

Communications I

Notes

DR. Mohammad Hawa
By: Saeed Mustafa



$m(t)$ is the baseband message signal

↳ a voltage or current or EM ... that represents the information.

① "Impairments: problems, limitations, ... noise, etc"

* Attenuation :- As the signal travels through the channel it loses some of its energy (power) as heat in the internal resistance of the channel. We say the signal was attenuated by the channel.

Attenuation \equiv power loss

eg $m(t) = 5V$ (a) $m'(t) = 4.5V$

(b) $m'(t) = 3V$ more loss or more attenuation

① Attenuation exists for all signal types

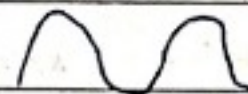
② " " " " " channel types but the level of attenuation depends on the channel type.

③ Attenuation increases as the length of the channel increases.

f_1 Hz

$f_2 > f_1$

$2\pi f_1$ rad/s



f_2

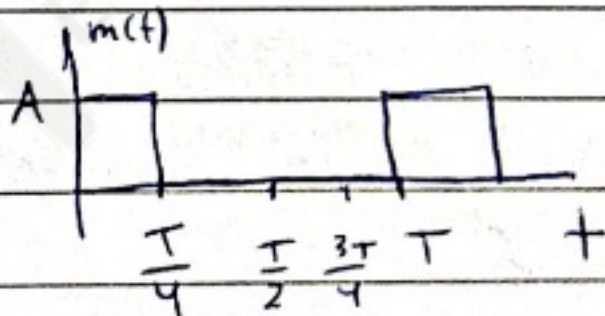
f_1

$P_{ave} = \frac{A^2}{2}$ $(V_{rms})^2 = \left(\frac{A}{\sqrt{2}}\right)^2$ for $A \cos(2\pi f t)$

$P_{ave} = V_{rms}^2$ we assumed that R is 1Ω

$R \rightarrow 1 \Omega$

HW)



$P_{ave} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_T m^2(t) dt$

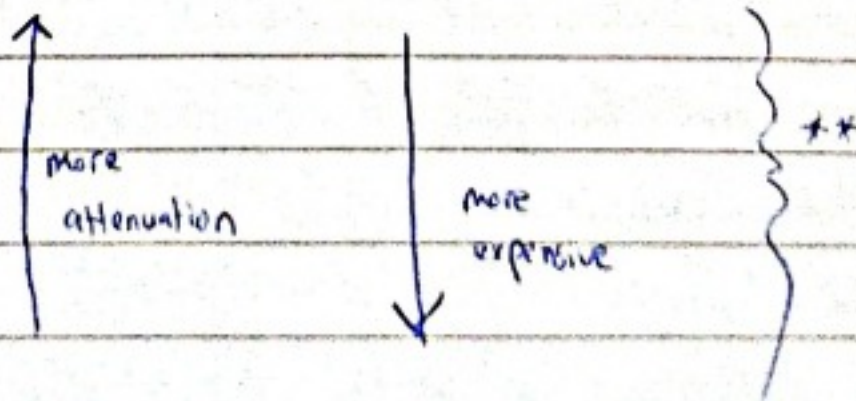
channels

* wireless

* copper wire

* Coaxial cable

* Optical fiber = wave guide



ch (a) $0.9/km$

ch (b) $0.8/km \rightarrow$ more attenuation.

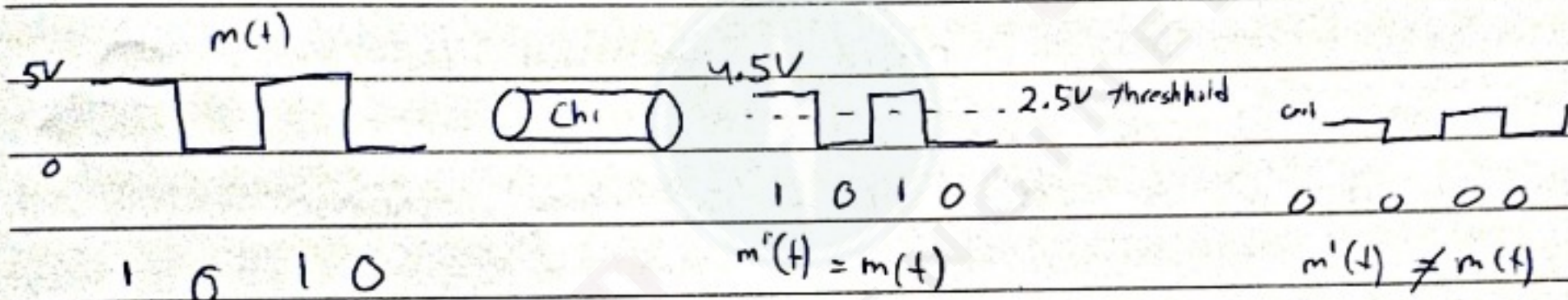
HW ch 10km calculate g

* If we use copper wires you need an amplifier every 5-10km

* If we use optical fibers you need an amplifier every 50-100km

Lecture 2

Tx



② Linear distortion

* The channel attenuation changes according to the transmitted signal frequency.

Usually, higher frequency is attenuated more. This means that the channel acts as a LPF with finite channel bandwidth, that blocks high frequencies, thus distorting the signal.

$$Z_R = R' \quad Z_C = \frac{1}{j\omega C} \quad Z_L = j\omega L$$

low freq $\rightarrow Z_C = \infty =$ open circuit

hi freq $\rightarrow Z_C = 0 =$ S.C. $m'(t) = 0$ heavy attenuation.

$H(f)$: ^{response} frequency transfer function

$H(\omega)$: means attenuation

For -ve DB curve

(lower att) red colour

ex) which channel has more attenuation coax 30m, or copper wire 30m

ex) which has more att coax 30m or coax 300m "lower curve"

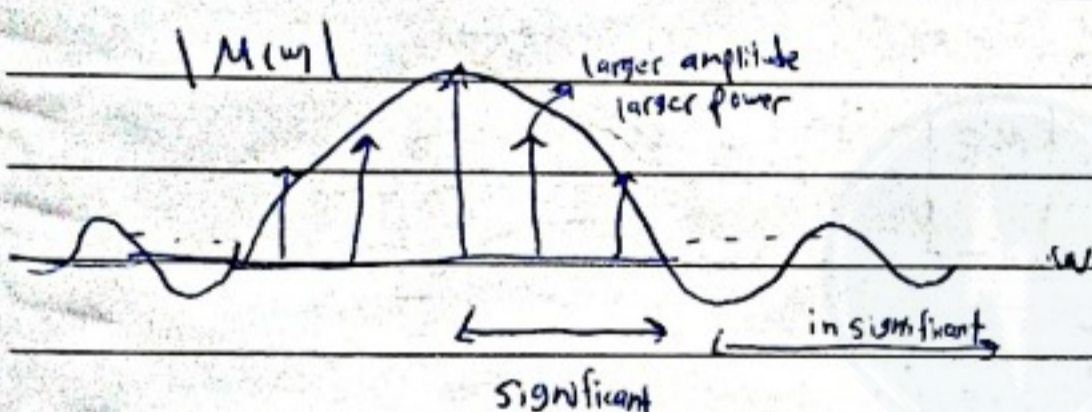
For the dB curves in the next slide
the higher curve has more attenuation.

Lecture 3

- Hz \rightarrow $B_{\text{channel}} =$
- ① It is a property of the channel itself
 - ② You read it from the datasheet of the channel.
 - ③ It is the frequency after which the channel presents heavy attenuation.

$$BW_{\text{channel}} = \frac{\text{rad}}{s} = 2\pi B_{\text{channel}}$$

* $B_{m(t)} =$ bandwidth of the message. "B" is used in Hz



- $B =$
- ① It is a property of the message not the channel.
 - ② You figure it out from $F\{m(t)\} = M(w)$
 - ③ The frequency after which $M(w)$ has insignificant components.

* Bandwidth of copper ch (1km) is 1-2 MHz

* Bandwidth of Coax (1km) is 1-2 GHz

ex) LPF $B_{\text{channel}} = 1.5 \text{ MHz}$ ① Bigger $B_{\text{channel}} = 1-2 \text{ GHz}$
 $B = 3 \text{ MHz}$ ② OR compress the frequency by expanding in time domain
 "Slower data rate"

* An equalizer is a complex expensive device. It works by opposing the effect of the channel.

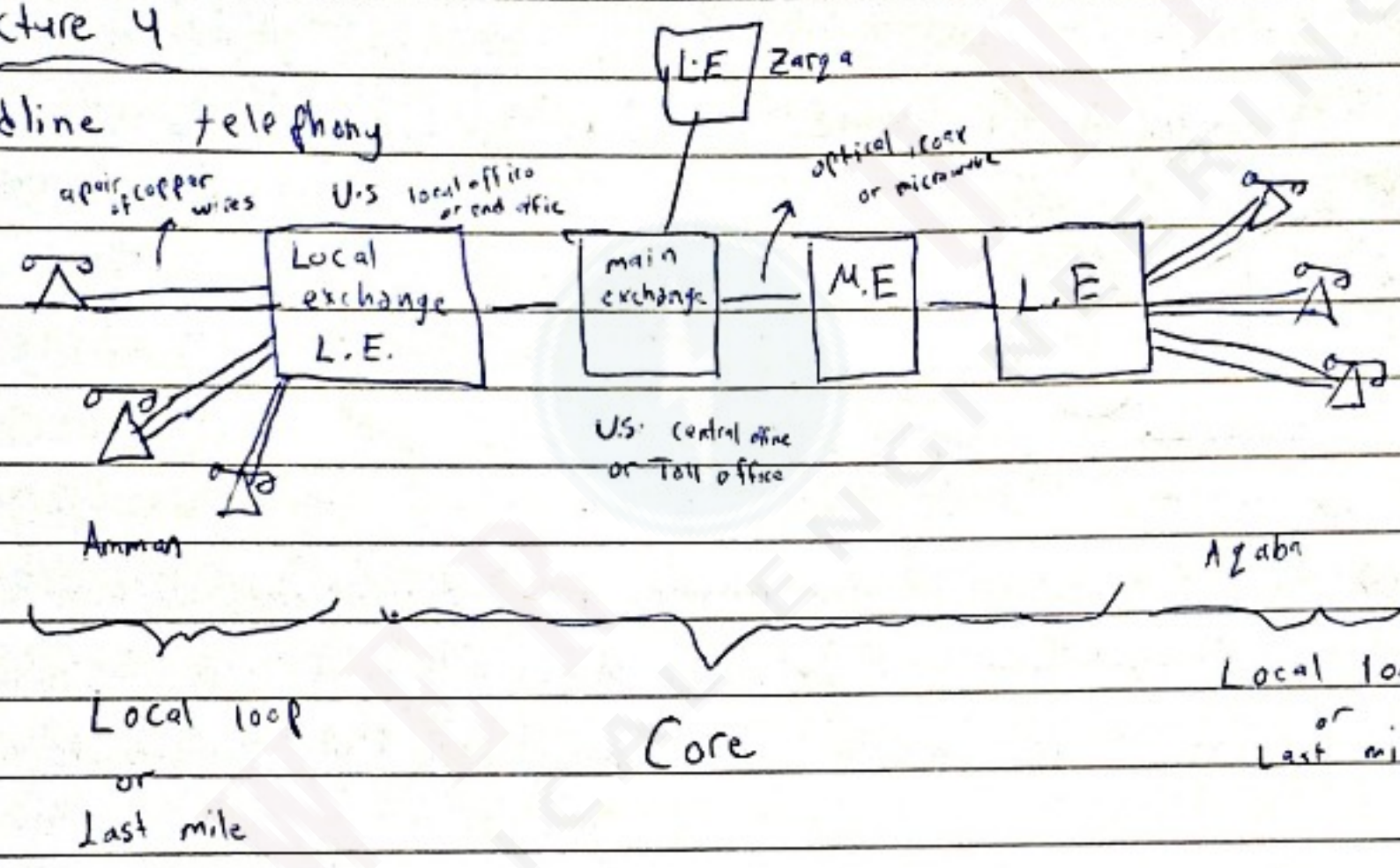
	Channel	Equalizer
low freq	less att	low gain amp
high freq	more att	high gain amplifier

③ Non-linear distortion :- The channel attenuation changes according to the transmitted signal amplitude and/or the transmitted signal frequency. Usually, higher amplitudes are attenuated more. This distorts the signal.

④ Noise :- all undesired signals that are not part of the transmitted signal which gets added by the channel. Noise is non-deterministic (random) and is generated by external and internal sources.

Lecture 4

Landline telephony



PSTN
public switched telephony network

POTS
plain-old telephony service

* Solutions for external noise :-

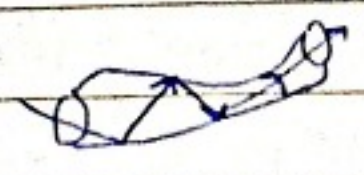
1) Shielding :- Very effective ~~and~~ but very expensive

STP :- shielding twisted pair.

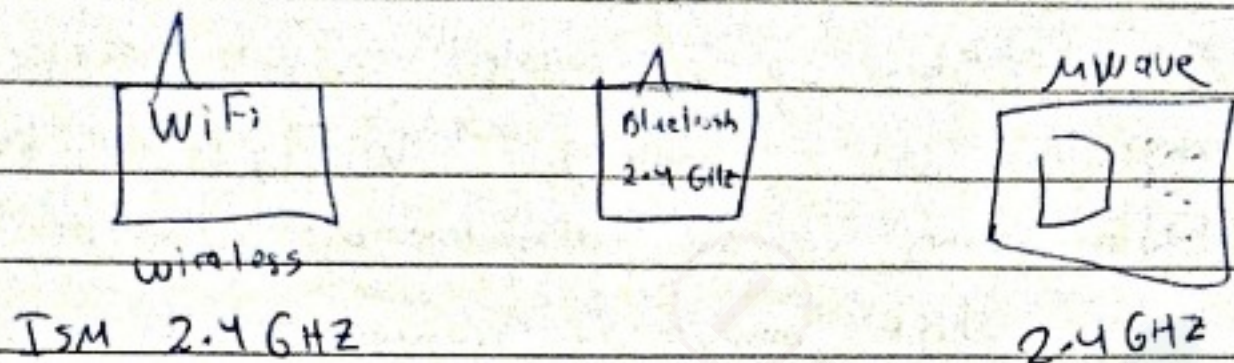
2) Twisting :- less effective but much cheaper

UTP :- unshielded twisted pair

3) Star design
different cable distances



9) Paper data

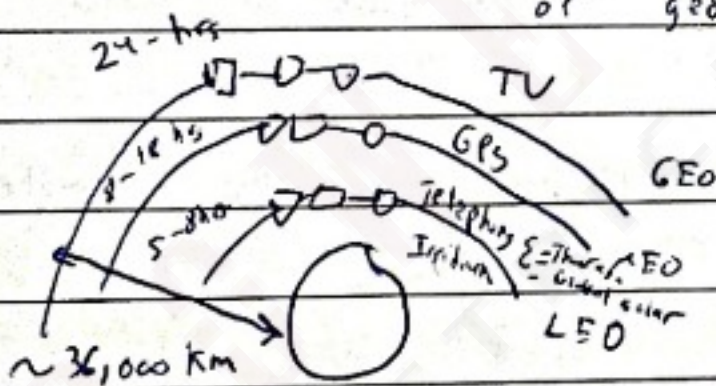


lecture 5

HW Find 3 ISM bandwidths "ex 2.4 GHz"

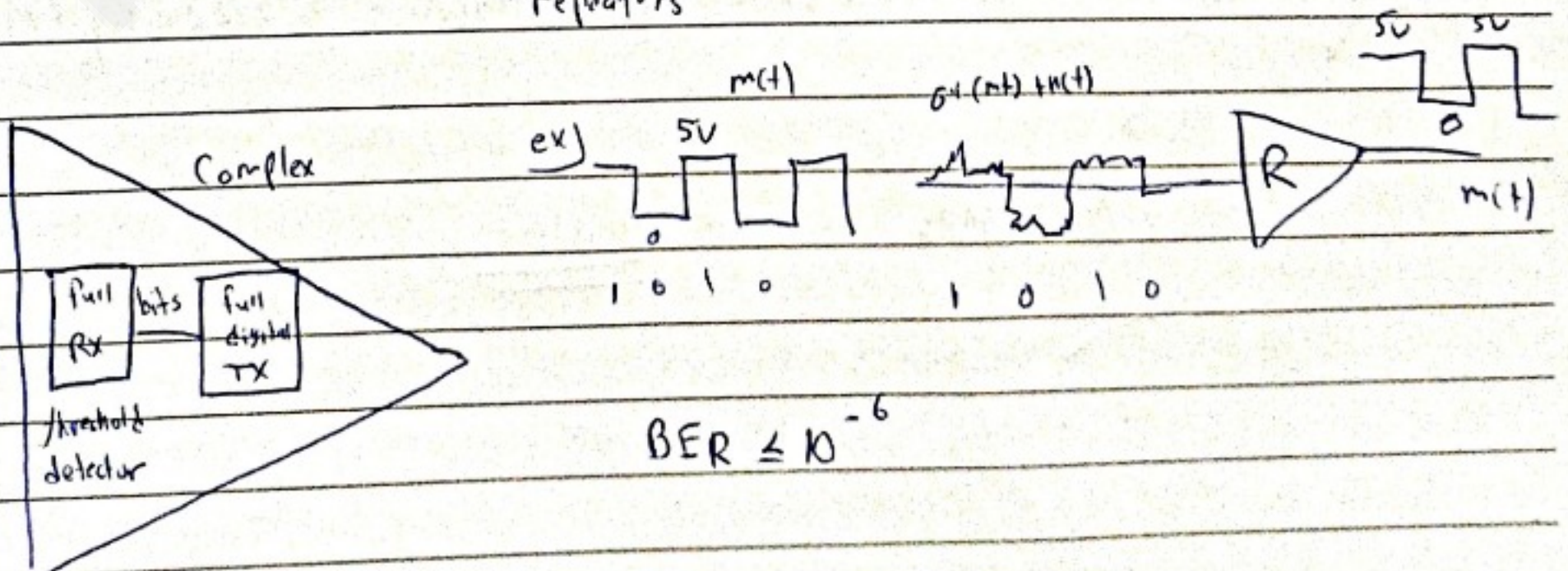
* Cooling :- $T \downarrow$ thermal energy \downarrow random motion of e^-
 very effective but very expensive

- * Satellite : LEO : low-earth orbit
- MEO : medium-earth orbit
- GEO : geosynchronous earth orbit or geostationary earth orbit



LNA: low-noise amplifier
 LNB: low-noise block.

Digital (trying to fix both noise and attenuation)
 \Rightarrow allows also regenerators to replace amplifiers
 'regenerators'



Other channel impairments

5. Fading: Shows up in wireless channels because of multipath propagation obstacles and shadowing. "WiFi, cellular, bluetooth, satellite"

Fading means variable attenuation with time or receiver position.

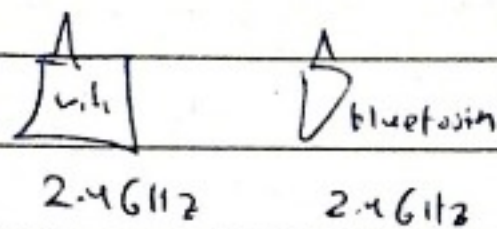
6. Doppler shift: means a frequency shift in the received signal.

Shows up in wireless channels where we have fast movement.

* satellite, cellular

7. Frequency reuse interference: we see when frequency is re-used

Looks similar to external noise



wifi, bluetooth, and cellular.

