Communications II

NoteBook

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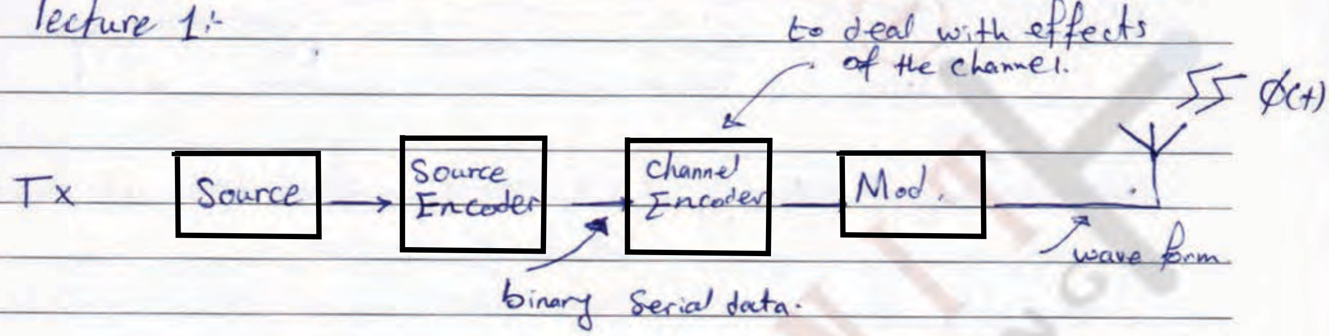
POWER UNIT
ELECTRICAL ENGINEERING

lecture 1:-



Mod:- convert the output of the channel Encoder to a wave form.

$\phi(t)$ it must carry power or energy. in order to move throug channel.

* Power signals are periodic and it doesn't carry any information.

$H_{s_i} = -\log(pr(s_i))$ in <u>bits</u>

* Digital Signal: finite and discrete set of signal.

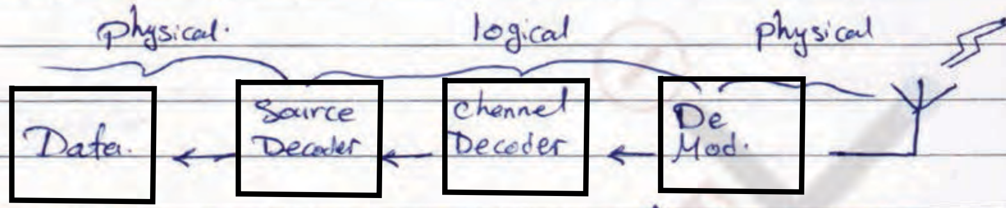
* analog signal:- infinity responce.

Mod: finite set of inputs into finite number of wave form. by using maping

* Errors occures in the channel so the channel Encoder is used by an added intellegence in the System in order to solve the Errors in the channel. [pre processing technique.]

No.

Rx:



aperiodic signals (Energy) : it carry information. it must be time limited.

* bit/s ⟶ cycles represent the speed rate
* the shape of the wave form represent the data So different shapes → different data.
* logical: i can manipulate the data using processors & coding. [Software]

* physical : where we put Energy.
  from source side: analog to digital conversion.
  from channel side: Digital to analog conversion.
  RF END : this stage holds the signal with a high frequency

* the source usually is a baseband signal.

No.

lecture 2:

Digital Source: finite set of signals
if it was infinite then it is an analog Source

* infinite like voltage for example let's say between $-5v$ & $+5v$ we have infinity number of levels, if the output represent the level of acertain voltage then we have infinity alphabits.

* the Source produces time and Signal
→ continous in time and amplitude: this is analog Source.
→ discrete in time and continous in amplitude: discrete Signal
→ discrete in both: digital Signal.

Source → $\boxed{A/D}$ → $x[n] \in \{a_m, m = 1, 2, \ldots M\}$

why n ?
to give an impression that this is discrete integer sampling periods

* the procedure of A/D conversion is sampling the Signal if it is analog then Quantazation.

Quantization: makes physical Quantity levels finite.

A/D: it is simply aconversion to binary Zero's & 1's
the information here are in logical format

No.

but here it will convert to [am's] to drop the meaning of voltage.

[am's]: are labels in binary form but that doesn't mean for example that [111] is greater than [101], we only concern that the first value is different from the second one, we don't care about the arithmatic meaning.

* letters must be different and every letter will be in respect to the Source kind, and it will have aprobability.

$Pr(a_m) = P_m$

$H(a_m) = -\log_2(P_m)$ bits

$\log_2$: to represent the Quantity in bits.

* every letter holds some information.
* if i define awell defined group i can map any Source of intelligent information into that Source.

to understand and study the Source:

$x[n]$: is a signal that has a physical source, the tools we use to study the signal are:

1. draw it in time domain
2. frequency domain.

No.

*Note: Source is random and it has aprobability so we can't predict what the future output might be.

Q:- do we have acertain technique to analyze and study those signals?
Yes, random theory.

random theory: it analyze some characteristics like:
1. Probability.
2. distribution function
3. moments (mean, variance).
4. corrolation
5. non linear corrolation.

Corrolation: relation between sequence of out comes
Non linear corrolation: third order, 4^{th} order ...
auto corrolation function: 2^{nd} order corrolation represent the relation between the sequence of the output of the source with respect to time and if there is arelation between the output of asignal now and the one that will come after it or the one before it.

Entropy of the Source:-

$$H = -\sum_{m=1}^{M} P_m \log_2 (P_m) \quad \text{bits.}$$

this is the minimum amount of bits per sample we can Quantize this Source such that No information loss

No.

Q: can we reach the minimum?
usually NO.

Entropy has a relationship with the data rate

$$\frac{\text{bits}}{\text{Sample}} * \frac{\text{Sample}}{\text{Sec}} = \text{bits/s} \equiv \text{data rate.}$$

* We normalize the data rate so we can easily work with it in a more simple way.

* Quantization
convert infinite number of levels into finite levels.

* 3-bit Source, what does that mean?
that means that the output of the Source is represented on average with three bits. it also means that the Source contains $2^3 = 8$ letters.

Q: how many bits we need to represent 10 letters?
4 bits, but this is not efficient because 4 bits can represent 16 letters so this is a sub optimal solution.

R ≡ Source Rate (bits/sample)

* the best system I can built is when R = H

Since R>=H always without data loss

No.

* baseband signals are cetered around Zero.
* Our application mainly concern in send a signal from A to B

(media)

$A \longrightarrow B$

* local communication, distance communication.
* all application are driven by the need.
* wire & wireless communication are our main concern.
* Unguided (wireless): our signal can't be guided in one direction but in certain level we can guid it
* guided (wired): stays between the two points A & B.

* media: 2 wires so we need a signal holding Energy (information) to move from A to B.

* Orthogonality: we can separate the signals at the reciver if they are mixed.
* each signal is on (2 wires) → orthogonal.

* in mathmatics: if we have 2 signals $\langle x_1 , x_2 \rangle$ the inner product is equal to Zero

* if we have time signals $x_1(t)$ & $x_2(t)$

$$\langle x_1 , x_2 \rangle = \int_{-\infty}^{\infty} x_1(t) \cdot x_2(t) \, dt$$

i need a certain technique to extract $x_1(t)$.

No.

lecture 3 :

in our speaking about orthogonality, if two signals are mixed we need the means to separate them.

from our study of signals we can use the inner product to separate the signals.

* if we have agroup of signals:-

x_1
x_2 $\quad$ $sum = \sum_{i=1}^{\infty} x_i$
x_3
$\vdots$
x_N

→ those signals are in one space & can be mixed
→ if the signals are voltage signals, they will be vectorialy added, if it was electric feild signals, they will also be added vectorialy.

So if this group of signals where vectorialy added can we separate them ?
yes, under one condition, if they are seprable.

$\langle x_i \cdot x_j \rangle = \delta_{ij}$

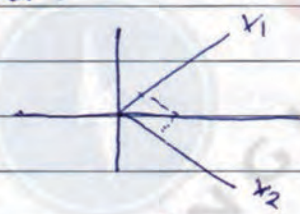
this is asimple mathmatical way to represent orthogonality
→ if $i \neq j$, the inner product will be zero
→ $\delta = 1$, only when i & j are the same
→ the inner product of a signal with itself is the energy of the signal.

No.

→ inner product is a second order measurment because it is the product of two things (voltage, current, ...) So it is related to energy or power.

what if the inner product is between two different signals , the inner product is related to what then ?
the answer is relations between the two signals.

* the concept of inner product is aprojection of acertain vector onto the other.



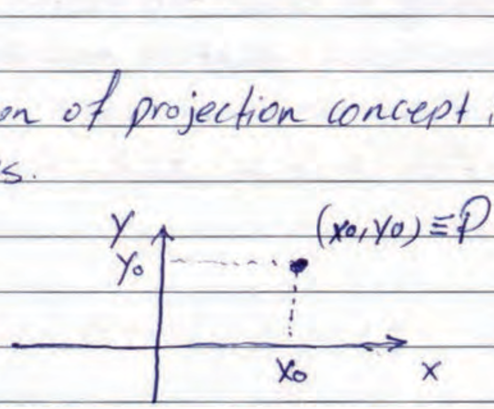
(representation of 2 signals in vector form)

* if x_1 & x_2 are time signals

$$\Rightarrow \int_{-\infty}^{\infty} s(t)\, e^{j\omega t}\, dt$$

this is an inner product (projection) $S(t)$ onto the phaser $e^{j\omega t}$ and that means, how much signal of $s(t)$ are at the same direction of ($e^{j\omega t}$) OR how much signal $s(t)$ can be represented by $e^{j\omega t}$

* asimple representation of projection concept is the cartesian coordinates.



No.

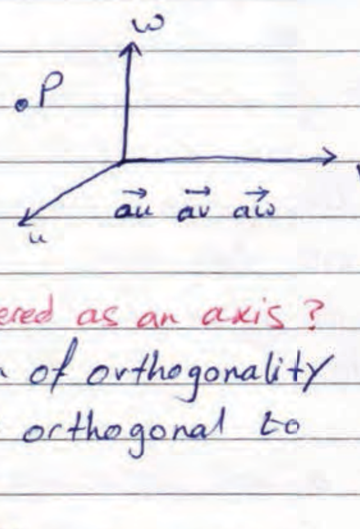
$P = x_0 \vec{ax} + y_0 \vec{ay}$

→ every axis are represented by basis functions ($\vec{ax}, \vec{ay}$)

→ here the product between $x_0 \vec{ax}$ is x_0 and that means that there are certain Quantity of points represent the direction of x-axis which is x_0 and the idea goes the same for signals, there is a certain Quantity of the Signal in the direction of $e^{j\omega t}$ at certain (ω), we keep it continous in order to project the signal $s(t)$ onto a continous axis which is the frequency axis.

* 2-dimentional space that means it has 2 axis,
3- dimentional space → 3-axis
So the system is defined by the projection of that point on each of their axis.

$P = p_u \vec{a_u} + p_v \vec{a_v} + p_w \vec{a_w}$
this is a mathmatical representation for the point (p) in this space.



Can i define a signal that can be considered as an axis?
here i am aiming to reach the definition of orthogonality
So the main condition is that it must be orthogonal to consider them axis.
So if we define (u, v, w) as functions:

$$\left\{ \begin{aligned} u &= u(t) \\ v &= v(t) \\ w &= w(t) \end{aligned} \right\} \text{ time representation of the axis}$$

No.

(u, v, w) they all must be orthogonal in order to consider them axis:

So:

$$\langle u \cdot v \rangle = 0$$

$$\langle v \cdot w \rangle = 0$$

$$\langle u \cdot w \rangle = 0$$

So N-Dimentional space is represented by N-orthonormal axis (basis function)

Orthonormal: it means that they are orthogonal & they are normalized (unit vector in the direction of each axis has one unit).

Ex:- $y = 3\vec{a_u} + 5\vec{a_v} - 2\vec{a_w}$

$$y(t) = \frac{3 \cdot u(t)}{\sqrt{\| u(t) \|^2}} + \frac{5\, v(t)}{\sqrt{\| v(t) \|^2}} - \frac{2\, w(t)}{\sqrt{\| w(t) \|^2}}$$

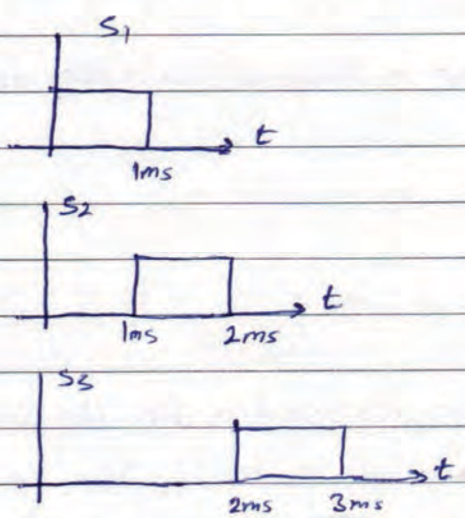
* Signals are orthogonal if they are Not over lapped in any space (time domain, frequency domain, space, polarization, code).

Space: XYZ space "coordiantes"

polarization: the direction of electric feild vector in space [EM definition we will talk about this later]

Code: representation of the shape of the signal

Ex:-



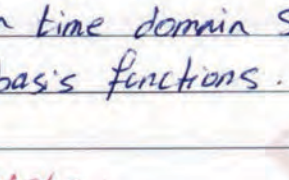
* they are orthogonal because they are not over lapping in time domain so these signals can be used as basis functions.

* Note:

if we have 2 channels (wires) and in each one of them we have a signal of $\sin(wt)$ are they orthogonal?

yes, different spaces because they are guided and can't be mixed unless they are on the same space.

Code domain : Simple example:



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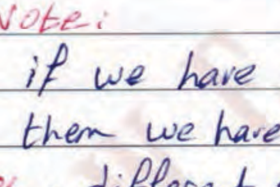

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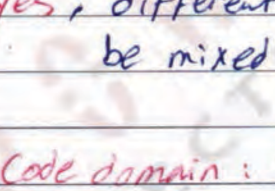

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* how to generate orthogonal basis functions [see website]

Walsh - Hadmard

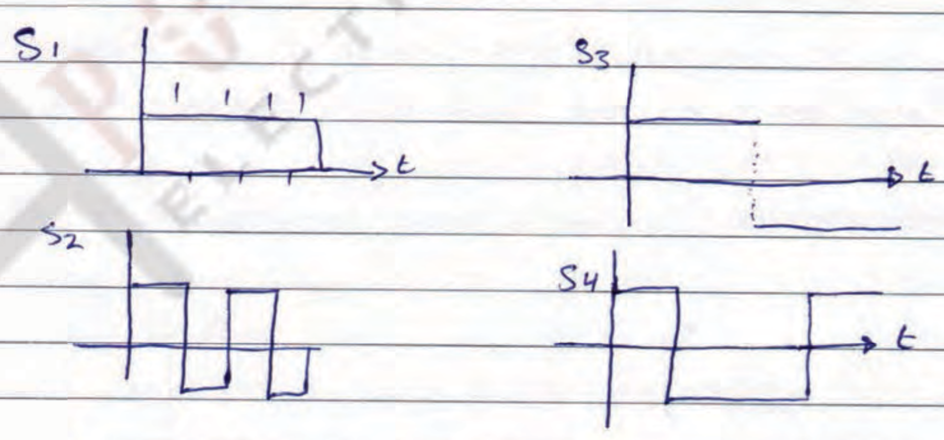
how to generate orthogonal signals from code domain

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$H_{2N} \quad \begin{bmatrix} H_N & H_N \\ H_N & -H_N \end{bmatrix}$$

$$H_4 = \begin{bmatrix} H_2 & H_2 \\ H_2 & -H_2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$



* those 4 orthogonal signal can represent a 4-dimentional space.

No.

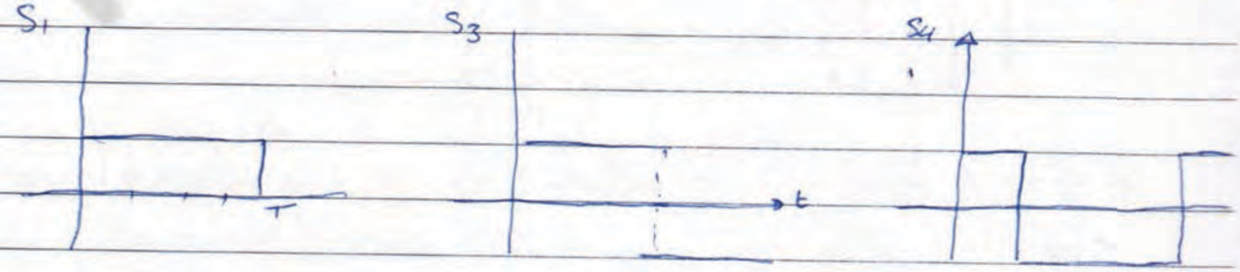
lecture 4

if we are required to design 5 orthogonal signals So we need N to be greater than 5 wich is 8 (H_8) & then we choose 5 of them.

in the previous example :-
the period is T , that means that the fundemental frequency is dependant on the period.

Note:
→ if we want to choose a signal with high frequency contents then we choose the signal that has more wrinkles.
→ if we want to choose a signal with low frequency contents then we choose the signal that has less wrinkles.

* Usually in communication we preserve the Band width. we alway aim to send the Data with the minimum Band width.
So from the previous example (walsh-Hadmard matrix) we choose:



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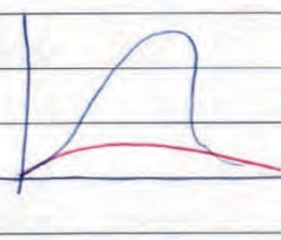
* the signal that has more zero crossing points, we can recieve it butter at the reciever side why?
→ because it is more stable for syncronization.
but this issue is not important as Bandwidth.

* sharp edges in the previous figures means infinity frequency content.

* if we took fourier transform of those signals, we will find that the power of those signals are localized. On the other hand, the signals with more wrinkles is spreaded over the frequency domain more.



* the red signal has more wrinkles
* they both have frequency content almost the same
* the blue signal is localized because it has a maximum peak where almost all Energy is around that point.

* in the red signal the Energy is almost spreaded equally over the frequency and there is No domenant frequency in it.

* the smothest signal is sinusoid why?
Because it is signal tone; cos(wt) is signal tone in frequency domain, it is an impulse (No width), so it is localized exactly at it's certain frequency.

No.

* in our designing of signals we prefer the localized signals why?

Because if we don't have enough bandwidth and we are limited to certain frequency that i can pass, so we directly choose the one at the peak, otherwise if it was spreaded signal if i took portion of it i still lose significant part.

* we have 3 definition to choose the bandwidth

→ 3-DB Bandwidth

→ Null-Null Bandwidth

→ % of Power Bandwidth

calculating

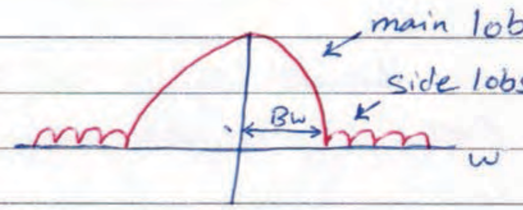
3-DB: Not used in communication it is rather used for Filtering

channel BW.

Digital Signals in general:

Sinc (?) . g(w)

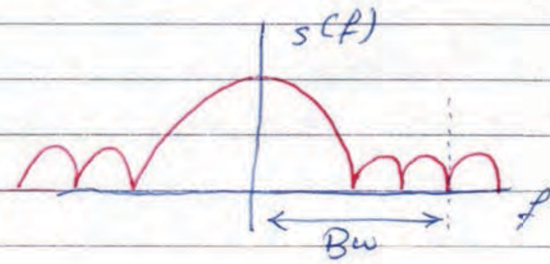
* most of the energy signal is around zero (main lob), so sidelobs can be neglected.



But, if we design a signal with large side lobs.

$$\% * P_t = 2\int_0^{Bw} |s(f)|^2 \, df$$

P_t: total signal power.

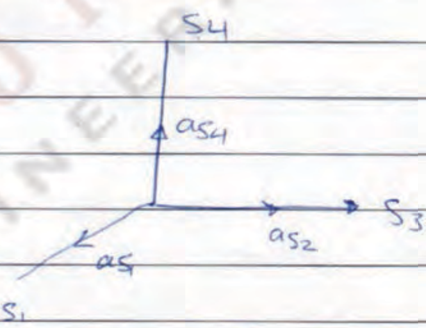


No.

* usually our signals are random process (Not determenistic)
* Null-Null Bandwidth for Digital signals are most commonly used.

Space Diagram: representation of a signal which is a function of time in point in x-y-z space.

$\overrightarrow{as_1} = \frac{s_1}{|s_1|}$



$\overrightarrow{as_3} = \frac{s_3}{|s_3|}$

$\overrightarrow{as_4} = \frac{s_4}{|s_4|}$

if we have a signal x

$x = [\longrightarrow]$

$E_{xx} = x \times t$

So $|s_1| = \sqrt{E_s} = \sqrt{a^2 T}$ assume $a = 1 \rightarrow |s_1| = \sqrt{T}$

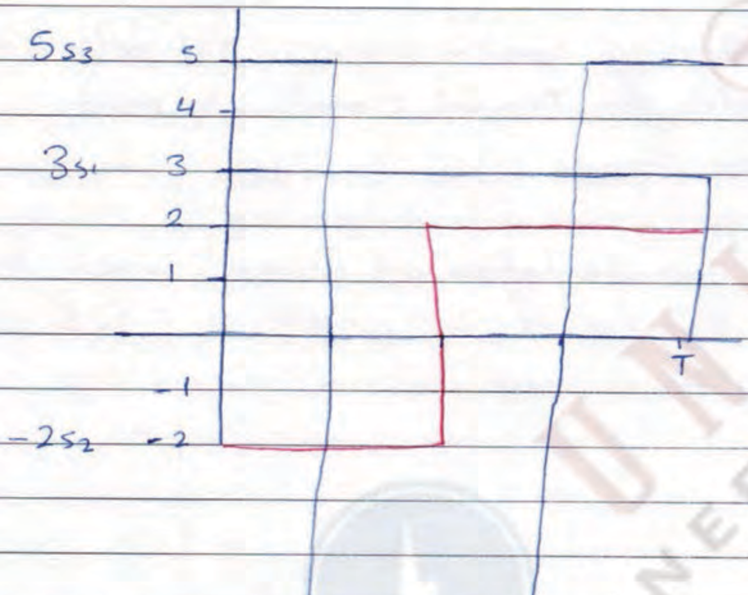
$\overrightarrow{as_1} = \frac{s_1}{\sqrt{T}}$, $\overrightarrow{as_3} = \frac{s_3}{\sqrt{T}}$, $\overrightarrow{as_4} = \frac{s_4}{\sqrt{T}}$

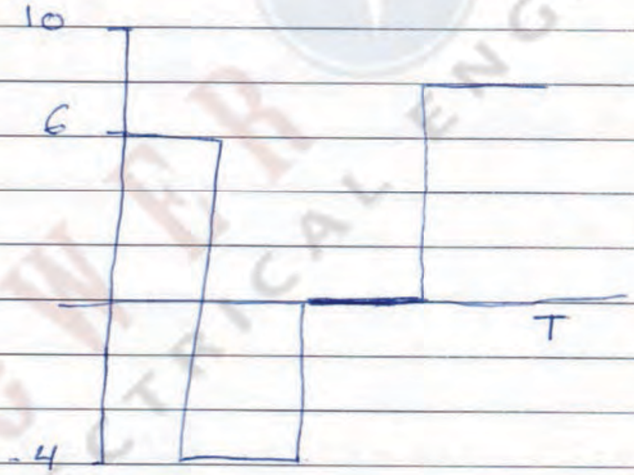
e.g. assume $T = 1$

$x = 3\overrightarrow{as_1} - 2\overrightarrow{as_2} + 5\overrightarrow{as_3}$

$x(t) = 3s_1(t) - 2s_2(t) + 5s_3(t)$

No.





lecture 5:-

Signal space: is a representation of a signal in time domain as a vector in space in order to use some tools to analyse the signal better.

* if two signals were linearly mixed we can separate them at the reciever side But if those signals were non-Linearly mixed, Distortion will happen and we won't be able to retrieve the original signal.

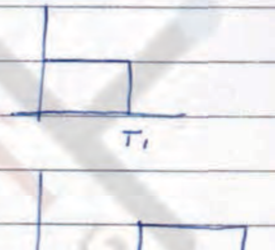
* in analog communication we represented orthogonality by sending several signals each have center frequency and Bandwidth so they are not over lapped in frequency domain that means they are orthogonal.

* the Basic concept of modulation is by multiplying the signal with Cos(wt) or sin(wt) to move it from base band to band pass on different frequency in such way it won't be mixed or over lapped with other signals.

* I can separate orthogonal signals either in frequency domain or in time domain depends on the current domain i have, in other words; if i have those signals in frequency domain i can put a Filter on each one of them and the out put of each Filter will be one of them separated from the others

No.

* if the signals were in time domain if we took Fourier transform to those signals they will over lap in frequency domain so using Filters won't work, so i want to separate them in time domain By knowing the period of each signal we start reading each of them based on the period of each one.



Note: in general inner product is:

$\vec{a}\cdot\vec{b} = |a|\cdot|b|\cos(\theta)$

if i have 20 vectors i need up to 20 plon (axis)

* if i have a matrix:

$A = [a_i]_{N \times N}$

* the rank is maximum N

* Full rank matrix:

→ it's rows and columns are independent

→ it has a non-zero determenent

→ Non-singular matrix A^{-1} exist.

* a more general way other than walsh-Hadmard matrix in the code domain to generate orthonormal basis function:

A is a full rank matrix

$\Phi = eig(A)$

No.

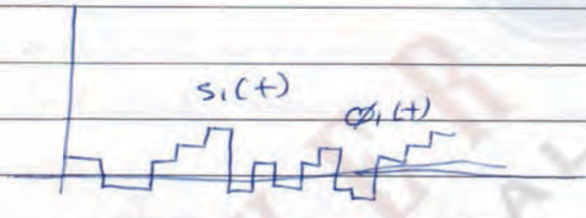
Eigen values : is the projection of this matrix (A) over the Eigen vector matrix

Eigen vector matrix : the space which span the whole (A) matrix

Span: can represent all the elements in the other Field.

$\Phi = [\phi_1 \ \phi_2 \ \phi_3 \ \phi_4 \ ... \ \phi_N]$

here i have N orthonormal vectors
those values can be represented graphically:



$s_2(t)$
⋮

how to find eig vectors:

$det(A - \lambda I) = 0$
another way is by using discret Fourier transform in digital domain
if i have a vector $S = [1, 2, ...]$
DFT (s):

$S' = S \, \Phi_{DFT}$

$$\Phi = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & e^{-j\frac{w_0}{N}} & & & \\ 1 & & & e^{\frac{-jnkw_0}{N}} & \\ 1 & & & & \end{bmatrix}$$

No.

* in discrete Farier transform, we project the signal on a complex sinusoid.

* eigen vector matrix we project the signal on an orthonormal space with N-Dimention every column represent different axis.

$A \Rightarrow eig(A) = \Phi$

$A = \sum_{i=1}^{N} \lambda_i \phi_i \phi_i^T$

I decomposed A to it's λ's

* any signal has certain corrolation, the more corrolation the more the power spectral density becomes localized

Power spectral density: Fourier transform of the auto corrolation function of the random process

* uncorrolated $\rightarrow$ white noise, the auto corrolation is an impulse and the power spectral density is constant.

* in this course most of the signals that we will deal with are localized (Limitted Bandwidth).

