"Chapter 5" Electric field in Materials.

- Material classification Based on its electrical p

- Conductanors. (Metals) $\sigma >>> 1$ Cu, AL
- semi-conductors: Si, Ge, Ga
- Dielectrics (Insulators) $0 < \sigma <<< 1$

$\sigma \equiv$ conductivity S/m $= (\Omega m)^{-1}$

~~Conduct~~

σ depends on 1- Temprature (°K)

2- frequency. (HZ)

T $\uparrow$ $\sigma \downarrow$

σ_{Cu} 4°k $= 10 * 10^{20}$ S/m

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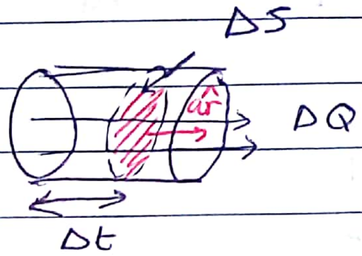
* Type of currents :-

① conduction Current
② Convection current
③ Displacement current $\rightarrow$ (CH. 9)



* $I = \Delta q / \Delta t$

$I = \dfrac{dq}{dt}$ (non-uniform)

$\vec{J} = \dfrac{I}{\Delta S}$ (current density) A/m^2

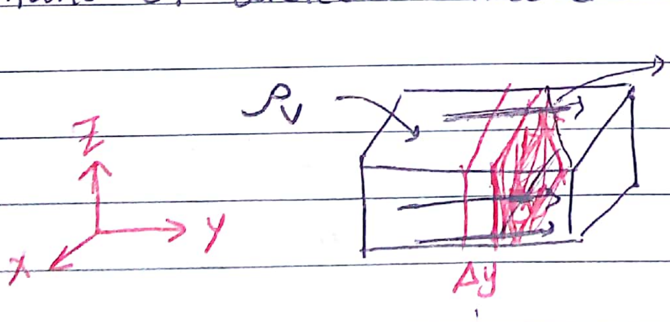
$I = \bar{J} \Delta S$

$\bar{I} = \int \bar{J} \cdot d\bar{s}$ $\leftarrow$ For non Uniform

$\psi = \int_S \bar{D} \cdot d\bar{s}$

* a Flimant of dielectric materail

$\vec{u}$ (velocity).



~~ΔS = ΔX Δy~~

$\Delta S = \Delta X \Delta Z$

$\Delta V = \Delta S \Delta y$

$\vec{u} = u_y \hat{a_y}$

$I = \dfrac{\Delta Q}{\Delta t}$ $\quad Q = \int \rho_v \, dv = \rho_v \Delta V$

$$I = \frac{\Delta Q}{\Delta t} = \frac{\rho_v \Delta V}{\Delta t} = \frac{\rho_v \Delta S \Delta y}{\Delta t} \vec{u}_y$$

$$I = \rho_v u_y \Delta S \quad (A)$$

$$j = \frac{I}{\Delta S} = \rho_v u_y$$

$$\boxed{\bar{J} = \rho_v \vec{u}}$$

Convection current dencity.

Relation between $(\bar{J})$ and $(\bar{E})$.

The force on (e) $q = -e$)

$$F = q\bar{E} = -e\bar{E} = m\bar{a}$$

$$\bar{a} = \frac{\bar{u}}{\tau}$$

τ = time between collisions. (s).

$$-eE = \frac{m\bar{u}}{\tau} \qquad \bar{u} = \frac{-e\tau}{m}\bar{E}$$

$$\bar{u} = \mu \bar{E}$$

μ = mobility $\left(\frac{cs}{kg}\right)$

$\bar{u} = V_d$ = drift velocity.

- for n-electrons

$\rho_v = -ne$

$\bar{J} = \rho_v \, \bar{u}$

$J = \rho_v \left(\frac{-e\tau}{m}\right)\bar{E}$

$J = \underbrace{-ne * \left(\frac{-e\tau}{m}\right)}_{\text{constant } (\sigma)}\bar{E}$

