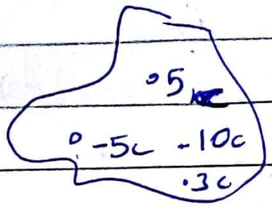
* Gauss's law :- used to find $\bar{E}$ or $\bar{D}$

$$\oint_S \bar{D} \cdot d\bar{s} = Q_{enc} = \psi$$



$\psi = Q_{enclosed} = 13C$

$$\oint_S D \cdot ds = Q$$ for point charge. $Q_{enc} = Q$

$$\oint_S D \cdot ds = \int_L \rho_L \, dl$$

$$\oint_S D \cdot ds = \int_S \rho_s \, ds$$

$$\oint_V D \cdot ds = \int_V \rho_v \, dv$$ $\Longleftrightarrow$ first maxwell's equation in integral form.

Appling divergence theorem.

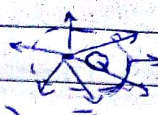
$$\oint_S D \cdot ds = \int \nabla \cdot \bar{D} \, dv = \int \rho_v \, dv$$

$$\boxed{\nabla \cdot \bar{D} = \rho_v}$$ $\Longleftrightarrow$ first maxwell's eq. in diff form / point form.

* Application of Gauss's law.

1. point charge. (find $\bar{E}$ or $\bar{D}$ due to point charge)

$\oint D \cdot ds = Q$



$\bar{D} = D_r \, \hat{a_r}$ (Electric field direction is a_r)

$\bar{ds} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a_r}$

$= \int_0^{2\pi}\int_0^{\pi} D_r \, r^2 \sin\theta \, d\theta \, d\phi = Q$

$= D_r \, r^2 \, 2\pi \, (-\cos\theta)\Big]_0^{\pi} d\phi = Q$

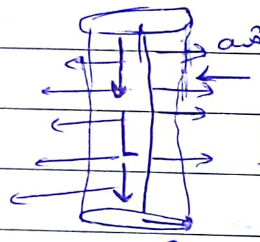
$2\pi r^2 D_r \, (-(-1-1)) = Q$

$4\pi r^2 D_r = Q$ $\qquad D_r = \dfrac{Q}{4\pi r^2}$

$D = \dfrac{Q}{4\pi r^2} \hat{a_r}$ $\qquad E = \dfrac{Q}{4\pi \epsilon_0 R^2}$

② infinite line of charge.

$\oint \bar{D} \cdot ds = \int_L \rho_L \, dl$



$\bar{D} = D_\rho \, a_\rho$

$ds = \rho \, d\phi \, dz \, a_\rho$

$dL = dz$

$\oint = \int_0^{L}\int_0^{2\pi} D_\rho \, \rho \, d\phi \, dz = \int_0^{L} \rho_L \, dz$

$D_\rho \, \rho \, (2\pi) \cancel{L} = \rho_L (\cancel{L})$

$D_\rho = \dfrac{\rho_L}{2\pi \rho}$

$\boxed{\bar{D} = \dfrac{\rho_l}{2\pi\rho} a_\rho}$

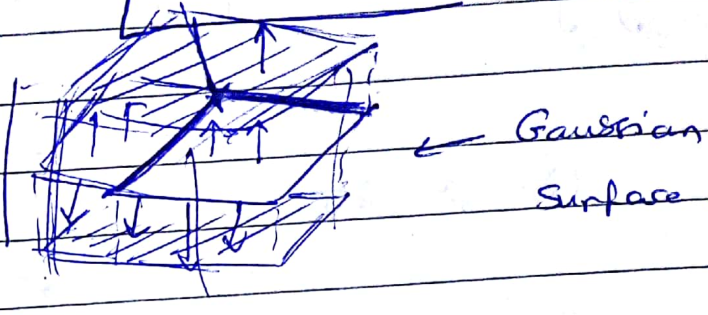
$\boxed{E = \dfrac{\rho_L}{2\pi\epsilon_0 \rho} a_\rho}$

③ infinite sheet of charge.

$$E = \frac{\rho_s}{2\epsilon_0}\,\hat{a_n}$$

$$D = \frac{\rho}{2}\,\hat{a_n}$$

$D.ds = \int_s \rho_s\, ds$



$\bar{D} = D_z\, a_z \quad , Z>0$

$D_z\,(-a_z) \quad Z<0$

$\bar{ds} = \begin{cases} dx\, dy\, a_z & , Z>0 \\ dx\, dy\, -a_z & Z<0 \end{cases}$

dot product.