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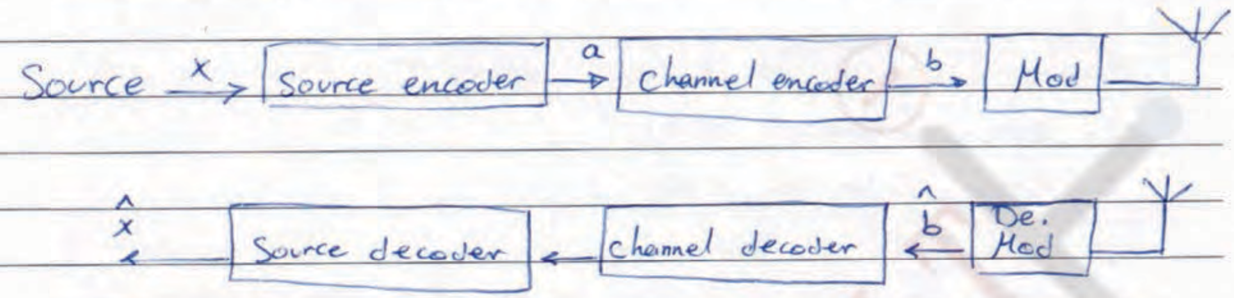


wire channel: it is local application, limitted use now adays

Source encoder: representation of the source signal in binary format, it's two stages:

→ A/D conversion

→ compression

the output of the source encoder is binary stream @ r_b bps

* in our design of the source encoder it must be the minimum data rate at best quality

* if we jumb to the reciever side directly from the source Encoder to the source decoder:

1. Source decoder will decompress the signal
2. D/A conversion
3. the output will be $\hat{x}$ because the A/D conversion & the compression may cause to lose portion of the signal.

mean square error (mse):

$|x-\hat{x}|^2 \Rightarrow$ indication of distortion.

No.

channel encoder: it is two types:

1. error correction codes

2. error detection codes (CRC)

* channel encoder will try to maintain the signal and correct the errors in the channel and the errors that happens in the modulation also.

* usually in channel Encoder & Decoder we don't lose information so (a) and ($\hat{a}$) are exactly the same if we directly put a feed back from ch. encoder to ch. decoder. Because in ch. encoder i add protection data.

* for butter performance of the system & more cabability of correction errors we have to add Control data Like parity checks.

* the Data rate at the channel encoder:

output of the

$$r_b' = r_b \cdot \frac{n}{k}$$

$$\frac{n}{k} = \frac{n \text{ bits output}}{k \text{ bits input}}$$

* if (a) is 10 bits for example aFter adding 3-bit parity to the signal (a) the time to output of those 13 bit must equal the time of input 10 bits

so $\frac{n}{k}$ will be $\frac{13}{10}$

$$r_b' = r_b \cdot \frac{13}{10}$$

No.

lecture 6:-

* the output of bits (output of the channel encoder) are formatted data added to it controll bits , so we don't care about the meaning of the data , all bits are equally likely and they have the same weight.

Blind modulation:- takes the stream of bits as it is
→ all bits are equally likely and have the same weight
→ the probability of 1 is equal the probability of zero on average.

* in the last stage (Modulation) the logical data is converted into channel wave forms $\phi(t)$

Wave form: Signal carrying Energy (information) in order to propagate through the channel.

* we have two types of channels:-
[1] wire [2] wireless

guided: we control the path between the transmitter and the reciever.

unguided: we can't control the path between the transmitter and the reciever.

partially guided:- Like spot light , i can light specific spot without physical guided , and the light is noticed away from that certain spot also.

No.

wired: copper, Coaxial cable, twistted pair wire ...
those are all channels that signals can go through

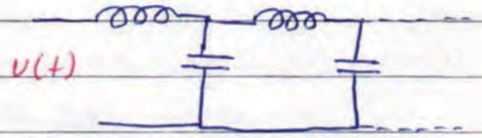
Note: No interferance will happen between two signals each on different wire unless they experienced a cross talk

Cross talk: adjacent channels will cause coupling and that will cause a small portion of signal interferance.

* Solutions of cross talk:
→ shielding
→ twisting.

* transmition line model:-

* if we apply a voltage signal at the input will propagate through according to



high frequency channel model and suffer reflections and delay.

in this LC circuit:

→ (L) is a reluctance component to the signal to energize the inductance

→ (C) needs time to charge so first charging the inductance and then discharge in the capacitor, then the capacitor will charge the inductor and so on, so the signal will eventually experience a delay.

No.

* Speed of propagation depends on the values of (C)&(L)

* the output of the transmition Line model is
$v(t-\tau) + n(t)$

$n(t)$: Could be white noise (thermal noise), $n(t)$ is Guasian noise.

* $n(t)$ characteristics:-
→ mean = Zero
→ Variance = power of the noise.

* in wired chanels case we can minimize $n(t)$ by shielding Or twisting the wires But if the noise is a thermal noise:-
→ Cooling
→ increase the cross sectional area of the conductor

* our mainly concern is reflection, if no matching is between the impedance of the Source and the impedance of transmition line, reflection of the signal will occure and that will cause loss in the signal or the signal will interfere with the reflected part in such away those parts will cancel each other causing what's known as Fading.

* Charactaristic impedance of the twested pair is 120 Ω
* charactaristics impedance of coaxial cable is 50 Ω

* those charactaristics defines the amount of power that will propagate through a transmition Line.

No.

* if the characteristic impedance of the next stage equal the charactaristic impedance of the previous stage matching impedance is obtained and the signal will not notice the differance between the two medias

* in wireless transmition channels the data is sent as electromagnatic waves, it will propagate through space and it doesn't need a matter to propagate.

* the impedance of pure space is 377 Ω

* in all communication devices the output will be on a coaxial cable for these reasons :-
→ No interference
→ Small amount of losses
→ Shielding.
and $Z_{T.L}$ for any communication device is 50 Ω
* in PCB the tracks are not straight and that is to maintain always a 50 Ω charactaristic impedance.

* Speed of propagation:-
$v = \lambda f$

* the antenna will do an impedance transformation between Space charactaristic impedance (377 Ω) and T.L (50 Ω) in order to send Energy signal without loss and this Operation is done gradually.
* the signal through space suffers from:-
→ white noise
→ multipath of the signal which cause fading.

No.

lecture 7: 4/3/2014

in this Course our mainly concern is about channels So if the channel suffers from additive white Guasian noise dealing with signals is usually easy But in fading channels dealing with signals require more attention and the modulation process also differ.

* in our speaking about modulation phase shift keying system is better than amplitude shift keying when the signal propagate through a fading channel why ?

because PSK is a constant evelope and it has a constant amplitude So as the signal propagate through the channel it experiance fading but the signal's amplitude is the one that will change not the phase.

* transmitted signal Can be written as a function of a certain amplitude and frequency and phase shift

$\phi(t) = A(t) \cos(\omega t + \theta(t))$.

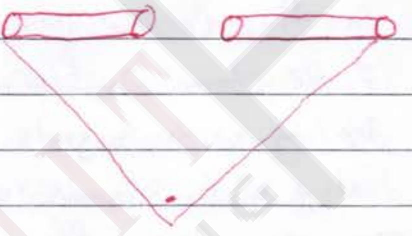
Cos and sin functions are used as a carrier why?
Because they are very good orthogonal basis function.

antennas Could be:

short antenna :- this antenna spread it's signal in all directions So it's called Omni directional antenna.

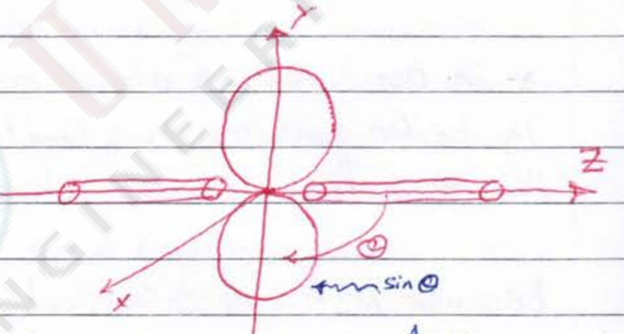
dipole antenna:-

* in the middle is the largest angle that we can receive the amount of power the Signal holds.



in Polar Coordinate:-

* radiation Pattern represent the direction of the antenna in transmiting and recieving.



(radiation pattern of the dipole)

* this trace represent how much power of the signal we can notice.
* if we move around from Z-axis to Y-axis we will see a figure looks like the number (8)

* radiation pattern in 3-D represent a figure looks like donut

* the transmitted signal is an electromagnatic wave this wave spread in space like a giant ball and expanding the more we get far away from the transmition point.

* that means that the energy of the signal is distributed Over the surface of that ball with radius equal the distance from the transmition Point

No.

* So, if the signal is spreaded to 10 m away from transmition point that means that the power is distributed on a ball surface it's radius is 10 m.

* the power Density ~~received~~ as a function of r is

$$P_D(r) = \frac{P_t}{4\pi r^2} \quad w/m^2$$

P_t: power transmited
r: radius.

power received as a function of r:

$$P_r(r) = \frac{P_t \cdot A_{eff}}{4\pi r^2}$$

A_{eff}: the effective area of the receiver.

ex: if the transmitted power is 100 watt and r = 36000 km assume $A_{eff} = 20\ m^2$ find P_r?

Sol:- $P_D(r) = \dfrac{100}{4\pi(36*10^6)^2} = 6*10^{-15}\ w/m^2$

$P_r(r) = P_D(r) * A_{eff}$

$P_r = 0.122\ pW$

* in Communication we prefer working with dbm.

So: $\dfrac{0.122\ pW}{1 mW} \Rightarrow$ ~~-90~~ $10 \log(.122*10^{-9}) = -99$ dbm

No.

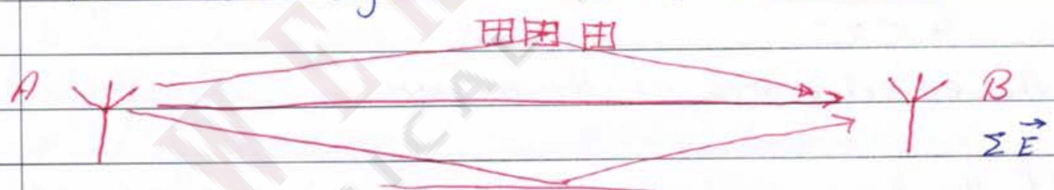
* in outer space the maximum power can be sent is 100 watt and for several reasons we can't exeed that number because of transponders and battery life time.

So, is there away i can receive the signal better?

yes, using coding technique, more complexity coding and more bandwidth will do the work specially in deep space communication ~~wer~~ where the power is very limitted

* this is similler to FM where the quality of FM receiver depends on the modulation index so increasing it will increase the bandwidth and increase the quality.

if we sent a signal from antenna A to antenna B



* this multi path each will cause an electric field at the receiver and they will add together vectorially.

* if they were out of phase $\rightsquigarrow \Sigma \vec{E} = 0$

* each electric field has noise in it different from the other depending on the channel (path) each has taken.

$$\vec{E}_T = \sum_{i=1}^{L} (\vec{E}_i + \vec{n}_i)$$

No.

* power density of the receiver:

$$P_D = \frac{|E_T|^2}{\eta}$$

in the case of fading:-

$$P_D = \frac{P_t}{4\pi r^{\gamma}}$$

γ: propagation constant $\quad 2 \leq \gamma \leq 4$

$$E_r = \sqrt{P_r * \eta}$$

$$= \sqrt{.122 * 50} = 2.4\ MV$$

* inside Communication system in general we always try to maintain the signal at zero dbm $\equiv$ 1mW $\equiv$ 200 mV

* the power at the receiver is random so

the pdf functions for it :

→ Rayleigh

→ Rician

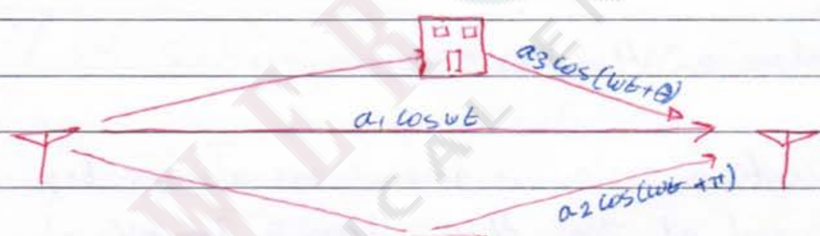
No.

lecture 8 :-   6/3/2014

* there are Some characteristics we should pay attention to Comes with wave propagation. which is the interaction between wave and matter, if the electromagnatic wave collide with a rough surface it will be scattered, if it passed through a knife edge it will diffracted.

* any Obstacle in the way from the transmitter to the receiver will reduce the power received.

* let's picture the following scenario :-



we have multi path here : 1. direct path
2. ground reflected
3. reflected from a building.

* Conductors reflect electromagnatic waves
* materials can be classified as conductors or not based on the wave length, Like glass, plastic both are not conductors yet they reflect the electromagnatic waves.

* here the ground if it is dry then it's not a conductor But if it was wet land it's then considered as a conductor usually ground contains moisture even in the desert

No.

* the wost case scenario is 2-ray model, two paths the first is direct from Tx to Rx and the second is ground reflected so they will cancel each other.

* ground reflected is aperfect conductor. (it will not consume energy)

* So the same energy that hit the ground will be reflected
* a_1 & a_2 the difference in amplitudes comes from distance, this difference is very small so the amplitudes are almost the same but still that's the reason they didn't cancel each other completely, still we reach very small amount of the signal at the receiver.

* Fading is due to multipath propagation.

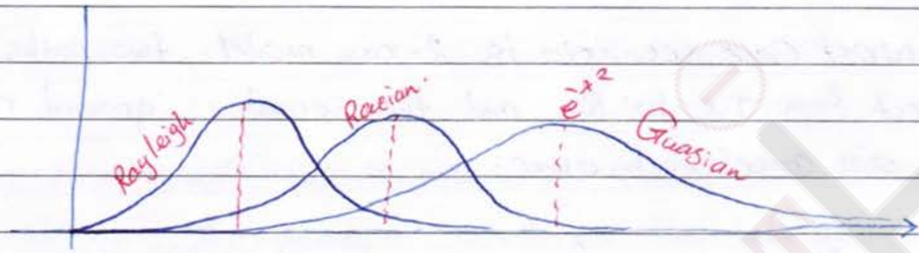
* power $= P_r \propto r^{-\gamma}$
  if single path $\gamma = 2$
  if 2 paths $\gamma = 4$
  other $2 \le \gamma \le 4$

* the received power as a magnitude is random

$y = |A + x|^2$

A: constant
x: Normal Guasian $N(0, \sigma_x^2)$

probability density function of y:
  if $A = 0 \rightsquigarrow$ Rayleigh
  if $A \neq 0 \rightsquigarrow$ Rician.



* Gusian Curve is shifted around A So the greater A the more the curve is shifted away from Zero.

* our goal is to make A as larg as possible & x which represent white nois then we want $|A+x|^2 \approx |A|^2$

* if the case is fading A becomes small related to x and the power of y becomes Rayleigh or Rician

* Rayleigh curve is close to Zero that means that the power is very small and it gives an indication that the signal experienced al lot of fading.

* Rician curve it's peak of power is greater than Rayleigh
* the worst case is Rayleigh fading channel.
** in Rayleigh we dont have adirect Line of sight between Tx & Rx.

* in Rician there is a direct line of sight between Tx & Rx.

* Receiver performance is related to SNR

$$SNR = \frac{\text{signal power}}{\text{noise power}}$$

No.

* in the case of fading we calculate the average SNR

Average means that we want to integrate over one of the curves depending on the channel case from zero to ∞ & the Avg. is around the peak (mean of the received power).

* the greater the mean value the greater the received power becomes.

So in fading channels the received power can be modeled as follows:-

$P_r = P_o \cdot \alpha$

α: random variable.

the noise in the case of fading is modeled as multiplicitive noise (α)

* this is practically can be seen in short wave radio.

* multipath can be seen in settaliate communication when the signal passes through the inosphere.

* inosphere's width is around 580 km which gives the signals enough distance to bounce back to earth in the case of broadcasting short waves in abroad contries.

* Charactaristics of the channel will be considered in our design of modulation and channel coding.

No.

* the receiver trys to isolate the noise from the signal.

* we can calculate BER depending on SNR

* SNR is the most important factor in quality of service.

* if the BER Limit in a system is 10% & the Calculated BER turns out to be greater than the Limit, this system then is insufficient

* if the BER is larger than the limit we can minimize it by using channel coding.

in general, quality measures must be passed.

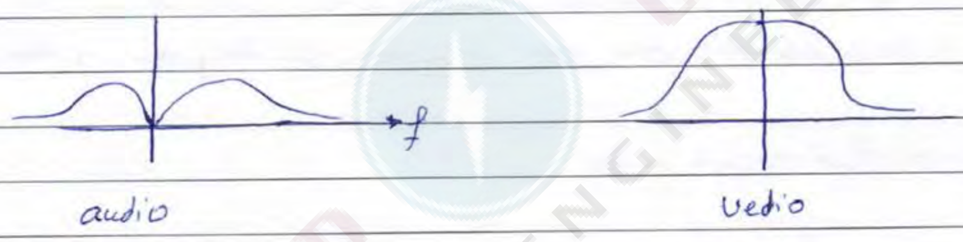
No.

lecture 9:   9/3/2014

Base-band Communication:-

Base-band : Centered around Zero.

is there any DC value in base-band signals?
⇒ it depends on the signal, for example:



* in time domain the signal goes to infinity so we can't actually see the whole signal, so we can't study the charactaristics of the signal accurately. for that reason we use frequency domain.

* fequency domain is the simplest tool to analyise & thensisize the signal.

* transformation between time domain and frequency domain is Linear transformation, that means that the signal doesn't lose any of it's charactaristics while transforming it from one domain to another.

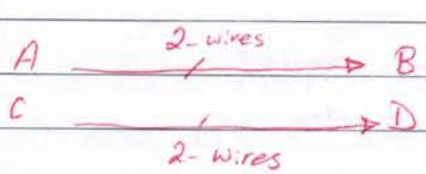
* transformation is a projection on basis functions (e^{-jwt}) in the case of fourier transform.

$$\int x(t)\, e^{-jwt}\, dt$$

No.

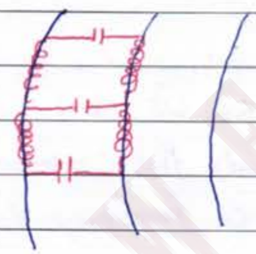
Band-pass: that means that the signal is transfered into frequency domain (centered around f_c HZ).

* To creat orthogonal signals in base-band, we need to make them orthogonal in space (each on separate twisted pair wire)



* those signals are not overlapped in space, so they are orthogonal.

transmition Line circuit model:-



* the more the wires close to each other with less sheilding between them the more mutual inductance will creat coupling (cross talk).

* if the signals are high frequency signals cross talk will increase.
⇒ the impedance of the capacitor is $\frac{1}{j\omega t}$. higher frequency means less impedance, Leakage between 2-wires will increase.

* if we have 2-signals:-

$S_1(t) = a(t)\cos(\omega t) + n(t)$
$S_2(t) = a\cos(\omega t + \theta(t)) + n(t)$

No.

* as mentioned before noise directly affect the amplitude
* in $S_2(t)$ the information are in the phase & the noise is in the amplitude.

* if the noise were a multiplicitive noise:-

$S_1(t) \rightsquigarrow$ $a(t)$ is multiplied by $n(t)$
$S_2(t) \rightsquigarrow$ a is multiplied by $n(t)$ but we don't really care because we are not intrested in amplitude information using discriminator we amplify the signal & then it does a clipping process to make a constant amplitude.

* So phase modulation in the case of multiplicative noise is better than amplitude modulation.

No.

lecture 10:

11/3/2014

* usually the type of modulation is defined by:

1. Channel ; sending data through a fading channel we prefer to send it in phase rather than amplitude because noise affect the amplitude direct.

2. the application ; if the application is point to point and for a close range communication we prefer to make it wired , Like ethernet connecting computer with a router.

* Base-band communication is point to point communication.

* Base-band communication is not applicable for RF transmittion for 2-reasons:-

1. all signals will interfere with each other
2. the size of transmition antenna.

for example: what is the wave length for 10 KHZ frequency?

$$\lambda f = c$$

$$\lambda = \frac{3*10^8}{3*10^3} = 3*10^4 \text{ m}$$

So we need here a very long antenna which is not practical.

* So base-band communication is a wired application.

Note: wired $\neq$ optical.

optical: is a wave guide I we can send an electromagnatic waves inside a conductor guiding the wave inside it Like under bridges.

No.

* here in order for the electromagnatic waves to propagate through this wave guide, the width of the wave guide must equal to $\lambda/2$

for example: we have Amman channel transmitted on those frequencies.

Amman	λ	$\lambda/2$	$\lambda/4$
612 KHZ	490	245	122.4
801 KHZ	374	187	93

* the transmitted power from the station is between (0.25-0.5) Mega watt.

* in Am, electromagnatic waves propagate on the surface of earth so it's called surface waves, So wherever you are those waves will reach you guided by the surface of earth

* the higher the transmitted power the far the waves can move.

* in figure 1:
* larg scales on small frequencies But on the other hand:

Figure 1

for FM:

99 MHZ $\rightsquigarrow$ $\frac{\lambda}{2} = 1.5\,m$

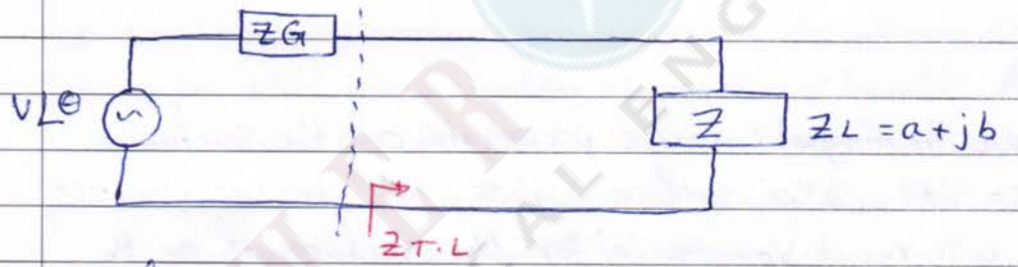
that's why radio antenna in cars are small

No.

* Sensitivity of the receiver:- the minimum power for the receiver to operate and it depends on :
1. power received (power density)
2. how much power the antenna can receive to drive it's own circuits.

example:- in mobile phones the sensitivity is -110 dbm. that means that if the power received reached 10^{-11} mW it can be receivd and distinguished.

* wired communication model:-



* for maximum power transfer
$Z_G = Z_{T.L}$
$Z_{T.L} = Z_{load}$
that's called impedance matching.
Note:
twisted pair 120 Ω
open wires 75 Ω
coaxial 50 Ω
different coaxial design can reach 75 Ω

Binary transmition:-

Modulation: maping between logical data into physical wave forms.

No.

* number of wave forms = the number of inputs, so we need to generate two wave forms (binary)

$S_0(t)$ = "0"
$S_1(t)$ = "1"

* the transmitted data is a serial of one's and zero's each have a certain time interval, and that defines the bit rate.

example: Calculate the bit rate for 8-bit mono adio at toll quality (PCM64)

Sol:-

bitrate $(r_b) = 8 * f_s$

f_s: Sampling frequency.

$f_s = 2 * \underbrace{f_m}_{4\,KHz} = 8\ KHz$

$r_b = 64$ k bps

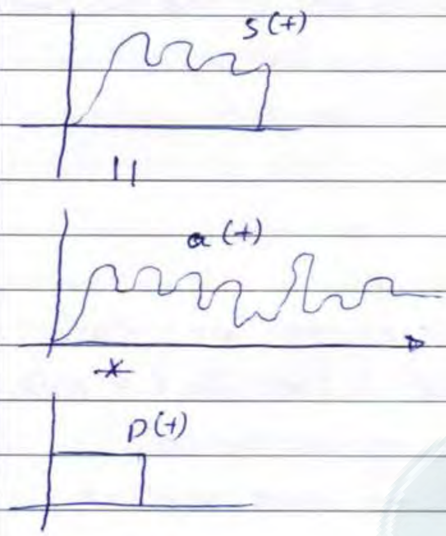
toll quality: telephone quality, which defines the characteristics of the human voice, so 4 KHz is enough for toll quality.

→ bit interval $T_b = \frac{1}{r_b}$

So, S_0 & S_1 can't be greater than T_b, so $S_0(t)$ & $S_1(t)$ are time limited.

Can we generate time limited wave forms?
it is impossible because we can't reach zero in no time.

* for all time limited signals band width



$$s(t) = a(t) \cdot p(t)$$
$$S(f) = a(f) * p(f)$$

$S_0(f)$, $S_1(f)$ } → band unlimited.

* any function convolved with a sinc the output will be something covering all the frequencies from $-\infty$ to ∞.

* all medias have limitations in band width
Band width depends on:-

$f_{max} = \frac{1}{2\pi\sqrt{LC}}$ for T.L

* infinity bandwidth signals will experience some loss at certain point because of bandwidth limitations and that means we distorted it.
* So we have white noise and distortion.

* So we must design a signal in such away it will over come the problems in the channel

* in our design of $S_0(t)$ & $S_1(t)$, the receiver must be able to distinguish each one of them clearly
⇒ less error [less BER].

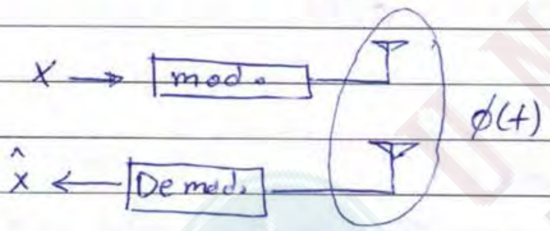
No.

lecture 11 :- 13/3/2014

Base-band binary transmition:-

Modulation:- maping from logical data into channel wave forms.

Channel wave forms:- physical signals carrying energy.



* in base band our channel is wired channel so the noise we have is additive white Guasian noise

Note: we can send modulated data on wires.

* Binary transmition:- that means that the logical data is either "1" or "zero"

input : $x \in \{0, 1\}$

output: of Mod. :- $\phi(t)$

* $\phi(t)$ is a wave form (continous signal always)

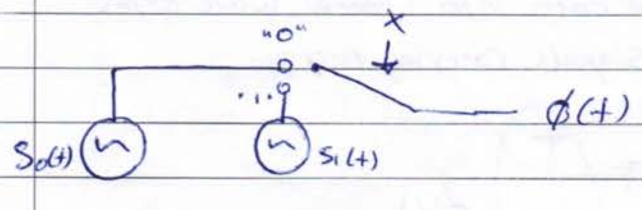
* if we have 2 input signals the output will be 2 also.

→ in order to represent that:-

$\phi(t)$ must be : $S_0(t)$ → represent "Zero"

$S_1(t)$ → represent "1"

* So $\phi(t)$ will be $S_0(t)$ if the input is zero, and $S_1(t)$ if the input is "1", so the modulation is like a multiplixer.

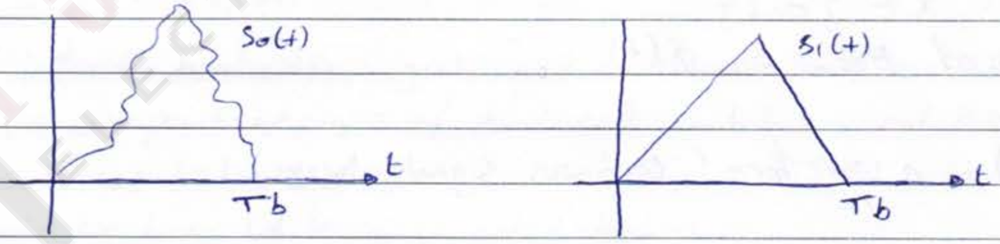


(functional diagram)

* the data changes every T_b, so the switch will be on the zero line for T_b period & then the switch will change to the "1" line for T_b period and so on.

So each signal needs T_b of time to change the state of $\phi(t)$

* is it a must that each of those signals must consume the whole T_b period?
No, maximum time needed for each signal is from $[0, T_b]$



* $S_0(t)$ & $S_1(t)$ must be different in order for the receiver to distinguish each of them.

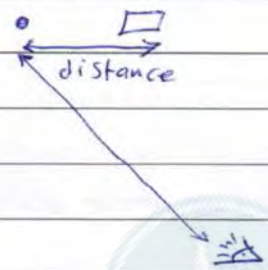
difference → represent distance in signal domain.
energy →

No.

