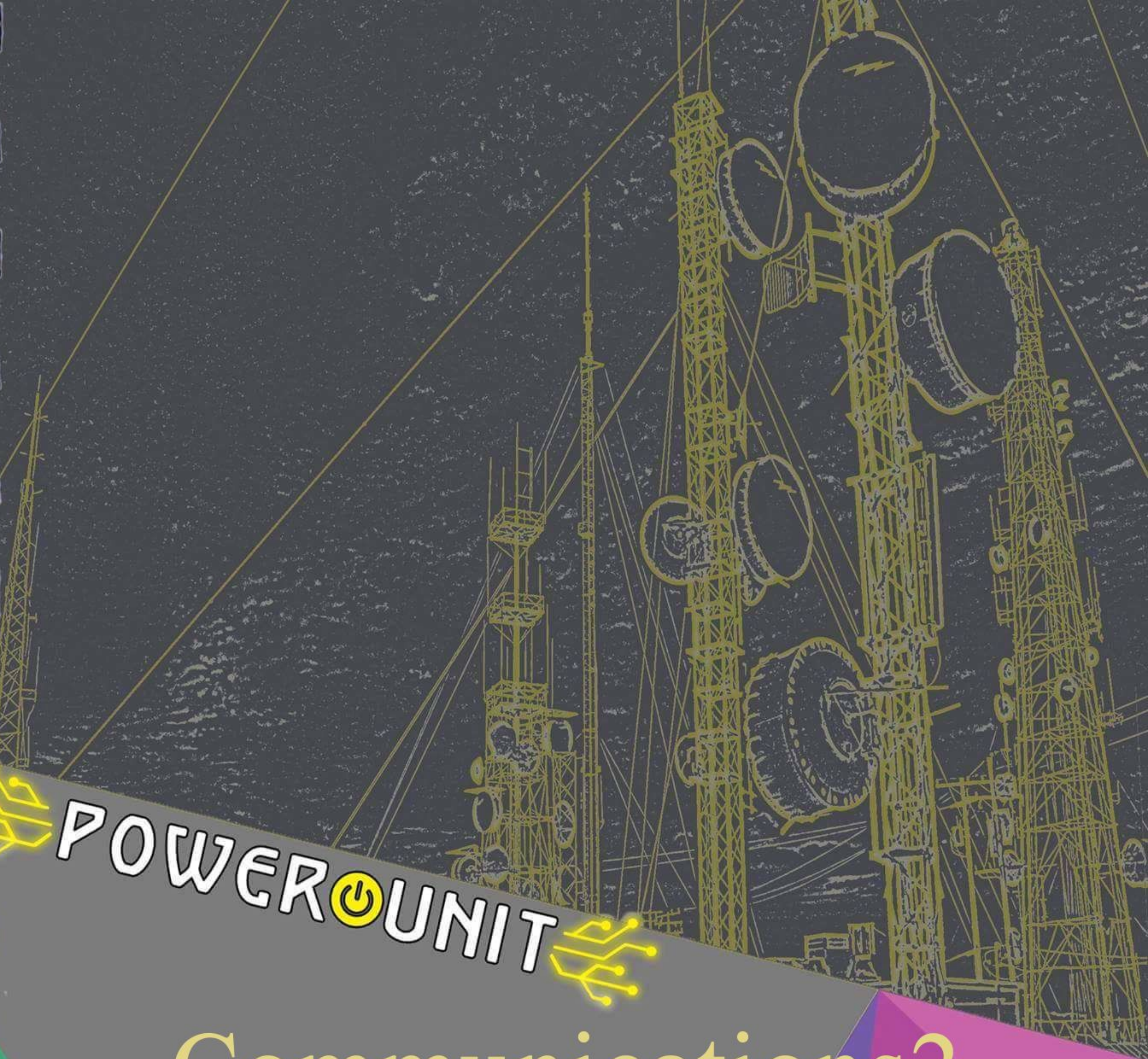

POWERUNIT

Communications2

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إعداد المهندس

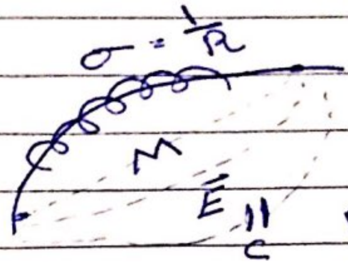
كريم محتسب

Fall2016

Communication II

Lecture.1

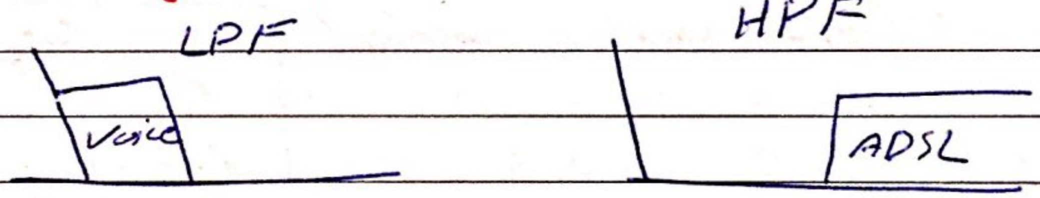
- Implimintation



as f increases ⇒ Design become difficalt

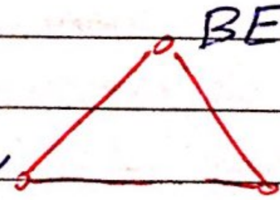
- Trade-off between cost & Quality
- Signal turns ratio
- Data sharing
- Logical channel (software man) & Physical channel (Engineer)
- Error correcting codes
- Tower ≡ Base station
- Resourse Allocation
- The BW of FM channels are 200 KHz ≡ wide American
  100 KHz ≡ narrow Japan & ...
- Toll Quality ≡ minimum Quality
- BW of AM channels are 9 KHz
- Old Internet system:

ADSL



High Digital subscriber Loop (HDSL) Tx & Rx
A " " " " (ADSL)

- Techniques always driven by cost
- Communication Tringle



orthogonal signal: They are on the same space, time & same BW

My no. 55

Lecture. 2

Block diagram for comm. systems usually second Answer

Audio < B.W ~ (4-15 K), Distortion

Video < " ~ (6 MHz)

Data (binary) serial stream

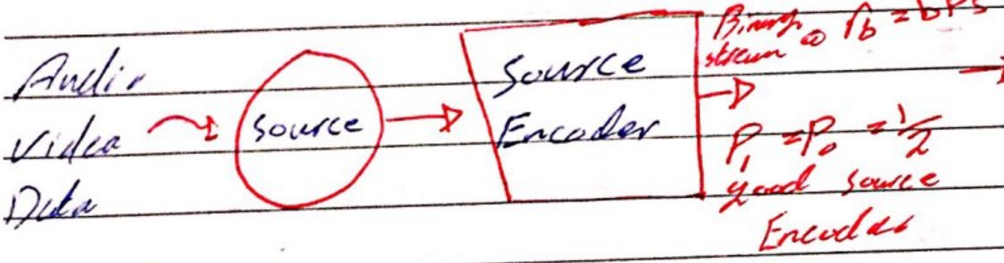
JPEG

MPEG

MP3

Binary R. P. → ~~Random~~ Time sequence of 1's & 0's mean Pr. levels

- Information Always be in random ~~[illegible]~~ Data
- 4 KHz : toll quality for Audio
- Voice, Audio & speech % comm. Systems usually study Audio
- 300 Hz - 15 KHz → Humanbeing Ability
- Engineering solution = cost effective solution
- As BW ↑ clearance ↑
- Linearity devices affect the quality of the signal % default = linear
- Luminance = intensity, chrominance = nature colors



- bit rate Direct Proportional with BW % ISM = unlicensed Bands

→ minimum bit rate as minimum distortion % Data compression

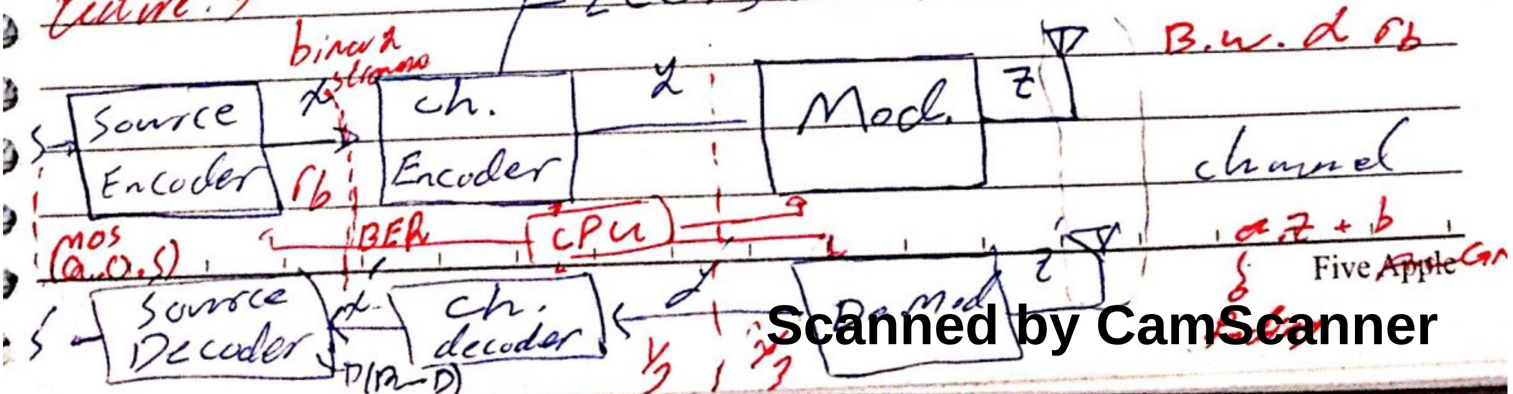
- Binary stream should be independent = uncorrelated

~~Disc as storage by [illegible]~~

Disc → storage by dipoles

CD →

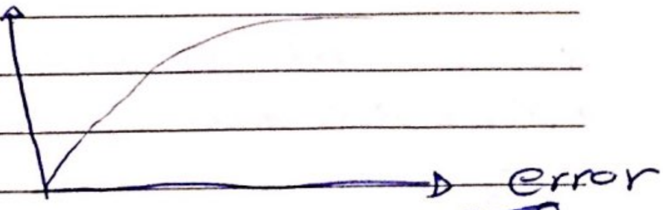
Lecture. 3



Tx → ch. → Rx decaly nulling → signal ≈ 0

fadding → as f ↑

- Distortion → + noise
→ × noise
→ Interference (capacity) → $C = B \cdot \log_2(1 + SNR)$

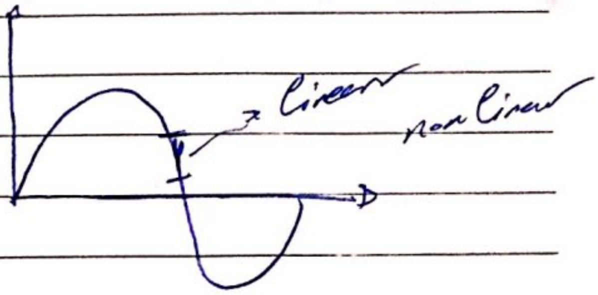


- B.W.
→ capacity

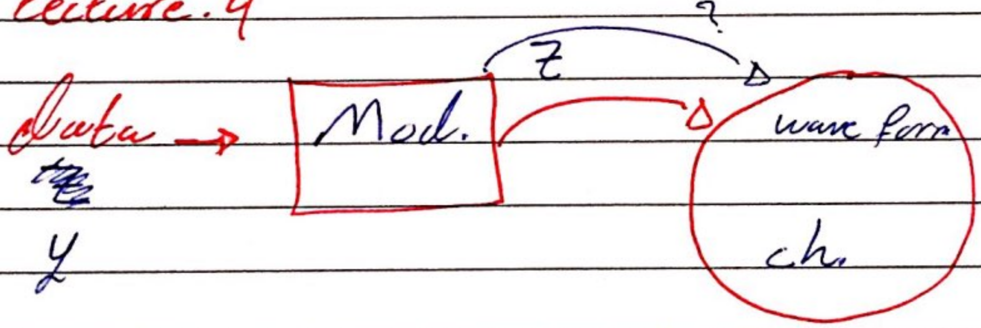
Spectral efficiency = b/s/Hz

mod. ??

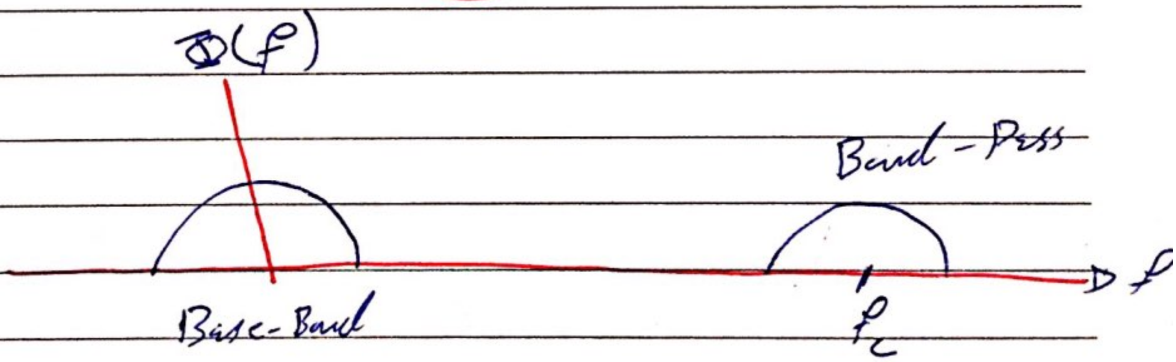
- Power
→ mod. (ch.)



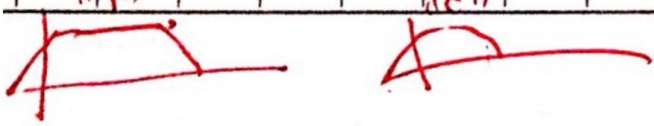
lecture. 4



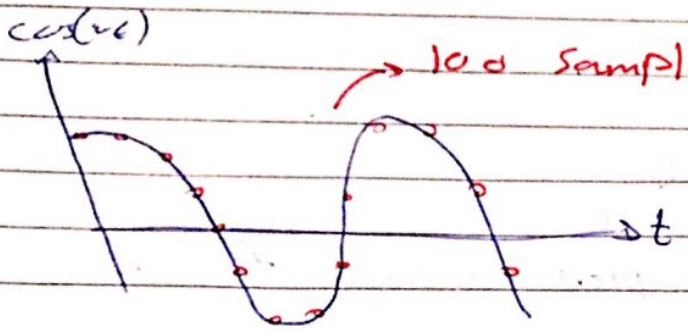
- mod.
→ Base - Band
→ Band - Pass

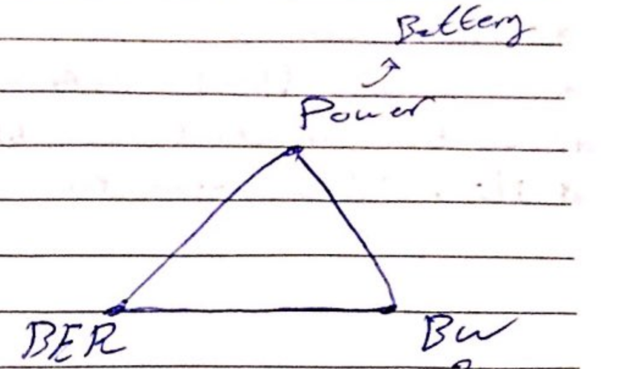


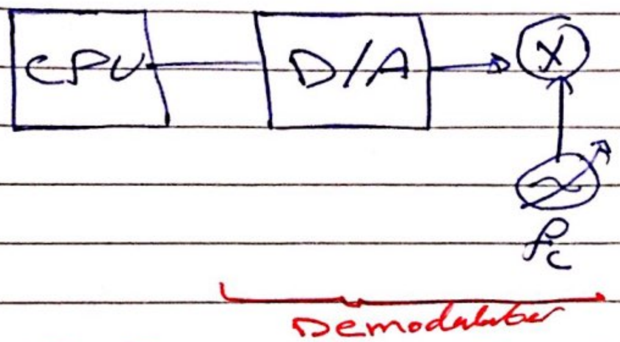
- mod. is mapping between logical Data into Physical wave form



- Base-Band → (Guided ch.) wire TX

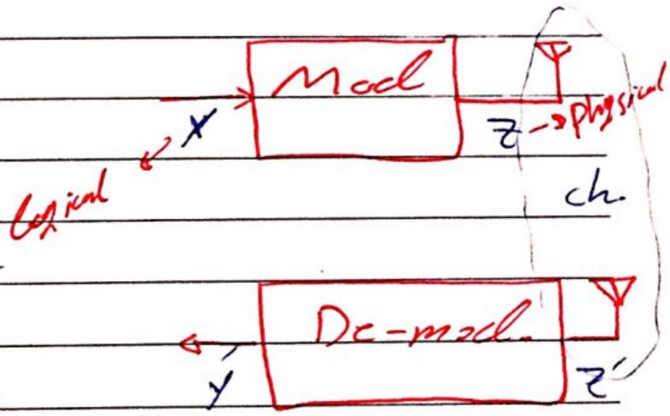






Lecture . 5

- Orthogonality



$Z'(t) = \alpha Z(t) + n(t)$

$\alpha(t)$ is R.V. ⇒ random process

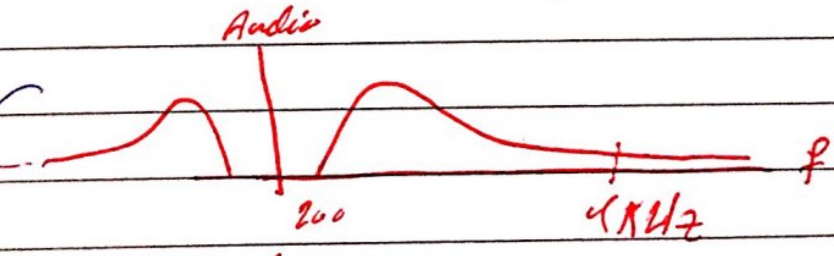
$n(t)$ is AWGN

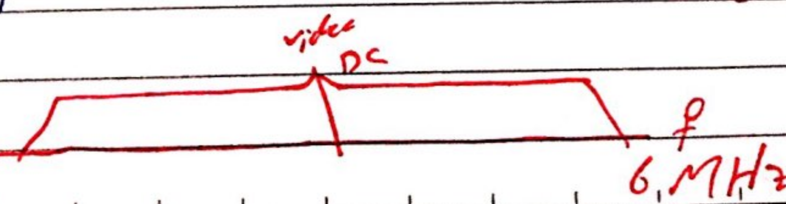
$M_Z = DC$? ⇒ Ergodic

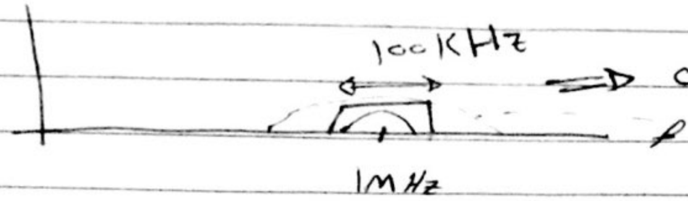
⇒ $Avg_{time} = Avg_{stat.}$ % signals in course are all ergodic

$\sigma_Z^2 = A.C$ Power

$P_{total} = \sigma_Z^2 + |M_Z|^2$



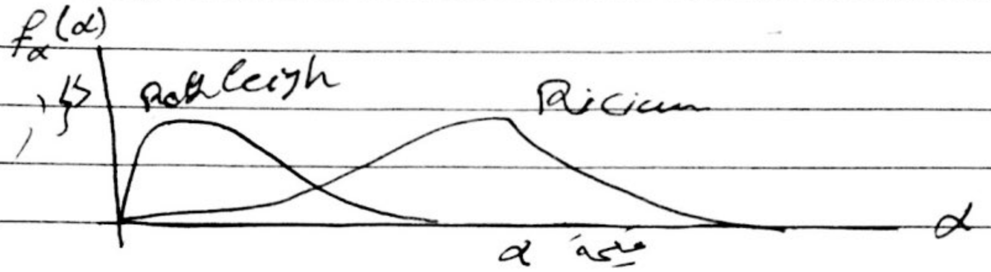




$$z'(t) = \alpha(t) z(t) \underbrace{+ n(t) + \hat{w}(t)}_{n(t)}$$

$S_{zz}(f) \Rightarrow$ PSD

$\alpha(t)$ → rely → worst
→ reisan → pdf



fading caused by multipaths of the same signal
→ can't use ASK

orthogonality

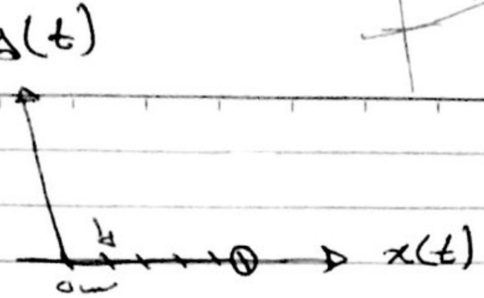
$x(t)$, $y(t)$

→ $\langle x(t), y(t) \rangle = 0$ ⇒ inner product

$$= \int_{-\infty}^{\infty} x(t)\, y^{H}(t)\, dt$$

lecture 6

⟶

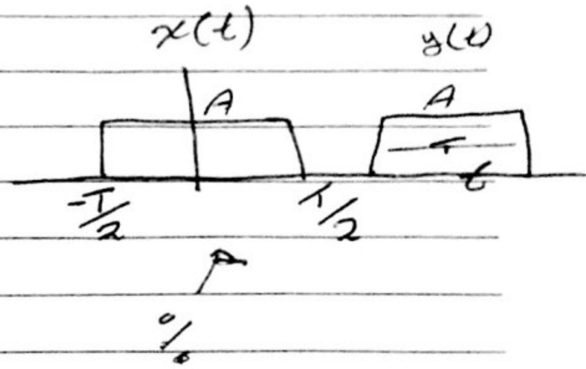


$x_1 = 5\,x(t)$

$x_2 = 7\,x(t)$

$x_3 = x_2 - x_1 = 2\,x(t)$

$\frac{1 \text{unit}}{|x(t)|^2} = 1$



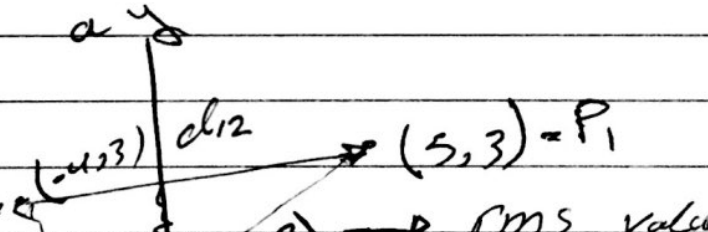
$v_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ $\langle v_1, v_2 \rangle = v_1^T \cdot v_2 = 0 \rightarrow$ orthogonal

$v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

if signals or waveform orthogonal in any space then they are orthogonal in all spaces

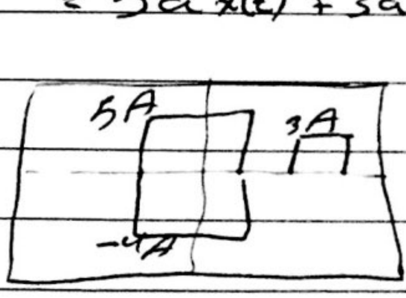
$E_x = A^2 T = 1$

- Any signals not overlabed in any domain are orthogo
- orthonormal space = orthonormal axis