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} D_z\, dx\, dy + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} D_z\, dx\, dy = \int\int_{-\infty}^{\infty} \rho_s\, dx\, dy.$$

$D_z + D_z = \rho_s$

$2D_z = \rho_s$ $\quad \boxed{\bar{D} = \frac{\rho_s}{2}\, a_z}$ $\quad \boxed{E = \frac{\rho_s}{2\epsilon_0}\, a_z}$

* * * * * * * * *

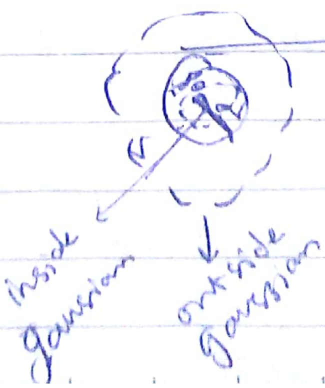
④ Uniform Voluome Charg distribution:-

Find $\bar{E}$, $\bar{D}$ Everywhere

for a sphere of radius a has a $\rho_v = \begin{cases} \rho_0 , & r<a \\ 0, & r>a \end{cases}$

find $\bar{D}, E$

$\oint \bar{D} \cdot ds = \int_v \rho_v\, dv$



$0 \le r \le a$

$0 \le \theta \le \pi$

$0 \le \phi \le 2\pi$

For $r < a$ $\qquad \bar{D} = D_r \hat{a_r}$

$ds = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a_r}$

$$\int_0^{2\pi}\int_0^{\pi} D_r \, r^2 \sin\theta \, d\theta \, d\phi = \int_0^{2\pi}\int_0^{\pi}\int_0^{r} \rho_0 \, r^2 \sin\theta \, dr \, d\theta \, d\phi$$

مساحة $\boxed{4\pi r^2} D_r = \boxed{\rho_0 \frac{r^3}{3} \cdot 2\pi \cdot (2)}$ ج كرة

$$\boxed{\bar{D} = \frac{\rho_0 r}{3} \hat{a_r}} \qquad \boxed{a \geq r}$$

$$\boxed{E = \frac{\rho_0 r}{3\epsilon_0} \hat{a_r}} \qquad \boxed{a \geq r}$$

For $r > a$

$$\oint_s D \cdot ds = \int_v \rho_v \, dv$$

$$= 4\pi r^2 D_r = \int_0^{2\pi}\int_0^{\pi}\int_0^{a} \rho_0 \, r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$4\pi r^2 D_r = \int_0^{2\pi}\int_0^{\pi}\int_a^{r} 0 \cdot r^2 \sin\theta \, dr \, d\theta \, d\phi = 0$$

$$= \boxed{4\pi r^2} D_r = \rho_0 \boxed{\frac{a^3}{3} 4\pi}$$ حجم

مساحة

$$\bar{D} = \frac{\rho_0 a^3}{3r^2} \hat{a_r} \qquad \boxed{a \leq r}$$

$$E = \frac{\rho_0 a^3}{3\epsilon_0 r^2} \hat{a_r} \qquad \boxed{a \leq r}$$

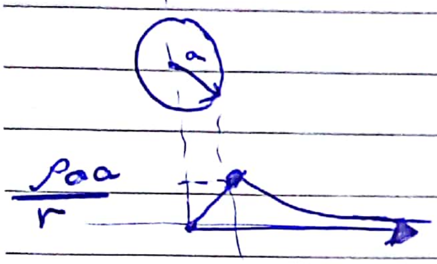
Five Apple

* For a uniform ~~shape~~ sphere of radius (a)

has $\rho_v = \begin{cases} \rho_0 & r \leq a \\ 0 & r > a \end{cases}$

$\bar{D} = \frac{\rho_0 r}{3} \hat{a_r} \quad , 0 \leq r \leq a$

$\bar{D} = \frac{\rho_0 a^3}{3r^2} \hat{a_r} \quad , r \geq a$



$\rho v = \begin{cases} \frac{\rho_0 r}{R} & , 0 \leq r \leq R \\ & , r > R \end{cases}$

$\oint D \cdot ds = \int_V \rho v \, dv$

$= \iiint \frac{\rho_0 r}{R} * r^2 * \sin\theta \, dr \, d\theta \, d\phi$

$= \frac{\rho_0}{R} \frac{r^4}{4} (2\pi)(2)$

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★ Ex:

Given $\bar{D} = z\rho\cos^2\phi\, \hat{a}_z$ C/m^2

Calculate (a) the charge density at $(1, \frac{\pi}{4}, 3)$

(b) the total charge enclosed by the cylinder

(c) of Radius 1m with $-2 \le z \le 2m$

(a) →

$\rho_s = D \cdot \hat{a}_n$?