* if SNR is larg , we can find the difference between them

SNR Concept is represented Like this:-

if i have two points:-



* being close to them will reduce the noise so finding the difference between them becoms easy unlike being away from them.
* if we in large the point's size (increased it's energy) we can then notice the difference even from a far point of veiw

* how to measure adistance:-

we need aspace (representation in mathmatics in certain domain)

one dimentional modulation $\vec{as}$

two dimentional modulation $\vec{as_1}$, $\vec{as_2}$

multi dimentional modulation $\vec{as_i}$ $i = 1, \dots M$

as's: wave forms that carries acertain characteristics in order to propagate through the channel & we represent $s_i(t)$ using these axis

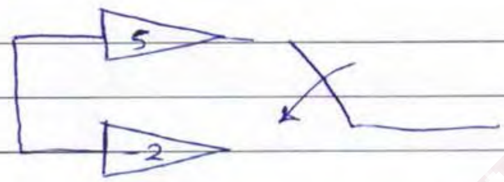
* So the problem in modulation is to define an orthonomal basis function

example:-

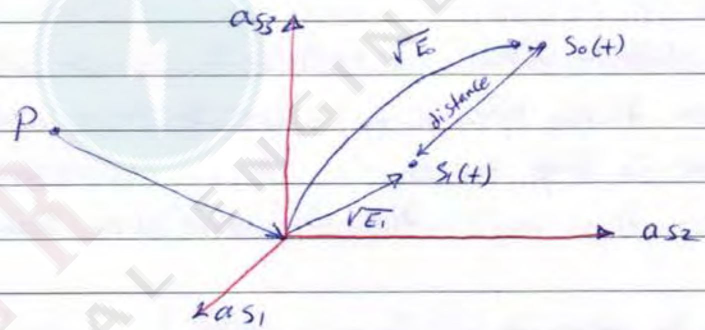
$S_0(t) = +5\,\vec{as}$

$S_1(t) = -2\,\vec{as}$

$as(t)$



* in Binary transmition if we define more axis we increase Complexity



prove that the distance from point (p) and the origin is $\sqrt{E_{avg}}$

assume point $p(2,3,4)$

Sol:-

$p(t) = 2\,\vec{as_1} + 3\,\vec{as_2} + 4\,\vec{as_3}$

distance to origin $= \sqrt{4+16+9} = \sqrt{29}$

$$Energy = \int |p(t)|^2\, dt$$

$$= \int |2\,\vec{as_1}|^2 + |3\,\vec{as_2}|^2 + |4\,\vec{as_3}|^2\, dt$$

$$= 29$$

So distance is $\sqrt{Energy}$.

No.

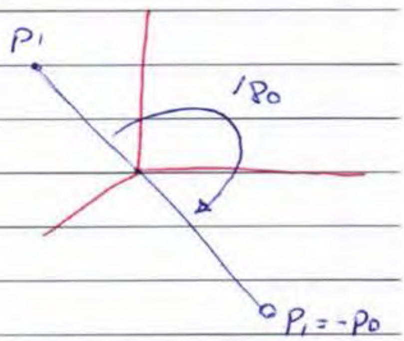
* we talked earlier that the distance is represented by difference and energy So based on that how can we say that those Signals ($s_0(t)$ & $s_1(t)$) are better than other Signals?
* distance should be as great as possible under the constrain of minimum energy.

So, the choice of $s_0(t)$ & $s_1(t)$:
* select the two signals such that they have maximum distance & minimum avarage ~~PE~~ Energy.

$$\left[\begin{array}{c} \text{Max } (d_{ij}) \\ \min E_{avg} \end{array}\right]$$

* this Solution is when $s_0(t)$ & $s_1(t)$ are antipodal (angle between them = π)

* here the optimal solution is when they are on the same line So we don't need to represent 3-axis, So in binary transmition we only need to define one axis.



* coorolation coofecient between antipodal signals = -1

lecture 12: 16/3/2014

let's talk about the receiver side:

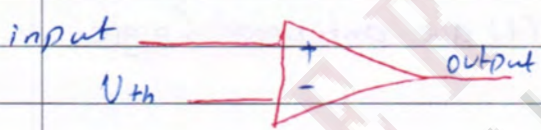
→ how to distinguish the difference between two signals?

if one of the signals is a positive pulse and the other is a negative pulse. we use the level detector

→ the simplest receiver is level detector.

→ here the difference of the two signals we created at the transmitter is in amplitude.

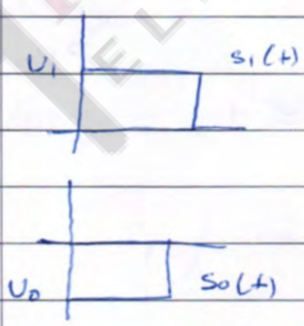
→ level detector is an op. amp



→ the output is high if the input is greater than the threshold.

→ the output is low if the input is less than the threshold.

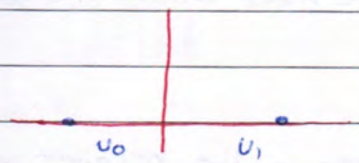
→ if we use the following antipodal signals:-



$$V_{th} = \frac{V_1 + V_0}{2}$$

* because we have the choice of creating antipodal signals at the transmitter, it is not an optimal solution to creat antipodal signals but with different amplitudes like for example $+5$ & -2, and here is why:

1. from energy point of view:
⇝ minimum average.
if we took 2 signals



$E_1 = V_1^2 \cdot T_b$

$E_2 = V_2^2 \cdot T_b$

$E_{avg} = \dfrac{E_0 + E_1}{2}$

2. distance point of view:-
⇝ maximum distance.
distance $= V_1 - V_0$

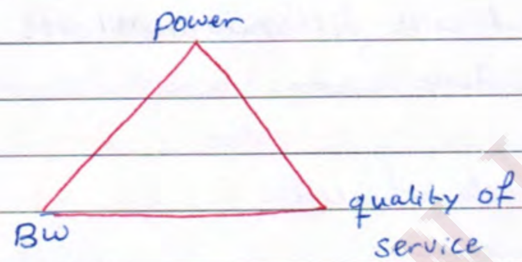
find the optimal solution of:

$$Y = \sum_{i=1}^{I} x_i \qquad x_i \geq 0$$

that means: the output = the summation of inputs such that those inputs are a non-negative numbers.

No.

3. quality point of view:



→ minimum transmition error (better quality)
→ when we send x & receive $\hat{x}$, we need minimum error between $(x, \hat{x})$

* we only assume the error might happen in the receiver.
→ the receiver analyse the signal and act like level detector.

* noise might change the signal charactaristics in such away it will look like the other transmitted signal.
→ assume the noise changed the signal from +ve pulse into -ve pulse, the receiver then will read zero instead of "1".

* when the receiver decide for a one or zero according to one point this is called hard decision.

→ creating a receiver that takes many samples of the signals will reduce the error the receiver might take.

No.

→ if in the first sample a spark happened & it switched that sample from +ve pulse to -ve pulse, the receiver yet can find the average of those samples & then decide.

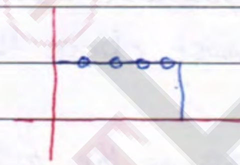
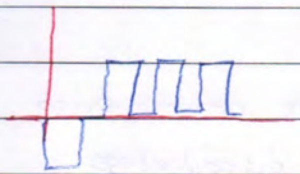


Figure 'a'

So figure 'a' becomes:-

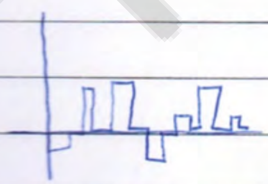


* do we count the number of positive pulses against negative pulses or do we take the average?

here the amplitude is constant so it doesn't matter which way to choose. But if the amplitudes differ, we take the average.

if the signals were:-

and at the receiver it was something like this:-

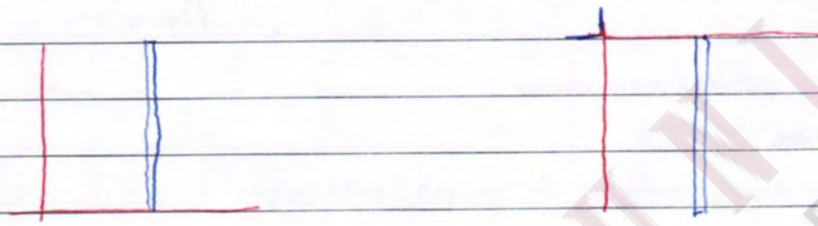


here it is a must to take the avg. of those pulses to make a decision.

No.

if you were asked to choose a hard decision for this signal you must choose the point in the middle.

→ assume the following antipodal signals:-



→ more suitable for hard decision but they are narrow pulse width that means more bandwidth & that is a disadvantage.

See ~~web~~ website for the following topics:-

1. LDO regulator
2. Charge pump
3. max 232.

No.

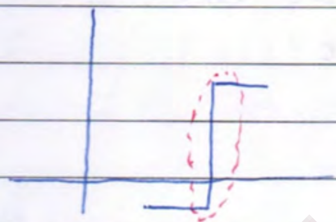
lecture: 13 18/3/2014

Base band binary transmition.

→ this assumption assumes that the channel is infinite (infinity bandwidth)

→ any signal with sharp edges means it has infinity bandwidth.

examples



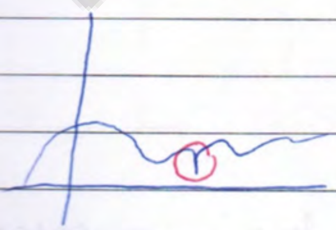
$\frac{dv}{dt} \propto$ frequency.

means that frequency goes to infinity.

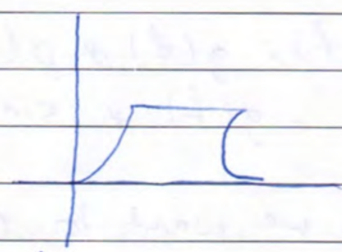
→ if it was angle (sudden change):-

$\frac{dv}{dt}$ = undefined.

more:-



frequency goes to infinity



frequency goes to infinity.

No.

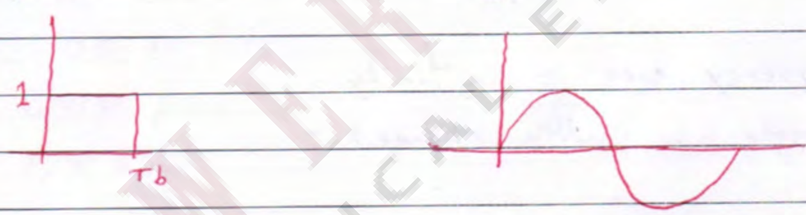
→ those signals need infinite bandwidth to protect it from distortion.

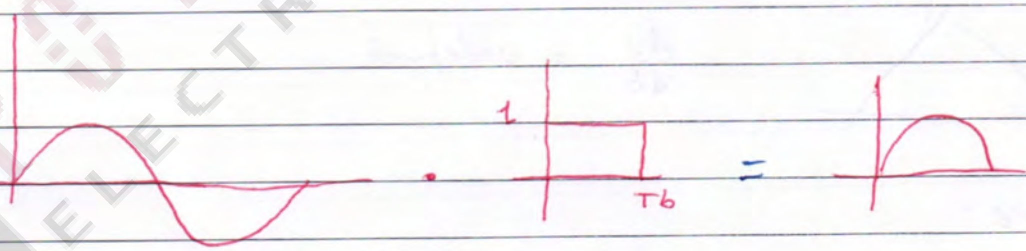
→ Basically we are sending digital data each have Certain period T_b

So : $g(t) \cdot p(t) = S(t)$

it means that : No matter what the signal is $g(t)$ multiplied by a pulse $p(t)$ ⇒ convolution between Certain frequency function with a sinc.

$p(t)$ $\qquad\qquad\qquad$ $g(t)$





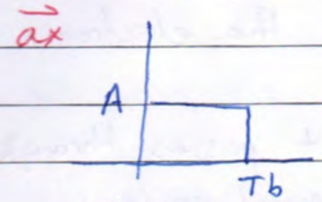
$$s(f) = g(f) * p(f)$$
$$= g(f) * \text{sinc}\,(\alpha f)$$

→ we want to creat a signal with minimum energy at maximum distance.

No.

→ maximum distance is when the signals are antipodal
→ we need one dimentional axis so we don't need more than one carrier.

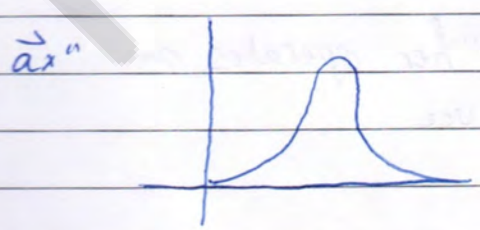
example:- binary antipodal.



$$E = \int_0^{Tb} |ax|^2 \, dt = A^2 Tb = 1 \text{ unit}$$

$$A = \frac{1}{\sqrt{Tb}}$$

we can choose another axis for example:-



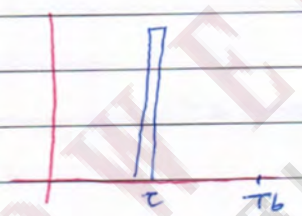
the best one to choose is the first one
→ in the previous lecture we wanted to use a narrow pulse for level detector receiver but it costs us rail to rail voltage.

→ usually amplifiers must have +ve supply & -ve Supply because when there is no signal no current will pass ⇒ 0 Voltage.

in single supply amplifier Q point = $\frac{V_{cc}}{2}$
so even when there is No signal there is still a DC current passes through the electronix elements

→ power is dissipated when a current passes through the resistors in the form of thermal noise
So it will consume the power supply.

→ most of communication devices uses rail to rail voltage. So Q-point must be at zero.



* high supply voltage is required.

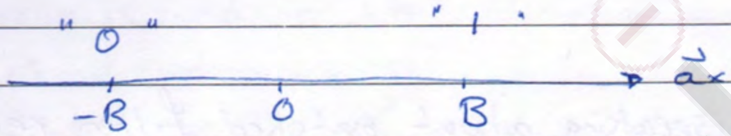
τ is smaller than T_b so high amplitude.

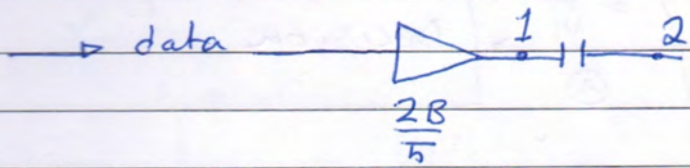
→ performance of the receiver depends on the power not the amplitude of the signal.

rail to rail amplifier:- the amplifier operates on linear region from $-V_{cc}$ to $+V_{cc}$

assume "1" → $B \cdot \vec{a}_x$
assume logic "0" → $-B \cdot \vec{a}_x$

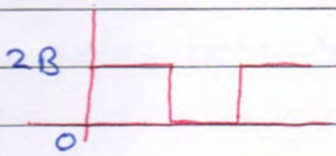
No.





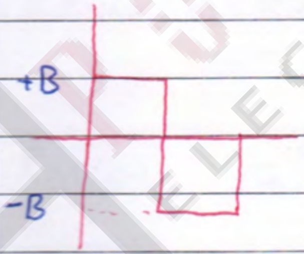
at point 1:-

the output is :



at point 2:

the output is



20/3/2014

No. ....

lecture 14

[1] We assumed that the Bandwidth is ∞

[2] In the receiver point of view the best choice is when the signals are antipodal

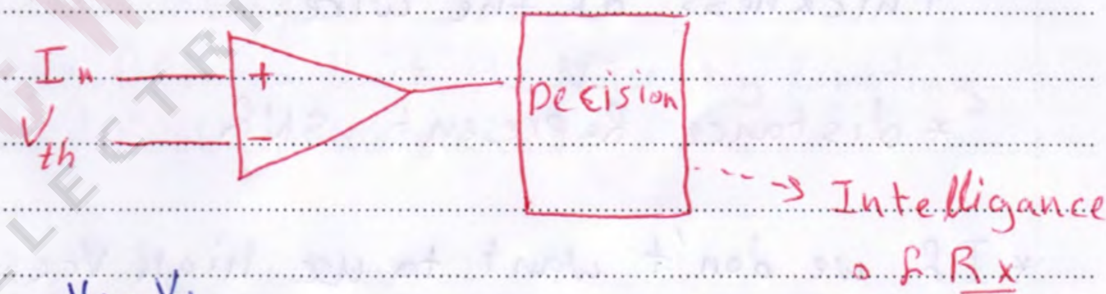
[3] Shape of the signals : voltage difference ($V_{Max} - V_{Min}$) should be large

Rx :-

① level detector :-

* simplest design

⇒ here we care about voltage



$$V_{th} = \frac{V_0 - V_1}{2}$$

, +ve → logic 1
-ve → logic 0

* If $V_{th} = 0$, The amplifier is called dual supply

No.

Low level signals (e.g Bluetooth, wifi ....)

* low power Transmitted

* The problem is that we need to amplify it and when we do that the Noise also will be amplified, so we use LNA in the first stage of the Front end.

* If LNA is too expensive we must increase the voltage of the signal.

* Voltage in Ethernet : ±1V

⇒ If we increase the distance the attenuation will increase, The solution is to increase the thickness of the wire.

* distance Represent SNR

* If we don't want to use high Vcc, the best solution is to use integration By using:-

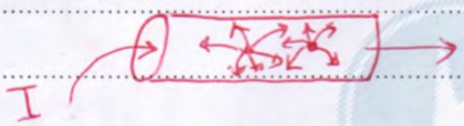
② Match filter Rx (correlator Rx):

* correlator Receiver is the best solution without considering Bandwidth

* To understant the match filter Rx we need to redefine Noise :-

Additive White Gaussian Noise :-

* Random process resulted from the collisions of electrons in the conductor



$I + I_r$ → Random current

$V + V_r$ → Noise

$\Rightarrow$ Gaussian $\rightarrow N(0, \sigma_n^2)$

* $V_{rms} = \sqrt{4KTB}$

B : Bandwidth of the BPF

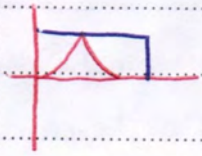
$\Rightarrow$ we use BPF to limit the Noise by limiting the Bandwidth of the signal

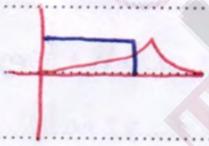
* All op-Amps have Bandwidth specifications

BW	G
1M	10
1000	10K

No.

* Raise time and fall time :-





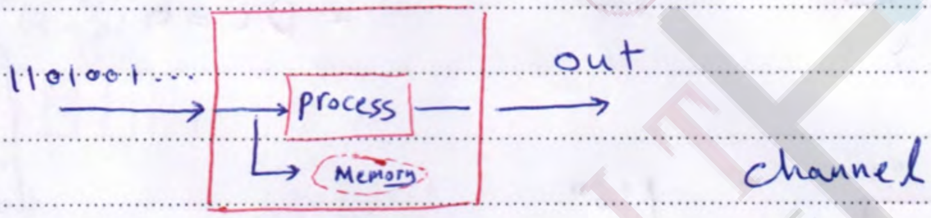
x bad ✓ good x bad

23/3/2014

No.

lecture 15:

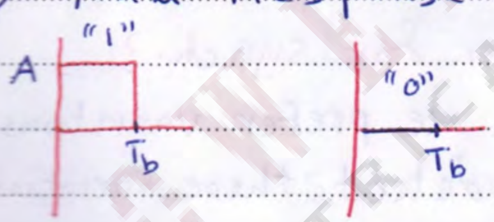
Tx :-



* Full Response: output depends only on current input [without memory]

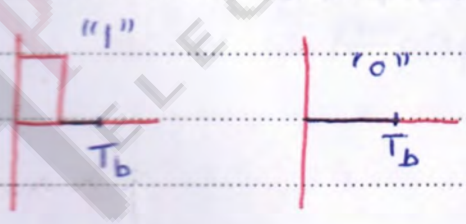
* partial Response: output depends on current input and previous inputs [with memory]

⇒ Full Response line codes (NRZ, RZ ...) :-



⟹ NRZ

$Dc = \frac{A}{2}$



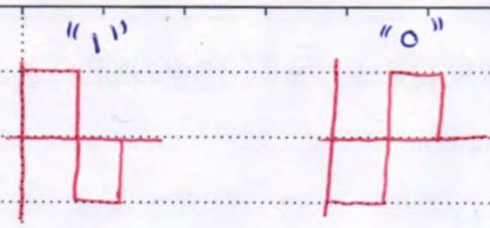
⟹ RZ

$Dc = \frac{A}{4}$

⇒ In RZ signal goes to zero to reduce the Dc

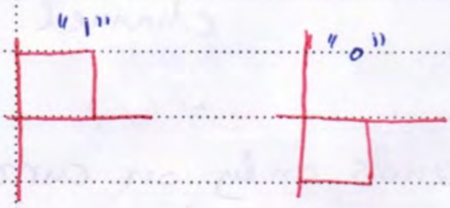
Dc ↑ → Noise ↑

No.



⇒ Manchester

* DC = 0



⇒ biphase

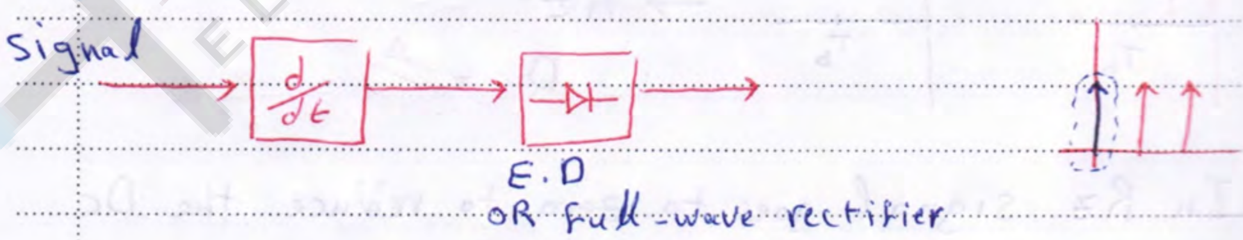
⇒ * both are antipodal
* both have the same energy

* Which is better Manchester or biphase??

Manchester is better, because we prefer a zero DC for synchronization and to reduce Noise.

⇒ why Manchester is better for synch.?

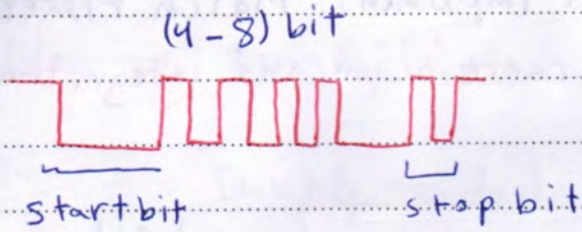
from sync. point of view we prefer variations in the signal and we can detect these variations using: differentiator + Envelop detector



No.

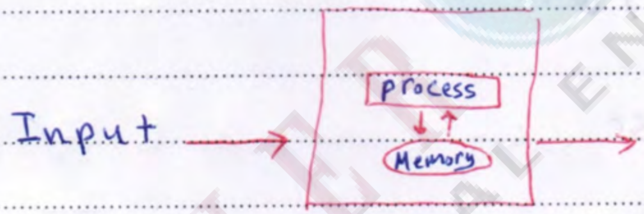
* async.

Example (serial communications in computer)



⇒ serial is async., but we must have at least synchronization at start bit

Rx : There are if statements (conditions) so we need memory



* If there is an error it will propagate, The solution is to do reset (for example every 10-bits) so the error will propagate only on these 10-bits

* Full Response :

⇒ Match filter Rx :

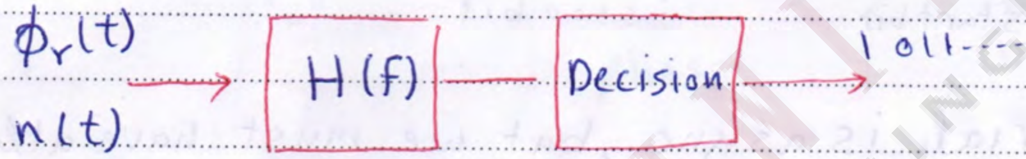
- coherent (from synch point of view)
- correlator (from Implementation point of view)

No.

coherent ≡ same phase and frequency

correlator ≡ How to implement Match filter RX

(we do correlation and integration)

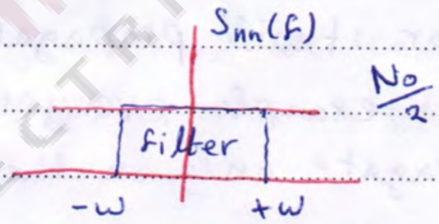


H(f) does convolution (Integration)

$$\int \phi_r(\tau)\, h^*(t-\tau)\, dt$$

If there is noise

$$\int n(\tau)\, h^*(t-\tau)\, dt$$



$\Rightarrow$ PSD of white Noise

$* \; \frac{N_0}{2} = 10^{-6}\ w/Hz$

after filtering

$$\int_{-w}^{+w} |H(f)|^2 \cdot \frac{N_0}{2}\, df = \int_{0}^{w} N_0\, |H(f)|^2\, df$$

No.

* Note

$N_o \longrightarrow$ single sided power spectral density

$\frac{N_o}{2} \longrightarrow$ Double sided power spectral density

Ex : $\underbrace{10^{-5}}$

$N_o = 10^{-5}$
If single sided

$\frac{N_o}{2} = 10^{-5}$
If double sided

No.

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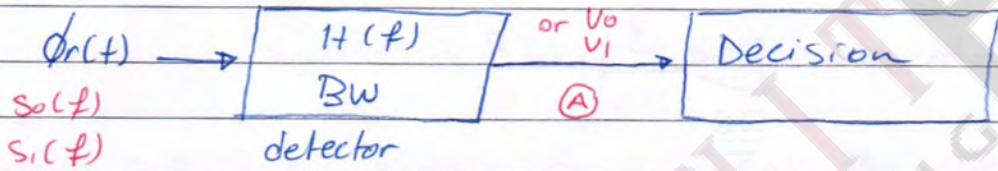
in our speaking about matched filter receiver



the idea of decision is to decide which one is sent "zero" or "1".

$S_0(f) =$ "0"
$S_1(f) =$ "1"

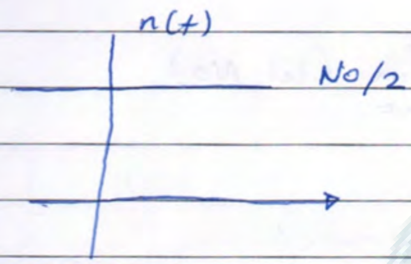
at point A:
we want to find SNR & we want to maximize it.

→ output of the detector
$V_0(t)$ if $S_0(t)$ is the input
$V_1(t)$ if $S_1(t)$ is the input.

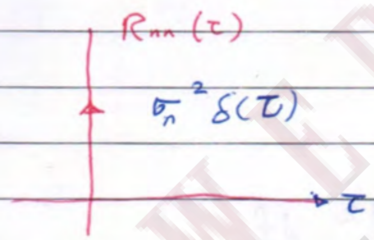
so $V_0(t) = S_0(t) * h(t)$

→ if there is a noise at the input does that mean that the noise at the output is the noise multiplied by $h(t)$?
yes.

- this noise is additive white Guasian noise
- noise is a random process.
- power spectral density of noise is constant which is the fourier transform of the auto corrolation function



auto corrolation function of the noise is an impulse.



$$R_{xx}(0) = \underbrace{\sigma_x^2}_{AC} + \underbrace{\mu_x^2}_{DC}$$

$$R_{nn}(0) = \sigma_n^2 + \cancel{\mu_n^2}^{\,0}$$

Usually in the white noise, it's mean is Zero with Variance $= \sigma_n^2$

δ: mathmatical function, it's amplitude is ∞ & width = Zero

No.

$$power = \int_{-\infty}^{\infty} S_{nn}(f)\, df = \sigma_n^{\ 2}$$

$$= \int_{-\infty}^{\infty} \frac{N_o}{2}\, df$$

$$\underset{channel}{\sigma_n^{\ 2}} = \lim_{w \to \infty} \int_{-w}^{w} N_o/2 \, df = \lim_{w \to \infty} (w \cdot N_o)$$

$$P_{out}\ noise = \int_{\substack{BW \\ \text{of the filter}}} \frac{N_o}{2} |H(f)|^2 \, df$$

$$SNR = \frac{\text{Signal power}}{\text{noise power}}$$

$$\text{Signal power} = \int_{-w}^{w} |S_o(f) \cdot H(f)|^2 \, df$$

$$SNR = \frac{\int |S_o(f) \cdot H(f)|^2 \, df}{\frac{N_o}{2} \int H(f)^2}$$

→ what is $H(f)$ to maximize this ratio?

$$h(t) = S_o(T - t)$$

See schwartz documintation on the web site

answer: when the signal is matched to the filter

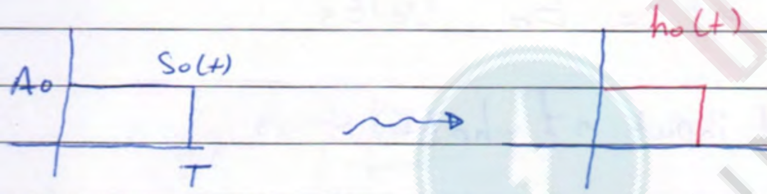
No.

why T?