8. Chromatic dispersion: Shows up in optical fiber, very similar to non-linear distortion.

Shannon's limit

C : maximum # of bits (information you can send over a channel) with reasonable BER

1 - Attenuation

2 - Noise

3 - Linear distortion. → Bch "limited"

$$C = B \text{ch} \times \log_2 (1 + \text{SNR})$$

↓ unitless

signal power

noise power

SNR = signal to noise ratio

~~log₁₀~~

log₂

$$\log_2 (x) = \log_2 (10) \log_{10} (x) = 3.3222 \times \log_{10} (x)$$

e.g. SNR = 30dB = 1000 unitless

lecture 6

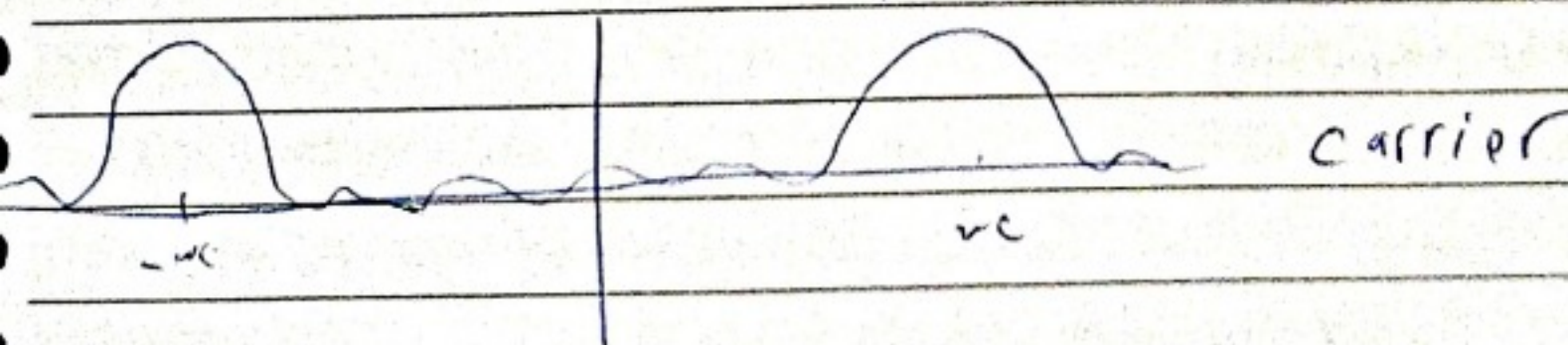
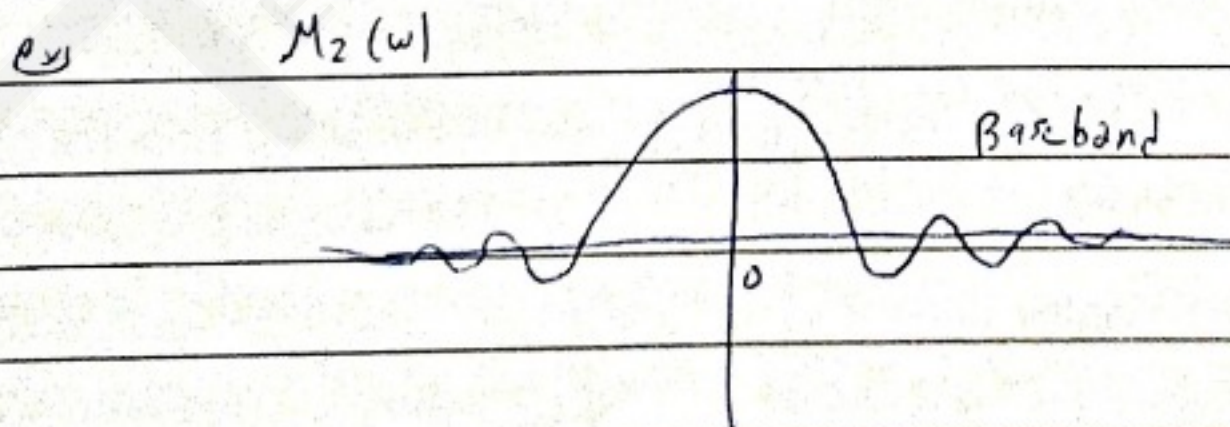
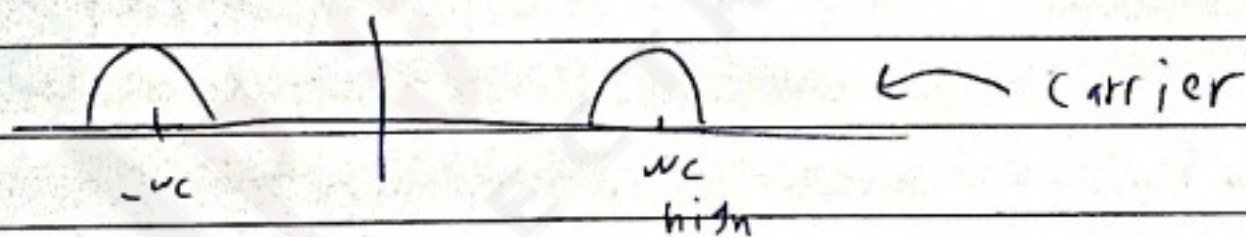
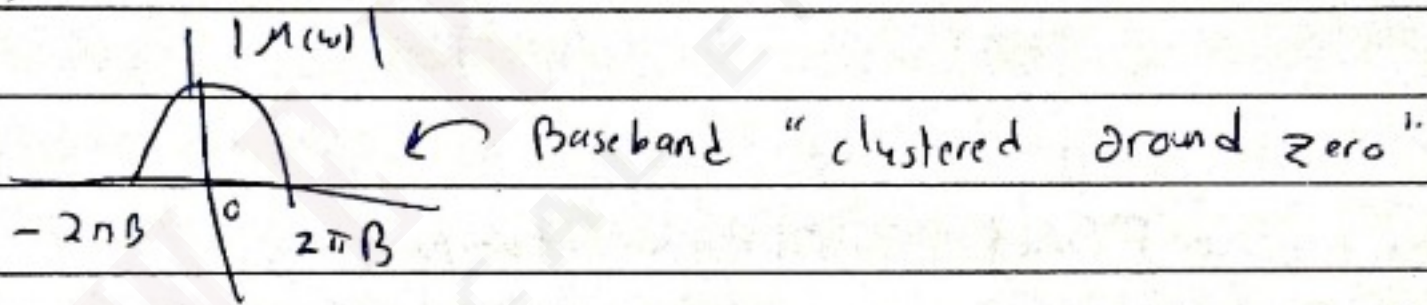
Analog baseband \rightarrow Digital carrier

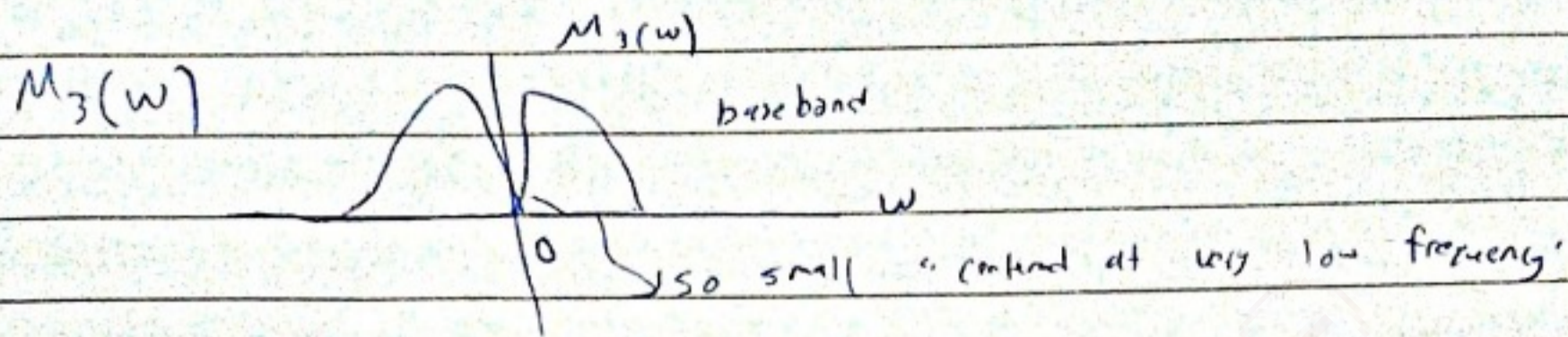
Digitization followed by Modulation.

Analog baseband systems

if $m(t)$ is continuous in voltage and time
if $m(t)$ is digital then it can assume only a finite set of voltages or shapes. } time domain

Baseband vs carrier } Frequency domain
 $F\{m(t)\} = M(\omega) \ll M(\omega)$

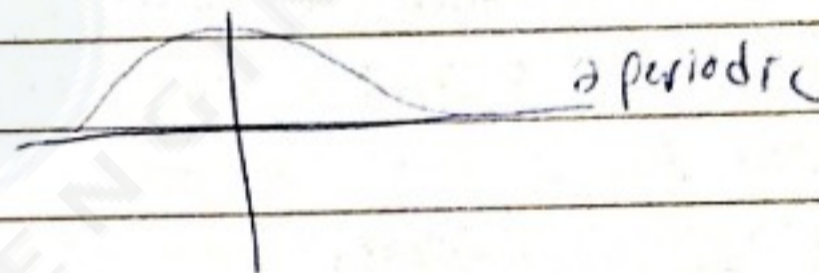
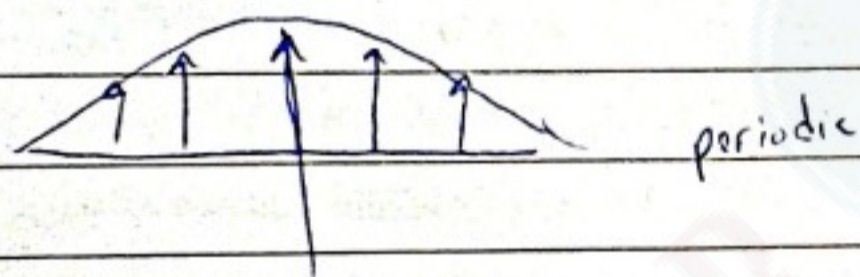
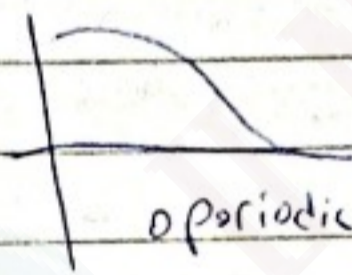
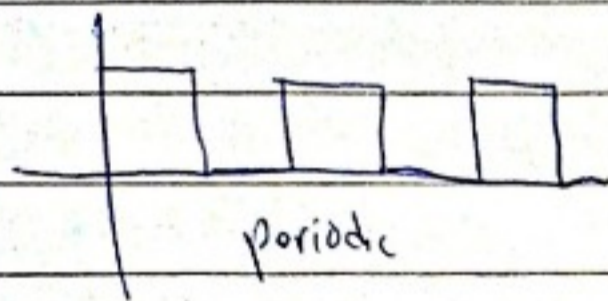




periodic / aperiodic \rightsquigarrow both

analog / Digital \rightsquigarrow t-domain

baseband / carrier \rightsquigarrow f-domain

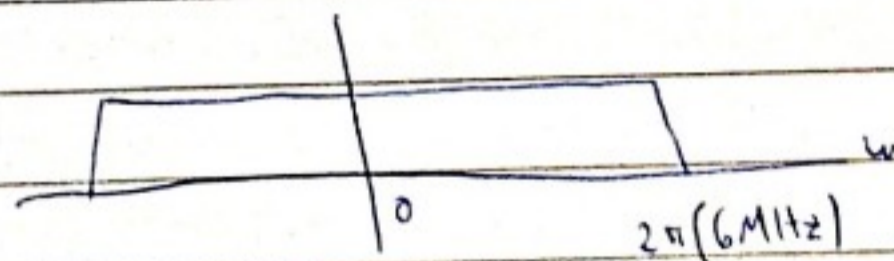


examples of analog baseband signals:

- ① human voice [voice]
- ② light [video]
- ③ Room temperature

$$B_{\text{voice}} = 4 \text{ kHz}$$

$M(w)$ PAL

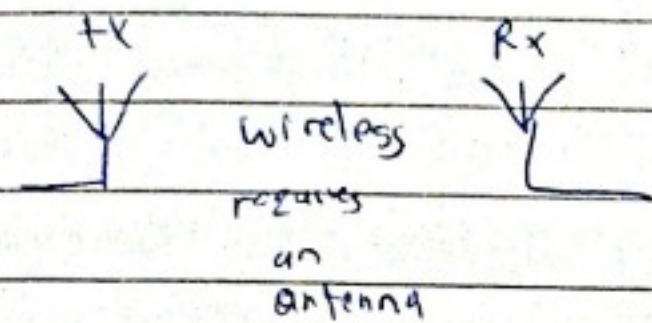


example of analog baseband systems

cheap phones

history + backwards compatibility.

Analog and digital carrier systems

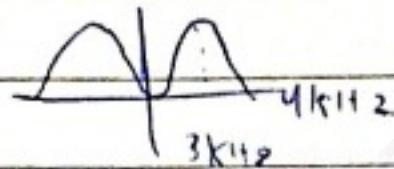


$$L = \frac{\lambda}{2}$$

$\lambda =$ wavelength of the signal

$$\lambda = \frac{c}{f} \approx \frac{300,000 \text{ km}}{f}$$

ex1 $m(t) = \text{voice}$



$$L_{\text{antenna}} = \frac{\lambda}{2} = \frac{1}{2} \frac{c}{f}$$

$$= \frac{1}{2} \frac{300,000 \text{ km/s}}{3 \text{ kHz}} = \frac{100 \text{ km}}{2} = 50 \text{ km}$$

ex2 $m(t) = \text{wifi}$ 3 GHz

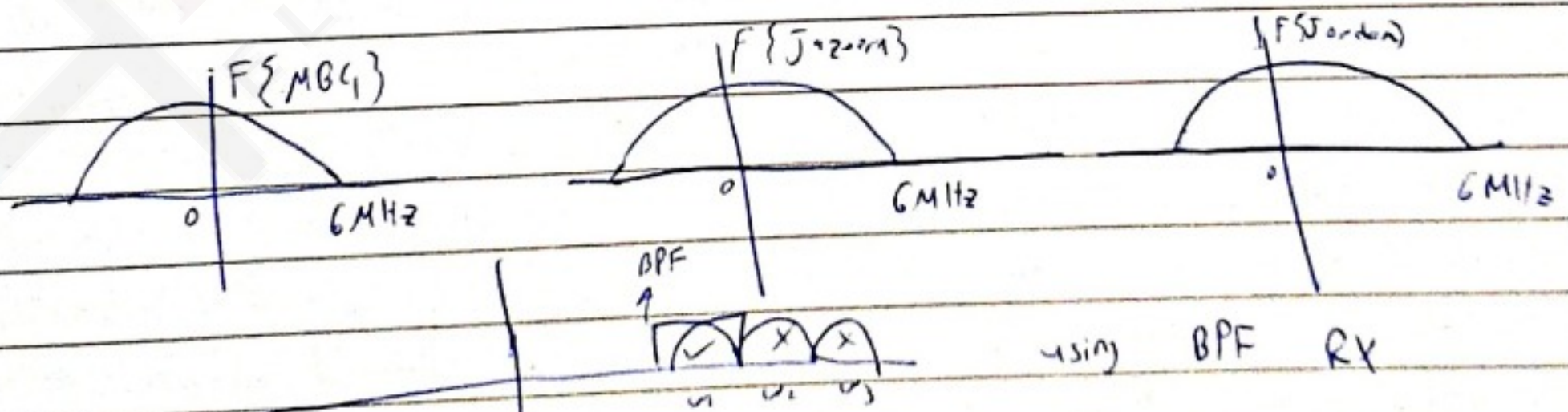
$$L_{\text{ant}} = \frac{1}{2} \lambda = \frac{1}{2} \frac{c}{f} = \frac{1}{2} \frac{300,000 \text{ km/s}}{3 \times 10^9} = \frac{0.1 \text{ m}}{2} = 5 \text{ cm}$$

H.W] wifi

Lecture 7

Multiplexing = sending multiple signals over one channel.

FDM = frequency division multiplexing (enabled by modulation)



- CDMA = code-division multiple access (3G)
 - OFDMA = Orthogonal frequency-division multiple access (4G)
- you need modulation and digital "digital carrier"

examples of carrier systems:

AM: Amplitude Modulation

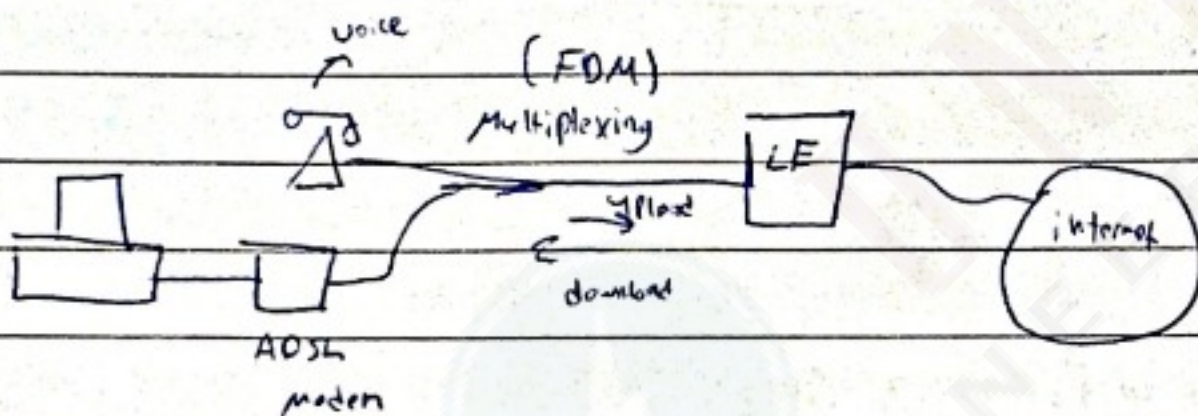
FM: Frequency Modulation

Digital carriers

* Digital radio ~~broadcast~~ broadcasting (DAB) digital audio broadcasting
± H.W " DVB-S, DVB-T, ATSC)

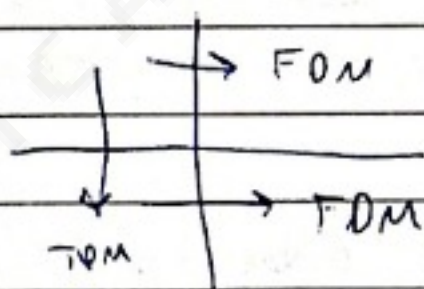
* Cellular 2G/GSM
3G

4G/Long term evolution "LTE"



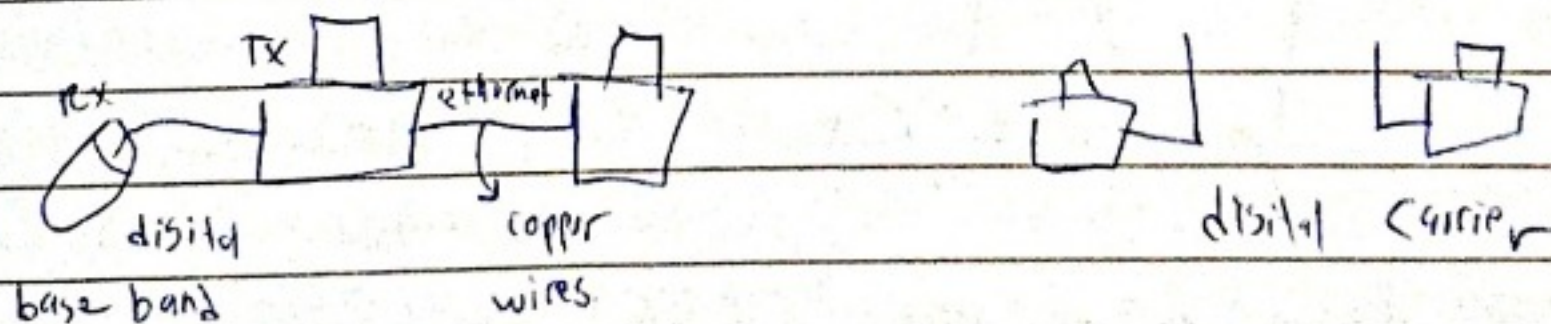
Digitization

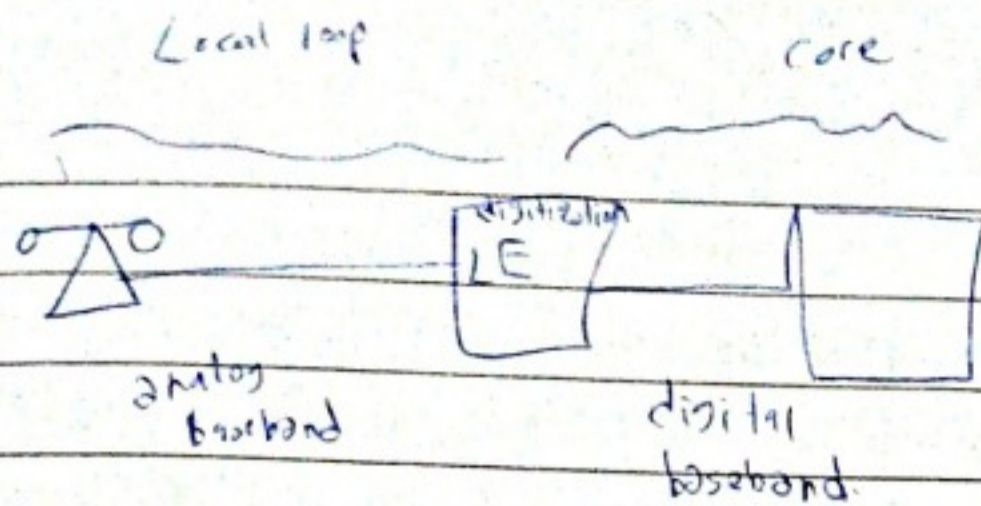
TDM = time-division multiplexing "enabled by digitization"



A/D converter only 3 steps - sampling
Quantization
mapping → 16-bit
bit stream

Digital base band examples





$$\frac{A \cos \omega t}{e^{j\omega t}} = 1$$

PDH \equiv Plesiochronous Digital Hierarchy
 SDH \equiv Synchronous = =

$x(t)$: analog baseband

info $\xrightarrow{\text{microphone}}$ $x(t)$ "transducer"
 voice

lecture 8 H.W for the following signals:

$a_n \equiv$ Fourier series coefficients

- (1) Find Fourier series (complex form)
- (2) Find $=$ (trigonometric form)
- (3) $=$ (compact form)
- (4) Find Bandwidth of $x(t)$
- (5) Find the average of $x(t) = \text{DC value}$
- (6) Find the average power of $x(t) = P_{ave}$
- (7) Find the Rms of $x(t) = \sqrt{P_{ave}}$

Answers for DC = $\frac{Az}{T}$ (3) 0

(4) $\frac{Az}{T}$ (5) 0 (6) 0

(1) $x(t) = \text{rep}_T \left\{ A \cos \left(\frac{t}{T} \right) \right\}$



$$a_n = \frac{1}{T} \int_T x(t) e^{-jn\omega t} dt = \frac{Az}{T} \sin \left(\frac{n\omega T}{2\pi} \right)$$

lecture 9

DC value (average value) for $x(t) = a_0$

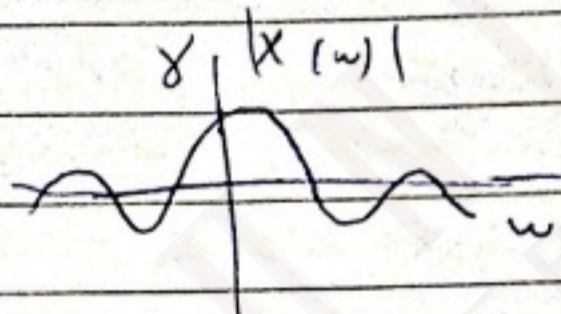
$$F\{\cos(\omega t)\} = \pi \delta(\omega - \omega_0) + \pi \delta(\omega + \omega_0)$$

$$F\{1\} = 2\pi \delta(\omega)$$

$$F\{a_0\} = 2\pi a_0 \delta(\omega)$$

↑ DC value

$$F\{\text{rect}(t)\} = \text{sinc}\left(\frac{\omega}{2\pi}\right)$$



DC value = 0 "no impulse at zero"

