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Mathematics Department Engineering Math. I (0301202)

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(1) Find the solution of

$$\frac{dy}{dx}=\frac{1}{2}\left(\frac{x}{y}+\frac{y}{x}\right), \qquad y(1)=1$$

$u = \frac{y}{x}$

$y = xu$

$y' = xu' + u$

$\frac{dy}{dx} = y' = xu' + u = \frac{1}{2}\left(\frac{1}{u} + u\right)$

$xu' = \frac{1}{2u} + \frac{u}{2} - u$

$xu' = \frac{1}{2u} - \frac{1}{2}u$

$u' = \frac{2-2u^2}{4u} \cdot \frac{1}{x}$

$\int \frac{4u}{2-2u^2}\,du = \int \frac{dx}{x}$

$-\ln|2-2u^2| = \ln|x| + c$

$\frac{1}{2-2u^2} = c^*|x|$

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$\frac{1}{2-2\frac{y^2}{x^2}} = c^*|x| \longrightarrow \boxed{\frac{1}{2-2} = c^* = \infty}$

$1 = 2c^*|x|\left(1 - \frac{y^2}{x^2}\right)$

$\frac{1}{2c^*|x|}$

$x^2\left(1 - \frac{1}{2c^*|x|}\right) = y^2$

$y = \pm\sqrt{x^2\left(1 - \frac{1}{2c^*|x|}\right)}$ $\quad y = \pm\sqrt{x^2}$

~~$1 = \pm\sqrt{1 - \frac{1}{2c}}$~~

~~$1 = 1 - \frac{1}{2c}$~~ $\quad -\frac{1}{2c} = 0$

$y = \pm x$. but $y(1) = 1$

$\therefore\ y = +x$

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(2) **Solve**

$$\frac{dy}{dx} = -\frac{e^x+3y^2}{y^3+2e^x y}$$

$$\cancel{\frac{dx}{dy} = -(y^2+2e^x)\, y \over e^x+3y^2}$$

$$(y^3+2e^x y)\,dy = -(e^x+3y^2)\,dx$$

$$\underbrace{(-e^x-3y^2)}_{M}\,dx + \underbrace{(-y^3-2e^x y)}_{N}\,dy = 0$$

$$M_y = -6y \neq N_x = -2ye^x$$

∴ not exact

$$R(x) = \frac{M_y - N_x}{N} = \frac{-6y+2ye^x}{-y^3-2e^x y} = \frac{y(-6+2e^x)}{y(-y^2-2e^x)} \quad X$$

$$R^*(y) = \frac{N_x - M_y}{M} = \frac{-2ye^x+6y}{-e^x-3y^2} = X$$

$$\frac{dx}{dy} = -\frac{y^3+2e^x y}{e^x+3y^2} = \frac{-y^3}{e^x+3y^2} + \frac{-2e^x y}{e^x+3y^2}$$

$$\frac{dy}{dx} = \frac{-e^x}{y^3+2e^x y} + \frac{-3y^2}{y^3+2e^x y}$$

$$\cancel{\left(\frac{y^3}{e^x}+2y\right)dy + \left(1+\frac{3}{e^x}y^2\right)dx = 0}$$

$N_x = -y^3 e^{-x}$ $\quad M_y = 6e^{-x}y$

$u = \frac{y}{x}$

$y = xu$

$y' = xu' + u$

$$\frac{\frac{e^x}{x}+3yu}{uy^2+2e^x u} = \frac{\frac{e^x}{x}+3u^2x}{x^2u^3+2e^x u}$$

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(3) **Find the general solution of**

$$y'' + (y')^2 = 2e^{-y}$$

$u = y' = \frac{dy}{dx}$

$u' = y'' = \frac{du}{dx} = \frac{du}{dy}\frac{dy}{dx}$

$y'' = u\frac{du}{dy}$

$u\frac{du}{dy} + u^2 = 2e^{-y}$

$u' + u = 2e^{-y}u^{-1}$ Bernoulli

$z = u^2$

$z' = 2uu'$

$2uu' + 2u^2 = e^{-y}u^{-1}(2u)$

$z' + 2z = 4e^{-y}$ 1st order linear

$F(y) = e^{\int 2dy} = e^{2y}$

$z = \frac{\int F(y)g(y)dy + c}{F(y)} = \frac{\int e^{2y}(2e^{-y})dy + c}{e^{2y}}$

