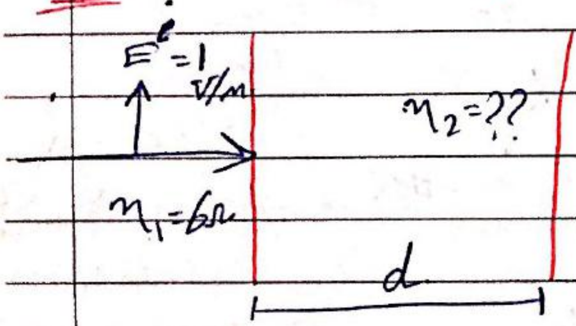
B.C
E^{tang} cont
H^{tang} cont
at $z = 0^{\pm}$

B.C
E^{tang} Cont
H^{tang}
at $z = d^{\pm}$

No.



Ex(1): find η_2, d such that $\Gamma_{overall} = 0$?



$\eta_3 = \frac{25}{6}\Omega$

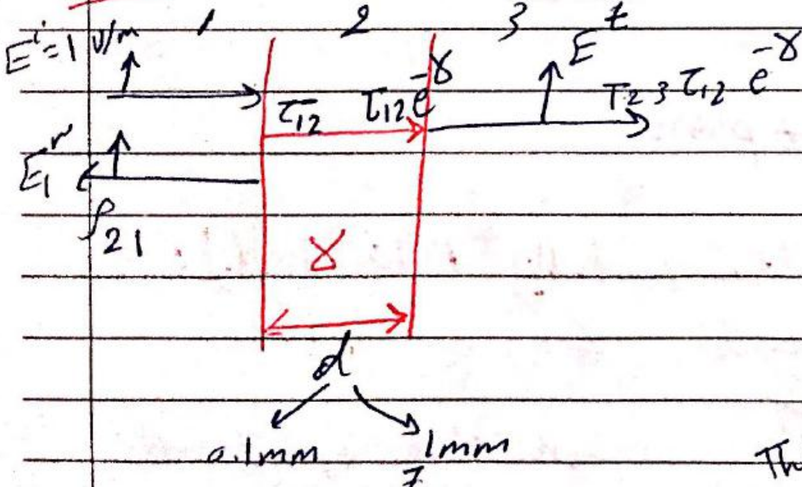
Home work

Ex(2):

find: $\Gamma_{over\,all}$ & $\tau_{over\,all}$

for $d = 0.1$ mm & 1 mm

$f = 100$ Hz, 1 GHz



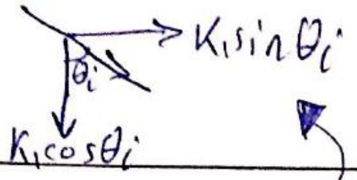
$\sigma = 10^7$

$\mu = \mu_0$

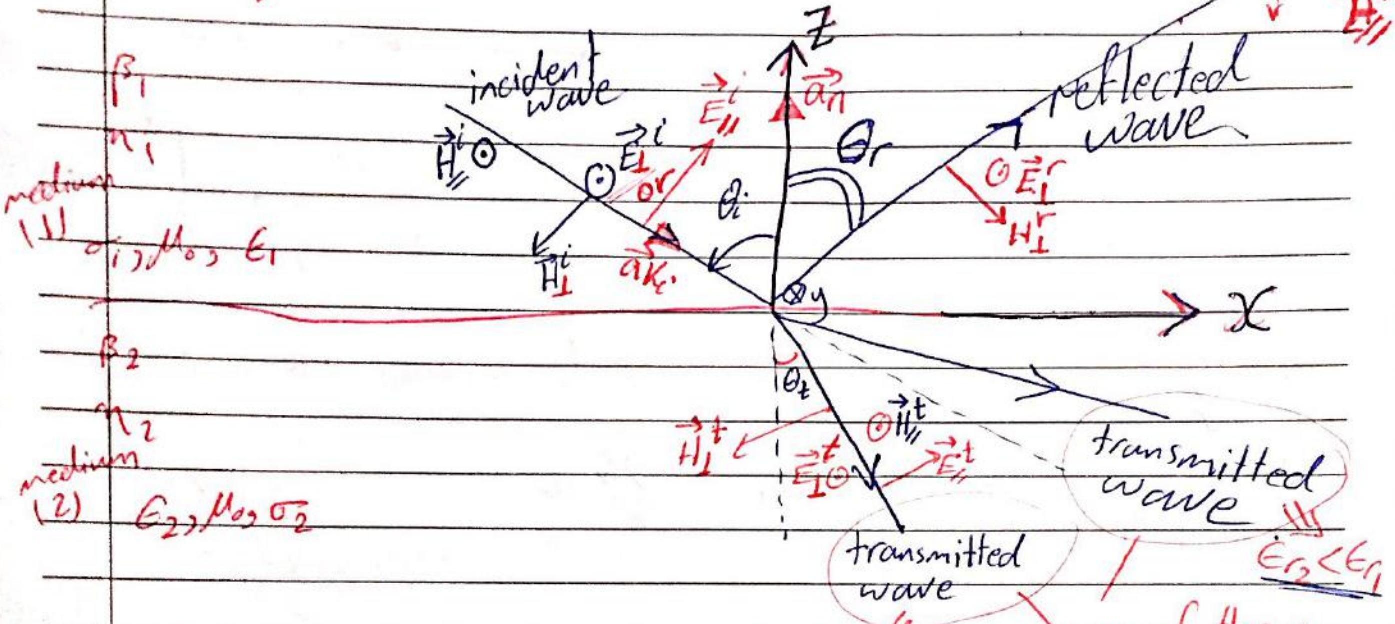
$10 \log_{10} |\tau_{overall}|^2 = ??$

This called $\equiv$ sheilding effectivness (SE)

* find SE ??



Oblique Incedence of em Wave:



⇒ * Reference Plane: interface

$\vec{a}_n$ (in the x-yplane) & $\vec{a}_{K_i}$

so in this case ⇒ x-z plane

* $\boxed{\perp \text{ polarized wave}}$ (Horizontally Polarized): ↳ ground plane

incident:

$$\vec{E}_{\perp}^{i} = E_1 \, e^{-jK_1 x \sin\theta_i + jK_1 z\cos\theta_i} \, \vec{a}_y$$

$$\vec{H}_{\perp}^{i} = \frac{E_1}{\eta}\left(-\cos\theta_i \vec{a}_x - \sin\theta_i \vec{a}_z\right) e^{-jK_1 x \sin\theta_i + jK_1 z\cos\theta_i}$$

transmitted:

$$\vec{E}_{\perp}^{t} = E_1 \tau_{\perp} \vec{a}_y \rightarrow e^{-jK_2 x \sin\theta_t + jK_2 z\cos\theta_t}$$

$$\vec{H}_{\perp}^{t} = \frac{E_1 \tau_1}{\eta_2}\left(-\cos\theta_t \, \vec{a}_x - \sin\theta_t \vec{a}_z\right) \nearrow$$

Reflected:

$$\vec{E}_{\perp}^{r} = -E_1 \rho_{\perp} \vec{a}_y \rightarrow e^{-jK_1 x \sin\theta_r - jK_1 z\cos\theta_r}$$

$$\vec{H}_{\perp}^{r} = \frac{E_1 \rho_{\perp}}{\eta_1}\left(\cos\theta_r \, \vec{a}_x - \sin\theta_r \vec{a}_z\right) \nearrow$$

No.

$\Rightarrow$ the four underlined variables are unknown:

from B.C:

(1) E_{tang} is Cont at $z = 0^{I}$ & (2) H_{tang} is con at this case at $z = 0^{+}$

$\Rightarrow$ (1) $-E_1 e^{-jK_1 x \sin\theta_i} - E_1 \rho_{\perp} e^{-jK_1 x \sin\theta_r} = -E_1 \tau_{\perp} e^{-jK_2 x \sin\theta_t}$

$\hookrightarrow$ true for all when:

$$\boxed{K_1 \sin\theta_i = K_1 \sin\theta_r = K_2 \sin\theta_t} \Rightarrow \theta_i = \theta_r \quad **$$

snell's law of Reflection.

~~$\theta_t \Rightarrow$ never equal to θ_i or θ_r~~ except in a trivial case $K_1 = K_2$

$$\Rightarrow \theta_t = \sin^{-1}\left[\frac{K_1}{K_2}\sin\theta_i\right] = \sin^{-1}\left[\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\sin\theta_i\right] \quad **$$

$\hookrightarrow$ snell's Law of Refraction.

also: $\boxed{1 + \rho_{\perp} = \tau_{\perp}}$ ** (1)

$\Rightarrow$ (2) $-\frac{E_1}{\eta_1}\cos\theta_i e + \frac{E_1 \rho_{\perp}}{\eta_1}\cos\theta_i e = -\frac{E_1 \tau_{\perp}}{\eta_2}\cos\theta_t e$

$$\Rightarrow \boxed{1 - \rho_{\perp} = \tau_{\perp}\frac{\eta_1 \cos\theta_t}{\eta_2 \cos\theta_i}} \quad ** \quad (2)$$

from (1) & (2):

$$\boxed{\rho_{\perp} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}}$$

* $\boxed{\text{// polarized wave}}$ (vertically polarized)

$\hookrightarrow$ w.r.t ground.

incident: $\vec{E}^{i}_{//} = E_1 [\cos\theta_i \vec{a}_x + \sin\theta_i \vec{a}_z] \rightarrow e^{-jK_1 x \sin\theta_i + jK_1 z \cos\theta_i}$

$\vec{H}^{i}_{//} = -\vec{a}_y \frac{E_1}{\eta_1} \longrightarrow e$

$\Rightarrow$

No.

Reflected:

$$\vec{E}_{\parallel}^{r} = E_1 \rho_{\parallel} [\cos\theta_r \vec{a}_x - \sin\theta_r \vec{a}_z] \rightarrow e^{-jk_1 x \sin\theta_r + jk_1 z \cos\theta_r}$$

$$\vec{H}_{\parallel}^{r} = \frac{E_1}{\eta_1} \vec{a}_y \rho_{\parallel} \longrightarrow$$

Transmitted:

$$\vec{E}_{\parallel}^{t} = E_1 \tau_{\parallel} [\cos\theta_t \vec{a}_x + \sin\theta_t \vec{a}_z]. \rightarrow e^{-jk_2 x \sin\theta_t + jk_2 z \cos\theta_t}$$

$$\vec{H}_{\parallel}^{t} = \frac{E_1}{\eta_2} \tau_{\parallel} \vec{a}_y \longrightarrow$$

* for the underlined variables:

B.C E_{tang} is cont at $z = 0^{+}$

$$\Rightarrow \cos\theta_i e^{jk_1 x \sin\theta_i} + \rho_{\parallel} \cos\theta_r e^{-jk_1 x \sin\theta_r} = \tau_{\parallel} \cos\theta_t e^{-jk_2 x \sin\theta_t}$$

$$\Rightarrow \boxed{\theta_r = \theta_i} \ ** \qquad \theta_t = \sin^{-1}\left[\frac{k_1}{k_2}\sin\theta_i\right] = \sin^{-1}\left[\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\sin\theta_i\right]$$

***

for a lossless nonmagnetic medium.

$$\boxed{\cos\theta_i (1+\rho_{\parallel}) = \tau_{\parallel} \cos\theta_t} \ **$$

$$\hookrightarrow 1+\rho_{\parallel} = \frac{\cos\theta_t}{\cos\theta_i} \tau_{\parallel}$$

$$\rho_{\parallel} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

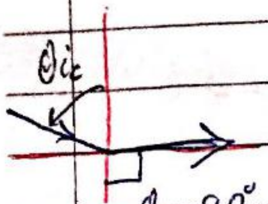
$$\Rightarrow \rho_{\parallel,\perp} = \frac{\eta_2 \cos\theta_{t,i} - \eta_1 \cos\theta_{i,t}}{\eta_2 \cos\theta_{t,i} + \eta_2 \cos\theta_{i,t}}$$

No. $\epsilon_{r_1} < \epsilon_{r_2}$

$$\theta_t = \sin^{-1}\left[\sqrt{\frac{\epsilon_{r_1}}{\epsilon_{r_2}}}\sin\theta_i\right] \rightarrow \theta_t \leq \theta_i \Rightarrow \theta_t = \theta_i \ (\theta_i = 0° \text{ or } 90°)$$

$\rightarrow \epsilon_{r_1} > \epsilon_{r_2} \Rightarrow \theta_t \geq \theta_i$

$\theta_t = \theta_i$ (just if $\theta_i = 0°$)



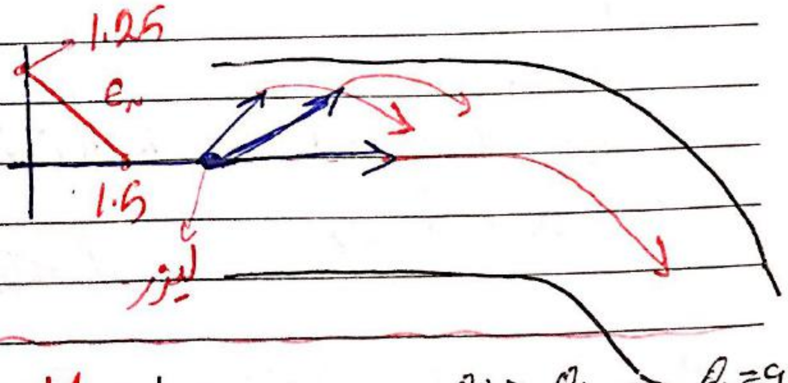
$\theta_t = 90° \Rightarrow$ when this case occure then $\theta_i \Rightarrow$ will be called "critical angle"

$\Rightarrow 1 = \sin\theta_i * \sqrt{\frac{\epsilon_{r_1}}{\epsilon_{r_2}}}$

$\Rightarrow \theta_{i_c} = \sin^{-1}\sqrt{\frac{\epsilon_{r_2}}{\epsilon_{r_1}}}$

so when $\theta_i \geq \theta_{ic} \Rightarrow \theta_t = 90° \Rightarrow$ No transmision in this case

* one of the application on the critical angle: (fiber optic)



$\rho_\perp = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_2\cos\theta_t}$
- $\rightarrow +1$ when $\eta_2 \rightarrow \infty$ or $\theta_i \geq \theta_{ic} \Rightarrow \theta_t = 90°$ (S.C)
- $\rightarrow 0$ only trivial case (ϵ_{r_1} to ϵ_{r_1})
- $\rightarrow -1$ $\eta_2 \rightarrow 0$ (O.C)

$\rho_\parallel = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$
- $\rightarrow +1$ $\eta_2 \rightarrow \infty$ (O.C)
- $\rightarrow 0$ $\theta_i = \theta_{iB}$ "Brewester angle"
- $\rightarrow -1$ $\eta_2 \rightarrow 0$ (S.C) or $\theta_i \geq \theta_{ic}$

full transmision will occure.

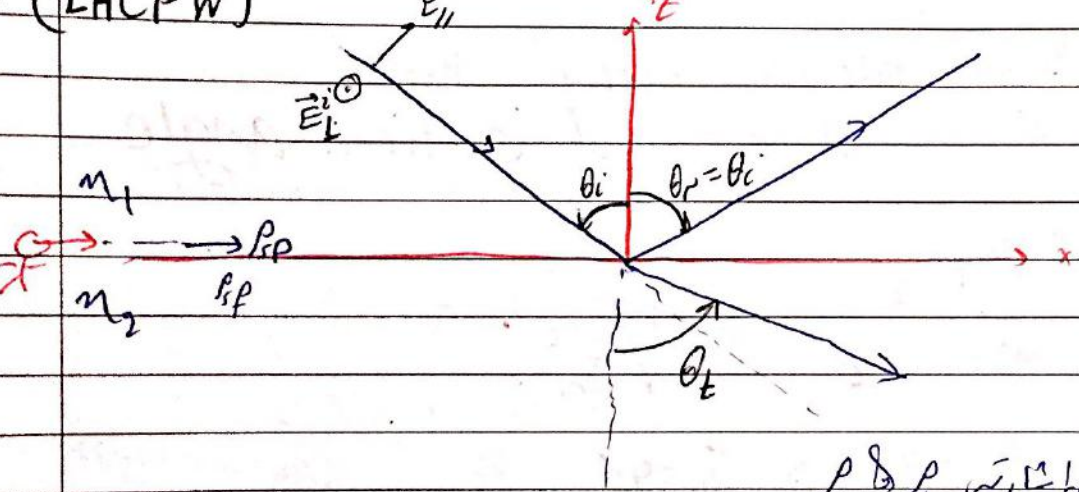
No.

$$\theta_{iB} = \tan^{-1}\left[\sqrt{\frac{\epsilon_{r_2}}{\epsilon_{r_1}}}\right]$$ ⇒ true just for parallel polarization.

$$\theta_{ic} = \sin^{-1}\left[\sqrt{\frac{\epsilon_{r_2}}{\epsilon_{r_1}}}\right]$$ ⇒ true for all.

→ No transmision for the wave.

Ex: (LHCPW)



* إذا توافقت اشارتين $\rho_\perp$ & $\rho_\parallel$
⇐ ينعكس الاستقطاب
* إذا اختلفت الاشارتين يبقى الاستقطاب كما هو

(i) $\eta_2 = 0$

$\vec{E}^t = 0 = \vec{H}^t$

$\rho_\perp = -1$, $\rho_\parallel = -1$

reflected signal is RHCPW.

$\vec{K} = \vec{a}_n \times \vec{H}\big|_{z=0^+}$, $\big|_{z=0^-}$

ρ_s < $\rho_{sf} = D_n$; $\rho_{sp} = E_n(\epsilon_1 - \epsilon_2)$

(ii) In general η_1 & η_2:

Reflected ⇒ RHEPW → same sign for $\rho_\perp$ & $\rho_\parallel$
LHEPW → different " " " " ".

transmitted → LHEPW ⇒ $K = 0$, $\rho_{sf} = 0$ ($\rho_{sp} \neq 0$)

(iii) if $\theta_i \geq \theta_{ic}$ ⇒ ($\eta_2 > \eta_1$).

$\vec{E}^t = 0 = \vec{H}^t$ $K = 0$, $\rho_{sf} = 0$ ($\rho_{sp} \neq 0$)

No.

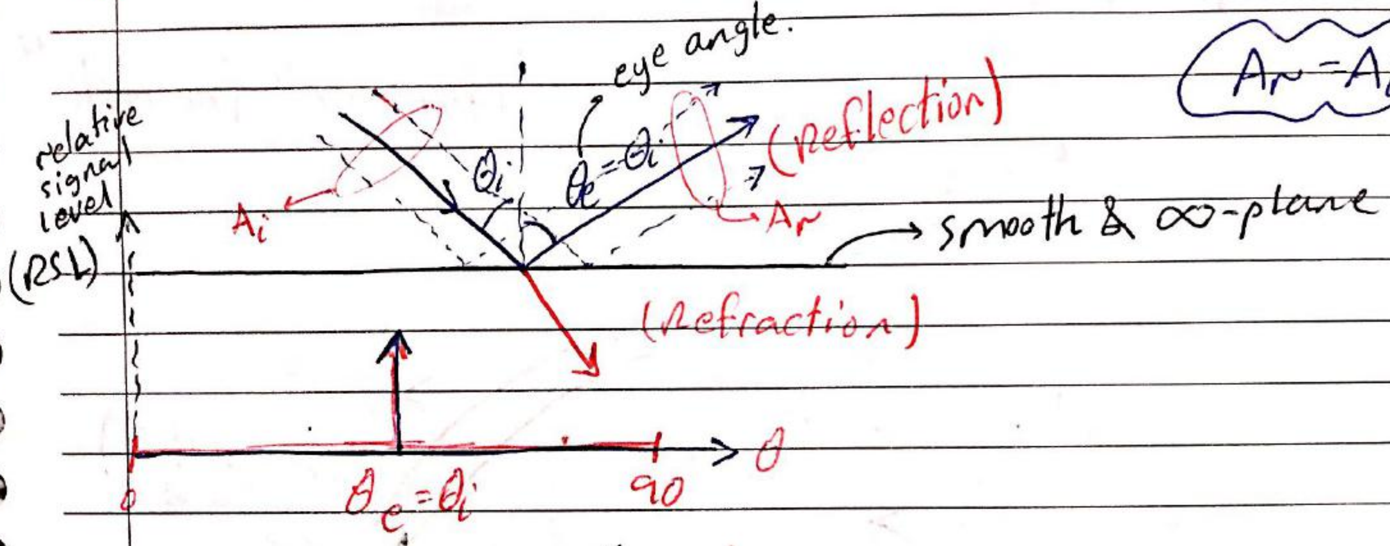
(iv) if $\theta_i = \theta_{iB}$.

Reflected signal LPW ($\perp$)

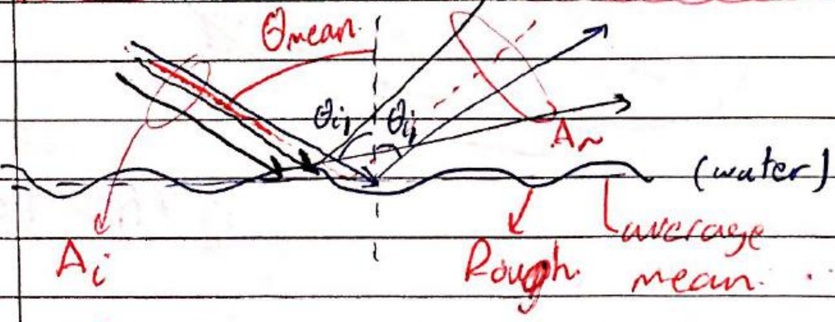
Transmitted signal LHEPW.

($\rho_{sp} \neq 0$)
$\rho_{sf} = 0$
$\kappa = 0$

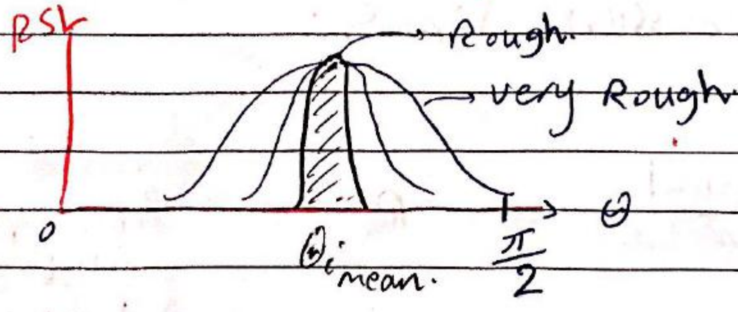
Reflection, Refraction, Scattering and Diffraction of EM wave:

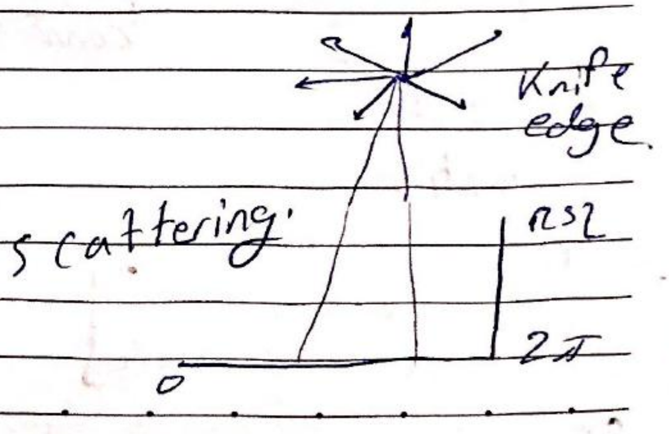


$A_r = A_i$

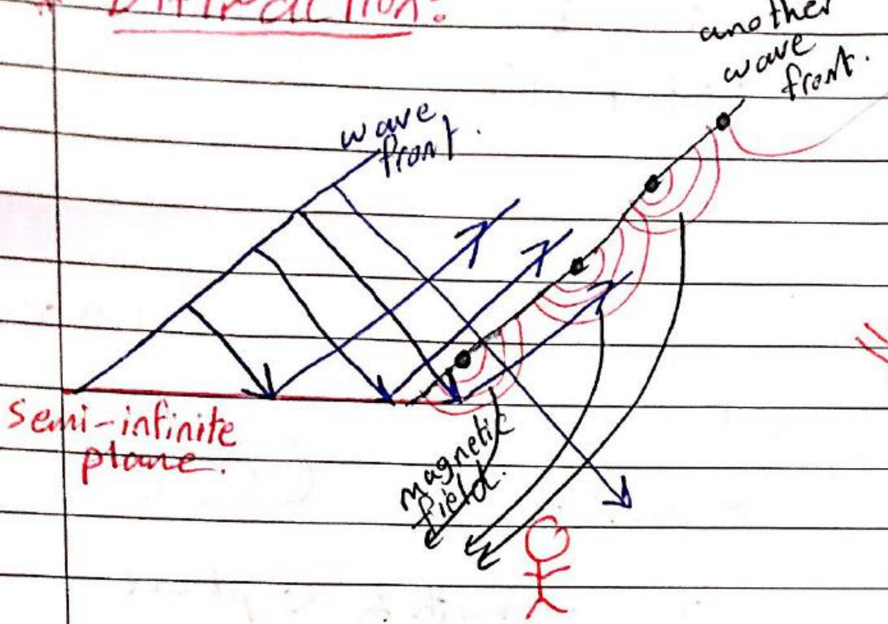


$A_i \neq A_r$





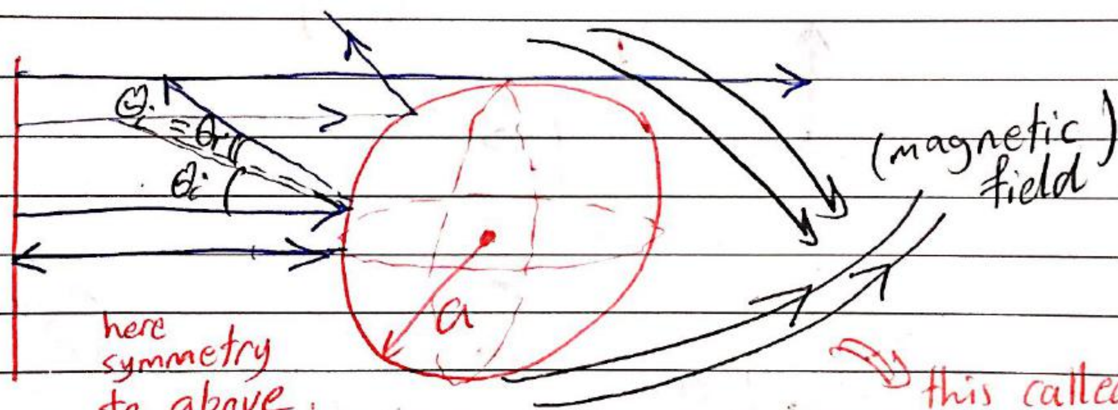
* Diffraction:



∞-number of secondary point sources.

⇒ This called:

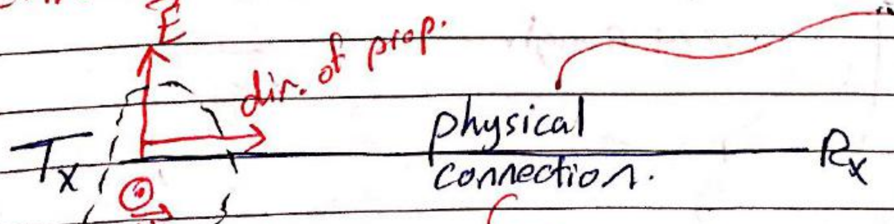
"Hueggen's Principle".



⇒ this called diffraction.

End of CH10

CHAPTER(11): Transmission lines:



could be fiber optic (solid or hollow.)

could be:

① Actual direction of wave propagation

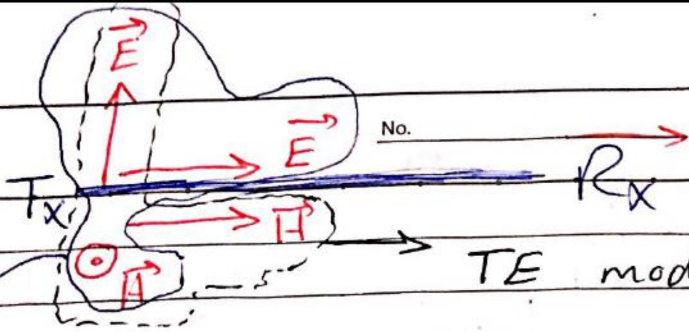
or

② Virtual direction of wave propagation.

This situation called:

[Transverse Electric & magnetic wave mode]

or "TEM mode"



TE mode ($E_z = 0$)

TM mode ($H_z = 0$)

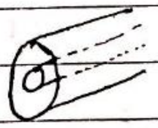
*if it is was a hollow connection: we just can use TE or TM mode.

* Types of Connection:

* Multi-conductors (≥ 2 wires) $\Rightarrow$ just used in High voltage (50 or 60 Hz)

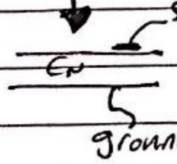
※ Two Conductors: (TEM mode)

(open wire) $\Rightarrow$ balanced line (few MHz)



(Coaxial) $\Rightarrow$ inside could be solid or hollow.

$\Rightarrow$ unbalanced line ($f \Rightarrow$ MHz).



(strip line) $\Rightarrow$ used when ($f \geq 1$ GHz)

$\Rightarrow$ used just inside equipments.

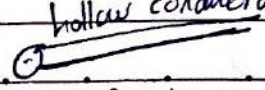
* Cost for the three types:

open wire
Coaxial
strip

$\downarrow$ Higher cost.

※ One connection: (TE or TM mode)

* doesn't use solid.

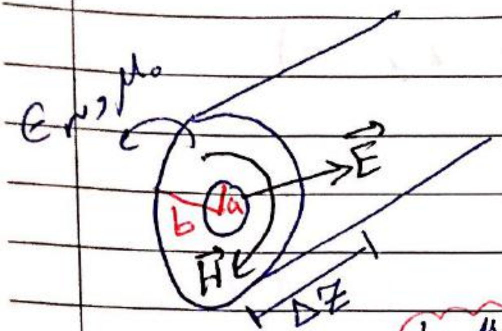


$f > 3$ GHz
(Highest cost)

or

fiber optic
$f = 10^{14}$ Hz
(cheapest one)
so used to connect network of a city

* Two Conductor (Transmission Line):



find $\underbrace{R_i \ \& \ R_o}_{AC}$ & $R_{leakage}$ & L & C (as obtaind in EM1).

⇓ Review.

$\boxed{L = \frac{\mu_o}{2\pi}\ln(b/a) \ H/m}$, $\boxed{C = \frac{2\pi\epsilon_o\epsilon_r}{\ln(b/a)} \ F/m}$.

$\boxed{G_{leakage} = \frac{2\pi\sigma}{\ln(b/a)} \ \mho/m}$, $\boxed{R_i = \frac{l}{\sigma 2\pi a\delta}}$, $\boxed{R_o = \frac{l}{\sigma 2\pi b\delta}}$

Wave Equation:

$\cancel{E} \Rightarrow V$

$\cancel{H} \Rightarrow I$

$\int_a^b \vec{E} \cdot d\vec{l} = K\ln(\frac{b}{a}) \Rightarrow f(Z,t) \Rightarrow \underline{V(Z,t)}$

$d\vec{l} = dr\,\vec{a_r}$

$\vec{E}(r, \cancel{\phi}, Z, t)$

the dot product will cancel it.

doesn't effect on $\vec{E}$

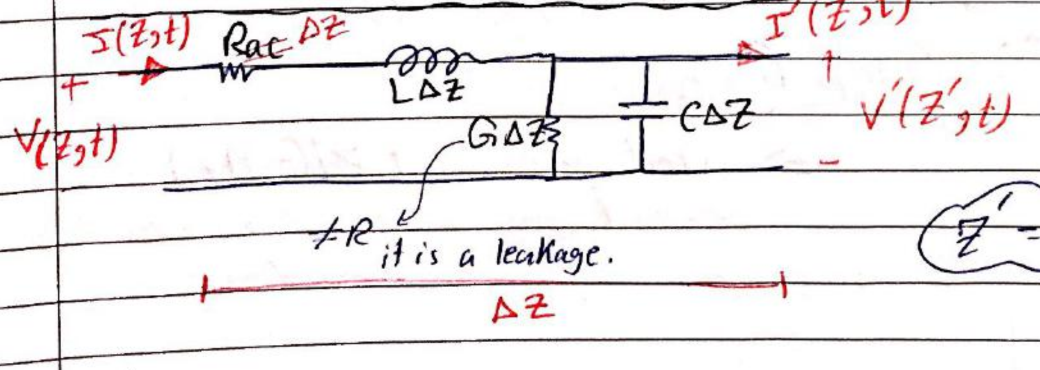
also: $\int_0^{2\pi} \vec{H} \cdot d\vec{l} \Longrightarrow I(Z,t)$

$d\vec{l} = r\,d\phi\,\vec{a_\phi}$

$H(r, Z, t)$

cancel each other since $H = \frac{I}{2\pi r}$

dot product will cancel ϕ-direction.



$\boxed{Z' = Z + \Delta Z}$

* KVL:

$V(Z,t) = V' + IR\Delta Z + L\Delta Z\frac{\partial I}{\partial t}$

Taylor series:

$V'(Z) = V(Z) + \frac{\partial V}{\partial Z}\frac{\Delta Z}{1!} + \frac{\partial^2 V}{\partial Z^2}\frac{\Delta Z^2}{2!} + \cdots$

$\cancel{V(Z)} \cong \cancel{V(Z)} + \frac{\partial V}{\partial Z}\cancel{\Delta Z} + \cdots + IR\cancel{\Delta Z} + L\cancel{\Delta Z}\frac{\partial I}{\partial t}$ ⟹

No.

$\Delta z \to 0$

$$\Rightarrow \boxed{\frac{\partial V}{\partial z} = -\left[I R + L \frac{\partial I}{\partial t} \right]} \quad \text{--- (1)}$$

* KCL:

Taylor series: $I'(z') = I(z) + \frac{\partial I}{\partial z} \frac{\Delta z}{1!} + \cdots$

do the same as before in KVL $\Rightarrow \boxed{\frac{\partial I}{\partial z} = -\left[G V + C \frac{\partial V}{\partial t} \right]}$ -- (2)

(1) & (2) are called: "Telegraphic Equations".