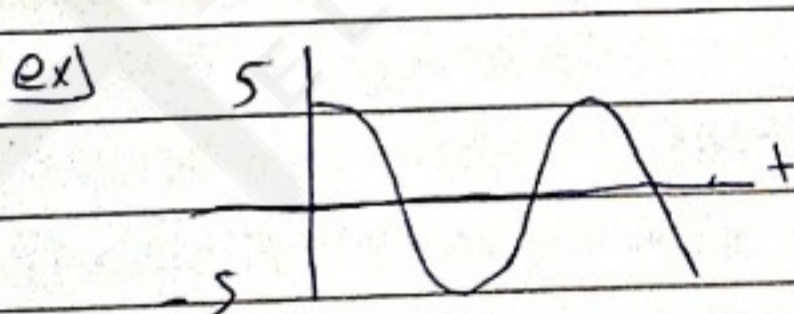
$$F\{\text{rect}(t) + 5\} = \text{sinc}\left(\frac{\omega}{2\pi}\right) + 2\pi 5\delta(\omega)$$

DC \equiv average of the signal $x(t)$

Average power \equiv average of instantaneous power.

$$\overline{x(t)} \begin{cases} \text{f-domain (impulse at zero)} \\ \text{t-domain } \left(\lim_{T \rightarrow \infty} \frac{1}{T} \int_T x(t) dt \right) \end{cases}$$

$$P_{\text{ave}} = \overline{x^2(t)} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_T x^2(t) dt$$

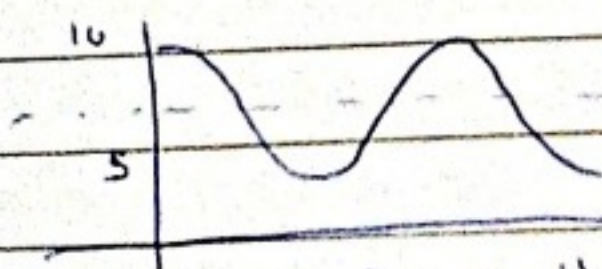


$$5 \cos(\omega t)$$

$$DC = 0 \quad (\text{area} = 0)$$

$$P_{\text{ave}} = \overline{x^2(t)} = \frac{5^2}{2} = \frac{A^2}{2} = (r_{\text{ms}})^2$$

ex 2)



$$DC = 7.5$$

$$P_{\text{ave}} = 7.5^2 + \frac{2.5^2}{2}$$

$$2.5 \cos(\omega t) + 7.5$$

Power from f-domain PSD "Power spectrum density"

$$PSD = S_x(\omega) = \lim_{T \rightarrow \infty} \frac{|X(\omega)|^2}{T}$$

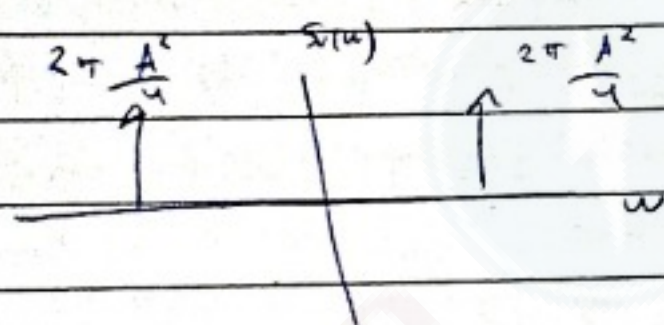
$$S_x(\omega) = F\{R_{xx}(t)\}$$

$$P_{ave} = \overline{x^2(t)} = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_x(\omega) d\omega$$

ex 1) Find PSD for $x(t) = A \cos(\omega_0 t)$

$$R_{xx}(\tau) = \langle \cos(\dots) \rangle$$

$$S_x(\omega) = F\{R_{xx}(\tau)\} = \text{two impulses}$$



$$P_{ave} = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_x(\omega) d\omega$$

$$\frac{1}{2\pi} \left(\frac{2\pi A^2}{4} + \frac{2\pi A^2}{4} \right) = \left(\frac{4\pi A^2}{4} \right) \frac{1}{2\pi}$$

$$= \frac{A^2}{2} \checkmark$$

H.W) Find PSD and average power for $x(t) = \text{rep}_T\{A \text{ rect}(t)\}$

Freq Response Function

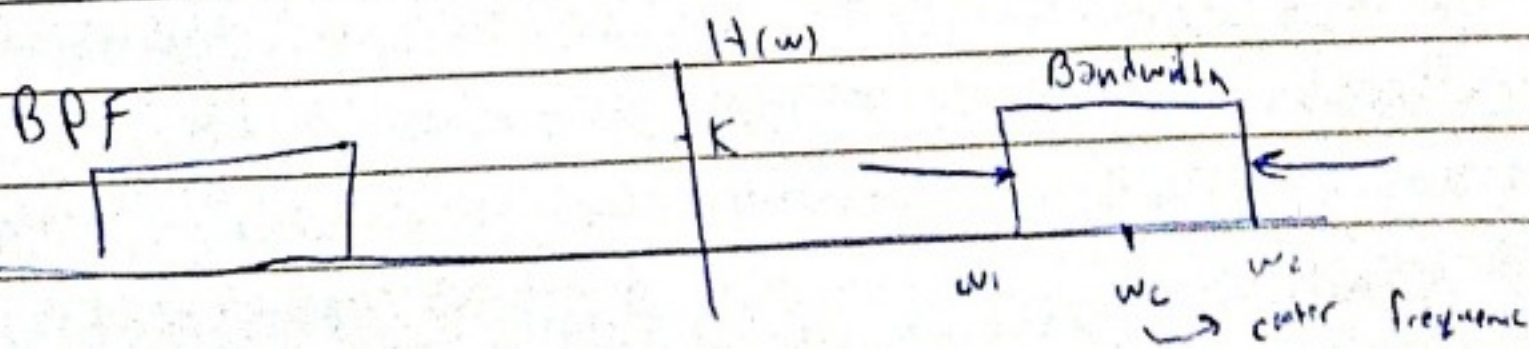
$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = \frac{F\{v_o(t)\}}{F\{v_i(t)\}}$$

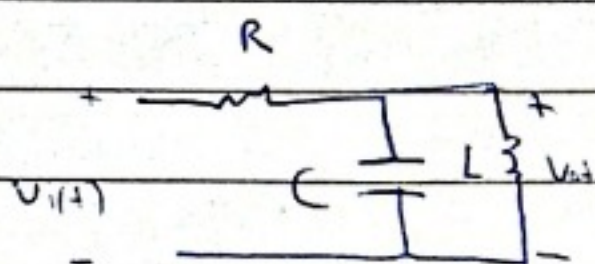
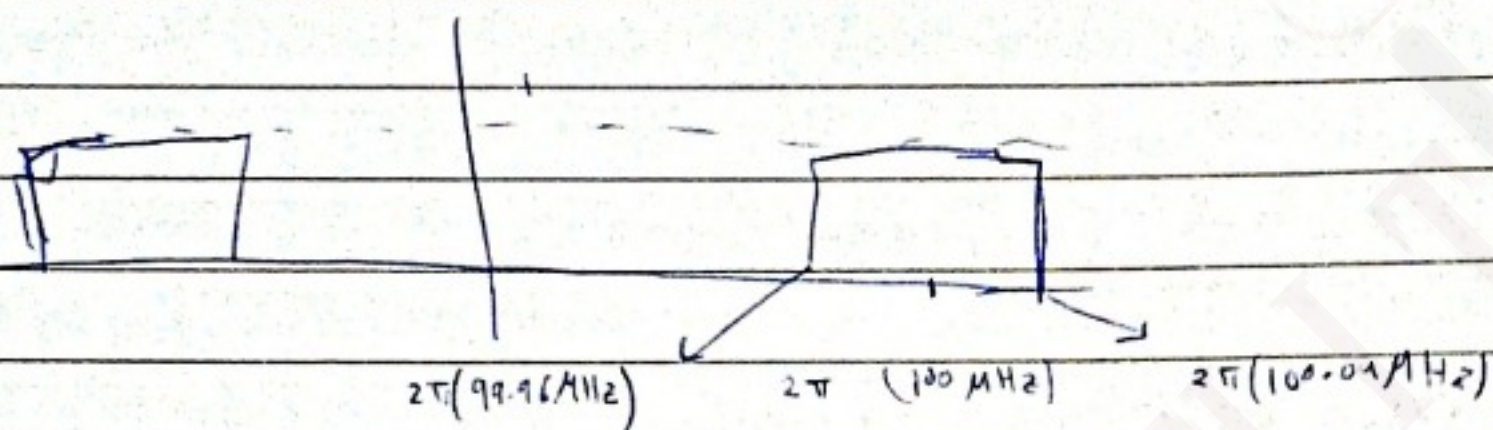
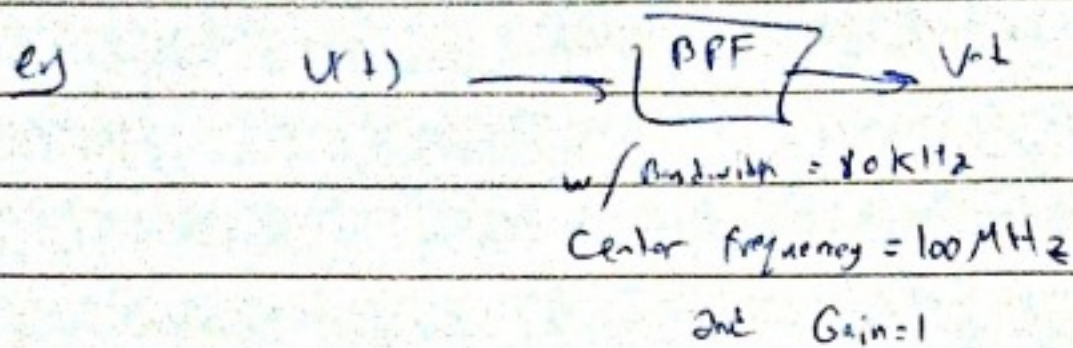
$$V_o(\omega) = H(\omega) V_i(\omega)$$

For bandwidth calculation, do not consider neg power

Lecture 10

F → low	C → open circuit	$V_o = V_i$	} LPF
F → hi	C → short	$V_o = 0$	





$$f_c = f_{res} = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

$$B_{BPF} = \Delta f = \frac{R}{2\pi L} \text{ Hz}$$

$$\text{Gain} = 1$$

Modulation: is the process of shifting frequency for a signal (from baseband \rightarrow carrier)

Modulation is performed by changing one or more of the basic parameters of a high-frequency periodic signal (called the carrier $c(t)$) in proportion to the baseband message $m(t)$.

$$c(t) = A \cos(\omega_c t + \phi_0)$$

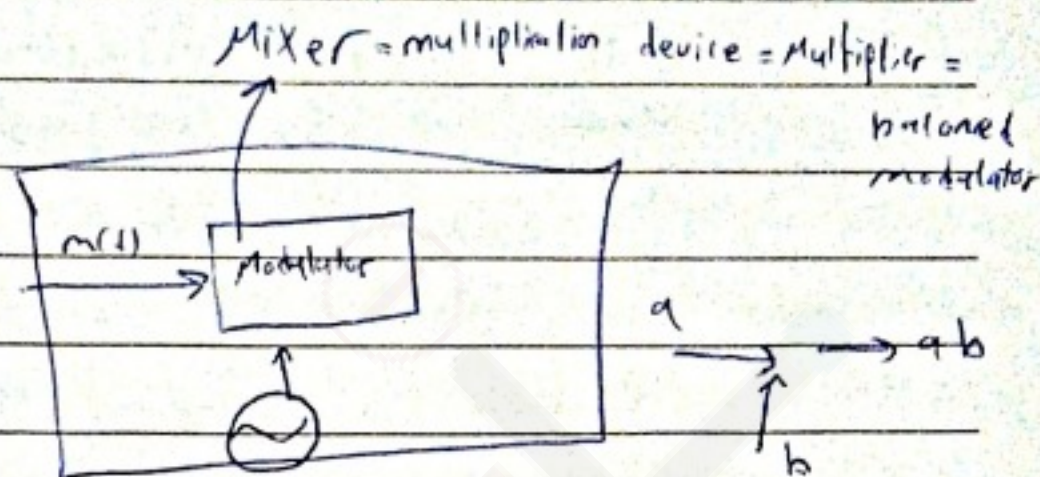
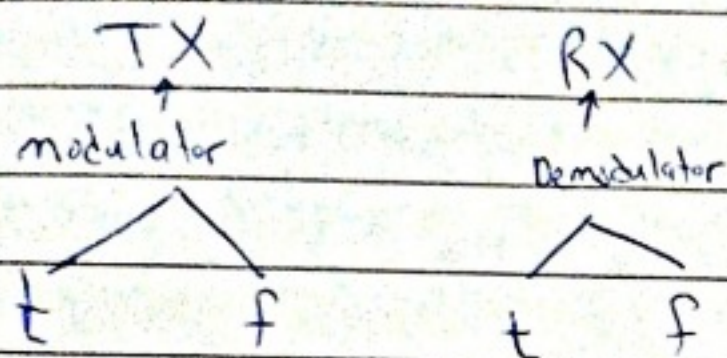
Labels: A (amplitude), ω_c (frequency), ϕ_0 (phase)

$A \propto m(t)$

* Amplitude modulation:

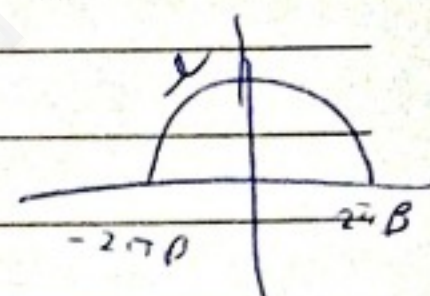
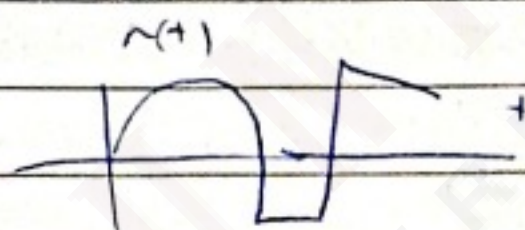
- (a) DSB-SC
- (b) DSB-LC (low carrier) [AM Radio]
- (c) SSB-SC
- (d) SSB-LC
- (e) VSB-SC
- (f) VSB-LC
- (g) QAM

DSB-SC



$$\phi = m(t) \cos(\omega_c t) = m(t) c(t)$$

DSB-SC



bandwidth of $m(t) = 2\pi B$ rad/s or B Hz

DC value of $m(t) = 0$ "no impulse"

Find and sketch $F\{\phi(t)\} = \Phi(\omega)$
DSB-SC DSB-SC

$$F\{x(t) \cdot y(t)\} = \frac{1}{2\pi} X(\omega) \otimes Y(\omega)$$

$$F\{c(t)\} = F\{\cos(\omega_c t)\} = \pi \delta(\omega - \omega_c) + \pi \delta(\omega + \omega_c)$$

\otimes Anything $\otimes \int(\omega \pm \omega_c)$

Anything shifted by ω_c * by its ~~freq~~ freq.

Sol #2 $F\{\phi(t)\} = F\{m(t) \cdot \cos(\omega_c t)\} = F\left\{ \frac{m(t)e^{j\omega_c t}}{2} + \frac{m(t)e^{-j\omega_c t}}{2} \right\}$
DSB-SC

$$= \frac{M(\omega - \omega_c)}{2} + \frac{M(\omega + \omega_c)}{2}$$

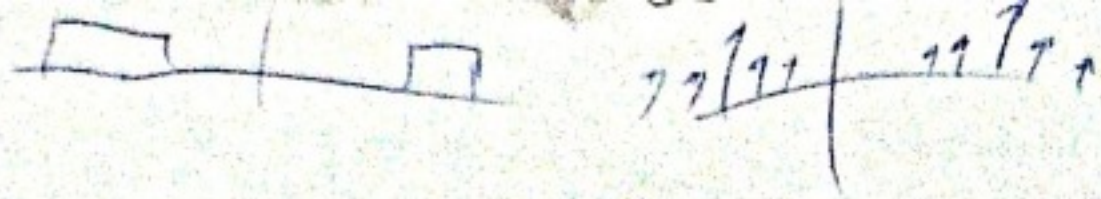
Sol #3 "graphical solution"

When multiplying by $\cos(\omega_c t)$

a) shift to right by ω_c

b) shift to left by ω_c

c) multiply by $1/2$



For $\phi(t)$ case
 a - The bandwidth is double that of $m(t)$
 b - The avg power is half that of $m(t)$.

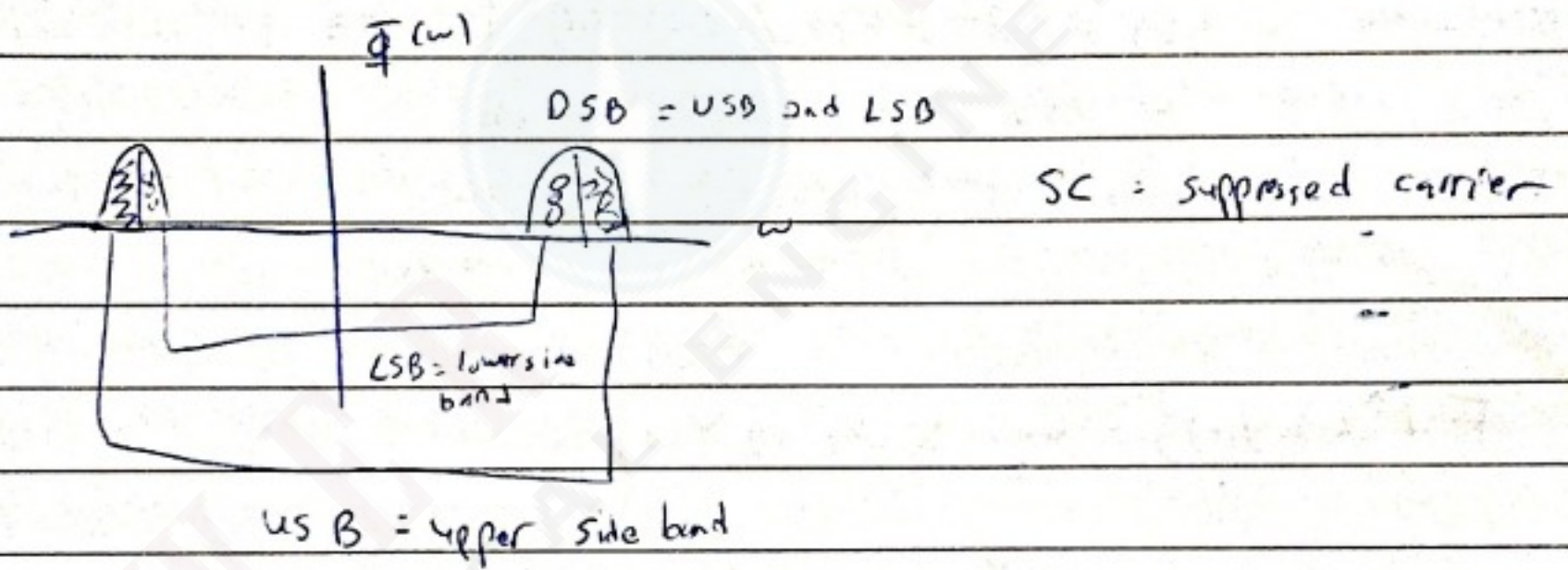
Bandwidth of $m(t) = 2\pi B$ rad/s
 or B Hz

Bandwidth of $\phi(t) = 4\pi B$ rad/s or $2B$ Hz

HW) assume $m(t) = a \cos(\omega t)$

(a) Find $\overline{m^2(t)}$ = avg power = $\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T m^2(t) dt = \frac{a^2}{2}$

