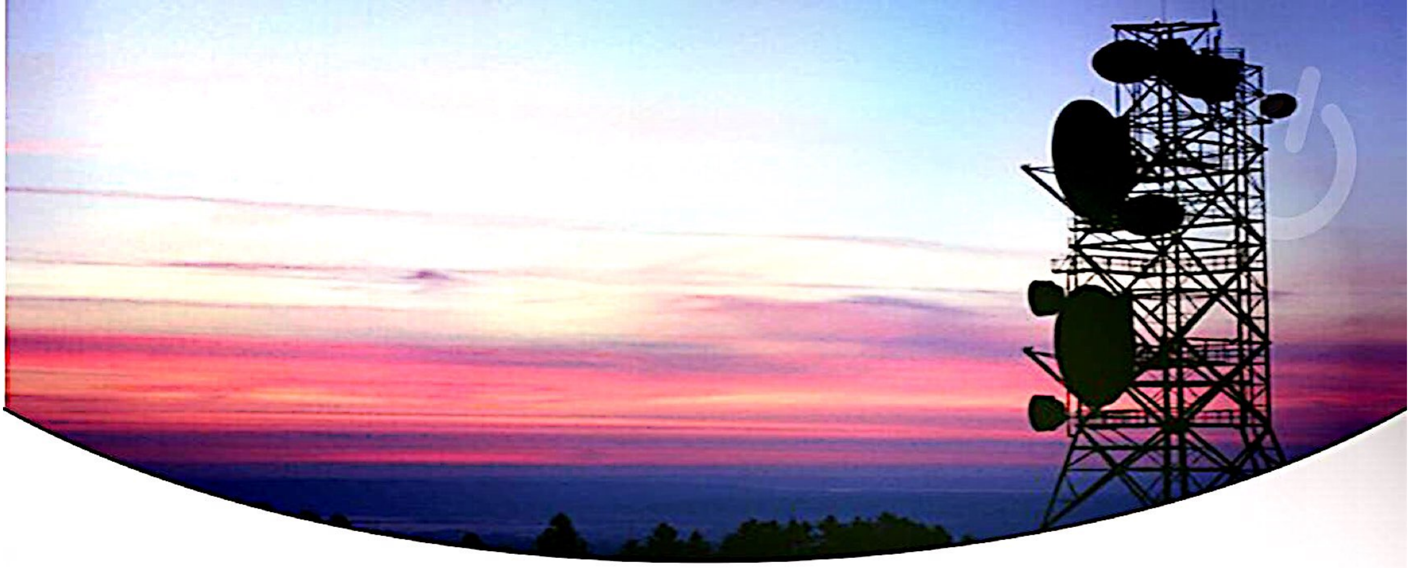

Communications1

Fall017

Dr.Mhmd Hawa

By: Mhmd Abuhashya

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Lecture 14: FDM, AM Radio, and the Superheterodyne Receiver

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EE421: Communications I: Lecture 14. For more information read Chapter 4 in your textbook or visit http://wikipedia.org/.

Multiplexing: FDM

- Frequency Division Multiplexing **(FDM)** is a process that allows the transmission of several signals over the same channel at the same time.
- This is achieved by modulating the different signals on different carriers with different carrier frequencies.
- The receiver isolates one signal from the rest using a **tuneable BPF.**

Tuneable BPF: is BPF with variable center freq.

TY Broadcasting (FDM)

using

Guard Band ≡ 0.25MHz.

one NTSC TV station.

Luminance (VSB+C)

Audio (FM).

PAL (8MHz)

BPF.

w/ center.

chrominance (QAM).

↳center freq $=\frac{60+54}{2}=57$

⇒ • For an FDM system, you need to know:

- ① – *Broadcast frequencies* for the stations (i.e., allocated spectrum). → center freq. of BPF.
- ② – *Bandwidth* of each station. → Bandwidth of BPF
- ③ – *Guardband* between adjacent stations. → standard ⇒ BPF roll off.

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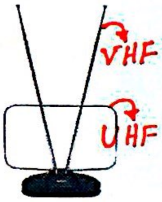
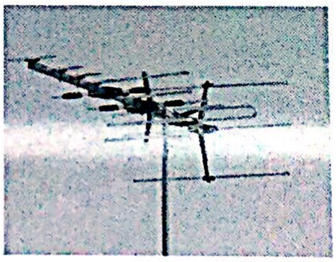
standard:
NTSC → 6MHZ
PAL → 8MHZ

GuardBand:
allow the use of practical BPF @ Rx.

TY Broadcasting



- Terrestrial TV uses broadcast frequencies within the ranges:
- VHF (Very High Frequency): 30 MHz to 300 MHz
- UHF (Ultra High Frequency): 300 MHz and 3 GHz.



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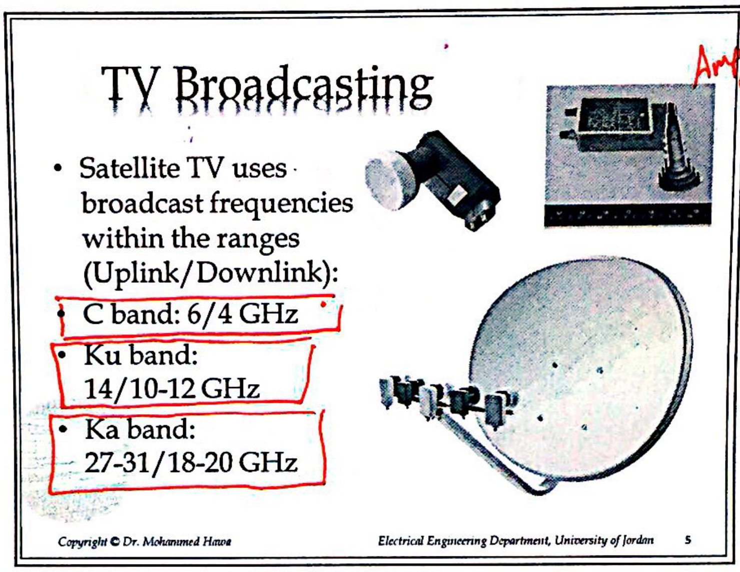
MEMORIZE:
VHF:
30MHZ–300MHZ.
UHF:
300MHZ–3GHZ.

BPF center freq. to watch 2nd TV station. =63MHZ.

TV station carrier freq. = 61.25 MHZ.

* The one with Lower Attenuation: VHF.

* The one which is shorter: UHF. due to: $\lambda\downarrow=\frac{c}{f\uparrow}\Rightarrow L_{ant.}\downarrow=\frac{\lambda\downarrow}{2}$



TV Broadcasting

- Satellite TV uses broadcast frequencies within the ranges (Uplink/Downlink):
- C band: 6/4 GHz
- Ku band: 14/10-12 GHz
- Ka band: 27-31/18-20 GHz

More Att.

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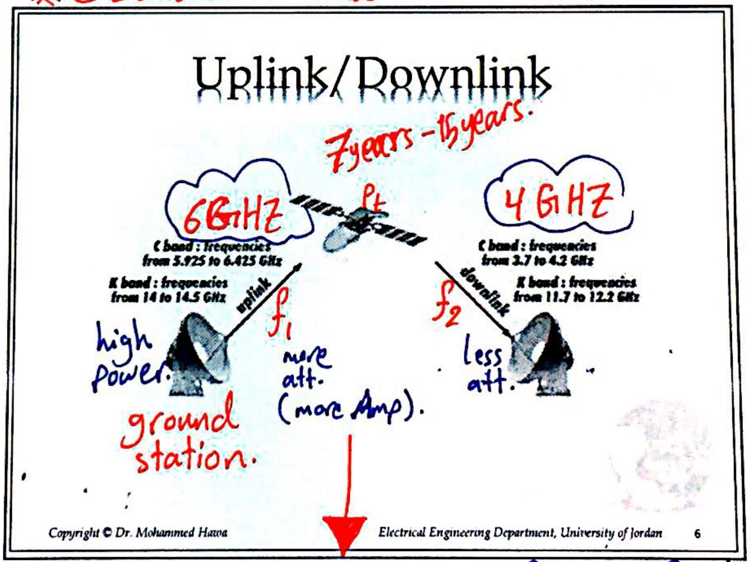
Amplifier & filter for Noise & ? ↳ down converter.

uplink is higher frequancy than downlink.

Because uplink is more att & downlink Is Less att.

※ C band & Ku band :



Uplink/Downlink

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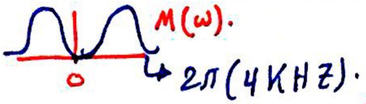
FDD ≡ Frequancy Division Duplex.

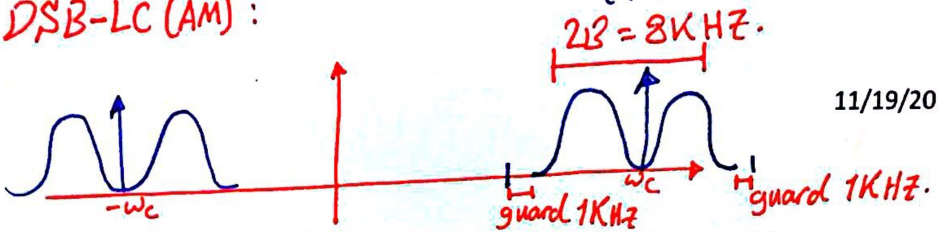
lower freq = lower att = reserve better power.

※AM Radio: m(t)=voice. [B=4KHz]



*Modulate using DSB-LC (AM):

2B = 8KHZ.

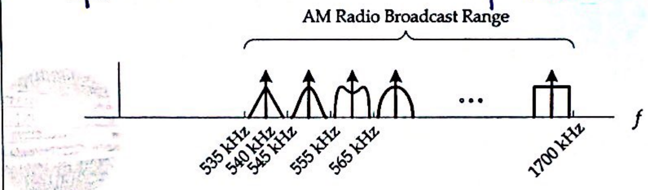


critical under
cheap ⇓
Rx
E, D

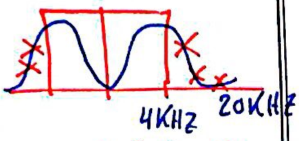
AM Radio Broadcasting

- Each station is an AM modulation of human voice.
- FDM is used to multiplex signals on the air waves.
- **US:** Each station occupies a bandwidth of 10 kHz.
- **Europe:** Each station occupies a bandwidth of 9 kHz.

→ Memorize [535KHz – 1700KHz] ≃ 1MHz



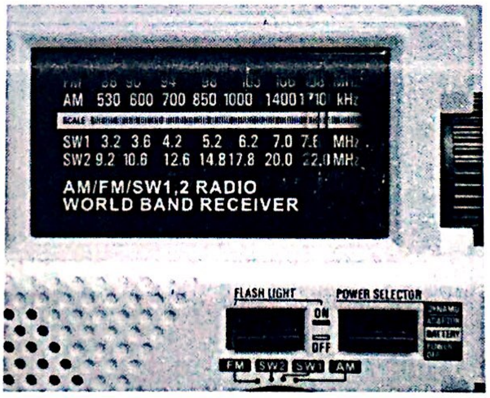
if m(t)=Music.
& B=4KHz.



most of it will be rejected.

so Music wouldn't be sent by AM radio due to 4KHz BW.

HW: Look at Your Radio Dial



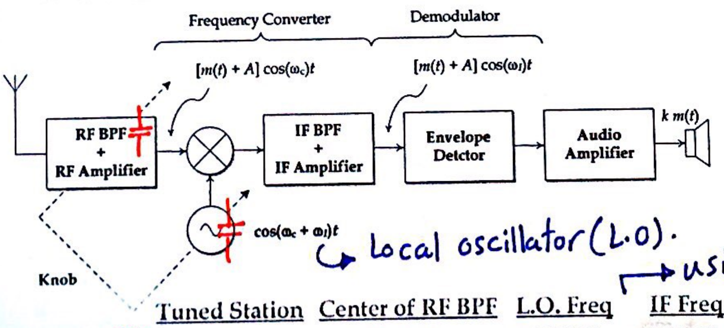
The Superheterodyne Receiver

- Receivers in FDM system require a BPF.
- It is extremely difficult (*expensive*) to design highly selective (*narrowband*) filters at **high** center frequencies.
- This is specially true if the filter is **tuneable.**
- Solution: Use a two-stage filtering process, one of which at lower frequency.

AM Superheterodyne Receiver

RF ≡ Radio Freq. ≡ High Freq.

IF ≡ Intermediate Freq.



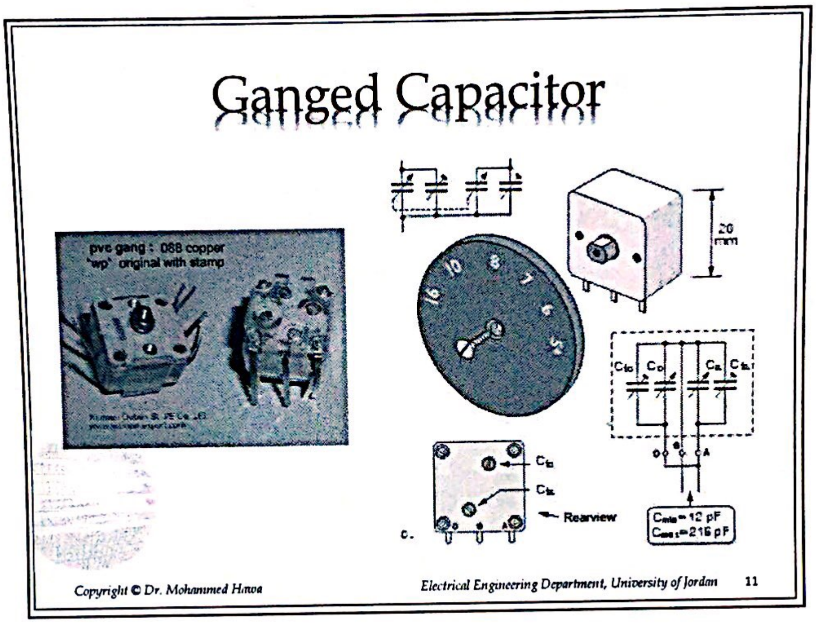
local oscillator (L.O).

using Sum converter.

	Tuned Station	Center of RF BPF	L.O. Freq	IF Freq
Ex.	1000 kHz	1000 kHz	1455 kHz	455 kHz
HW1	1020 kHz	1020 kHz	1475 kHz	455 kHz
HW2	1500 kHz	1500 kHz	1955 kHz	455 kHz

This is U.S standard since it is 10KHZ step.

Memorize: f_I used for AM Radio = 455KHZ.



Ganged Capacitor

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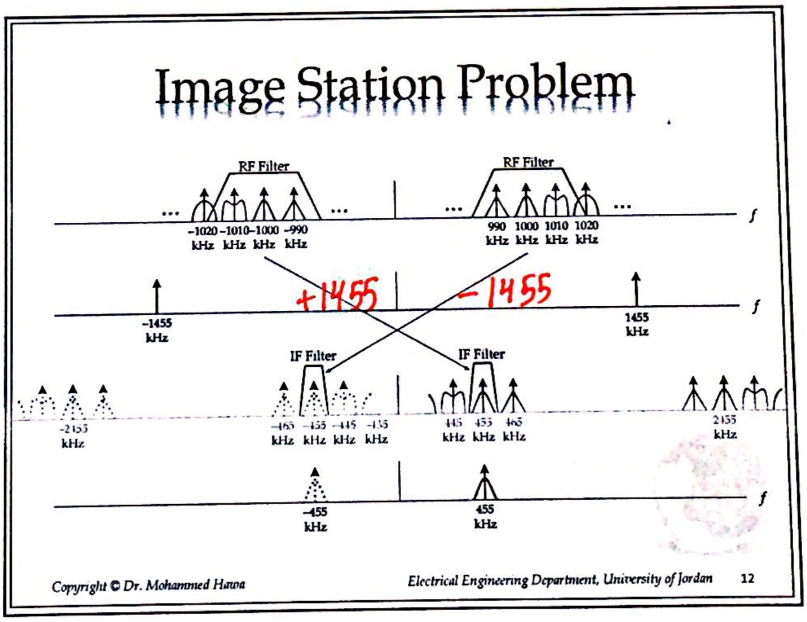

Image Station Problem

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*What is the Maximum allowable RF BPF Bandwidth?

Image Station (*Part* 2)

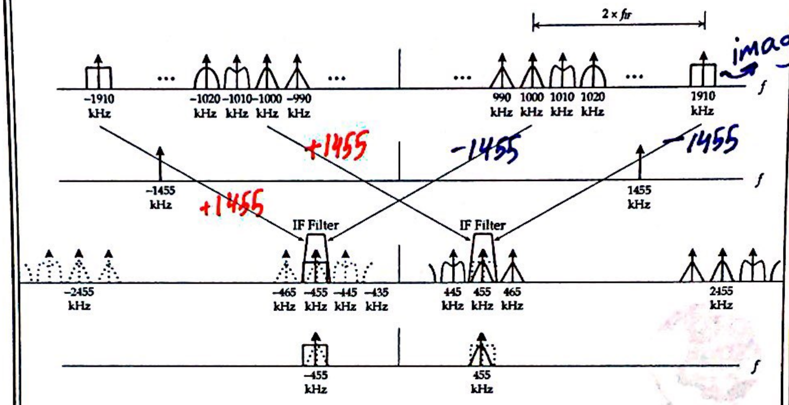


image station for station @ 1000KHZ

Superheterodyne *Why*'s

- *Why* the RF Filter?
  - Eliminates the *image station*.
  - Reduces the amount (power) of noise that enters the receiver.
- *Why* the IF Stage (heterodyning)?
  - With its high-selectivity and lower price, the IF filter isolates the desired radio station from all others sent using FDM.
  - Since the IF frequency does not change with the tuned station, it is easier to design the E.D.

Superheterodyne *Why*'s

- *Why* the sum, not difference?
- The sum (as opposed to the difference) in the receiver results in a smaller tuning range ratio, which requires a smaller tuning capacitor for the local oscillator.
- Hence, this solution is cheaper.

Homework

- Now design a superheterodyne receiver, but this time using the difference for L.O.:
  - If you want to listen to the station at 1000 kHz what settings should you choose for the RF BPF, the oscillator, and the IF BPF?
  - Repeat the same problem if you want to listen to the 1020 kHz and 1500 kHz stations.
  - What is the frequency of the image station if you are listening to the station at 1000 kHz?

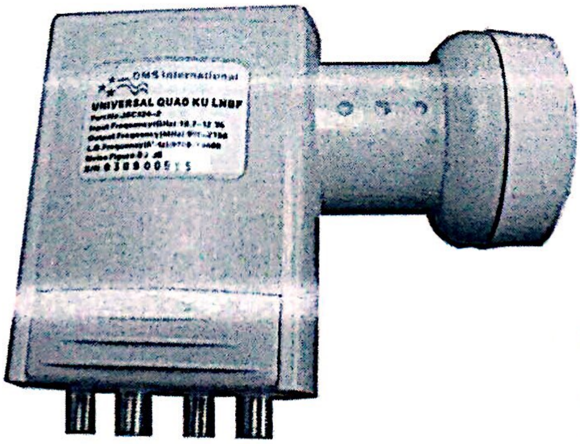
Superheterodyne *Everywhere!*

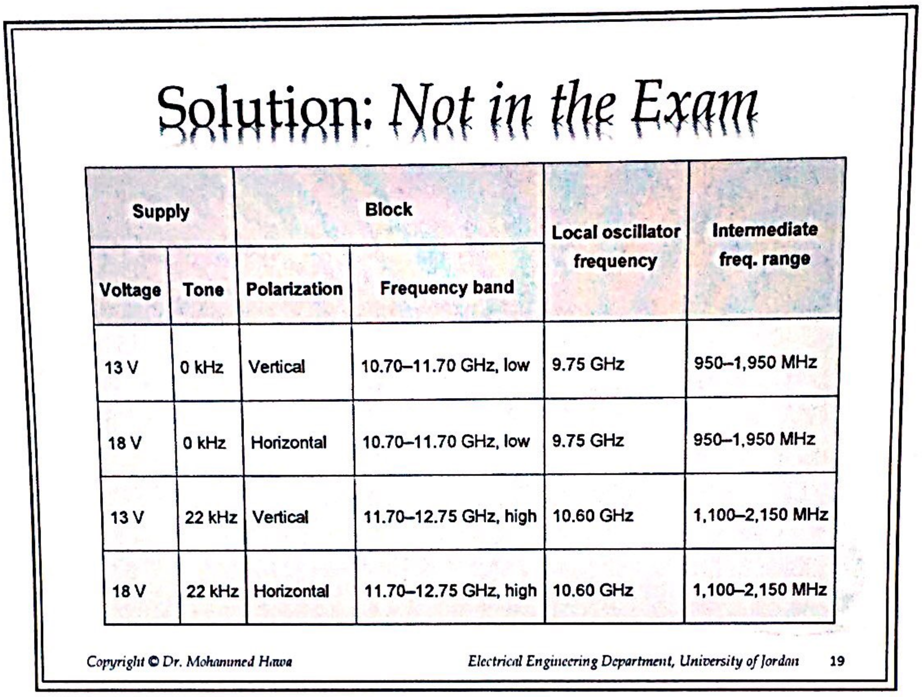
- The superheterodyne receiver is much more popular nowadays compared to the homodyne receiver.
- It is used in many communication systems including: FM Radio, Analog and Digital TV broadcasting, Cellular phones, WiMAX, Satellite and Microwave systems, GPS, etc.
- Some popular IF frequencies:
  - AM radio receivers: 455 kHz
  - FM radio receivers: 10.7 MHz
  - Analogue television receivers: 45.75 MHz

} Memorize.