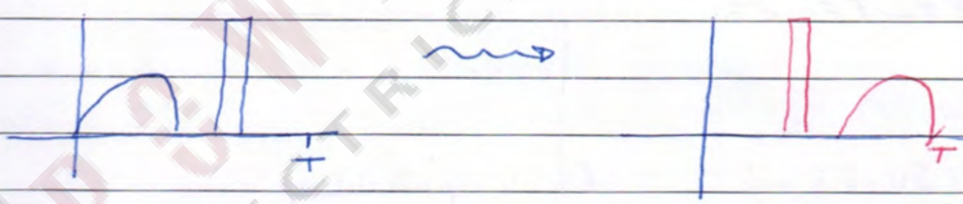
any signal must be bounded in time from Zero to T

T : maximum time to define one signaling interval.
for binary $T = T_b$

examples:-

1. 

2. 

3. 

in these examples those filters are designed to receive those signals to give maximum output, yet choosing those filters for other signals will make the decision process easier.

for second figure:-

$$V_o = \int_{-\infty}^{\infty} S_o(t) \, . \, h_o^{*}(\tau - T)\, dt \qquad 0 \leq \tau \leq 2T$$

No.

$\leadsto V_0 = \int_0^T s_0(t) \cdot h_0^*(\tau - T)\, dt$

maximum V_0 when $\tau = T$

$\leadsto V_{0\,max} = \int_0^T s_0(t) \cdot s_0^*(t)\, dt$

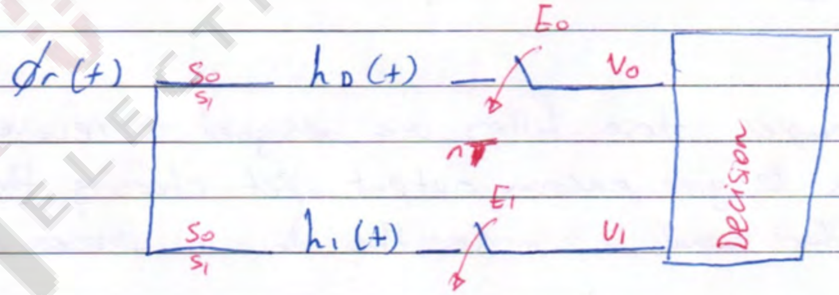
$= \int_0^T |s_0(t)|^2\, dt = E_0$ Volts

$\leadsto$ if $s_1(t)$ at input of $h_0(t)$:-
@ $\tau = T$

Output $= \int_0^T s_1(t) \cdot s_0^*(t)\, dt$

$= |s_1|\,|s_0| \cos(\theta) = \rho_{01}$
$= \sqrt{E_0}\sqrt{E_1}\,\rho_{01}$

Receiver :-



takes a sample every nT

if $\phi_r(t)$ is s_0
$V_0 = \sqrt{E_0}\sqrt{E_1}\,\rho_{01} \leadsto E_0$

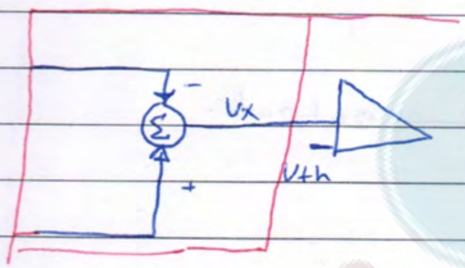
No.

if $\phi_r(t)$ is S_1

$V_1 = \sqrt{E_0}\,\sqrt{E_1}\,\rho_{01}$ no E_1

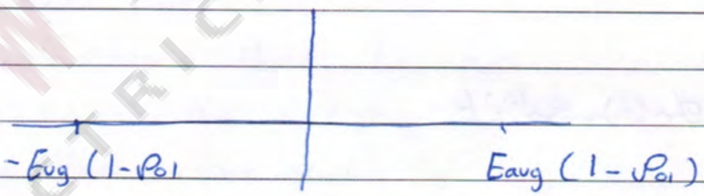
each time Decision will obtain two Values assume No noise.

let : $E_0 = E_1 = E_{avg}$. at input of Decision :-



$V_0 = E_{avg}$ $\quad V_1 = E_{avg}\rho_{01}$ $\quad$ So input, $V_x = -E_{avg}(1-\rho_{01})$

$\quad E_{avg}\rho_{01}$ $\quad E_{avg}$ $\quad S_1$ input, $V_x = E_{avg}(1-\rho_{01})$



$$V_{th} = \frac{V_1 + V_0}{2}$$

No.

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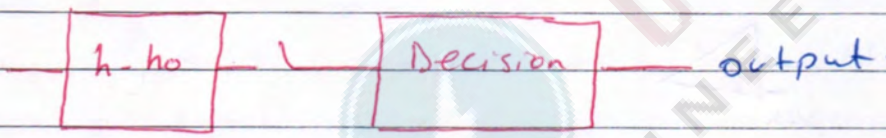
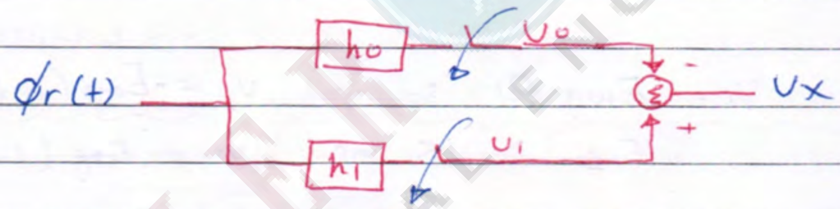
$$Ux = \begin{cases} E_{avg}(1-\rho_{01}) & \text{if "1" was transmitted} \\ -E_{vg}(1-\rho_{01}) & \text{if "0" was transmitted} \end{cases}$$

in the case of binary we can put one filter for both inputs

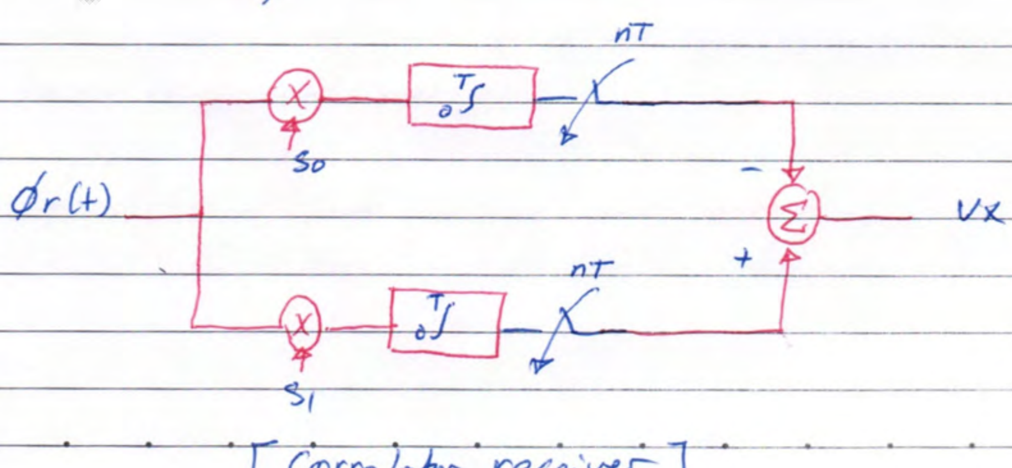




$$U_0 = \int_0^T \phi r(t)\, s_0(t)\, dt$$

$$U_1 = \int_0^T \phi r(t)\, s_1(t)\, dt$$

this is equivelent to:-



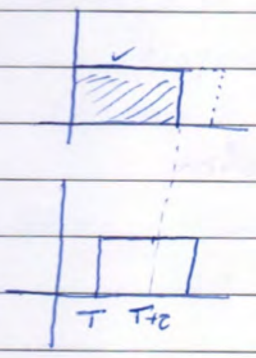
[Corrolator receiver]

No.

→ equivelant to a matched filter at the end of the interval.

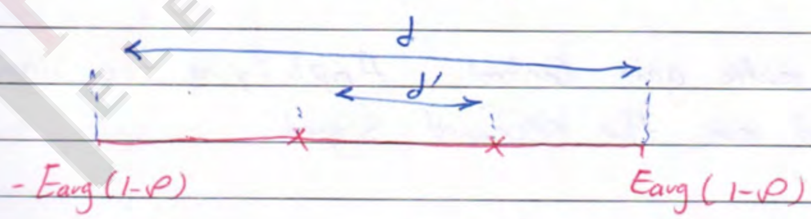
→ i need to regenerate S_0 & S_1 at the receiver & they must be synchronized with Tx

→ if there was amiss syncronization by (τ) inter symbol interferance will happen.



→ it means that we integrated the symbol + part of the next one.

→ there is a chance that the next symbol we are integrating on is the antipodal of the signal which means that the output might be less than maximum.



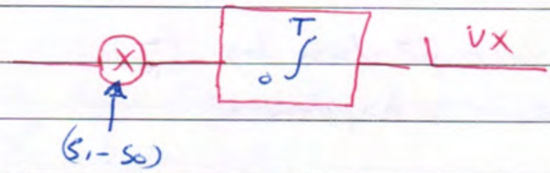
due to miss synchronization the distance is reduced & the chance of error between them is higher so SNR is less.

No.

→ So the optimal solution is when ρ is (-1)

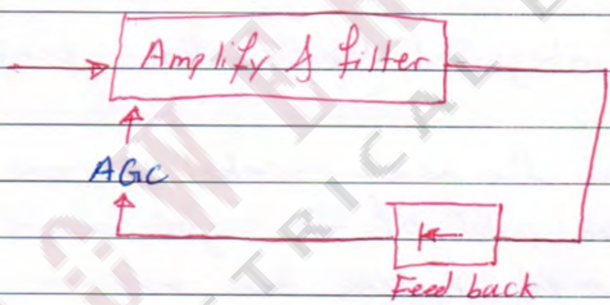
→ for antipodal signals

→ more simplified model.



for antipodal → $2 s_1$

→ Block diagram for practical implementation of a receiver :-



AGC: automatic gain control, Amplifying the signal based on the received signal

→ Controlling the gain based on the dynamic range of the receiver

dynamic range :- the receiver can adapt it self from 1 μV to 1 mV for example ($G_{max} - G_{min}$)

No.

dynamic range $= 20 \log \left(\frac{V_{max}}{V_{min}}\right)$

→ receiver sensitivity is when maximum gain happen
→ minimum input signal.

→ receiver sensitivity $= 20 \log (V_{min})$ dbm.

what does it mean :
dynamic range = 80 dbm ?
it means that the gain changes 100×10^6 times.

Note:- −120 db receiver sensitivity is better than −110 receiver sensitivity.

→ what is the limit of sensitivity?
noise floor :- it's natural noise at certain temprature.

→ signals level must be higher than nois floor

$V_{rms} = \sqrt{4kTb}$

noise floor $= 20 \log \left(V_{rms}^{noise}\right)$

No.

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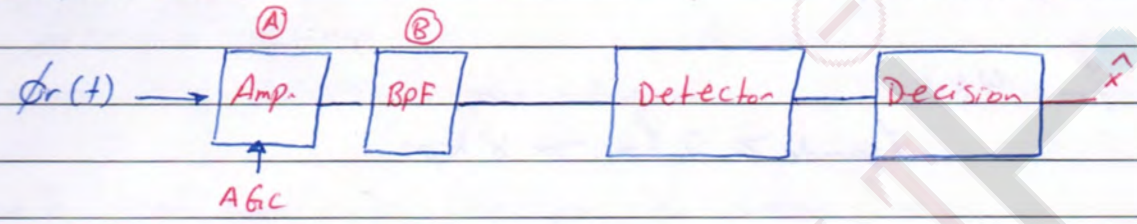
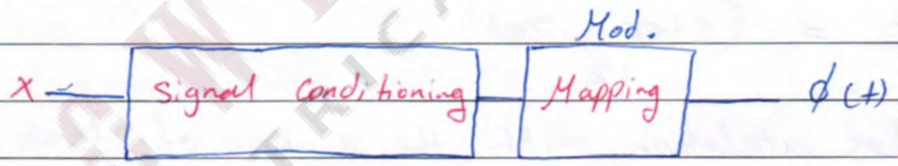


at poin B : BPF for Band pass
LPF for base band

Detection:- will detect the "1" signal & the "0" signal.

Decision:- decide if Zero or one based on the output Signal from the detection.

transmitter:-



x: binary serial of data (logical data)

Signal conditioning: Limit the amplitude & frequency at this stage.

Note:
x is fine quantized.

fine quantization:- over A/D conversion for the signal to satisfy toll quality.

No.

→ bandwidth for human voice = 4 KHz

$f_m = 4\ KHz$

$f_{sample} \geq 2 f_m \geq 8\ Kps$

→ Quantization at Certain m bits

bit rate $r_b = m \cdot f_s$

→ we found that 8 bits is enough to satisfy toll quality

→ if we took 16 bits ⇒ over quantized, distortion in A/D conversion is less.

quantization noise:-

$$\overline{\epsilon^2} = \frac{\Delta^2}{12} = \frac{(\text{step size})^2}{12}$$

Δ: it has a relation with the number of levels

assume a signal limited from $(V_{min} - V_{max})$

$$\Delta = \frac{V_{max} - V_{min}}{2^m}$$

So fine quantization is an over sampling of over quantization.

No.

if $f_s > 2 f_m$

$m >$ min required

then this is fine quantization.

** read Line code file on the web site.

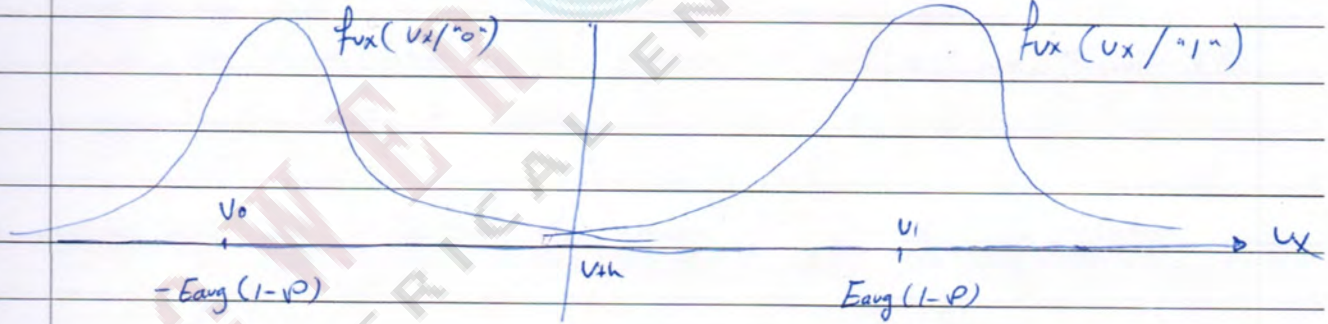
→ after fine quantization → data compression.

→ for synchronization we can add some control bits

So we assume x is binary of logical data

equally likely & compressed.

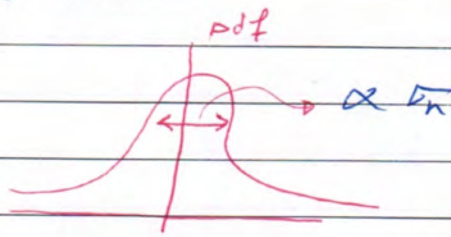
→ V_x has 2 possibilities :



V_x has some noise which is Guasian

$\mu_n = 0$

$\sigma_n^2 = N_0 BW$



Guasian distribution function.

$$f_n(n) = \frac{1}{\sqrt{\sigma_n^2 \, 2\pi}} \cdot e^{-\frac{(n-\mu_n)^2}{2\sigma_n^2}}$$

No.

→ so the output of the detection is either $V_1 + n$ or $V_0 + n$

as shown it's random becaue of noise.

probability of error $= \int_{-\infty}^{V_{th}} f_{V_x}(V_x / \text{"1"})\, dV_x$
$pr(e)$

error event $= pr(V_x \leq V_{th} / \text{"1"})$ or $pr(V_x \geq V_{th} / \text{"0"})$

AVG BER $= pr(V_x \leq V_{th} / \text{"1"}) \cdot pr(\text{"1"}) + pr(V_x \geq V_{th} / \text{"0"}) \cdot pr(\text{"0"})$

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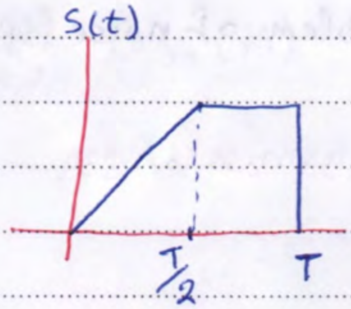
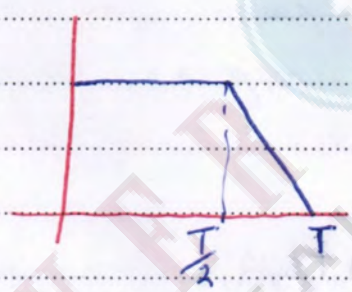
Quiz:-

Find the Matched filter reciever for:-



Solution:-

$$h(t) = s(T-t)$$



#

*Binary:-

"0" $\longrightarrow$ $S_0(t)$

"1" $\longrightarrow$ $S_1(t)$

⇒ optimal solution for $S_0(t)$ & $S_1(t)$ when:

$S_0(t) = -S_1(t)$ (antipodal)

*This solution will minimize the energy with maximum distance.

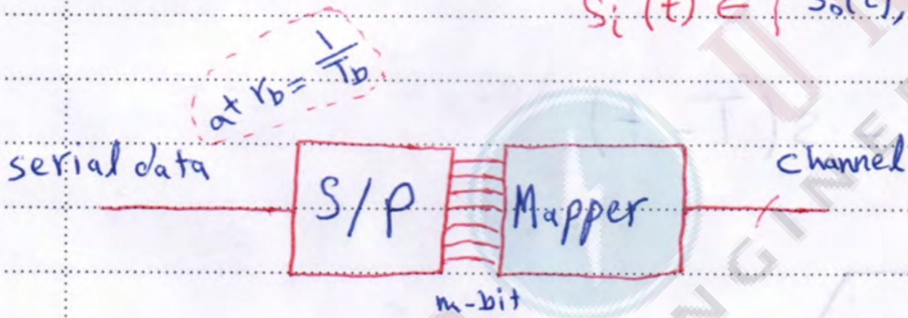
⇒ optimal reciever is Matched filter reciever

* We can use level detector which depends on voltage but there is the problem of noise (spiky noise)

* Non-binary Base band Transmission:-

⇒ symbols : group of 1's & 0's
($S_i(t)$)

$$S_i(t) \in \{ S_0(t), S_1(t) \ldots S_{M-1}(t) \}$$



$M = 2^m$ possible Inputs

⇒ we need M signals (waveforms) to output

Mapper

Binary Input	Sout
0 0	$S_0(t)$
0 1	$S_1(t)$
1 0	$S_2(t)$
1 1	$S_3(t)$

symbol Interval $= T_s$

$T_s = 2\, \underbrace{T_b}_{\text{bit Interval}}$

* $T_s = m\,T_b$
* $T_s = \log_2(M) \cdot T_b$

Ex: ADSL sends 292 symbols

$\log_2(292) \equiv$ number of bits in parallel

Ex:- 1-D space (1D PAM)

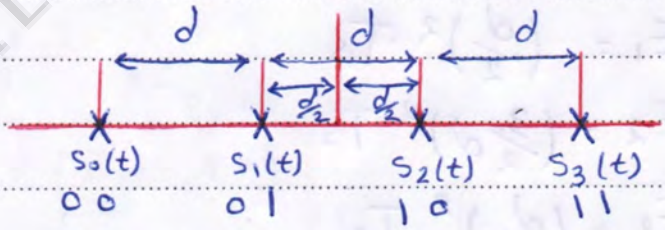
$\vec{a}_x = a_x(t)$, s_0, s_1, s_2, s_3

* Remember we want min. energy at max. distance

⇒ we will use gray code

* Gray code:

⇒ adjacent symbols differs in ONLY one bit



s_0	0	0	0	
s_1	0	0	1	
			0's ↑ 1's ↓	mirror
s_2	0	1	1	
s_3	0	1	0	
	0's ↑ 1's ↓			mirror
s_4	1	1	0	
s_5	1	1	1	
s_6	0	0	1	
s_7	0	0	0	

* Note: If we want to protect an Important data we can change distances between symbols and make our Important data away from other symbols

No.

calculating the average energy :-

$$E_{av} = \frac{E_0 + E_1 + E_2 + E_3}{4}$$

assuming They are equally likely

$$S_0(t) = \frac{3}{2} d \quad a_x(t)$$

$$E_0 = \int |S_0(t)| \, dt$$

$$= \int_0^{T_s} \left(\frac{3}{2} d\right)^2 |a_x(t)|^2 \, dt$$

$$= \left(\frac{3}{2} d\right)^2 \cdot \underbrace{\int_0^{T_s} |a_x(t)|^2 \, dt}$$

1

$$E_0 = \left(\frac{3}{2} d\right)^2 \cdot T_s$$

$$E_1 = \left(\frac{d}{2}\right)^2 T_s$$

$$E_2 = \left(\frac{3}{2} d\right)^2 T_s$$

$$E_3 = \left(\frac{d}{2}\right)^2 T_s$$

$$E_{av} = \frac{d^2 T_s}{4} \left(\frac{9}{4} + \frac{1}{4} + \frac{9}{4} + \frac{1}{4}\right)$$

$$= \frac{20 \, d^2 T_s}{16} = \boxed{\frac{5}{4} d^2 T_s}$$

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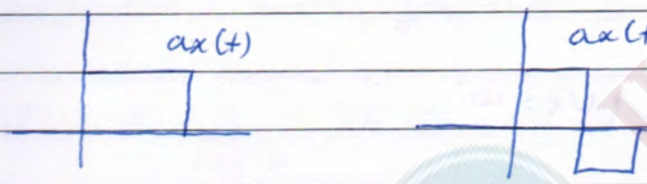
non-binary base band transmition:-

→ one dimention.
only one signal (axis)

example:-



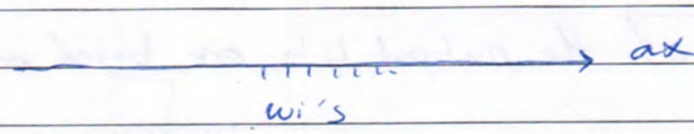
effeciency in energy is the same in both, but they differ in Bandwidth.

→ for base band wired transmition we assume Bandwidth is infinite for now

→ we can implement the matched filter as corrolator

* if we have
M signals $= 2^m$ m bits.

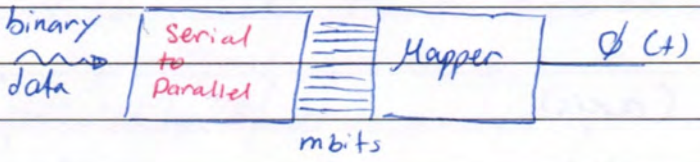
$S_i(t) = w_i \, ax(t)$
w_i = weight



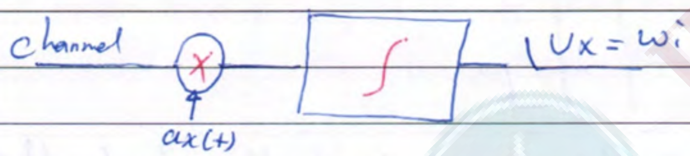
wi's are point along (x axis)

Transmitter & Receiver:-

TX:



Rx



amplitude represent the data.

Inputs	w_i
0000	w_0
0001	w_1
⋮	⋮
	w_{M-1}

one's & zero's are sequence of binary data it doesn't mean numbers

in Non-binary ⇝ M-ary ($\log_2 M$) bits

$\log_2 M$ is prefered to be integer

⇝ Selection of the output bits are based on gray code.

⇝ most likely to have an error is between adjacent symbols

No.

find BER:-

$\hat{x}$ is received.

there is one bit difference between them that means that there are still 3 bits correct

→ if we choose gray code:-

$$BER = \frac{1}{\log_2 M} SER$$

SER: symbol error rate.

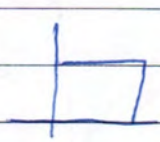
→ performance depends on the BER not SER

→ in channel we sent symbols and having an error in certain symbols doesn't mean there is an error in all bits inside that symbol.

→ 2 dimention:-

→ needs 2 axis:-

$a_x(t)$:



$a_y(t)$ →

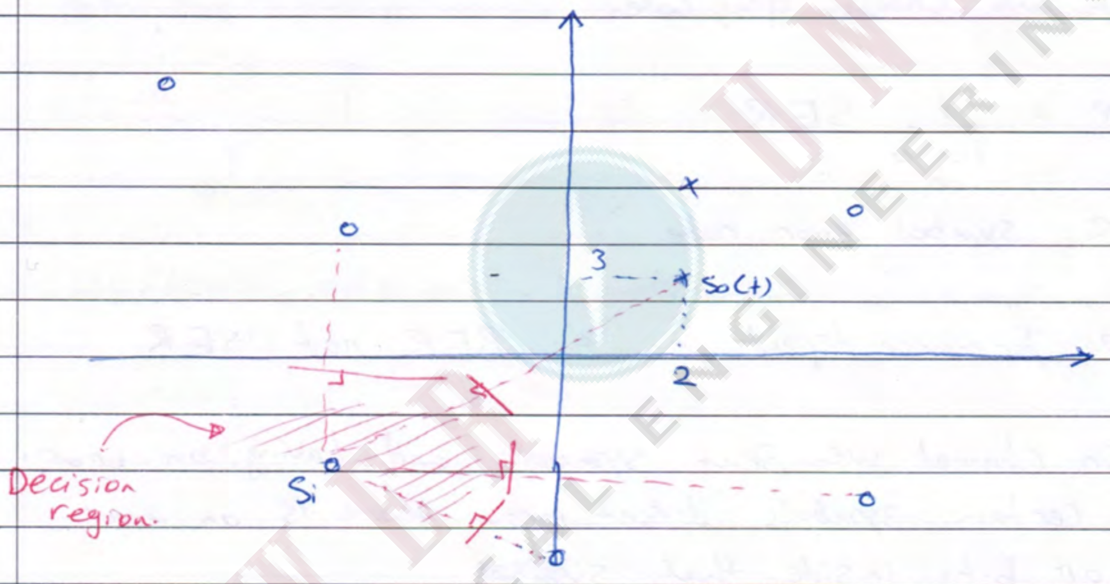
No.

example: if we want to send 8 symbols

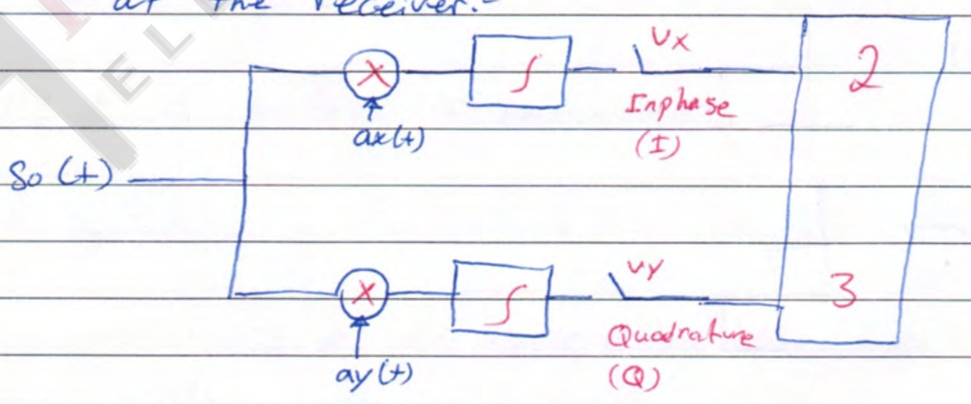
$M = 8$

$\log_2 8 = 3$ bits

the 8 points were distributed randomly. as follow:



distance between those signals does it mean any thing?
yes;
at the receiver:



No.

→ any point in the decision region is S_i

→ error happens when it moves to another region.