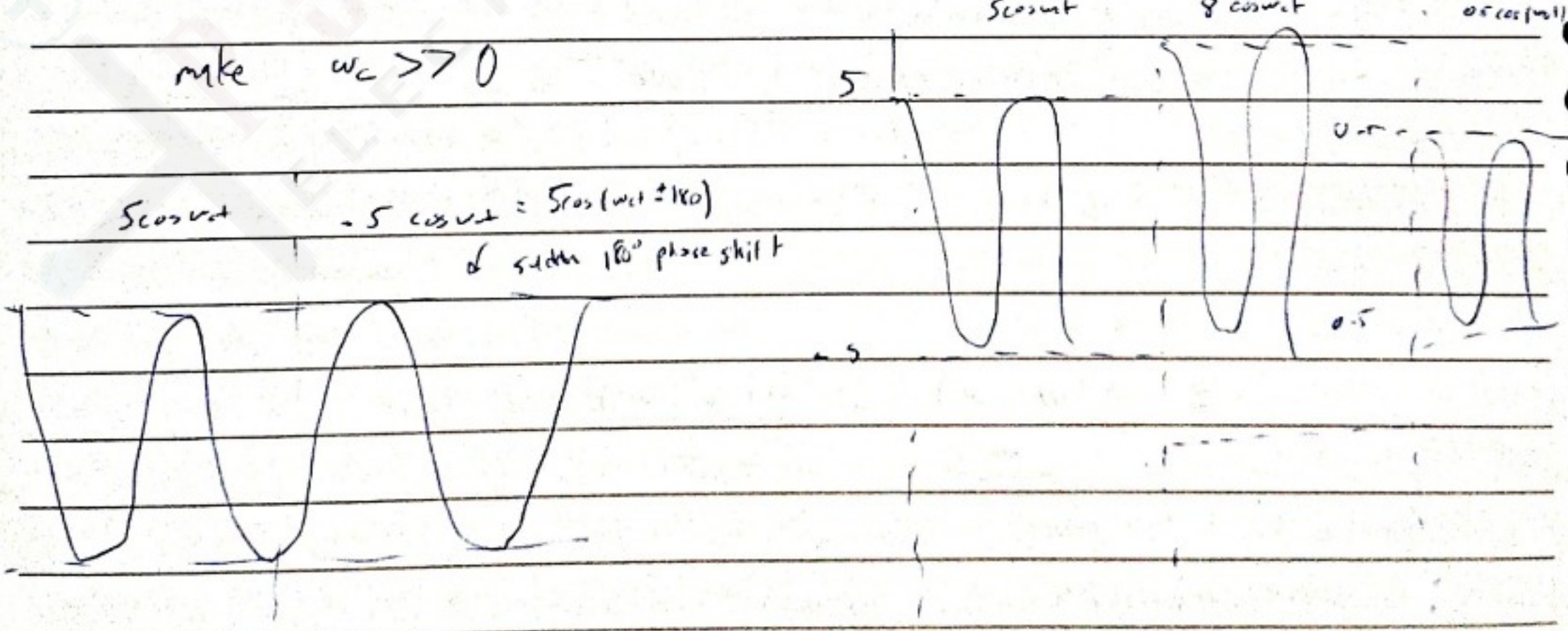
(b) Find $\overline{\phi^2(t)}$ = avg power of modulated signal = $\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \phi^2(t) dt = \frac{a^2}{4}$



Sketch $\phi(t)$ for $m(t) = \text{rect}\{t\}$ $\rightarrow \times \times \times$

make $\omega_c \gg 0$

$5 \cos \omega t = 5 \cos(\omega t + 180^\circ)$
 of sudden 180° phase shift



1

$\phi(t) = m(t) \cos(\omega_c t)$ "demodulation"

Q. Find the output $y(t)$ from the circuit shown & the input is $\phi(t)$

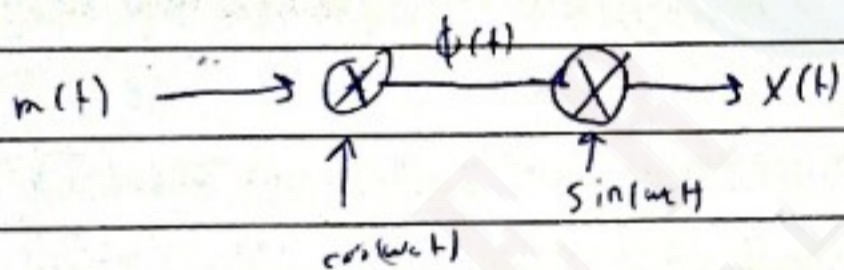
The oscillator at the RX should be fully synchronized with the oscillator at TX [Same freq.] "Same phase"

Multiplying by $\cos(\omega_c t)$

1. Shift the whole signal to the right by ω_c
2. --- -- -- -- --
3. multiply by $1/2$

Q. How many copies of $M(\omega)$ does $X(\omega)$ contain? 3 Not 4

HW: Find $X(\omega)$ in the following



Draw the phase spectrum $X(\omega)$

$$\begin{aligned} \text{Find } X(\omega) &= F\{x(t)\} = F\{\phi(t) \cdot \cos(\omega_c t)\} = F\{m(t) \cdot \cos(\omega_c t)\} \\ &= F\left\{\frac{1}{2}m(t) + \frac{1}{2}m(t) \cos(2\omega_c t)\right\} = F\left\{\frac{1}{2}m(t) + \frac{1}{4}m(t)e^{j2\omega_c t} + \frac{1}{4}m(t)e^{-j2\omega_c t}\right\} \\ &= \frac{1}{2}M(\omega) + \frac{1}{4}M(\omega - 2\omega_c) + \frac{1}{4}M(\omega + 2\omega_c) \end{aligned}$$

when gain of LPS is = 1 $y(t) = \frac{1}{2}m(t)$

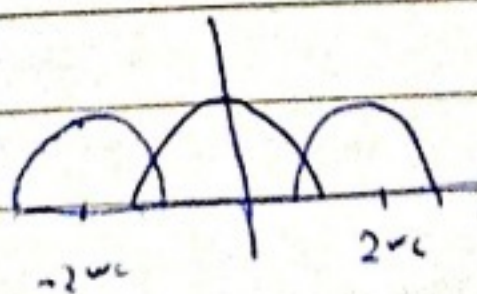
gain = 2 $y(t) = m(t)$

gain = 5 $y(t) = \frac{5}{2}m(t)$

What gain should you use to get $y(t) = 5m(t)$

ans gain = 10

if ω_c is small



$2\omega_c \geq 4\pi B = (2 \times 2\pi B)$

$\omega_c \geq 2\pi B$

$f_c \geq B$ ideally

practically $f_c \gg B$

*** Sketch $\phi(t)$, $y(t)$, and $y'(t)$.

LPF removes sharp edges \equiv smoothing device

HWJ Draw the slides again but use $m(t) = \text{rep}\{\Delta(t)\}$

HWJ Do the ~~the~~ example in slide #13

example "13"

Tone modulation

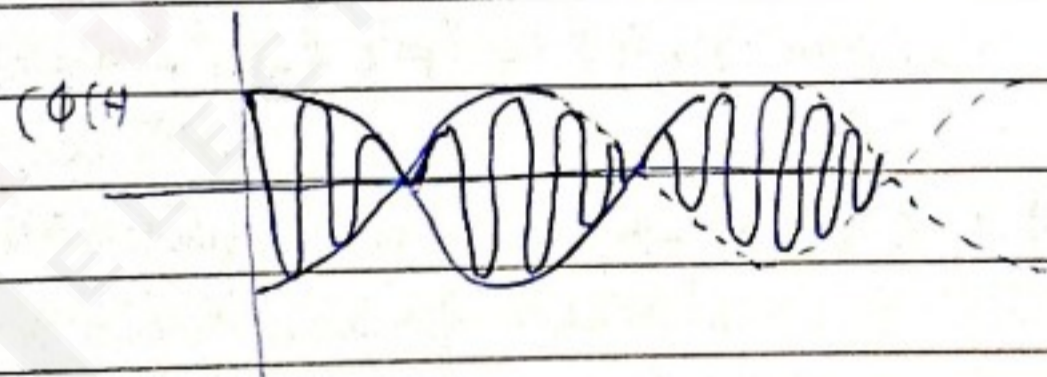
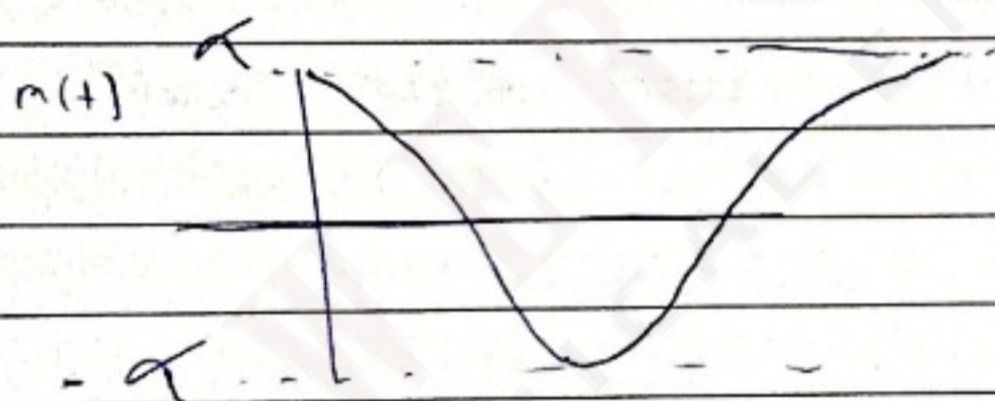
$$\Rightarrow m(t) = \alpha \cos(\omega_m t)$$

$$c(t) = \cos(\omega_c t)$$

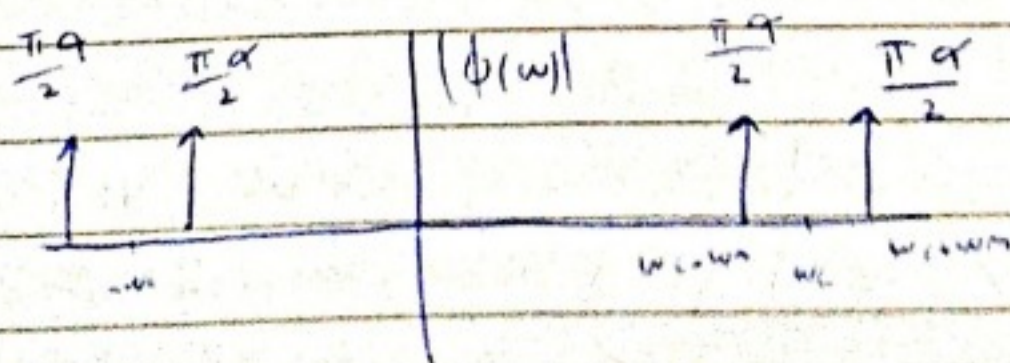
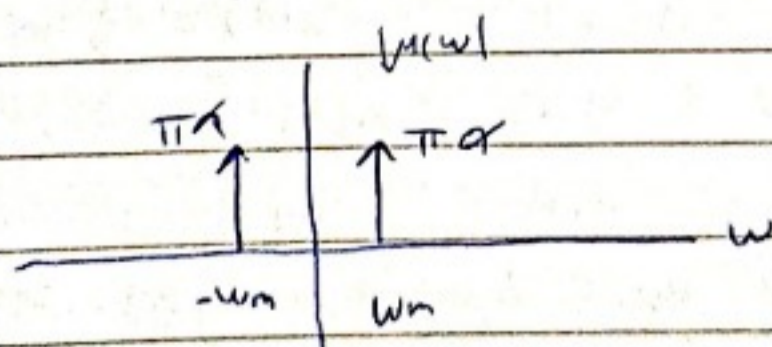
\uparrow very high

$$\omega_c \gg \omega_m$$

(a) Sketch $\phi(t)$



(b) $M(\omega)$



$$\textcircled{c} \beta_{m(t)} = \frac{\omega_m}{2\pi} \text{ Hz} = f_m \text{ Hz} = \frac{1}{T_m} \text{ Hz} = \text{fundamental frequency (special case)}$$

$$\beta_{d(t)} = \frac{2\omega_m}{2\pi} \text{ Hz} = 2 \beta_{m(t)}$$

$$\textcircled{d} P_{av} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{T_m} x^2(t) dt = \frac{\alpha^2}{2}$$

$$x(t) = \alpha \cos(\omega_m t) \cos(\omega_c t)$$

$$\overline{x^2(t)} = \frac{\alpha^2}{4}$$

HW = Repeat the above example but for the following $m(t)$
"15"

Transceiver = TX + RX

GSM = 2G cellular

W-CDMA = 3G cellular

Gilbert cell "23"

Gilbert cell = 3 differential amplifiers + 1 current source

3 x 2 transistors + 2 transistors = 8

Amplifier $V_m = V_2 - V_1$

$V_{out} = V_{o2} - V_{o1}$

$$V_{out} = G V_{in}$$

$$V_{o2} - V_{o1} = \left(\frac{-I_{EE} R_L}{2V_T} \right) (V_2 - V_1)$$

$$V_T = 0.026 \text{ V at } 25^\circ\text{C} \\ = \frac{kT}{q}$$

G

k: Boltzmann const = $1.38 \times 10^{-23} \text{ J/K}$

T: temp (K)

q: electron charge 1.6×10^{-19}

I_x controls the gain of Q_3 and Q_6

R_x controls I_x and therefore controls the gain

Gilbert cell

Diff Amp $V_{o2} - V_{o1} = \frac{-I_{EE} R_L}{2V_T} (V_2 - V_1)$

Using superposition

$$V_{o2} - V_{o1} = \underbrace{V_{o2} - V_{o1}}_{\text{due to } Q_1 - Q_2} + \underbrace{V_{o2} - V_{o1}}_{\text{due to } Q_3 - Q_4}$$

$$= \frac{-I_{EE} R_L}{2V_T} (X_1 - X_2) + \frac{-I_{EE} R_L}{2V_T} (X_2 - X_1)$$

gain

$$V_{o2} - V_{o1} = \frac{-R_L}{2V_T} (X_1 - X_2) \cdot (I_{E1} - I_{E2}) \quad \text{--- (1)}$$

↑ multiplication

We can prove that for $Q_1 - Q_2$

$$\underbrace{I_{E2} - I_{E1}}_{\text{out}} = \frac{-2}{R_X} \underbrace{(m_2 - m_1)}_{\text{gain in}} \quad \text{--- (2)}$$

$$V_{o2} - V_{o1} = \frac{R_L}{R_X V_T} (X_2 - X_1) (m_2 - m_1) = G(f) m(f)$$

Q_1, Q_2

Q_3, Q_4

Gain $\times -c(f)$

gain $\times c(f)$

$$3c(f) = 4c(f) + (1)(-c(f))$$

$$5c(f) = 6c(f) + (1)(-c(f))$$

$$0 = 1c(f) + (1)(-c(f))$$

$$ab = ba$$

In practice, make sure that carrier goes to the top
amp Q_1, Q_2 and Q_3, Q_4 allows us to saturate

the hp diff amp

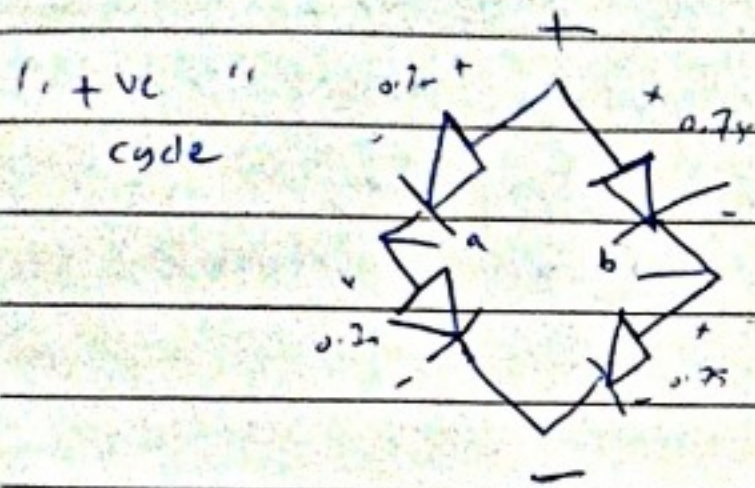
\Rightarrow Gilbert cell works in switching mode "recommended mode"

allows us to unbalance the top for diff amp

\Rightarrow Gilbert cell works in unbalanced mode DSB-SC "AM"

Switching Modulator

$$C \gg m(t) \quad C \cos(\omega_c t)$$

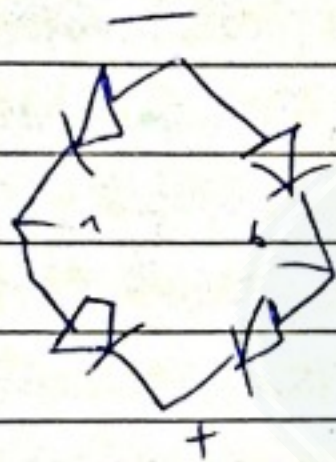


all diodes are forward biased

$V_{ab} = 0$ volt looks like short ckt.

$$X(t) = m(t)$$

-ve cycle



all diodes are reversed biased

\Rightarrow open ckt

$$X(t) = 0$$

$X(t)$ is called switched $m(t)$ "natural sampling"

Q. In the system shown (slide #28)

(shunt bridge diode modulator) find $y(t) = ?$

(b) Repeat but for BPF gain = 2 bandwidth = 2B Hz
 Center = $2\omega_c$ rad/s

$C \cos(\omega_c t)$ is positive cycle \rightarrow diodes ON $\rightarrow X(t) = 0$

$C \cos(\omega_c t)$ is neg. cycle \rightarrow diodes OFF $\rightarrow X(t) = m(t)$

$X(t) = 0; m(t); 0; m(t); 0; m(t) \dots$

$$\alpha_1 = \frac{1}{\pi} e^{-j\pi}$$

$$\alpha_{-1} = \frac{1}{\pi} e^{j\pi}$$

$$y(t) = 2 |\alpha_1| m(t) \cos(\omega_c t - \pi) = \frac{2}{\pi} m(t) \cos(\omega_c t - \pi)$$

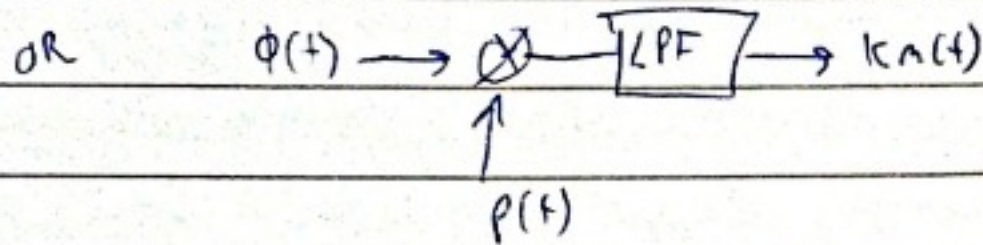
$Y(\omega) =$ two copies of $M(\omega)$

Ring modulator

$$X(t) = m(t); -m(t); m(t)$$

4th Design We can use the Gilbert cell as a switching modulator by pushing it into switching mode (preferred mode) by saturating the two top differential amplifiers.

Gilbert linear mode



For you to be able to demodulate $m(t)$ from DSB-SC the RX should be perfectly synchronized with the TX

same freq
same phase

Part (a) : Phase error $\phi(t)$ $y(t)$ $y(t)$

$$\phi(t) = m(t) \cos(\omega_c t)$$

$$X(t) = \phi(t) \cos(\omega_c t + \Delta\theta)$$

$$= m(t) \cos(\omega_c t) \cos(\omega_c t + \Delta\theta) = \frac{1}{2} m(t) [\cos(2\omega_c t + \Delta\theta) + \cos(\Delta\theta)]$$

$$y(t) \rightarrow X(t) \rightarrow \text{LPF}$$

$$y(t) = \frac{1}{2} \cos(\Delta\theta) m(t) \neq \frac{1}{2} m(t)$$

ex $\Delta\theta = 60^\circ$

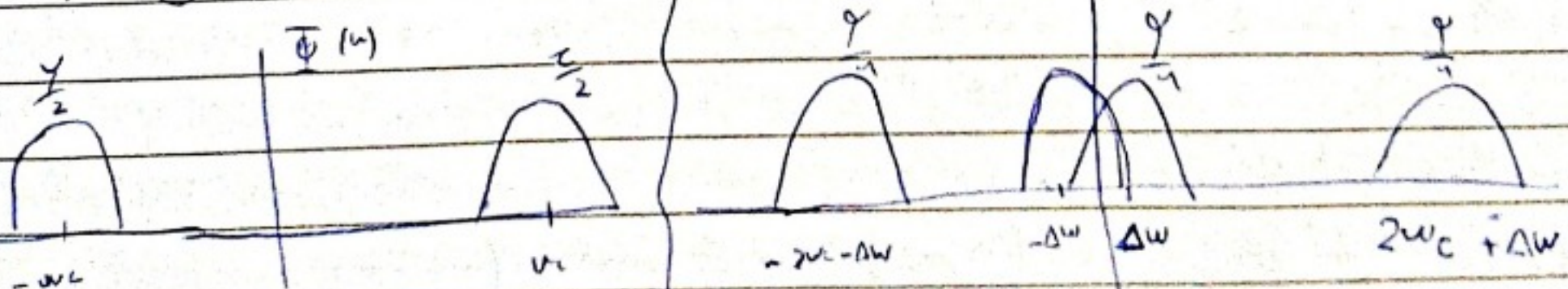
$$\cos(\Delta\theta) = \frac{1}{2}$$

Attenuation

$$y(t) = \frac{1}{4} m(t) \quad \text{attenuation}$$

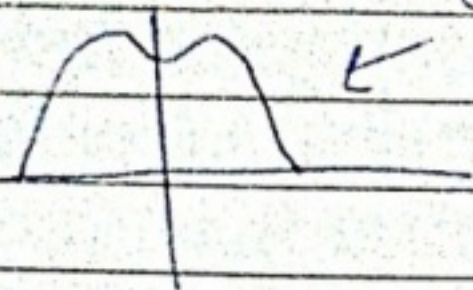
demodulation

Part (b) Frequency error $\Delta\omega$



modulation

distorted $M(\omega)$



"Distortion" frequency error

To avoid distortion and attenuation then maintain a perfect synchronization

Solutions to maintain perfect synchronization?

1- Old solution

Invent another modulation technique

that does not require perfect sync.

