

# Circuits 2

POWER UNIT

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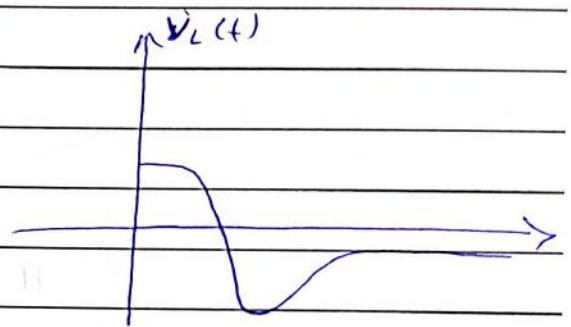
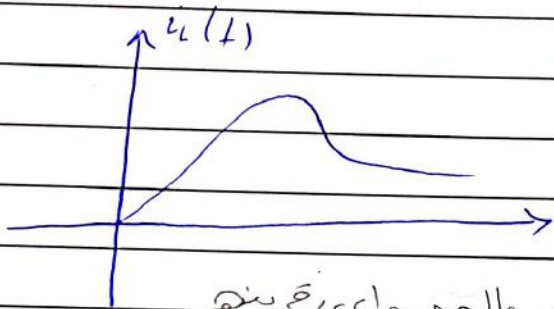
Dr. Mohd Al-haj

Spring 16

\* Calculate & sketch  $v_L(t)$  when  $i_L(t) = 20t e^{-2t}$  &  $L = 0.1 \text{ H}$  ?

$$v_L(t) = L \frac{di_L(t)}{dt} = 0.1 \times \frac{d(20t e^{-2t})}{dt}$$

$$v_L(t) = 0.1 (20 e^{-2t} - 40t e^{-2t}) = \underline{2 e^{-2t} (1 - 2t)}$$



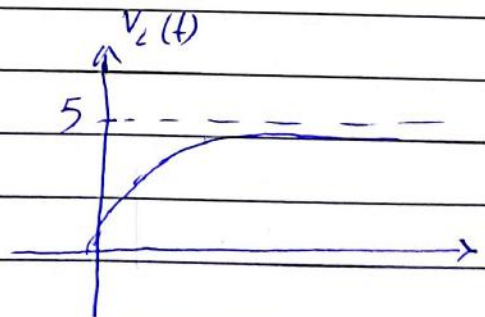
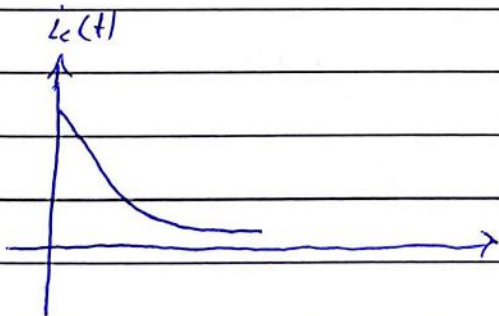
بنوعه الصفر والحد واحد تقريباً  
وترفع (مع تقريباً).

\* Inductor voltage can be changed instantaneously,  
Inductor current could not.

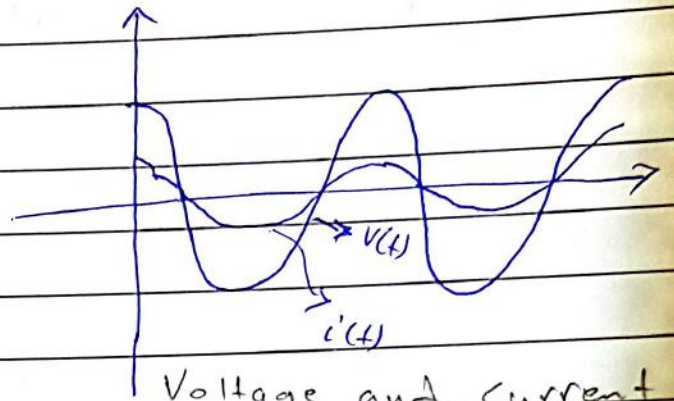
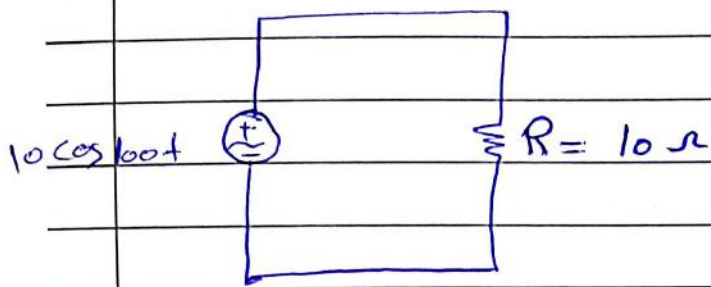
\* Calculate & sketch  $i_C(t)$  when  $v_C(t) = 5(1 - e^{-t/10^{-6}})$

$$i_C(t) = C \frac{dv_C(t)}{dt} = 10^{-7} \times \left[ 5 \times \frac{1}{10^{-6}} e^{-t/10^{-6}} \right]$$

$$= 0.5 e^{-t/10^{-6}}$$



\* Voltage of the capacitor can not be changed  
instantaneously, the current can

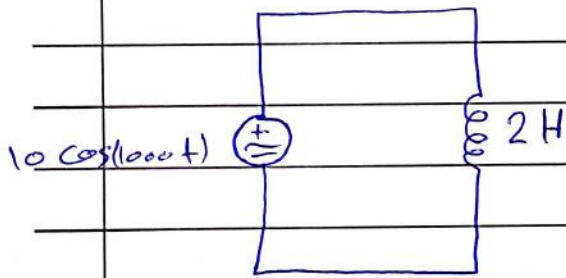


$$i_R(t) = \frac{V_R(t)}{R} = \cos(100t)$$

Voltage and current across a resistor are in phase

$$i_L(t) = ?$$

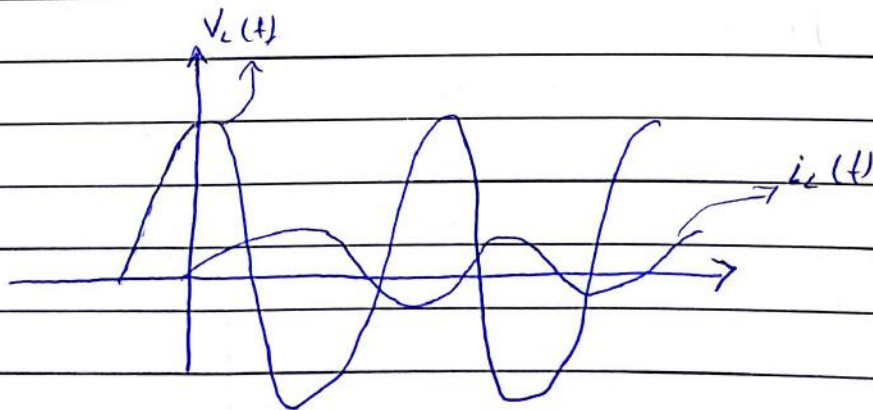
$$i_L(0^-) = 0$$



$$V_L(t) = L \frac{di_L(t)}{dt} \Rightarrow i_L(t) = \frac{1}{L} \int_0^t 10 \cos(100\omega t) dt$$

$$i_L(t) = \frac{1}{200} \sin(100\omega t)$$

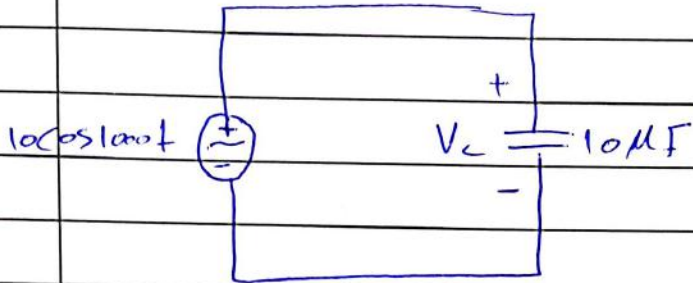
$$\sin(\omega t) = \cos(\omega t - \frac{\pi}{2})$$



$V(t)$  leads  $i_L(t)$  by  $90^\circ$

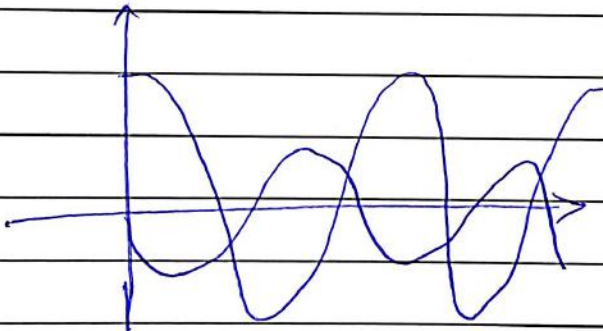
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$$i_c(0^-) = 0 \quad V_c(0^-) = 0$$



$$i_c = C \frac{dv_c}{dt} = 10^{-5} * 10 * -1000 * \sin(1000t)$$

$$i_c(t) = \frac{-1}{10} \sin(1000t)$$



$i_c(t)$  Leads  $V_c(t)$  by  $90^\circ$

in capacitor  $v(t) = L \frac{di}{dt}$

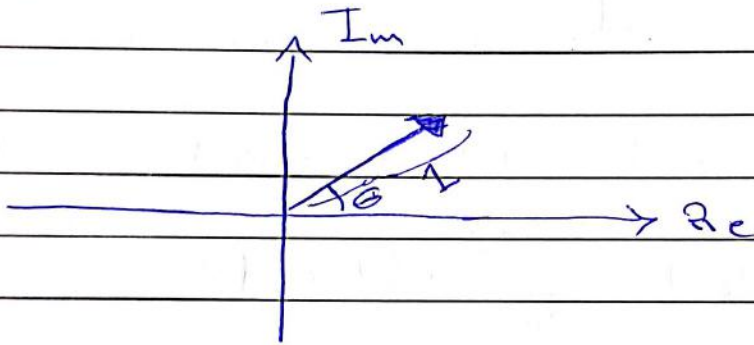
} differential form

in inductor  $i(t) = C \frac{dv}{dt}$

Euler's:

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}, \quad \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2}$$



$$\operatorname{Re}\{e^{j\theta}\} = \cos\theta$$

$$\operatorname{Im}\{e^{j\theta}\} = \sin\theta$$

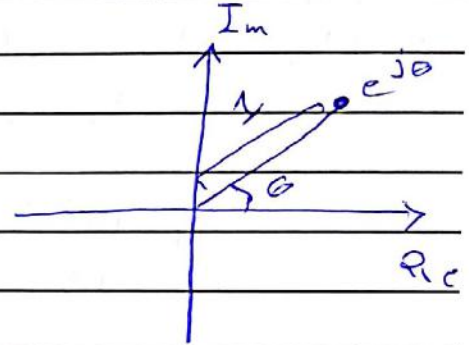
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$$e^{j\theta} = \cos\theta + j\sin\theta \quad (\text{Euler's identity})$$

$$\operatorname{Re}\{e^{j\theta}\} = \cos\theta$$

$$\operatorname{Im}\{e^{j\theta}\} = \sin\theta$$

$$e^{j(\omega t + \theta)} = \cos(\omega t + \theta) + j\sin(\omega t + \theta)$$



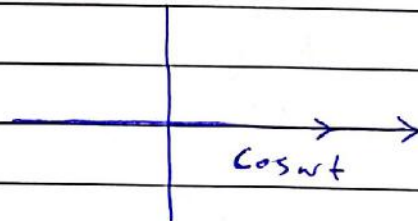
$$e^{j(\omega t + \phi)} = \cos(\omega t + \phi) + j\sin(\omega t + \phi)$$

$$e^{j(\omega t + \gamma)} = \cos(\omega t + \gamma) + j\sin(\omega t + \gamma)$$

$$\begin{aligned} e^{j(\omega t + \theta)} &= e^{j\theta} \boxed{e^{j\omega t}} \\ e^{j(\omega t + \phi)} &= e^{j\phi} \boxed{e^{j\omega t}} \\ e^{j(\omega t + \gamma)} &= e^{j\gamma} \boxed{e^{j\omega t}} \end{aligned}$$

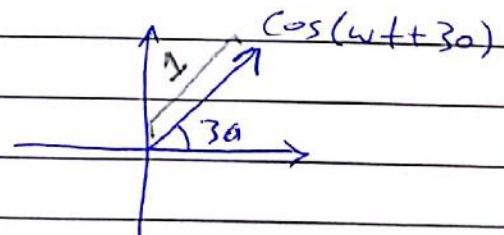
①  $\cos \omega t$

$$\Rightarrow \cos(\omega t + 0) = \boxed{e^{j0}} = \operatorname{Re}\{e^{j0}\} = 1$$



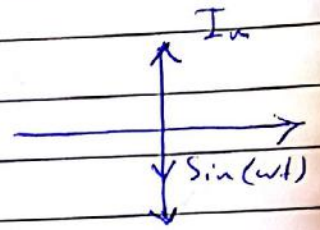
②  $\cos(\omega t + 30)$

$$= \boxed{e^{j\pi/6}}$$



No

$$\textcircled{3} \sin(\omega t) = \cos(\omega t - \pi/2) \\ = -j$$



$$V_1(t) = 15 \cos(377t + \pi/4)$$

$$V_2(t) = 15 \cos(377t + \pi/6)$$

Find  $V_1 + V_2$  ??

Sol

$$V_1 = 15 \angle \pi/4 = 15 \cos \pi/4 + j15 \sin \pi/4$$

$$V_2 = 15 \angle \pi/6 = 15 \cos \pi/6 + j15 \sin \pi/6$$

$$V_1 + V_2 = (15 \cos \pi/4 + 15 \cos \pi/6) + j15(\sin \pi/4 + \sin \pi/6)$$

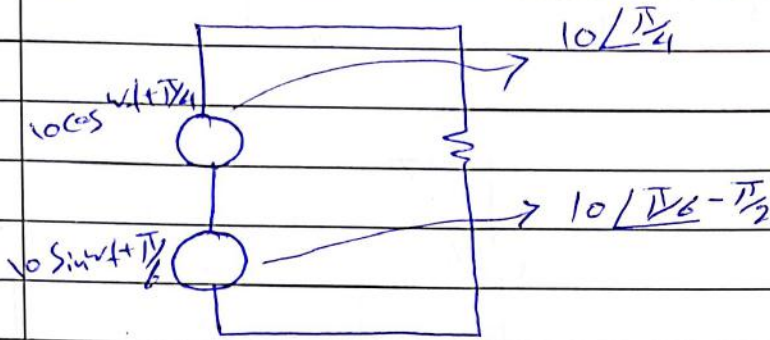
Time	Phasor domain
$10 \cos(\omega t)$	$10 \angle 0$
$10 \cos(\omega t - 30)$	$10 \angle -30$
$10 \sin \omega t$	$10 \angle -90$
$10 \sin(\omega t + 40)$	$10 \angle -50$

For  $v(t) = R i(t)$   $V = RI$

For  $L \Rightarrow v(t) = L \frac{di(t)}{dt}$   $V = j\omega L I$

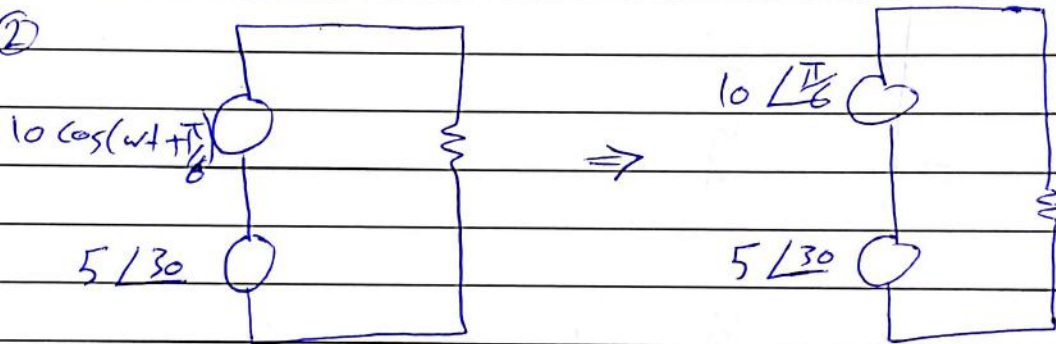
For  $C \Rightarrow i(t) = C \frac{dv(t)}{dt}$   $I = j\omega C V$

Note (1)

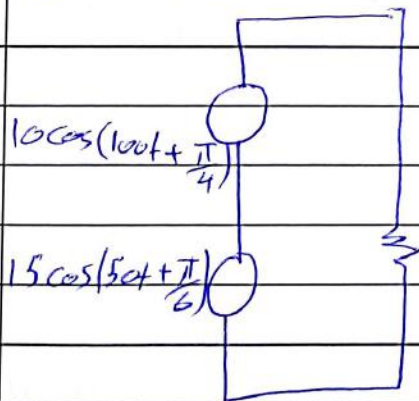


در کسینوس الی ال Cos  
ولیس ال Sin

Note (2)



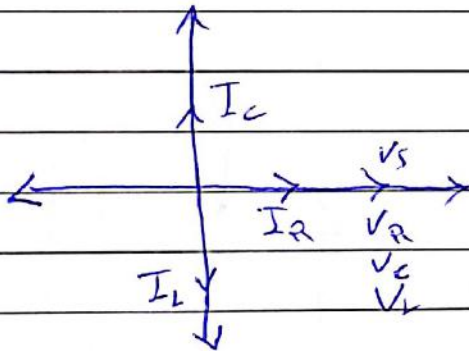
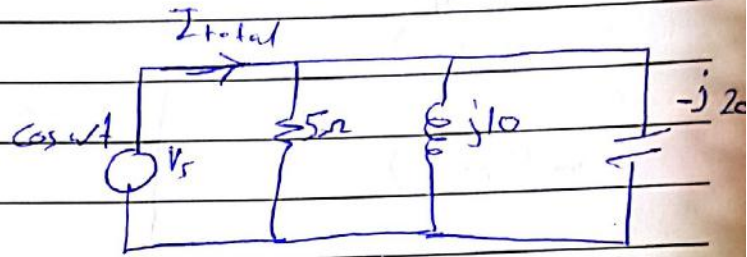
Note (3)



We use Superposition



Draw the phasors for Voltages & currents :-



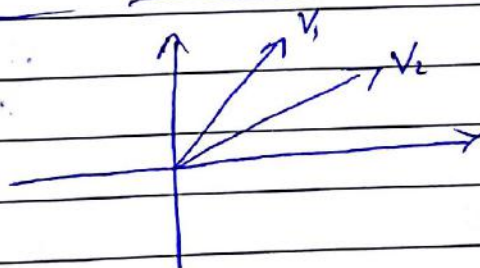
$$I_{total} = I_R + I_C + I_L = 0.2 - j0.05$$

$$I_R = \frac{V_R}{5} = \frac{V_s}{5} = \frac{1 \angle 0}{5} = 0.2 \angle 0$$

$$I_L = \frac{V_L}{X_L} = \frac{1 \angle 0}{j10} = -0.1j$$

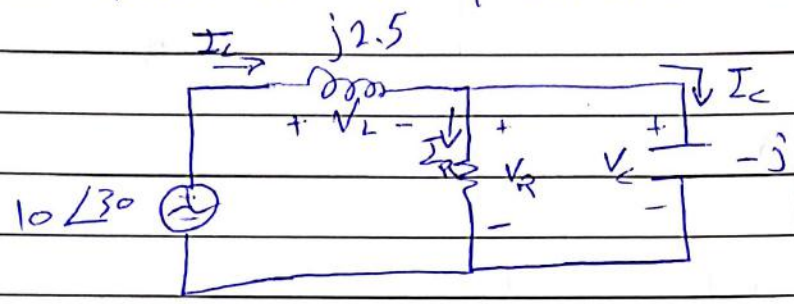
$$Z = jX + R$$

$$I_C = \frac{V_C}{X_C} = \frac{1 \angle 0}{-j20} = j0.05$$



$V_1$  leads  $V_2$

ex: Draw all the phasors :-



Sol :

$$I_L = \frac{10 \angle 30}{j2.5 + \frac{-j}{2-j}}$$

$$\neq \begin{pmatrix} 4.17 \\ -3.92 \end{pmatrix}$$

$$I_L = 3.92 - j4.17 \quad \angle \tan^{-1}\left(\frac{-4.17}{3.92}\right)$$

$$= \sqrt{(3.92)^2 + (-4.17)^2}$$

$$= 5.72 \angle -46.75$$

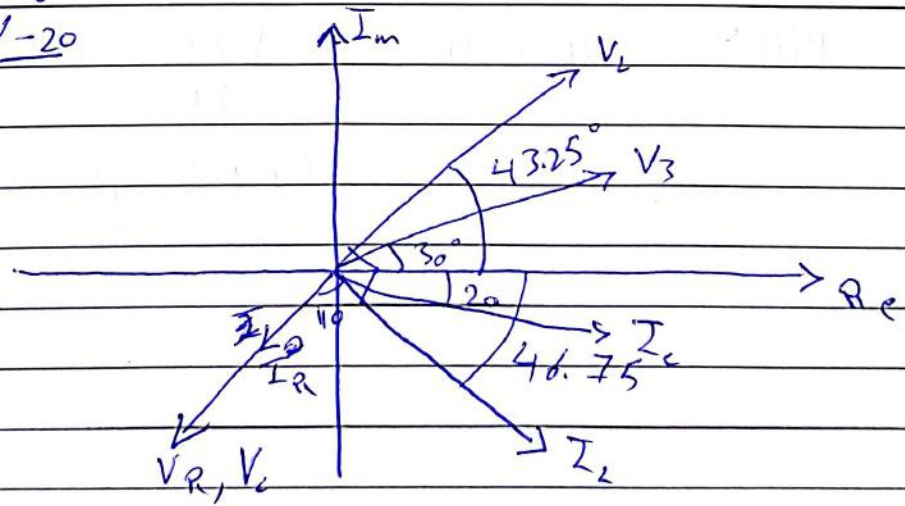
$$V_L = j2.5 * I_L = 14.3 \angle 43.25$$

$$I_R = \left(\frac{-j}{2-j}\right) (3.92 - j4.17) = 2.56 \angle -110$$

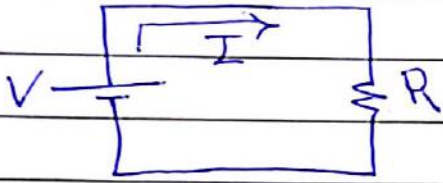
$$V_R = I_R * R = 5.12 \angle -110, \quad V_C = V_R$$

$$I_C = \frac{V_C}{-j}, \quad I_C = \frac{2}{2-j} * I_L, \quad I_C = I_L - I_R$$

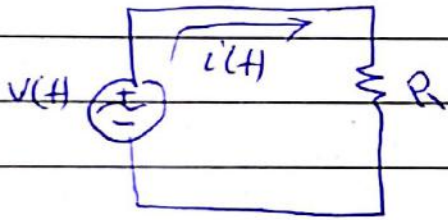
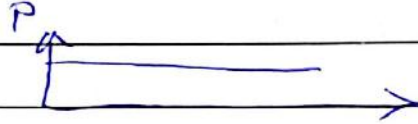
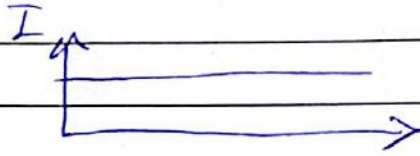
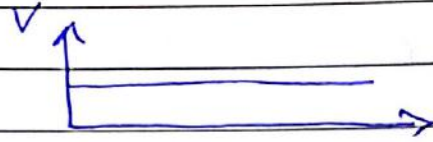
$$I_C = 5.12 \angle -20$$



Power :-



$$P = VI$$



$$P(t) = i^2(t) R = \frac{v^2(t)}{R}$$

$$= i(t) v(t)$$

In Inductor :-

$$P(t) = i(t) v(t)$$

$$= i(t) L \frac{di(t)}{dt}$$

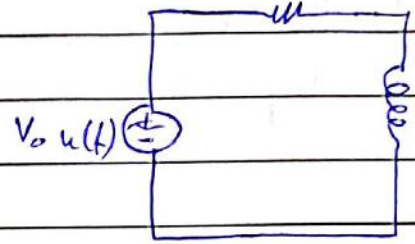
$$= v(t) * \frac{1}{L} \int_{-\infty}^t v(\tau) d\tau$$