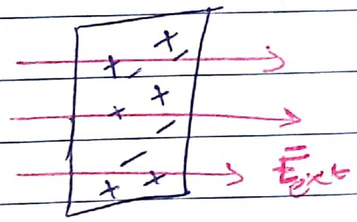
$\bar{J} = \sigma E \quad (A/m^2)$

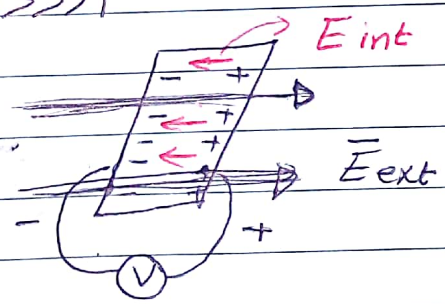
$$\boxed{\frac{\sigma}{\mu} = \rho_v}$$

↳ ~~pon~~ point form of ohm's law

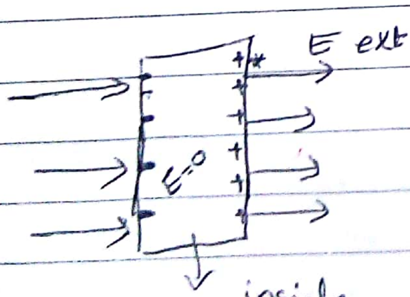
* Conductors:-

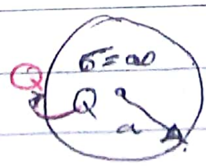
$\sigma >>>>> 1$





$E = -\nabla V$





E inside = 0

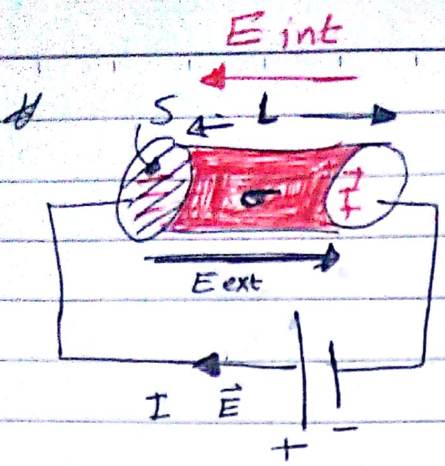
inside

E=0

D=0

Q=0

(equipotential surface)



$$R = \frac{V}{I} \Rightarrow \frac{L}{\sigma S} = \frac{\rho L}{S}$$ (uniform object)

$$R = \frac{V}{I} = \frac{\int_L -\bar{E}\cdot dL}{\int \sigma \bar{E}\cdot \bar{ds}}$$ ((for nonuniform object))

$$R = \frac{\cancel{E} L}{\sigma \cancel{E} S} \qquad \boxed{R = \frac{L}{\sigma S} = \frac{\rho L}{S}}$$

* power = $\frac{\text{Work}}{\text{Time}}$ (watt) (J/s).

$$P = \frac{W}{t} = \frac{F\cdot L}{t} = \frac{Q E\cdot L}{t}$$

$$P = Q\bar{E}\cdot u \Rightarrow P = \int_V \rho_V \bar{E}\cdot\bar{u}\, dv = \boxed{\bar{J} = \rho_V \bar{u}}$$

$$P = \int_V J\cdot\bar{E}\, dv = \int_V \sigma (E)^2\, dV$$

joul's Law

$P = IV = I^2 R = \frac{V^2}{R} = \frac{I^2}{G} = GV^2$

For Uniform Object

~~Uniform obj.~~

$dv = ds\, dL$

$P = \int_L \int_S \vec{E}\cdot\vec{J}\, ds\, dl$

$P = \int_L \vec{E}\cdot dl \int_S \vec{J}\, ds$

$\boxed{P = VI}$

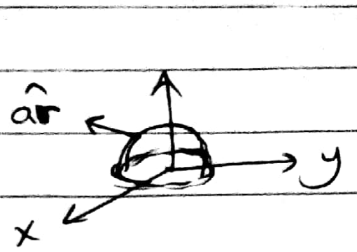
Ex if $J = \frac{1}{r^2}\left(2\cos\theta\, \hat{a}_r + \sin\theta\, \hat{a}_\theta\right)$ A/m^2

Find the current passing through.

