Communications II

Second Semester
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Notebook:

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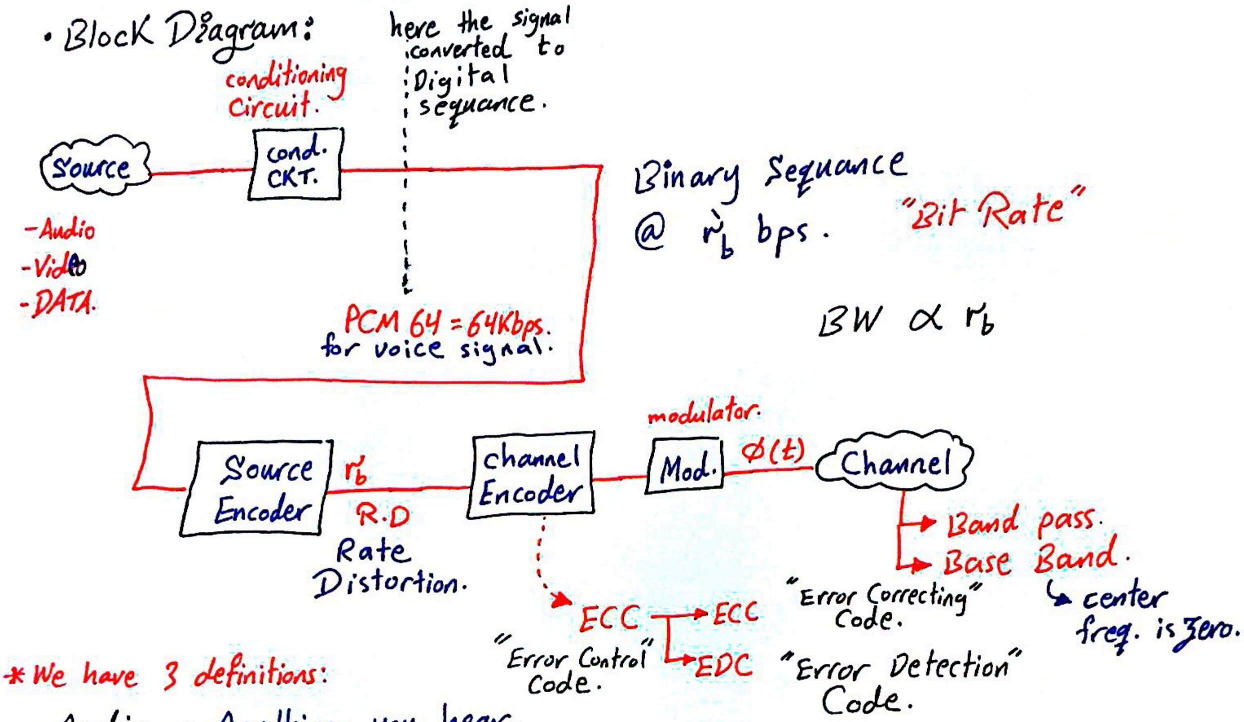
※ Digital Communication :

• Block Diagram:



* We have 3 definitions:

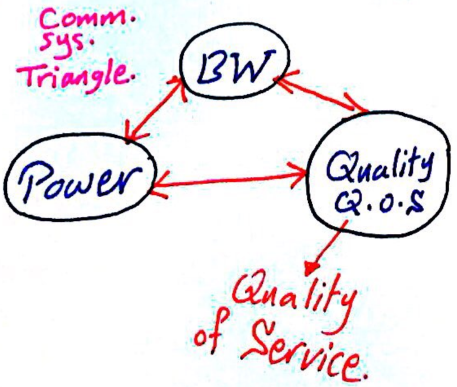
- Audio → Anything you hear.
- Voice → Anything came out from the mouth.
- Speech → for anything has a meaning.

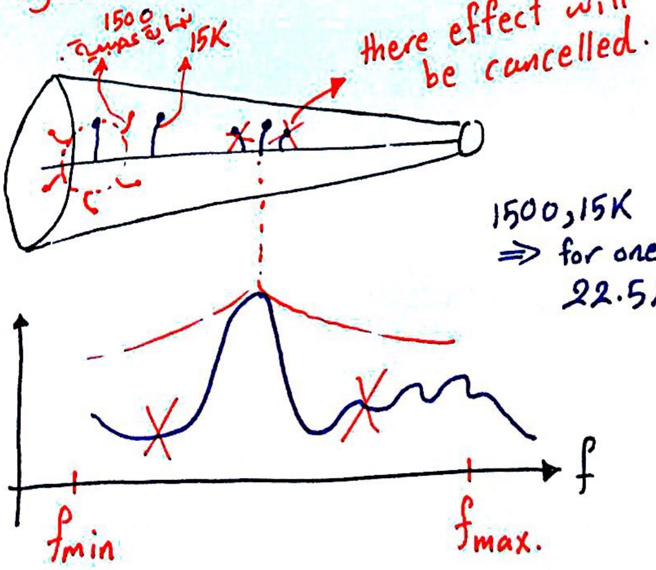
⇒ we use comm. systems for AUDIO signals.

* Voice Signal has a BW 4KHZ. this 4KHZ called "toll quality."

Toll Quality : minimum quality required for Telephony Signals.

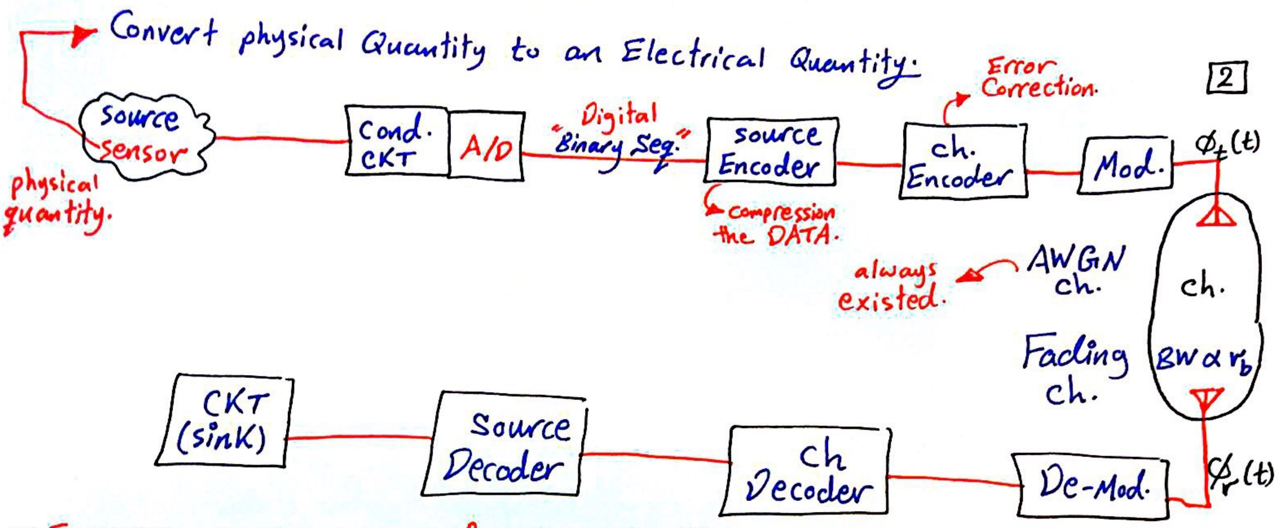
• The most expensive part in the system is the Bandwidth.





1500, 15K
⇒ for one ear 22.5M sample.

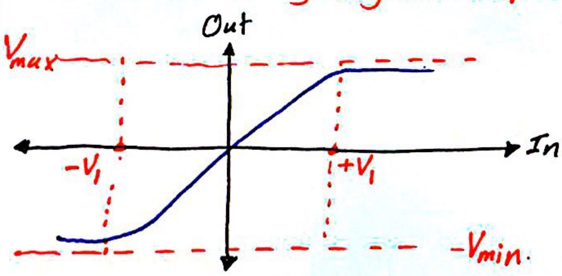
$\boxed{\lambda f = u}$ $\lambda \downarrow \Rightarrow f \uparrow$



* For every signal we care for two main things:

1) in time domain: peak-to-peak value.
2) in freq. domain: Bandwidth.

* The following figure represent a conditional circuit:



the -ve voltage existed To make the DC value equal to zero.

* Grey level of the signal "intensity": white — Black.

* The three main colors: R G B.
Red Green Blue

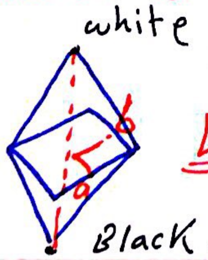
* Color Matrix:

$$I = 0.3R + 0.59G + 0.11B$$

intensity.

⇒ this is the electrical analysis of the color.

* Human Eyes:



La^*b^*

⇒ This is the model of the vision of the eye (L a b).

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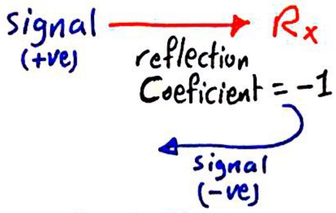
• **Contrast:**

it is the divergent in intensity between two pixels beside eachother that the human eye could recognize.

* **Two Ray Model:**

This is the worst case senario.



These two signals will be added "Coherently" & give zero "null".

*when R_x is a perfect conductor.

* **The Relation between $\phi_t(t)$ & $\phi_r(t)$:**

$\phi_r(t) = \alpha \, \phi_t(t)$ where α is a R.V "Random Variable". — this is a Flat Fading.

$\Rightarrow \phi_r(t) = \sum \alpha_i \, \phi_t(t - t_i)$ this is freq. Selective Fading.

also it is a replica to: $\phi_r(t) = h(t) \star \phi_t(t)$ — convolve.

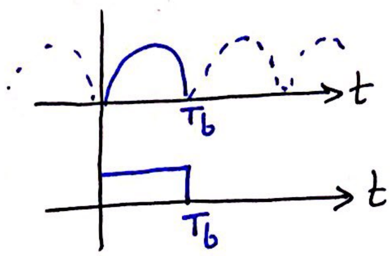
※ **Modulation:**

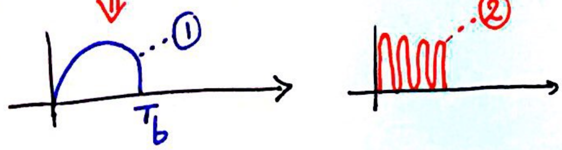
it is aperiodic signal has an infinite Bandwidth.

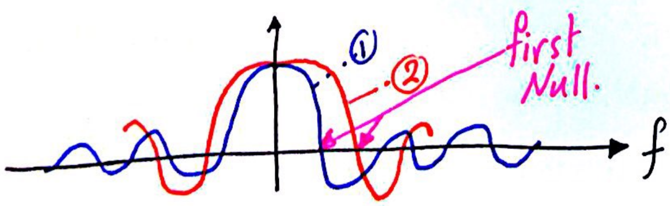
Data rate $\equiv r_b$

Period $\equiv T_b = \frac{1}{r_b}$

$$\boxed{\text{first Null} = \frac{1}{2T_b} = \frac{r_b}{2}}$$







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* The projection of x(t):

$$X(f) = \int_{-\infty}^{\infty} \underbrace{x(t)}_{\text{Signal}} \cdot \underbrace{e^{-j\omega t}}_{\text{Basis (Axis) function.}} \, dt = \langle x(t), e^{-j\omega t} \rangle$$

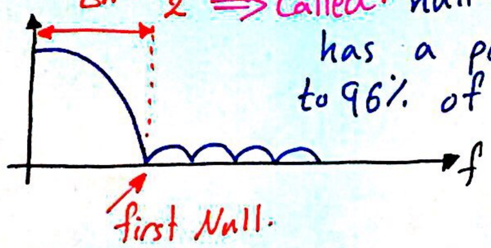
↳ "inner product"

* Entropy: it is a method to measure the information content of a certain signal.

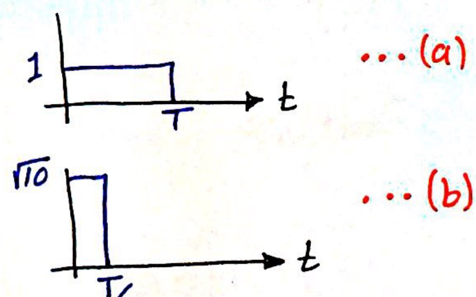
$$H_i = -\log_2(P_i) \xrightarrow[\text{Data}]{\text{for all}} H = -\sum_{i=1}^{N} P_i \cdot \log_2 P_i \text{ bits.}$$

• **Deterministic Signals: Carry NO Information.**

$BW = \frac{r_b}{2}$ ⇒ Called: null-to-null Bandwidth has a percentage of 95% to 96% of the signal.



* Consider the following two signals (a) & (b). Which one is Better? justify your answer.



Both signals have the same Energy = T

(a) is better than (b) due to Utilization. since (a) need 1 Voltage source over T period, where (b) need $\sqrt{10}$ voltage source over shorter period $\frac{T}{10}$ which is a Bad Utilization.

* Synchronization:

• Bit synchronization.

every 10 bits ≡ word ⇒ word synchronization.

* The Requirements for Communication System:

1) Minimum Bandwidth.
2) Minimum Energy.
3) Best Performance (Q.o.S).

* The Requirements for Better Design Between two signals:

1) Minimum mean Energy.
2) Maximum difference between the two signals. "Max. Distance"

⇒ correlation = -1

* when the correlation coeficient between two signals = 0

⇒ "Orthogonality."

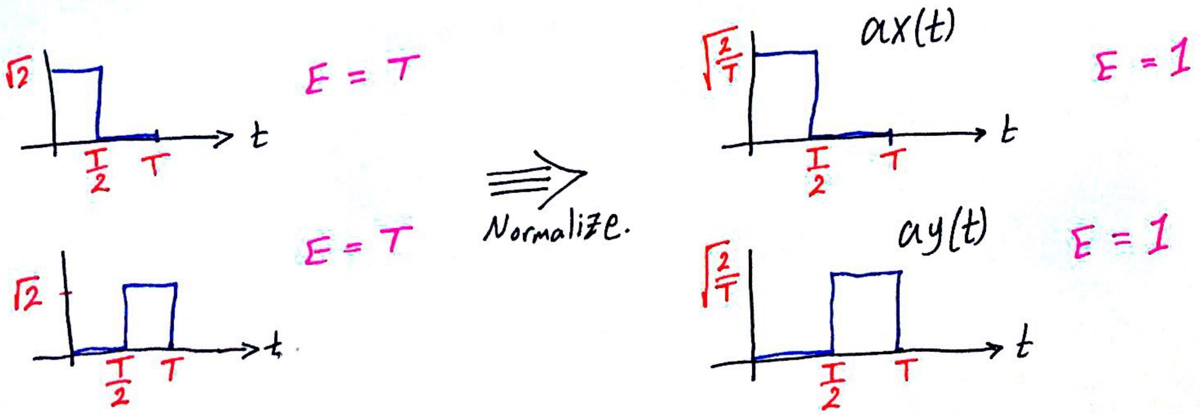
* To represent a space of signals we need:

#of axis = size of that space.