In capacitor :-

$$P(t) = i(t) v(t) = C \frac{dv_c(t)}{dt} * v(t)$$

$$= i(t) * \frac{1}{C} \int_{-\infty}^t i(\tau) d\tau$$

ex 
$$i(t) = \frac{V_0}{R} \left(1 - e^{-\frac{tR}{L}}\right) u(t)$$



Power generated by the source

$$P(t) = i(t) v(t)$$

$$= \frac{V_0}{R} \left(1 - e^{-\frac{tR}{L}}\right) u(t) \cdot V_0 u(t)$$

$$u(t) \cdot u(t) = u(t)$$

Power dissipated by  $R$  or delivered to the resistor:

Ans

$$P(t) = i^2(t) R = \frac{V_0^2}{R} \left(1 - e^{-\frac{tR}{L}}\right)^2 u(t)$$

Power stored in the inductor or absorbed by

$$P_2(t) = v(t) i(t)$$

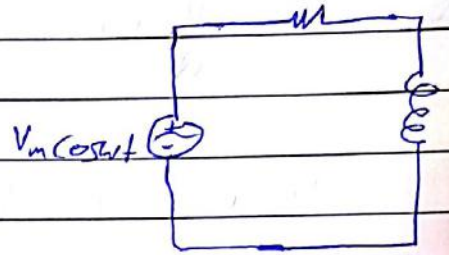
$$= L \frac{di(t)}{dt} \cdot i(t)$$

$$\begin{aligned} \text{ex } L \frac{di(t)}{dt} &\Rightarrow P_2(t) = L \left( \frac{V_0 (-1) \left(\frac{R}{L}\right) e^{-\frac{tR}{L}} \right) \left( \frac{V_0}{R} \left(1 - e^{-\frac{tR}{L}}\right) \right) \\ &= \frac{V_0^2}{R} e^{-\frac{tR}{L}} \left(1 - e^{-\frac{tR}{L}}\right) u(t) \end{aligned}$$

$\Rightarrow$  Instantaneous Power

$$\frac{d u(t)}{dt} = 0$$

(ex) The power generated by the source :-



$$P(t) = V_m \cos(\omega t) * I_m \cos(\omega t + \phi)$$

$$= \frac{V_m I_m}{2} [\cos(2\omega t + \phi) + \cos\phi]$$

$$v(t) = V_m \cos \omega t$$

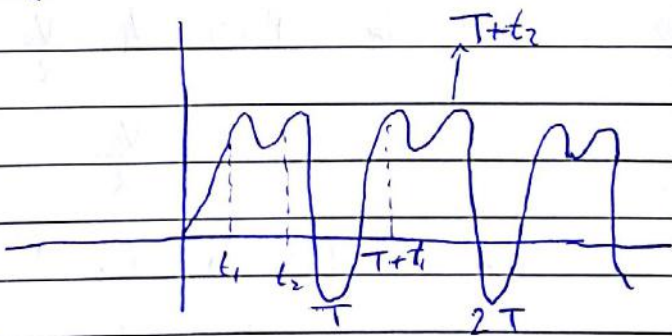
$$i(t) = I_m \cos(\omega t + \phi)$$

⇒ we need more efficient & accurate power representation :-

⇒ average power :-

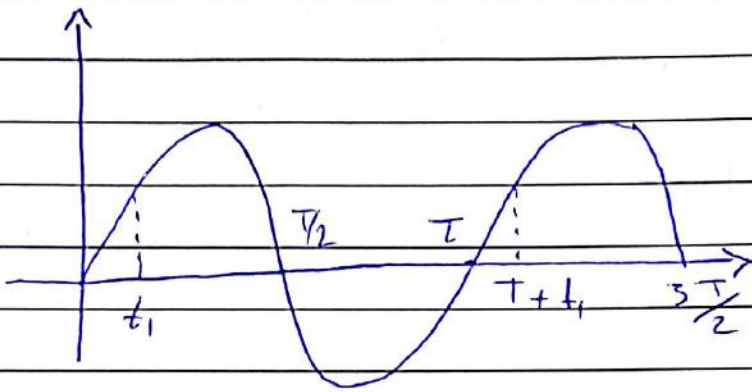
$$P = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} P(t) dt$$

For a periodic signal :-



$$\frac{1}{T} \int_{t_1}^{t_2} P(t) dt = \frac{1}{T} \int_{T+t_1}^{T+t_2} P(t) dt$$

No \_\_\_\_\_



$$\int_0^T P(t) dt = \int_{t_1}^{t_1+T} P(t) dt = \int_{T/2}^{5T/2} P(t) dt$$

912 → Let's assume : (for any element  $v(t) = V_m \cos(\omega t + \theta)$   
 $i(t) = I_m \cos(\omega t + \phi)$ )

$$P(t) = v(t) i(t)$$

$$= I_m V_m \cos(\omega t + \theta) \cos(\omega t + \phi)$$

$$P(t) = \frac{1}{2} V_m I_m \cos(\theta - \phi) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta + \phi)$$

$$P = \frac{1}{T} \int_0^T P(t) dt = \frac{1}{T} \int_0^T \frac{1}{2} V_m I_m [\cos(\theta - \phi) + \cos(2\omega t + \theta + \phi)] dt$$

$$= \frac{1}{T} \int_0^T \frac{1}{2} V_m I_m \cos(\theta - \phi) dt$$

$$= \frac{1}{T} * T * \frac{1}{2} V_m I_m \cos(\theta - \phi)$$

$$P = \frac{1}{2} V_m I_m \cos(\theta - \phi)$$

No

ex:  $v(t) = 4 \cos\left(\frac{\pi}{6}t\right)$ ,  $Z = 2 \angle 60^\circ$

$\Rightarrow v(t) \Rightarrow 4 \angle 0$

$I = \frac{4 \angle 0}{2 \angle 60} = 2 \angle -60^\circ$

$P = \frac{1}{2} * 4 * 2 * \cos(0 - -60)$

$P = 2W$

$\cos a \cos b = \frac{\cos(a+b)}{2} + \frac{\cos(a-b)}{2}$

$P(t) = v(t)i(t)$

$= 4 \cos\left(\frac{\pi}{6}t\right) * 2 \cos\left(\frac{\pi}{6}t - 60\right)$

$P(t) = 2 + \cos\left(\frac{\pi}{3}t - 60\right)$

\* The average power absorbed by a resistor is :-

$\rightarrow 1 \sim 1$  because  $V \& I$  are in phase

$P = \frac{1}{2} V_m I_m \cos(\theta - \phi) = \frac{1}{2} V_m I_m$  for R  
 $= \frac{1}{2} I_m^2 R = \frac{V_m^2}{2R}$   
for sinusoidal

\* average power "absorbed" by an inductor :-

$P = \frac{1}{2} V_m I_m \cos(\theta - \phi)$

$P_{avg} = 0$  for Inductor

\* for a capacitor :-

$P = \frac{1}{2} V_m I_m \cos(\theta - \phi)$

$P_{avg} = 0$  for C

ex: Find the average power delivered to an impedance  
 $Z = 8 - j11 \Omega$  &  $I = 3 + j4$   
 $\rightarrow p = 0$

$$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta - \phi)$$

$$= \frac{1}{2} (I_m)^2 R$$

$$I = \sqrt{3^2 + 4^2} \angle \tan^{-1}\left(\frac{4}{3}\right)$$

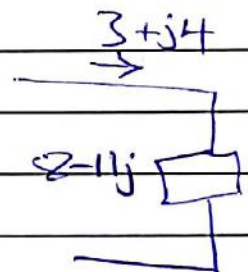
$\hookrightarrow 5$

$$P = \frac{1}{2} 5^2 * 8 = 100 \text{ W}$$

$$V = (3 + j4)(8 - j11)$$

$$= (68 - j)V$$

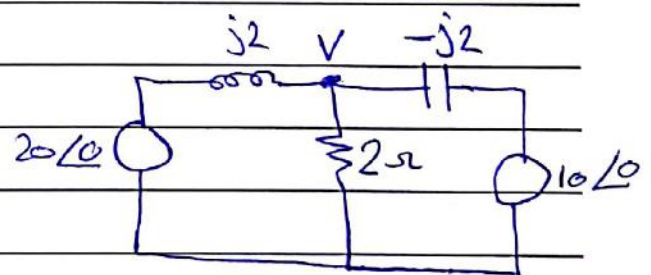
$$= 68.1 \angle -0.84$$



$$P(t) = v(t) \cdot i(t)$$

$$= 68.1 \cos(\omega t - 0.84) * 5 \cos(\omega t + 53^\circ)$$

Find the average power absorbed by the elements & generated by sources:-



at node V:-

$$\frac{20 \angle 0 - V}{j2} + \frac{0 - V}{2} + \frac{10 \angle 0 - V}{-j2} = 0$$

$$-j10 + \cancel{j0.5V} - 0.5V + j5 - \cancel{0.5V} = 0$$

$$V = \frac{j5}{-0.5} = 10 \angle -90^\circ$$



$$P_R = \frac{1}{2} \frac{V_m^2}{R} = \frac{1}{2} (10)^2 * \frac{1}{2} = 25 \text{ W (absorbed)}$$

$$P_{20} = \frac{1}{2} V_m I_m \cos(\theta - \phi)$$

$$I_1 = \frac{20 \angle 0}{j2} = \frac{10 \angle -90}{1} = 5 - j10$$
$$= 11.18 \angle -63.43$$

$$P_{20} = \frac{1}{2} * 20 * 11.18 \cos(0 + 63.43) = 50 \text{ W (generated)}$$

$$P_{10} = \frac{1}{2} V_m I_m \cos(\theta - \phi)$$

$$I_2 = \frac{5 - j5}{1} = 7.071 \angle -45$$

$$P_{10} = \frac{1}{2} * 10 * 7.071 * \cos(0 + 45) = 25 \text{ W (absorbed)}$$

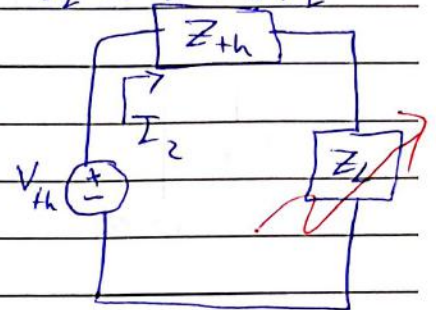
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Maximum Power transfer:- give =  $Z_{th} = R_{th} + jX_{th}$  &  $V_{th}$   
 $Z_L = R_L + jX_L$

$V_{oc} ??$   $I_{sc} ??$   $Z_{th} ??$

$$I_L = \frac{V_{Th}}{Z_{Th} + Z_L}$$

$$= \frac{V_{Th} \angle 0}{R_{Th} + jX_{Th} + R_L + jX_L}$$



$$V_L = V_{th} \times \frac{Z_L}{Z_L + Z_{th}} = V_{th} \times \frac{R_L + jX_L}{R_L + jX_L + R_{th} + jX_{th}}$$

$$|I_L| = \frac{|V_{th}|}{\sqrt{(R_L + R_{th})^2 + (X_L + X_{th})^2}}$$

$$\angle I_L = \angle V_{th} - \tan^{-1} \left( \frac{X_L + X_{th}}{R_L + R_{th}} \right)$$

$$|V_L| = \frac{|V_{th}| \times \sqrt{R_L^2 + X_L^2}}{\sqrt{(R_L + R_{th})^2 + (X_L + X_{th})^2}}$$

$$\angle V_L = \angle V_{th} + \tan^{-1} \left( \frac{X_L}{R} \right) - \tan^{-1} \left( \frac{X_L + X_{th}}{R_L + R_{th}} \right)$$

$$P = \frac{1}{2} I_m V_m \cos(\theta - \phi)$$

$$P = \frac{1}{2} |V_L| |I_L| \cos(\theta - \phi)$$

→ Follow

No \_\_\_\_\_

$$P = \frac{\frac{1}{2} V_{th}^2 \sqrt{R_L^2 + X_L^2}}{(R_{th} + R_L)^2 + (X_{th} + X_L)^2} \cos\left(\tan^{-1} \frac{X_L}{R_L}\right)$$

$$\left. \frac{dP}{dR_L} \right|_{R_L=0} \Rightarrow R_L = R_{th}$$

$$R_L = 0$$

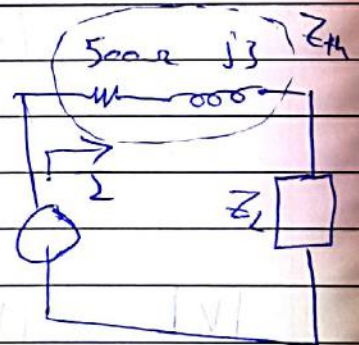
$$\left. \frac{dP}{dX_L} \right|_{X_L=0} \Rightarrow X_{th} = -X_L$$

In conclusion :-

Max Power transfer to  $Z_L = Z_{th}^*$  to have maximum power transfer.  
 $Z_{th}$  is conjugate of  $Z_L$

(ex) what value of  $Z_L$  to have Maximum Power transfer

$$Z_L = 500 - j3$$



Max Power transfer current

$$I = \frac{V_{th}}{Z_L + Z_{th}} = \frac{V_{th}}{2R_L} = \frac{V_{th}}{2R_{th}}$$

$$I = \frac{V_{th}}{R_L + jX_L + R_L - jX_L}$$

Power for signals with multiple frequencies

$$i(t) = \sin t + \sin \pi t, R = 1$$

find the average dissipated by R?

$$P = \frac{R}{T_1} \int_{T_1} \sin^2 t dt + \frac{R}{T_2} \int_{T_2} \sin^2 \pi t dt \quad \text{follow}$$

$$\text{Sol: } T_1 = 2\pi, \quad T_2 = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2$$

$$P = \frac{1}{2\pi} \int_0^{2\pi} \sin^2 t \, dt + \frac{1}{2} \int_0^2 \sin^2 \pi t \, dt$$

$$= \frac{1}{2\pi} \int_0^{2\pi} \sin^2 t \, dt = \frac{1}{2\pi} \int_0^{2\pi} \left[ \frac{1}{2} - \frac{1}{2} \cos 2t \right] dt + \frac{1}{2} \int_0^2 \left[ \frac{1}{2} - \frac{1}{2} \cos 2\pi t \right] dt$$

$$= 2\pi \times \frac{1}{2\pi} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times 2 = 1 \text{ W} = \left( \frac{1}{2} (1)^2 + \frac{1}{2} (1)^2 \right) R$$

⇒ For sources with multiple frequency

$$P_{\text{avg}} = \frac{1}{2} (I_{m_1}^2 + I_{m_2}^2 + I_{m_3}^2 + \dots) R$$

(ex)  $i(t) = 2 \cos 10t - 3 \cos 20t$      $R = 4$

Find the average Power:-

Sol:

$$P_{\text{avg}} = \frac{1}{2} I_{m_1}^2 R + \frac{1}{2} I_{m_2}^2 R$$

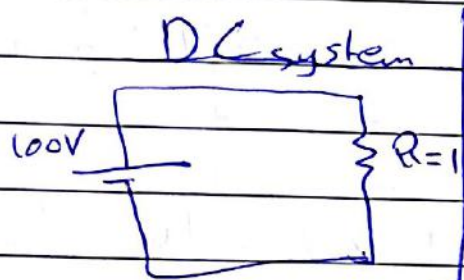
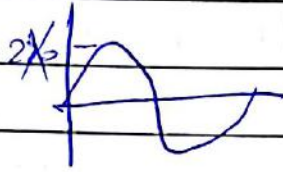
$$= \frac{1}{2} (2)^2 4 + \frac{1}{2} (3)^2 4 = 26 \text{ W}$$

(ex)  $i(t) = 2 \cos 10t - 3 \cos 10t$  ,  $R = 4$   
 $= -\cos 10t$

$$P_{\text{avg}} = \frac{1}{2} (1)^2 \times 4 = 2 \text{ W}$$

# # Effective values :-

220V



$$P = \frac{(100)^2}{1} = 10^4 \text{ W}$$

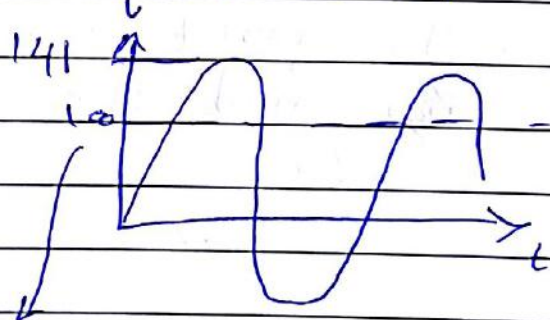
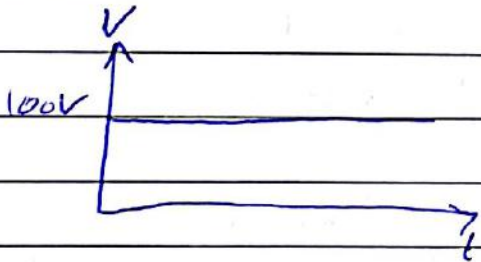


AC ?  $\Rightarrow 10^4 \text{ W}$

$$P = \frac{1}{2} \frac{(V_m)^2}{R}$$

$$10^4 = \frac{1}{2} (V_m)^2 \Rightarrow (V_m)^2 = 2 \times 10^4$$

$$V_m = \sqrt{2} \times 100 = 141$$



Effective Value  
(Root mean square value)

$\leftarrow$   $\leftarrow$   $\leftarrow$   $\leftarrow$   
 $\frac{1}{T} \int_0^T v^2 dt$

$$P_{av (DC)} = \frac{(V_{DC})^2}{R}$$

$$P_{av (AC)} = \frac{1}{T R} \int_0^T v^2(t) dt$$

$\Rightarrow$   
Follow

No \_\_\_\_\_

$$\frac{(V_{DC})^2}{R} = \frac{1}{TR} \int_0^T v(t)^2 dt$$

$$V_{DC} = V_{eff} = V_{RMS} = \sqrt{\frac{1}{T} \int_0^T v^2(t) dt}$$

⇒ RMS Value for a sinusoidal signal

$$i(t) = I_m \cos(\omega t + \phi)$$

$$T = \frac{2\pi}{\omega}$$

sol

$$I_{RMS} = \sqrt{\frac{\omega}{2\pi} \int_0^{\frac{2\pi}{\omega}} I_m^2 \cos^2(\omega t + \phi) dt}$$

$$\frac{1}{2} + \frac{1}{2} \cos(2\omega t + 2\phi)$$

$$= \sqrt{\frac{\omega}{2\pi} \int_0^{\frac{2\pi}{\omega}} \frac{I_m^2}{2} + \frac{I_m^2}{2} \cos(2\omega t + 2\phi) dt}$$

$$= \sqrt{\frac{\omega}{2\pi} \int_0^{\frac{2\pi}{\omega}} \frac{I_m^2}{2} dt} = \sqrt{\frac{\omega}{2\pi} \times \frac{2\pi}{\omega} \times \frac{I_m^2}{2}}$$

$$I_{RMS} = \frac{I_m}{\sqrt{2}}$$

⇒ 1/√2 rms Value  
Peak Value  
2

For Sinusoidal &

$$P: P_{avg} = \frac{1}{2} V_m I_m = \frac{1}{2} (\sqrt{2} V_{RMS}) (\sqrt{2} I_{RMS}) = V_{RMS} I_{RMS}$$

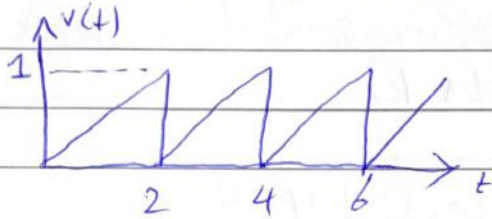
$$= \frac{1}{2} \frac{V_m^2}{R} = \frac{(V_{RMS}^2)}{R} = \frac{1}{2} I_m^2 R = (I_{RMS})^2 R$$

$$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta - \phi) = V_{RMS} I_{RMS} \cos(\theta - \phi)$$

$$= V_{eff} I_{eff} \cos(\theta - \phi)$$

$$\begin{aligned}
 v(t) &= 300 \cos(\omega t - 30^\circ) \\
 &= \sqrt{2} 220 \cos(\omega t - 30^\circ) \\
 &\quad \swarrow \text{RMS Value}
 \end{aligned}$$

(ex) Find the RMS value :-



$$v(t) = \frac{1+t}{2}$$

↙  
for 0 ≤ t ≤ 2

$$T = 2$$

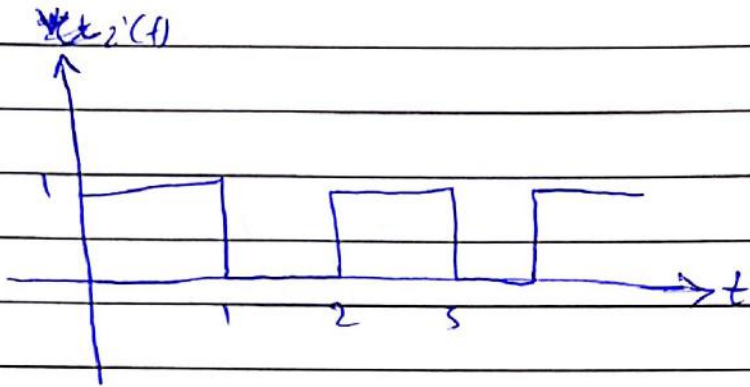
$$V_{\text{eff}} = \sqrt{\frac{1}{2} \int_0^2 \left(\frac{1+t}{2}\right)^2 dt}$$

$$= \sqrt{\frac{1}{2} \int_0^2 \left(\frac{1}{2} + \frac{t}{2}\right)^2 dt} = \sqrt{\frac{1}{2} * \left[\frac{1}{2}t + \frac{t^3}{6}\right]_0^2} = \frac{1}{\sqrt{3}}$$

In general :- For smooth signals :-  $V_{\text{rms}} = \frac{V_m}{\sqrt{3}}$

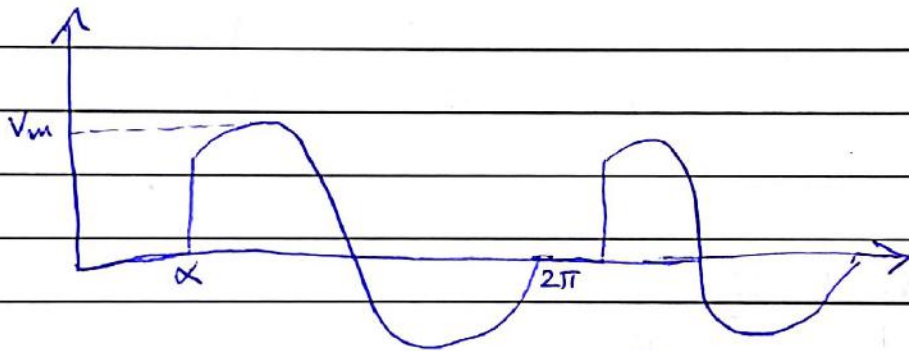
$$\begin{aligned}
 P_{\text{avg}} &= \left( \frac{1}{2} I_{m1}^2 + \frac{1}{2} I_{m2}^2 + \frac{1}{2} I_{m3}^2 + \dots \right) R \\
 &= \left( I_{\text{rms}1}^2 + I_{\text{rms}2}^2 + I_{\text{rms}3}^2 + \dots \right) R
 \end{aligned}$$

$$I_{\text{eff}} = \sqrt{(I_{\text{eff}1})^2 + (I_{\text{eff}2})^2 + (I_{\text{eff}3})^2}$$



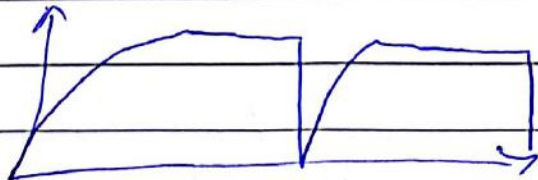
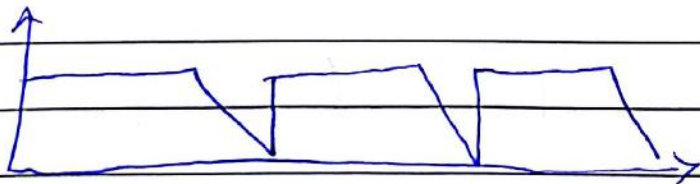
$$i(t) = \begin{cases} 1 & 0 \leq t < 1 \\ 0 & 1 \leq t \leq 2 \end{cases}$$

$$I_{\text{eff}} = \sqrt{\frac{1}{2} \int_0^2 (i)^2 dt}$$



$$T = 2\pi, \quad v(t) = \begin{cases} 0 & 0 < t < \alpha \\ V_m \sin \omega t & \alpha < t < 2\pi \end{cases}$$

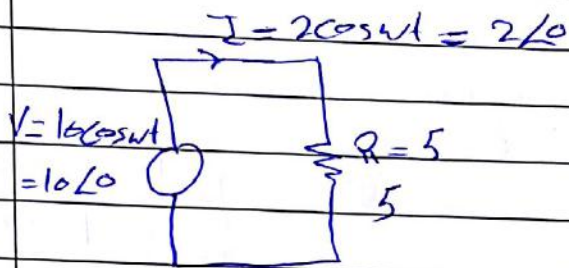
$$V_{\text{rms}} = \sqrt{\frac{1}{2\pi} \int_{\alpha}^{2\pi} V_m^2 \sin^2 \omega t dt}$$



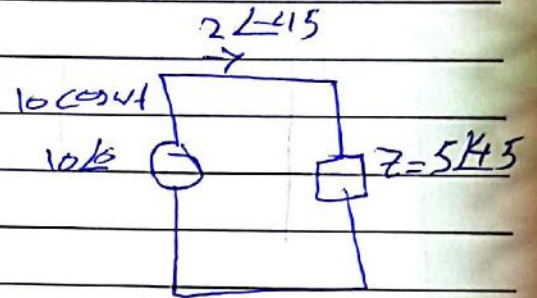
T?? f(t)??



