ملخص قوانين مادة :

"Control"

Power Unit

حمزة وقاد

* MatLab Commands:

```
roots( )
Poly( )
```
تدخل ال roots تعطي ال fun

```
PolyVal(n,-5)
```
تعويض رقم

```
Conv(n,q)
```
ضرب 2 polys

num: بسط
den: مقام

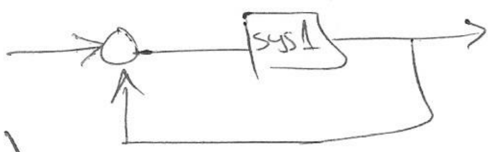
```
sys = tf(num,den)
```
→ transfer function

```
pole(sys)
```
→ poles after system

```
Zero(sys)
```
→ Zeroes.

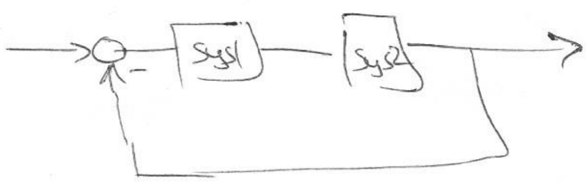
```
[P,Z] = PZmap(sys)
```

```
[sys] = feedback(sys1,[1],sign)
```



unity feed back

```
sys3= series(sys1, sys2)
```




```
[Y,T] = step(sys,t);
plot(t,y);
```

Steady state Error (e_{ss}):

① step input

$e_{ss} = \frac{1}{1+K_p}$ $K_p = \lim_{s \to 0} G(s)$

$e_{ss} = 0$ if $N \geq 1$

② Ramp input

$N = 0$ $e_{ss} = \infty$

$N = 1$ $e_{ss} = \frac{1}{K_v}$, $K_v = \lim_{s \to 0} s\,G(s)$

$N \geq 2$ $e_{ss} = 0$

③ parabolic input

$N = 0, 1$ $e_{ss} = \infty$

$N = 2$ $e_{ss} = \frac{1}{K_a}$ $K_a = \lim_{s \to 0} s^2 G(s)$

$N \geq 3$ $e_{ss} = 0$

$$e_{ss_{O.L}} = 1 - K$$

$$e_{ss_{C.L}} = \frac{1}{1+K}$$

$$\boxed{\frac{Y}{R} = \frac{1}{\Delta} \sum_{K=1}^{n} P_K \Delta_K}$$ $\Rightarrow$ Maison's gain formulla

P: path from input to output
n: number of paths
Δ: graph determinent

$\Delta = 1 - \sum$ (all individual loops) $+ \sum$ (all possiable product of non touching two loops)
$- \sum$ (all product of non-touching 3 loops) + ---

$\Delta_K \equiv \Delta$ (with remove all loops toching P_K)

State variable:

$$\frac{Y(s)}{U(s)} = \frac{b_0 + b_1 s + b_2 s^2 + \cdots}{a_0 + a_1 s + a_2 s^2 + a_3 s^3 + \cdots}$$

$$\left.\begin{aligned} \dot{X} &= \begin{bmatrix} -a_3 & 1 & 0 & 0 \\ -a_2 & 0 & 1 & 0 \\ -a_1 & 0 & 0 & 1 \\ -a_0 & 0 & 0 & 0 \end{bmatrix} X + \begin{bmatrix} b_3 \\ b_2 \\ b_1 \\ b_0 \end{bmatrix} u \\ Y &= [1 \; 0 \; 0 \; 0] X + [0] u \end{aligned}\right\}$$ observable

---

$$\left.\begin{aligned} \dot{X} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 & -a_3 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u \\ y &= [b_0 \; b_1 \; b_2] X \end{aligned}\right\}$$ Controlable.

tranfer function from the state Variable: $\begin{bmatrix} \circ \end{bmatrix} \times \begin{bmatrix} 0 \end{bmatrix}$

$$\dot{X} = AX + BU$$
$$Y = CX + DU$$
$$\Longrightarrow \underbrace{G(s)}_{\text{transfer function}} = C\,[SI - A]^{-1} B + D$$

state transition Matrix $\equiv \phi(s) = [SI - A]^{-1}$

Zero state Response $\equiv (SI - A)^{-1} B U(s)$

إذا كان المطلوب step

بدل ال $u \rightarrow \frac{1}{s} \rightarrow$ unitstep

Charactaristic equation $\underline{\underline{\Delta = 0}}$ gives you eigen values!