(a) A hemi-spherical shell of Radius 20 cm with $0 \le \theta \le \pi/2$, $0 \le \phi \le 2\pi$

(b) A spherical shell of Radius 10 cm.

$$I = \int_S J \cdot ds$$



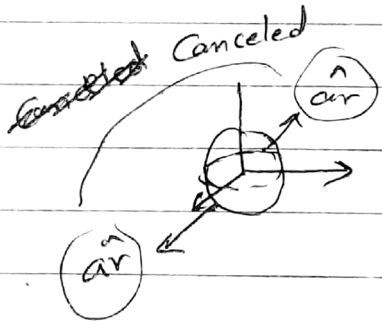
$$ds = r^2 \sin\theta \, d\theta \, d\phi \ \hat{ar}$$

$$I \int_0^{2\pi}\int_0^{\pi/2} \frac{1}{r} 2\sin\theta\cos\theta \, d\theta \, d\phi \Big|_{\boxed{r = .2 \text{ m}}}$$

$$I = 5 \ (2\pi) \ \left. -\frac{\cos 2\theta}{2} \right]_0^{\pi/2}$$

$$= 10\pi \left(-\tfrac{1}{2}\right) (-1-1) \qquad = \underline{10\pi \text{ A}}$$

(b)



$$I = \int_S J . ds =$$

$$I = \int_0^{2\pi}\int_0^{\pi} \frac{1}{R} \sin 2\theta \, d\theta \, d\phi \Big|_{R=0.1}$$

$$I = 10 \ (2\pi) \ \left(-\tfrac{1}{2}\right) \left(\overset{0}{\cancel{1}} - 1\right) \quad = 0 \text{A}$$

Method 2 for part (b)

$$I = \oint_S J \cdot ds = \iiint_V \nabla \cdot \bar{J} \, dv$$

just for a closed surfaces

$$\nabla J = \frac{1}{r^2} \frac{d}{dr}(r^2 J_r) + \frac{1}{r\sin\theta} \frac{d}{d\theta}(\sin\theta J_\theta) +$$

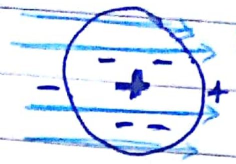
$$\nabla J = 2\cos\theta \left(\frac{-1}{r^2}\right)\left(\frac{1}{r^2}\right) + \frac{1}{r^4 \cancel{\sin\theta}} (2\cancel{\sin\theta}\cos\theta)$$

$$\nabla J = \frac{-2\cos\theta}{r^4} + \frac{2\cos\theta}{r^4} = 0$$

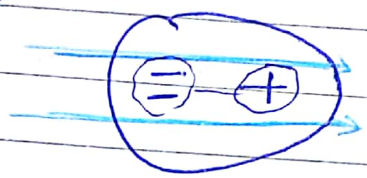
* polarization in Dielectric

(Q)

- free to move (few)
- point charges (a lot)



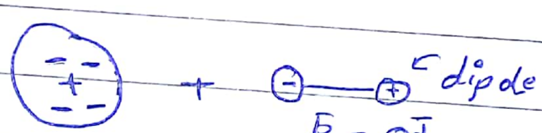
Normal charge distribution



$F_e = Q\,E_{ext}$

Distorted charge distribution

$\equiv$

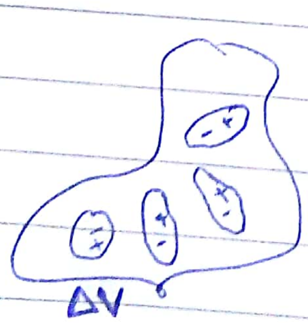


$\bar{P} = Q\bar{d}$

polarized dieclectric. $\bar{P} \neq 0$

* Polaraization ($\bar{P}$)

* other types of Dielectric



$\bar{P} = 0$

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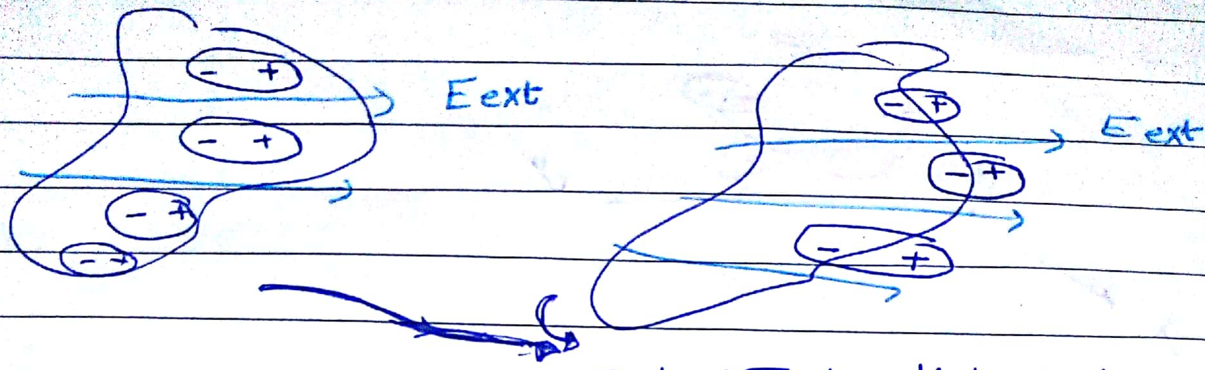


polarized dielectric

Types of Dielectrics

non-polar dielectrics

(dielectric material that have Dipoles after $\bar{E}$ is applied)