Ex. we represent a 3-dimensional space with 3 axis.

• Note: Non-overlap signals in any domain are Orthogonal.

* Inner Product: $<x_1, x_2> = \int_{-\infty}^{\infty} x_1 \cdot x_2^* \, dt$ → "Energy Function".



- **Discriminant:** it is the global meaning of Difference.

if we have considered: $y = f(x) + w$

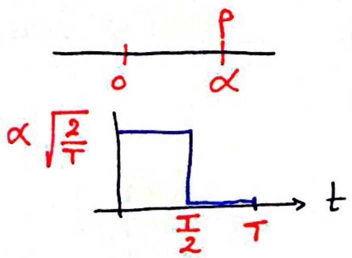
Then if $w = 0$: $y = f(x) \Rightarrow$ "dependency."

if $w \neq 0$: $y = f(x) + w \Rightarrow$ "Correlation".

* One-D:

using $a_x(t)$ axis

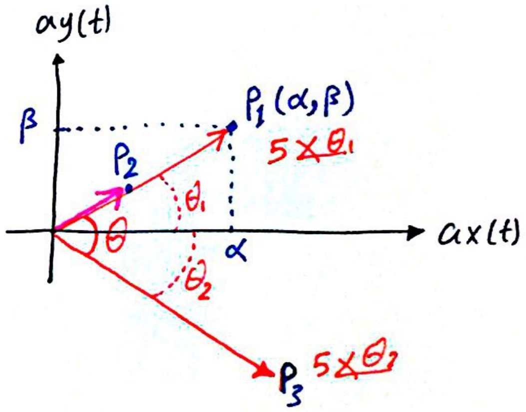
$P(t) = \alpha \, a_x(t)$



* 2-D:

There is multiple forms to represent it.

for e.g: $\rightarrow$ complex.
$\rightarrow$ point.
$\rightarrow$ vector.



Let: $\alpha = 3$, $\beta = 4$ $\quad P_1 = (3,4)$
$P_3 = (4,-3)$

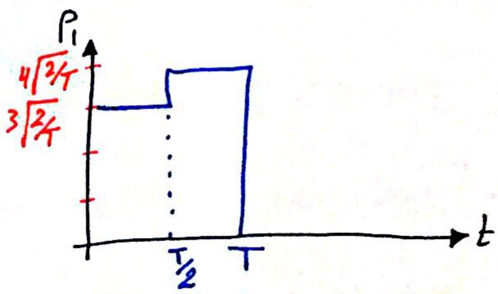
$\Rightarrow P_1 = 3a_x(t) + 4a_y(t)$

we could write P_1 as: $P_1 = 3 + j4$

- P_2 is different with P_1 by a scale.

we could write as vectors: $P_1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, $P_3 = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$

$\langle P_1, P_3 \rangle = (3)(4) + (4)(-3) =$ Zero



$L_2 = |x|^2 \rightarrow$ second-order-Norm.
$L_\infty = |x| \rightarrow$ infinite Norm.

if the signal is Random:

$L_2 = MSE = E\{|x - \hat{x}|^2\}$
$L_\infty = E\{|x|\}$

$MSE \equiv$ Mean Square Error.

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$<P_1, P_2> = |P_1| \cdot |P_2| \cdot \cos\theta_{12}$

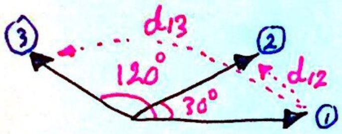
⇒ Correlation Coeficient:

$$\boxed{\rho_{12} = \frac{<P_1, P_2>}{|P_1| \cdot |P_2|} = \cos\theta_{12}}$$

if $\theta_{12} = 90^\circ$

⇒ ρ_{12} = Zero "orthogonal"

* Consider a 3-vector:



The best choice between two signals for better work ⇒ 1 & 3 "since we care for Max. Distance".

* Criteria of Modulation:

1) Minimum Energy. 2) Maximum Difference "Distance".

• HomeWork: Find the optimal α_i's for $y = \sum_{i=1}^{I} \alpha_i^2$?

α is optimal when all α_i equaled.

• Note: sine & cosine are orthogonal in Code-domain.

* We have 4 domains:
→ Time Domain.
→ Frequancy Domain.
→ Code Domain.
→ Space Domain.

※ BW Unlimited Base-band Tx:

$Tx \underset{\text{.........}}{\overset{ch.}{\text{———}}} Rx$

Base-band ≡ means the signal has a center frequancy around Zero.

• Note: the wireless communication for Low frequancy systems is not feasible.

Low freq. will need very Larg Antenna.

i.e $L_{ant} = \frac{\lambda}{2}$ or $\frac{\lambda}{4}$ ⇒ $\lambda = \frac{c}{f}$

• for Extremely short communication distances we will use Wire-communication "Baseband".

* Number of signals in the channel equal the number of input DATA. (ex. for Binary ⇒ input 0 & 1 so we have 2-signals).

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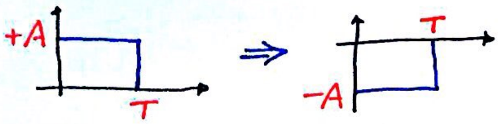
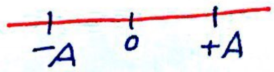
* Binary Transmission :

"0" "1" $\Rightarrow$ $S_0(t)$ $S_1(t)$

T_x $\xrightarrow{ch.}$ R_x

we can select that:



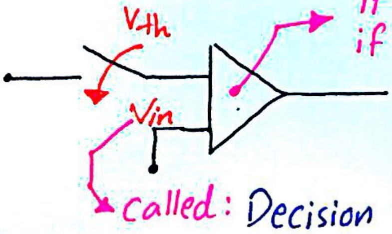


single space Diagram.

"Constellation Diagram".

@ R_x :

- threshold detector:



if $V_{in} > V_{th} \Rightarrow$ "0"
if $V_{in} < V_{th} \Rightarrow$ "1"

This called: "Decision Rule".

called: Decision Variable.

- threshold: is the average of the two signals.

$$\boxed{V_{th} = \frac{V_0 + V_1}{2}}$$

- Note:

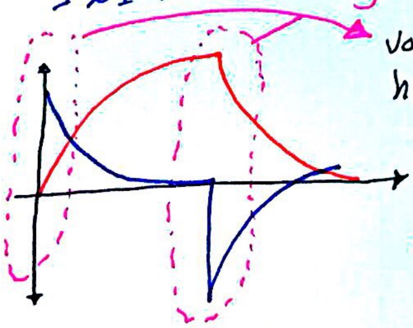
  * Hard Decision: we only decide @ the R_x according to one sample.

  * for multiple samples it called "Soft Decision".

- First requirement for wire communication is the average value "DC value" to be zero.
- 1's & 0's are "equally Likely", which means that optimal solution when the distribution is equaled.

- Note: Miss-synchronization will cause an effect on the "ISI".

ISI ≡ Inter Symbol Interference.



voltage & current here are NOT Orthogonal.

- Note: smooth signal is the signal that has less freqnancy content.

* Power loss in magnetic material is related to f^2.

* We reduce the voltage $V\downarrow$ To reduce the power dissapation inside the digital circuits.

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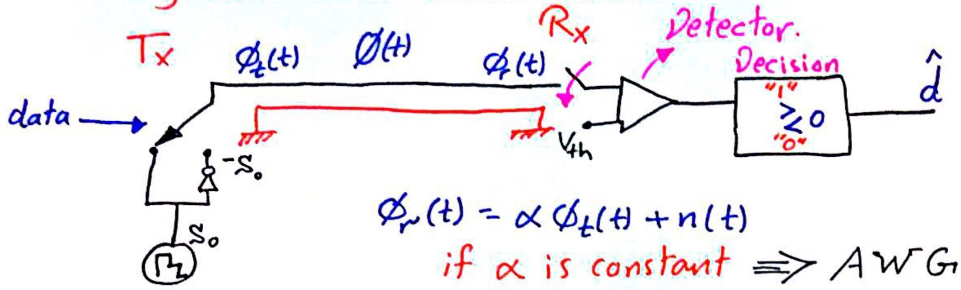
※ Binary Base-band Communication:



$$\phi_r(t) = \alpha\, \phi_t(t) + n(t)$$

if α is constant $\Rightarrow$ AWGN.

if α is R.V $\Rightarrow$ Fading.

- *Note*:

  Available BW has No relation with the used BW.

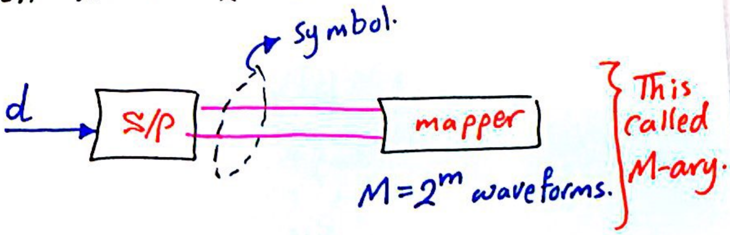
Ex. in Cellular: Available (25 KHZ) $\Rightarrow$ used (200 KHZ) "$\frac{1}{8}$ of the time period."

- *Note*: We must have energy to do communication. "To transfer information from Tx to Rx".

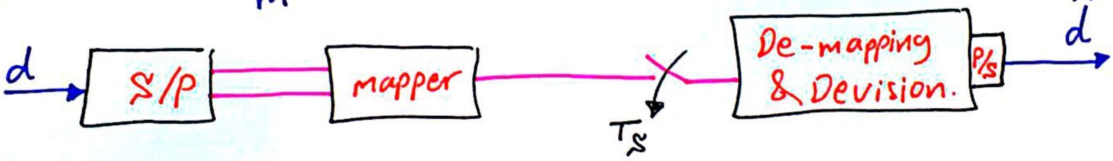
* For Non-Binary:

for 2 bits: $T_s = 2\, T_b$

for m bits: $T_s = m\, T_b$

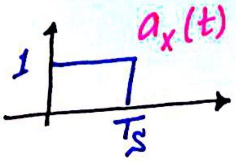


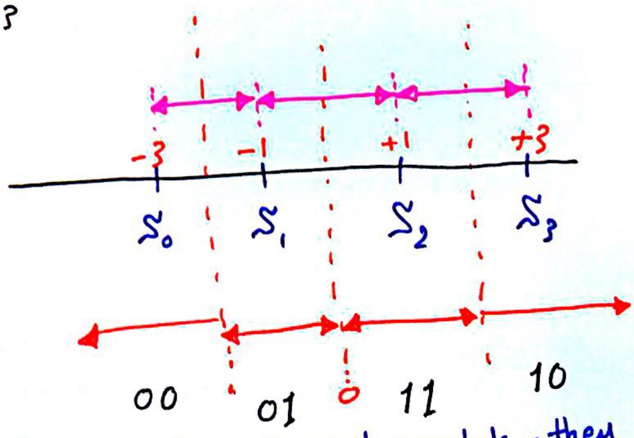
$$BW \propto r_s = \frac{r_b}{m}$$



* Consider the following: S_0 S_1 S_2 S_3

$\Rightarrow$ Take at first one axis:





- We need to maximize the distance with minimum Energy.

- *Note*: every two adjacent symbols, they differ only in one bit. "Gray Code".

Energy here is equal to:

$E_{av} = 5$.

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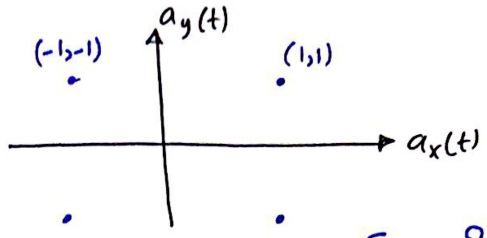
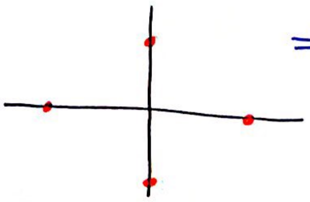
* Gray Code:

0	0	0
0	0	1
0	1	1
0	1	0
1	1	0
1	1	1
1	0	1
1	0	0

* Consider two axis:



$E_{av} = 2$



⇒ This is better to use due to easier detection.

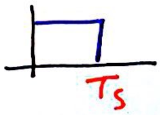
we can send one axis at a time

Example:

Consider M-ary with $m = 4$ bits.

always: $BW \propto r_s$

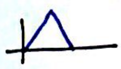
$\boxed{T_s = m\,T_b}$; $\boxed{r_s = \frac{r_b}{m}}$

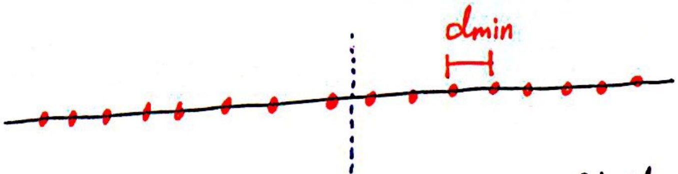


increasing # of bits ⇒ interval ↑ ⇒ BW ↓

• **For 1-D:**

$a_x(t)$





* **Note:** PSK (Phase shift Keying) vs. ASK (Amplitude shift Keying).

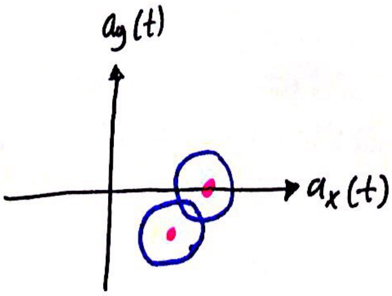
⇒ PSK is better in immunity to Fading.

* Fading occur when distructive interference occur for signals.

• **For 2-D:**

$a_x(t)$

$a_y(t)$



⇒ "Sphere Pack Problem"

it is a mathematical problem.

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* Criteria: Minimize the Average Energy @ Maximum minimum distance.