※ Lossless T.L:

* Transient Analysis: ($R = 0$ & $G_{leakage} = 0$)

only There is

L & C: $L = \frac{\mu_0}{2\pi} \ln(b/a)$ & $C = \frac{2\pi \epsilon_0 \epsilon_r}{\ln(b/a)}$

※ $\sqrt{\frac{L}{C}} = \frac{1}{2\pi} \underbrace{\sqrt{\frac{\mu}{\epsilon}}}_{\frac{120\pi}{\sqrt{\epsilon_r}}} \ln(b/a) \Rightarrow \sqrt{\frac{L}{C}} = \frac{60}{\sqrt{\epsilon_r}} \ln(b/a)\ \Omega \equiv$ characteristic impedance $Z_0\ (\Omega)$

** $(\sqrt{LC})^{-1} = (\sqrt{\mu \epsilon})^{-1} = \frac{3 \times 10^8}{\sqrt{\epsilon_r}}$ m/s.

Now: The equations become:

$$\frac{\partial V}{\partial z} = -L \frac{\partial I}{\partial t} \quad \& \quad \frac{\partial I}{\partial z} = -C \frac{\partial V}{\partial t}.$$

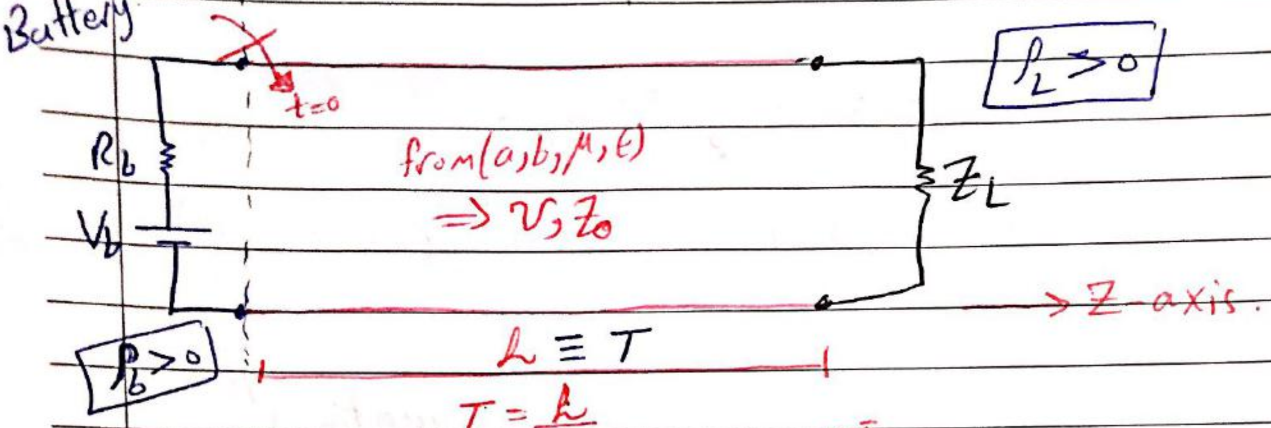
$$\Rightarrow \frac{\partial^2 V}{\partial z^2} = \underbrace{LC}_{\frac{1}{v^2}} \frac{\partial^2 V}{\partial t^2} \Rightarrow \frac{\partial^2 I}{\partial z^2} = LC \frac{\partial^2 I}{\partial t^2}$$

$\frac{V}{I}(z,t) \xrightarrow{\text{with } v = \frac{1}{\sqrt{LC}}}$ V & I with impedance $\equiv Z_0$

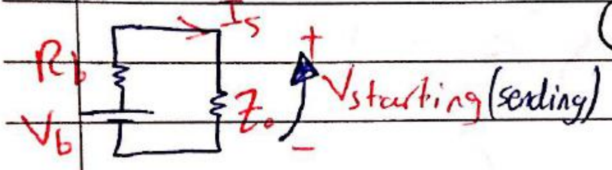
dir of prop. $\longrightarrow$

No.

* open wire → 150Ω, 300Ω, 600Ω, 1200Ω.
* Coaxial → 50Ω, 75Ω

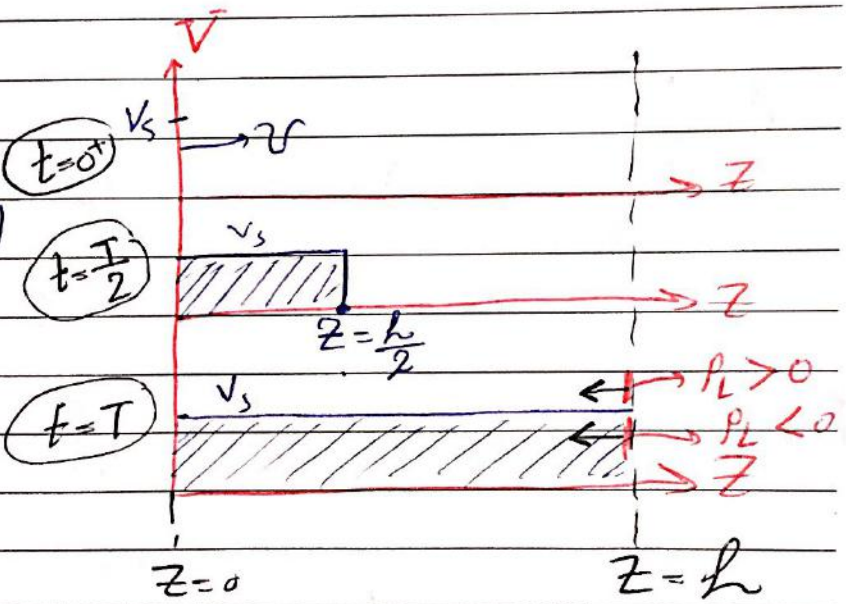


at ($t=0^+$):



$$V_s = \frac{V_b Z_0}{Z_0 + R_b}$$

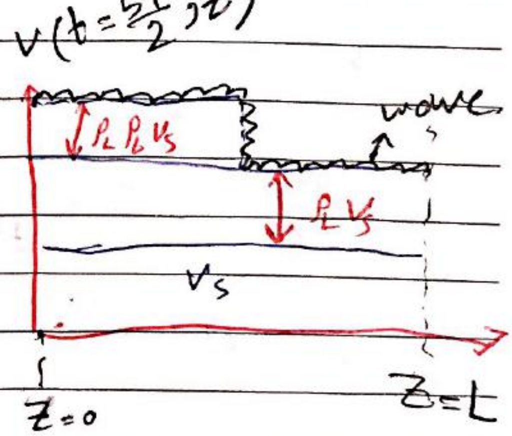
$$I_s = \frac{V_b}{Z_0 + R_b}$$



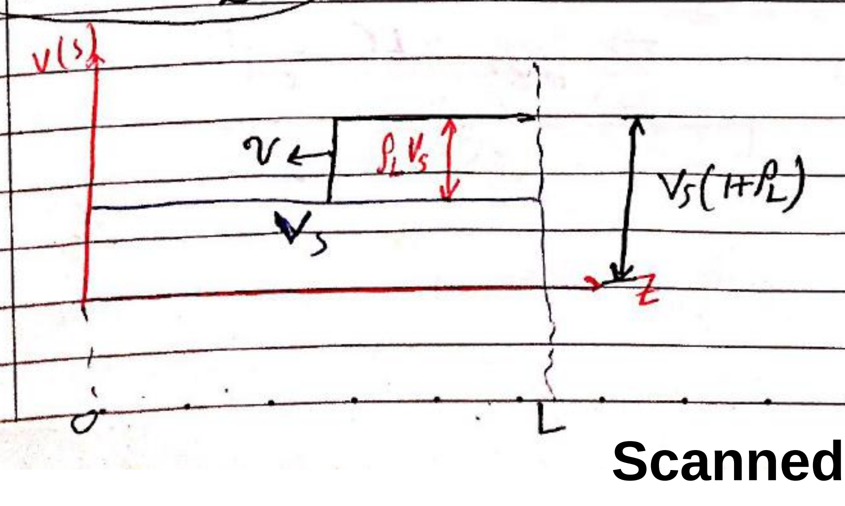
Reflected signal (V^-) ⇒ at t equal exactly T.

$$\Rightarrow \boxed{V^- = \rho_L V^+ = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

$v(t = \frac{5T}{2}, z)$



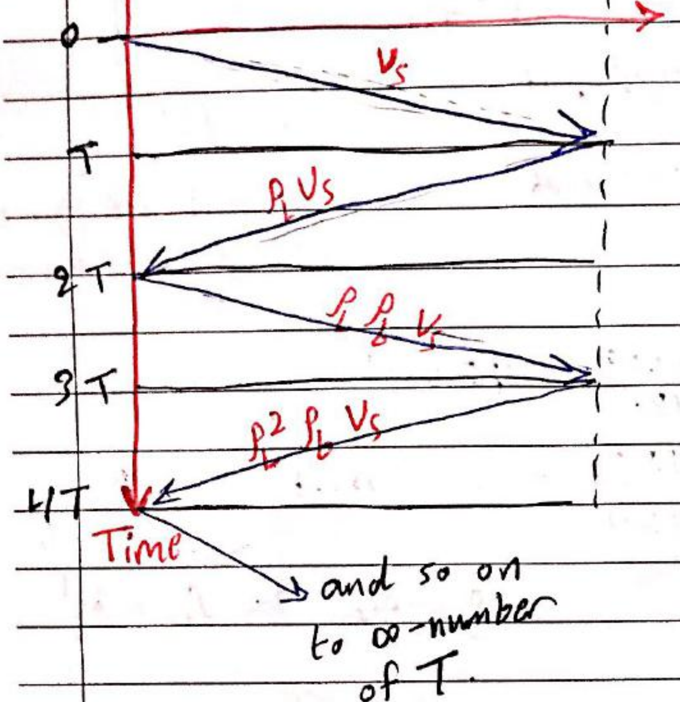
at $t = \frac{3T}{2}$ for $\rho_L > 0$!



No.

※ Voltage Space-Time Diagram (Zig-Zag Diagram) or (Bernouli-Diagram).

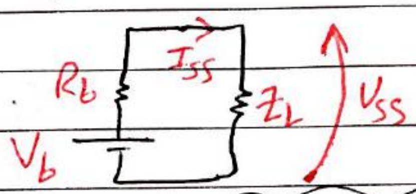
$$\rho_{b\,volt} = \frac{R_b - Z_0}{R_b + Z_0} \qquad z = L \qquad \rho_{L\,volt} = \frac{Z_L - Z_0}{Z_L + Z_0}$$



** steady state:

$(t \to \infty) \Rightarrow (f \to 0)$

$\Rightarrow (\lambda \to \infty)$

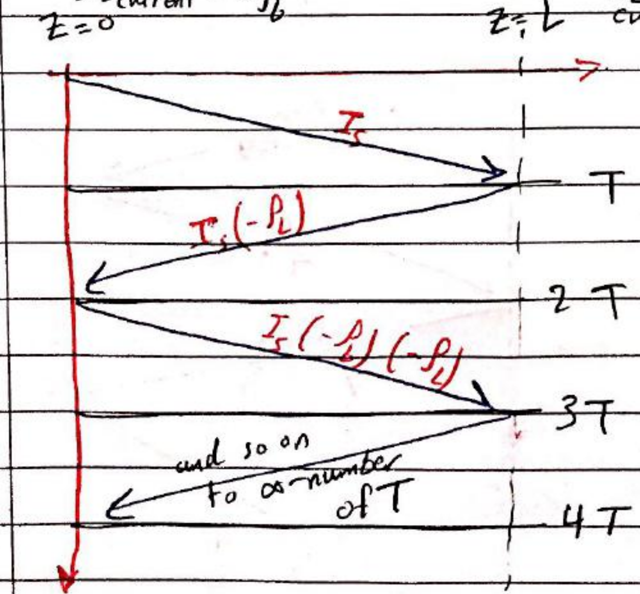


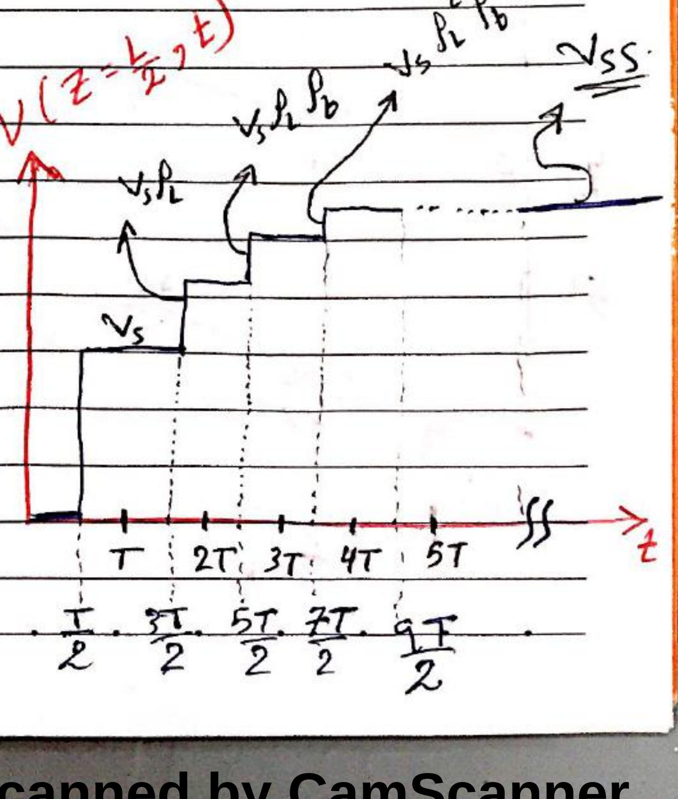
$$V_{ss} = V_b \frac{Z_L}{Z_L + R_b}$$

$$I_{ss} = \frac{V_b}{Z_L + R_b}$$

※ Current space-Time Diagram:

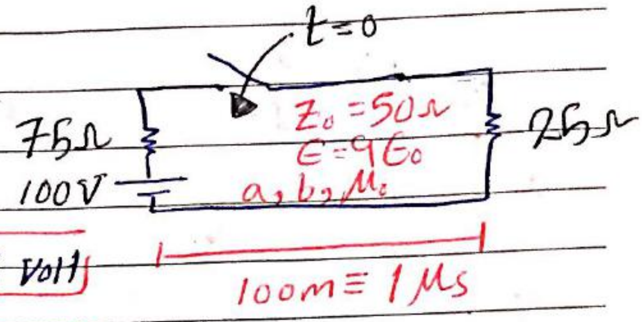
$\rho_{b\,current} = -\rho_b \qquad \rho_{L\,current} = -\rho_L$



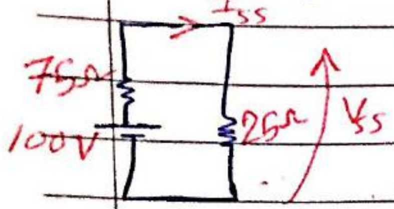


* we always write I_{ss} & I_s in (mA).

Ex/. find $v(z=\frac{3L}{4}, t)$, $I(z=L, t)$ for $0<t<5T$?
also find V_{ss} & I_{ss}?



at steady state: ($t=\infty$)

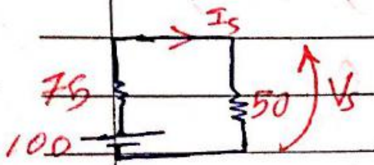


$\Rightarrow V_{ss} = 100\,\frac{25}{100} = \boxed{25 \text{ volt}}$

$I_{ss} = \frac{100}{100} = \boxed{1000\,mA}$

a, b will be given if Z_0 not given.

at ($t=0^+$):

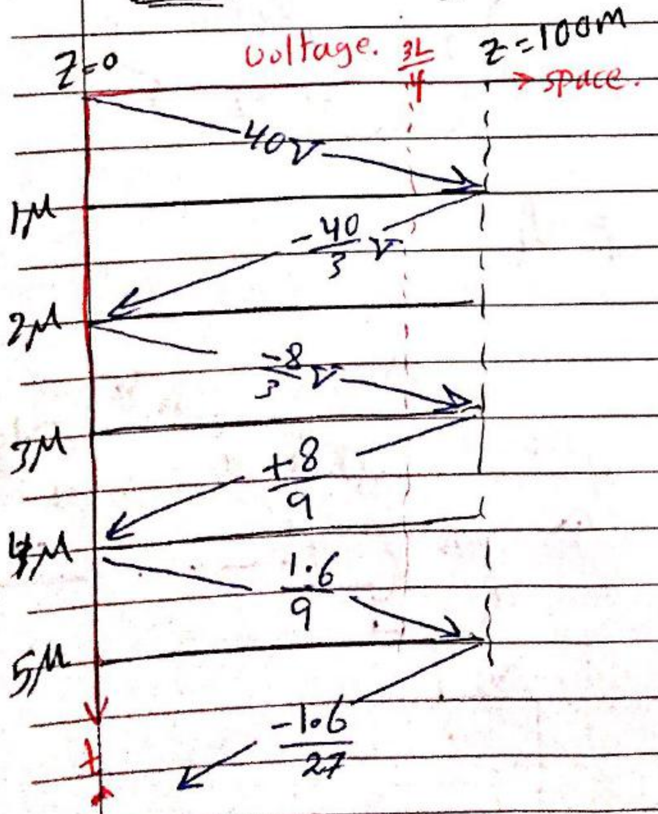


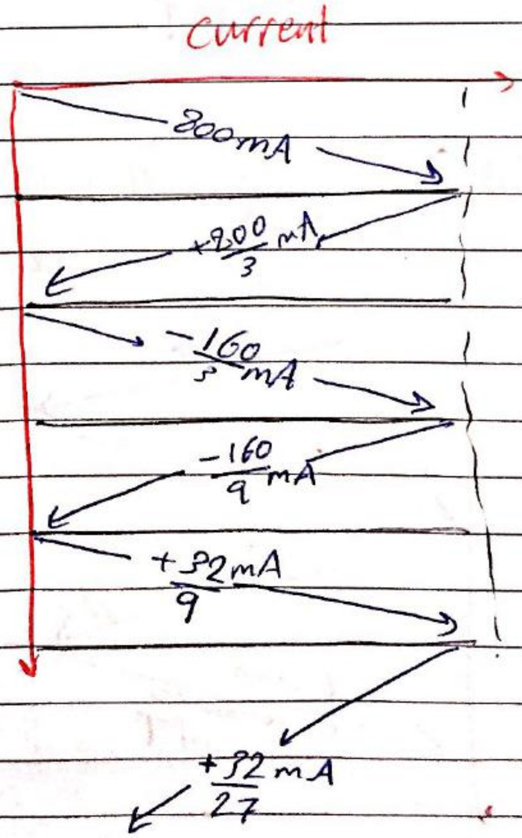
$\Rightarrow V_s = \frac{100\,(50)}{125} = \boxed{40 \text{ volt}}$

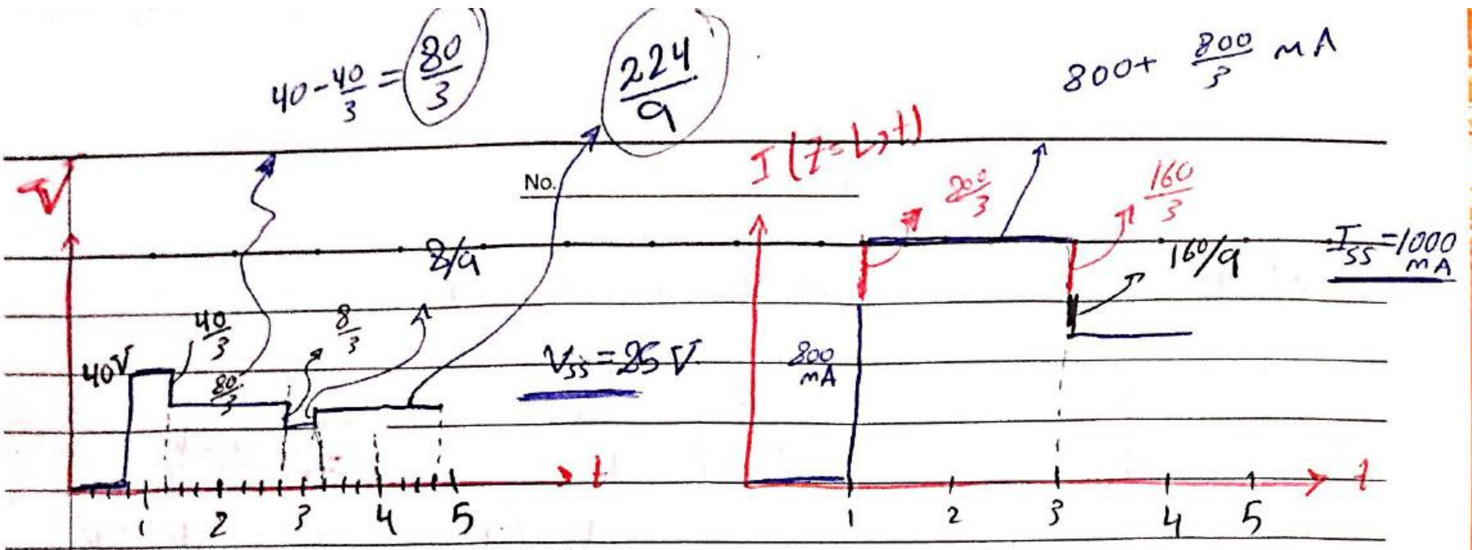
$I_s = \frac{100}{125} = \boxed{800\,mA}$

$\rho_g = 0.2$ & $\rho_L = \frac{-1}{3}$

$\rho_g = -0.2$ & $\rho_L = \frac{+1}{3}$



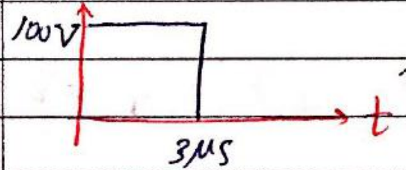




Ex.

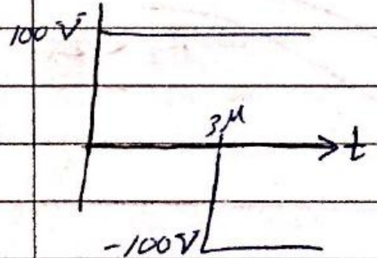
$a=1mm,\ b=2.5mm$
$\epsilon=4\epsilon_0$

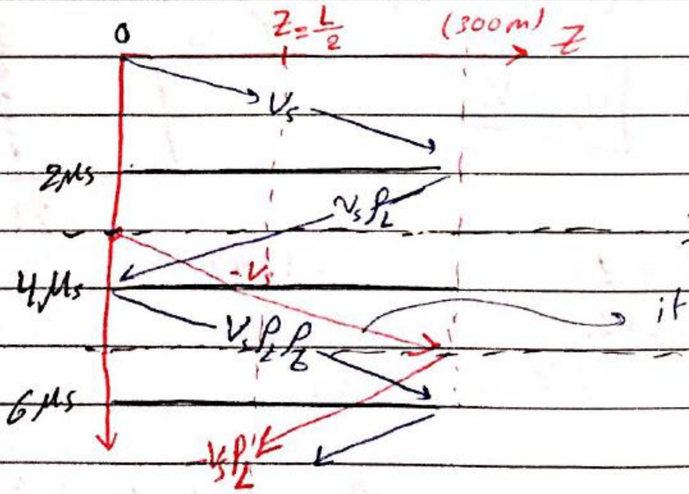
F.G:



Draw $V(z=\frac{L}{2},t)$
$I(z=\frac{L}{4},t)$
V_{ss} & I_{ss} ?

V_{ss} & I_{ss} = Zero

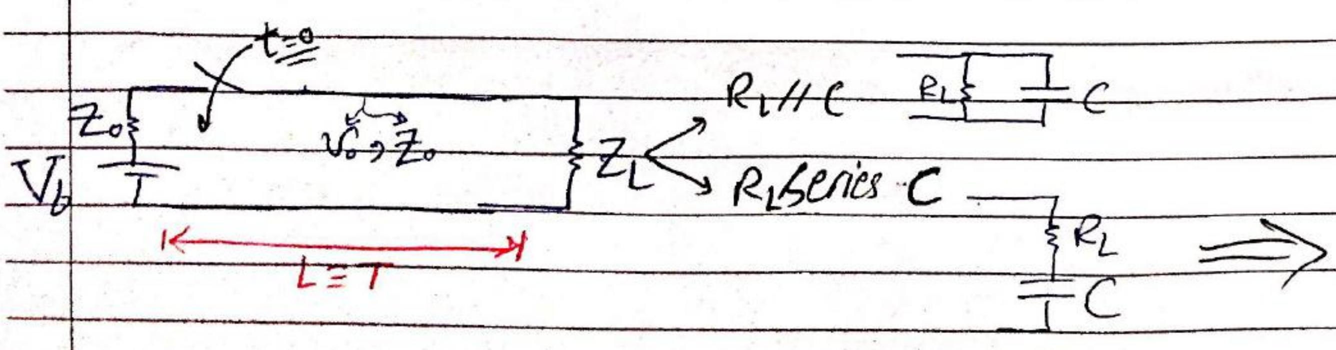




it is look like a shift with a minus sign.

.....etc.

Loads with R parallel with C & series with C:



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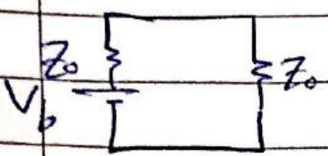
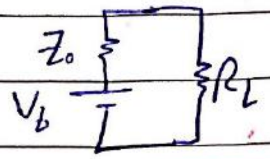
No.

at $t=0^+$:



$$\boxed{V_s = \frac{V_b}{2}}$$

at $t=\infty$: ($R_L // C$)

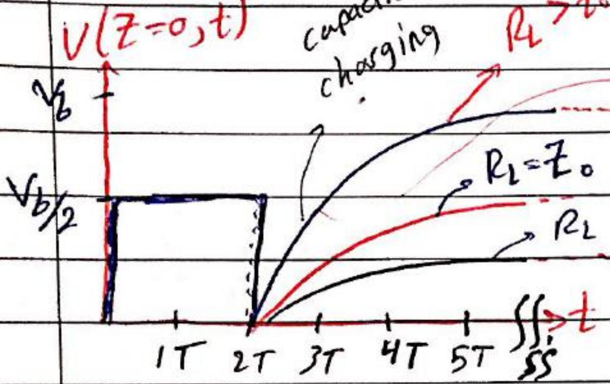


$$\boxed{V_{ss} = V_b \frac{R_L}{Z_0+R_L}}$$

$R_L = Z_0$ $V_{ss} = \frac{V_b}{2}$

$R_L > Z_0$ $V_{ss} = V_b$

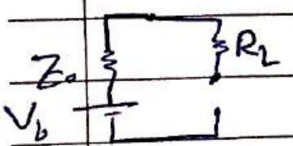
$R_L < Z_0$



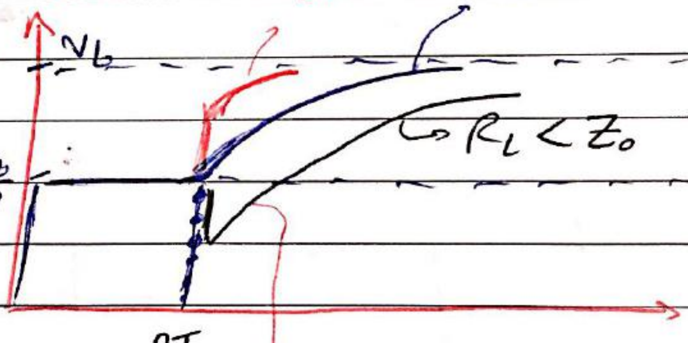
$e^{-t/\tau}$; $\tau = R_{eq}C$

$R_{eq} = R_L // Z_0$

at $t=\infty$ (R_L series C)



$$\boxed{V_{ss} = V_b}$$



$e^{-t/\tau}$

; $\tau = R_{eq}C$

$R_L + Z_0$

$\rho_L = \frac{R_L - Z_0}{R_L + Z_0}$

$R_L > Z_0$ (+ve)

$R_L = Z_0$ (0)

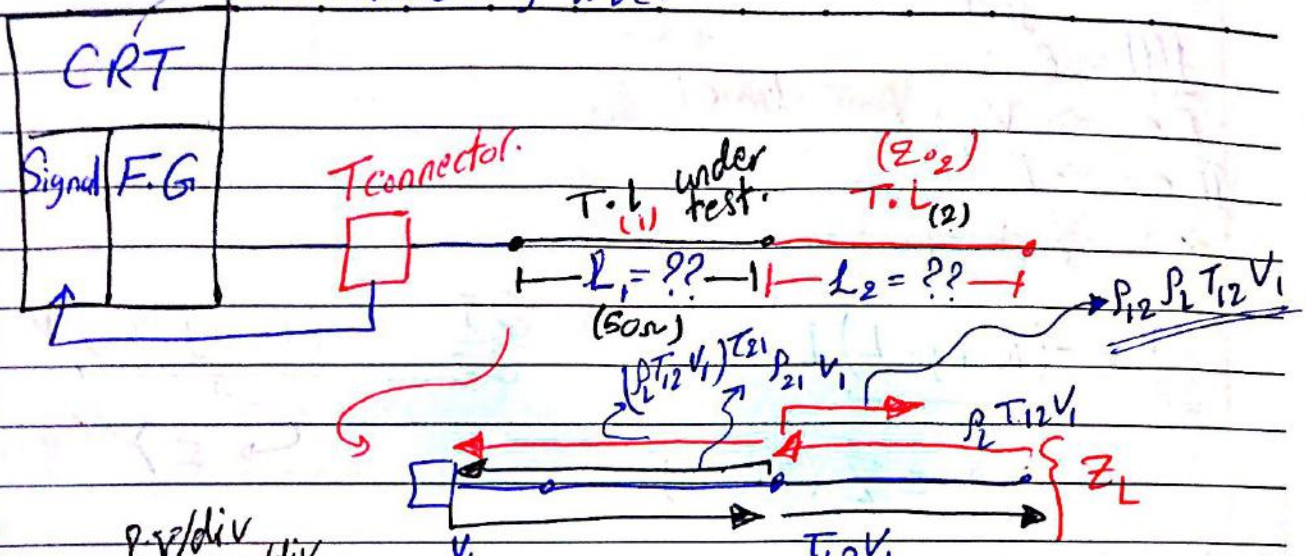
$R < Z_0$ (-ve)

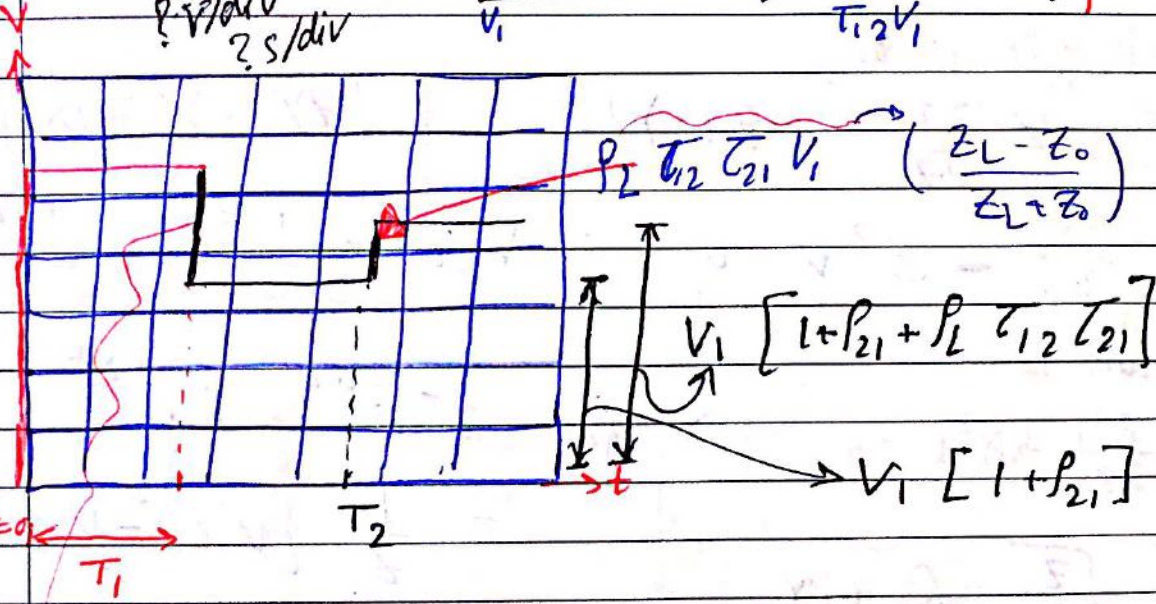
※ Do the same for inductive Connection.

After First.

Time Domain Reflectometer: (TDR)

* cathode ray tube.





$$T_1 = \frac{2L_1}{v_1} \qquad T_2 = \frac{2L_1}{v_1} + \frac{2L_2}{v_2}$$

$\rho_{21} V_1$

$$\frac{Z_{02} - Z_0}{Z_{02} + Z_0}$$

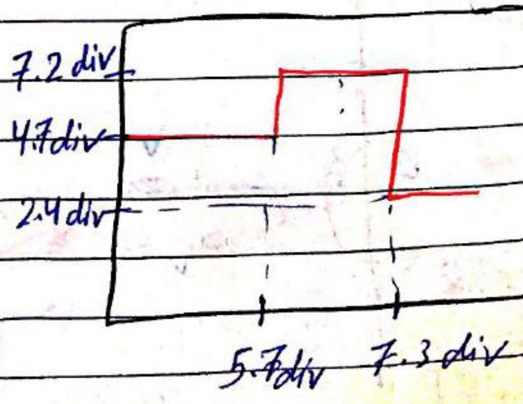
H.W:

$\epsilon_1 = 2.25\epsilon_0$

$\epsilon_2 = 9\epsilon_0$

$Z_0 = 50\,\Omega$

CRT reading: 1V/div, 1μs/div



find: Z_{02}, L_1, L_2, Z_L ??

Steady State Analysis of T.L:

No.