\* The Power Factor :-



$$P = \frac{1}{2} V_m I_m = \frac{1}{2} (10)(2) = 10 \text{ W}$$



$$P = \frac{1}{2} * 10 * 2 * \cos(45)$$

$$= \frac{10}{\sqrt{2}} \text{ W}$$

سرقت کار ایکس سے ہوا ہے اس لیے یہ ناقص ہے اس لیے  
\* Power

$PF \triangleq \frac{\text{Real Power}}{\text{Apparent Power}}$

$$= \frac{V_{rms} I_{rms} \cos(\theta - \phi)}{V_{rms} I_{rms}} = \boxed{\cos(\theta - \phi)}$$

$$\text{Apparent Power} = V_{rms} I_{rms} = \frac{1}{2} V_m I_m$$

$(\theta - \phi) \rightarrow PF \text{ angle}$

$$\Rightarrow R \text{ Loads} \Rightarrow PF = \cos(\theta_r - \theta_r) \Rightarrow PF = 1$$

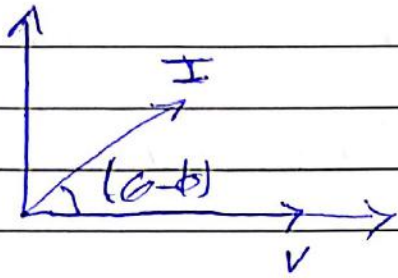
$$\Rightarrow X_L \text{ Loads} \Rightarrow PF = \cos(\theta - \phi) = 0$$

$$\Rightarrow R-X \text{ Loads} \Rightarrow PF = \cos(\theta - \phi)$$

$$0 \leq PF < 1$$

No \_\_\_\_\_

Capacitive

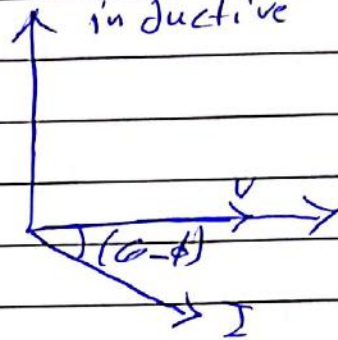


$$PF = \cos(\theta - \phi)$$

(Leading)

- Capacitive case

inductive



$$PF = \cos(\theta - \phi)$$

(Lagging)

- inductive case

$$PF = \frac{\text{Real Power (P)}}{\text{Apparent Power (|S|)}}$$

$$= \frac{V_{rms} I_{rms} \cos(\theta - \phi)}{V_{rms} I_{rms}} = \cos(\theta - \phi)$$

$$R \rightarrow PF = 1$$

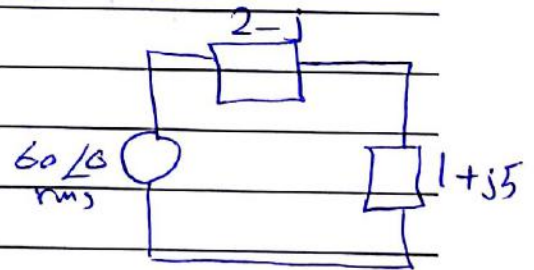
$$X_L, X_C \rightarrow PF = 0$$

$$R_L, R_C \Rightarrow 0 < PF < 1$$

$$V = 60 \angle 0$$

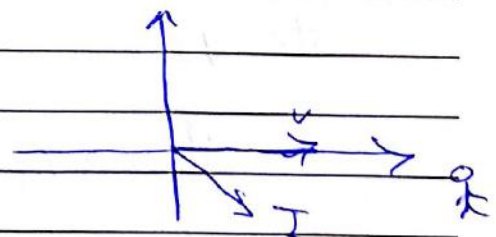
$$v(t) = \sqrt{2} 60 \cos(\omega t)$$

PF of the source



$$I_{rms} = 60 \angle 0$$

$$\begin{aligned} & (2-j) + 1 + j5 \\ & = 12 \angle -53.13 \end{aligned}$$



$$P_{avg}(2-j) = (I_{rms})^2 R = (12)^2 * 2 = 288 \text{ W}$$

$$P_{avg}(1+j5) = (I_{rms})^2 R = (12)^2 * 1 = 144 \text{ W}$$

Apparent Power Produced by the source :-

$$|S| = V_{rms} I_{rms} = 60 * 12 = 720 \text{ (Voltampere)} \\ \text{(VA)}$$

$$PF_{(1+j5)} = \frac{V_{(1+j5)}}{V_{(-1+j5)}} = \frac{(1+j5)(12 \angle -53.13)}{60 \angle 25.56} = 61 \angle 25.56 \text{ V}$$

$$PF_{(1+j5)} = \cos(25.56 + 53.13) = 0.19$$

## \* Complex Power (S)

↳ is introduced to represent the power in both R & X

Complex Power  $\begin{cases} \rightarrow \text{Real Power (P)} \\ \rightarrow \text{Reactive Power (Q)} \end{cases}$

is introduced  $\downarrow$   
to represent the charging & discharging in L & C  
[rate of energy exchange]

$$\# V = V_{eff} \angle \theta \quad I = I_{eff} \angle \phi$$

$$P = |V_{eff}| |I_{eff}| \cos(\theta - \phi)$$

$\Rightarrow$  follow

No

$$P = |V_{eff}| |I_{eff}| \operatorname{Re} [e^{j(\theta - \phi)}]$$

$$= \operatorname{Re} [ |V_{eff}| e^{j\theta} * |I_{eff}| e^{-j\phi} ]$$

$$= \operatorname{Re} \{ V I^* \}$$

$$S = V_{eff} I_{eff}^*$$

Follow

$$S = V_{eff} I_{eff} e^{j(\theta - \phi)} = P + jQ$$

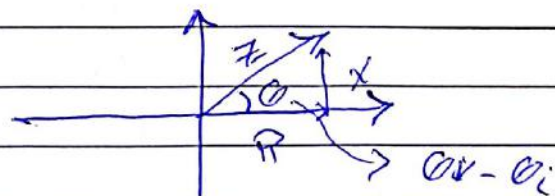
$$= \underbrace{|V_{eff}| |I_{eff}| \cos(\theta - \phi)}_P + j \underbrace{|V_{eff}| |I_{eff}| \sin(\theta - \phi)}_Q$$

Complex Power  $V_{eff} I_{eff}$  (VA)  $\rightarrow$  Reactive Power  $|V_{eff}| |I_{eff}| \sin(\theta - \phi)$  (VARS)

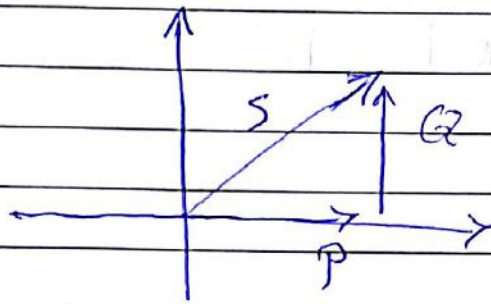
Real Power (average Power)  $|V_{eff}| |I_{eff}| \cos(\theta - \phi)$  (W)

$$|S| = |I_{eff}| |V_{eff}|$$

$$Z = R + jX$$



⇒ Power triangle :-

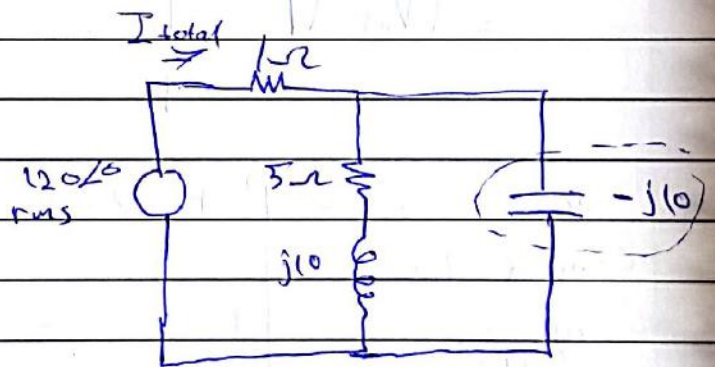


$$S = P + jQ$$

ex) Find the complex Power produced/absorbed by the sources & the elements :-

$$I_{total} = \frac{120 \angle 0}{1 + \frac{-j10(5+j10)}{5}}$$

$$= 5.16 \angle 25.46^\circ \text{ A}$$



$$P_{avg(1\Omega)} = (I_{rms})^2 R$$

$$= (5.16)^2 (1) = 26.6 \text{ W}$$

$$S_{(1\Omega)} = V_{rms} I_{rms}^*$$

$$= 5.16 \angle 25.46^\circ * 5.16 \angle -25.46^\circ$$

$$= 26.6 \angle 0$$

$$\Rightarrow 26.6 + j0 \text{ (VA)}$$

No

$$I_{(-j10)} = ?? \rightarrow V_{rms}, I_{rms}$$

$$I_{(-j10)} = 5.16 \angle 25.46 \times \frac{5+j10}{5} \quad \left. \vphantom{I_{(-j10)}} \right\} \text{Current divider}$$
$$= 11.53 \angle 88.89 \text{ A}$$

$$V_{(-j10)} = (-j10) 11.53 \angle 88.89 = \underline{115.3} \angle -1.11$$

ohm's law  $V=IZ$

$$S_{(-j10)} = V_{eff} I_{eff}^*$$
$$= 115.3 \angle -1.11 \times 11.53 \angle -88.89$$
$$= 1331 \angle -90 = 0 - j1331 \text{ VA}$$

$$I_{(5+j10)} = ?? \quad V_{rms}, I_{rms}$$

$$I_{rms} = 5.16 \angle 25.46 \times \frac{-j10}{5} = 10.31 \angle -64.53 \text{ A} \quad \left. \vphantom{I_{rms}} \right\} \text{Current divider}$$

$$V_{rms} = (5+j10) (10.31 \angle -64.53)$$
$$= 115.27 \angle -1.095 \text{ V} \quad \left. \vphantom{V_{rms}} \right\} \text{ohm's law } V=IZ$$

$$S = 115.27 \angle -1.095 \times 10.31 \angle 64.53$$
$$= 531.48 + j1062.96 \text{ VA}$$

No \_\_\_\_\_

$$S_{\text{source}} = V_{\text{rms}} I_{\text{rms}}^*$$

$$= (120 \angle 0) (5.16 \angle -25.46)$$

$$= 558.98 - j266$$

$$P(11) = 26.6 \text{ W}$$

$$\rightarrow V_L = 60 \cos(\omega t - 10) \text{ V}$$

$$i_L = 1.5 \cos(\omega t + 50) \text{ A}$$

Complex Power :-

$$S = V_{\text{rms}} I_{\text{rms}}^*$$

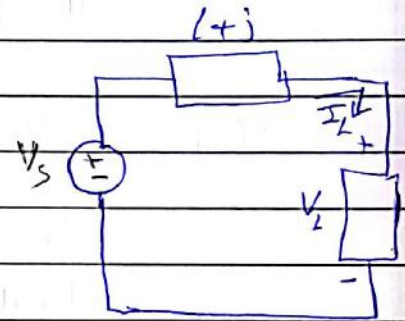
$$= \frac{60}{\sqrt{2}} \angle -10 \times \frac{1.5}{\sqrt{2}} \angle -50$$

$$= 45 \angle -60$$

$$= 45 (\cos(-60) + j \sin(-60))$$

$$= 45 \cos(-60) + j 45 \sin(-60)$$

$$= (22.5 - j 39) \text{ VA}$$



if  $Q < 0 \Rightarrow$  capacitive load

if  $Q > 0 \Rightarrow$  Inductive load

$$PF = \cos(-10 - 50) = 0.5 \text{ Leading}$$

$\Rightarrow$  Find complex power (S) source :-

$$\text{KVL} \Rightarrow V_s = \frac{60}{\sqrt{2}} \angle -10 + \left( \frac{1.5}{\sqrt{2}} \angle 50 \right) * (1+j) = 42 \angle -8.01$$

$$S = 42 \angle -8 \times \frac{1.5}{\sqrt{2}} \angle -50 = 44.5 \angle -58$$

$$PF_{\text{source}} = \cos(-8 - 50) = 0.53 \text{ leading}$$

\* PF correction :-

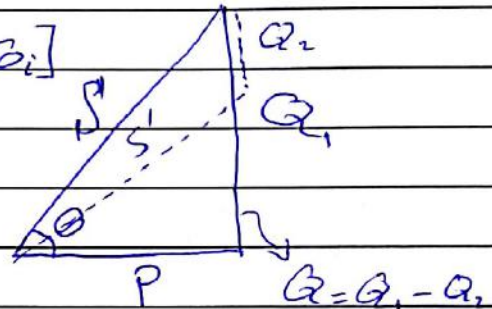
its an Increasing of the load PF while while maintaining Same load Voltage & current

$PF = \cos(\theta)$

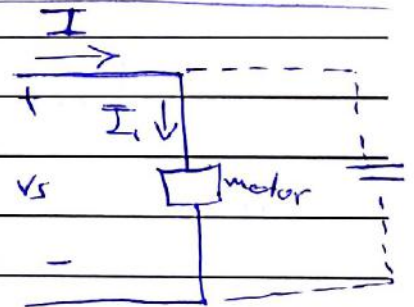
$[\theta = \theta_v - \theta_i]$

$PF' = \cos(\theta')$

$PF' > PF$



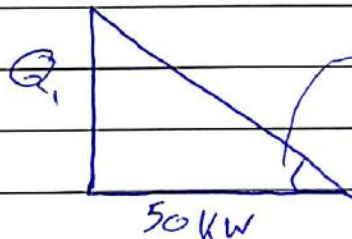
Induction motor operating at 50 kW & 0.8 lagging PF &  $V_s = 230$  Vrms.



We need to make the PF = 0.95

Sol :-

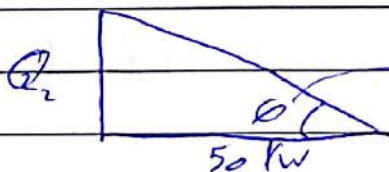
⇒ add a corrective device [capacitor]



$\cos^{-1}(0.8) = 36.9^\circ$

$\frac{Q_1}{P} = \tan(36.9^\circ)$

$Q_1 = 37.5 \text{ KVAR}$



$\cos^{-1}(0.95)$

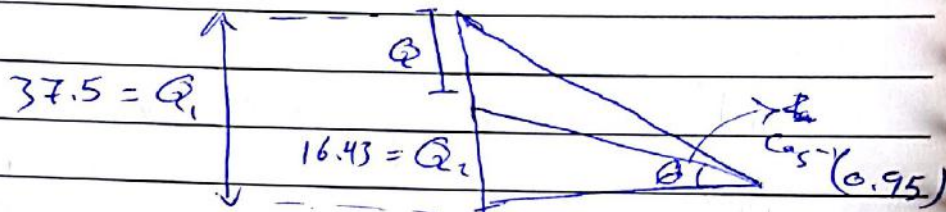
~~Q1~~

→ follow



$$\frac{Q_2}{50 \text{ kW}} = \tan(\cos^{-1}(0.95)) = \tan(18.19)$$

$$Q_2 = 16.43 \text{ kVAR}$$



$$S_1 = P + jQ, \quad |S_1| = \sqrt{P^2 + Q^2}$$

$$|Q_c| = Q_1 - Q_2$$

$$C = ??$$

$$|Q_c| = |V_{rms}| |I_{rms}| \sin(\theta - \phi) \xrightarrow{90^\circ}$$

$$= |V_{rms}| |I_{rms}|$$

$$= |V_{rms}| |V_{rms}| \omega C$$

$$= |V_{rms}|^2 \omega C$$

$$\left. \begin{aligned} |V_{rms}| &= |I_{rms}| |X_c| \\ &= |I_{rms}| \frac{1}{\omega C} \end{aligned} \right\}$$

$\omega = 2\pi f$   
rad/sec  
1 Hz =  $2\pi$  rad/sec

$$C = \frac{|Q_c|}{|V_{rms}|^2 \omega} = \frac{P(\tan \theta_1 - \tan \theta_2)}{|V_{rms}|^2 \omega}$$

Review

$$V(t) = 5 + 5 \sin(314t)$$

(جس کا ایک حصہ  $\pi/8$  کا  $\Delta$ )

Find  $V_{avg}$  &  $V_{rms}$  ??

sol:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{314}$$

$$V_{rms} = \left( \frac{314}{2\pi} \int_0^{2\pi/314} (5 + 5 \sin(314t))^2 dt \right)^{1/2}$$

$$V_{rms} = \sqrt{25 + \frac{25}{2}}$$

No \_\_\_\_\_

Suggested Problems : (hyat Fadithr)

14, 16, 20, 26, 31, 36, 39, 42, 44, 45, 52

$$v(t) = 5 + 5 \sin(31.4t)$$

$$V_{eff} = \sqrt{V_{eff_1}^2 + V_{eff_2}^2} = \sqrt{25 + \left(\frac{5}{\sqrt{2}}\right)^2}$$

$$V_{avg} = 5$$

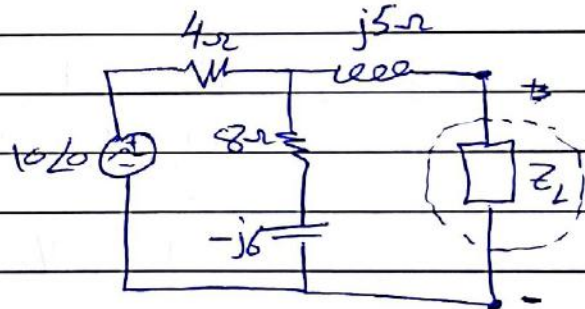
\* Determine the load impedance that maximize the average power drawn from the ckt & what is the maximum average power.

sol

$Z_{th} :$

$$Z_{th} = \frac{j5 + 4(8 - j6)}{4 + 8 - j6}$$

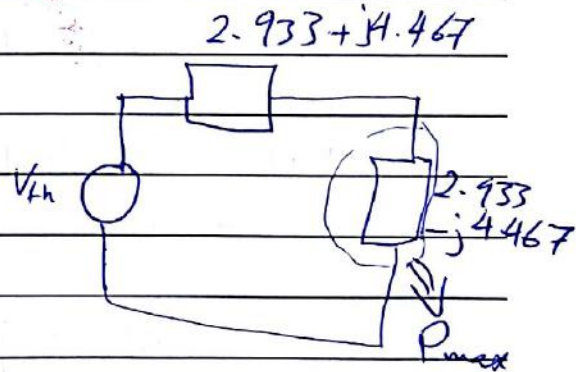
$$= 2.933 + j4.467 \Omega$$



$$Z_L = Z_{th}^* = 2.933 - j4.467 \Omega$$

$$V_{th} = 10 \angle 0 \times \frac{(8 - j6)}{8 - j6 + 4}$$

$$= 7.454 \angle -0.3$$



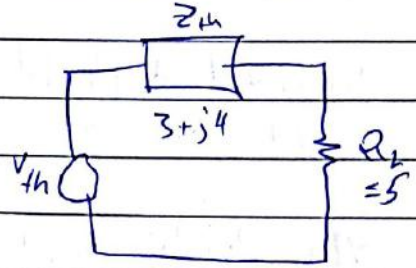
(average power)  $\frac{1}{2}$

$$P_{max} = \frac{1}{2} V_L I_L \cos(\phi - \phi) = \frac{1}{2} \times \frac{V_{th}}{2} \times \frac{V_{th}}{2R}$$

$$= \frac{|V_{th}|^2}{8R} = 2.368 \text{ W}$$

$$\frac{dP}{dX_L} \Rightarrow X_{th} = -X_L$$

$$\frac{dP}{dR_L} \Rightarrow R_{th} = R_L$$



$$R_L = \sqrt{R_{th}^2 + (X_{th} + X_L)^2}$$

if  $Z_L$  is pure resistive:

$$R_L = \sqrt{R_{th}^2 + X_{th}^2} = \|Z_{th}\|$$

Q) Find the average power across a 4  $\Omega$  resistor when:

①  $V(t) = 8 \sin(100t) + 6 \cos(200t - 45^\circ)$

Sol:

$$P_{total} = P_{avg1} + P_{avg2}$$

$$= \frac{1}{2} (I_{m1}^2 + I_{m2}^2) R$$

$$* = \frac{1}{2} (V_{m1}^2 + V_{m2}^2 + \dots) / R = \frac{1}{2} \frac{(8)^2}{4} + \frac{1}{2} \frac{(6)^2}{4} = 12.5 \text{ W}$$

②  $V(t) = 8 \sin 200t - 6 \cos(200t - 45^\circ)$

Sol:

$$v(t) = 8 \sin 200t - 6 [\cos 200t \cos 45^\circ + \sin 200t \sin 45^\circ]$$

$$= 8 \sin 200t - \frac{6}{\sqrt{2}} \cos 200t - \frac{6}{\sqrt{2}} \sin 200t$$

$$= \left(8 - \frac{6}{\sqrt{2}}\right) \sin 200t - \frac{6}{\sqrt{2}} \cos 200t$$

$$= 3.75 \sin 200t - 4.24 \cos 200t$$

$$= \sqrt{(3.75)^2 + (4.24)^2} \cos(200t - \tan^{-1}\left(\frac{-4.24}{3.75}\right))$$

$$P_{avg} = \frac{((3.75)^2 + (4.24)^2)}{4} = 4.01$$

$$A \sin \omega t + B \cos \omega t = C \cos(\omega t - \phi)$$

$$C = \sqrt{A^2 + B^2}$$

$$\phi = \tan^{-1}\left(\frac{B}{A}\right)$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

