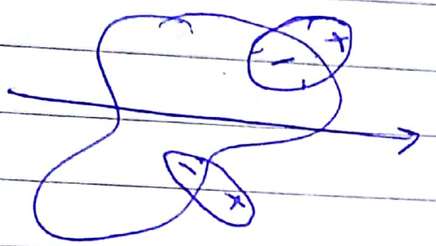
☆ Dielectric Breakdown:
depends on:
1- Temprature
2- humidity
3- Applied $\bar{E}$- field
4- Time of applied E-field

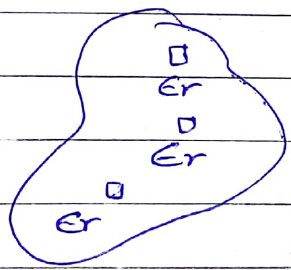
$$P = \epsilon_0(\epsilon_r - 1)E$$



Dielectric strength
The maximum value of Electric field the dielectric can tolerate without being- breaking down by fixing T, humidity, Time.

* Linear Isotropic homogeneous materials:

for non-isotropic materials
$\epsilon = [\epsilon]$



ϵ_r constant everywhere (Homogeneous)

$\epsilon_r = 2(x,y)$
non-homogeneous

$$\bar{D} = \epsilon E$$

$$\bar{D} = \begin{bmatrix} E_{xx} & E_{xy} & E_{xz} \\ E_{yx} & E_{yy} & E_{yz} \\ E_{zx} & E_{zy} & E_{zz} \end{bmatrix} \vec{E}$$

Five Apple

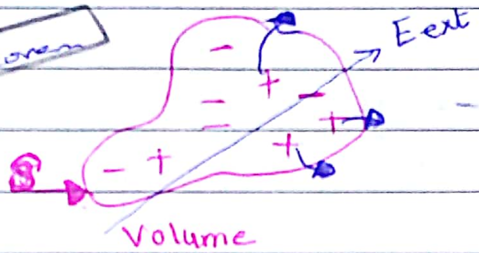
if all elements $= 2\epsilon_0$

$\vec{D} = 2\epsilon_0 \vec{E}$ (isotropic).

Continuity equ. and Relaxation Time.

$I_{in} = I_{out}$

Divergence theorem



$I = \oint_S J \cdot ds$

I_{out} (on surface)

$J = \sigma \vec{E}$

$I_{in} = -\frac{(dq)_{in}}{dt}$

① negative charges
② decreasing.

$I_{in} = I_{out}$

$$\boxed{-\frac{(dq)_{in}}{dt} = \oint J \cdot ds}$$

$$\iiint_V \nabla \cdot J \, dv = \frac{-d}{dt} \iiint \rho_v \, dv$$

$$\boxed{\nabla \cdot J = -\frac{d\rho_v}{dt}} \Rightarrow \text{Continuity Equ}$$

* $\rho_v(t)$?

$$\nabla \cdot J = -\frac{d\rho_v}{dt}$$

$J = \sigma E$ $\quad \nabla D = \rho_v$

$$\nabla \cdot E = \frac{\rho_v}{\epsilon} \Rightarrow \text{If } \epsilon \text{ is homogeneous.}$$

$$\nabla \cdot J = \sigma \frac{\rho_v}{\epsilon}$$

$$\frac{-d\rho_v}{dt} = \frac{\sigma \rho_v}{\epsilon}$$

$$\int \frac{d\rho_v}{\rho_v} = \int_0^t \frac{-\sigma\, dt}{\epsilon}$$

$\boxed{\text{at } t=0 \quad \rho_v = \rho_{v0}}$

$$\ln \rho_v \Big]_{\rho_{v0}}^{\rho_v} = \frac{-\sigma}{\epsilon} t$$

$$= \ln \rho_v - \ln \rho_{v0} = \frac{-\sigma t}{\epsilon}$$

$$\ln\left(\frac{\rho_v}{\rho_{v0}}\right) = \frac{-\sigma t}{\epsilon}$$

$$\frac{\rho_v}{\rho_{v0}} = e^{\frac{-\sigma t}{\epsilon}}$$

$$\boxed{\rho_v = \rho_{v0} * e^{-t/\tau_r}}$$

$$T_r = \tau = \frac{\epsilon}{\sigma} \rightarrow \text{Relaxation time.}$$

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* if $t = T_r$ $\boxed{\rho_v = \rho_{vo} e^{-1}}$

$e^{-1} = 0.368$ 36.8% in

63.2% out.

$t = 2T_r$ → 13%

e^{-3} 5%

⋮

e^{-5} ⇒ less than 1% ⇒ (become aconduct

* For copper (good conductor)

$\epsilon_r = 1$ $\sigma = 5.8 * 10^{7}$ S/m.

* $T_r = \frac{\epsilon}{\sigma} = \frac{\epsilon_0 \epsilon_r}{\sigma} = \frac{\frac{10^{-9}}{36\pi}}{5.8 \times 10^{7}}$

$\boxed{T_r = 1.53 * 10^{-19}\ s}$

* for fused quartz (good dielectric)

$\epsilon_r = 5$ $\sigma = 10^{-17}$ S/m

$T_r = \frac{\epsilon_0 \epsilon_r}{\sigma} = \frac{5 * 8.85 * 10^{-12}}{10^{-17}}$

$\boxed{51.2 \text{ days.}}$

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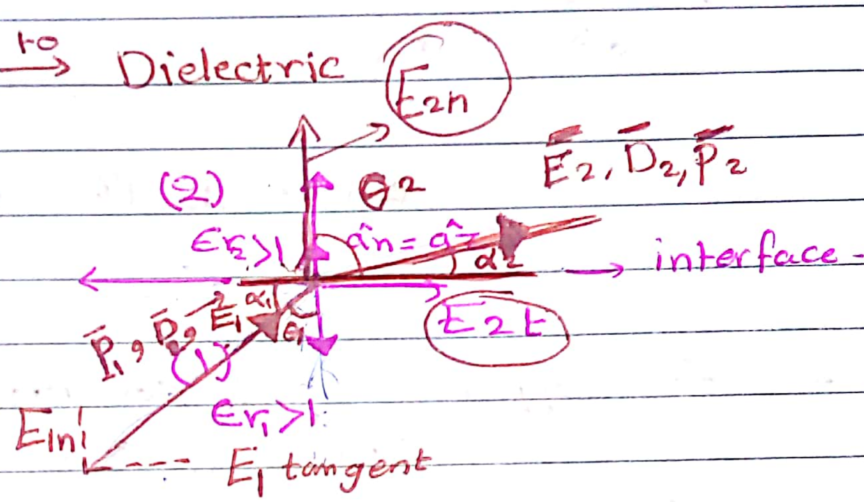
* **Boundary conditions:-**

General Case:-

Dielectric $\xrightarrow{\text{to}}$ Dielectric



(Z>0)

(Z=0)

(Z<0)

Start with $\vec{E}_1$

$$\vec{E}_1 = \vec{E}_1(\text{normal}) + \vec{E}_1(\text{tangent})$$

$$\hat{a}_n = \hat{a}_z$$

$$\bar{E}_{1n} = (\bar{E}_1 \cdot \hat{a}_n)\,\hat{a}_n$$

$$E_{1t} = (\bar{E}_1 - E_{1n})$$

or $(E_{1t} = \vec{E}_1 \times \hat{a}_n)$

$$\theta_1 = \sin^{-1}\left(\frac{E_{1t}}{E_1}\right) = \cos^{-1}\left(\frac{E_{1n}}{E_1}\right) = \tan^{-1}\left(\frac{E_{1t}}{E_{1n}}\right)$$

$\alpha_1 = 90^\circ - \theta_1$

Using maxwell's equations

$$\oint \bar{D} \cdot ds = Q_{enc} = \int \rho_s ds \quad \text{— (1)}$$

$$\oint E \cdot dl = 0 \rightarrow (2)$$

Using Maxwell's eq to find E_{2t}, E_{2n}

$$\oint_S D \cdot ds = \int_S \rho_s ds$$

$$D_{2n} \Delta s - D_{1n} \Delta s = \rho_s \Delta s$$

$$\rightarrow \boxed{D_{2n} - D_{1n} = \rho_s} \rightarrow \quad 1A$$

if $\rho_s = 0 \Rightarrow \boxed{D_{2n} = D_{1n}}$

$$\epsilon_{r_2} \epsilon_0 E_{2n} = \epsilon_{r_1} \epsilon_0 E_{1n}$$

$$E_{2n} = \frac{E_{1n} \epsilon_{r_1}}{\epsilon_{r_2}}$$

$\Rightarrow$ for dielectrics

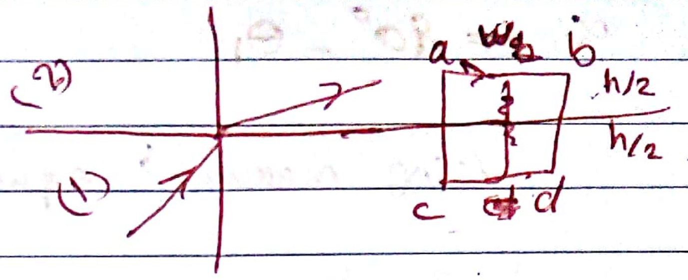
$$\epsilon_0 \epsilon_{r_2} E_{2n} - \epsilon_0 \epsilon_{r_1} E_{1n} = \frac{\rho_s}{\epsilon_0}$$

