$\frac{16}{20}$

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Digital Signal Processing

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Q1. For the signal $y[n] = \left[5\left(\frac{1}{2}\right)^n - 2\left(\frac{1}{4}\right)^n\right]\mu[n]$,Find the energy and power of the signal. Is it energy or power signal? Justify your answer.

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$$\mathcal{E}_x = \sum_{n=-\infty}^{\infty} |Y[n]|^2 = \cancel{5\sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{2n} - 2\sum_{n=0}^{\infty}\left(\tfrac{1}{4}\right)^{2n}}$$

$$\cancel{= 5 * \frac{1}{1-\frac{1}{4}} - 2 * \frac{1}{1-\frac{1}{16}}}$$

$$\cancel{= \frac{20}{3} - \frac{32}{15}}$$

$$\cancel{= \frac{68}{15}}$$

$$P_x = \lim_{k\to\infty} \frac{1}{2k+1} \sum_{n=-k}^{k} |Y[n]|^2 = 0$$

(finite energy
zero Power)

$\Rightarrow$ ~~Energy Signal~~

$\boxed{\text{Energy Signal}}$

$$\lim_{k\to\infty} \frac{516/35}{2k+1} = \text{zero}$$

$$\mathcal{E}_x = \sum_{n=-\infty}^{\infty} |Y[n]|^2 = \sum_{n=0}^{\infty}\left(5\left(\tfrac{1}{2}\right)^n - 2\left(\tfrac{1}{4}\right)^n\right)^2 = \sum_{n=0}^{\infty} 25\left(\tfrac{1}{4}\right)^n + 4\left(\tfrac{1}{16}\right)^n - 20\left(\tfrac{1}{2}\right)^n\left(\tfrac{1}{4}\right)^n$$

$$= 25\sum_{n=0}^{\infty}\left(\tfrac{1}{4}\right)^n + 4\sum_{n=0}^{\infty}\left(\tfrac{1}{16}\right)^n - 20\sum_{n=0}^{\infty}\left(\tfrac{1}{8}\right)^n$$

$$= \frac{25}{1-\frac{1}{4}} + \frac{4}{1-\frac{1}{16}} - \frac{20}{1-\frac{1}{8}} = \frac{516}{35}$$

Q2. For the system:

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$$y[n] = 4x[n] + 2x[n-1] + 0.8y[n-1]$$

1) Find the transfer function $H(e^{j\omega})$.
2) Find y[n] in a compact form if $x[n] = (0.4)^n \mu[n]$.

1] $Y(e^{jw}) = 4X(e^{jw}) + 2X(e^{jw})e^{-jw} + 0.8Y(e^{jw})e^{-jw}$

$Y(e^{jw})(1 - 0.8e^{-jw}) = X(e^{jw})(4 + 2e^{-jw})$

$$\boxed{H(e^{jw}) = \frac{4 + 2e^{-jw}}{1 - 0.8e^{-jw}}}$$

2] $x[n] = (0.4)^n \mu[n]$

$X(e^{jw}) \cancel{X(e^{jw})} = \dfrac{1}{1 - 0.4e^{-jw}}$

$Y(e^{jw}) = X(e^{jw}) \cdot H(e^{jw})$

$$= \frac{4 + 2e^{-jw}}{(1 - 0.8e^{-jw})(1 - 0.4e^{-jw})}$$

$$= \frac{4 + 2e^{-jw}}{1 - 0.8e^{-jw} - 0.4e^{-jw} + 0.32e^{-j2w}}$$

$-1.2e^{-jw}$

$$1 - 1.2e^{-jw} + 0.32 \,\overline{\big)\, 4 + 2e^{-jw}}\quad e^{j2w} \quad 4e^{jw}$$