Not included:

Homework





Solution: *Not in the Exam*

Supply		Block		Local oscillator frequency	Intermediate freq. range
Voltage	Tone	Polarization	Frequency band		
13 V	0 kHz	Vertical	10.70–11.70 GHz, low	9.75 GHz	950–1,950 MHz
18 V	0 kHz	Horizontal	10.70–11.70 GHz, low	9.75 GHz	950–1,950 MHz
13 V	22 kHz	Vertical	11.70–12.75 GHz, high	10.60 GHz	1,100–2,150 MHz
18 V	22 kHz	Horizontal	11.70–12.75 GHz, high	10.60 GHz	1,100–2,150 MHz

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Amplitude Modulation
- DSB-SC $m(t) \cdot \cos(\omega_c t)$
- DSB-LC $[m(t)+A] \cos(\omega_c t)$
- SSB
- VSB
- QAM

Related to each other.

Lecture 15: Frequency and Phase Modulation (FM and PM)

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EE421: Communications I: Lecture 15. For more information read Chapter 5 in your textbook or visit http://wikipedia.org/.

Just like *earlier*…

- Time domain expression.
- Time domain *sketch*.
- Average power of modulated signal.
- Frequency domain representation.
- *Bandwidth* of modulated signal.
- Signal-to-Noise Ratio and Quality.
- Practical Applications.
- Modulators and Demodulators (hardware).

Carrier freq. vs Instantaneous freq.

↓ Always Constant.

↓ changeable $\propto m(t)$.

Angle Modulation (FM and PM)

$$\varphi_{unmodulated}(t) = A\cos(\omega_c t + \theta_0)$$

→ Argument or angle

$$\varphi_{FM\ or\ PM}(t) = A\cos\theta(t)$$

$$\boxed{\omega_i(t) \triangleq \frac{d\theta(t)}{dt}}$$

$$\boxed{\theta_i(t) \triangleq \theta(t) - \omega_c t}$$

- $\theta(t)$ is **generalized angle** of the modulated signal.
- $\omega_i(t)$ is **instantaneous frequency** of modulated signal.
- $\theta_i(t)$ is **instantaneous phase** of modulated signal.

Frequency Modulation (FM)

- The *instantaneous frequency* of the carrier changes in proportion to the message.

$$\omega_{i_{FM}}(t) = \omega_c + k_f m(t)$$

$$\theta_{FM}(t) = \omega_c t + k_f \int_{-\infty}^{t} m(t)dt$$

$$\boxed{\varphi_{FM}(t) = A\cos\left(\omega_c t + k_f \int_{-\infty}^{t} m(t)dt\right)}$$

$$\theta_{i_{FM}}(t) = k_f \int_{-\infty}^{t} m(t)dt$$

$\omega_i = \frac{d\theta(t)}{dt}$

$\int \omega_i dt = \int d\theta(t)$

$\theta(t) = \int \omega_i(t)\,dt$

$= \int[\omega_c + K_f m(t)]dt$

$= \omega_c t + K_f \int m(t)dt$

$\theta_i(t) = \theta(t) - \omega_c t$

$= K_f \int_{-\infty}^{t} m(t)\,dt.$

unit of the constant K_f: $\frac{rad/s}{V}$

from $\phi_{FM}(t)$ find $\omega_i(t)$ & $\theta_i(t)$.

solution: $\theta(t) = \omega_c t + K_f \int m(t)dt$

$\Rightarrow \omega_i(t) = \frac{d\theta(t)}{dt} = \underline{\omega_c + K_f m(t)}$; so $\omega_i(t)$ changes depending on the original signal $m(t)$.

Example:

For the carrier signal $c(t) = A\cos(w_c t + \theta_0)$ find $w_i(t)$ & $\theta_i(t)$?

Solution: $\theta(t) = w_c t + \theta_0 \Rightarrow w_i(t) = \frac{d\theta(t)}{dt} = w_c \Rightarrow \boxed{w_i(t) = w_c}$

ONLY for the case of carrier.

$\theta_i(t) = \theta(t) - w_c t = w_c t + \theta_0 - w_c t = \theta_0$

$\Rightarrow \boxed{\theta_i(t) = \theta_0}$

These values for Unmodulated

⇒ They don't apply for the Modulated.

Summarize:

$$\phi(t)_{\text{unmodulated}} = A\cos(w_c t + \theta_0)$$

$$\phi(t)_{FM} = A\cos\left(w_c t + K_f \int_{-\infty}^{t} m(t)\,dt\right) \quad , w_{i_{FM}}(t) = w_c + K_f m(t).$$

Phase Modulation (PM)

- The *instantaneous phase* of the carrier changes in proportion to the message.

$\theta_i(t) = \cancel{\theta_0} + k_p m(t)$

$$\downarrow \theta_{i_{PM}}\uparrow(t) = k_p m(t)\uparrow \downarrow$$

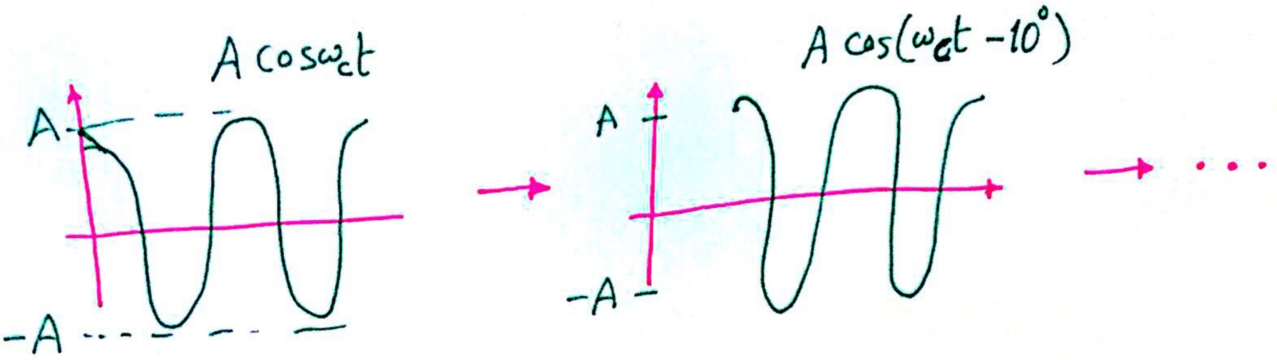
$$\theta_{PM}(t) = \omega_c t + k_p m(t)$$

$$\varphi_{PM}(t) = A\cos\left(\omega_c t + \underbrace{k_p m(t)}_{\theta_i}\right)$$

$$\omega_{i_{PM}}(t) = \omega_c + k_P \frac{dm(t)}{dt} = \boxed{\omega_c + k_P m'(t)}$$

FM and PM Equivalence

- FM
  - Constant amplitude A
  - Constant carrier frequency ω_c
  - Variable *instantaneous* frequency $\omega_i \propto m(t)$
  - Variable *instantaneous* phase $\theta_i \propto \int m(t)dt$
- PM
  - Constant amplitude A
  - Constant carrier frequency ω_c
  - Variable *instantaneous* frequency $\omega_i \propto m'(t)$
  - Variable *instantaneous* phase $\theta_i \propto m(t)$



- Hard to do this, so we use:

$$\omega_{i_{FM}} = \omega_c + K_f\, m(t)$$

* if $m(t)\uparrow \Rightarrow \omega_{i_{FM}}\uparrow$ "Do Compresion for the signal".

* if $m(t)\downarrow \Rightarrow \omega_{i_{FM}}\downarrow$ "Do Expansion for the signal".

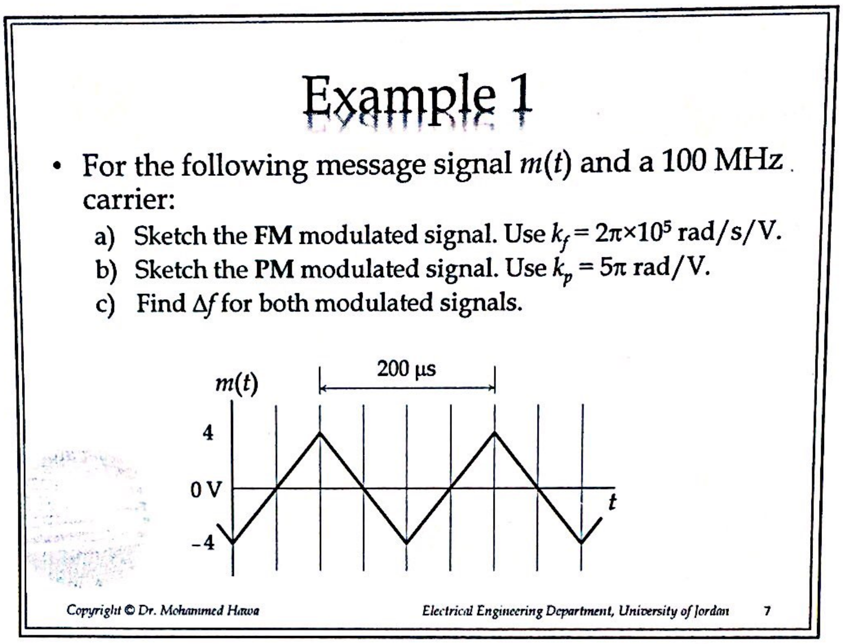
*Max. & Min values of $\underline{\underline{f}}$:

$$\omega_{max} = \omega_c + K_f\, m(t)_{max}.$$

Memorize:

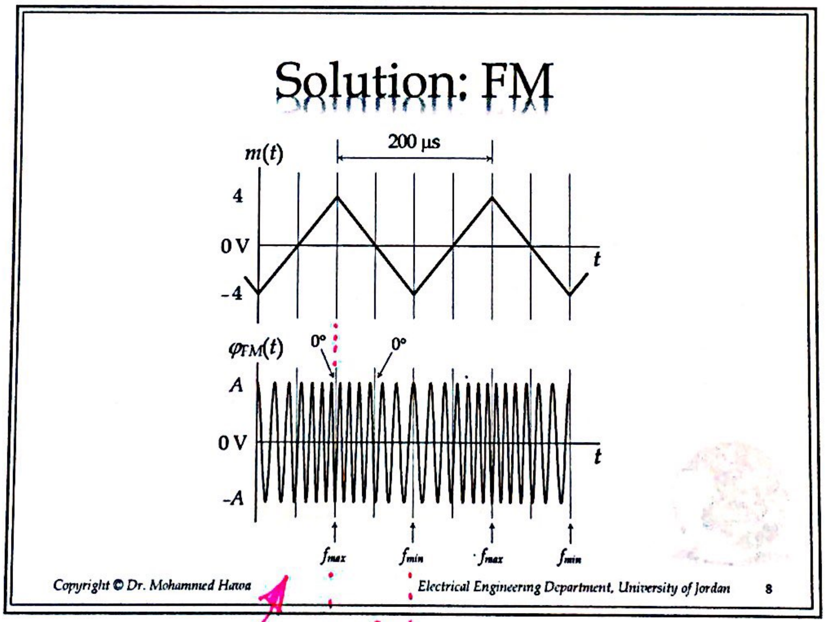
$$\boxed{f_{max} = f_c + \frac{K_f}{2\pi} m(t)_{max}.}$$

$$\boxed{f_{min} = f_c + \frac{K_f}{2\pi} m(t)_{min}}$$



Example 1

- For the following message signal $m(t)$ and a 100 MHz carrier:
  - a) Sketch the **FM** modulated signal. Use $k_f = 2\pi \times 10^5$ rad/s/V.
  - b) Sketch the **PM** modulated signal. Use $k_p = 5\pi$ rad/V.
  - c) Find Δf for both modulated signals.



Solution: FM

Compresion

Expansion.

for this Example:

$$f_{max} = 100MHZ + \frac{2\pi*10^5}{2\pi}\,\frac{rad/s}{V} * 4V = 100MHZ + 0.4MHZ = \boxed{100.4MHZ}$$

$$f_{min} = 100MHZ + \frac{2\pi*10^5}{2\pi} * (-4) = \boxed{99.6MHZ}$$

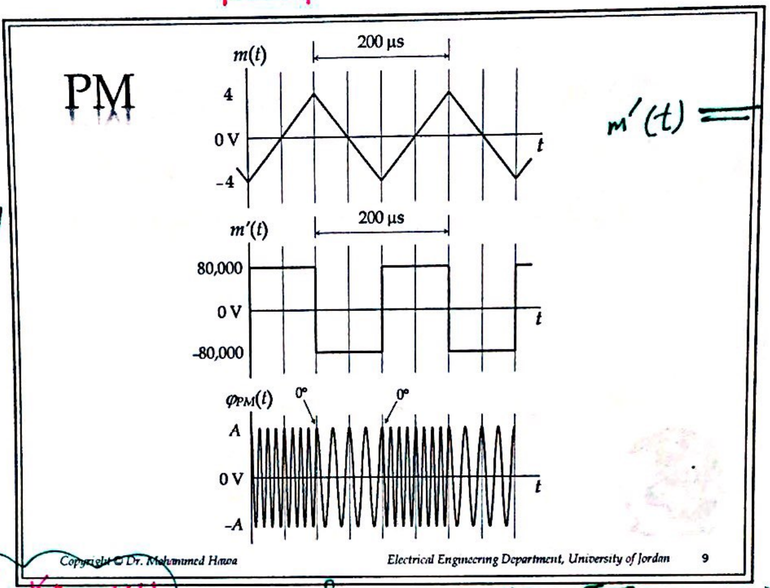
• Remember: Derivative of Triangle is Rect.

$\omega_{i_{PM}} = \omega_c + K_p m'(t)$ ⇒ since $m'(t)$ is Constant ⇒ $\omega_{i_{PM}}$ constant.

@ max of $m'(t)$ ⇒ f_{max}.

@ min of $m'(t)$ ⇒ f_{min}

100µs.



PM

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$m'(t) = \frac{8}{100\mu s} = 80000\ V/s$.

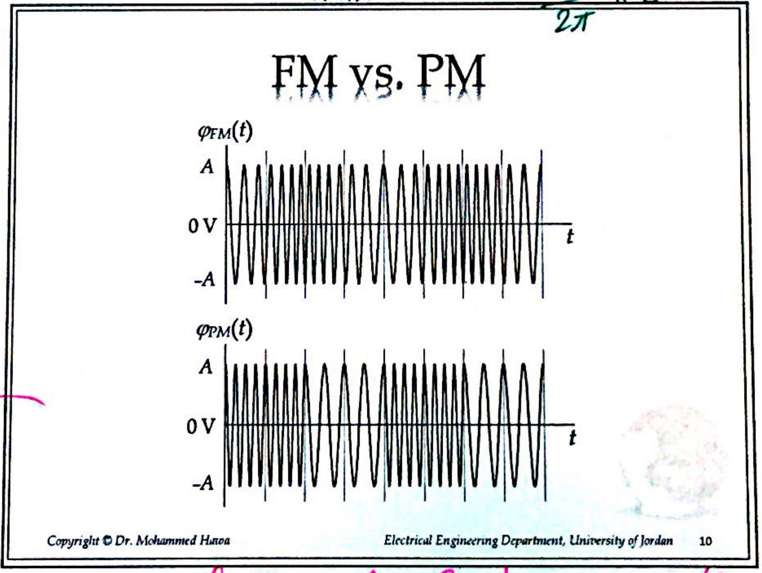
$\phi_{PM}(t)$ ⇒ changing freq & changing phase.

But it is PM since m(t) changing the phase.

$f_{max} = f_c + \frac{K_p}{2\pi} m'(t)_{max}$ ⇒ $f_{max} = 100MHz + \frac{5\pi}{2\pi} \times 80000 = \underline{100.2MHz}$.

$f_{min} = f_c + \frac{K_p}{2\pi} m'(t)_{min}$

$f_{min} = 100MHz + \frac{5\pi}{2\pi} * 80000 = \underline{99.8MHz}$.



FM vs. PM

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$\phi_{FM}(t)$ & $\phi_{PM}(t)$ ⇒ Both of them freq & phase variations.

BUT different in the Control of the message:

$\phi_{FM}(t)$ ⇒ $m(t) \propto f_i$.

$\phi_{PM}(t)$ ⇒ $m'(t) \propto f_i$.

$\Delta f \equiv$ Peak freq. Division.

Peak Frequency Deviation

$$\Delta f = \frac{\cancel{f_c} + \frac{K_f}{2\pi} m(t)_{max} - \cancel{f_c} - \frac{K_f}{2\pi} m(t)_{min}}{2}$$

- For FM:

$$\Delta f \triangleq \frac{f_{max}-f_{min}}{2} = \frac{k_f}{2\pi}\times\frac{m(t)_{max}-m(t)_{min}}{2}$$

$$\boxed{\Delta f = \frac{k_f}{4\pi}\times m(t)_{pk-pk} \quad [Hz]}$$

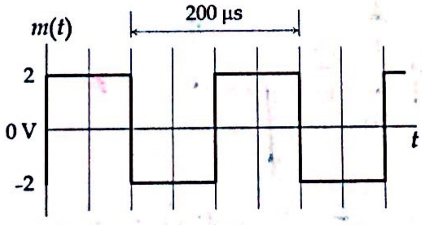
- For PM:

$$\Delta f \triangleq \frac{f_{max}-f_{min}}{2} = \frac{k_p}{2\pi}\times\frac{m'(t)_{max}-m'(t)_{min}}{2}$$

$$\boxed{\Delta f = \frac{k_p}{4\pi}\times m'(t)_{pk-pk} \quad [Hz]}$$

Example 2

- For the following message signal $m(t)$ and a 100 MHz carrier:
  a) Sketch the FM modulated signal. Use $k_f = 2\pi\times10^5$ rad/s/V. → solved before same part (b) in example(1).
  b) Sketch the PM modulated signal. Use $k_p = \pi/4$ rad/V.
  c) Find Δf for both modulated signals.



You Can think about this Signal as:

→ Analog as a rep{rect}.

OR

→ Digital as m(t) ⇒ Polar NRZ.

For Example (1): Find Δf FM & PM ?

- FM: $\Delta f = \frac{f_{max} - f_{min}}{2} = \frac{100.4-99.6}{2}\ MHZ = \boxed{0.4MHZ.}$

$\updownarrow \updownarrow$

OR

$\Delta f = \frac{K_f}{4\pi} m(t)_{PK-PK} = \frac{2\pi * 10^5}{4\pi} * 8 = \boxed{0.4\ MHZ.}$

#

- PM: $\Delta f = \frac{K_p}{2\pi} m'(t)_{AC-PK} = \frac{5\pi}{4\pi} * 160000 = \boxed{0.2MHZ.}$

#

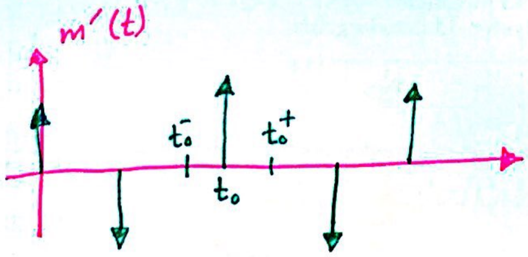
- if m(t) = polar NRZ + FM = FSK.
- if m(t) = polar NRZ + PM = BPSK.
- if m(t) = polar NRZ + AM = ASK.

