The University of Jordan
Department of Mathematics

Second Exam
Math 201

First Semester
05.12.2010

Name: .... Section:

Student Number: ... Instructor:..............

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1. Reparametrize the curve $\vec{r}(t)=\langle e^{t}, e^{-t}, \sqrt{2}t\rangle$ with respect to arc length parameter s, with $(1,1,0)$ as reference point. (5 points)

$r'(t) = \langle e^{t}, -e^{-t}, \cancel{\frac{1}{2\sqrt{2t}}} \rangle$

$L = \int_a^b |r'(t)|\,dt$

$|r'(t)| = \sqrt{(e^{t})^2 + (-e^{-t})^2 + \left(\frac{1}{2\sqrt{2t}}\right)^2} = \sqrt{e^{2t} + e^{-2t} + \frac{1}{8t}}$

$|r'(t)| = \sqrt{e^{2t} + \frac{1}{e^{2t}} + \frac{1}{8t}}$

$L = \int_0^1 \sqrt{e^{2t} + \frac{1}{e^{2t}} + \frac{1}{8t}}$

$L = \int_0^1 \left(e^{2t} + \frac{1}{e^{2t}} + \frac{1}{8t}\right)^{\frac{1}{2}}$

$\left[\frac{2}{3}\left(e^{2t} + \frac{1}{e^{2t}} + \frac{1}{8t}\right)^{\frac{3}{2}}\right]_0^1 =$

2

2. Let $\vec{r_1}(t)=\langle 2e^{-t}, \cos t, t^2+3\rangle$ and $\vec{r_2}(t)=\langle 1-t, t^2, t^3+4\rangle$ be two curves. Find the acute angle between the tangent lines to the graph of $\vec{r_1}(t)$, $\vec{r_2}(t)$ at the point of intersection $p(2,1,3)$. (4 points)

$$\vec{r_1} \times \vec{r_2} = \begin{vmatrix} i & j & k \\ 2e^{-t} & \cos t & t^2+3 \\ 1-t & t^2 & t^3+4 \end{vmatrix}$$

$\cos\theta = \frac{\overrightarrow{r_1 \times r_2}}{|\overrightarrow{r_1 \times r_2}|}$

$$\kappa = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3}$$

3. $\vec{r}(t) = \langle t - \sin t, 1 - \cos t, t \rangle$

Find the curvature κ at $t = 0$. **(4 points)**

4

$* \; r'(t) = \langle 1 - \cos t, \sin t, 1 \rangle \qquad r'(t=0) = \langle 0, 0, 1 \rangle$

$* \; r''(t) = \langle \sin t, \cos t, 0 \rangle \qquad r''(t=0) = \langle 0, 1, 0 \rangle$

$$* \; r'(t) \times r''(t) = \begin{vmatrix} \oplus i & \ominus j & \oplus k \\ 1-\cos t & \sin t & 1 \\ \sin t & \cos t & 0 \end{vmatrix} = -\cos t\, i - \sin t\, j + (\cos t - \cos^2 t - \sin^2 t)\, k$$

$* \; r'(t=0) \times r''(t=0) = \langle -1, 0, 0 \rangle$

$|r'(t) \times r''(t)| = \sqrt{(-1)^2 + (0)^2 + (0)^2} = \sqrt{1}$

$|r'(t)| = \sqrt{(0)^2 + (0)^2 + (1)^2} = \sqrt{1}$

$$\kappa = \frac{\sqrt{1}}{\sqrt{1}} = \underline{\underline{1}}$$ Final Ans

4. Find and sketch the domain of $f(x,y) = \dfrac{\ln(4 - x^2 - y^2)}{\sqrt{y}}$. **(4 points)**

① $\ln(4 - x^2 - y^2) \Rightarrow 4 - x^2 - y^2 > 0 \Rightarrow -x^2 - y^2 > -4 \;\; (\cdot -1)$