$z = \frac{2\int e^{y}dy + c}{e^{2y}} = \frac{2e^{y} + c}{e^{2y}}$

$u = \pm\sqrt{\frac{2e^{y}+c}{e^{2y}}}$

$\frac{dy}{\pm\sqrt{\frac{2e^{y}+c}{e^{2y}}}} = dx$

$x = \pm\int\left(\frac{2e^{y}+c}{e^{2y}}\right)^{-\frac{1}{2}}dy$

$x = \pm\int(2e^{-y} + ce^{-2y})^{-\frac{1}{2}}dy$

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(4) **Given that $y_1 = e^x$ is a solution for**

(A) $xy'' - (x+1)y' + y = 0$

Find the general solution of

(B) $xy'' - (x+1)y' + y = x^2$

$\int \frac{x+1}{x}$

$\int 1 + \frac{1}{x}$

$= x + \ln|x| + c$

$y'' - \frac{(x+1)}{x} y' + \left(\frac{1}{x}\right) y = 0$

$u(x) = \int \frac{e^{-\int \frac{-x-1}{x}}}{y_1^2} = \int \frac{c\, e^x |x|}{e^{2x}} = c \int |x| e^{-x}$

$= -c\left(x e^{-x} + e^{-x}\right)$

~~$x^2 y'' - x(x+1) y' + x y = x^3$~~

(C) $y'' - \frac{(x+1)}{x} y' + \left(\frac{1}{x}\right) y = x$

$p(x)$ $\quad q(x)$ $\quad g(x)$

$u = x \qquad dv = e^{-x}$

$du = dx \qquad v = -e^{-x}$

$-x e^{-x} + \int e^{-x} dx$

$-x e^{-x} - e^{-x}$

$y_2 = (x e^{-x} + e^{-x})(e^x)$

$y_2 = x + 1$ ✓

G.S of (A) : $c_1 e^x + c_2 (x+1) = y_c$ ✓

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$y_p = (Ax + B) x^2 = Ax^3 + Bx^2$

$y_p' = 3Ax^2 + 2Bx$

$y_p'' = 6Ax + 2B$

sub.. in (C) :

$6Ax + 2B - \frac{(x+1)}{x}(3Ax^2 + 2Bx) + \frac{1}{x}(Ax^3 + Bx^2) = x$

$6Ax - (3Ax^2 + 2Bx + 3Ax + 2B) + Ax^2 + Bx + 2B = x$

$x(6A - 2B - 3A + B) + x^2(-3A + A) = x$

$6A - 2B - 3A + B = 1$

$\boxed{3A - B = 1}$

$-3A + A = 0$

$-2A = 0$

$\boxed{A = 0}$

$\boxed{B = -1}$

$\therefore y_p = -x^2$

.. G.S of B is : $y = y_c + y_p$

$= c_1 e^x + c_2(x+1) - x^2$

$r = \frac{-b \pm \sqrt{\;}}{2a}$

(5) **Determine a suitable form for the particular solution if the method of undetermined coefficients is to be used for the D.E.**

$y'' + 2y' + 2y = 3e^{-x} + 2x^2 e^{-x} \cos x$

$= \cancel{e^{-x}(3 + 2x^2 \cos x)}$

$y'' + 2y' + 2y = 0$

$r^2 + 2r + 2 = 0$

$4 - 8 = -4 < 0$

$\alpha = \frac{-b}{2a} = \frac{-2}{2} = -1$

$\beta = \frac{\sqrt{4ac - b^2}}{2a} = \frac{2}{2} = 1$

$y_p = y_1 + y_2$

$y_1 = z e^{-x}$

$y_2 = x\left((Ax^2 + Bx + c)e^{-x}\cos x + (Ex^2 + Fx + g)e^{-x}\sin x\right)$

~~$y_p = Ae$~~

$y_c = c_1 e^{-x}\cos x + c_2 e^{-x}\sin x$

$y_p = x\left((Ax^2 + Bx + c)e^{-x}\cos x + (Ex^2 + Fx + g)e^{-x}\sin x\right) + ze^{-x}$

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