No

$$\textcircled{3} \quad v(t) = 8 \sin 200t - 6 \cos (200t - 45^\circ) + 4$$

$$4.0 \text{ W}$$

$$\frac{V^2}{R} = \frac{16}{4} = 4$$

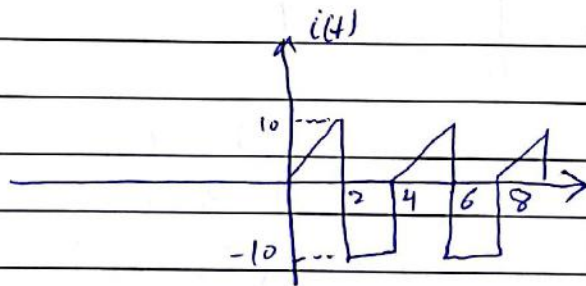
$$4.0 \text{ W} + 4 = 8.0 \text{ W}$$

$$v(t) = 100 \cos(\omega t - 10^\circ)$$

$$i(t) = 2 \sin(\omega t + 30^\circ) = 2 \cos(\omega t + 30^\circ - 90^\circ) = 2 \cos(\omega t - 60^\circ)$$

$$\text{PF} = \cos(-10^\circ + 60^\circ) = \cos(50^\circ) \text{ lagging}$$

Find the rms value :-



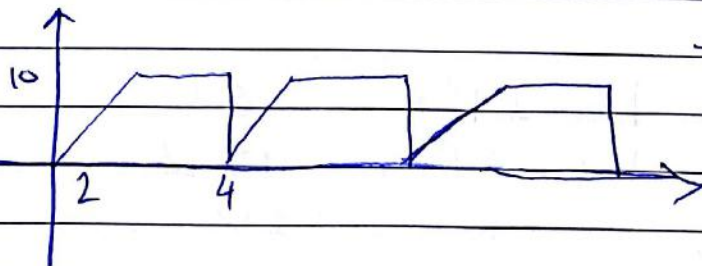
Sol 3

$$T = 4$$

$$i(t) = \begin{cases} 5t, & 0 < t < 2 \\ -10, & 2 < t < 4 \end{cases}$$

$$I_{\text{rms}} = \sqrt{\frac{1}{4} \left[ \int_0^2 (5t)^2 dt + \int_2^4 (-10)^2 dt \right]}$$

$$= 8.165 \text{ A}$$



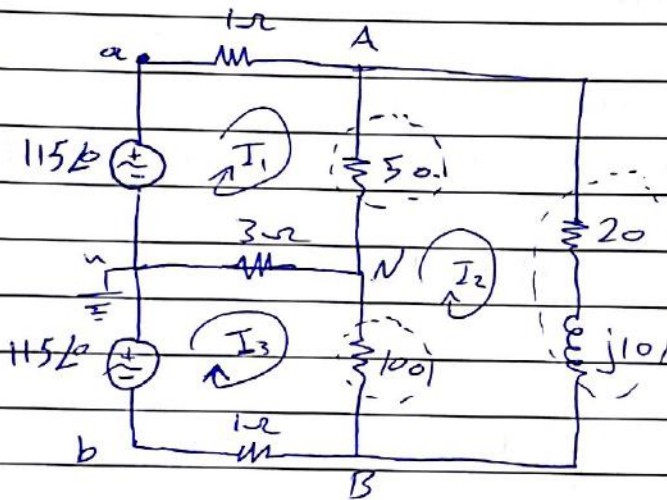
-: 5 A, 4

No \_\_\_\_\_

دوائر كهرلبيه

Poly phase systems :-

Single phase 3-wire system :-



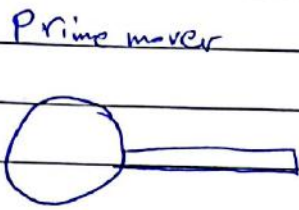
$$\begin{aligned}
 -115 \angle 0 + I_1 + 50(I_1 + -(I_2)) + 3(I_1 - I_3) &= 0 \\
 (20 + j10)I_2 + 100(I_2 - I_3) + 50(I_2 - I_1) &= 0 \\
 -115 \angle 0 + 3(I_3 - I_1) + 100(I_3 - I_2) + I_3 &= 0
 \end{aligned}$$

mesh currents

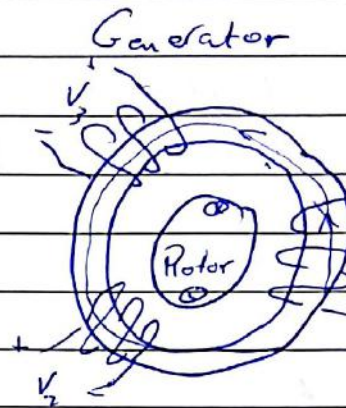
$$\begin{cases}
 I_1 = 11.24 \angle -19.83^\circ \text{ A} \\
 I_2 = 9.38 \angle -24.47^\circ \text{ A} \\
 I_3 = 10.37 \angle -21.8^\circ \text{ A}
 \end{cases}$$

$$\begin{aligned}
 I_{aA} &= \text{current flows from a to A} = I_1 = 11.24 \angle -19.83^\circ \\
 I_{bB} &= \text{" " " " b " B} = -I_3 = 10.37 \angle 158.2^\circ \\
 I_{nN} &= \text{" " " " n " N} = I_3 - I_1 = 0.9459 \angle -177^\circ
 \end{aligned}$$

$$\begin{aligned}
 P_{(50)} &= |I_1 - I_2|^2 R = 206 \text{ W} \\
 P_{(100)} &= |I_3 - I_2|^2 R = 117 \text{ W} \\
 P_{(20+j10)} &= I_2^2 * R = 1763 \text{ W}
 \end{aligned}$$



Prime mover



Generator

$$\phi = \sin \omega t$$

$$V = -\frac{dN\phi}{dt}$$

کوس  
دائے  
تاریخ

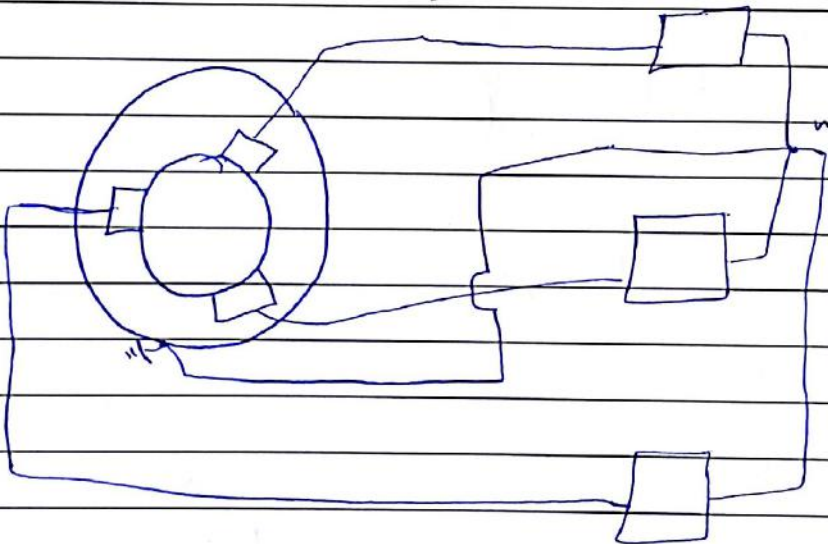
$$|V_1| = |V_2| = |V_3|$$

STATOR: Non-moving part

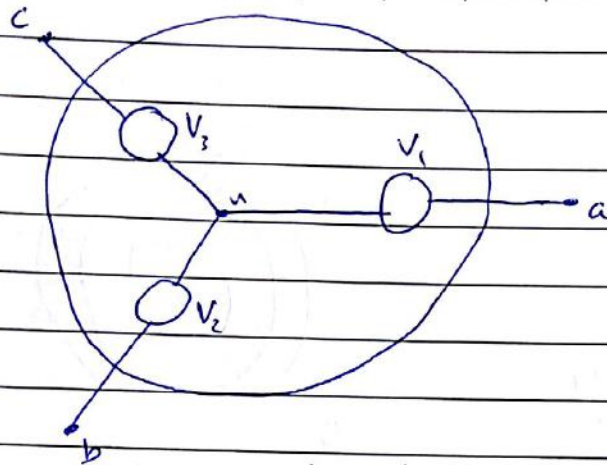
3 phase system:-

- ① overall system is more economical
- ② Less generates vibration (net torque = 0)

⇒ more stable system  
جائزہ



No \_\_\_\_\_



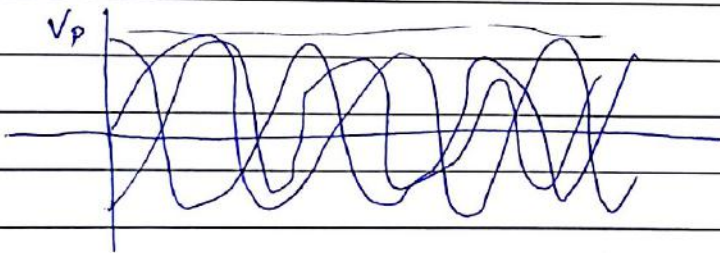
$$|V_{an}| = |V_{bn}| = |V_{cn}|$$

$$V_{an} = V_p \sin \omega t$$

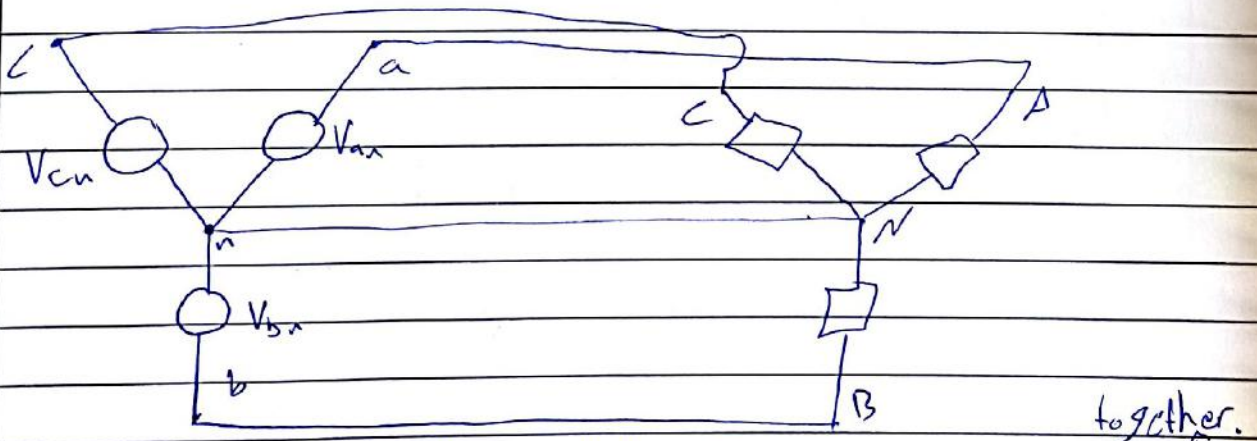
$$V_{bn} = V_p \sin(\omega t - 120^\circ)$$

$$V_{cn} = V_p \sin(\omega t - 240^\circ) = V_p \sin(\omega t + 120^\circ)$$

Phase shift between them is  $120^\circ$



Y-Y Balanced 3 $\phi$  system:-



Y-source : the neutral of the 3 sources are connected together.

Y-load : the neutral of the 3 loads are connected together

No \_\_\_\_\_

$$V_a = V_p \angle 0$$

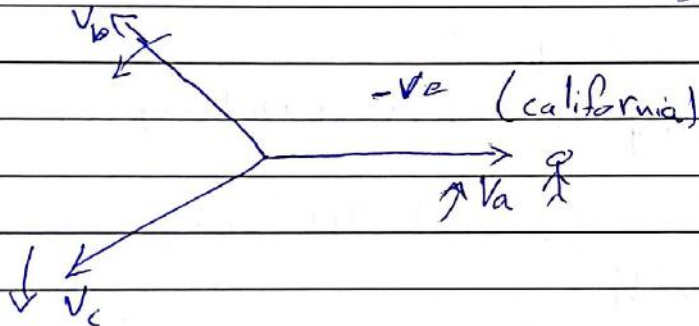
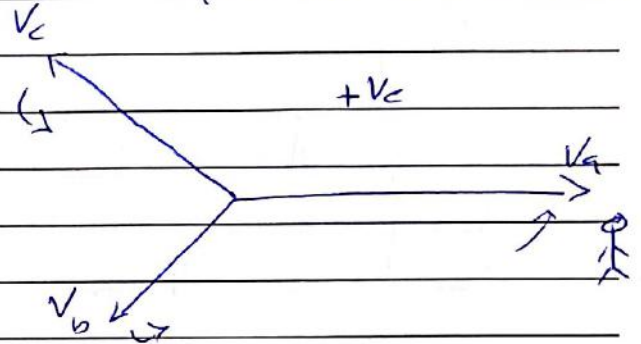
$$V_b = V_p \angle -120$$

$$V_c = V_p \angle +120$$

(the rest of the world)

$$|V_a| = |V_b| = |V_c|$$

$$V_a + V_b + V_c = 0$$



$$V_{an} = V_p \angle 0$$

$$V_{bn} = V_p \angle -120$$

$$V_{cn} = V_p \angle +120$$

$$V_{ab} = V_{an} - V_{bn} = V_p \angle 0 - V_p \angle -120$$

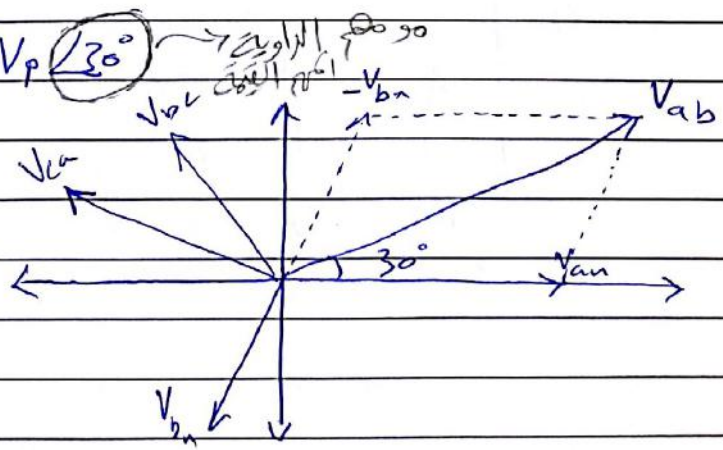
$$= V_{an} + (-V_{bn}) = V_p - V_p [\cos(-120) + j \sin(-120)]$$

$$= V_p \left[ 1 + \frac{1}{2} + j \frac{\sqrt{3}}{2} \right] = V_p \left[ \frac{3}{2} + j \frac{\sqrt{3}}{2} \right]$$

$$= \sqrt{3} V_p \left[ \frac{\sqrt{3}}{2} + j \frac{1}{2} \right] = \sqrt{3} V_p [\cos 30 + j \sin 30]$$

$$= \sqrt{3} V_p \angle 30^\circ$$

Load 1) (the rest of the world)  
 Load 2) (california)  
 Load 3) (elevator)





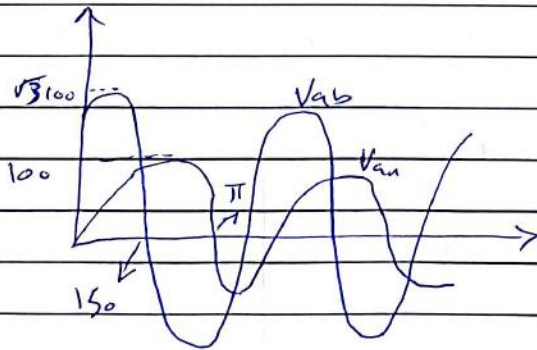
No \_\_\_\_\_

$V_{an}$  } phase voltages phase-to-neutral voltages  
 $V_{bn}$  }  
 $V_{cn}$  }

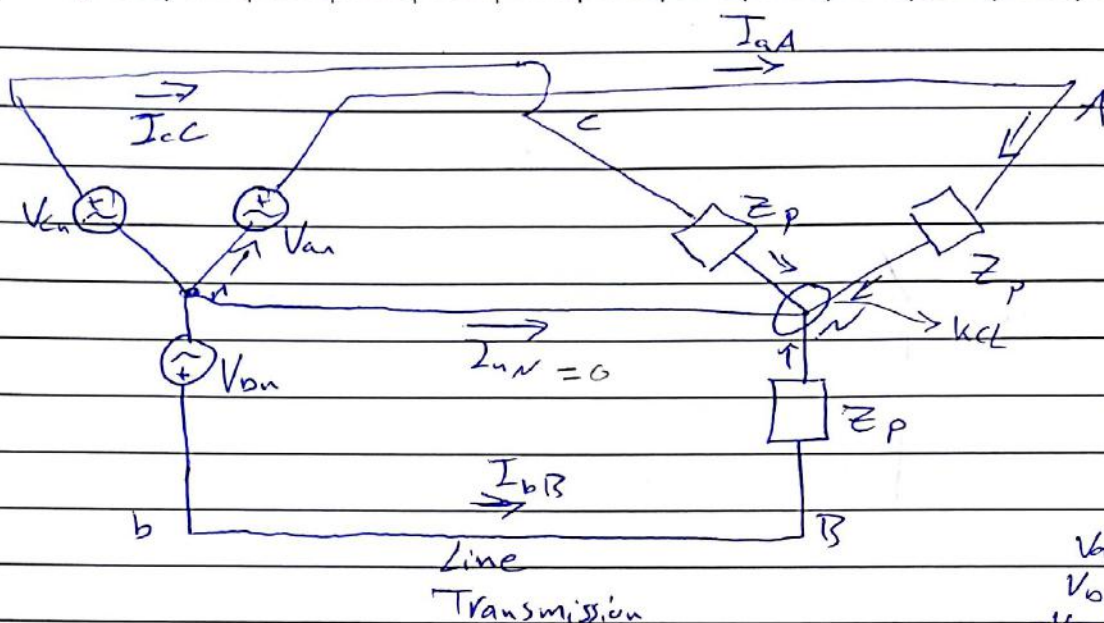
$V_{ab}$  } phase-to-phase  
 $V_{ca}$  } line-to-line  
 $V_{ba}$  } voltages

$$\begin{aligned} V_{bc} &= V_{bn} - V_{cn} = V_p \angle -120^\circ - V_p \angle 120^\circ \\ &= V_p [\cos(-120^\circ) + j\sin(-120^\circ)] - V_p [\cos(120^\circ) + j\sin(120^\circ)] \\ &= V_p \left(-\frac{1}{2}\right) - jV_p \left(\frac{\sqrt{3}}{2}\right) - V_p \left(-\frac{1}{2}\right) - jV_p \left(\frac{\sqrt{3}}{2}\right) \\ &= -j\sqrt{3}V_p \\ &= \sqrt{3}V_p \angle -90^\circ \end{aligned}$$

$$\begin{aligned} V_{ca} &= V_{cn} - V_{an} = V_p \angle +120^\circ - V_p \angle 0^\circ = V_p \angle 120^\circ - V_p \angle 0^\circ \\ &= V_p [\cos 120^\circ + j\sin 120^\circ] - V_p \\ &= -\frac{1}{2}V_p + j\frac{\sqrt{3}}{2}V_p - V_p = -\sqrt{3}V_p \left[\frac{-\sqrt{3}}{2} + j\frac{1}{2}\right] \\ &= \sqrt{3}V_p \angle 150^\circ \end{aligned}$$



No



$$\begin{aligned} V_{cn} &= V_p \angle 0 \\ V_{bn} &= V_p \angle -120 \\ V_{cn} &= V_{cn} \angle +120 \end{aligned}$$

$$\begin{aligned} V_{an} &= I_{aA} Z_p \\ I_{aA} &= \frac{V_{an}}{Z_p} \end{aligned} \quad \left. \begin{aligned} I_{bB} &= \frac{V_{bn}}{Z_p} = \frac{V_{an} \angle -120}{Z_p} = I_{aA} \angle -120 \end{aligned} \right\}$$

$$I_{cc} = \frac{V_{cn}}{Z_p} = \frac{V_{an} \angle +120}{Z_p} = I_{aA} \angle +120$$

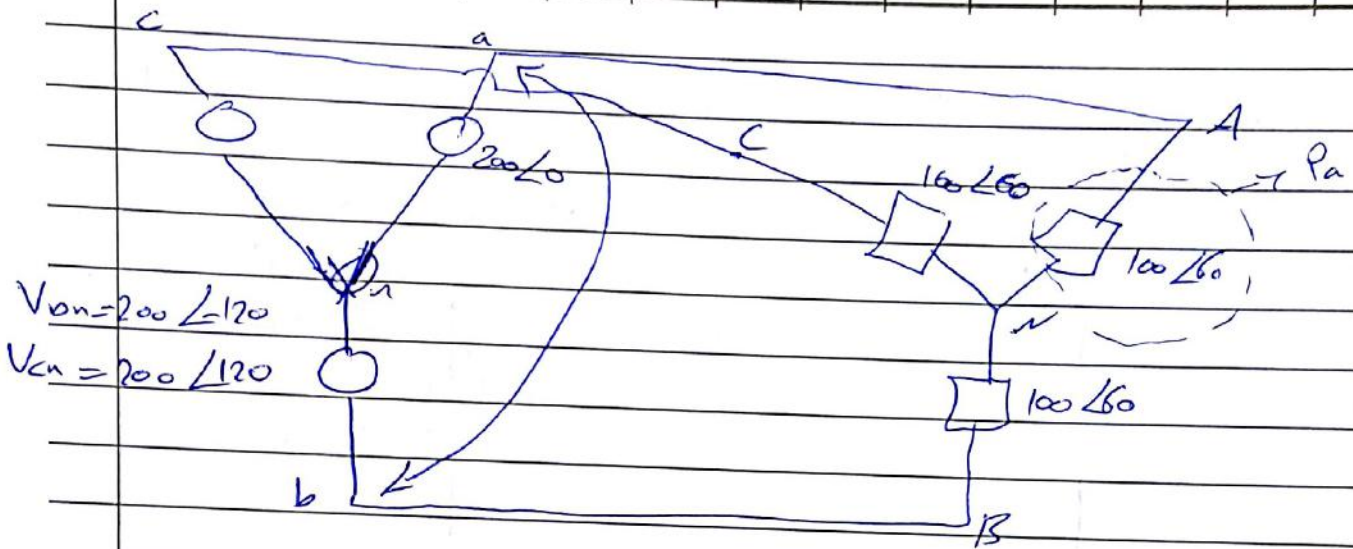
$$I_{aA} = I_{max} \angle 0 \quad / \quad I_{bB} = I_{max} \angle -120 \quad / \quad I_{cc} = I_{max} \angle +120$$

$$\begin{aligned} I_{nN} &= -(I_{aA} + I_{bB} + I_{cc}) = -(I_{max} + I_{max} \angle -120 + I_{max} \angle +120) \\ &= -(I_{max} + I_{max} \cos(-120) + j I_{max} \sin(-120) + I_{max} \cos(120) + j I_{max} \sin(120)) \\ &= 0 \quad \text{[For Balanced system } I_{nN} = 0 \text{]} \end{aligned}$$

Q) A balanced 3φ Y-Y connected system with  $V_{an} = 200 \angle 0$  (RMS)  
 $Z_p = 100 \angle 60$ , find the line and phase current & the line & phase voltages:

→ follow

No \_\_\_\_\_



$$V_{ab} = \sqrt{3} V_{an} \angle 30^\circ = \sqrt{3} * 200 \angle 30^\circ = 346 \angle 30^\circ$$

$$V_{bc} = 346 \angle 90^\circ$$

$$V_{ca} = 346 \angle 150^\circ$$

Mag. of line is  $\sqrt{3}$  times,  $30^\circ$  (phase angle,  $30^\circ$  phase angle)

$$I_{aA} = \frac{V_{an}}{Z_p} = \frac{200 \angle 0^\circ}{100 \angle 60^\circ} = 2 \angle -60^\circ \text{ A}$$

$$I_{bB} = \frac{V_{bn}}{Z_p} = \frac{200 \angle -120^\circ}{100 \angle 60^\circ} = 2 \angle -180^\circ = 2 \angle 180^\circ = I_{aA} \angle -120^\circ$$

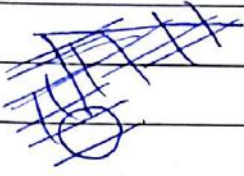
$$I_{cC} = I_{aA} \angle 120^\circ = 2 \angle 60^\circ \text{ A}$$

$$= \frac{V_{cn}}{Z_p} = \frac{200 \angle 120^\circ}{100 \angle 60^\circ} = 2 \angle 60^\circ \text{ A}$$

