$\frac{23}{30}$

The Jordan University — **Engineering Math. I (0301202)** — **Second Exam**

Mathematics Department — **14/12/2013**

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Section: .... **Instructor:** ......

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(1) Determine a Suitable form for the particular solution if the method of undetermined coefficients is to be used for the D.E.

$$y''' - y'' + 2y = e^{-x} + xe^{x}\sin(x)$$

$y_{p_1} = Ae^{-x}$ ✗

$y_{p_2} = xe^{x}(Ax+B)\sin x + xe^{x}\cos x\,(Cx+D)$??

why?

$y_c = ??$

2

$y''' - y'' + 2y = 0$

$r^3 - r^2 + 2 = 0$

$(r+1)(r-2) = 0$

$r = -1$, $r = 2$

$y_c = c_1e^{-x} + c_2e^{2x}$

(2) Find the general solution of

$$y^{(5)} + 4y''' + 5y'' = 0$$

$r^5 + 4r^3 + 5r^2 = 0$

$r^2(r^3 + 4r + 5) = 0 \rightarrow (r+1)(r^2 - r + 5) = 0$

1	0	4	5
	-1	1	-5
1	-1	5	0

$r_1 = 0$, $r_2 = 0$, $r_3 = -1$

$r^2 - r + 5$

$r_4 = \frac{1}{2} - \frac{\sqrt{19}}{2}i$

$r_5 = \frac{1}{2} + \frac{\sqrt{19}}{2}i$

$\alpha = \frac{1}{2}$, $\beta = \frac{\sqrt{19}}{2}$

$D = 1 - 20 = -19$

G.S.: $y = c_1e^{0} + c_2e^{0}x + c_3e^{-x} + \left(c_4e^{\frac{1}{2}x}\cos\frac{\sqrt{19}}{2}x + c_5e^{\frac{1}{2}x}\sin\frac{\sqrt{19}}{2}x\right)$

5.5

(3) Use Cramer's rule to find the value of x_2 for the following system of linear equations

$$x_1 + 2x_2 = 6$$
$$-3x_1 - 6x_3 = 3$$
$$x_1 - 2x_2 + 3x_3 = 2$$

$$\begin{pmatrix} 1 & 2 & 0 \\ -3 & 0 & -6 \\ 1 & -2 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix}$$

$$\vec{A} \cdot \vec{x} = \vec{b}$$

$$\det(A) = 1(0-12) - 2(-9+6) + 0$$
$$= -12 + 6 = -6$$

$$A_1 = \begin{pmatrix} 6 & 2 & 0 \\ 3 & 0 & -6 \\ 2 & -2 & 3 \end{pmatrix}$$

$$\det(A_1) = 6(0-12) - 2(9+12) + 0$$
$$= -72 - 2(21)$$
$$= -72 - 42$$
$$= -114$$

$$A_2 = \begin{pmatrix} 1 & 6 & 0 \\ -3 & 3 & -6 \\ 1 & 2 & 3 \end{pmatrix}$$

$$\det(A_2) = 1(9+12) - 6(-9+6) + 0$$
$$= 21 - 6(-3)$$
$$= 21 + 18$$
$$= 39$$

$$x_2 = \frac{\det(A_2)}{\det(A)} = \frac{39}{-6}$$

6

(4) Find the general Solution of the following system

$$Y' = \boxed{\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}} Y \quad \textcircled{1} \longrightarrow \vec{A}$$

$$\begin{pmatrix} 3-\lambda & -4 \\ 1 & -1-\lambda \end{pmatrix}$$

$(3-\lambda)(-1-\lambda) + 4 = 0$

$-3 - 3\lambda + \lambda + \lambda^2 + 4 = 0$

$\lambda^2 - 2\lambda + 1 = 0$

$(\lambda-1)(\lambda-1) = 0$

$\lambda_1 = 1$

$\lambda_2 = 1$

$X\lambda_1 = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ (2)

$$\begin{pmatrix} 2 & -4 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$2a_1 - 4a_2 = 0$

$2a_1 = 4a_2$

$\boxed{a_1 = 2a_2}$

$a_1 - 2a_2 = 0$

$a_1 = 2a_2$

$X\lambda_1 = \begin{pmatrix} 2a_2 \\ a_2 \end{pmatrix} = a_2\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

for $a_2 = 1$ $\quad X\lambda_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