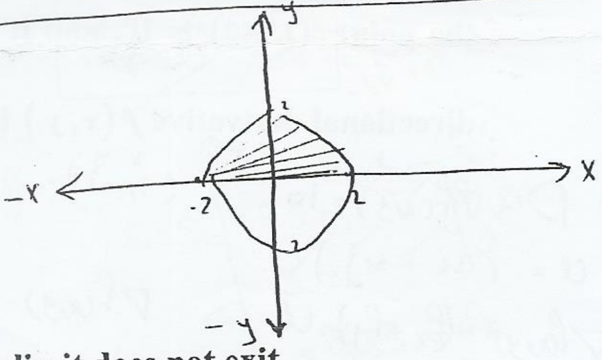
$x^2 + y^2 < 4$

② $- \sqrt{y} \Rightarrow y > 0$

$$\boxed{\text{The Domain is } \{(x,y) \mid x^2 + y^2 < 4,\ y > 0\}}$$

Final Ans

4



5. Find the limit, if it exits, or show that the limit does not exit. **(4 points)**

$$\lim_{(x,y) \to (0,0)} \frac{xy^2}{x^2 + y^4}$$

$$\lim_{r \to 0} \frac{(r\cos\theta)(r\sin\theta)^2}{(r\cos\theta)^2 + (r\sin\theta)^4} = \cancel{\text{does not exist}} \quad \cancel{\text{does not exist}}$$

along y-axis put $x = 0$

$$\lim_{(x,y) \to (0,0)} \frac{0}{y^4} = 0$$

along x-axis put $y = 0$

$$\lim_{(x,y) \to (0,0)} \frac{0}{x^2 + 0} = 0$$

2

along put $y = x$

$$\lim_{(x,y) \to (0,0)} \frac{x \cdot x^2}{x^2 + x^4} = \frac{x^3}{x^2 + x^4} = \frac{x^3}{x^2(1 + x^2)} = \frac{x}{(1 + x^2)} \quad \boxed{0}$$

final

6. Let $z = f(x, y)$, $x = r\cos\theta$, $y = r\sin\theta$. Find $\frac{\partial^2 z}{\partial r\,\partial\theta}$. **(4 points)**

$f_{\theta r} = ?$

2

$\frac{\partial x}{\partial \theta} = -r\sin\theta$

$Z = Z_x \times \frac{\partial x}{\partial \theta} + Z_y \times \frac{\partial y}{\partial \theta}$

$\frac{\partial y}{\partial \varphi} = r\cos\theta$

$= Z_x \times (-r\sin\theta) + Z_y \times r\cos\theta$

$(-r\sin\theta) \times (-r\sin\theta) + (r\cos\theta)(r\cos\theta)$

$Z_x = \frac{dx}{dy} = -r\sin\theta$

$Z_y = \frac{dy}{d\theta} = r\cos\theta$

$= Z$

final Ans $\boxed{Z = r^2\sin^2\theta + r^2\cos^2\theta}$

$Z_{\theta r} = \left(Z_{xy} \cdot \frac{\partial y}{\partial r} + Z_{xx} \cdot \frac{\partial x}{\partial r}\right) \cdot \frac{dx}{d\varphi} + \left(Z_{yy} \cdot \frac{dy}{\partial r} + \partial_{yx} \cdot \frac{dx}{\partial r}\right)\frac{dy}{d\theta}$

7. **The maximum value of the directional derivative of the function $f(x, y)$ at the point $(1, -2)$ is 10, and it occurs in the direction of $3\hat{i} - 4\hat{j}$. Find the directional derivative $f(x, y)$ in the direction of $\hat{i} - \hat{j}$.** **(5 points)**

$D_u \nabla f(x,y) = 10$ $\quad (1, -2)$

$u = (3i - 4j)$

$\nabla f(x,y) = (f_x + f_y) \cdot \vec{u} \Rightarrow \nabla f(x,y) = f_x \cdot a + f_y \cdot b$

$10 = (f_x + f_y) \cdot \langle 3, -4 \rangle$

$10 = 3f_x - 4f_y$

$\nabla f(x,y) =$

$V_\theta = \langle 1, -1 \rangle \langle \frac{3}{5} i - \frac{4}{5} j \rangle$

$\langle i - j \rangle$

$D_u = |\nabla f(x,y)|$

$u = \frac{\langle 3, -4 \rangle}{\sqrt{(3)^2 + (-4)^2}}$

$u = \frac{3}{5} i - \frac{4}{5} j$

$D_u = |D| \Rightarrow 10$

$= 10 \cdot \frac{3}{5} + 10 \cdot \frac{-4}{5}$

$= -2$

final Ans

$= \boxed{6i - 8j}$

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25

Dept. of Math. Math. 0301201b Calculus 3 Exam.I
Univ. Of Jordan Sun.31-7-2005 6:40-7:40 P.M.