O_2, H_2, N_2

polar Dielectrics

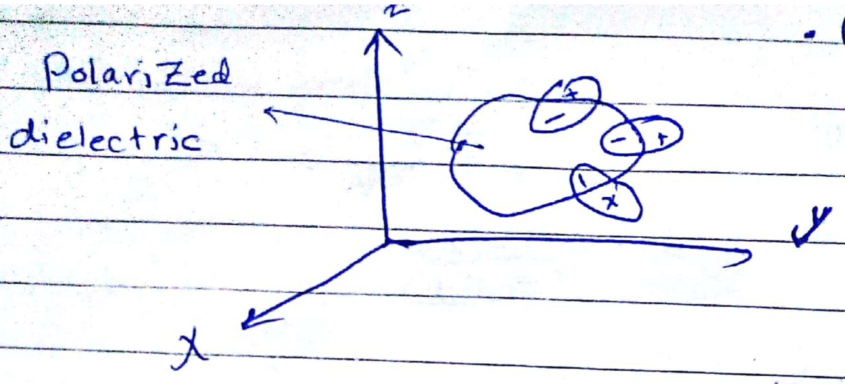
(permenant dipoles) but the dipoles will be arrange in $\bar{E}$ ext -field Direction

dipole moment

$$\underline{P} = \lim_{\Delta V \to 0} \sum_{k=1}^{N} \frac{\bar{p}_k}{\Delta V}$$

$\boxed{C/m^2}$

* How to find (V) and ($\bar{E}$) from polarized Dielectric



$$V = \frac{P \cdot \hat{a_r}}{4\pi\epsilon_0 r^2}$$ ← for one dipole.

$$dV = \frac{\bar{P} \cdot \hat{a_r}}{4\pi\epsilon_0 r^2} dv'$$

$$\bar{r} = (x-x')\hat{a_x} + (y-y')\hat{a_y} + (z-z')\hat{a_z}$$

$$\bar{r}' = (x'-x)\hat{a_r} + (y'-y)\hat{a_y} + (z'-z)\hat{a_z}$$

$$\frac{\hat{a_r}}{r^2} = \frac{\bar{r}}{r^3} = \underset{\partial/\partial r}{-\nabla\left(\frac{1}{r}\right)} = \underset{\partial/\partial r'}{\nabla'\left(\frac{1}{r}\right)}$$

$$\nabla\left(\frac{1}{r}\right) = \frac{-1}{r^2}\hat{a_r}$$

$$\int dV = \int_{v'} \frac{\bar{P} \cdot \nabla'\left(\frac{1}{r}\right)}{4\pi\epsilon_0} dv'$$

$V=$

$$\nabla \cdot (V\bar{A}) = \underbrace{\bar{A} \cdot (\nabla V)} + V(\nabla \cdot \bar{A})$$

$A = \bar{P}$, $V = \frac{1}{r}$, $\nabla = \nabla'$

$$\bar{P} \cdot \nabla'\left(\frac{1}{r}\right) = \nabla' \cdot \left(\frac{1}{r}\bar{P}\right) - \frac{1}{r}(\nabla' \cdot \bar{P})$$

$$V = \int \frac{\vec{\nabla}' \cdot \left(\frac{\bar{P}}{r}\right)}{4\pi\epsilon_0} dv' - \int \frac{\frac{1}{r}(\vec{\nabla}' \cdot \bar{P})}{4\pi\epsilon_0} dv'$$

(1) (2)

Applyind Divergence theorem on (1)

$ds = ds\,\hat{a}_n$

$$V = \oint_{s'} \frac{\frac{\bar{P}}{r} \cdot \bar{ds}}{4\pi\epsilon_0} - \int \frac{(\vec{\nabla} \cdot \bar{P})}{4\pi\epsilon_0 r} dv'$$

$$V = \oint_{s'} \frac{\bar{P} \cdot \hat{a}_n}{4\pi\epsilon_0 r} ds' - \int \frac{\nabla \cdot \bar{P}}{4\pi\epsilon_0 r} dv'$$