+ve is,  $I_{aA}$  is  $120^\circ$  phase plus  $3$  small times  $120^\circ$   $I_{cc}$ ,  $-120^\circ$   $I_{bb}$  is

line current / phase current for Y- $\Delta$  connection

No \_\_\_\_\_



$$P_a = V_{rms} I_{rms} \cos(\theta - \phi)$$

$$= 200 \times 2 \times \cos(60 - (-60)) = 200 \text{ W average Power}$$

$$P_b = 200 \times 2 \times \cos(-120 - (-180)) = 200 \text{ W}$$

$$P_c = 200 \text{ W}$$

$$P_{3\phi} = P_a + P_b + P_c = 600 \text{ W} \Rightarrow 3P_{1\phi}$$

$$V_{AN}(t) = 200\sqrt{2} \cos(\omega t)$$

$$i_{AN}(t) = 2\sqrt{2} \cos(\omega t - 60)$$

$$P_{AN}(t) = V_{AN}(t) i_{AN}(t) = 200 \cos \omega t \cos(\omega t - 60)$$

$$= 200 + 400 \cos(2\omega t - 60) \text{ W}$$

$$P_{BN}(t) = V_{BN}(t) i_{BN}(t) = 200\sqrt{2} \cos(\omega t - 120) \times 2\sqrt{2} \cos(\omega t - 180)$$

$$= 200 + 400 \cos(2\omega t + 60) \text{ W}$$

$$P_{CN}(t) = V_{CN}(t) i_{CN}(t) = 200 + 400 \cos(2\omega t + 180)$$

$$P_{3\phi}(t) = P_A(t) + P_B(t) + P_C(t) = 600 + 400 \cos(2\omega t - 60) + 400 \cos(2\omega t + 60)$$

$$+ 400 \cos(2\omega t + 180)$$

$$= 600 \text{ W} \} \Rightarrow \text{less generator vibration}$$

For comparison:-

if it was single phase system:

$$600 = 200 \times I_{rms} \times \cos(60) \Rightarrow I_{rms} = 6$$

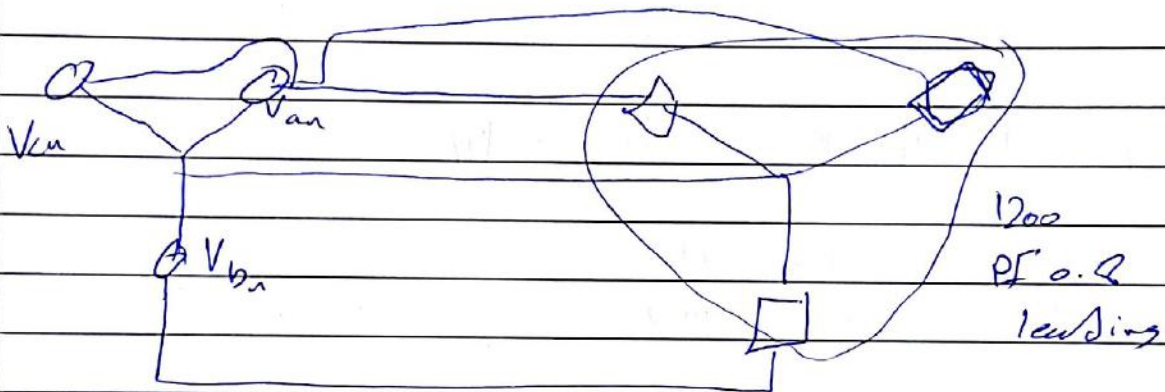
$$P_{losses 3\phi} = I_a^2 R + I_b^2 R + I_c^2 R =$$

$$4 \times R + 4 \times R + 4 \times R = 12 \text{ RW}$$

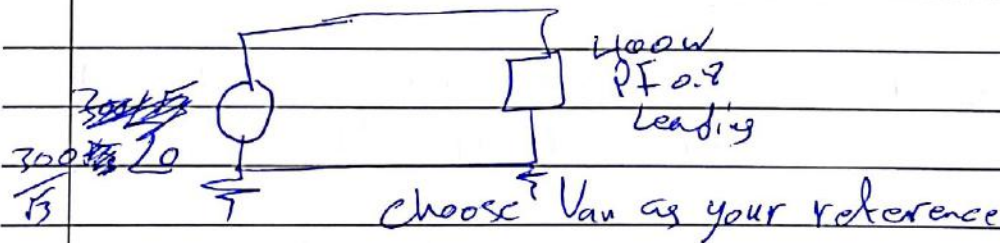
$P_{\text{losses}} = I^2 R = 36R$

(if not specified in Q)  
Line-to-line

3φ (if not specified) → a balanced 3φ system with  $V_L = 300V$  & supplying a balanced Y connected load with 1200w, leading PF of 0.8  
Find the line current & the per-phase load impedance



Single phase equivalent circuit



$P = V_{rms} I_{rms} \cos(\theta) \rightarrow PF$   
 $400 = \frac{300}{\sqrt{3}} \times I_{rms} \times 0.8 \Rightarrow I_{rms} = 2.89 A$

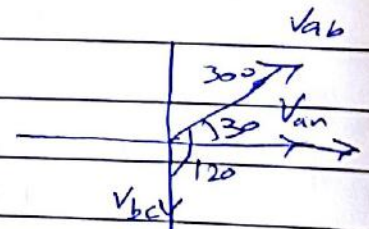
$I_{aA} = I_{rms} = 2.89 / \cos^{-1}(0.8) = 36.27$

$Z = \frac{V_{a_n}}{I_{a_n}} = \frac{300/\sqrt{3}}{2.89 / 36.87} = 60 \angle -36.9^\circ \Omega$

$I_{bB} = 2.89 \angle 36.9 - 120^\circ$  (phase shift)

$I_{cC} = 2.89 \angle 36.9 + 120^\circ$

$V_{a_n} = \frac{300}{\sqrt{3}} \angle -120^\circ \quad V_{bc} = 300 \angle -90^\circ$

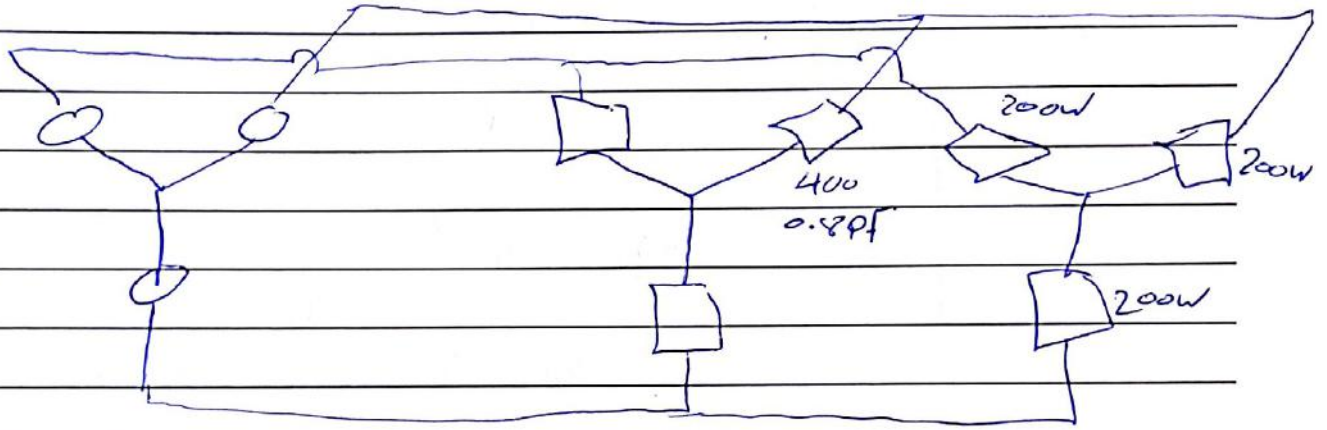


No \_\_\_\_\_

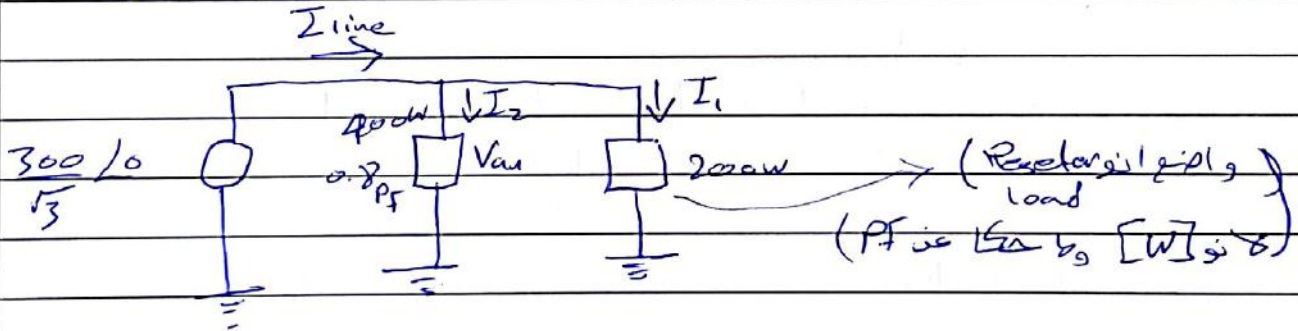
روادة →

(P<sub>3φ</sub>)

ex) a balanced load is added in parallel to the system, find the new line current. (مثال لزيادة الحمل)



sol :



$$I_{line} (A) = I_1 + I_2 = 2.89 \angle 36.9^\circ + I_1$$

$$I_1 = \frac{P}{V} = \frac{200}{300/\sqrt{3}} = 1.155 \angle 0^\circ$$

$$P = IV \cos \theta \rightarrow 1$$

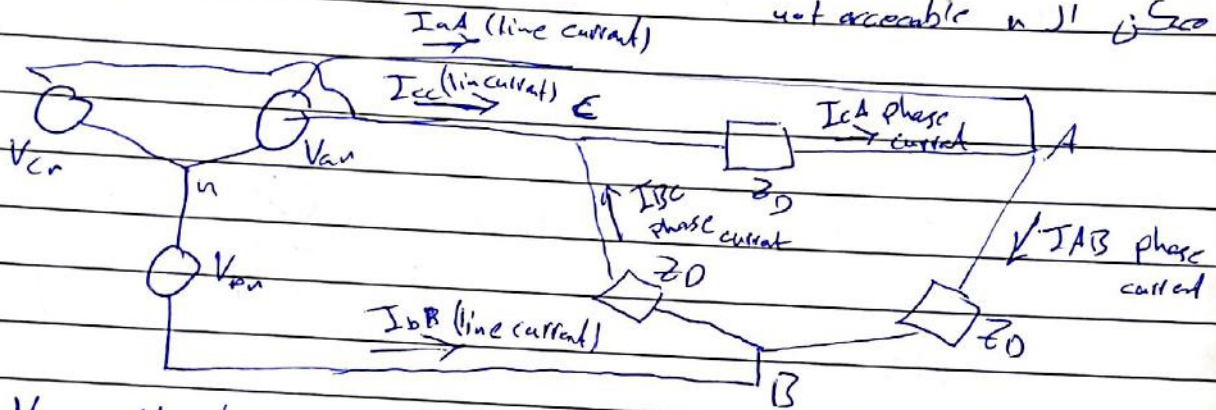
$$200 = \frac{300}{\sqrt{3}} I_1$$

$$\rightarrow I_{line} (A) = 2.89 \angle 36.9^\circ + 1.155 = 3.87 \angle 26.6^\circ A$$

$$I_{LB} = 3.87 \angle -93.4^\circ A, \quad I_{LC} = 3.87 \angle 146.6^\circ A$$

Y-source Delta load :-

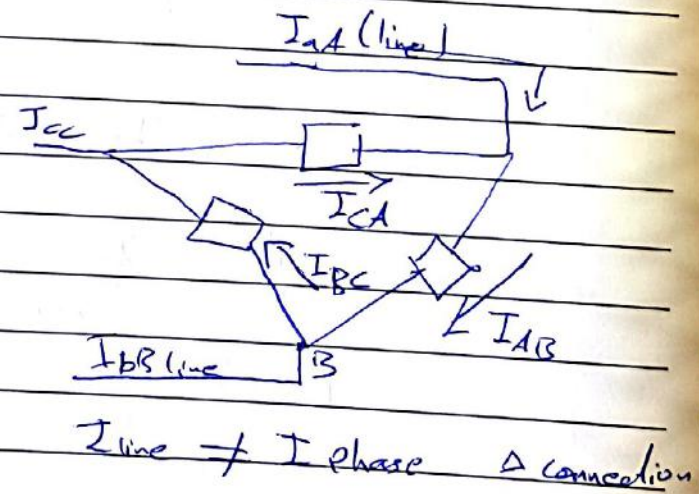
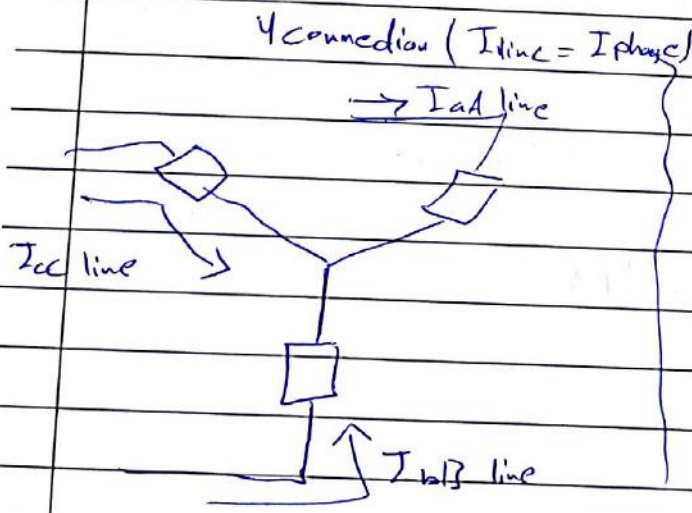
- زيادة الفولتج في load  
- not accessible n) I<sub>sc</sub>



$V_{an} = V_p \angle 0$  ,  $V_{bn} = V_p \angle -120$  ,  $V_{cn} = V_p \angle +120$   
 $V_{AB} = \sqrt{3} V_p \angle +30$

We use  $\Delta$ -load :-

- if the neutral point is not accessible
- Increase the load Voltage



⇒

$$|V_L| = |V_{AB}| = |V_{BC}| = |V_{CA}|$$

$$|V_p| = |V_{an}| = |V_{bn}| = |V_{cn}|$$

$$\neq V_L = \sqrt{3} V_p \angle +30^\circ \quad [\Delta \neq Y]$$

$V_L$   
 $\Rightarrow |V_L| = \sqrt{3} |V_p|$

$$I_{AB} = \frac{V_{AB}}{Z_A}, \quad I_{BC} = \frac{V_{BC}}{Z_A}$$

$\Rightarrow$  phase current  
~~phase current~~

[p = phase]

$$I_{CA} = \frac{V_{CA}}{Z_D}$$

$$I_{AB} = I_p \angle 0^\circ$$

$$I_{BC} = I_p \angle -120^\circ$$

$$I_{CA} = I_p \angle +120^\circ$$

$$I_{aA} = I_{AB} - I_{CA} = I_p \angle 0^\circ - I_p \angle +120^\circ$$

$$= I_p - I_p \cos(120^\circ) - j I_p \sin(120^\circ)$$

$$= I_p + \frac{1}{2} I_p - j \frac{\sqrt{3}}{2} I_p$$

$$= \left( \frac{3}{2} I_p - j \frac{\sqrt{3}}{2} I_p \right)$$

current line  $\rightarrow$   $= \sqrt{3} I_p \left( \frac{\sqrt{3}}{2} - j \frac{1}{2} \right) = \sqrt{3} I_p \angle -30^\circ$

$$I_{aA} = \sqrt{3} I_p \angle -30^\circ, \quad I_{bB} = \sqrt{3} I_p \angle -150^\circ, \quad I_{cC} = \sqrt{3} I_p \angle 90^\circ$$

Y connections (loads)

$\Delta$  connections: (loads)

$$V_{LL} = \sqrt{3} V_p \angle +30^\circ$$

$$V_{L(\text{line})} = V_p \text{ phase}$$

$$I_{L \text{ line}} = I_p \text{ phase}$$

$$I_L = \sqrt{3} I_p \angle -30^\circ$$



No \_\_\_\_\_

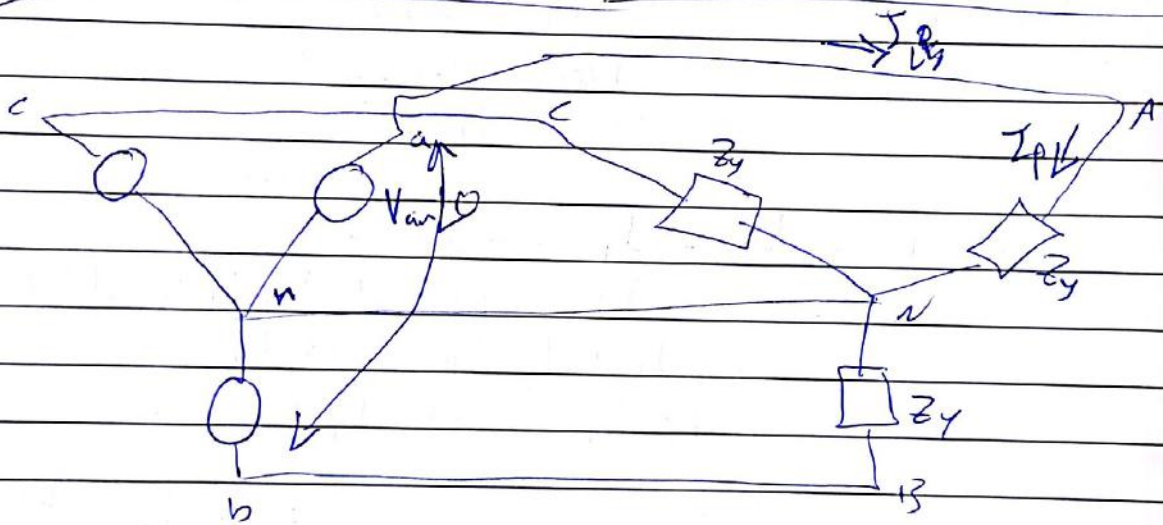
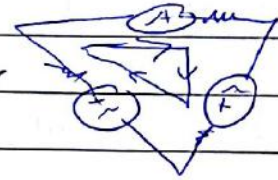
⇒ 4 types of connections :-

① Y Y Connection

② Y Δ //

③ Δ Y } circulating currents

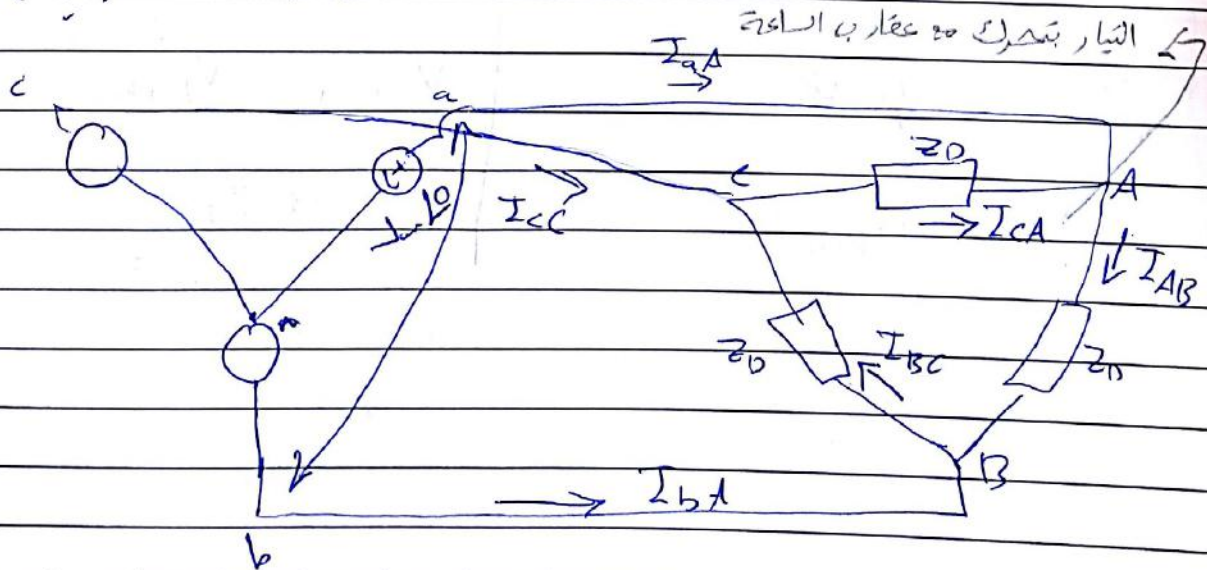
④ Δ Δ } source & load



$$I_{line} = I_{phase} (load) = \frac{V_{ph}}{Z_Y}$$

$$|V_{AR}| = \sqrt{3} V_{an} \angle +30^\circ$$

$$|V_{LL}| = \sqrt{3} V_p \angle +30^\circ$$



$$V_{line} = V_p(\text{Load}) \Rightarrow V_{line} = V_{phase}$$

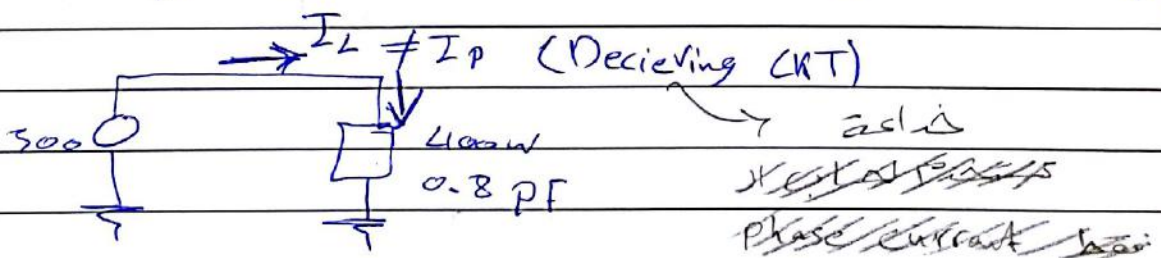
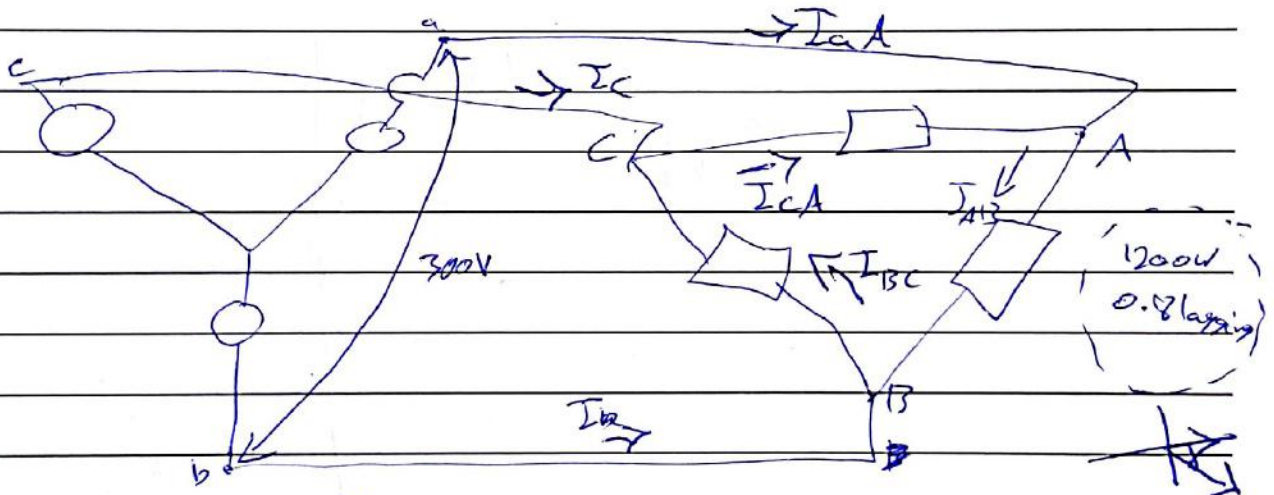
$$I_L = \sqrt{3} I_p \angle -30^\circ$$

$$I_{nA} = \sqrt{3} I_{AB} \angle -30^\circ$$

$$I_{AB} = \frac{V_{AB}}{Z_D}$$

$$I_{CA} = \sqrt{3} \left( \frac{V_{AB}}{Z_D} \right) \angle -30^\circ$$

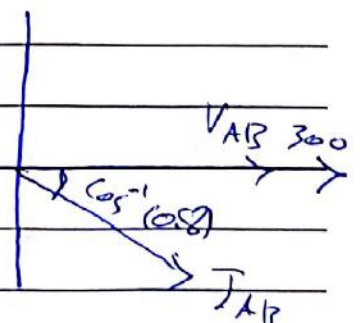
ex) what are the line currents in a 3φ system with line voltage of 300V & supplies 1200W to a Δ load at lagging PF of 0.8, & then find the phase impedance



