$\frac{15}{30}$

University of Jordan
School of Engineering
Department of Electrical Engineering
Spring Term – A.Y. 2016-2017, Second Exam
Digital Signal Processing
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Q1. For the causal system defined by the pole-zero pattern shown in the Figure.

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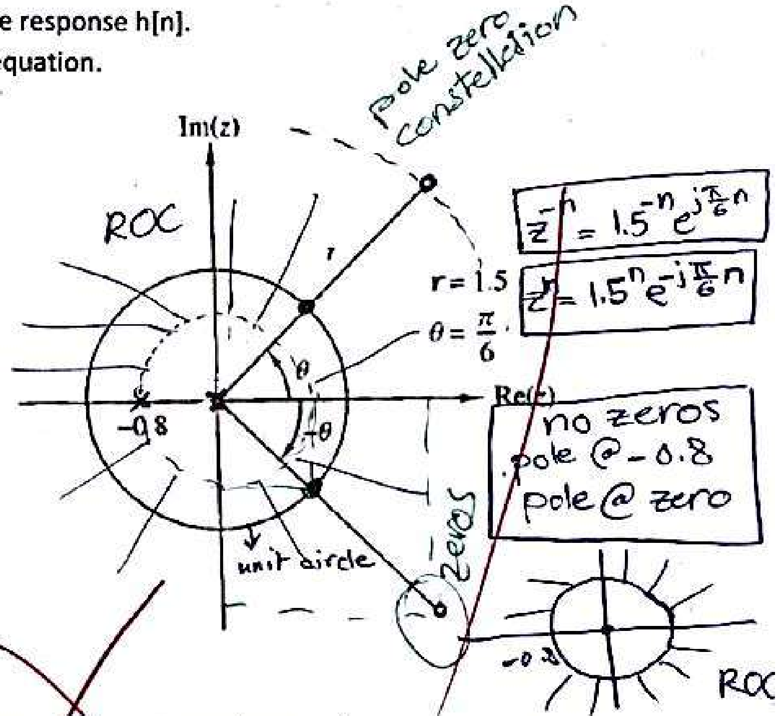
- a) Determine the transfer function H(z) and the impulse response (right sided) given that $H(z)|_{z=1} = 2$.
- b) Is the system stable? Why?
- c) Determine the corresponding impulse response h[n].
- d) Write the corresponding difference equation.



$z^{-n} = 1.5^{-n} e^{j\frac{\pi}{6}n}$

$z^{n} = 1.5^{n} e^{-j\frac{\pi}{6}n}$

no zeros
pole @ −0.8
pole @ zero

(a)

$$H(z) = \frac{9/2.5}{(1+0.8z^{-1})}$$

$$H(1) = \frac{9/2.5}{1+\frac{0.8}{1}} = 2$$

(b) since system is causal then all sequences are right sided giving ROC of

$|z| > 0.8$

which is not bounded; so the system is not stable

(c) $h[n] = \frac{9}{2.5}(-0.8)^n u[n] + 0$

+ 0

(d) input is $\delta[n]$ so $H(z) = \frac{Y(z)}{\delta(z)} = \frac{Y(z)}{1}$

so since $Y(z) = H(z)$

then $y[n] = -0.8y[n] + \frac{9}{2.5}x[n]$

$z(z+0.8)$ $\qquad$ $z^2+0.8z$

$$= \frac{C\,[1-2.6z^{-1}+2.52z^{-1}]}{1+0.8z^{-1}}$$

$H(z)\big|_{z=1}=2$

$$H(1)=\frac{C(1-2.6+2.25]}{1+0.8}=2 \qquad C=5.54$$

2 zeros → complex $\qquad$ 2poles → real

ⓑ ROC : $|z|>0.8$ causal

it has unit circle so system is unstable

ⓒ by long division (num or > dem or)

$$H(z)=5.54\left[2.8125z^{-1}-6.77+\frac{7.77}{1+0.8z^{-1}}\right]$$

$$h[n]=5.54\left[2.815\,\delta[n-1]-6.77\,\delta[n]+7.77(-0.8)^{n}u[n]\right]$$

ⓓ $H(z)=\dfrac{Y(z)}{X(z)}$

$$y[n]=-0.8\,y[n-1]+5.54\,x[n]-5.54(2.6)x(n-1)$$
$$+\,5.54\,(2.25)\,x[n-2]$$

Q2. Consider an LTI discrete time system with impulse response h[n] whose pole-zero-pattern shown,

a) Determine the ROC of the system H(z) if the system is known to be stable. → bounded

b) Is it possible for the given pole-zero plot to corresponds to a causal and stable system at the same time? If so, what is the appropriate ROC?

c) How many possible systems can be associated with the pole-zero pattern, what are their h[n]s and their corresponding ROCs?

Note: neglect the multiplicative constant of the transfer function.

