University of Jordan
Elect. Eng. Dept.

Mid Exam. **Electromagnetics 0903351** **29/7/2016**

26/30

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Q.1 (6 Points)

Find the emf voltage generated at the terminals of a 250 turns square coil of area 0.1m^2 centered at the origin in x-y plane if the magnetic flux density is given by:

$$B = 4e^{-|x|}\cos(2\pi 10^3 t)\,a_z$$

$\rightarrow -N \cdot \frac{dB}{dt} \cdot S$

$V_{emf} = -N\frac{d\phi}{dt} = -N\int \frac{\partial B}{\partial t}\cdot ds$

$\frac{dB}{dt} = -4e^{-|x|}\sin(2\pi\times10^3 t) \times 2\pi\times10^3\, a_z$

area $= \pi r^2$

r=0.31

x=0.158

$V_{emf} = 250 \times 0.1 \times 4e^{-|x|} \times 2\pi\times10^3 \sin(2\pi\times10^3 t)$

$= 2\pi\times10^5 e^{-|x|}\sin(2\pi\times10^3 t)$

~~$= 2\pi\times10^5 e^{-0.158}\sin(2\pi\times10^3 t) = 536489.64\sin(2\pi\times10^3 t)$ V~~

at $x=0 \rightarrow 2\pi\times10^5\sin(2\pi\times10^3 t)$ V

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Q.2 (6 Points)

In free space at 1.5GHz the direction of propagation is in (0.6**ax** -0.8**ay**) if the power density is 100mW/m^2 and the electric field is in the +z direction, Find E and H.

$E = E_0\cos(\omega t - \beta z)\,a_z$

$\omega = 2\pi\times1.5\times10^9 = 3\pi\times10^9$ rad/s

$r = (0.6a_x - 0.8a_y)$

$\beta = \frac{\omega}{u}$ but in free space $u = c = 3\times10^8$

$\beta = \frac{3\pi\times10^9}{3\times10^8} = 10\pi$ rad/m

η in free space $= 120\pi$

a_E, a_r, a_H, a_x, a_z, a_y

4

$P_{avg} = Re(E\times H^*) = \frac{1}{2}\frac{E^2}{\eta} \Rightarrow 100\times10^{-3} = \frac{1}{2}\frac{E_0^2}{120\pi} \rightarrow E_0^2 = 24\pi$

$E_0 = 8.683$ V/m

$E = 8.683\cos(3\pi\times10^9 t - 10\pi(0.6x - 0.8y))\,a_z$

$H = \frac{E}{\eta} \rightarrow H = 0.023\cos(3\pi\times10^9 t - 10\pi(0.6x - 0.8y))(-0.6a_y - 0.8a_x)$ A/m

$a_r \times a_E = a_H \rightarrow (0.6a_x - 0.8a_y)\times a_z = -0.6a_y - 0.8a_x$ direction of H

cross product

Q.3 (8 Points)

A plane wave at 6GHz is incident normally from air into a dielectric with relative permittivity of 4. If the incident electric field amplitude is 10^{-6} V/m. Draw the standing wave in the first media.

$\epsilon_r = 4 \quad \mu_r = 1$

$|E_i| = 10^{-6}$

Free space: $\eta_1 = 120\pi\ \Omega$

$\eta_2 = \sqrt{\frac{\mu}{\epsilon}} = 120\pi \times \frac{1}{2} = 60\pi\ \Omega$

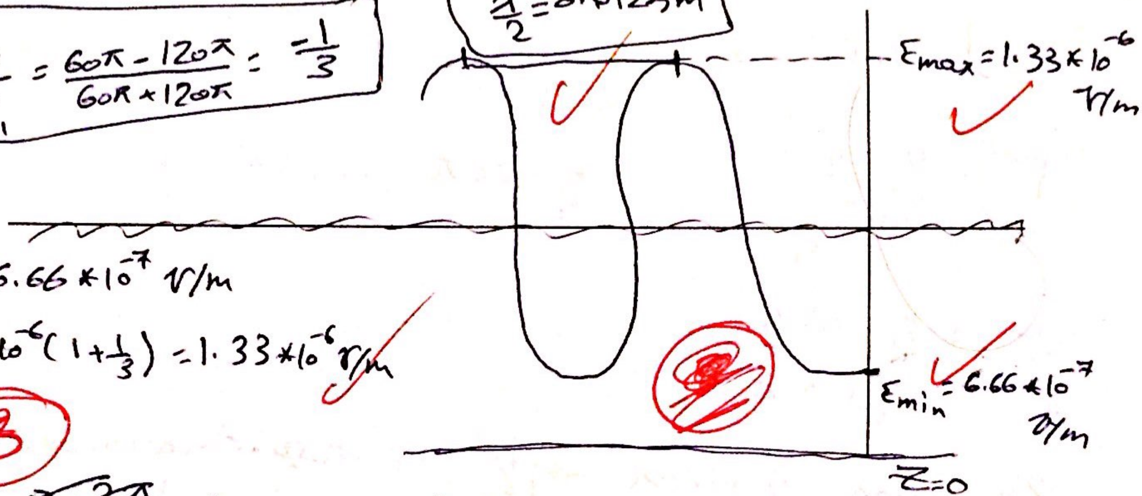
$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{60\pi - 120\pi}{60\pi + 120\pi} = \frac{-1}{3}$

$E_{min} = |E_i|(1+\Gamma) = 10^{-6}(1 + -\frac{1}{3}) = 6.66 \times 10^{-7}$ V/m

$E_{max} = |E_i|(1-\Gamma) = 10^{-6}(1+\frac{1}{3}) = 1.33 \times 10^{-6}$ V/m

$|E_i| = 10^{-6}$

8



$\lambda = \frac{u}{f}$, $u = \frac{1}{\sqrt{\mu\epsilon}} = \frac{3\times10^8}{2} = 1.5\times10^8$