[can use a synchronous RX]

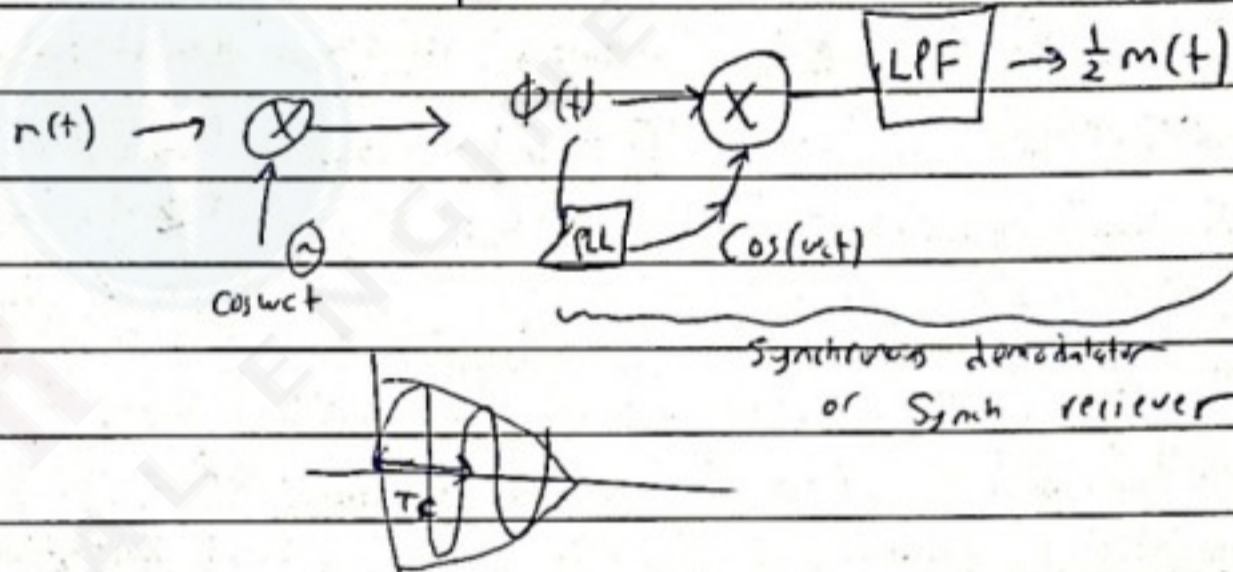
ex DSB-LC (AM)

2- New solution: use a carrier recovery

circuit (PLL) phase-locked-loop @ RX

to recreate a perfectly sync cosine.

very complex



Synchronous demodulator or Sync receiver

switched $m(t)$

Sampling: Ideal vs practical vs natural sampling

Nyquist rate and anti-aliasing filter

Slide 6 "uniform sampling" - time intervals are all equal and are equal in sampling time " T_s "

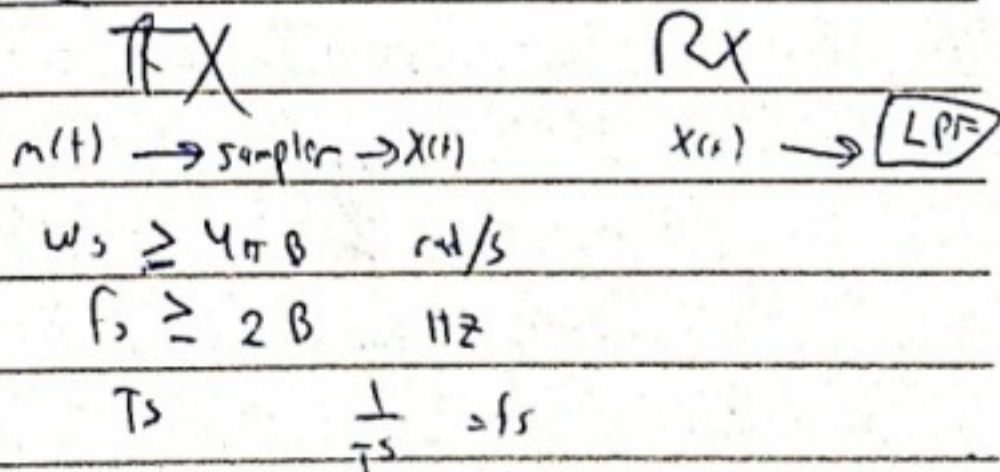
Practical (PAM) $Z = \frac{T_s}{2}$

Practical (PAM) (S/H) $Z = T_s$

Pulse amplitude modulation

sample and hold

Practical sampling



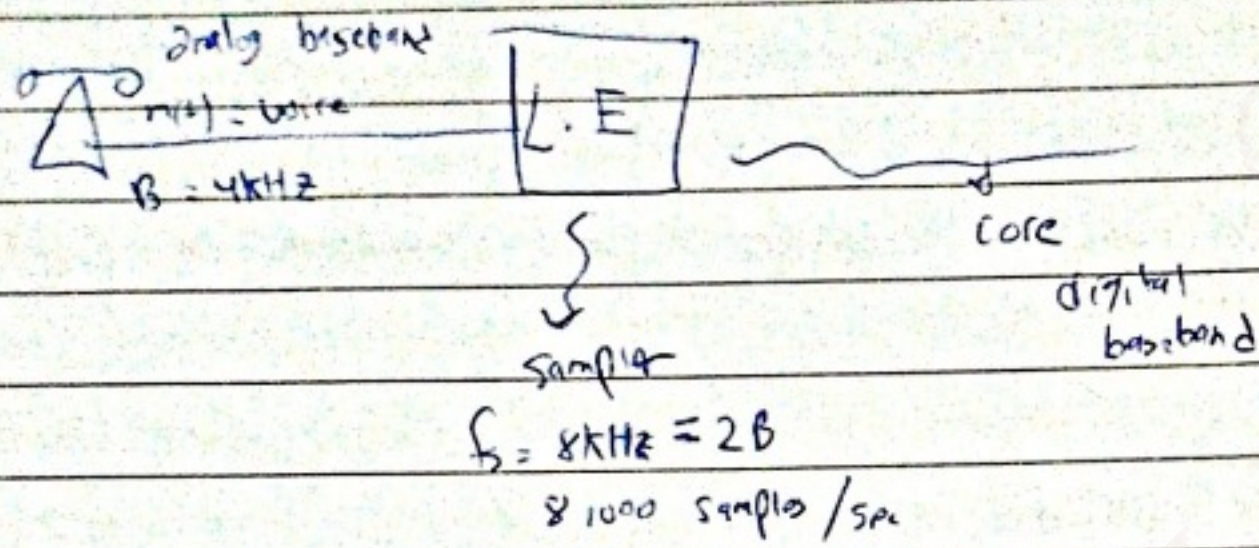
$$\omega_s \geq 4\pi B \quad \text{rad/s}$$

$$f_s \geq 2B \quad \text{Hz}$$

$$T_s = \frac{1}{f_s} = T_s$$

$$f_{max} f_s \gg 2B$$

2) Telephony



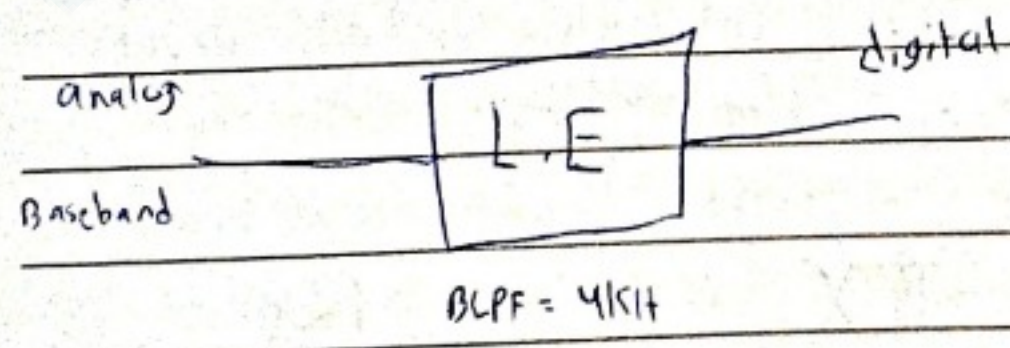
$f_s \geq 2B$ Solution #1 against the problem of aliasing
increase f_s

Solution #2 against aliasing: anti-aliasing LPF (at TX before the sampler)
reduces B into $B_{new} = \frac{f_s}{2}$
 \Rightarrow distorting $m(t)$ but no aliasing
 $f_s = 2B_{new}$
 $f_s < 2B$

#1	no distortion; no aliasing	↑ better
#2	distortion; ~ aliasing	

aliasing

Practical example "Telephony"
 voice $B = 4kHz \Rightarrow f_s = 8kHz$
 music $B = 20kHz$ still $f_s = 8kHz$



20V
4

Quantization

$m(t) = [-1, 2]$

2 V

11

1 V

10

2 bits per sample

0 V

01

-1 V

00

$1.55 V \Rightarrow 2 V$

$\Delta V =$ error in quantization

2 V

111

1.5 V

110

$1.55 \Rightarrow 1.5 V$

1 V

101

less error

0.5 V

100

but more bits per sample

0 V

011

-0.5 V

010

-1 V

001

-1.5 V

000

ex) Telephony

8 bits/sample

$L = 2^8 = 256$ levels

CDs

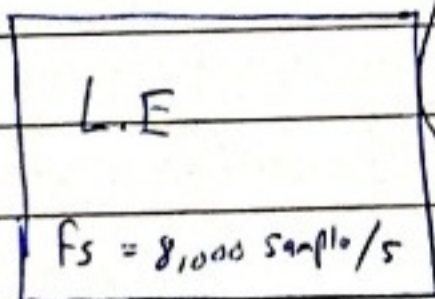
$L = 65,536$

16 bits/sample

small error better quality

P1)

$m(t) =$ voice



* 8 bits/sample

data rate bit rate = $R_0 = 64,000$ bit/sec

= 64 kbit

Sec

PCM stream pulse code modulation

$\uparrow L$ SNR \uparrow better quality

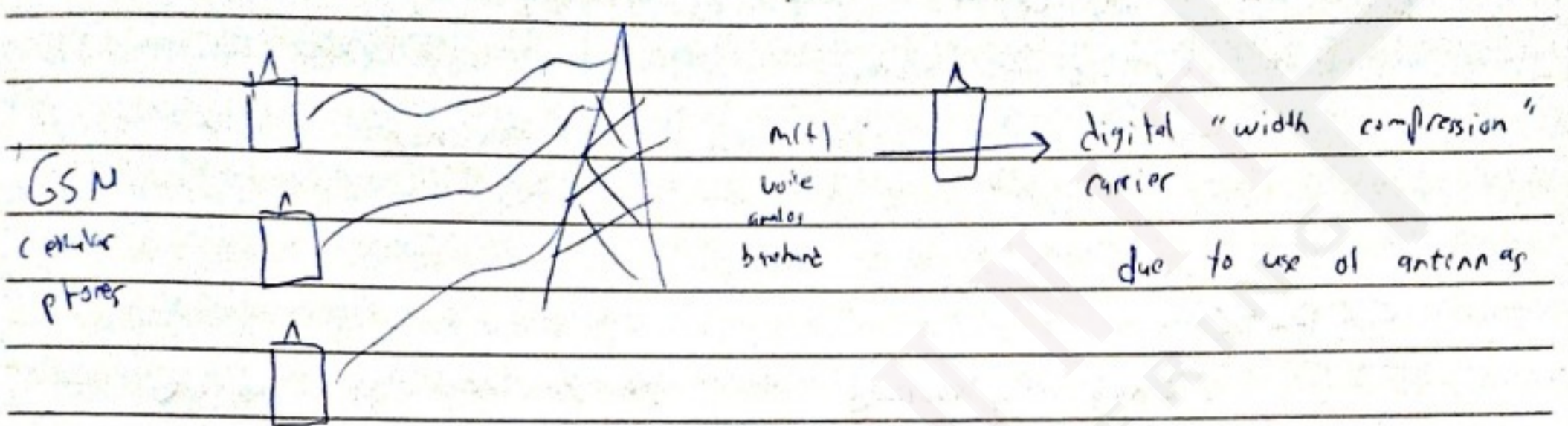
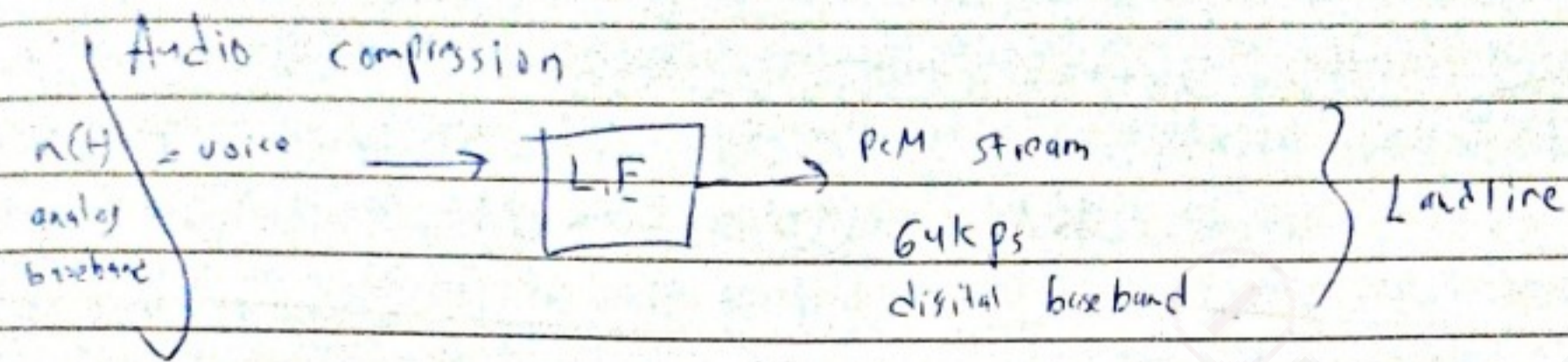
\downarrow error

$K_m^2 =$ crest factor or peak / rms ratio

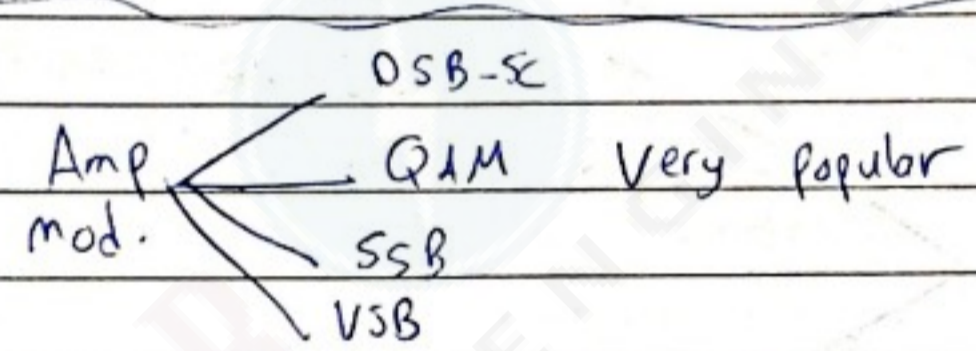
$= \frac{m_p^2}{m(t)^2} = \frac{P_{peak}^2}{rms^2}$

\uparrow avg. power

$$\frac{1}{2} (\sin(\omega t) - \sin(\omega - \omega_c))$$



lecture 6



$$F\{m_2(t) \sin(\omega_c t)\} = F\left\{-j m_2(t) \frac{e^{j\omega_c t} - e^{-j\omega_c t}}{2}\right\}$$

$$= \frac{-j}{2} M_2(\omega - \omega_c) + \frac{j}{2} M_2(\omega + \omega_c)$$

→ right ← left * 1/2 "phase shift of 90°"

Bandwidth of $\phi_{10}(t)$ is $2B$ Hz
 where we sent 2 messages

Q. Find $y_1(t) =$

$$m_1(t) \cos(\omega_c t) + m_2(t) \sin(\omega_c t)$$

$$\times \cos(\omega_c t)$$

$$= m_1(t) \cos^2(\omega_c t) + \frac{1}{2} m_2(t) \sin(2\omega_c t)$$

$$= \frac{1}{2} m_1(t) + \frac{1}{2} m_1(t) \cos(2\omega_c t) + \frac{1}{2} m_2(t) \sin(2\omega_c t)$$

Low F High F High F

$x_1(t) \rightarrow$ LPF $\times = \frac{1}{2} m_1(t)$

H.W] Find $y_1(t) = ?$

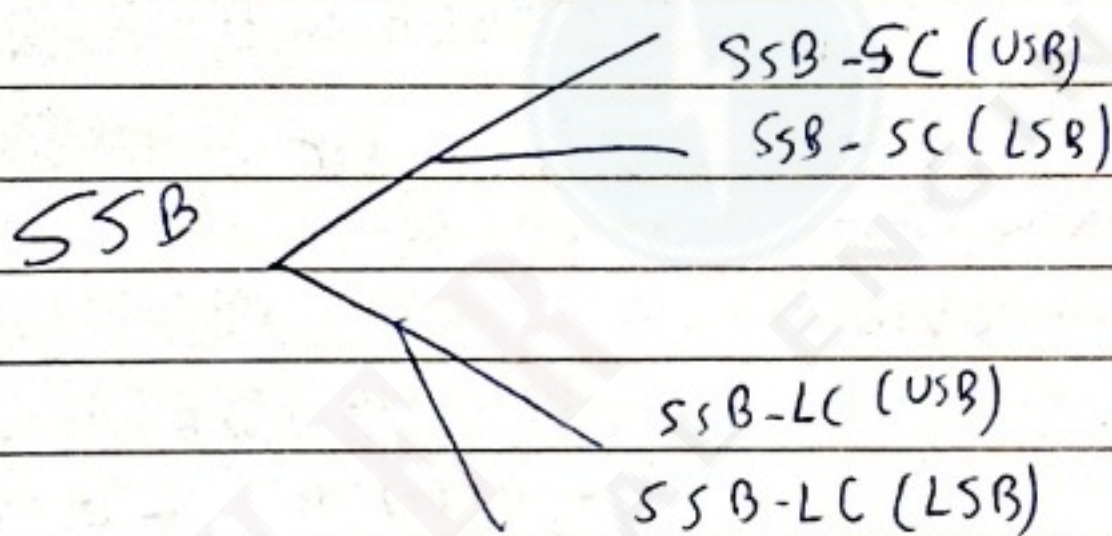
RX $\approx \cos(\omega t + 10^\circ)$

DSB-SC and QAM $\approx \cos(\omega t + 10^\circ)$

both require synchronizer
demodulation (do not work with 2 synchron RX)

SSB-SC

SSB-SC (Single side band) allows us to send one message
modulated in B bandwidth



SSB was limited in real life to voice application (MCM)

	Bandwidth	Avg power in $\phi(t)$
DSB-SC	$2B$	$\frac{1}{2} m^2(t)$
SSB-SC	B	$\frac{1}{4} m^2(t)$
QAM	$2B$	$\frac{1}{2} m_1^2(t) + \frac{1}{2} m_2^2(t)$
VSB-SC	$(1+\epsilon)B$ $= B + B\epsilon$ $0 \leq \epsilon \leq 1$ theoretical $0.1 \leq \epsilon \leq 0.25$ practical	$\frac{1}{4} m^2(t)$

Vestigial sideband (VSB)

VSB-SC (USB)

VSB-SC (LSB)

VSB-LC (USB)

VSB-LC (LSB)

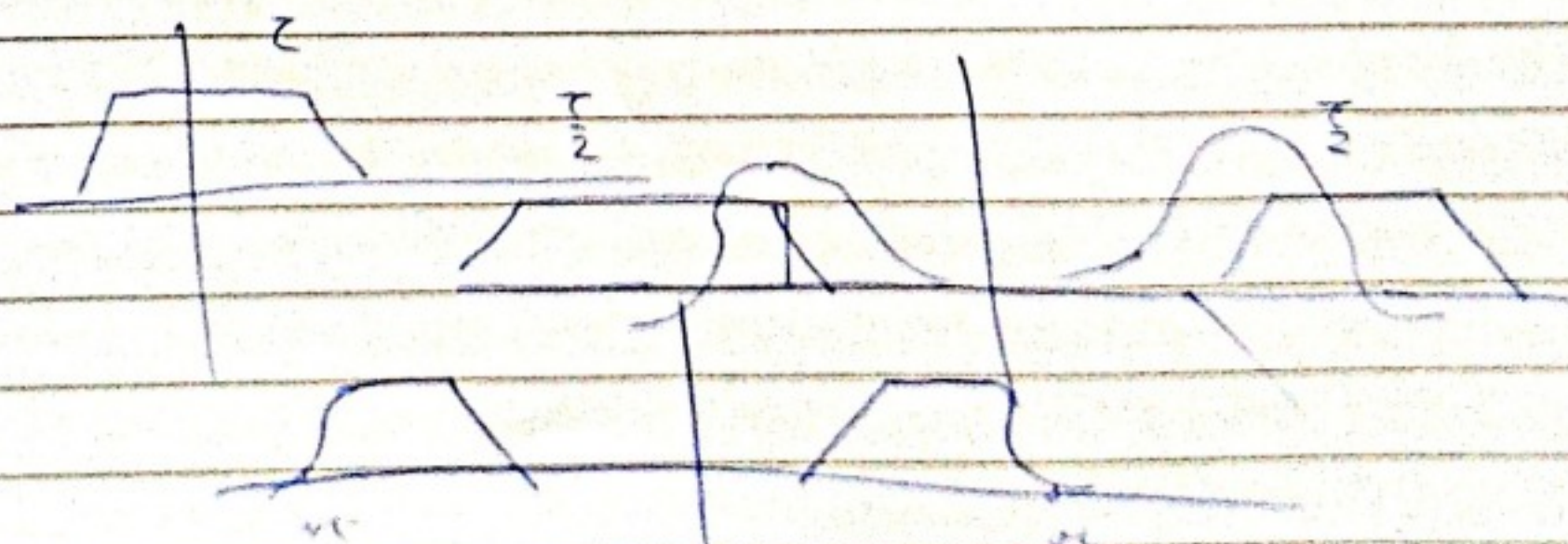
(1) Passes some of the lower side band "LSB"

(2) But also reject some of the USB

Perfectly symmetric

USB 0.1 0.9

USB 0.9 0.8



Higher bandwidth more more fading

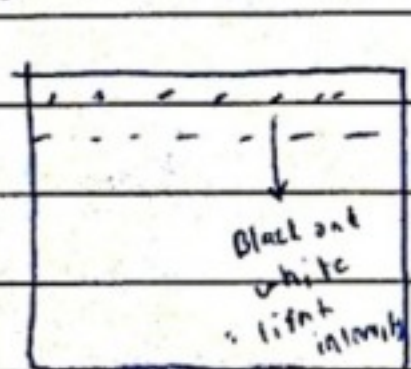
Slide 18

in terms of attenuation and distortion

SSB-SC suffers less than VSB-SC less than DSB-SC less than OAM

Envelope detection \equiv asynchronous demodulator.