$$D_{1n} = \epsilon_0 \epsilon_{r_1} E_{1n}$$

$$D_{2n} = \rho_s + D_{1n}$$

$$E_{2n} = \frac{D_{2n}}{\epsilon_0 \epsilon_{r_2}}$$

To find E_{2t}



$$\oint E \cdot dl = 0$$

$$\oint E \cdot dl = \int_a^b E \cdot dl + \int_b^c E \cdot dl + \int_c^d E \cdot dl + \int_d^a E \cdot dl = 0$$

$$= E_{2t} w + \left(\cancel{-E_{2n}\left(\frac{h}{2}\right)} - E_{1n}\left(\frac{h}{2}\right)\right) +$$

$$- E_{1t} w \left(\cancel{E_{2n}\left(\frac{h}{2}\right)} + E_{2n}\left(\frac{h}{2}\right)\right) = 0$$

$$\boxed{\bar{E}_{2t} = \bar{E}_{1t}} \rightarrow 2B \text{ Always.}$$

$$E_2 = \bar{E}_{2t} + \bar{E}_{2n}$$

$$E_{1t} = E_{2t} \Rightarrow \frac{\bar{D}_{1t}}{\cancel{\epsilon_0} \epsilon_{r1}} = \frac{\bar{D}_{2t}}{\cancel{\epsilon_0} \epsilon_{r2}}$$

$$\bar{D}_{2t} = \frac{\epsilon_{r1}}{\epsilon_{r2}} \bar{D}_{1t}$$

$E_{1t} = E_{2t}$

$\sin\theta_1 \bar{E}_1 = \sin\theta_2 \bar{E}_2 \leftarrow (1)$

$\bar{D}_{1n} = \bar{D}_{2n} \quad (\rho_s = 0)$

$\cancel{\epsilon_0}\epsilon_{r1}\bar{E}_{1n} = \cancel{\epsilon_0}\epsilon_{r2}\bar{E}_{2n}$

$\epsilon_{r1}\cos\theta_1 E_1 = \epsilon_{r2}\cos\theta_2 E_2 \rightarrow (2)$

$(1) \div (2)$

$$\frac{\tan\theta_1}{\epsilon_{r1}} = \frac{\tan\theta_2}{\epsilon_{r2}}$$

$\rho_s = 0$

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_{r1}}{\epsilon_{r2}}}$$

$\Rightarrow$

* * * * * * *

always

$D_{2n} - D_{1n} = \rho_s$

$E_{1t} = E_{2t}$

$\Rightarrow$

for dielectric

$D_{2n} = D_{1n}$

$E_{2n} = \frac{\epsilon_{r1}}{\epsilon_{r2}} E_{1n}$

$E_1 \sin\theta_1 = E_2 \sin\theta_2$

$$\frac{\tan\theta_1}{\epsilon_{r1}} = \frac{\tan\theta_2}{\epsilon_{r2}}.$$

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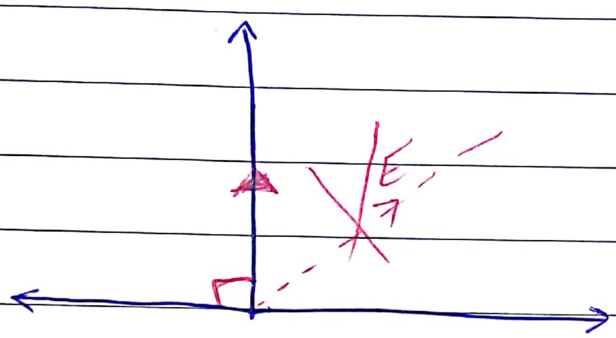
$E_{1t} = E_{2t}$

$D_{1n} - D_{2n} = \rho_s$

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_{r1}}{\epsilon_{r2}}$$

* conductor to dielectric.



$E_{2t} = 0 = E_{1t}$

$E_2 = E_{2n}$

$D_{2n} - D_{1n} = \rho_s$

$D_{2n} = \rho_s$

$$E_{2n} = \frac{D_{2n}}{\epsilon_0 \epsilon_{r2}}$$

$$\bar{E} = \frac{\rho_s}{\epsilon_0 \epsilon_{r2}} \hat{a_n} = E_n \qquad \theta_2 = 0^\circ \qquad \alpha_2 = 90^\circ$$

* * * * * *

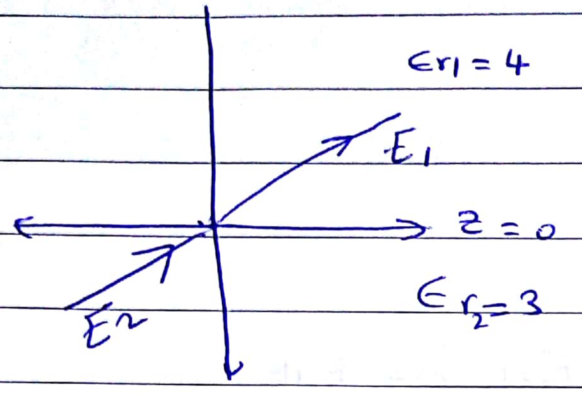
Example :- Two homo. dielectrics meet on plane $z = 0$

$\epsilon_{r1} = 4$ / $z < 0$ $\epsilon_r = 3$

if $\bar{E} = (5, 2, -3)$ kV/m

find ① E_2 ② $\theta_1, \theta_2, \alpha_1, \alpha_2$

③ W_{E1}, W_{E2} ④ the energy in the cube of side 2m centered in $(3, 4, -5)$



$E_1 = E_{1n} + E_{1t}$

$E_{1n} = (E_1 \cdot \hat{a_n}) \cdot \hat{a_n} = \cancel{} 2\hat{a_z}$

$E_{1t} = E_1 - E_{1n} = 5\hat{a_x} - 2\hat{a_y}$

$E_{1t} = E_{2t}$ $\quad E_{2t} = 5\hat{a_x} - 2\hat{a_y}$

$\boxed{E_2 = (5, -2, 4) \text{ kV/m}}$

$D_{1n} = D_{2n}$.

$\cancel{\epsilon_0} \epsilon_{r1} E_{1n} = \cancel{\epsilon_0} \epsilon_{r2} E_{2n}$

$E_{2n} = \frac{\epsilon_{r1} E_{1n}}{\epsilon_{r2}} = 4\hat{a_z}$ kV/m

(b) $\theta_1 = \cos\theta_1 = \frac{E_{1n}}{E_1}$ $\quad \boxed{\theta_1 = \cos^{-1}\frac{3}{\sqrt{38}} = 60.9^\circ}$

$\boxed{\alpha_1 = 90^\circ - \theta_1 = 29.1^\circ}$

$\theta_2 \not\rightarrow \frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_{r1}}{\epsilon_{r2}}$ $\quad \boxed{\begin{array}{l}\theta_2 = 53.4^\circ \\ \alpha_2 = 36.6^\circ\end{array}}$

(c) $w_{E1} = \frac{1}{2} \epsilon_0 \epsilon_{r1} (E_1)^2 = 672 \mu J/m^3$

$w_{E2} = \frac{1}{2} \epsilon_0 \epsilon_{r2} (E_2)^2 = 597 \mu j/m^3$

(D) $= W = \int_V w_{E2} \, dv = 597 \times 10^{-9} \times 4$

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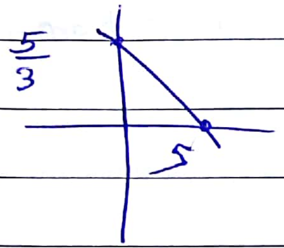
if the interface ~~has~~ is oblique or has an equation

$x+3y=5$ $\rightarrow$ ① $x+3y<5$

$\rightarrow$ ② $x+3y>5$



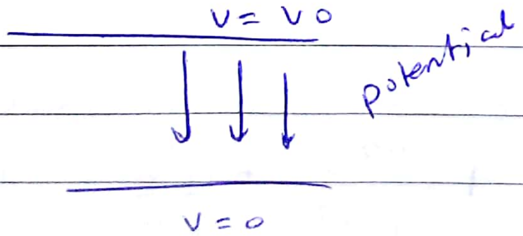
$$\nabla = \frac{\langle 1,3,0\rangle}{\sqrt{10}}$$

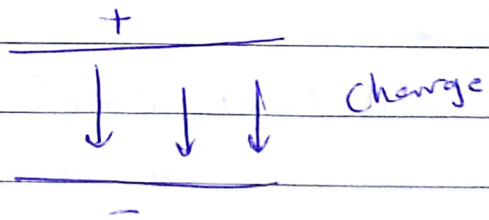
$$\hat{a}_n = \frac{\nabla F}{|\nabla F|} \qquad \hat{a}_n = \left(\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right)$$

CH6 Boundary value problems. (B.V.P)

poissons and laplace equations.