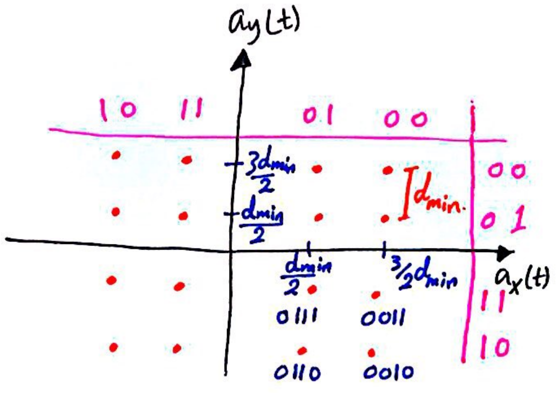


Fig. 1

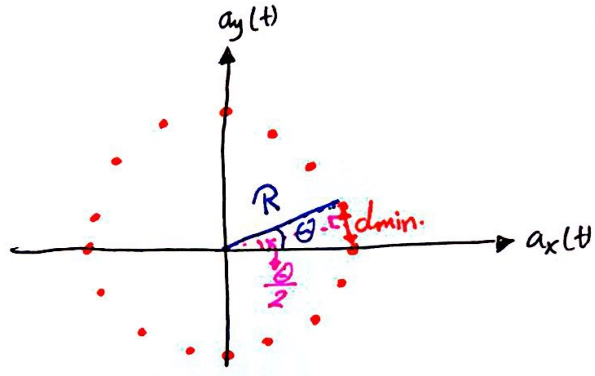


Fig. 2

⇒ for Fig. 2:

$E_{av.} = R^2 \cdot T_s$

$d_{min} = 2R\sin\left(\frac{\theta}{2}\right)$; $\theta = \frac{2\pi}{M}$ ⇒ $d_{min} = 2R\sin\left(\frac{\pi}{M}\right)$ ⇒ $R = \frac{d_{min}}{2\sin\left(\frac{\pi}{M}\right)}$

$$\Rightarrow \boxed{E_{av.} = \frac{d_{min}^2 T_s}{4\sin^2\left(\frac{\pi}{M}\right)}} \quad \cdots (1)$$

⇒ for Fig. 1:

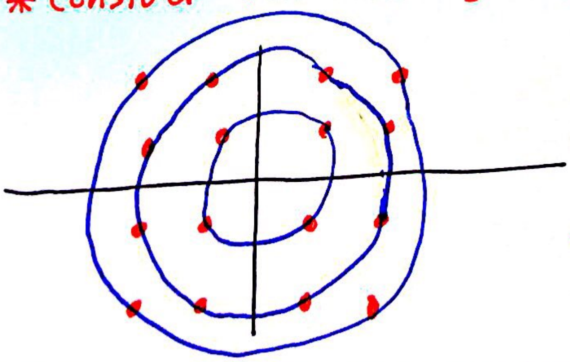
$$E_{av.} = \frac{1}{4}\left(\frac{d_{min}^2}{2} + \frac{9d_{min}^2}{2} + 2 \times \frac{10 d_{min}^2}{4}\right) \Rightarrow \boxed{E_{av.} = \frac{20}{8} d_{min}^2 T_s} \cdots (2)$$

Evalnating (1) & (2) using $M = 2^4 = 16$:

$E_{av._{Fig.2}} = 6.57\, d_{min}^2 T_s$

$E_{av._{Fig.1}} = 2.5\, d_{min}^2 T_s$

* Consider the following representation:



⇒ This Called APSK.

* PAPR:

PAPR ≡ Peak-to-Average Power Ratio.

⇒ for previous Example:

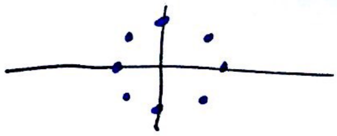
Fig. 1: $PAPR = \frac{9/2\, d^2_{min} T_s}{2.5\, d^2_{min} T_s} = \frac{9}{5}$

Fig. 2: $PAPR = 1$ ⇒ optimal case.

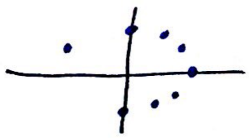
- Always Peak power ≥ Average Power, so PAPR = 1 is the optimal case, we need it to be small as possible.

* Note: usually we calculate for the worst case.

* Consider the following 2-Dim.:

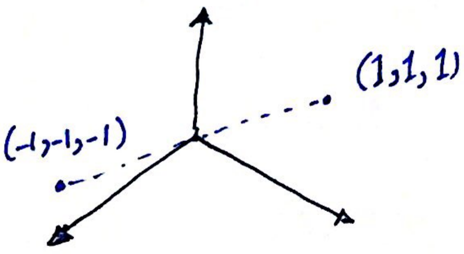


This is a regular representation.



This is called UEP ≡ Unequal Error Protection.

- 3-Dim.:



if we add $X \sim N(\mu_1, \sigma_1^2)$
$Y \sim N(\mu_2, \sigma_2^2)$

they give: ⇒ $r \angle \theta$

where $r = \sqrt{x^2 + y^2}$

This (r) called:

- Reyleigh: "if all μ's are zeros". "worst case". or "when the points centered @ zero".
- Rician: "if one of μ's is zero" or "when the points centered around zero."

* In Multi-dimension system we can't use grey code since the purpose of the grey code is to have two adjacent symbols differ only in one bit. and we don't have this in multi-dim. system. Just used for 1-Dim or 2-Dim. systems.

* How to generate (MD) "Multiple Dimension" Modulation:

using N-axis ⇒ N-Dim.

※ Some Types of Transformation:

$$\int x(t) \cdot y(t)\, dt = \vec{X}[n] \cdot W$$

↳ it is a Transformation Matrix.

* DFT "Discrete Fourier Transform":

$e^{-jn\omega t}$ $\qquad W = \frac{1}{\sqrt{N}} e^{-jnm\frac{2\pi}{N}}$ $\qquad$ where $n, m = 0, \cdots, \infty$

* LT "Laplace Transform":

e^{st} $\qquad W = \frac{1}{\sqrt{N}} e^{st}$

* DCT "Discrete Cosine Transform":

$\cos(n\omega_0 t)$ $\qquad W = \frac{1}{\sqrt{N}} \cos\left(\frac{n\omega 2\pi}{N}\right)$

* DST "Discrete Sine Transform":

$\sin(n\omega_0 t)$ $\qquad W = \frac{1}{\sqrt{N}} \sin\left(\frac{n\omega 2\pi}{N}\right)$

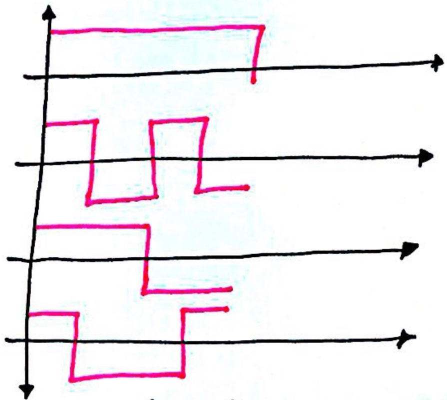
* W-H Transform "Walsh-Hadamard Transform":

it is an example of a generalized class of fourier transforms.

⟹ Continue.

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad H_{2N} = \begin{bmatrix} H_N & H_N \\ H_N & -H_N \end{bmatrix}$$

$$H_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$



* KLT "Karhunen Loeve Transform":

it also has other Names Hotelling Transform/Eigenvector Transform.

$$W^{-1} = W^H = W^{*T}$$

consider a matrix: $A \Rightarrow W = eig(A)$

Eigenvector $\Rightarrow$ orthonormal space.

• covariance Matrix: C

$C \rightarrow$ Rank.

$C \rightarrow$ C.N "condition Number." $\Rightarrow C.N = \dfrac{\lambda_{max}}{\lambda_{min}}$

where λ is the eigenvalue of that matrix.

Proparties of Covariance Matrix:

$\boxed{1}$ it is a full rank matrix.

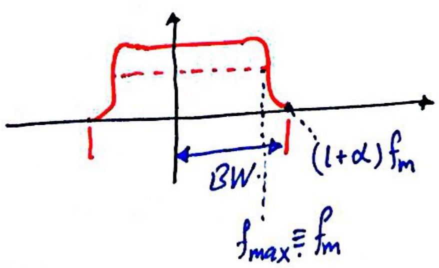
$\boxed{2}$ it is a good conditioning matrix.

$\boxed{3}$ it is positive definite Matrix. (i.e All eigenvalues are +ve and nonzero.)

* Band Limited Base-band Tx:

$\Rightarrow$ only one dimensional signal we can produce.

Raised Cosine Funcition: $\Rightarrow$ $P(t) = \dfrac{\text{sinc}(t/\tau)\ \cos\left(\frac{\pi\alpha t}{\tau}\right)}{1-\left(\frac{2\alpha t}{\tau}\right)^2}$ pulse.



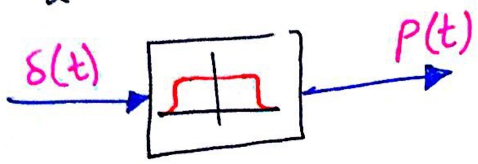
$\alpha \equiv$ Roll-off Factor.

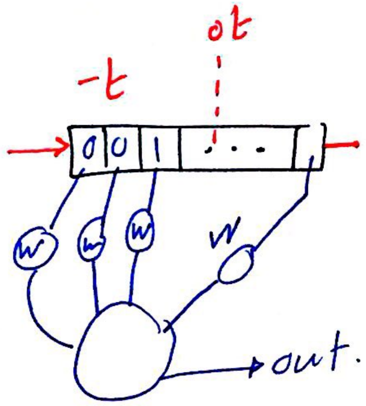
α is a parameter.

* impulse response: $\delta(t) \rightarrow \boxed{H(f)} \rightarrow h(t)$

• Note: RC stand for Raised Cosine.

* RC_α - Filter:

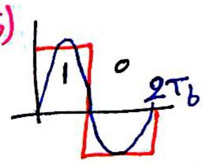




* the relation between Bandwidth & the Bitrate : $BW = \frac{r_b}{2}(1+\alpha)$ → this for Binary.

$BW = \frac{r_s}{2}(1+\alpha) = \frac{r_b}{2m}(1+\alpha)$ → this is for M-ary.

• Note: Typical value of $\alpha = (0.12 - 0.25)$
• smaller α: better utilization for the BW.
• Bigger α: easier to design the system filter.



$f_m = \frac{r_b}{2}$

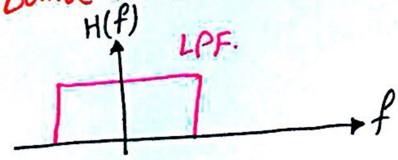
• Example: $BW = 10KHZ$
$r_b = 100K$ bps.
$\alpha = ?$

$10K = \frac{100K}{2m}(1+\alpha)$ $m = 6 \rightarrow \frac{100}{12}(1+\alpha) = 10$

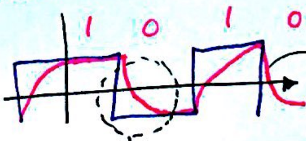
$\alpha = 0.2$

... Continue

* Band limited Base-band TX:



* consider the following square wave: → $BW = \infty$



→ this is smoothing for the signal.

→ This called ISI ≡ Inter symbol Interference.

• Central Limit Theorem:

$V_{ISI} = \sum_{\infty} V_i = N(\sigma^2_{ISI}, \mu_{ISI})$ → R.V.

• Base band:

1D ⇒ raised cosine $(RC)_\alpha$.

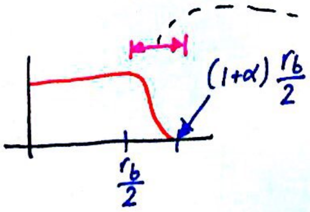
$BW = \frac{r_b}{2}(1+\alpha)$; for Binary.

$BW = \frac{r_s}{2}(1+\alpha)$; for M-ary.

$= \frac{r_b}{2m}(1+\alpha)$ ⇒ m & α are two degrees of freedom.

→ m: for size of the constellation diagram (2^m)

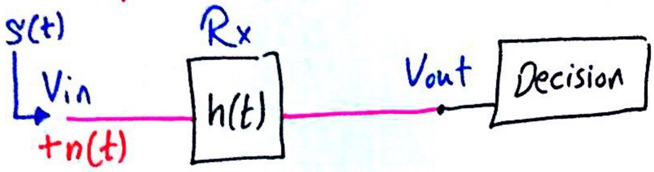
→ α: Complexity of designing the filter; it is easier for bigger α.



→ for Shorter period freq. (smaller α) you will need more number of components to Build the corresponding filter. → (more complex)

rolloff factor ≡ High to low.

* Optimal Receiver:



. optimal R_x : find the optimal $h(t)$ such that SNR_{out} is MAX.

$$V_{out} = V_{in} \star h(t) = \int s(t)\, h^{*}(\tau - t)\, dt$$

$$SNR_{out} = \frac{\text{Signal Power}}{\text{Noise Power.}} \Rightarrow \left[\begin{aligned} E_{out} &= \iint |s(t)\, h^{*}(\tau-t)|^2\, dt\, d\tau \\ E_{noise} &= \sigma_n^2 \int_{BW} |H(f)|^2 df \\ &= \sigma_n^2 \int |h(t)|^2 dt . \end{aligned} \right]$$

$$\Rightarrow SNR_{out} = \frac{\iint |s(\tau - t)\, h(t)|^2\, dt\, d\tau}{\sigma_n^2 \int |h(t)|^2\, dt}$$

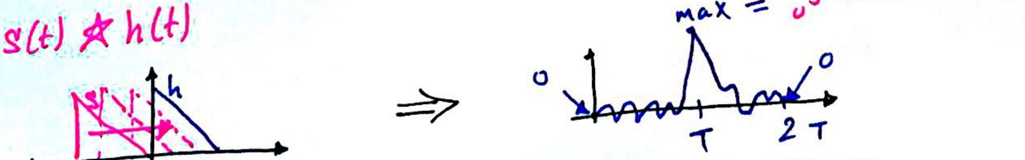
. remember the following: $|V_1 \cdot V_2|^2 \leq |V_1|^2 \cdot |V_2|^2$; called: "Cauchy-Schwarz" Inequality:

so SNR_{out} will be Maximum when: $\boxed{h(t) = s^{*}(T - t)}$

This called: Matched Filter.