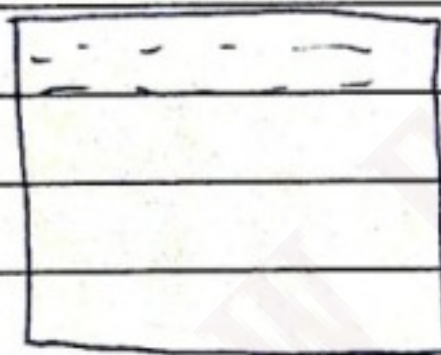
NTSC



525 lines of resolution

Standard definition
SDTV

PAL



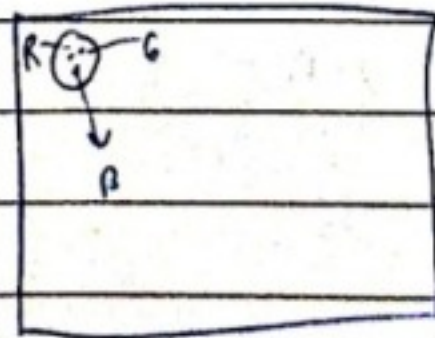
625 lines of resolution

1080 P lines of resolution

720 P

HDTV

Coloured



RGB



Luminance and Chrominance

Y

U & V

Reasons

#1: Maintains backwards compatibility

#2: Save bandwidth \because human eye is

less sensitive to chrominance to luminance

Slide 22 Right side change 4 MHz to 1 MHz

Luminance = - USB+C bandwidth = $(1+\epsilon)B = 5 \text{ MHz}$

Chrominance U & V : QAM Bandwidth = $2B = 2 \times 1 = 2 \text{ MHz}$
↓ because we send 2 signals simultaneously

In general, you cannot send USB or QAM on top of each other except in TV - '(interference)'

Audio is FM modulated bandwidth = $0.5 \text{ MHz} = 500 \text{ kHz}$

* 0.25 MHz between audio and video " the gap is called guard band

The bandwidth for one NTSC TV station

0.25 MHz	guard
1 MHz	LSB
4 MHz	USB
0.25 MHz	guard
0.5 MHz	audio
6 MHz	xxx

for Pal station = 8 MHz

QAM "chrominance occupies 2 MHz "

~~Total bandwidth for one station = 8 MHz~~

total bandwidth for one DVB-T 7 station = 8 MHz
digital

Lecture 7 Homodyne \rightarrow single frequency

Heterodyne \rightarrow multiple frequency

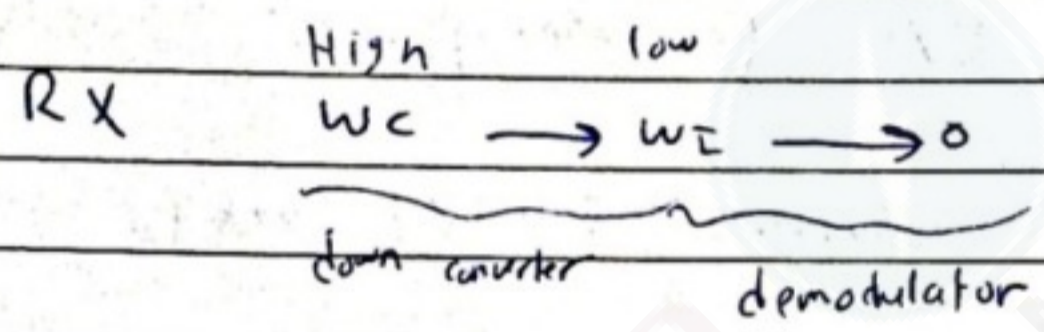
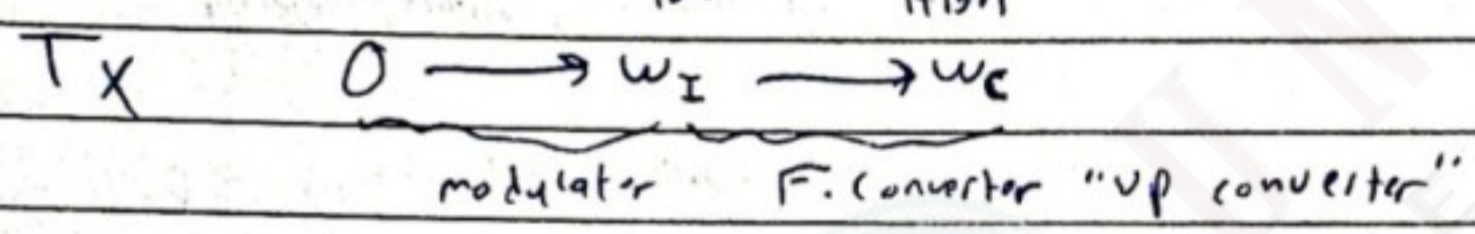
Frequency conversion

freq conv: modulated \rightarrow modulated

$$\omega_1 \rightarrow \omega_2$$

$$m(t) \cos(\omega_1 t) \rightarrow m(t) \cos(\omega_2 t)$$

low High



Slide 4:-

$\cos(?)$

the local oscillator (L.O) in the converter has a frequency of:.

$\omega_{LO} = \omega_{in} + \omega_{out}$ up converter

or $\omega_{LO} = |\omega_{in} - \omega_{out}|$ down converter

ex) $\omega_{in} = \omega_c$ $\omega_{out} = \omega_c$ ω_{LO}

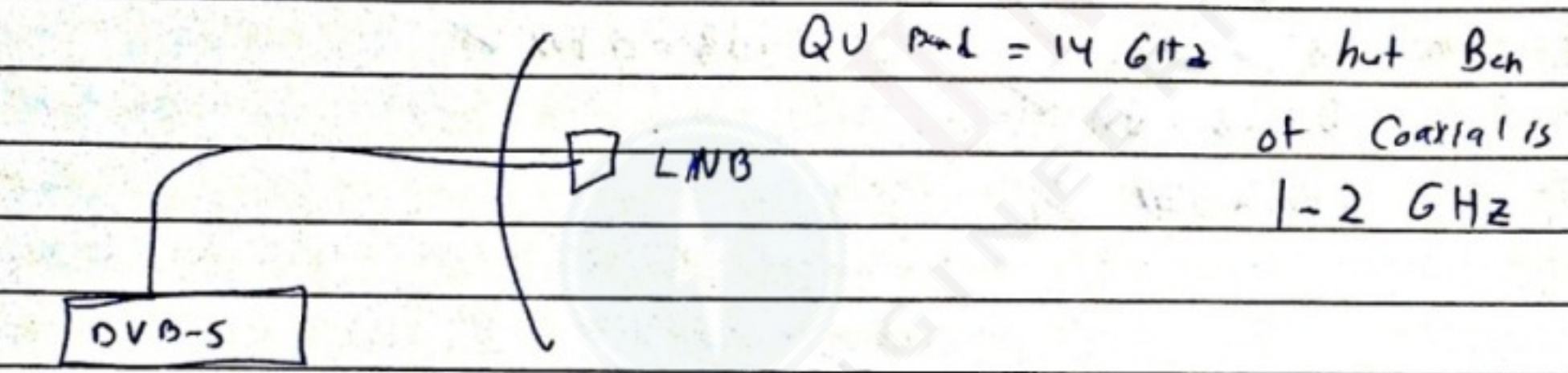
- (1) $(2\pi \times 100 \text{ MHz}) \text{ rad/s} \rightarrow (2\pi \times 200 \text{ MHz}) \text{ rad/s}$
100 MHz \rightarrow 200 MHz 300 MHz "up-conv"
- (2) 200 MHz \rightarrow 600 MHz 300 MHz "up-conv"
- (3) 100 MHz \rightarrow 200 MHz 100 MHz "down-conv"
- (4) 700 MHz \rightarrow 100 MHz 100 MHz "down-conv"

BPF = center frequency ω_I
bandwidth = 20 kHz or 40 B rad/s

gain = 1
 $y(t) = \frac{1}{2} m(t) \cos(\omega_I t)$

if gain = 2 $y(t) = m(t) \cos(\omega_I t)$

Practical Application for comu ?



Lecture 8

Performance \rightarrow SNR \rightarrow Quality

if SNR \downarrow bad Quality

For analog

To get good quality voice you need SNR ≥ 1000 unitless

For digital

SNR \rightarrow BER \rightarrow Quality
bit error rate

BER $\leq 10^{-6}$ for good quality digital

we don't use unitless, we use dB

$$\text{SNR} = 1000 \text{ unitless}$$

$$\text{SNR} \geq 30 \text{ dB}$$

$$0 \text{ unitless} \Rightarrow -\infty \text{ dB}$$

$$\text{dBm} + \text{dB} = \text{dBm} \quad \checkmark \quad \text{dBW} + \text{dB} = \text{dBW} \quad \checkmark$$

$$\text{dB} + \text{dB} = \text{dB} \quad \checkmark$$

$$\text{dBm} + \text{dBW} \quad \times$$

SNR_{out} \rightarrow at the receiver

Noise in time domain is gaussian distributed.

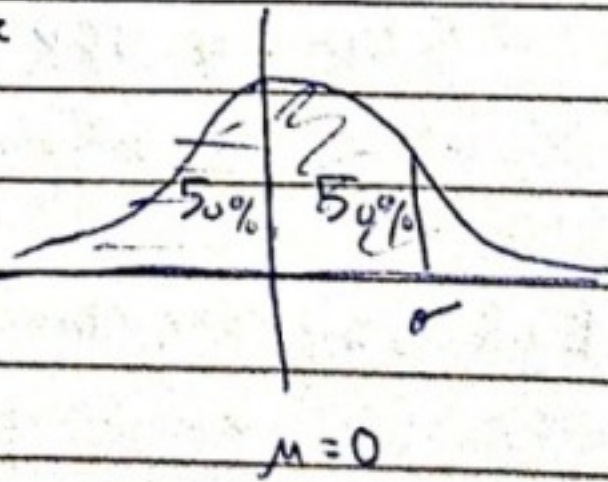
AWGN noise = The most popular mathematical model for noise (fits nicely with thermal internal noise)

Additive white gaussian noise "AWGN"

Mean = $\mu = 0$ = DC value

Variance = σ^2 = avg. power in noise

Standard deviation = σ rms value



Noise in Frequency Domain " Power spectral density "

$S_n(\omega)$

$N_0/2$

ω

$$N_0 = 4KT$$

$K = \text{boltzmann's const}$

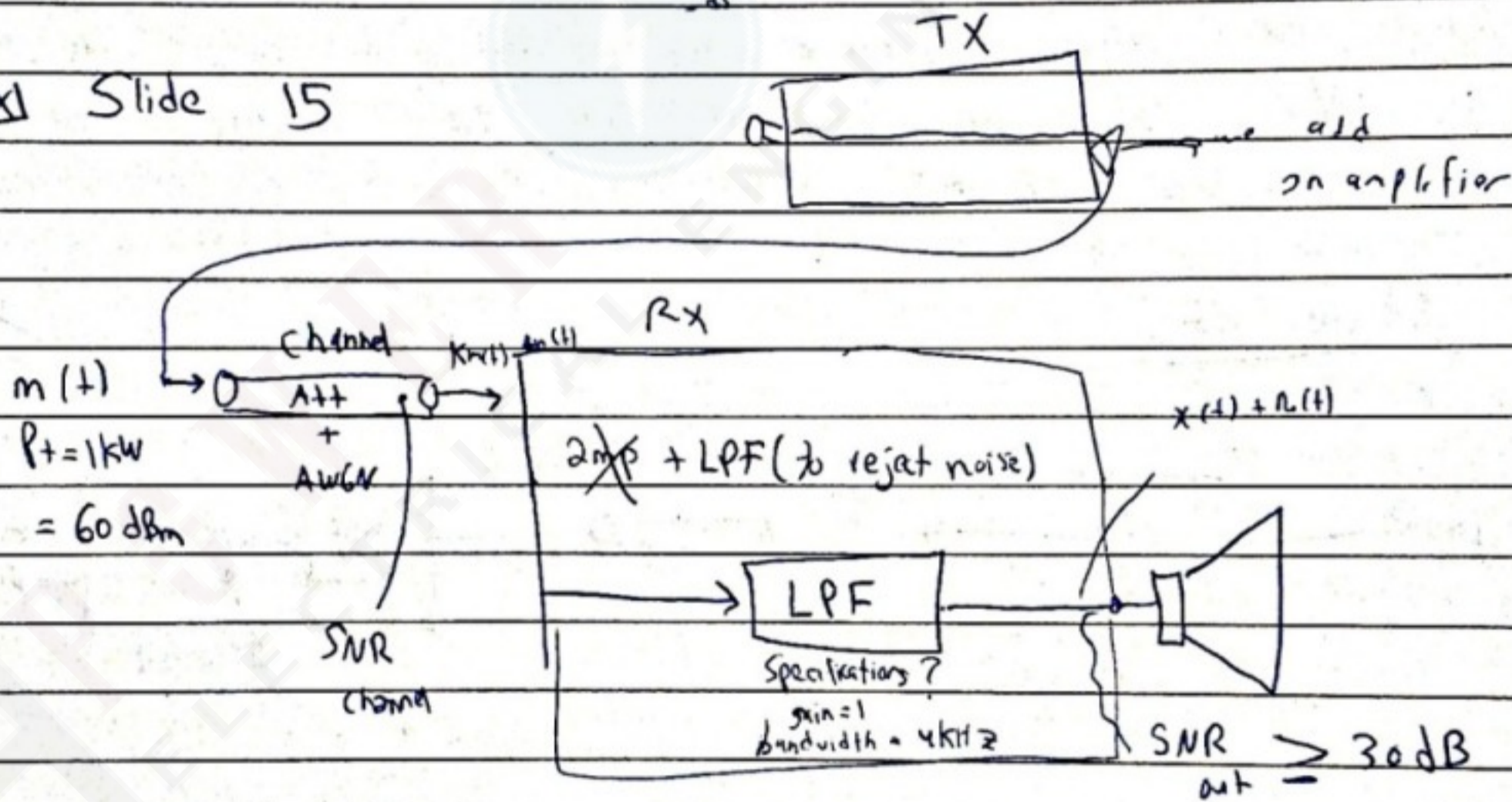
$$= 1.38 \times 10^{-23} \text{ J/K}$$

$T = \text{temp (K)}$

white noise has constant PSD

9/4 ~~Power Spectral Density~~ $= \frac{1}{2\pi} \int_{-\infty}^{\infty} S_n(\omega) d\omega$

ex1 Slide 15



$$SNR_{\text{channel}} = \frac{\text{signal power}}{\text{noise power}} = \frac{10\text{mW}}{\frac{1}{2\pi} \int_{-\infty}^{\infty} S_n(\omega) d\omega} = 0 \text{ unitless}$$

$$P_t + 60 \text{ dBm} + (-50 \text{ dB}) = 10 \text{ dBm} = 10 \text{ mWatt}$$

worst
value
possible

$$P_{\text{unitless}} = 10^6 \text{ nW} \times 10^{-5} = 10 \text{ nW}$$

$$P_{\text{dBW}} = 30 \text{ dBW} + (-50 \text{ dB}) = -20 \text{ dBW} \Rightarrow 0.01 \text{ W}$$

$$\text{output noise power} = \overline{n_o^2(t)} = \frac{1}{2\pi} \int_{-2\pi B}^{2\pi B} S_{n_o}(\omega) d\omega$$

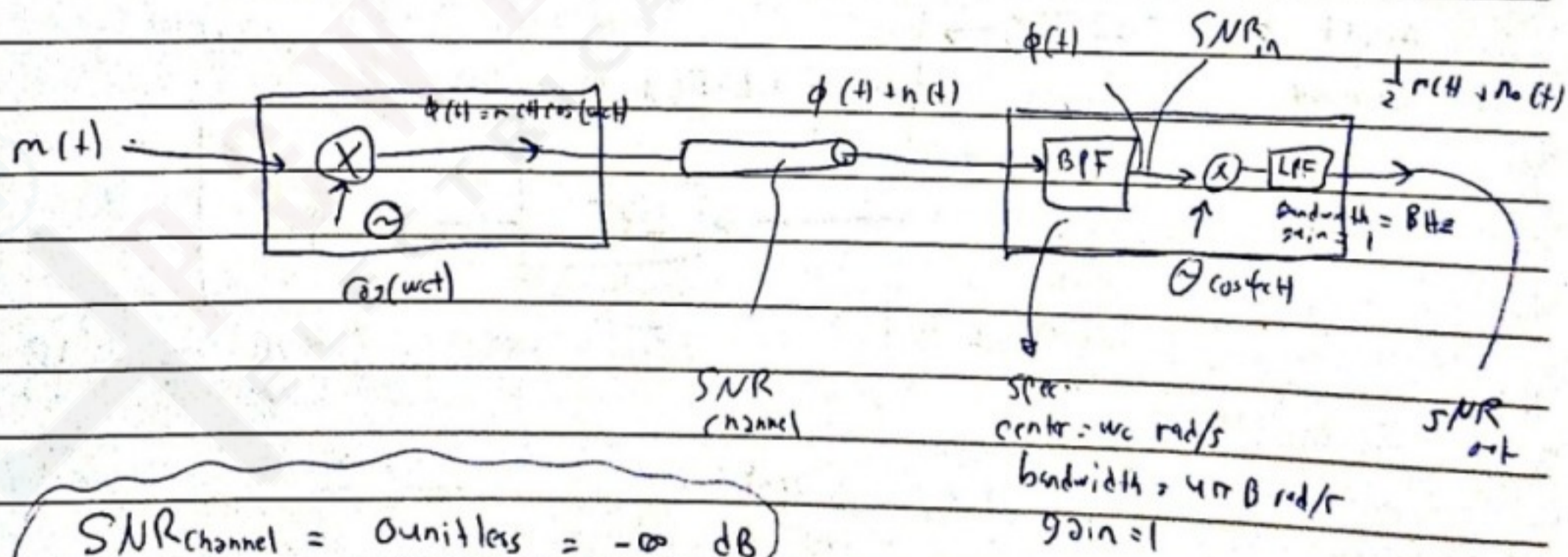
$$\frac{1}{2\pi} \frac{N_o}{2} (4\pi B) = N_o B = \left(4 \times 10^{-9} \frac{W}{Hz} \right) 4000 \text{ Hz} = 16 \mu W$$

$$SNR_{out} = \frac{\text{signal power}}{\text{noise power out}} = \frac{10 \text{ mW}}{16 \mu W} = 625 \text{ unitless} = 27.96 \text{ dB}$$

still
bad
quality

* $x(t)$ will have the same power, because gain of LPF = 1
 if gain = 4 then power is 16 times input power. so does
 the noise increase by a factor of 16 "No use"
 We add an amplifier at the transmitter before
 the channel

Slide 17



$$SNR_{channel} = 0 \text{ unitless} = -\infty \text{ dB}$$

$$SNR_{in} = \frac{\text{signal power}}{\text{noise power}} = \frac{\overline{\phi^2(t)}}{\text{noise } \rho} = \frac{\frac{1}{2} \overline{m^2(t)}}{N_o B}$$

$$SNR_{in} = \frac{\frac{1}{2} \overline{m^2(t)}}{\frac{1}{2\pi} (2\pi B)} = \frac{\frac{1}{2} \overline{m^2(t)}}{\frac{1}{2\pi} \left(\frac{N_o}{2} 4\pi B \right)} = \frac{\overline{m^2(t)}}{4 N_o B} = \frac{S_{in}}{2 N_o B}$$

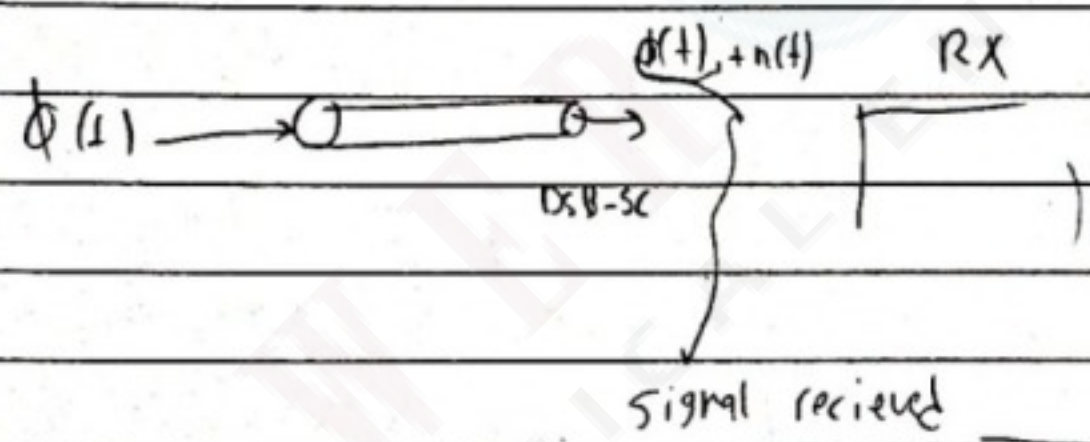
27 $\frac{1}{2}$ $10 \log \left(\frac{1}{4} \right) = -3 \text{ dB}$

SNR_{out} ∴ for noise power we multiply by $\frac{1}{4}$
 when it enters the mixer instead of $\frac{1}{2}$
 since its power

$$SNR_{out} = \frac{S.P (out)}{N.P (out)} = \frac{\frac{1}{4} m^2(t)}{\frac{1}{2\pi} \left[4\pi B \frac{N_0}{4} \right]}$$

$$SNR_{out} = \frac{m^2(t)}{2 N_0 B} = \frac{S_{in}}{N_0 B}$$

We usually express SNR values in terms of Avg ^{signal} power of the
 RX S_{in}



$$S_{in} = \phi^2(t) = \frac{1}{2} m^2(t)$$

SNR_{out} > SNR_{in}
 "better quality"