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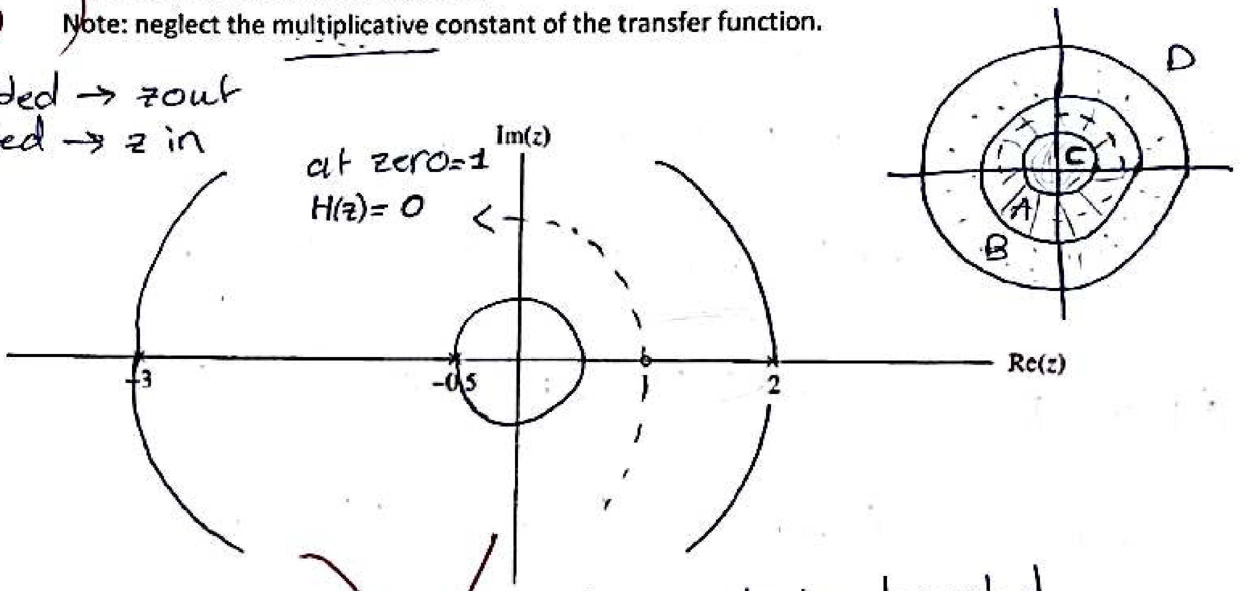
rightsided → zout
Leftsided → z in



ⓐ for the system to be stable it has to be bounded thus we have 3 possible ROC's
ROC A → between pole -0.5 & pole 2 $(0.5<|z|<2)$
ROC B → between pole 2 & pole -3 $(2<|z|<3)$
ROC C → less than pole -0.5 $(|z|<0.5)$

ⓑ the ~~causal~~ system is composed of right sided sequences the three possible ROC's for stability are all composed of two sided sequences thus the system can't be stable & causal at the same time.

ⓒ the system can have 4 possible ROC's shown in the back ⇒

⊛ ROC C → ~~less~~ than -0.5 (left sided)

$h[n] = -(-0.5)^n \cancel{\mu[-n-1]} - (2)^n \mu[-n-1] - (-3)^n \mu[-n-1]$

where $\cancel{|z|} < 0.5$

⊛ ROC A → $0.5 < |z| < 2$ (two sided)

$h[n] = \cancel{(-0.5)^n} \mu[n] - (2)^n \cancel{\mu[-n-1]} - (3)^n \cancel{\mu[-n-1]}$

⊛ ROC B → $2 < |z| < 3$ (two sided)

$h[n] = \cancel{(-0.5)^n} \mu[n] + \cancel{(2)^n} \mu[n] - (-3)^n \cancel{\mu}[-n-1]$

⊛ ROC D → $|z| > 3$ (right sided) $\underline{\underline{\text{causal}}}$

$h[n] = (-0.5)^n \mu[n] + \cancel{(2)^n \mu[n]} + (-3)^n \mu[n]$

LTI system

ⓐ ROC $\frac{1}{2} < |z| < 2$

ⓑ no far causality $|z| 73$

ⓒ ⇑

$$x[\langle n-n_0\rangle_N] \xleftrightarrow{DFT} W_N^{kn_0} X[k]$$

Q3. Consider the sequence

real seq. $x[n] = \{\underset{\uparrow}{\overset{0}{0}}\ \overset{1}{1}\ \overset{2}{2}\ \overset{3}{3}\ \overset{4}{4}\}$ $\quad N = 5$

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a) Find the sequence y[n] for the six-point DFT

$$Y[k] = W_2^k X[k] \qquad k = 0,1,2\cdots,5$$

b) Determine the sequence r[n] with six-point DFT R[n]=real(X[k]).

c) Determine the sequence v[n] with six-point DFT V[n]=im(X[k]).