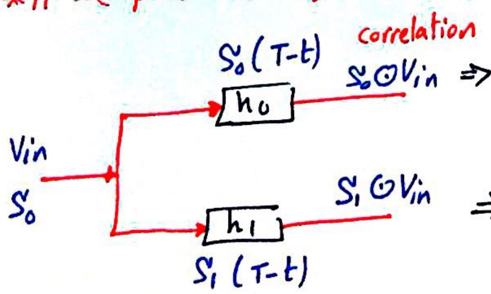
Example:

$\Rightarrow$ $s(t) \star h(t)$



max occur @ T.

*if we pass the signal through the following:



$S_0 \odot V_{in} \Rightarrow$ E_0 if $V_{in} = S_0$; $\rho_{01} \sqrt{E_0} \cdot \sqrt{E_1}$ if $V_{in} = S_1$

$S_1 \odot V_{in} \Rightarrow$ E_1 if $V_{in} = S_1$; $\rho_{01} \sqrt{E_0} \sqrt{E_1}$ if $V_{in} = S_0$

. remember:

$$\rho_{01} = \frac{\langle S_0, S_1 \rangle}{\sqrt{E_0}\sqrt{E_1}}$$

$\Longrightarrow$

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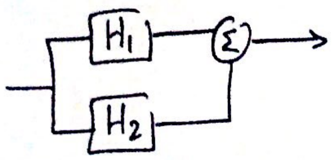
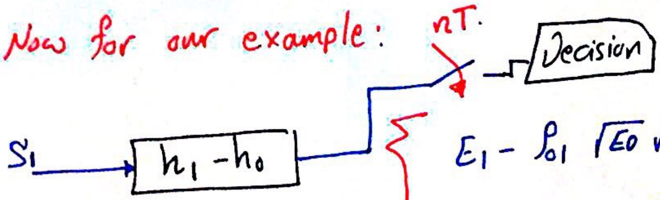
* 

$\Rightarrow S_1 \cdot (H_1 + H_2)$ in freq. domain.

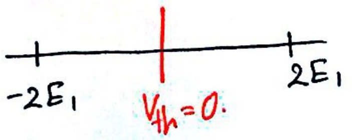
$S_1 \star (h_1 + h_2)$ in Time domain.

Now for our example:



$$\begin{cases} E_1 - \rho_{01}\sqrt{E_0}\sqrt{E_1}, & \text{when } S_1 \text{ is received.} \\ \rho_{01}\sqrt{E_0}\sqrt{E_1} - E_0, & \text{when } S_0 \text{ is received.} \end{cases}$$

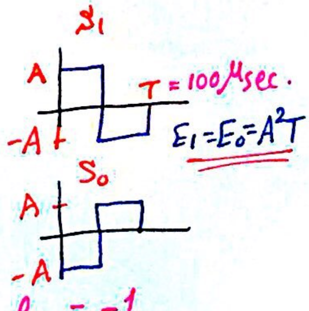
when $S_0 = -S_1$: $\begin{cases} 2E_1, & \text{when } S_1 \text{ is received.} \\ -2E_1, & \text{when } S_0 \text{ is received.} \end{cases}$



$\Rightarrow$ in Voltage.

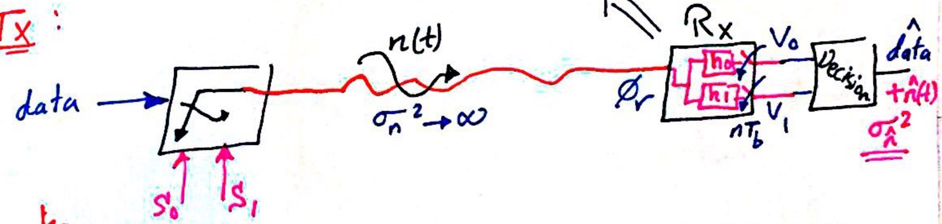
Example: a 10Kbps data is to be transmitted over a wire comm. channel is baseband. Design the Tx & optimal Receiver.

* Need 2 filters, one for S_0 & one for S_1.



$\rho_{01} = -1$

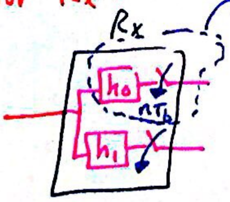
Tx:



Note: Matched filter is a coherent Receiver.

Coherent Rx ≡ Synchronous Rx.

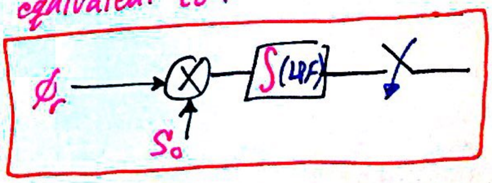
for Rx:



period of T $\quad V_0 = \int_0^T \phi_r(t)\, S_0(t)\, dt$

↳ This is equivalent to:

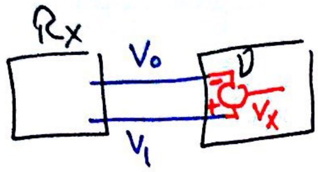
** Correlator Rx (coherent Rx)



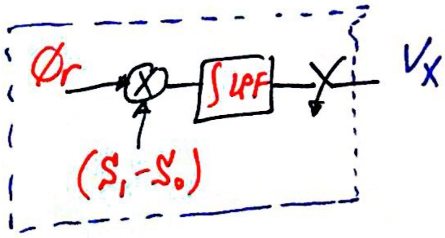
⇓

- implementation of the matched filter at the end of T. (integral period).

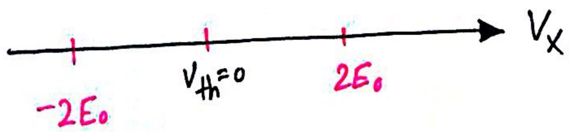
⇒ if we do this @ the Decision:



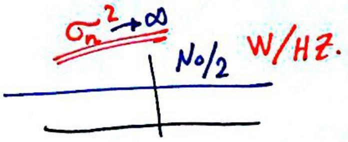
$$V_x = \begin{cases} 2E_0 , & \text{"1"} \\ \cdots V_{th}=0 \cdots & \\ -2E_0 , & \text{"0"} \end{cases}$$



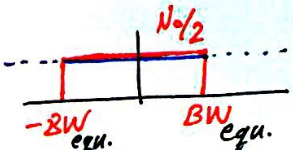
↓ it will simplify the Rx so that it will integrate once.



· **Note**: the receiving filter has limited the Noise



⇒



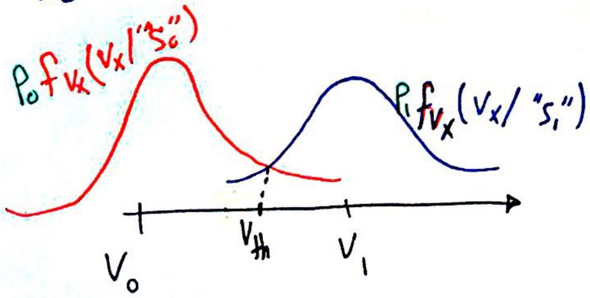
$\sigma_{\hat{n}}^2 = BW_{equ} \cdot N_0$

$\sigma_n^2 / \sigma_{\hat{n}}^2 \equiv$ Total power Before/After the filter.

$BW_{equ} \equiv$ Noise Equivalent BW.

***** Square Integrable Functions:**

it is Power/Energy function, if we integrate over the square of this function, Then it would be Bounded.



$$\boxed{f_{V_x/S_i} = \frac{1}{\sqrt{2\pi\sigma_{\hat{n}}^2}} e^{\frac{-1}{2\sigma_{\hat{n}}^2}(V_x - V_i)^2}} \quad ; \ i = 0, 1$$

conditional Pdf.

*** Error Event:**

Bit Error Rate ≡ BER

⇒ $BER = P_r(1/0) \cdot P(0) + P_r(0/1) \cdot P(1)$

$P_r(1/0)$ ↳ probability of receiving one given that 0 is transmitted.

for 1 M bit file

we need: $BER \leqslant 0.1 * \frac{1}{size} = 10^{-7}$

.where:

$$\Rightarrow P_r(1/0) = \int_{V_{th}}^{\infty} f_{V_x}(V_x/0)\, dV_x$$

$$\Rightarrow P_r(0/1) = \int_{-\infty}^{V_{th}} f_{V_x}(V_x/1)\, dV_x .$$

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- Case I : Antipodal Equally Likely.

$P(0) = P(1) = \frac{1}{2}$

$P(0/1) = Pr(1/0)$

• remember: $Q(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{\alpha}^{\infty} e^{-t^2/2} \, dt$

$V_0 = -2E$

$V_1 = 2E$

$\Rightarrow BER = \int_{V_{th}}^{\infty} f_{V_x}(V_x/0) \, dV_x \; ; \; V_{th} = P(0) \cdot V_0 + P(1) \cdot V_1$

$\Rightarrow$ in our example we have $V_{th} = 0$.

$$BER = \int_0^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma_n} e^{\frac{-1}{2\sigma_n^2}(V+V_0)^2} \, dV \qquad t = \frac{V+V_0}{\sigma_n} \Rightarrow dt = \frac{dV}{\sigma_n}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{\frac{V_0}{\sigma_n}}^{\infty} e^{-t^2/2} \, dt = Q\left(\frac{V_0}{\sigma_n}\right) = Q\left(\sqrt{\frac{4E^2}{N_0 \cdot BW \cdot E}}\right)$$

$= \frac{r_b}{2}(1+\alpha)$

$r_b = \frac{1}{T_b}$

$\sigma_n^2 = N_0 \cdot BW$

$\sqrt{E}$

$\Rightarrow n(t) * h(t) = n(f) \cdot H(f)$

gain due to match filter.

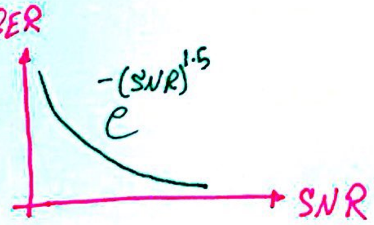
$\Downarrow$

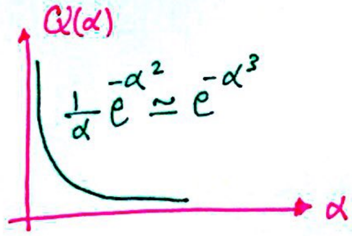
$\sqrt{E}$ represent the contribution of $H(f)$.

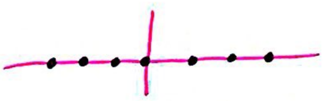
$$\boxed{BER = Q\left(\sqrt{\frac{8E\,T_b}{N_0(1+\alpha)}}\right)}$$

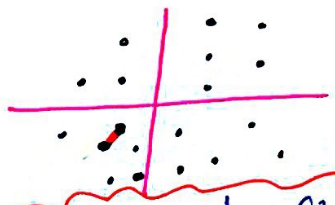
$$BER \propto Q\left(\sqrt{\frac{SNR}{r_b}}\right)$$

$\hookrightarrow$ so there are 2-options to enhance the Quality of Transmission (BER) :
we either increase SNR or reduce r_b.









$K_1 = \frac{d_{min}}{2}$

$$\boxed{BER = \frac{1}{2\log_2 M} Q\left(\sqrt{\frac{d_{min}^2}{4\sigma_n^2}}\right)}$$

$\rightarrow$ for Coherent Rx & Gray Coded.

* the only solution for enhance the current & future digital comm. systems is using ECC (Error Control Coding).

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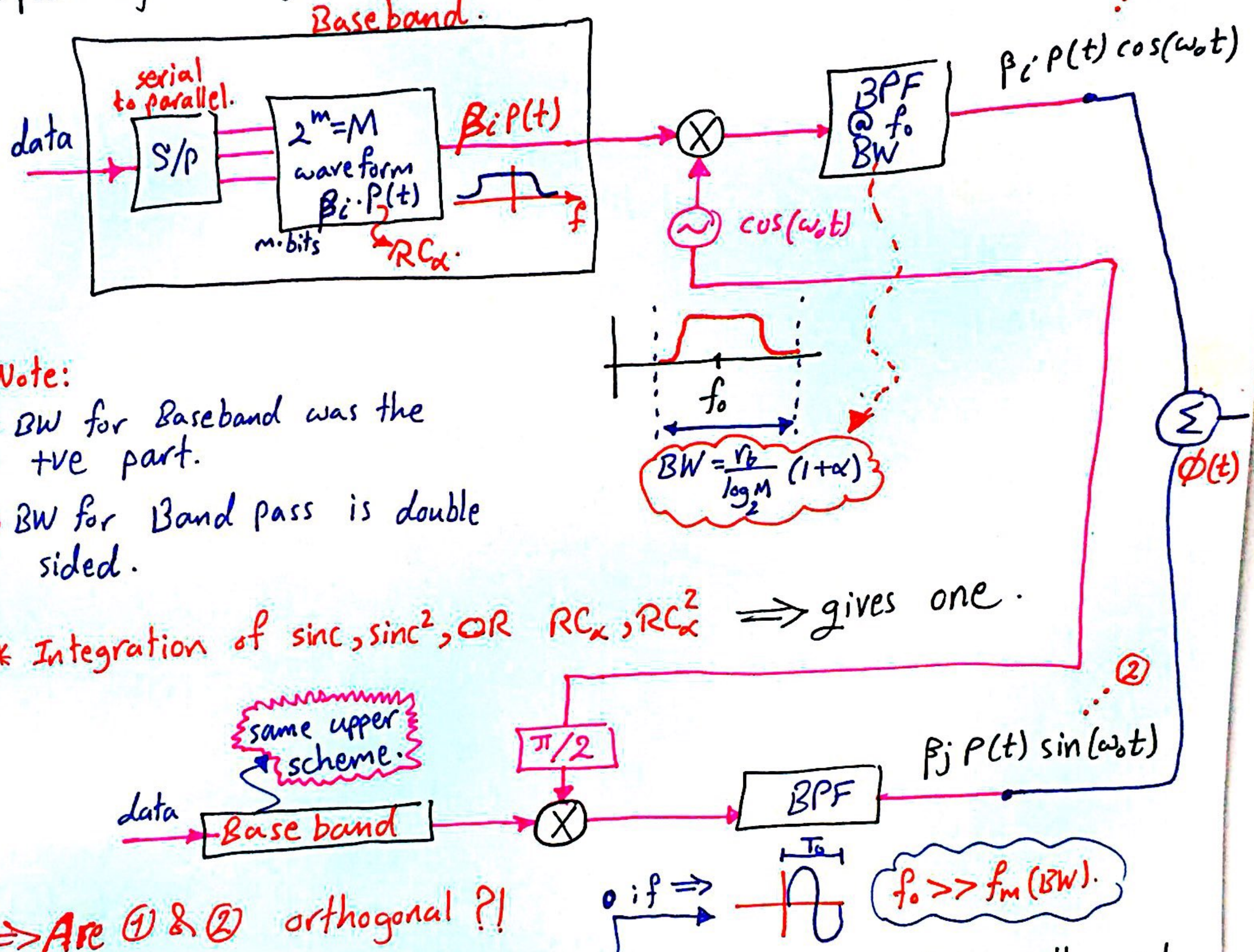
$$BER \simeq \frac{1}{2\log_2 M} e^{\frac{-d_{min}}{8\sigma_n^2}} \rightarrow \text{for Non-Coherent Rx.}$$

* Band Pass:

- previously the only choice for Baseband was using 1-D.