Name of Student :
ID.# : 0041160
Section : 1,50 – 2,50

CIRCLE the Correct answer or write the final answer .

* Consider the vectors : $\vec{a} = 2i - j + k, \vec{b} = 3i + 2j - 2k$, and
answer questions : 1,2,and 3 .

1- $\mathrm{Pr}\,oj_{\vec{a}}\vec{b}$ is the vector :

(a) $\frac{1}{3}(3i+2j-2k)$ (b) $\frac{2}{\sqrt{17}}(3i+2j-2k)$

(c) $\frac{1}{3}(2i-j+k)$ (d) $\frac{2}{\sqrt{17}}(2i-j+k)$

2- $\|\overrightarrow{2a}\times\vec{b}\| =$ $\boxed{14\hat{j} + 14\hat{k}}$

3-A vector orthogonal to both $\vec{a},\vec{b}$ and of norm 3 is : $\boxed{\frac{3}{\sqrt{98}}(7\hat{j}+7\hat{k})}$

$\|a\|^2 = 9+4+4$

$6 + -2 - 2 = \frac{2}{6} = \frac{1}{3}$

$\sqrt{98} \cdot x = 3$

$x = \frac{3}{\sqrt{98}}$

* Consider the the vectors $\vec{a},\vec{b}$ such that $\|\vec{a}\| = 5, \|\vec{b}\| = 3, \|\vec{a}-\vec{b}\| = 2$
then answer questions : 4 and 5 .

4- $\|\overrightarrow{2a}+\vec{b}\| =$ $\boxed{}$

5- $\overrightarrow{3b}\bullet(\overrightarrow{2a}\times\vec{b}) =$ $\boxed{0}$

$n = <1, 2, -1>$

6- Parametric equations of the line passing through the point $P(2,3,1)$ and perpendicular to the plane $x+2y-z=10$ are :

(a) $x=3-t, y=-2+2t, z=-4+t$ (b) $x=2+t, y=3+2t, z=1-t$

(c) $x=1+2t, y=2+3t, z=-1+1t$ (d) $x=1+3t, y=2-2t, z=1-4t$

* Consider the points : $P(2,1,0), Q(4,0,1), R(5,3,-2), S(2,3,1)$, and answer questions : 7,8, 9 and 10 .

$<2, -1, 1>$

7- Parametric equations of the line passing through the points P,Q are :

(a) $x=2+3t, y=1+2t, z=1-2t$ (b) $x=2+2t, y=1-t, z=+t$

(c) $x=2+3t, y=1-2t, z=-2t$ (d) $x=2+3t, y=1+2t, z=-2t$

PQ

8- An equations of the plane determined by the points P,Q,S is :

(a) $3x+2y-4z=8$ (b) $3x-4z=1$

(c) $y+z=1$ (d) $3x+2y=8$

(P X Q)

$PQ = <2, -1, 1>$

$PS = <0, 2, 1>$

$<3, 2, -4>$ $\sqrt{9+4+16}$

9-The area of the triangle with vertices P,Q,S is : $\boxed{\dfrac{\sqrt{29}}{2}}$

10 - Determine whether the points P,Q,S are collinear (justify your answer)