$\rho_v = \nabla \cdot \bar{D} \Leftarrow = \frac{\partial D_z}{\partial z} = \rho\cos^2\phi$ C/m^3

$\rho_v(1, \frac{\pi}{4}, 3) \Rightarrow 1 \times \frac{1}{2} = \boxed{\frac{1}{2}\ C/m^3}$

(b) $Q = \oint_s \bar{D} \cdot \bar{ds}$

$Q = \int_v \rho_v\, dv$

$= \int_{-2}^{2}\int_{0}^{2\pi}\int_{0}^{1} \rho^2\cos^2\phi\ \cancel{\rho}\, d\rho\, d\phi\, dz$

$\frac{1}{3}(4)\int_{0}^{2\pi} \frac{1}{2} + \frac{1}{2}\cos 2\phi\, d\phi$

$= \frac{1}{3}(4)\left(\frac{1}{\cancel{2}}\right)\left[(\cancel{2}\pi) + 0\right]$

$= \boxed{\frac{4\pi}{3}}\ C$

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$$Q = \oint D \cdot ds$$

$$Q = \iint_S D \cdot ds_{bot.} + \int_S \bar{D} \cdot \bar{ds}_{top} + \int_S \cancel{D \cdot ds_{side}}$$

0, $\otimes$ $a_z \cdot a_\rho$

$$Q = \int_0^{2\pi}\int_0^1 z \rho^2 \cos^2\phi \, d\rho \, d\phi + \int_0^{2\pi}\int_0^1 -z \rho^2 \cos^2\phi \, d\rho \, d\phi$$

$z = 2$

$$= \boxed{\frac{2\pi}{3} + \frac{2\pi}{3} = \frac{4\pi}{3} \, C}$$

* * * *

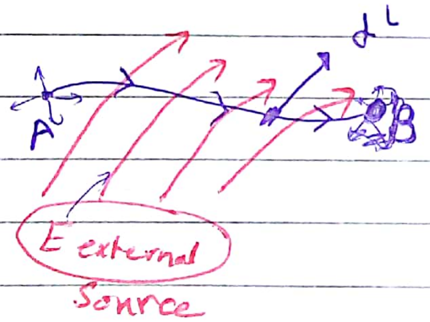
Electric Potential (V) (scalar)

(another way to find $\bar{E}$ or $\bar{D}$)

1- coloumb, 2- Gauss → You need a charge

3- potential → you need voltage.

To move a point charge from point (A) to point (B)



W ≡ work joul (N.m)

$$W = F L$$

$$W = -\int_L \bar{F} \cdot d\bar{L}$$

→ The work is done by an Eext (external field)

$$\frac{W}{Q} = \int_L -E \cdot dL$$

$$V_{AB} = V_B - V_A = \int_L -\bar{E} \cdot d\bar{L}$$

~~V_{AB}~~ ↳ $V_{AB} = EL$

$$\boxed{E = \frac{V_{AB}}{L} = \frac{V}{m}}$$

if V_{AB} is positive ⇒

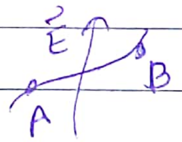
- ↳ gain in potential.
- ↳ $\int_L \bar{E} \cdot d\bar{L}$ is $-ve$.
- ↳ The work is done by the external field.

if V_{AB} is negative

- ↳ drop in potential
- ↳ $\int_L \bar{E} \cdot d\bar{L}$ is positive
- ↳ ~~the~~ work is done by the field itself.

* * * * * *

For a point charge



$$\boxed{V_B - V_A = \frac{Q}{4\pi\epsilon_0 r_B} - \frac{Q}{4\pi\epsilon_0 r_A}}$$

$$\bar{E} = \frac{Q}{4\pi\epsilon_0 R^2} \hat{a_r}$$

$$V_{AB} = -\int_L E \cdot dL \Rightarrow -\int \frac{Q}{4\pi\epsilon_0 r^2} \cancel{\hat{a_r}} \, (dr \cancel{\hat{a_r}})$$

$$= \frac{-Q}{4\pi\epsilon_0} \int \frac{dr}{r^2} = \frac{-Q}{4\pi\epsilon_0} \cdot \left(\frac{-1}{R}\right)$$

if point A is at ~~infinity~~ ∞

$r_A = \infty$

$$V_{AB} = V_{\infty B} = V_B - V_\infty = \boxed{\frac{Q}{4\pi\epsilon_0 r_B}}$$