Example(2):

b) for the zero: $\omega_{i_{PM}} = \omega_c + K_p \overset{0}{\cancel{m'(t)}} \Rightarrow \omega_{i_{PM}} = \omega_c$ "Constant".

for the impulse: $\omega_{i_{PM}} = \omega_c + \infty \Rightarrow \omega_{i_{PM}} = \infty$

$\Rightarrow$ Will Cause a Sudden Phase-Shift "180°"



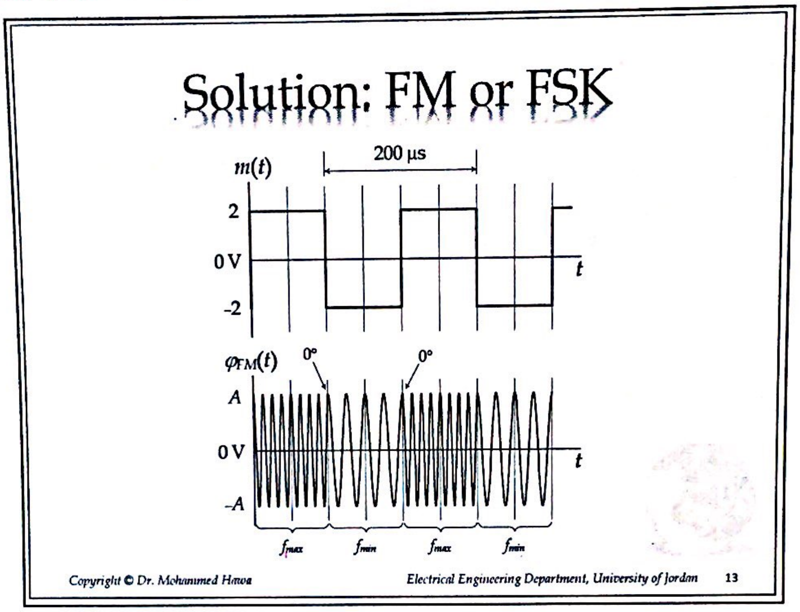
$\phi_{PM}(t_0^-) = A\cos(\omega_c t_0^- + K_p m(t_0^-)) = A\cos(\omega_c t_0^- + \frac{\pi}{4} * -2)$
$= A\cos(\omega_c t_0^- - \frac{\pi}{2})$

$\phi_{PM}(t_0^+ = A\cos(\omega_c t_0^+ + K_p * m(t_0^+))$
$= A.\cos(\omega_c t_0^+ + \frac{\pi}{2})$

HW: solve for $K_p = \frac{\pi}{2}$, $K_p = \frac{\pi}{8}$.

$\sin(\omega_c t) = \cos(\omega_c t - 90^\circ)$

$-\sin(\omega_c t) = \cos(\omega_c t + 90^\circ)$



Solution: FM or FSK

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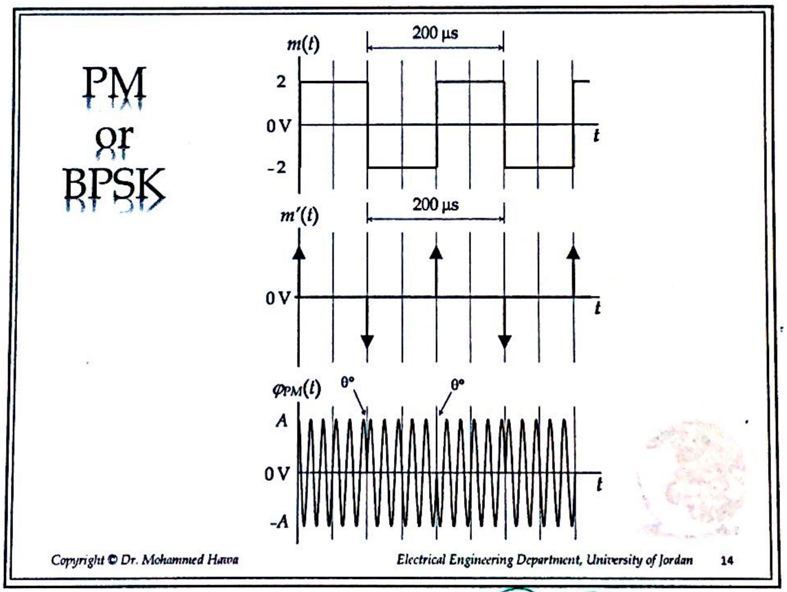

PM or BPSK

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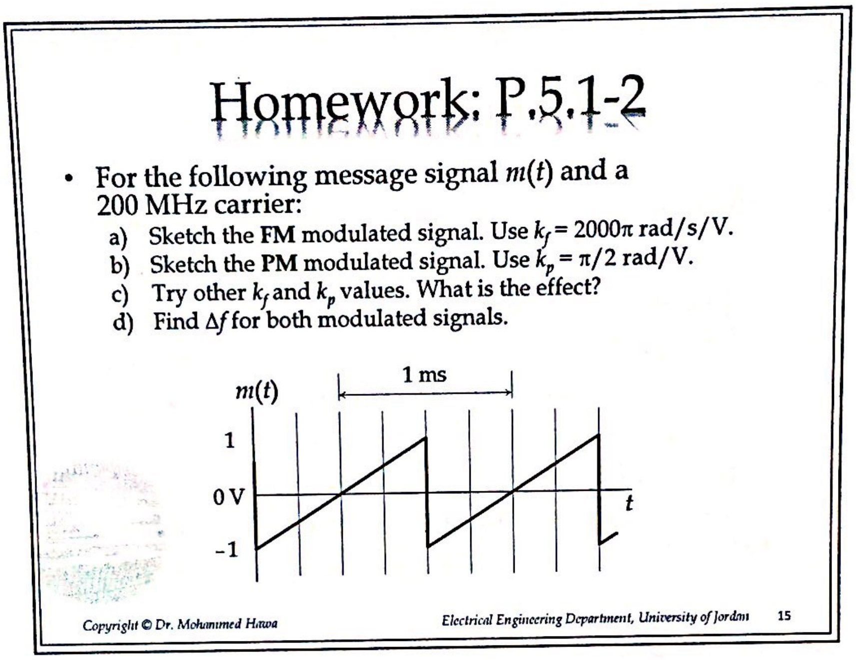
Memorize:

Δf for $\varphi_{PM}(t)$ for BPSK: $\Delta f = 0$

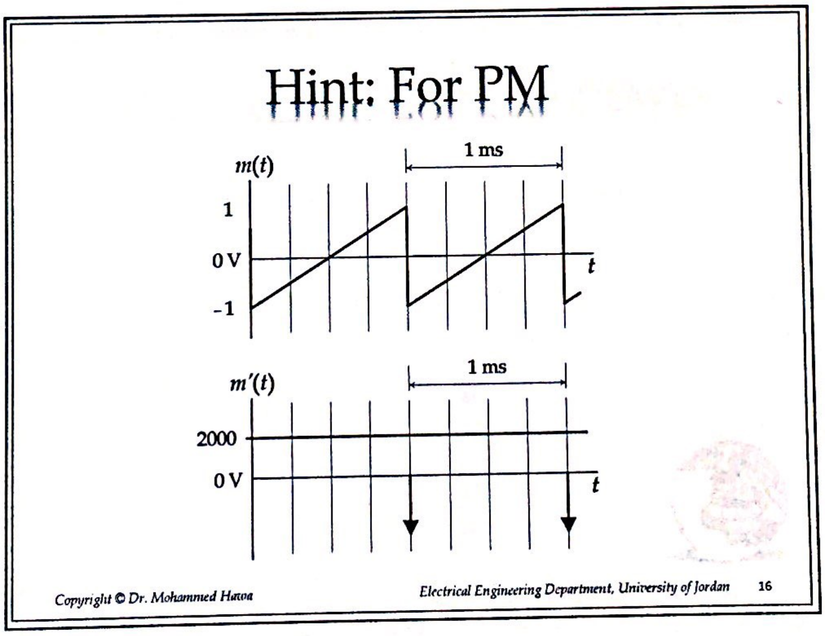
$f_{max} = f_c + K_p(0) = f_c \Rightarrow \Delta f = 0.$

$f_{min} = f_c + K_p(0) = f_c$



Homework: P.5.1-2

- For the following message signal $m(t)$ and a 200 MHz carrier:
  a) Sketch the **FM** modulated signal. Use $k_f = 2000\pi$ rad/s/V.
  b) Sketch the **PM** modulated signal. Use $k_p = \pi/2$ rad/V.
  c) Try other k_f and k_p values. What is the effect?
  d) Find Δf for both modulated signals.



Hint: For PM

Rules of Thumb

- Smooth change in frequency means smooth change in phase *always*.
- Sudden change in frequency (i.e., unit step change) *does not* mean a sudden change in phase, i.e., it means 0° phase shift.
- *Impulse* change in frequency (i.e., infinity frequency) *might* cause a sudden change in phase. To determine the phase shift (or lack thereof) see $k_p m(t)$ for PM or $k_f \int m(t)dt$ for FM.

FM and PM Average Power

$$\phi_{FM}(t) = A\cos\left(\omega_c t + k_f \int_{-\infty}^{t} m(t)dt\right)$$

$$\phi_{PM}(t) = A\cos\left(\omega_c t + k_p m(t)\right)$$

$$\overline{\varphi_{FM}^2(t)} = \frac{A^2}{2}$$

$$\overline{\varphi_{PM}^2(t)} = \frac{A^2}{2}$$

in FM & PM power depend ONLY on $\boxed{A}$!

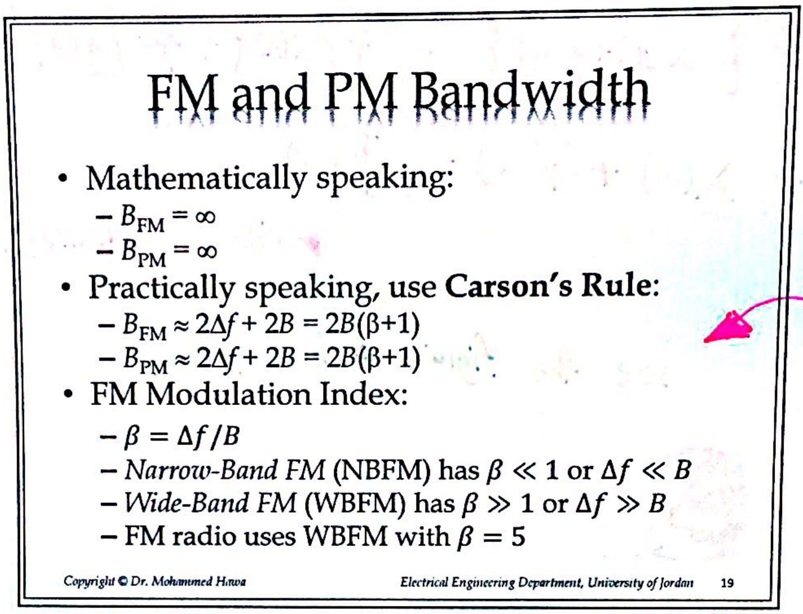
$$\Rightarrow \overline{\phi_{FM}^2(t)} = \overline{\phi_{PM}^2(t)} = \frac{A^2}{2}$$

"independent of m(t)"

- AM Modulation Index: $\boxed{m = \frac{-m_{min}(t)}{A}}$.
- FM Modulation Index: $\boxed{\beta = \frac{\Delta f}{B}}$

11/29/2016

$$B_{FM} = 2\Delta f + 2B = 2B\left(\frac{2\Delta f}{2B} + \frac{2B}{2B}\right) \Rightarrow \boxed{B_{FM} = 2B(\beta+1)}$$



FM and PM Bandwidth

- Mathematically speaking:
  - $B_{FM} = \infty$
  - $B_{PM} = \infty$
- Practically speaking, use **Carson's Rule**:
  - $B_{FM} \approx 2\Delta f + 2B = 2B(\beta+1)$
  - $B_{PM} \approx 2\Delta f + 2B = 2B(\beta+1)$
- FM Modulation Index:
  - $\beta = \Delta f / B$
  - *Narrow-Band FM* (NBFM) has $\beta \ll 1$ or $\Delta f \ll B$
  - *Wide-Band FM* (WBFM) has $\beta \gg 1$ or $\Delta f \gg B$
  - FM radio uses WBFM with $\beta = 5$

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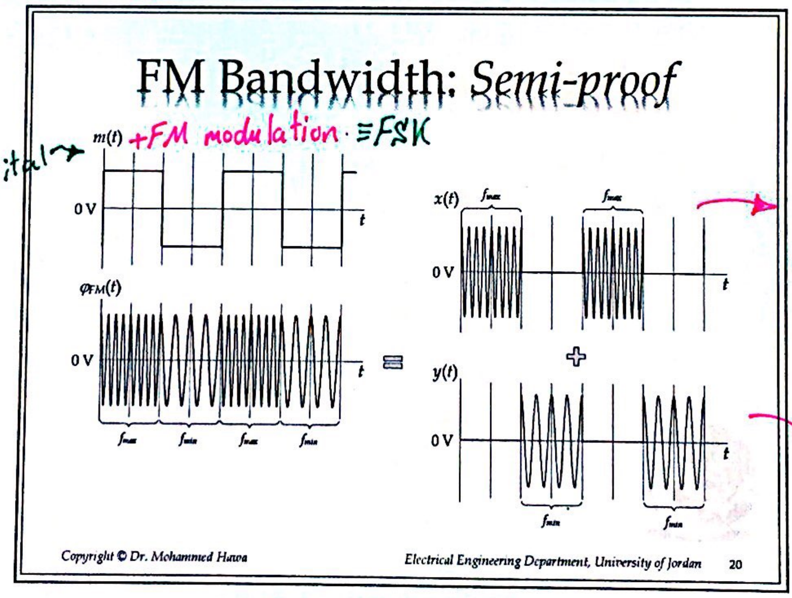
Memorize.

Sketch $\phi_{FM}(t)$ & $\Phi_{FM}(\omega)$ & find Bandwidth?



FM Bandwidth: *Semi-proof*

+FM modulation ≡ FSK

digital

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this is AM @ ω_{max}. Critical Modulation.

AM @ ω_{min}.

B_{AM} vs B_{FM}:

$B_{AM} = 2B$ ↳ $m(t)$ = Voice
$B_{m(t)} = 4KHz$.

"No control on B_{AM}"

$\Delta f_{FM} = \frac{K_f}{4\pi} m(t)_{pk\text{-}pk}$. $\Rightarrow B_{FM} = 2\Delta f + 2B$

"There is Control on B_{FM}"

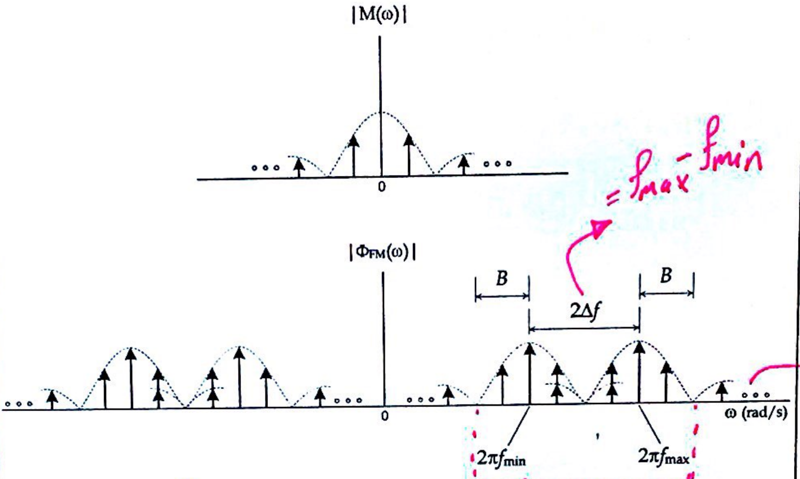
$\mathcal{F}\{A\cos(\omega_c t + K_f \int m(t)\,dt)\}$ Not easy to find

$$\Rightarrow \mathcal{F}\{x(t)+y(t)\} = \mathcal{F}\{x(t)\} + \mathcal{F}\{y(t)\}$$

$$= X(\omega) + Y(\omega) = \Phi(\omega)$$

↳ with ∞ impulses.

see the figures on the slides.



$= f_{max} - f_{min}$

insignificant.

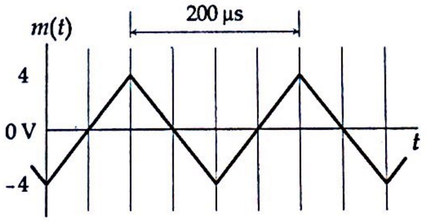
significant.

$$\Delta f = \frac{f_{max} - f_{min}}{2}$$

Bandwidth: Example 1

- Estimate the bandwidth B_{FM} and B_{PM} for the modulating signal $m(t)$ shown below. Assume $k_f = \pi\times10^4$ rad/s/V and $k_p = \pi/4$ rad/V.

Answers: $B_{FM} = 60KHZ$. $B_{PM} = 40KHZ$.



$$B = \frac{1}{\tau} = \frac{1}{100*10^{-6}} = 10KHZ.$$

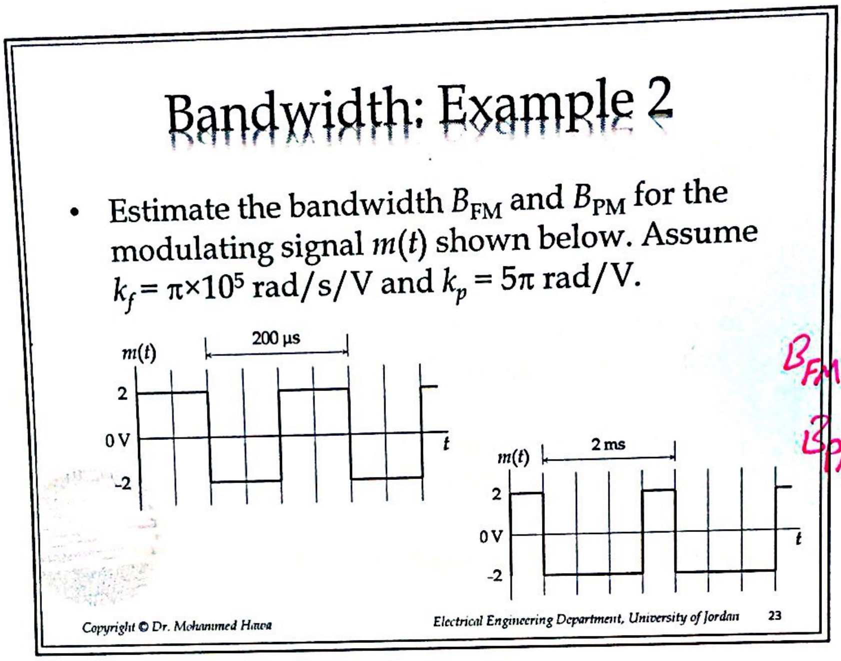
$$\Delta f = \frac{f_{max} - f_{min}}{2}$$

$$= \frac{K_f}{4\pi} m(t)_{pk\text{-}pk}$$

$$= \frac{\pi*10^4}{4\pi} * 8 = 20KHZ$$

$$B_{FM} = 2\Delta f + 2B = 2B(\beta+1)$$

$$= 2(20K) + 20K = 60KHZ.$$



Bandwidth: Example 2

- Estimate the bandwidth B_{FM} and B_{PM} for the modulating signal $m(t)$ shown below. Assume $k_f = \pi\times10^5$ rad/s/V and $k_p = 5\pi$ rad/V.