So, the synchronous demodulator in DSB-SC has the ability to improve quality factor of 2 (because of orthogonality)

What is NF for our earlier example (DSB-SC demod)

$$NF = \frac{SNR_{in} (\text{unitless})}{SNR_{out} (\text{unitless})} = \frac{1}{2} \text{ unitless} \Rightarrow NF = -3 \text{ dB}$$

AM modulation

DSB-SC $\Rightarrow \phi(t) = m(t) \cos(\omega_c t)$

f-domain

DSB-LC "AM" $\Rightarrow \phi(t) = m(t) \cos(\omega_c t) + \underbrace{A}_{\text{constant}} \cos(\omega_c t)$

extra carrier

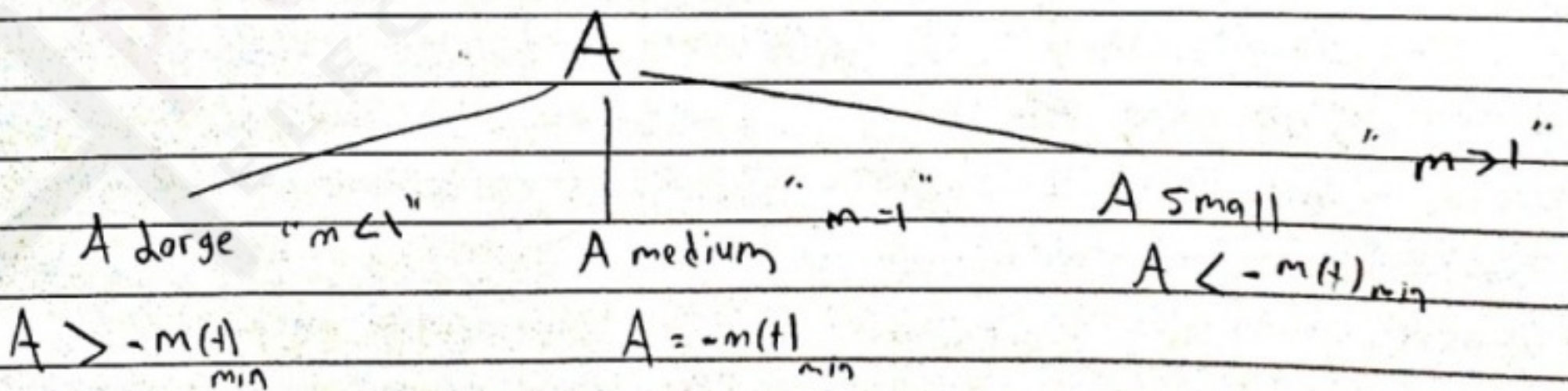
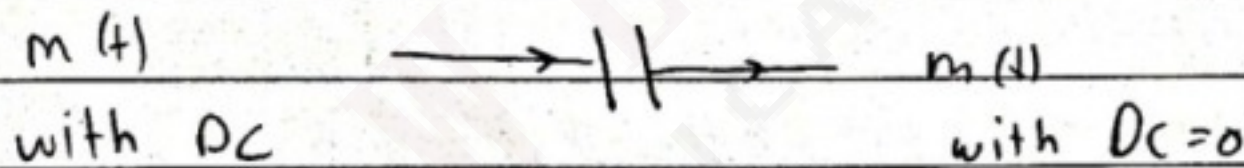
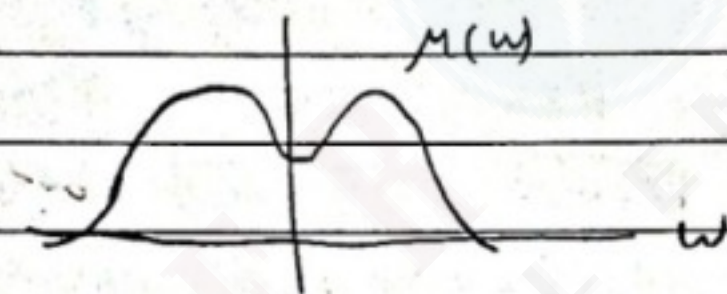
$\phi(t) = (m(t) + A) \cos(\omega_c t) \dots \textcircled{2} \dots \textcircled{1}$

f-domain

* In practical AM systems, we don't like $m(t)$ with a DC value.

Sol # 1 : AM Radio

$m(t) = \text{voice}$



"No sudden 180° Shift"

"No sudden 180° shift"

"Sudden 180° shift"

"Under modulation"

"critical modulation"

"Over modulation"

DSB-SC is

a special case when $A=0$

Bandwidth of $\phi(t)$ is $2B$ Hz or $4\pi B$ rad/s

Average power in $\phi_{AM}(t)$

$$\phi_{AM}(t) = m(t) \cos(\omega_c t) + \underbrace{A \cos(\omega_c t)}_{\frac{A^2}{2}}$$

* If we have no sudden phase shifts we could be able to use 2 synchronous demodulators.

$E(t)$ = envelope of $\phi(t)$ = ^{positive} pos. peaks = $m(t) + A$ for under and critical

$E(t) = |m(t) + A| \neq m(t) + A$ in over modulation.

* We usually define an AM modulation index for $\phi(t)$ as follows :-

$$m = \frac{-m(t)_{\min}}{A} = \text{AM modulation index.}$$

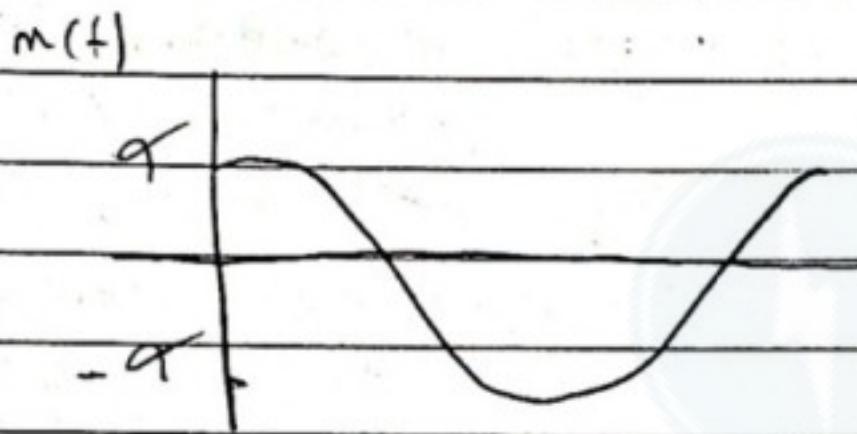
Case	Avg. Power	RX	180°	B
$m < 1$	large	2synch and Synch better preferred	NO	$2B$
$m = 1$	medium	2synch and Synch	NO	$2B$ ✓ best case
$m > 1$	small	Synch only	Yes	$2B$

$m=0.5$ under modulation "No sudden phase shift"

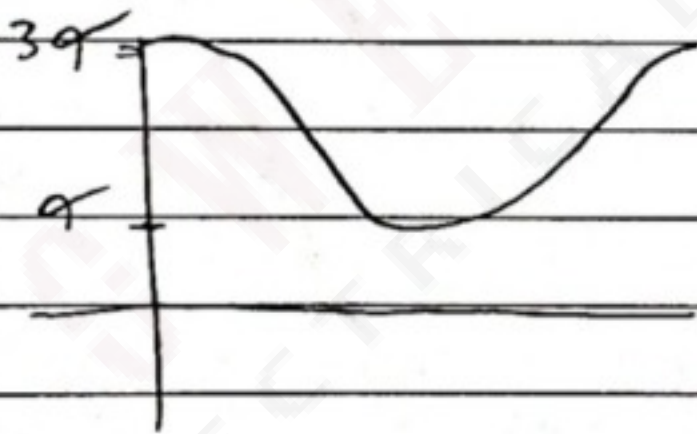
$$m = \frac{-m(t)_{\min}}{A} = 0.5$$

$$m = \frac{-(-\alpha)}{A} = 0.5$$

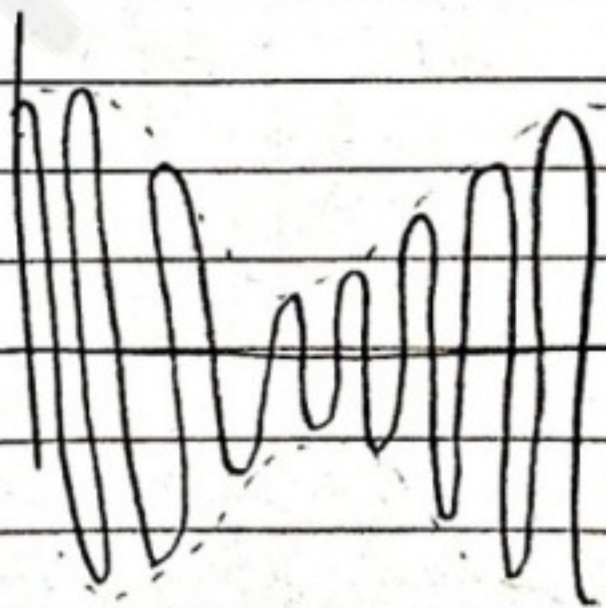
$$A = \frac{\alpha}{0.5} = 2\alpha$$



$$m(t) + A = m(t) + 2\alpha = Z(t)$$



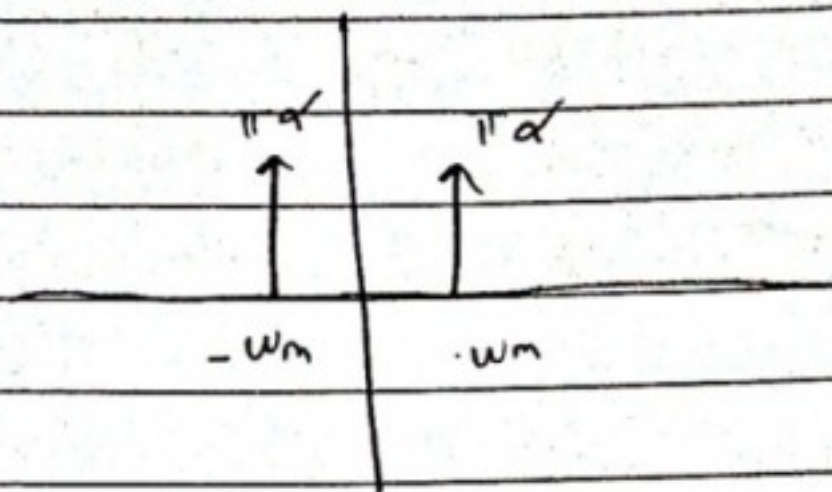
$$Z(t) \cos(\omega_c t)$$



"No 180° shift"

$$\phi_{AM}(t)$$

sketch Φ_{AM} $M(\omega)$



$$\text{bandwidth of } \Phi_{AM}(t) = 2\omega_m \text{ rad/s}$$

$$= \frac{2\omega_m}{2\pi} \text{ Hz}$$

slide 15 term #2

$$A^2 \cos^2(\omega_c t) = A^2 \left[\frac{1}{2} + \frac{1}{2} \cos(2\omega_c t) \right]$$

$$= A^2 \left[\frac{1}{2} + \frac{1}{2} \cos(2\omega_c t) \right]$$

$$= \frac{A^2}{2} = P_c = \text{Power in extra carrier.}$$

term #1

$$m^2(t) \cos^2(\omega_c t) \approx m^2(t) \cos^2(\omega_c t) = m^2(t) \cdot \frac{1}{2} = P_s$$

$$\int x(t) \cdot y(t) dt \neq \int x(t) dt \cdot \int y(t) dt$$

$m(t)$ is slowly varying with respect to the carrier

$$B \ll f_c$$

$$4 \text{ kHz} \ll 1 \text{ MHz}$$

$$\approx 100 \text{ MHz}$$

Power in sideband
"DSB-SC"

$$\text{term \#3} \quad 2A m(t) \overline{\cos^2(\omega_c t)} \approx 2A m(t) \overline{\cos^2(\omega_c t)} \cdot \frac{1}{2}$$

$$= A \overline{m(t)}$$

* In practical AM system we make sure that $m(t) = DC = 0$

$$P_t = P_s + P_c + 0 = \frac{1}{2} \overline{m^2(t)} + \frac{A^2}{2} + A \overline{m(t)}$$

$$m(t) = \alpha \cos(\omega_m t)$$

$$A = 2\alpha$$

$$P_t = \frac{\alpha^2}{4} + 2\alpha^2 + 0$$

$$\eta = \frac{\text{useful power}}{\text{Total power}} = \frac{P_s}{P_t} = \frac{P_s}{P_s + P_c + 0}$$

(for modulation) $\eta = \frac{m^2}{m^2 + 2}$

$$\eta_{\text{DSB-SC}} = \frac{\frac{1}{2} \overline{m^2(t)}}{\frac{1}{2} \overline{m^2(t)}} = 1 = 100\% \quad "A=0"$$

$m < 1$	large A	↑ worse ↓
$m = 1$	mod A	
$m > 1$	small A	
$m = \infty$	$A = 0$	

Back to the
example

$$\eta = \frac{P_s}{P_s + P_c} = \frac{\frac{1}{2} m^2 (I)}{\frac{1}{2} m^2 (I) + \frac{A^2}{2}} = \frac{\frac{1}{2} \left(\frac{q^2}{2}\right)}{\frac{1}{2} \left(\frac{q^2}{2}\right) + \frac{A^2}{2}}$$

$$m = \frac{-m(4) \pm \sqrt{m^2(16) - 4(1)(-A^2)}}{2(1)} \quad m = \frac{q}{A}$$

$$A = \frac{q}{m}$$

$$\eta = \frac{\frac{q^2}{4}}{\frac{q^2}{4} + \frac{q^2}{2m^2}} = \frac{\frac{4m^2}{q^2}}{\frac{4m^2}{q^2} + \frac{2m^2}{q^2}}$$

$$\eta = \frac{m^2}{m^2 + 2}$$

$$m = 0.5 \quad \eta = 11.111\%$$

$$m = 1 \quad \eta = 33.333\%$$

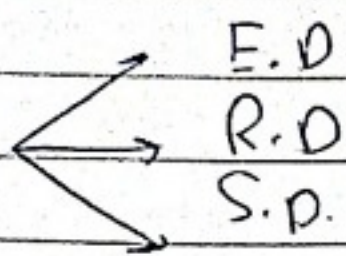
$$m = 2 \quad \eta = 66.666\%$$

$$m = \infty \quad \eta = 100\%$$

For voice when $m=1$ $\eta = 25\%$

Missing Lecture

AM TX
AM RX



E.D
R.D
S.P.

} 2 synch demodulators
① No L.O at the RX side

② which means there is no need for perfect synch.

$V_o(t) = E(t) + \text{ripple} = [m(t) + A] + \text{ripple}$ assuming $\frac{d(t)}{A_m}$
under or critically modulated.

Increase $(\tau - RC)$ to minimize ripple. X

If we increase τ , there is a risk that we don't follow the envelope

High τ : don't follow the envelope

low τ : too much ripple

Intermediate τ is the desired.

In practice we $\Rightarrow \omega_m \gg \omega_c$

You want $T_c \ll \tau \ll T_m$

$$\frac{1}{f_c} \ll \tau \ll \frac{1}{f_m}$$

$$\frac{1}{f_c} \ll \tau \ll \frac{1}{B}$$

\uparrow Bandwidth of the message

in Design B

$$\alpha_1 = \frac{1}{\pi}$$



unless you use a DC coupling capacitor

unless you use LIF

E.D output = $E(t) + \text{ripple}$

R.D output = $\frac{1}{\pi} E(t) < 0.333 E(t)$

unless you use 2 DC coupling capacitor.

Asynch	<u>RX</u> cheap	<u>TX</u> 2	is very small ($\frac{\text{wasted power}}$)
--------	--------------------	----------------	--

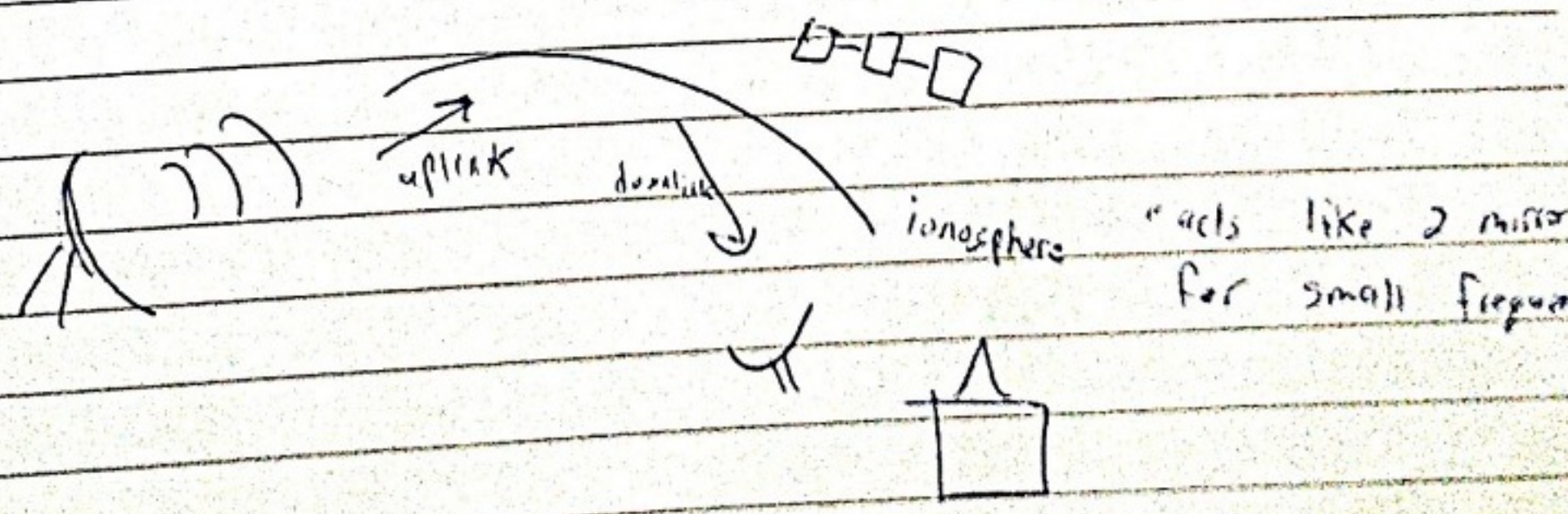
synch	expensive	2	→ 100% (DSD-SC)
-------	-----------	---	-----------------

* TV Broadcasting frequencies

VHF: very high frequency	30 - 300 MHz
UHF: ultra high freq	300 - 3 GHz
	MHz

+ Satellite TV Broadcasting

AB	AC	C Band	uplink 6 GHz	Downlink 4 GHz
		Ku Band	14 GHz	11 - 12 GHz
		Ka Band	30 GHz	18 - 20 GHz



AM Radio Broadcasting also bandwidth of the tunable BPF

US: 10 kHz
AM voice = 4 kHz $\xrightarrow{\text{AM modulation}}$ 8 kHz = 2B
Guard Band = 1 kHz

Euro: 9 kHz $0.5 + 8 \text{ kHz} + 0.5$ $\xrightarrow{\text{guard band}}$
Bandwidth of tunable BPF

AM Super heterodyne Receiver

RF \equiv Radio frequency $\sim 1 \text{ MHz}$ "high frequency"

IF \equiv intermediate frequency smaller than ω_c
"455 kHz"

Frequency Modulation "FM"

\rightarrow FM and PM are equivalent "duality"

Angle modulation

FM :- Frequency of $c(t)$ varies with $m(t)$

PM :- Phase of $c(t)$ varies with $m(t)$

$$c(t) = A \cos(\underbrace{\omega_c t}_{\text{carrier frequency}} + \theta_0)$$

frequency $\left\{ \begin{array}{l} \omega_c: \text{carrier freq: constant decided by the oscillator} \\ \omega_i(t): \text{instantaneous frequency } \omega_i \text{ is variable} \end{array} \right.$

- 1) Generalized angle $\phi(t)$ $c(t) = A \cos\{\omega_c t + \phi(t)\}$
- 2) Instantaneous phase $\phi_i(t)$ $\downarrow \phi(t)$
- 3) Instantaneous frequency $\omega_i(t)$

$$c(t) = A \cos(\phi(t))$$

$$\phi_i(t) = \phi(t) - \omega_c t$$

$$\omega_i(t) = \frac{d}{dt} \phi(t)$$

example $c(t) = A \cos(\omega_c t + \phi_0)$

$$\left. \begin{array}{l} \phi(t) = \omega_c t + \phi_0 \\ \phi_i(t) = \phi_0 \\ \omega_i(t) = \omega_c \end{array} \right\} \text{only specified for unmodulated carrier.}$$

$$\omega_i(t) \propto m(t)$$

FM $\left\{ \begin{array}{l} A \text{ constant} \\ \omega_c \text{ constant} \\ \omega_i(t) = \omega_c + k_f m(t) \\ \phi_i(t) = k_f \int_{-\infty}^t m(t) dt \end{array} \right.$