Used if charge or potential are known on same part of the Geometry.





$$\nabla\cdot\bar{D} = \rho_v$$

$$\nabla\cdot\epsilon E = \rho_v \qquad \text{if } \epsilon \text{ is homo.}$$

$$\nabla\cdot\bar{E} = \frac{\rho_v}{\epsilon} \qquad \nabla(-\nabla V) = \frac{\rho_v}{\epsilon}$$

$$\boxed{\nabla^2 V = -\frac{\rho_v}{\epsilon}}$$

$$\nabla^2 V = \frac{-\rho_v}{\epsilon}$$ | poisson's eq for homo. media

(if $\rho_v = 0$) $\boxed{\nabla^2 V = 0}$ laplace eq for homo. media

if ϵ is non-homo.

$$\nabla \cdot \epsilon(-\nabla V) = \rho_v$$

$$-\nabla(\epsilon \nabla V) = \rho_v \rightarrow$$ poisson's eq for non-homo media

$\boxed{\rho_v = 0}$

$$-\nabla(\epsilon \nabla V) = 0$$ laplace eq for non-homo media

in Cart coordinates

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

in cylindrical

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

in Spherical

$\rightarrow$

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$$= \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dv}{dr}\right) + \frac{1}{r^2 \sin\theta}\frac{d}{d\theta}\left(\sin\theta \frac{dv}{d\theta}\right)$$

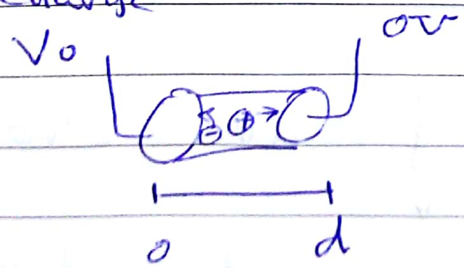
$$+ \frac{1}{r^2 \sin\theta}\left(\frac{d^2 v}{d\phi^2}\right).$$

* Boundary value problems + 2 initial conditions

Steps to follow in solving B.V.P

1. solve laplace or poisson's eq by :-
   a. Direct integration if v is function of one variable.
   b. sepration of variable if V is Fun of two or more ✗
2. Apply B. condition to find the Unique solution.
3. $E = -\nabla V$
4. Find the Required from the question.

Ex) an electrohydrolic (EHD) Pumping by Uniform charge



Find pressure

$$\text{pressure} = \frac{|F|}{A}$$

if

$\rho_o = 25 \ mc/m^3$ $\quad V_o = 22 kV$

Poisson's eq

$$\nabla^2 V = \frac{-\rho_v}{\epsilon} \Rightarrow \frac{d^2V}{dz^2} = -\frac{\rho_v}{\epsilon}$$

$$dv = -\frac{\rho_v}{\epsilon} z + A \Rightarrow V = -\frac{\rho_v}{\epsilon} \frac{z^2}{2} + Az + B$$

I.C $\quad V(z=0) = V_o$

$V(z=d) = 0$

$\Rightarrow$

$B = V_o$

$$A = \left(\frac{\rho_v}{\epsilon} \frac{d^2}{2} - V\right) \div d$$

$$V = \frac{-\rho_v z^2}{2\epsilon} + \left(\frac{\rho_v d^2}{2\epsilon} - V_o\right) \frac{1}{d} z + V_o$$

$$E = -\nabla V = -\frac{dv}{dz} \hat{a_z}$$

$$E = -\frac{\rho_v z}{\epsilon} + \left(\frac{\rho_v d^2}{2\epsilon} - V_o\right) \frac{1}{d} \hat{a_z} \quad V/m$$

$\bar{F} = Q\vec{E} \qquad Q = \int_V \rho_v \, dv \qquad \rho_v = \rho_o$

$f = \int -\rho_v E \, dv \qquad dv = ds \times dl$

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$$\bar{F} = -\rho_v s \int \frac{-\rho_v z}{\epsilon} + \left(\frac{\rho_v d^2}{2\epsilon_0} - V_0\right)\frac{1}{d} \, \hat{a_z}$$

$$\bar{F} = \rho_v * S \left(\frac{-\rho_v z^2}{2\epsilon} + \left(\frac{\rho_v d^2}{2\epsilon} - V_0\right)\frac{z}{d}\right)\Bigg|_0^d \hat{a_z}$$

$$\hat{F} = \rho_v \, S \, V_0 \qquad \text{Pressure} = \frac{F}{A} = \boxed{\rho_v V_0} \; N/m$$

Resistance and Capacitance

$$R = \frac{I}{V} = \frac{\int_L E \cdot dl}{\int_s \sigma E \, ds}$$

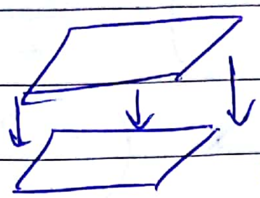
$$C = \frac{Q}{V} = \frac{\int_s \epsilon E \cdot \bar{d}s}{-\int E \cdot dl}$$

Steps to find ~~the~~ Capacitance.

① choose a suitable coordinates.

B.V.P	Assume Q gausses' method
① assume $V = V_0$	
③ solve B.V.P	② find $\vec{E}$ Using $\int_s \epsilon \vec{E} \cdot ds = Q$
④ $E = -\nabla V$	
⑤ $Q = \int \epsilon E \cdot ds$	$V = -\int_L E \cdot dl$
$C = \frac{Q}{V_0}$ (F)	$C = \frac{Q_0}{V}$ (F)

types of Capacitors



(A) parallel plate capacitor

(1) Gaussés method

$$Q = \int \epsilon \vec{E} \cdot ds$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \epsilon E_x \, (-\hat{a}_x) \cdot dy \, dz \, (-\hat{a}_x) = Q$$

$$Q = \epsilon E_x S$$

$$\vec{E} = \frac{Q}{\epsilon S} (-\hat{a}_x)$$

$$V = \int -E \cdot dl \qquad V = -\frac{Q}{\epsilon s} \int_0^d -\hat{a}_x \cdot \hat{a}_x \, dx$$

$$= \frac{Q}{\epsilon s} (d) = \boxed{\frac{Qd}{\epsilon s}}$$

$$\boxed{C = \frac{Q}{V} = \frac{\epsilon s}{d} \, (F)}$$

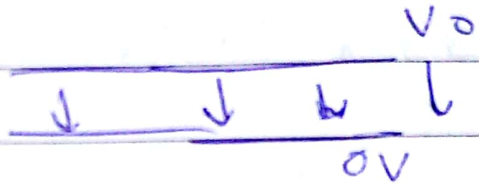
$$I = \int_s \sigma \vec{E} \cdot ds \qquad I = \frac{\sigma Q}{\epsilon}$$

$$R = \frac{V}{I} =$$

$$W_E = \frac{1}{2}\int_V \epsilon E^2 \, dv = \frac{1}{2}\cancel{\epsilon}\,\frac{Q^2}{\epsilon^{\cancel{2}} s^{\cancel{2}}} * \cancel{s} * d$$

$$= \frac{Q^2 d}{2\epsilon s} = \frac{Q^2}{2C} = \frac{1}{2}QV = \frac{1}{2}CV^2$$

Using B.V.P



$$\nabla^2 V = 0$$

$$\frac{d^2V}{dx^2} = 0 \rightsquigarrow \frac{dV}{dx} = A \rightsquigarrow V = Ax + B.$$

$V(x=d) = V_o \rightsquigarrow \boxed{B = 0} \rightsquigarrow \boxed{A = \frac{V_o}{d}}$

$V(x=0) = 0$

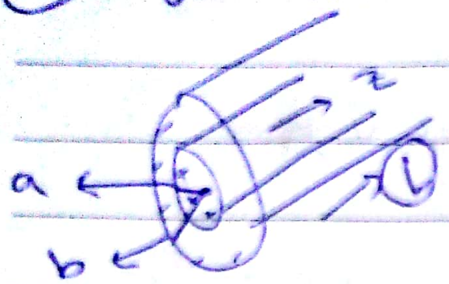
$$V = \frac{V_o}{d}x \qquad \vec{E} = \frac{-V_o}{d} \text{ V/m } \hat{a}_x$$

$$Q = \int_s \epsilon \vec{E} \cdot ds$$

$$Q = \frac{\epsilon V_o}{d}(s)\,\hat{a}_x$$

$$\boxed{C = \frac{Q}{V} = \frac{\epsilon}{d}s}$$

② Cylindrical Capacitor



$\vec{E} = E_\rho$

$\oint \epsilon E \cdot ds = Q$

$\epsilon E_\rho \rho * 2 * \pi * L = Q$

$\vec{E} = \frac{Q}{2\epsilon \pi L \rho} \rho_L \hat{a_\rho}$

$V = \int_L - E \cdot dl = \frac{-Q}{2\pi\epsilon L} \int_b^a \frac{1}{\rho} \hat{a_\rho} d\rho \hat{a}$

$$\boxed{V = \frac{Q}{2\pi\epsilon L} \ln\left(\frac{b}{a}\right) V}$$

$C = \frac{Q}{V}$ $\qquad$ $C = \frac{2\pi\epsilon L}{\ln\left(\frac{b}{a}\right)} \; (f)$

