$\frac{25}{30}$

University of Jordan
School of Engineering
Department of Electrical Engineering
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Digital Signal Processing , Second Exam
Number of questions :4

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Q1. Consider the sequence $g[n]$ with DTFT $G(e^{j\omega})$. Making use of DTFT theorems, determine the inverse DTFTs of the following functions of $G(e^{j\omega})$ in terms of $g[n]$:

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a) $X_1(e^{j\omega}) = e^{-j6w}G(e^{jw})$
b) $X_2(e^{j\omega}) = G(e^{j(w+0.5\pi)})$
c) $X_3(e^{j\omega}) = 3G(e^{jw}) + 4G(e^{-jw})$
d) $X_4(e^{j\omega}) = \frac{dG(e^{jw})}{dw}$ $\rightarrow \frac{1}{j} n g[n].$

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n] e^{-jwn}$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{jw}) e^{jwn} dw.$$

ⓐ $X[n] \xleftrightarrow{DTFT} X(e^{jw}).$

$g[n-n_0]. \xleftrightarrow{DTFT} e^{-j6w} G(e^{jw})$

$\boxed{n_0 = 6.}$ time shift theorem $\boxed{g[n-6] = x_1[n]}$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-j6w} G(e^{jw}) e^{jw} dw.$$

$$= \frac{1}{2\pi}\int_{\pi}^{\pi} G(e^{jw}) e^{j(w-6\omega)} dw.$$

$\boxed{b}$ $e^{-jwn} g[n] \xleftrightarrow{DTFT} G(e^{j(w+0.5\pi)})$ frequency shift

$x_2[n] = g[n] e^{-jn(0.5\pi)}$

$\boxed{c}$ $ag[n]+bg[n]. \xleftrightarrow{DTFT} aG(e^{jw}) + bGe(^{-jw}).$ linear theroem

$a = 3$
$b = 4.$

$\boxed{g[-n]. \xleftrightarrow{DTFT} G(e^{-jw})}$

$x[n] = a\,g[n] + b\,g[-n].$

$\boxed{x_3[n] = 3g[n] + 4g[-n].}$

$\sum x[n]$

Q2. For the sequence $x[n] = \{-1 \quad 3\}$,

a) Find the D_2 matrix.
b) Find X[k], k= 0, 1. (The DFT of x[n]).
c) Find the D_2^{-1} matrix.
d) Find the IDFT of X[k].

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$\boxed{a}$ $D_2 = \begin{bmatrix} 1 & 1 \\ 1 & W_2^1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$W_2^1 = e^{\frac{-j2\pi kn}{N}} = e^{-j\frac{2\pi(1)(1)}{2}} = e^{-j\pi} = (-1)$

$\boxed{b}$ $X[K] = D_N \, x[n].$

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \end{bmatrix} \quad \begin{matrix} -1+3 \\ -1-3 \end{matrix}$$

$\boxed{c}$ $D_2^{-1} = \frac{1}{N} D_2^* = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{bmatrix}$

$\boxed{d}$ $x[n] = D_2^{-1} X[K]$

$$\begin{bmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{bmatrix} \begin{bmatrix} 2 \\ -4 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$\frac{1}{2} \times 2 + \frac{1}{2} \times -4 = 1 - 2 = 1$

$\frac{1}{2} \times 2 + -\frac{1}{2} \times -4 = 1 + 2 = 3$

RSS .

$\frac{10}{5}$ $5\sqrt{10}$

Q3. The transfer function of a causal LTI discrete-time system is given by

$$H(z) = \frac{0.5 - 0.3z^{-1}}{1 - 0.6z^{-1}}$$

a) Determine the impulse response h[n] of the system.

b) Determine the output of the system y[n] if the input $x[n] = (0.2)^n \mu[n]$.