→ for the optimal solution for distribution around zero: maximum distance with minimum energy.

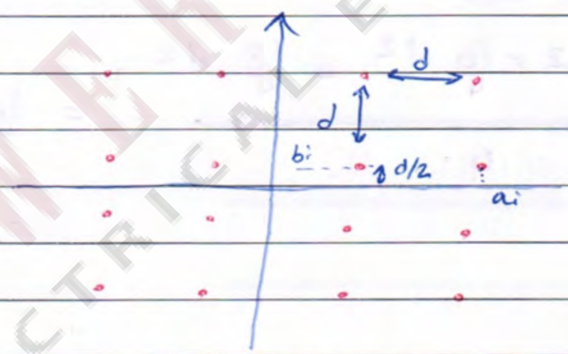
→ how to solve for the optimal solution.

sphere packing.

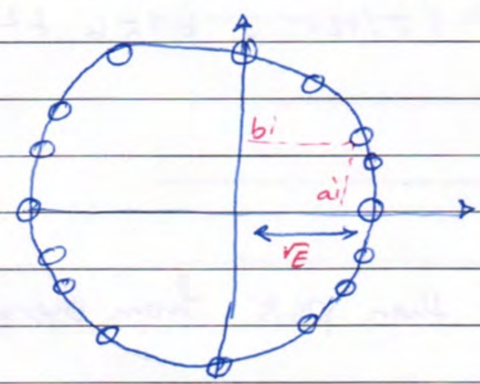
Center of the sphere is the signal

packing represent the energy

→ 16 points distributed on a grid the distances between them is fixed = (d)



we can distribute the same number of points on a circle.



No.

→ the amplitude of the symbol it self is constant they only differ in phaze.

→ for the best constalation they must be equally likely & uniformlly distributed.

→ the closest points to each other represent d min.

for figure 1: calculate the avg energy

$$E_{avg} = \sum_{i=1}^{16} \frac{(a_i)^2 + (b_i)^2}{16} = \frac{1}{M} \sum_{i=0}^{M-1} (a_i^2 + b_i^2)$$

$$E_{avg} = \frac{\frac{d^2}{2} + 2 \times \frac{10}{4} d^2 + \frac{18}{4} d^2}{4} = \frac{10 d^2}{4} = 2.5 d^2$$

second figure:-

$R^2 = E_{avg}$

$$E_{avg} = \frac{d^2}{4} / \sin^2(\pi/16) = 6.156 \, d^2$$

$$* \; R = \frac{d}{2 \sin(\pi/M)}$$

QAM is better than PSK from energy point of view.

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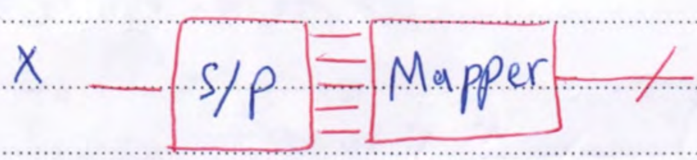
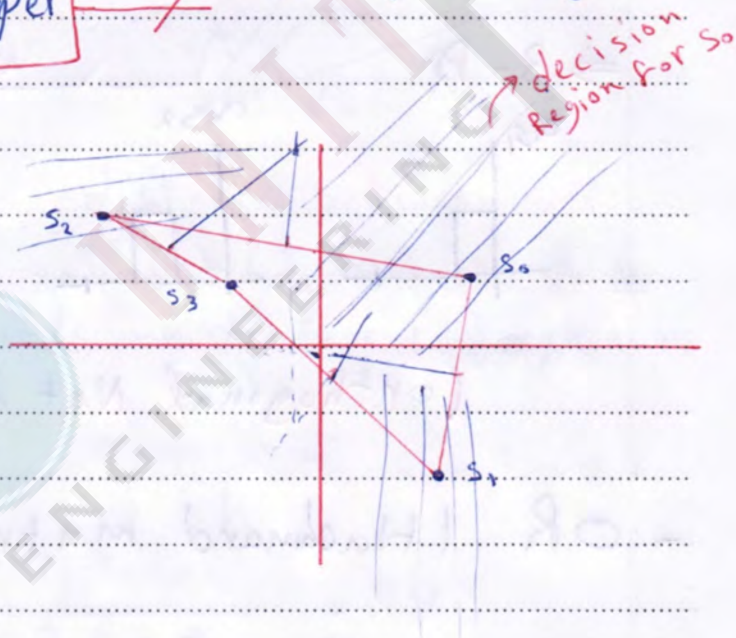
No. ....

* Non-Binary Baseband Tx :-

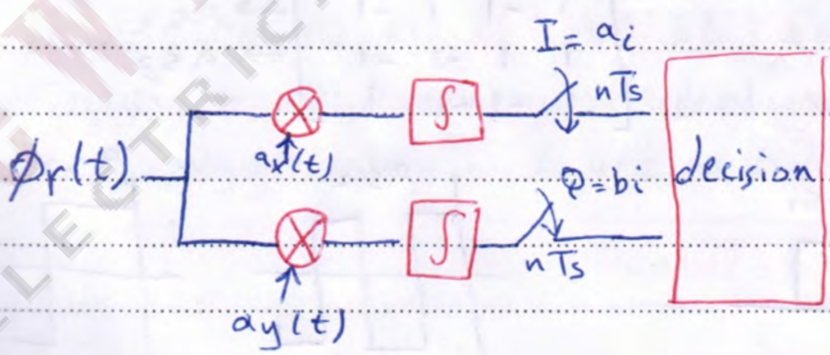
⇒ 2D space



output = $\phi_t(t)$



* Rx :-



$$\phi_r(t) = \phi_t(t) + n(t)$$

$$\phi_r(t) = a_i\, a_x(t) + b_i\, a_y(t) + n(t)$$

⇒ 2D space is more efficient in Energy

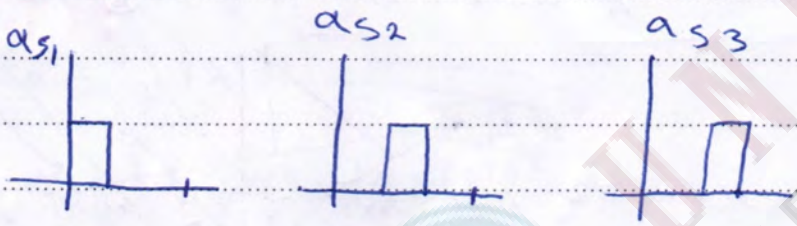
No.

* N-D space :-

⇒ axises are : $a_{sj}(t) \equiv j = 1, 2, \dots\dots N$

$\langle a_{sj}, a_{sk} \rangle = \delta_{jk}$ ⇒ orthonormal

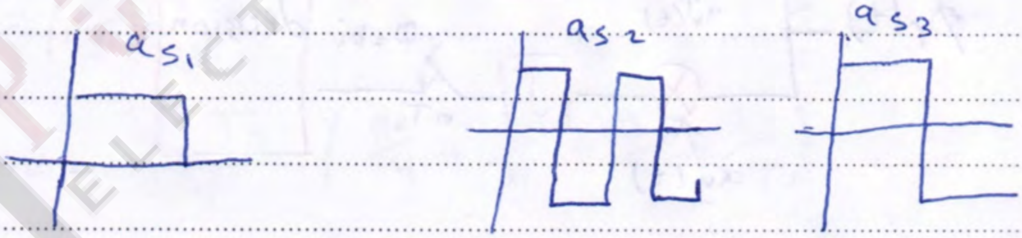
⇒ 3-D



(orthogonal Not overlapped in time)

- OR (Hadamard Matrix) :

$$H_4 = \begin{array}{cc} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} & \begin{array}{l} \to a_{s1} \\ \to a_{s2} \\ \to a_{s3} \\ \\ \end{array} \end{array}$$



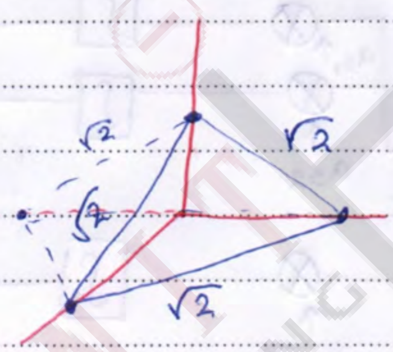
(orthogonal Not overlapped in code)

* we can use any orthogonal signals.

No.

EX:- 4-D , $M = 4$ symbols

$S_0(t) \rightarrow a_{s_1}$
$S_1(t) \rightarrow a_{s_2}$
$S_2(t) \rightarrow a_{s_3}$
$S_3(t) \rightarrow a_{s_4}$



* distance between all symbols are equal

$\Rightarrow$ we can send signals on different frequencies

Ex:

$a_{s_1} \Rightarrow \cos(w_1 t)$
$a_{s_2} \Rightarrow \cos(w_2 t)$
$\vdots \qquad \vdots$

} FSK

FSK is Multidimentional Modulation

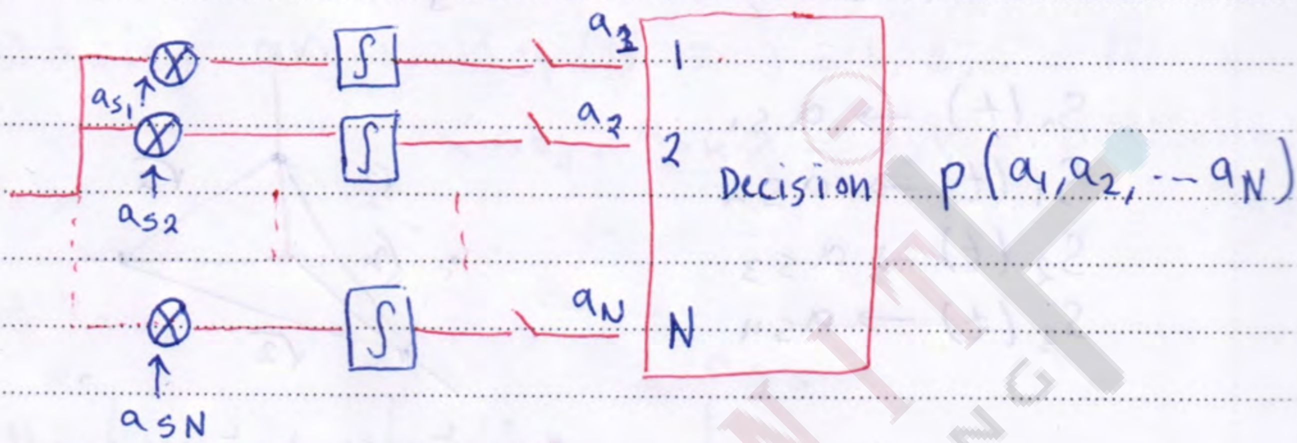
* This gives us the minimum selection Of Energy
* lowest Energy design will use multidimentional space

$$\boxed{\text{Number of dim.} = \frac{\text{Number of signals}}{2}}$$

* This equation because we need them antipodal

No.

* Rx In Multidimentional space :-



* Quality of the Reception depends ONLY on distance

* distance represents SNR

$$SNR = \frac{\text{signal power}}{\text{noise power}}$$

* we can't control noise power
* we can control signal power by changing ~~dist~~ the position of the points.

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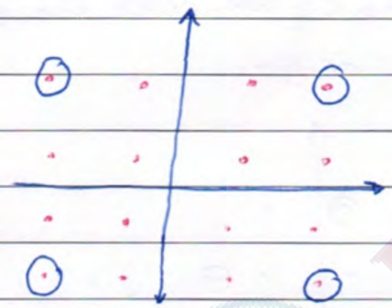
peak to average power atio:-

example:-



those 4 points represent the peak power

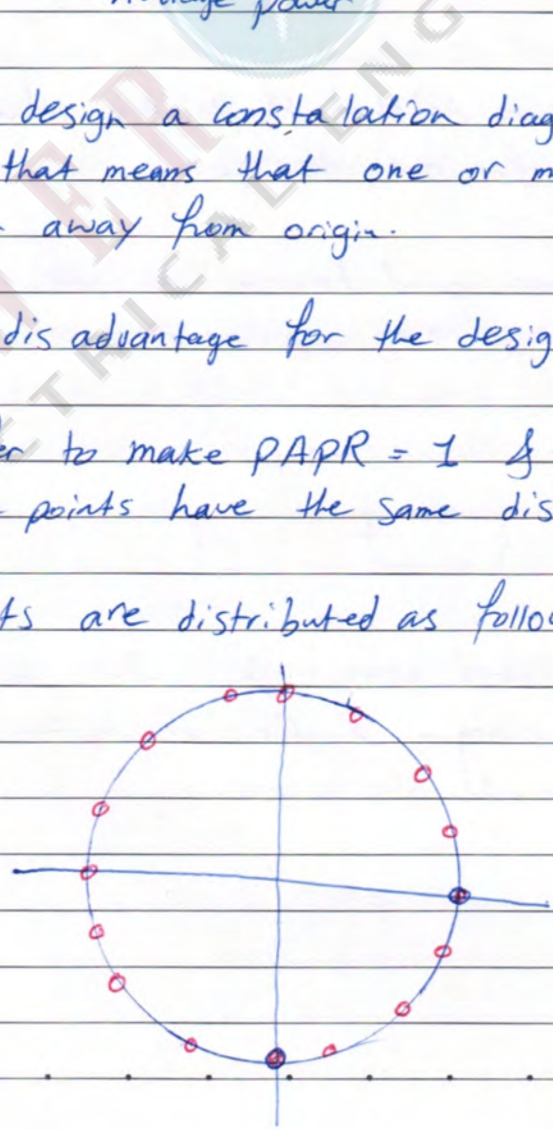
this ratio = $\frac{\text{peak power}}{\text{Average power}}$

So if we design a constalation diagram & the PAPR is high that means that one or more of the points are far away from origin.

so it's a disadvantage for the design to have a high PAPR

so we prefer to make PAPR = 1 & that is satisfied when the points have the same distances from origin.

if the points are distributed as follow



No.

the average power is constant & the peak power is equal to it.

So, to reduce the PAPR we delet the points that are farthest from origin

$M=12$

8 data $\Rightarrow$ 3 bits

4 control bits (circled)

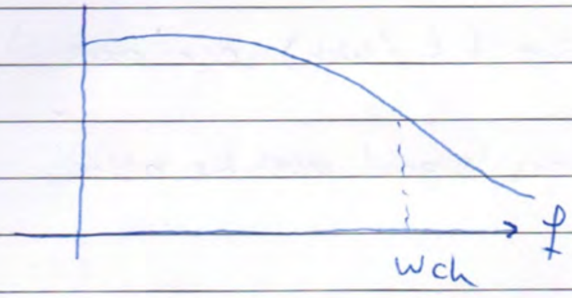
in this case we deleted 4 points which represent a worse case for our design.

$\rightarrow$ even in base band transmition we are limited in bandwidth

Model of transmition line.

if the signal speed was high, there will be No time for charging & dis charging to take place so it will attenuate the signal.

No.



$w_{ch} = \frac{L}{l\sqrt{Lc}}$

L: inductance / m

l: length

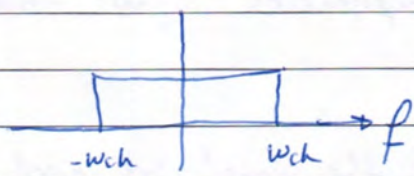
transmition line $Z_{in} = \sqrt{L/c}$

delay: speed of propagation (v) = $\frac{1/\sqrt{Lc}}{l}$ m/s

→ higher (l) smaller Bandwidth.

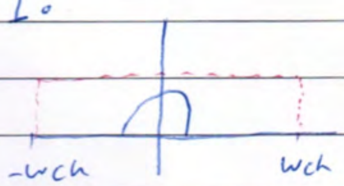
~~Base~~ Band limited channels:

becase we are talking about base band, the band limited channel is alway low pass Filter

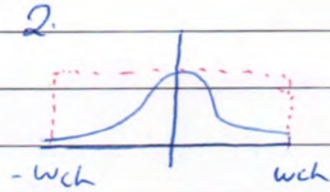


→ if you have this band limited what shape of signal would you choose?

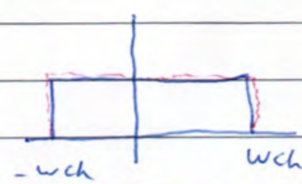
1.



2.



3.

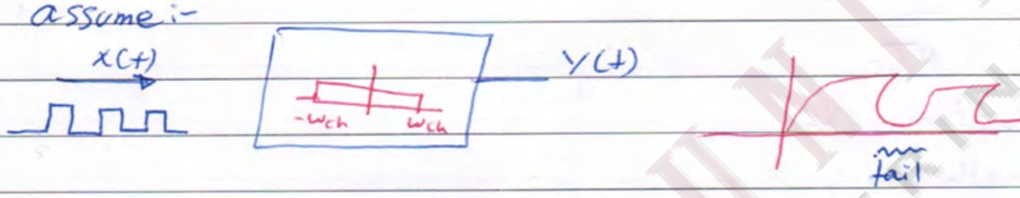


No.

So the best option is sinc (t/W_{ch}) our axis must be sinc

→ from bandwidth limitations signal must be a sinc signal.

assume :-



the output is due to limitation in bandwidth.

→ tail will have signifacant value so contribution will sum (sum of tails) which will increase the attenuation on the output creating inter symbol interferance.

→ this is due to memory system, so we alway can implement memory system as a delay line.

→ at different frequencies signals travel in different speeds.

→ to fully utalize the signal we need to have a sinc function at the input.

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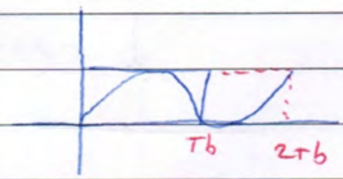
lecture :- 23 10/4/2014

if we have asignal in frequency domain :-

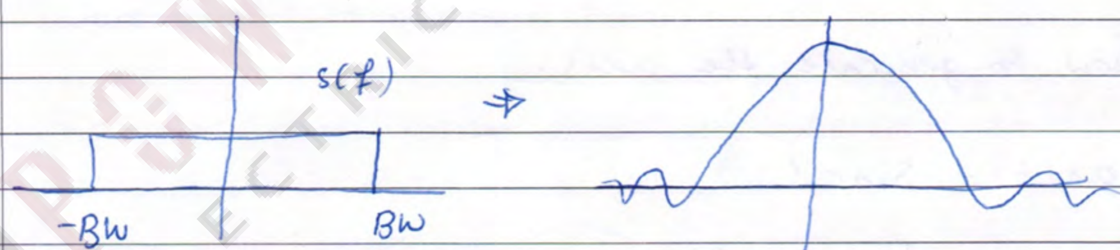


find the relation between BW & Bit rate?

fundemental frequency of this signal is sinosoide with period = 2 Tb

$f = \frac{1}{2Tb} = 1/2\, rb$

$B = 1/2\, rb$



Zero Crossing points

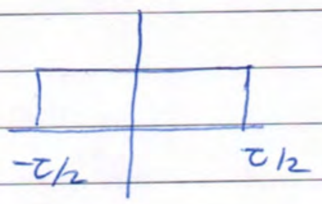
$Tb = \frac{1}{2BW}$

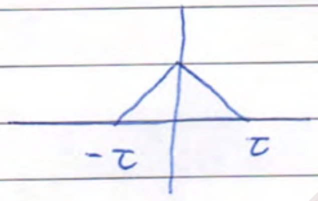
$BW = \frac{1}{2Tb} = \frac{1}{2} rb$

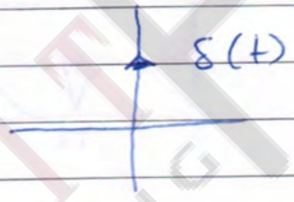
No.

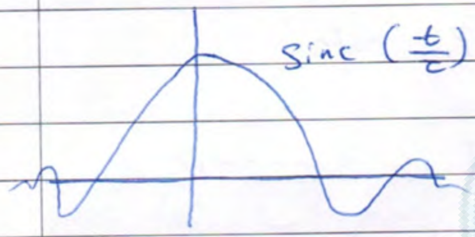
→ required for mid term exam:-

fourier transform pairs for the following signals







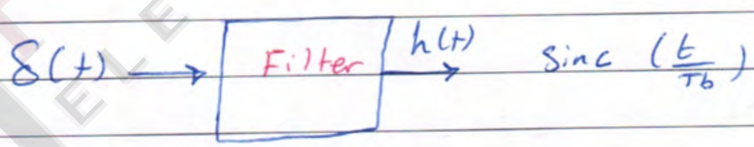


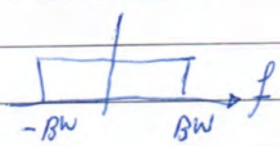
* $\cos(w_c t)$
* $\sin(w_c t)$
* $\cos^2(w_c t)$
* $\sin^2(w_c t)$
* $\cos(w_c t) \cdot \sin(w_c t)$

→ sinc function is the only function that has exactly Bandwidth in rectangular shape.

how to generate the axis.

$ax(t) = \text{Sinc}(\ )$

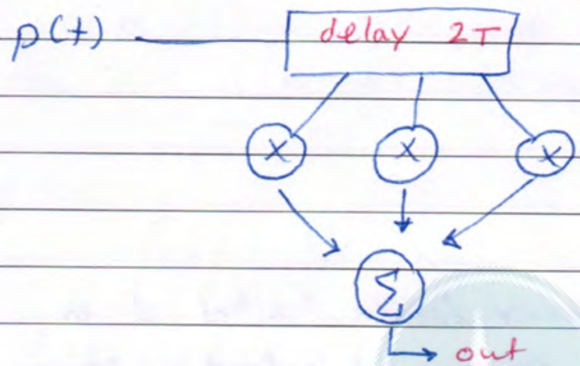




→ Can we implement an ideal filter like this ?
NO
Sinc function is non causal ; it starts from $-\infty$

No.

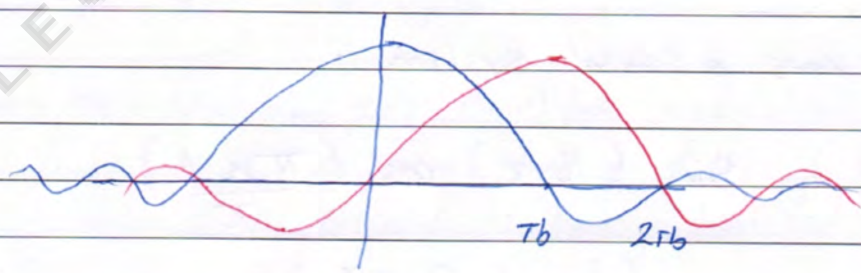
So, how to generate the sinc function?
we take a pulse & move it into a delay line
then multiply it by weight & then add.



→ we delay the signal for certain period to generate
an approximated sinc.

→ 2 characteristics that is for sinc functions that
makes us want to generate it which is BW &
inter symbol interferance free.

Sinc has zero crossing points at integer number of T_b



So, if we are limited in BW we only use the
sinc so that it takes us to 1D modulation.

No.

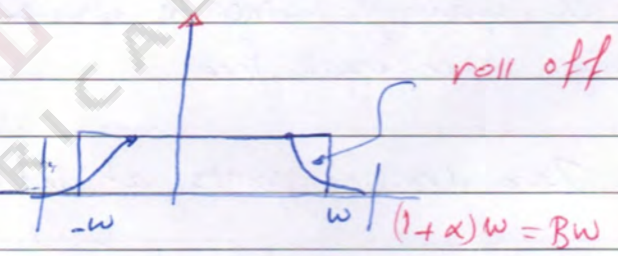
one of the advantages of the sinc that it has apeak at the center

$$\int_{-\infty}^{\infty} \text{sinc}(t)\, dt = 1$$

$$\int_{-\infty}^{\infty} \sin^2(t)\, dt = 1$$

is there asignal in frequency domain limited & in time domain has a Zero crossing at integer number of T_b? & the slope at the edges of the Band is not vertical?

Yes, raised cosine.



it becomes a causal system.

$$p(t)_{New} = \frac{\text{Sinc}(t/\tau)\cos\left(\frac{\pi \alpha t}{\tau}\right)}{\left(1 - \left(\frac{2\alpha t}{\tau}\right)^2\right.}$$

α: roll off factor

$$BW = \frac{r_b}{2}(1+\alpha)$$

No.

$\rightsquigarrow$ at Zero it becomes sinc

$\rightsquigarrow$ at integer multiple of τ $\rightsquigarrow$ Zero

So we maintaind the Zero crossing point

we maintained the peak at the origin.

the BW required is larger by factor of $(1+\alpha)$

Note:-

$$RC_2 = RC_{0.2}$$

it means that raised cosine at point 2 with roll off factor of 0.2

Non- binary base band (band limited)

$$x \rightarrow \boxed{\text{mapping into } a_i\text{'s}} \xrightarrow{a_i \delta(t)} \boxed{\begin{matrix} h(t) \\ RC\alpha \end{matrix}} \rightarrow \phi(t)$$

$\phi(t) = a_i p(t)$

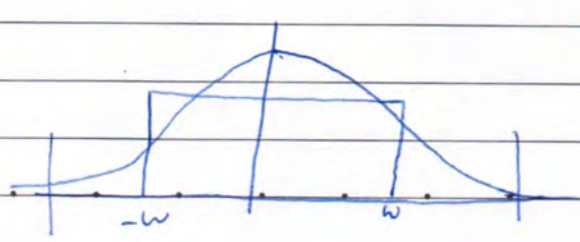
with raised cosine.

$\rightsquigarrow$ the filter shaped the signal.

Non-binary base band $\rightsquigarrow$ pulse Amplitude modulation

$\rightsquigarrow$ this filter is wave form shaping filter (generation filter, called filter because it has a transfer function)

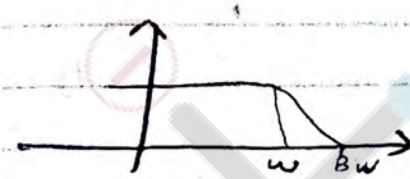
Note: we can use similar filters in Band pass which is Guasian filter used in celu lares.



* Binary Baseband & non binary baseband

$$\boxed{BW = \frac{1}{2} r_b (1+\alpha)}$$

α : roll off factor



$w = \frac{r_b}{2}$

$BW = w(1+\alpha)$

① [redacted] binary

$\phi(t) = \underbrace{a_i}_{\text{amplitude}} \underbrace{p(t)}_{\text{raised cosine}}$

a = ones & zeros

[redacted]

② Non - binary

Bandwidth $= \frac{r_s}{2}(1+\alpha)$

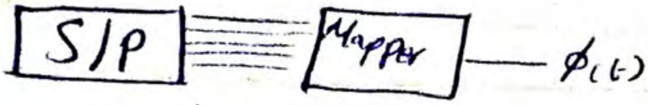
$r_s = \frac{r_b}{\log_2 M}$

$\therefore$ Bandwidth $= \frac{r_b}{2\log_2 M}(1+\alpha)$

$\phi(t) = a_i\, p(t) \qquad i = 1, 2, 3 \; .... \; M$

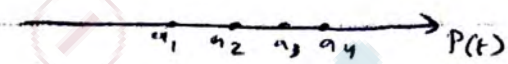
* The difference between binary and non-binary is in the value of [a] which represents the amplitude of the raised cosine.

* The amplitude doesn't affect the bandwidth of the raised cosine



S/P : serial to parallel

$$m = \log_2 M$$
$$r_s = \frac{r_b}{m}$$
$$T_s = m\,T_b$$



* This is a one dimensional modulation Technique, where we have only one axis (raised cosine)

<u>Example</u>:

~~I~~If the channel BW is 20 kHz, we need to transmit a data at r_b 100 kbps. design the non-binary base band system

<u>→ sol</u>

assume a raised cosine with $\alpha = .2$

$$BW = \frac{r_b}{2\log_2 M}(1+\alpha)$$

$$m = \log_2 M = \frac{100\,k}{2 \times 20\,k}(1+.2) = 3\,\text{bits}$$

sending 3 bits in parallel

* Increasing the roll off factor makes the signal implementation easier.