$\frac{C}{L} = \frac{2\pi\epsilon}{\ln\left(\frac{b}{a}\right)} \; (F/m)$

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Resistance

$R_d = \frac{V}{I}$ $I = \int_s \sigma E\, ds$

$I = \frac{Q \ast \sigma_d}{2\pi \epsilon L} \int_0^c \int_0^{2\pi} \frac{1}{\rho} a_\rho \, \rho \, d\phi \, dz$

$I = \frac{Q \sigma_d}{\epsilon} \; A \Rightarrow \boxed{R = \frac{\ln\left(\frac{b}{a}\right)}{2\pi \sigma_d L}}$

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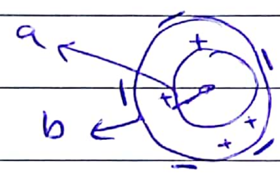
③ Spherical Capacitor



$E = E_r a_r$

$ds = r^2 \sin\theta \, d\theta \, d\phi \, a_r$

$\oint \epsilon \vec{E} \cdot ds = Q \Rightarrow \epsilon \cdot E_r r^2 (2)(2\pi) = Q$

$\boxed{E = \frac{Q}{4\pi \epsilon r^2}} \rightarrow$ $V = -\int E \cdot dl$

$V = -\frac{Q}{4\pi\epsilon} \int_b^a \frac{1}{r^2} dr$

$V = \frac{Q}{4\pi\epsilon} \left(\frac{1}{a} - \frac{1}{b}\right) V$

$\boxed{C = \frac{Q}{V} = \frac{4\pi\epsilon_0}{\frac{1}{a} - \frac{1}{b}} \; F}$

NOTEBOOK

$R = \frac{V}{I}$ $= \int_s \sigma_d E = \frac{\sigma_d \; Q}{4\pi\epsilon} \int \frac{1}{\cancel{r^2}} \cancel{r^2} \sin\theta \, d\theta \, d\phi$

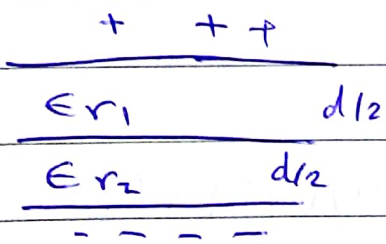
$= \frac{\sigma_d \; Q}{4\pi\epsilon} (2)(2\pi)$

$I = \frac{\sigma_d \; Q}{\epsilon}$ A $\qquad \boxed{R = \frac{\frac{1}{a} - \frac{1}{b}}{4\pi \sigma_d} \; \Omega}$

(4) isolated capacitor , isolated sphere $\quad b \to \infty$

$C = 4\pi\epsilon a$ F $\qquad R = \frac{1}{a(4\pi\sigma_d)}$ Ω

Example:- find



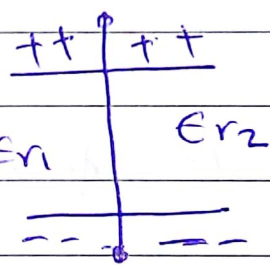
↓

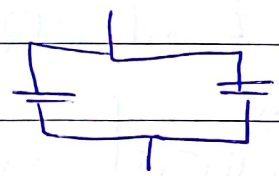
Series Capacitors

$C_1 = \frac{\epsilon_o \epsilon_{r1} S}{d/2}$

$C_2 = \frac{\epsilon_o \epsilon_{r2} S}{d/2}$

$\Rightarrow \boxed{C_{eq} = \frac{C_1 \, C_2}{C_1 + C_2}}$





(parallel

$\boxed{C_{eq} = C_1 + C_2}$

Ex) Coaxial cable has $a = 1\,cm$ $b = 2.5\ cm$ and $\epsilon_r = \frac{10+\rho}{\rho}$ where ρ is in cm. Find C/L

$$Q = \oint_S \epsilon E \cdot ds \qquad Q = \int_0^L \int_0^{2\pi} \epsilon E \rho\, \rho\, d\phi\, dz$$

$$\vec{E} = \frac{Q}{2\pi \epsilon \rho L} a_{\hat{\rho}}$$

$$V = -E \cdot dL \qquad dL = d\rho\, a_{\hat{\rho}}$$

$$V = \frac{-Q}{2\pi \epsilon_0 L} \int_b^a \left(\frac{1}{\cancel{\rho}\left(\frac{10+\rho}{\cancel{\rho}}\right)} \right) d\rho$$

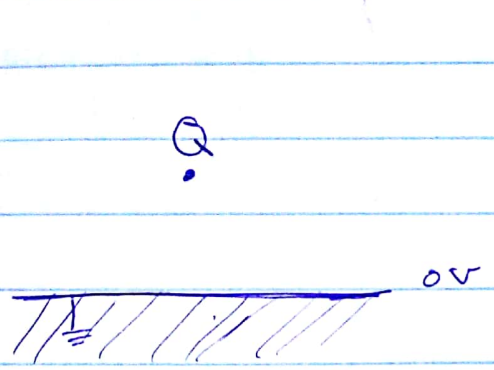
$$V = \frac{-Q}{2\pi \epsilon_0 L} * Ln\left(\frac{10+a}{10+b}\right) = \frac{Q}{2\pi \epsilon_0 L} Ln\left(\frac{10+b}{10+a}\right)$$

$$C = \frac{Q}{V} = \frac{2\pi \epsilon}{Ln\left(\frac{10+b}{10+a}\right)} \Rightarrow \frac{C}{L} = 434.6\ pF/m$$

Line charge, surface, volume charge

Method of Images :-

Used to find $\vec{E}$, $\vec{D}$, $\vec{V}$ for <u>a charge</u> located above a perfect ground conducting plane

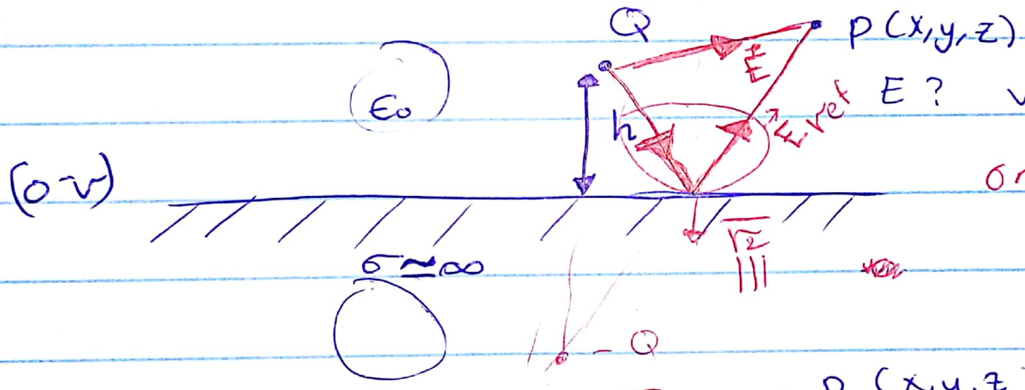


$\sigma = \infty$

$\boxed{V = 0}$

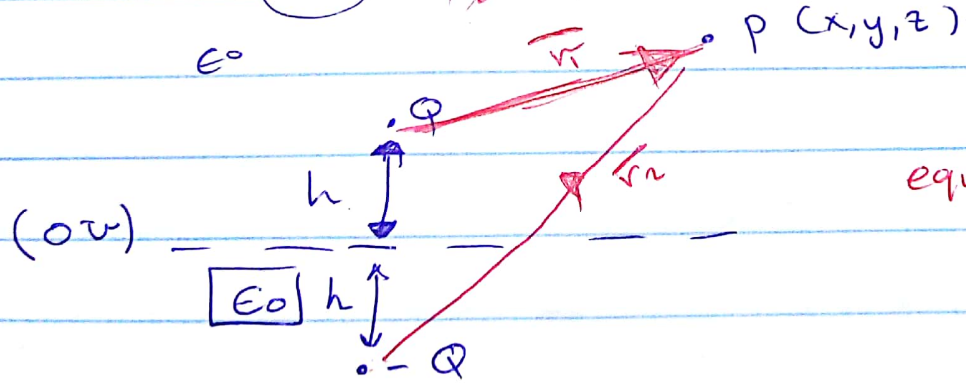
$\begin{cases} \vec{D} = 0 \\ \vec{E} = 0 \end{cases}$

* point charge located above a perfect conducting ground plane



E ? V? $\vec{D}$?

original problem



equivilent problem

$V = V_+ + V_-$

$V = \frac{Q}{4\pi\epsilon_0 r_1} - \frac{Q}{4\pi\epsilon_0 r_2}$

$r_1 = x, y, z - (0, 0, h) = x\hat{a_x} + y\hat{a_y} + (z-h)\hat{a_z}$

$r_2 = \langle x, y, z\rangle - (0, 0, h) = x\hat{a_x} + y\hat{a_y} + (z+h)\hat{a_z}$.