$P_1(t) = 5a_x(t) + 3a_y(t)$

$= 5a\,x(t) + 3a\,y(t)$



$P_2(t) = -4a\,x(t) + 3a\,y(t)$

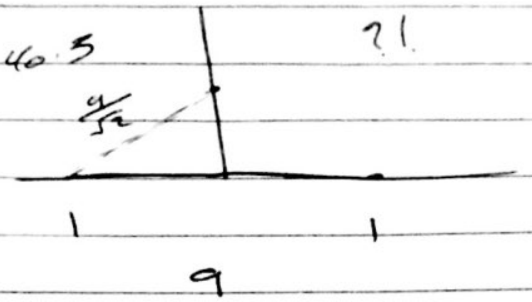
$P_1 = |P_1| \angle \theta_{P_1}$

$P = 34 + 2.5 + 5a$

$P = 40.5$

Five Apple

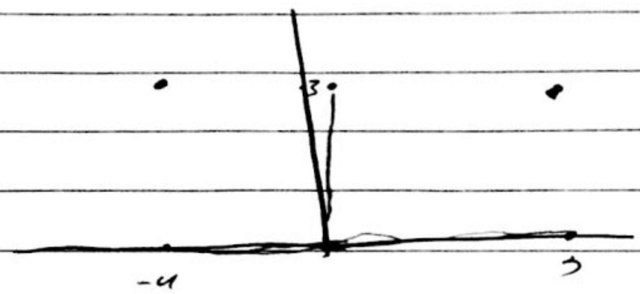
$d_{12} = \sqrt{(5-(-4))^2 + (3-3)^2} = 9 \longrightarrow$ diffrence between wave forms



⇒ anti Bandl = [illegible]

⇒ Pases origin

$\underbrace{\text{Max. Gain}}_{\text{distance}}$ $\underbrace{\text{min. cost}}_{\text{Energy}}$ = Engineering solution

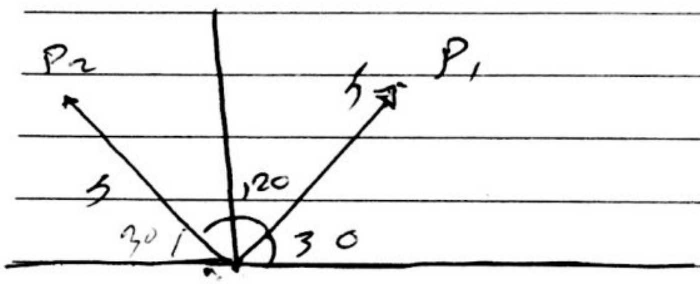


$\frac{x_1+x_2}{2}$, $\frac{y_1+y_2}{2}$

$r = \left(+\frac{1}{2}, 3\right)$

$P_1^* = P_1 - r$

$P_2^* = P_2 - r$



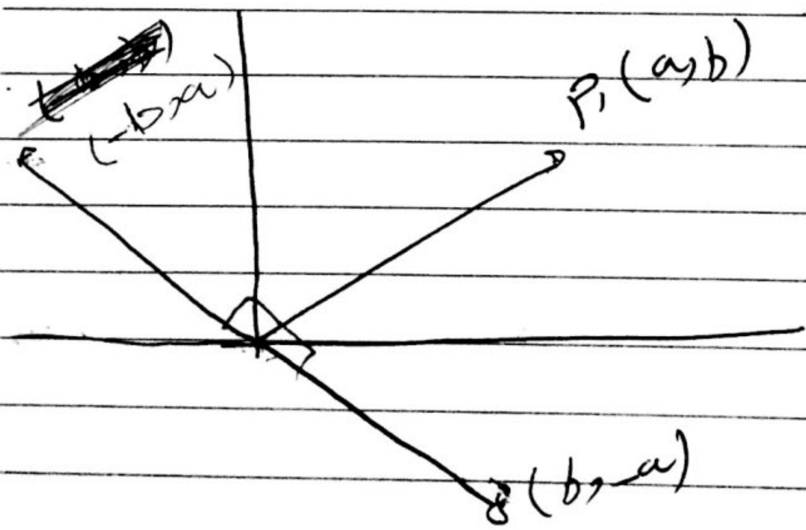
time

frequency

space

code

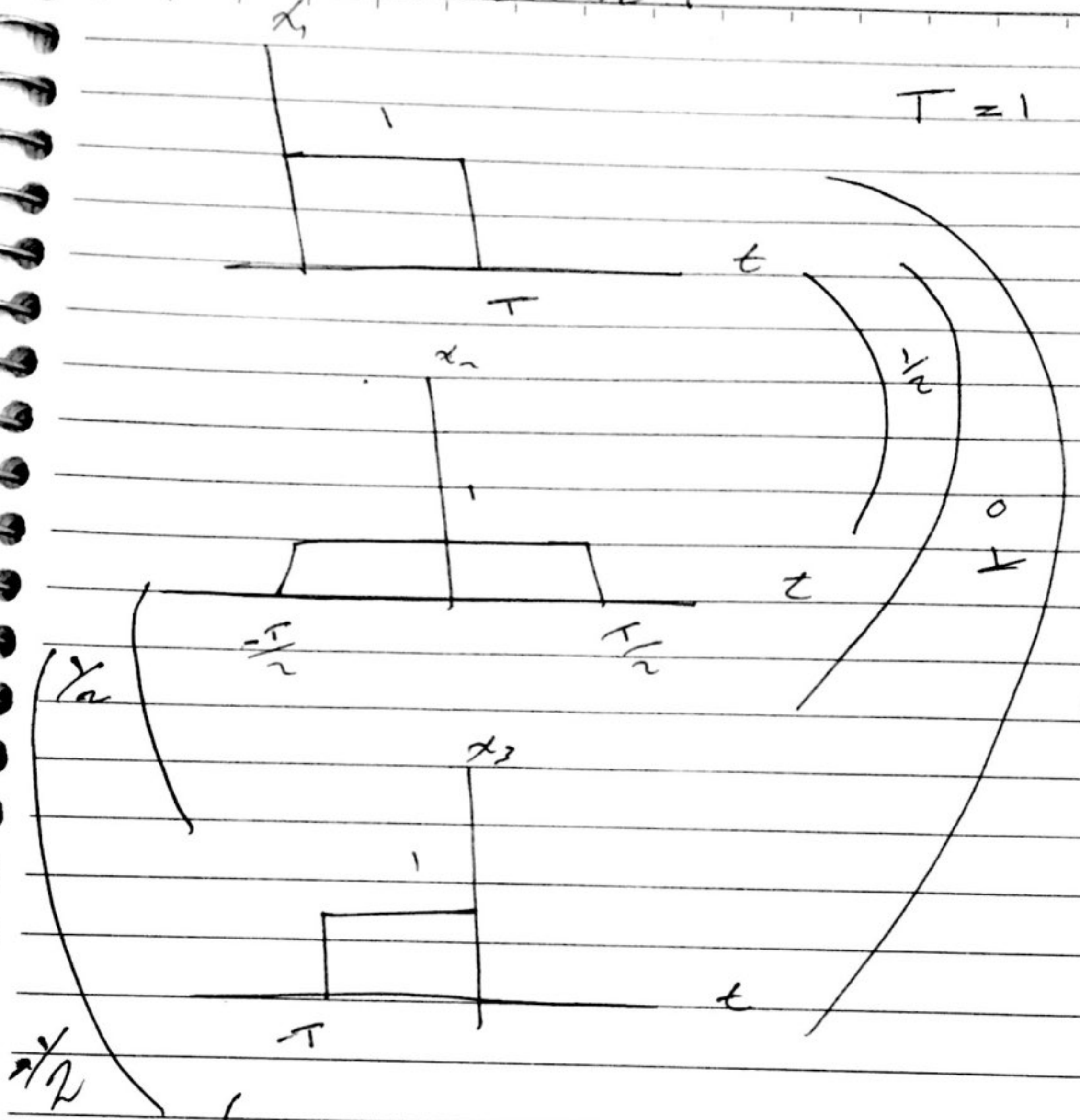
Polarization



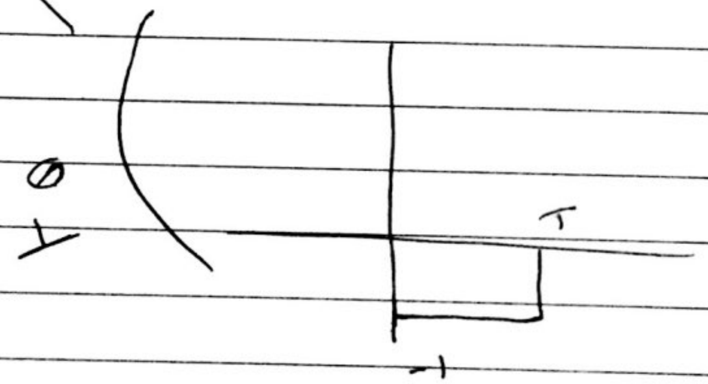
Q1

Lecture 7

$T = 1$



-1 anti Podal



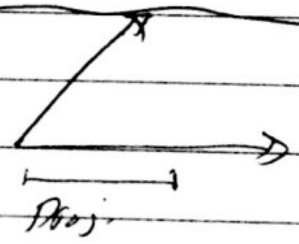
Find $\langle x_i, x_j \rangle \ \forall\, i,j = 1,2,3,4$

$$= |x_i||x_j| \cos(\theta_{ij})$$

$$\cos(\theta_{ij}) = \frac{\langle x_i, x_j \rangle}{|x_i||x_j|} \ ?$$

$$? \quad = \rho_{ij} \rightarrow \text{correlation coefficient}$$

$$\triangleq \sqrt{E_i} \cdot \sqrt{E_j}$$



Five Apple

$\max_{i} \left(\langle x_{re}, x_i \rangle \right)$ $i = 1, 2, 3, 4$

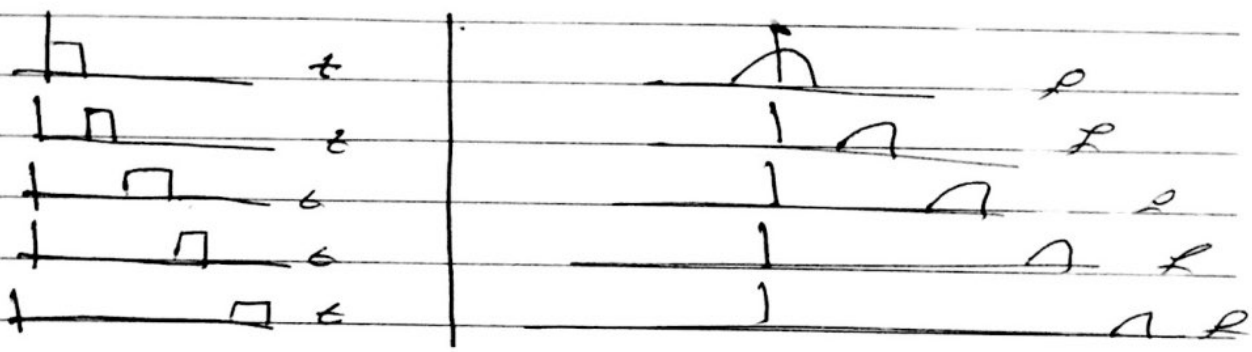
$(: \ i = k)$

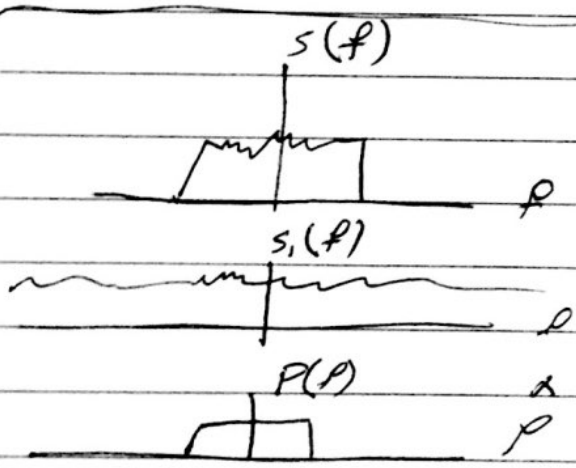
Signal Space

↓

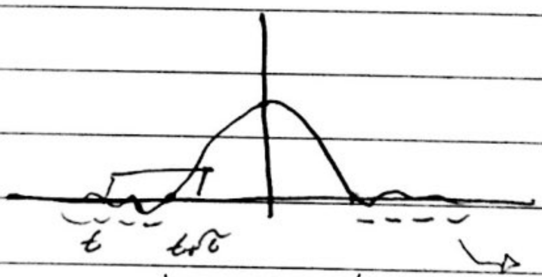
n-dimontions

to create an ~~space~~ N-dimontional space (N-axis (basis functions)) wave forms





$s(t) = s_1(t) * \underbrace{p(t)}_{\text{sinc}(\cdot)}$



→ same distance

→ most ~~Energy~~ Bulk is here

$\int \text{sinc}(t)\, dt = 1$

$\text{sinc}(t) = \dfrac{\sin(\pi t)}{\pi t}$

H.W. $f(x) = e^{\frac{-t^2}{\alpha^2}}$; find $F(f)$?

not causal

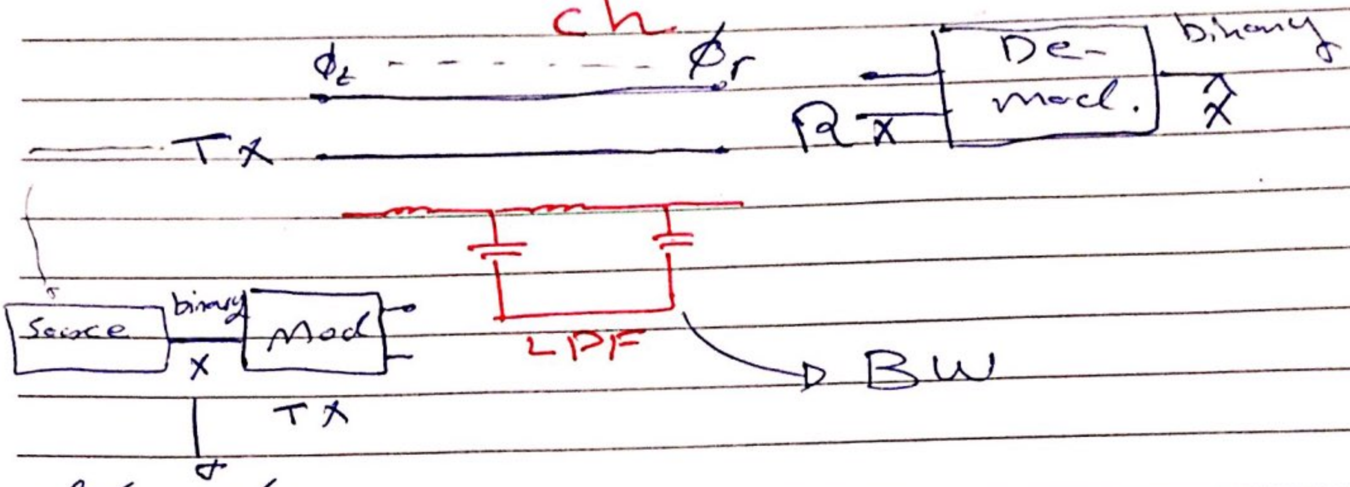
$\alpha \rightarrow$ large → sharp(f)

wired

lecture 8

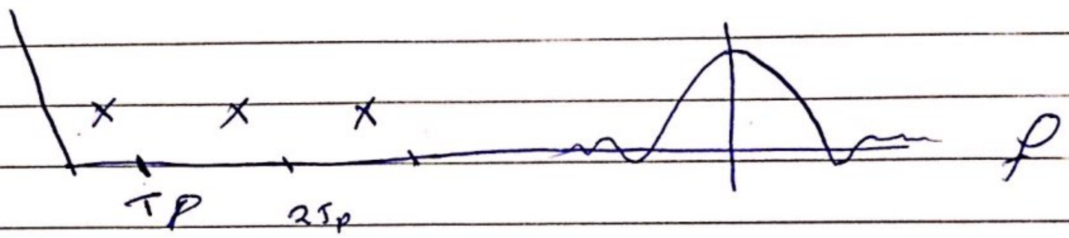
Base-band → BW unlimited

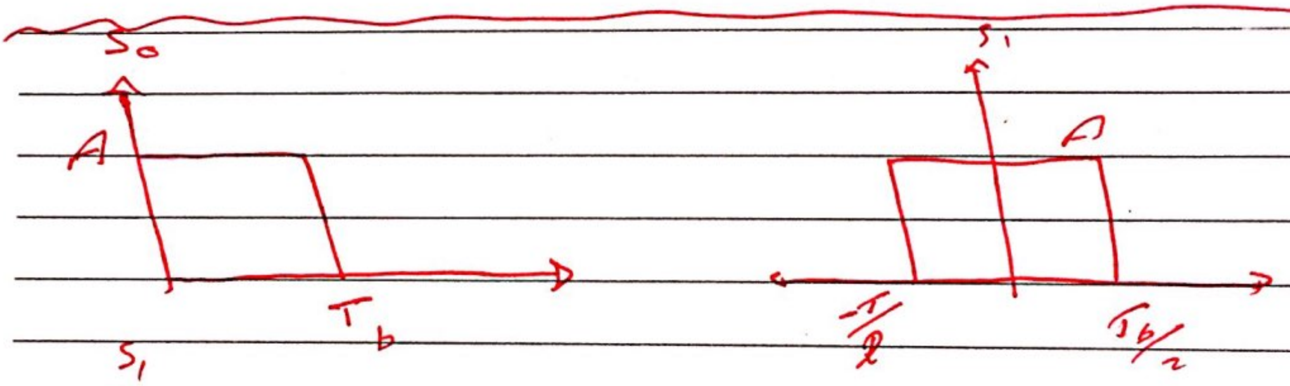
Base-band → BW limited

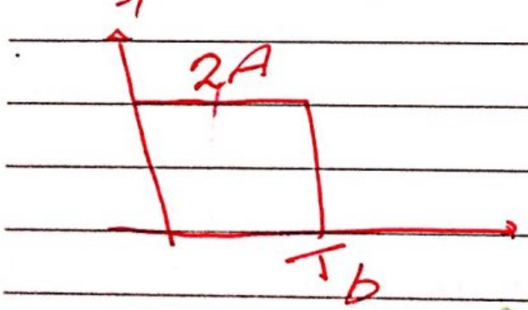


data rate

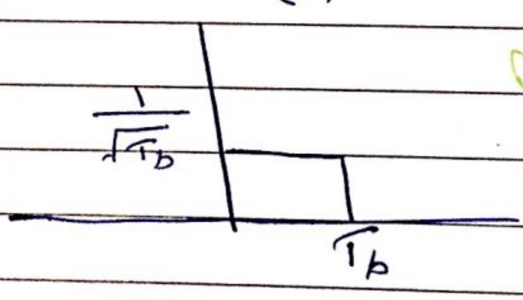
$r_b = \frac{1}{T_p}$ [bPS]

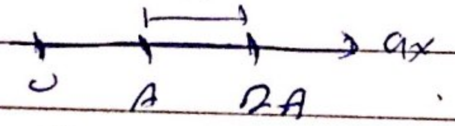






$a_x(t)$



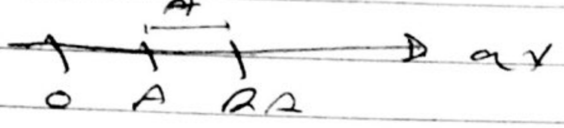


$S_0 = A \cdot \vec{a_x}$

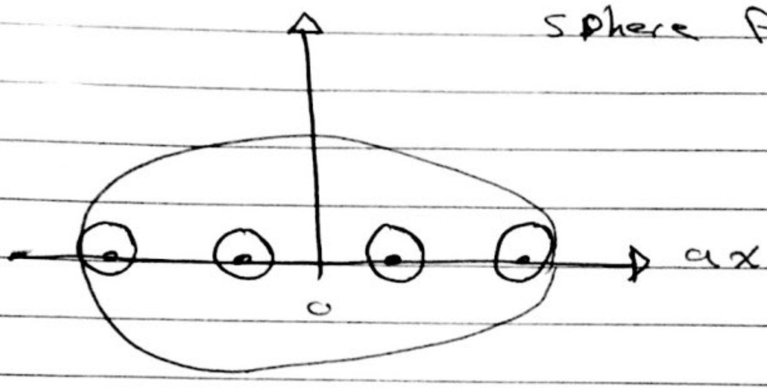
$S_1 = 2A \cdot \vec{a_x}$

$E_{avg} = \frac{5A^2}{2}$

$s_0 = A/2$, $S_1 = a/2$



sphere packing



case of m-bits

$T_s = m T_b$

$r_s = \frac{r_b}{m}$

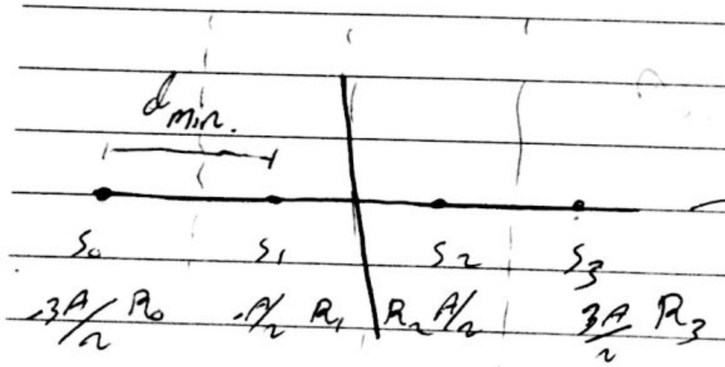
M-array

Example: $m = 2$

→ 4 signal 2^m

mapping table

00 →	S_0
01 →	S_1
10 →	S_2
11 →	S_3



→ signal space diagram

Gray code

00	s_0
01	s_1
11	s_2
10	s_3

$$\begin{cases} S_0(t) = \frac{-3A}{2} a x(t) \\ S_1(t) \\ S_2(t) \end{cases}$$

0	0	0
0	0	1
0	1	1
0	1	0
1	1	0
1	1	1
1	0	1
1	0	0

Gray code

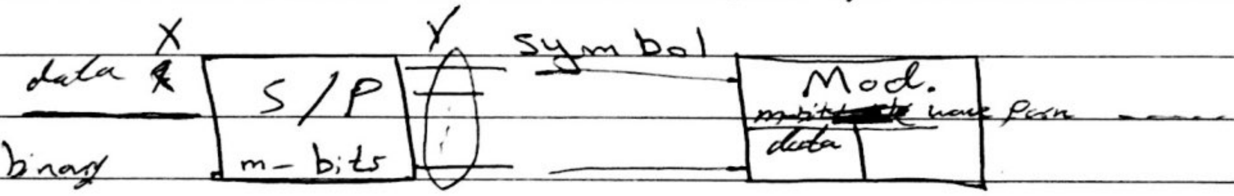
Lecture. 4

Binary TX → $m = 1$

Non-binary $M = 2$

→ (M-ary)

→ m-bits , $2^m = M$ (combination)



r_b bps

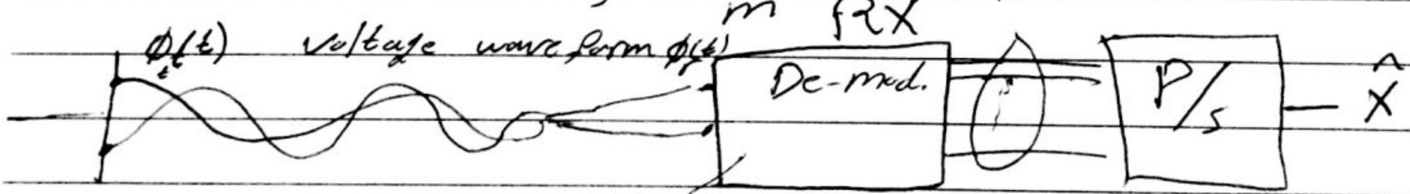
$\left(T_b = \frac{1}{r_b}\right)$

m-bits

$T_s = mT_b$

$r_s = \frac{r_b}{m}$ m-bits ??!