$V_\infty = 0$ $\boxed{\text{will be taken as Ref}}$

location   potential

* if Ref at ∞ * $\boxed{V = \dfrac{Q}{4\pi\epsilon_0 r}}$

field - source

*** For N-charges**

$$V = \frac{1}{4\pi\epsilon_0} \sum_{k=1}^{N} \frac{Q_k}{|\bar{r} - \bar{r}_k|}$$

field

✓ **For Line Charge:-** $\displaystyle\int_L \frac{\rho_L \, dl}{4\pi\epsilon_0 r}$

✓ **For surface charge:** $\displaystyle\int \frac{\rho_s \, ds}{4\pi\epsilon_0 r}$

✓ **For volume charge** $\displaystyle V = \int \frac{\rho_v \, dv}{4\pi\epsilon_0 r}$

volume

Example two point charges $-4\mu C$ / $5\mu C$ are located

sources $(2,-1,3)$, $(0,4,-2)$ find the potential at

$A(1,0,1)$ (Assuming Zero potential at ∞)

field.

$r_1 = |(-1,1,-2)| = \sqrt{6}$

$r_2 = |(1,-4,3)| = \sqrt{26}$

$V = V_1 + V_2$.

$$= \frac{Q}{4\pi\epsilon_0 r_1} + \frac{Q}{4\pi\epsilon_0 r_2}$$

$$V_{\infty A} = \frac{-4 \times 10^{-6}}{4\pi \times \frac{10^{-9}}{36\pi} \times \sqrt{6}} + \frac{5 \times 10^{-6}}{4\pi \times \frac{10^{-9}}{36\pi} \times \sqrt{26}} =$$

$V_{\infty A} = -5.872\ kV = V_A - V_\infty$.

$\boxed{V_A = -5.872\ kV}$ Drop in potential.

* * * * * * *

Example... A point charge of $5nc$ located at $(-3,4,0)$

and aline $y=1$, $z=1$ has $\rho_L = 2nc/m$

@ find V at $A(5,0,1)$ if V at $O(0,0,0)$ is $0V$

$V = V_Q + V_L$

find $V_{OA} = V_A - V_O = V_A$

field - source

$r_A = |(5,0,1) - (-3,4,0)|$

$r_A = 9$

$r_O = |(0,0,0) - (-3,4,0)| =$

$$V_{OA_Q} = \int \frac{Q}{4\pi\epsilon_0 r^2}\, dr$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_O}\right)$$

$$V_{OA} = \frac{5 * 10^{-9}}{4\pi * \frac{10^{-9}}{36\pi} * \cancel{8}} \left(\frac{1}{9} - \frac{1}{5}\right)$$

$$= 45 \left(\frac{1}{9} - \frac{1}{5}\right) V = -4\ V$$

$$V_{OA_L} = -\int \frac{\rho_L}{2\pi\epsilon_0 \rho} \hat{a_\rho} \cdot d\vec{\rho}\ \hat{a_\rho}$$

$$V_{OA_L} = \frac{-\rho_l}{2\pi\epsilon_0} \int_{\rho_0}^{\rho_A} \frac{d\rho}{\rho} \qquad V_{OA} = \frac{-\rho_L}{2\pi\epsilon_0} \ln \rho \Big|_{\rho_0}^{\rho_A}$$

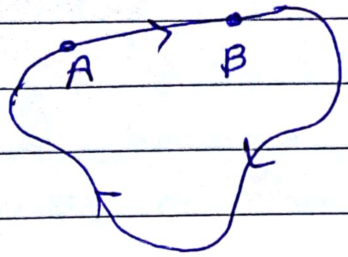
$$V_{OA} = \frac{-\rho_L}{2\pi\epsilon_0} \ln \frac{\rho_A}{\rho_0}\ V$$

$$\rho_A = |(5,0,1) - (x,1,1)| \quad \boxed{x = 5} \quad (0,-1,0) = \boxed{1}$$

$$\rho_0 = |(0,0,0) - (0,1,1)| = \sqrt{2}$$

$$V_{OA} = V_A = 8.477\ V$$

* Moving a charge on a closed path.



$$V = -\oint \bar{E}\cdot\bar{dl} = -\int_A^B \bar{E}\cdot\bar{dl} + -\int_B^A \bar{E}\cdot\bar{dl}$$

$$= \int_A^B \bar{E}\cdot\bar{dl} + \int_A^B \bar{E}\cdot\bar{dl} = 0$$

$\oint E\cdot dl = 0$ "2nd maxwells equ. in integral form.

equipotential surface

by applying stock's theorem.

$$\oint_l \bar{E}\cdot dl = \int \nabla\times\bar{E}\cdot \bar{ds} = 0$$

$\nabla\times\bar{E} = 0$ // second maxwell's eq. in differential form

* any Vector satisfy the Relation

$\nabla\times\bar{E} = 0$

Vector is called $\rightarrow$ irrotational (conservative).