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Answers:

B_{FM} : 220 K, 20K

B_{PM} : 104K, 4K

FM Signal-to-Noise Ratio

$$SNR_{out} = \left(\frac{3\beta^2}{k_m^2}\right)\frac{S_{in}}{N_0B}$$

$$\beta = \frac{\Delta f}{B}$$

$$S_{in} = \overline{\varphi^2(t)} = \frac{A^2}{2}$$

Rxed power.

Crest Factor: $\equiv k_m^2 = \frac{m_p^2}{\overline{m^2(t)}}$

"peak-to-RMS Ratio"

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DSB-SC:

$$SNR_{out} = \frac{S_{in}}{N_0B}$$

DSB-LC:

$$SNR_{out} = \eta\frac{S_{in}}{N_0B}$$

$$SNR_{out} = \frac{3\beta^2}{K_m^2}\left(\frac{S_{in}}{N_0B}\right) \to \text{increasing } \beta$$

$\propto$ Quality

↳ advantage: Quality↑

↳ disadvantage: B_{FM}↑

$$B_{FM} = 2B(\beta+1)$$

Bandwidth ↑ ⇒ Fading ↑

$B_{FM_{WB}} > B_{AM}$; WBFM suffers more Fading.

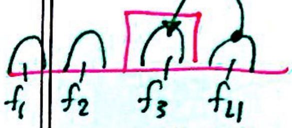
But can be fixed @ Rx Using AGC ≡ Automatic Gain Control.

FM (and PM) vs. AM

- FM (and PM) Advantages: → all of them for Constant Amplitude of FM.
  - The constant amplitude of FM makes it less susceptible to nonlinearities.
  - The constant amplitude of FM gives it a kind of immunity against rapid fading (even with the larger bandwidth).
  - Due to the constant amplitude, FM is less vulnerable than AM to adjacent-channel interference.
  - FM is capable of exchanging SNR for bandwidth.

In total: FM suffers Less Fading than AM or QAM.

for Non Linear channels.



$\cos\omega_c t \xrightarrow{(\ )^2} \cos^2\omega_c t = \frac{1}{2}+\frac{1}{2}\cos 2\omega_c t$

⇒ adjacent channel interference due to change in the freq.

FM (and PM) vs. AM

- FM (and PM) Disadvantages:
  - WBFM (which provides better quality) requires large transmission bandwidth.
  - FSK FM modulators and demodulators are relatively more expensive that AM hardware.
  - PSK PM demodulation requires synchronous detection (relatively expensive).

AM Radio:
m(t) - voice ⇒ $B_{m(t)}$ = 4KHZ.
can't send music by AM Radio.

mainly designed for Music.

Applications: FM Radio

- FM + FDM
  - The baseband message is 15 kHz
  - With β = 5, the bandwidth of each station is 200 kHz (both U.S. and Europe).
  - * The broadcast range is 88 – 108 MHz. *
- * FM radio sounds better than AM radio:
  - m(t) has a larger bandwidth.
  - WBFM: exchanging SNR for bandwidth.
  - Pre-emphasis/De-emphasis improves SNR.
  - Stereo FM.

m(t) = voice + Music ⇒ 15 KHZ = $B_m(t)$.

@Tx @Rx

Purpose: Better SNR for same transmitted Power.

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B_{AM} = 2B
= 2 * 4KHZ
= 8KHZ

0.5KHZ 1KHZ guard

0.5KHZ 1KHZ guard

0.5KHZ (Europe).
1 KHZ. (U.S).

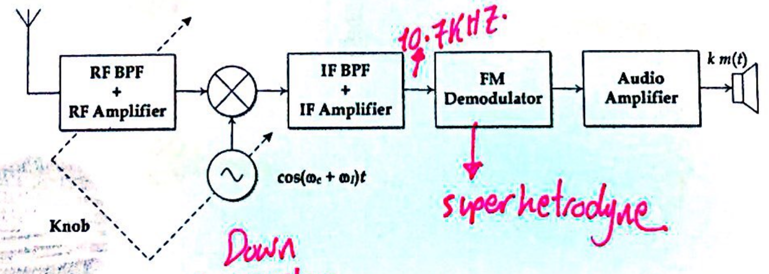
Total (U.S) = 10KHZ.
(Europe) = 9KHZ.

B_{FM} = 2B + 2Δf
= 2B(β+1)
= 2 * 15K (5+1)
= 180KHZ.
Disadvantage

180KHZ
10KHZ guard 10KHZ guard.
US & Europe.
Total = 200KHZ.

FM Superhetrodyne Receiver

- IF frequency = 10.7 MHz
- L.O. frequency = 88 + 10.7 MH to 108 + 10.7 MHz



10.7KHZ.
superhetrodyne
Down Converter

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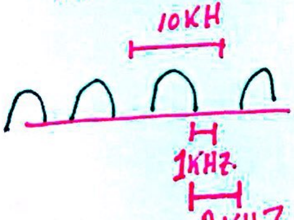
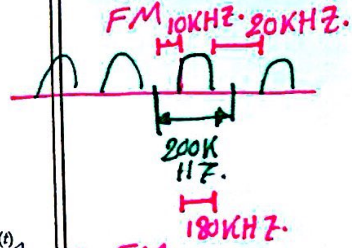
AM (U.S).



in AM distance between two stations 2KHZ.

FM



in FM distance between two stations 20KHZ.

* stereophonic vs. monophonic FM

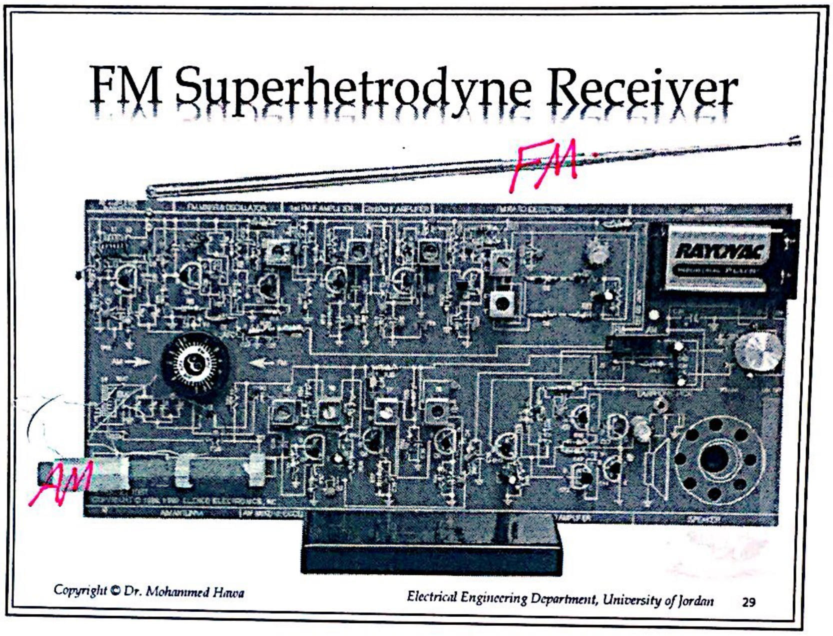
two Signals Right & Half.	one Signal:

signal that send through stereo <u>better</u>.

length of antenna for AM & FM:

FM: $(88 \rightarrow 108)\,MHZ \Rightarrow \lambda\downarrow \Rightarrow L_{ant}\downarrow$

AM: $535KHZ \rightarrow 1700KHZ \Rightarrow \lambda\uparrow \Rightarrow L_{ant}\uparrow$



FM Superhetrodyne Receiver

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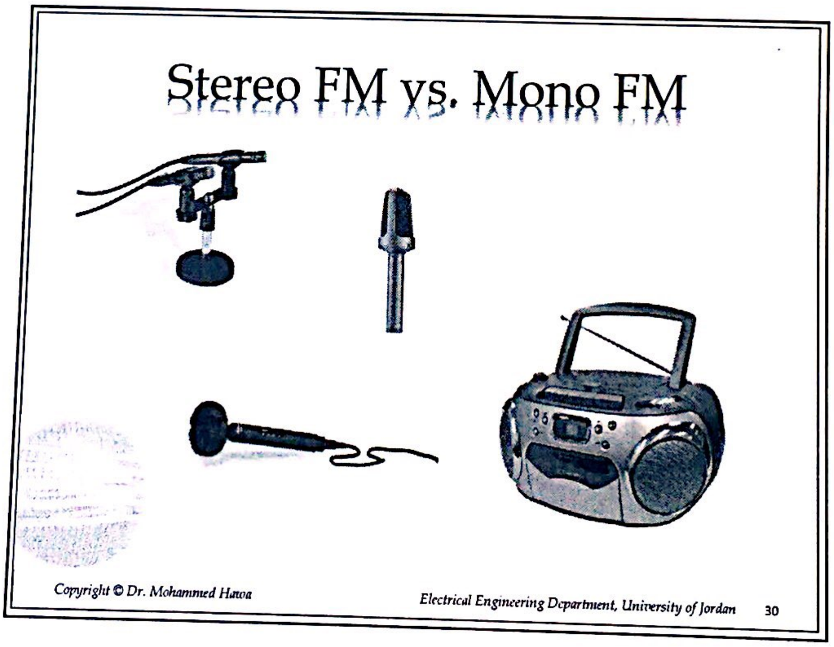

Stereo FM vs. Mono FM

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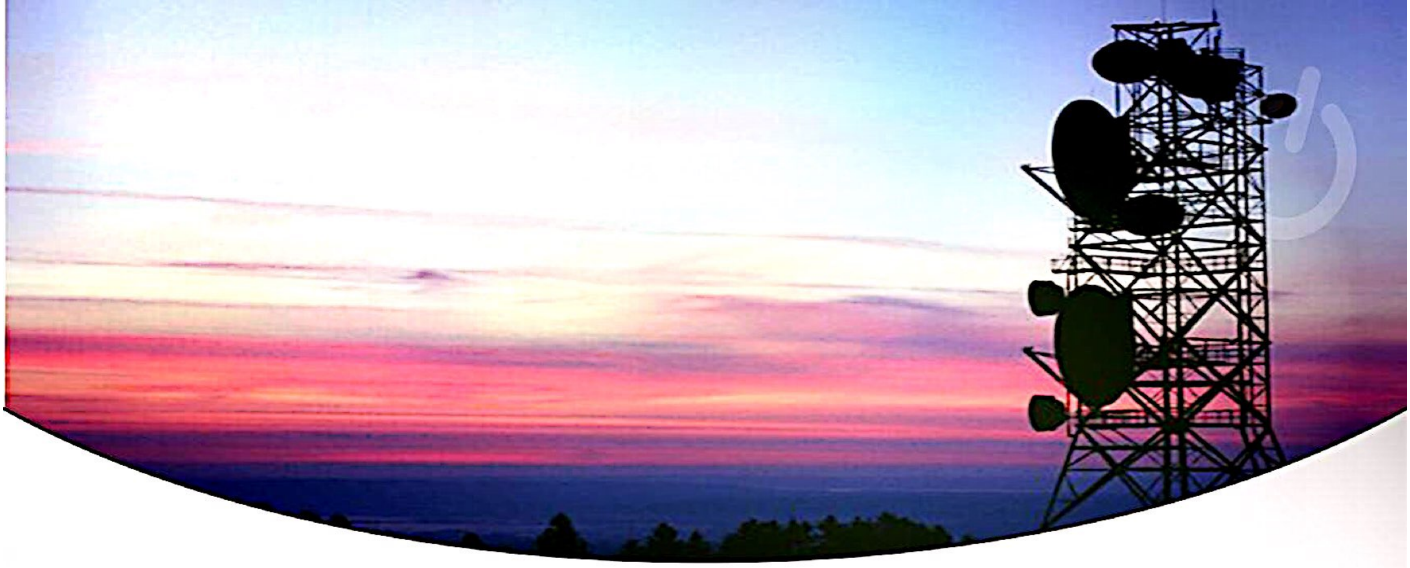

Communications1

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Dr. Mhmd Hawa

By: Mhmd Abuhashya

www Powerunit-ju.com

- Bandwidth of Mono-FM Radio Station =200KHZ.
- " " Stereo-FM " " =200KHZ. → we send $m_R(t)$ & $m_L(t)$

for Backwards Compatibility [old compitable with new].

Lecture 16: FM Modulators and Demodulators

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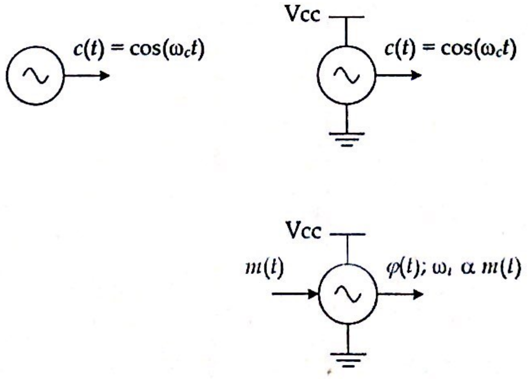
EE421: Communications I: Lecture 16. For more information read Chapter 5 in your textbook or visit http://wikipedia.org/.

FM Modulator: VCO

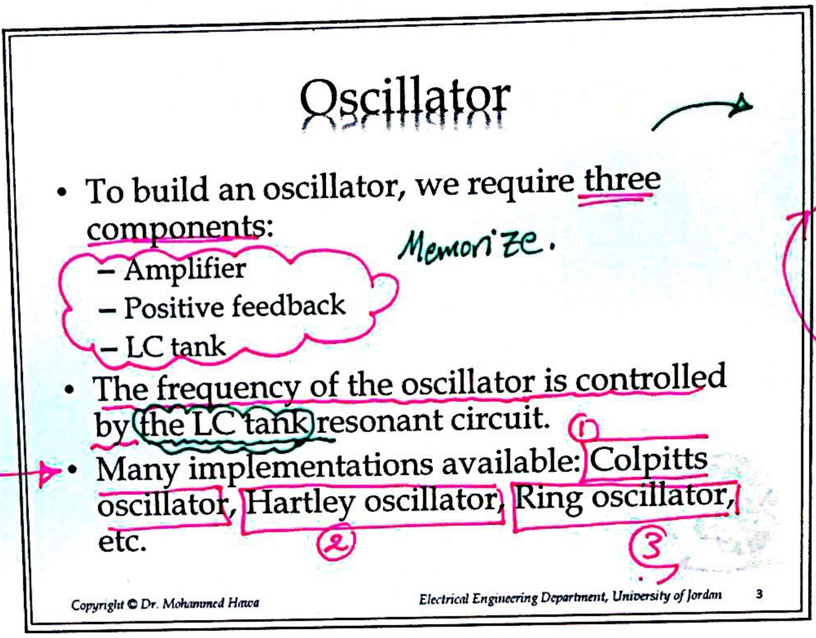


⟹ $m(t)\uparrow$

VCO ≡ Voltage Controlled Oscillator.

$f_c = 100 MHz.$

$v_c(t) = \cos w_c t$

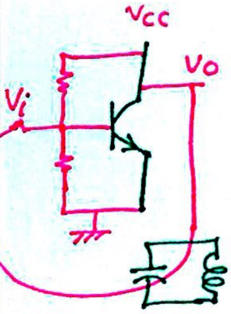


Oscillator

- To build an oscillator, we require three components: Memorize.
  - Amplifier
  - Positive feedback
  - LC tank
- The frequency of the oscillator is controlled by the LC tank resonant circuit.
- Many implementations available: Colpitts oscillator ①, Hartley oscillator ②, Ring oscillator ③, etc.

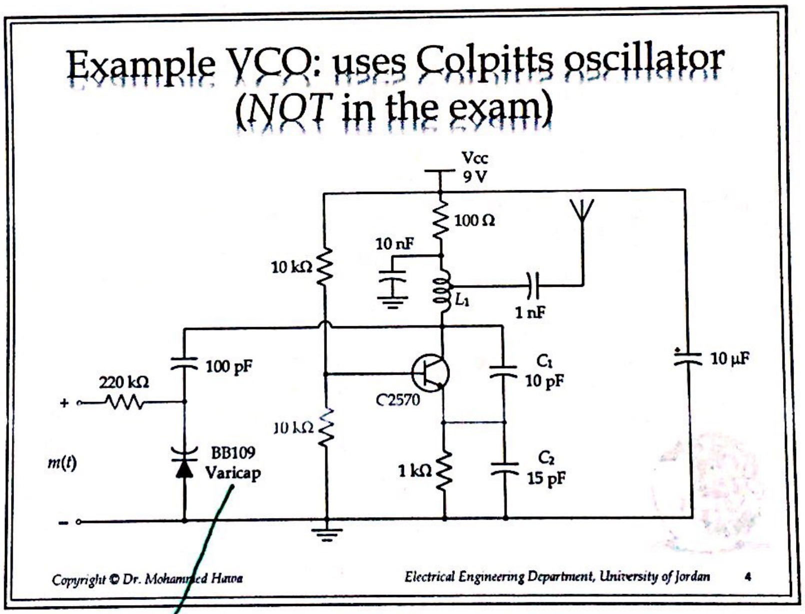
important.

Memorize.



BPF or LC Tank. or resonant LC cct.

$w_{res} = \frac{1}{\sqrt{LC}}$



Example VCO: uses Colpitts oscillator (*NOT* in the exam)

Varicap OR Varactor.

FM Demodulators

- #1 FM Discriminator (also called Slope Detector or Ratio Detector):
  - Convert frequency variations into amplitude variations, then use an envelope detector.
- #2 Quadrature Detector:
  - Convert frequency variations into phase-difference variations, then use a phase detector.
- #3 Phase-Locked Loop:
  - A phase-difference feedback system.

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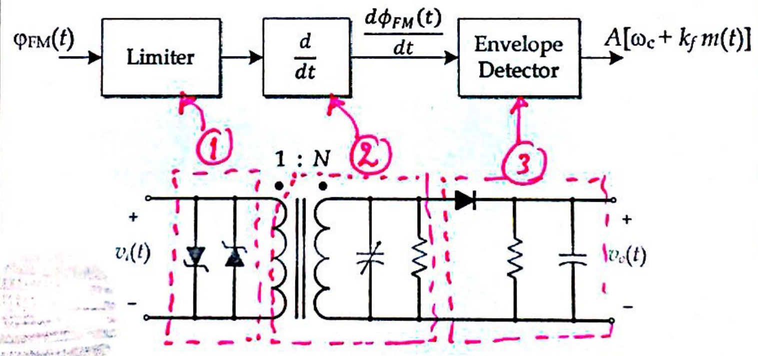
- What do we need to Build FM Discriminator?

1) Limiter.
2) Differentiator.
3) Envelope Detector.