<u>Ex</u>:

what if we assumed $\alpha = .25$, in the previous example?

$$20\,k = \frac{100\,k}{2\log_2 M}(1+.25)$$

$$m = \log_2 M = 3.125 \text{ bits}$$

→ in order not to lose data

$m = 4$ bits

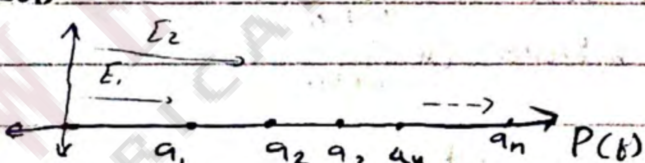
∴ we increase the Roll off factor to decrease complexity of implementation

$20k = \frac{100k}{2 \times 4}(1 + \alpha_{new})$

$1.6 = 1 + \alpha_{new}$

$\boxed{\alpha_{new} = .6}$

* The number of bits in each symbol doesn't affect the bandwidth of our message. On the other hand, increasing the number bits in each symbol, would increase the number of symbols

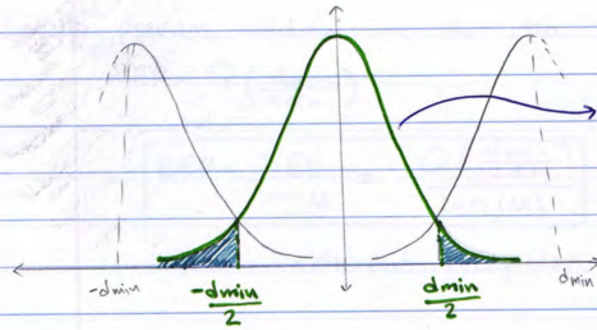


And increasing the number of symbols on the constellation would mean consuming more Energy to represent those Symbols.

* To get less energy consumption we decrease the value of (m) to get less # of symbol of the constellation axis.

The End of the midterm material

20/4/2014



→ we take the general case, which is a symbol (S_i) that is located between two other symbols.

$$BER = P\left(v_i > \frac{d_{min}}{2} \,\middle|\, \text{"}s_i \text{ was sent"}\right) \times \frac{1}{2} + P\left(v_i < -\frac{d_{min}}{2} \,\middle|\, \text{"}s_i \text{ was sent"}\right) \times \frac{1}{2}$$

equally likely

$$= \underbrace{\left(\frac{1}{2} + \frac{1}{2}\right)}_{1} P_r\left(v_i > \frac{d_{min}}{2} \,\middle|\, \text{"}s_i \text{ was sent"}\right)$$

$$BER = \int_{\frac{d_{min}}{2}}^{\infty} \frac{1}{\sigma_n\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma_n^2}} dx \quad \ldots (1)$$

$$Q(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{\alpha}^{\infty} e^{-t^2/2} dt$$

$$\approx \left[\frac{1}{(1-a)\alpha + a\sqrt{\alpha^2 + b}}\right] \frac{e^{-\frac{\alpha^2}{2}}}{\sqrt{2\pi}} \quad \ldots (2) \Rightarrow a = \frac{1}{\pi}, \; b = 2\pi$$

let $\frac{x}{\sigma_n} = t \longrightarrow dx = \sigma_n dt$

plug in (1)

$$BER = \int_{\frac{d_{min}}{2\sigma_n}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = Q\left(\frac{d_{min}}{2\sigma_n}\right)$$

actualy the previous relation was for the Symbol Error Rate (SER)

$$SER = Q\left(\frac{d_{min}}{2\sigma_n}\right)$$

in general:-

$$\therefore \boxed{BER = \frac{SER}{\log_2 M} = \frac{Q\left(\frac{d_{min}}{2\sigma_n}\right)}{\log_2(M)}}$$

← this formula can be used as an approximated calculation for all types of constellation (I just need d_{min}).

↳ under coherent reception.

→ for non-coherent R_x (e.g: level detector, envelope detector, differential detector, ...):

$$\boxed{BER = \frac{1}{2\log_2 M} e^{-\left(\frac{d_{min}}{2\sigma_n}\right)^{\beta}}}$$

$\beta \approx (1.1, 2)$ — 1.1: envelope det., 2: coherent R_x, β's range!

↳ non-coherent R_x

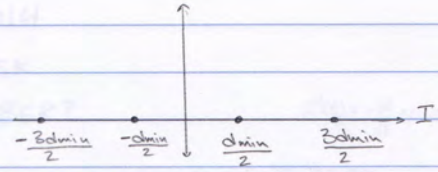
↳ also, $Q(\alpha) = \frac{1}{2} erfc\left(\frac{\alpha}{\sqrt{2}}\right)$, erfc : complimentary error function.

$$\therefore BER = \frac{1}{2\log_2 M} erfc\left(\frac{d_{min}}{2\sqrt{2}\sigma_n}\right)$$

* maximum value of $Q = \frac{1}{2}$, ∴ max. $BER = \frac{1}{2}$
since if $BER > \frac{1}{2}$, you can put an inverter & your new BER will become $(1 - BER_{old})$ which is less than $\frac{1}{2}$!

$BER = 10^{-6}$ ≡ Probability of error in each bit is $\frac{1}{1000000}$ (exact meaning).
≡ if I transfer 1000000 bit, It's expected to have 1 bit error. (ergodic meaning).

to enhance the quality, we decrease BER using error detecting & correcting codes.
(e.g: CRC "Cyclic Redundancy Check" ← error detecting code used to count # of errors).



$E_{av} = \frac{5}{4} d^2_{min}$

if $T_s \neq 1$ (in general)

$E_{av} = \underbrace{\frac{5}{4}}_{\xi} d^2_{min} T_s$

$P_{av} = \frac{5}{4} d^2_{min}$

$$BER = \frac{Q\left(\sqrt{\frac{d^2_{min}}{4\sigma_n^2}}\right)}{\log_2 M} = \frac{Q\left(\sqrt{\frac{d^2_{min}}{4 BW N_0}}\right)}{\log_2 M}$$

$, BW = \frac{r_b}{2\log_2 M}(1+\alpha)$

$, d^2_{min} = \underbrace{\frac{4}{5}}_{1/\xi} P_{av}.$

$$= Q\left(\frac{P_{av}}{4\xi \frac{r_b}{2\log_2 M}(1+\alpha) N_0}\right) \Big/ \log_2 M$$

$$= Q\left(\sqrt{\frac{P_{av}\log_2(M)}{2\xi\, \underbrace{r_b}_{\cancel{Hz}}(1+\alpha)\underbrace{N_0}_{Watt/\cancel{Hz}\,\rightarrow\, power}}}\right) \Big/ \log_2 M$$

$r_b = \frac{1}{T_b}$

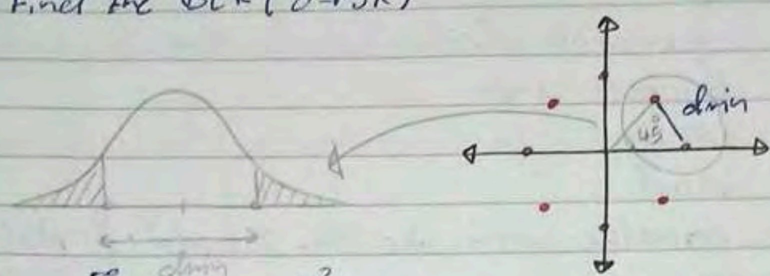
$\& \ P_{av} = \frac{E_{av}}{T_s} = \frac{E_{av}}{(\log_2 M)T_b}$

$$= \frac{Q\left(\sqrt{\frac{E_{av}}{2\xi(1+\alpha)N_0}}\right)}{\log_2(M)}$$

Lecture #   22/4/2014

Ex:

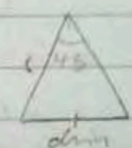
Find the BER (8-PSK)



$$BER = \int_{\frac{d_{min}}{2}}^{\infty} \frac{1}{\sigma_n\sqrt{2\pi}} e^{\frac{-x^2}{2\sigma_n^2}} dx = \frac{1}{\log_2 M} Q\left(\sqrt{\frac{d_{min}}{4\sigma_n^2}}\right)$$

↳ This is a general, approximated formula regardless of the constellation form.

$$\left[E_{Avg} = \xi d_{min}^2 T_s \right] \rightarrow \xi = \frac{1}{4\sin^2(22.5)}$$



$$\Rightarrow BER = \frac{1}{\log_2 M} Q\left(\sqrt{\frac{E_{Av}}{2\xi(1+\alpha)N_0}}\right) \qquad r = \frac{d_{min}}{2\sin(22.5)}$$

$$= \frac{1}{3} Q\left(\sqrt{\frac{E_{Avg}}{2\times1.707(1+\alpha)N_0}}\right)$$

* *Recall:*

Ergodicity indicates that we can use the time Avg. to represent the statistical Avg.

* Simulation of comm. system:

- Loop:
  - generate data (usually Binary)
- Tx
  - Encode data
  - (S/P)
  - mod.
- Ch.
  - convolve with the ch. & add: { Nois effect / fading.
- Rx
  - Demod.
  - (P/S)
  - Decode data
  - check for errors
- go to loop

* Good to know:

In practical, modern systems, we use adaptive modulation, such that we can use software to pick up whichever constellation form we wish.

$$BER = \frac{\#\text{ of errors}}{\#\text{ of transmitted bits}}$$

- As the data rate ↑ SNR ↓

"خطوط الهوائي بتنتج عن [illegible]، وعشان كدا 2000 خط [illegible]
و[illegible] بتعمل crosstalk يعني interference، هل الـ interference
[illegible] بـ [illegible] أو بتكبس؟! لا بيزيد الـ interference [illegible]
الـ channel بتتأثر كيف، فبقلل الـ rate، وهذا بسموه Adaptive
rate transmission"

* Band pass transmission :

→ wireless comm.

→ $p(t)$ is no longer voltage waveform, it's electromagnetic waveform (plane wave)

Mathematical formalism of the plane wave :

$$Ae^{-\alpha r}\cos(\omega t - kr)$$ → its projection over the orthonormal axes, $a_x(t)$ & $a_y(t)$, can be represented using F. T.

$k \equiv$ the wave number

$r \equiv$ distance, it's a vector

→ represents the direction of propagation

$$\omega t = kr \rightarrow \omega = \frac{kr}{t}$$

$$\rightarrow \omega = 2\pi f, \quad k = \beta 2\pi, \quad \beta = \frac{1}{\lambda}$$

$$\therefore f = \beta \frac{r}{t} \rightarrow \text{speed}$$

$$\boxed{f = \beta v}$$, $v \equiv$ velocity of propagation.

$$v = \frac{1}{\sqrt{LC}}$$, $L, C \equiv$ inductance & capacitance per unit length.

For unguided mode, $LC = \mu_0 \epsilon_0 \rightarrow v = C \equiv$ speed of light.

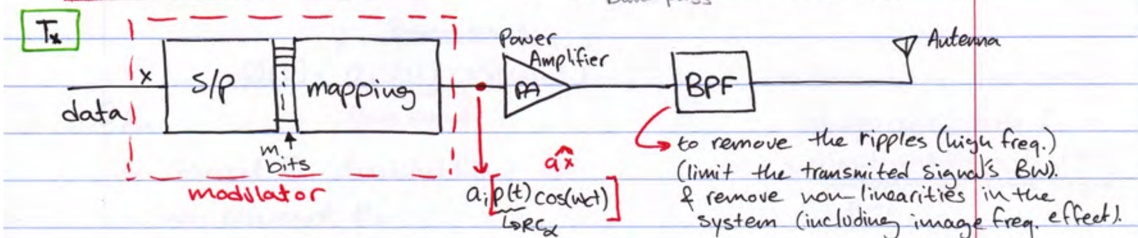
$$[\lambda f = v]$$

→ For wireless we use $[\lambda f = C]$

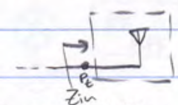
24/4/2014

Band pass Transmission:

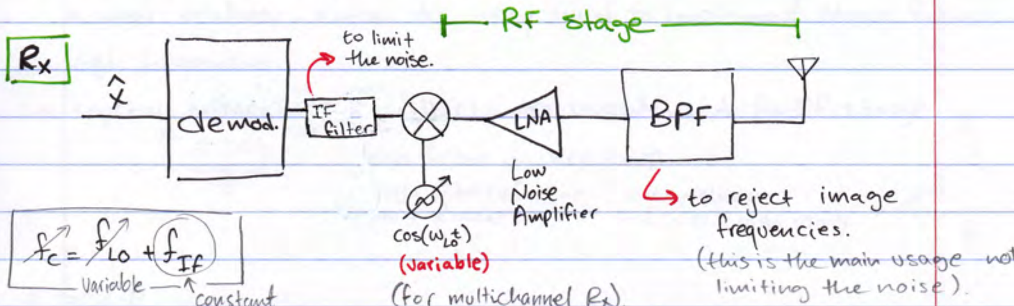
$*\ BW_{Band\ pass} = 2BW_{base\ band}$



- size of the Antenna $\propto \lambda$.
- Antenna's characteristics that we care about:-
  - Z_{in} ($Z_{in} \propto P_{Loss}$)
  - Gain (Gain $\propto$ directivity).



- Antenna is a passive element (sometimes it's simulated by a resistor).



$$\boxed{f_c = f_{LO} + f_{IF}}$$

variable — constant

- the Limit of the recieved power in cellular is $-90\,dBm$

  $-90\,dBm = 10^{-9}\,mW = 10^{-12}\,W = 1\,pWatt$

  for GPS $\rightarrow$ $-174\,dBm$

- $V_{rms\,(noise)} = \sqrt{4KTB}$

  T: effective temp.

Amplitude Shift Keying (ASK):

$$\phi(t) = \underbrace{\overbrace{a_i\, p(t)\cos(\omega_c t)}^{\text{Band pass}}}_{a_i\,p(t) \text{ : Base band}}$$

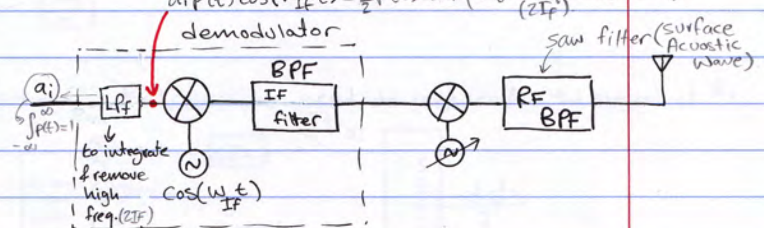
** from now on, $p(t) = RC\alpha$ always exists even if we didn't mention it (pulse shaping to limit signal's BW).

@ Rx:

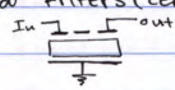
- Coherent Rx (matched filter ...)
- non coherent Rx

$\boxed{Rx}$

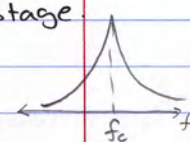
$a_i p(t)\cos^2(\omega_{IF} t) \equiv \frac{1}{2} p(t) a_i + (\text{high freq. component } (2I_F))$



- If filter's BW is exacly the same as the recieved signal's BW.
- While RF's BW is wider relatively, since it's very hard to implement sharp filters at high frequencies.
- crystal filters (or ceramic filter) are usually used for If stage.

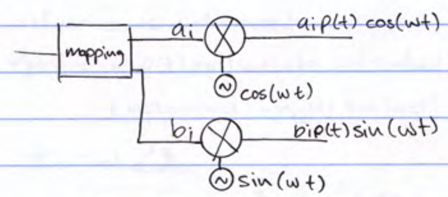


this filter is very sharp
- high Quality factor
- very linear
- very cheap
- $f_c = 10.7\,MHz$ usually.



- for 2-D:

T_x :



2D mod. but not ASK it's type depends on the constelation.

$BW_{1\text{-}D} = BW_{2\text{-}D}$

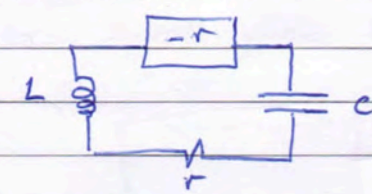
27-4-2014

ASK: in one dimentional modulation.

$\phi(t) = a_i\, p(t) \cos(\omega_c t)$

it differs from base band that here
We are Carrying the Signal on a Carrier.

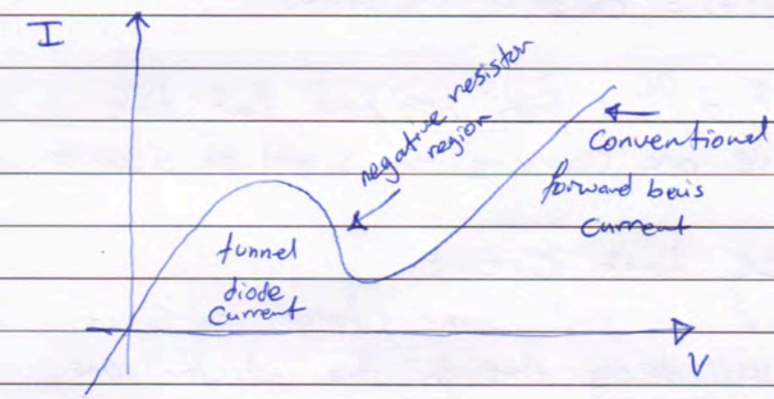
LC tank circuit:
if we put some charges on C it
will decay through the inductor and
the inductor will charge the capacitor
back, in away if we measure the
Voltage it will be sinosoidal.



$r \equiv$ omic loss
if we placed a negative resistor the
circuit will not suffer any loss

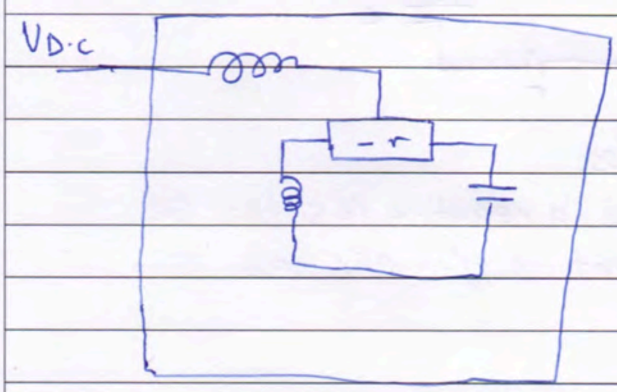
* negative resistor can be represented as "tunnel diode"

* characteristics of the diode :-

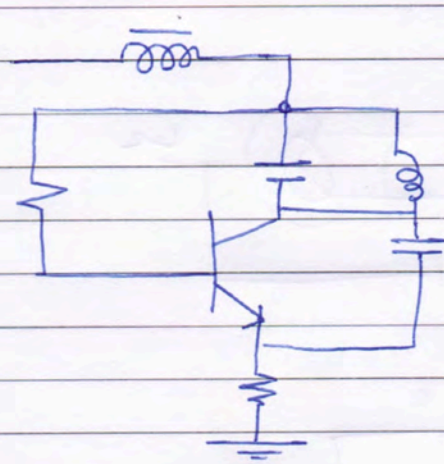


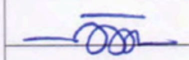
* it's a component that generate energy.

* in order to generate that energy we first should push that energy through a coil

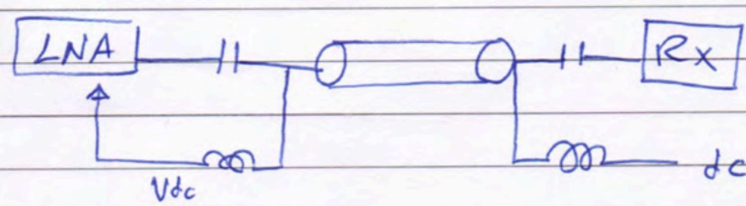


* Simplest oscilator:-

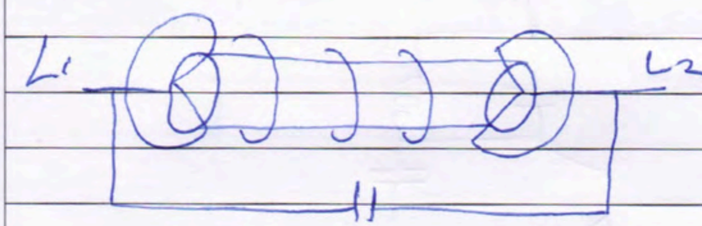


 : the bar means that it has a core.

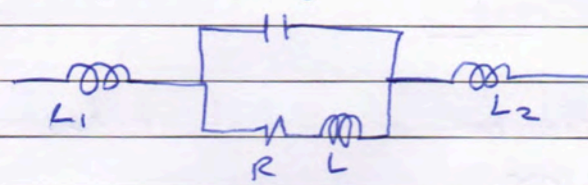
* negative feed back will try to sustain the Q- point.



* high frequency model of the resistor.



equivelant to:-



the domenent factor here is R.

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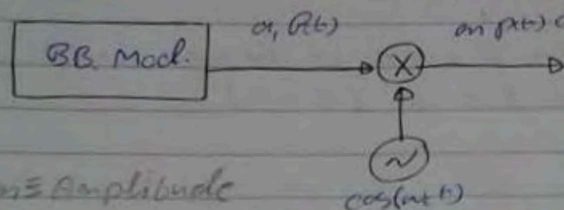
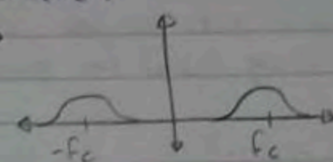
* Band Pass Tx:

The best axis we can use is cosine (or sine)
[from implementation and quality pt of view]



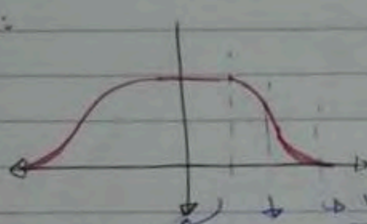


$a_n \equiv$ Amplitude
$p(t) \equiv$ raised cosine

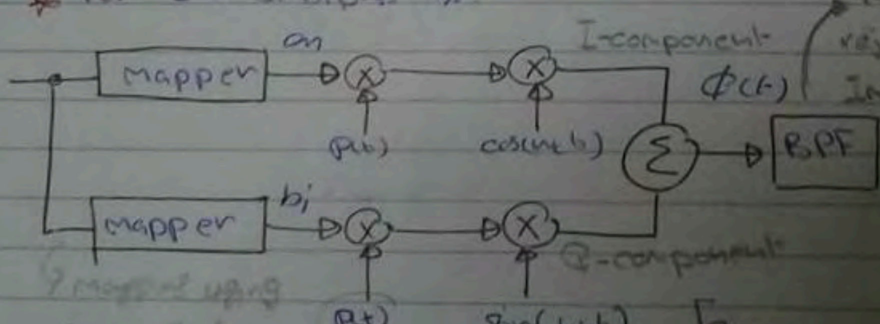
* $BW_{(a_n\,p(t))} = \dfrac{r_b}{2\log_2 M}(1+\alpha)$

$BW = \dfrac{r_b(1+\alpha)}{\log_2 M}$

* Note:



* For 2-D Bandpass Tx:



RF BPF to reject the output Image freq. due non-linearity of the mixers

↳ mapped using Gray Code

R.Cos, it limits the signal B.W

$$\boxed{BW_{\phi(t)} = \frac{r_b(1+\alpha)}{\log_2 M}}$$

→ B.P., 1-D mod. ⇒ ASK : we shift the amplitude according to the data

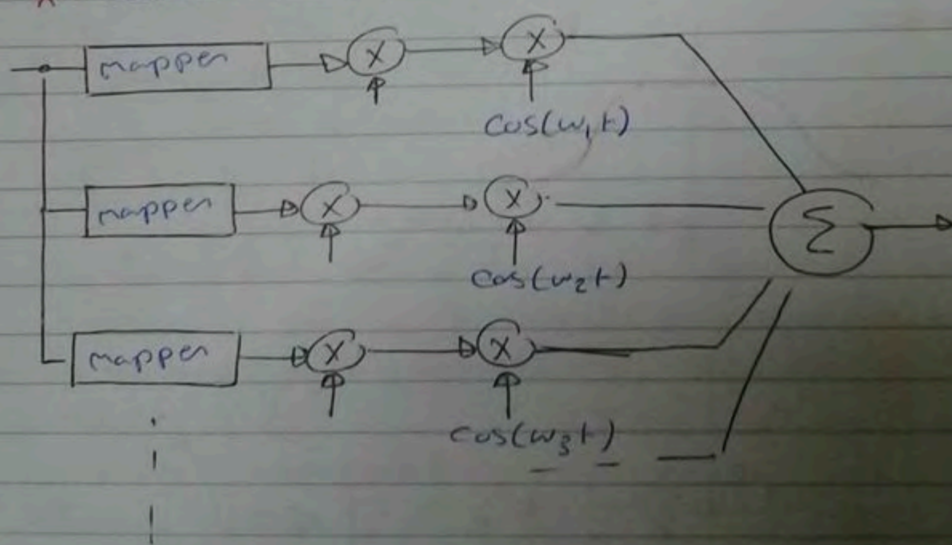
→ B.P., 2-D mod ⇒ PSK

→ B.P., multi-D mod ⇒ FSK

→ in 2-D we use sine & cosine at the same carrier freq., and the same B.W as 1-D mod.

→ in multi-D we use either sine or cosine but at different freq. for each dimension.

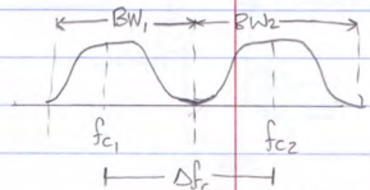
* Multi-D. B.P Tx:



3/5/2014

- **Band pass T_x**

$$BW = \frac{r_b}{\log_2 M}(1+\alpha)$$



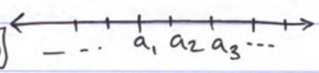
$= \frac{1}{2}(BW_2 + BW_1)$

↳ minimum distance to avoid aliasing

$\boxed{\text{1-D}}$

$$\phi(t) = a_i \, \overset{RC_\alpha}{p(t)} \cos(w_c t)$$

my axis is $[p(t)\cos(w_c t)]$



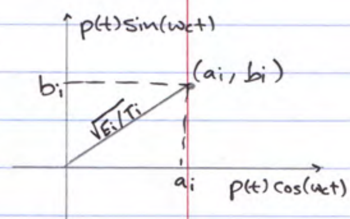
PAM

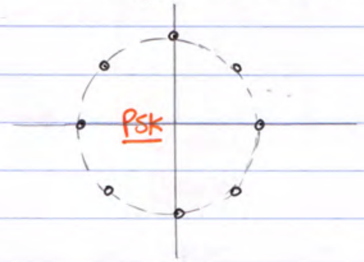
$\boxed{\text{2-D}}$

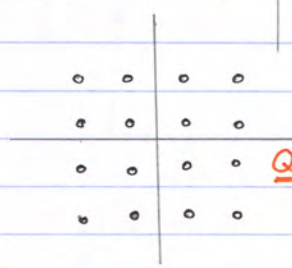
$$\phi(t) = a_i \, p(t) \cos(w_c t) + b_i \, p(t) \sin(w_c t)$$

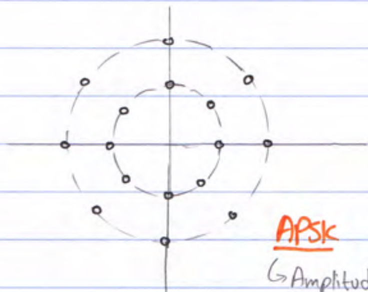
من تكامل ال $(\cos^2)$

$$E_i = \frac{1}{2}(a_i^2 + b_i^2) T_s$$



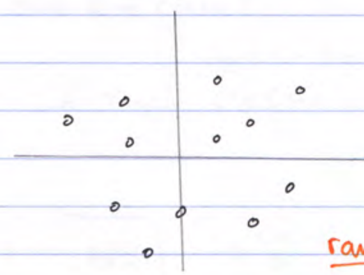






APSK

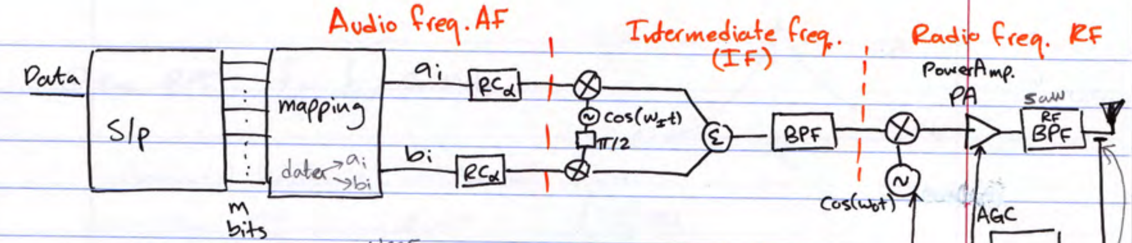
↳ Amplitude Phase Shift Keying.



random

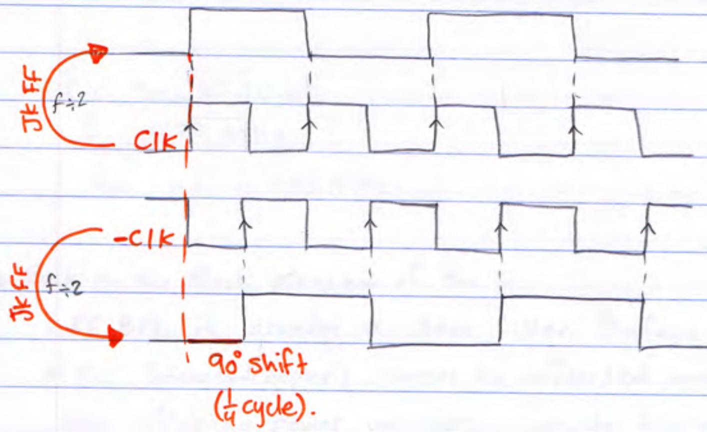
all of them are 2-D & all have the same $BW = \frac{r_b}{\log_2 M}(1+\alpha)$

type of constelation affects mapping process only.