Compare with the potential of free charges

$$V = \int_s \frac{\rho_s\, ds}{4\pi\epsilon_0 r} \qquad V = \int_v \frac{\rho_v\, dv}{4\pi\epsilon_0 r}$$

$\boxed{\begin{array}{l} \rho_{ps} = \bar{P} \cdot \hat{a}_n \\ \text{bound (polarized) surface density charge} \\ \rho_s = \bar{D} \cdot \hat{a}_n \end{array}}$

polarized volume charge density

$\boxed{\begin{array}{l} \rho_{pv} = -\nabla \cdot \bar{P} \\ \rho_v = \nabla \cdot \bar{D} \end{array}}$

★ bound Charge

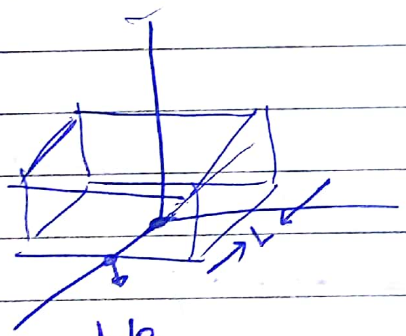
$Q_{b+} = \int \rho_{ps}\, ds = \int \bar{P} \cdot \hat{a}_n\, ds = \int_s \bar{P} \cdot \bar{ds}$.

$Q_{b-} = \int_v \rho_{pv}\, dv = -\int_v \nabla \cdot \bar{P}\, dv$ ..

$Q_{b\ total} = Q_{b+} + Q_{b-} = 0$

★ Example :- A dielectric Cube of length L Centered at the Origin has $\bar{P} = a\bar{r}$ where $\bar{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$ and (a) is constant

Find ρ_{ps}, ρ_{pv}, Q_{b+}, Q_{b-}, $Q_{b\ total}$.



$\rho_{ps} = \bar{P} \cdot \hat{a}_n$

$a(x\hat{a}_x + y\hat{a}_y + z\hat{a}_z) \cdot \hat{a}_n$

$\rho_{ps\ front}\Big|_{x = L/2} = ax = \boxed{\frac{aL}{2}}$ c/m^2

ρ_{ps} right $\Big|_{y=L/2} = a\,y = \boxed{\frac{aL}{2}}\ C/m^2$

ρ_{ps} bot. $\Big|_{z=-L/2} = -a z = \boxed{\frac{aL}{2}}\ C/m^2$

$\boxed{\text{All surfaces has } \rho_{ps} = aL/2 \ C/m^2}$

$\rho_{pv} = -\nabla \cdot \bar{P} \qquad = \qquad \bar{P} = \langle ax, ay, az \rangle$

$= \boxed{-3a \ C/m^3}$

$Q_{b+} = 6 \int_S \rho_{ps}\, ds = 6 \; \frac{aL}{2} * L^2 = \boxed{3aL^3 \ C}$

constants.

$Q_{b-} = \int_V \rho_{pv}\, dv = -3a * L^3 = -3aL^3 \ C$

$Q_{tot} = 0$

The Cube is electrically nutral

- for some dielectrics

$\bar{P} = \chi_e \epsilon_0 \bar{E}$ (linear Dielect

χ_e → electrical susceptability (Unitless)

$$\boxed{\bar{D} = \epsilon_0 \bar{E} + \bar{P} \quad \text{in General}}$$

$\chi_e = \epsilon_r - 1$

ϵ_r = relative permittivity (Dielectric Constant)

$\epsilon = \epsilon_0 \epsilon_r$ → Unitless

ϵ ↓ F/m, ϵ_0 ↓ F/m

$\epsilon_r = \dfrac{\epsilon}{\epsilon_0}$

$$\boxed{\begin{array}{l}\text{for free space} \\ \epsilon = \epsilon_0 \quad \sigma = 0 \\ \epsilon_r = 1\end{array}}$$

$\bar{P} = (\epsilon_r - 1)\, \epsilon_0 \bar{E}$

$\boxed{\text{Conductors} \quad \epsilon_r = 1}$ $\quad \sigma >>> 1$

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$$\bar{D} = \epsilon_0 E + \bar{P}$$

$$\bar{D} = \cancel{\epsilon_0 E} + \epsilon_r \epsilon_0 \bar{E} - \cancel{\epsilon_0 \bar{E}}$$

$$\bar{D} = \epsilon_0 \epsilon_r \bar{E}$$

$$\boxed{\bar{D} = \epsilon \bar{E}}$$