RX

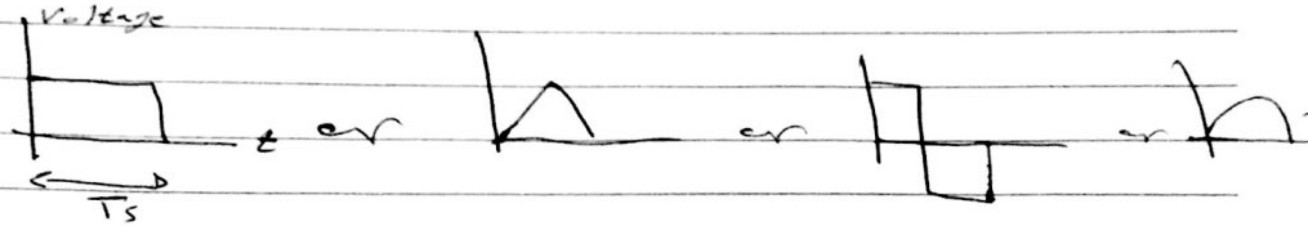


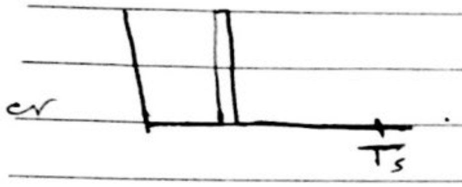
table^{-1}

wave form	data
$\phi_i(t)$	Y

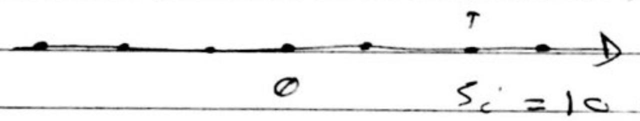
- How many dimensions we need to implement modulation

1-D modulation ⇒ one axis $a_x(t)$



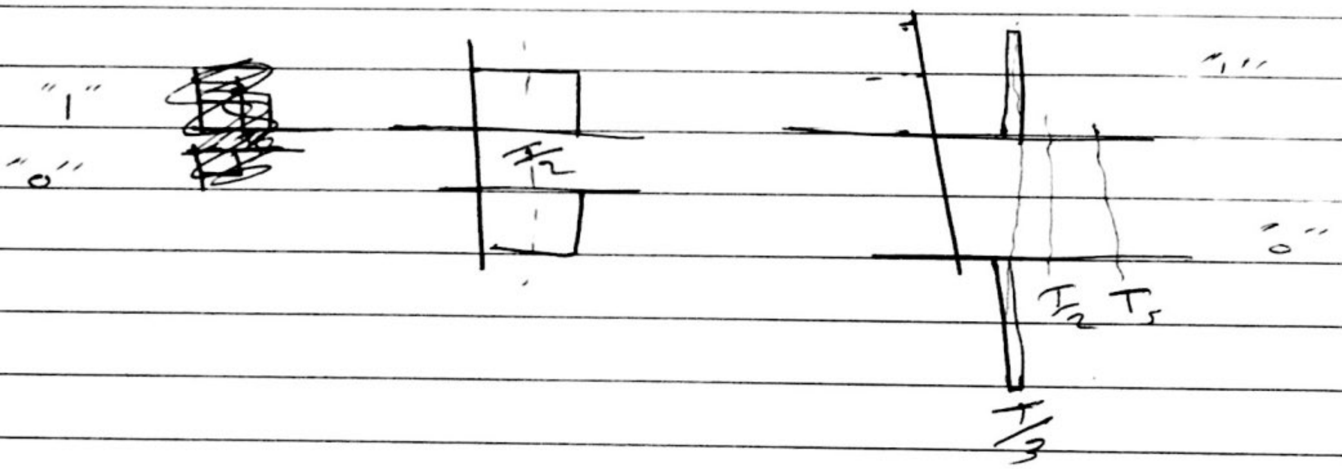


10 units ⇒ signal space diagram



"constellation"

$S_i = 10\,\vec{a_x}$

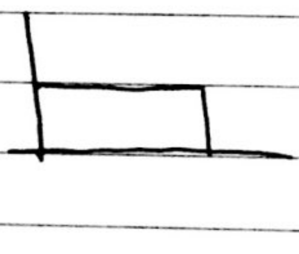


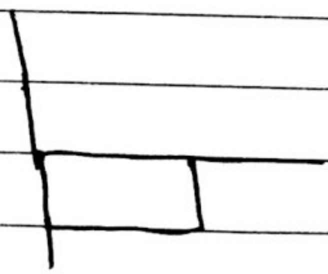
Lecture . 10

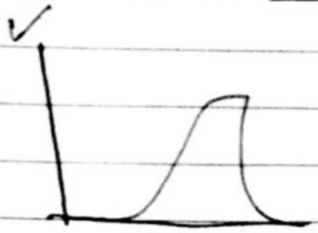
Synchronism

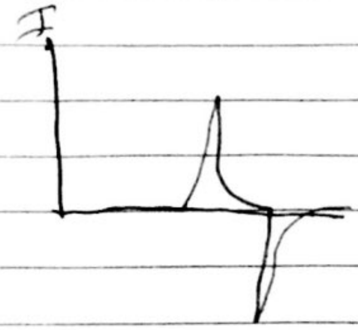
larger bias voltage

$i^2R \equiv$ noise

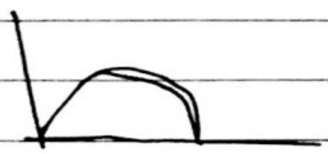


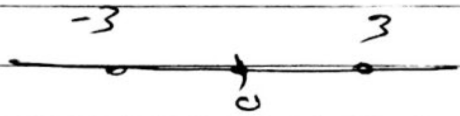


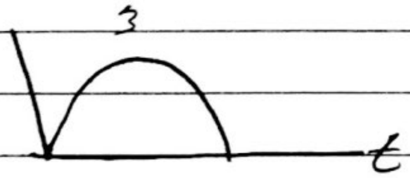


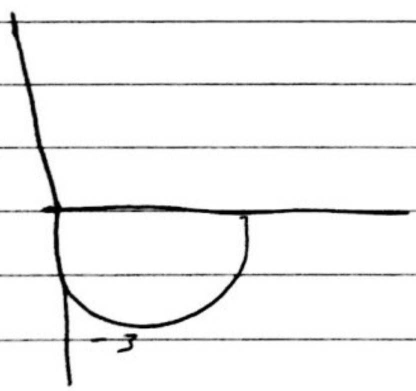


(Power $\propto \Delta V \cdot f^2$)



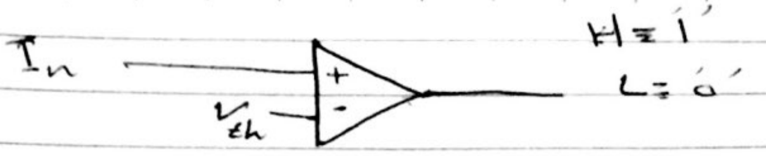




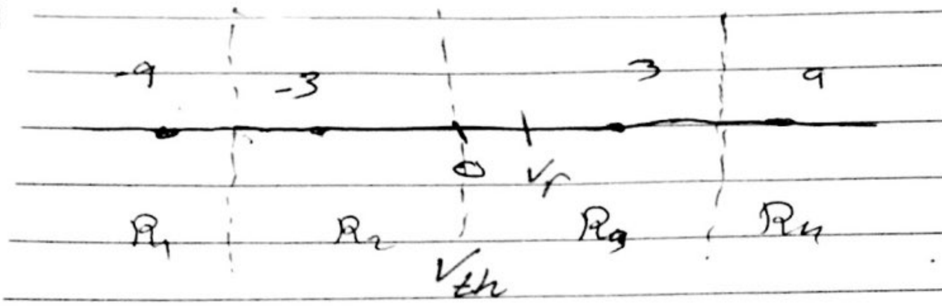


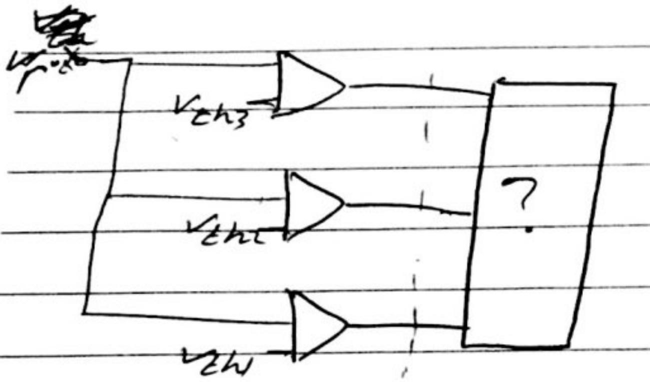
Rx:-

1- Level detection Rx



$V_{th} =$



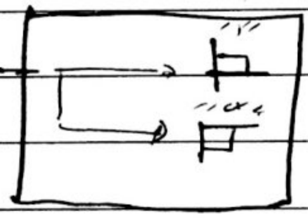


Detection | Decision

one sample = hard Decision
multible samples = Soft "

Lecture 11

- Rx: $\phi_r(t)$



diff: $v_0 = \int (\phi_r - S_0)\, dt$

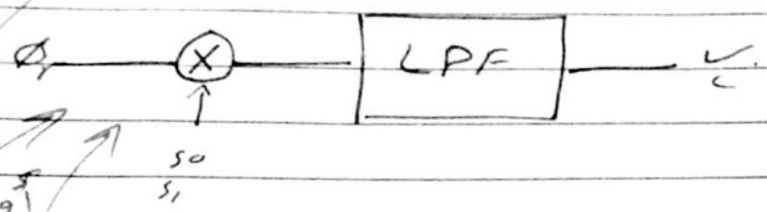
or $v_1 = \int (\phi_r - S_1)\, dt$

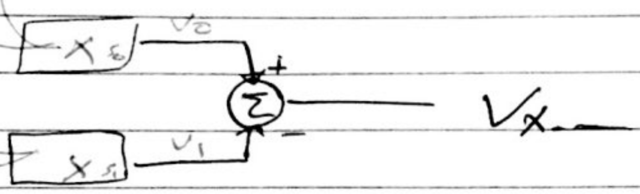
corr: $v_0 = \int \phi_r \cdot S_0 \, dt$

$v_1 = \int \phi_r \cdot S_1 \, dt$

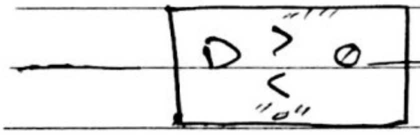
Corr better than LPF.

$\Rightarrow$ coherent





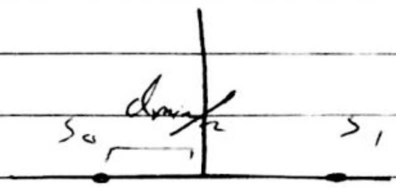
	$v_0 v_1$	V_x
$s_0 \rightarrow$	0 1	−1
$s_1 \rightarrow$	1 0	+1



$\rightarrow$ threshold

$\rightarrow d_{min}/2$

Maximum likelyhood detector
ML (Rx)

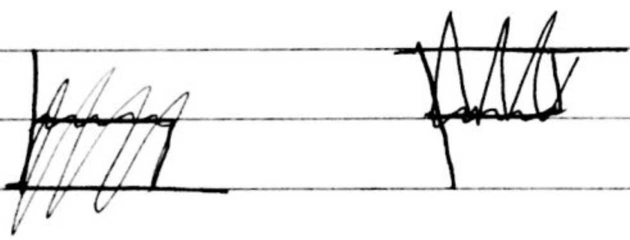


$$V_0 = \int \phi_r s_0 \, dt = |s_0||s_0| = E_0$$

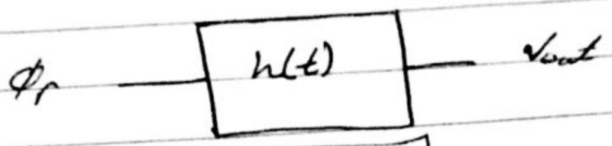
$$V_1 = \int \phi_r s_1 \, dt = |s_0||s_1| \cos\theta_{01} = \sqrt{E_0} \cdot \sqrt{E_1} \; \rho_{01}$$

correlator RX

$$\phi_r(t) = \phi_t(t) + n(t)$$



Rx?



$\boxed{\text{max. } SNR_{out}}$

$\begin{cases} \phi_r(t) = s(t) \longrightarrow V_{out} = \int \\ n(t) = \end{cases}$

$\uparrow$

σ_n^2

output noise

$$\sigma_n^2 = \int_{-\infty}^{\infty} PSD_n \cdot |H(f)|^2 \, df$$

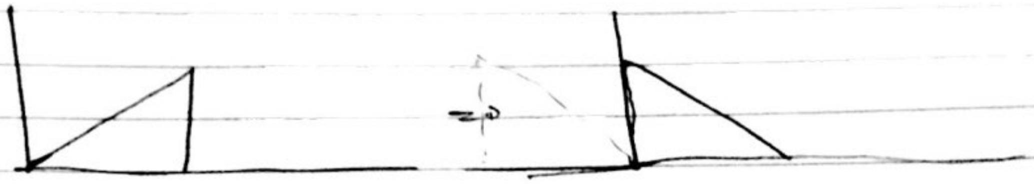
$$PSD_n \downarrow \frac{\sigma_n^2 \cdot T}{2}$$

$$V_{out} = \int_{-\infty}^{\infty} s(t) \cdot h(\tau - t) \, dt$$

$$\text{max SNR} = \max \frac{\left| \int s(t) \cdot h(\tau - t) \, dt \right|^2}{\frac{\sigma_n^2 T}{2} \cdot \int_{-\infty}^{\infty} |H(f)|^2 \, df}$$

$$= \max \frac{\left| \int S(f) \cdot H(f) \, df \right|^2}{\frac{\sigma_n^2 T}{2} \int |H(f)|^2 \, df}$$

max. when $\Rightarrow h(t) = s^*(T - t)$



Lecture 12 Matched Filter RX:-

Maximize (SNR)

$$\phi_r(t) = s(t) \longrightarrow \boxed{h(t)} \longrightarrow V_{out}$$

(T)

$h(t) = s^*(T-t)$ $\qquad \tau = T$

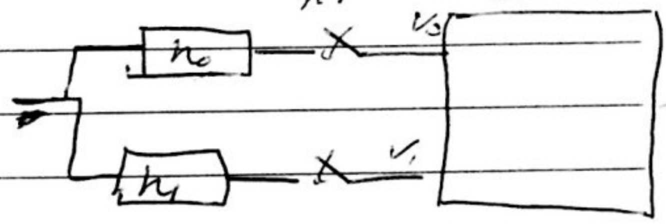
$V_{out} = \int s(t)\, h^*(\tau - t)\, dt$ $\qquad V_{out}(\tau) = \int |s(t)|^2\, dt$

$V_{out}(\tau) = \int s(t)\, s(\tau - T + t)\, dt$

For binary TX:- $\qquad$ @ RX:

"0" $\longrightarrow s_0(t)$

"1" $\longrightarrow s_1(t)$



$$V_0 = \int_0^T \phi_r(t)\, s_0(t)\, dt \longrightarrow \int |s_0(t)|^2\, dt = E_0$$

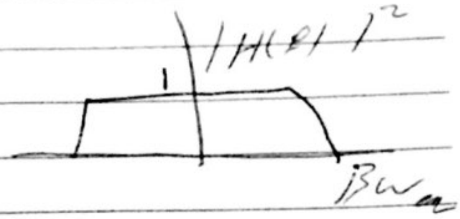
$$\text{or} \longrightarrow \int s_1(t)\, s_0(t)\, dt = \rho_{01} \sqrt{E_1} \sqrt{E_2} + n$$

out of $h(t)$ $\qquad$ PSD noise

$$\sigma_n^2 = \int_{-\infty}^{\infty} \frac{N_0}{2} \cdot |H(f)|^2\, df$$

$= \frac{N_o}{2} * 2 \int_0^{\infty} |H(f)|^2 \, df$

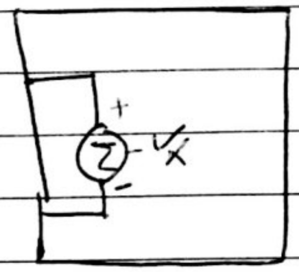
$= \cancel{N_o \int} \; N_o \cdot BW_{eq}$



$V_1 \rightarrow E_1 + n$

$\;\;\;\; \rightarrow P_{01} \sqrt{E_1} \sqrt{E_2} + n$

Decision



$n + n = n$

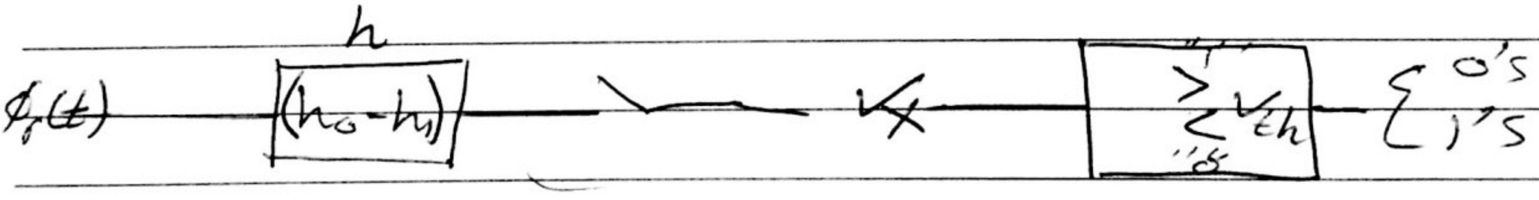
$n - n = n$?!

$n \rightarrow n \rightleftarrows$

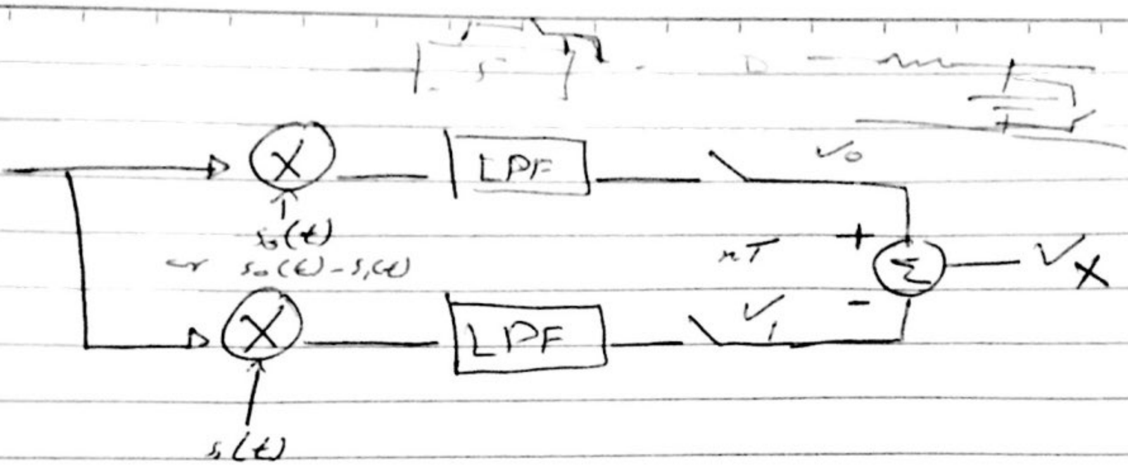
$n \leftarrow n$

"0" $\rightarrow V_x = E_0 - P_{01} \sqrt{E_1} \sqrt{E_0} + n$?

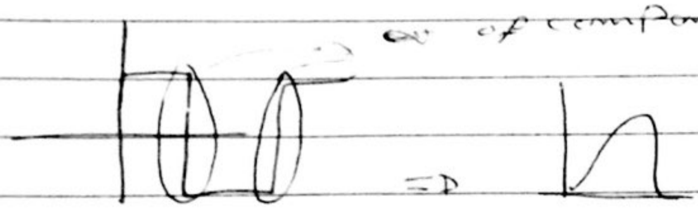
"1" $\rightarrow V_x = P_{01} \sqrt{E_1} \sqrt{E_0} - E_1 + n$?

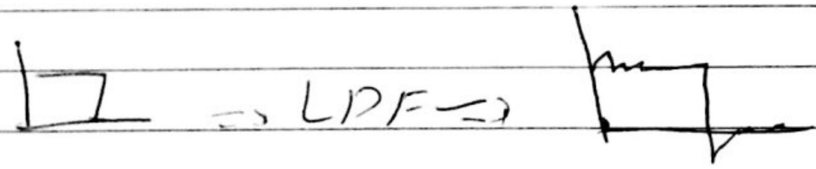


Correlator Rx :

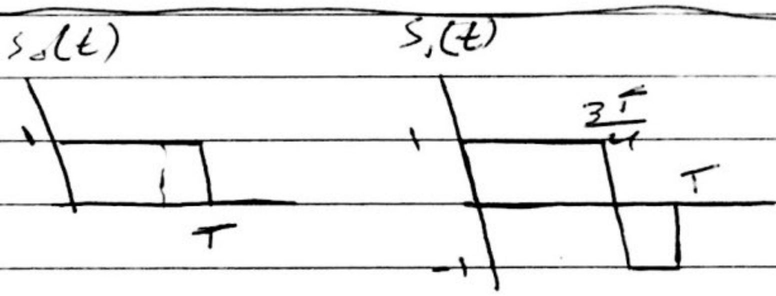


of components

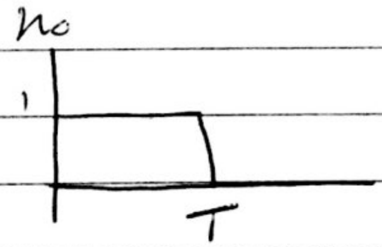


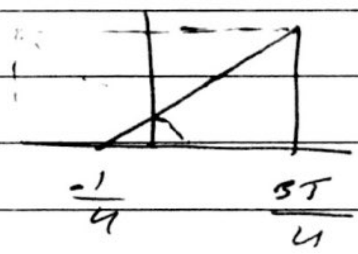


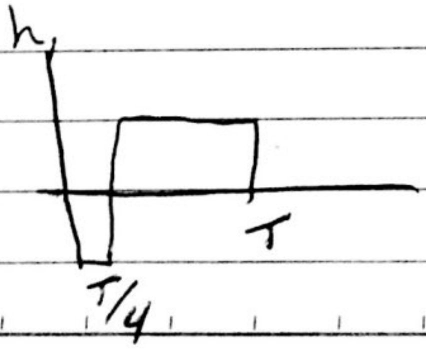
Ex:- Asume

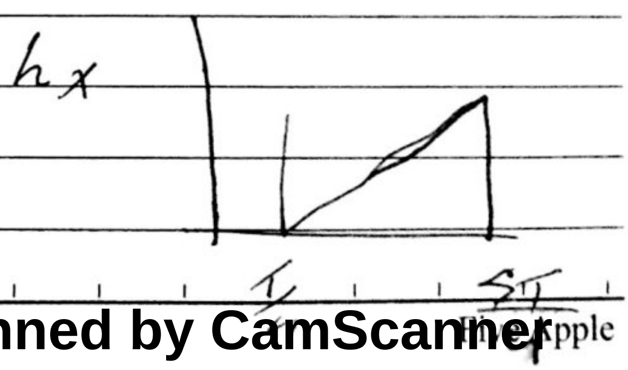


M.F. Rx

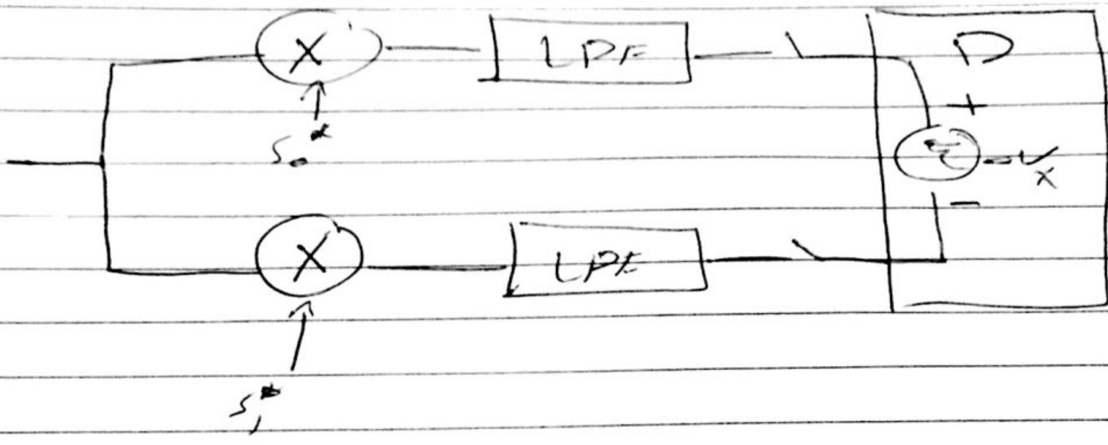








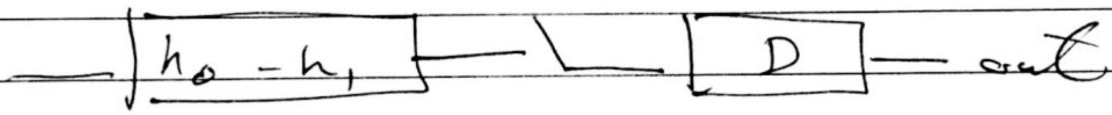
Correlator



$E_0 = T$

$E_1 = T$

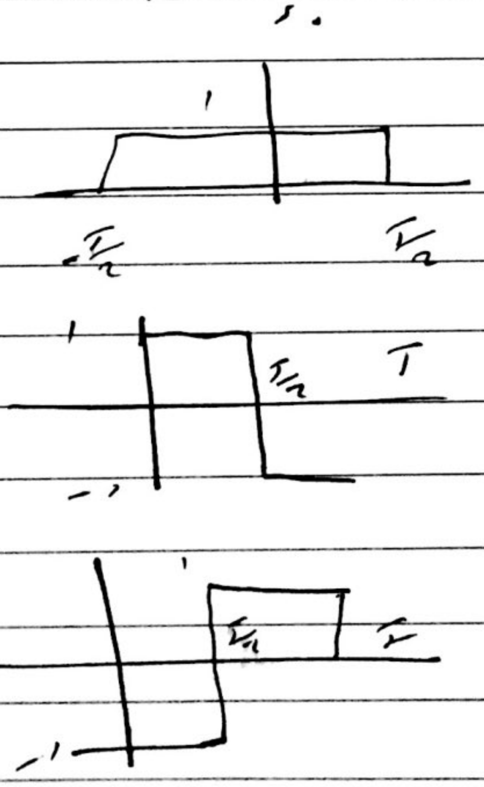
$$\rho_{01} = \frac{\int s_0 s_1 \, dt}{T} = \frac{1}{2}$$