ⓐ $Y[K] = W_2^K X[K]$ ⇝ circular time shift

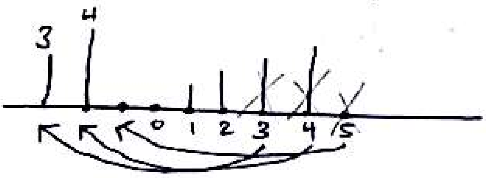
$W_N^{nk} = e^{-j\frac{2\pi nk}{N}} \Rightarrow W_6^{nk} = e^{-j\frac{2\pi nk}{\cancel{6}_3}}$

$W_2^K = e^{-j\frac{2\pi K}{2}}$

$-j\frac{2\pi n_0 k}{6} = -j\frac{2\pi k}{2} \Longrightarrow n_0 = \frac{6}{2} = \boxed{3}$

$y[n] = x[\langle n-3\rangle_6]$

$= \{3\ 4\ 0\ 0\ 1\ 2\}$



ⓑ for a real sequence $x[n] = x^*[n]$.

$X[K] = \sum_{n=0}^{5} x[n] e^{-\frac{j2\pi nk}{6}}$

$= 0 + 1 \times e^{-j\frac{\pi}{3}k} + 2e^{-j\frac{2}{3}\pi k} + 3e^{-j\pi k} + 4e^{-j\frac{4}{3}\pi k} + 0$

$X^*[\langle -k\rangle_N] = e^{j\frac{\pi}{3}K} + 2e^{j\frac{2}{3}\pi k} + 3e^{j\pi k} + 4e^{j\frac{4}{3}\pi k}$

$R[K] = Re\{X[K]\} = \frac{1}{2}\left[X[K] + X^*[\langle -k\rangle_N]\right]$

$r[n] = \frac{1}{2}\left[x[n] + x^*[n]\right] = x[n] = \{0\ 1\ 2\ 3\ 4\ 0\}$

ⓒ $V[K] = Im\{X[K]\} = \frac{1}{2j}\left[X[K] - X^*[\langle -k\rangle_N]\right]$

$v[n] = \frac{1}{2}\left[x[n] - x^*[n]\right] = 0$

ⓑ $r[n] = \text{IDFT}\left\{ \frac{X[K] + X^*[K]}{2} \right\}$

$= \frac{1}{2}\left\{ x[n] + x^*[\langle -n \rangle_6] \right\}$

$= \left\{ 0, \frac{1}{2}, 3, 3, 3, \frac{1}{2} \right\}$

ⓒ $r[n] = \text{IDFT}\left\{ \frac{X[K] - X^*[K]}{2j} \right\}$

$= \frac{1}{2j}\{ x[n] \mp x^*[\langle -n \rangle_6]$

$= \{$

Q4. For an LTI system with impulse response $h[n] = \{1 \quad -2 \quad -2.5\}$, if the input of the system is $x[n] = \{1 \quad 2\}$. Use the DFT and the IDFT to implement the linear convolution between the impulse response and the input of the system.

$x[n] \circledast h[n]$ where $N_x = 2$ $N_h = 3$ $N_L = N_x + N_h - 1 = 4$

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using DFT & IDFT

$x[n]$ ③ $h[n]$ will give $N_c = 3$

thus we must extend both sequences so that $N_c = 4$

$x_e[n] = \{1 \quad 2 \quad 0 \quad 0\}$

$h_e[n] = \{1 \quad -2 \quad -2.5 \quad 0\}$

$x_e[n]$ ④ $h_e[n]$ will give $N_c = 4$

n	0	1	2	$\langle 3\rangle_4$	$\langle 4\rangle_4$		
x	1	2	0	0			
h	1	-2	-2.5	0			
	1	2	0	0			
	0	-2	-4	0	0		
	0	0	-2.5	-5	0	0	
	0	0	0	0	0	0	0
y	1	0	-6.5	-5			

$x_e[n]$ ④ $h_e[n] = y_L[n] = \{1 \; 0 \; -6.5 \; -5\}$

$h[n] = \{1 \quad -2 \quad -2.5\}$

$x[n] = \{1 \quad 2\}$

$h_e[n] = \{1 \quad -2 \quad -2.5 \quad 0\}$

$x_e[n] = \{1 \quad 2 \quad 0 \quad 0\}$

$$D_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & -j \end{bmatrix}$$

$$H_e[K] = \begin{bmatrix} -3.5 \\ 3.5+j2 \\ 0.5 \\ 3.5-j2 \end{bmatrix} \qquad X_e[K] = \begin{bmatrix} 3 \\ 1-j2 \\ -1 \\ 1+j2 \end{bmatrix}$$

$$y[n] = \begin{bmatrix} 1 \\ 0 \\ -6.5 \\ -5 \end{bmatrix}$$