⊛ Steady state Error:

$\boxed{\text{Closed loop with unity feed back}}$

① step input $\left(\frac{1}{s}\right)$

$$\underset{C.L}{e_{ss}} = \frac{1}{1 + K_p} \qquad K_p = \lim_{s \to 0} G(s) \Longleftrightarrow N = 0$$

$N > 0 \quad e_{ss} = 0$

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② ~~close~~ ramp input $\left(\frac{1}{s^2}\right)$

$$\underset{C.L}{e_{ss}} = \frac{1}{K_V} \qquad K_V = \lim_{s \to 0} s\, G(s) \Longleftrightarrow N = 1$$

$N < 1 \quad e_{ss} = \infty$

$N > 1 \quad e_{ss} = 0$

---

③ parabolic input

$$e_{ss} = \frac{1}{K_a} \qquad K_a = \lim_{s \to 0} s^2 G(s) \Longleftrightarrow N = 2$$

$N < 2 \quad e_{ss} = \infty$

$N > 2 \quad e_{ss} = 0$

Root Locus

1. $\Delta(s) = 1 + K p(s) = 0$

2. factor $p(s)$ for poles & zeroes

3. locate the poles & zeroes on the s-plane.

4. the root that lies on the Real axis always lies in the section of the Real-axis to the left of an odd number of zeroes & poles.

5. Determine the # of seperate loci (s.l. = # poles)

6. Root locus must be symmetrical around Real axis.

7. the loci proceeds to the zero at ∞ along asymptotes centered at (σ_A) with angle(ϕ_A) if $N = n_p - n_z$, then an N section of locies proceed to the zero at ∞ along asymptote as $K \to \infty$, where:

a) $$\sigma_A = \frac{\sum_{i=1}^{n_p} (P_i) - \sum_{j=1}^{n_z} (z_j)}{n_p - n_z}$$

b) $$\phi_A = \frac{(2q+1)}{n_p - n_z} \times 180^\circ \quad q = 0, 1, 2, \cdots (n_p - n_z - 1)$$

8. Evaluate K at which the root locus crosses the imaginery axis (creterion from previous chapter)

9. Determine the (Break away point) where the root leaves the Real axis.

$\boxed{0 = \frac{dK}{ds}}$

~~(K) write the characteristic equation.~~

(10) Determine the angle of Departure from the complex ~~plan~~ poles

Steps :

①Draw

② assymptote $\phi_A = \frac{(2q+1)}{n_p - n_z} \times 180°$ $\sigma_A = \frac{\sum P_i - \sum z_j}{n_p - n_z}$

③ departure angle.

④ locus Crosses Imaginary-axis points. (Routh array)

⑤ Break away points

⑥ Complete the skitch

"Root Locus"

first order Response :

$$G(s) = \frac{K}{1+\tau s}$$

$$y(t) = K\left(1 - e^{-t/\tau}\right) \longrightarrow \text{for step}$$

$$\text{DC gain} = G(0) \Longrightarrow \text{steady state}$$

Response of second order System

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

ω_n : natural freq.

ζ = damping factor.

$$y(t) = 1 - \frac{1}{\beta} e^{-\zeta\omega_n t} \sin(\omega_d t + \theta)$$

$\beta = \cancel{\omega_n} \sqrt{1-\zeta^2}$

$\theta = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$

$\omega_d = \omega_n \sqrt{1-\zeta^2}$

$\zeta > 1$ over damped

$\zeta = 1$ Critically

$\cancel{\zeta}\ 0 \leq \zeta < 1$ under damped

$\zeta = 0$ pure Imaginary

$\zeta < 0$ unstable.

$$y(t) = \frac{\omega_n}{\beta} e^{-\zeta\omega_n t} \sin(\omega_d t \cancel{+\theta})$$

2^{nd} order:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$ → Characteristic polynomial

$$\boxed{s_1, s_2 = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}}$$

Step Response

$$y(t) = 1 - \frac{1}{\beta} e^{-\zeta\omega_n t} \sin(\omega_n \beta t + \theta)$$

$$\tau = \frac{1}{\zeta\omega_n}$$

where:

$\beta : \sqrt{1-\zeta^2}$

$\theta = \cos^{-1}(\zeta)$

ζ conditions:

① if $\zeta > 1$ overdamped.

② if $\zeta = 1$ critically damped.

③ if $0 < \zeta < 1$ under damped.

④ if $\zeta < 0$ unstable.

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settling time $t_s = 4\tau = \frac{4}{\zeta\omega_n}$

time to peak $\equiv t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$

the maximum % overshoot $P.O = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100\% = e^{\frac{-\pi}{\tan(\alpha)}} \times 100\%$

$M_p \equiv$ Peak Value $= 1 + e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$

$$\alpha = \cos^{-1}(\zeta) = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

Cont. second order system

$t_s = 4\tau$ where $\tau = \frac{1}{\zeta \omega_n}$ (settling time)

$t_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}$ (time to peak)

$\%\text{overshoot} \equiv PO = 100\, e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \%$

$M_{pt} = 1 + e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \longrightarrow$ Peak value

typical $\zeta = \frac{1}{\sqrt{2}} = 0.707$