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How to find $\bar{E}$ from (V)

$$V = -\int_L E \cdot dL$$

$$dV = -E \cdot dL.$$

if $E = E_x a_x + E_y a_y + E_z a_z$

$dL = dx\, a_x + dy\, a_y + dz\, a_z$

$E \cdot dL = E_x dx + E_y dy + E_z dz.$

$$dV = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz$$

by equating both sides.

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

$$E = -\left(\frac{\partial V}{\partial x} a_x + \frac{\partial V}{\partial y} a_y + \frac{\partial V}{\partial z} a_z \right)$$

$$\boxed{E = -\nabla V}$$

the direction of ~~the~~ $\bar{E}$ is the same direction with the decreasing of (V)

$\nabla \times \bar{E} = 0$

$\boxed{\nabla \times (-\nabla V) = 0}$. Always ✓

Example Given $V = \frac{10}{r^2} \sin\theta \cos\phi$

Find

(a) $\bar{D}$ at $(2, \pi/2, 0°)$

(b) the work done in moving a $10\,\mu c$ charge from A $(1, 30°, 120°)$ to B $(4, 90°, 60°)$

$$\bar{D} = \epsilon * \bar{E} = \epsilon_0 \cdot -\nabla V$$

$$E = -\left(\frac{\partial V}{\partial r}\hat{a_r} + \frac{\partial V}{r\,d\phi}a_\theta + \frac{\partial V}{r\sin\theta}\hat{a_\phi}\right)$$

$$= -\left(\frac{-20}{r^3}\sin\theta\cos\phi\,\hat{a_r} + \frac{10}{r^3}\cos\theta\cos\phi\,a_\theta + \frac{-10\sin\phi}{r^3}\hat{a_\phi}\right)$$

$$\bar{D} = \epsilon_0 \times E$$

$$\bar{D}\Big|_{(2,\pi/2,0)} = -\epsilon_0\left(\frac{-20}{8}\hat{a_r}\right)$$

$$\boxed{\bar{D} = 22.1\ \hat{a_r}\ \ pc/m^2}$$

* * * *

$$W = -Q\int_L E\cdot dl = Q\,V_{AB} = -\int_L F\cdot dl$$

$$W = Q\,V_{AB} = Q * V_B - V_A$$

$$= 10 * 10^{-6} * \left(\cancel{\frac{10}{16}}\ \cancel{\frac{11}{2}} \frac{5}{16} + \frac{10}{4}\right)$$

$$= W = 28.125\ \mu J$$

Example Given $V = \frac{10}{r^2} \sin\theta \cos\phi$

Find

ⓐ $\bar{D}$ at $(2, \pi/2, 0°)$

ⓑ the work done in moving a $10\mu C$ charge from ~~A~~ $A(1, 30°, 120°)$ to $B(4, 90°, 60°)$

$$\bar{D} = \epsilon * \bar{E} = \epsilon \cdot -\nabla V$$

$$E = -\left(\frac{\partial V}{\partial r} \hat{a_r} + \frac{\partial V}{r\partial\phi} \hat{a_\theta} + \frac{\partial V}{r\sin\theta} \hat{a_\phi}\right)$$

$$= -\left(\frac{-20}{r^3} \sin\theta \cos\phi \hat{a_r} + \frac{10}{r^3} \cos\theta \cos\phi \hat{a_\theta} + \frac{-10 \sin\phi}{r^3} \hat{a_\phi}\right)$$

$$\bar{D} = \epsilon_0 \times E$$

$$\bar{D}\Big|_{(2, \pi/2, 0)} = -\epsilon_0 \left(\frac{-20}{8} \hat{a_r}\right)$$

$$\boxed{\bar{D} = 22.1 \ \hat{a_r} \ pc/m^2}$$

* * * *

$$W = -Q \oint_L E \cdot dl = Q V_{AB} = \oint_L F \cdot dl$$

$$W = Q V_{AB} = Q * V_B - V_A$$

$$= 10 * 10^{-6} * \left(\cancel{\frac{10}{16}} \cancel{\frac{1}{2}} \frac{5}{16} + \frac{10}{4}\right)$$

$$= W = 28.125 \ \mu J$$

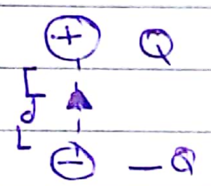
$W = -Q \int_L E \cdot dL$

$= -10^{-5} \left(\int E_r \, dr + \int E_\theta \, r \, d\theta + \int E_\phi \, r \sin\theta \, d\phi \right)$