$f(t) = e^{j\omega t}$ $\quad e^{j\omega t}$

$\vec{E} \longleftrightarrow V$ (space, time)

$\vec{H} \longleftrightarrow I$

* Telegraph equations:

$e^{j\omega t} *$ $\left(\frac{\partial V}{\partial z} = -\underbrace{(R + j\omega L)}_{\equiv Z\ (\Omega/m)} I\right)$ $\qquad$ $\left(\frac{\partial I}{\partial z} = -\underbrace{(G + j\omega C)}_{\equiv Y\ (\mho/m)} V\right) * e^{j\omega t}$

$\Rightarrow$ By diffrentiating:

$$\frac{\partial^2 V}{\partial z^2} = -Z\frac{\partial I}{\partial z} = -Z(-Y)V \qquad \sqrt{ZY} = \gamma \equiv \alpha + j\beta$$

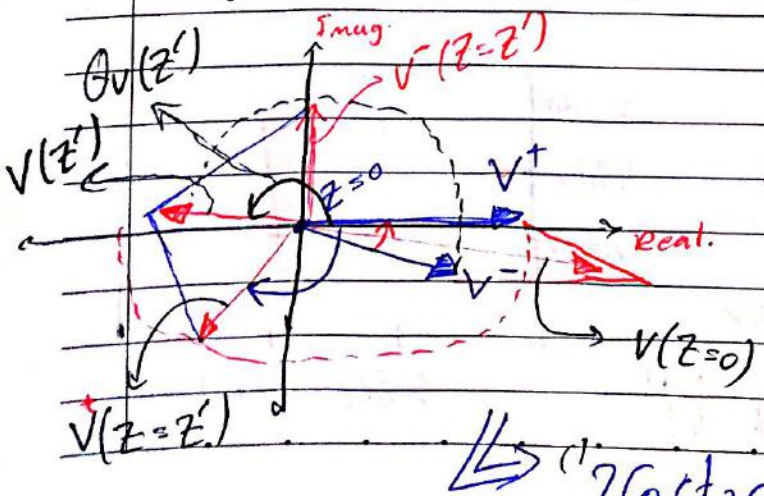
$$\Rightarrow \frac{\partial^2 V}{\partial z^2} - \gamma^2 V = 0 \Longrightarrow \boxed{V(z) = V^+ e^{-\gamma z} + V^- e^{+\gamma z}}$$

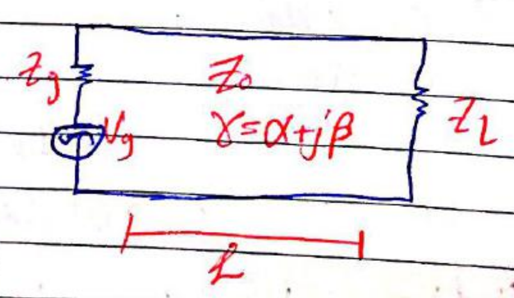
the same for:

$$\frac{\partial^2 I}{\partial z^2} - \gamma^2 I = 0 \Longrightarrow \boxed{I(z) = I^+ e^{-\gamma z} + I^- e^{+\gamma z} = \frac{1}{Z_0}\left[V^+ e^{-\gamma z} - V^- e^{+\gamma z}\right]}$$

$$Z_0 = \sqrt{\frac{Z}{Y}} = R_0 + jX_0$$

* for $V(z) = \underbrace{(V^+ e^{-\alpha z})\, e^{-j\beta z}}_{\text{incident}} + \underbrace{(V^- e^{+\alpha z})\, e^{+j\beta z}}_{\text{Reflected}}$

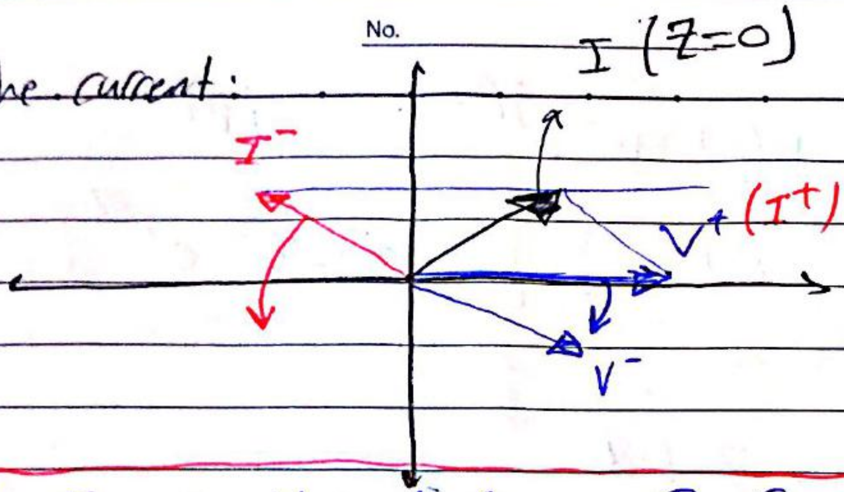




$\Longrightarrow$ "Vector Diagram"

No.

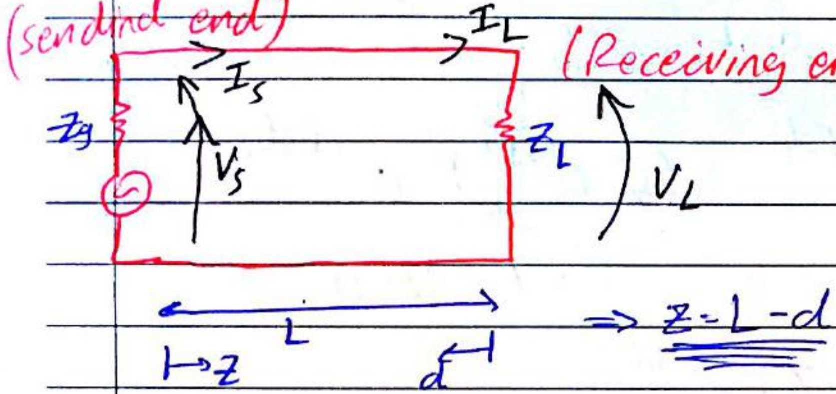
for the current:



$$\frac{1}{Z_0} = \left|\frac{1}{Z_0}\right| \angle\theta$$

V^+ & V^- are obtained from B.C

$\Rightarrow$ At sending end & At Receiving end.



$\Rightarrow$ $\underline{\underline{z = L - d}}$

we prefer B.C at $Z = L$

$V|_{Z=L} = V_L$

$I|_{Z=L} = I_L$

$$Z_{in}(Z) = \frac{V(Z)}{I(Z)} = Z_0 \, \frac{V^+ e^{-\gamma Z} + V^- e^{+\gamma Z}}{V^+ e^{-\gamma Z} - V^- e^{+\gamma Z}}$$

$$\rho_{(Z)} = \frac{V^-(Z)}{V^+(Z)}$$

$$= Z_0 \, \frac{e^{-\gamma Z} + \rho_L e^{+\gamma Z}}{e^{-\gamma Z} - \rho_L e^{+\gamma Z}}$$

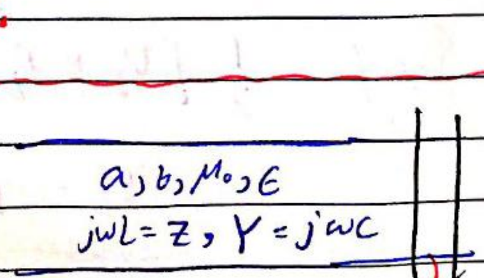
$$\hat{P} = \frac{1}{2} V I^* = \frac{|I|^2}{2} Z_{in} \quad \text{Watt.}$$

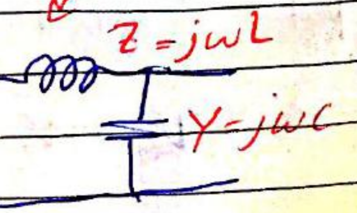
※ Lossless T.L:

$\sigma_c \to \infty \Rightarrow R_{ac} \to 0$

$\sigma_d \to 0$ \} $\Rightarrow R_{leakage} = \infty$

$\epsilon'' \to 0$ \} or $G = 0$





$\alpha \cong 0$

$\Rightarrow \alpha L \sim 0$ (practically).

No.

$$\Rightarrow V(z) = V^+(z) + V^-(z) = \boxed{V^+ e^{-j\beta z} + V^- e^{+j\beta z}}$$

At receiving end:

$$\boxed{I(z) = \frac{1}{Z_0}\left[V^+ e^{-j\beta z} - V^- e^{+j\beta z}\right]}$$

B.C

$V\big|_{z=L} = V_L$

$I\big|_{z=L} = I_L$

$\underline{Z = L - d}$

incident $\rightarrow$ Reflected $\leftarrow$

$$V(d) = V^+\Big|_{\substack{z=0\\ d=L}} e^{-j\beta(L-d)} + V^-\Big|_{\substack{z=0\\ d=L}} e^{+j\beta(L-d)}$$

$$\Rightarrow V(d) = \underbrace{\left(V^+ e^{-j\beta L}\right)}_{[V^+(d=0)]} e^{j\beta d} + \underbrace{\left(V^- e^{+j\beta L}\right)}_{[V^-(d=0)]} e^{-j\beta d}$$

$$\Rightarrow V(d) = V^+(0)\, e^{j\beta d} + V^-(0)\, e^{-j\beta d}$$

$$I(d) = \frac{1}{Z_0}\left[V^+ e^{j\beta d} - V^- e^{-j\beta d}\right]$$

implicitly means at $d=0$

B.C.

$V_L = V(d)\big|_{d=0} = V^+ + V^- \Rightarrow Z_0 I_L = V^+ - V^-$

$$\Rightarrow \boxed{\begin{aligned} V^+\Big|_{d=0} &= \frac{1}{2}\left[V_L + I_L Z_0\right] \\ V^-\Big|_{d=0} &= \frac{1}{2}\left[V_L - I_L Z_0\right] \end{aligned}}$$

$$\Rightarrow V(d) = \frac{1}{2}\left[V_L + I_L Z_0\right] e^{j\beta d} + \frac{1}{2}\left[V_L - I_L Z_0\right] e^{-j\beta d}$$

$$\rightarrow V(d) = V_L \cos\beta d + j\, I_L Z_0 \sin\beta d$$

the same for current

$$I(d) = \frac{1}{Z_0}\left[j V_L \sin\beta d + I_L Z_0 \cos\beta d\right]$$

No.

for $Z_{in}(d) = \dfrac{V(d)}{I(d)} = Z_0 \dfrac{V_L \cos\beta d + j I_L Z_0 \sin\beta d}{j V_L \sin\beta d + I_L Z_0 \cos\beta d}$

$\Rightarrow \; = Z_0 \dfrac{1 + j \frac{Z_0}{Z_L} \tan\beta d}{j \tan\beta d + \frac{Z_0}{Z_L}} = \boxed{Z_0 \dfrac{Z_L + j Z_0 \tan\beta d}{Z_0 + j Z_L \tan\beta d}}$

at $d=0$ $Z_{in} = Z_L$ as obtained.

for thevenin impedance:

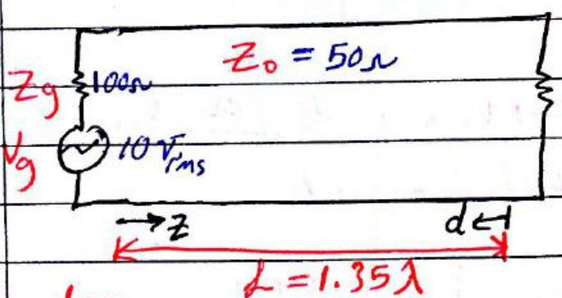
$Z_{th(in)}(d) = Z_{in}(d)$.

$\rho(d) = \dfrac{V^-(d)}{V^+(d)}$. at $d=0$ $\rho(d=0) = \rho_L = \dfrac{V^-(0)}{V^+(0)} = \dfrac{V_L - I_L Z_0}{V_L + I_L Z_0}$

$\Rightarrow \boxed{\rho(d=0) = \dfrac{Z_L - Z_0}{Z_L + Z_0}}$

※ Vector Diagram:

Ex.



find the total V, I, Z_{in} at $d = 0, 0.25\lambda, l$

$V_L = V_L^+ + V_L^-$

for $d=0$

$V(d) = V^+(0)\, e^{+j\beta d} + V^-(0)\, e^{-j\beta d}$

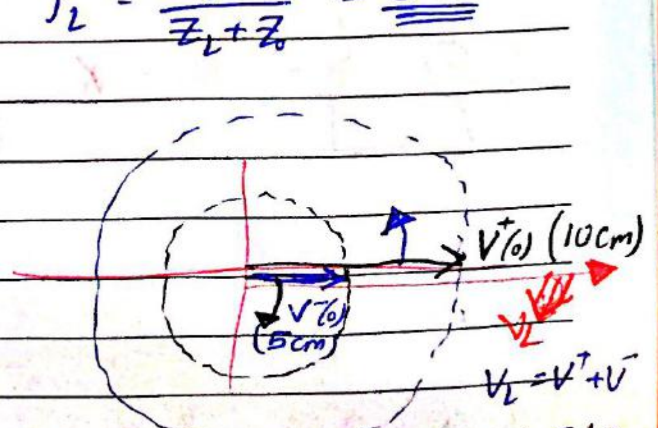
$I(d) = \dfrac{1}{Z_0}\left[V^+(0)\, e^{+j\beta d} - V^-(0)\, e^{-j\beta d}\right]$

$\rho_L = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \underline{0.5}$

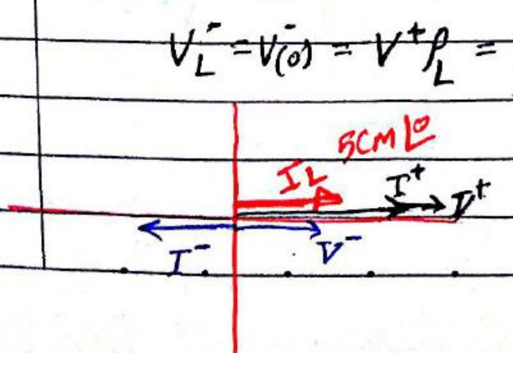
Assume $V_L^+ = V^+_{(0)} = \underline{10 \text{ volt} \angle 0^\circ}$

choose a scale $1V/cm$.

$V_L^- = V^-_{(0)} = V^+ \rho_L = \underline{5\angle 0^\circ}$ volt.



$V_L = V^+ + V^-$
$= 15\angle 0$ cm.



$$\Rightarrow Z_{in}(d=0) = Z_0 \frac{\text{Length of } V_L \times \text{scale}}{\text{Length of } I_L \times \text{scale}} = \boxed{150\,\Omega}$$

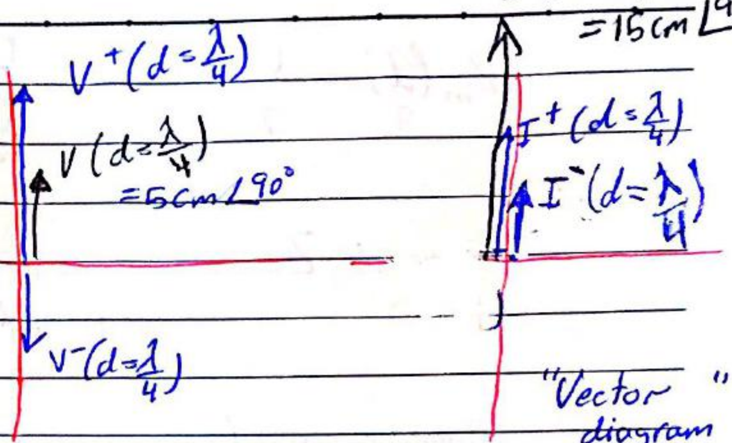
$50\,\Omega$

No. 5 1

for $d = 0.25\lambda$

$\lambda \equiv 2\pi$

$\frac{\lambda}{4} \Rightarrow \frac{2\pi}{4}$



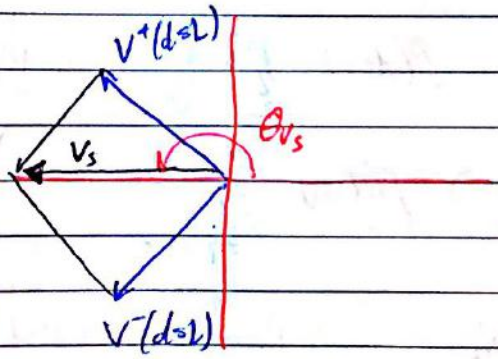
$$Z_{in}(d=\tfrac{\lambda}{4}) = Z_0 \times \frac{5}{15} = \boxed{\frac{50}{3}\,\Omega}$$

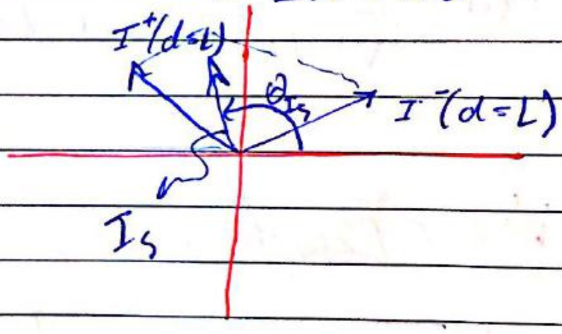
for $d = L$:

$1.35\lambda \Rightarrow 1.35 \times 2\pi$

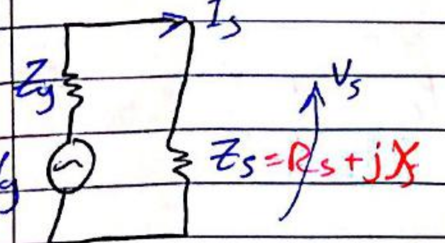
$2\pi + 0.35 \times 2\pi$

$= 2\pi + 126°$





$$Z_{in}(d=L) = Z_s = Z_0 \frac{\text{Length of } V_s \times \text{scale}}{\text{length of } I_s \times \text{scale}} \angle \theta_{V_s} - \theta_{I_s}.$$



it has phase.

$$\Rightarrow |V_s| = \left| \frac{V_g \times Z_s}{Z_g + Z_s} \right|$$

* لإيجاد مقياس الرسم الصحيح نقوم بقسمة V_s الفعلية على V_s التي تم إيجادها بالرسم.

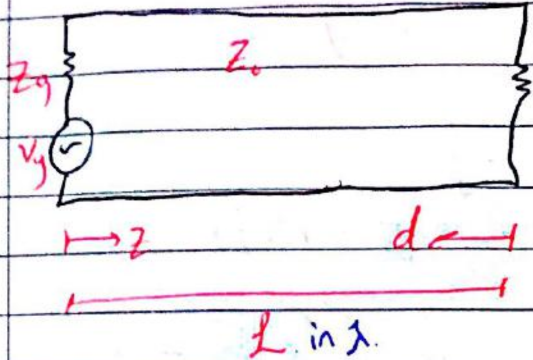
if we asked to find P_s & P_L:

$P_s = P_L = I_L^2 R_L$

if Not rms multiply By $\frac{1}{2}$.

Crank Diagram:

No.



$V(d) = V^+ e^{+j\beta d} + V^- e^{-j\beta d}$

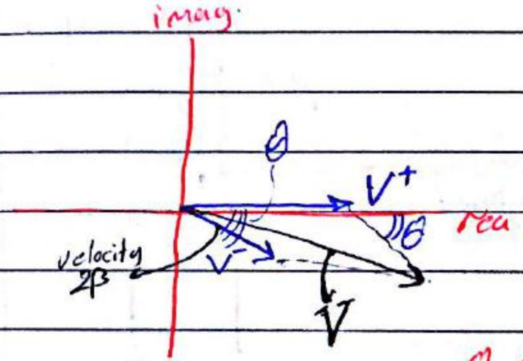
$I(d) = \frac{1}{Z_0}\left[V^+ e^{+j\beta d} - V^- e^{-j\beta d}\right]$

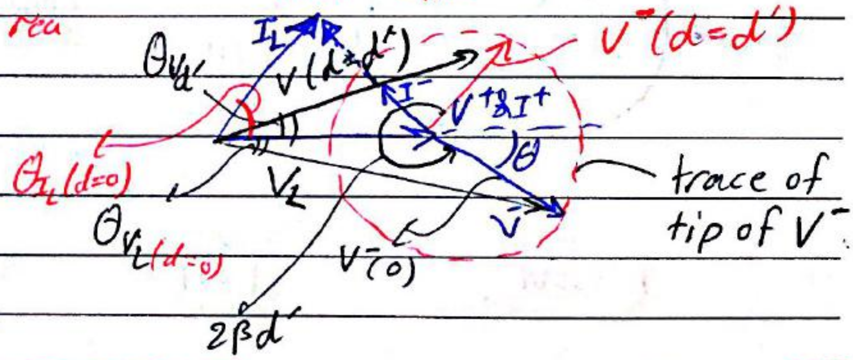
$V(d) = \cancel{e^{j\beta d}}\left[V^+ + V^- e^{-2j\beta d}\right]$

$I(d) = \frac{1}{Z_0} \cancel{e^{j\beta d}}\left[V^+ - V^- e^{-2j\beta d}\right]$

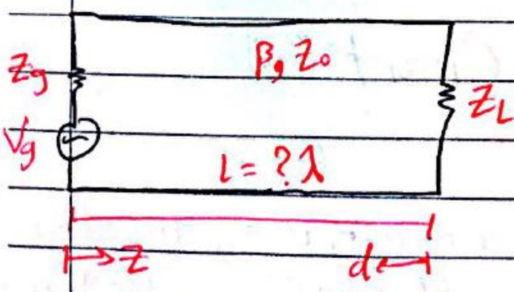
so we disregard $e^{+j\beta d}$ in V & I

$\Rightarrow$ this doesn't affect $\hat{\rho}$ or load Z_{in}.





يختلف ال V عن I هنا بأنه
I معه $\frac{1}{Z_0}$ وأيضا هنالك
إضافة $\pm 180°$ على الزاوية.



$V(d) = V^+(d) + V^-(d) = V^+ e^{+j\beta d} + V^- e^{-j\beta d}$

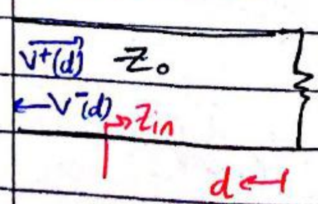
$I(d) = \frac{1}{Z_0}\left[V^+ e^{j\beta d} - V^- e^{-j\beta d}\right]$

$\Rightarrow V(d) = V^+ + V^- e^{-2j\beta d}$

$I(d) = \left[V^+ - V^- e^{-j2\beta d}\right] * \frac{1}{Z_0}$

$\hookrightarrow$ always we could write it if we want Z_{in} or $\hat{\rho}$.

T.L charts:



$\rho(d) = \frac{V^-(d)}{V^+(d)} = \frac{Z_{in}(d) - Z_0}{Z_{in}(d) + Z_0}$

$\rho(d) = \frac{Z - Z_0}{Z + Z_0} \Rightarrow Z = Z_0 \frac{1+\rho}{1-\rho}$

we will replace the physical length [L] By $\left(\frac{L}{\lambda}\right)$

$\Rightarrow$

replace Z by $\frac{Z}{Z_0} \equiv$ Normalized charchteristic impedance (z)

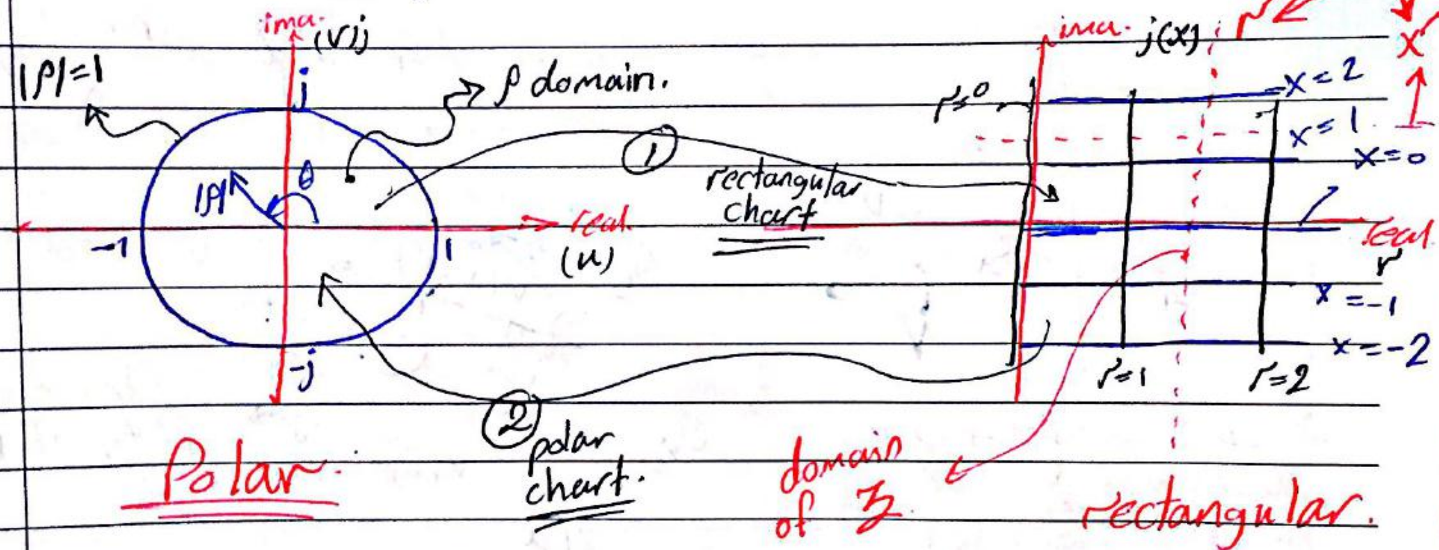
No.

$$\Rightarrow \rho(d) = \frac{z-1}{z+1} \Rightarrow z = \frac{1+\rho}{1-\rho}$$

$\rho = u + jv = \sqrt{u^2+v^2}\, e^{j\theta}$ $\qquad$ $z = r + jx$

$|\rho| \leq 1$ & θ = could be any value $\qquad$ $\infty \geq r \geq 0$

$\infty \geq x \geq -\infty$



$*$ Polarchart $\xrightarrow[\text{called}]{\text{which is}}$ "Smith Chart"

※ Smith Chart:

$$z = r + jx = \frac{1+\rho}{1-\rho} = \frac{1+u+jv}{1-u-jv} * \frac{(1+u)+jv}{(1-u)-jv} * \frac{(1-u)+jv}{(1-u)+jv}$$

$$\Rightarrow \left(u - \frac{r}{1+r}\right)^2 + v^2 = \left(\frac{1}{1+r}\right)^2 \Rightarrow \text{family of circles.}$$

radius $= \frac{1}{1+r}$, Centre $= \left(\frac{r}{1+r}, 0\right)$

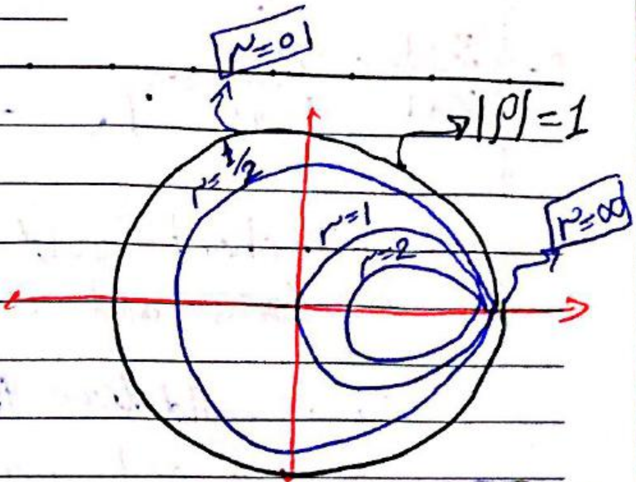
$$\Rightarrow (u-1)^2 + \left(v - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2$$

family of circles

radius $= \left|\frac{1}{x}\right|$, centre $\left(1, \frac{1}{x}\right)$

No.

r	rad (ρ)	centre
0	1	(0,0)
0.5	2/3	(1/3, 0)
1	1/2	(1/2, 0)
2	1/3	(2/3, 0)
∞	0	(1, 0)

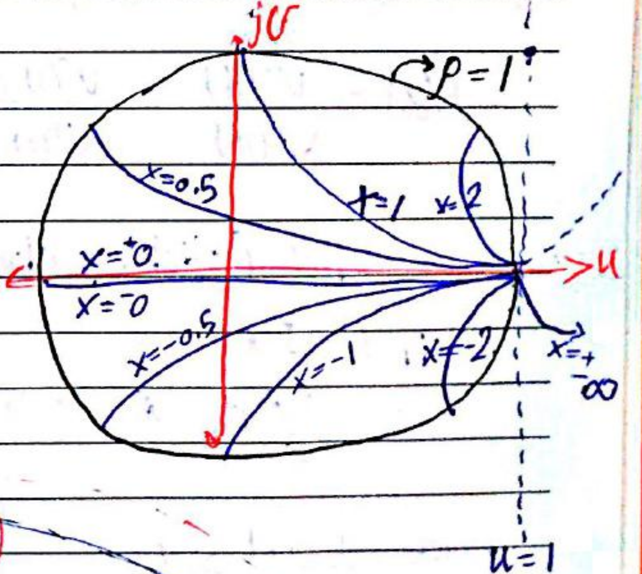


inductive.

capacitive.

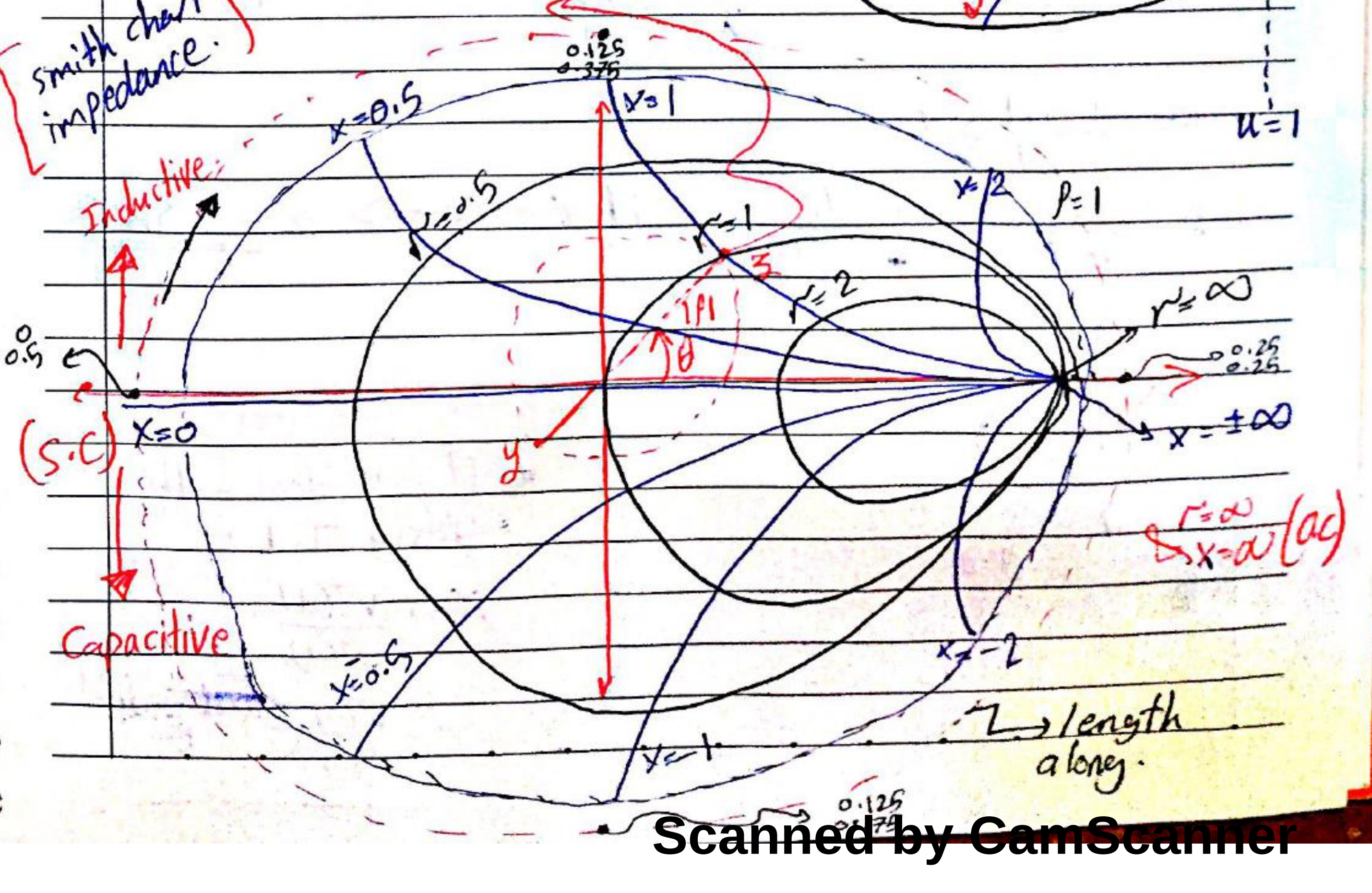
x	rad (ρ)	centre
± 0	∞	$(1, \pm\infty)$
± 0.5	2	$(1, \pm 2)$
± 1	1	$(1, \pm 1)$
± 2	0.5	$(1, \pm 0.5)$
$\pm\infty$	0	$(1, \pm 0)$

it is shrinking down to $r=\infty$.



$z=r+jx$ $\quad \rho=|\rho|e^{j\theta}$

smith chart impedance.



$z = r + jx = \frac{Z}{Z_0}$ r: resistance g: conductance

x: reactance b: susceptance

$y = g + jb = \frac{Y}{Y_0}$

No.

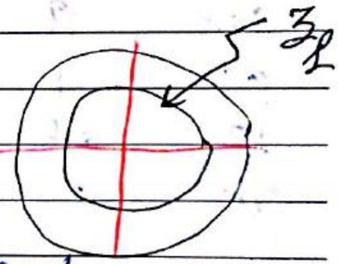
$\rho = \frac{z-1}{z+1} = \frac{1/y - 1}{1/y + 1} = - \frac{y-1}{y+1} \Rightarrow \rho = \frac{y-1}{y+1} \angle \pm 180^\circ$

$\Rightarrow$ smithchart could be impedance and admittance.

$\hookrightarrow$ we could draw the smith chart without putting r or g and x or b on it $\Rightarrow$ just put numbers like the actual smith chart.