$V = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{\sqrt{x^2+y^2+(z-h)^2}} - \frac{1}{\sqrt{x^2+y^2+(z+h)^2}}\right) \quad V, \; z>$

$\boxed{\overset{\uparrow}{z=0} \text{ Sub}}$ $\quad V = 0V \quad \boxed{z=0}$

~~z/<0~~

$0 \quad z<0 \rightarrow$ ~~[illegible]~~ (grounded)

* * * * * * * * * * * * * * * *

$\vec{E} = E_+ \oplus E_-$

$= \frac{Q \, r_1}{4\pi\epsilon_0 (r_1)^3} - \frac{Q \, r_2}{4\pi\epsilon_0 (r_2)^3}$

$\frac{Q}{4\pi\epsilon_0}\left(\frac{\langle x, y, z-h\rangle}{(x^2+y^2+(z-h)^2)^{3/2}} - \frac{\langle x, y, z+h\rangle}{(x^2+y^2+(z+h)^2)^{3/2}}\right) \quad V, \; z$

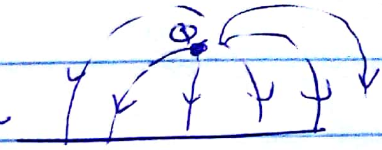
$-\frac{2hQ}{4\pi\epsilon_0}\left(\frac{1}{(x^2+y^2+h^2)^{3/2}}\right)\hat{a_z} \quad z=0 \quad V/m$

$0 \quad z/0$

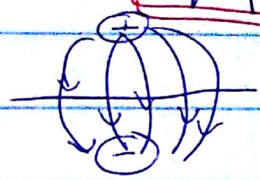
$\boxed{\bar{D} = \epsilon_0 \bar{E}}$

original problem

or



eq. prob



$\Rightarrow$

$\rho_s = D \cdot \hat{a}_n$ at $(z=0)$

$$= \boxed{\frac{-\cancel{z}\,h\,Q}{4\pi\,[x^2+y^2+h^2]^{3/2}} \; c/m^2}$$

$$Q_{induced} = \int_s \rho_s \, ds$$

if $Q_{induced} = -Q$

eq problem is correct.

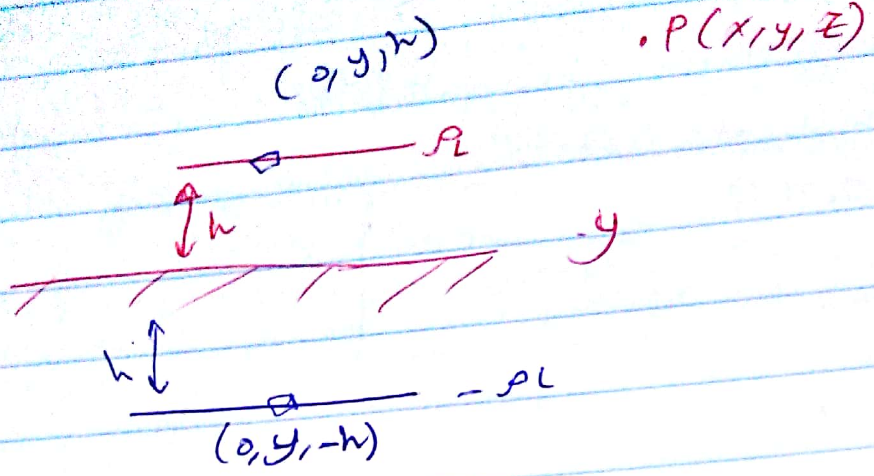
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \cancel{\rho_s} \frac{(-2hQ}{4\pi\,[x^2+y^2+h^2)^{3/2}} \, dx\,dy$$

$$Q_{induced} = \frac{-2hQ}{4\pi} \int_0^{2\pi}\int_0^{\infty} \frac{\rho\,d\rho\,d\phi}{(\rho^2+h^2)^{3/2}}$$

$$= \frac{-hQ}{2} \left(\rho^2 + h^2 \right)^{-\frac{1}{2}} \times \cancel{-2} \Big]_0^{\infty}$$

$$= \frac{\cancel{2}\,h\,Q}{\sqrt{\rho^2+h^2}} \Big]_0^{\infty} = \cancel{h}Q \left(0 \; \frac{-1}{\cancel{h}} \right)$$

$$= \boxed{-Q}$$



$$\bar{E} = E_+ + E_-$$

$$\frac{\rho_L \ \bar{\rho}_1}{2\pi\epsilon_0(\rho_1)^2} + \frac{\rho_L \ \bar{\rho}_2}{2\pi\epsilon_0(\rho_2)^2}$$

$\rho_1 = (x,y,z) - (0,y,h)$ $\langle x, 0, z-h\rangle$

$\rho_2 =$ $\langle x, 0, z+h\rangle$

* * * * *

$$\boxed{Q_i = \int \rho_s \, ds = \int \rho_{L_i} \, dl_i}$$

$$\boxed{\rho_i = -\rho_L}$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \rho_s \, dx \, dy = \int_{-\infty}^{\infty} \rho_{L_i} \, dy$$

$$\rho_L = \int_{-\infty}^{\infty} \rho_s \, dx \rightsquigarrow$$ cylindrical

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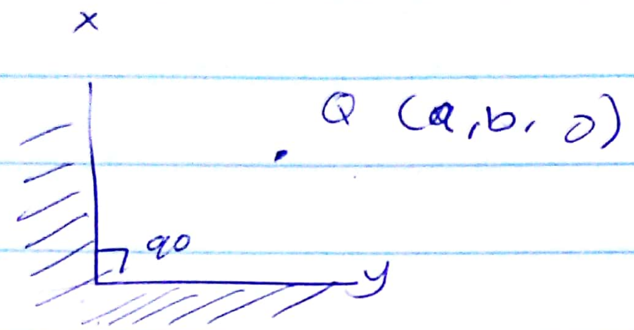
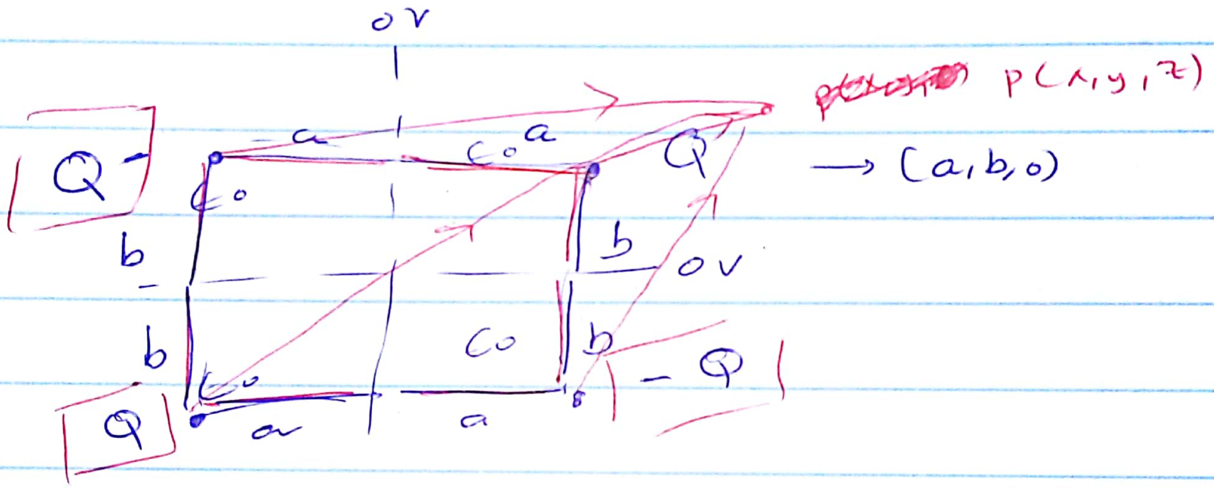


$$N = \frac{360}{\phi} - 1$$

number of images



CH7 Magnatic field

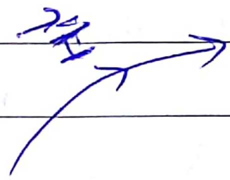
sources of Magnatic field :-

— x ① permenat magnatic.