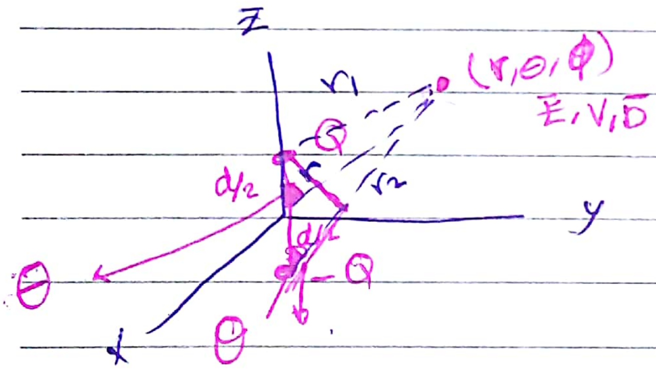
$= -10^{-5} \left(\int_1^4 \frac{20}{r^3} \sin\theta \cos\phi + \int_{30^\circ}^{90^\circ} \frac{10}{r^2} \cos\theta \cos\phi \, d\theta + \int \right.$

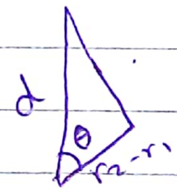
$\theta = 30^\circ$, $\phi = 120^\circ$ $\qquad$ $r = 4$, $\phi = 120^\circ$ $\qquad$ $r = 4$, $\theta = 90^\circ$

* Electric Dipoles



how to find E, $\bar{D}$, V





$\boxed{\cos\theta = \frac{r_2 - r_1}{d}}$

$V = V_+ + V_-$

$V = \frac{Q}{4\pi\epsilon_0 r_1} + \frac{-Q}{4\pi\epsilon_0 r_2} = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$

$\boxed{V = \frac{Q \, (r_2 - r_1)}{4\pi\epsilon_0 r_1 r_2}} \; V \longrightarrow (1)$

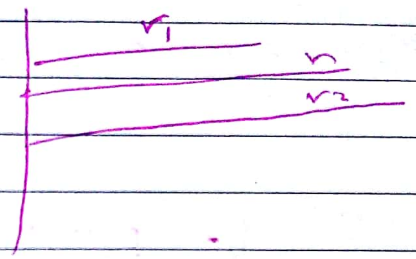
Five Apple

Q is unknown.

$\left.\begin{matrix} r_1 \\ r_2 \end{matrix}\right\}$ is unknown.

$(r_1, r_2) \longrightarrow r$ $\qquad r =$ field $-$ center of Dipole.

$$r_2 - r_1 = d\cos\theta$$



$$r_1 r_2 \approx r^2$$

$$\boxed{V = \frac{Q\, d\cos\theta}{4\pi\epsilon_0 r^2}} \longleftarrow (2)$$

~~define~~

Define $\bar{p} = Q\bar{d}$ $\quad$ C.m $\qquad p = Qd$

Dipole moment

$$\boxed{V = \frac{p\cos\theta}{4\pi\epsilon_0 r^2}} \longleftarrow (3)$$

$$\bar{p}\cdot\hat{a_r} = p\,(1)\cos\theta).$$

$$V = \frac{\bar{p}\cdot\hat{a_r}}{4\pi\epsilon_0 r^2}$$

$$V = \frac{\bar{p}\cdot(\bar{r} - \bar{r}')}{4\pi\epsilon_0\, |\bar{r} - \bar{r}'|^3}$$ for dipole centered at $(\bar{r}')$

for N dipoles

$$V = \frac{1}{4\pi\epsilon_0}\sum_{K=1}^{N} \frac{\bar{p}_k\cdot(\bar{r} - \bar{r}_k)}{(\bar{r} - \bar{r}_k)^3}$$