* Length & smithchart:

$\rho(d) = \frac{V^-(d)}{V^+(d)} = \frac{V^-(0)\, e^{-j\beta d}}{V^+(0)\, e^{+j\beta d}} = \rho_L \, e^{-2j\beta d}$



Constant ρ_L circle.

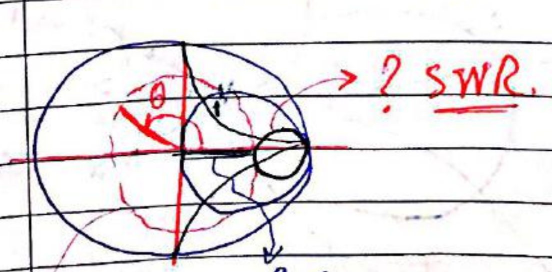
$\Rightarrow 2\beta d$ (clicking clockwise)

$2\beta d = 2\pi$

$\Rightarrow \frac{2\pi}{\lambda} d = \pi \Rightarrow \boxed{\frac{d}{\lambda} = 0.5}$

$SWR = \frac{1+|\rho|}{1-|\rho|} \geq 1$

$z = \frac{1+\rho}{1-\rho} = \frac{1+|\rho|e^{j\theta}}{1-|\rho|e^{j\theta}}$, if $\theta \to 0 \Rightarrow z \to SWR$.



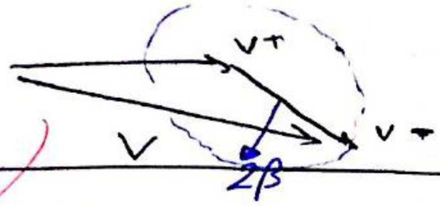
* if we deal with lossy T.L:

$\rho(d) = \frac{V^-(d)}{V^+(d)}$

$= \frac{V^-(0)}{V^+(0)} e^{-2\alpha d} e^{-2j\beta d}$.

$V(d) = V^+ + V^- e^{j2\beta d}$

$I(d) = \frac{1}{Z_0}\left[V^+ - V^- e^{-2j\beta d}\right]$



No.

for the voltage & current we put them on smithchart.

* خطوات الحل على smith chart :

① نقوم بتحديد z من خلال تقاطع x مع r حيث $z = \frac{Z}{Z_0}$

② نقوم بتحديد ρ من خلال الخط الواصل من centre إلى نقطة التقاطع ونقوم بقياس طول ذلك الخط الذي سيمثل الـ $|\rho|$ ونقوم بإيجاد الزاوية بالمنقلة.

③ إذا أردنا تحديد SWR ستكون عند نقطة تقاطع الدائرة التي أوجدناها مع المحور الحقيقي SWR

④ قيمة $\frac{\lambda}{2}$ تمثل لفة كاملة $360°$ أو 2π لذلك λ تمثل $720°$ أو 4π.

⑤ * المقياس الأخير المحيط بـ smith chart يمثل المسافة باتجاه المصدر بدلالة λ.

* " قبل الأخير " " " " " الحمل " " " .

* " الداخلي " " يمكن استخدامه بدلا من المنقلة لتحديد زاوية reflection coefficient.

⑥ لتحديد قيم الموقع لـ V_{max} و V_{min} عند نقطة معينة بدلالة λ بنشوف لـ V_{max} كم مرة قطع المحور الموجب الحقيقي وبنحسبها بكل مرة وآخر اشي بنجمع ونفس الإشي لـ V_{min} ولكن للتقاطع مع المحور السالب الحقيقي.

Ex. T.L (lossless) with $l = 2.05\lambda$, $V^+(d=0) = 10$ Volt, $Z_g = 100+j50$, $Z_L = 50+j50$, $Z_0 = 50\Omega$, using smith chart find $Z_{in}(d=0, l)$, $Y_{in}(d=0, l)$, V_L, I_L, V_g, I_g, P_L & P_s ? Plot $|V|$ & $|I|$ vs d ?

* نقاط الـ imped. تمثل نقاط voltage
* نقاط الـ addm. تمثل نقاط current

$P_L = P_s$ (lossless)

$z_L = \frac{50+j50}{50} \equiv Z_L/Z_0 = (1+j)\,\Omega$

$\beta d = 2\left(\frac{2\pi}{\lambda} * d\right) = 2 * \frac{(2\pi)}{\lambda} * 2.05\lambda$

$= 4\pi * 2.05 = 8\pi + (0.05 * 4\pi) * \frac{180}{\pi}$ rad

$= 8\pi + 36°$

$\mathfrak{Z}_s = 2 + j$

$Z_s = Z_0 \mathfrak{Z}_s = 100 + j50\ \Omega$

Do the same for I_s & V_s

$|\rho_L| = \dfrac{3.3\text{cm}}{8.7\text{cm}} = \boxed{0.38}$

$\theta_{\rho_L} = 63.8°$

The Complete Smith Chart

Black Magic Design

10 volt → 7.6cm

V_L ?? → 9.5cm

$\Rightarrow \boxed{V_L = 12.5\ \text{volt}}$

10 volt → 7.6cm

I_L ?? → 6.8cm

$I_L = 8.95A$

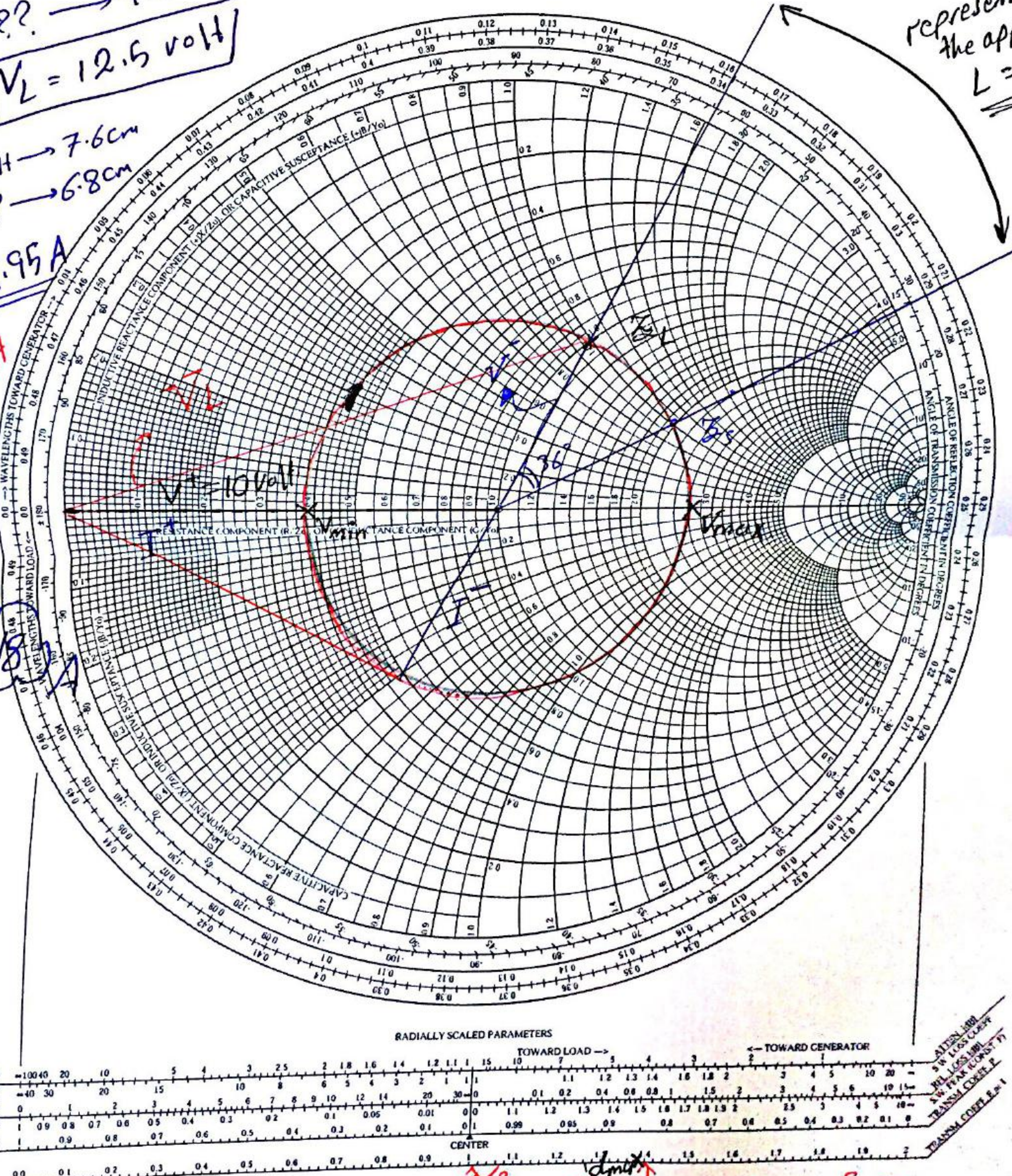
Apparent

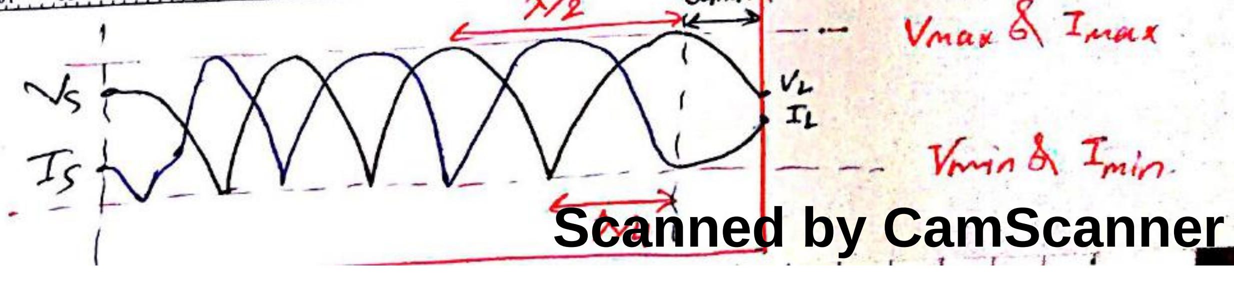
⇓

true: $\dfrac{8.95}{Z_0}$

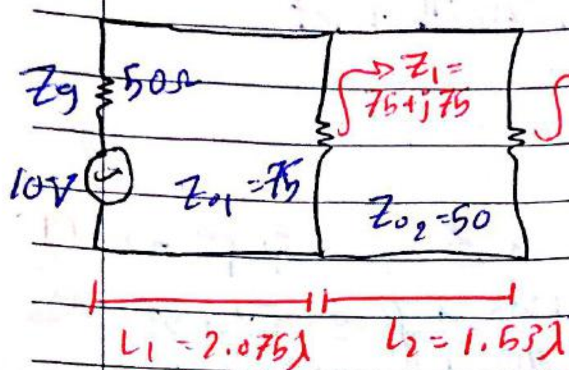
$I_L = 0.18\ A$

represent the apparent $L = 0.05\lambda$





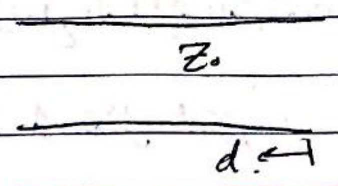
H.W:



Need: $Z_S, Z_3, Z_2, \rho_S, \rho_1, \rho_2$?

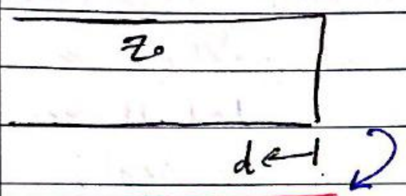
Matching in T.L:

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan\beta d}{Z_o + jZ_L \tan\beta d}$$



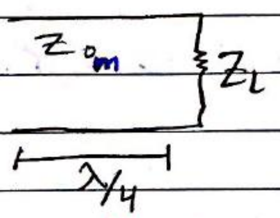
$$\boxed{Z_{o.c} = -jZ_o \cot\beta d}$$

$= -jX$



$$\boxed{Z_{s.c} = jZ_o \tan\beta d}$$

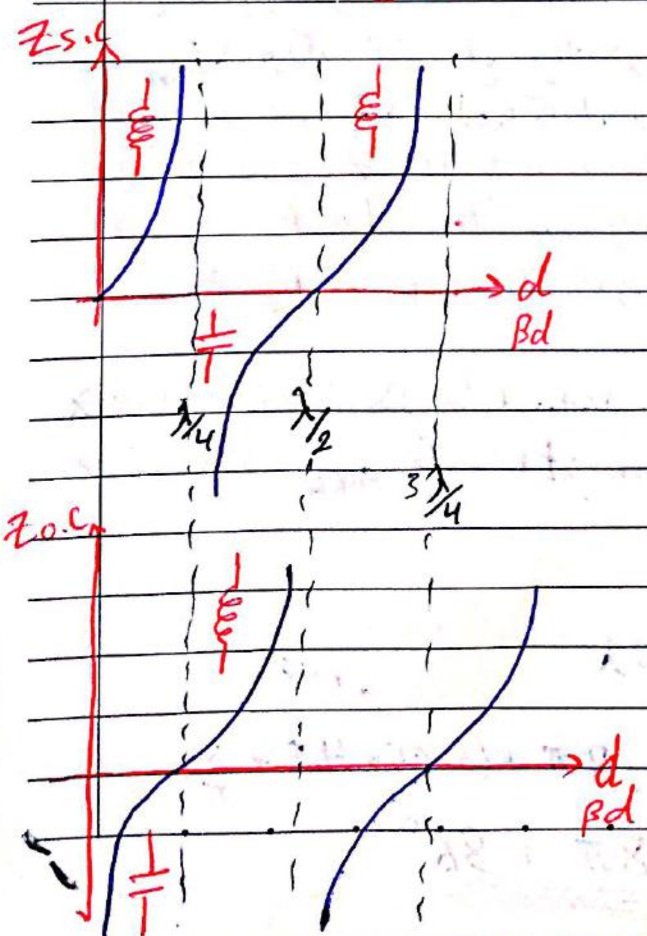
$\Rightarrow = jX$

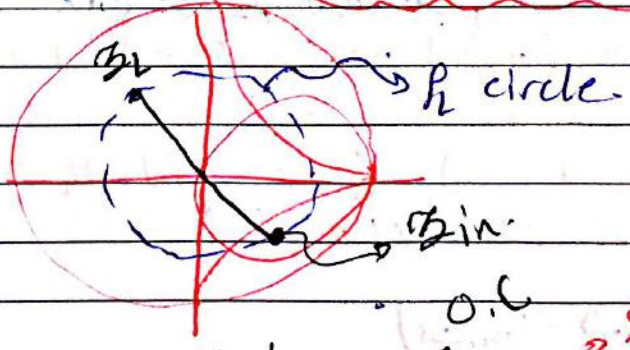


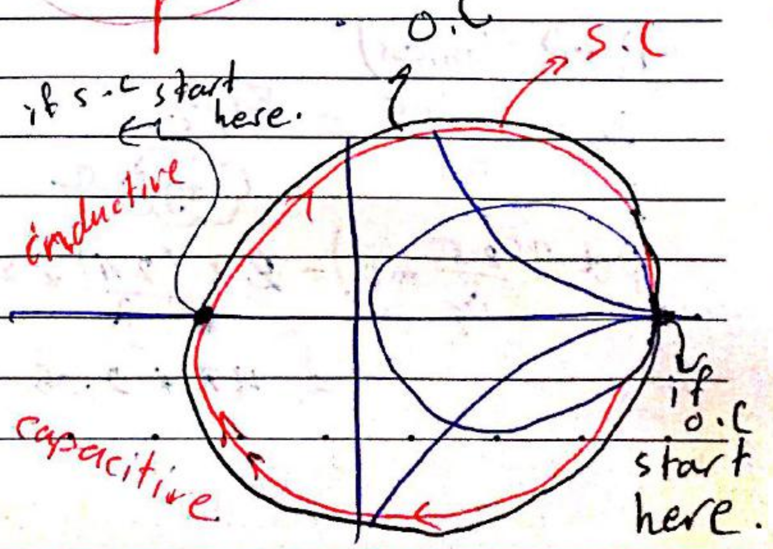
$$\Rightarrow \boxed{Z_{in} = \frac{Z_{om}^2}{Z_L}}$$

$\frac{\lambda}{4}$ - Transformer

$$\Rightarrow Z_{om} = \sqrt{Z_L Z_{in}}$$







if $Z_0 = R_L$. stub $\Rightarrow$ Not used in High frequancy (GHz)

Z_0 $\quad Z_L = R_L + jX_L$

Hf ≡ 3–30 MHz

if $Y_0 = G_L$

Y_0 $\quad Y_L = G_L + jB_L$

* Matching using series & parallel stubs:

$\Rightarrow$ we will study parallel stubs. $\quad y_L' = g_L' + j b_L'$

$y_{in} = 1 + j0$

$z_0 = 1 \Rightarrow Z_0 \Leftrightarrow Y_0 \Rightarrow y_0 = 1$ $\quad Z_L \Leftrightarrow Y_L$

$\Rightarrow y_L = g_L + j b_L$

$z_L = \frac{Z_L}{Z_0}$

Assuming $g \neq 1$. $\quad y_{st_1} = j b_{st_1}$ $\quad$ o.c

$y_L = g_L + j b_L$ with $g_L \gtrless 1$

$\Rightarrow$ move on ρ_L circle till making $g_L \Rightarrow g_L' = 1$

* Need $y_{in} = 1 + j0 = y_L' + y_{st_1} = g_L' + j b_L' + j b_{st_1}$

$g_L' = 1$ $\Rightarrow g = 1$ circle is the locuse of y_L'

$\Rightarrow b_{st_1} = -b_L'$

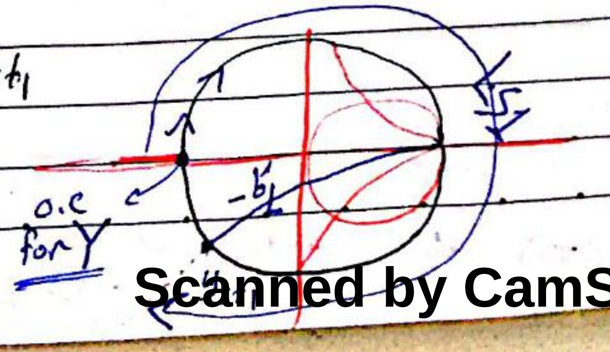
at $d = 0 \Rightarrow z_L \rightarrow y_L \rightarrow \rho_L$ circle $\quad$ move toward generator.

ρ_L is the other locuse of $y_L' \Rightarrow (y_L' = 1 + j b_L')$

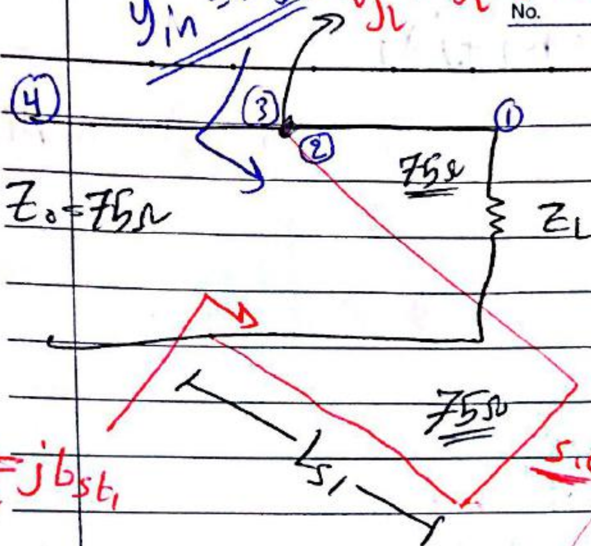
* stub: $y_{st_1} = 0 + j b_{st_1}$

$j b_{st_1}$

L_{s_1} ??



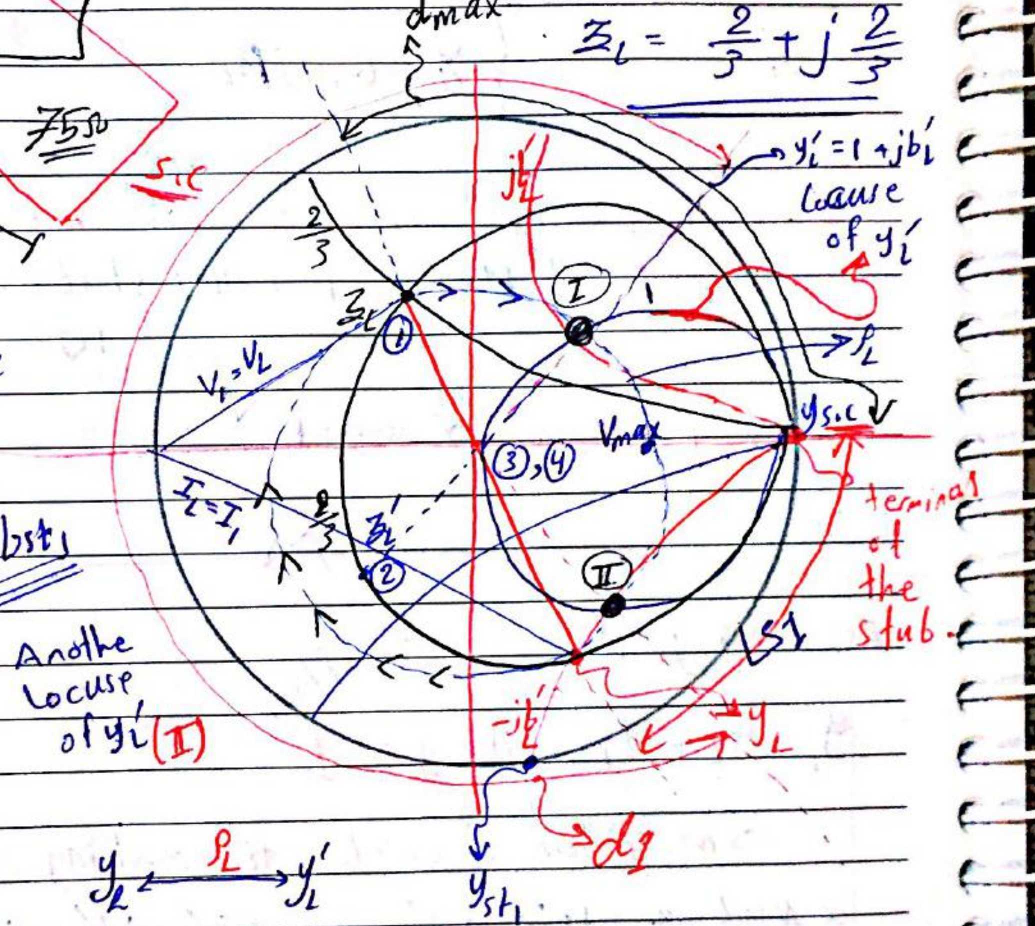
Ex. Match the load for the T.L: (using matching parallel stub.)



$y_{st_1} = jb_{st_1}$

$y_L = \frac{Y_L}{Y_o} = \frac{1}{z_L}$

$z_L = \frac{2}{3} + j\frac{2}{3}$



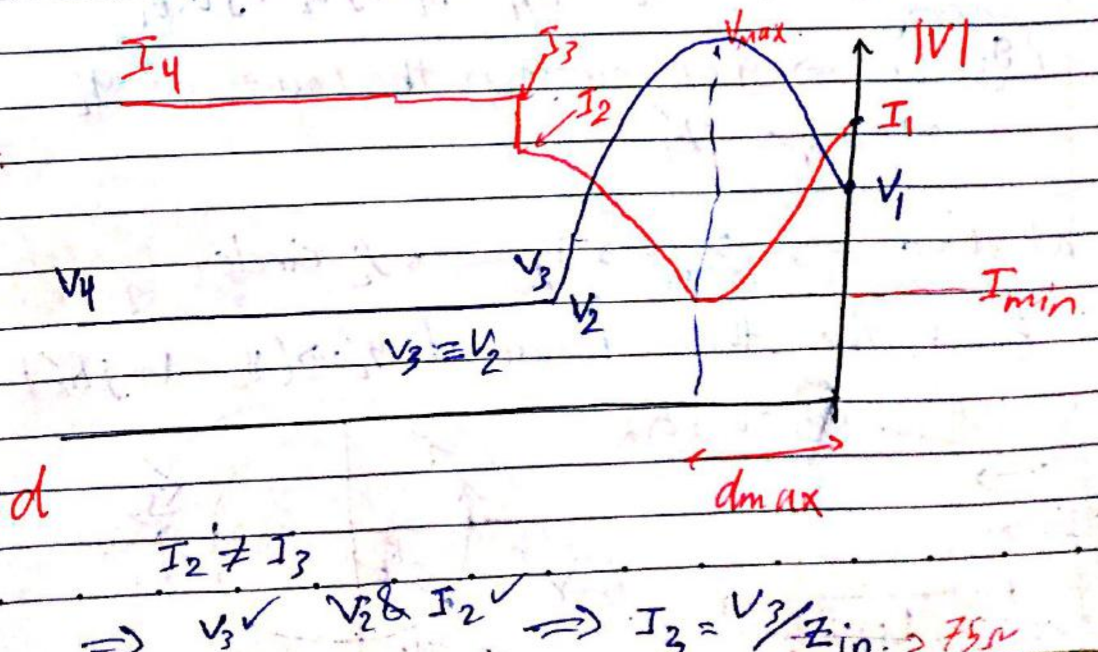
if the two locuses intersect there is matching.

$y_{in} = 1+j0 = g_L' + jb_L' + jb_{st_1}$

$g_L' = 1$

$b_{st_1} = -b_L'$

$y_L \xleftrightarrow{\rho_L} y_L'$

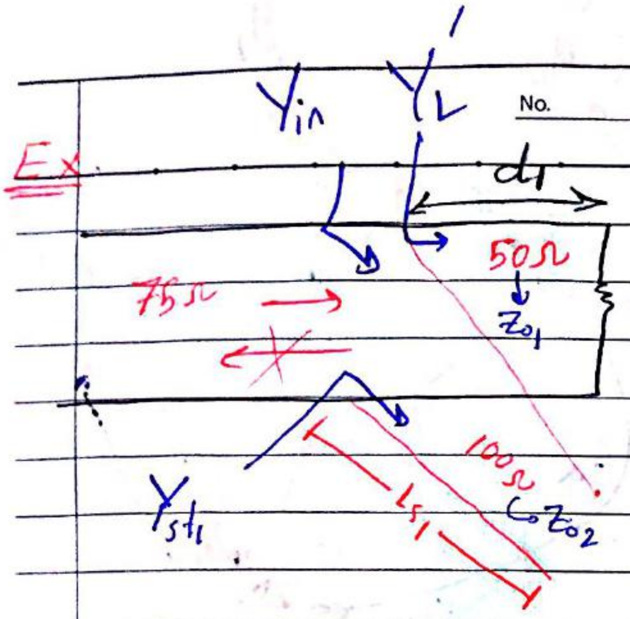


for power:
$P_s = P_3 = P_2 = P_1$
$= |I_1|^2 R_L$

$I_2 \neq I_3$

$\Rightarrow V_3$ ✓ V_2 & I_2 ✓ $\Rightarrow I_3 = V_3/Z_{in} \Rightarrow 75Ω$

Ex



$Z_L = 75 + j50 \Rightarrow z_L = \frac{Z_L}{50} \Rightarrow \rho_L \Rightarrow y_L$

Locuse of y_L'

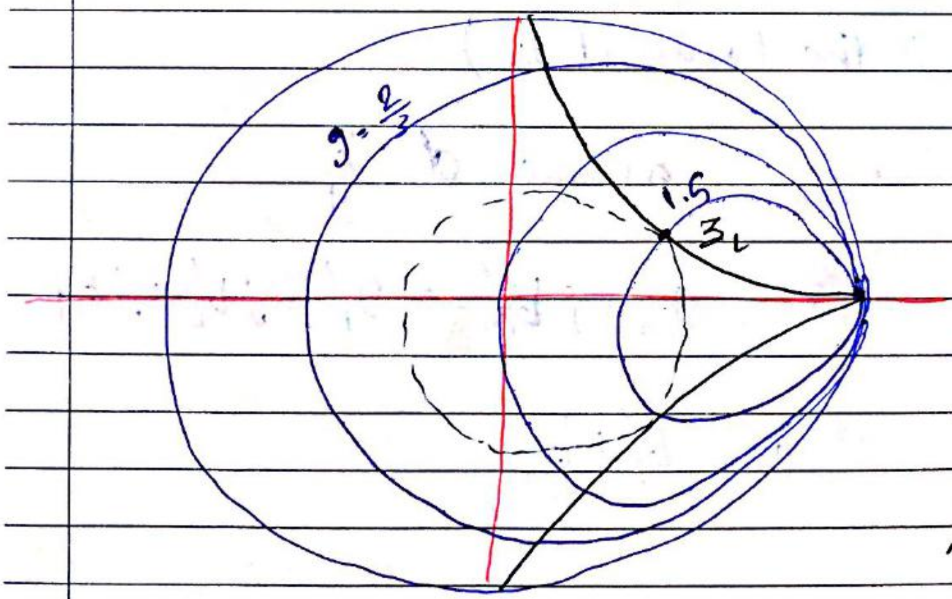
$$Y_{in} = \frac{1}{75} = Y_L' + Y_{st_1}$$

$$= G_L' + jB_L' + jB_{st_1}'$$

Need to match $Y_{in} = \frac{1}{75} (\Omega)^{-1}$

$$G_L' = \frac{1}{75}, \quad B_{st_1} = -B_L'$$

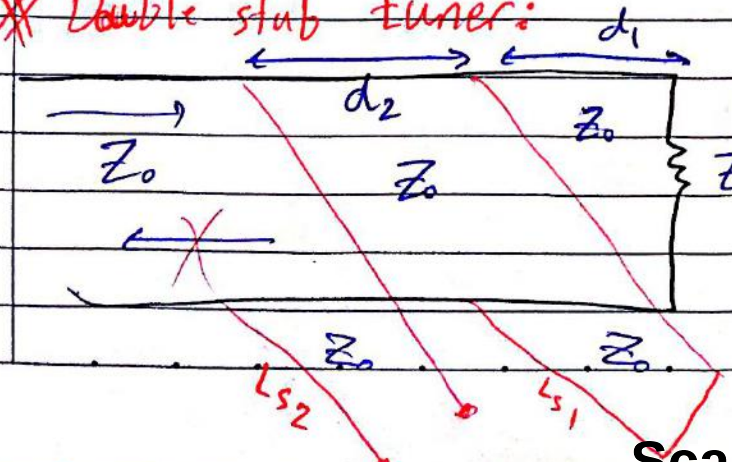
$$\Rightarrow g_L' = \frac{G_L'}{1/Z_{o_1}} = \frac{1}{75} * \frac{50}{1} = \frac{2}{3}$$ Locuse of y_L'



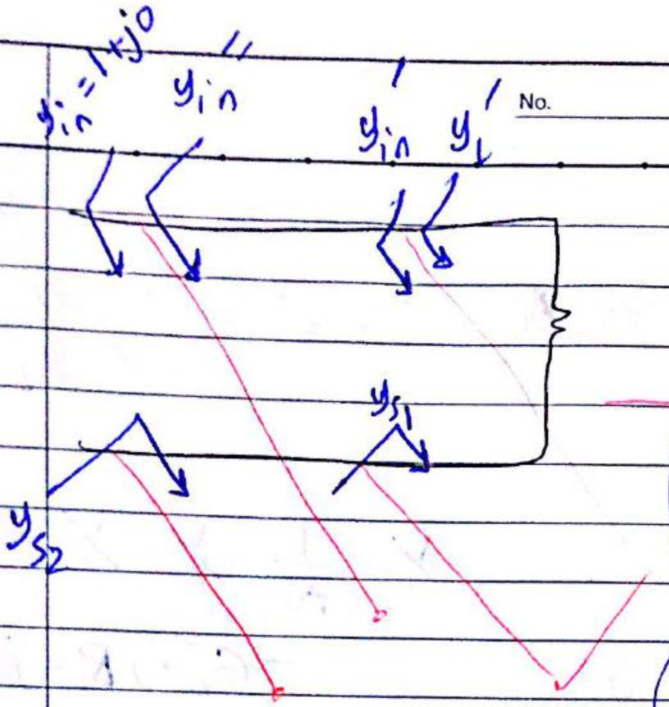
$$b_{st_1} = \frac{B_{st_1}}{1/100}$$

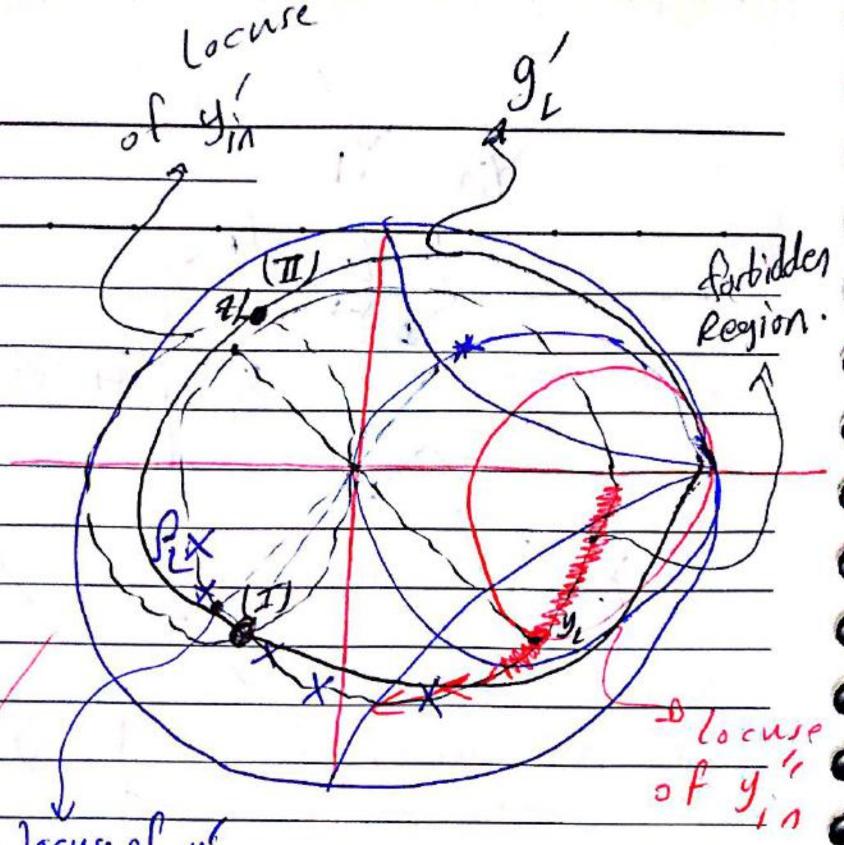
in this example No matching will occure.