↳ senthesized Tx. (uses PLL to senthesize carrier).
(Quadrature modulator).

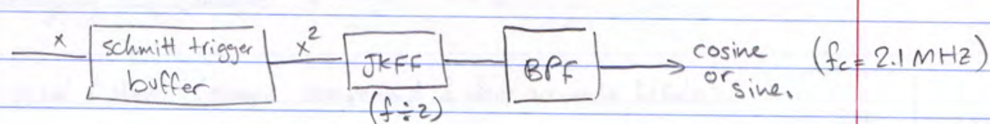
in the project:- (generation of sine & cosine):

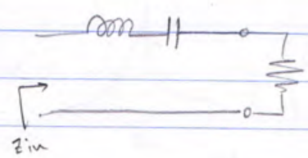


after the BPF we'll get a sine & a cosine! (rising/falling) حسب نوعه

toggle for the state every rising edge.

* in the JK FF, if $J=1$, $K=1$, → Toggle flip flop.





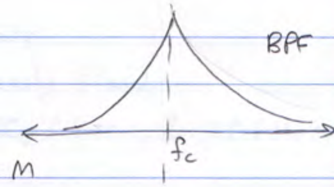
$Z_{in} = j\omega L + \frac{1}{j\omega C} + R$

for pure R:

$\omega L = \frac{1}{\omega C}$, $\omega = \frac{1}{\sqrt{LC}}$

$f = \frac{1}{2\pi\sqrt{LC}}$ ...(1)

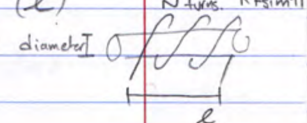
@ the BPF $f_{res.} = f_c = 2.1\,MHz$



fundamental harmonic will be at 2.1 M
while the 2^{nd} will be at $\approx 4M$ $\left(\frac{\text{clk freq}}{2}\right)$
so the BPF will pass f_c and highly attenuate other harmonics.

to find L & C, choose C to be one of the standard values, & calculate L from ① then using simulation software find the length & # of turns for the given (L)

RFsim99



* for the RF unit

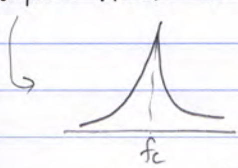
$f_c = 433\,MHz$

$f_{res.} = f_c = 433\,MHz.$

→ Back to the Block diagram of the Tx:

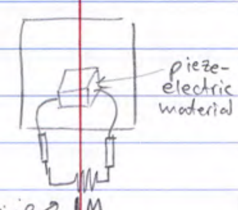
* RF BPF is usualy a SAW filter (Surface Acoustic wave)
* D.C (directional coupler) senses the reflected wave. (feed back) when the reflected power increases, ∴ we do tuning for the BPF (means that there's mismatch).
* BPF is used to reject internal harmonics. (after PA & other devices).
* Changing the type of constellation changes only mapping processes. (software)
* changing the channel changes w_o.

crystal filter: (single component & very accurate filter)



very sharp at f_c.

$\boxed{Q \propto \frac{f_c}{BW}}$



ضفناها بالمشروع حتى نزّل ال Quality factor ← لحتى نزيد ال BW

* RC_α is implemented by software.
* $\therefore$ design of the IF stage is usually fixed.
* I can exchange the places of RC_α filter & IF filter ("هل أحمق")

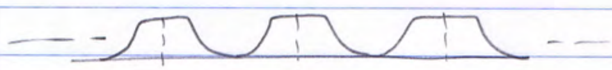
$$\phi(t) = \underbrace{a_i\, p(t) \cos(\omega t)}_{\text{In-phase component}} + \underbrace{b_i\, p(t) \sin(\omega t)}_{\text{Quadrature component}}$$

for PSK:

$$\phi(t) = A \angle \theta_i = A\, p(t) \cos(\omega_c t + \theta_i)$$

for multi-dimensional mod:

it's called
Wide band modulation



$$BW = \frac{M\, r_b}{\log_2 M}(1+\alpha)$$

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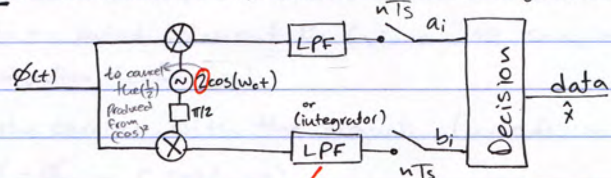
$$\phi(t) = a_i\, p(t)\cos(w_c t) + b_i\, p(t)\sin(w_c t)$$
$$= A_i \angle \theta_i$$

if $= \underbrace{A}_{\text{constant}} \angle \theta_i$: PSK

$$A_i = \sqrt{a_i^2 + b_i^2}$$
$$\theta_i = \tan^{-1}\left(\frac{b_i}{a_i}\right)$$

at $\underline{R_x}$: (coherent R_x); since we need to regenerate the signal at R_x



* only from mod. point of view (without filtering...)

to remove the double freq. & limit the BW.

- mixer is easier than multiplier to implement.
- what about decision?

① we can divide $\frac{b_i}{a_i} \rightarrow \tan^{-1}\left(\frac{b_i}{a_i}\right) = \theta_i$

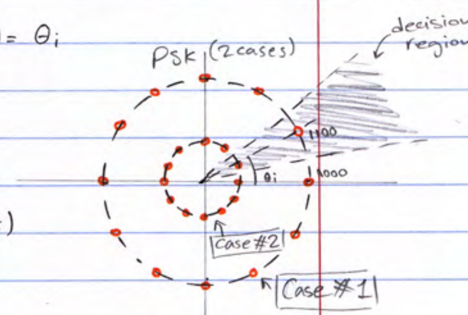
→ amplitude's (A) error doesn't affect the reception.

→ if noise is introduced:

$$\phi_{PSK} = a_i\, p(t)\cos(w_c t) + b_i\, p(t)\sin(w_c t) + n_I(t)\cos(w_c t) + n_Q(t)\sin(w_c t)$$

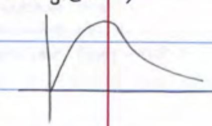
$$\vec{n} = |n| \angle \zeta_n$$

$$|n| = \sqrt{n_I^2 + n_Q^2}\ , \quad \zeta_n = \tan^{-1}\left(\frac{n_Q}{n_I}\right)$$



→ in case #2 error is more likely to happen $d_{min_2} < d_{min_1}$

- Pdf of the noise: n_I, n_Q gaussian — $\overset{\text{envelope}}{\boxed{\text{detect.}}}$ — Ricien Pdf. (with uniform angle.)



  if the mean $=0$ → Rayleigh

  else → Ricien. (if n_I or n_Q (or both) have a mean).

  Uniform Pdf for (B_n) $\Rightarrow \tan^{-1}\left(\frac{\text{gaussian}}{\text{gaussian}}\right) = \text{uniform}$.

  * $\theta_i = \tan^{-1}\left(\dfrac{b_i + n_Q}{a_i + n_I}\right)$

  ↳ Pdf of the phase angle of PSK signal (H.W)!

- coherent R_x is the optimal R_x.
- techniques to aviod coherent R_x (since it's complex to build):
  ↳ (synchronization techniques):

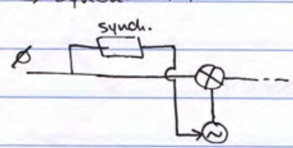
  1. Send the carrier with the signal. (needs more power).

     $\phi = \phi_{old} + C\cos(w_c t)$

     if $C > A$ → dominent

     if $C < A$ → pilot carrier

     ↳ synch. CKT



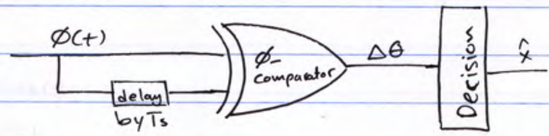
  2. Use differential PSK (DPSK)

     e.g.:

data	$\Delta\theta$ ← $\theta_n - \theta_{n+1}$
0	0
1	π

• DPSK:

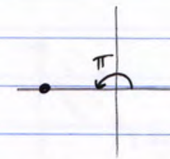
Rx



I don't need to regenerate the carrier. (resynch).

e.g:

data	$\Delta\theta$
0	0
1	π

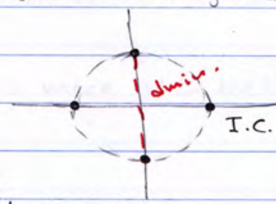


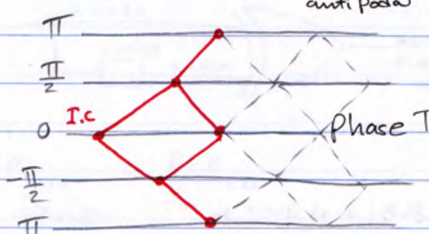
* in this type of mod. we use the constelation diagram to define the possible phases (don't label the dots).
↳ (absolute shape of constelation doesn't give direct info. about the dimensionality of the transmitted signal).

e.g 2:

data	$\Delta\theta$
0	$\frac{\pi}{2}$
1	$-\frac{\pi}{2}$

(minimum DPSK)
(MSK) since they're anti podal $\Delta\theta = \pi$

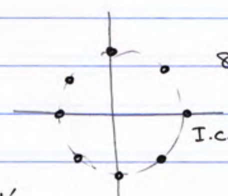




as if I'm working on a cylinder.

e.g 3:

data	$\Delta\theta$
0	$\frac{\pi}{4}$
1	$-\frac{\pi}{4}$



8 possible phases.

# of possible phase $= \frac{2\pi}{(\frac{\pi}{4})} = 8$ (since it's uniform).

$\Delta(\Delta\theta) \propto d_{min}$ between symbols. , to implement DPSK, we need system with memory

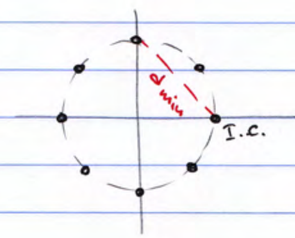
DPSK: partial response signaling.

PSK, QAM, .. : full-response signaling.

non-binary:

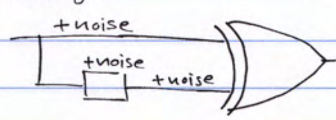
4-DPSK

data	$\Delta\theta$
00	$\pi/4$
01	$-\pi/4$
10	$3\pi/4$
11	$-3\pi/4$



this is called $\frac{\pi}{4}$-Quadrature DPSK.

- When # of possible phases is more it's better (better to choose from).
- disadvantage:-



double the noise.

- $BER_{Dxxx} \approx BER_{xxx}$ at SNR less 1.5dB.

  (D = differential)

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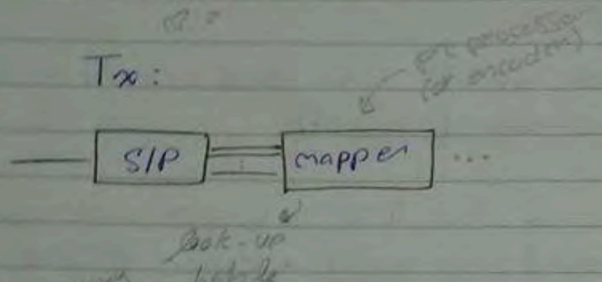
DPSK: ⇒ Data is carried over the differences between symbols, and distributed over many transmissions

T_x:



Ex;

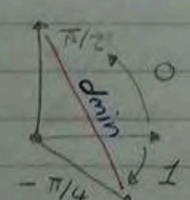
For BDPSK ⇒ we have only 2-bits

mapper ⇒

data	DG
0	$\pi/2$
1	$-\pi/4$

• The choice of DG depends on the min distance (which is related to the difference between DG's i.e D(DG))

Suppose we're at specific phase, we either send 0 $(+\pi/2)$ or $1(-\pi/4)$



$\cancel{d_{min} = \frac{\pi}{2} - -\frac{\pi}{4} = \frac{3\pi}{4}}$

at R_x we need to distinct between $\pi/2$ shift or $-\pi/4$ with respect to the current phase.

"مش كل constellation diagram مكانه (1's & 0's) موجودين كل DG"

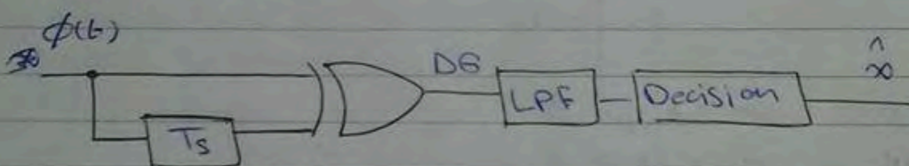
C.D.

→ The constellation diagram for DPSK represents the possible phases not the data itself.

→ The # of points on the C.D. does not depend on the # of transmitted signals

→ From quality pt. of view, PSK is better than DPSK, but practically, it's harder to build a coherent Rx for PSK, so to reduce the complexity we tend to use DPSK.

* DPSK Rx:



• In comm. systems we need the info. mapping to be one-to-one so we can reverse the operation, but, in dealing with w.f. we must have linear devices (system) such as mixer, osc, Amp....

• إذا استخدمنا BDPSK بتكون الفرق بين النقاط هي π

π/2 " " " QDPSK " "

→ quadrature

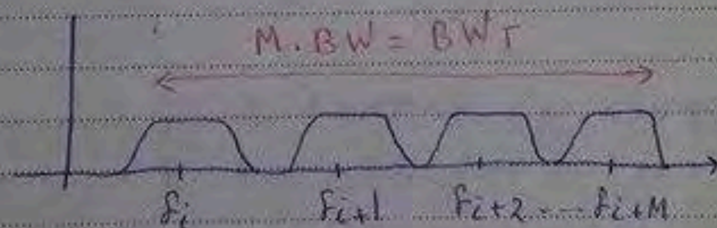
• DPSK is better than ASK, since it's more immune to fading

* Multidimentional Tx:

$\underline{\underline{FSK}}$: $\phi_i(t) = a_i\, p(t) \cos(\omega_i t) = \underbrace{a}_{\text{Constant}}\, p(t) \cos(\omega_i t)$

* 1-D & 2-D

$$BW = \frac{r_b}{\log_2 M}(1+\alpha)$$



* Multi Dim.

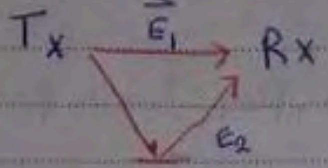
$$BW_{FSK} = \frac{M\, r_b}{\log_2 M}(1+\alpha)$$

⇒ Here information are on frequency NOT the amplitude

⇒ FSK is wide band transmission

No.

* Multipath fading :-



$\vec{E}_1 = E \measuredangle 0^\circ$

$\vec{E}_2 = E \measuredangle \Delta\theta$

⇒ Reflection = 180° phase shift

$$E_t = \vec{E}_1 + \vec{E}_2$$

$$= E + E(\cos(\Delta\theta) + j\sin(\Delta\theta))$$

$$\boxed{E_t \approx E \cdot \Delta\theta}$$

$$\Delta\theta = \frac{2\pi\Delta d}{\lambda} \Rightarrow \lambda f = c$$

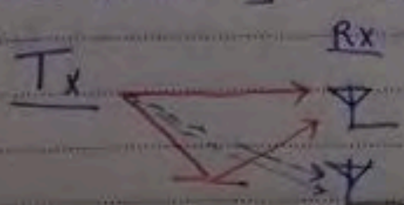
$$\lambda = \frac{f}{c} \Rightarrow \boxed{E_t = E \cdot 2\pi\Delta d \cdot \frac{f}{c}}$$

⇒ Frequency selective fading: fading occurs at a certain frequency not all frequencies

* The ONLY solution for fading is:-

Diversity:

① space diversity : Multiple Rx antennas



⇒ cheap solution

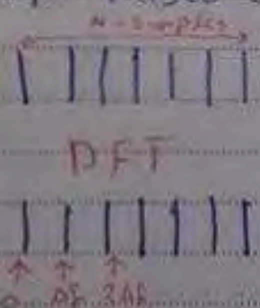
② Time diversity: Multiple time (sending the Same signal more than one time)

⇒ more expensive

③ Frequency diversity: Multiple frequency

⇒ FSK has this solution.

* If we want to decrease the Bandwidth we use DFT (Discreate Fourier transform) instead of raised cosine.



$X[K] = DFT(X[N])$

$$\boxed{\Delta f = \frac{f_s}{N}}$$

⇒ Bandwidth effictive modulation

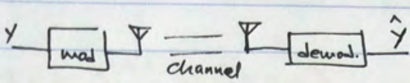
* advantages:-

① frequencies are orthogonal (fading occurs on one frequency Not all of them)

② BW are more ~~[illegible]~~ efficient

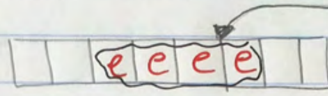
13/5

- Channel coding:

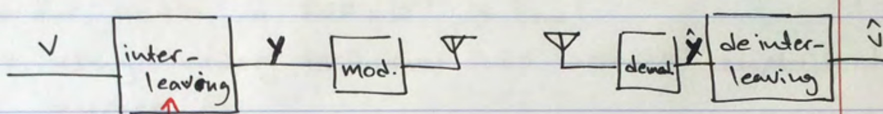


- In AWGn channels errors are randomly distributed among the sequence.
- In fading channels errors come in burests groups.

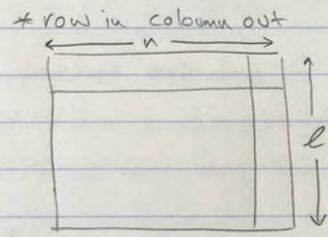
since changes in channel need more than (T_b)



→ that's why we use interleaving / coding.



special type of scramblers, such that consecutive data are far from each others in the output.

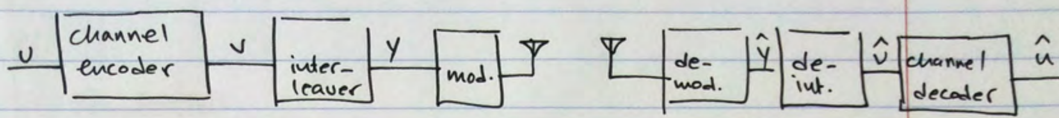


V= | 1 | 2 | 3 | 4 | 5 | ---- |

Y= | 1 | n+1 | 2n+1 | 3n+1 | --- | 2 | n+2 | 2n+2 | ---- | 3 | n+3 | 2n+3 | --- | - |

$\longleftarrow l \longrightarrow$

- Interleaver randomizes the data. to avoid burrest of errors.
- Channel coding:-
  - error detection.
  - error correction: can correct (t) errors in (n) bits in $Y \to \hat{Y}$



- $(BER_y)^{t+1} \cong (BER_u)$.
  ↳ channel's BER

e.g: $BER_y = 0.01$, $t = 3$
$BER_{coded} = (0.01)^4 = 10^{-8}$

→ $BER_{\hat{v}} = BER_{\hat{y}}$ only data is de-scrambled.

* for audio a $BER = 10^{-3}$ is fine!
* using coding techniques will increase complexity & might increase BW.
* Error detection: (CRC)
* T_x in digital systems is packet oriented

"يعني هيك هيك رح تستنى كل ال Packet توصل قبل لا تبعتها فغالباً ما بكون في مشكلة delay نتيجة ال interleaving"

increasing # of errors (length of burrest error) ⟶ increases ℓ ⟶ increases size of memory we need $(n \times \ell)$ ⟶ increases the delay $(n \times \ell \times T_b)$ ← time we have to wait before scrambling (interleaving) data.

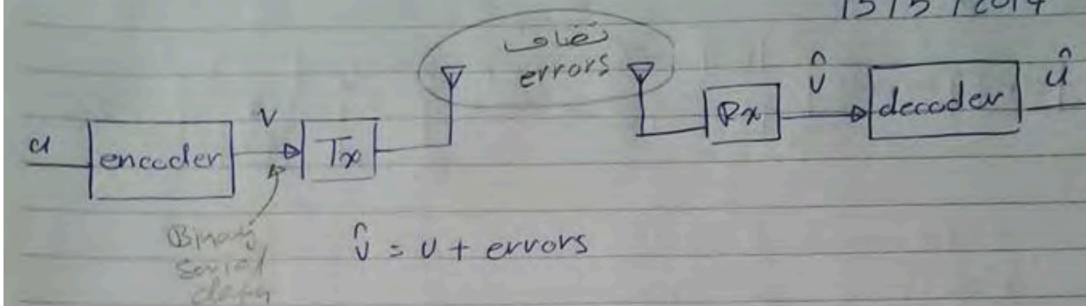
- every digital comm. system has an interleaver & de interleaver.
- Binary field arithmetics:

A field is a set of elements (c) that has two binary operators (+, x)

Binary field GF(2)

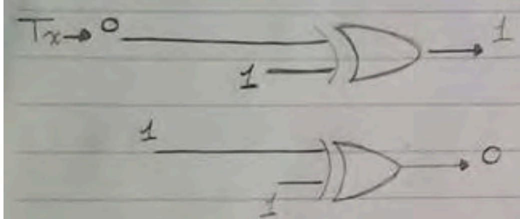
$C = \{0, 1\}$

15/5/2014



$\hat{v} = v + \text{errors}$

* Errors in Binary are defined by recieving 1 when 0 has been transmitted and vice versa, which ~~[illegible]~~ can be implemented using XOR



let's say we have:

$v = [n\text{-bits}]$

$e = [n\text{-bits}]$

↳ should have the same length as v, so we can sum them up

Ex.
$v = [1011]$
$e = [0010]$ ⇒ $\hat{v} = v + e$ → bitwise XOR

يعني مش جمع عادي لأن مافيه carrier

$\hat{v} = [1001]$ ↳ ERROR!

$0+0=0$, $0+1=1$

$1+0=1$, $1+1=0$

- There are 2-ways to represent the numbers:
  - vector form
  - polynomial form

For Binary. $GF(2)$, $G = [0, 1]$ → elements set → 2-elements for Binary

represented by m-bits ← $GF(2^{m})$ ⇒ Extended field

↳ Binary

Ex:

[1011] ⇒ vector form, $GF(2^4)$

\# of digits (extension)

$GF(2^4)$ is equivalent to $GF(16)$ → 1-digit has a value between 1–15 (in hexa)

Extended field	not extended
Binary with 4-bits	we represent the number using a set of 16-elements

∴ The difference is in the representation.

$GF(10^4)$ → Decimal with 4 digits
(any number lies between 0000 – 9999)

"We can represent fields with different formats, we can, for instance, map any field to the binary field"

* Polynomial form:

Ex:

$$
\begin{aligned}
f(x) &= 1 + x + x^3 \\
+\ b(x) &= 1 + 5x^2 + 2x^3 \\
\hline
s(x) &= 2 + 5x^2 + x + 3x^3
\end{aligned}
$$

← Addition in the decimal field

In binary field, ~~[illegible]~~ factors are either 1's or 0's, (فيه x ولا لأ)
the exponent of x represents the location of the bit

Ex: $\overset{0}{1}\ \overset{1}{0}\ \overset{2}{1}\ \overset{3}{1} \xrightarrow{\text{polynomial}} x^0 + 0 + x^2 + x^3$

$1 + x^2 + x^3$

position zero

$101001 \Rightarrow 1 + x^2 + x^5$

Ex: (Binary)

$$\begin{array}{r} 1 + x^2 + x^3 \\ + \quad x^2 \\ \hline 1 + x^3 \end{array} \quad \equiv \quad \begin{array}{r} 1011 \\ +\ 0010 \\ \hline 1001 \end{array}$$

For coding:

→ represented using vector or polynomial form

$u \rightarrow$ [encoder] $\rightarrow v$

↳ certain function, has to be linear

* Codes are classified into 2-main families:
  1- Linear Block Codes (LBC)
  2- Convolutional codes (C.C)

• Block ≡ يعني باخذها أطوال محددة every k-bits at inp. وبطلعها n-bits at the output.

k-bits, 2^k possible inputs ← $u \rightarrow$ [encoder] $\rightarrow v$ → n-bits, 2^n possible outputs.

↳ linear : $v = u \cdot G$

$G \equiv$ constant mtx., Generator mtx. (it generates the code)

→ usually $n > k$

→ Coding is one-to-one process

- Subclasses of LBC.
  - Hamming codes → the simplest
  - cyclic codes → more general than hamming.
  - BCH codes
  - LDPC codes

- Convolutional codes → better than LBC
  - Turbo codes
  - C.C (regular conv. code)

  → Convolves the input stream with a certain fcn.

The LBC is to select 2^k code words out of possible 2^n, such that the selected code words have max., min. hamming distance

* Suppose → $k = 4$ → 16 - possible inputs
  $n = 7$ → 128 - possible outputs

LBC codes pick 16 out of 128 possibilites, each of wich is 7-bits code word, those code words are the alphabets of the used code

→ Hamming distance ⇒ the difference between the code words; for instance, let's say an 1st chosen code word is:

1st → 1 0 1 1 1 0
2nd → 1 1 1 0 1 1 $d^H = 3$
difference

• كلما كبرت d^H_{min} بنكون أقدر
كى نصحح الـ error، فلو صار
error في واحد من الـ bits،
بنكون لسا أقرب للداتا الصحيحة [d=1]
وأبعد عن باقي الـ code-words [d=2]

$$t = \left\lfloor \frac{d^H_{min} - 1}{2} \right\rfloor$$

→ # of errors the Rx can correct for n-bits

No.

lecture 18/5/2014.

Binary field arithmatic :

$GF(2^m)$

let $m = 4$

$V_1 = [1011] = 1 + x^2 + x^3$

$V_2 = [1100] = 1 + x$

add

$0111 \rightsquigarrow x + x^2 + x^3$

multiplication:-

polonomial multiplication:-

$(1+x)(1+x^2+x^3)$

$= 1 + x^2 + x^3 + x + x^3 + x^4$

$= 1 + x + x^2 + x^4$

Division:

let $V_1 = 1 + x^2 + x^3$

$V_2 = 1 + x$

find $V_1 \div V_2$

$$\begin{array}{r|l} & x^2 \\ \hline 1+x & x^3 + x^2 + 1 \\ & \underline{x^3 + x^2} \\ & 1 \end{array}$$

Note that we dont subtract here we only add & multiply in Binary field.

x^2 represent the quatient
1: represent the remainder

$$\frac{V_1}{V_2} = x^2 + \frac{1}{1+x} = x^2 \text{ rem } [1]$$

$V_3 = \frac{V_1}{V_2} \rightsquigarrow V_2 V_3 = V_1 \rightsquigarrow q * V_2 = \text{remainder}.$

another example:-

let $v_1 = 1 + x^7$

$v_2 = 1 + x + x^3$

$$\begin{array}{r|l}
 & x^4 + x^2 + x + 1 \\
\hline
1 + x + x^3 & x^7 + 1 \\
 & \underline{x^7 + x^5 + x^4} \\
 & x^5 + x^4 + 1 \\
 & \underline{x^5 + x^3 + x^2} \\
 & x^4 + x^3 + x^2 + 1 \\
 & \underline{x^4 + x^2 + x} \\
 & x^3 + x + 1 \\
 & \underline{x^3 + x + 1} \\
 & 0
\end{array}$$

No.

any polonomial i can find its roots.

find the roots of v_2:

$0 = 1 + x + x^3 \rightsquigarrow x \in \{0, 1\}$

try $x = 0 \rightsquigarrow 1$

$x = 1 \rightsquigarrow 1$

$\rightsquigarrow$ there is No roots.