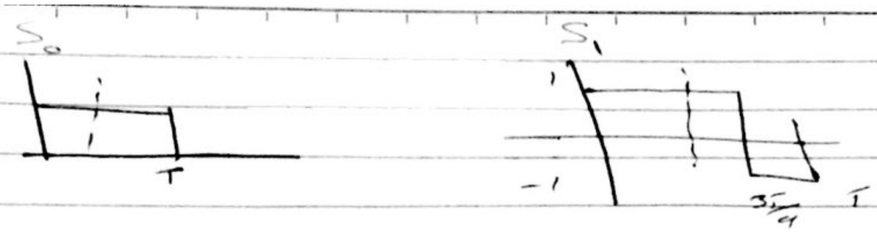


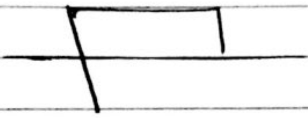
Lecture .13

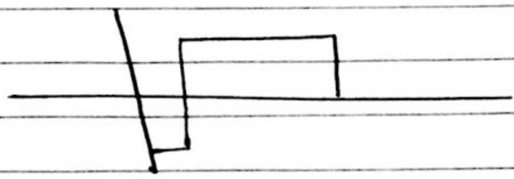
Quiz

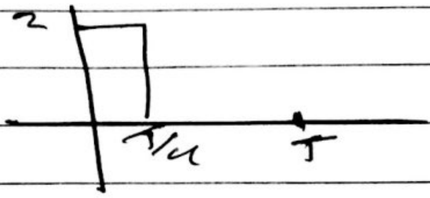
E.M

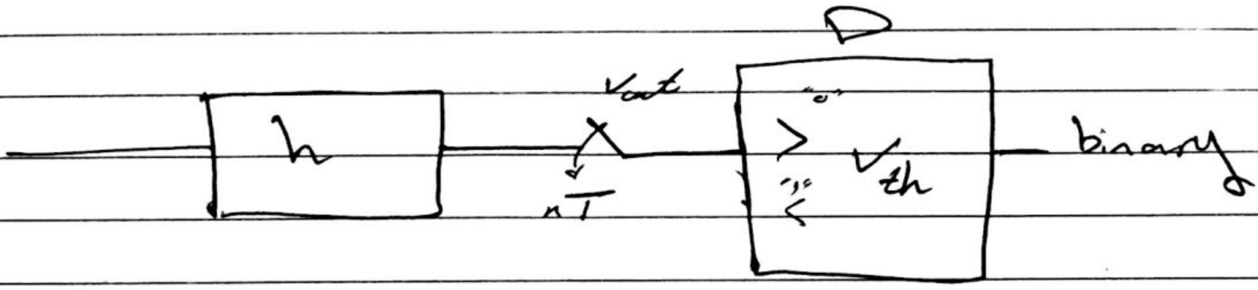




$h_0 =$ 

$h_1 =$ 

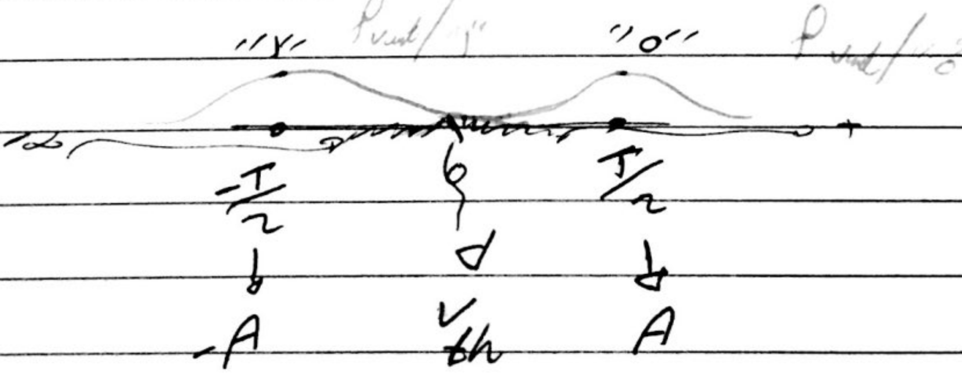
$h = h_0 - h_1 =$ 



$$V_0 = E_0 - \rho_{01}\sqrt{E_0}\sqrt{E_1} = \frac{T}{2} \text{ Volts}$$

$$; \rho_{01} = \frac{1}{2}$$

$$V_1 = -E_1 + \rho_{01}\sqrt{E_1}\sqrt{E_0} = \frac{-T}{2} \text{ volts}$$



$$f_{V_{out}/``0"} = \frac{1}{\sqrt{2\pi}\,\sigma_n} e^{\frac{-1}{2\sigma_n^2}(V_{out}-A)}$$

$$f_{V_{out}/``1"} = \frac{1}{\sqrt{2\pi}\,\sigma_n} e^{-\frac{1}{2\sigma_n^2}(V_{out}+A)}$$

$$\sigma_n^2 = N_0 \cdot BW$$

$$P_r(``1"/``0") = \int_{\infty}^{V_{th}} f_{V_{out}/``0"}\, dV_{out}$$

$$P_r(``0"/``1") = \int_{V_{th}}^{\infty} f_{V_{out}/``1"}\, dV_{out}$$

error event

$$BER = P_r(``0"/``1") \cdot \underbrace{P_r(``1")}_{1/2} + P_r(``1"/``0") \cdot \underbrace{P_r(``0")}_{1/2}$$

$$= P_r(``0"/``1") = \int_{V_{th}}^{\infty} \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{\frac{-1}{2\sigma_n^2}(x+A)^2} dx$$

$$= P_r(``1"/``0") = \int_{\infty}^{V_{th}} \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{\frac{-1}{2\sigma_n^2}(x-A)^2} dx$$

$$Q(\alpha) \triangleq \frac{1}{\sqrt{2\pi}} \int_{\alpha}^{\infty} e^{\frac{-t^2}{2}} dt$$

$$u = \frac{x - A}{\sigma_n} \quad , du = \frac{dx}{\sigma_n}$$

$$= \int_{\frac{V_{th} - A}{\sigma_n}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-u^2}{2}} du$$

$$= Q\left(\frac{V_{th} - A}{\sigma_n}\right)$$

$$V_{th} = \frac{V_0 + V_1}{2}$$

$$\frac{V_{th} - \overset{= V_0}{A}}{\sigma_n} = \frac{\frac{V_0 + V_1}{2} - V_0}{\sigma_n}$$

$$\alpha = \frac{V_1 - V_0}{2\sigma_n}$$

$$\therefore BER = Q\left(\frac{V_1 - V_0}{2\sigma_n}\right) \overset{?}{=} Q\left(\sqrt{\frac{(V_1 - V_0)^2}{4\sigma_n^2}}\right)$$

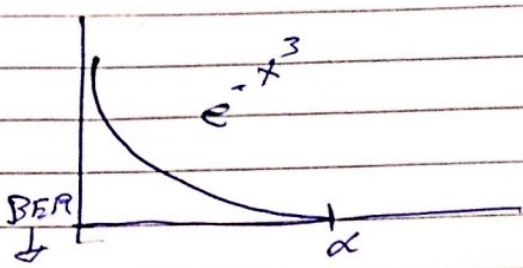
$$V_0 = E_0 - \rho_{01}\sqrt{E_0}\sqrt{E_1}$$

$$V_1 = -E_1 + \rho_{01}\sqrt{E_0}\sqrt{E_1}$$

~~BER~~ $V_1 - V_0 = -E_1 + \rho_{01}\sqrt{\cdot}\sqrt{\cdot} - E_0 + \rho_{01}\sqrt{\cdot}\sqrt{\cdot}$

$$= -(E_1 + E_0) + 2\rho_{01}\sqrt{E_0}\sqrt{E_1}$$

$$BER = Q\left(\sqrt{\frac{\left[2\rho_{01}\sqrt{E_0}\sqrt{E_1} - (E_1+E_0)\right]^2}{4\,\sigma_n^2}}\right) \rightarrow SNR$$

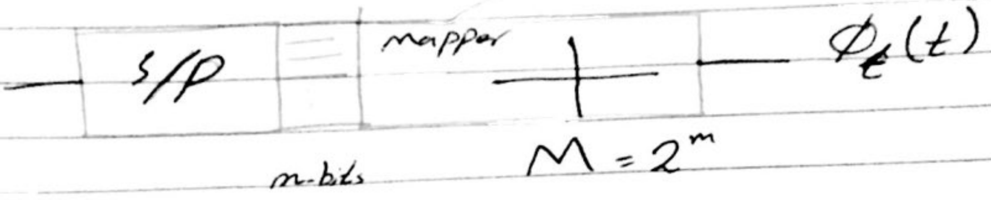


Simulation : no. of loops in simulation

$$10 * \frac{1}{BER}$$

SNR

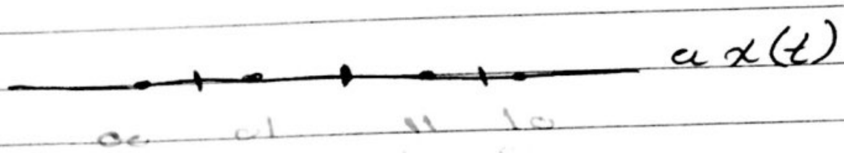
→ Non-binary base band TX.



$M = 2^m$

Ex: $M=4 \Rightarrow S_0(t), S_1(t), S_2(t), S_3(t)$

1-D:

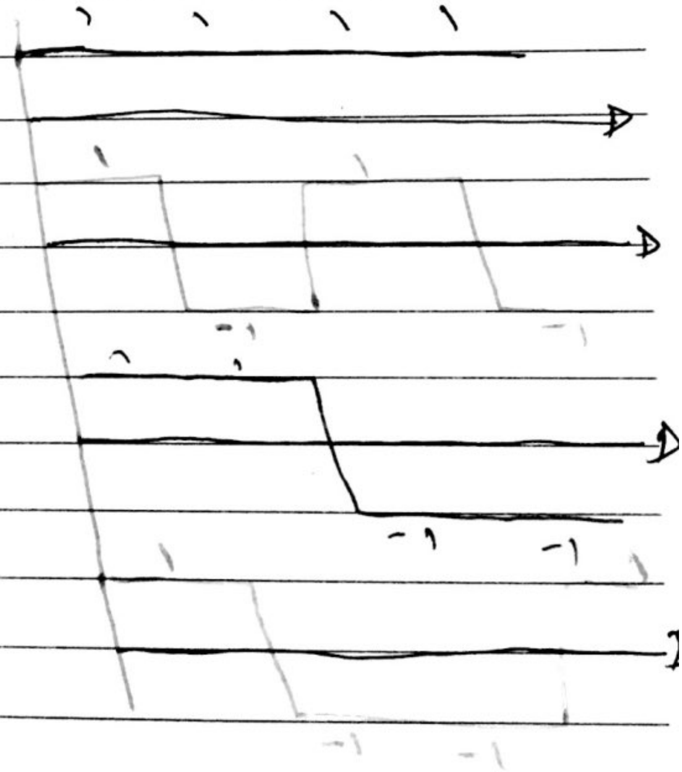


* How to generate orthogonal wave form:

$\boxed{I}$ W-H matrix

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \longrightarrow H_{2N} = \begin{bmatrix} H_N & H_N \\ H_N & -H_N \end{bmatrix}$$

$$H_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$



$\boxed{11}$ Full Rank square matrix : A

* condition number = $\frac{\lambda_{max}}{\lambda_{min}}$ +ve defends

eigenvalue

Φ = eig(A) orthogonal matrix → each component of A has eig vector
↳ orthogonal space of A

* we find eig by : $\det(A - \lambda I) = 0$

* Any eig. vector space and Any +ve defenet matrix Full Rank matrix this matrix will be orthogonal matrix

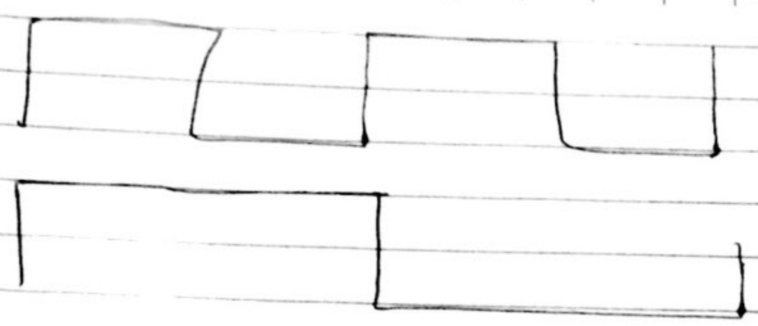
* If $A_{N \times N} \Rightarrow \Phi = [\phi_1 \ \phi_2 \ \cdots \ \phi_N]$

↳ each one is eig vector has N number

* good condition number ≃ 1

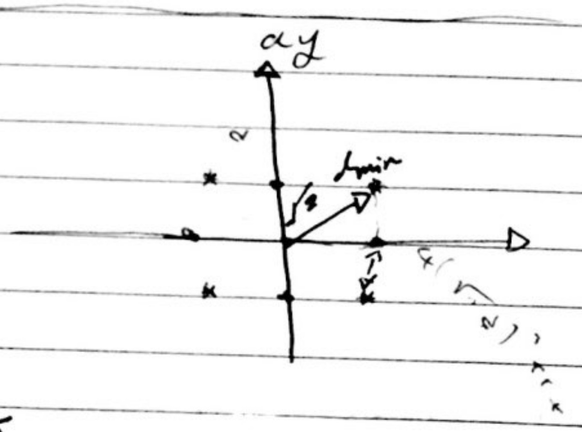
↳ stable system

$$A = \sum_{i=1}^{N} \lambda_i \phi_i \phi_i^{H}$$



T-flip flop

2-D



4-Points

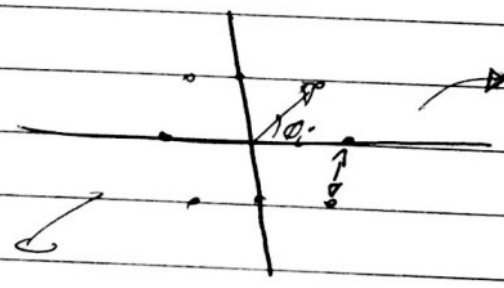
H.w :-

$$L = \sum_{i=1}^{I} \alpha_i^2$$; find α_i's for optimal y

max. min.

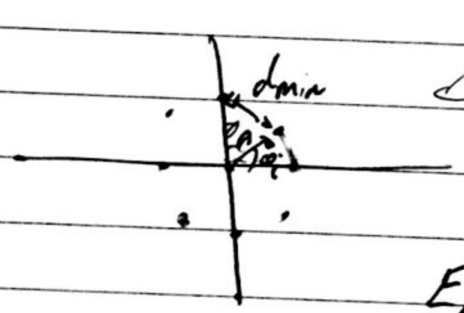
Sol: $\alpha_i = \alpha_j$

BER – Performance



→ grid is better than circle

→ for 8-Points



$$E_{avg} = A^2 T_s = \frac{d_{min}^2 \sin^{-2}\left(\frac{\pi}{M}\right) T_s}{4}$$

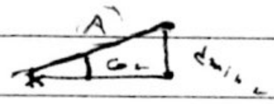
$$\theta = \frac{2\pi}{M}$$

$$A = \frac{d_{min}}{2\sin(\theta/2)}$$

$$\frac{4\, d_{min}^2 T_s + 2(4)\, d_{min}^2 T_s}{8} = \frac{3}{2} d_{min}^2 T_s$$

reunidlan: $\frac{d_{min}^2 T_s}{4 \sin^2\left(\frac{\pi}{m}\right)}$ $m = 8$

$= 1.7\, d_{min}^2 T_s$



$A^2 T_s$

$\sin\theta = \frac{d}{A}$

$A = \left(\frac{d}{\sin\theta}\right)$ $\frac{2\pi}{m}$

$= 1.5\, d_{min}^2 T_s$

grid = QAM

circle = PSK ⟶ looks like PSK → Base band

sin & cos → BandPass

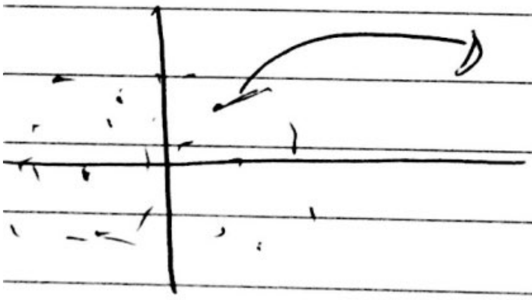
⟶ more immune against fading than QAM

4-D $a_x \to S_0$ best system against fading → multi dimen

$a_y \to S_1$

$a_z \to S_2$

$a_w \to S_3$ FSK is the best



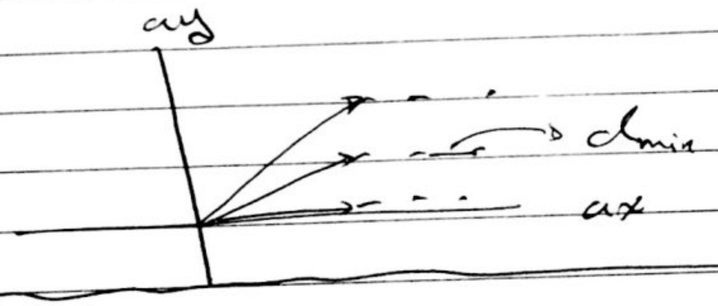
$\cong$ $d_{min}/2$ | $d_{min}/2$ Bringing → Partially

d_{min}

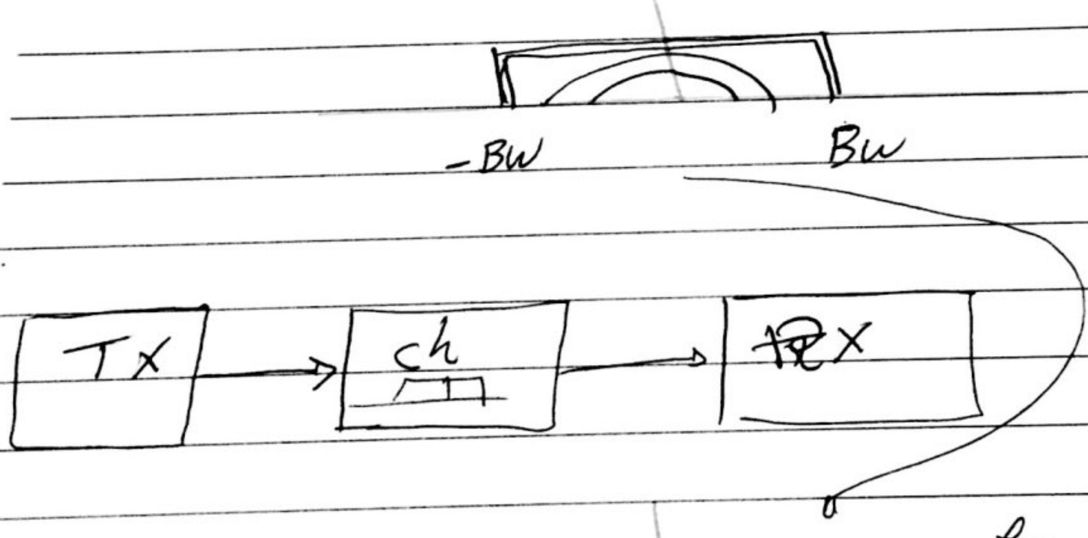
$$BER = Q\left(\frac{2V_{th} - A}{2\sigma_n}\right) \rightarrow \frac{d_{min}}{2}$$

$$\boxed{BER \cong \frac{1}{\log_2 M} Q\left(\frac{d_{min}}{2\sigma_n}\right)}$$

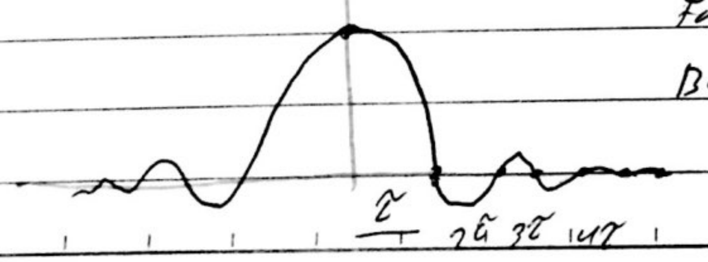
$$BER \cong \frac{1}{\log_2 M} Q\left(\frac{d_{min}}{2\sigma_n}\right) \rightarrow$$ لازم نحفظها



Band limited Tx:

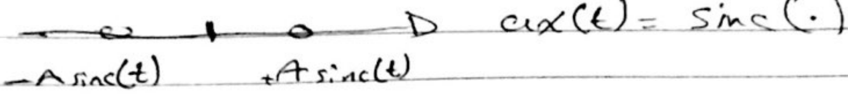


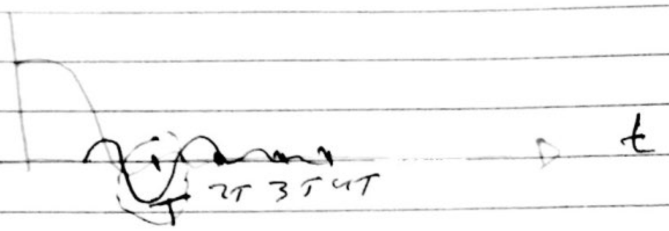
$$\frac{\sin(\pi t)}{\pi t}$$



for Band limited ~~Tx~~
Base band TX we have
t only sinc as
axis

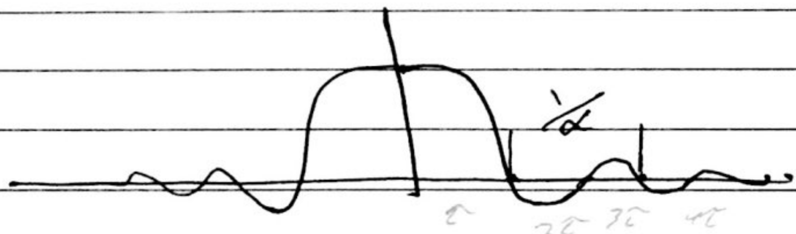
1-D mod.