※ Double stub tuner:



$Z_L = R_L + jX_L \rightarrow z_L \rightarrow \rho_L \rightarrow y_L$





$y_{in} = y''_{in} + y_{st_2}$

$1+j0 = g''_{in} + j\,b''_{in} + j\,b_{s_2}$

$g''=1$ ($g=1$ is the Locuse of y''_{in})

$y''_{in} \longrightarrow y'_{in}$: assume $d_2=\frac{\lambda}{4}$

$y'_{in} = y'_L + y_{st_1} \Longrightarrow g'_{in} + j\,b'_{in} = g'_L + j\,b'_L + j\,b_{s_1}$

$\boxed{g'_L = g'_{in}}$ $\Rightarrow$ $b'_{in} = b'_L + b_{s_1}$

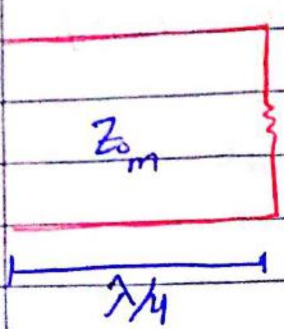
solve for

$Z_L = 50+j50\,\Omega$

$Z_0 = 50\,\Omega$

$d_2 = \lambda/4$

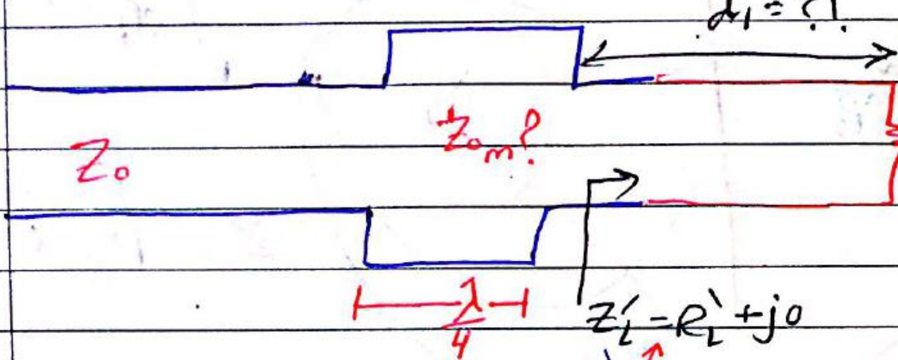
$d_1 = ?$



$$Z_{in} = \frac{Z_{o_m}^2}{Z_L} \Rightarrow Z_{o_m} = \sqrt{Z_{in} Z_L}$$

if Z_{in} is real $\Rightarrow$ Z_L should be real.

※ Matching using $\frac{\lambda}{4}$ Transformer :

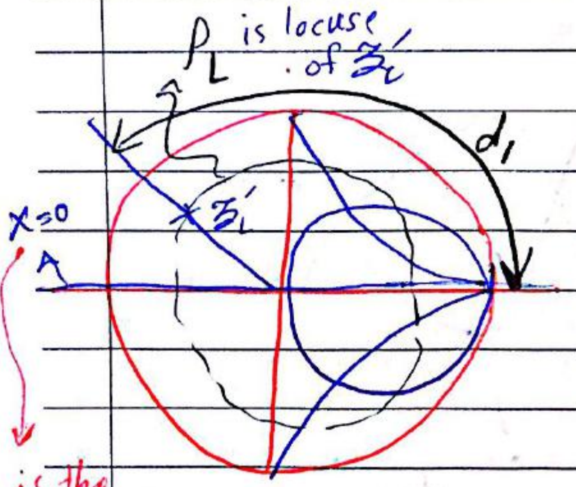


$z_L = \frac{Z_L}{Z_o}$

should be

$z_L' = \frac{Z_L'}{Z_o}$

ρ_L circle.



$x = 0$ is the Locuse of z_L'

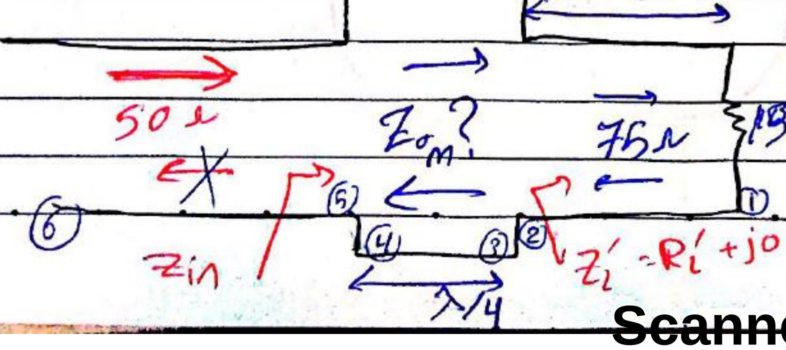
intersection gives $\underline{\underline{d_1}}$

$z_L' = r_L' + j0$

is the other locuse of z_L'

Ex. Match a T.L with Z_o 50Ω using $\frac{\lambda}{4}$ with a load $150 + j150\Omega$:

find Z_{o_m} & d_1? plot V & I on the line.

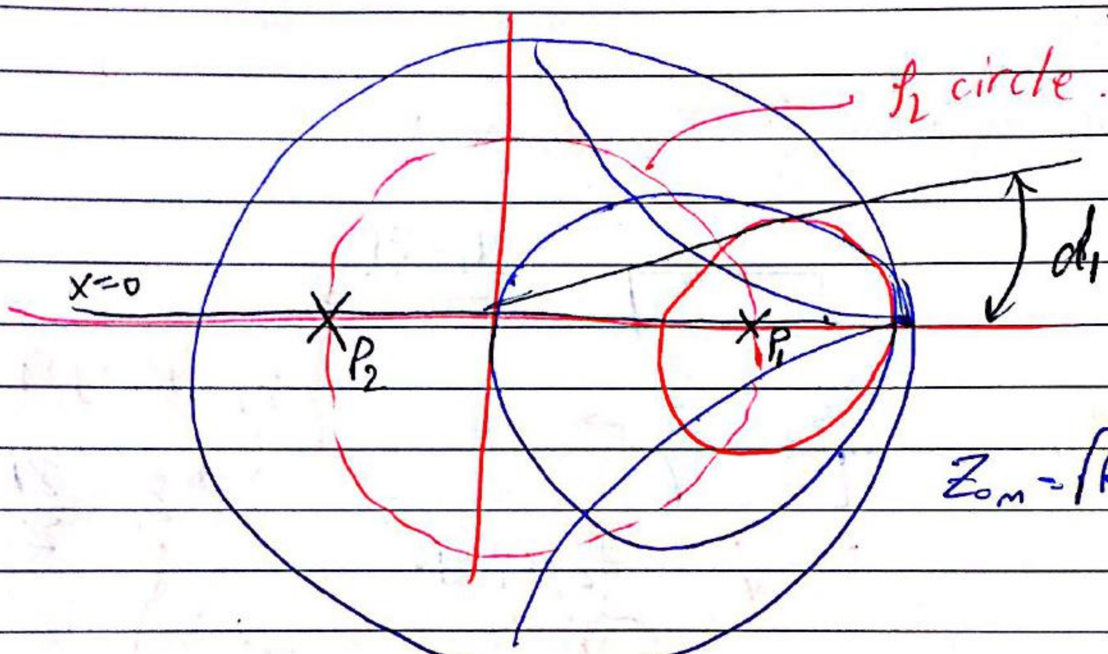


$Z_{in} \xrightarrow{\text{need}} Z_0 = 50\,\Omega$, $Z_L' \Rightarrow z_L' = \dfrac{Z_L'}{75} = r_L' + j0$

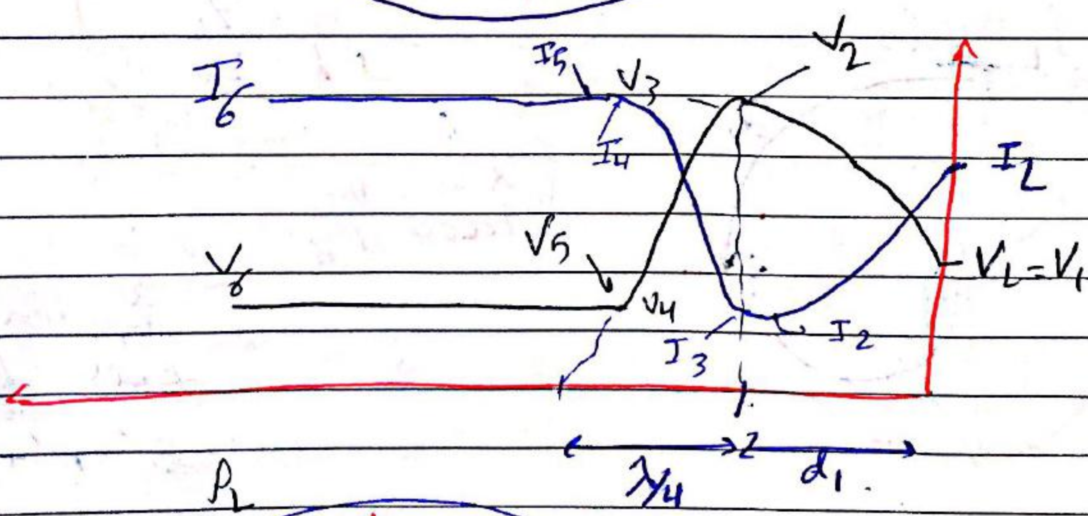
$z_L \xrightarrow{\rho_L} z_L'$

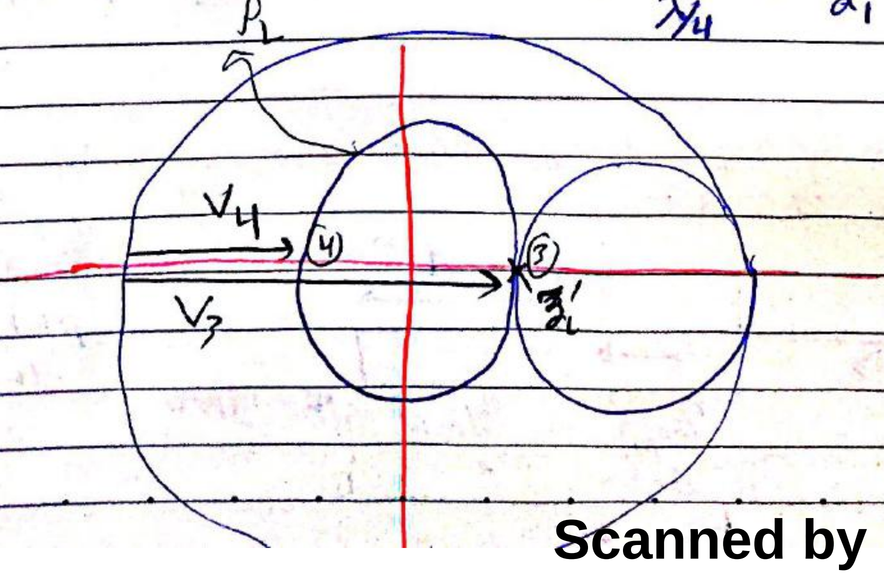
ρ_L circle.

$x=0$ is the locus of z_L'

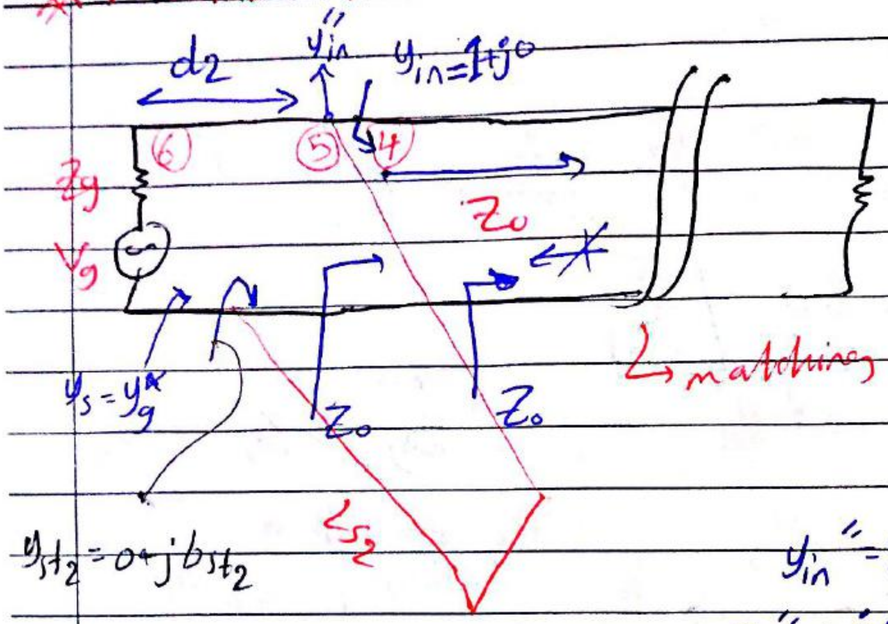


$Z_{0M} = \sqrt{R_L' \times 50}$





* Maximum Power Transfer:



$Z_s = Z_g^*$ max power.

↳ matching has ~~been~~ occured

$y_{st_2} = 0 + j b_{st_2}$

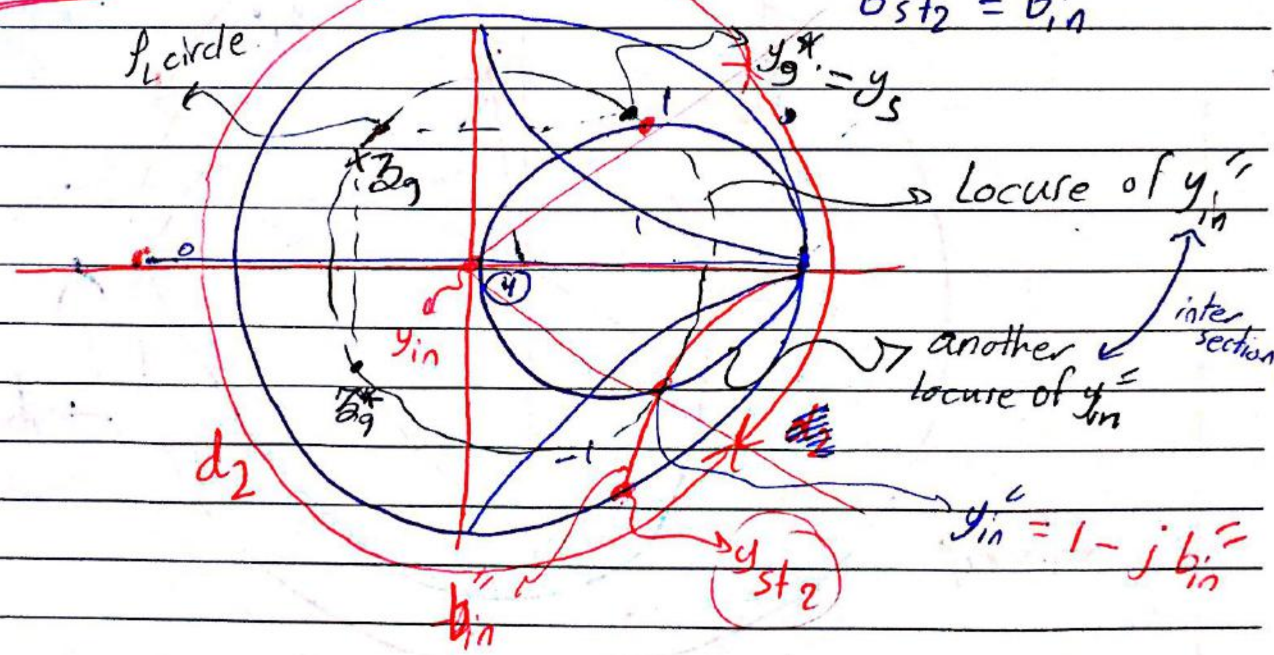
$y_{in}'' = y_{in} + y_{st_2}$

$g_{in}'' + j b_{in}'' = 1 + j0 + j b_{st_2}$

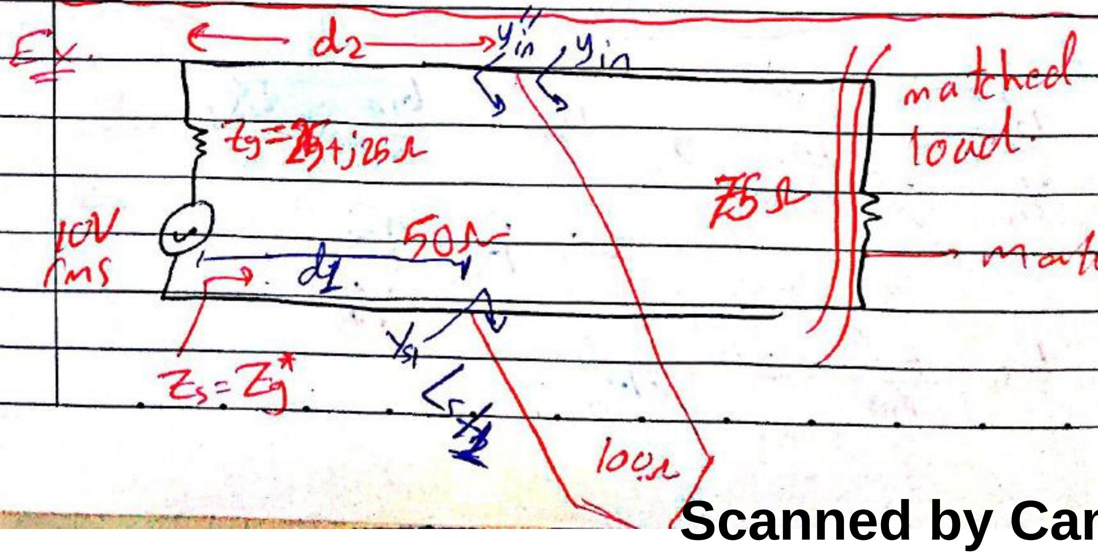
(1) using single stub.

$\Rightarrow \boxed{g_{in}'' = 1}$ is the locuse of y_{in}''

$b_{st_2} = b_{in}''$



Ex.



⟹ Need to satisfy max power transfer requirement:

$Z_s = Z_g^*$ $\qquad Y_{in} = 1/75$ ℧

No.

$Y_{in}'' = Y_{s_1} + Y_{in}$

$G_{in}'' + jB_{in}'' = jB_{s_1} + \frac{1}{75} \Rightarrow G_{in}'' = \frac{1}{75}$, $B_{s_1} = B_{in}''$

$g_{in}'' = \frac{G_{in}''}{Y_{o_1}} = \frac{1}{75} \times 50 = \boxed{\frac{2}{3}}$

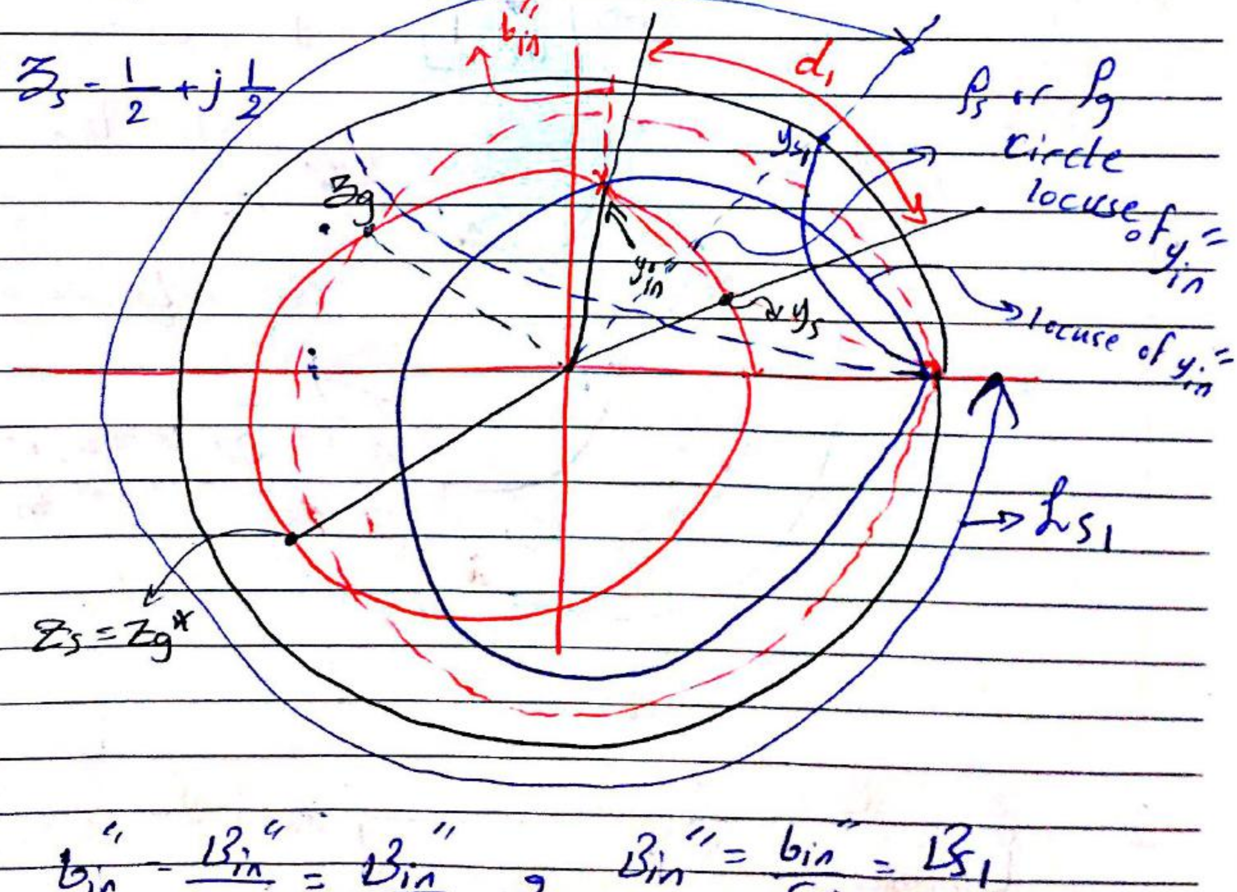
$g = \frac{2}{3}$ is the locuse of y_{in}''.

from $Z_g \rightarrow Z_g' = Z_s$

is the locuse of y_{in}''. ⟵ ρ_s ⟶ y_s

y_{in}'' ∈ intersection.

$Z_s = \frac{1}{2} + j\frac{1}{2}$

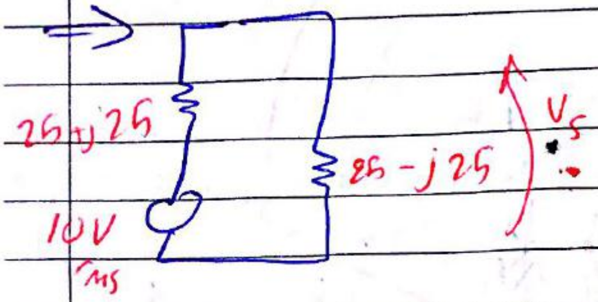


$b_{in}'' = \frac{B_{in}''}{Y_{o_1}} = \frac{B_{in}''}{1/50}$, $B_{in}'' = \frac{b_{in}''}{50} = B_{s_1}$

$\Rightarrow b_{s_1} = \frac{B_{s_1}}{Y_{os}}$

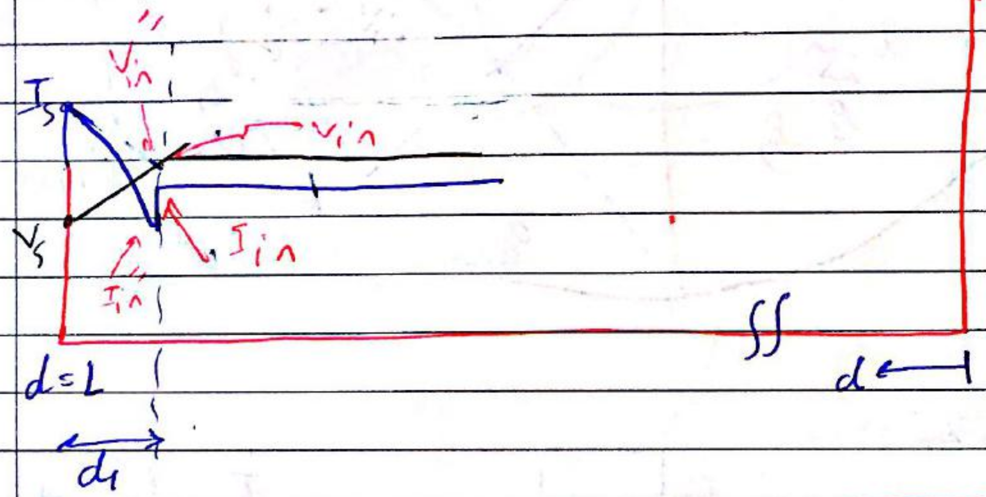
$b_{s_1} = \frac{b_{in}''}{50} \times 100 = 2b_{in}''$

$I_s = \frac{1}{5} = 0.2\,A$, $V_s = 5\sqrt{2}$

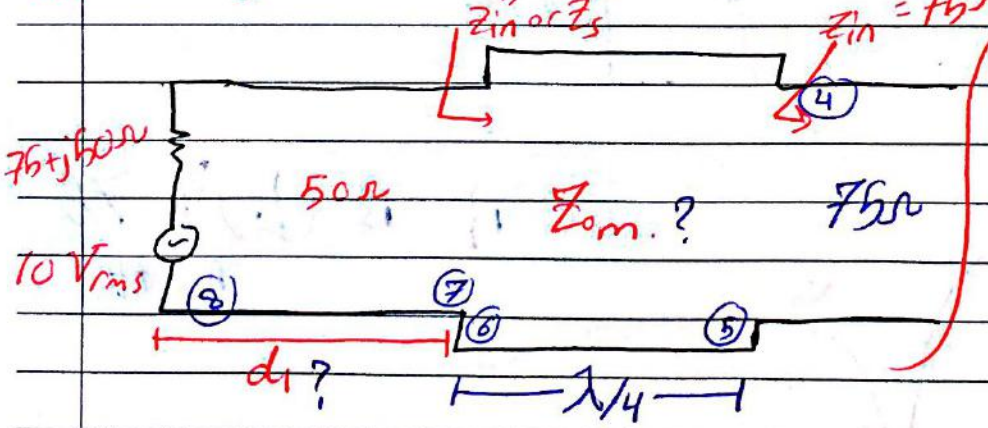


$P_s = P_L = I_s^2 \times R_s$

$= \frac{4}{100} \times 25$ W.



using λ/4 Transformer:



Matched has been achieved ~~Left hand side~~ LHS.

Need d_1, Z_{om} find P_L, P_s ; Plot $|V|$ & $|I|$ vs d ?

$Z_{om} = \sqrt{Z_{in} Z''_{in}}$

$Z''_{in} = R''_{in} + j0$

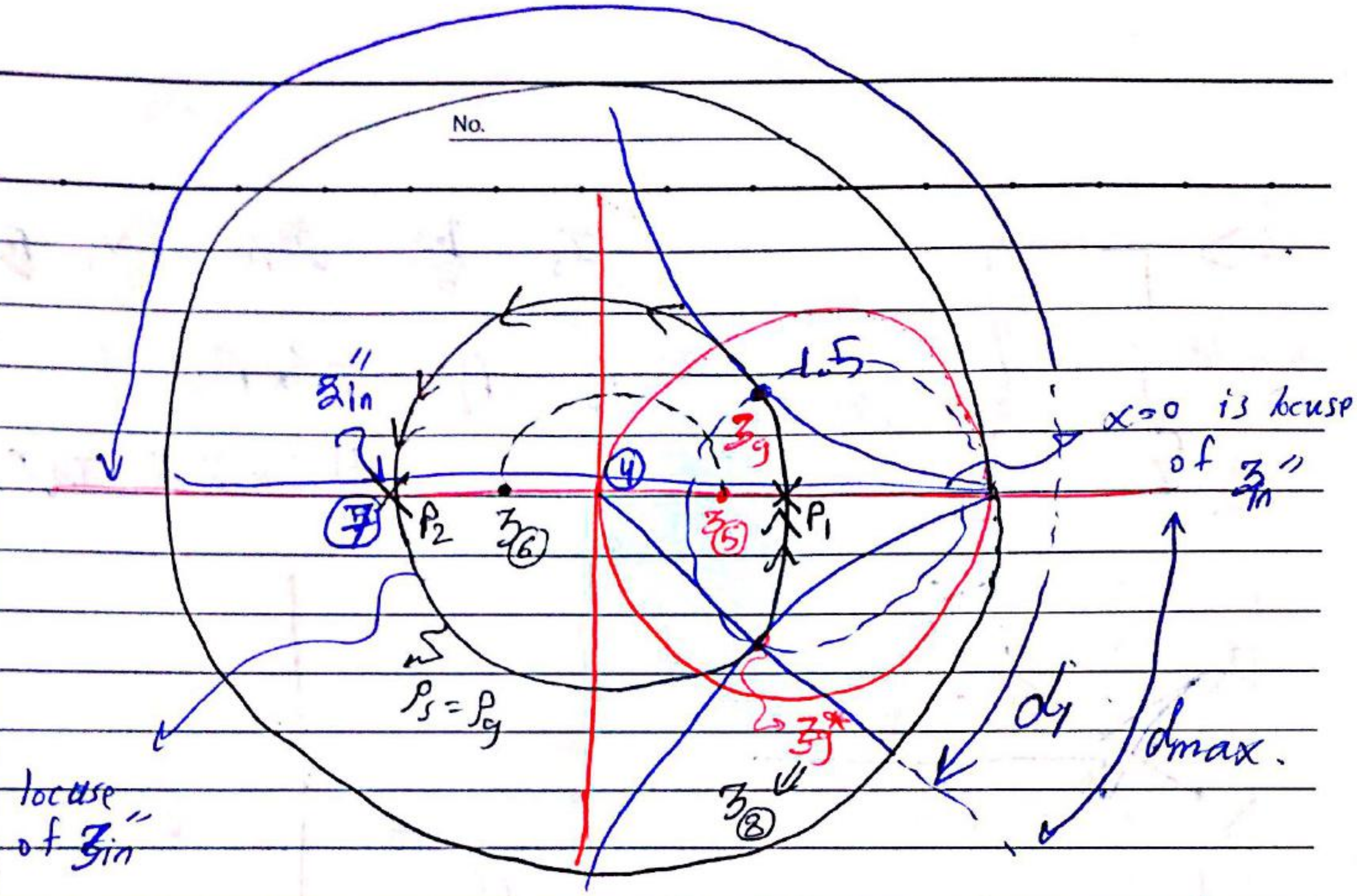
$z''_{in} = r''_{in} + j0 \Rightarrow \underline{x=0}$ circle is the locus of z''_{in}.

