$\frac{23.5}{25}$

The University of Jordan Mathematics Department

Math 202 / First Exam
Date: 02/04/2014

رقم المقعد

Name: علي بسام صبحي الرفاعي Student Number: 0130355 Section: $\boxed{44}$

---

(1) Solve the initial value problem: $xy' = y + xe^{y/x}$, $y(1) = 0$.

$$xy' = y + x e^{\frac{y}{x}}$$

homogenous

$$y' = \frac{y}{x} + e^{\frac{y}{x}}$$

5

$$u = \frac{y}{x} \Rightarrow u'x + u = y'$$

$$u'x + u = u + e^{u}$$

$$u'x = e^{u}$$

$$x\frac{du}{dx} = e^{u}$$

$$\int \frac{dx}{x} = \int \frac{du}{e^{u}}$$

$$\cancel{\ln|x| + c} = \int \frac{dx}{x} = \int e^{-u}\,du$$

$$\ln|x| + c = -e^{-u}$$

$$\ln|x| + c = -e^{-\frac{y}{x}}$$

$$y(1) = 0 \Rightarrow \ln|1| + c = -e^{-\frac{0}{1}}$$

$$\boxed{c = -1}$$

$$\therefore \ln|x| - 1 = -e^{-\frac{y}{x}}$$

(2) Find the general solution of the given differential equation:
$2y^2y'' + 2y(y')^2 = 1$.

$u = y'$ $\quad$ x missing

$u\dfrac{du}{dy} = y''$

5

$2y^2 u\dfrac{du}{dy} + 2yu^2 = 1$

$2y^2 u\dfrac{du}{dy} = 1 - 2yu^2$

$2y^2 u\,du = (1 - 2yu^2)\,dy$

$\cancel{2y^2 u\,du - 1 + 2yu^2 dy = 0}$

$\cancel{2y^2 u\,du}$

$2y^2 u\,du - (1 - 2yu^2)\,dy = 0$

$\dfrac{\partial M}{\partial y} = 4yu \qquad \dfrac{\partial N}{\partial u} = 4uy$

$\underbrace{\qquad\text{Exact}\qquad}$

$u(u, y) = \int M(u,y)\,du + \int N(y)\,dy = c$

$\int 2y^2u\,du + \int 0\,dy = c$

-1

$y^2u^2 - y = c$

$u^2 = \dfrac{c}{y^2}$

$(y')^2 = \dfrac{c}{y^2}$ $\Rightarrow \dfrac{y^2}{2} = \sqrt{c}x + c_1$

$y' = \dfrac{\sqrt{c}}{y}$ $\qquad y = \sqrt{2x\sqrt{c} + 2c_1}$

$\dfrac{dy}{dx} = \dfrac{\sqrt{c}}{y}$

$\int y\,dy = \int \sqrt{c}\,dx$

(3) Let $4y'' - 16y' + 17y = x\cos(\frac{x}{2}) + e^{2x}\sin^2(\frac{x}{4})$. Find a suitable form of the particular solution y_p if the method of undeterminate coefficients is to be used.

y_c:

$4\lambda^2 - 16\lambda + 17 = 0$

$\alpha = \frac{-b}{2a} = \frac{16}{8} = 2$

$\beta = \frac{\sqrt{|D|}}{2a} = \frac{4}{8} = \frac{1}{2}$

$\lambda = 2 \pm \frac{1}{2} i$

$D = \sqrt{b^2 - 4AC} = \sqrt{-16}$

$\cancel{D = \sqrt{(16)^2 - 4 * 17 * 4}}$

$\cancel{= \sqrt{27}}$

$\lambda = \pm\frac{1}{2}i, \pm\frac{1}{2}i, 2, 2, 2 \pm \frac{1}{2}i$

$* \; e^{2x}\sin^2(\frac{x}{4}) = e^{2x}\left(\frac{1}{2} - \frac{1}{2}\cos(\frac{x}{2})\right) = \frac{1}{2}e^{2x} - \frac{1}{2}e^{2x}\cos(\frac{x}{2})$

$y_p = (Ax+b)(C\cos x + D\sin x) + Fe^{2x} + xe^{2x}(N\cos(\frac{x}{2}) + R\sin\frac{x}{2})$

$\therefore y_p = (Ax+b)(C\cos x + D\sin x) + Fe^{2x} + xe^{2x}(N\cos(\frac{x}{2}) + R\sin(\frac{x}{2}))$

4

(4) Find the differential equation whose general solution is given by $y = c_1\cos(\ln x) + c_2\sin(\ln x)$.