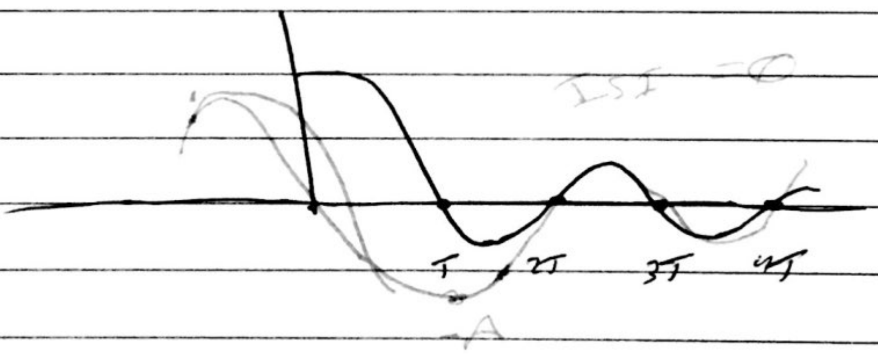


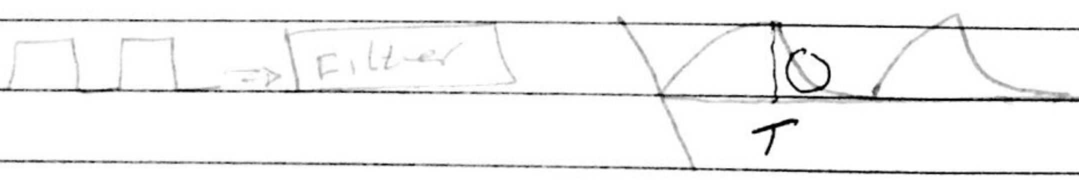
(ISI) old signal have some Parts ~~ve~~ when new one have arrived

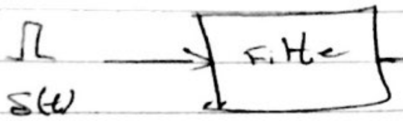
$$\frac{\sin(\pi t)}{\pi t} \Rightarrow \frac{\sin(2\pi\alpha t)}{\pi t}$$



$$\tau = \frac{1}{2\alpha} = T$$





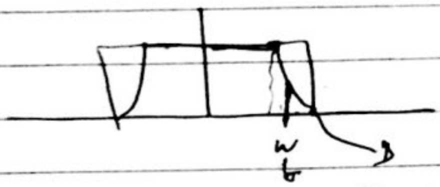


$s(t) = h(t)$

$h(t) = \text{sinc}(\cdot)$

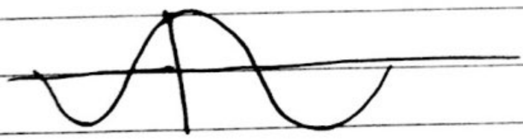
Slope $\rightarrow \alpha$

no of components needed $\propto$ slope



$(1+\alpha)W$; α = roll-off factor

wanted BW



Raised cosine filter

RC_α

$RC_\alpha = \text{Sinc}(\cdot)(\text{cosine})$

wave form shaping function

if signal has r_b bPS and ch. has W Hz of B.W

$BW = \frac{r_b}{2}(1+\alpha)$

$$\boxed{BW = \frac{r_s}{2}(1+\alpha) = \frac{r_b}{m\,2\log_2 M}(1+\alpha)}$$

2^m

Five Apple

Ex:

$BW = 10\ KHz$

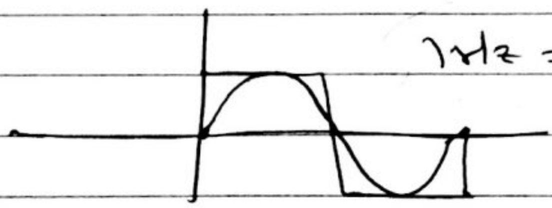
$r_b = 120\ Kbps$

$$10 = \frac{120}{2m}(1+\alpha)$$

$\alpha = 0.13 - 0.23$

$m = 7$

$$10 = \frac{120}{2.7}(1+\alpha) \Rightarrow \alpha = 0.16$$



$$1\,Hz = \frac{1}{2T_b} = \frac{r_b}{2}$$

RC_α → Smaller is better

$$BW = \frac{r_b}{2m}(1+\alpha)$$

Base band

$m \to \log_2 M$

$(1+\alpha)\,w$

Constelation Points : $M = 2^m$ → integer

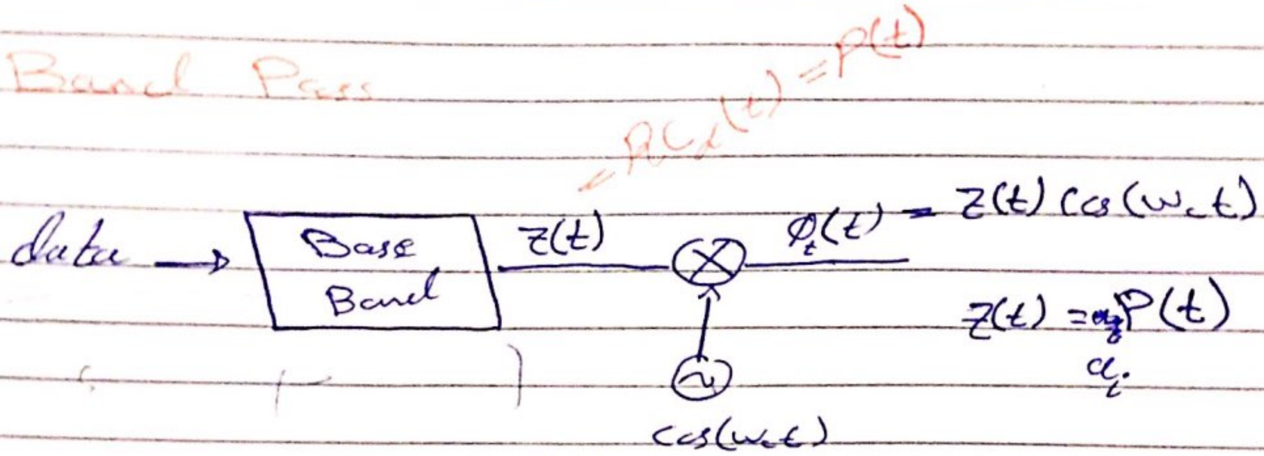
more cost → more E_{avg} ↓

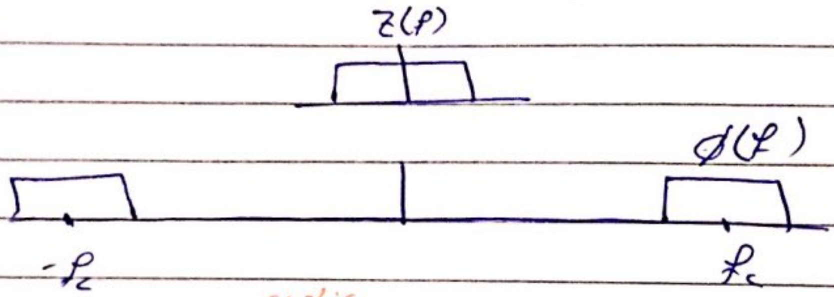
$$h(t) = \sum_{i=0}^{I} d_i Z^{-i}$$

FEC : Forward Error correction

TCM : to be con't.

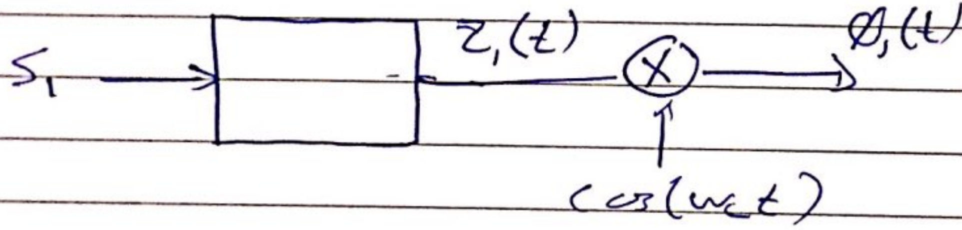
Band Pass

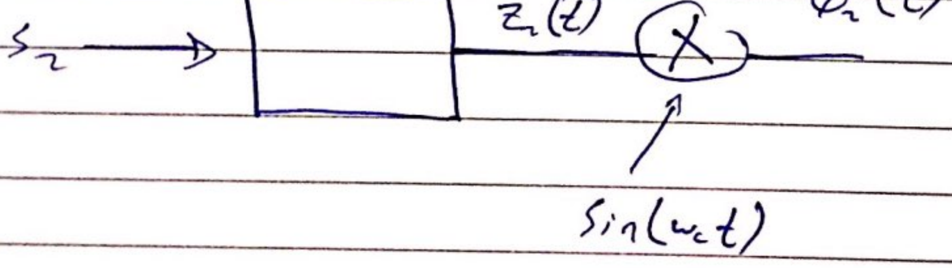




axis

$$\phi_c(t) = a_i \, P(t) \cos(\omega_c t)$$ ASK Signal





$$\int_{-\infty}^{\infty} \phi_1 \phi_2 \, dt$$

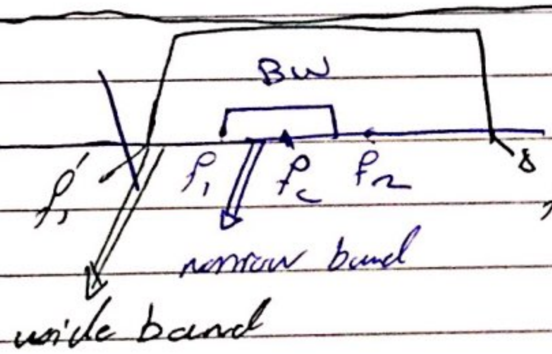
$$= \int z_1(t)\, z_2(t) \cos(\omega_c t) \sin(\omega_c t)\, dt$$

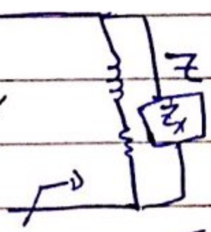
$$\boxed{f_c >> f_m}$$

$$= z_1(t)\, z_2(t) \int \cos(\omega_c t) \sin(\omega_c t)\, dt$$

$$\frac{BW}{f_c} << 1 \longrightarrow \text{narrow band}$$

wide band for implementation is very difficult

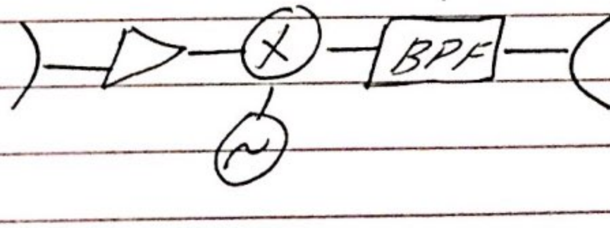




$Z = R + j\omega L$

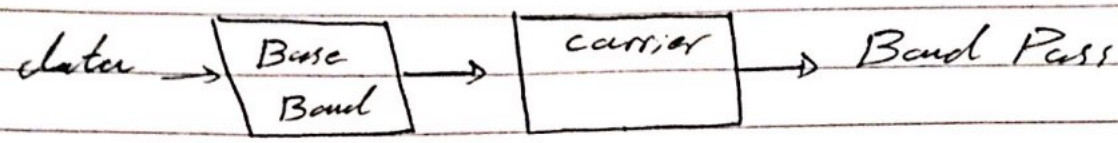
$\omega \to 2\pi f$

$Z_{in} = 50\,\Omega$



T.P.

Band Pass TX:

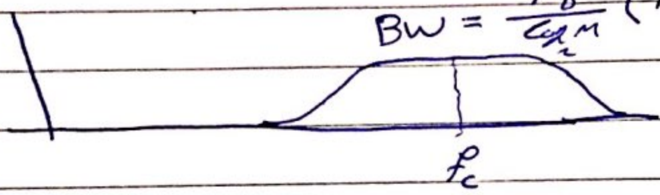


1-D

A one axis F P

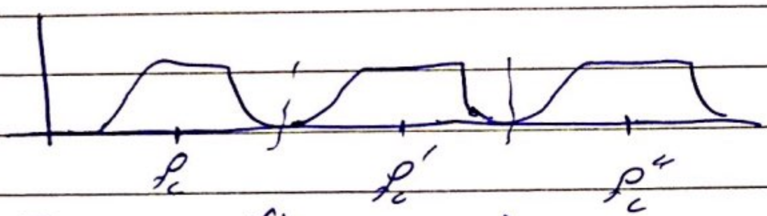
$A \; P(t) \cos(\omega_c t + \emptyset)$ → ASK (1-D mod.) / PSK (2-D mod.) / FSK (multi-Dimensional mod.)

$BW = \frac{r_b}{\log_2 M}(1+\alpha)$



A & w are constants

$\emptyset = 0$



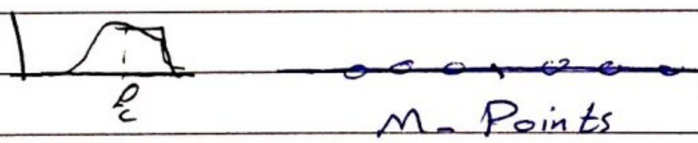
$BW = \frac{r_b}{\log_2 M}(1+\alpha) \cdot M$ ⇒ FSK (wide band)

WBM

⇒ ASK & PSK (narrow band)

NBM

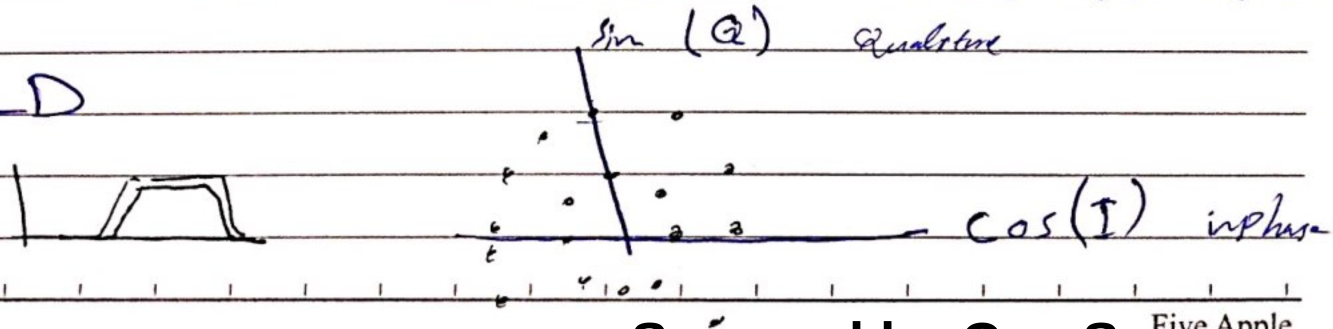
1-D



ASK

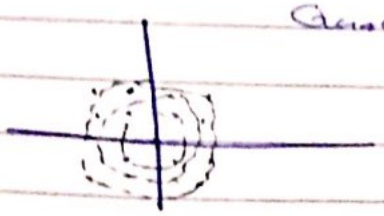
M-Points → $A_i \, P(t) \cos(\omega_c t)$

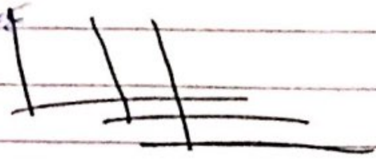
2-D



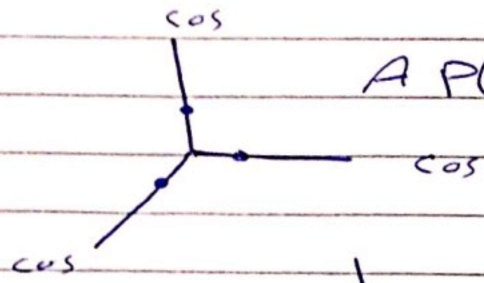
$I_i \, P(t) \cos(\omega_c t) + Q_i \, P(t) \sin(\omega_c t)$

Quadrature Amplitude modulation

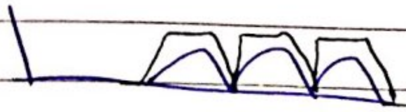




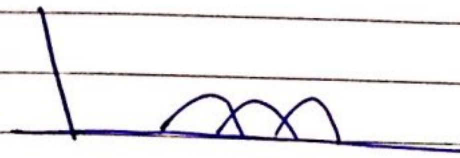
3-D



$A\,P(t)\cos(w_i t)$



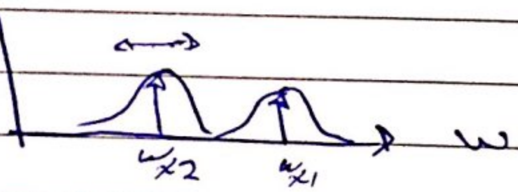
Solve selectivity of frequency & in fading



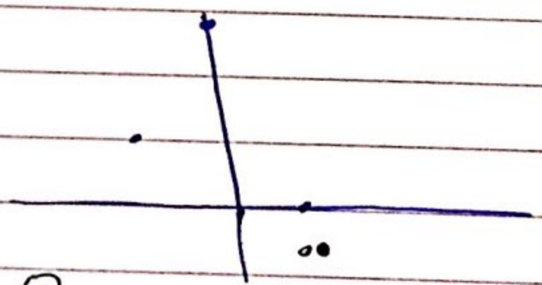
OFDM

Lecture 19

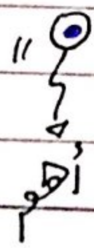
$P(t)\cos(w_x t)$



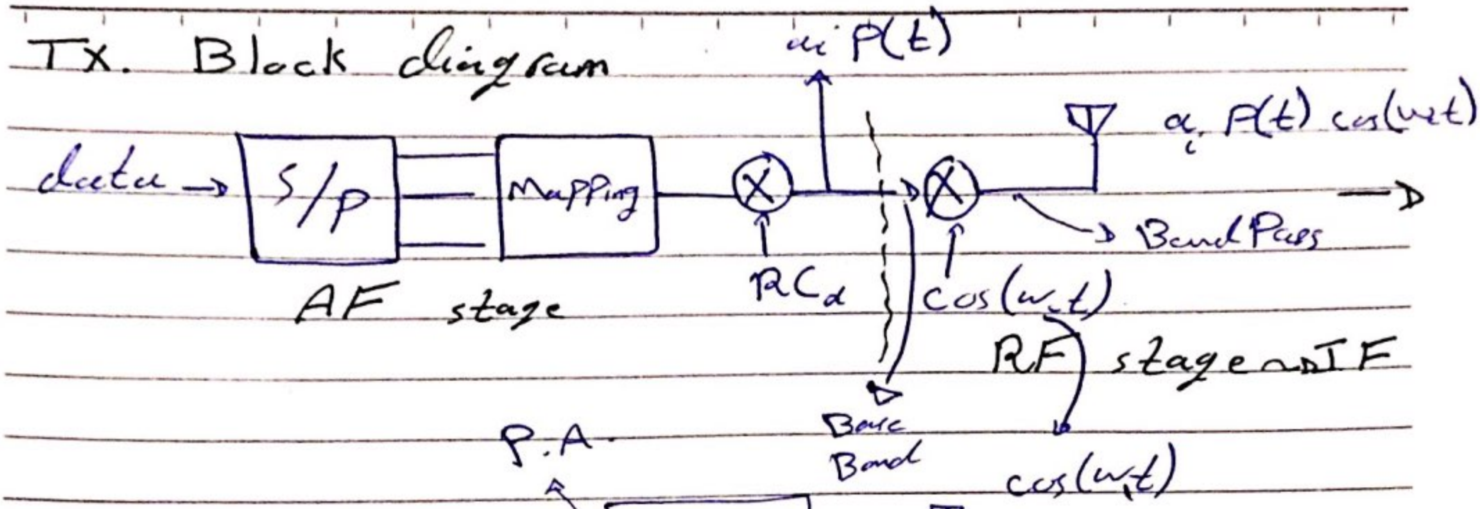
$BW = \frac{r_b}{\log_2 M}(1+\alpha)$?!

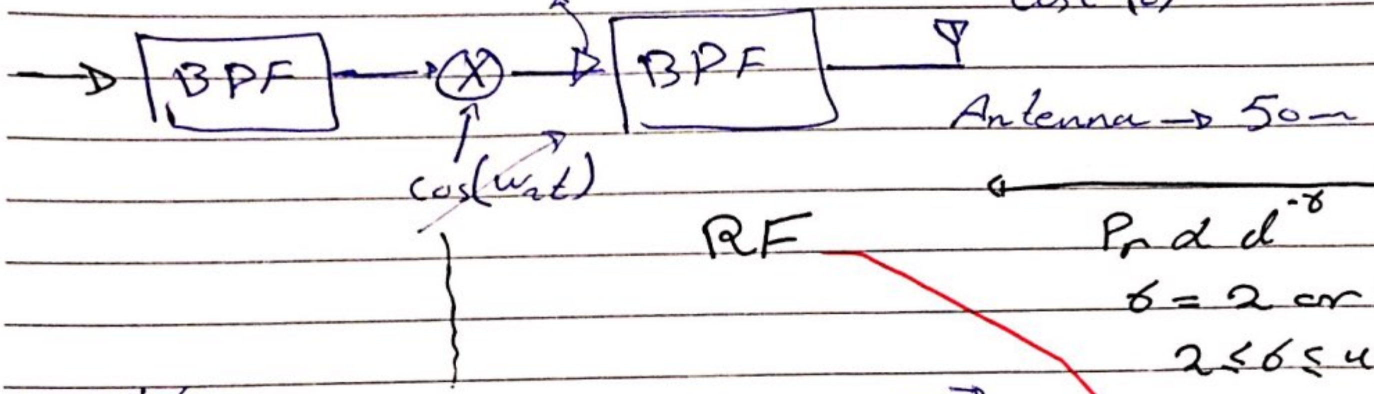


⇒ axis: sin & cos



Tx. Block diagram





P_r

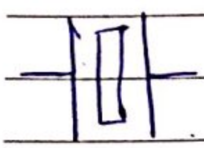
RF

$P_r \propto d^{-\gamma}$

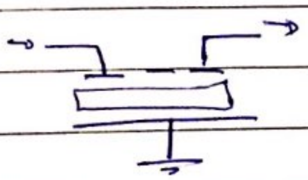
$\gamma = 2$ or 4

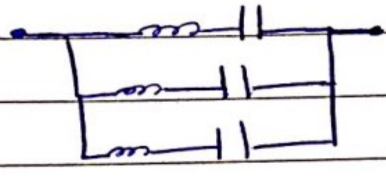
$2 \leq \gamma \leq 4$

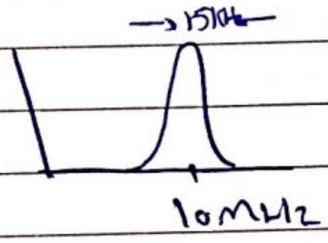
oscillator

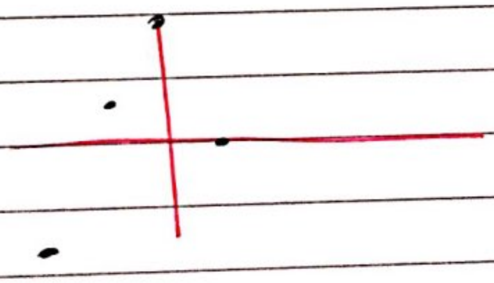


→ crystal filter









$$PAPR = \frac{\text{Peak Power}}{\text{Average Power}} \geq 1$$

1 is (optimum)

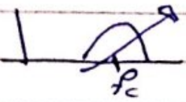
↳ Implementation

PSK & FSK → PAPR = 1

ASK → " ≠ 1

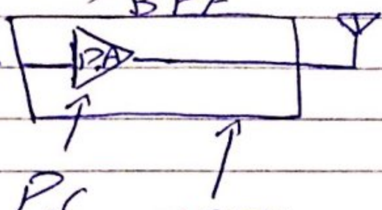
lecture. 20

PAPR:



Front End → TX.

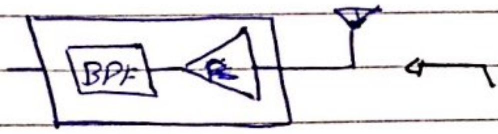
① Design Amplifiers
② Dominant interference



P.C.   AFC

$P_t = 1W$
$R = 50\Omega$

RX.