$Z_s = Z_g^* \to P_s$

locuse of z''_{in}

X (gives d_1 & z''_{in})

$z_g = 1.5 + j1$

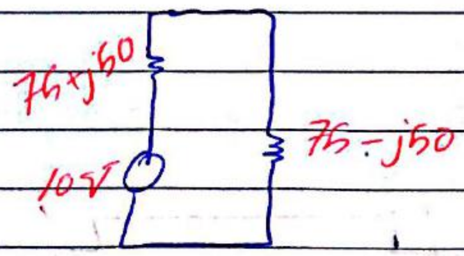


$Z_{om} = \sqrt{Z_{in}'' Z_{in}}$

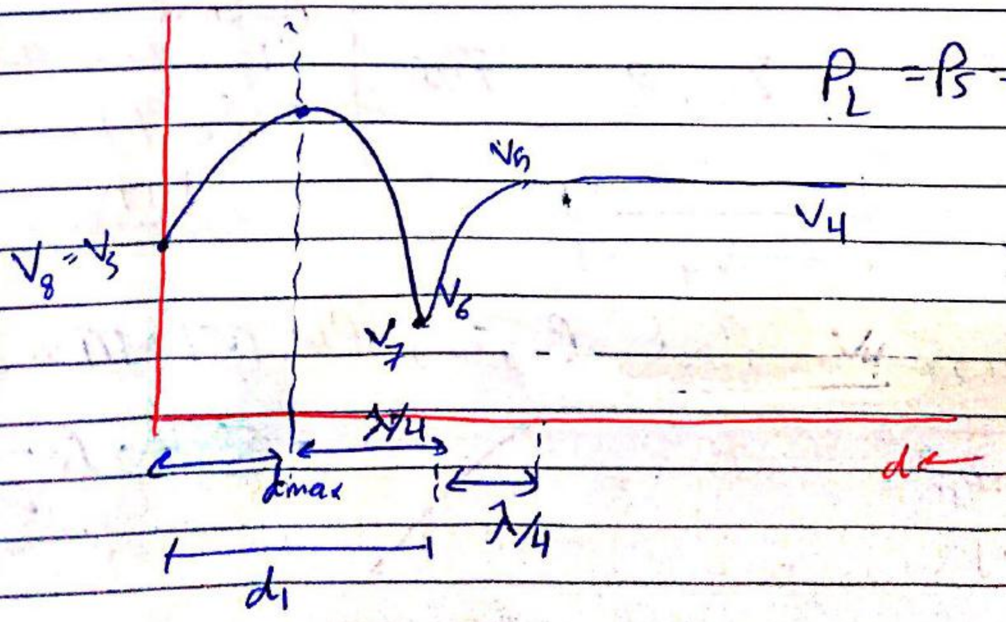
if it was 0.6

60 × 0.6

75

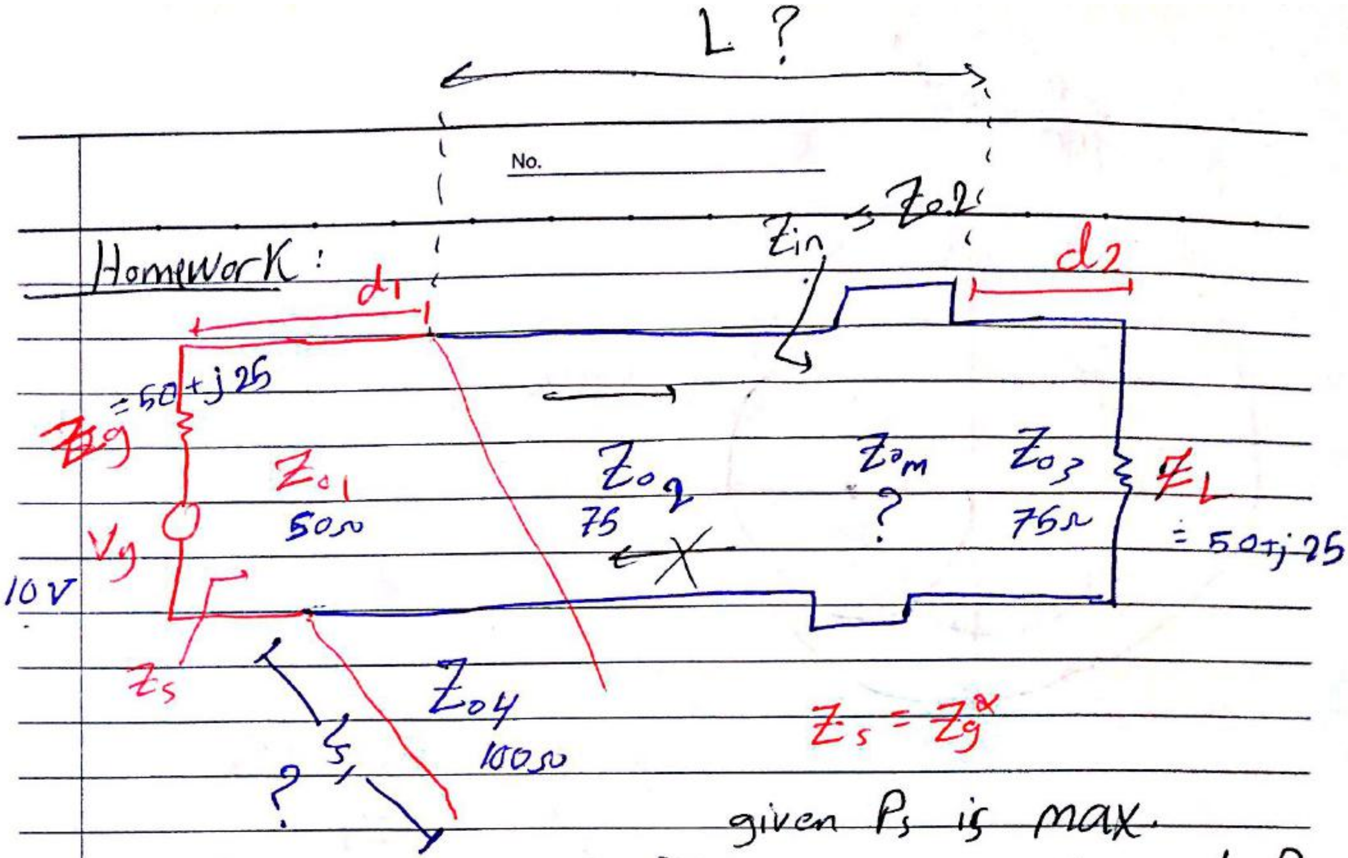


$I_s = \frac{10}{150}$ A.



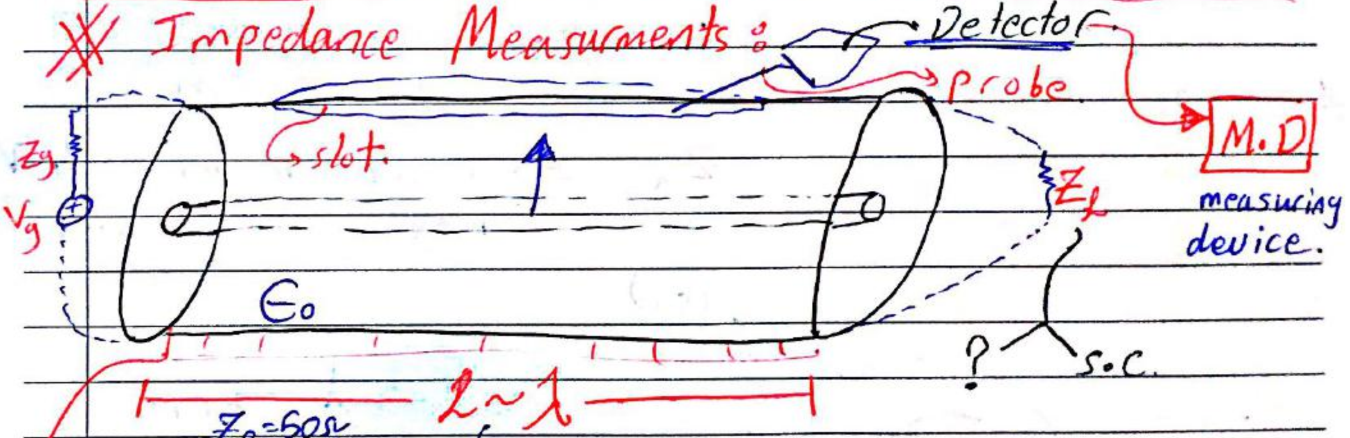
$P_L = P_s - \left(\frac{1}{15}\right)^2 \times 75$ Watt.

Homework:

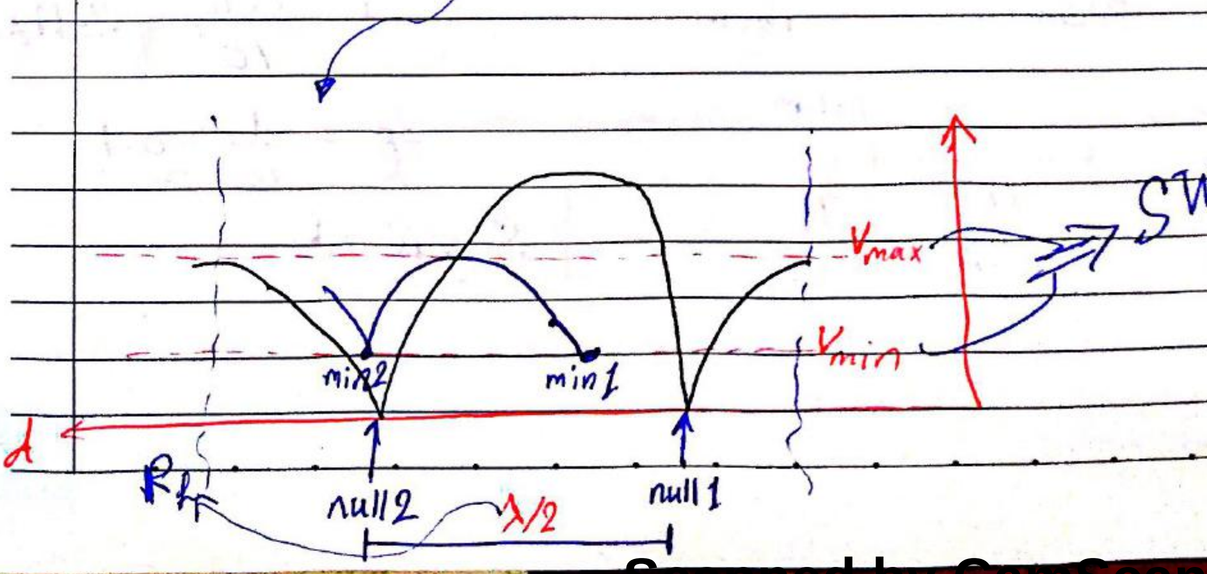


given P_s is max.
plot $|V|$ & $|I|$ vs d ?

※ Impedance Measurements:



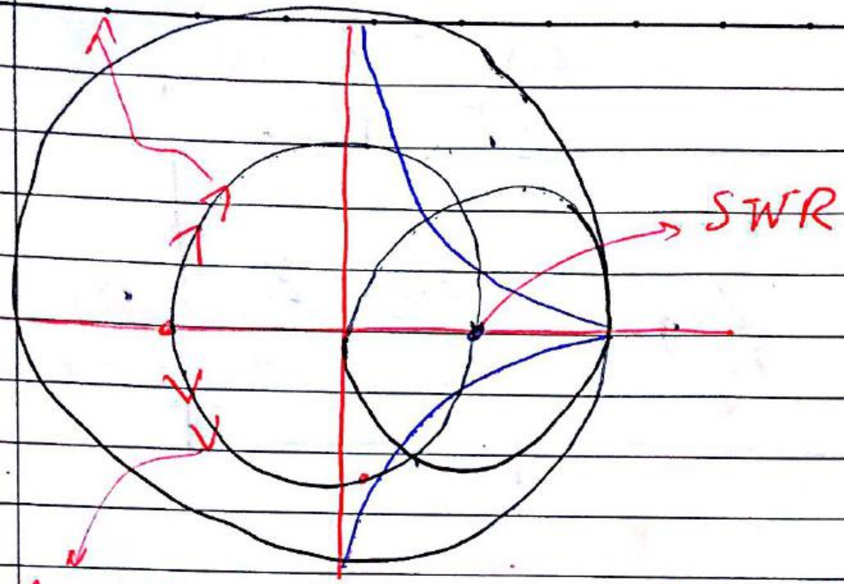
slotted line used to measure the impedance.



$SWR = \frac{V_{max}}{V_{min}}$

if we move to left of min1.

No.



if move from to right of min1

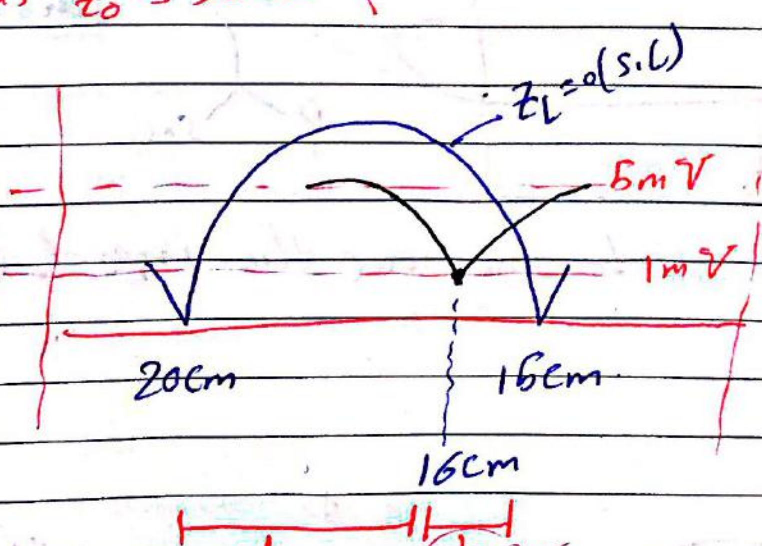
Ex. using slotted line to determine Z_a.

The following measurements are obtained.

$Z_L = 0 \Rightarrow$ null1 15 cm.
null2 20 cm.

$Z_a ? \Rightarrow$ min1 16 cm $\Rightarrow$ 1mV.
max $\longrightarrow$ 5mV.

✱ find f, Z_a assuming that slotted line has $Z_o = 50\Omega$?



$\Rightarrow \frac{\lambda}{2} = 5\text{cm}$.

$\lambda = 10\text{ cm}$

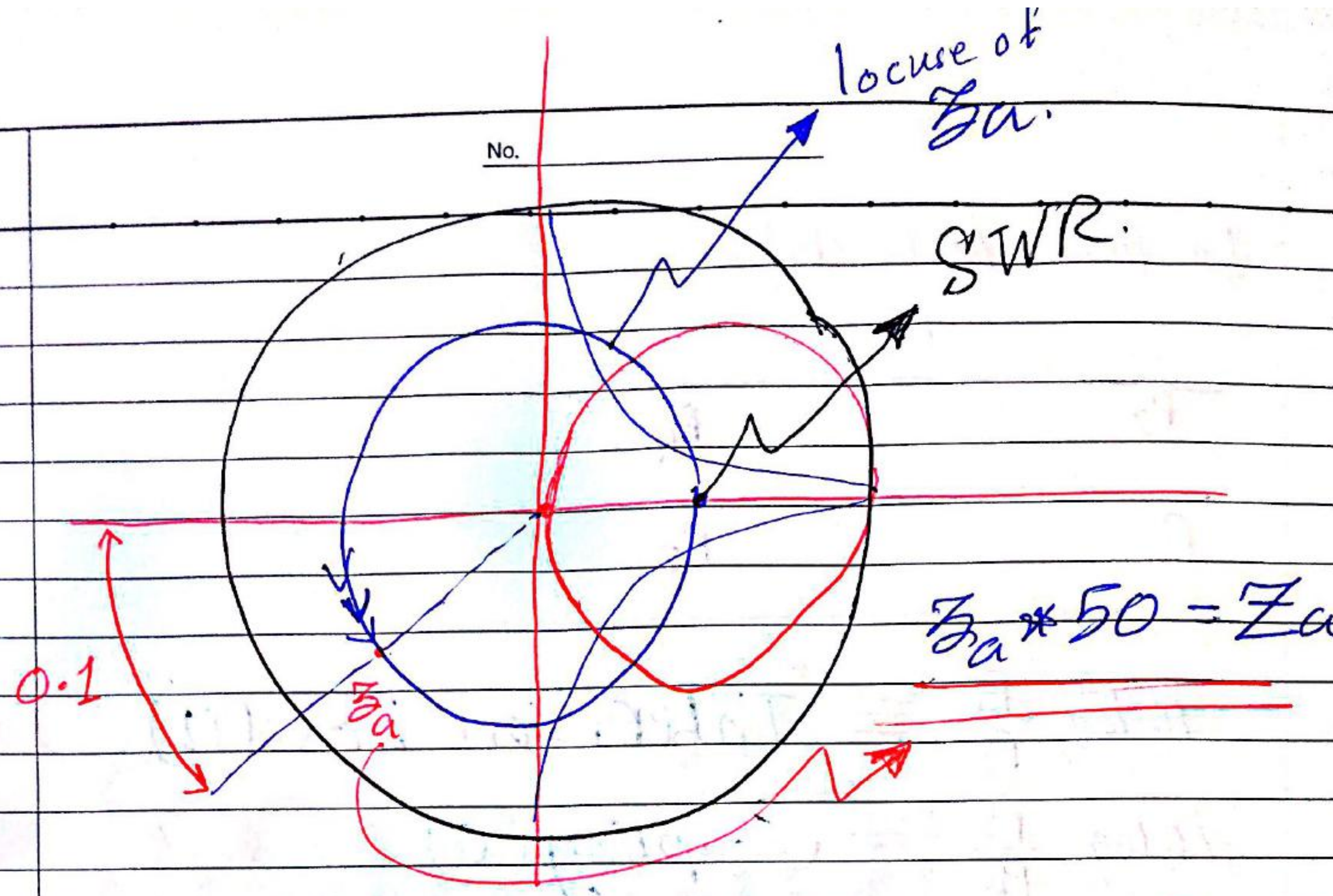
$f = \frac{3\times10^{10}}{10} = \boxed{3GHz}$

$\frac{d_R}{\lambda} = \frac{1}{10} = \underline{0.1}$

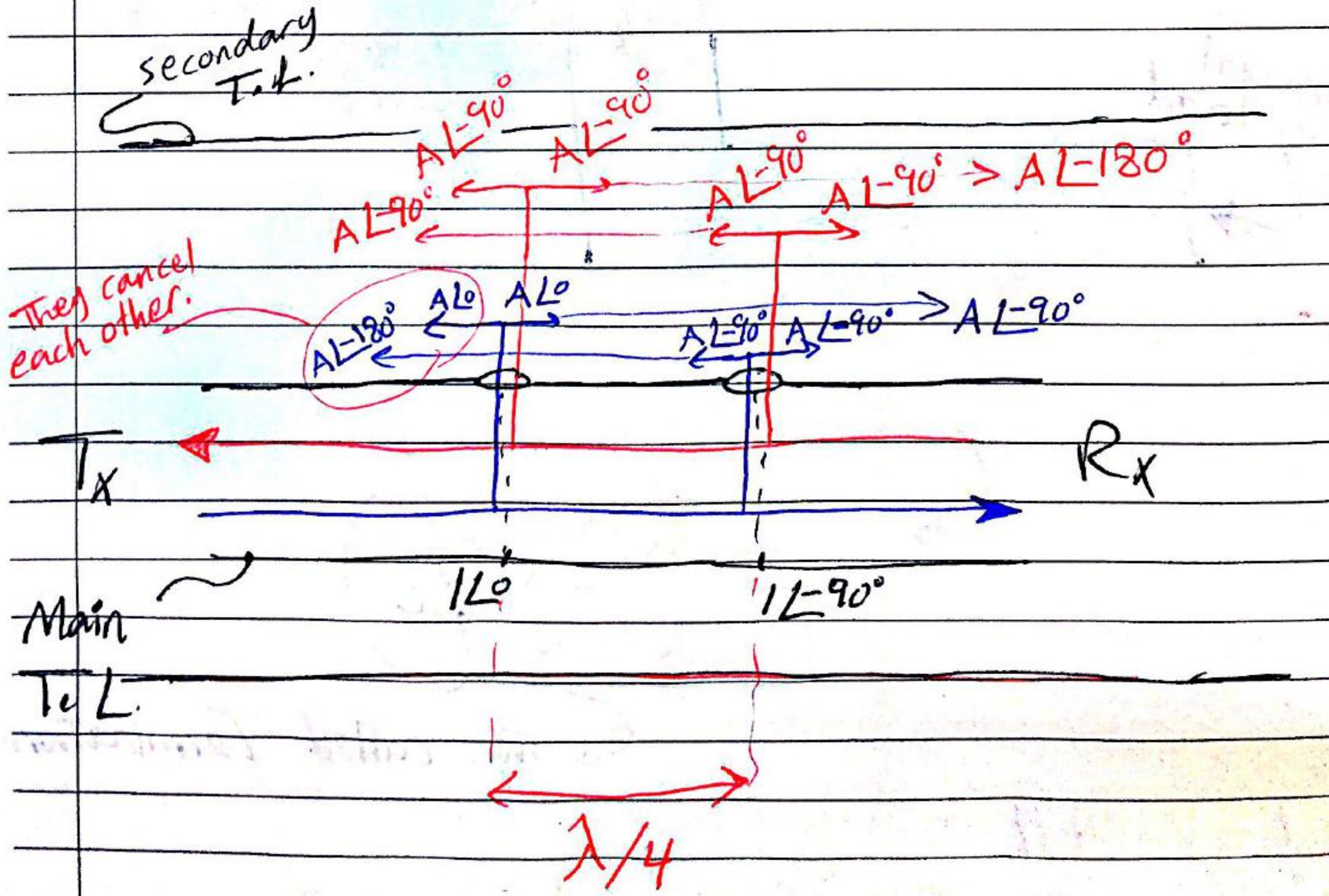
$\boxed{SWR = 5}$

Scanned by CamScanner

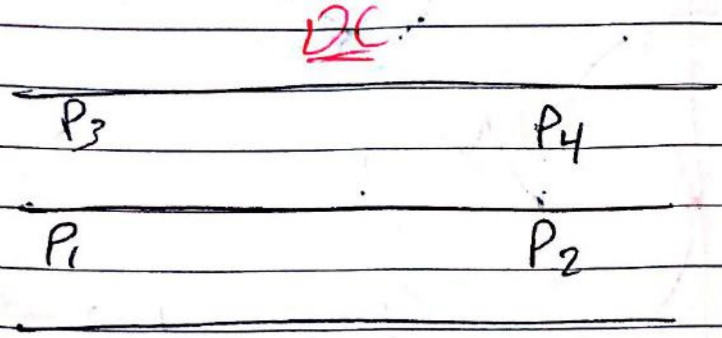
No.



Directional Coupler (DC):



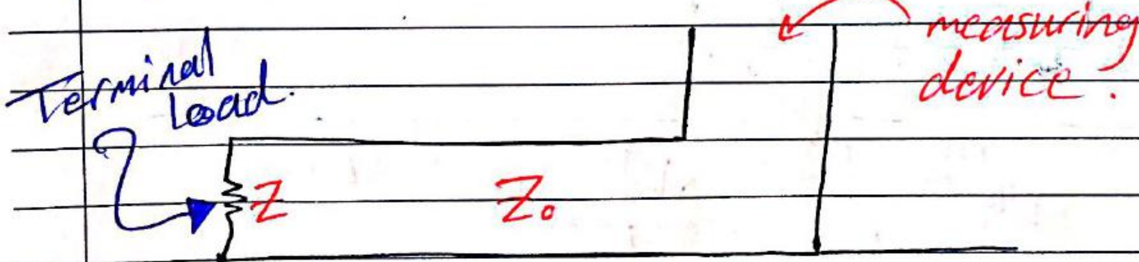
(4-ports Network):



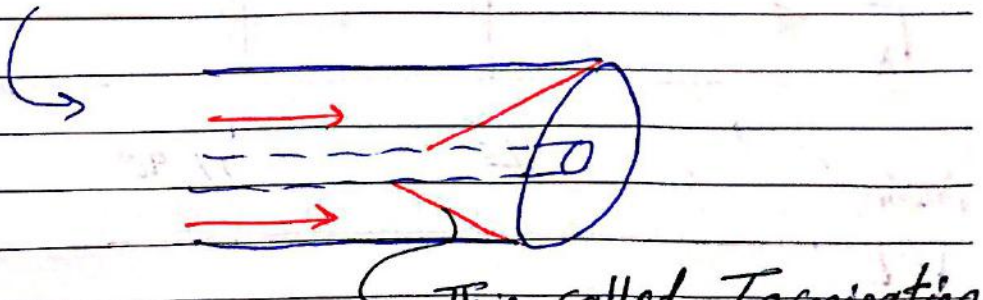
$10\log_{10}\frac{P_1}{P_2} \equiv$ "Inbiersion Loss(IL)" $< 1dB$

$10\log_{10}\frac{P_1}{P_4} =$ "Coupling (C)" $\geq 10dB$

$10\log_{10}\frac{P_1}{P_3} \equiv$ "Isolation (I)" $\geq 30dB$



Tx



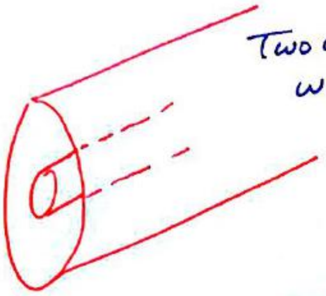
This called Termination.

End of CH11

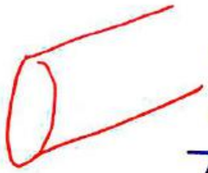
* Second Material *

CHAPTER (12)

Waveguide



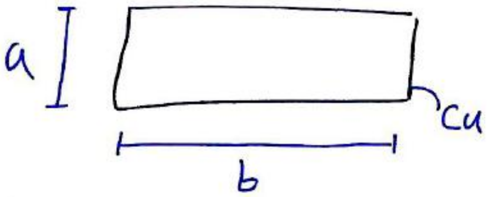
Two conductor
we use TEM $\Longrightarrow$



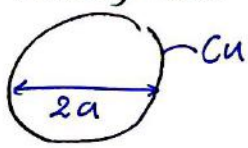
one conductor
we use
TE OR/AND TM.

* Waveguides take the following shapes:

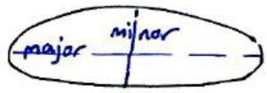
exactly rectangle



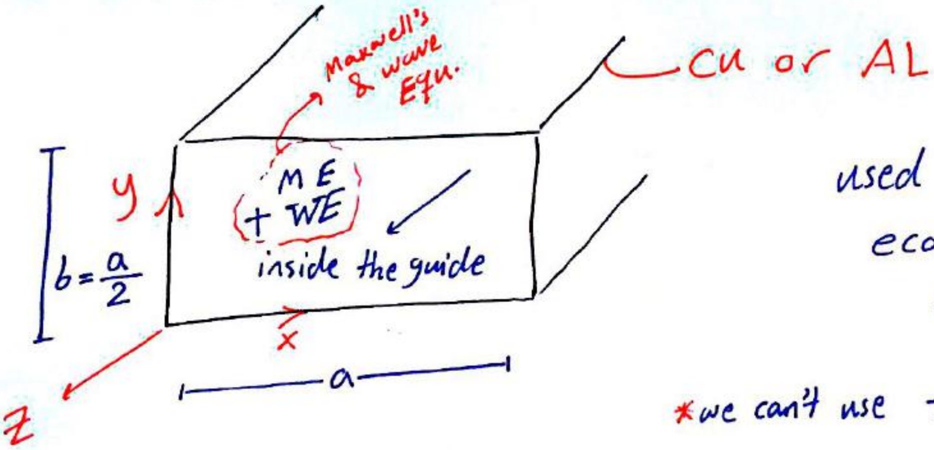
exactly circle.



Ellipse.



* Rectangular Waveguides (RWG):



used $f > 1\,GHz$.
economically:
$f > 3\,GHz$.

*we can't use TEM.

using
TE & TM
$\downarrow$ $\downarrow$
$E_z = 0$ $H_z = 0$

* ME:

$\nabla \times \vec{E} = -j\omega\mu\vec{H}$ ---(1)

$\nabla \times \vec{H} = 0 + j\omega\epsilon\vec{E}$ ---(2)

* Taking (1):

$$\Longrightarrow \begin{bmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \partial/\partial x & \partial/\partial y & -j\beta_g \\ E_x & E_y & 0 \end{bmatrix} = -j\omega\mu \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix}$$

$T_x \xrightarrow{e^{\pm j\beta_g z}} R_x$ → The propagation constant for the guidewave.

$\xrightarrow{e^{-j\beta z}}$

$\xleftarrow{e^{+j\beta z}}$

part(1):

$\Rightarrow +j\beta_g E_y = -j\omega\mu H_x \Rightarrow \boxed{H_x = \frac{-E_y}{Z_{TEWG}}}$ ---- ①

$Z_{TEWG} \equiv$ it is the waveguide charcterstic impedance.

$\Rightarrow Z_{TEWG} = \frac{\omega\mu}{\beta_g}$ **

part(2):

$-j\beta_g E_x = -j\omega\mu H_y \Rightarrow \boxed{H_y = \frac{E_x}{Z_{TEWG}}}$ --- ②

* Taking (2):

$$\begin{bmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & -j\beta_g \\ H_x & H_y & H_z \end{bmatrix} = j\omega\epsilon \begin{bmatrix} E_x \\ E_y \\ 0 \end{bmatrix}$$

$= \frac{E_x}{Z_{TEWG}}$, $Z_{TEWG} = \frac{\omega\mu}{\beta_g}$

$\Rightarrow \frac{\partial H_z}{\partial y} \mp j\beta_g H_y = j\omega\epsilon E_x$

$\Rightarrow \frac{\partial H_z}{\partial y} + j\beta_g^2 \frac{E_x}{\omega\mu} = j\omega\epsilon E_x$

$\rightsquigarrow \omega\mu\frac{\partial H_z}{\partial y} + j\beta_g^2 E_x = j\underbrace{\omega^2\epsilon\mu}_{K^2} E_x$

$\Rightarrow \boxed{E_x = \frac{-j\omega\mu}{K^2-\beta_g^2} \frac{\partial H_z}{\partial y}}$ --- ③

as before also:

$\boxed{E_y = \frac{+j\omega\mu}{K^2-\beta_g^2} \frac{\partial H_z}{\partial x}}$ -- ④

※ W.E : we Need a relation for H_z.

$\nabla^2 H_{\cancel{x,y,z}} + K^2 H_{\cancel{x,y,z}} = 0$

$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + (K^2-\beta_g^2) H_z = 0$

$H(x,y,z,t)$: $z \to e^{-j\beta_g z}$, $t \to e^{j\omega t}$

$\Rightarrow H_z(x,y) = X(x)\, Y(y)$

$\underbrace{\frac{X''}{X}}_{\pm K_x^2 \text{ or } 0} + \underbrace{\frac{Y''}{Y}}_{\pm K_y^2} + (K^2-\beta_g^2) = 0$

This true for:
$0 \le x \le a$
$0 \le y \le b$

$\Rightarrow$

$$\Rightarrow \left.\begin{aligned} X &= A e^{-K_x X} + B e^{+K_x X} \\ X &= A \cos K_x X + B \sin K_x X \\ X &= A x + B \end{aligned}\right] \Rightarrow$$

The Acceptable equation:

$$X = A \cos K_x X + B \sin K_x X$$

due to B.C.

**

$$\Rightarrow H_z = [A \cos K_x X + B \sin K_x X] * [C \cos K_y y + D \sin K_y y] * e^{j(\omega t - \beta_g z)}$$

where $E_y \propto \frac{\partial H_z}{\partial x} \Rightarrow [-A K_x \sin K_x X + B K_x \cos K_x X] \cdots\cdots$

$E_y = 0$ @ $x = 0$ $(B = 0)$

$E_y = 0$ @ $x = a$ $(\sin K_x a = 0) \Rightarrow$ if $K_x a = m\pi$

$\Rightarrow$ do The same work for (y):

$\boxed{K_x = \frac{m\pi}{a}}$, $m = 0, 1, 2, \cdots$ & $\boxed{K_y = \frac{n\pi}{b}}$, $n = 0, 1, 2, \cdots$

m & n can't be together equal zero
(Then it will be No fields components)
due to the following equations.

$$\Rightarrow H_z = A_{mn} \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b}$$

$E_x = n A'_{mn} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \Longrightarrow (E_x \overset{\text{with}}{\Leftrightarrow} H_y) \Rightarrow Z_{TE_{mn}}$

$E_y = m A''_{mn} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \Longrightarrow (E_y \Leftrightarrow H_x) \Rightarrow Z_{TE_{mn}}$

$E_z = \text{zero}.$

$$\nabla^2 H_z + K^2 H_z = 0$$

$-K_x^2 - K_y^2$

$$\Rightarrow -\left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2 - \beta_g^2 + K^2 = 0$$

So,

$$\boxed{\beta_{g_{mn}} = \sqrt{K^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}}$$

where $K = \omega\sqrt{\mu\epsilon}$

$$\beta_{g_{mn}} = \sqrt{K^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

$K > \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$

$\Rightarrow \beta_g$ is real.

Propagation is Possible.

$K < \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$

$\Rightarrow \beta_g$ is imaginary.