$\cancel{\lambda = 0}$ $\quad \lambda = +i, +i$

$\cancel{(\lambda+1)(\lambda+1) = 0}$

$\lambda^2 \quad \cancel{+\lambda}$ $\quad (m+1)(m+1) = 0$

$m^2 + 2m + 1 = 0$

$\cancel{m^2 + m}$ $\quad m^2 - m + m + 2m + 1 = 0$

$m(m-1) + 3m + 1 = 0$

$a = 1 \quad b = 3 \quad 1 \quad c = 1$

$\times$

$\therefore \boxed{x^2y'' + 3xy' + y = 0}$

The D.E

3.5

(5) Solve the differential equation:
$y'' + 4y' + xy = e^{-2x}\ln x + y(x-4)$.

$$y'' + 4y' + xy = e^{-2x}\ln x + yx - 4y$$

$$y'' + 4y' + 4y = e^{-2x}\ln x$$

6

y_c:

$$y'' + 4y' + 4y = 0$$

$$\lambda^2 + 4\lambda + 4 = 0$$

$$(\lambda+2)(\lambda+2) = 0$$

$$\lambda = -2, -2$$

~~G.S $\Rightarrow$ $\boxed{c_1 e^{2x} + c_2 e^{-2x}}$~~

G.S: $c_1 e^{-2x} + c_2 x e^{-2x}$

y_p: $y_1 = e^{-2x}$ $\quad y_2 = x e^{-2x}$

$$y_p = -y_1 \int \underbrace{\frac{y_2\, g(x)}{w(y_1,y_2)}\,dx}_{\text{I}} + y_2 \int \underbrace{\frac{y_1\, g(x)}{w(y_1,y_2)}\,dx}_{\text{II}}$$

$$\begin{vmatrix} e^{-2x} & xe^{-2x} \\ -2e^{-2x} & e^{-2x} - 2e^{-2x}x \end{vmatrix}$$

~~$w(y_1,y_2) = e^{-2x}(e^{-2x} - 2e^{-2x}x) - -2e^{-2x}$~~

$$w(y_1,y_2) = e^{-2x}(e^{-2x} - 2e^{-2x}x) - (-2e^{-2x})(xe^{-2x})$$

$$= e^{-4x} - 2xe^{-4x} + 2xe^{-4x}$$

$$= e^{-4x}$$

$\Rightarrow$ I: $-e^{-2x}\int \frac{xe^{-2x}e^{-2x}\ln x}{e^{-4x}}\,dx$

$$= -e^{-2x}\int x\ln x\,dx$$

$u = \ln x \quad \int dv = \int x$

$du = \frac{1}{x}dx \quad v = \frac{x^2}{2}$

$$= -e^{2x}\left(\frac{x^2}{2}\ln x - \int \frac{x^2}{2x}\right)$$

$$= -e^{2x}\left(\frac{x^2}{2}\ln x - \frac{x^2}{4}\right) = -e^{2x}\frac{x^2}{2}\ln x + e^{-2x}\frac{x^2}{4}$$

II: $xe^{-2x}\int \frac{e^{-2x}e^{-2x}\ln x}{e^{-4x}}$

$u = \ln x \quad du = dx$

$du = \frac{1}{x} \quad v = x$

$$= xe^{-2x}\int \ln x = xe^{-2x}\left(x\ln x - \int 1\right) = x^2e^{-2x}\ln x - x^2e^{-2x}$$

$\therefore y = y_c + y_p$

$$\therefore y = c_1 e^{-2x} + c_2 x e^{-2x} + -\frac{e^{2x}x^2}{2}\ln x + \frac{e^{-2x}x^2}{4} + x^2e^{-2x}\ln x - x^2e^{-2x}$$