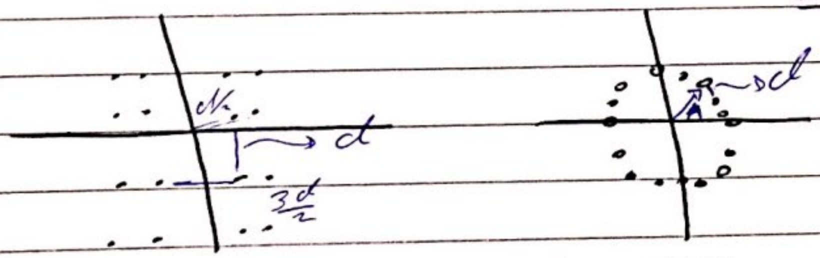
$0dB_m = 1mW$

$P = \frac{V^2}{50}$

$V = \sqrt{50(1mW)}$

$=$

Ex:



16-QAM   16-PSK

$E_{avg} = \frac{1}{4}\left[\frac{d^2}{2} + \frac{9}{2}d^2 + 2\cdot\frac{10}{4}d^2\right]$

$E_{avg} = \frac{5}{2}d^2 = 2.5\, d_{min}^2$

$E_{peak} = 9/2\, d_{min}$

$PAPR = \frac{9}{5}$

$PAPR = 1$ ✓

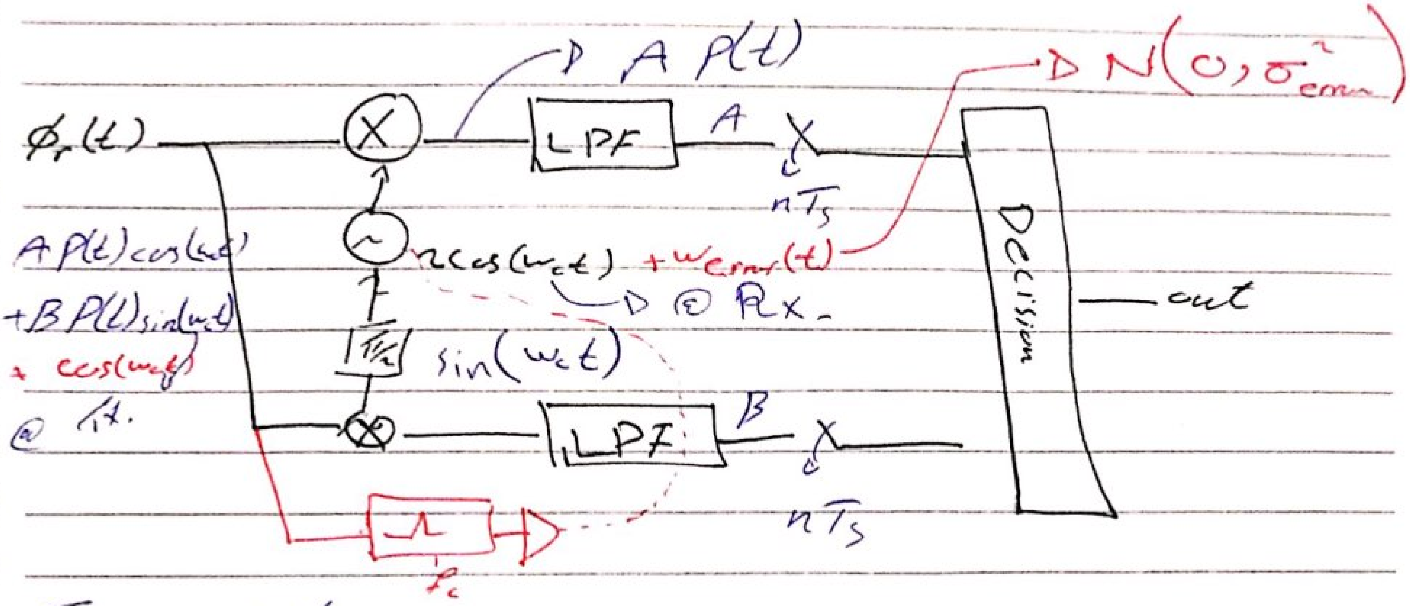
$E_{avg} = A^2$

$A^2 = \frac{d^2}{4\sin^2\left(\frac{\pi}{16}\right)} = 6.5\, d_{min}$

Five Apple

(9→10) — 15/11/2016

lecture. 21 RX.

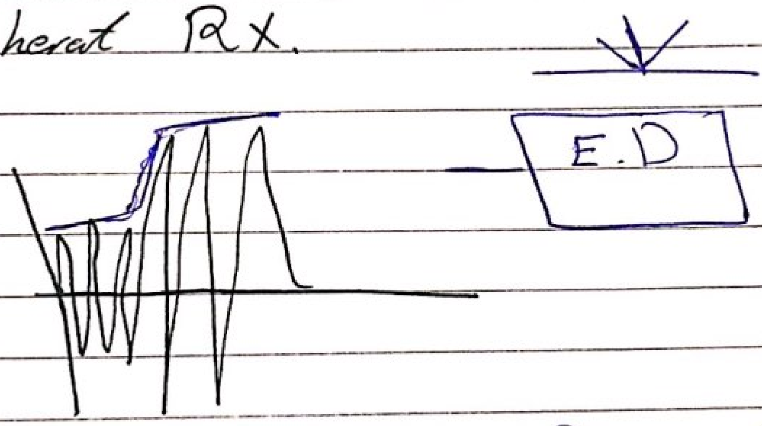


Syncronization

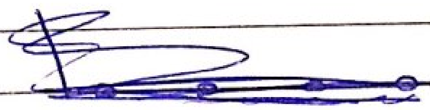
- Carrier synch.
- bit synch.
- Frame synch.

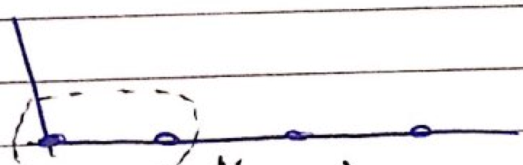
Non- coherat RX.

1_D



if we use non-cohent or En-D in ASK -one Dimansiunal so we cant use -ve sympols because we lose them





(on-off Keying)

ASK = OOK

"0" → 0 volt

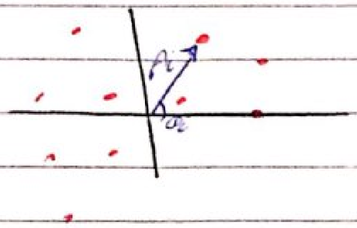
"1" → A volt

Lenovo → (Android → Java / ↳ smooth interface)

2-D

$A_i \, P(t) \cos(w_c t + \theta_i)$

$= A_i \measuredangle \theta_i$



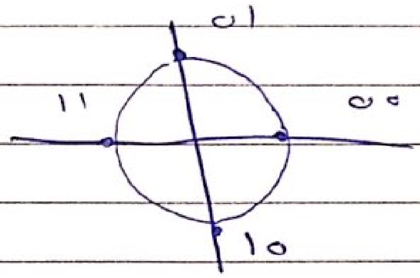
Differential Modulation

DPSK

$PSK = A\, P(t) \cos(w_c t + \theta_i)$

@ RX

data	θ_i
00	0
01	$\pi/2$
11	π
10	$3\pi/2$

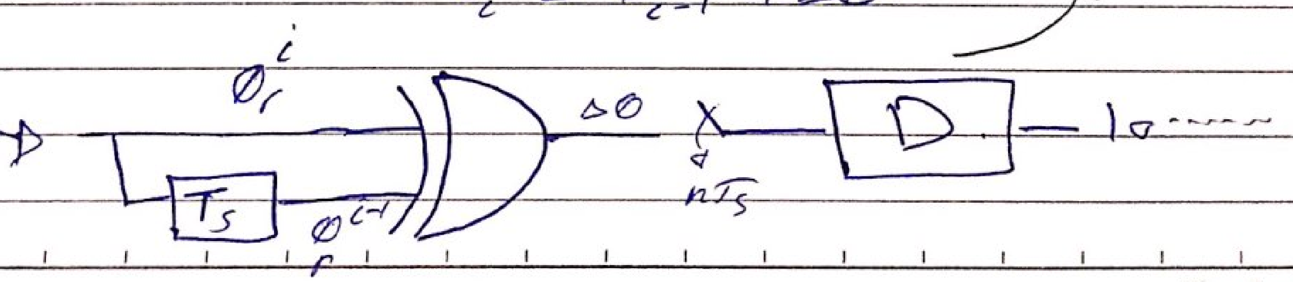


DPSK:

data	$\Delta\theta$
00	0
01	$\pi/2$
11	$-\pi/2$
10	$3\pi/2$

Differentially coherent RX.

$\theta_i = \theta_{i-1} + \Delta\theta$



Five Apple

Full Response signaling: we use the signal Fully to transmit the data

↳ Memoryless channels

Partial Response signaling: when we transmit data between time slots

↳ Memory channels

Lecture 22 DPSK

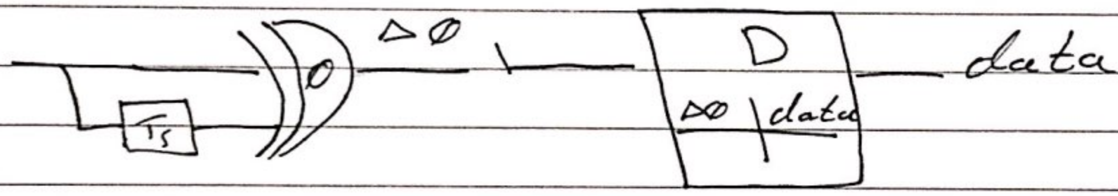
$A\,p(t)\cos(w_c t + \phi_i)$

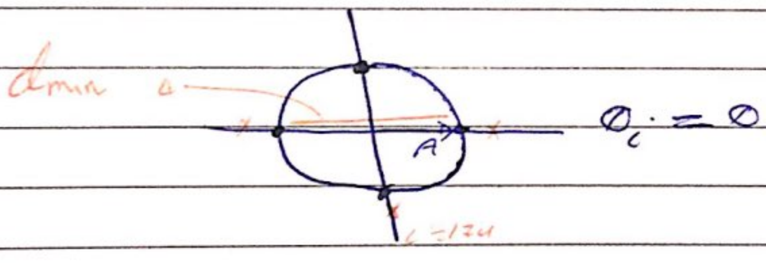
data	$\Delta\phi = \phi_i - \phi_{i-1}$
0	$+\pi/2$
1	$-\pi/2$

min. shift keying

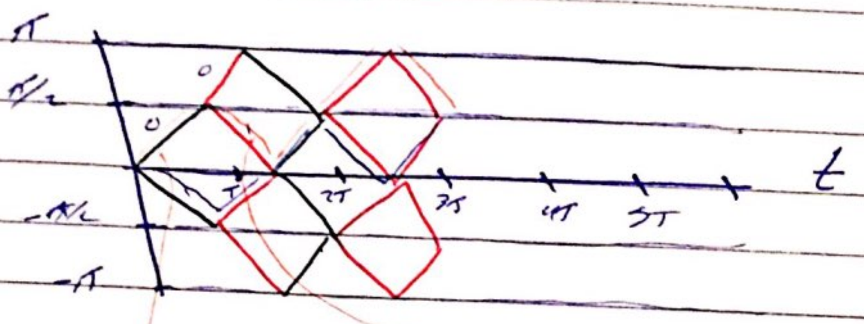
↳ GSM

MSK





Phase Tree (Trellis diagram)



0 1 0 0 1

$A\,p(t)\cos(w_c t + \pi/2)$

$A\,p(t)\cos(w_c t + \phi)$

1 0 0 1 0

$$BW = 2(f_m + \Delta f)$$

$\quad \hookrightarrow \alpha\, \Delta V$

$\phi(t) \rightarrow f = \phi^\circ$

$$\Delta f = \frac{\Delta \phi}{\Delta t = T}$$

CPFSK

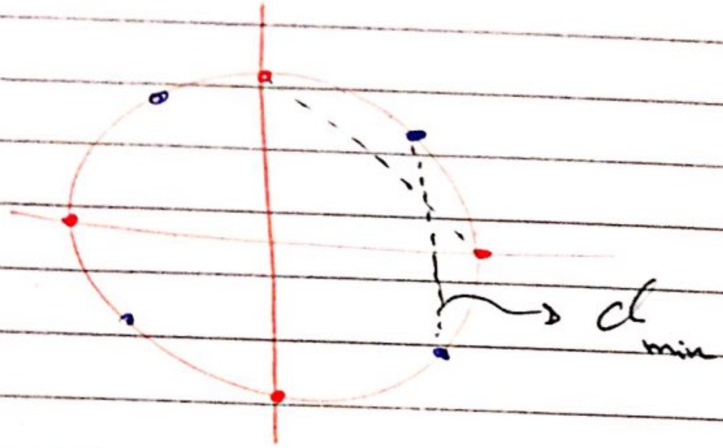
d_{min} @ $min(\Delta(\Delta\phi))$

BW $\downarrow$ when $\Delta\phi \downarrow$

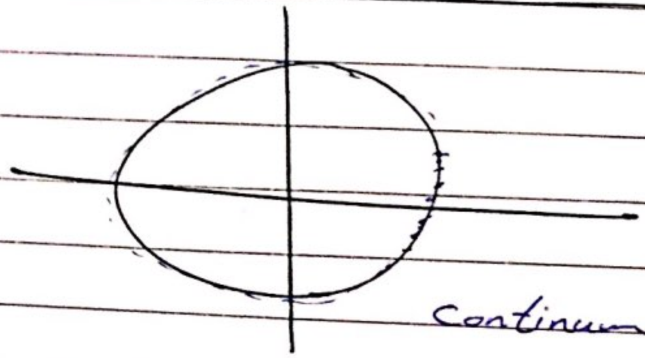
$d_{min} \uparrow$

data	$\Delta\Phi$
0 0	$\pi/4$
0 1	$3\pi/4$
1 0	$-\pi/4$
1 1	$-3\pi/4$

$\pi/4$ − QDPSK

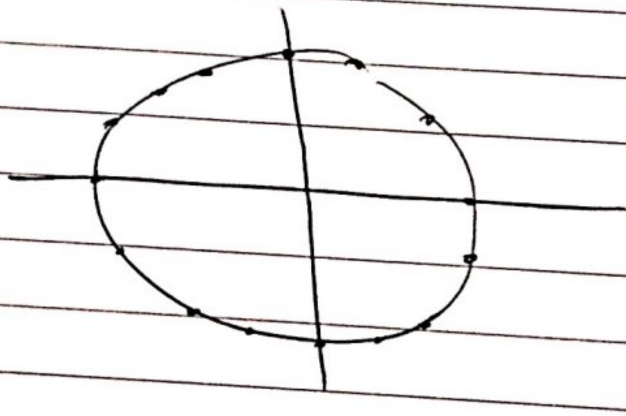


data	$\Delta\Phi$
0	$\frac{\pi}{17}$
1	$\frac{-5\pi}{37}$



Quiz 4.

0 0	0
0 1	$\pi/4$
1 0	$-\pi/8$
1 1	$\pi/2$



$$BER_{AWGN}(SNR) = P_e(\gamma)$$

$SNR \to \gamma$

in fading channel with $f_\gamma(\gamma)$

$$BER_{faded} = \int_0^\infty P_e(\gamma) \cdot f_\gamma(\gamma)\, d\gamma$$

DPSK:

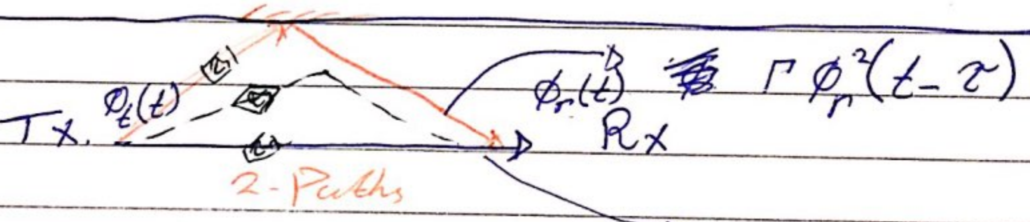
$$BER = \frac{1}{\log_2 M} Q\left(\sqrt{\frac{d^2}{4\sigma_n^2} \cdot \zeta^2}\right)$$

$$\boxed{\zeta = 0.8 \Rightarrow DPSK}$$

$(1.2 - 1.5)$ dB diff from coherent.

Fadding

Multi-Path channels



$$\phi_r(t) = \alpha\, \phi_t(t) \qquad \phi_r'(t)$$

R.V.

$$\phi_r(t) = \sum_{i=0}^{I} \Gamma_i\, \phi_r^i(t - \tau_i)$$

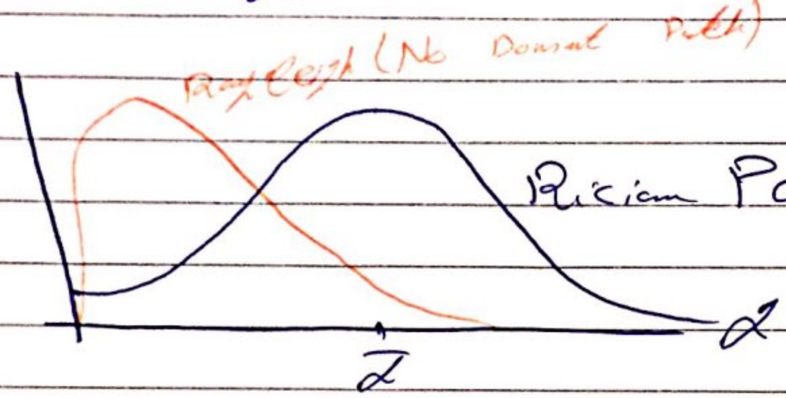
ref. direct Path $\quad i = 0 \to \Gamma_0 = 1 \quad \tau_0 = 0$

$\tau_i \longrightarrow$ Phase shift @ w_c carrier

$$\boxed{2\pi \frac{\tau}{T} = \text{Phase shift}}$$

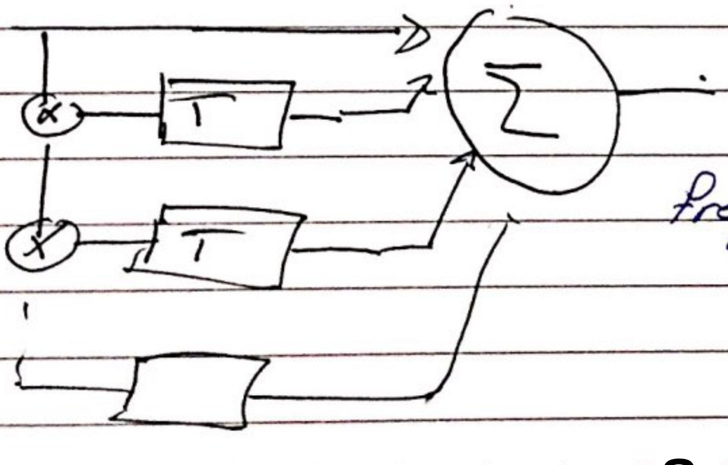
→ iff τ_i's are very small

$$\phi_r(t) = \phi_r(t) \cdot \underbrace{\sum \Gamma_i}_{\alpha}$$



when dominant Path (Direct Path) exist

FIR Filter



Multi Path Fading channel

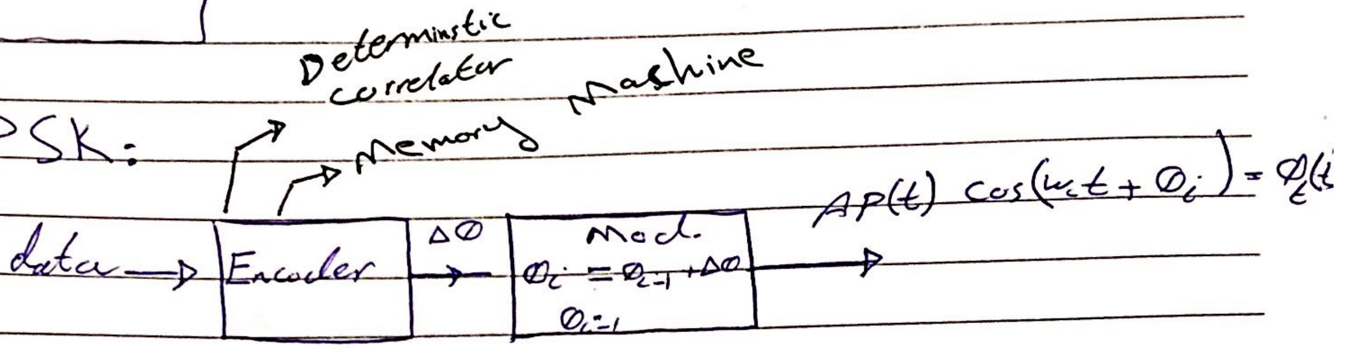
memoryless   memory

Frequency selectivity

lecture . 25

$\boxed{\text{DXX SK}}$

DPSK:



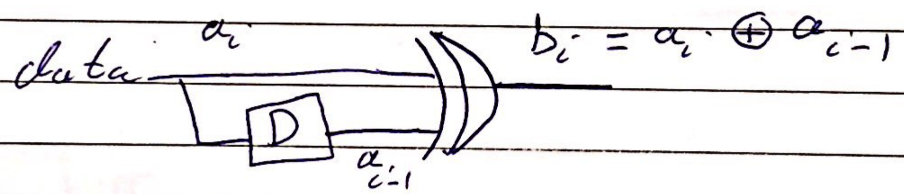
Ex: $\phi_i(t) = 4P(t)\cos(w_c t) + 3P(t)\sin(w_c t)$

$$= 5P(t)\cos(w_c t + \theta_i)$$

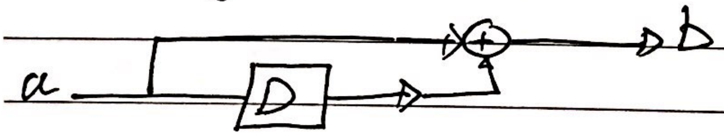
Q.5

00	0
01	$\pi/2$
10	$-\pi/2$
11	π

$\sqrt{2}\,.\,R$



Memory Machine (state Machine)



out = f(Input, Memory)

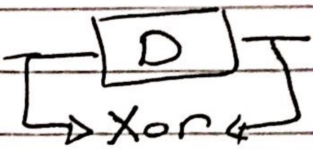
= f(I, state)

we can impliment any Encoder by a state Machine

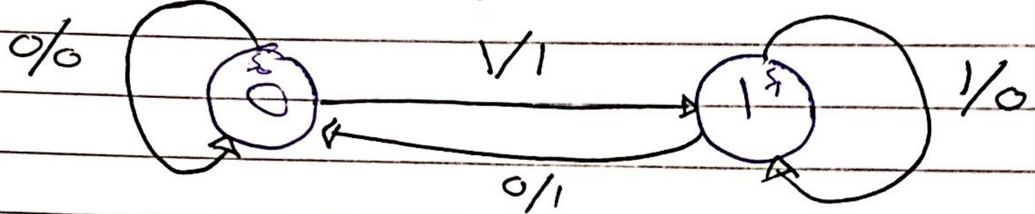
So, states = contents of Memory

no. of states = 2^{L} $L \to$ Memory length

Graph Theory



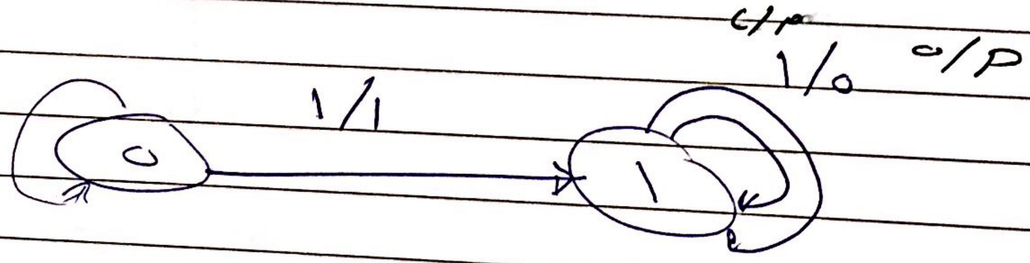
state diagram



states = Nodes

data : ~~[illegible]~~

In : 0111
out : 0100



$$\boxed{\Pi_n = \Pi_0 \Pi^n}$$ → state transition Matrix

مهم

$$\pi = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\pi = \sum_{i=0}^{1} P_i T_i$$

Probability of state

$$T_i = \begin{array}{cc} & \text{state} \\ \text{state} & \begin{bmatrix} 1 & 0 \\ \vdots & \vdots \end{bmatrix} \end{array}$$