$$P = V_{AB} I_{AB} \cos(\theta) \Rightarrow 400 = 300 \times I_{AB} \times 0.8$$

$$\Rightarrow I_{AB} = 1.667 \text{ A}$$

⇒ taking  $V_{AB}$  as your reference



$$I_{AB} = 1.667 \angle -36.9^\circ$$

$$I_{BC} = 1.667 \angle -36.9 - 120 = 1.667 \angle -156.9$$

$$I_{CA} = 1.667 \angle -36.9 + 120 = 1.667 \angle 83.1 = 1.667 \angle -277.9$$

} phase currents

Line currents :-

$$I_L = \sqrt{3} I_p \angle -30^\circ$$

$$I_a = \sqrt{3} I_{AB} \angle -30^\circ = 2.89 \angle -66.9^\circ$$

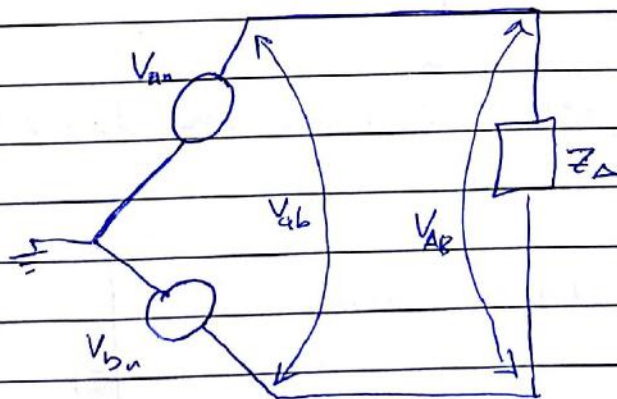
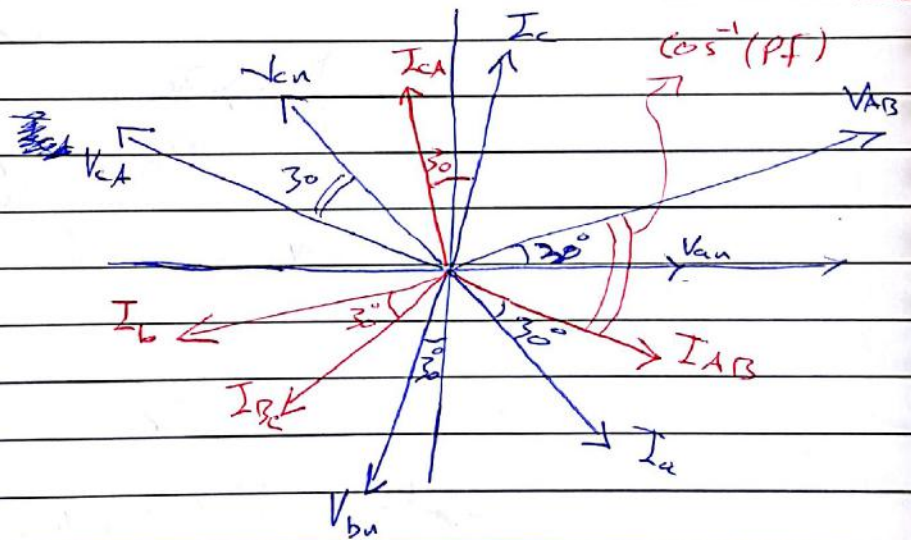
$$I_b = 2.89 \angle -66.9 - 120$$

$$I_c = 2.89 \angle -66.9 + 120$$

$\angle 120^\circ$  as  $\Delta$

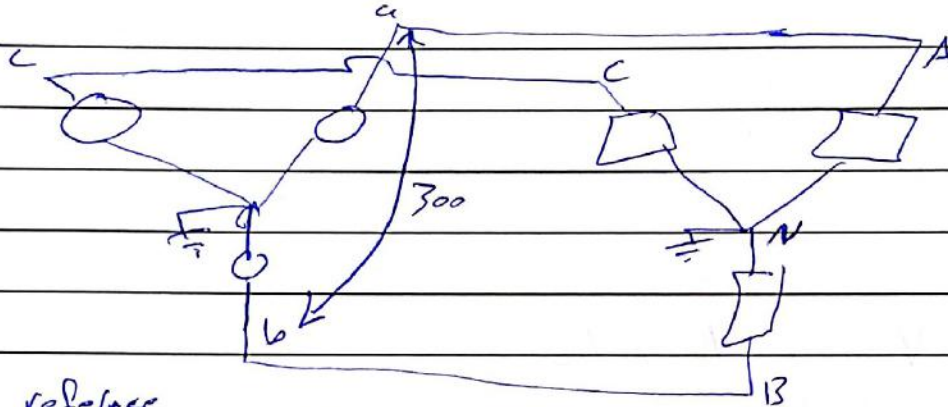
$\Delta$ -connected

$\cos^{-1}(PF)$

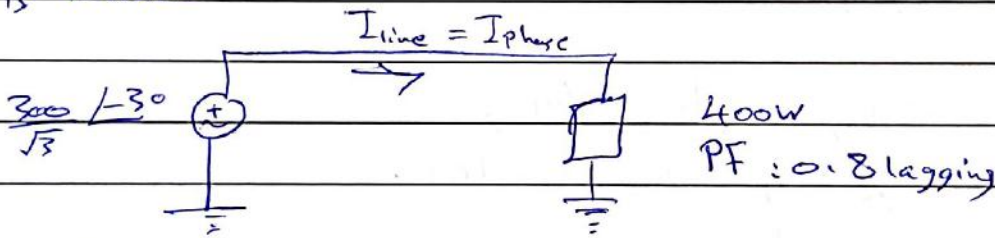


$$Z_p = \frac{V_{AB}}{I_{AB}} = \frac{300 \angle 0}{1.667 \angle -36.9} = 180 \angle 36.9$$

⇒ what if the load is connected Y



$V_{AB}$  is reference



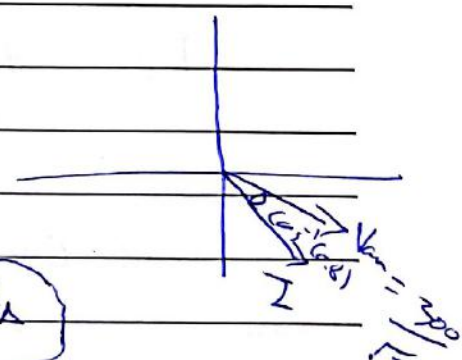
$$P = V_{an} I_a \cos(\theta) \rightarrow 400 = \frac{300}{\sqrt{3}} I_a (0.8)$$

$$I_a = 2.89 \text{ (phase)} = \text{(line)}$$

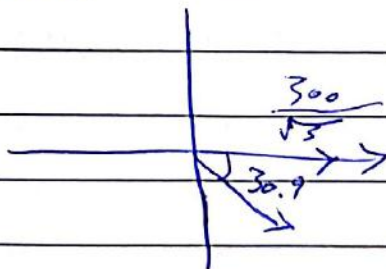
$$I_a = 2.89 \angle -30 - 36.9^\circ$$

$$Z_Y = \frac{V_{AN}}{I_{aA}} = \frac{300/\sqrt{3} \angle -30}{2.89 \angle -66.9} = 60 \angle 36.9$$

$$Z_{\Delta} = 3Z_Y \text{ OR } Z_Y = \frac{1}{3} Z_{\Delta}$$

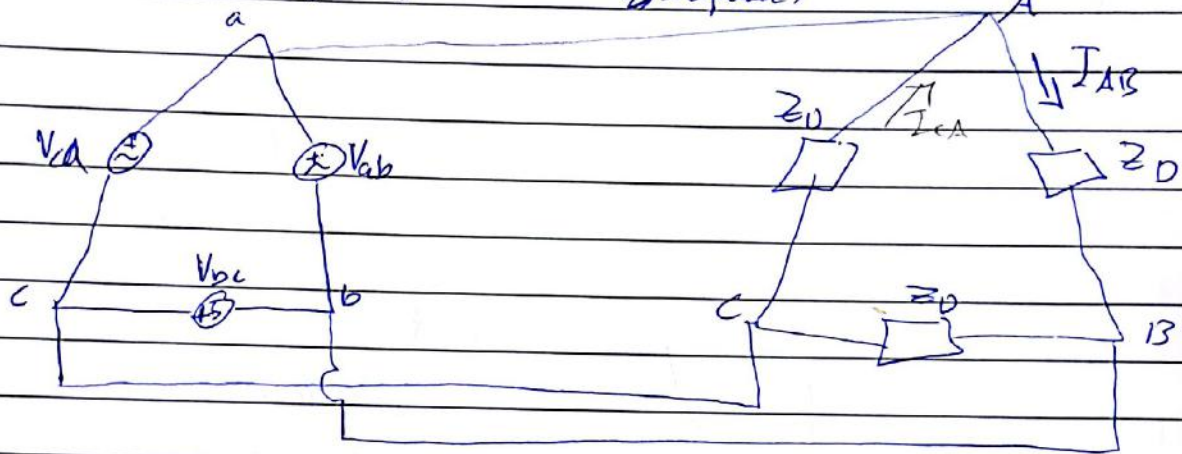


if  $V_{an}$  is the reference



No \_\_\_\_\_

$\Delta - \Delta$  Connection : ( +ve sequence )



انا دايع  
بهمي سنا  
load  
والقولنج

$$V_{ab} = V_{AB} , V_{bc} = V_{BC}$$

$$V_{ca} = V_{CA}$$

$\Rightarrow$  line voltage = phase voltage

$$\text{line-line} \leftarrow \frac{V_L}{\text{Load}} = V_{p||} \text{ (phase neutral)}$$

phase current

$$I_{AB} = \frac{V_{AB}}{Z_{\Delta}}$$

$$I_{BC} = \frac{V_{BC}}{Z_{\Delta}} = I_{AB} \angle -120^\circ$$

$$I_{CA} = \frac{V_{CA}}{Z_{\Delta}} = I_{AB} \angle +120^\circ \text{ } \left. \vphantom{I_{CA}} \right\} \text{ phase current}$$

line currents :-

$$I_L = \sqrt{3} I_p \angle -30^\circ$$

$$I_a = \sqrt{3} I_{AB} \angle -30^\circ$$

$$I_b = \sqrt{3} I_{BC} \angle -30^\circ = I_a \angle -120^\circ$$

$$I_c = \sqrt{3} I_{CA} \angle -30^\circ = I_a \angle +120^\circ$$

(ex) A  $\Delta$  system with a load impedance =  $20 - j15$  is connected to a source of with  $V_{ab} = 330 \angle 0$ , find  $I_{\text{phase}}$  &  $I_{\text{line}}$  solve phase currents :-

$$I_{AB} = \frac{V_{AB}}{Z_0} = \frac{330 \angle 0}{20 - j15} = 13.2 \angle 36.87^\circ \text{ A}$$

$$I_{BC} = 13.2 \angle -83.2 \angle -83.13^\circ \text{ A}$$

$$I_{CA} = 13.2 \angle 156.87^\circ$$

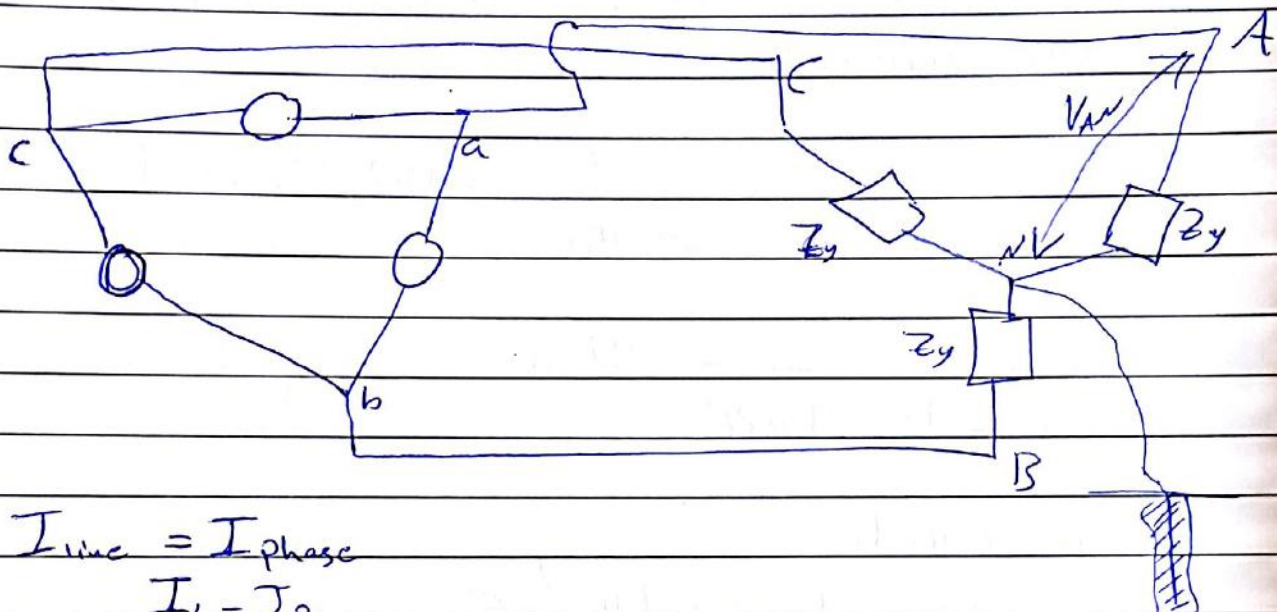
Line currents :-

$$I_a = I_{aA} = \sqrt{3} I_{AB} \angle -30^\circ$$
$$= 22.86 \angle 6.87^\circ$$

$$I_b = 22.86 \angle -113.13^\circ$$

$$I_c = 22.86 \angle +126.87^\circ$$

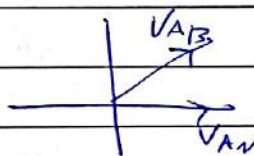
# \* Δ-Y Connections :-



$$I_{line} = I_{phase}$$

$$I_L = I_p$$

$$I_a = I_{an} = \frac{V_{AN}}{Z_y}$$

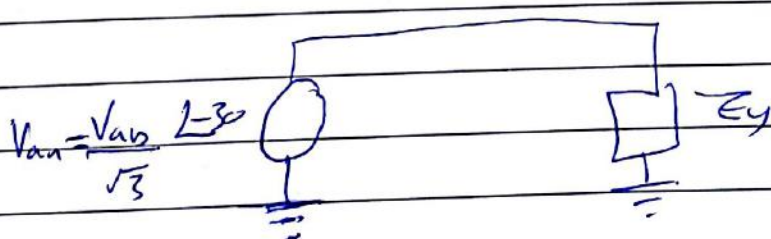


⇒ if given  $V_L = V_{ab} / 0$

$$V_p = \frac{V_{ab}}{\sqrt{3}} \angle -30^\circ$$

taking  $V_{AB}$  as your reference

$$I_a = \frac{V_{AN}}{Z_y} = \frac{V_{AB} / \sqrt{3} \angle -30^\circ}{Z_y}$$



(ex) A balanced Y connected load with  $Z_Y = 40 + j25$  is supplied by  $\Delta$ -source with a line voltage of 210V. calculate the phase currents  $\Rightarrow$  use  $V_{AB}$  as your reference. Sol ->

$$\Rightarrow V_L = V_{AB} = 210 \angle 0$$

$$\Rightarrow V_{AN} = \frac{210}{\sqrt{3}} \angle -30 = 121.2 \angle -30^\circ \text{ V}$$

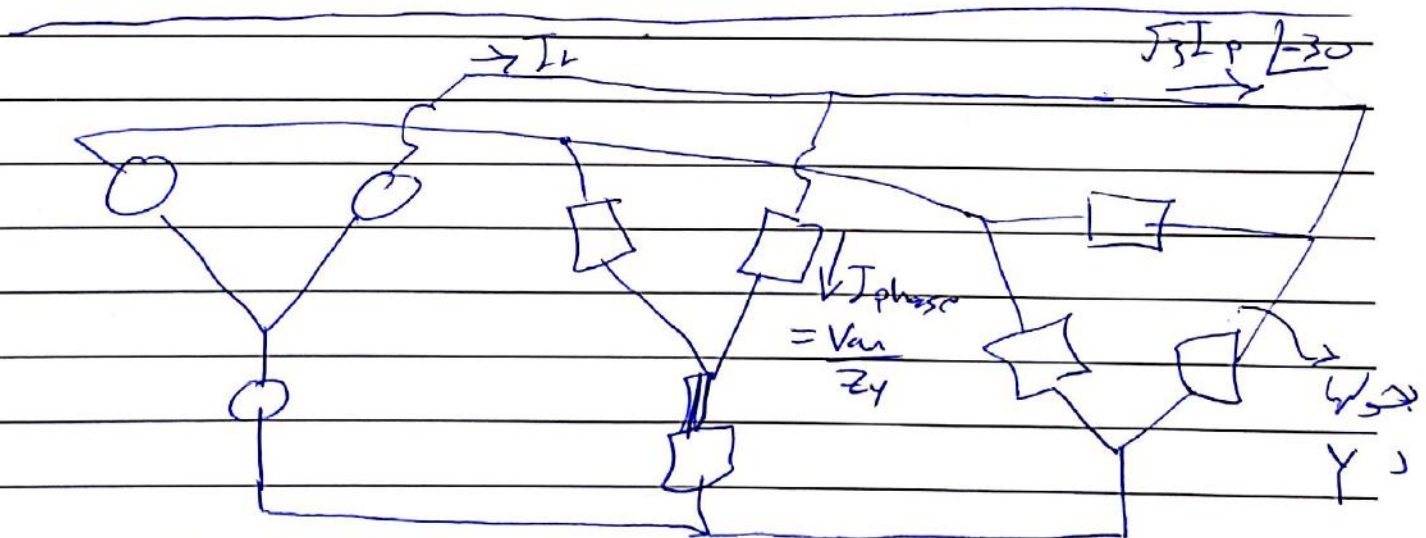
$$I_a = \frac{V_{AN}}{Z_Y} = \frac{121.2 \angle -30}{40 + j25} = 2.57 \angle -62^\circ$$

phase currents

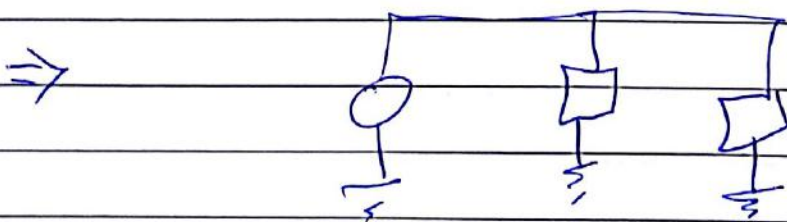
$$I_b = 2.57 \angle +178^\circ \text{ A}$$

$$I_c = 2.57 \angle +58^\circ \text{ A}$$

$$I_{line} = I_{phase}$$

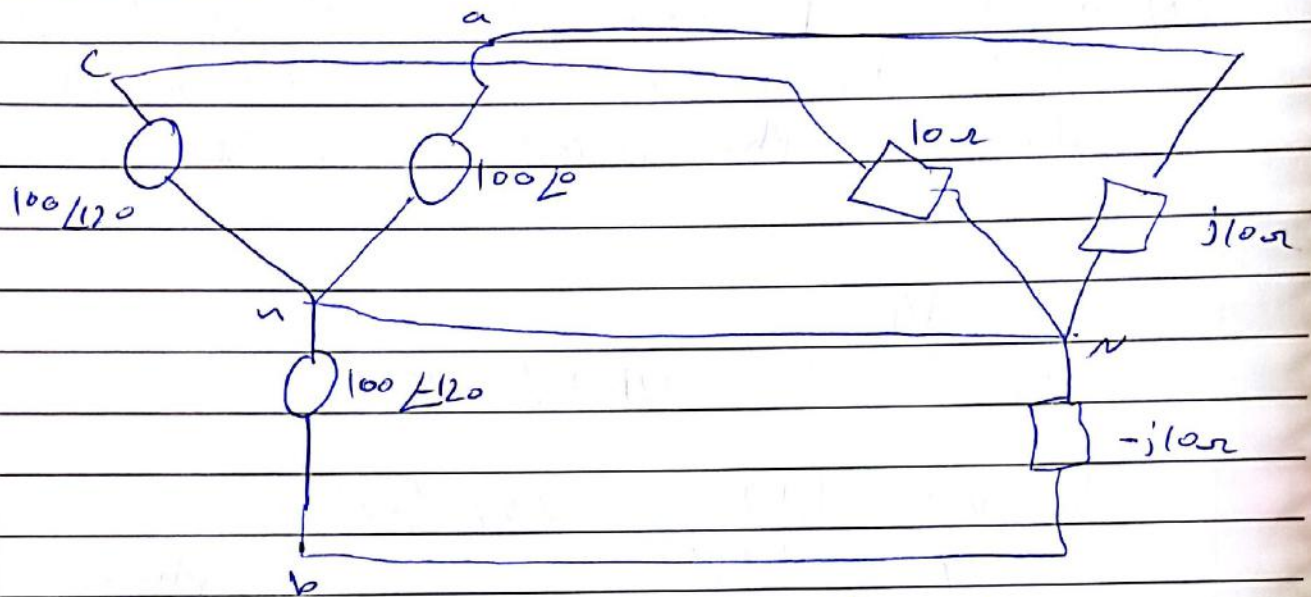


$$\Rightarrow I_L = I_{p1} + \sqrt{3} I_{p2} \angle -30$$





No \_\_\_\_\_



$$I_a = \frac{V_{an}}{j10}$$

$$I_b = \frac{V_{bn}}{-j10}$$

$$I_c = \frac{V_{cn}}{10}$$

$$I_{In} = I_a + I_b + I_c$$

~~As per answer~~

\* Power in 3 $\phi$  systems :-

$$V_{AN} = \sqrt{2} V_p \overset{\text{rms}}{\cos \omega t}$$

$$V_{BN} = \sqrt{2} V_p \cos(\omega t - 120^\circ)$$

$$V_{CN} = \sqrt{2} V_p \cos(\omega t + 120^\circ)$$

assume :-  $Z_Y = Z/\phi$

$$i_{AN} = \sqrt{2} I_p \cos(\omega t - \theta)$$

$$i_{BN} = \sqrt{2} I_p \cos(\omega t - 120^\circ - \theta)$$

$$i_{CN} = \sqrt{2} I_p \cos(\omega t + 120^\circ - \theta)$$

average =

$$P_A = V_p I_p \overset{\text{rms}}{\cos \theta}$$

$$P_B = V_p I_p \cos \theta$$

$$P_C = V_p I_p \cos \theta$$

$$P_{3\phi} = P_A + P_B + P_C = 3V_p I_p \cos \theta$$

$$P_{3\phi} = 3 P_{1\phi}$$

Power calculations in 3 $\phi$  system :-

$$P_A = V_p I_p \cos \theta = P_B = P_C$$

$$Q_A = V_p I_p \sin \theta = Q_B = Q_C$$

$$S_A = V_p I_p^* = S_B = S_C = P_p + j Q_p$$

$$S_{3\phi} = 3 S_{1\phi} = 3 V_p I_p^*$$

$$P_{3\phi} = 3 P_{1\phi} = 3 V_p I_p \cos \theta$$

$$Q_{3\phi} = 3 Q_{1\phi} = 3 V_p I_p \sin \theta$$

$$|S_A| = \|V_p\| \|I_p\|$$

$$P_{3\phi} = 3 V_p I_p \cos \theta \quad (\Delta \& Y)$$

But

in Y connections  $V_L = \sqrt{3} V_p$

in  $\Delta$  "  $I_L = \sqrt{3} I_p$

$$\rightarrow P_{3\phi} = \sqrt{3} V_L I_L \cos \theta \quad (\Delta \& Y)$$

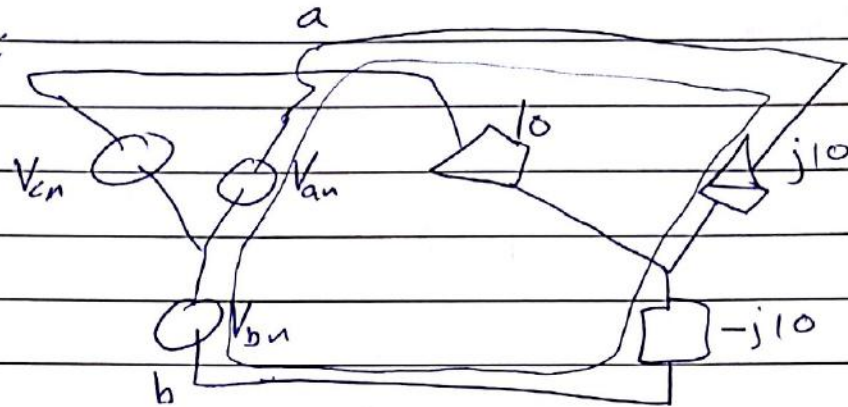
$$Q_{3\phi} = \sqrt{3} I_L \Rightarrow V_L \sin \theta$$