FM Discriminator



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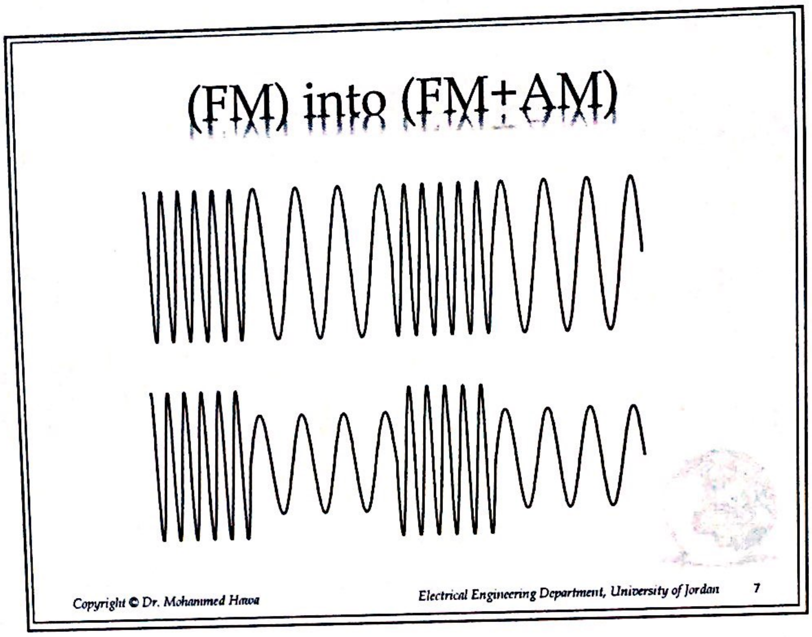
$$\phi_{FM}(t) = A\cos(\omega_c t + K_f \int_{-\infty}^{t} m(t)\,dt)$$

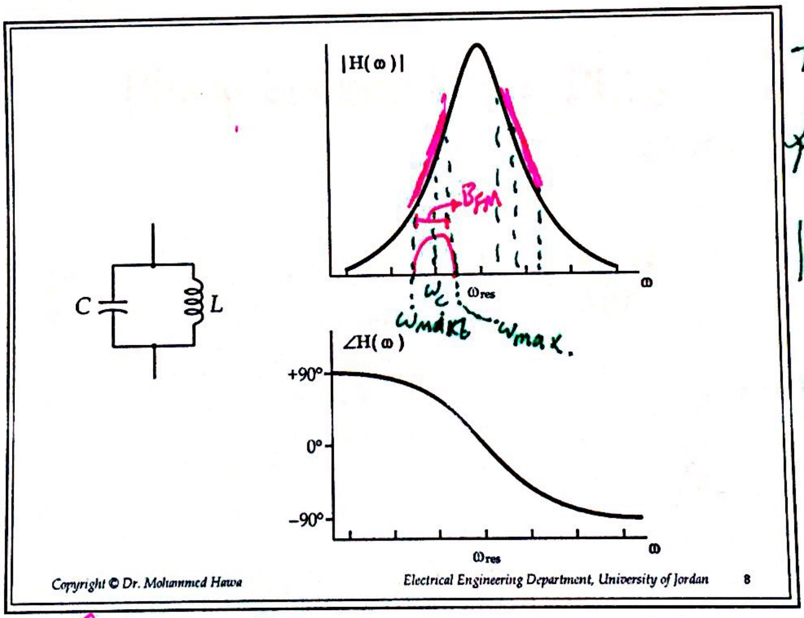
$$\frac{d}{dt}\phi_{FM}(t) = -A[\omega_c + K_f m(t)]\,\sin(\omega_c t + \int m(t)\,dt)$$

$$= \underbrace{A[\omega_c + K_f m(t)]}_{\text{Envelope.}}\,\sin(\omega_c t + \int m(t)\,dt - 180^\circ).$$

$E(t)$. $\Rightarrow E(t) = K_1 + K_2 m(t) \xrightarrow{||} E(t) = K_2 m(t)$.



(FM) into (FM+AM)



$\mathcal{F}\{x(t)\} = X(\omega)$

$\mathcal{F}\left\{\frac{dx(t)}{dt}\right\} = j\omega X(\omega)$

$\left|\mathcal{F}\left\{\frac{dx(t)}{dt}\right\}\right| = \overset{H(\omega)}{|\omega|}\,|X(\omega)| = |Y(\omega)|$

$\Rightarrow H(\omega) = \omega$

So for $H(\omega) = \omega$ it is Differentiator.

"always Linear line in freq. domain represent diff." d/dt.

Applications of LC Tank:

① BPF.

② Inside Oscillator or VCO to control Freq..

③ $\frac{d}{dt}$. "differentiator".

④ Converts freq. Variations into phase variations.

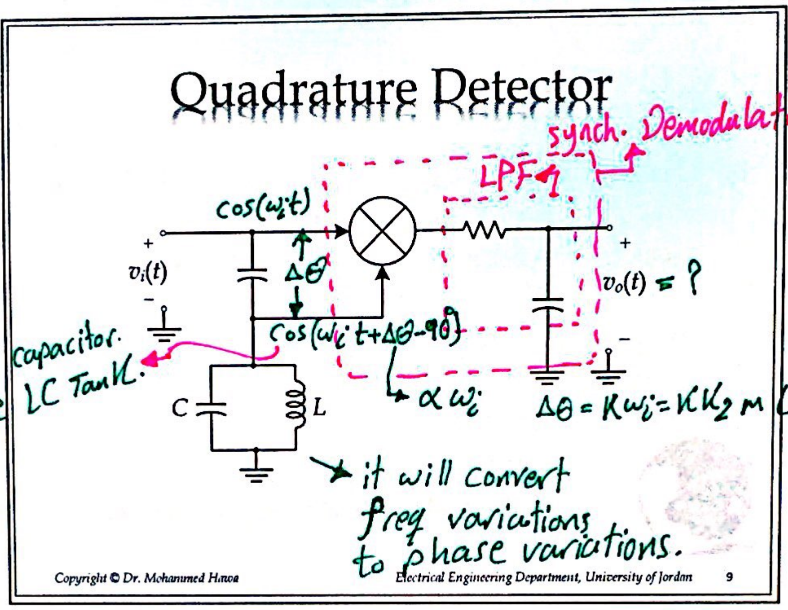
$x(t) = \cos(\omega_i t)\cos(\omega_i t + \Delta\theta - 90) = \frac{1}{2}\cos(2\omega_i t + \Delta\theta - 90) + \frac{1}{2}\cos(\Delta\theta - 90°)$.

$\frac{1}{2}\cos(2\omega_i t + \Delta\theta - 90)$: rejected ⇒ By LPF. High freq.

$\frac{1}{2}\cos(\Delta\theta - 90°)$: low freq. ⇒ passes.

$\Rightarrow V_o(t) = \frac{1}{2}\cos(\Delta\theta - 90°)$

$\boxed{V_o(t) = \frac{1}{2}\sin(\Delta\theta)} \Rightarrow \boxed{V_o(t) = \frac{1}{2}\Delta\theta}$ if $\Delta\theta \ll 1$



Quadrature Detector

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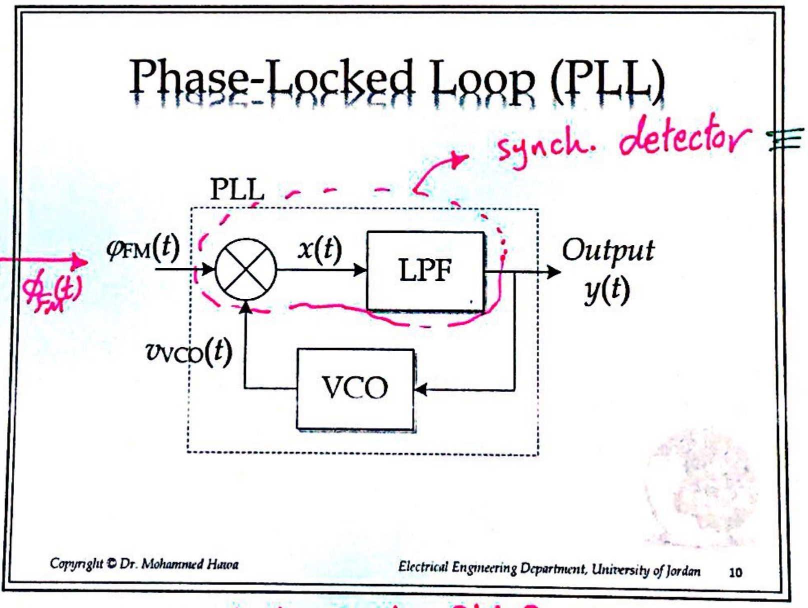
synch. Demodulator. ≡ it measure phase difference.

−90° from the capacitor.
$\Delta\theta$ from the LC Tank.

$\propto \omega_i$ $\Delta\theta = K\omega_i = KK_2\, m(t)$.

$\Delta\theta \propto \omega_i \propto m(t)$.

it will convert freq variations to phase variations.



Phase-Locked Loop (PLL)

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synch. detector ≡ Phase Difference Detector

$m(t) \rightarrow$ VCO $\rightarrow \varphi_{FM}(t)$

- What we need to Build PLL?

① Mixer ② LPF. ③ VCO.

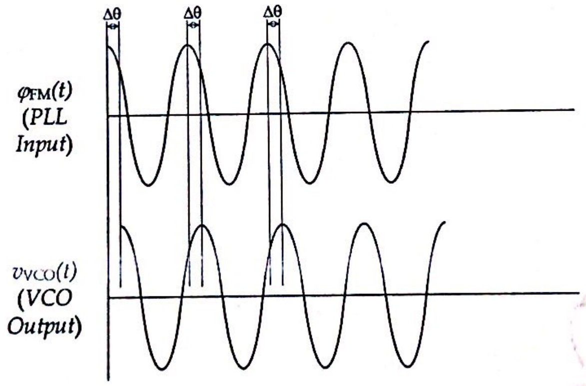
PLL Terminology

- A PLL is said to be **in-lock** if the frequency of the inner VCO $v_{VCO}(t)$ is exactly the *same* as the frequency of the received signal $v_{in}(t) = \varphi_{FM}(t)$.
- If the PLL stays in-lock, then the output voltage $v_{out}(t)$ is proportional to the baseband message signal $m(t)$.
- We want the PLL to stay in-lock.
- See the datasheet for MC4046 for details.

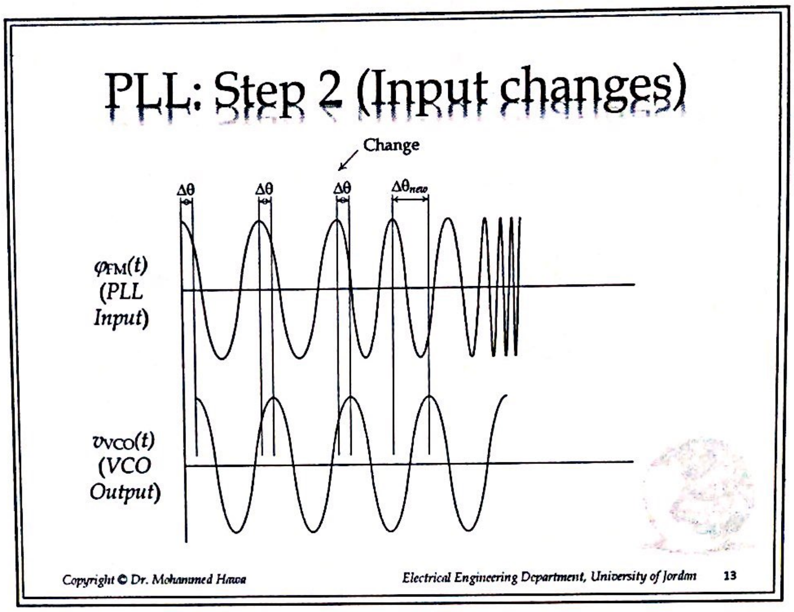
PLL: Step 1 (In Lock)

→ same freq.

But there is phase difference.



$\omega_i \uparrow \Rightarrow \Delta\theta \uparrow$



PLL: Step 2 (Input changes)

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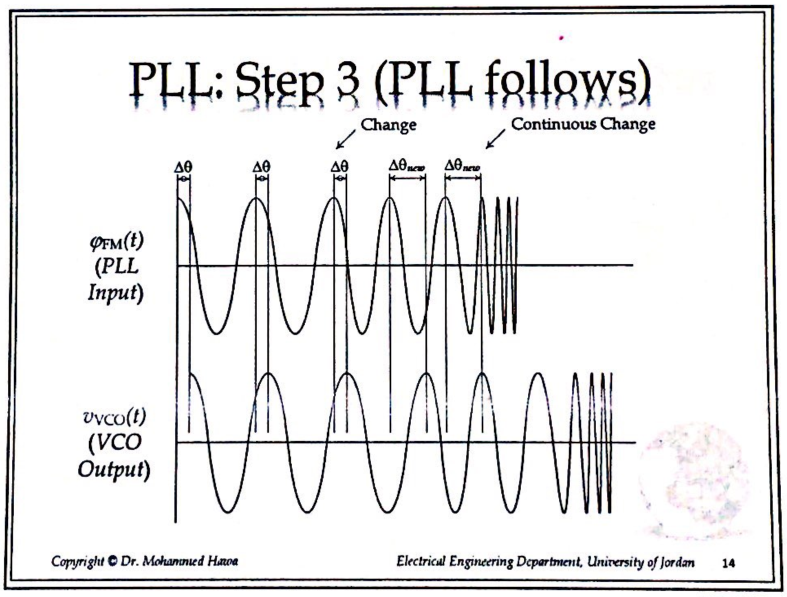

PLL: Step 3 (PLL follows)

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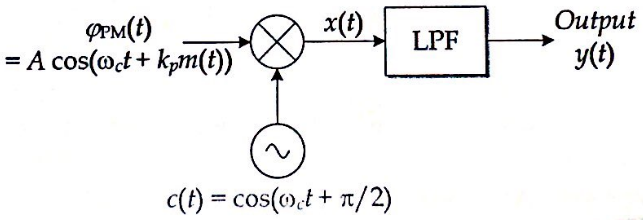
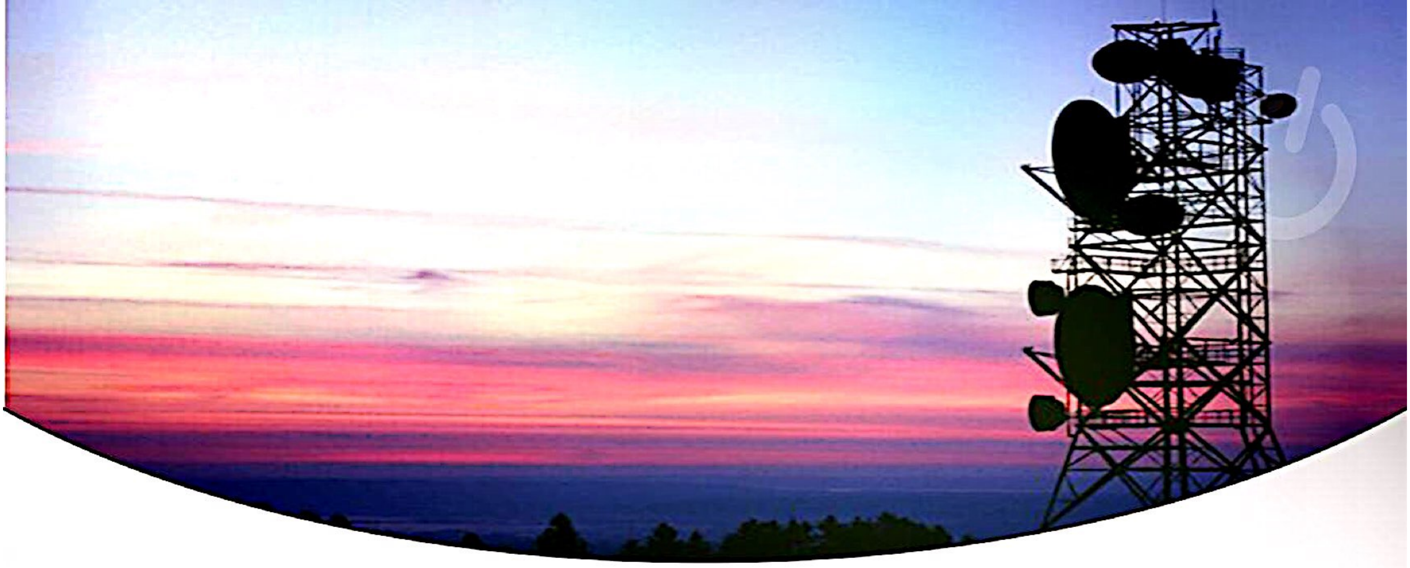
PLL Applications

- Carrier recovery for Synchronous detectors.
- Clock recovery for digital baseband receivers.
- Stabilizing VCO frequencies in FM transmitters.
- FM Demodulator.

PM Demodulator: Synchronous





Communications1

Fall017

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By: Mhmd Abuhashya

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Lecture 17: Time Division Multiplexing and Telephony

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EE421: Communications I. For more information read Chapter 6 in your textbook or visit http://wikipedia.org/.

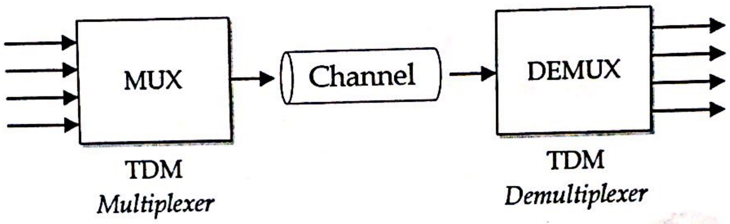
TDM used in Digital.

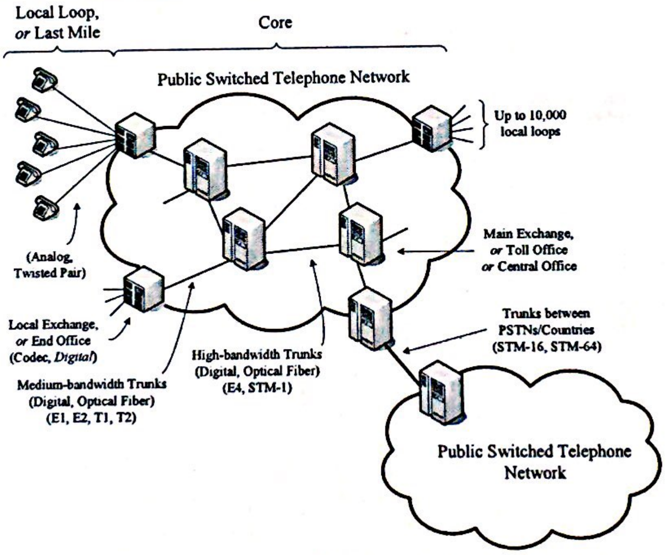
F.D.M used in Analogue & Digital.

Multiplexing: TDM

- Time Division Multiplexing (**TDM**) is a process that allows the transmission of several signals over the same baseband channel.
- Achieved by interleaving the bits of the different signals using different **time instants.**
- The receiver isolates one signal from the rest using a **time demultiplexer.**
- TDM is ***not*** limited to PCM or telephony.

Multiplexer/Demultiplexer





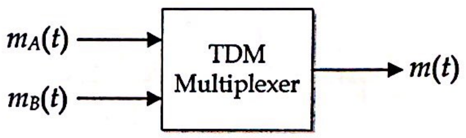
- Always TDM used inside the Core.

one phone call → 1 PCM stream $f_s = 8KHZ$ 8 bits/sample $\Rightarrow f_o = 64Kbps$. } in Telephony.

* Type of line coding: Bipolar. $1 \to +v -v +v -$
$0 \to$ zero.

5/10/2016

TDM Multiplexer

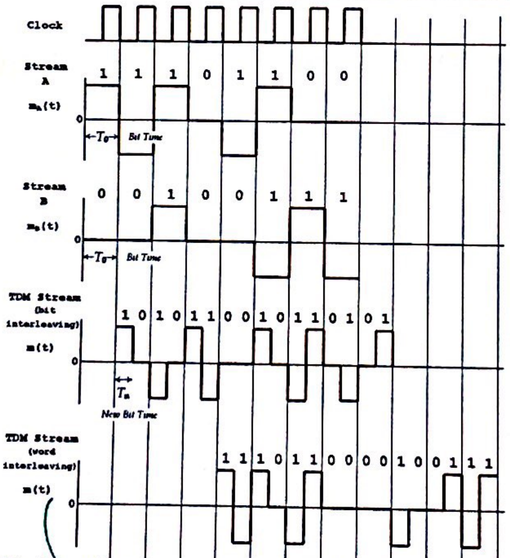


** TDM works on bits.

NOT voltages.

Because if we use 0V&5V & -5V,... we will violate the Bipolar so we must use 1,1,0,1,0,... "use bits".

in the final step we convert to voltage.



Bipolar (AMI)
E.g., telephony

$T_o = \frac{1}{f_o} = \frac{1}{64Kbps}$.

New Bit Time:

$\boxed{T_n = \frac{T_o}{2}}$

$f_n = 2f_o \Rightarrow f_n = 128Kbps$.

word = sample = 8 bits.

Taking 8bits @atime called:
word interleaving;

more narrower bits ⇒ more Bandwidth.

so we are limited by the Bandwidth of channel.