(3) $(\vec{A} - \lambda_2 I)X_{\lambda_2} = X_{\lambda_1}$ $\quad X_{\lambda_2} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$

$$\begin{pmatrix} 2 & -4 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$2b_1 - 4b_2 = 2 \Rightarrow b_1 - 2b_2 = 1$

~~$b_1 = 2b_2 = 1$~~

$b_1 = 2b_2$

$X\lambda_2 = \begin{pmatrix} 2b_2 \\ b_2 \end{pmatrix} = b_2\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

for $b_2 = 1$ $\quad X\lambda_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

5

$\vec{y} = c_1 X_{\lambda_1} e^{\lambda t} + c_2(t X_{\lambda_1} e^{\lambda t} + X_{\lambda_2} e^{\lambda t})$

$\vec{y} = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{t} + c_2(t \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{t} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{t})$

$y_1 = 2c_1 e^t + 2c_2 t e^t + 2e^t$

$y_2 = c_1 e^t + c_2 t e^t + e^t$

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(5) Find the general Solution of the following system using variation of parameters

$$Y' = \begin{pmatrix} 1 & -4 \\ -4 & 1 \end{pmatrix} Y + \begin{pmatrix} e^t \\ t \end{pmatrix} \quad \text{--- (1)} \quad (*)$$

$\begin{pmatrix} 1 & -4 \\ -4 & 1 \end{pmatrix} \to A$

1

$$\begin{pmatrix} 1-\lambda & -4 \\ -4 & 1-\lambda \end{pmatrix}$$

$(1-\lambda)^2 - 16 = 0$

$(1-\lambda)^2 = 16$

$1-\lambda = \pm 4$

$1-\lambda = 4 \Rightarrow \boxed{\lambda_1 = -3}$

$1-\lambda = -4 \Rightarrow \boxed{\lambda_2 = 5}$

$\cancel{-\lambda = \pm 4}$

2

$X_{\lambda_1} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$

$$\begin{pmatrix} 4 & -4 \\ -4 & 4 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$4a_1 - 4a_2 = 0$

$4a_1 = 4a_2$

$a_1 = a_2$

$X_{\lambda_1} = \begin{pmatrix} a_1 \\ a_1 \end{pmatrix} = a_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

for $a_1 = 1$ $\boxed{X_{\lambda_1} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}}$

3

$X_{\lambda_2} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$

$$\begin{pmatrix} -4 & -4 \\ -4 & -4 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$-4b_1 - 4b_2 = 0$

$-4b_1 = 4b_2$

$b_2 = -b_1$

$X_{\lambda_2} = \begin{pmatrix} b_1 \\ -b_1 \end{pmatrix} = b_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

for $b_1 = 1$ $\boxed{X_{\lambda_2} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}}$

4

$\vec{y}_c = c_1 e^{\lambda_1 t} X_{\lambda_1} + c_2 e^{\lambda_2 t} X_{\lambda_2}$

$\vec{y}_c = c_1 e^{-3t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{5t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

5

$g = \begin{pmatrix} e^t \\ t \end{pmatrix} = \begin{pmatrix} 0 \\ t \end{pmatrix} + \begin{pmatrix} e^t \\ 0 \end{pmatrix}$

$\begin{pmatrix} 0 \\ 1 \end{pmatrix} t \qquad \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^t$

$\vec{y}_{p_1} = \vec{A} t$

$\vec{y}_{p_2} = \vec{B} e^t$

$\boxed{\vec{y}_p = \vec{A} t + \vec{B} e^t}$

$\boxed{\vec{y}_p' = \vec{A} + \vec{B} e^t}$ فسداي

4.5

sub in --(*):

$\vec{A} + \vec{B} e^t = A(\vec{A} t + \vec{B} e^t) + \boxed{\begin{pmatrix} 0 \\ 1 \end{pmatrix} t + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^t}$

$\begin{pmatrix} 1 & -4 \\ -4 & 1 \end{pmatrix} \qquad \begin{pmatrix} e^t \\ t \end{pmatrix}$

$\vec{A} + \vec{B} e^t = \boxed{A\vec{A} t} + A\vec{B} e^t + \boxed{\begin{pmatrix} 0 \\ 1 \end{pmatrix} t} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^t$

$-A\vec{A} t + \vec{B} e^t (1 - A) + \vec{A} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} t + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^t$

تابع خلف الورقة