Propagation is Not Possible.

$$K_{c_{mn}} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

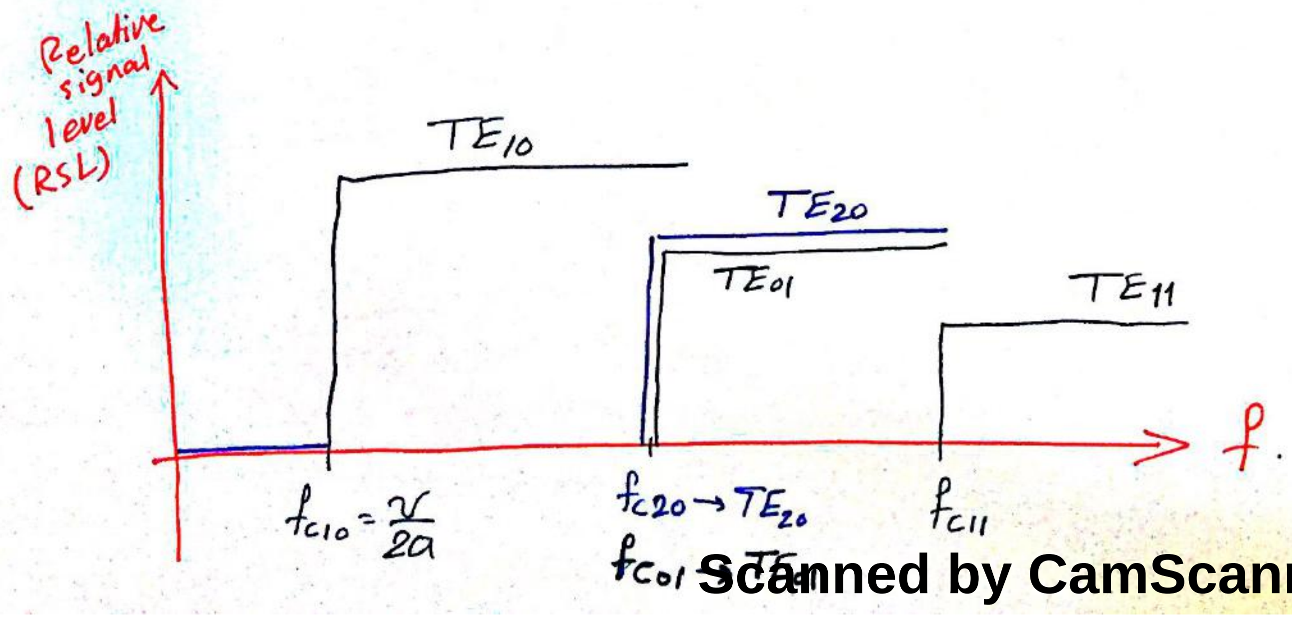
where $K = \frac{\omega}{v}$

so $\omega = 2\pi f = K v$

$$\Rightarrow f_{c_{mn}} = \frac{v}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \quad b \to a/2$$

$$\Rightarrow ** \boxed{f_{c_{mn}} = \frac{v}{2a}\sqrt{m^2 + 4n^2}} **$$

m \ n	0	1
0		TE_{01} $f_{c_{01}} = \frac{v}{a}$
1	TE_{10} $f_{c_{10}} = \frac{v}{2a}$	TE_{11} $f_{c_{11}} = \frac{v\sqrt{5}}{2a}$
2	TE_{20} $f_{c_{20}} = \frac{v}{a}$	TE_{21} $f_{c_{21}} = \frac{v\sqrt{2}}{a}$



$$\beta_{g_{mn}} = \sqrt{K^2 - K^2_{c_{mn}}}$$

$$f_{c_{mn}} = \frac{v}{2a}\sqrt{m^2 + 4n^2}$$

$b = \frac{a}{2}$

TM:

$E_z \neq 0$

$H_z = 0$

$E_{x,y}$

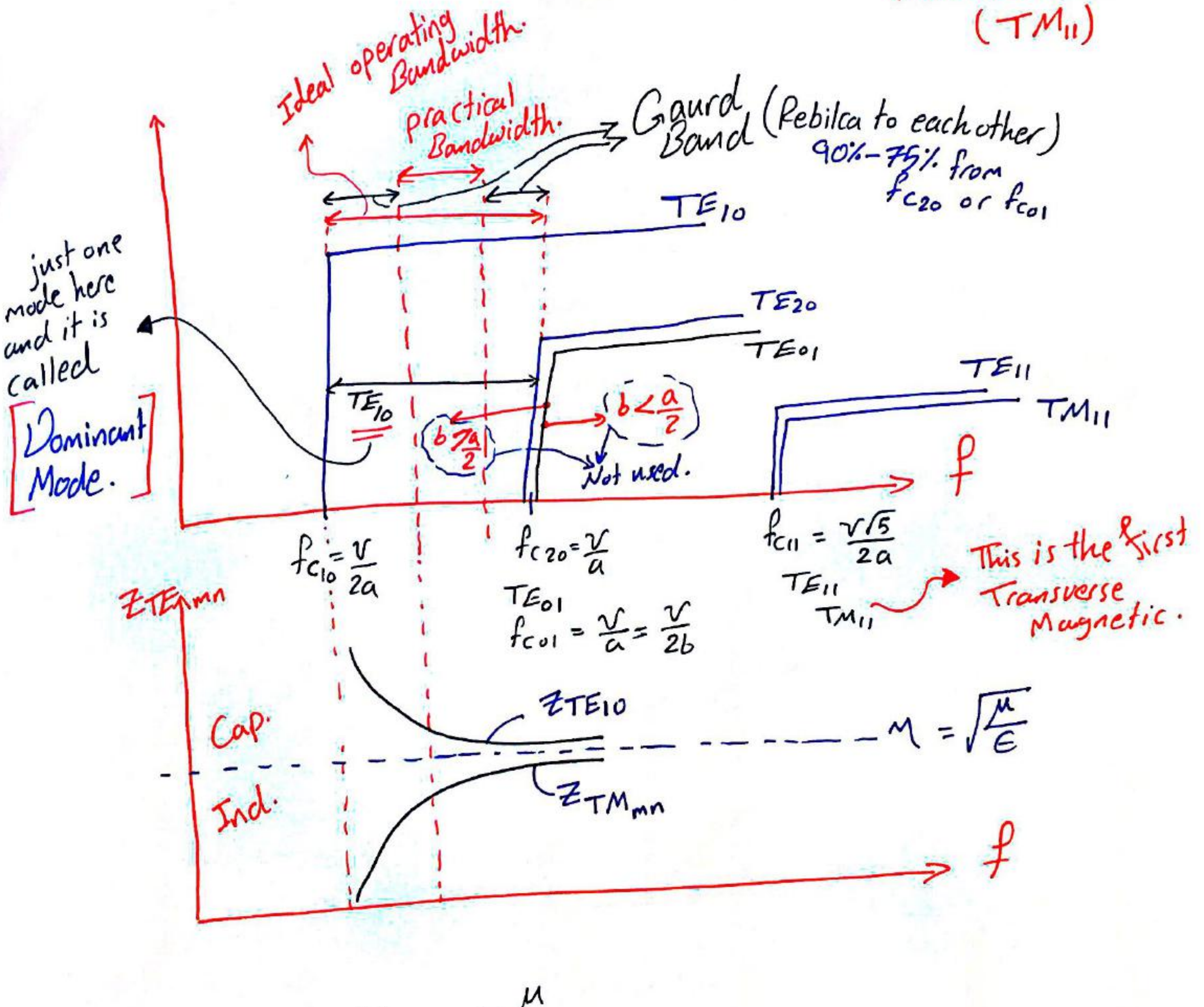
$H_{x,y}$

$E_z \Rightarrow A_{mn} \sin\frac{m\pi}{a}x \sin\frac{n\pi}{b}y \, e^{-j\beta_{g_{mn}} z}$

$m = 1, 2, \dots$

$n = 1, 2, \dots$

$\Rightarrow$ so TM starts from $m=1$ & $n=1$ (TM_{11})

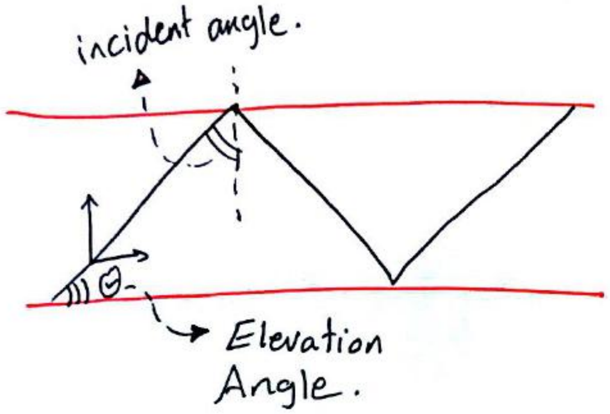


$$Z_{TE_{mn}} = \frac{\omega\mu}{\beta_{g_{mn}}} = \frac{\omega\mu}{\sqrt{K^2 - K^2_{c_{mn}}}}$$

T.L.

$TEM: Z_0 \longrightarrow \lambda = \frac{2\pi}{K}$

$TE: Z_{TE_{10}} \longrightarrow \lambda_{g_{10}} = \frac{2\pi}{\beta_{g_{10}}} = \frac{2\pi}{\sqrt{K^2 - \left(\frac{\pi}{a}\right)^2}}$

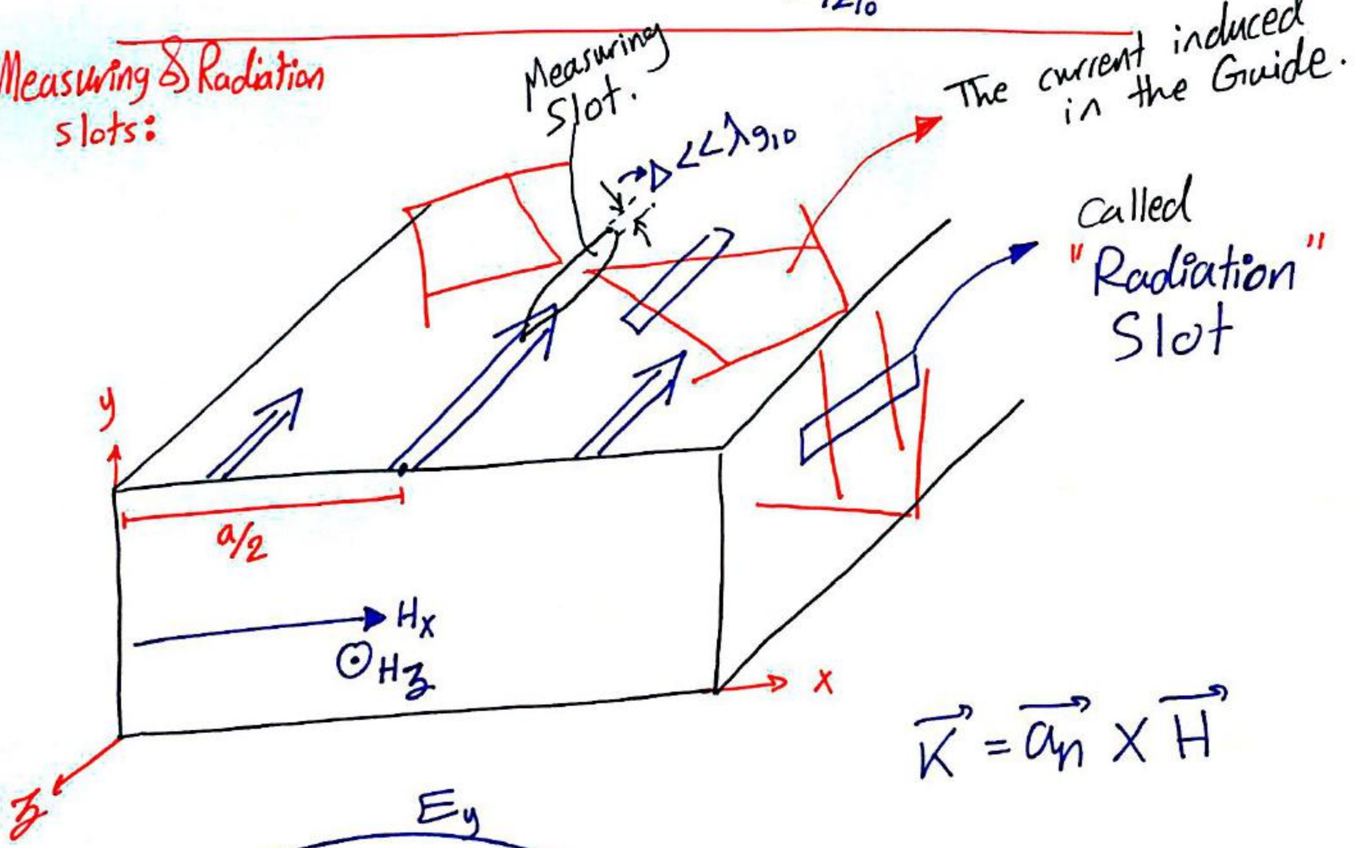


$$TE: \quad f \rightarrow f_{c_{10}} \Rightarrow \theta = 90^\circ$$
$$f \rightarrow \infty \Rightarrow \theta = 0$$

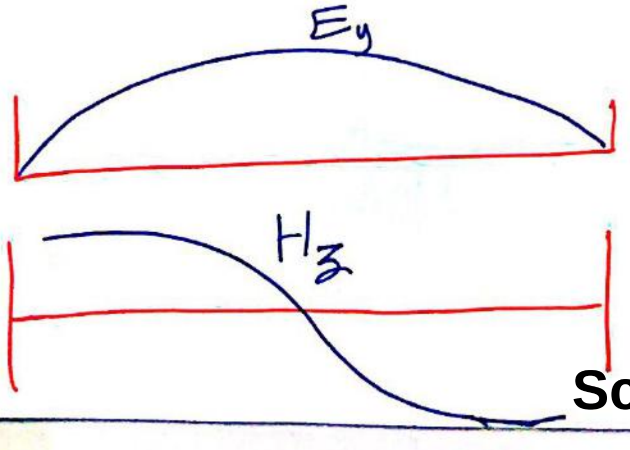
※ Dominant Mode: (TE_{10} Mode):

$$b = \frac{a}{2} \quad \Rightarrow \quad \left.\begin{aligned} H_z &= A_{10} \cos \frac{\pi}{a} x \\ E_x &= 0 \\ H_y &= 0 \\ E_y &= A'_{10} \sin \frac{\pi}{a} x \\ \Rightarrow H_x &= -\frac{E_y}{Z_{TE_{10}}} \end{aligned}\right\} * e^{j(\omega t - \beta_{g_{10}} z)}$$

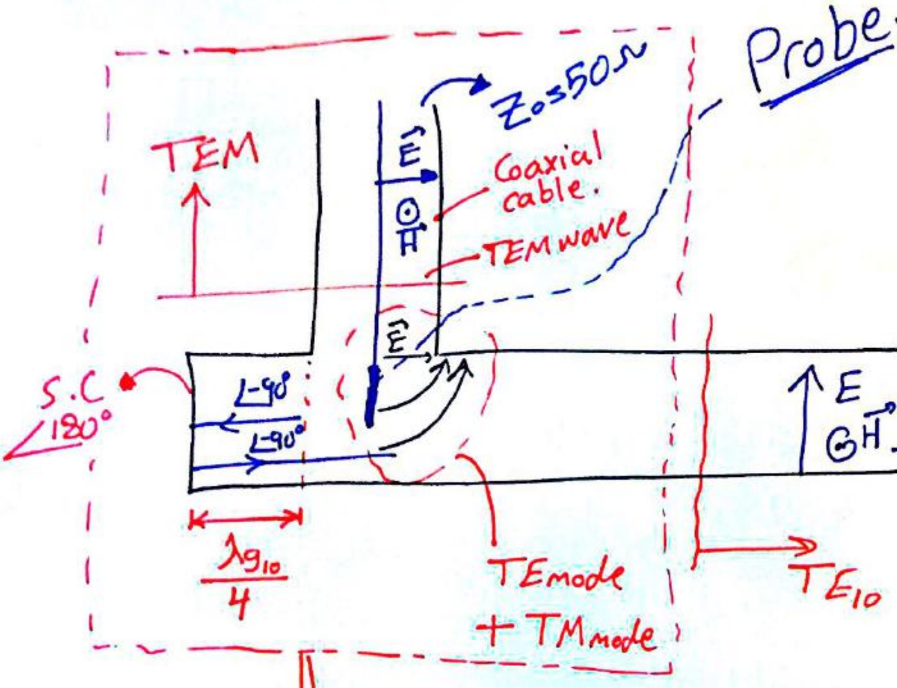
※ Measuring & Radiation slots:



$$\vec{K} = \vec{a_n} \times \vec{H}$$



※ Wave Launcher:



Probe: has L & D

⇒ we adjust them for matching purposes.

RWG

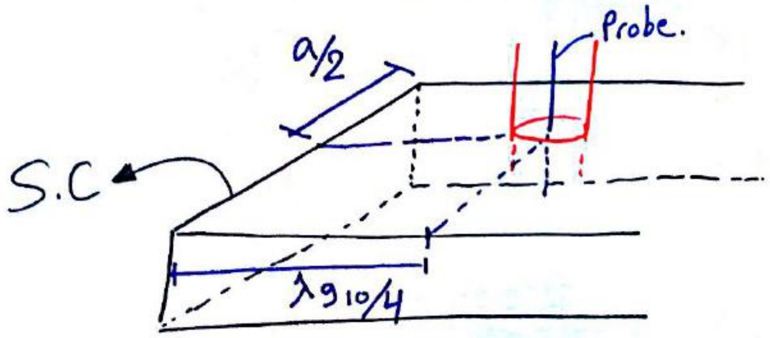
$a, b = \frac{a}{2}$

$f_{c_{10}} < f < f_{c_{20}}$

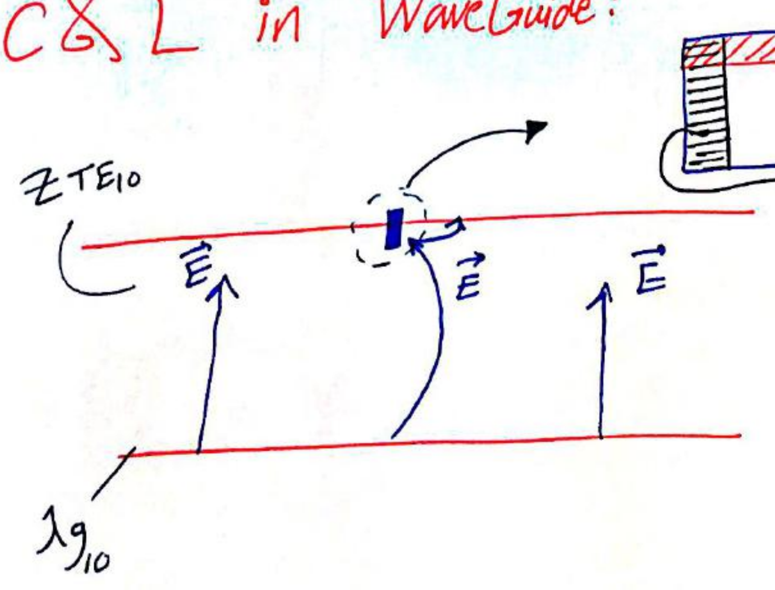
ZE_{10} ~~mode~~ wave.

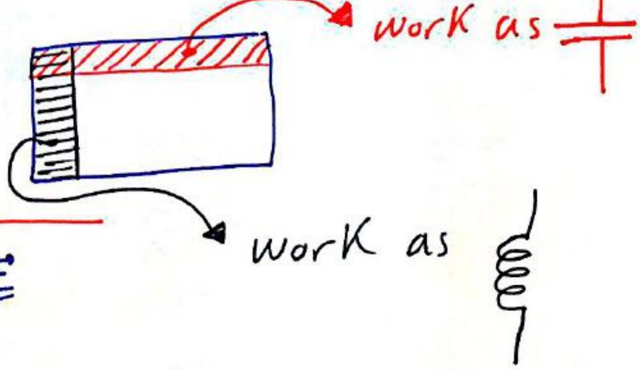
$ZTE_{10} \sim 500\Omega$

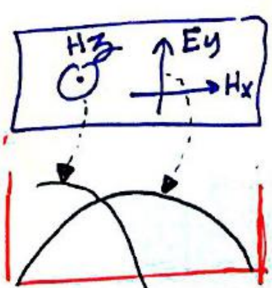
⇒ This called: "Wave Launcher"



※ C & L in WaveGuide:

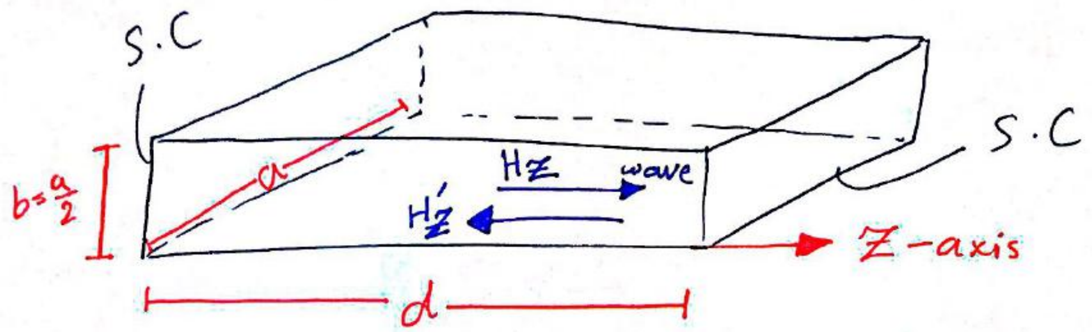






※ RWG Cavities:

Cavity:



$$H_z = A_{mn}\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\, e^{-j\beta_{g_{mn}} z}$$

$$H'_z = A'_{mn}\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\, e^{+j\beta_{g_{mn}} z}$$

* B.C:

$H_z = 0$ → @ $z=0 \Rightarrow A'_{mn} = -A_{mn}$.

→ @ $z=d$ $\quad H_z\big|_{z=d} = 2j A_{mn}\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\sin\beta_{g_{mn}} d$

$$\Rightarrow \beta_{g_{mn}} d = P\pi, \quad P = 1,2,3,\dots$$

$$\beta_{g_{mn}} = \sqrt{K^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

$$\Rightarrow \frac{P\pi}{d} = \sqrt{K^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

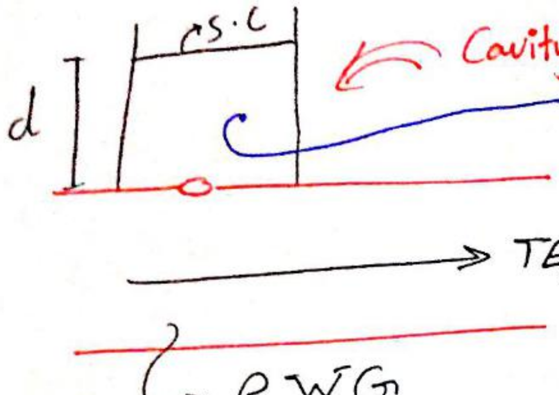
square both sides $\Rightarrow$

$$K_{mnp} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{P\pi}{d}\right)^2} = \frac{2\pi f_{mnp}}{v}$$

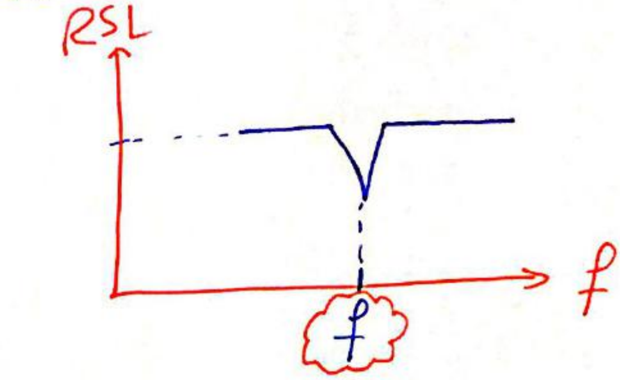
The resonance freq.:

$$\boxed{f_{r_{mnp}} = \frac{v}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{2n}{a}\right)^2 + \left(\frac{P}{d}\right)^2}}$$

$$\Rightarrow \boxed{f_{r_{10p}} = \frac{v}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{P}{d}\right)^2}} \quad P = 1,2,\dots$$

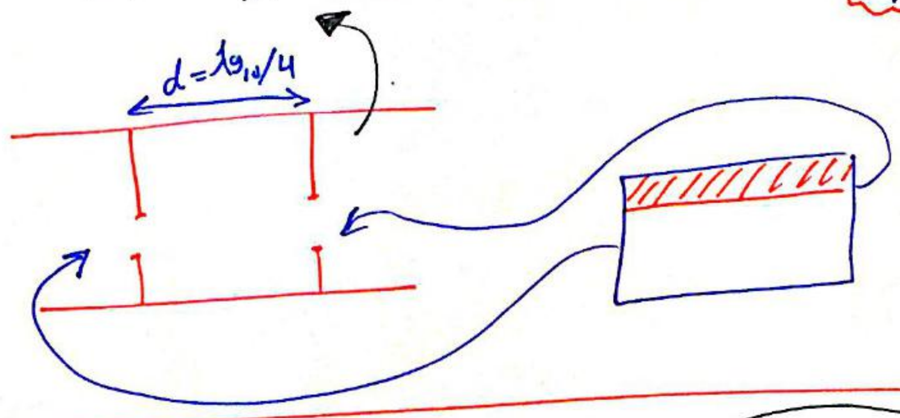


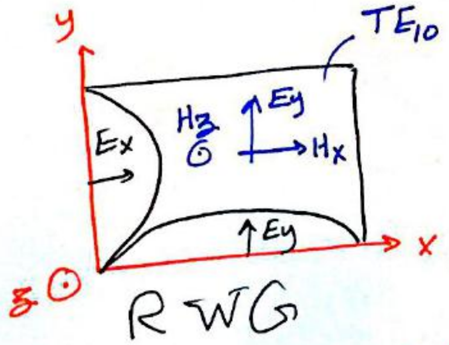
Cavity. ⇒ Called: "Q meter"



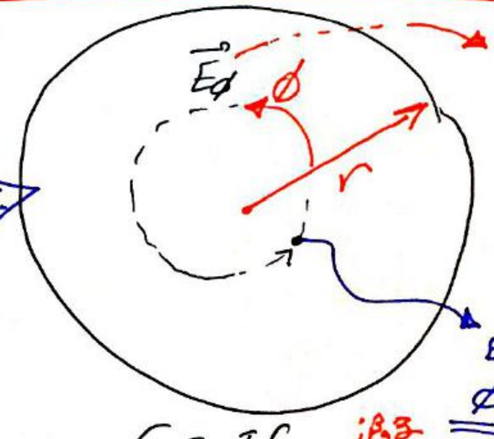
$f_{c10} < f < f_{c20}$

$a, b = \frac{a}{2}$





⇒



This is the only one which has B.C

$E|_{\phi=0}$ must $= E|_{\phi=360°}$

CWG

$\underset{H_{x,y,z}}{E}(x, y, z, t)$ — $e^{j\omega t}$, $e^{-j\beta_g z}$

$\underset{H_{r,\phi,z}}{E}(r, \phi, z, t)$ — $e^{j\beta z}$, $e^{j\omega t}$, $\sin n\phi$, $\cos n\phi$

⇓

TE ⇒ $E_z = 0$, $H_z \neq 0$

TM ⇒ $H_z = 0$, $E_z \neq 0$

⇒

⇒ for ϕ:

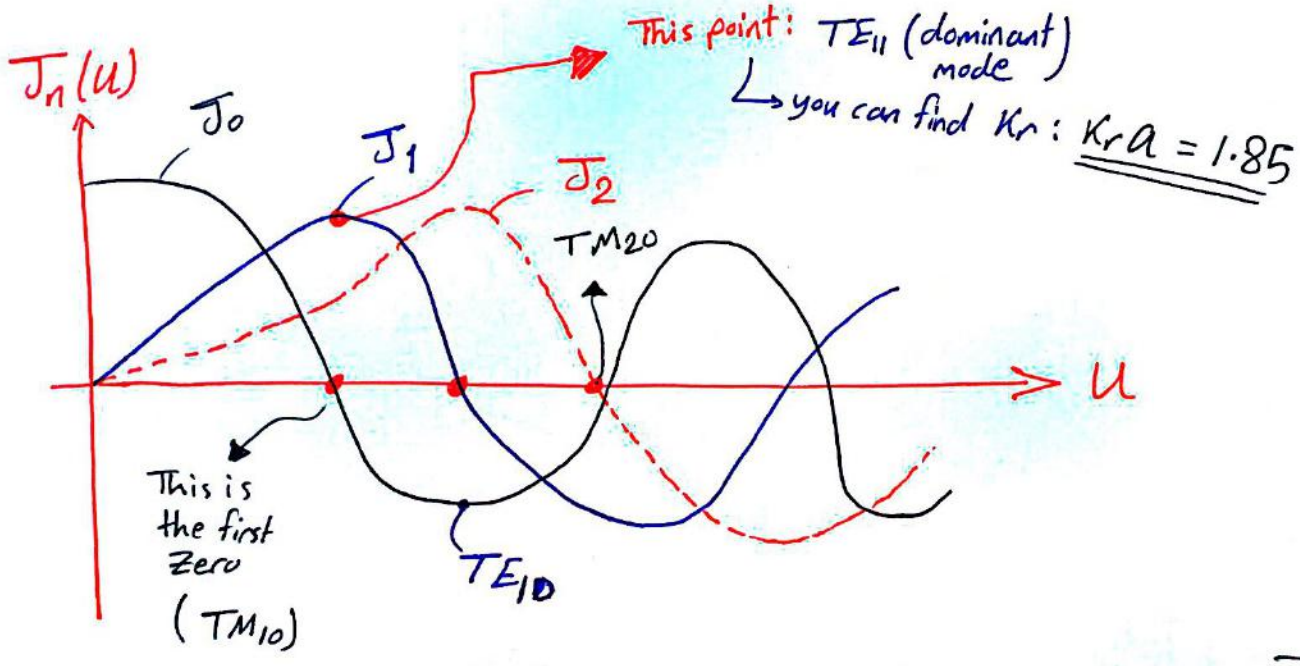
$$\begin{matrix}E\\H\end{matrix}(\phi_0) = \begin{matrix}E\\H\end{matrix}(\phi_0 + 2n\pi)$$

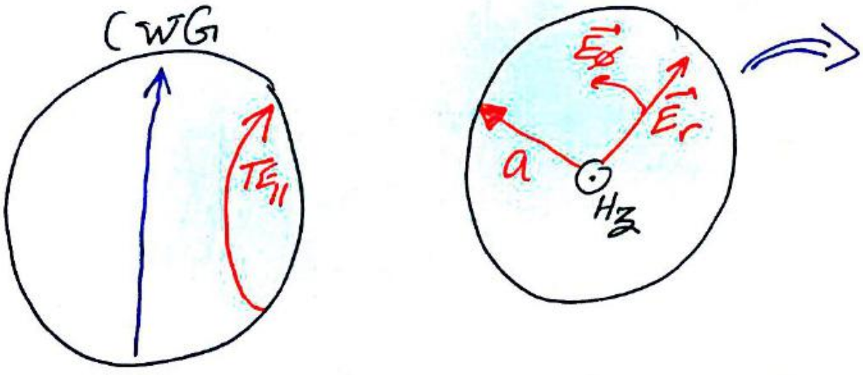
where ϕ:
$\sin n\phi$ or $\cos n\phi$ or combination
can't be otherwise.

⇒ for r:

$$\begin{matrix}E_z(r)\\H_z(r)\end{matrix} = A\, J_n(K_r r) + B_n\, Y_n(K_r r)$$

if $r=0$ is included
Then: ⇒ $\boxed{B_n = 0}$





$$H_z(r) \begin{cases} \xrightarrow{\partial/\partial\phi} \vec{E_r} \\ \xrightarrow{\partial/\partial r} \vec{E_\phi} \end{cases}$$

just $\vec{E_\phi}$ has B.C
$\vec{E_\phi} = 0$ @ $r=a$

in circular waveguide:

$$\beta g_{mn} = \sqrt{K^2 - K_r^2}$$

End of CH12.

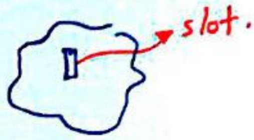
CHAPTER (13)

Antenna.

* Antenna:

1. Types
   - ① wire — dipole. monopole.

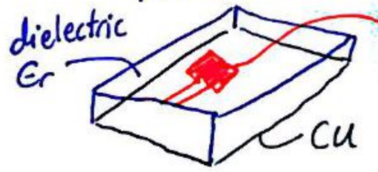
   - ② Aparture



   - ③ Reflector

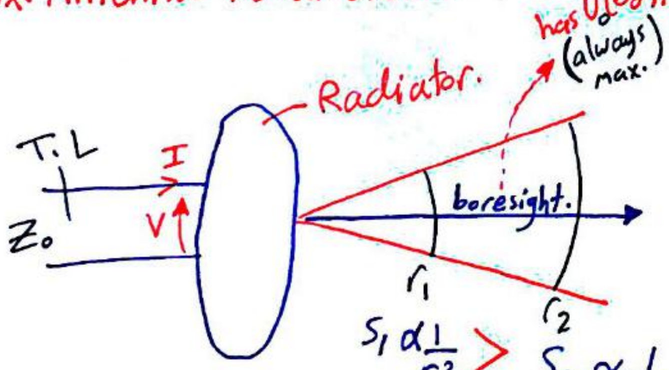
   - ④ Array

   - ⑤ printed circuit.



2. Parameters
3. Analysis

※ Antenna Parameters:



$S_1 \propto \frac{1}{r_1^2} > S_2 \propto \frac{1}{r_2^2}$

$\Rightarrow r_1^2 S_1 = r_2^2 S_2 \Rightarrow$ "power intensity" $\equiv U_{(\theta,\phi)}$

$$\begin{matrix} E^{r} \\ H^{r} \end{matrix}{}_{,\theta,\phi}(r,\theta,\phi) \propto \frac{e^{-jkr}}{r} f(\theta,\phi)$$

(r → radiator.)

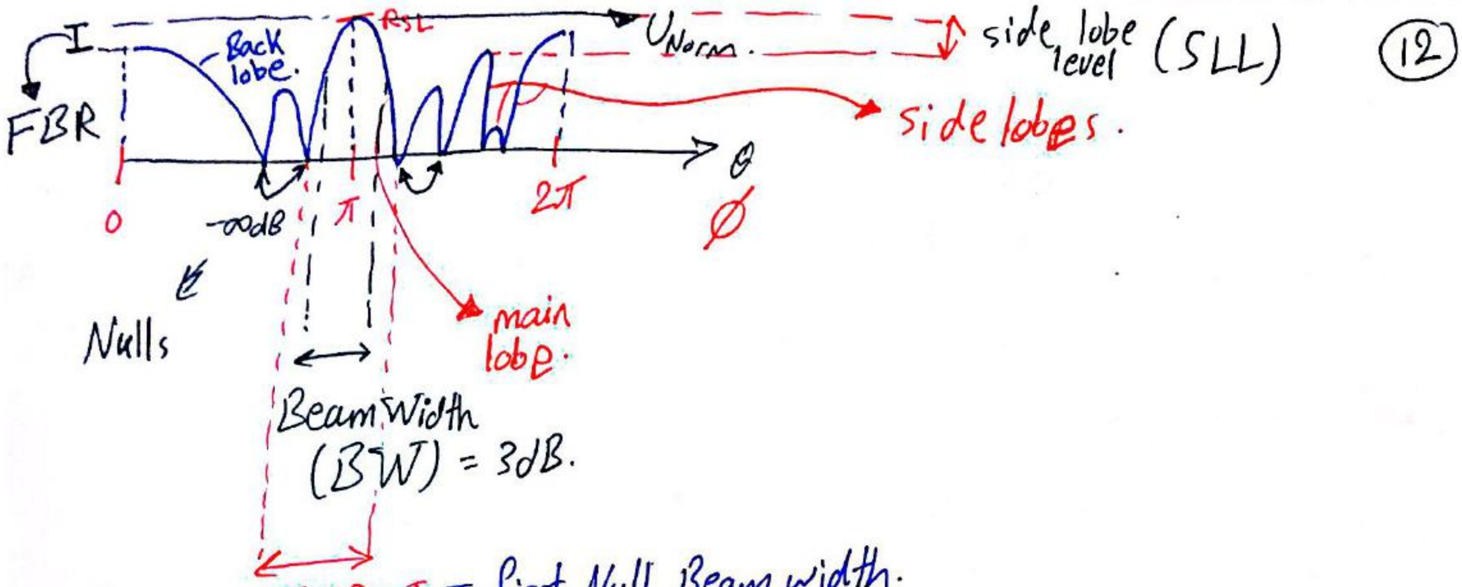
$$\overline{S}_{av} = \frac{1}{2}\vec{E} \times \vec{H}^{*} (r,\theta,\phi)$$

⇓ "power density"

* Radiation Pattern:

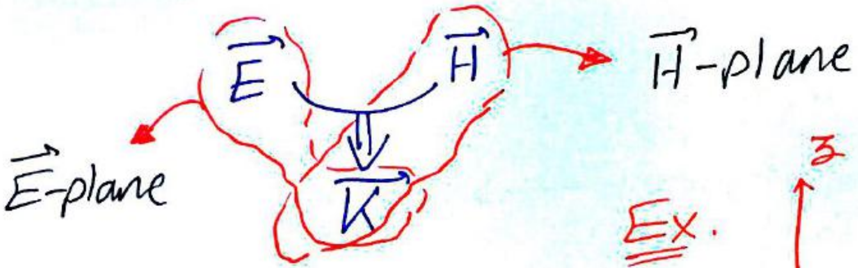
↳ Variation of $U(\theta,\phi)$ in space.