Ex. A copper channel (B_{ch}=1MHZ) over 1Km, you want to send PCM streams using TDM, what is the max. phone calls you can send?

solution:

$n_{max} f_o \le B_{ch}$

$n_{max} = \left\lfloor \frac{1 MHZ}{64 KHZ} \right\rfloor$

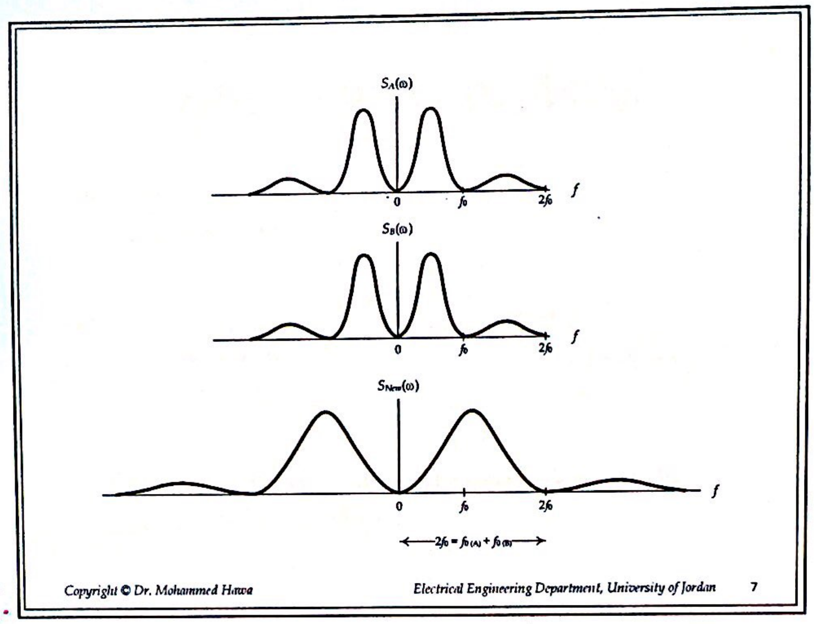
we take the floor.

$= \lfloor 15.625 \rfloor$

so $\boxed{n_{max} = 15}$

if we use Coaxial channel.

$n_{max} f_o \le B_{ch} = 1GHZ$.



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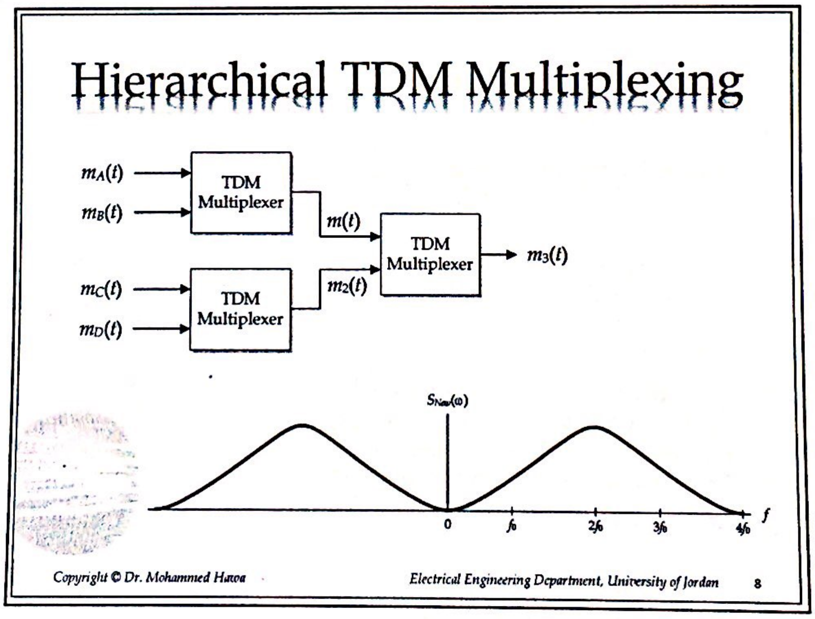
Hierarchical TDM Multiplexing



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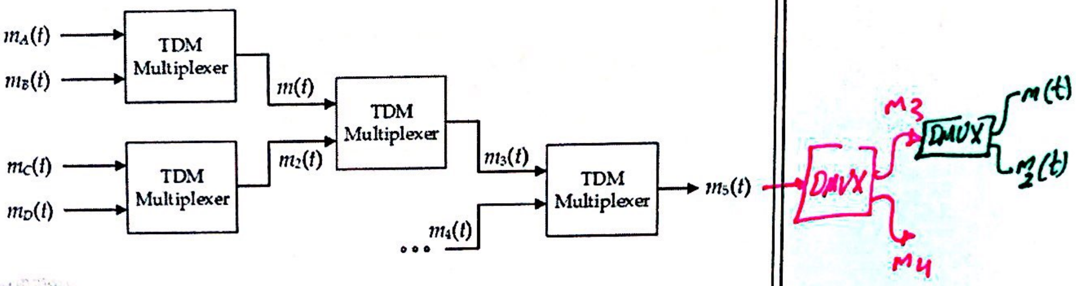
Any number of steps!



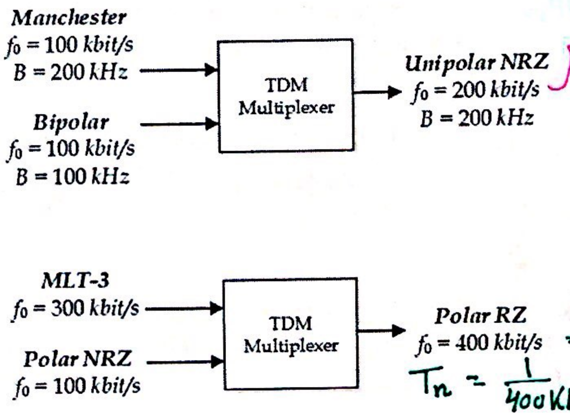
- *Homework*: Show the hardware blocks at the receiver side (Aqaba).

MEMORIZE:

TDM Hierarchy Standard (Tel.)

	U.S. Standard	*European Standard*
Low bit-rate *PDH: Plesiochronous Digital Hierarchy*	***T1***	***E1***
High bit-rate *SDH: Synchronous Digital Hierarchy*	***SONET*** (Synchronous Optical Network)	***SDH*** (Synchronous Digital Hierarchy)

TDM works on bits *NOT* voltages!



Manchester
$f_0 = 100$ *kbit/s*
B = 200 *kHz*

Bipolar
$f_0 = 100$ *kbit/s*
B = 100 *kHz*

TDM Multiplexer

Unipolar NRZ
$f_0 = 200$ *kbit/s*
B = 200 *kHz*

→ $f_{0\,manchester} + f_{0\,bipolar}$
= 200Kbps.

$B = f_0$ (for unipolar)
= 200KHZ.

MLT-3
$f_0 = 300$ *kbit/s*

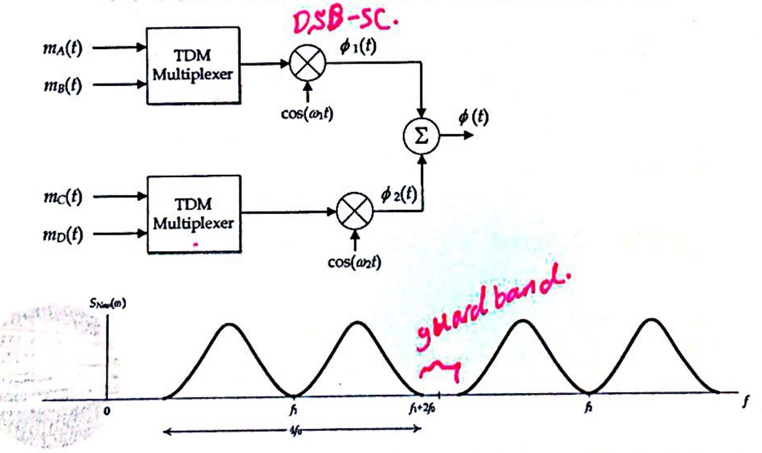
Polar NRZ
$f_0 = 100$ *kbit/s*

TDM Multiplexer

Polar RZ
$f_0 = 400$ *kbit/s* ⇒ $B = 2f_n = 800KHZ$.

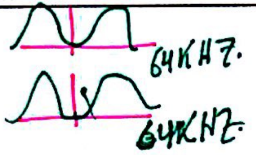
$T_n = \frac{1}{400Kbps}$

TDM combined with FDM

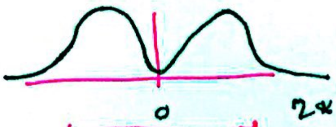


Assume: m_A ⇒ PCM 64Kbps.

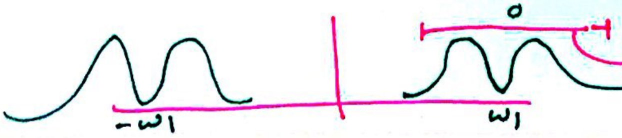
m_B ⇒ PCM 64K bps



TDM of m_A & m_B



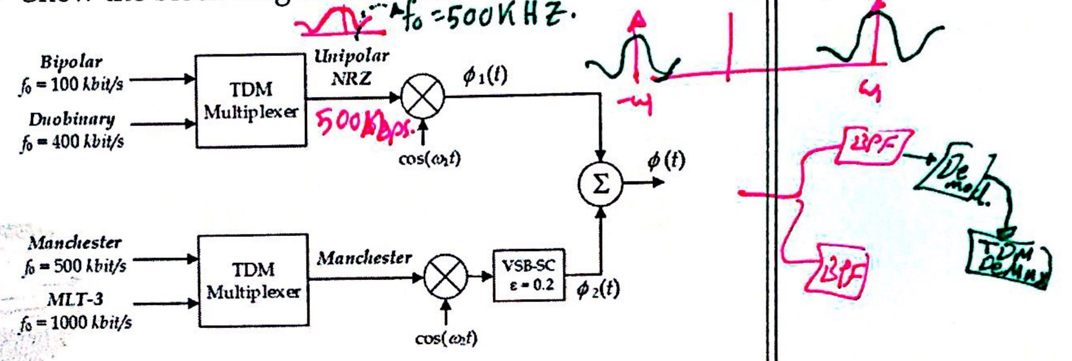
2 * 64 = 128 KHZ



→ 2B = 2 * 128K HZ.

Homework

- Sketch the PSD for the output signal $\phi(t)$ below.
- Show the block diagram of the receiver.



Examples on TDM with FDM

- GSM cellular communications system.
  - Every 8 phone calls are combined using TDMA into one 200 kHz channel.
  - The 200 kHz channels are multiplexed using FDMA.
- ATSC and DVB digital TV broadcasting systems.
  - Anywhere between 6 and 12 TV stations are multiplexed in one 6 MHz or 8 MHz channel using TDM.

Memorize. ⟹

2G cellular $\xrightarrow{use}$ GSM.

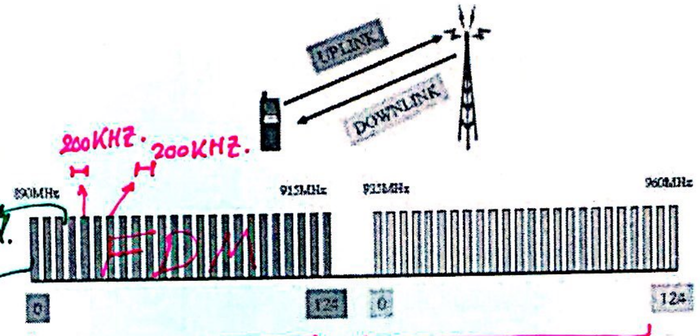
3G cellular ⟹ CDMA.

4G+5G ⟹ OFDMA.

Freq. Division Duplex

GSM uses TDMA/FDMA/FDD

DONOT Memorize the Numbers.



Uplink.

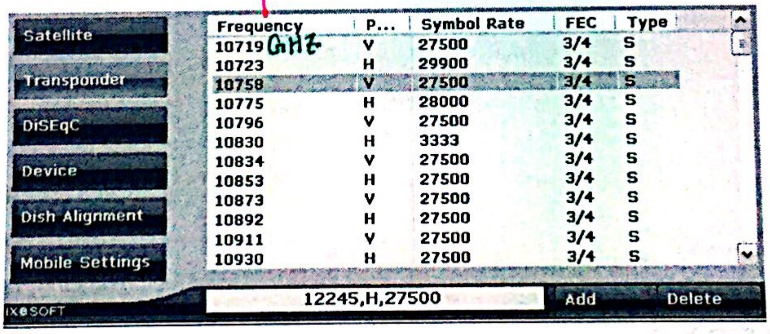
Downlink.

inside each one it send 8 phone calls using TDMA

Lower freq. ⇒ smaller Att. ⇒ to save the battery.

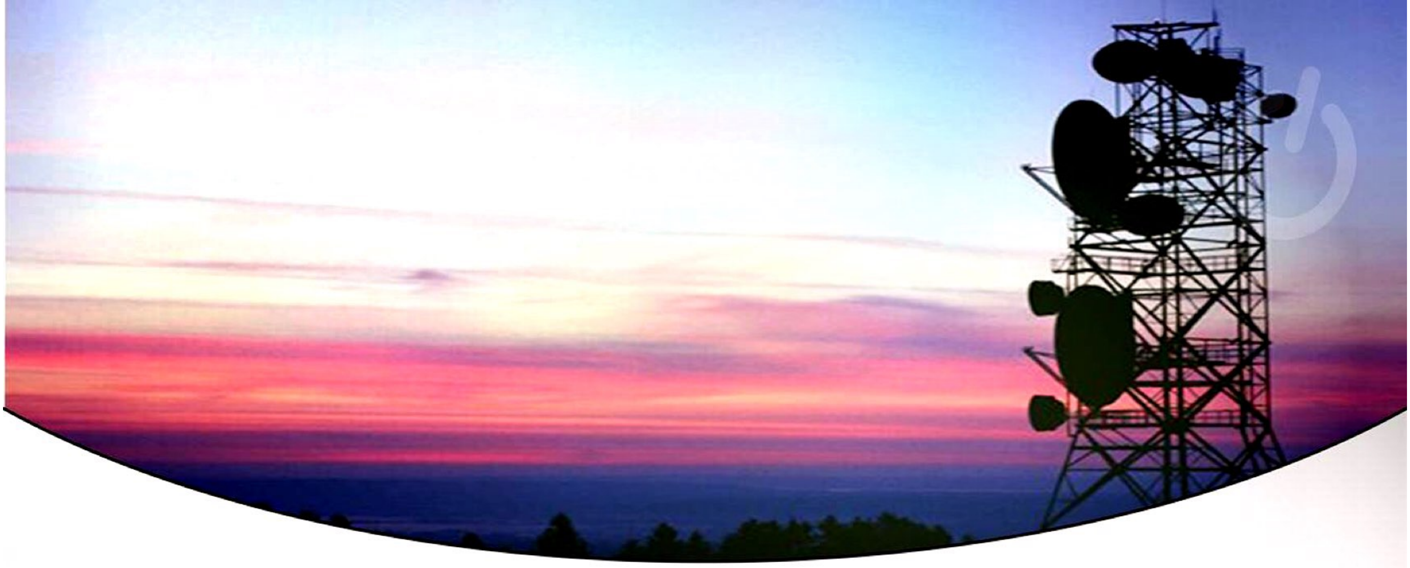
Homework: Satellite Receiver

center freq. of FDM. → then use TDM to send more than one.



Frequency	P...	Symbol Rate	FEC	Type
10719 GHZ.	V	27500	3/4	S
10723	H	29900	3/4	S
10758	V	27500	3/4	S
10775	H	28000	3/4	S
10796	V	27500	3/4	S
10830	H	3333	3/4	S
10834	V	27500	3/4	S
10853	H	27500	3/4	S
10873	V	27500	3/4	S
10892	H	27500	3/4	S
10911	V	27500	3/4	S
10930	H	27500	3/4	S

12245,H,27500



Communications1

Fall017

Dr. Mhmd Hawa

By: Mhmd Abuhashya

Lecture 19: Digital Modulation

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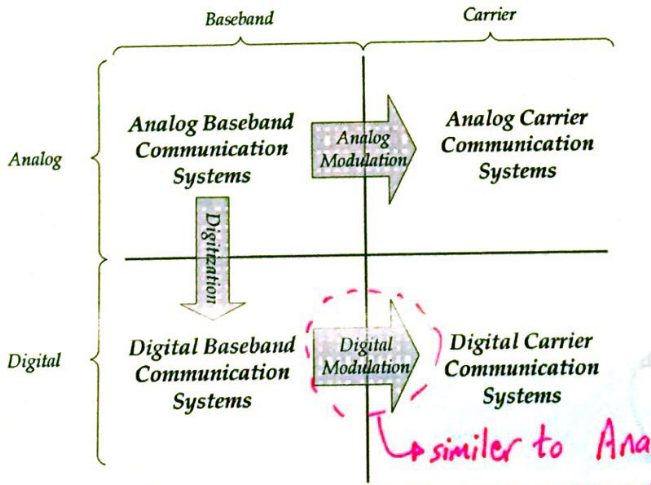
University of Jordan

EE421: Communications I. For more information read Chapters 7 & 10 in your textbook or visit http://wikipedia.org/.

The *Last Piece* of the Puzzle!



Baseband

Carrier

Analog

Analog Baseband Communication Systems

Analog Modulation

Analog Carrier Communication Systems

Digitization

Digital

Digital Baseband Communication Systems

Digital Modulation

Digital Carrier Communication Systems

→ similer to Analog Modulation.

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Digital Modulation

- *Four* main modulation techniques:
  - – Amplitude-Shift Keying (**ASK**).
  - – Frequency-Shift Keying (**FSK**).
  - – Phase-Shift Keying (**PSK**).
  - – Quadrature Amplitude Modulation (**QAM**).
- PSK and QAM are the most popular nowadays because of their *smaller* bandwidths.
- PSK and QAM require synchronous detection, which is easier nowadays (PLLs).

