University of Jordan
Elect. Eng. Dept.

Control Systems 903441 — Second Exam — Wednesday 12/4/2017

Student Name: [redacted] Reg. No [redacted] Mark out of 30

**Answers should be written in ink** **Exam Duration: 70 min**

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## Q1

a) In **two lines or less**, what is meant by the root locus?

the root locus is a pictorial exposition of the possible poles & zeros of a system with the variations of the parameter (k).

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b) To continue the Routh's method in the case of a zero in the first column, a student multiplies the characteristic equation by $(s-1)$. Is he/she right or wrong and why?

wrong. he/she must multiply by (s+1) so as to get the C.E without change in its coefficient's signs. after multiplying by (s+1) the problem of zero in first column will be solved.

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c) Prove why a second order system having a characteristic equation of the different coefficient signs is always unstable?

when using such CE in Routh's method we will get a change in signs of the first column meaning we will get +ve real poles → unstable

0

ex. CE = $s^3+2s^2-2s+1$ ; ex. CE = $s^3+2s^2+2s+1$

$$\begin{array}{c|cc} s^3 & 1 & -2 \\ s^2 & 2 & 1 \\ \hline s^1 & -\frac{1}{2} \rightarrow -ve & \\ s^0 & & \end{array} \qquad \frac{2-2-1}{2} = -\frac{1}{2}$$

$$\begin{array}{c|cc} s^3 & 1 & 2 \\ s^2 & 2 & 1 \\ \hline s^1 & \frac{3}{2} \rightarrow +ve & \\ s^0 & & \end{array} \qquad \frac{2+2-1}{2} = \frac{3}{2}$$

d) A system having $G(s) = \frac{2s+5}{s^2+10s}$ and $H(s) = 1$ is excited by a unit step. Write down matlab **commands** ( not Simulink blocks) to determine its **open loop poles and zeros**, its **root locus**, and its **open loop response to two signals** having Laplace transforms $\frac{1}{s}$ and $\frac{1}{s^2}$.

```
>> n=[2 5]; d=[1 10 0]; roots(n,d); rlocus(CE)
```

~~>> C=(15~~

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**Q2** Consider a system having the following characteristic equation $s^4 + Ks^3 + s^2 + s + 1 = 0$.

What values of ***K*** are certain to make the system unstable?.

K=0 → will result in CE with missing power (unstable)

K<0 → will result in CE with different coefficient signs (also unstable)

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Determine if any positive value of ***K*** make the system stable.

by applying Routh's method

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 1 | 1 |
| $s^3$ | K | 1 | 0 |
| $s^2$ | K-1 | K | 0 |
| $s^1$ | $\frac{K^2-K+1}{K-K}$ | 0 | |
| $s^0$ | K | | |

$\frac{K-1}{1}$ $\frac{K-0}{1}$

$\frac{K-1-K^2}{K-1} = -\frac{(K^2-K+1)}{K-1}$

so $K-1>0 \Rightarrow K>1$

$K^2-K+1>0$

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If the root locus is to be sketched for this system , what are the $Z(s)$ and $P(s)$ for this system?.

$CE = S^4+S^2+S+1+KS^3=0$

$CE = 1+K\frac{S^3}{S^4+S^2+S+1}=0$

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zeros : $S=0,0,0$

poles : $S=-1, \ldots$

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**Q3** a unity gain feedback system has $G(s) = \frac{2s+5}{s^2+10s}$ . $H(s) = 1$

Use the **error coefficient concepts** to calculate the steady state errors due to

i) A unit step set point.

↳ position error coefficient

$$K_p = \lim_{s \to 0} G(s)H(s) = \lim_{s \to 0} \frac{2s+5}{s^2+10s} = \frac{5}{0} = \infty$$

$$e_{ss} = \lim_{s \to 0} sE(s) = \frac{1}{1+K_p} = \frac{1}{\infty} = 0$$

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this result is expected because :

$$G(s)H(s) = \frac{2(s+2.5)}{s(s+10)} \qquad k=2 \quad N=1$$

only 1 integrator (at least) is needed in this case to give zero steady state error

ii) A unit ramp set point.

↳ velocity error coefficient

$$K_v = \lim_{s \to 0} s\,G(s)H(s) = \lim_{s \to 0} \frac{2s+5}{s+10} = \frac{5}{10} = \frac{1}{2}$$

$$e_{ss} = \lim_{s \to 0} sE(s) = \frac{1}{K_v} = \frac{1}{\frac{1}{2}} = 2$$

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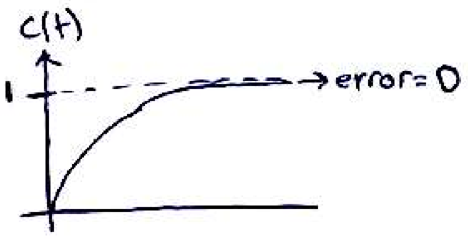
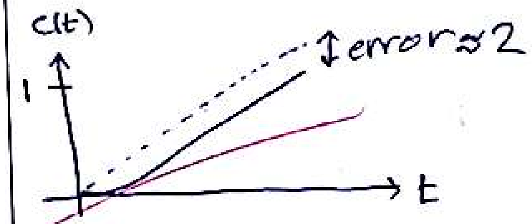
this result is expected beacause $k=2$ , $N=1$ and 2 integrators or more are needed in this case to result in zero steady state error.