$\cancel{\dfrac{4 + 2e^{-jw}}{e^{-j2w}}}$

$\cancel{y[n] = 4(0.4)^n \mu[n] + 2\,(0.4)^{n-1}\mu[n-1] + 0.8\,y[n-1]}$

$\cancel{y[n]}$

$\cancel{h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{4 + 2e^{-jw}}{1 - 0.8e^{-jw}} e^{jwn}\, dw}$

$\cancel{y[n] = x[n] \circledast h[n]}$

$$Y(e^{jw}) = \frac{4 + 2e^{-jw}}{1 - 1.2e^{-jw} + 0.32e^{-j2w}}$$

$$y[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} Y(e^{jw})\, e^{jwn}\, dw$$

Q3. a) If $x[n] = \cos(0.55\pi n + 30°)$. Is x[n] periodic or not? If yes, what is the fundamental period N_f?

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b) Evaluate the inverse DTFT of $X(e^{j\omega}) = 3 + 2\sum_{l=0}^{N}\cos(\omega l)$.

a) $X[n] = A\cos(\omega_o n + \phi)$

$$\frac{2\pi}{\omega_o} = \frac{N_f}{r}$$

* Yes it's periodic with $N_f = 40$

$$\frac{2\pi}{0.55\pi} = \frac{N_f}{r}$$

$$\frac{40}{11} \quad \frac{N_f}{r} = \frac{40}{11}$$

b) $$X[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi}\left(3 + 2\sum_{l=0}^{N}\frac{e^{j\omega l} + e^{-j\omega l}}{2}\right) e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} 3e^{j\omega n} + \left(\frac{1-e^{j\omega(N+1)}}{1-e^{j\omega}} + \frac{1-e^{-j\omega(N+1)}}{1-e^{-j\omega}}\right) e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left[\frac{3e^{j\omega n}}{jn}\Big|_{-\pi}^{\pi} + \int_{-\pi}^{\pi}\left(\frac{e^{j\omega n}}{1-e^{j\omega}} - \frac{e^{j\omega(n+N+1)}}{1-e^{j\omega}} + \frac{e^{j\omega n}}{1-e^{-j\omega}} - \frac{e^{-j\omega(N+1-n)}}{1-e^{-j\omega}}\right) d\omega\right.$$

~~X[n] = {3, n=0~~

$$X(e^{j\omega}) = 3 + 2\sum_{l=0}^{N}\frac{e^{j\omega l} + e^{-j\omega l}}{2} = 3 + \frac{1-e^{j\omega(N+1)}}{1-e^{j\omega}} + \frac{1-e^{-j\omega(N+1)}}{1-e^{-j\omega}}$$

$$X[n] = 3 + \sum_{n=0}^{N}(e^{j\omega n} + e^{-j\omega n})$$

~~X[n] = 2{ n=0~~

X[n] = {

Q4. If the impulse response of a system $h[n] = (2)^{n-5}\mu[n-5]$. What can you tell about the following properties of the system?

1) Linear or nonlinear.
2) Causal or noncausal.
3) Time invariant or time varying.
4) Stable or unstable.

Justify your answers.

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$\alpha^n \mu[n], \alpha < 1$

$y[n] = h[n] \circledast x[n]$

$h[n-k]\ x[n-k]$

1] $h[n] = \sum_{n=5} 2^{n-5}\ \mu[n-5]$

$\alpha h_1[n] + \beta h_2[n] = \alpha T[x_1[n]] + \beta T[x_2[n]]$

linear

2] ~~Non Causal~~

$h[0] = 2\ 2^{-5}$

2] Causal, since $h[n] = 0$ for $n < 0$

3] $y[n-k] \stackrel{?}{=} T[x[n-k]]$

$h[n-k] = 2^{n-5}\ \mu[n-5-k]$

Time Invariant

4] $\sum_{k=0}^{n} |h[k]| < \infty$

Un Stable

$\sum \alpha^n \mu[n] \quad |\alpha| < 1$ to be stable

⇒ ~~linear, Causal, Time Invariant, Stable~~

⇒ Causal, Un Stable, linear, Time Invariant