$\boxed{\text{not Collinear}}$ $\boxed{PQ \times PS \neq 0}$

$\hat{k} + 2\hat{i} + 2\hat{j}$

$\hat{i}$ $\hat{j}$ $\hat{k}$ $\hat{i}$ $\hat{j}$

2 -1 1 2 -1

0 2 1 0 2

$\hat{i} + 4\hat{k}$

$-3\hat{i} + 4\hat{k} - 2\hat{j}$

$<-3, -2, 4>$

-6 -2 4

$\overrightarrow{P_1P_2} = v = \langle 1, 1, -2 \rangle$

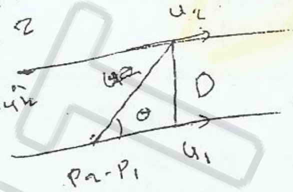
$P_1 \langle 1, 0, 1 \rangle$

$P_2 \langle 2, 1, -1 \rangle$

$\sin\theta = \frac{\|D\|}{\|u_1\|}$

$D = \|v\|$

$\frac{\|v \times u_1\|}{\|u_1\|}$



6

parallel

- Consider the lines :

$L_1 : x = 1 + t, y = 2t, z = 1 - t; L_2 : x = 2 - 2t, y = 1 - 4t, z = -1 + 2t$,

and answer questions : 1`1,and12 .

$u_1 \langle 1, 2, -1 \rangle, u_2 = \langle -2, -4, 2 \rangle$

11-The distance between the two lines L_1, L_2 is equal to :

$\frac{\sqrt{35}}{6}$

12- An equations of the plane determined by the lines L_1, L_2 is :

(a) $8x - y + 5z = 21$ (b) $3x + y + z = 4$

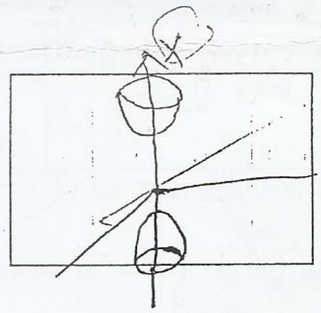
(c) $3x - y + z = 0$ (d) $3x - y + z = 4$

In questions 13,14 Identify and sketch .

13- (a) $z^2 + y^2 = x^2 - 4$

$x^2 - z^2 - y^2 = 4$

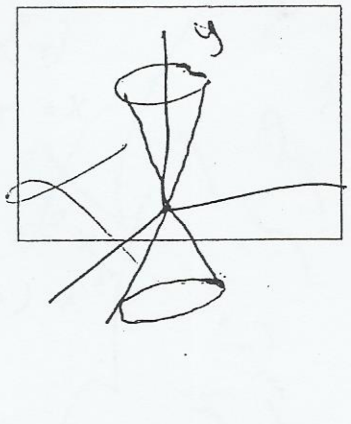
Hyp. of two sheet



$y = \frac{x^2}{2} + \frac{z^2}{2}$

(b) $2y = x^2 + z^2$

Circular cone



$\pm y$

$x = \rho \quad , y = 0 \quad , z$

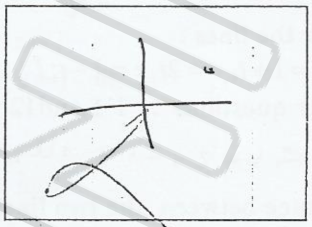
$= \rho \cos \frac{\pi}{6}$

$\cos \phi =$

$\rho = 0 \; , \theta = 0 \quad \phi = \frac{\pi}{6} \qquad \frac{z}{\rho} \qquad \frac{\sqrt{3}}{2}$

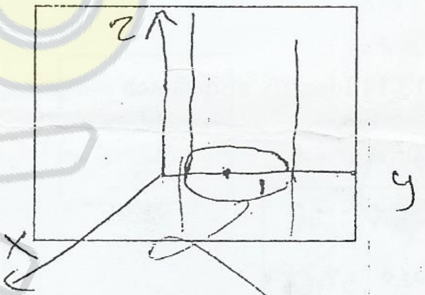
14- (a) $\varphi = \dfrac{\pi}{6}$

~~plane~~ point



(b) $r = 2\sin\theta$

cylindrical surface



$r = 2\sin\theta$

$x = r\cos\theta \quad , \quad y = \sin\theta$

$r^2 = 2r\sin\theta$

$r^2 =$

$\boxed{r^2} = 2y$

$r = \sqrt{x^2 + y^2}$

$x = 2y$

$\frac{z}{z} + \frac{z}{z} = z$

at $x = 0$

$x^2 + y^2 - 2y = 0$

$x^2 + (y - 1)^2 = 1$