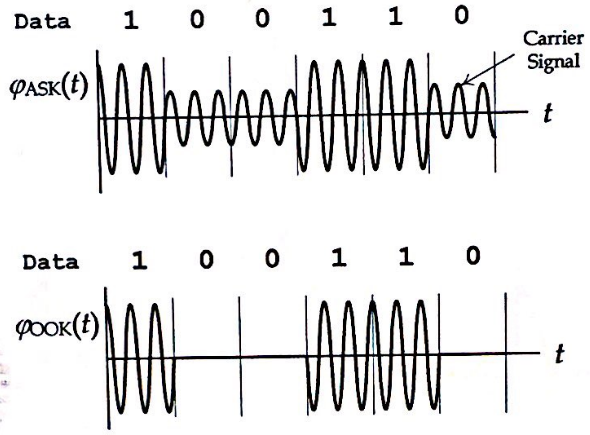
Analog vs. *Digital* Modulation

- $m(t)$ *is* Polar NRZ + AM = **ASK**
- $m(t)$ *is* Polar NRZ + FM = **FSK**
- $m(t)$ *is* Polar NRZ + PM = **BPSK**
- $m(t)$ *is* Q-ary NRZ + PM = **QPSK**
- $m(t)$ *is* M-ary NRZ + PM = **M-PSK**
- $m(t)$ *is* M-ary NRZ + QAM = **QAM**
- $m(t)$ *is* M-ary NRZ + AM = **M-ASK**

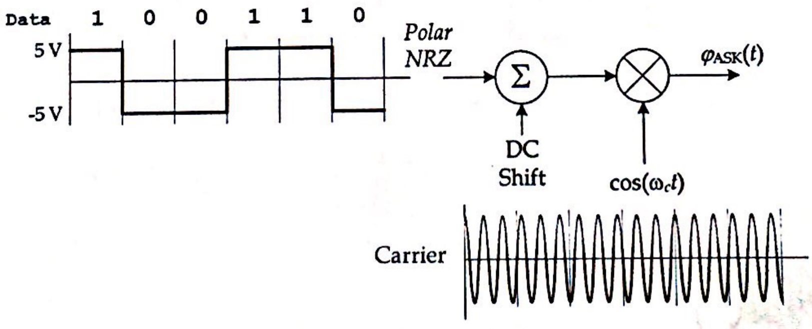
Bandwidth of $\underset{FSK}{\phi(t)}$: $= 2\Delta f + 2B$

↳ Bandwidth of polar NRZ $= f_0$

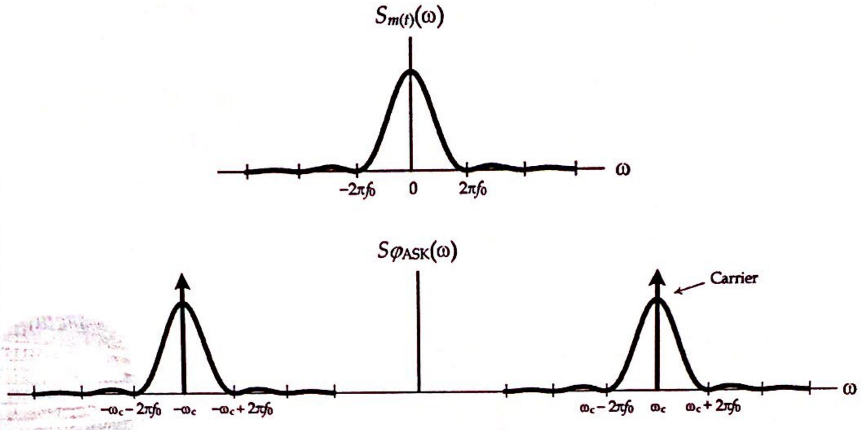
ASK and OOK



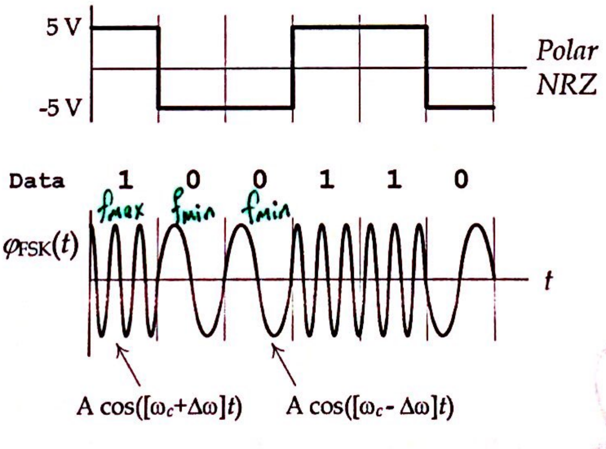
ASK is AM



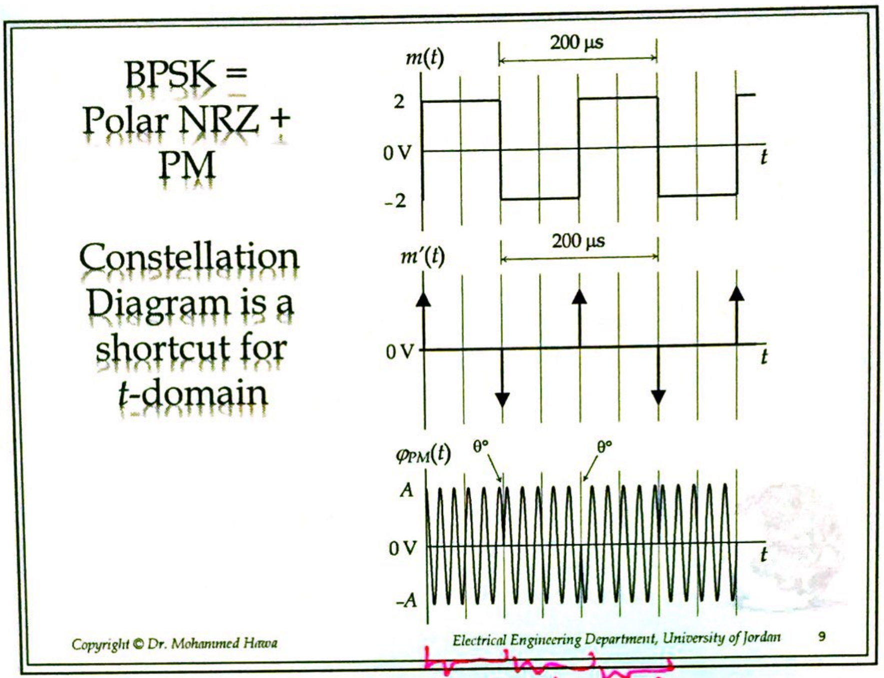
PSD of an ASK Signal



FSK



SNR ↓ BER → Quality



BPSK = Polar NRZ + PM

Constellation Diagram is a shortcut for *t*-domain

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sketch $\phi(t)$ BPSK in t-domain for 10101010

⇓

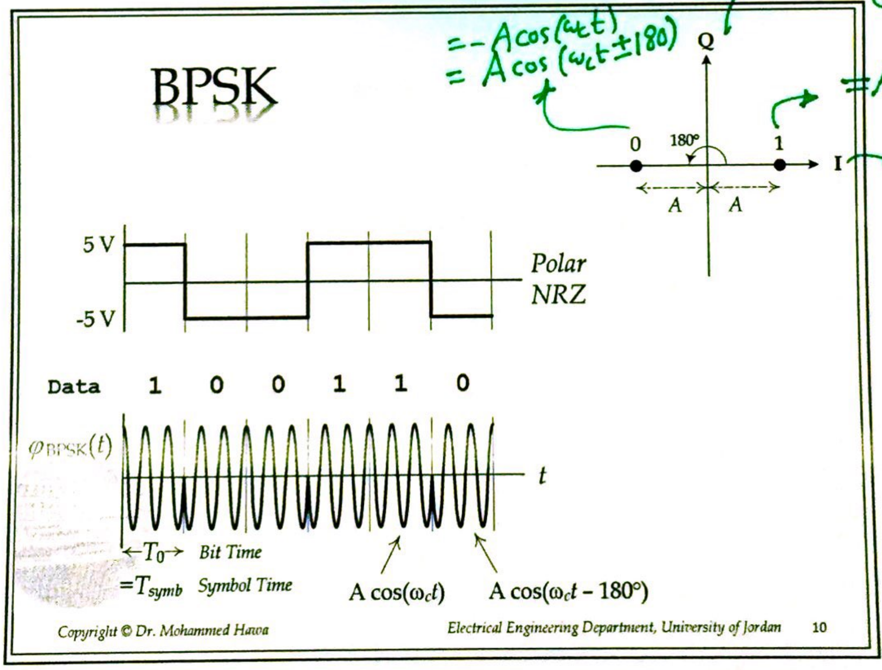
$m(t)$

$\phi(t)_{PM}$

$\omega_{i_{PM}} = \omega_c + K_p m'(t)$ 0

$\omega_i = \omega_c$.

$A\cos\omega_c t$ $-A\cos\omega_c t$ $A\cos\omega_c t$



BPSK

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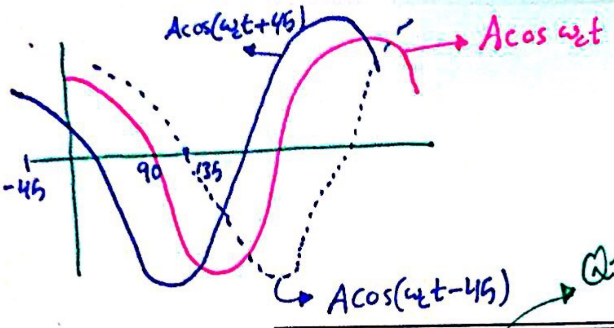
quadrature $Q = \sin(\omega_c t)$.

$= -A\cos(\omega_c t)$

$= A\cos(\omega_c t \pm 180)$

$= A\cos(\omega_c t)$

in phase $= \cos(\omega_c t)$

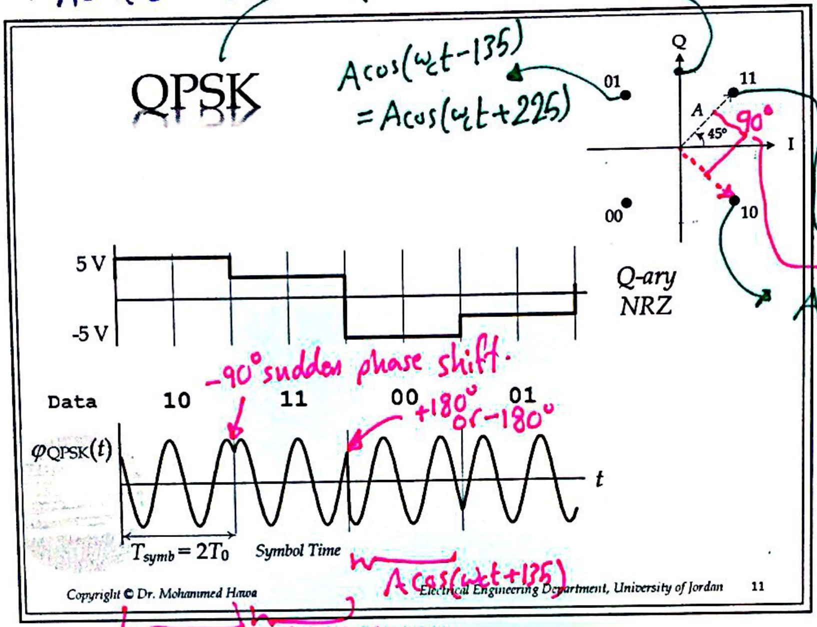


Q-ary + PM

$Q\text{-ary} = \overset{m(t)}{4\text{-ary}} + PM$

if we have point here: $A\sin\omega_c t = A\cos(\omega_c t-90)$

C.C.W ⇒ -ve angle in Constellation



QPSK

$A\cos(\omega_c t-135) = A\cos(\omega_c t+225)$

90°

$A\cos(\omega t-45°)$

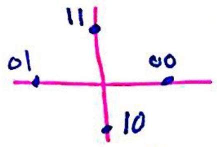
$A\cos(\omega t+45)$

Sudden phase shift.

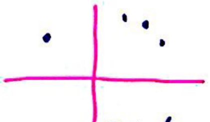
-90° sudden phase shift.

+180° or -180°

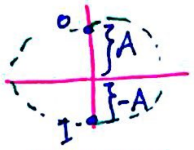
$A\cos(\omega_c t+45)$ $A\cos(\omega_c t-45)$ $A\cos(\omega_c t+135)$



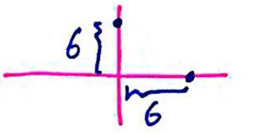
⇒ QPSK.



⇒ QPSK.

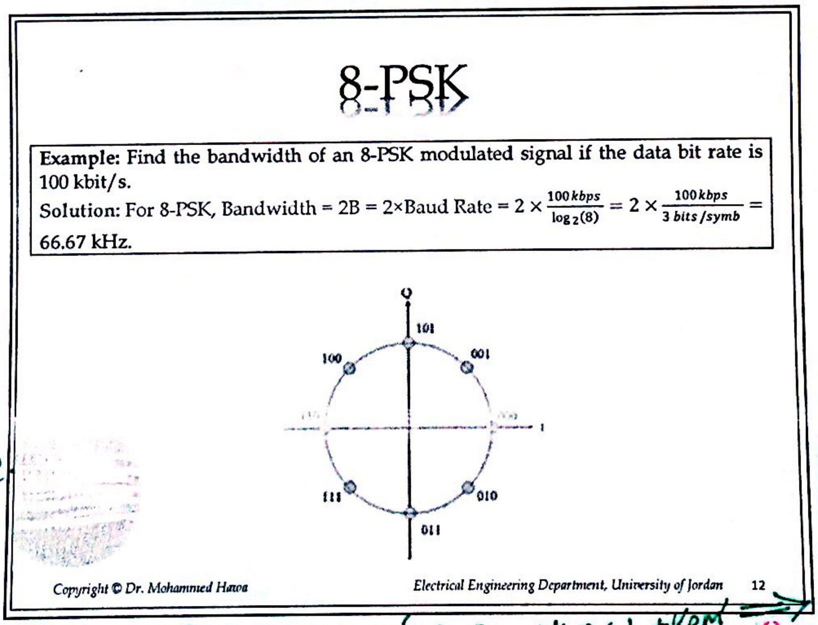


⇒ BPSK
2-PSK



⇒ BPSK.

always PSK has the same freq. & the same Amplitude



8-PSK

Example: Find the bandwidth of an 8-PSK modulated signal if the data bit rate is 100 kbit/s.

Solution: For 8-PSK, Bandwidth = 2B = 2×Baud Rate = $2 \times \frac{100\,kbps}{\log_2(8)} = 2 \times \frac{100\,kbps}{3\ bits/symb} =$ 66.67 kHz.

$f_{max} = f_{min} = f_c$ so $\Delta f = 0$.

$\omega_i = \omega_c$

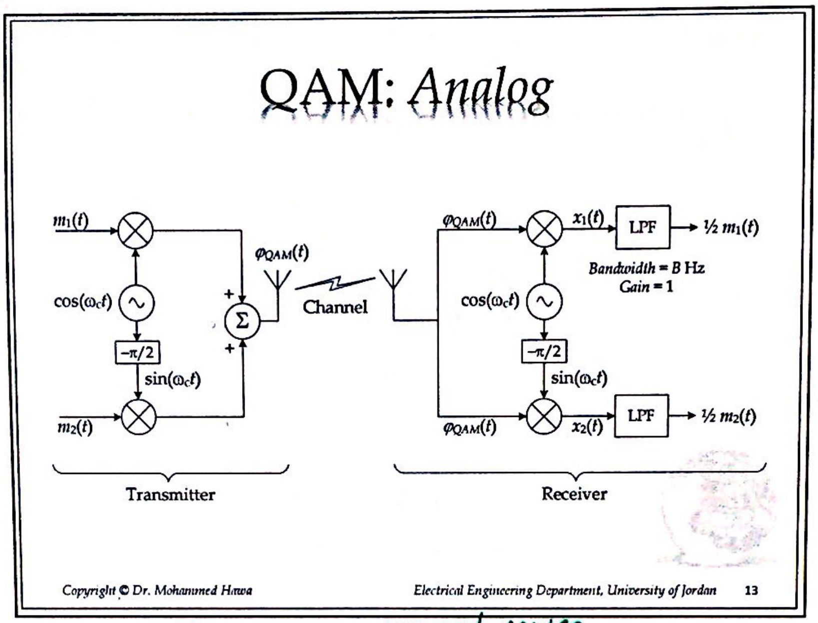
always $\Delta f_{PSK} = 0$ ⇒ $m' = 0$ so $\omega_i = \omega_c + k_p \overset{0}{\cancel{m'}}$

Bandwidth of 8-PSK = BandWidth of PM $= 2\overset{0}{\cancel{\Delta f}} + 2B$

= 2 Baud Rate.

Ex. sketch $\phi(t)$ for 100101110000 assuming constellation in slide(14)?
16-QAM

5/10/2016



QAM: *Analog*

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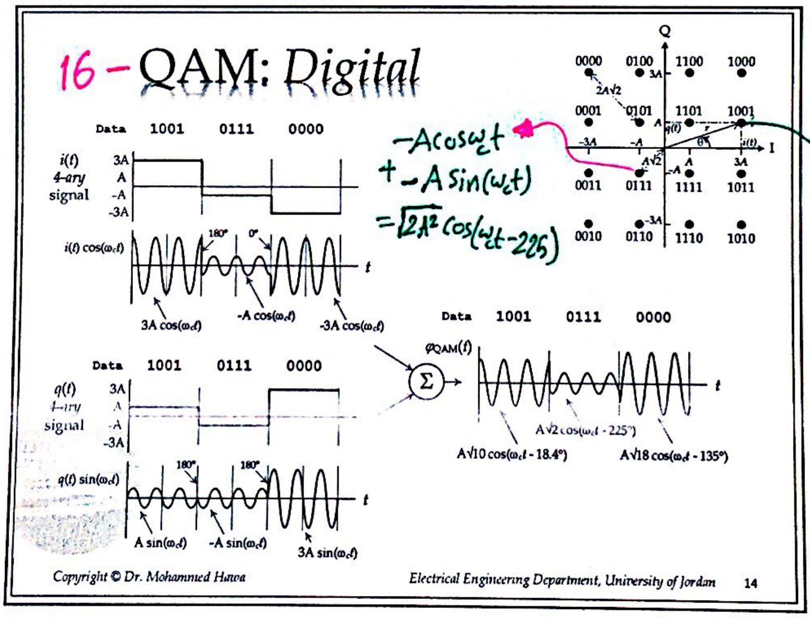
Quality ∝ BER ∝ SNR = $\dfrac{\text{signal power}}{\text{Noise power}}$



16 – QAM: *Digital*

$-A\cos\omega_c t$

$+ -A\sin(\omega_c t)$

$= \sqrt{2A^2}\cos(\omega_c t - 225)$

$r\cos(\omega_c t - \theta)$

$= \sqrt{(3A)^2 + A^2}\cos\left(\omega_c - \tan^{-1}\frac{A}{3A}\right)$

$= A\sqrt{10}\cos(\omega_c t - 18.4^\circ)$

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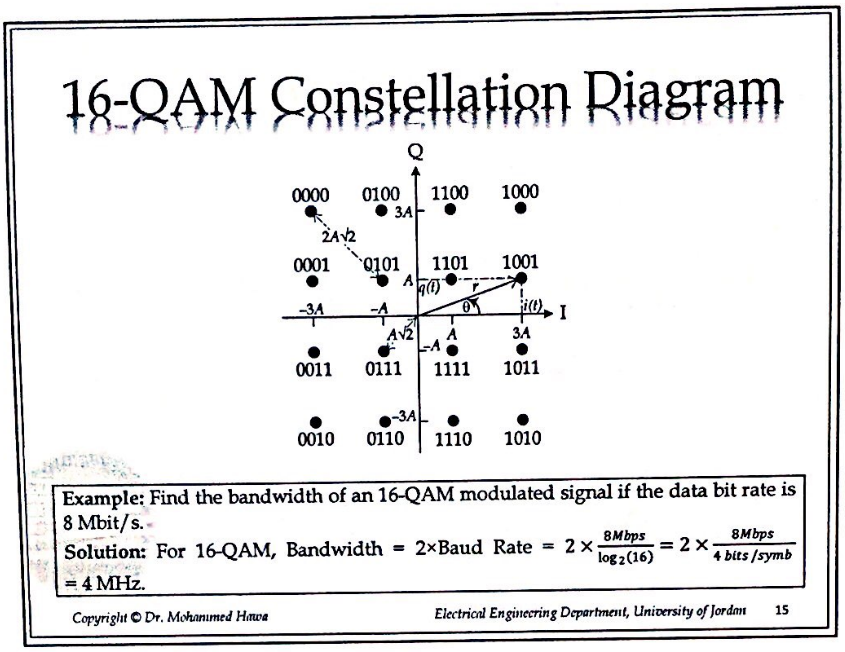
remember: $\alpha\cos(\omega_c t) + \beta\sin(\omega_c t) = C\cos(\omega_c t - \theta)$

$C = \sqrt{\alpha^2 + \beta^2}$, $\theta = \tan^{-1}\left(\frac{\beta}{\alpha}\right)$

for QAM: $m_1(t)\cos(\omega_c t) + m_2(t)\sin\omega_c t = 3A\cos\omega_c t + A\sin\omega_c t$?

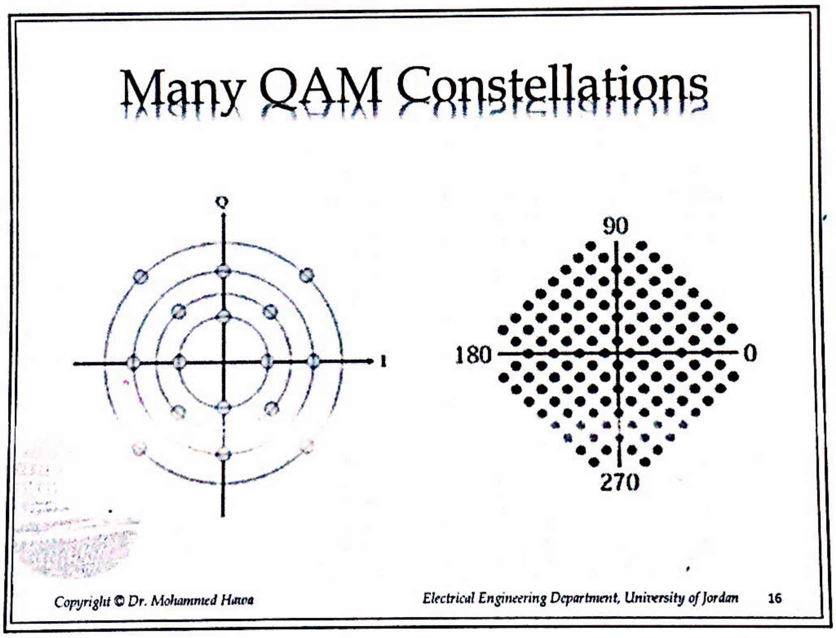
will give the same answer.