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ⓐ

$$\begin{array}{r|l} & 0.5 \\ \hline 1 - 0.6z^{-1} & 0.5 - 0.3z^{-1} \\ & 0.5 - 0.3z^{-1} \; ⊖ \\ \hline & 0 \end{array}$$

$$H(z) = 0.5 + \frac{0}{1 - 0.6z^{-1}} = 0.5.$$

$$\boxed{h[n] = 0.5\,\delta[n].}$$

ⓑ $y[n] = h[n]\,x[n].$ $\quad X(z) = \frac{1}{1 - 0.2z^{-1}}$

$$Y(z) = H[z]\,X[z] = \left(\frac{0.5 - 0.3z^{-1}}{1 - 0.6z^{-1}}\right) \cdot \left(\frac{1}{1 - 0.2z^{-1}}\right).$$

$$Y(z) = \frac{0.5 - 0.3z^{-1}}{(1 - 0.6z^{-1})(1 - 0.2z^{-1})}$$

$$Y(z) = \frac{A_1}{(1 - 0.6z^{-1})} + \frac{A_2}{(1 - 0.2z^{-1})}$$

$$\boxed{z^{-1} = \frac{10}{6}} \quad A_1\Big|_{\substack{z = 0.6 \\ \frac{6}{10}}} = \frac{0.5 - (0.3)\left(\frac{10}{6}\right)}{\left(1 - 0.2 \cdot \frac{10}{6}\right)} = 0$$

$$\boxed{z^{-1} = \frac{10}{2}} \quad A_2\Big|_{z = 0.2} = \frac{0.5 - 0.3\left(\frac{10}{2}\right)}{\left(1 - \left(0.6 \cdot \frac{10}{2}\right)\right)} = \frac{-1}{-2} = 0.5$$

$$Y(z) = \frac{0}{1 - 0.6z^{-1}} + \frac{0.5}{1 - 0.2z^{-1}} = \frac{0.5}{1 - 0.2z^{-1}}$$

RSS $|z| > |a|$

$|z| > 0.2$

$$\boxed{y[n] = 0.5\,(0.2)^n\,\mu[n].}$$

$$x(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-1}$$

$$x[n] = \frac{1}{N} \sum_{z=-\infty}^{\infty} x[z]\, z.$$

Q4. The continuous-time signal $x_a(t) = 2\cos(10\pi t) + 3\sin(24\pi t)$ is sampled at a 40 Hz rate. The generated sequence $x[n]$ is an input to a system with a transfer function

$H(z) = (z - r1)(z - r1^*)(z - r2)(z - r2^*)$. →

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Find $r1$ and $r2$ such that the output of the system $y[n]$ is equal zero.

$H(z) = \frac{Y(z)}{X(z)}$, $H[n] = \frac{y[n]}{x[n]}$.

$H[n] =$

~~$x[n] =$~~

$H(z) \cdot X(z) = y(z)$

$= 0.$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{4} x[n]\, e^{-j\omega n}$$

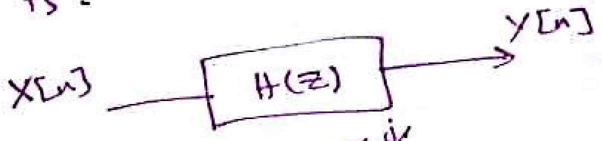
$H(e^{j\omega}) = H(z)\Big|_{r=1} = (z-r1)(z-r1^*)(z-r_2)(z-r^{2*})$ $z = e^{j\omega}$

$X(e^{j\omega}) = 2\left[e^{+j\omega_0} + e^{-j\omega_0}\right] + 3\left[\frac{e^{+j\omega_0} - e^{-j\omega_0}}{}\right] = e^{j10\pi} + e^{-j10\pi} + \left[\frac{3}{2j} e\right.$

Q4: Sol:

$x_a(t) = 2\cos(10\pi t) + 3\sin(24\pi t)$.

$F_s = 40$ Hz.



~~H(z) = (z~~

$\omega_1 = 10\pi \cdot \frac{1}{40} = \frac{\pi}{4} = 0.25\pi$ rad/sample.

$\omega_2 = 24\pi \cdot \frac{1}{40} = 0.6\pi$ rad/ Sample

$r_1 = e^{jw_1} = e^{j0.25\pi}$

$r_2 = e^{jw_2} = e^{j0.6\pi}$.

$H(z) = (z - \overset{e^{jw_1}}{r_1})(z - r_1^*)(z - r_2^{j\omega_2})(z - r_2^*)$

$=(z - r^{e^{j(0.25)}})(z - r_1^*)(z - r_2^{j(0.6)})(z - r_2^*)$

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