$\lambda = \frac{1.5\times10^8}{6\times10^9} = 0.025$m

$\frac{\lambda}{2} = 0.0125$m

Q.4 (10 Points)

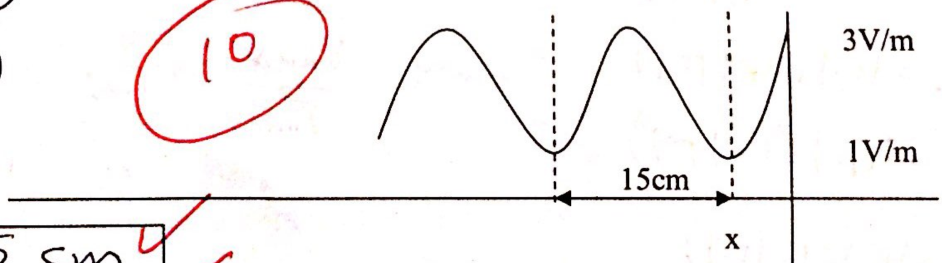
Given the following standing waves, find **x** and the characteristic impedance for the second media.

Free space

$E_{max} = 3 = |E_i|(1+\Gamma)$

$E_{min} = 1 = |E_i|(1-\Gamma)$

10

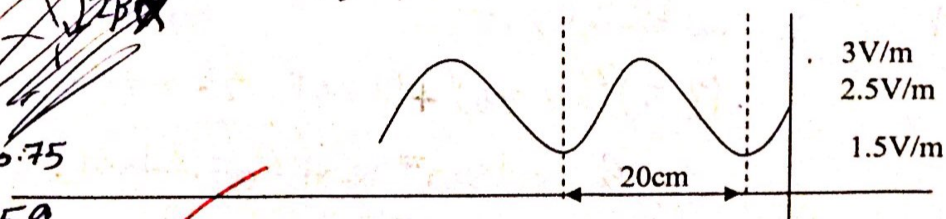


x= 7.5 cm

$\eta_2 = 360\pi\ \Omega$

$\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \Gamma = \frac{1}{3}\angle -100.67°$

$|E_i| = 2.25$, 2.5, $|E_i||\Gamma| = 0.75$



x= 5.59 Cm

$\eta_2 = 337.228\angle -36.39\ \Omega$

$|E_i| = \frac{9}{4} = 2.25$, 2.5, $|E_i||\Gamma| = 0.75$

Q4) divide $\frac{E_{max}}{E_{min}}$:-

1

$$\frac{3}{1} = \frac{|E_i|(1+\Gamma)}{|E_i|(1-\Gamma)}$$

$$3 - 3\Gamma = 1 + \Gamma$$

$$2 = 4\Gamma \rightarrow \boxed{\Gamma = \frac{1}{2}}$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$ $\eta_1 = 120\pi$ in free space

$$\frac{1}{2} = \frac{\eta_2 - 120\pi}{\eta_2 + 120\pi}$$

$$2\eta_2 - 240\pi = \eta_2 + 120\pi \rightarrow \boxed{\eta_2 = 360\pi}$$ → characteristic impedance of the second media

$$\frac{\lambda}{2} = 15\text{cm} \longrightarrow \boxed{x = \frac{\lambda}{4} = 7.5\text{ cm}}$$

2 $|\Gamma| = \frac{|\eta_2| - |\eta_1|}{|\eta_2| - |\eta_1|}$, $\eta_1 = 120\pi$ in free space

$E_{max} = |E_i|(1+|\Gamma|)$ divide $\frac{E_{max}}{E_{min}} \rightarrow \frac{3}{1.5} = \frac{1+|\Gamma|}{1-|\Gamma|}$

$E_{min} = |E_i|(1-|\Gamma|)$

$$2 = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$2 - 2|\Gamma| = 1 + |\Gamma| \rightarrow 3|\Gamma| = 1 \quad \boxed{|\Gamma| = \frac{1}{3}}$$

$\Rightarrow E_{max} = |E_i|(1+|\Gamma|)$

$3 = |E_i|(1+\frac{1}{3})$ ($\frac{4}{3}$) $\rightarrow \boxed{|E_i| = \frac{9}{4}\ V/m}$

$\boxed{|E_i||\Gamma| = \frac{3}{4}}$ to find θ_x :-

$$(2.5)^2 = (0.75)^2 + (2.25)^2 - 2 * 0.75 * 2.25 \cos\theta_x$$

$$\cos\theta_x = 0.1851 \rightarrow \boxed{\theta_x = ~~79.31~~}\ 100.67$$

$-\theta_x = 180 - 79.31 = 100.67 \rightarrow \boxed{\theta_\Gamma = ~~100.67°~~\ -79.31°}$

to find x :- $2\beta x = \theta_x$

$$2 * \frac{2\pi}{\lambda} x = \frac{2\pi * \theta_x}{360°}$$

$\frac{\lambda}{2} = 20\text{cm} \rightarrow \lambda = 40\text{cm}$

$$\frac{2 * x}{40} = \frac{~~79.31~~\ 100.67}{360} \rightarrow \boxed{~~4.4~~\ \text{cm} = x}$$

Q4 الجزء الثاني

$$\frac{1}{3}\angle -100.67 = \frac{Z_2 - 120\pi}{Z_2 + 120\pi}$$

$$\frac{1}{3}\angle -100.67 \, Z_2 + 125.66\angle -100.67 = Z_2 - 120\pi$$

$$1.111\angle -162.85 \, Z_2 = 374.66\angle 160.755$$

$$Z_2 = 337.228\angle -36.39^\circ$$