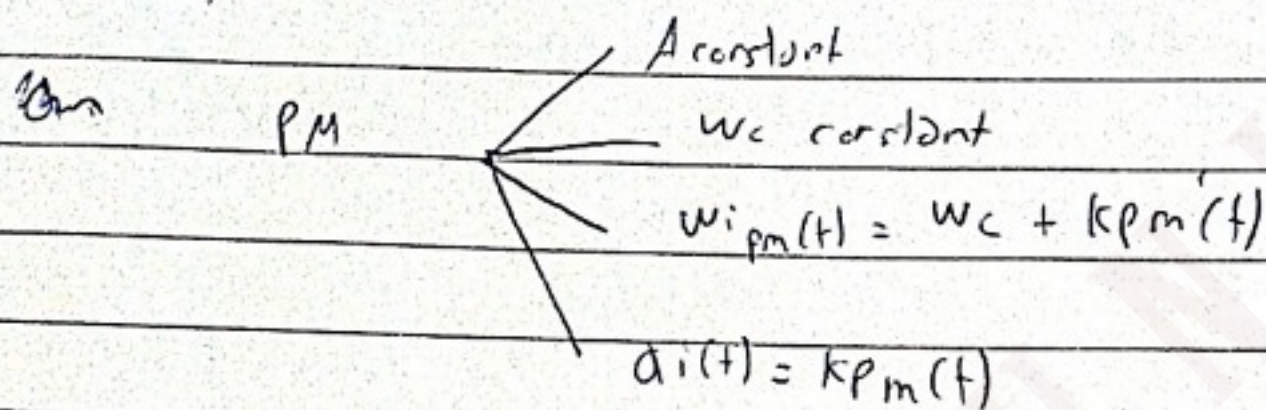
$$\phi(t) = \int_{-\infty}^t \omega_i(z) dz$$

For FM $\phi_{FM}(t) = \int_{-\infty}^t [\omega_c + k_f m(t)] dt$

$$\phi_{FM}(t) = \omega_c t + k_f \int_{-\infty}^t m(t) dt$$

PM modulation

$$\phi_i(t) = \phi_0 + k_p m(t) = k_p m(t)$$



$$\omega_i(t) = \frac{d}{dt} \phi_{PM}(t) = \frac{d}{dt} [\omega_c t + k_p m(t)] = \omega_c + k_p \frac{d}{dt} m(t)$$

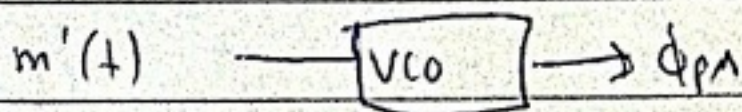
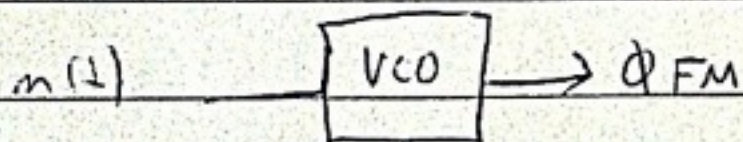
$$\phi_{FM}(t) = A \cos \left(\omega_c t + k_f \int_{-\infty}^t m(t) dt \right)$$

$$\omega_i = \omega_c + k_f m(t)$$

$$\phi_{PM}(t) = A \cos (\omega_c t + k_p m(t))$$

$$\omega_i = \omega_c + k_p m'(t)$$

VCO :- voltage controlled oscillator "FM modulator"



ex 11 $f_c = 100 \text{ MHz}$

Slide "7"

$$\omega_c = \frac{2\pi (100 \text{ M}) \text{ rad}}{\text{s}}$$

(a) $\omega_i(t) = \omega_c + k_f (4) = 100 \text{ MHz}$
max

$$\omega_{i \text{ min}}(t) = \omega_c + k_f (-4)$$

$$f_{i \text{ max}} = 100 \text{ MHz} + \frac{2\pi \times 10^5 (4)}{2\pi} = 100.4 \text{ MHz}$$

$$f_{i \text{ min}} = f_c + \frac{k_f}{2\pi} (-4) = 100 \text{ MHz} + \frac{2\pi \times 10^5 (-4)}{2\pi} = 99.6 \text{ MHz}$$

* $\Delta f =$ peak frequency deviation

$$\Delta f = \frac{f_{\text{max}} - f_{\text{min}}}{2}$$

↙ inst frequency

For part (a) $\Delta f = \frac{100.4 - 99.6}{2} = 0.4 \text{ MHz}$

For FM (general eq),

$$f_{\text{max}} = f_c + \frac{k_f m(t)_{\text{max}}}{2\pi}$$

$$f_{\text{min}} = f_c + \frac{k_f m(t)_{\text{min}}}{2\pi}$$

$$\Delta f = \frac{k_f}{4\pi} (m(t)_{\text{max}} - m(t)_{\text{min}}) = 0.4 \text{ MHz}$$

Δf for PM

$$f_{\max} = f_c + \frac{k_f}{2\pi} m'(t)_{\max}$$

$$f_{\min} = f_c + \frac{k_f}{2\pi} m'(t)_{\min}$$

$$\Delta f = \frac{k_f}{4\pi} \left(m'(t)_{\max} - m'(t)_{\min} \right)$$

* $\Delta \phi$ = peak phase deviation ~~of the wave~~

$$\Delta \phi = \frac{\phi_{i\max} - \phi_{i\min}}{2} \quad \rightarrow \text{inst phase}$$

$\Delta \phi$ FM

$$\phi_{i\max} = k_f \left[\int_{-\infty}^t m(t) dt \right]_{\max}$$

$$\phi_{i\min} = k_f \left[\int_{-\infty}^t m(t) dt \right]_{\min}$$

$$\Delta \phi = \frac{k_f}{2} \left[\int_{\max} m(t) dt - \int_{\min} m(t) dt \right]$$

$\Delta \phi_{PM}$

$$\phi_{\max} = k_f m(t)_{\max}$$

$$\phi_{\min} = k_f m(t)_{\min}$$

$$\Delta \phi_{PM} = \frac{k_f}{2} \left(m(t)_{\max} - m(t)_{\min} \right) \quad \begin{array}{l} \text{in rads} \\ \text{if degrees } \times 180 \end{array}$$

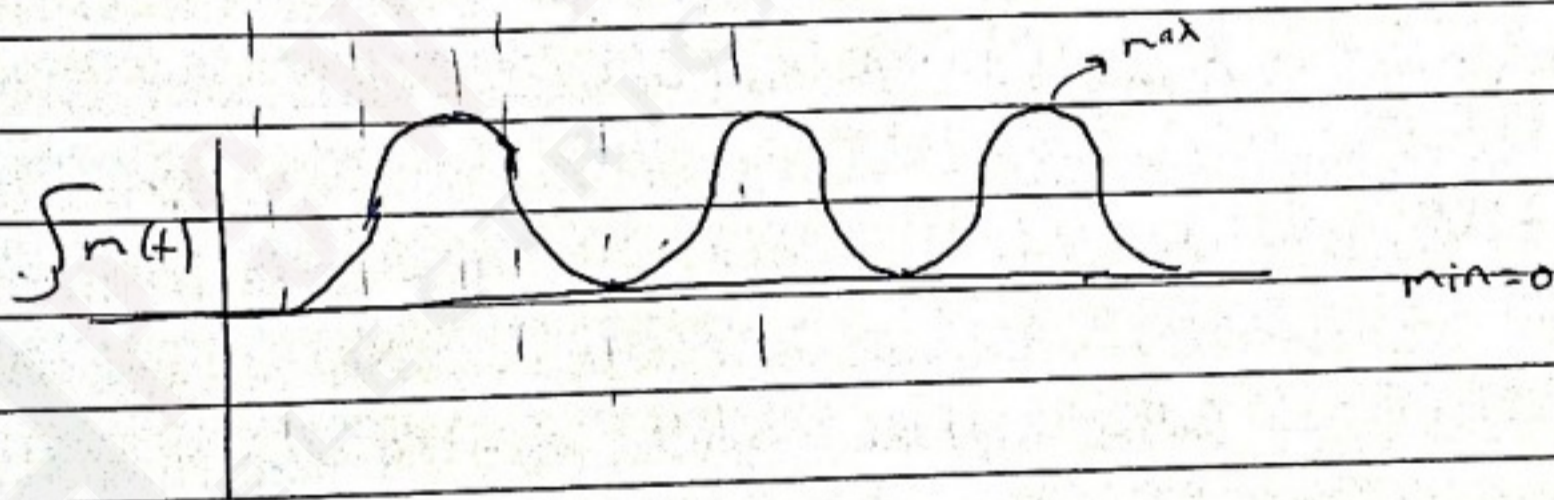
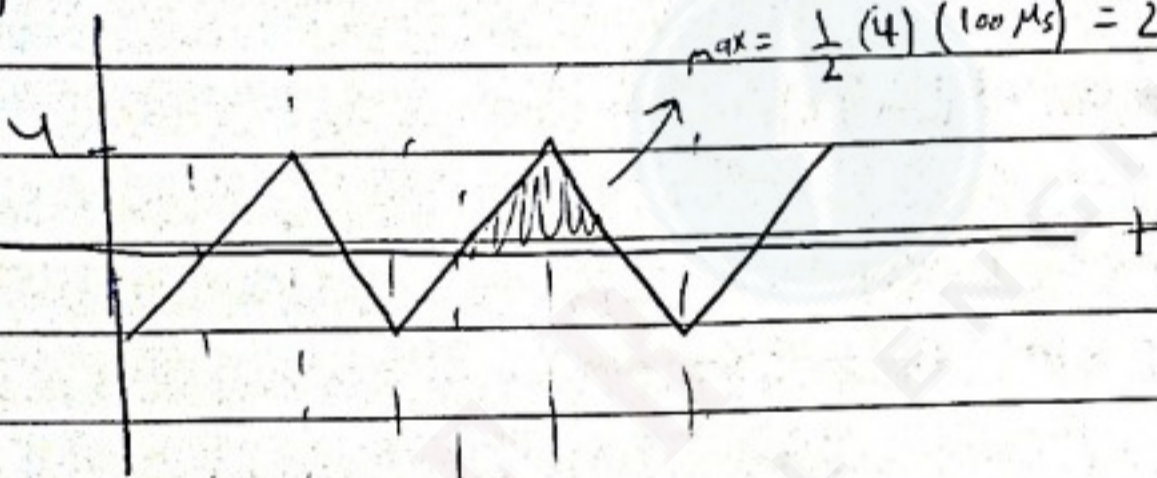
$$(b) f_{\max} = 100 \text{ MHz} + \frac{5\pi}{2\pi} (80,000) = 100.2 \text{ MHz}$$

$$f_{\min} = 99.8 \text{ MHz}$$

$$\Delta f_{\text{FM}} = \frac{100.2 - 99.8}{2} = 0.2 \text{ MHz}$$

$$\underline{\text{OR}} \quad \Delta f_{\text{FM}} = \frac{5\pi}{4\pi} (80,000 - -80,000) = 0.2 \text{ MHz}$$

$$m(t) \quad \text{max} = \frac{1}{2} (4) (100 \mu\text{s}) = 200 \text{ V} \cdot \mu\text{s} = 200 \times 10^{-6} \text{ V} \cdot \text{s}$$



For FM

$$\Phi_{\max} = k_f \left[\int m(t) dt \right] = 2\pi \times 10^5 \times 200 \times 10^{-6} = 40\pi \text{ rad}$$

$$\Phi_{\min} = k_f \left[\int m(t) dt \right] = 0 \text{ rad}$$

$$\Delta \Phi = \frac{40\pi - 0}{2} = 20\pi \text{ rad}$$

$$\text{If degree} \times \frac{180}{\pi} = 3,600^\circ$$

$$\Delta \phi_{pm} = \frac{5\pi}{2} (4 - (-4)) = 20\pi$$

if $m(t) = \text{Polar NRZ} + \text{FM} = \text{FSK}$ "digital modulation"



if $m(t) = (\text{Polar NRZ}) + \text{PM} = \text{BPSK}$ ~ Binary phase shift keying.

ex 2

$$\phi_{pm}(t) = A \cos(\omega_c t + k_p m(t))$$

$$\omega_{ipm}(t) = \omega_c + k_p m'(t)$$

if $m(t) = 0 \Rightarrow \omega_{ipm}(t) = \omega_c$

if $m(t) = \infty \Rightarrow \omega_i = \infty$ for $\geq 40 + \pi$

$$m(t_0^-) = 2V$$

$$\phi_{pm}(t_0^-) = A \cos(\omega_c t_0^- + \frac{\pi}{4}(2))$$

$$m(t_0^+) = -2V$$

$$\phi_{pm}(t_0^+) = A \cos(\omega_c t_0^+ + -\frac{\pi}{4}(2))$$

Sudden phase shift = -180°

FM modulation index

if β large ($\beta > 1$)
wideband FM

in which case the bandwidth of $\phi_{pm}(t)$ is large

if β is small ($\beta < 1$)

Slide 22 2nd exam 2' not included

Special cases $\Phi_{FM}(\omega)$

① Fsk $\Rightarrow m(t) = \text{Polar NRZ} + \text{FM}$

② Tone modulation

$\Rightarrow m(t) = \alpha \cos(\omega_m t) + \text{FM}$

Sketch $\phi_{FM}(t)$, $\mathcal{F}_{FM}(\omega)$, bandwidth

Slide 24

$J_n(\beta)$ \equiv Bessel function of the 1st kind and the n^{th} order.

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} dx$$

ex) $J_2(\beta=2) = 0.353$ from table

ex) $J_{-3}(\beta=2) = -J_3(\beta=2) = -0.129$

Find Pave in $\phi_{FM}(t)$

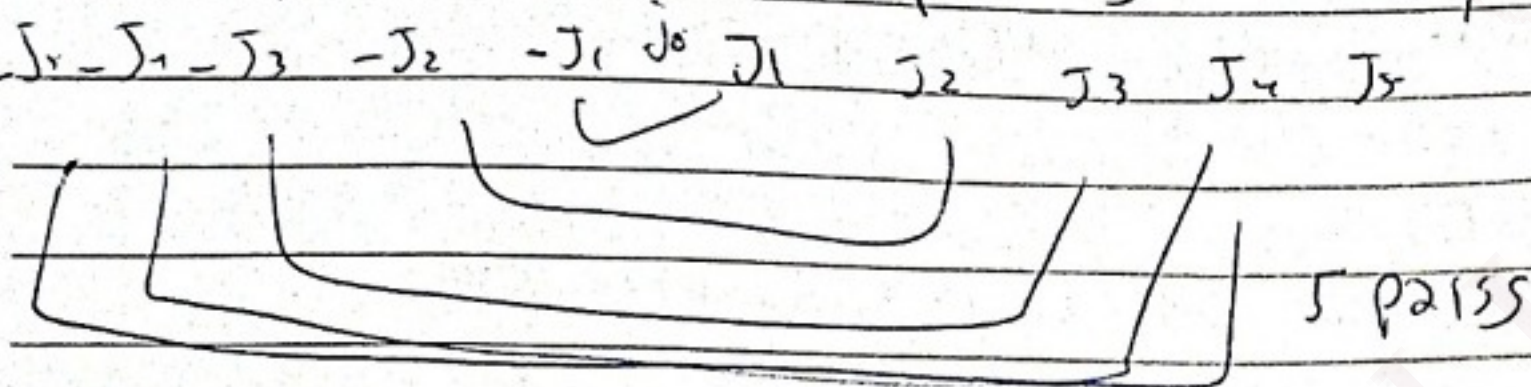
Sol #1 = $\overline{\phi^2_{FM}(t)} = \frac{A^2}{2}$

Sol #2 = $\sum_{n=-\infty}^{\infty} \left(\frac{[A J_n(\beta)]^2}{2} \right) = \frac{A^2}{2} \times 1 = \frac{A^2}{2}$

$|J_n| < 0.1$ then J is insignificant

$\beta = 2 \Rightarrow$ SSP: significant sideband pairs = 3 = $\beta + 1$

$\beta = 4 \Rightarrow$ SSP = 5 = $\beta + 1$



Bandwidth for $\beta = 2$ 3 pairs (2 um) = 6 um rad/s

$\beta = 4$ 5 (2 um) = 10 um rad/s

Bandwidth FM = $2B(\beta + 1)$ Hz or $2\beta + 2\Delta F$

* To overcome fading in FM we use AGC (automatic gain control)

* Adjacent-channel interference shows up in FDM systems

* Pre-emphasis in FM radio TX } improves SNR
* De-emphasis in FM Radio RX }

Stereophonic FM

Monophonic FM

Lecture 16 :- VCO: voltage - controlled oscillator = FM modulator.
= modified oscillator

→ amplitude variations

$$\phi_{AT} = \frac{d\phi_{FM}}{dt}$$

$$\phi_{FM}(t) = A \cos(\omega_c t + k_f \int m(t) dt)$$

$$\frac{d}{dt} \phi_{FM}(t) = +A(\omega_c + k_f m(t)) \sin(\omega_c t + k_f \int m(t) dt + \pi)$$

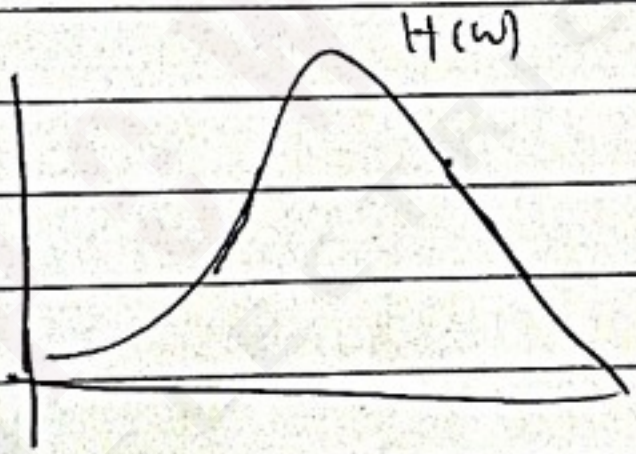
↓

$$F_{envelope} = F(t) = \text{const} + \text{const } m(t)$$

An LC tank can be used as a differentiator $\frac{d}{dt}$

An LC resonant circuit can also be used as a BPF

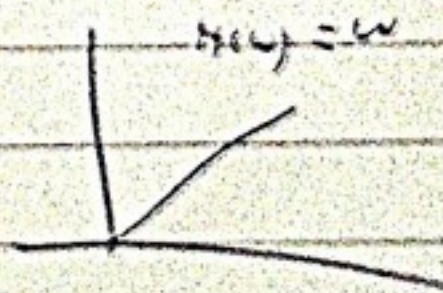
$$\omega_{center} = \omega_{res}$$



Time \leftrightarrow Freq
 $\frac{d}{dt} \leftrightarrow$ linear curve

$$F\left\{\frac{d}{dt} x(t)\right\} = j\omega X(\omega)$$

$$|j\omega| = \omega$$



Lecture 17

FDM $\xrightarrow{\text{requires}}$ modulation
 TDM \longrightarrow digital baseband

Why do we need delay in TDM
 and how much is it for bit interleaving
 was interleaving.

ll. w assume

mA : bipolar $f_0 = 100 \text{ kbit/s}$

mB : Manchester $f_0 = 300 \text{ kbit/s}$

mC : Duobinary $f_0 = 500 \text{ kbit/s}$

mD : Polar NRZ $f_0 = 1000 \text{ kbit/s}$

using slide #8

Sketch PSD for $m_3(t)$.

Assuming unipolar RZ

Typical guard band = 5% of bandwidth

Slide 11

$E_1 = 32$ PCM streams

Synch 211 TX/RX \rightarrow sine master clock ^{^Very expensive}

ex) Systems that use TDM and FDM

- GSM cellular

- DVB

digitization \rightarrow modulation

BPF \rightarrow demod \rightarrow TDM \rightarrow de-mux \rightarrow de-digitization

lecture 19

Constellation diagram (PSK and QAM): easy way to sketch time-domain

BER example.

Constellation diagram

I axis: $\cos(\omega t)$

Q axis: $\sin(\omega t)$

ccw \rightarrow -ve in c.d

Slide 11 :-

10 $\rightarrow A \cos(\omega t + 45^\circ)$

11 $\rightarrow A \cos(\omega t - 45^\circ)$

Slide 14:

$$r = \sqrt{(3A)^2 + (A)^2} = \sqrt{10} A$$

$$\theta = -\tan^{-1}\left(\frac{A}{3A}\right) = -18.4^\circ$$

$$1001 \rightarrow A\sqrt{10} \cos(\omega t - 18.4^\circ)$$

$$\text{QAM} \rightarrow 3A \cos(\omega t) + A \sin(\omega t)$$

BER example (16-QAM)

means quality "BER $\leq 10^{-6}$ good quality"

Slide 22

$K = \text{bits per symbol} = 4$ for 16-QAM

$M = \text{number of symbols} = 16$

$E_b = \text{average energy per bit} = \frac{E_s = \text{average energy per symbol}}{K}$

ex $E_s = 4J$ $K = 4$ $E_b = 1J$

$E = \text{Power} \times \text{time} = P_{\text{symbol}} \times T_{\text{symbol}}$

for QAM $E = \frac{(A_{\text{symbol}})^2}{2} \times T_{\text{symbol}}$

Take average of all points then take its power.

$E_s = E_{s1} \cdot Pr[s_1] + E_{s2} \cdot Pr[s_2] + \dots + E_{s16} \cdot Pr[s_{16}]$

ex $\frac{(1.414)^2}{2} \cdot T_{\text{symbol}} \cdot \frac{1}{16} + \text{Symmetric point} + \dots$

4 symmetric "equal" points on slide

$T_{\text{symbol}} = 4T_0 = kT_0$

$\frac{1}{T_{\text{symbol}}} = \text{band frequency}$ $\frac{1}{T_0} = f_0$

if given data rate $\rightarrow f_0$
if = band rate $\rightarrow f_{\text{symbol}}$

linear interpolation

$$3.15 \longrightarrow 8.16 \times 10^{-4}$$

$$3.162 \longrightarrow x$$

$$3.20 \longrightarrow 6.87 \times 10^{-4}$$

$$\frac{3.2 - 3.15}{3.2 - 3.162} = \frac{6.87 \times 10^{-4} - 8.16 \times 10^{-4}}{6.87 \times 10^{-4} - x}$$