$$S_{3\phi} = \sqrt{3} V_L I_L^*$$

$$I_a = \frac{V_{an}}{j10}$$

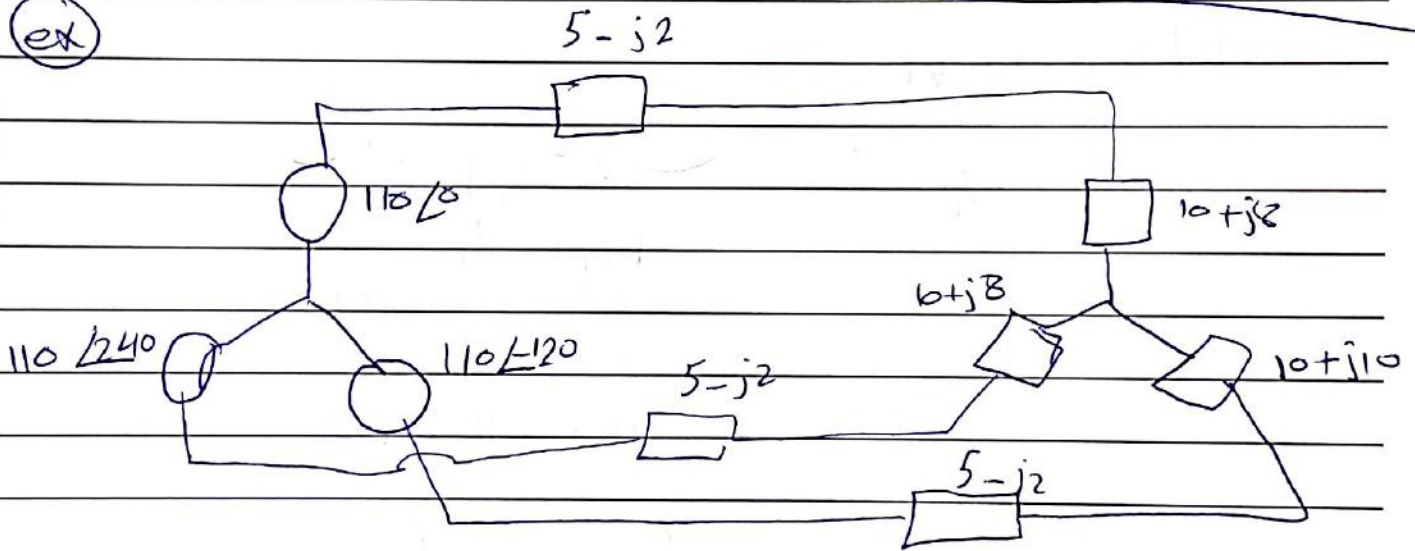
$$I_b = \frac{V_{bn}}{-j10}$$

$$I_c = \frac{V_{cn}}{10}$$



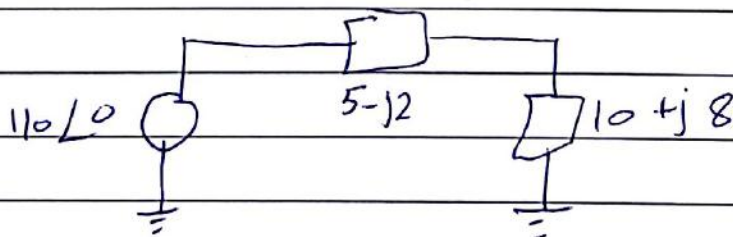
$$-V_{an} + j10 I_a - j10 I_b + V_{bn} = 0$$

ex

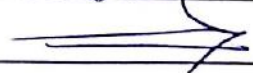


Find total complex Power supplied by source & drawn by load  $5-j2$ . ← 3-source

Sol - e 3p r j r 2 = 4 p a s 3 l o a d s 2 0  
Single phase equ. ct.



Follow



No \_\_\_\_\_

$$I_p = I_{line} = \frac{110 \angle 0}{5 - j2 + 10 + j8} = 6.8 \angle -21.8^\circ A$$

$$S_{source} = 3V_p I_p^* = \sqrt{3} V_L I_L^*$$

$$= 3 \times 110 \angle 0 \times 6.8 \angle +21.8^\circ$$

$$= 2247 \angle 21.8 = \underbrace{2087}_{P_{3\phi}} + j \underbrace{834.6}_{Q_{3\phi}}$$

$$|S| = 2247 \text{ VA}$$

$$S_{load} = 3V_p I_{p_{load}}^*$$

$$= 3(Z_p I_p) I_p^*$$

$$= 3|I_p|^2 Z_p$$

$$= 3 \times (6.8)^2 \times (10 + j8)$$

$$= 1782 \angle 38.66 \text{ VA}$$

$$= \underbrace{1392}_{P_{3\phi}} + j \underbrace{1113}_{Q_{3\phi}} \text{ VA (load)}$$

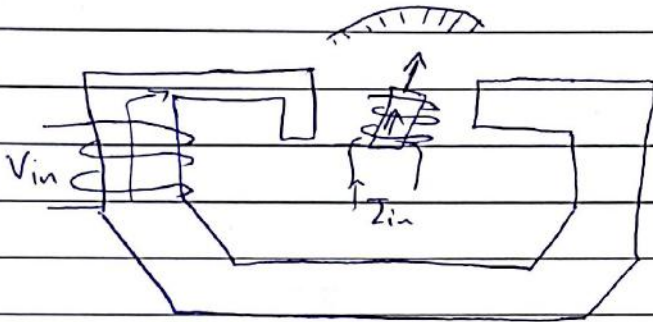
$$S_{losses} = 3|I_L|^2 Z_{TL}$$

$$= S_{source} - S_{load}$$

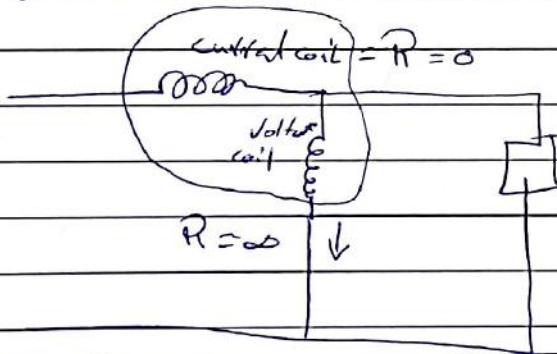
$$= 695.6 - j278.3$$

**\* Power measurement :-**

⇒ **Wattmeter** : device that is used to measure that average power.

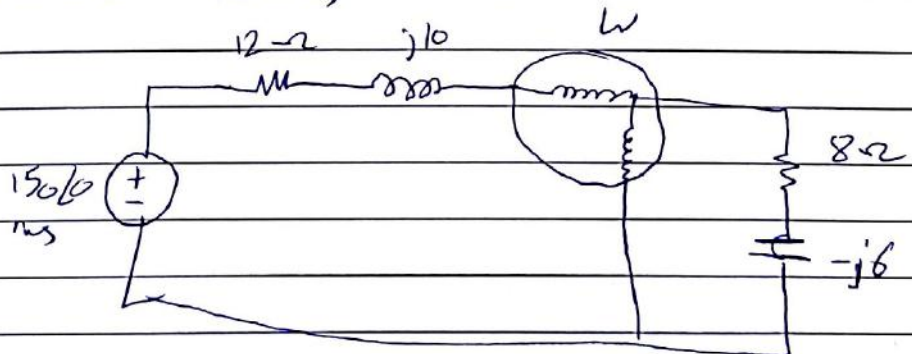


$$P \approx VI \cos \phi$$



$$P = |V_{rms}| |I_{rms}| \cos(\phi_v - \phi_i)$$

⊙ **ex** Find the wattmeter reading



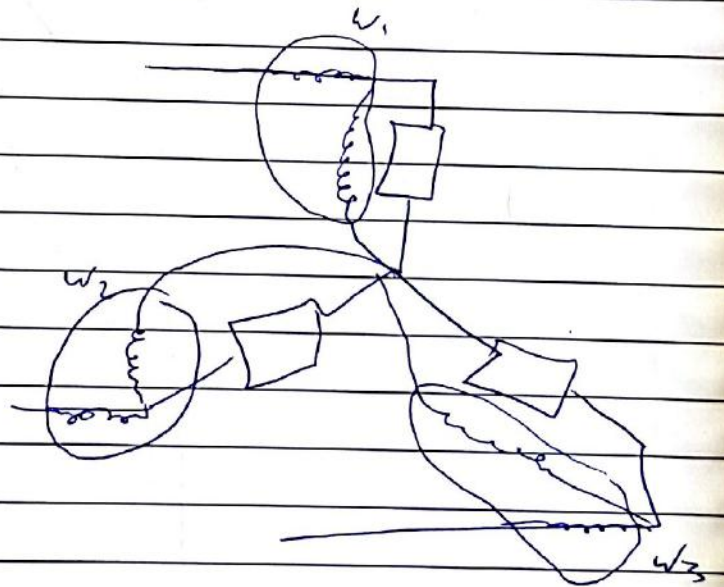
⇒ **Solve**

No \_\_\_\_\_

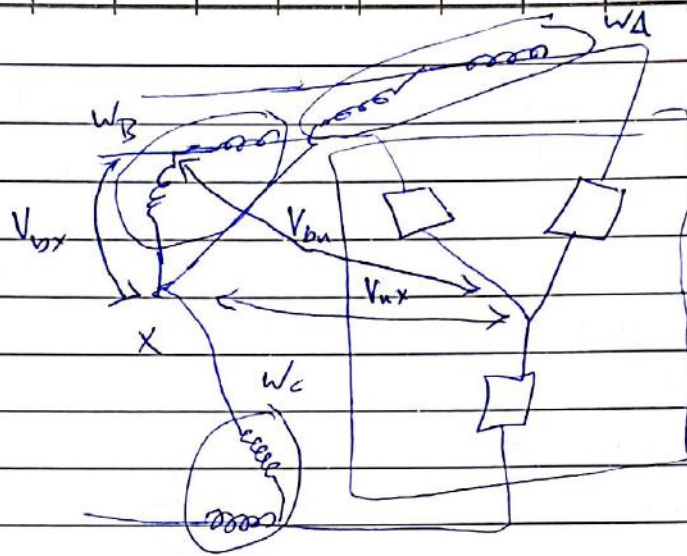
$$I_{rms} = \frac{150 \angle 0}{12 + j10 + 8 - j6} = \frac{150 \angle 0}{20 + j4} = 7.35 \angle -11.3$$

$$V_{rms} = 150 \angle 0 * \frac{8 - j6}{12 + 8 + j10 - j6} = 73.5 \angle 48$$

$$P = |V_{rms}| |I_{rms}| \cos(\theta_v - \theta_i) = 432.15 \text{ W}$$



No \_\_\_\_\_



$$P_A = \frac{1}{T_0} \int_0^{T_0} V_{ax} i_{aA} dt$$

$$P_B = \frac{1}{T_0} \int_0^{T_0} V_{bx} i_{bB} dt$$

$$P_C = \frac{1}{T_0} \int_0^{T_0} V_{cx} i_{cC} dt$$

$$\begin{aligned} V_{ax} &= V_{an} + V_{nx} \\ V_{bx} &= V_{bn} + V_{nx} \\ V_{cx} &= V_{cn} + V_{nx} \end{aligned}$$

$$P_{3\phi} = P_A + P_B + P_C = \frac{1}{T_0} \int_0^{T_0} (V_{ax} i_{aA} + V_{bx} i_{bB} + V_{cx} i_{cC}) dt$$

$$= \frac{1}{T_0} \int_0^{T_0} (V_{an} i_{aA} + V_{nx} i_{aA} + V_{bn} i_{bB} + V_{nx} i_{bB} + V_{cn} i_{cC} + V_{nx} i_{cC}) dt$$

$$P_{total} = \frac{1}{T_0} \int_0^{T_0} (V_{an} i_{aA} + V_{bn} i_{bB} + V_{cn} i_{cC}) dt$$

$$+ \frac{1}{T_0} \int_0^{T_0} V_{nx} (i_{aA} + i_{bB} + i_{cC}) dt$$

$$= \frac{1}{T_0} \int_0^{T_0} V_{an} i_{aA} + V_{bn} i_{bB} + V_{cn} i_{cC}$$



Conclusion: choice of location of point X will not change the total power measured.

(ex) Find the 3 wattmeters readings when X is at the neutral of the source.

$$V_{ab} = 100 \angle 0^\circ$$

$$V_{bc} = 100 \angle 120^\circ$$

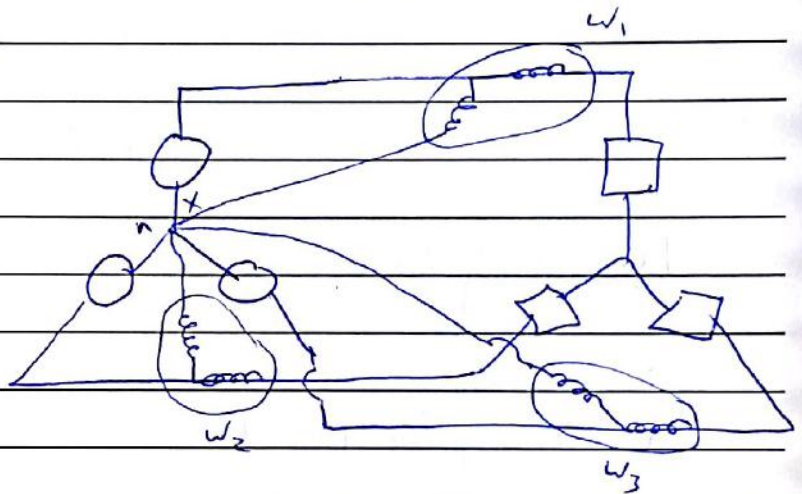
$$V_{ca} = 100 \angle -120^\circ$$

Sol:

$$I_{aA} = 19.32 \angle 15^\circ$$

$$I_{bB} = 19.32 \angle 165^\circ$$

$$I_{cC} = 10 \angle -90^\circ$$



$$\text{or } \frac{V_{an} - V_n}{j10} + \frac{V_b - V_n}{-j10} + \frac{V_c - V_n}{10} = 0$$

$$\text{Find } V_n \Rightarrow I_a = \frac{V_a - V_n}{j10}, \quad I_b = \frac{V_b - V_n}{-j10}$$

$$I_c = \frac{V_c - V_n}{10}$$

$$\begin{aligned} W_A = P_A &= V_{an} i_{aA} \cos(\theta_V - \theta_i) \\ &= \frac{100}{\sqrt{3}} * 19.32 \cos(-30 - 15) \\ &= 788.7 \text{ W} \end{aligned}$$

$$\begin{aligned} W_B = P_B &= \frac{100}{\sqrt{3}} * 19.32 \cos(-150 - 165) \\ &= 788.7 \text{ W} \end{aligned}$$

$$\begin{aligned} V_{ab} &= 100 \angle 0^\circ \\ V_{bc} &= 100 \angle 120^\circ \end{aligned}$$

$$V_{an} = \frac{100}{\sqrt{3}} \angle -30^\circ$$

$$V_{bn} = \frac{100}{\sqrt{3}} \angle -150^\circ$$

$$V_{cn} = \frac{100}{\sqrt{3}} \angle 90^\circ$$

$$W_c = P_c = \frac{100}{\sqrt{3}} * 10 * \cos(90 + 90) = -577.4 \text{ W}$$

$$P_T = P_A + P_B + P_c = 1000 \text{ W}$$

## \* 2-wattmeter method :-

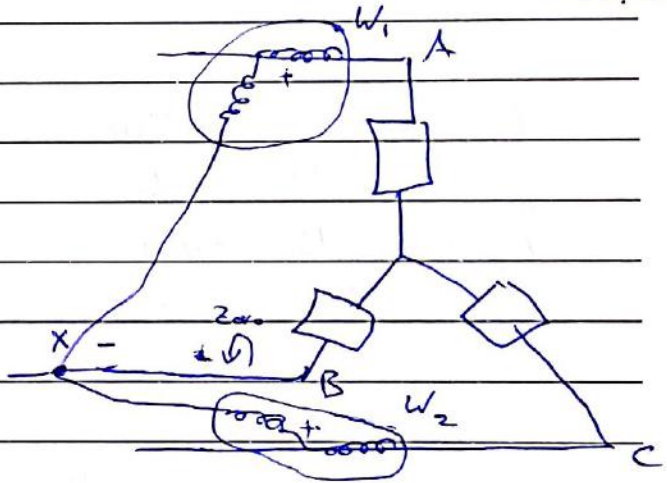
What if we put X on one of the phases.

في الـ 11  
من الـ 11

$$W_1 = I_{AA}, V_{AB}$$

$$W_2 = I_{CC}, V_{CB}$$

$$W_1 = P_1 = 100 * 19.32 * \cos(0 - 15) \\ = 1866 \text{ W}$$



$$W_2 = P_2 = 100 * 10 \cos(60 + 90) \\ = -866 \text{ W}$$

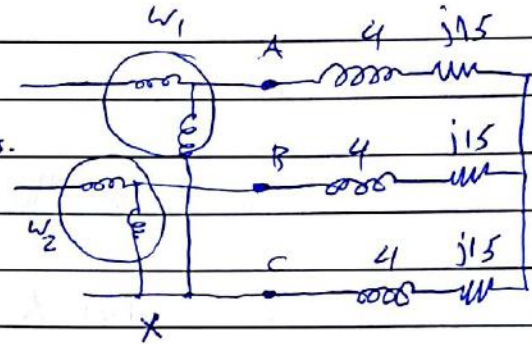
$$V_{CB} = 100 \angle +60$$

$$P_T = P_1 + P_2 = 1000 \text{ W}$$

\* Conclusion :- ~~For~~ For power measurements in 3 phase systems, 2 wattmeters are enough to measure the total power. #

⊗ given  $V_{ab, rms} = 230 \angle 0$

Find the total power using the readings at the two wattmeters.



Sol:-

$$W_1 = I_{aA} \cdot V_{AC}$$

$$W_2 = I_{bB} \cdot V_{BC}$$

$$V_{ab} = 230 \angle 0 = V_{AB}$$

$$V_{bc} = 230 \angle -120 = V_{BC}$$

$$V_{ca} = 230 \angle +120 = V_{CA}$$

$$V_{AC} = 230 \angle -60$$

$$I_{aA} = \frac{V_{an}}{Z_Y} = \frac{230 \angle -30}{\sqrt{3} (4 + j15)}$$

$$I_{aA} = 8.554 \angle -105.1$$

$$I_{cC} = 8.554 \angle +14.9$$

$$I_{bB} = 8.554 \angle -226.1$$

$$= 8.554 \angle +134.9$$

$$P_1 = V_{AC} I_{aA} \cos(\theta_{V_{AC}} - \theta_{I_{aA}})$$

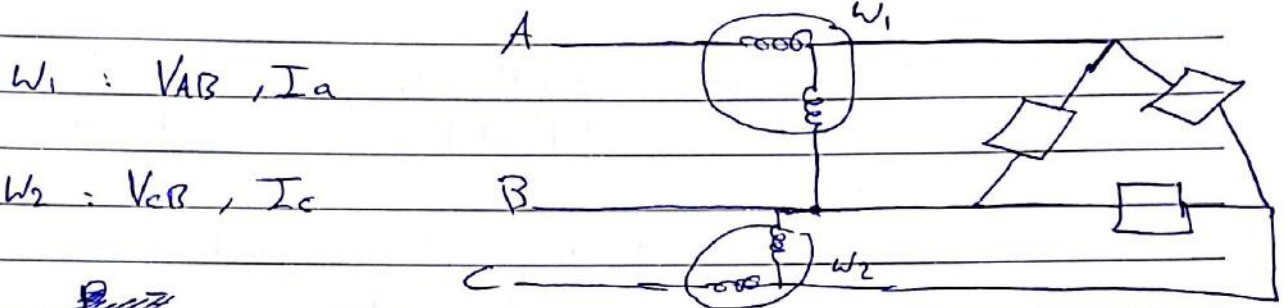
$$= 230 * 8.554 \cos(-60 + 105.1) = 1389 \text{ W}$$

$$P_2 = V_{BC} * I_{bB} * \cos(\theta_{V_{BC}} - \theta_{I_{bB}})$$

$$= 230 * 8.554 * \cos(-120 - 134.9) = -512 \text{ W}$$

$$P_T = P_1 + P_2 = 876.5 \text{ W}$$

How to calculate power factor from the readings of the two wattmeters:

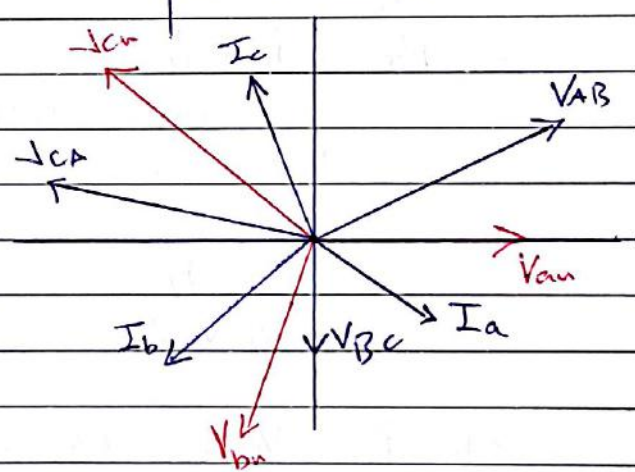


$W_1 : V_{AB}, I_a$

$W_2 : V_{CB}, I_c$

$P_1 = V_{AB} I_a \cos(\theta - \phi)$   
 $P_2 = V_{CB} I_c \cos(\theta - \phi)$

assume an inductive load



$P_1 = V_{AB} I_a \cos(30 + \theta)$   
 $= V_L I_L \cos(30 + \theta)$

$P_2 = V_L I_L \cos(30 - \theta)$

$\frac{P_1}{P_2} = \frac{V_L I_L \cos(30 + \theta)}{V_L I_L \cos(30 - \theta)}$

$= \frac{\cos(30 + \theta)}{\cos(30 - \theta)}$

$= \frac{\cos 30 \cos \theta - \sin 30 \sin \theta}{\cos 30 \cos \theta + \sin 30 \sin \theta}$

$= \frac{\frac{\sqrt{3}}{2} \cos \theta - \frac{1}{2} \sin \theta}{\frac{\sqrt{3}}{2} \cos \theta + \frac{1}{2} \sin \theta} \times \frac{2}{\cos \theta} = \frac{\sqrt{3} - \tan \theta}{\sqrt{3} + \tan \theta}$

$$\Rightarrow \text{cont... } \sqrt{3} P_1 + \tan \theta P_1 = \sqrt{3} P_2 - P_2 \tan \theta$$

$$\tan \theta (P_1 + P_2) = \sqrt{3} (P_2 - P_1)$$

$$\therefore \tan \theta = \frac{\sqrt{3} (P_2 - P_1)}{(P_2 + P_1)} \Rightarrow \begin{array}{l} P_2 > P_1 \text{ inductive} \\ P_2 < P_1 \text{ capacitive} \\ P_2 = P_1 \text{ Resistive} \end{array}$$