iii) Make rough ( approximate ) sketches for the outputs in every case in the table shown .

| | |
|---|---|
|   (case i) |   (case ii) |

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Q4 A closed loop system is given by $G(s) = \frac{1}{(s+2-j\sqrt{3})(s+2+j\sqrt{3})}$ and $H(s) = \frac{K}{s-1}$.
Sketch the root locus with the appropriate arrows indicated, and write down the values of **K** at the **points** of intersection of the root locus with the **real and imaginary axes**. In your calculations, leave square root of three as $\sqrt{3}$ or approximate it with highest precision your calculator permits.

$$CE = 1 + G(s)H(s) = 1 + \frac{K}{(s-1)(s+2-j\sqrt{3})(s+2+j\sqrt{3})}$$

→ no zeros
→ poles : $s=1$, $s=-2\pm j\sqrt{3}$ (at these poles $K=0$)

$$\text{Asymptote Angles} = \frac{(1\pm 2h)180^\circ}{3} = 60^\circ,\ 180^\circ,\ 300^\circ$$

$$\sigma = \frac{\Sigma P - \Sigma Z}{n_p - n_z} = \frac{1 - 2 - \cancel{j\sqrt{3}} - 2 + \cancel{j\sqrt{3}}}{3} = -1$$

departure angle : $\Sigma\phi_z - \Sigma\theta_p + \theta_d = 180 \Rightarrow \theta_d = -180 - 150 - 90 = -420$

break away point $-\frac{dK}{ds} = 0$

$$-\frac{d}{ds}(s^3+3s^2+3s-7) = 3s^2+6s+3 = 0$$

$s = -1$, $-1$

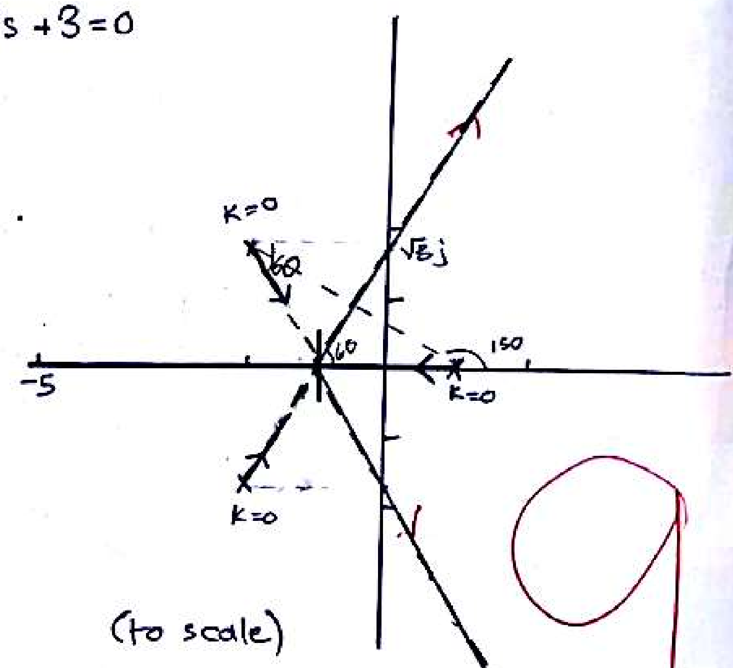
Routh's → intersection with Im axis:

$CE = s^3 + 3s^2 + 3s - 7 + K = 0$

| | | |
|---|---|---|
| $s^3$ | 1 | 3 |
| $s^2$ | 3 | $-7+K$ |
| $s^1$ | $\frac{16-K}{3}$ | 0 |
| $s^0$ | $K-7$ | |

$K = 16$ $\quad A(s) = 3s^2 + 9$

$s = \mp j\sqrt{3}$




Use the magnitude condition to calculate the value of **K** where **-5** is a closed loop pole.

$$\left|\frac{K(1)}{6\times\sqrt{9}\times\sqrt{9}}\right| = 1 \qquad K = 54\ 72$$

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Use the angle condition to decide if $j\sqrt{3}$ is a closed loop pole or not.

angle condition : $\Sigma\phi_z - \Sigma\theta_p = -120 - 60 = -180$

angle from $s=1$ to $s=j\sqrt{3}$ ; angle from $s=-2-j\sqrt{3}$ to $s=j\sqrt{3}$

thus $j\sqrt{3}$ is a closed loop pole

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