Note: polonomial with no roots is called prime polonomial. in $GF(2)$

$$\left.\begin{array}{l} 1+x+x^3 \\ 1+x+x^2 \\ 1+x \end{array}\right\} \text{prime polonomials}$$

$1+x$ is aspecial case.

Note: for each [m] we have a certain number of prime polonomials. see website for more info.

* the prime polonomials are the roots of $1+x^n$

where $n = 2^m - 1$

m: the order of the polonomial.

in $GF(3)$ $\rightsquigarrow$ the prime polonomials are the roots of $1+x^7$

$n = 2^m - 1$

$7 = 2^3 - 1 \rightsquigarrow \boxed{m=3}$

each prime polonomial can span the whole field

Span: يولد

example:

$v = 1 + x^2 + x^3$

find elements of $(GF)^m$

sol.

$1 + x^2 + x^3 = 0$

$*X \quad 1 + x^2 = x^3$

$x + x^3 = x^4$

$*X \quad x + 1 + x^2 = x^4$

$x^2 + x + x^3 = x^5$

$x^2 + x + 1 + x^2 = x^5$

$*X \quad 1 + x = x^5$

$x + x^2 = x^6$

0	$= 0$	000	0
1	$= 1$	100	1
x	$= x$	010	α
x^2	$= x^2$	001	α^2
x^3	$= 1 + x^2$	101	α^3
x^4	$= 1 + x + x^2$	111	α^4
x^5	$= 1 + x$	110	α^5
x^6	$= x^2 + x$	011	α^6

No.

* each time we multiply by x we are doing a rotation (cyclic) & generating all the elements in the field.

→ if we want to <u>add</u> we use the binary field
if we want to multiply we use the GF field (α)

$111 + 110 = 001 \rightsquigarrow \alpha^2$

$\alpha^2 * \alpha^4 = \alpha^6$

$\alpha^4 * \alpha^3 = \alpha^7 = 1$ Not 0 becase Zero is the identity of addition not multiplication.

$\alpha^k \cdot \alpha^j = \alpha^{k+j=l}$

if $l > n$ then $l = l - n$

Linear block code:

Cyclic codes:

$u(x) \to \boxed{g(x)} \to v(x)$

$v(x) = \underbrace{u(x)}_{\text{order } k} \cdot \underbrace{g(x)}_{\text{primitive}}$

g(x): generating potonomial.

No.

Note: No ~~pt~~ polonomial with order less than n devide the primitive polonomial.

the constrain: $\boxed{m + k \leq n}$

m+k+1: is the number of bits the polonomial holds.

20/5

LBC

* cyclic codes:

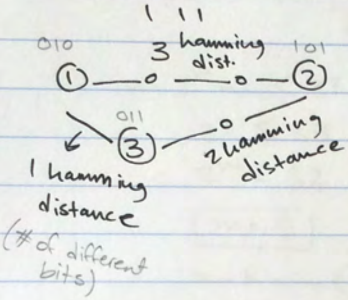
000
001
010 1
011 3
100
101 2
110
111

$k=2$

$2^k = 4$ inputs

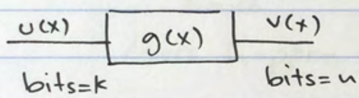
$n = 3$

$2^k = 4$ outputs



$d^H_{min} \geq 3$

$t = \frac{d_{min}}{2}$



* it's a single error correcting code.

$t=1, \quad d^H_{min} = 3$

$$t = \left\lfloor \frac{d^H_{min} - 1}{2} \right\rfloor$$

- Special for cyclic codes:

parameters of any cyclic code (we control m).

$$\begin{cases} n = 2^m - 1 \\ n - k = mt \\ t = 1 \quad \leftarrow \text{can correct 1 bit in every(n) bits} \\ d^H_{min} = 3 \end{cases}$$

ex: let $BER = 10^{-2}$, find the single error correcting code to make $BER = 10^{-8}$

→

Solution:-

$$BER_{coded} = \left(BER_{uncoded}\right)^{t+1}$$

$$10^{-8} = (10^{-2})^{t+1}$$
$$= 10^{-2t'+2} \quad (t'=3)$$

the code correct 1 bit in every $(10^{-2})^{-1} = 100$ bits.

required (n) $\quad n \leq \frac{100}{3} \leftarrow t'$

$$n = 2^m - 1 \leq 33$$
$$31 \leq 33$$
$$\therefore 2^m = 32$$
$$\boxed{m=5}$$
$$n-k = mt \leftarrow t=1$$
$$k = 26$$

errors happen in the coded data (n not m).

- Shortened code: $n' \rightarrow n-\ell$
  $k' \rightarrow k-\ell$

  "I can remove (ℓ) bits from the input & this will remove (ℓ) bits from the output"

  for (e.g) the previous: let $\ell = 6$
  $n = 25$
  $k = 20$

→ $g_{cyclic}(x)$ is a prime factor of $1+x^n$ ← this g_{cyclic} will satisfy the previous condition.

ex: (n,k) code
$m=3$, $\therefore k=4$
& $n=7$
$n-k = mt$
input bit rate r_b
output bit rate r'_b
$$\frac{n}{k} r_b = r'_b$$

$$BW = \frac{r_b'}{\log_2 M}(1+\alpha)$$
$$= \left(\frac{n}{k}\right)\frac{r_b}{\log_2 M}(1+\alpha)$$
BW expansion factor.

we must get maximum t at minimum $\left(\frac{n}{k}\right)$, n is constant
$$\therefore k \propto \frac{1}{t}$$

$\boxed{\frac{k}{n}: \text{code rate}}$ usually <1

cont →

Check tables on the website!

$1+x^7 = (1+x)(1+x+x^3)(1+x^2+x^3)$

$g(x) = 1+x+x^3$

order

$m=3 \qquad (0,1,3) \quad , (0,1) \quad , (0,2,3)$

$1+x+x^3 \qquad 1+x \qquad 1+x^2+x^3$

I want the order of g_{cyclic} to be $m=3$

- $g(x)$ has a reciprocal polynomial, $g^{-1}(x)$, where

$$\boxed{g^{-1}(x) = x^m g(x^{-1})}$$

take $g(x) = 1+x^2+x^3$

$$g^{-1}(x) = x^3(1+x^{-2}+x^{-3}) = \underline{\underline{x^3+x+1}}$$

*** Any code & it's reciprocal are equivelant.

$\therefore$ we can choose $g(x)$ to be either $1+x^2+x^3$ or $1+x+x^3$

$\rightarrow$ let $g(x) = 1+x+x^3$

remember

$u(x) \rightarrow \boxed{g(x)} \rightarrow v(x)$

ex: let $u(x) = 1+x^3$

$$\begin{aligned} v(x) &= u(x)g(x) \\ &= (1+x^3)(1+x+x^3) \\ &= 1+x+x^3+x^3+x^4+x^6 \\ v(x) &= 1+x+x^4+x^6 \end{aligned}$$

let's introduce an error $e(x) = x^3$

$\therefore \hat{v}(x) = 1+x+\underline{\underline{x^3}}+x^4+x^6$

due to error

if no error has occured $v(x)$ must divide $g(x)$

$$\frac{v(x)}{g(x)} = \frac{u(x)\,g(x)}{g(x)} = u(x)$$

وإلا ما بقدر أصلحه

+error: (single error)

$$\frac{\hat{v}(x)}{g(x)} = \frac{u(x)g(x) + e(x)}{g(x)} = u(x) + \frac{e(x)}{g(x)}$$

remainder

→ max order of $e(x)$ is $m-1$

* let's take all remainder probabilities

syndrome table.

	$b(x)$	$r(x)$
1	0	1
x	0	x
x^2	0	x^2
x^3	1	$1+x$
x^4	x	$1+x^2$
x^5	$1+x^2$	$1+x+x^2$
x^6	$x+x^3$	$1+x^2$

بحدد مكان ال error من خلال الجدول.

$b(x)$: quotient

$r(x)$: remainder

← notice that any remainder is unique (not repeated)

Syndrome table 22/5/2014

For $g(x) = 1 + x + x^3$ ← The code

error	quotient	remainder	
$e(x)$	$b(x)$	$r(x)$	H
1	0	1	100
x	0	x	010
x^2	0	x^2	001
x^3	1	$1+x$	110
x^4	x	$x+x^2$	011
x^5	$1+x^2$	$1+x+x^2$	111
x^6	$1+x+x^3$	$1+x^2$	101

error at position zero

100 → I; rows 110 to 101 → P

← $r(x)$ in Binary form

$$g(x) \overline{\big)\, e(x)}\quad \text{quotient } b(x),\ \text{remainder } r(x)$$

$$H = \Bigg[\underbrace{\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}}_{I}\ \Bigg|\ \underbrace{\begin{matrix}1&0&1&1\\1&1&1&0\\0&1&1&1\end{matrix}}_{P^T}\Bigg] \Rightarrow \text{parity check}$$

Another method to find the P-mtx. :

$$\underbrace{1+x+x^3}_{g(x)} = 0 \longrightarrow \begin{aligned} x^3 &= 1+x \\ x^4 &= x+x^2 \\ x^5 &= x^2+x^3 = x^2+(1+x) \\ x^6 &= x^3+x^4 = \cancel{x}+x^2+1+\cancel{x} = 1+x^2 \end{aligned}$$

$$P = \begin{bmatrix} 1&1&0\\ 0&1&1\\ 1&1&1\\ 1&0&1 \end{bmatrix}$$

From previous ex:

$u(x) = 1 + x^3$

$v(x) = 1 + x + x^4 + x^6$

assume the error in position 4 $\Rightarrow e(x) = x^3$

$\hookrightarrow \hat{v}(x) = 1 + x + x^3 + x^4 + x^6 \rightarrow$ بنطلع الداتا الصحيحة من $\hat{v}(x)$

* devide by $g(x)$:

$$
\begin{array}{r|l}
 & x^3 \Rightarrow q(x) \\
\hline
g(x) = 1 + x + x^3 & x^6 + x^4 + x^3 + x + 1 \\
 & x^6 + x^4 + x^3 \\
\hline
 & 1 + x \Rightarrow r(x)
\end{array}
$$

$\Rightarrow \dfrac{\hat{v}(x)}{g(x)} = \underbrace{a(x) + b(x)}_{q(x) = x^3} + \dfrac{r(x)}{g(x)}$

من الـ table حسب معرفتي لـ $r(x)$

$[b(x) = 1]$

$\downarrow$

لأنو الـ remainder بيجي من الـ error

$a(x) = q(x) + b(x) = x^3 + 1$ ✓

مكانه صحيح

* Ex 2:

$u(x) = 1$

$v(x) = 1 + x + x^3 \xrightarrow{\text{ch.}} e(x) = x^5$

$\hookrightarrow \hat{v}(x) = 1 + x + x^3 + x^5 = [1\ 1\ 0\ 1\ 0\ 1\ 0]$

$$
\begin{array}{r|l}
 & x^2 \Rightarrow q(x) \\
\hline
1 + x + x^3 & x^5 + x^3 + x + 1 \\
 & x^5 + x^3 + x^2 \\
\hline
 & 1 + x + x^2 \rightarrow r(x)
\end{array}
$$

$\therefore b(x) = 1 + x^2$

$\hat{u}(x) = x^2 + x^2 + 1 = 1$

• In mtx. form. ⇒ in this method we don't need the syndrome table

$V = u.G \rightarrow$ Generator mtx.

u ↓ the message

$G = [I \mid P]$ → 4×3 ∴ $I = 4 \times 4$

$H = [I \mid P^T]$ → 3×4 ∴ $I = 3 \times 3$

$$G = \left[\begin{array}{cccc|ccc} 1&0&0&0&1&1&0 \\ 0&1&0&0&0&1&1 \\ 0&0&1&0&1&1&1 \\ 0&0&0&1&1&0&1 \end{array}\right]$$

↓ parity check mtx.

Ex.1*:

→ Parity check bits

$u(x) = 1 + x^3 = 1001$

$$\Rightarrow V = u.G = [\underbrace{1\ 0\ 0\ 1}_{u.[I] = u}\ \underbrace{0\ 1\ 1}_{u.[P]}]$$

u^T

$1 \times 1\ 1\ 0$
$0 \times 0\ 1\ 1$
$0 \times 1\ 1\ 1$
$1 \times 1\ 0\ 1$

$0\ 1\ 1$

$1{\times}1 + 0{\times}0 + 0{\times}1 + 1{\times}1$
$= 1 + 0 + 0 + 1 = 0$

$\Rightarrow e(x) = x^3 = [0001000]$

$\hat{V}(x) = [1000011]$

* At Rx: we find what we call it "syndrome"

$S = \hat{V} H^T = [1\ 1\ 0]$

$$H = \left[\begin{array}{ccc|cccc} 1&0&0&1&0&1&1 \\ 0&1&0&1&1&1&0 \\ 0&0&1&0&1&1&1 \end{array}\right]$$

$S^T = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ → same as the 4th col of H mtx.

∴ error has occured in the 4th bit.

$$H^T = \begin{bmatrix} 1&0&0 \\ 0&1&0 \\ 0&0&1 \\ 1&1&0 \\ 0&1&1 \\ 1&1&1 \\ 1&0&1 \end{bmatrix}$$

* If S is non zero, then it represents the position of the error, if zero, then no error has occured.

Ex 2:

$g(x) = 1 = [1000]$

$U(x) = [1000110]$

$\rightarrow e(x) = x^5 = [0000010]$

$\therefore \hat{U}(x) = 1000100] \Rightarrow S = \hat{U} H^T$

$S = [1\ 1\ 1] \rightarrow S^T = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

↳ 6th bit

$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$

$V = [\underbrace{1000}_{g(x)\text{ our data}}\ 110]$ Determine the error's position then correct the data

* CRC (cyclic redundancy check)

↳ only for error detection, can detect $(d^H_{min}-1)$ errors every n-bits.

- The code that used for correction can't be used for error detection at the same time.

$x \rightarrow$ CRC $\rightarrow$ ECC $\rightarrow$ mod. $\rightarrow$ de-Mod. $\rightarrow$ de-coder $\rightarrow$ CRC^{-1} $\rightarrow \hat{x}$

CRC: adds parity check bits to the data

CRC^{-1}: finds "S"
if $S = 0 \rightarrow$ pass
if $S \neq 0 \rightarrow$ drop the data, or retransmit it.

For CRC detection, start with cyclic code:

(m+1)th order

$\therefore$ m+2 bits

$$\underset{CRC}{g(x)} = \underset{cyclic}{g(x)} \cdot (1+x)$$

$\rightarrow$ mth order

$\hookrightarrow$ can detect d^H_{min} errors

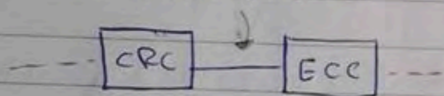
How does CRC work?

it takes the data, $u(x)$, ~~devidesth~~ devides it by $\underset{CRC}{g(x)}$:

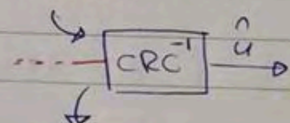
$$\frac{u(x)}{\underset{CRC}{g(x)}} = q(x) + \frac{r(x)}{g(x)}$$

$\rightarrow$ there'll be always remainder, 'cuz $\underset{CRC}{g(x)} \rightarrow [m+1]$ is of higher order than $u(x) \rightarrow$ [kth-order]

$[u(x) \vdots r(x)]$



$[\hat{u}(x) \vdots \hat{r}(x)]$



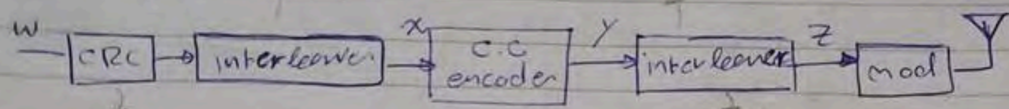
devides $\hat{u}(x)$ by $\underset{cyclic}{g(x)}$ once again.

$$\frac{\hat{u}(x)}{\underset{cyclic}{g(x)}} = \hat{q}(x) + \frac{\hat{r}(x)}{\underset{cyclic}{g(x)}}$$

if $r(x) = \hat{r}(x) \rightarrow$ No errors.

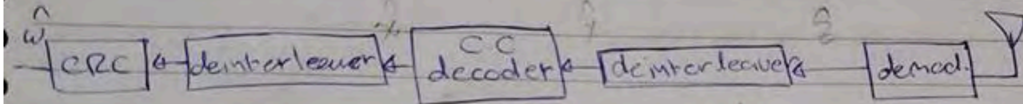
26/5/2014

Tx:



adds error-check bits to the input data [adds m+1 bits]

double-code



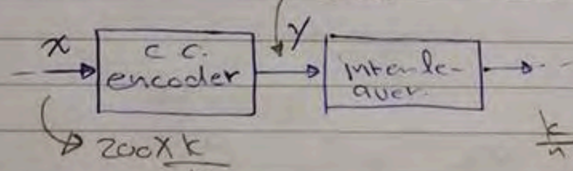
→ Ex.

If a packet of length 200 bits is to be used in transmission, find the CRC code (let C.C of 1/2 rate)

$\frac{k}{n}$

Sol: $L_p = 200$

packet of length 200 bits



$\frac{k}{n} \rightarrow$ rate

$\frac{n}{k} \rightarrow$ B.W. expansion factor.

$w \equiv$ row data $= L_p \cdot \frac{k}{n} - (m+1)$

CRC → m?

$g(x) = 1 + \dots + x^m$

$g(x)_{CRC} = g(x)(1+x) = 1 + \overbrace{\dots + x^{m+1}}^{m+1}$

$[n = 2^m - 1]$, $L_p = \frac{n}{k}(L_i + CRC)$

$200 = 2(L_i + m + 1) \rightarrow \underbrace{L_i + m}_{n} = 99 = 2^m - 1$

$2^m - 1 \geq L_i + m \geq 99$ $\boxed{CRC_{bits} = 8}$

$\Rightarrow m = 7$

$\therefore L_i = 99 - 7 = 92$ bits

$g(x) = 1 + x + x^7$ → For this code, how many error bits can we detect ? ⇒ out of 100 bits.

$g_{CRC}(x) = g(x)(1+x)$

انزل تحت

For most m we find ⇒ $g(x) = 1 + x + x^m$

or ⇒ $g(x) = 1 + x^{m-1} + x^m$

هدول الغالبا بالعموم prime polynomial الكامل

$t' = d_{min}^H = 3$

يعني ما أساسًا غير ه

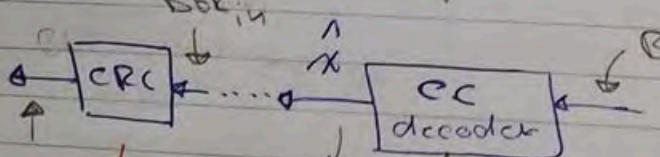
$\therefore$ t ← دايمًا 1

d ← دايمًا 3

$\xrightarrow{92}$ CRC $\xrightarrow{~~100~~}$ 100

الـ200bits الها علاقة بالـ c.c

• let $BER = 10^{-2}$, $t = 5$ (for c.c)



$BER = (10^{-2})^{t+1} = 10^{-12} = BER_{in}$

⇒ can detect $t = 3$ errors [3 every 100bits]

$\therefore$ probability of undetected errors:

$BER = (BER_{in})^{d_{min}+1} = (BER_{in})^{4}$ ⇒ بنقل احتمال وجود الـerrors

3 بقدر اكتشفهم، 4 بقلة &

الـ output ؟

$t \equiv$ # of errors that can be corrected

$t+1 \equiv$ 〃 〃 〃 detected

$d_{min} \equiv$ niether detected not corrected

we said

* Summary:

How to use CRC?

take $w(x)$, devide it by $g(x)_{CRC}$:

$$\frac{w(x)}{g(x)_{CRC}} = \text{sol}.(x) + \frac{r(x)}{g(x)_{CRC}}$$

→ [illegible]

→ max. order is m

$m+1$ bits

Packet ⇒ | $w(x)$ | $m+1$ |

for the last e.g. → $w(x) \approx 92$ bits

$m+1 = 8$ bits

the max. capability for this CRC is 119 bit → $[2^7 - 1 = 127 = 119 + 8]$

Ex:

let $g(x)_{CRC} = 1 + x^3 + x^{17}$

what's the max. data length we can use?

Sol: $m+1 = 17 \rightarrow m = 16$

$2^{16} - 1 - (m+1) = $ Max. Capability.

$= \underline{65518}$

→ we can correct 3-bits of them :\ !

→ for such amount of data, we need to increase d^{H}_{min}.

For higher d_{min} ($d_{min}^{H} > 3$) we can cascade:

$\rightarrow \boxed{CRC} \rightarrow \boxed{\text{interleaver}} \rightarrow \boxed{CRC} \rightarrow \boxed{\text{interleaver}} \rightarrow \cdots$

$g(x) = g_1(x) \cdot g_2(x) \cdots g_\ell(x)$

$g(x) = 1 + \cdots + x^{\ell \cdot m}$

↳ has $d_{min} = \underline{3} \cdot \ell \rightarrow \ell$ is a design factor

$g(x)_{CRC} = (1+x)\, g(x)$ → d_{min} of each $g_i(x)$

Ex.

let $\ell = 3$, $m = 5 \rightarrow g(x)_{CRC} = 1 + \cdots + x^{3 \times 5 + 1}$

$g(x)_{CRC} = 1 + \cdots + x^{16}$

→ additional bits to the data = 16-bits

→ max. data = $2^{m} - 1 - 16 = 2^{5} - 1 - 16 = 15$

بنعمل مرة وحدة

مش الـ cascading

So we can detect $\underset{m \times \ell}{15}$ bits out of $\underset{n}{31}$ bits

---

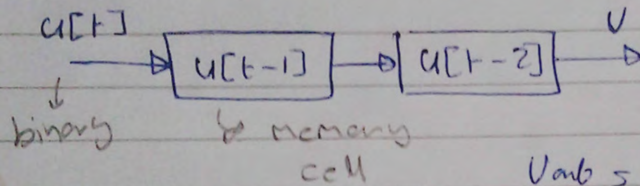
C.C :

↳ convolves the data with a certain transfer fcn.

$u \rightarrow \boxed{C.C.} \xrightarrow{v}$ → correlated

deterministic correlation

Common correlator → memory machine (state machine)
↓
linear shift Register



v → fcn. of u & the machine state (content of the memory)

$V_{out} = f(u, state)$

↳ Surely the memory machine might have more than 2-cells.

$V = f(u, state)$

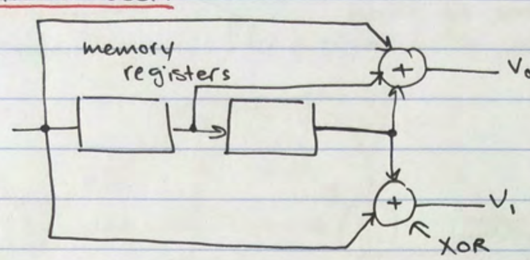
n-~~bits~~ outputs   k-~~bits~~ inputs

→ there's no relation between n & K.

# of states $= 2^c$ $c \equiv$ # of memory cells

27/5/2014

- Convolutional coder:



output $[V_0 V_1]$

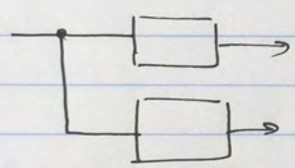
notation:

$\boxed{C.C\,(1, 2, \underset{\sim}{\ell})}$ input, output, memory length (# of memory cells).

↳ in this case = 2.

* States:- (contents of the memory).

00	S_0
01	S_1
10	S_2
11	S_3



in this design we can only have (00) or (11)

so parallelism is redundance.

≡ 1 memory cell

→ Serial shift register is the best way to represent c.c.

→ this is a digital binary filter.

↳ it represents a correlation.

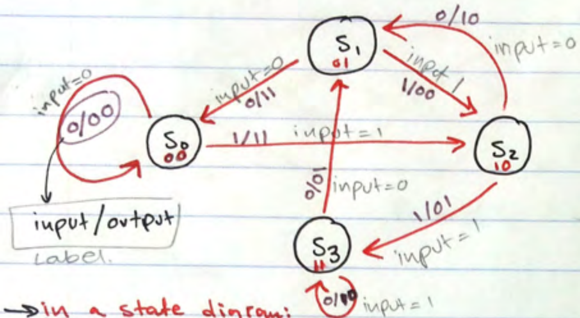
$V = f(U, \underset{\sim}{S})$

Previous inputs or (function of the previous inputs).

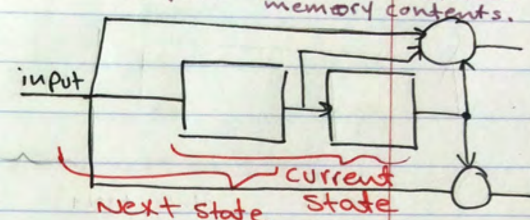
to analyze this machine:

- Graph theory

→ state diagram: (for systems with memory) easier to analyze



* state isn't the output.
output = XOR ~~twee~~ between memory contents.



→ in a state digram:

- # of possible inputs = # of emitting arrows from each state.
- nodes represent the states (memory contents)
- the arrows connecting nodes represents the transition from the current state into next states.
- transition happens when the input changes (every clock cycle).

ex: if $u = [1100101]$

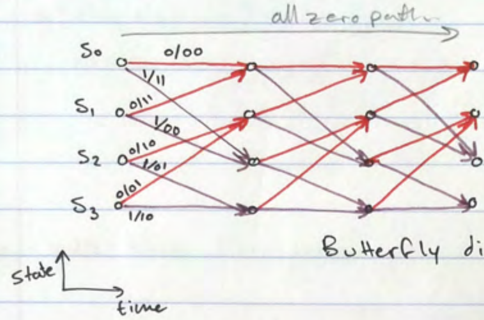
find $V = ?$

** state S_0 is our reference state (initial).

$$V = [\underbrace{11}_{S_2}\ \underbrace{01}_{S_3}\ \ 01_{S_1}\ \ \underbrace{11}_{S_0}\ \underbrace{11}_{S_2}\ \underbrace{10}_{S_1}\ \underbrace{00}_{S_2}]$$

S_0 I.C (initial)

- **Tree diagram:**



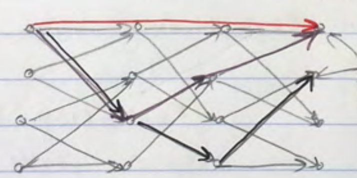
Butterfly diagram

output & input are represented as a Path.
as if we are sending a path. (its length increases with time).

* there are (2^7) possible paths of length 7.

~~* error in a seq~~ .

* error in recieved signal will block the path, thus error will be detected.
* minimum error event / the min. # of errors such that we can't detect them.
  let the all-zero path be the reference



possible paths.

min error event = 11 10 11

$d_{free}^{min} = 5$
the sum of ones in the min error event

$t = \left\lfloor \frac{d_{free}^{min} - 1}{2} \right\rfloor$

3 transmitions
6 bits at the output , 3 bits input
in every 6 bits we can correct 2 bits.

when # of bits $< d_{free} - 1$ it will always detect the error.

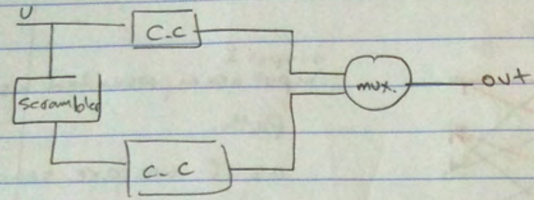
* State diagram depends only on the connections of memory. (# of memory cells) what changes is input/output relation.

* حكم بزيد عدد ال registers أو عدد ال outputs

* good C.C depends on the connection Polynomial.
* performance of the code depends on d_{free}

* to $\uparrow d_{free}$, change the code rate K/n ($\downarrow \frac{K}{n}$, $\uparrow d_{free}$) , increase memory length (# of states)

• turbo code:



to end at zero state in the end, I end the data (input) with (00)