- Note:
  - BW for Baseband was the +ve part.
  - BW for Band pass is double sided.

* Integration of sinc, $sinc^2$, OR RC_α, RC_α^2 $\Rightarrow$ gives one.

$\Rightarrow$ Are ① & ② orthogonal ?!

- if $\Rightarrow$ $\overset{T_o}{\longleftrightarrow}$ $f_o >> f_m (BW)$.

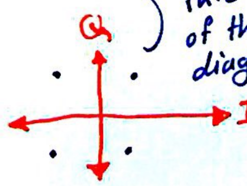
$$\int \underbrace{\beta_i \beta_j |P(t)|^2}_{\text{if this constant}} \cos(\omega_o t) \sin(\omega_o t)\, dt = 0 \quad \text{Then.}$$

so they will be orthogonal for $1\,T_o$ (within $1\,T_o$).

Since they are orthogonal, Now we can add them to eachother.

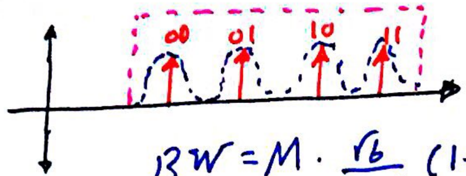
- using 1 cosine or 1 sine This is 1-D.
- Using sine & cosine This is 2-D
- Using more than that. This is Multi-D.

* resulted $\phi(t) = \underbrace{\beta_i P(t)\cos(\omega_o t)}_{I} + \underbrace{\beta_j P(t)\sin(\omega_o t)}_{Q}$



the distribution of the points rule the name of the constellation diagram.

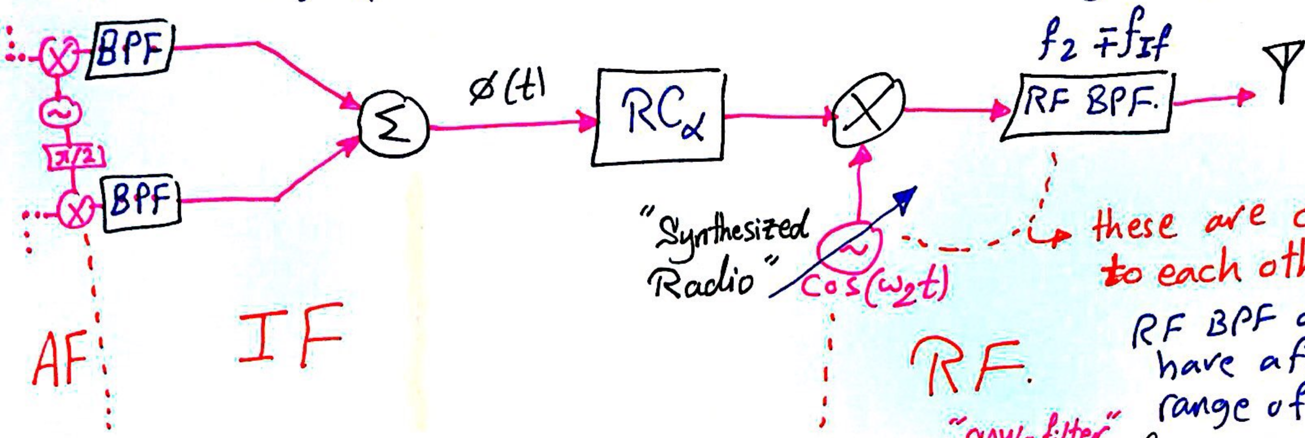
* for FSK:



$$BW = M \cdot \frac{r_b}{\log_2 M}(1+\alpha)$$

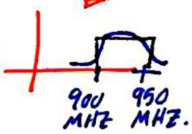
here No need for the Gray Code.
Because the distances are equaled between any two points.

replace each ω_0 by ω_{If}



RF BPF will have a fixed range of BW.
for ex. 900-950 MHZ.
& we change in the $\cos\omega_2 t$.

"SAW-filter"



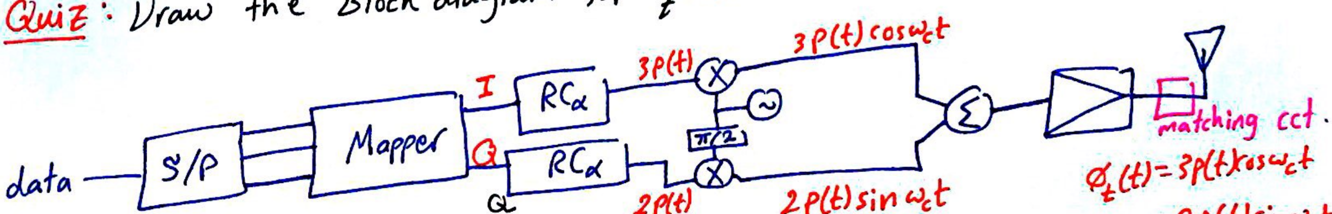
AF ≡ Audio Frequency Stage.
IF ≡ Intermediate Freq. stage.
RF ≡ Radio Freq. stage.

* RF BPF could be:
- Tunable filter (expensive solution).
- SAW-Filter ≡ Surface Acoustic Wave Filter. (cheaper solution).

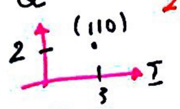
for 1-D & 2-D:
* Bandwidth Efficiency $= \log_2 M$ (bit/S/HZ). $\equiv \frac{\text{data rate}}{BW}$.

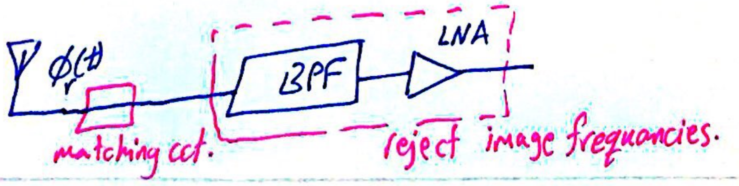
↳ the most important feature we look @ it.

· Quiz: Draw the Block diagram for quadrature modulator with m=3bits.



$\phi_t(t) = 3p(t)\cos\omega_c t + 2p(t)\sin\omega_c t$





Between Rx & Tx:
$2 \le \gamma \le 4 \Rightarrow r^{-\gamma}$
Ex. $\gamma = 3$, $(5Km)^{-3} \times 10W = 80pW$
for voltage $\sqrt{80\times10^{-12}} \simeq 9\mu V$.

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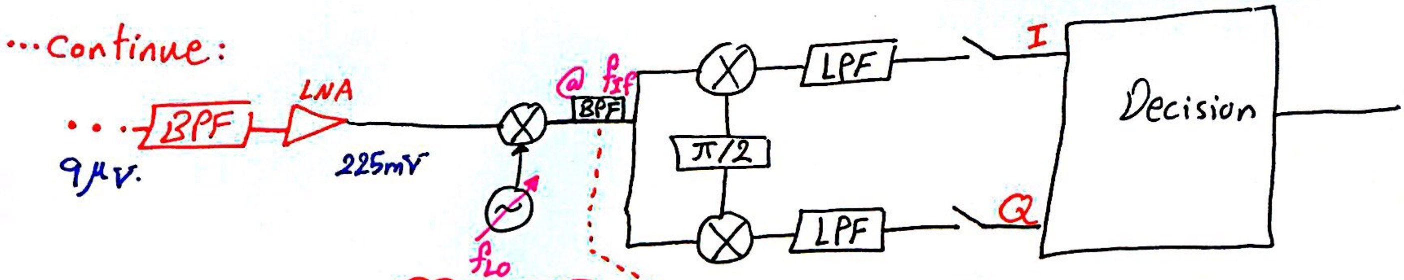
* Characteristics of LNA:

(1) Low Currents. (2) Fits Technology. (3) Zero Q-point.
(4) Linearity. (5) Rail-to-Rail output voltage. ⇒ i.e output voltage will swing between $+V_{cc}$ & $-V_{cc}$.

...continue:



$f_{IF} = f_{LO} - f_c$

this BPF work as a Limiter (Limit the SNR).

$$\phi_i(t) = K\left(3\,p(t)\cos\omega_c t + 2p(t)\sin\omega_c t + n(t)\right)$$

$n_I + n_Q$

usually it is a ceramic Filter.

the (225mV) calculated as follows:

consider $R = 50\Omega$ & 0dBm

0dBm ⇒ 1mW $\quad P = \frac{V^2}{R} \Rightarrow V = \sqrt{1\times10^{-3}\times 50} = 0.2236 \simeq 225\text{ mV}.$

for the gain of LNA: $\text{Gain} = \frac{225\text{m}}{9\mu} = 25\text{K} \equiv (88\text{dB}).$

* * *

End of Mid Material.

* * *

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Communications II

Second Semester
2018

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Abu Hashya.

* * *

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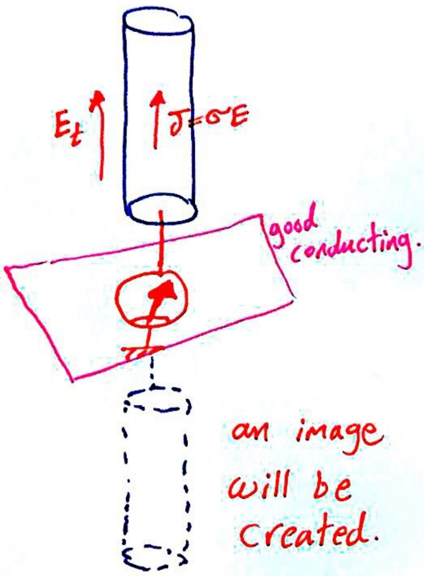
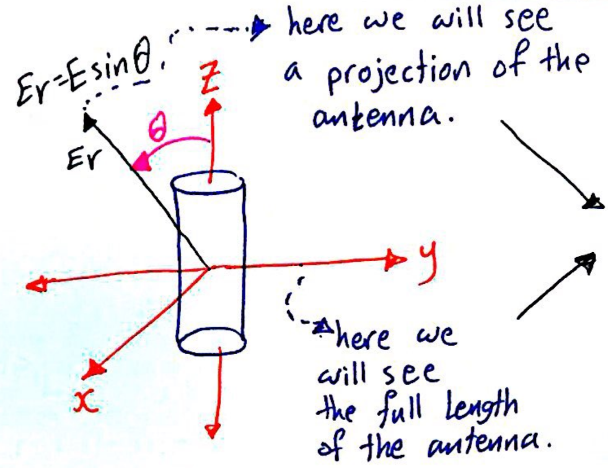
"Lecture 4/4/2018"

※ Antenna:

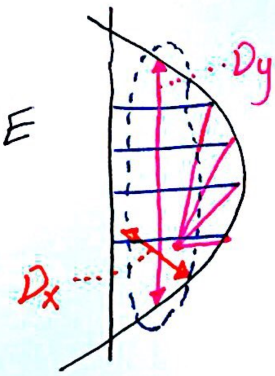




it is a function of space & it is called:
Voltage/Electric Field Radiation Pattern

$$\boxed{E_r = E\sin\theta} \begin{cases} \text{min: } \theta = 0 \\ \text{max: } \theta = 90^\circ \end{cases}$$

* Radiation Pattern:
it is a characteristic of the Antenna & it descibe for the antenna how it is transmit & how it receive.

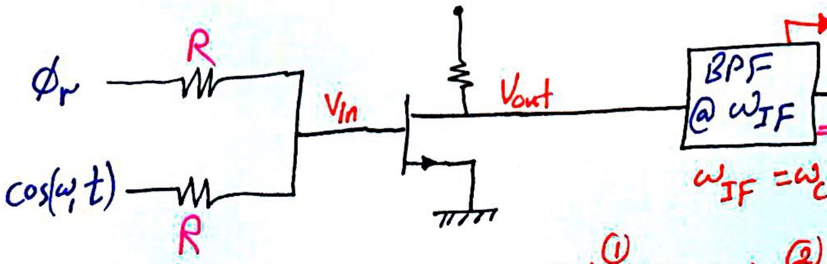


$$\Rightarrow G \propto \frac{D_x D_y f^2}{(22)^2}$$

$$\lambda = c/f \Rightarrow \boxed{G = \frac{D_x D_y C^2}{\lambda^2 \cdot (22)^2}}$$

※ Mixer:

"it is a non-linear device passes the signal."



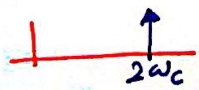
$BW = \frac{r_b}{\log_2 M}(1+\alpha)$

it limits the SNR.

$\omega_{IF} = \omega_c - \omega_1$

$$V_{out} = K V_{in}^2 = K(\phi_r + \cos\omega_1 t)^2 = \underbrace{K\phi_r^2}_{①} + \underbrace{2K\phi_r\cos\omega_1 t}_{②} + \underbrace{K\cos^2\omega_1 t}_{③}$$

① $\Rightarrow$ since ϕ_r is squared, it is distorted.