② Charge moveng with Univform velocity

(zero acceleration)

③ Dc current flowing in a conductor



two major laws

1- Biot - savarts low (general)

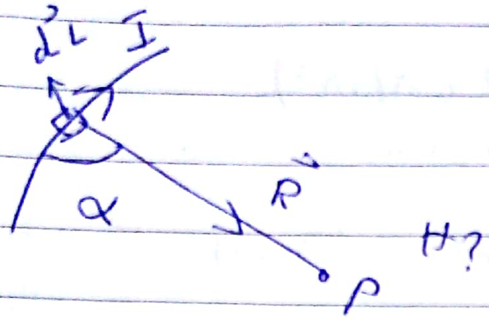
2- Amperé's law (special case)

* symetry + Duality

electrostatics	Magnetostatic
	~~$\vec{H}$ (A/m~~
$\vec{E}$ V/m	$\vec{H}$ (A/m
$\vec{D}$ C/m^2	$\vec{B}$ (wb/m^2) (T)
$\epsilon = \epsilon_0 \epsilon_r$ (f/m)	$\mu = \mu_0 \mu_r$ (H/m)
$w_E = \frac{1}{2} \int \vec{E} \cdot \vec{D} \, dv$ (J)	$W_m = \frac{1}{2} \int_v \vec{H} \cdot \vec{B} \, dv$
$\vec{D} = \vec{E} \epsilon$	
$\vec{E} = -\nabla V$	$\vec{H} = -\nabla V_m$
$\vec{D} = \epsilon \vec{E}$	$\vec{B} = \mu \vec{H}$

Subject Date No.

Biot savart law.



$$dH \propto \frac{I\, dL \sin\alpha}{R^2}$$

$$dH = \frac{k I\, dL \sin\alpha}{R^2}$$

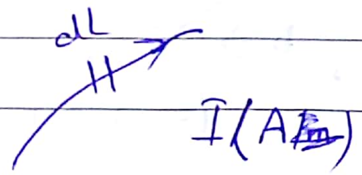
$$\boxed{k = \frac{1}{4\pi}}$$

$$dH = \frac{I\, dL \sin\alpha}{4\pi R^2}$$

$$\boxed{H = \int_L \frac{I\, \vec{dl} \times \hat{a_r}}{4\pi R^2} \quad A/m}$$

Types of current distribution.

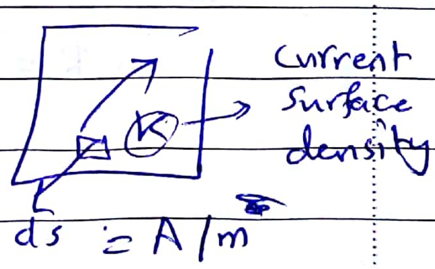
① Line current



$$\vec{H} = \int \frac{I\, dL \times \hat{a_r}}{4\pi r^2}$$

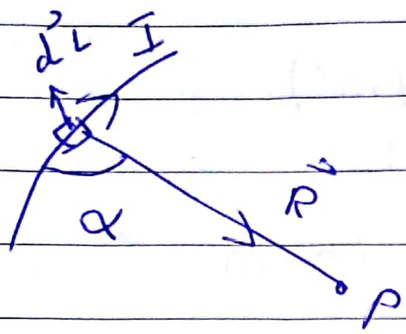
② Surface current.

$$H = \int \frac{\bar{K}\, ds \times \hat{a_r}}{4\pi R^2}$$



Biot savart law.

$$dH \propto \frac{I\,dL\,\sin\alpha}{R^2}$$



$$dH = \frac{kI\,dL\,\sin\alpha}{R^2}$$

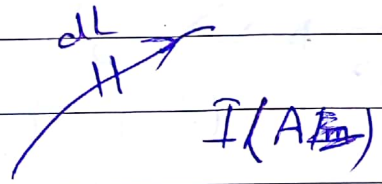
$$\boxed{k = \frac{1}{4\pi}}$$

$$dH = \frac{I\,dL\,\sin\alpha}{4\pi R^2}$$

$$\boxed{H = \int_L \frac{I\,\vec{dL} \times \hat{a_r}}{4\pi R^2} \quad A/m}$$

Types of current distribution.

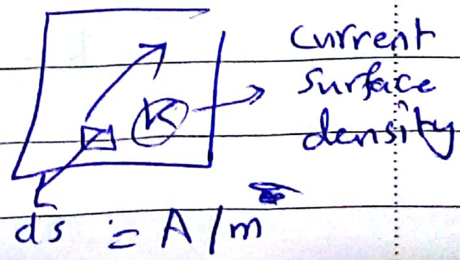
① Line current



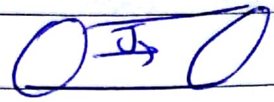
$$\vec{H} = \int \frac{I\,dL \times \hat{a_r}}{4\pi r^2}$$

② Surface current.

$$H = \int \frac{\vec{k}\,ds \times \hat{a_r}}{4\pi R^2}$$



③ volume current



$$\bar{H} = \int \frac{J\, dv \times \hat{a_r}}{4\pi r^2}$$

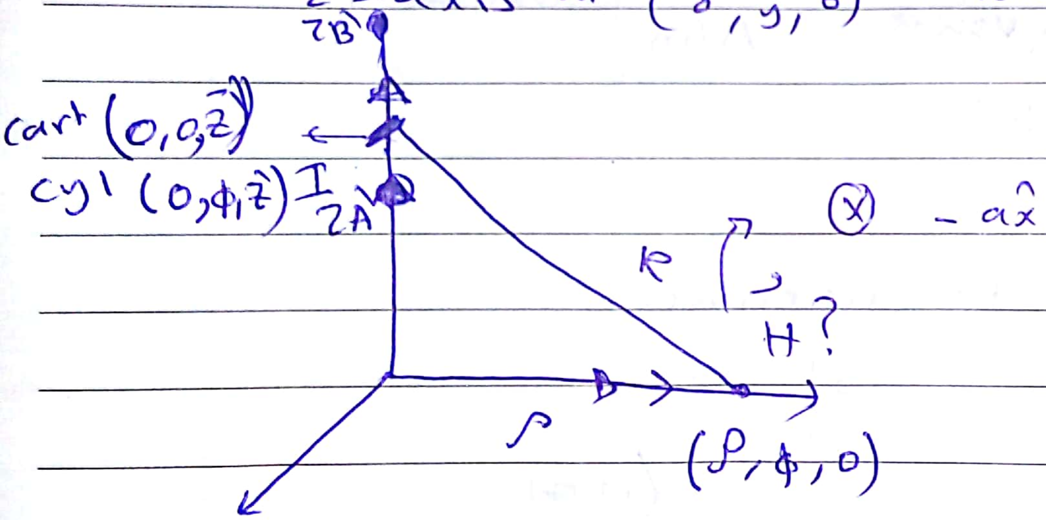
$J\ (A/m^2)$

$\bar{J}\, dv \to (A.m)$

$\downarrow$ scalar $(A.m)$

$*\quad I\, d\bar{L} = \bar{K}\, ds = \bar{J}\, dv$

Ex = $\bar{H}$ due to finitien stright line along Z-axis at $(0, y, 0)$ along the Y-axis.

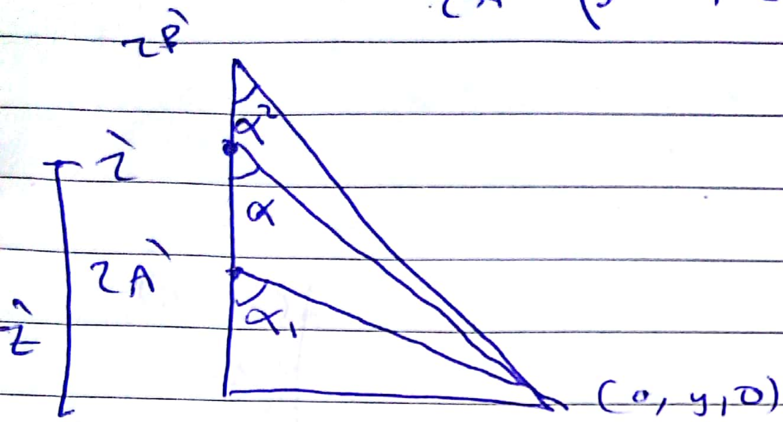


$$\vec{H} = \int_L \frac{I\, d\bar{L} \times \bar{R}}{4\pi R^2} \qquad dL = dz\, \hat{a_z}$$

$$\bar{R} = \rho\, \hat{a_\rho} - z'\, \hat{a_z}$$

$$R = \sqrt{\rho^2 + (z')^2}$$

$$\bar{H} = \frac{I}{4\pi} \int_{zA'}^{zB} \frac{dz\, \hat{a}_z \times (\rho \hat{a}_\rho - z \hat{a}_z)}{(\rho^2 + z^2)^{3/2}}$$