Ex: when i/P = 0

$$\pi = P_0 T_0 + P_1 T_1$$

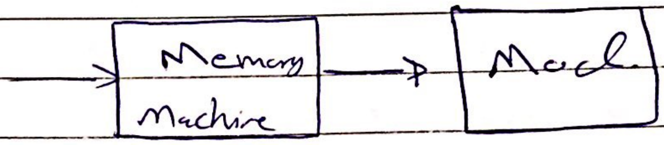
$$T_0 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \qquad T_1 = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\pi = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

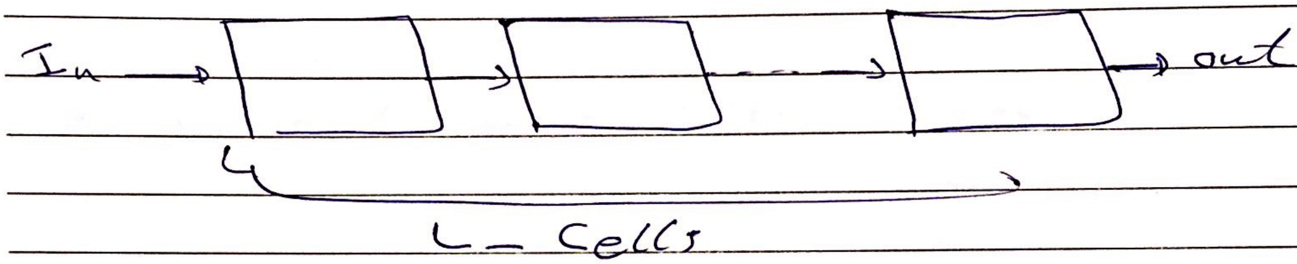
To build comm. System

0- Antenna (Sensor)
1- oscillator
2- Amplifier
3- Filter
4- Mixer
5- controller

Lecture 26

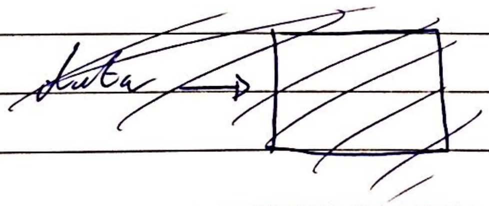


Memory Machine

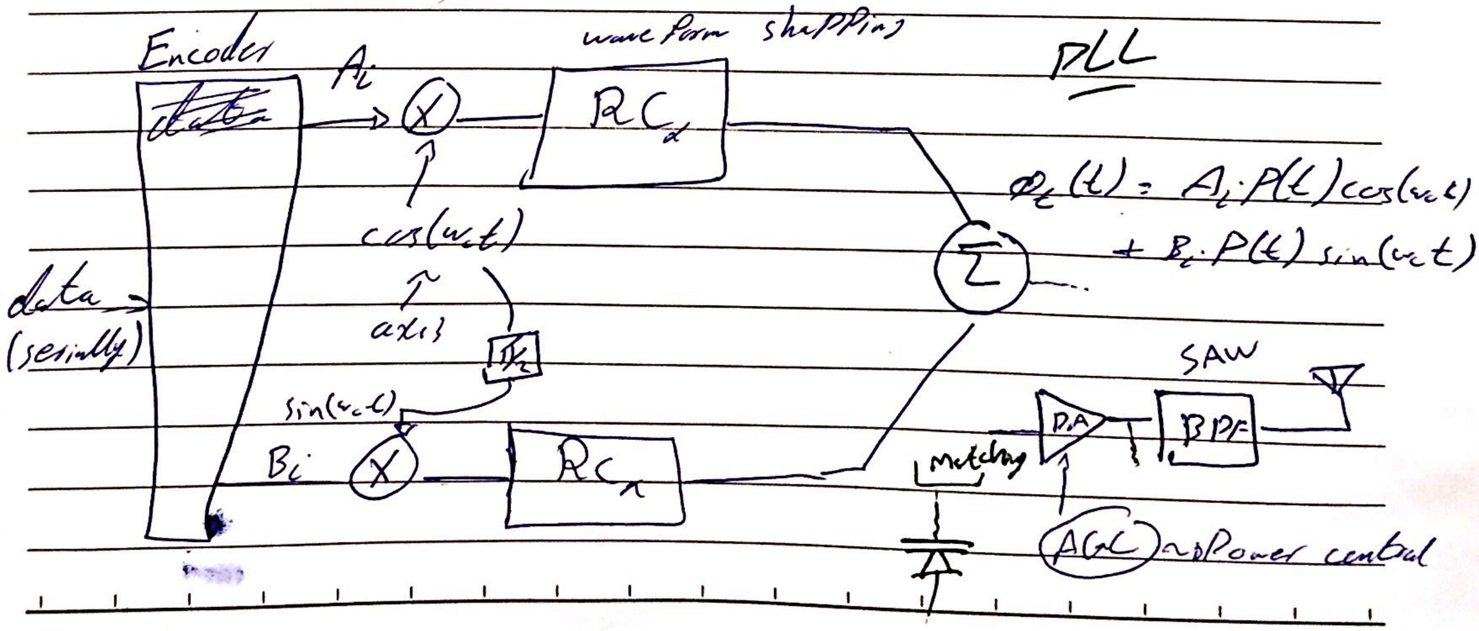


$$out = f(In, state)$$

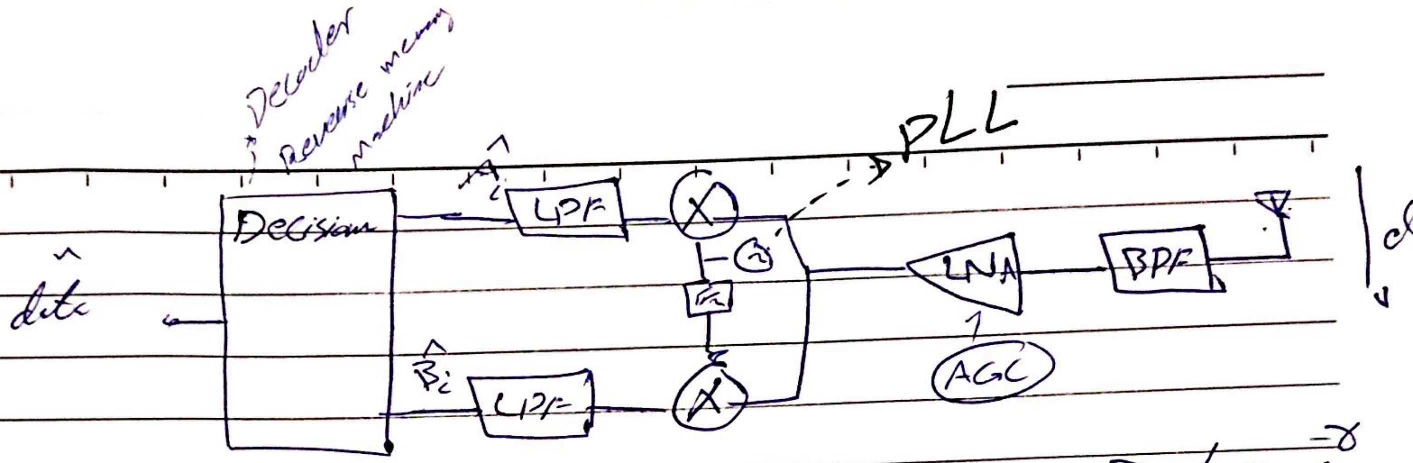
$$state = 2^L$$



Quadrature Modulator



$$\Phi_i(t) = A_i \cdot P(t)\cos(\omega_c t) + B_i \cdot P(t)\sin(\omega_c t)$$

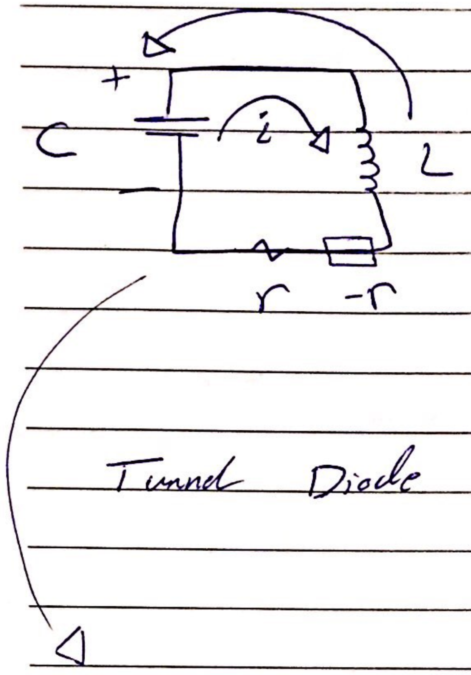


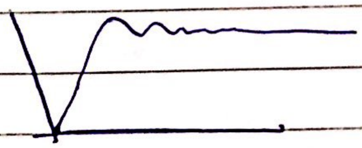
$\hat{data}$ = data + error
↓
BER

$P_r \alpha d$ $P_t d^{-\gamma}$
$2 \leq \gamma \leq 4$

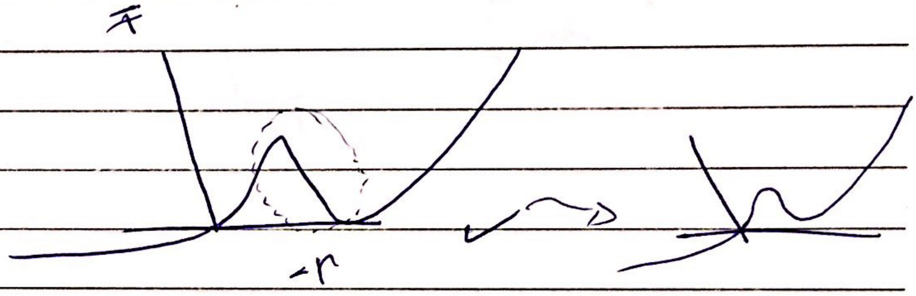
Service Acostic wave filter = SAW

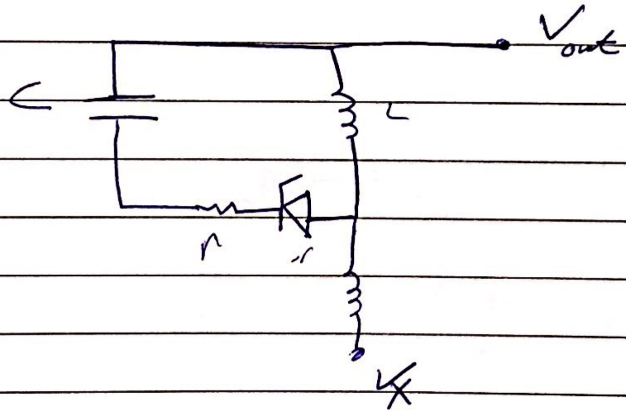
oscillators:

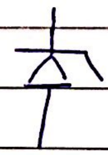


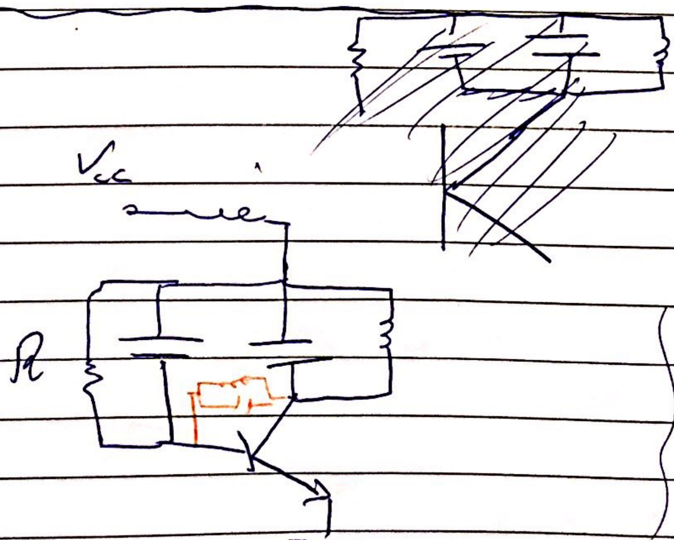


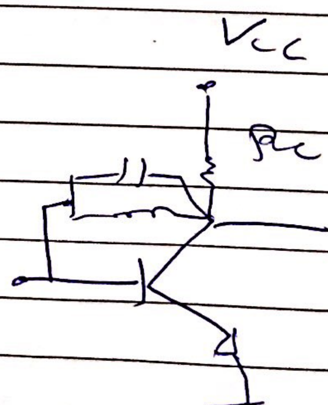
Tunnel Diode



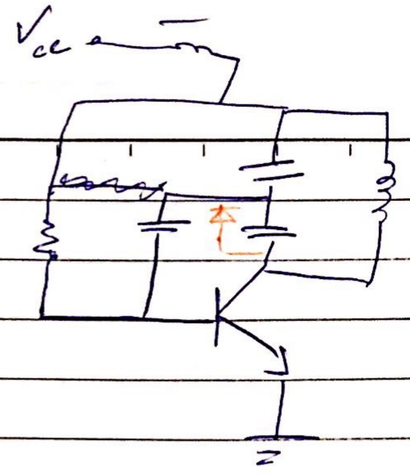




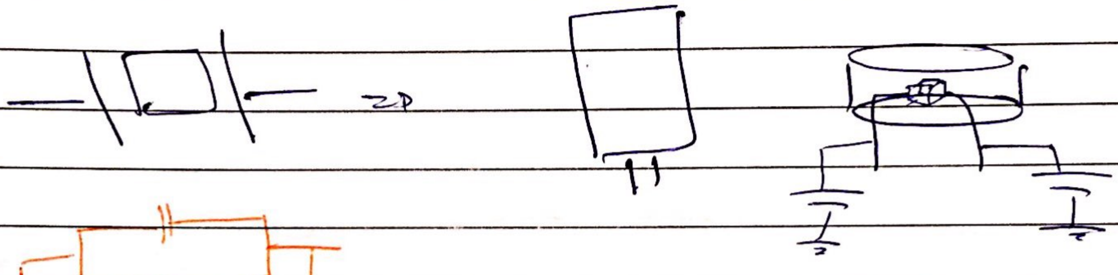


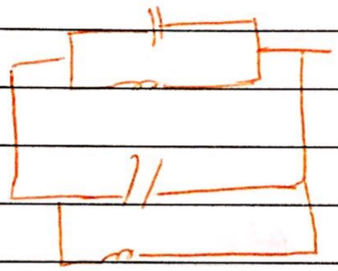


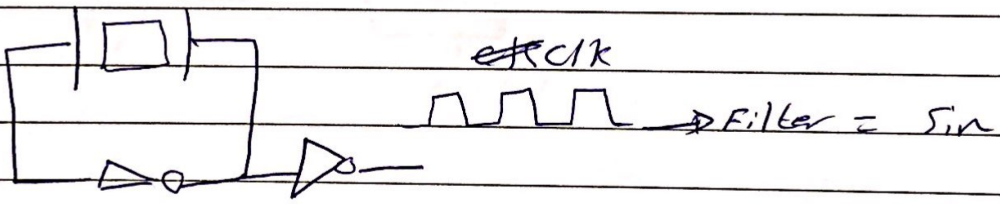
Five Apple



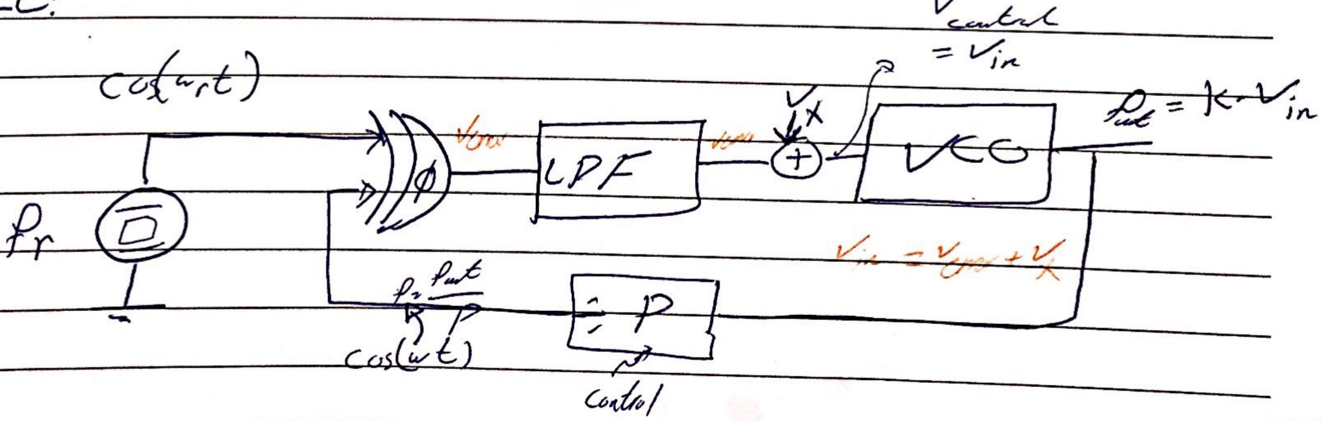
$$f = \frac{1}{2\pi\sqrt{LC}}$$







PLL:

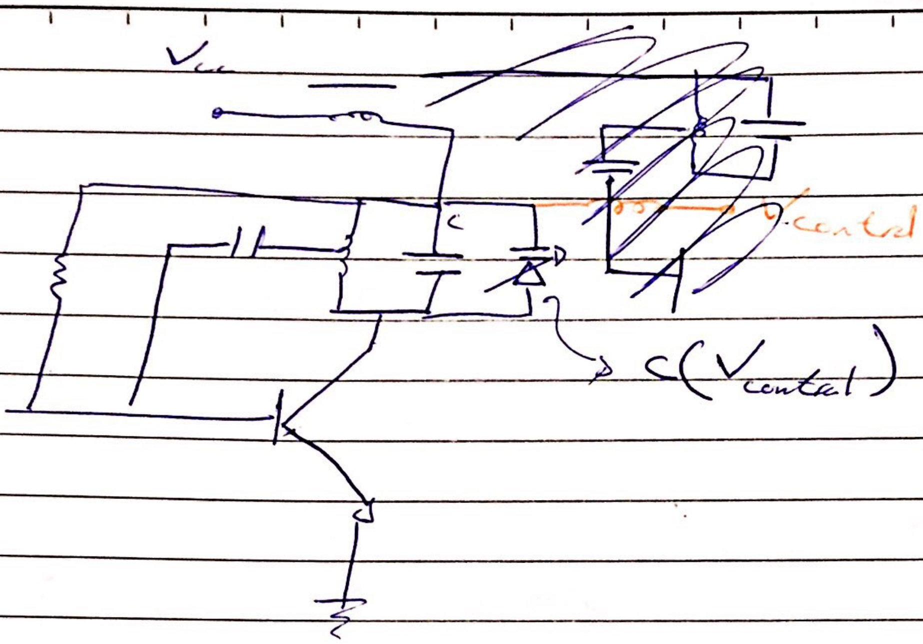


lock state:

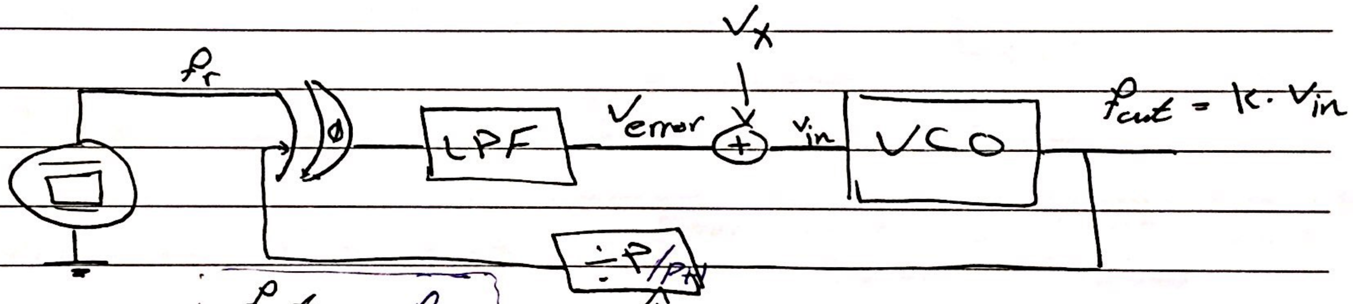
$$f_r = f = \frac{f_{out}}{P} \Rightarrow f_{out} = P \cdot f_r$$

settling time $\propto \dfrac{1}{BW_{LPF}}$

Five Apple



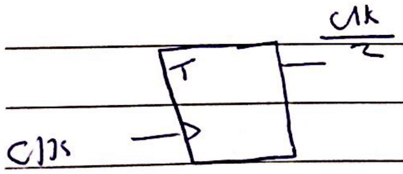
lecture 29

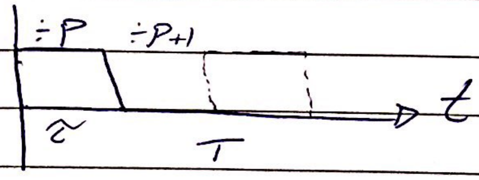


$$\boxed{\frac{f_{out}}{P_{eff}} = f_r}$$

Hi $\rightarrow \div P$

Low $\rightarrow \div P+1$





$$\frac{\tau}{T}$$

$$P_{eff} = \frac{P\tau + (P+1)(T-\tau)}{T} \quad ; \frac{1}{T} < BW_{LPF}$$

$$= \frac{PT + T - \tau}{T} = P + \left(1 - \frac{\tau}{T}\right)$$

$$\frac{\tau}{T} * f_r = 200\ \text{KHz}$$

$$f_{s\,max} = f_r \cdot (P+1)$$

$$f_{min} = f_r \cdot P$$

890 – 915 MHz

$\Delta f = 0.2$ MH

$f_r = 25$ MHz

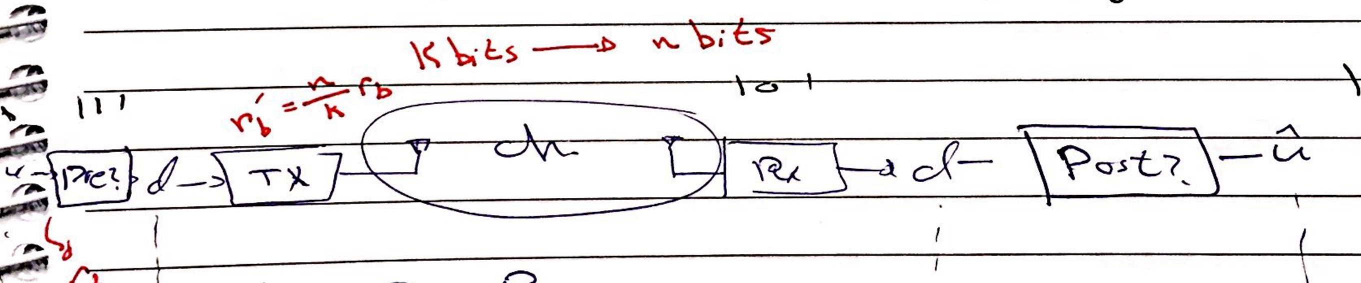
$$\frac{\tau}{T} = \frac{0.2}{25} = 0.8\%$$

C.C → c.c, Turbo codes

LBC → Hamming code, cyclic code, BCH code, LDPC code

GSM: 900 MHz

Channel Coding: Error correcting codes → LBC, C.C.
Detecting codes



$BER_u = P_e$

t → errors we can fix

$$BER_c \cong (BER_u)^{t+1}$$

$$10^{-6} = (0.1)^{t+1}$$

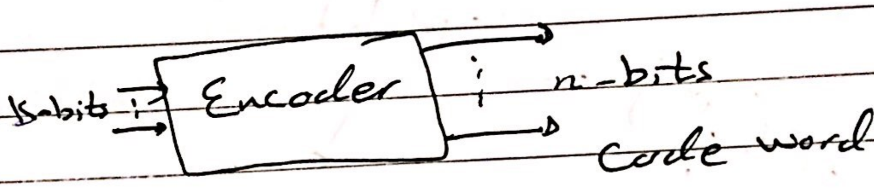
$\frac{n}{K}$ BW expansion factor

$\frac{K}{n}$ code rate

when Comm. sys. is interference limited the only solution to inhance the BER is using ch. coding

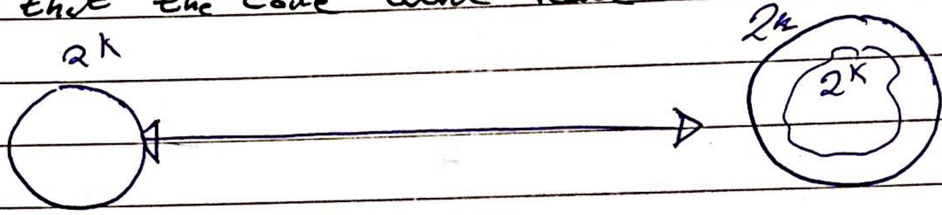
Channel Coding

n > K

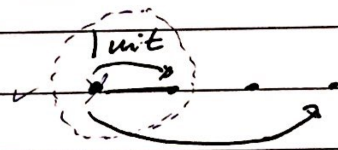


?

select 2^K code words out of 2^n Possible combination
such that the code word have max min hamming distance



d^H_{min}



0 → 1 or 1 → 0

k = 1

1
0 } 2

k = 3

000
001
⋮
111 } 8

Hammin distance

?

