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University of Jordan
School of Engineering
Department of Electrical Engineering
Fall Semester – A.Y. 2016-2017
Digital Signal Processing , First Exam

Number of questions :5

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Q1. Determine the range of values of *a* and *b* for which the linear time-invariant system with impulse response

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$$h[n] = \begin{cases} a^n & n < 0 \\ b^n & n \geq 0 \end{cases}$$

a) is causal.
b) is stable.

→ condition of causality $h[n] = 0 \quad n < 0$.

When $n < 0$ $h[n] = a^n$ $a = 0$

→ Condition of stability

$|h[n]| \leq B_x$

$b^n \leq B_x < \infty$

$\lim_{n \to \infty} b^{n < \infty}$ when $b = 0$.

$\lim_{n \to -\infty} a^n$ $a \neq 0$.

Q2. A continuous- time signal $x_a(t)$ is composed of a linear combination of sinusoidal signals of frequencies 300 Hz, 500 Hz, 1.2 kHz, 2.5 kHz, and 3.5 kHz. The signal $x_a(t)$ is sampled at 2.0 kHz rate, and the sampled sequence is passed through an ideal low pass filter with a cutoff frequency of 1.0 kHz, generating a continuous-time signal $y_a(t)$. What are the frequency components present in the reconstructed signal $y_a(t)$?

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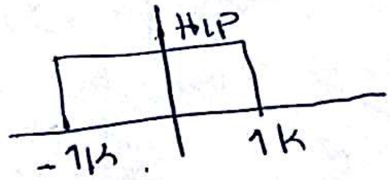
we do sampling



300 HZ

300 + 2000 = 2300 X

300 - 2000 = -1700 X

500 HZ

500 + 2000 = 2500 X

500 - 2000 = -1500 X

1200 HZ

1200 + 2000 = 3200 X

1200 - 2000 = -800 ✓

2.5 KHZ

2500 + 2000 = 4500 x

2500 - 2000 = +500 ✓

3.5 KH

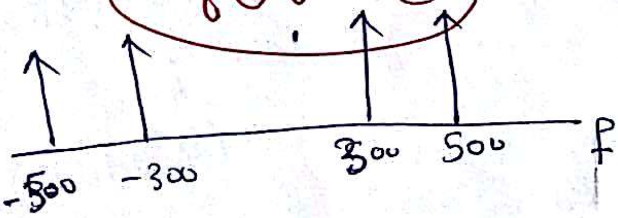
3500 + 2000 = 5500 X

3500 - 2000 = 1500 X

the frequency Component is that will pass.

300 HZ ~~500 HZ~~

800 HZ



Q3. a) The odd part of a real-valued sequence $x[n]$ is given by $x_{od}[n] = (\frac{1}{2})^3 u[n]$. If the average power of x[n] is P_x= 5, determine the average power of its even part $x_{ev}[n]$.

b) Compute the energy of length-N sequence $x[n] = \sin(2\pi kn/N)$, $0 \le n \le N-1$.

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avarage Power = $\lim_{K\to\infty} \frac{1}{2K+1} \sum_{K=0}^{\infty} (X_{od} + X_{even})^2$

avarge power odd $= \lim_{K\to\infty} \frac{1}{2K+1} \sum_{0}^{K} \left(\left(\frac{1}{2}\right)^3\right)^2$

$= \lim_{K\to\infty} \frac{1}{2K+1} \sum_{0}^{K} \frac{1}{2^6}$

$= \lim_{K\to\infty} \frac{K+1}{2^6 * 2(K+1)} = \frac{1}{2^7} = \frac{1}{128}$

$5 = \frac{1}{128} + P_{even}$

$P_{even} = 4.992$

b) energy $= \sum_{-\infty}^{\infty} |\sin(2\pi kn/N)|^2$

$= \sum_{-\infty}^{\infty} \frac{1}{2} - \frac{1}{2}\cos(4\pi kn/N)$

$= \sum_{0}^{N-1} \frac{1}{2} - \sum_{0}^{N-1} \cos(4\pi kn/N)$ 1

$= \frac{1}{2}(N) - \sum_{0}^{N-1} 1$

$= \frac{1}{2}N - 1N = -N$

0

cos 2n = 1 - 2...

sin . cos2...

Q4. Determine the fundamental period of the following periodic sequences:

1) $\tilde{x}[n] = \sin(0.8\pi n + 0.8\pi)$
2) $\tilde{y}[n] = 5\cos(1.5\pi n + 0.75\pi) + 4\cos(0.6\pi n) - \sin(0.5\pi n)$

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1) $\tilde{X}[n] \longrightarrow \frac{N}{r} = \frac{2\pi}{\omega} = \frac{2\pi}{0.8\pi} = \frac{2}{8} * 10 = 0.4$ ra

2) $\tilde{y}[n] =$

→ For more than Cosines .

$\frac{N}{r} = \frac{2\pi}{1.5\pi} = \frac{4}{3}$ integer. $= T_1$ $\quad \frac{T_1}{T_2} = \frac{4}{3} * \frac{6}{20} = \frac{2}{5}$

$\frac{N}{r} = \frac{2\pi}{0.6\pi} = \frac{\cancel{20}\ 10}{\cancel{6}\ 3}$ integer $\quad \frac{T_1}{T_3} = \frac{4}{3} * \frac{1}{4} = \frac{1}{3}$

$\frac{N}{r} = \frac{2\pi}{0.5\pi} = \frac{4}{1}$ integer.

$T_P = T_1 * K$
$= \frac{4}{3} * 15 = 20$

Q5. a) Find the DTFT of $x[n] = \alpha^n(\mu[n] - \mu[n-8]) \quad , |\alpha| < 1$

b) Find the IDTFT of $H(e^{j\omega}) = \begin{cases} j & -\pi < \omega < 0 \\ -j & 0 < \omega < \pi \end{cases}$

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a) $\alpha^n (\mu[n]) - \mu[n-8])$

$$\sum_{n=-\infty}^{\infty} \alpha^n(\mu[n] - \mu[n-8]) =$$

$$\sum_{n=-\infty}^{\infty} \alpha^n \mu[n] \Rightarrow \sum_{n=0}^{\infty} \alpha^n e^{-j\omega n} = \sum_{n=0}^{\infty} (\alpha \cdot e^{-j\omega})^n$$

$$= \frac{1}{1 - \alpha \cdot e^{-j\omega}}$$

DTFT for $\alpha^n \mu[n-8]$

$$= \frac{1}{1 - \alpha e^{-j\omega}} * e^{-j\omega 8} \quad \text{Shiftting Proparity}$$

$$= \frac{1}{1 - \alpha \cdot e^{-j\omega}} - \frac{e^{-j\omega 8}}{1 - \alpha e^{-j\omega}} = \frac{1 - \alpha^8 e^{-j\omega 8}}{1 - \alpha e^{-j\omega}} \quad \#$$

b) $H(e^{j\omega}) \longrightarrow \frac{1}{2\pi}\int_{-\pi}^{0} j \cdot e^{+j\omega n} \, d\omega + \frac{1}{2\pi}\int_{0}^{\pi} -j \cdot e^{+j\omega n}$

$$\frac{j}{2\pi} \frac{e^{j\omega n}}{jn} \Big|_{-\pi}^{0} + \frac{j}{2\pi} \frac{e^{+j\omega n}}{jn} \Big|_{0}^{\pi}$$

$e^{-\pi j n} =$ zero.

$$\frac{1}{2\pi n}[1 - -1] - \frac{1}{2\pi n}[1 - 1]$$

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