Variation of $U_{Norm.} = \dfrac{U(\theta,\phi)}{U_0(\theta_0,\phi_0)}$ or $10\log_{10} U_{Norm.} \Rightarrow$ in dB

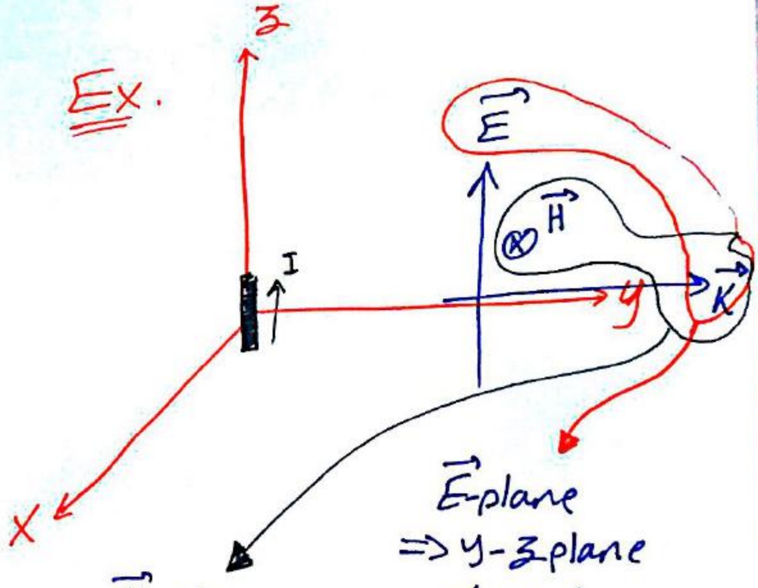


FNBW ≡ first Null Beam width.

* Radiation planes:



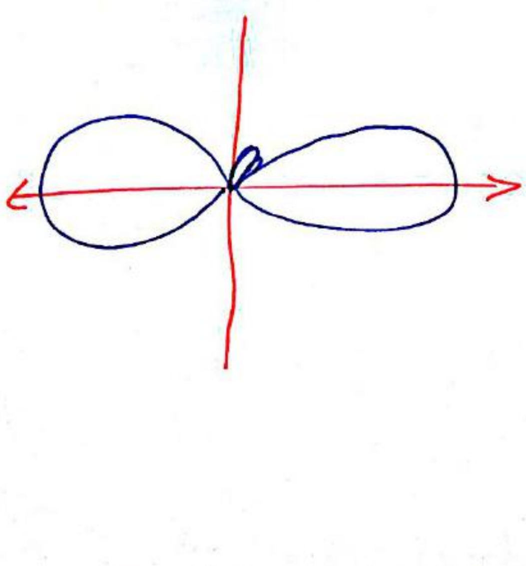
Ex.



$\vec{E}$-plane
$\Rightarrow$ y-z plane
$\phi = 90°$
θ $(0 \le \theta \le 180°)$

$\vec{H}$-plane
$\Rightarrow$ x-y plane
$\theta = 90°$
ϕ $(0 \le \phi \le 2\pi)$

* Radiation pattern in Polar:



* radiation pattern $\Longrightarrow$ directivity.

$D_0(\theta_0, \phi_0)$

$$D \equiv \frac{4\pi\, U(\theta,\phi)}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} U(\theta,\phi)\, d\Omega}$$

could be normalized.

$\sin\theta\, d\theta\, d\phi$

since it is for all the sphere.

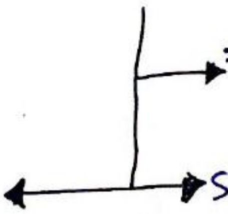
$$\iint_{sphere} d\Omega = \underline{\underline{4\pi}}$$

$\vec{ds} = r^2 \sin\theta\, d\theta\, d\phi$

$\Rightarrow \frac{\vec{ds}}{r^2} = \sin\theta\, d\theta\, d\phi$

$\equiv d\Omega$

Solid Angle.

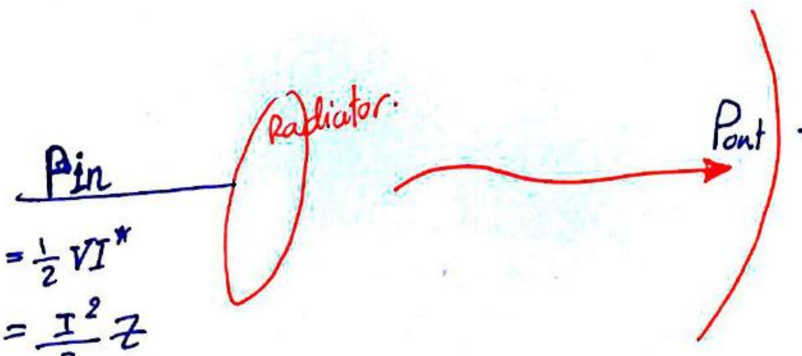


D

$= 1$ point source $\overset{or}{\Rightarrow}$ $10 \log D_0 = \underline{\underline{0 dB}}$ in dB

30 m dipole @ 4 GHz $= \underline{\underline{10^6}} = \underline{\underline{60 dB}}$

Short dipole $= \frac{3}{2} \sim 2.15 dB$.



Radiator.

P_{in}

$= \frac{1}{2} V I^*$

$= \frac{I^2}{2} Z$

Watt.

$P_{out} = \frac{|E|^2}{2\eta}$

Always. $P_o < P_i$

called: effeciency.

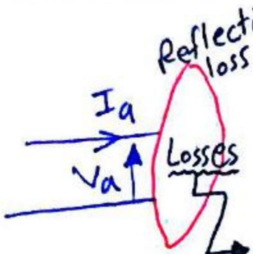
$\eta = \frac{P_o}{P_i} < 1$

$D_0 * \eta \equiv gain(G)$

$G = D_0 \eta$

$G = D_0 + \eta$ in dB

*Antenna Impedance:



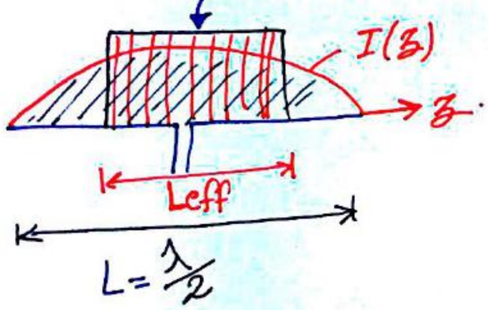
$$Z_{in} = Z_a = \frac{V_a}{I_a} = R_a + jX_a = \oiint_s \vec{S}_{av.} \cdot \vec{ds} = \frac{1}{2} \oiint \vec{E} \times \vec{H}^* \cdot \vec{ds}$$

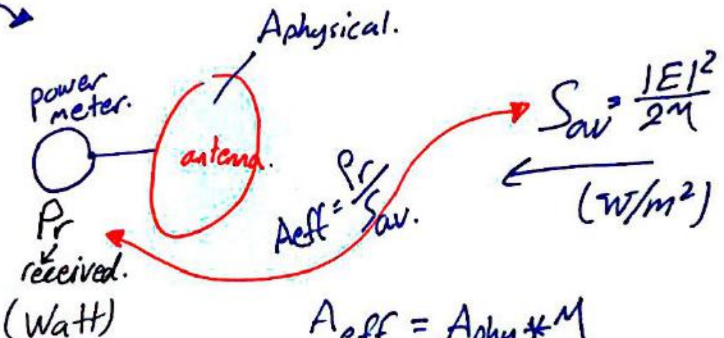
- ohmic loss.
- dielectric loss.
- gap loss.
- Blockage loss.
- scattering loss.
- polarization loss.

R_r radiation, R_L loss

$$\eta = \frac{R_r}{R_L}$$

* Effective length & area:



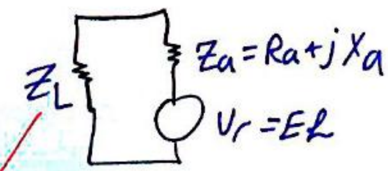


$A_{eff} = A_{phy} * \eta$

efficiency.

*short dipole:

$\Rightarrow$

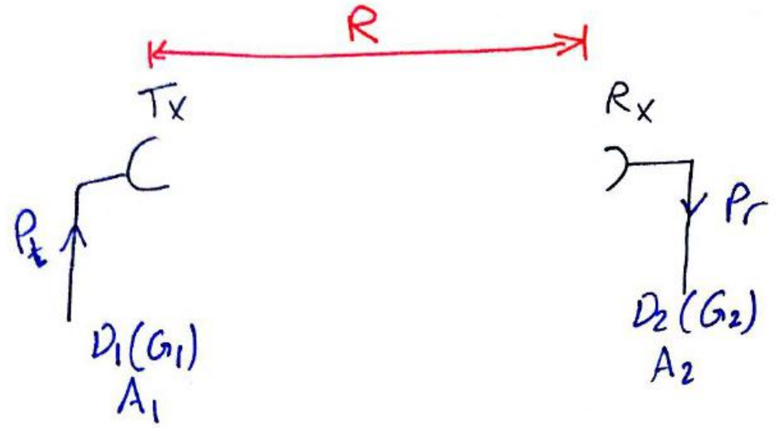


$\Rightarrow R_a = 80\pi^2 \left(\frac{l}{\lambda}\right)^2 \Omega$

we take it to maximize the power $(Z_a - jX_a)$

$$P_r = \frac{1}{2}\left(\frac{El}{2R_a}\right)^2 * R_a = \frac{E^2 l^2}{8R_a}$$

$$A_{eff} = \frac{P_r}{S_{av}} = \frac{|E^2| l^2}{8 * 80\pi \frac{l^2}{\lambda^2}} * \frac{2 * 120\pi}{|E|^2} = \frac{3\lambda^2}{8\pi} \ m^2$$



$$P_r = \frac{P_t D_1}{4\pi R^2} A_2 \quad \text{---(1)} \qquad , \qquad P_r = \frac{P_t D_2}{4\pi R^2} A_1 \quad \text{---(2)}$$

from (1) & (2):

$$D_1 A_2 = D_2 A_1$$

	Point source (P.S)	short dipole
D	1	$3/2$
A_{eff}	?	$\frac{3\lambda^2}{8\pi}$

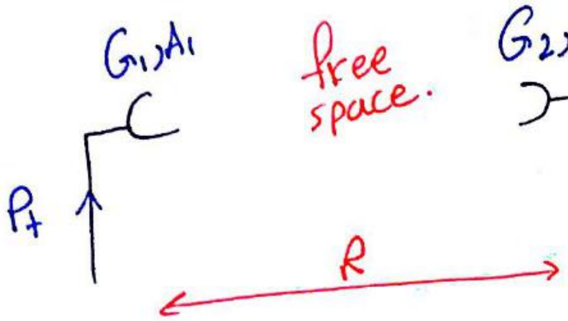
$$\Rightarrow \quad D_1 A_2 = D_2 A_1 \Rightarrow 1 \cdot \frac{3\lambda^2}{8\pi} = \frac{3}{2} A_1 \Rightarrow \boxed{\underset{(P.S)}{A_{eff}} = \frac{\lambda^2}{4\pi}} \ m^2$$

	P.S	Any antenna
D	1	D or G
A_{eff}	$\frac{\lambda^2}{4\pi}$	A_{eff}

$A\uparrow \ \lambda\downarrow \ D\uparrow \Rightarrow A_{eff}$ constant.

$$\boxed{\underset{\text{for any Antenna}}{A_{eff}} = \frac{\lambda^2}{4\pi} D} \rightarrow \text{Gain}$$

we prefere here to deal with gain.



$$P_r = \frac{P_t G_1}{4\pi R^2} A_2 \rightarrow \frac{\lambda^2}{4\pi} G_2$$

in dB:

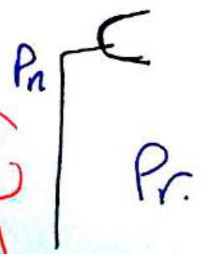
$$\underline{P_r = P_t + G_1 + G_2 - FSPL}$$

where

FSPL: Free Space Path Loss.

$$FSPL = 20\log_{10}\left(\frac{4\pi R}{\lambda}\right)$$

※ Antenna Temprecture:



$$P_n = K T_a B$$

P_n: noise.

K: Boltzman Constant.

T_a: in Kelvin.

B: Bandwidth.

※ Magnetic Vector Potential:

$$\vec{B} = \nabla \times \vec{A}$$

$$\vec{A} = \frac{\mu}{4\pi} \iiint_{V} \frac{\vec{J}(\vec{r}\,')}{R}\, dv'$$

$$\Rightarrow M.E: \quad \nabla \times \vec{E} = -j\omega \underbrace{\mu \vec{H}}_{\vec{B}} = -j\omega\, \nabla \times \vec{A}$$

$$\Rightarrow \nabla \times [\underbrace{\vec{E} + j\omega\vec{A}}_{\equiv -\nabla V}] = 0$$

$$\underline{\vec{E} + j\omega\vec{A} = -\nabla V} \Rightarrow \boxed{\vec{E} = -j\omega\vec{A} - \nabla V} \quad (*)$$

Now $\nabla \times \vec{H} = \nabla \times \left(\frac{\vec{B}}{\mu}\right)$

$$= \frac{1}{\mu}\nabla[\nabla \times \vec{A}] = \frac{1}{\mu}\left\{\nabla(\nabla\cdot\vec{A}) - \nabla^2\vec{A}\right\} = \vec{J} + j\omega\epsilon\left[-j\omega\vec{A} - \nabla V\right]$$

$$\Rightarrow \nabla(\nabla\cdot\vec{A}) - \nabla^2\vec{A} = \mu\vec{J} + \underbrace{\omega^2\mu\epsilon}_{K^2}\vec{A} - j\omega\mu\epsilon\,\nabla V$$

$$\Rightarrow \underline{\underline{\nabla(\nabla\cdot\vec{A})}} - \left\{\nabla^2\vec{A} + K^2\vec{A}\right\} = \mu\vec{J} - \underline{\underline{j\omega\mu\epsilon\,\nabla V}}$$

$$\rightarrow \nabla\cdot\vec{A} = -j\omega\mu\epsilon V$$

from (*)

$$\vec{E} = -j\omega\vec{A} - \frac{j}{\omega\mu\epsilon}\nabla(\nabla\cdot\vec{A}) \quad \text{-----} \quad \boxed{1}$$

$$\Rightarrow \left[\nabla^2 A + K^2 \vec{A} = -\mu \vec{J}\right] * e^{j\omega t}$$

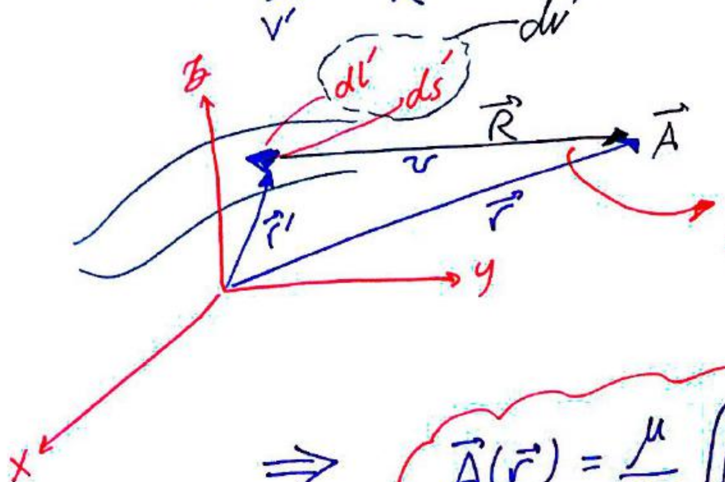
$$\Downarrow \qquad e^{j\omega(t-\frac{R}{v})}$$

$$\vec{A}(\vec{r},t) = \frac{\mu}{4\pi}\iiint_{V'} \frac{\vec{J}(\vec{r}',t)}{R}\, dv'$$

$$\nabla^2 A + K^2 \vec{A} = -\mu \vec{J} \quad \text{---} \quad \boxed{2}$$

$\boxed{1}$ & $\boxed{2}$ called:

Auxiliary Equations.



$$t' = t - \frac{R}{v}$$

$$K \equiv \frac{\omega}{v}$$

$$\Rightarrow \quad \vec{A}(\vec{r}) = \frac{\mu}{4\pi}\iiint_{V'} \frac{\vec{J}(\vec{r}')}{R}\, e^{-jKR}\, dv'$$

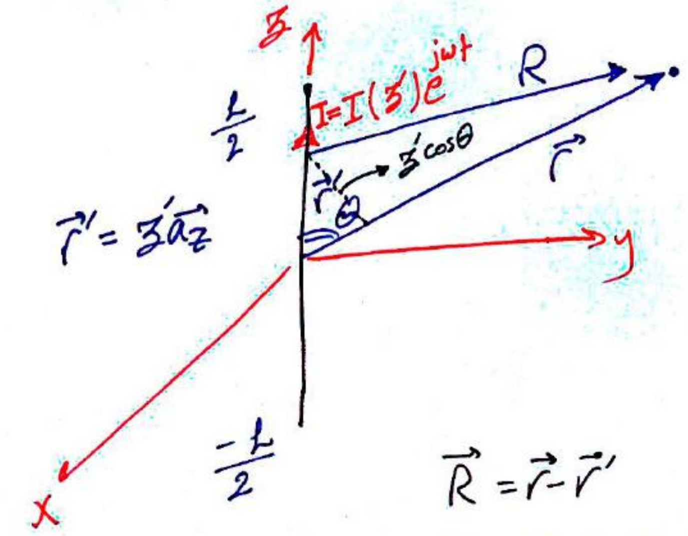
---

* Dipole:

L.

*Taylor Series:

$$f(x) = \sum_{n=0}^{\infty} f^{(n)}(x)\Big|_{x=x_0} \frac{(x-x_0)^n}{n!}$$



$\vec{r}' = z'\vec{a}_z$

$A_z ? (\vec{r})$

z since the current in z direction.

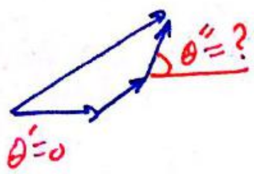
$$A_z(\vec{r}) = \frac{\mu}{4\pi}\int_{-L/2}^{L/2} \frac{I(z')}{R}\, e^{-jKR}\, dz'$$

$$\vec{R} = \vec{r} - \vec{r}'$$

$$R = [r^2 + r'^2 - 2rr'\cos\theta]^{1/2} \Rightarrow \text{"cosine rule"}$$

$$= r\left[1 + \left(\frac{z'}{r}\right)^2 - 2\frac{z'}{r}\cos\theta\right]^{1/2}$$

$\frac{z'}{r} \to X \qquad \frac{z'}{r} \to X$



if $\theta'' \le \frac{\pi}{8}$ $\Longrightarrow$

$\Rightarrow r \gg L \Rightarrow r \parallel R$ (far field)

$R \sim r$ in the amplitude. $R \cong r - z' \cos\theta$

$KR = Kr - Kz' \cos\theta$

$$A_z(\vec{r}) = \frac{\mu}{4\pi}\int_{-L/2}^{L/2} \frac{I(z')\, e^{-jKR}}{R}\, dz' \cong \frac{\mu\, e^{-jKr}}{4\pi r}\int_{-L/2}^{L/2} I(z')\, e^{jKz'\cos\theta}\, dz' = A_z(r)$$

$A_z(r) \rightarrow A_\theta, \; A_r$

$$R = r\underbrace{\left(1 + \left(\frac{z'}{r}\right)^2 - 2\frac{z'}{r}\cos\theta\right)^{\frac{1}{2}}}_{f(x)}$$

$f(x)$: $x = \frac{z'}{r}$, $\underline{\underline{x_0 = 0}}$

$$f(x) = f(x_0) + \left.\frac{\partial f}{\partial x}\right|_{x=0}\frac{x}{1!} + \left.\frac{\partial^2 f}{\partial x^2}\right|_{x=0}\frac{x^2}{2!} + \cdots$$

$f(x_0)$, $x_0 = 0$ $\Rightarrow 1$

$\left.\frac{\partial^2 f}{\partial x^2}\right|_{x=0}\frac{x^2}{2!} \Downarrow * K \le \frac{\pi}{8}$

$\Rightarrow \boxed{r \ge \frac{2L^2}{\lambda}}$

R in the amplitude $\sim r$

R in the phase: KR

$\Rightarrow KR \cong Kr - Kz'\cos\theta$

if this condition is true

$$A_z(\vec{r}) = \frac{\mu\, e^{-jKr}}{4\pi r}\int_{-L/2}^{L/2} I(z')\, e^{jKz'\cos\theta}\, dz'$$

$$\vec{E} = -j\omega\vec{A} - \frac{j}{\omega\mu\epsilon}\nabla(\nabla\cdot\vec{A})$$

$A_z \rightarrow A_r, \; A_\theta, \; \cancel{A_\phi} \rightarrow 0$

$A_r \rightarrow E_r \rightarrow H_\theta$

$A_\theta \rightarrow E_\theta \rightarrow H_\phi$ → poynting vector.

$\cancel{A_\phi} \rightarrow \cancel{E_\phi} \rightarrow 0$

$$E_\theta \sim f(\theta,\cancel{\phi})\,\frac{e^{-jKr}}{r}\int_{-\frac{L}{2}\to-\infty}^{L/2\to\infty} J(z')\,e^{+jKz'\cos\theta}\,dz'$$

$\Longrightarrow$ fourier transform.

$I(z')$ = Inverse F.T of $E(\theta)$

$$= f(\theta)\,\frac{e^{-jKr}}{r}\int_{0}^{\pi,\,2\pi} E_\theta(\theta)\,e^{-jKz'\cos\theta}\,d(K\cos\theta)$$

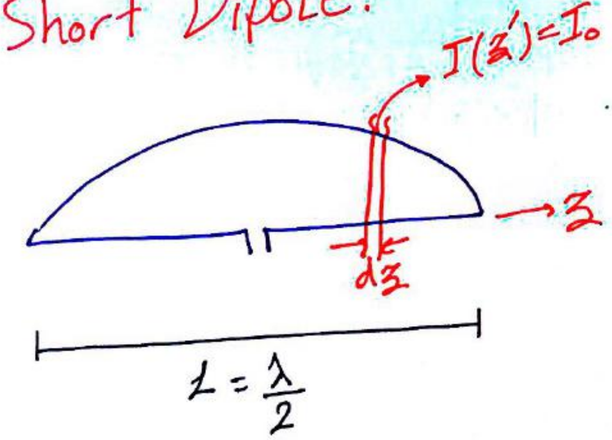
could be ϕ.

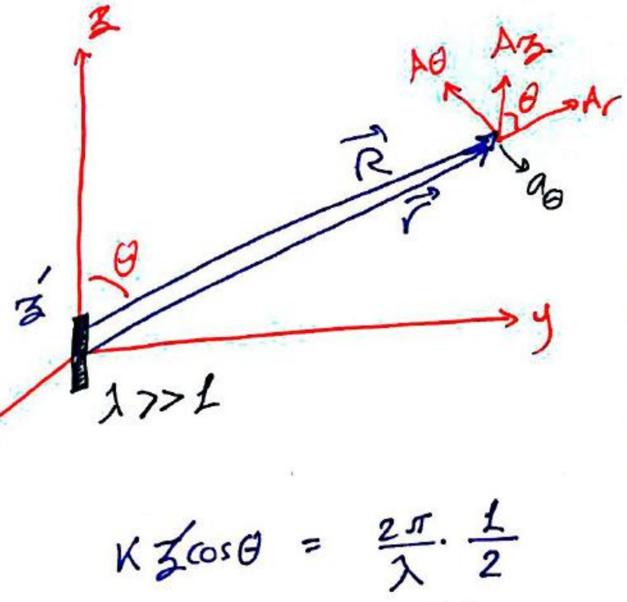
θ (or/and ϕ)

$\Rightarrow$ This called:

Radiation Pattern is F.T of the spactial variation of the current on the Antenna.

※ Short Dipole:





$\lambda \gg L$

$A_\theta = -A_z \sin\theta$

$A_r = A_z\cos\theta$

$$A_z = \frac{\mu I_0 e^{-jkr}}{4\pi r}\int_{-\frac{L}{2}}^{\frac{L}{2}} \underbrace{e^{+jkz'\cos\theta}}_{1}\,dz'$$

$$= \boxed{\frac{\mu I_0 L\, e^{-jkr}}{4\pi r}}$$

$$K z'\cos\theta = \frac{2\pi}{\lambda}\cdot\frac{L}{2} = \pi\left(\frac{L}{\lambda}\right)$$

$L \sim \frac{1}{100}\lambda$

$\simeq 1.8^\circ$

$e^{j0} \sim e^{j1.8} \simeq 1$

$$\vec{E} = -j\omega\vec{A} - \frac{j}{\omega\mu\epsilon}\nabla(\nabla\cdot\vec{A})$$

$$= \text{Term}\left(\frac{1}{r}\right) + \cancel{\text{Terms}\left(\frac{1}{r^2} \text{ or more}\right)}$$

$$\left.\begin{aligned} E_\theta &= \frac{\eta\, jkl\, I_o\, e^{-jkr}}{4\pi r}\sin\theta \\ H_\phi &= \frac{E_\theta}{\eta} \end{aligned}\right\} \text{far field Component}$$

$$r > \frac{2l^2}{\lambda}$$

$\vec{S}_{av}$:

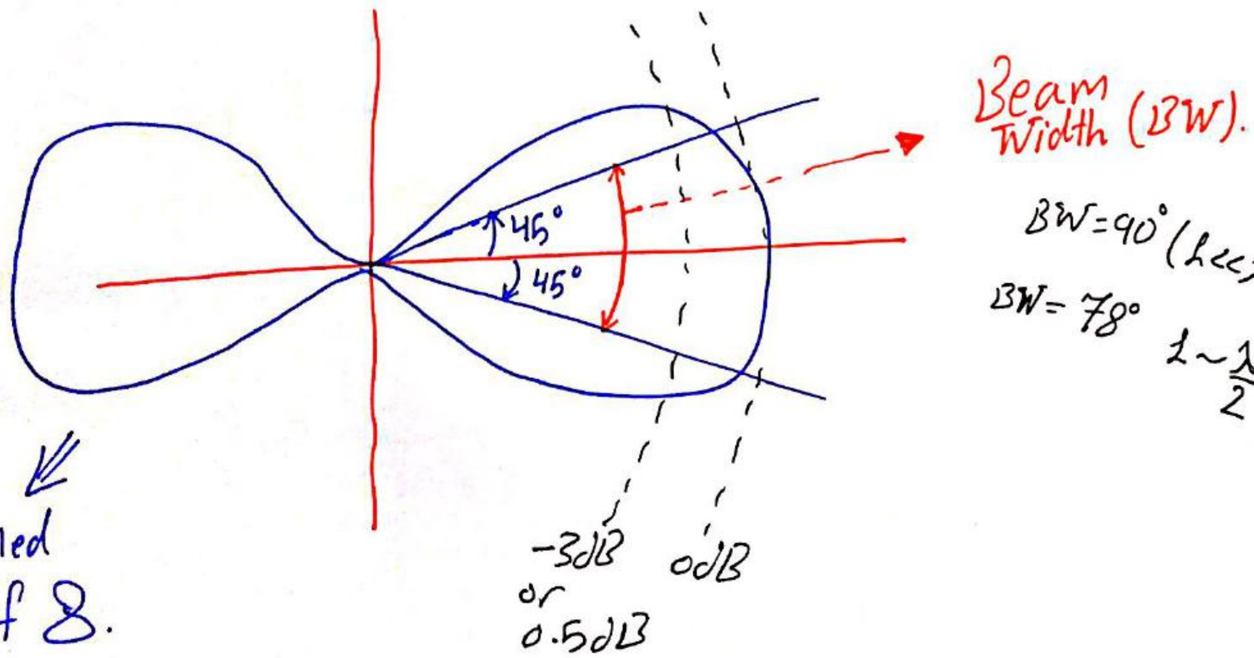
$$\vec{S}_{av} = \frac{1}{2}\vec{E}\times\vec{H}^*$$

$$= \frac{|E|^2}{2\eta} = \frac{\eta\,(kl)^2 I_o^2}{32\pi^2 r^2}\sin^2\theta$$

$$U(\theta\phi) = r^2\vec{S}_{av} = \frac{\eta\,(kl)^2 I_o^2}{32\pi^2}\sin^2\theta$$

***Radiation Pattern:**

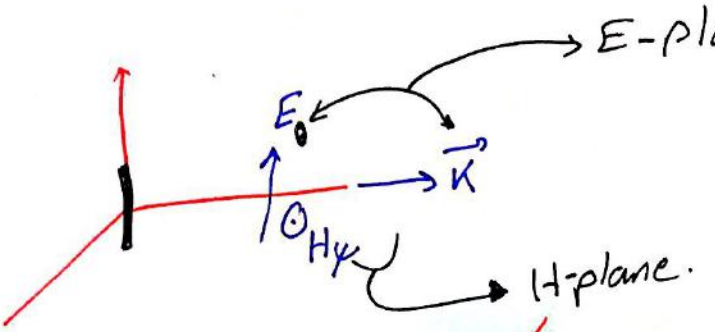
$$U_{normalized} = \frac{U(\theta,\phi)}{U(\theta_o,\phi_o)} = \sin^2\theta \quad \text{or} \quad = 10\log_{10}(\sin^2\theta)$$



$BW = 90°$ $(l \ll \lambda)$

$BW = 78°$ $\quad l \sim \frac{\lambda}{2}$

This called figure of 8.



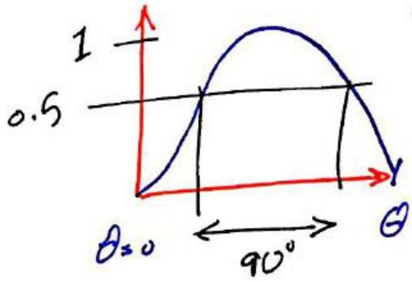
y-z plane

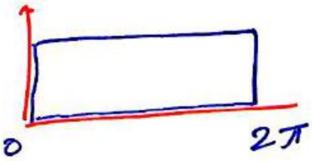
$\phi = \frac{\pi}{2}$, $\phi \nearrow$

y-x plane

$\theta = 90°$, $\phi \rightarrow$

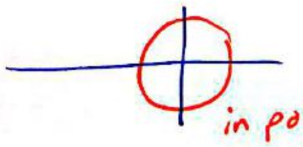
BW = 90°





BW = 360°

this called: omnidirectional antenna.



in polar.

*Directivity:

$$D_0 = \frac{U(\theta_0, \phi_0)}{\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} U(\theta, \phi) \sin\theta \, d\theta \, d\phi} * 4\pi = \frac{4\pi \left. \sin^2\theta \right|_{\theta = \frac{\pi}{2}}}{\int_0^{\pi}\int_0^{2\pi} \sin^2\theta \sin\theta \, d\theta \, d\phi}$$

$$= \frac{4\pi * 1}{\frac{4}{3} * 2\pi} = \boxed{\frac{3}{2}} \text{ or } (1.76) \text{ dB}$$

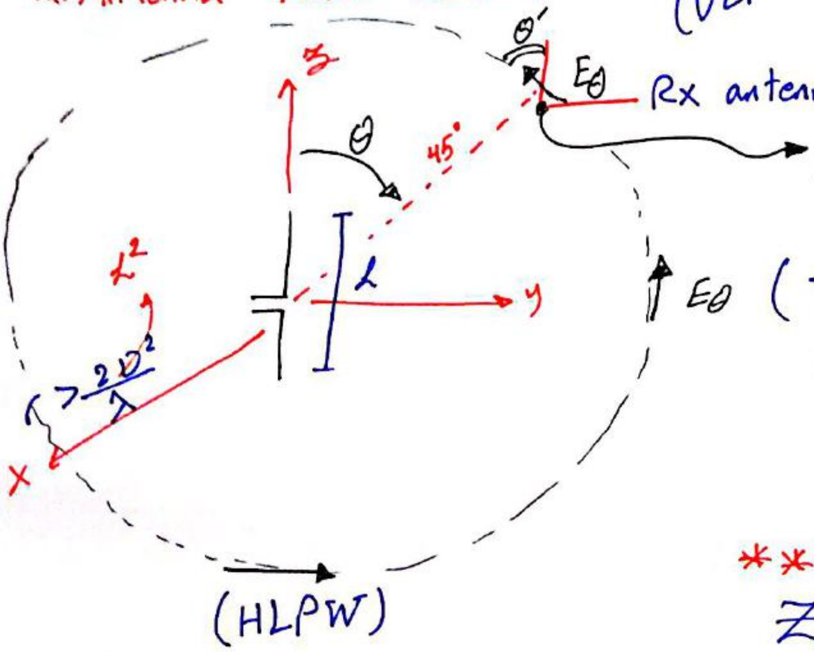
$$G = D * \eta$$

$\eta \rightarrow$ efficiency.

***

※ Antenna Polaraization:

(VLPW) with angle 45°



E_θ (VLPW)

verticly Linearly polarized wave.

** $Z_a = \frac{1}{|I|^2} \oiint_s \vec{E} \times \vec{H}^* \cdot d\vec{s}$

$= R_a + jX_a$

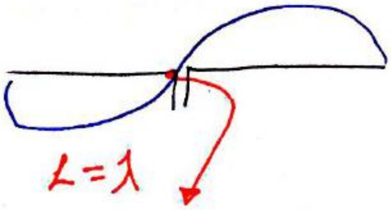
X_a (-ve) if $\frac{\lambda}{2} > l$

* in far field:

$\boxed{Z_a = R_a} \Rightarrow R_r$

radiation R.

for (R_r):

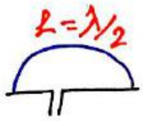


current idealy = 0
(very small value)

⇓

This will make the value Of R_r very Large.

(so we use $\frac{\lambda}{2}$)



$\Rightarrow R_r = \frac{1}{|I_o|^2} \int_0^{2\pi} \int_0^{\pi} \frac{\eta^2 (Kl)^2 I_o^2}{\eta \, 32\pi^2 \cancel{r^2}} \sin^2\theta$

$\cancel{r^2} \sin\theta \, d\theta \, d\phi$

$\Rightarrow \boxed{R_r = 80\pi^2 \left(\frac{l}{\lambda}\right)^2}$

$A_{eff} = \frac{\lambda^2}{4\pi} D = \frac{\lambda^2}{4\pi} * \frac{3}{2}$

$A_{eff} = \frac{3\lambda^2}{8\pi}$ m^2.

END of CH13 ***