$d^H(001101, 101000)$

= sum (XOR)

= 3

001101
101000

Number System.

Field. → group of elements & two binary operations → Addition Identity element = 0

→ Multiblication " " = 1

$a, b, c, d \in \varepsilon$

$a + b = c$

$a \cdot b = d$

Fields

GF (9)

GF(10)

GF(2)

Extended Fields

$GF(9^m)$

$GF(2^3)$

000

⋮

111

GF(2) $+$ = XOR

$\cdot$ = AND

+	0	1
0	0	1
1	1	0

·	0	1
0	0	0
1	0	1

Extended Fields:

$GF(2^m)$

ex: $m = 4$ vector Polynomial

$a = [1\,0\,1\,1]$ $a(x) = 1 + x^2 + x^3$

$b = [1\,1\,0\,0]$ $b(x) = 1 + x$

$[0\,1\,1\,1]$

1

$$a+b = 1+x^2+x^3+1+x$$

$$= x+x^2+x^3$$

$$= [0\ 1\ 1\ 1]$$

2

$$a.b = (1+x^2+x^3)(1+x) = c$$

$$= 1+x^2+x^3+x+x^3+x^4$$

$$= 1+x+x^2+x^4$$

3 $\frac{c}{a} = ?$

$x+1 \longrightarrow$ Quotient

$$1+x^2+x^3 \,\overline{)\, x^4+x^2+x+1}$$

$$x^4+x^3+x$$

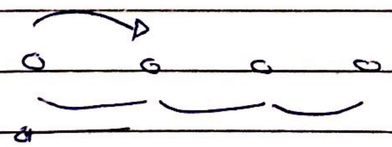
$$x^3+x^2+1$$

$$x^3+x^2+1$$

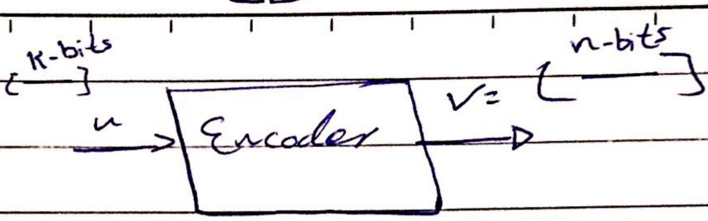
$0 \longrightarrow$ Reminder

Lecture: 31

$GF(2^m)$



$$t = \left\lfloor \frac{d^H_{min}-1}{2} \right\rfloor \longrightarrow \text{floor}$$



$$\vec{v} = \vec{u}\,G$$

$1\times n$ $\quad$ $1\times K$ $\quad$ $\to K\times n$

$$A^{-1}_{K\times n} = \left[A_{K\times n} A^{H}_{n\times K}\right]^{-1} A_{K\times n}$$

$$A^{t}_{K\times n}$$

$$\left(G^{t}_{K\times n} V^{T}_{n\times 1}\right)^{T} = u_{1\times K}$$

Solution:-

$\boxed{\text{Prime members}}$

?

1. Find $\vec{S} = \vec{V} H$ $\quad$ Syndrom

Hamming Code :-

K $\qquad$ $GF(2^{m}) \to$ (m-bits field)

$n = 2^{m} - 1$

$n - K = m$ $\qquad$ $m \geq 3 \to$ non-trivial codes

$d^{H}_{min} = 3$

$t = 1$

$GF(2^{3}) = GF(8)$

Prime members : 2, 3, 5

Ex: $m = 3$

$n = 7$, $K = 4$

(n, K) Hamming Code

① Find a Prime Polynomial in $GF(2^m)$ with order $= m$

$\Rightarrow$ factor of $x^n + 1$

$x^7 + 1 = (1+x)(1+x+x^3)(1+x^2+x^3)$

$1+x+x^3 = 0 \Rightarrow x^3 = 1+x$

Lecture. 32

Ex: $m=3$, $n = 2^3 - 1 = 7$

$k = d = n - m$

$d^H_{min} = 3 \Rightarrow t = 1 = \left\lfloor \frac{d_{min}-1}{2} \right\rfloor \rightarrow$ floor

?!

LBC

$\vec{V} = \vec{u}\, G \rightarrow$ Generating Matrix

$GF(2^m) = GF(8)$

$x^{n \to 7} + 1 = (x+1)(1+x+x^3)(1+x^2+x^3)$?

?!

$1+x+x^3 = 0$

$x^3 = x+1 \quad \dots \quad x^m = (x+1)$

$x^4 = x^3(x) = (x+1)x = x^2 + x$

$x^5 = x^4(x) = x^3 + x^2 = 1 + x + x^2$

$x^6 = x^5(x) = x + x^2 + x^3 = x + x^2 + 1 + x$

$x^7 = x^6(x) = x^3 + x = x + x + 1$

?

	1	x^1	x^2	
0	0	0	0	
1	1	0	0	I
x^1	0	1	0	
x^2	0	0	1	
x^3	1	1	0	
x^4	0	1	1	
x^5	1	1	1	P
x^6	1	0	1	
x^7	1	0	0	

$$G = [I_{k\times k} : P] = \left[\begin{array}{cccc:ccc} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$

Send $u = [1100]$

$$v = uG = [1101101]$$

4th	3rd	2nd	1st
0001	0010	0111	1000
1100	1100	1100	1100
1101 = 3	1110 = 3	1011 = 3	0100 = 1
"	"	"	
4 !?	?!		
X	X	X	X

@ RX

$I_{n-k * n-k} = I_{m\times m}$

$$H = \left[\begin{array}{ccc:c} 1 & 0 & 0 & \\ 0 & 1 & 0 & P^T \\ 0 & 0 & 1 & \end{array}\right]$$

$$H = \left[\begin{array}{ccc:cccc} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{array}\right]$$

$$S = vH^T = [0\,0\,0] \Rightarrow u = [1100] \text{ ?!}$$

$$H^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

error

assume $\hat{V} = [1110101]$

$S = [0\,0\,1] \Rightarrow S^T = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \rightarrow$ look in H

$\hat{V} = [1100\,101] \Rightarrow u = [1100]$

assume $\hat{V} = [1100\,001]$

$S = [0\,0\,1] \Rightarrow S^T = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ look in H

$\hat{V} = [1100\,101] \Rightarrow u = [1100]$

Ex: In a digital comm. sys the BER in channel was 10^{-2} use $(7,4)$ H.C to (approxim) get $BER_{coded} = 10^{-8}$

$(\underset{n}{7}, \underset{k}{4})$ 10^{-10}

$BER_c = (BER_{uncoded})^{t+1}$

$t+1 = 4 \rightarrow t = 3$

$4 \rightarrow 7$

$100 \rightarrow 175$

$\frac{1}{BER} = 100 \rightarrow 100 \times \frac{7}{4} = 175$

$10^{-2} \rightarrow \frac{1}{100}$, 100 bit in channel $\Rightarrow$ we have 14 of (7) packet

$\boxed{14 \times 7 + 2}$ غلط

$(10^{-2})^{15} = 10^{-30}$

$\boxed{14 \times 4 + 2}$ صحيح

⊛ cyclic codes

$$g(x) \cdot u(x) = v(x)$$

generator uncoded coded

$$\frac{v(x)}{g(x)} = u(x) \rightarrow (x^n - 1) \text{ or less}$$

can't divide $g(x)$

$$(1, x^1, x^2, x^3, x^4, x^5, x^6) = a(x) + \frac{r(x)}{g(x)}$$

from Previous example

$$g(x) = 1 + x + x^3$$

$$\begin{array}{r|l} 1+x+x^3 & x^3 \\ & x^3+1+x \\ \hline & 1+x \end{array}$$

(1)

lecture 33

cyclic codes:-

m

$n = 2^m - 1$, $n - k = m$, $d_{min} = 3 \rightarrow t = 1$ H ?! ?!

generator Polynomial $g(x)$

$$v(x) = g(x) \cdot u(x)$$

$$Tx + e(x) \longrightarrow \quad \hat{v}(x) = v(x) + e(x)$$

ⓐ Rx

$$\frac{\hat{v}(x)}{g(x)} = u(x) + \frac{e(x)}{g(x)} = u(x) + a(x) + \frac{r(x)}{g(x)}$$

$e(x) = \{1, x, x^2, \ldots, x^{n-1}\}$

$g(x)$ is Prime in $GF(2^m)$

Ex: $m = 3$, $g(x) = 1 + x^2 + x^3$

$n = 7$, $K = 4$

$u(x) = 1 + x^3$ 1001

$v(x) = (1+x^3)(1+x^2+x^3)$

Syndrom table

$$\frac{e(x)}{g(x)} = a(x) + \frac{r(x)}{g(x)}$$

$e(x)$	$a(x)$	$r(x)$
1	0	1
x	0	x
x^2	0	x^2
x^3	1	$1+x^2$
x^4	$1+x$	$1+x+x^2$
x^5	x^2+x+1	$1+x$
x^6	x^3+x^2+x	$x+x^2$
$\cancel{x^7}$		

$e(x) = x^4$

$\hat{v}(x) = 1 + x^2 + x^4 + x^5 + x^6$?!

Five Apple

$$\begin{array}{r} x^3 + x \quad = u(x) + a(x) = \hat{u}(x) \\ 1 + x^2 + x^3 \overline{) x^6 + x^5 + x^4 + x^2 + 1} \\ \underline{x^6 + x^5 + x^3} \\ x^4 + x^3 + x^2 + 1 \\ \underline{x^4 + x^3 + x} \\ 1 + x + x^2 \end{array}$$

$1 + x + x^2 = r(x) \rightarrow$ refer to table

$u(x) = \hat{u}(x) + a(x) = x^3 + x + 1 + x = x^3 + 1$

$a(x) \rightarrow$ from table $= 1 + x$

ex: $m = 4$

$n = 15$ $\quad (d^H_{min} = 3 \;, t = 1)$

$k = 11$

Shortened codes

$(n - \ell, k - \ell)$ | $(7, 4)$ $\ell = 1 \rightarrow (6, 3)$

$(13, 9)$

$\frac{k-\ell}{n-\ell} < \frac{k}{n}$ = code rate

Ex:- $BER_{ch} = 10^{-2}$

$\frac{1}{BER} = 100 = \ell$

So we need $t \geq 3$ $\quad \frac{100}{3} = 33.3 \approx 33$

$\left(n < 33 = \frac{1}{BER} \right)$

$(m=5 \rightarrow n=31\ ,\ k=26)$

another Practical implimintation

$n'=25$ $\qquad$ $(l=6)$

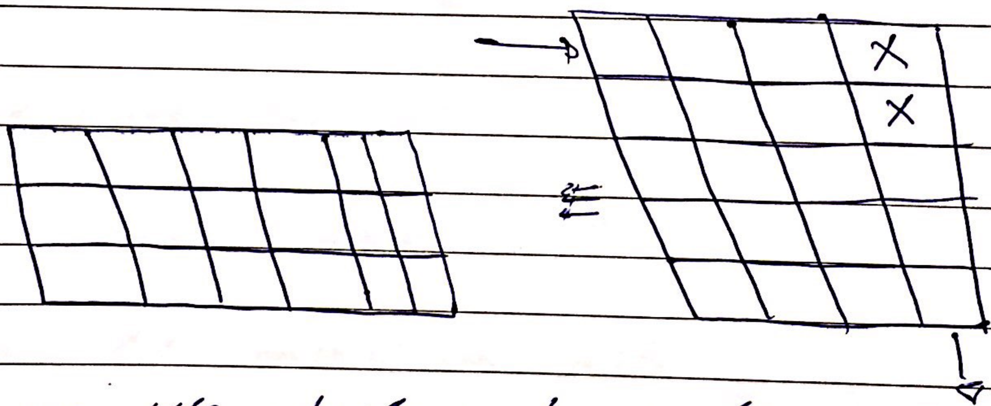
$(25, 20)$ $\qquad$ $BER = 10^{-10} \approx 10^{-8}$

---

use $(15, 11)$ to generate a 3 error correcting code

$[\ [n\ ,\ 1\ ]\ [n\ ,\ 1\ ]\ [n\ ,\ 1\ ]\ ]$

111222 ...



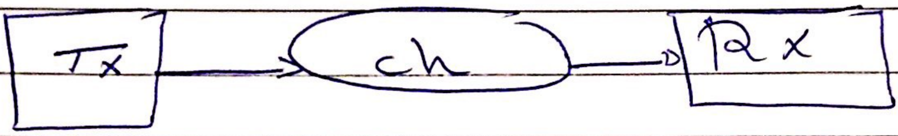
interleaving

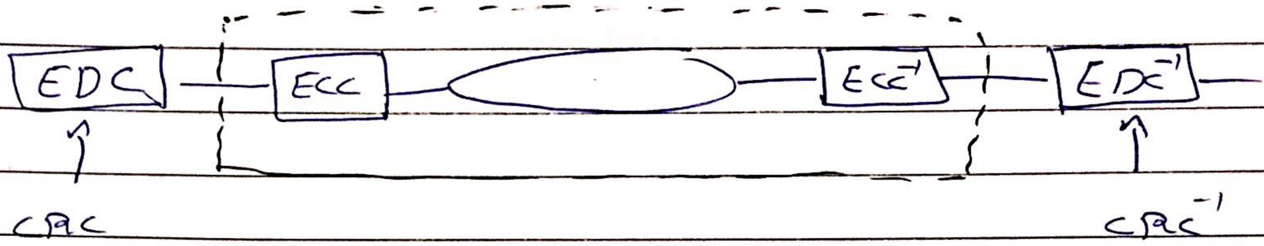
so we use interleaving to randomize Bursts of error

Lecture. 34

CRC:

cyclic redundancy check (Error detection codes)





$g(x)_{C.C.}$ in $GF(2^m)$

$\rightarrow d_{min}^{H} = 3$

$$\boxed{g(x)_{CRC} = (1+x)\, g(x)_{C.C.}} \qquad \text{can detect } d_{min}^{H} \quad ?$$

$1+\dots+x^m$

$\rightarrow 1 + \dots + x^{m+1}$

any code can detect $(d_{min}^{H} - 1)$ errors ?

Maximum Packet length $= L_p = 2^m = (n+1)$

Ex: let $m=4 \Rightarrow n=15$

$g(x)_{C.C.} = 1 + x + x^4$, $\begin{pmatrix} 1+x+x^m \\ 1+x^{m-1}+x^m \end{pmatrix}$

$g(x)_{CRC} = (1+x)(1+x+x^4)$
$= 1 + x + x^4 + x + x^2 + x^5 = 1 + x^2 + x^4 + x^5$

How to use:

$u(x) \cdot g(x) = V(x) \xrightarrow{+e(x)}$

$\hat{v}(x) = v(x) + e(x)$

$\dfrac{\hat{v}(x)}{g_{CRC}(x)}$ = if reminder $\sim\!\!\to$ errors

Method II

at Tx : $\dfrac{u(x)}{g_{CRC}(x)} = a(x) + \dfrac{r(x)}{g(x)}$

$\overset{n+1}{\longleftrightarrow}$

$[\underbrace{u(x)}_{k} \; ; \; \underbrace{r(x)}_{m+1 \text{ bits}}]$

at Rx

$\dfrac{\hat{u}(x)}{g_{CRC}(x)} = \hat{a}(x) + \dfrac{\hat{r}(x)}{g_{CRC}(x)}$

if $r(x) = \hat{r}(x) \longrightarrow$ no errors

Ex: $L_p = 1500$, Find the CRC to ? detect 3 errors

$L_p \overset{?}{\leq} n+1 \sim\!\!\to m = 11 \longrightarrow n = 2047$

$l = 548$ shorted

$g_{C.C}(x) = 1 + x + x^{11}$

$\cancel{g(x)}$

$g_{CRC}(x) = (1+x)(1+x+x^{11})$
$= 1+x^2+x^{11}+x^{12}$

data length $= k' = L_p - 1 - m = 1500 - 1 - 11$
$= 1488$ bits

Lecture 35

BCH codes:

$t > 1$
$n = 2^m - 1$
$n-k = mt$
$d^H_{min} = 2t+1$

Ex: $t=2$ $GF(2^5)$

$g_{BCH}(x) = LCM(\phi_i's ; i=1, .. 2t)$
$= \phi_1 . \phi_3 = (1+x^2+x^5)(1+x^2+x^3+x^4 ...$

$\phi_1 = \phi_2 = \phi_4 = \phi_8$
$\phi_3 = \phi_6$
$\phi_5 = \phi_{10}$

$\phi_i = \phi_{i \cdot 2^c}$

$m = 5$
$n = 31 \Rightarrow k = 21$

Ex: $g_{CRC}(x) = 1 + x^2 + x^{11} + x^{12}$

$\Rightarrow m = 11 \Rightarrow GF(2^{11})$

$n = 2^m - 1 = 2047$, $\boxed{L_p = 2048}$

$[2036 : 12] \Rightarrow [1036 : 12]$
data Parity

$\Rightarrow [1988 : 12]$

$L_p \Rightarrow 2000$ bit

Punctured Codes :-

$u = [\, 1\, 0\, X - X - - X \,]$

$\underbrace{\qquad\qquad}_{9 \text{ bits} \Rightarrow X}$

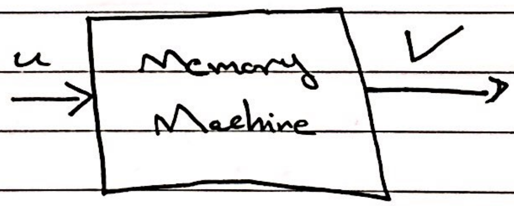
LDPC codes

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 1 & 0 & 0 & 1 & \cdots \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & \cdots \end{bmatrix}$$

→ Sova (Sum Product)

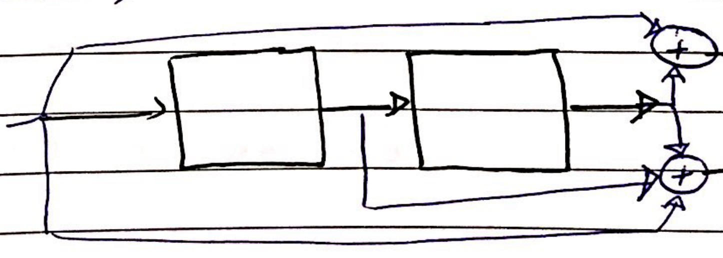
$H = [\qquad]$

* Convolutional Codes:



$v_i = f_i(u, \underbrace{\text{Memory}}_{\text{states}})$, $i = 0, 1, 2, \cdots, n-1$

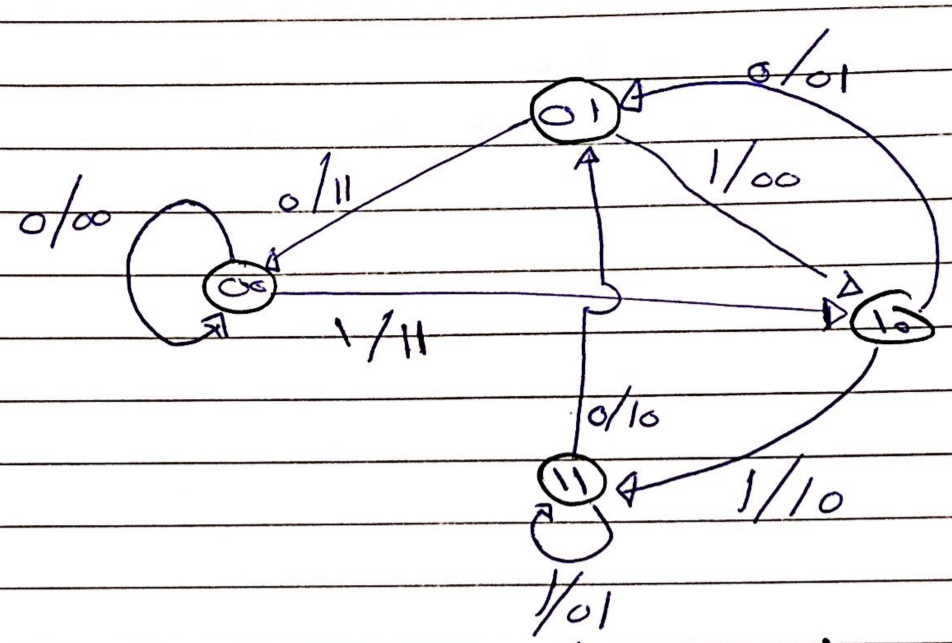
Ex:- $K=1$, $n=2$, $n > K$



$v_0 = u + u_{-2}$ $\boxed{P/5}$

$v_1 = u + u_{-1} + u_{-2}$

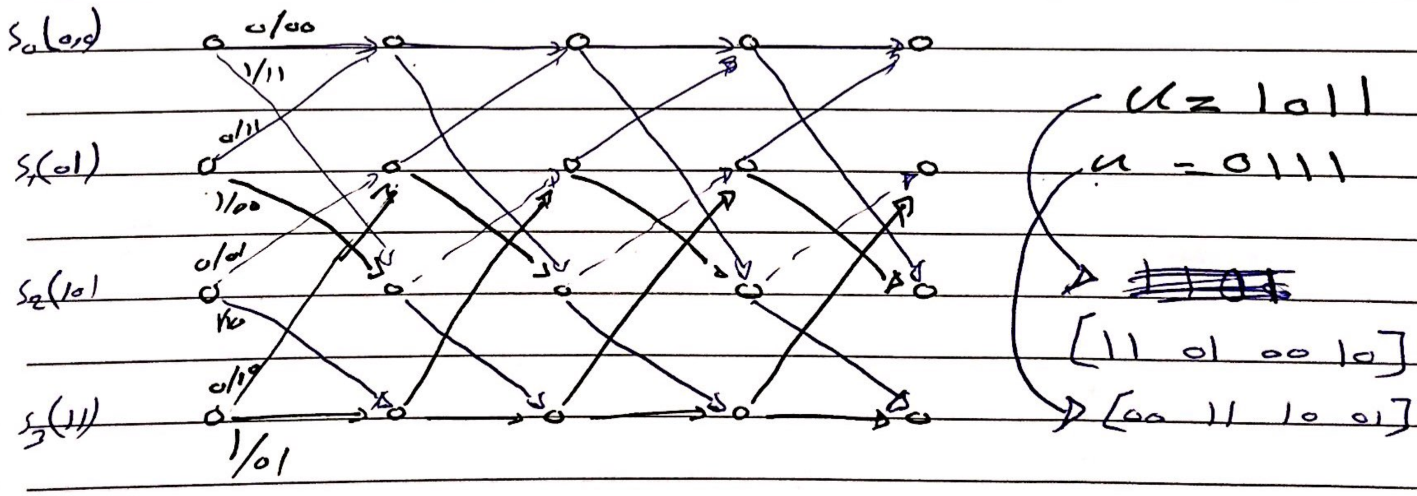
state diagram



convolutional codes ⇒ (2, 1, 2) C.C

n k Memory

Tree diagram



u = 1011 ⇒ [11 01 00 10]

11

Minimum error event happens when an error diverts the correct Path at a certain state & reImage with the correct Path at the later state in minimum number of transitions

$e = [11 \quad 01 \quad 11] \;,\; [00 \quad 00 \quad 00 \quad 00]$

?!

5 errors

$\boxed{d_{free} = 5} \;,\; t = \frac{d_{free} - 1}{2} = 2$