$$E = E_+ + E_-$$

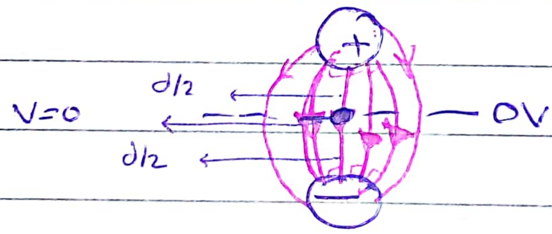
$$E = -\nabla V$$

$$E = \left(\frac{-\partial V}{\partial r}\hat{a}_r + \frac{-\partial V}{r\,d\theta}\hat{a}_\theta + \frac{-\overset{0}{\cancel{\partial V}}}{r\sin\theta\,\partial r}\hat{a}_\phi \right)$$

eq ③ for V ↓

$$E = -\left(\frac{-2p\cos\theta}{4\pi\epsilon_0 r^3} + \frac{-p\sin\theta}{4\pi\epsilon_0 r^3}\hat{a}_\theta + 0 \right)$$

$$\boxed{E = \frac{p}{4\pi\epsilon_0 r^3}\left(2\cos\theta\,\hat{a}_r + \sin\theta\,\hat{a}_\theta\right) \quad V/m}$$



⊕ Example :- Two dipoles with dipole moments $-5\hat{a}_z$ nC.m and $9\hat{a}_z$ nC.m are located at (0,0,-2) and (0,0,3). Find the potential at the origin.

$$V = \frac{1}{4\pi\epsilon_0}\sum_{k=1}^{2}\frac{\bar{P}_k\cdot(\bar{r}-\bar{r}_k)}{(r-r_k)^3}$$

$$V = \frac{P_1(r-\bar{r}_1)}{4\pi\epsilon_0|\bar{r}-\bar{r}_1|^3} + \frac{P_2(\bar{r}-\bar{r}_2)}{4\pi\epsilon_0|r-r_2|^3}$$

$\bar{r}-\bar{r}_1 = (0,0,0)-(0,0,-2) = 2\hat{a}_z$

$\bar{r}-\bar{r}_2 = (0,0,0)-(0,0,3) = -3\hat{a}_z$

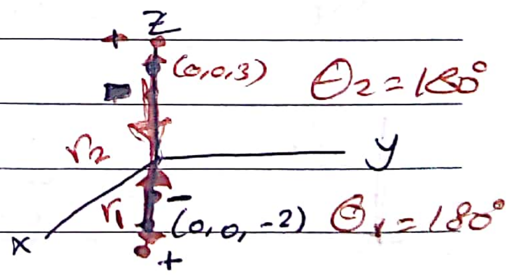
$$V = \frac{-5\times10^{-9}(2)}{4\pi\times\frac{10^{-9}}{36\pi}\times(8)} + \frac{9\times10^{-9}(-3)}{4\pi\times\frac{10^{-9}}{36\pi}(27)} = -20.25\,V$$

* * * *

$E = E_1 + E_2$

$$E_1 = \frac{P}{4\pi\epsilon_0 r^3}\left(2\cos\theta_1\,\hat{a}_r + \sin\theta_1\,\hat{a}_\theta\right)\ V/m$$

$$E_1 = \frac{5\times10^{-9}}{4\pi\epsilon_0(8)}$$



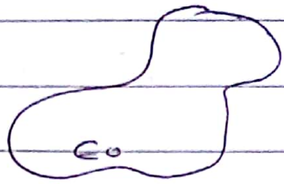
Energy Density in Electrostatic field

W_E = Electric energy.

Q_1

$Q_2 \Rightarrow$ moving $Q_1 \to Q_2 \to Q_3$

Q_3



Space - Volume

$$W_E = W_1 + W_2 + W_3.$$

~~now~~ $$= Q_1(0) + Q_2(V_{21}) + Q_3(V_{31} + V_{32}) \longrightarrow (1)$$

Moving $Q_3 \to Q_2 \to Q_1$

$$W_E = W_1 + W_2 + W_3$$

$$W_E = Q_1(V_{12} + V_{13}) + Q_2 V_{23} + Q_3(0) \longrightarrow (2)$$

(1) + (2)

$$2W_E = Q_1\overbrace{(V_{12} + V_{13})}^{V_1} + Q_2\overbrace{(V_{21} + V_{23})}^{V_2} + Q\overbrace{(V_{31} + V_{32})}^{V_3}$$

$$W_E = \frac{1}{2} Q_1 V_1 + \frac{1}{2} Q_2 V_2 + \frac{1}{2} Q_3 V_3$$

$$\boxed{W_E = \frac{1}{2} \sum_{k=1}^{N} Q_k V_k \ (J)}$$

→ اذا عارفة شكل ال Volume

For line charge

$$\boxed{W_E = \frac{1}{2} \int_L \rho_L V \cdot dl}$$

for surface charge.

