University of Jordan

Faculty of Engineering & Technology

Department of Electrical Engineering

Course Title:	Exam:	Date:
Electromagnetics I	1st Exam	Nov/03/2015
Course No.:	**Semester:**	**Time Period:**
0903251	1st Term 2015-2016	1:00 Hr.

Instructors:
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Q1	Q2	Total 20
8.5	5	13.5/20

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Section:

$$\nabla \cdot \mathbf{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$d\mathbf{S} = \rho\, d\phi\, dz\, \mathbf{a}_\rho \qquad d\rho\, dz\, \mathbf{a}_\phi \qquad \rho\, d\phi\, d\rho\, \mathbf{a}_z$$

$$\nabla \times \mathbf{A} = \frac{1}{\rho}\begin{vmatrix} \mathbf{a}_\rho & \rho\mathbf{a}_\phi & \mathbf{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

$$d\mathbf{l} = d\rho\, \mathbf{a}_\rho + \rho\, d\phi\, \mathbf{a}_\phi + dz\, \mathbf{a}_z$$

$$\varepsilon_0 = \frac{10^{-9}}{36\pi}\ F/m$$

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Note that bold letters are vectors

Q1)

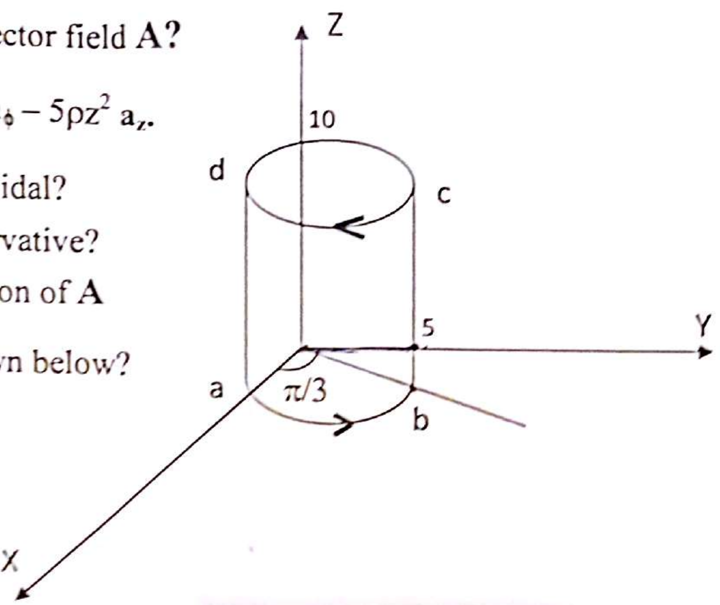
Find the following for the vector field **A**?

$\mathbf{A} = \rho^2 cos\,(\phi)\, \mathbf{a}_\rho - \rho sin\,(\phi)\, \mathbf{a}_\phi - 5\rho z^2\, \mathbf{a}_z.$

a) Is the vector **A** solenoidal?

b) Is the vector **A** conservative?

c) Calculate the circulation of **A** along the path **abcd** shown below?



(a)

if $\nabla \cdot A \neq 0 \rightarrow$ sino

$$\nabla \cdot A = \frac{1}{\rho}\frac{\partial \rho A_\rho}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$= \frac{1}{\rho}\left[\frac{\partial \rho^3 \cos\phi}{\partial \rho}\right] + \frac{1}{\rho}\left[\frac{\partial -\rho\sin\phi}{\partial \phi}\right] + \frac{\partial -5\rho z^2}{\partial z}$$

$$= \frac{1}{\rho}\left[3\rho^2\cos\phi\right] - \frac{\rho}{\rho}\cos\phi - 10\rho z$$

$$= 3\rho\cos\phi - \cos\phi - 10\rho z \neq 0$$

$\boxed{\text{So vector A is not solenoidal}}$ (2)

(b)

$$\nabla \times A = \frac{1}{\rho}\begin{vmatrix} a_\rho & \rho a_\phi & a_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

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$\frac{1}{\rho}(0 - 0)\, \vec{a_\rho} - (-5z^2 - 0)\, \rho a_\phi + (-2\rho\sin\phi + \rho^2\sin\phi)\, a_z$

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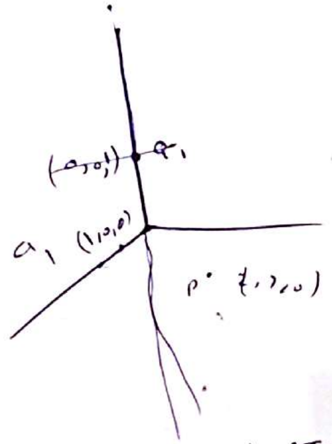
Q2)

A uniform line charge density $\rho_\ell = 1\ \text{nCm}^{-1}$ of an infinite length is placed along the z-axis in free space and an electric charge $Q_1 = 1$ nC is placed at (1, 0, 0). Find the electric flux density at point P(2, 3, 0).

$D = \epsilon_0 E$



$E_l = \dfrac{\rho_l}{2\pi\epsilon_0 \rho} a_\rho$

$R = 2a_x + 3a_y$

$|R| = \sqrt{4+9} = \sqrt{13}$

$a_R = \dfrac{(2a_x + 3a_y + 0a_z)}{\sqrt{13}}$

$\dfrac{\rho_l}{2\pi\epsilon_0\sqrt{13}} \dfrac{(2a_x+3a_y)}{\sqrt{13}} = \dfrac{\rho_l}{2\pi\epsilon_0 13}(2a_x+3a_y)$

$= \dfrac{10^{-9}(2a_x+3a_y)}{2\pi \times \frac{10^{-9}}{36\pi}\ 13} = \dfrac{18(2a_x+3a_y)}{13} =$

$E = (2.768a_x + 4.152a_y)$ V/m

$R = 1a_x + 3a_y$

$|R| = \sqrt{1+9} = \sqrt{10}$

$a_R = \dfrac{(1a_x+3a_y)}{\sqrt{10}}$

$E_{Q_1} = \dfrac{Q_1 a_R}{4\pi\epsilon_0 |R|^2}$

$= \dfrac{10^{-9}(1,3,0)}{4\pi \times \frac{10^{-9}}{36\pi}(\sqrt{10})^3} = \dfrac{9(1,3,0)}{(\sqrt{10})^3} = (0.192a_x + 0.576a_y)$