$dz' \Rightarrow d\alpha$

$\tan\alpha = \frac{\rho}{z'}$

$z' = \rho \cot\alpha$

$dz' = -\rho \csc^2\alpha \, d\alpha$

$$R^2 = \rho^2 + z^2 = \rho^2 + \rho^2 \cot^2\alpha$$

$$= \rho^2 \left(1 + \frac{\cos^2\alpha}{\sin^2\alpha}\right) = \frac{\rho^2}{\sin^2\alpha} = \rho^2 \csc^2\alpha$$

$$R^3 = \rho^3 \csc^3\alpha$$

$$\bar{H} = \int_{\alpha_1}^{\alpha_2} \frac{I}{4\pi} \frac{(\rho)(-\rho \csc^2\alpha)\, d\alpha\, \hat{a}_\phi}{\rho^3 (\csc^3\alpha)}$$

$$\boxed{H = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\, \hat{a}_\phi} \Rightarrow \text{for infinite stright line}$$

Special Case (semi infinite) $zA = (0,0,0)$ $\alpha_1 = 90^\circ$

$zB \to \infty \Rightarrow \alpha_2 = 0$

$\bar{H} = \frac{I}{4\pi\rho} \hat{a}_\phi$ ⟶ not always

for infinite line $\boxed{\bar{H} = \frac{I}{2\pi\rho} \hat{a}_\phi \text{ always}}$

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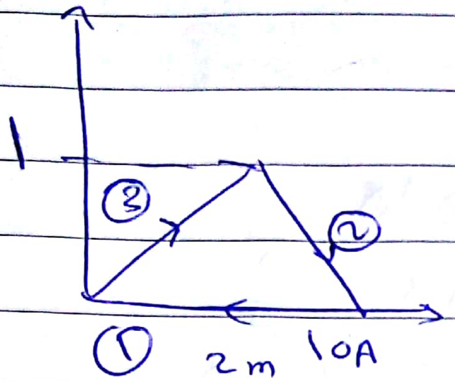
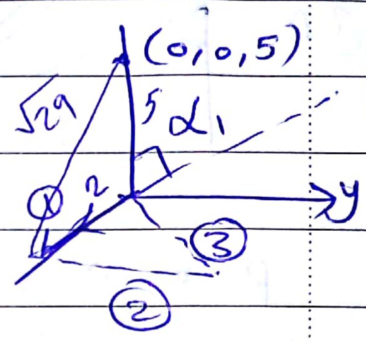
Find $\vec{H}$ at $(0,0,5)$ due to side(1) of loop.





$$\vec{H} = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\,\hat{a}_\phi$$

$\alpha_1 = 90°$ $\qquad \cos\alpha^2 = 2/\sqrt{29}$

$\rho = 5$ $\qquad \hat{a}_\phi = \hat{a}_L \times \hat{a}_\rho$

$\hat{a}_x \times \hat{a}_z = -\hat{a}_y$

$$\boxed{\vec{H} = -59.4\,\hat{a}_y \quad mA/m}$$

side ③

$\rho = 5m \qquad \alpha_1 = 90° \qquad \cos\alpha_2 = \frac{\sqrt{2}}{\sqrt{27}}$

$\hat{a}\phi = \rightarrow$

$\vec{L} = \langle -1, -1, 0 \rangle$

$||L|| = \sqrt{2}$

$\hat{a}_L = \frac{1}{\sqrt{2}}\hat{a}_x - \frac{1}{\sqrt{2}}\hat{a}_y$

then $\hat{a}_L \times \hat{a}_z$

side 2

$\vec{BA} \cdot \vec{BC} =$

$|BA||BC| \cos \alpha_2$

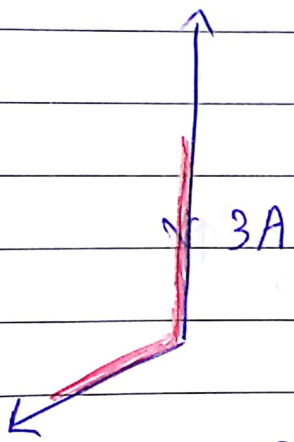
$\alpha_1 = \langle -1, -1, 5 \rangle \langle 1, -1, 0 \rangle$ $= \sqrt{2} * \sqrt{27} * \cos \alpha_2$

$\alpha_2 = 90$ $\rho = 90$

$\rho = BC = \sqrt{27}$

$\vec{AB} \cdot \vec{AC} = |AB| |AC| \cos \theta_1$

$\cos \alpha_1 = -\cos \theta_1$



$(-3, 4, 0)$ $\bar{H} = H_1 + H_2$

$H_1 = \frac{I}{4\pi\rho} ((\cos \alpha_2 - \cos \alpha_1) \hat{a\phi}$

$\boxed{\alpha_2 = 0}$ $\boxed{\cos \alpha_1 = \frac{-3}{5}}$ $\boxed{\rho = 4}$

$\hat{a\rho} = \hat{ay}$ $\hat{a\phi} = aL \times \hat{a\rho} = \boxed{\hat{az}}$

$H_1 \neq \frac{I}{4\pi\rho} a\phi$

$H_2 = \frac{I}{4\pi\rho} (\cos \alpha_2 - \cos \alpha_1) \hat{a\phi}$

$\alpha_1 = 90^\circ$ $\alpha_2 = 0^\circ$ $\rho = 5$

$\hat{a\phi} = aL \times \hat{a\rho} = -\hat{az} \times \frac{\langle -3, 4, 0 \rangle}{\sqrt{5}}$

$\bar{H} = \bar{H}_1 + \bar{H}_2$

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Ampere's Law .

$\oint H \cdot dL = I_{enc}$ ⟵ 3rd Maxwell's eq in integral form.

Applystoke's theorem

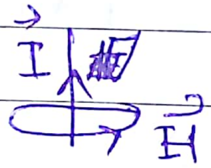
$$\oint_L H \cdot dL = \int_S \nabla \times \bar{H} \, ds = I_{enclosed} = \int_S J \cdot ds .$$

$\boxed{\nabla \times H = J}$ → 3rd maxwell's eq in diff form .

Application of Amperés low.

① infinite line of current. ($H = \frac{I}{2\pi\rho} \hat{a}_\phi$)

$\oint H \cdot dL = I_{enclosed}$



$$\vec{H} = H_{\phi} \hat{a}_\phi$$

$$\int_0^{2\pi} H_\phi \, \rho \, d\phi = I$$

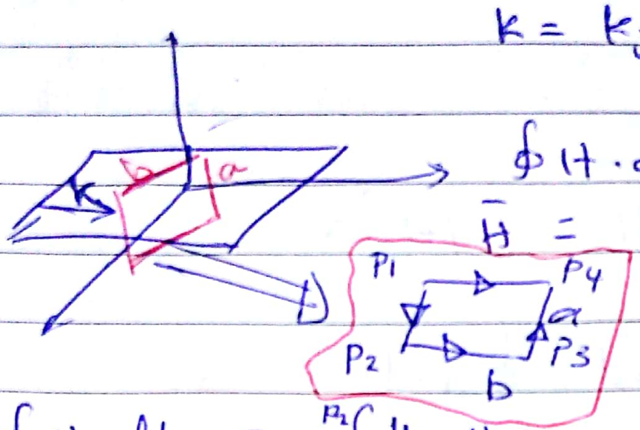
$$2\pi H_\phi = I$$

$$\boxed{\bar{H} = \frac{I}{2\pi\rho} \hat{a}_\phi}$$

N O T E B O O K

② infinite sheet of current

$K = K_y \, \hat{a_y}$



$\oint H \cdot dl = I_{enc}$

$\bar{H} = \begin{cases} H_o \, \hat{a_x} & Z > 0 \\ H_o(-\hat{a_x}) & Z < 0 \end{cases}$

$\oint H \cdot dl = \int_{P_1}^{P_2} H \cdot dl_1 + \int_{P_2}^{P_3} H \cdot dl_2 + \int_{P_3}^{P_4} H \cdot dl_3$

zero zero

$+ \int_{P_4}^{P_1} H \cdot dl_4 \Rightarrow H_o(-a_x)\cancel{b}(-\hat{a_x}) + H_o \hat{a_x}(\cancel{b}\hat{a_x}) = K_y \cancel{b}$

$2H_o = K_y \qquad H_o = \frac{K_y}{2}$

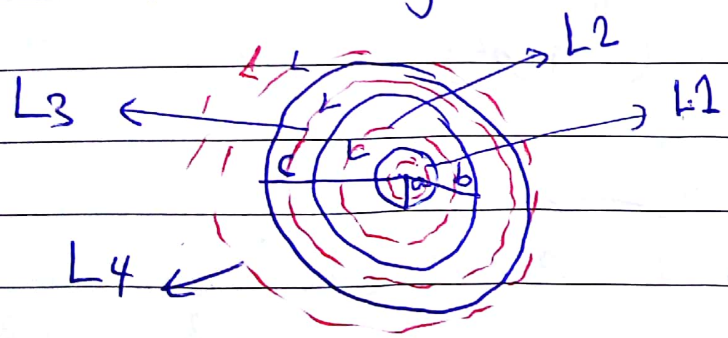
$\vec{H} = \left.\begin{cases} \frac{K_y}{2} \hat{a_x} & Z > 0 \\ \frac{K_y}{2}(-\hat{a_x}) & Z < 0 \end{cases}\right\} \Rightarrow \frac{1}{2} K \hat{\times} \hat{a_n}$

cross product.