$$\boxed{W_E = \frac{1}{2}\int_S \rho_s V \, ds}$$

for volume charge

$$\boxed{W_E = \frac{1}{2}\int_V \rho_v V \, dv}$$

to find W_E Energy from ~~the~~ $\bar{E}$

from $W_E = \frac{1}{2}\int_V \rho_v V \, dv$

$\rho_v = \nabla\cdot\bar{D}$

$$W_E = \frac{1}{2}\int_V (\nabla\cdot\bar{D}) V \, dv$$

$$(\nabla\cdot\bar{D}) V = V(\nabla\cdot\bar{D}) + \bar{D}\cdot\nabla V$$

$\nabla\cdot(V\bar{D})$

$$W_E = \frac{1}{2}\int (\nabla\cdot\bar{D}) V \, dv - \int \frac{1}{2}\bar{D}\cdot\nabla V \, dv$$

$\Uparrow$

applying div theorem

0 as $r\to\infty$

$$W_E = \frac{1}{2}\oint_S V\bar{D}\cdot d\bar{s} - \frac{1}{2}\int_V \bar{D}\cdot\nabla V \, dv$$

$\bar{D} \propto \frac{1}{r^2}$ $\quad$ dr^2

$V \propto \frac{1}{R}$

$$W_E = -\frac{1}{2}\int_V \bar{D}\circ \underbrace{\nabla V}_{-E}\; dV$$

$$W_E = \frac{1}{2}\int_V D\circ E\, dv = \boxed{\frac{1}{2\epsilon_0}\int_V D^2\, dv}$$

$$\boxed{W_E = \frac{1}{2}\int_V \epsilon_0 E^2\, dv}$$

إذا عطى ال Volume

* * * *

Energy Density $\boxed{\text{joul}/m^3}$

$$w_E = \frac{W_E}{\text{Volume}}$$

$$\boxed{w_E = \frac{1}{2}\epsilon_0 E^2} = \frac{1}{2\epsilon_0} D^2 = \frac{1}{2}\bar{E}\cdot\bar{D}.$$

$$W_E = \int w_E\, dv \quad (\text{jowl}).$$

* * * *

Example :- the point charge $-1nc$, $4nc$ and $3nc$ are located at $(0,0,0)$, $(0,0,1)$ and $(1,0,0)$ find the Energy in the system.

$$W_E = \frac{1}{2} \sum_{k=1}^{3} Q_k V_k$$

$$= \frac{1}{2} Q_1 V_1 + Q_2 V_2 + Q_3 V_3.$$

$$= \frac{1}{2} \left[Q_1 (V_{12} + V_{13}) + Q_2 (V_{21} + V_{23}) + Q_3 (V_{31} + V_{32}) \right]$$

$$V_{12} = \frac{Q_2}{4\pi\epsilon_0 (1)} \qquad V_{23} = \frac{Q_3}{4\pi\epsilon_0 (\sqrt{2})}$$

$$V_{13} = \frac{Q_3}{4\pi\epsilon_0 (1)} \qquad V_{31} = \frac{Q_1}{4\pi\epsilon_0 (1)}$$

$$V_{32} = \frac{Q_2}{4\pi\epsilon_0 \sqrt{2}}$$

$$V_{21} = \frac{Q_1}{4\pi\epsilon_0 (1)}$$

$$W_E = \frac{1}{2} \left(\frac{1}{4\pi\epsilon_0} \right) \left(2 Q_1 Q_2 + 2 Q_1 Q_3 + \frac{2 Q_2 Q_3}{\sqrt{2}} \right) = 13.37 \text{ n}$$

* * * *

"Chapter 5" Electric field in Materials.

- Material classification Based on its electrical properties

→ Conductanors. (Metals) $\sigma >>> 1$ Cu, AL, Ag, Au

→ Semi-conductors: Si, Ge, Ga

→ Dielectrics (insulators) $0 < \sigma <<< 1$

$\sigma \equiv$ Conductivity S/m $= (\Omega m)^{-1}$

~~Conduct~~

σ depends on 1- Temprature (°K)

2- frequency. (HZ)

$T \uparrow \quad \sigma \downarrow$

σ_{Cu} 4°K $= 10 * 10^{20}$ S/m

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