16-QAM Constellation Diagram

Example: Find the bandwidth of an 16-QAM modulated signal if the data bit rate is 8 Mbit/s.

Solution: For 16-QAM, Bandwidth = 2×Baud Rate = $2 \times \frac{8Mbps}{\log_2(16)} = 2 \times \frac{8Mbps}{4\ bits/symb}$ = 4 MHz.

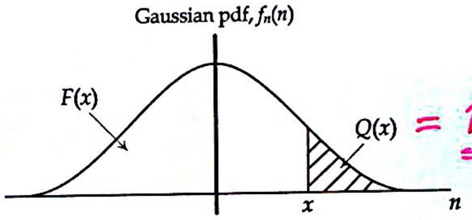


Many QAM Constellations

AWGN Noise

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$Q(x) = 1 - F(x) = 1 - \int_{-\infty}^{x} f(\alpha)d\alpha = \int_{x}^{\infty} f(\alpha)d\alpha = \frac{1}{\sqrt{2\pi}}\int_{x}^{\infty} e^{-\frac{\alpha^2}{2}} d\alpha$$



$= 1 - F(x).$

Quantile Function $Q(x)$

x	$Q(x)$	x	$Q(x)$	x	$Q(x)$	x	$Q(x)$
2.00	2.28E-02	3.00	1.35E-03	4.00	3.17E-05	5.00	2.87E-07
2.05	2.02E-02	3.05	1.14E-03	4.05	2.56E-05	5.05	2.21E-07
2.10	1.79E-02	3.10	9.68E-04	4.10	2.07E-05	5.10	1.70E-07
2.15	1.58E-02	3.15	8.16E-04	4.15	1.66E-05	5.15	1.30E-07
2.20	1.39E-02	3.20	6.87E-04	4.20	1.33E-05	5.20	9.96E-08
2.25	1.22E-02	3.25	5.77E-04	4.25	1.07E-05	5.25	7.60E-08
2.30	1.07E-02	3.30	4.83E-04	4.30	8.54E-06	5.30	5.79E-08
2.35	9.39E-03	3.35	4.04E-04	4.35	6.81E-06	5.35	4.40E-08
2.40	8.20E-03	3.40	3.37E-04	4.40	5.41E-06	5.40	3.33E-08
2.45	7.14E-03	3.45	2.80E-04	4.45	4.29E-06	5.45	2.52E-08
2.50	6.21E-03	3.50	2.33E-04	4.50	3.40E-06	5.50	1.90E-08
2.55	5.39E-03	3.55	1.93E-04	4.55	2.68E-06	5.55	1.43E-08
2.60	4.66E-03	3.60	1.59E-04	4.60	2.11E-06	5.60	1.07E-08
2.65	4.02E-03	3.65	1.31E-04	4.65	1.66E-06	5.65	8.02E-09
2.70	3.47E-03	3.70	1.08E-04	4.70	1.30E-06	5.70	5.99E-09
2.75	2.98E-03	3.75	8.84E-05	4.75	1.02E-06	5.75	4.46E-09
2.80	2.56E-03	3.80	7.23E-05	4.80	7.93E-07	5.80	3.32E-09
2.85	2.19E-03	3.85	5.91E-05	4.85	6.17E-07	5.85	2.46E-09
2.90	1.87E-03	3.90	4.81E-05	4.90	4.79E-07	5.90	1.82E-09
2.95	1.59E-03	3.95	3.91E-05	4.95	3.71E-07	5.95	1.34E-09

x

2.37

Do interpolation:

$2.35 \longrightarrow 1.07\times10^{-2}$

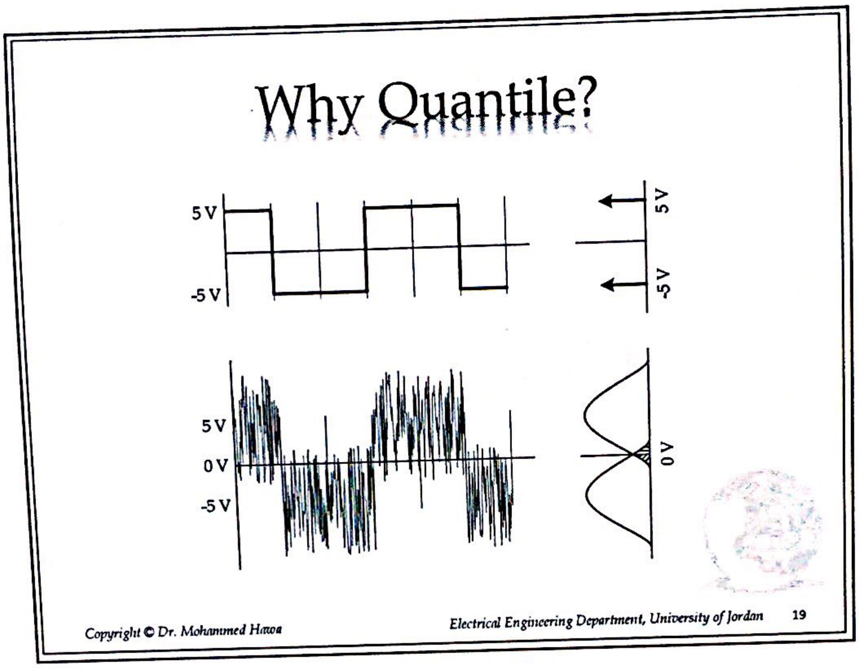
$2.37 \longrightarrow X$

$2.40 \longrightarrow 8.2\times10^{-2}$

so

$$\frac{2.37-2.35}{2.4-2.35} = \frac{X-1.07\times10^{-2}}{(8.2-1.07)\times10^{-2}}$$

Then find K.



Why Quantile?

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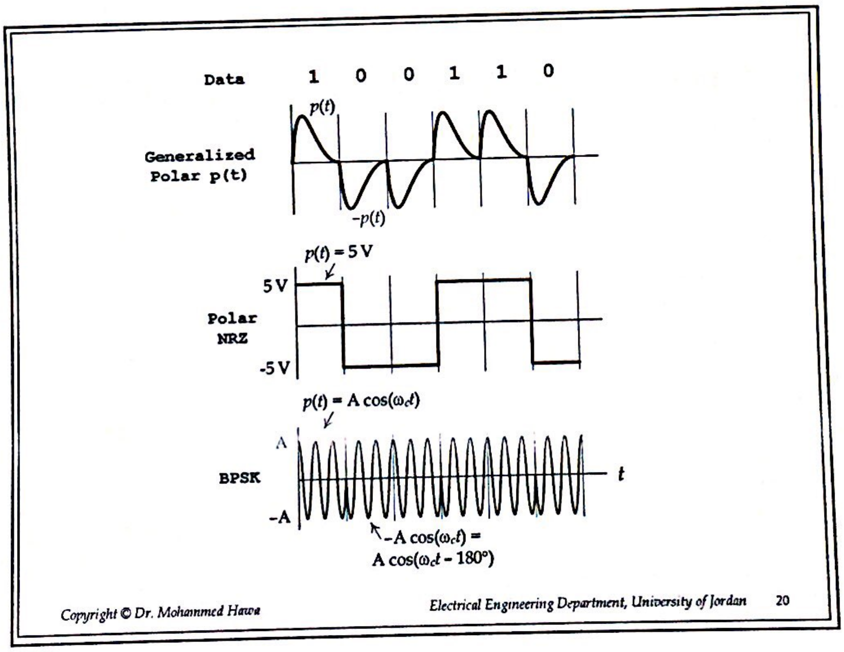

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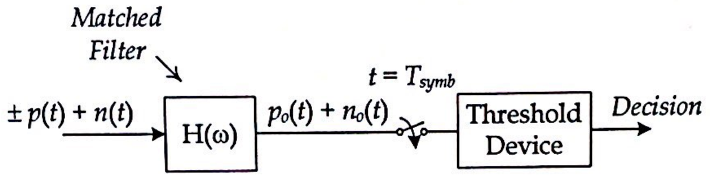
Matched Filter



Perfromance of Digital Systems

Memorize.

Modulation with AWGN	Error Probability
ASK	$BER = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$
FSK	$BER = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$
BPSK	$BER = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$
QPSK	$BER = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$
M-PSK (order M)	$BER \cong \frac{2}{k} Q\left(\sqrt{2k\frac{E_b}{N_0}} \times \sin\left(\frac{\pi}{M}\right)\right)$
M-QAM (order M) *(Rectangular QAM)*	$P_{bc} = \frac{4}{k}\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3k}{M-1}\frac{E_b}{N_0}}\right)$ $BER = 1 - (1 - P_{bc})^2$

if he asked about them it would be given.

Definitions

For the rest of this document, we will use the following notation:

- M = Number of possible symbols that the modulated signal can assume. → 16-QAM $M=16$
- k = the number of bits sent per transmitted symbol = $\log_2(M)$. → $K=4$
- E_s = *Average* energy-per-transmitted-symbol in the modulated signal (Joule).
- E_b = *Average* energy-per-transmitted-bit in the modulated signal (Joule) = E_s/k.
- $S_n(\omega) = \frac{N_0}{2}$ = Double-sided noise power spectral density (*in* W/Hz = Joule).
- T_o = Bit duration. → $f_o = \frac{1}{T_o}$ = bit rate.
- T_{symb} = Symbol duration = $k\,T_o$
- BER = Probability of bit-error = bit error rate.

$T_{symbol} = KT_o$

Example:
Find the BER for BPSK if we use an *optimal* detector (a matched filter). Assume the amplitude of the carrier is $A = 0.5$ V, data rate is 2 bps, and $N_0 = 2 \times 10^{-2}$ W/Hz.

Solution:
In BPSK there is one symbol per bit (i.e., a total of two symbols that the modulated signal can assume). The two symbols can be written as:

$$s_1 = A\cos(\omega_c t) \qquad s_2 = -A\cos(\omega_c t) = A\cos(\omega_c t - \pi)$$

The energy-per-symbol here is the same as the energy-per-bit and is equal for both possible symbols. Hence, its average is:

$$E_b = E_s = \left(\frac{A^2}{2}T_{symb}\right)\Pr[1] + \left(\frac{A^2}{2}T_{symb}\right)\Pr[0] = \frac{A^2}{2}T_{symb} = \frac{A^2}{2}T_o = \frac{A^2}{2}\frac{1}{f_0}$$

Hence,

$$\text{BER} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{\frac{A^2}{N_0 f_0}}\right) = Q\left(\sqrt{\frac{0.5^2}{2\times10^{-2}\times 2}}\right) = Q(\sqrt{6.25}) = Q(2.5)$$
$$= 6.21 \times 10^{-3}$$

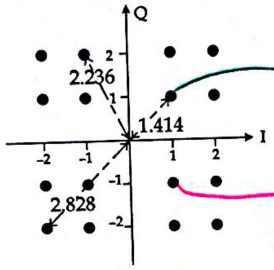
$$E_b = \frac{E_s}{K} = \frac{\text{average energy per symbol}}{K} = \frac{\text{average power per symbol} \times \text{Tsymb.}}{K}$$

$$\underset{\substack{\uparrow\\ \text{all symbols}}}{E_s} = E_{s_1}\overset{1/16}{P[\text{symb}1]} + E_{s_2}\overset{1/16}{P[\text{Symb}2]} + \dots + E_{s_{16}}\underset{1/16}{P[\text{Symb}16]}$$

$$\sum_{i=1}^{16} P[\text{symb}\,i] = 1$$

Example

Find the BER for the 16-QAM constellation shown below if we use an *optimal* detector (a matched filter). Assume the data rate is 4 bps, and $N_0 = 5 \times 10^{-2}$ W/Hz.



$1.414\cos(\omega_c t - 45°)$

avg. power $= \frac{1.414^2}{2}$

$1.414\cos(\omega_c t + 45°)$

Solution:

In this system there are 16 possible symbols, which we assume to be equally probable, i.e., each occurs with a probability of 1/16. Hence, the energy-per-symbol is:

$$E_s = \left(\frac{1.414^2}{2}T_{symb}\right)\left(\frac{4}{16}\right) + \left(\frac{2.236^2}{2}T_{symb}\right)\left(\frac{8}{16}\right) + \left(\frac{2.828^2}{2}T_{symb}\right)\left(\frac{4}{16}\right)$$

$$E_s = [0.25 + 1.25 + 1]T_{symb} = 2.5(T_{symb})$$

$$E_b = \frac{E_s}{k} = 2.5\left(\frac{T_{symb}}{k}\right) = 2.5(T_0) = \frac{2.5}{f_0} = \frac{2.5}{4} = 0.625 \text{ J}$$

$$P_{bc} = \frac{4}{k}\left(1 - \frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{3k}{M-1}\frac{E_b}{N_0}}\right) = \frac{4}{4}\left(1 - \frac{1}{\sqrt{16}}\right)Q\left(\sqrt{\frac{3\times 4}{16-1}\times\frac{0.625}{0.05}}\right) = \frac{3}{4}Q(\sqrt{10})$$

$$= \frac{3}{4}Q(3.162) = \frac{3}{4}\times 8\times 10^{-4} = 6\times 10^{-4}$$

$$BER = 1 - (1 - P_{bc})^2 = 1 - (1 - 6\times 10^{-4})^2 = 1.2\times 10^{-3}$$

"Bad Quality".

$T_0 = \frac{T_{symb}}{K}$

Not in the table
Find it
By interpolation.

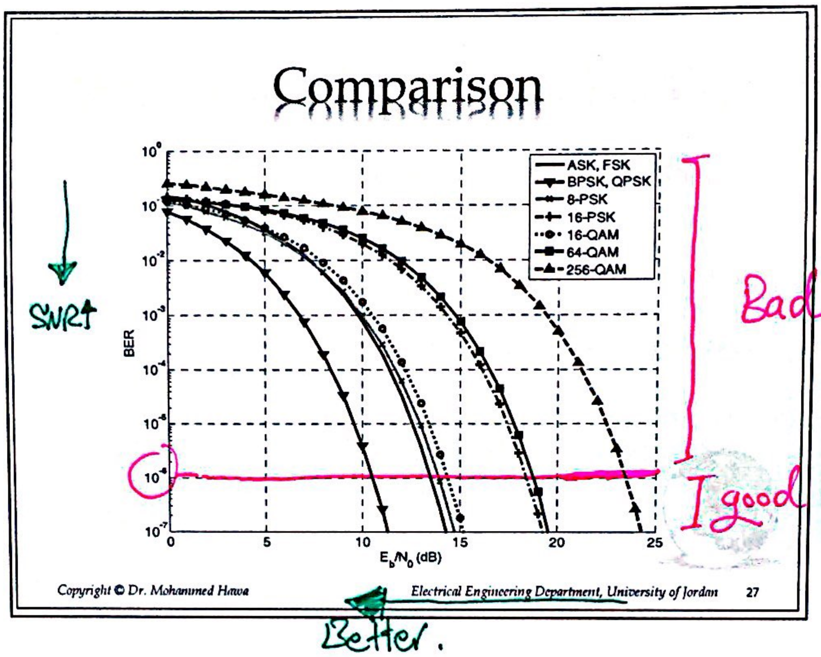
Note: if S_n was given $\Rightarrow$ $N_0 = 2S_n$

AM, FM vs. PM

Not popular any more.

ASK, FSK vs BPSK, QPSK.

↳ Better since smaller BW.



Comparison

SNR↑

Bad Quality

good Quality.

Better.

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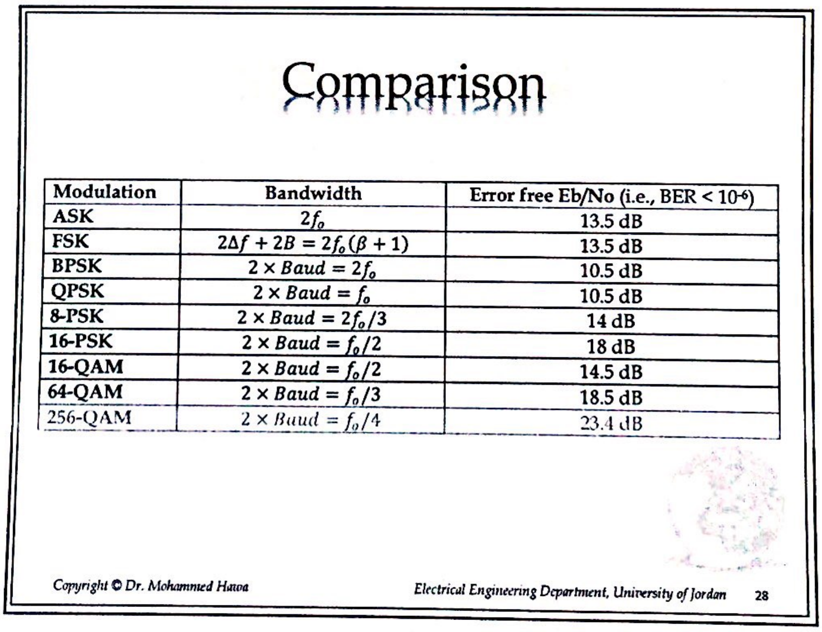

Comparison

Modulation	Bandwidth	Error free Eb/No (i.e., BER < 10^{-6})
ASK	$2f_o$	13.5 dB
FSK	$2\Delta f + 2B = 2f_o(\beta + 1)$	13.5 dB
BPSK	$2 \times Baud = 2f_o$	10.5 dB
QPSK	$2 \times Baud = f_o$	10.5 dB
8-PSK	$2 \times Baud = 2f_o/3$	14 dB
16-PSK	$2 \times Baud = f_o/2$	18 dB
16-QAM	$2 \times Baud = f_o/2$	14.5 dB
64-QAM	$2 \times Baud = f_o/3$	18.5 dB
256-QAM	$2 \times Baud = f_o/4$	23.4 dB

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$$BW\downarrow = 2\ \text{Baud Rate}\downarrow = 2\ \text{Symbol Rate}\downarrow = 2\frac{f_o}{\log_2 M\uparrow}$$

Remember: Digital Modulation

- **Bandwidth** of the channel decides the **baud rate** (symbols per second) you can send.
- **Signal-to-noise ratio** (E_b/N_0) decides the level of modulation you can use while still maintaining a small bit error rate. In other words, it decides the number of **bits you can send per symbol**.
- Hence, the two factors together (bandwidth and SNR) decide the **total bit rate** you can achieve over any single channel.
- Shannon's Limit!

Shannon's Limit

$$C = B_{ch} \times log_2(1 + SNR)$$

- C: Capacity of the channel in bits/second (bps)
- B_{ch}: Channel bandwidth (units of Hz)
- SNR: Signal-to-Noise Ratio (unitless)(*not dB)*

Memorize

Applications

- **IEEE 802.11 (Wi-Fi):** BPSK, QPSK, 16-QAM, 64-QAM and CCK (Complementary Code Keying) (CCK is an extension of QPSK).
- **IEEE 802.16 (Wi-MAX):** BPSK, QPSK, 16-QAM, and 64-QAM. It uses these modulation schemes in combination with OFDM (Orthogonal Frequency division multiplexing) (OFDM is an extension of FDM).
- **DVB (Digital Video Broadcasting):** DVB-S (for satellite broadcasting) uses QPSK or 8-PSK; DVB-C (for cable) uses 16-QAM, 32-QAM, 64-QAM, 128-QAM or 256-QAM; and DVB-T (for terrestrial television broadcasting) uses 16-QAM or 64-QAM.
- **DAB (Digital Audio Broadcasting):** DQPSK (Differential QPSK) (DQPSK is a variation of QPSK).
- **ADSL:** QAM in a scheme called DMT (Discrete Multi-Tone modulation).