* * * *

infinitly long Coaxial cable :

find H every where



for $0 < \rho < a$ on path L1

$$\oint_{L_1} H \cdot dl_L = I_{enc}$$

$H = H\phi \hat{a}_\phi$ $\qquad dl = \rho \, d\phi \hat{a}_\phi$

$2\pi \rho H\phi = I_{enclosed} = \oint_S J \cdot ds \Rightarrow (1)$

at $\rho = a$ $\qquad I_{enclosed} = I = \int_S J \cdot ds$

$$I = \int_0^{2\pi}\int_0^{a} J_Z \cancel{\hat{a}_z} (\rho \, d\rho \, d\phi \cancel{\hat{a}_z})$$

$$I = \frac{J_Z}{2} a^2 (2\pi) \; I$$

$I = j_Z \pi a^2$ $\qquad J = \frac{I}{\pi a^2} = \frac{I}{S}$

$\Downarrow$ Sub in (1)

$$2\pi \rho H_\phi = \int_S J \cdot ds.$$

$$2\pi \rho H_\phi = \int_0^{2\pi}\int_0^{\rho} \frac{I}{\pi a^2} \cancel{\hat{a}_z} (\rho \, d\rho \, d\phi \hat{a}_z)$$

$$2\pi \rho H_\phi = \frac{I}{\pi a^2} (\pi \rho^2)$$

$$\boxed{\vec{H} = \frac{I \rho}{2\pi a^2} \hat{a}_\phi \qquad 0 < \rho \leq a}$$

for $0 < \rho < b$ on path L_2

$$\oint \vec{H} \cdot dL_2 = I_{enc}$$

$$2\pi \rho H_\phi = I$$

$$\vec{H} = \frac{I}{2\pi \rho} \hat{a_\phi} \quad a \leq \rho \leq b$$

(3) for $b < \rho < c$ path L_3

$$\oint H \cdot dL_3 = I_{enc}$$

$$2\pi \rho H_\phi = I_{enc} = I - \int_s J \cdot ds \Longrightarrow (2)$$

at $\rho = c$

$I_{enc} = I$

$$I = \int_s \vec{J} \cdot ds = \int_0^{2\pi} \int_b^c J_z (-\hat{a_z}) \rho \, d\rho \, d\phi \, (-\hat{a_z})$$

$$I = J_z (c^2 - b^2) \pi$$

$$J = \frac{I}{(c^2 - b^2)\pi} \hat{a_z}$$

$\rho \, d\rho \, d\phi$

Sub in (2) $\Longrightarrow$ $2\pi \rho H_\phi = I - \int_0^{2\pi} \int_0^{\rho} \frac{I}{(c^2 - b^2)\pi}$

$$2\pi \rho H_\phi = I - \frac{I}{\cancel{\pi}(c^2 - b^2)} \frac{(\rho^2 - b^2)\cancel{(2\pi)}}{\cancel{2}}$$

$$\vec{H} = \frac{I}{2\pi \rho} \left(1 - \frac{(\rho^2 - b^2)}{(c^2 - b^2)} \right) \hat{a_\phi} \quad b \leq \rho \leq c$$

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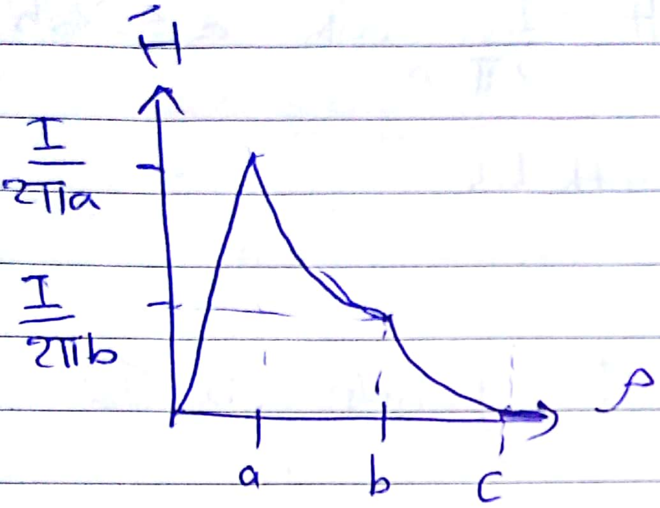
for $\rho > c$ on Path $L4$

$$\oint H \cdot dl_4 = I_{enc} = I - I = 0.$$



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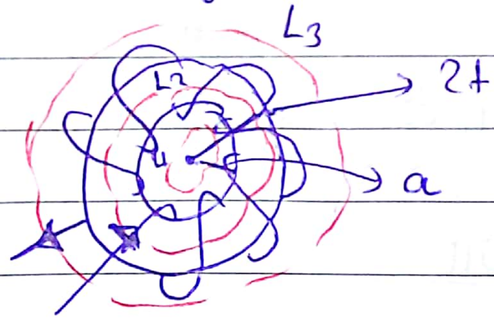
Toroid (find $\bar{H}$ everywhere)



for $0 < \rho < a$ on Path L_1

$\bar{H} = 0$

for $a + 2t < \rho$ on path $L3$

$2\pi\rho H_\phi = I - I = 0$.

$\vec{H} = 0$

$a < \rho < a + 2t$ on path L_2

$$\oint H \cdot dL_2 = I_{enc} = NI$$

$$2\pi\rho H_\phi = NI$$

$$\vec{H} = \frac{NI}{2\pi\rho}\hat{a}_\phi \qquad \rho_o - t < \rho < \rho_o + t$$

ρ_o in middle of Toroid.

$$\vec{H} = \frac{NI}{2\pi\rho_o} a_\phi = \frac{NI}{L_c} = n I \hat{a}_\phi$$

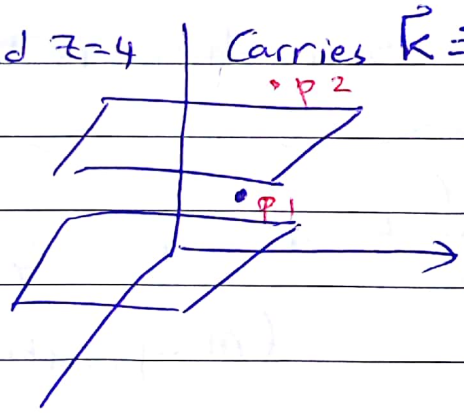
$L_c = 2\pi\rho_o$, $n = \frac{N}{L_c}$ turns density

$$\boxed{\vec{H} = n I a_\phi \text{ for Toroid and infinite Solenoid}}$$

Example :- planes $z = 0$ and $z = 4$ Carries $\vec{K} = 10\hat{a}_x$ and $10\hat{a}_x$ Respectively

Find $\vec{H}$ at $p_1(1,1,1)$

$p_2(0,-3,10)$



① $H = H_0 + H_4$

$$H = \frac{1}{2} K \times \hat{a}_n = \frac{1}{2}(-10)\hat{a}_x \times \hat{a}_z + \frac{1}{2}(10\hat{a}_x) \times -\hat{a}_z = \boxed{10\hat{a}_y \ A/m}$$

at point $p_2 = 0$ $(5\hat{a}_y - 5\hat{a}_y = 0)$.

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Magnatic flux Density ($\bar{B}$)

H A/m²

$B = \mu_0 \bar{H}$ $\quad \mu = \mu_0 \mu_r$ permetivity (H/m^2)

μ_0 = free space permability $\mu_0 = 4\pi * 10^{-7}$ H

$\mu_r = \frac{\mu}{\mu_0}$ (unitless)

for air and conductors ~~gases~~ $\mu_r = 1$

$\sigma = 0$ $\quad \sigma = \infty$

Magnatic flux $\psi_m = \int_s \bar{B} \cdot d\bar{s}$. (Wb)

for closed surface

$$\psi_m = \oint B \cdot ds = 0$$

since there is no single magnatic pole

$\oint B \cdot ds = 0$ 4th Maxwell's eq in integral form
(magnatic gauss's law)

$$\int \nabla \cdot B \, dv = 0$$

$\nabla \cdot B = 0 \rightarrow$ 4th Maxwell's eq in diff form.

Subject Date No.

Maxwell's eq for static fields.

integral form diff form

$\oint D \cdot ds = Q_{enc}$ $\nabla \cdot \bar{D} = \rho_V$

$\oint E \cdot dl = 0$ $\nabla \times \bar{E} = 0$

$\oint H \cdot dl = I_{enc}$ $\nabla \times \bar{H} = J$

$\oint B \cdot ds = 0$ $\nabla \cdot \bar{B} = 0$

Magnatic potential

scalar vector

$(V_m) \to V$ Vector A

$\nabla \cdot (\nabla \times A) = 0$

$\nabla \times (\nabla \cdot V) = 0$

Just if $\underbrace{J = 0 \quad \rho_V = 0}_{\text{source free region}}$.