② $\Rightarrow$ $A_i P(t)\cos(\omega_c t) + B_i P(t)\sin(\omega_c t)$ which could be written as: $W_i P(t)\cos(\omega_c t + \theta_i)$

$\omega_c \pm \omega_1$

③ $\Rightarrow$ $\frac{K}{2}(1 + \cos(2\omega_1 t))$

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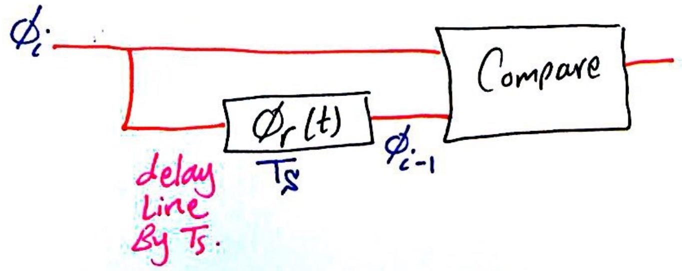
⇒ the output of the BPF will be:

$$2KW_i\,p(t)\cos(\omega_{IF}t+\theta_i)$$

* Differential Transmission:

Ex. ASK. Mapper

data	$\Delta\phi=\phi_i-\phi_{i-1}$
0	Δ Amplitude $=+10$
1	Δ Amp. $=-10$



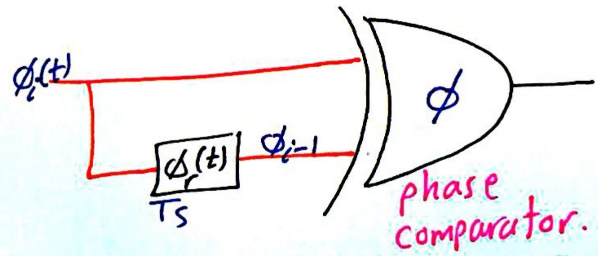
** Differential Phase Shift Keying (DPSK):

$$\phi(t)=A\,p(t)\cos(\omega_c t+\theta_i)$$

Mapper

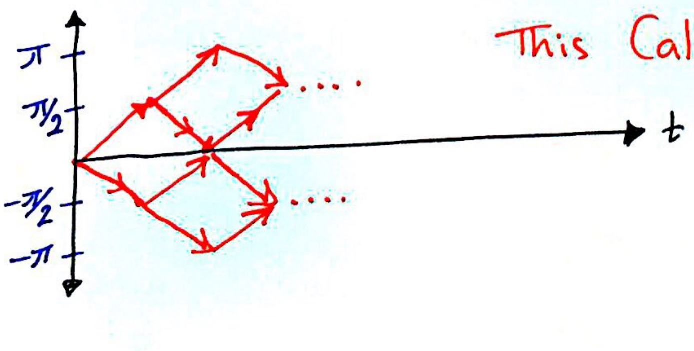
data	$\Delta\theta$
0	$-\pi/2$
1	$\pi/2$

This Called: "MSK" Minimum Shift Keying.



• This is Differentially Coherent Rx

* Possible phases:



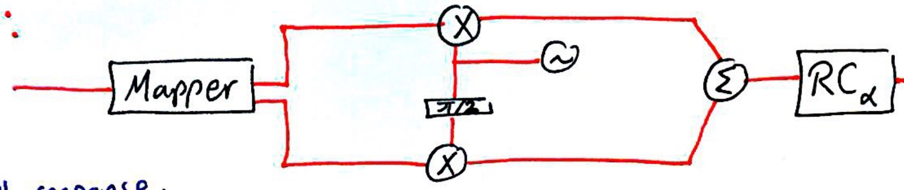
This Called: Tree Diagram.
Phase Tree.
Phase Trellis.

• **Note**:

Memory Machine: Any memory that Keep the previous information & change the output depending on the previous information & current information input.

... Continue (DPSK):

※ Tx:

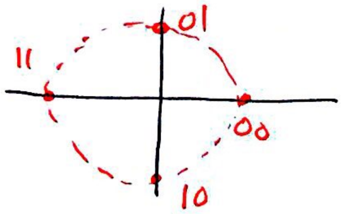


$\phi(t) = A\,p(t)\cos(\omega t + \theta_t)$

BW ∝ $f_m + \Delta f$

$\dot{\theta}$ (first) derivative

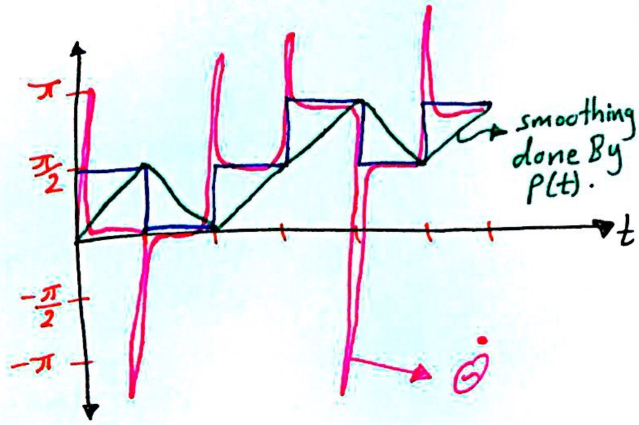
Full response.

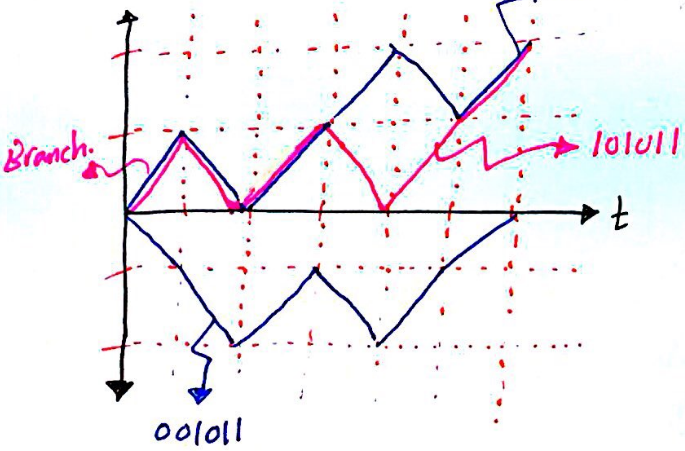


• MSK:

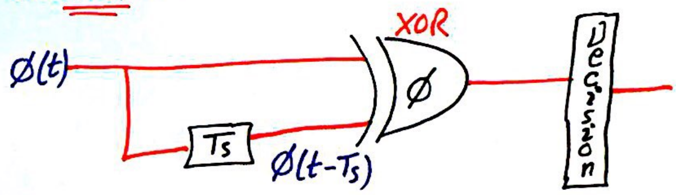
d	$\Delta\theta$
0	$-\pi/2$
1	$\pi/2$

101101
001011
101011





※ Rx:



001011
101011 } one bit difference.

• Hamming distance = 1 Bit.

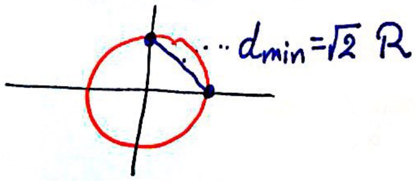
• **Note**: DPSK also called → Multi-h Modulation.
→ CPFSK ≡ Continuous Phase Frequancy Shift Keying.

* π/4 QDPSK:

d	$\Delta\theta$
00	$\pi/4$
01	$3\pi/4$
10	$-\pi/4$
11	$-3\pi/4$

• **Note**: d_{min} found @ $\min(\Delta(\Delta\theta))$.

in this example: d_{min} @ $\pi/2$.



for MSK: d_{min} @ π ⇒ $d_{min} = 2R$.

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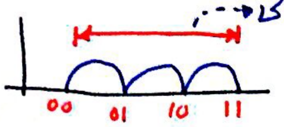
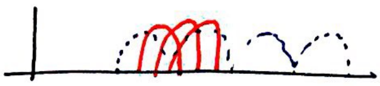
※FSK :

$BW = M \frac{r_b}{\log_2 M}(1+\alpha)$.

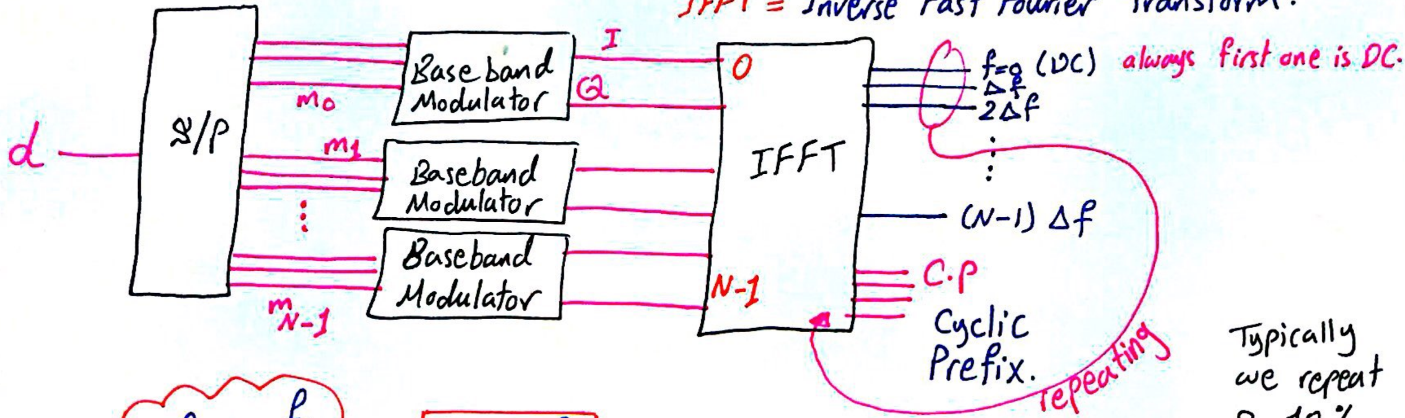


$\Rightarrow$



Becomes OFDM

IFFT ≡ Inverse Fast Fourier Transform.



Typically we repeat 8-10%.

$\Delta f = \frac{f_s}{N}$, $\boxed{BW = f_s}$

similar to α.

$BW = (N + C.P)\frac{f_s}{N} = 2 f_m \left(1 + \frac{C.P}{N}\right)$

• **Note**:

Due to Doppler Effect it cause ICI ≡ Inter Carrier Interference.

solved by using guardbands & mangement for the used frequancies.

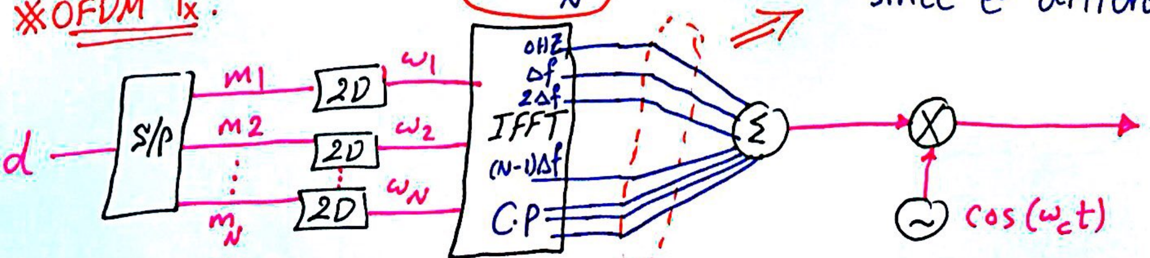
⇒ "LTE systems" ≡ Long Term Evolution Systems.

"Lecture 16/4/2018".

※OFDM Tx :

$\Delta f = \frac{f_s}{N}$

outputs are orthogonal since @ different frequancies.



* number of bits @ the input $= m_{Total} = \sum_{i=1}^{N} m_i$

the BW required is:

$$\boxed{\begin{aligned} BW &= \Delta f\,(N + C.P) \\ &= f_s\left(1 + \frac{C.P}{N}\right) \end{aligned}}$$

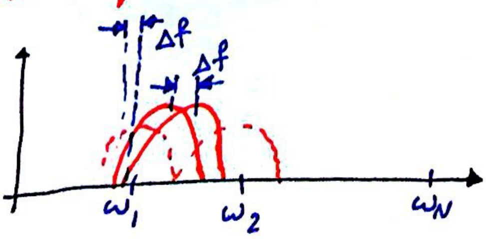
fastest signal @ input.

$f_s = N \times \frac{1}{T_{symbol}}$

$T_{symbol} = 2T_b$ ($m_i = 2$)

$= m\,T_b$

• in freq. domain:



* C.P is added To remove the ICI (Aliasing).

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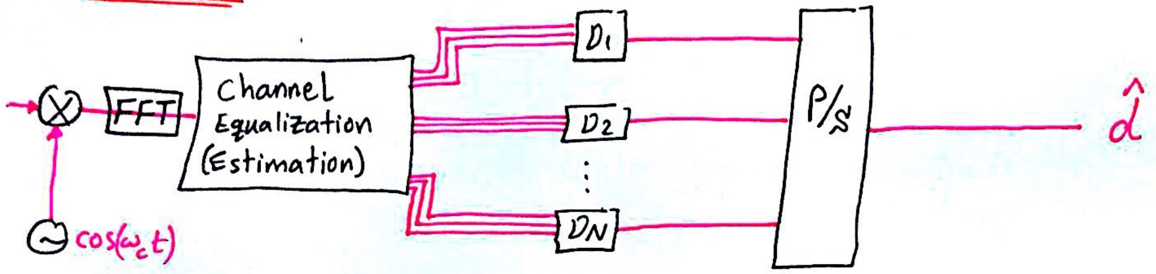
*OFDM R_x :



* Water Filling Algorithm: To have all levels equaled @ the R_x.

* we will call the carrier "pilot carrier" ; it is the reference point send with the data.

  usually we send one pilot every 32 or 64 carriers.

* Some of the sub carriers could have Nulling ≡ Deep Fading in this case we can't send data over these sub carriers.

* The OFDM used To Convert Freq. Selective Fading into Freq. Non Selective Fading.

• SS (Spread Spectrum) → FH (Frequancy Hopping).
  → DS (Direct Sequance).

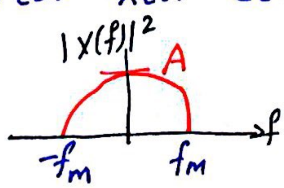
⇒ HF mainly used for: Military Applications & communication security.

* Direct Sequance:

Sequance ≡ $C(t)$ ≡ PN ; $\in \{\overset{\text{"1"}}{+1}, \overset{\text{"0"}}{-1}\}$

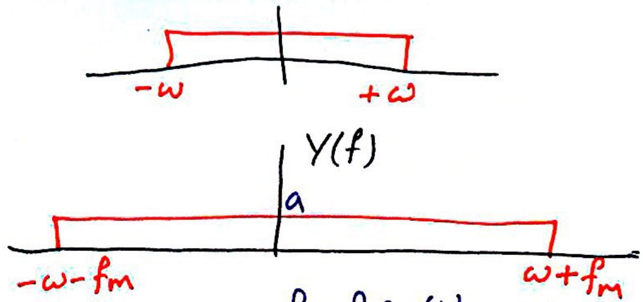
↳ Pseudo Noise.

$Y(t) = X(t) \cdot \underbrace{C(t)}_{\text{like a carrier}}$



$P_s = 2Af_m$

• for $\omega \gg f_m$:



$P_y = 2a\,\omega$

$$\frac{P_s}{P_y} = \frac{2A\,f_m}{2a\,\omega} = 1$$

$$A = \frac{a\,\omega}{f_m}$$

where $\frac{\omega}{f_m} \equiv \underset{\text{Coding Gain}}{C.G} \text{ or } \underset{\text{spreading factor}}{(S.F)}$

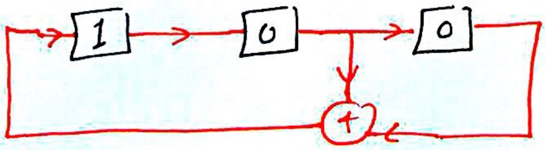
C_i , $i = 1, 2, \dots$

$\langle c_i, c_j \rangle = \delta_{ij}$

1- Hadmard Sequance. $H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

2- m-sequance. ⇒ it is a Pseudo Noise generated By F.B.S.R ≡ Feed Back shif Register.

Ex. $m = 3$



$g(x) = 1 + X + X^3$

1	0	0
0	1	0
1	0	1
1	1	0
1	1	1
0	1	1
0	0	1
1	0	0

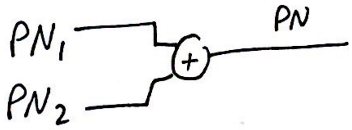
This is the output.

$C_1 = 0010111$
$C_2 = 1001011$
$C_3 = 1100101$
$C_4 = 1110010$

& so on we can generate up to C_7 in this example.

so $N = 2^m - 1$

* Pseudo Noise:

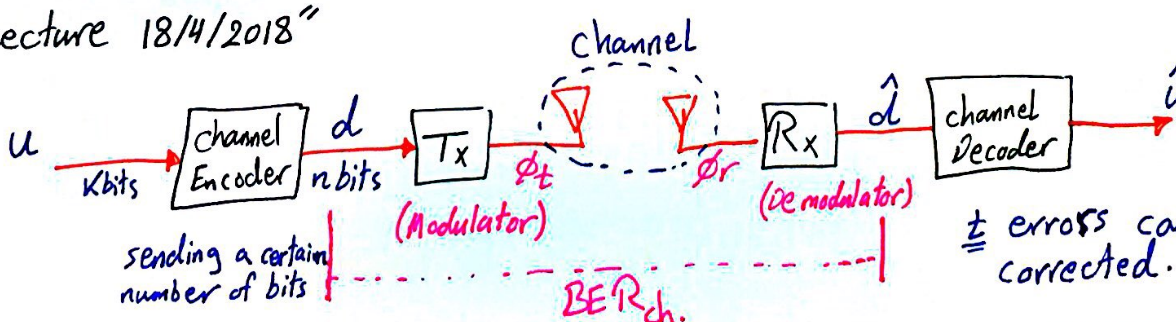


Gold Code.

$g_1(x) = 1 + X + X^3$
$g_2(x) = X^m g_1(x^{-1})$
$= 1 + X^2 + X^3$

- Gold Code: it is a Number.
- Turbo Code: it is a system (Error correction Code).

"Lecture 18/4/2018"



t errors can be corrected.

Ex. sending 3bits, $t = 1$ error will be corrected.

$$BER_{u-\hat{u}} \cong (BER_{ch})^{t+1}$$

↳ This will give a good performance.
since it is a power relation.

Ex. $BER_{ch} = 0.01$, required $BER_{coded} = 10^{-8}$. Find t?

$BER_{coded} = 10^{-8} = (0.01)^{t+1} \Rightarrow \underline{\underline{t = 3}}$

for $BER_{coded} = 10^{-9} \Rightarrow \underline{\underline{t = 4}}$ $\left((BER_{ch})^{t+1} \leq BER_{ch} \right)$

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• Note: * we only use one code to detect or to correct.
* All types of correction can be detected.

$\frac{n}{K} \equiv$ BW Expansion Factor , $\frac{K}{n} \equiv$ Code Rate   n & K are integers.

* ECC (Error Control Code):
  - → EDC (Error Detection Code).
  - → ECC (Error Correction Code). ⟹ FEC.

* Error Correction Code:

I- BLocK Codes.   simpler.
- Hamming Code.
- Cyclic Codes.
- BCH.
- LDPC "Low Density Parity Check Code"

• Note: * Cyclic Code include the Hamming Code
* BCH include the cyclic & the Hamming Codes.

II - Convolutional Codes.
- Regular Convolutional Codes (CC) → Symmetrical / → Non-symmetrical.
  - ↳ TCM "Trellis Coded Modulation".
- Turbo Codes (T.C) ⟹ Best Performance.

III - STC "Space Time Code" ⟹ To give Diversity Gain.

*** Idea of Coding: To creat a deterministic dependancy among transmitted Data stream.

• Fields, Arithmatic operations.

* Fields:
- it is a set of elements or ALphabets. $G = \{a, b, c, \dots\}$
- Two Binary Operations:

[1] Addition "0" (+). $a + b = c \; ; \; a, b, c \in G$

[2] Multiplication "1" (*). $a * b = d \; ; \; a, b, c, d \in G$

$G = \{0, 1, a_1, a_2, \dots\}$

• Min Field:

$G = \{0, 1\}$ "Binary Field"

*** Notes:**

- Any Mathematical operation can be performed in any field.
- All fields are equivalent.
- Addition work as XOR gate $\oplus$.
- Multiplication work as AND Gate.
- All other operations are constructed using these two gates XOR, AND.

*** Extended Field:**

Binary field is a part of Galileo Field.

$GF(2) \longrightarrow$ Binary Field
"Galois Field"

$GF(10) \longrightarrow$ Decimal Field.
$GF(2^m) \longrightarrow$ Extended Field.

Ex. $m=3 \Rightarrow GF(2^3) = GF(8)$
$\hookrightarrow$ But this notation will be used.

*** Extended Binary Fields:**

- represent elements:
  1- Using Vectors. $V_1 = [1\ 1\ 0\ 1\ 1]$, $V_2 = [1\ 0\ 1\ 0\ 0]$
  2- Using Polynomials. $V_1 = 1 + X + X^3 + X^4$, $V_2 = 1 + X^2$

$\Rightarrow$ These used to simplified the incoming mathematics.

- Addition: $V_1 \oplus V_2 = [0\ 1\ 1\ 1\ 1] \Rightarrow V_1 + V_2 = X + X^2 + X^3 + X^4$
- Multiplication: $V_1 \cdot V_2 = 1 + X + \cancel{X^3} + X^4 + X^2 + \cancel{X^3} + X^5 + X^6$
  $= 1 + X + X^2 + X^4 + X^5 + X^6$

- No subtraction Technique.

"Lecture 23/4/2018"

Ex. Find $V_1 * V_2$ in Polynomial Form?

$V_1 = [1\ 0\ 1\ 0\ 0] \longrightarrow 1 + X^2$
$V_2 = [1\ 1\ 0\ 0\ 1] \longrightarrow 1 + X + X^4$

$\Rightarrow V_1 * V_2 = 1 + X + X^2 + X^3 + X^4 + X^6$

$$
\begin{array}{r|l}
 & X^2 + 1 \text{ (quotient)} \\
\hline
1 + X + X^4 & X^6 + X^4 + X^3 + X^2 + X + 1 \\
 & X^6 + X^3 + X^2 \\
\hline
 & X^4 + X + 1 \\
 & X^4 + X + 1 \\
\hline
 & 0 \text{ (remainder)}
\end{array}
$$

$$
\begin{array}{r|l}
 & X^4 + X + 1 \\
\hline
1 + X^2 & X^6 + X^4 + X^3 + X^2 + X + 1 \\
 & X^6 + X^4 \\
\hline
 & X^3 + X^2 + X + 1 \\
 & X^3 + X \\
\hline
 & X^2 + 1 \\
 & X^2 + 1 \\
\hline
 & 0
\end{array}
$$

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Ex. extended field $GF(2^3) \equiv GF(8)$

$0 \; 1 \; \alpha \; \alpha^2 \; \alpha^3 \; \alpha^4 \; \alpha^5 \; \alpha^6$ • we define "α" to ease the process of multiplication.

* primitive Polynomials:

"generate all of the other elements"

- any primitive Polynomial (prime number) can span the whole field.

Ex. (3) ⇒ 3 6 9 2 5 8 1 4 7 0

$GF(2^m)$ Key number has a non-zero element.

$n = 2^m - 1$

- prime polynomial are the factors of $X^n + 1$

Ex. $m=3 \Rightarrow n = 7$, $X^7 + 1 = (1+X)(1+X+X^3)(1+X^2+X^3)$

* Generating the Extended Field:

for $m=3$: $g(x) = 1 + X + X^3 = 0$

$\Rightarrow X^3 = 1 + X$

$$\left\{ \begin{array}{l} \alpha^K \cdot \alpha^j = \alpha^q \\ q = (K+j) \bmod n \end{array} \right\} \Rightarrow \text{ex. } \alpha^4 \cdot \alpha^5 = \alpha^2$$

$\boxed{\alpha^{K-1} \cdot \alpha^{j-1}}$ choose: r, P.

$y = X^r \bmod n$

$X = y^P \bmod n$

where: $r \Rightarrow$ public. $P \Rightarrow$ private.

m=3

0	0	0	0	
1	1	0	0	
α	0	1	0	
α^2	0	0	1	
α^3	1	1	0	$1+\alpha$
α^4	0	1	1	$\alpha+\alpha^2$
α^5	1	1	1	$1+\alpha+\alpha^2$
α^6	1	0	1	$1+\alpha^2$

This called $I_{m \times m}$

we span the whole field.

this called dual Space. ⇒ P

* * *

Coding:

Hamming Code:

- u, v are vector.
- G is a matrix called **Generator** Matrix.

$V = u \cdot G$ — output, input, generator matrix.

$BW = \frac{n}{K} \cdot BW_{old}$

* linear Block code is selecting 2^K code word from 2^n possible code word.

$u = [10 \cdots]$ (K bits) 2^K possible combinations.

$V = [10 \cdots]$ (n bits) 2^n " " .

G matrix $[\cdots]$ $K \times n$.

u & v are with 1's & 0's with K & n Bits respectively;

* Hamming Distance: Number of 1's & 0's that differ in position.

Ex.

V_1 1 0 1 1

V_2 0 0 1 0

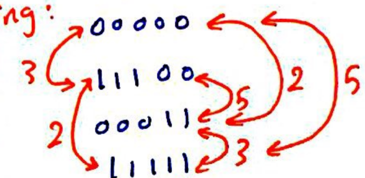
$d_H = 2$

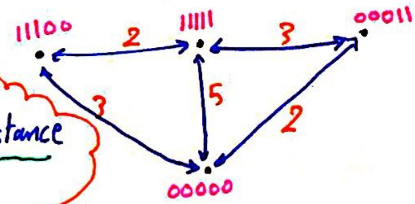
$\Rightarrow d_H = \mathrm{sum}(V_1 \oplus V_2)$

Input:

$$4: \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \qquad 32: \begin{bmatrix} 00000 \\ 00001 \\ \vdots \\ 11111 \end{bmatrix}$$

choosing: 4 points





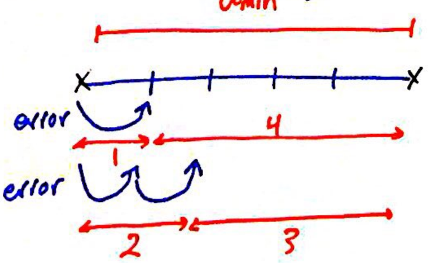
• Memorize:

* Coding is to select 2^K code word from 2^n possible code word with ① maximum minimum hamming distance & ② with maximum code rate $\left(\frac{K}{n}\right)$.

③ also maximum t; error correcting capability.

$\frac{K}{n} < 1$

$d^H_{min} = 5$



$\Rightarrow$ we can correct up to t such that:

$$\boxed{t = \frac{d^H_{min} - 1}{2}}$$

it is better if d^H_{min} was odd number.

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* Hamming code: it is a single error correcting code. $\boxed{t=1}$
which means that $\boxed{d^{H}_{min}=3}$.

under conditions: $\begin{cases} n=2^m-1 \\ n-K=m \end{cases}$ $\Rightarrow$ (n,K) Hamming Code.

• for $m=3$: $n=7$, $K=4$ $\Rightarrow$ This is "$(7,4)$ H.C" or "$\frac{4}{7}$ rate H.C."

* for G: $G=\left[I_{K\times K} \vdots P\right]$ — This P was obtained before for $m=3$.

$$K=4 \Rightarrow G=\begin{array}{c} \begin{array}{cc} I & P \end{array} \\ \left[\begin{array}{cccc:ccc} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right] \end{array}$$

Ex:
let $u=[1\ 1\ 0\ 1] \Rightarrow v=uG=[1\ 1\ 0\ 1\ 0\ 0\ 0]$

@ R_x: $\hat{v}=v+e$ (it fix one error).

* Parity Check Matrix: $H=\left[I_{m\times m} \vdots P^T\right]$

$$m=3 \Rightarrow H=\left[\begin{array}{ccc:cccc} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{array}\right] \Rightarrow [S \text{ errors}].$$

@ R_x: Find $S=\hat{v}H^T$ (if $S=0$, No errors)

$\Rightarrow S=[0\ 0\ 0]$

if $\hat{v}=[1\ 1\ 0\ 1\ 0\ 0\ 1] \Rightarrow S=[1\ 0\ 1]$

↳ it represent the last column in H which means the error in V is in the last bit.

another example:

$\hat{v}=[1\ 1\ 1\ 1\ 0\ 0\ 0]$

$\Rightarrow S=[0\ 0\ 1]$

• Note: Different codes are methods, mathematically well-defined input & output relations to satisfy coding criteria.

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"Lecture 30/4/2018"

※ Cyclic Codes: → used for Error Detection & Correction.

$V(x) = u(x) \cdot g(x) \longrightarrow$ Rx $\hat{V}(x) = V(x) + e(x)$

• for cyclic code:

$n = 2^m - 1$, $d^H_{min} = 3$
$n - K = m$, $t = 1$

$g(x) \longrightarrow m$ order.

$r(x) \longrightarrow$ max order $m-1$

$$\frac{\hat{V}(x)}{g(x)} = \frac{V(x)}{g(x)} + \frac{e(x)}{g(x)}$$

$$= \underbrace{u(x) + a(x)}_{\text{quotient}} + \frac{r(x)}{g(x)}$$

$e(x)$	$a(x)$	$r(x)$
1	0	1
x	0	x
x^2	0	x^2
$\vdots$		$\vdots$
x^{m-1}	0	x^{m-1}
x^{n-1}	?	?

for $m \neq 5, 8, 10$

$\begin{cases} 1 + x + x^m \\ 1 + x^{m-1} + x^m \end{cases}$ these two forms always existed for any $m \neq 5, 8, 10$

• Example: (7,4) Cyclic code.

Let $g(x)$ from $GF(2^3)$

$g(x) = 1 + x^2 + x^3$

$u(x) = 1 + x$

$V(x) = 1 + x^2 + \cancel{x^3} + x + \cancel{x^3} + x^4$

$= 1 + x + x^2 + x^4$

$\hat{V}_1(x) = 1 + x^2 + x^4$

$\hat{V}_2(x) = 1 + x + x^4$

$\hat{V}_3(x) = 1 + x + x^2 + x^4 + x^5$

@ Rx: generate syndrom table: $g(x) = 1 + x^2 + x^3$

$e(x)$	$a(x)$	$r(x)$
1	0	1
x	0	x
x^2	0	x^2
x^3	1	$1 + x^2$
x^4	$1 + x$	$1 + x + x^2$
x^5	$1 + x + x^2$	$1 + x$
x^6	$x^3 + x^2 + x$	$x + x^2$

Note that this is the P matrix. (rows x^3 to x^6)

* sample for the division:

$$\begin{array}{r|l} & x^2 + x + 1 = a(x) \\ \hline 1 + x^2 + x^3 & x^5 \\ & x^5 + x^4 + x^2 \\ \hline & x^4 + x^2 \\ & x^4 + x^3 + x \\ \hline & x^3 + x^2 + x \\ & x^3 + x^2 + 1 \\ \hline & x + 1 = r(x) \end{array}$$

if there is No error:

$$\Rightarrow \frac{\hat{V}(x)}{g(x)} = b(x) + \frac{r(x)}{g(x)}$$

$r(x) = 0$
$u(x) = b(x)$

$r(x) \neq 0$
$u(x) = b(x) + a(x)$

* for $\hat{V}_1(x)$:

$$\begin{array}{r|l} & \underbrace{x + 1}_{b(x)} \\ \hline 1 + x^2 + x^3 & x^4 + x^2 + 1 \\ & x^4 + x^3 + x \\ \hline & x^3 + x^2 + x + 1 \\ & x^3 + x^2 + 1 \\ \hline & x \rightarrow r(x) \end{array}$$

$u(x) = 1 + x$

$r(x) \neq 0$

so $u(x) = b(x) + a(x)$

for $r(x) = x \Rightarrow a(x) = 0$

so $u(x) = 1 + x$

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* for $\hat{V}_2(x)$:

$$\begin{array}{r|l} & x+1 \\ \hline 1+x^2+x^3 & 1+x+x^4 \\ & x^4+x^3+x \\ \hline & x^3+1 \\ & x^3+x^2+1 \\ \hline & x^2 \end{array}$$

$\Rightarrow u(x) = 1+x$

* for $\hat{V}_5(x)$:

$$\begin{array}{r|l} & x^2 \rightarrow b(x) \\ \hline 1+x^2+x^3 & x^5+x^4+x^2+x+1 \\ & x^5+x^4+x^2 \\ \hline & x+1 \end{array}$$

$$u(x) = \underbrace{x^2}_{b(x)} + \underbrace{1+x+x^2}_{\text{from table } a(x)}$$

$$= \underline{1+x} \checkmark$$

• **Example**: Packet length = 200 Bits, $BER_{ch} = 10^{-2}$

target $BER = 10^{-6} \Rightarrow t=2$

pkt (200 Bits)

100	100

in the 200 Bits we fix 2 errors.

$\frac{1}{BER_{ch}} = 100$ (1 error every 100 Bits).

$m=6 \longrightarrow n=63$

$m=7 \longrightarrow n=127 \Rightarrow K=120$ (we need $n'=100 \Rightarrow K'=93$)

* shortened codes:

$(n-\ell, K-\ell)$ code.

100	100

$(127-\ell, 120-\ell)$

$(100, 93)$ cyclic code.

$g(x) = 1+x+x^7$

$\ell = 27$

• **Example**: for the same previous example, let $t=3$

$BER = 10^{-8}$

63	63	74
(63,57)	(63,57)	(74,67)
$1+x+x^6$	$1+x+x^6$	$1+x+x^7$

* Burrest error: errors happen beside each other.

$\Rightarrow$ solved by using interleaver.

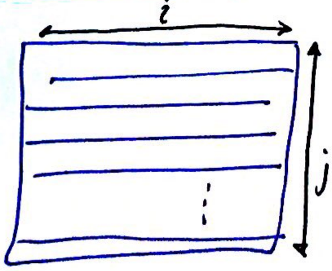
• Burrest error $\Rightarrow$ due to Fading.

• Random error $\Rightarrow$ due to White noise.

100	100

$\rightarrow$ Interleaving $\rightarrow$

write rows.



read columns.

* Burrest length $= \left\lceil \frac{j}{t} \right\rceil \rightarrow$ ceiling.

• All communication systems are interleaved systems. Because of Fading. so the interleaver is needed in every system.

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※ CRC : "Cyclic Redundancy Check."

→ for Error Detection.

$V(x) = u(x) \cdot g_{CRC}(x)$ ⇒ This can detect d^H_{min} errors.

$g_{CRC}(x) = (1+x)\, g_{cyclic}(x)$

• **Example:**

Length $= 255$

from tables. ($m=8$, choose $i=1$)

$n = 2^m - 1 \Rightarrow m = 8 \Rightarrow g_{cyclic}(x) = 1 + x^2 + x^3 + x^4 + x^8$

$\Rightarrow g_{CRC}(x) = (1+x)(1+x^2+x^3+x^4+x^8) = 1 + \dots + x^{m+1}$

detect 3 errors every $(2^m - 1) = n$

$$\frac{u(x)}{g_{CRC}(x)} = \underset{\text{value}}{xxx} + \frac{r_{CRC}(x)}{g_{CRC}(x)}$$

Packet $= [u \,\vdots\, r]$

※ BCH :

$g_{BCH}(x) = LCM\{\phi_i \,;\, i = 1, 2, \dots, 2t\}$, $g_{CRC}(x) = (1+x)\, g_{BCH}(x)$

Ex. $t = 3 \overset{m=8}{\Rightarrow} LCM\{\phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6\}$; Note that $\phi_1 = \phi_2 = \phi_4$ so take one of them due to the equation $\phi_i = \phi_{i \cdot 2^l}$; $l = 1$ here.

also $\phi_3 = \phi_6$

From Tables: $\phi_1 = 1 + x^2 + x^3 + x^4 + x^8$

$\phi_3 = 1 + x + x^2 + x^4 + x^5 + x^6 + x^8$

$\phi_5 = 1 + x + x^4 + x^5 + x^6 + x^7 + x^8$

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• **Quiz:** a PKt of length 400 bits. Design a CRC code that detects 6 errors.

$g_{CRC}(x) = (1+x)\, g_{BCH}(x)$

$g_{cyclic}(x) \to d^H_{min} = 3$

$\to 2t+1$

$g_{CRC}(x) = (1+x)(1+x+x^9)(1+x^8+x^9)$

※ Convolutional Codes: (n, K, L) C.C.

$V = f(\underset{\text{input}}{u}, \underset{\text{state}}{S})$

state machine.

memory length $= L$

possible combinations $= 2^L$

#bits in V > # of Bits in u

$n > K$.

n & K are integers.

for Ex. shift register. $(2, 1, 2)$ C.C

$n = 2$

$L = 2$

⇒ 4 possible comb. (states).

S_0	00
S_1	01
S_2	10
S_3	11

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$\Rightarrow V$ is a convolution function between the input & the state.

code rate $\uparrow$ $\Rightarrow$ SNR received is Better.

correlation Data $\uparrow$ $\Rightarrow$ more protection.

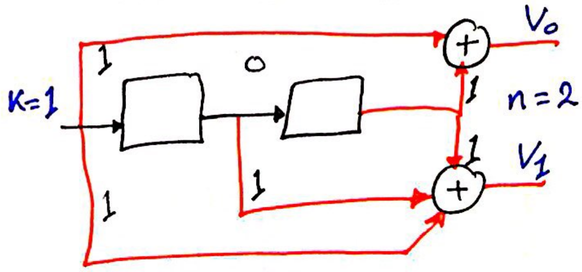
• Convolutional Encoder: it will convolve the data with the states.

$V = f(u, S)$

$\Rightarrow u = f^{-1}(V, S)$

memory Length $\uparrow$ $\Rightarrow$ depth of memory $\uparrow$ $\Rightarrow$ protection is Better.

... continue the Example:



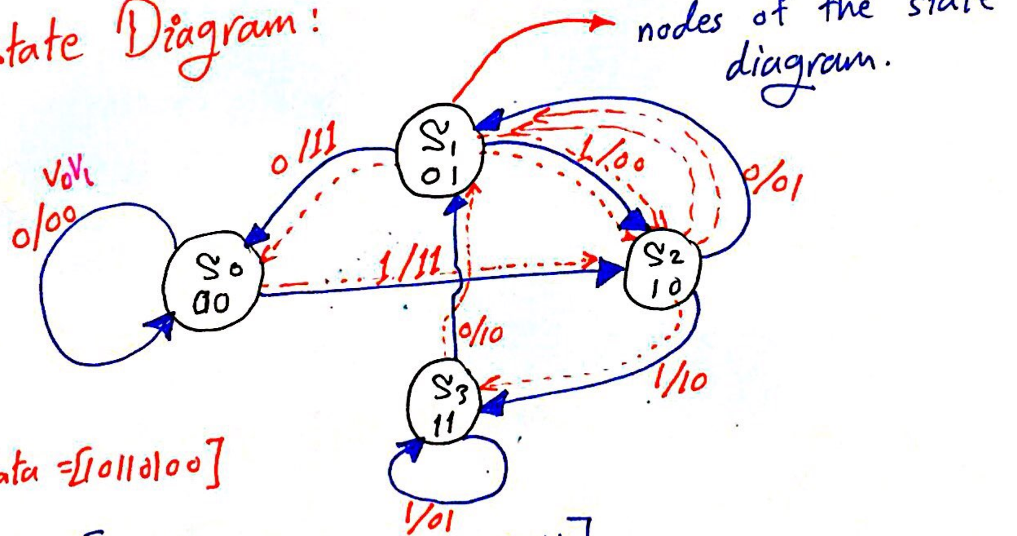
$\Rightarrow V_0 = u + u_{-2}$

$\Rightarrow V_1 = u + u_{-1} + u_{-2}$

* connection vectors: $g_0 = [1\ 0\ 1] = 1 + X^2$

$g_1 = [1\ 1\ 1] = 1 + X + X^2$

• State Diagram:



for data = [1011 0100]

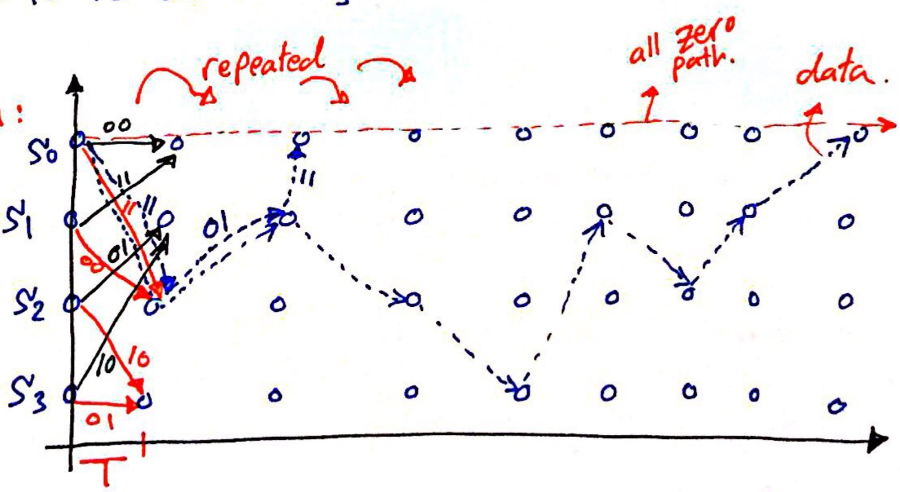
$\Rightarrow V = [11\ 01\ 00\ 10\ 10\ 00\ 01\ 11]$

for easier method:

* Tree Diagram:

or

Trellis Diagram.



*** Error Event:** is defined as forking from the correct path & re-emerging to the correct path without detecting the error.

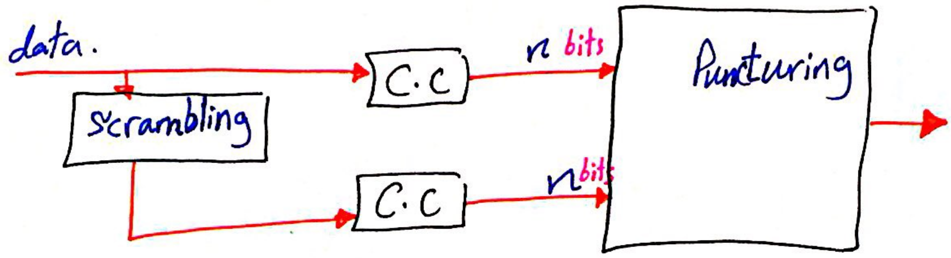
- **free distance:** number of ones in the error event path.

$\boxed{d_{free} = 5}$ in 6 output bits.

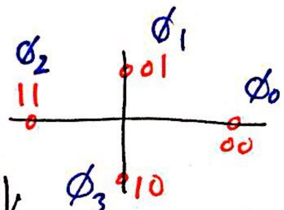
it can fix 2 errors.

$t = \left\lfloor \frac{d_{free} - 1}{2} \right\rfloor = 2$. → floor.

*** Turbo Codes:**



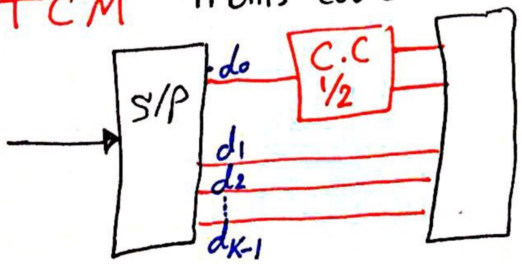
*** The Problem of Turbo Code that:** the process of decoding is Complex & iterative.



- rather than sending 11 ⇒ we send $-\cos \omega t$.

⇓

*** TCM "Trellis Code Modulation":**



*** The idea of TCM:** Modulation integrated with the code.

⇒ This give two features:

1) it can corrects error.
2) Rotationally invarient.

* *** * *** * *** * *** *

End of Material.

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