↳ you can find  $\theta$  by this equation  
 , then you can find PF

$$P_T = \sqrt{3} V_L I_L \cos \theta$$

$$\Rightarrow P_T = P_1 + P_2$$

$$= V_L I_L \cos(30 + \theta) + V_L I_L \cos(30 - \theta)$$

$$= V_L I_L \cos 30 \cos \theta - V_L I_L \sin 30 \sin \theta$$

$$+ V_L I_L \cos 30 \cos \theta + V_L I_L \sin 30 \sin \theta$$

$$= 2 V_L I_L \cos \theta \cos 30$$

$$= \sqrt{3} V_L I_L \cos \theta$$

$$\Rightarrow P_1 - P_2 = V_L I_L \cos(30 + \theta) - V_L I_L \cos(30 - \theta)$$

$$= V_L I_L (\cos 30 \cos \theta - \sin 30 \sin \theta - \cos 30 \cos \theta + \sin 30 \sin \theta)$$

$$= -2 V_L I_L \sin 30 \sin \theta$$

$\hookrightarrow 1/2$

$$P_1 - P_2 = -V_L I_L \sin \theta$$

$$P_2 - P_1 = V_L I_L \sin \theta$$

$$\sqrt{3} (P_2 - P_1) = \sqrt{3} V_L I_L \sin \theta = Q_T$$

$$Q_T = \sqrt{3} (P_2 - P_1)$$

$$S_T = \sqrt{P_T^2 + Q_T^2}$$

No \_\_\_\_\_

ex) the two wattmeters produce reading of  $P_1 = 1560 \text{ W}$  and  $P_2 = 2100 \text{ W}$  when connected to a  $\Delta$  load Find  $P_{1\phi}$ ,  $\phi_{1\phi}$ , PF

Sol:

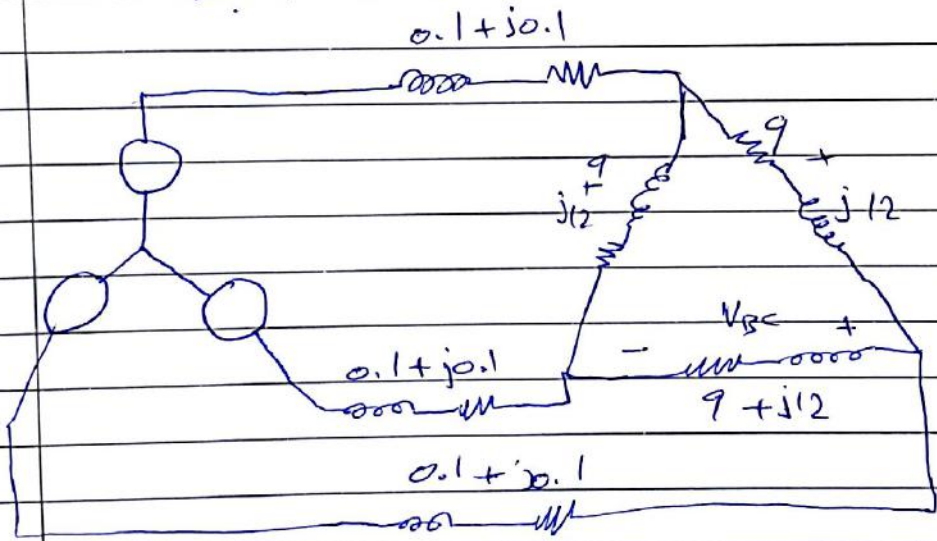
$$P_{1\phi} = \frac{P_T}{3} = \frac{P_1 + P_2}{3} = \frac{3660}{3} = 1220 \text{ W}$$

$$Q_{1\phi} = \frac{Q_T}{3} = \frac{\sqrt{3}(P_2 - P_1)}{3} = 311.77 \text{ VAR}$$

$$\theta = \tan^{-1} \left( \frac{Q_{1\phi}}{P_{1\phi}} \right) = 14.33^\circ$$

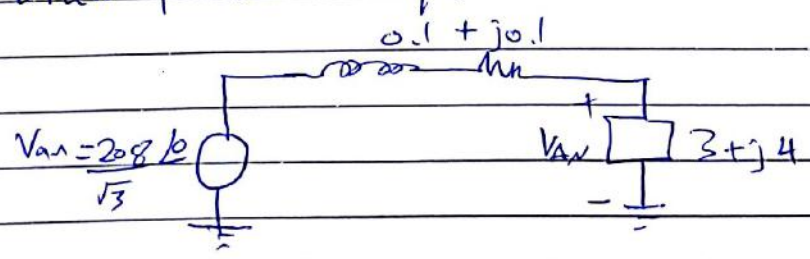
$$\cos(\theta) = 0.968 \text{ lagging} \quad \#$$

⇒ Find  $V_{bc}$ ? ,  $V_{ab} = 208$



$$\text{Sol: } Z_Y = \frac{1}{3} Z_{\Delta} = \frac{1}{3} (9 + j12) = 3 + j4$$

⇒ Draw phase a equivalent CKT ,  $|V_{ab}| = 208$



No \_\_\_\_\_

taking  $V_{an}$  as the reference :-

$$V_{AN} = \frac{3+j4}{3.1+j4.1} * \frac{208}{\sqrt{3}} \angle 0 = 117 \angle 0.22^\circ \text{ V}$$

$$V_{AB} = \sqrt{3} * 117 \angle 0.22 + 30^\circ = 203 \angle 30.22^\circ$$

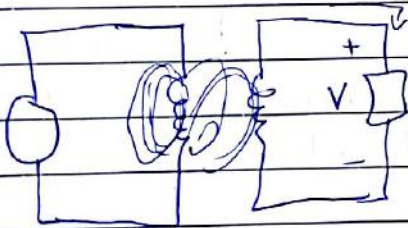
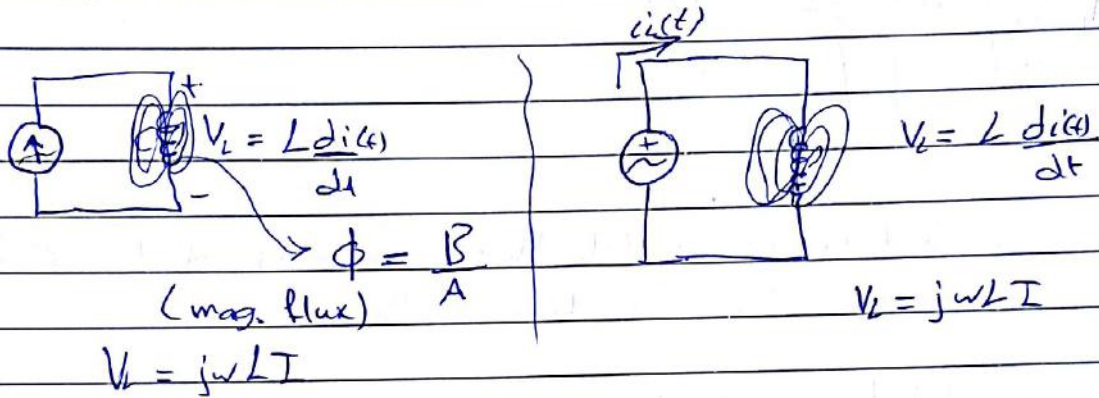
$$V_{BC} = 203 \angle 30.22 - 120^\circ$$

# Chapter 13

No \_\_\_\_\_

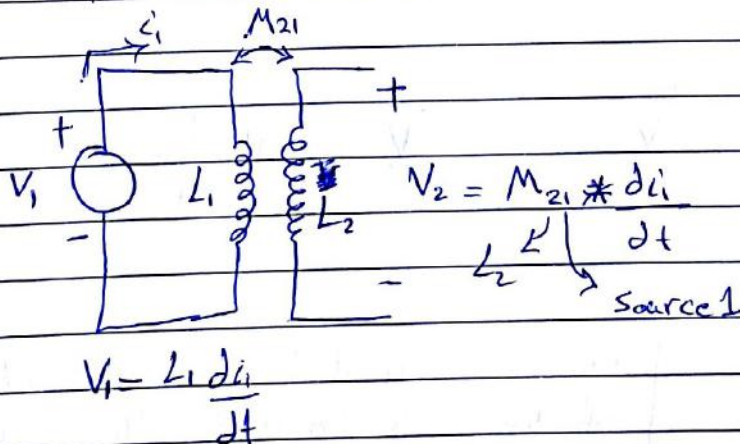
## \* Magnetically coupled CKTs :-

→ Self inductance :-

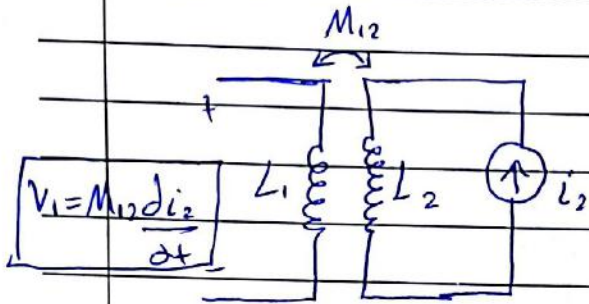


## Mutual inductance :-

دالة الـ  $M_{21}$



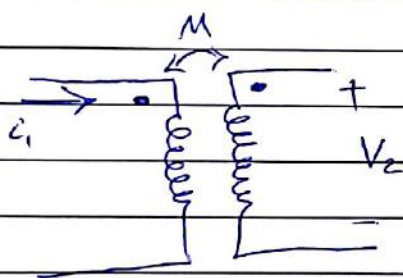




⇒ what if we switch the terminals of  $L_2$  OR change the direction of the current.

⇒ Dot convention :-

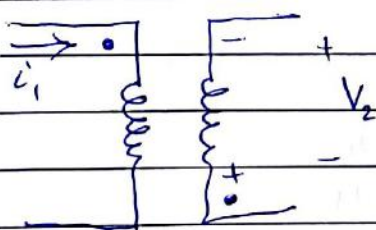
①



if the current enters the dot in one coil, it makes the dotted terminal on the other coil positive

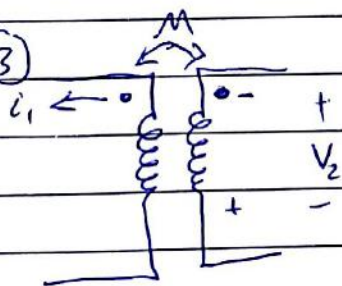
$$V_2 = M \frac{di_1}{dt}$$

②



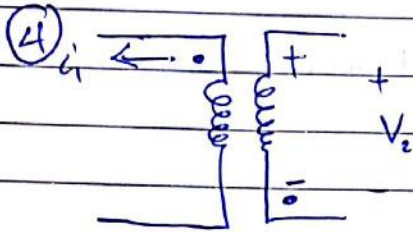
$$V_2 = -M \frac{di_1}{dt}$$

③



if the current leaves the dot in one of the coils it makes the dotted terminal on the other coil negative

$$V_2 = -M \frac{di_1}{dt}$$



$$V_2 = M \frac{di_1(t)}{dt}$$

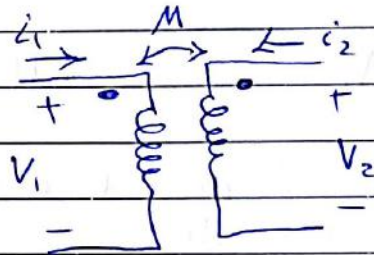
ex)  $M = 2H$

Determine  $V_2$  when  
 $i_1 = 5 \sin 10t$  &  $i_2 = 0$

Sol:

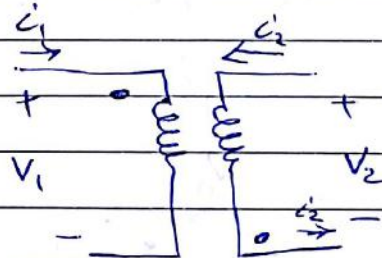
$$V_2(t) = M \frac{di_1(t)}{dt}$$

$$= 2 \times 5 \times 10 \cos(10t) = 100 \cos 10t \text{ V}$$



⇒ determine  $V_1$  if

$i_2 = 5 \sin 45t \text{ A}$ ,  $i_1 = 0$



$$V_1 = (-) M \frac{di_2}{dt}$$

$$= -2 \times 5 \times 45 \cos(45t)$$

$$= -450 \cos(45t) \text{ V}$$

⇒ determine  $V_2$  if  $i_1 = -8e^{-t}$  &  $i_2 = 0$

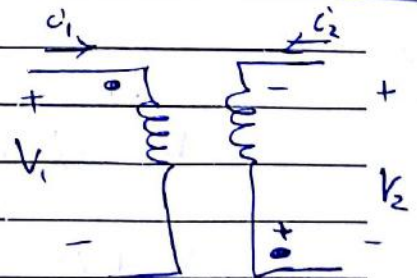
Sol:

$$V_2(t) = (-) M \frac{di_1}{dt}$$

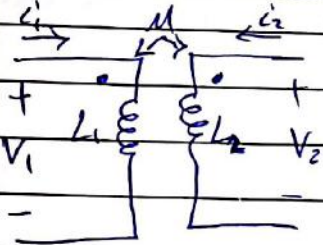
$$= -M \frac{di_1}{dt}$$

$$= -2 \times (-8) \times (-1) e^{-t} \text{ V}$$

$$= -16 e^{-t} \text{ V}$$

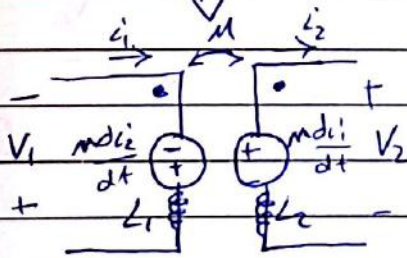
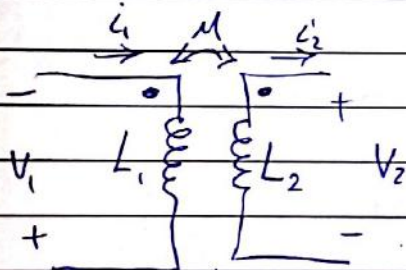


⇒ Combined mutual & self inductance :-



$$V_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$V_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$



اذا افادت  
بجانب  
مصدر الجهد  
بجانب  
التيار

$$\left. \begin{aligned} &+ V_1 - M \frac{di_2}{dt} + L \frac{di_1}{dt} = 0 \end{aligned} \right\}$$

$$V_1 = -L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$V_2 + L_2 \frac{di_2}{dt} - M \frac{di_1}{dt} = 0$$

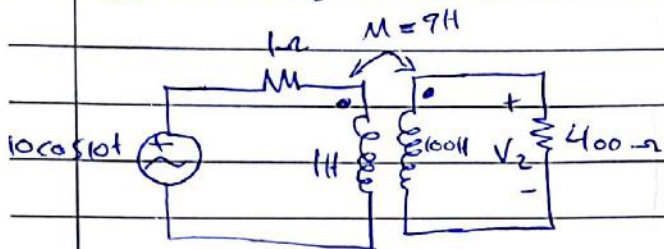
$$V_2 = -L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

$$V_1 = -j\omega L_1 I_1 + j\omega M I_2$$

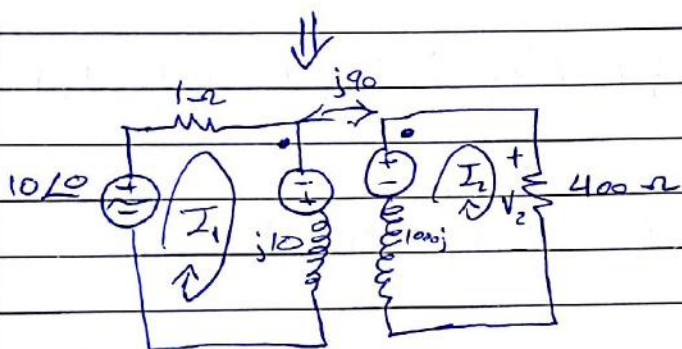
$$V_2 = -j\omega L_2 I_2 + j\omega M I_1$$

No \_\_\_\_\_

(ex) Find  $V_2$



$\omega = 10 \text{ rad/sec}$



$-10/0 + 1I_1 + j10I_1 - j90I_2 = 0$  — (1)

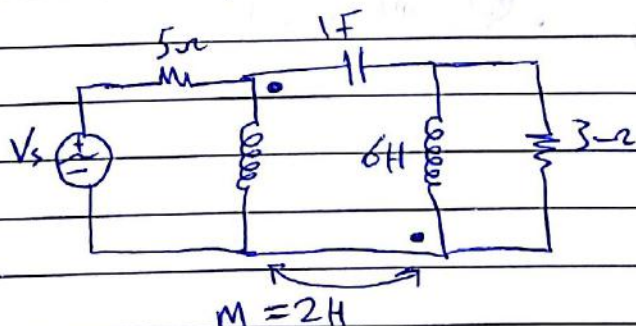
$400I_2 + j1000I_2 - j90I_1 = 0$  — (2)

$V_2 = 400I_2$

$\Rightarrow I_2 = 0.172 \angle -16.7^\circ \text{ A}$

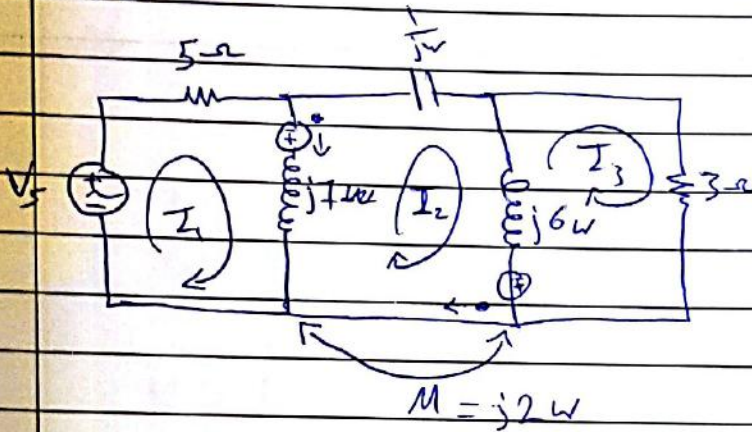
$V_2 = 68.8 \angle -16.7^\circ \text{ V}$

(ex) write the equation for meshes in the CKT:-



Follow solution

Sol :-



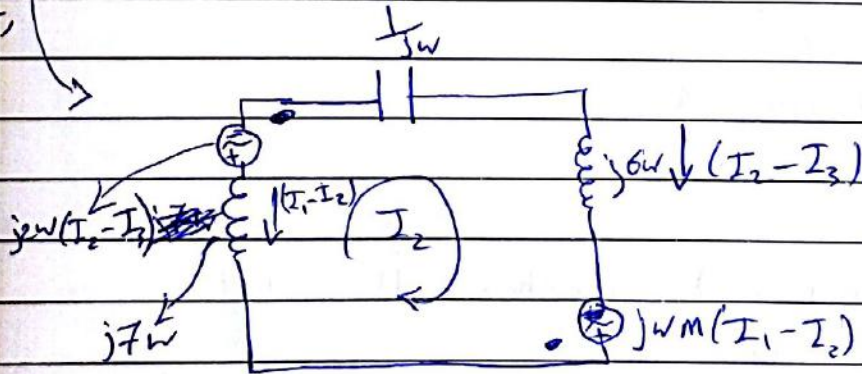
Mesh 1 :-  $-V_s + 5I_1 + j7W(I_1 - I_2) - j2W(I_2 - I_3) = 0$

Mesh 3 :-  $3I_3 + j6W(I_3 - I_2) + j2W(I_1 - I_2) = 0$   
 $\Rightarrow 3I_3 + j6W(I_3 - I_2) - j2W(I_2 - I_1) = 0$

Mesh 2 :-

$\frac{1}{jW} I_2 + j6W(I_2 - I_3) + j7W(I_2 - I_1) - j2W(I_1 - I_2) + j2W(I_2 - I_3) = 0$

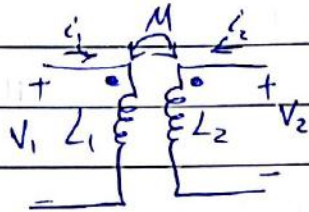
Mesh 2 :-



Energy stored in a circuit with mutual inductance:

[Prove that  $m_{12} = m_{21}$ ]

coil  $\swarrow$   $\searrow$  Source  
Source



① set  $I_2 = 0$

set  $i_1 : 0 \rightarrow I_1$

$t : 0 \rightarrow t_1$

$$V_1 i_1 = L_1 \frac{di_1}{dt} i_1$$

$$\Rightarrow \int_0^{t_1} V_1 i_1 dt = \int_0^{I_1} L_1 i_1 di_1$$

$$\text{energy} \Rightarrow W_1 = \int_0^{t_1} V_1 i_1 dt = \frac{1}{2} L_1 I_1^2, \text{ energy stored from } t=0 \text{ to } t=t_1$$

② set  $i_1 = I_1$

$i_2 : 0 \rightarrow I_2$

$t : t_1 \rightarrow t_2$

$$V_2 i_2 = L_2 \frac{di_2}{dt} i_2$$

$$\int_{t_1}^{t_2} V_2 i_2 dt = \int_0^{I_2} L_2 i_2 di_2$$

$$\text{energy} : W_2 = \frac{1}{2} L_2 I_2^2$$

~~$\Rightarrow$  Energy stored~~



No

Set an upper limit for  $M$ :-

assume that  $i_1$  &  $i_2$  are positive.

$w > 0$

$$\frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 - M I_1 I_2 > 0$$

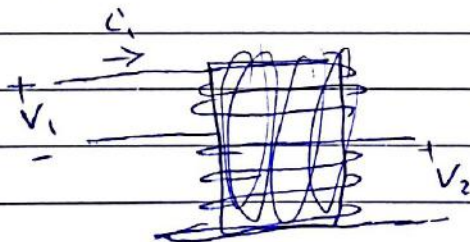
$$\frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 - M I_1 I_2 - I_1 I_2 \sqrt{L_1 L_2} + I_1 I_2 \sqrt{L_1 L_2} > 0$$

$$\frac{1}{2} (I_1 \sqrt{L_1} - I_2 \sqrt{L_2})^2 + I_1 I_2 (\sqrt{L_1 L_2} - M) > 0$$

$$I_1 I_2 (\sqrt{L_1 L_2} - M) > 0 \Rightarrow \sqrt{L_1 L_2} - M > 0$$

$$M < \sqrt{L_1 L_2}$$

Coupling Coefficient :-

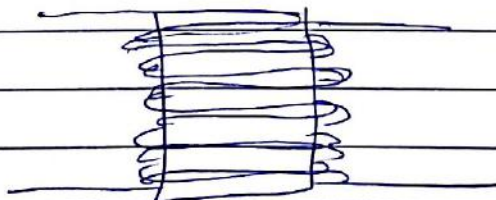


Magnetic Core

$$0 < K < 1$$

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

→ coupling coefficient



Coupling coefficient

سواء الجبراس



(ex) Find  $V_1(t)$ ,  $w(t)$

$$k = 0.6$$

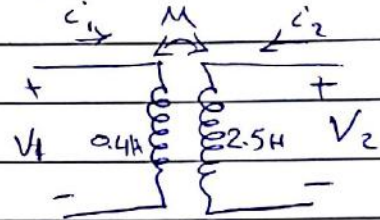
sol

$$M = k \sqrt{L_1 L_2}$$

$$= 0.6 \sqrt{0.4 * 2.5}$$

$$= 0.6$$

عرضنا انو  $M$  موجبة كسب الفولتس  $i_1$  و  $i_2$    
 Dot's



$$i_1 = 4i_2 = 20 \cos(300t - 20^\circ) \text{ mA}$$

$$V_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$V_1(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt}$$

$$= 0.4 * [-10 \sin(-20)] + 0.6 [-2.5 \sin(-20)]$$

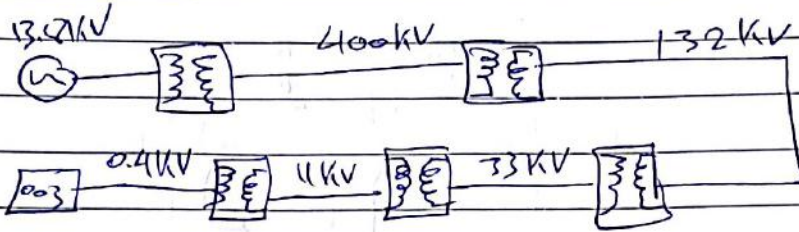
$$= 1.881 \text{ V}$$

$$w(t) = \frac{1}{2} L_1 i_1^2(t) + \frac{1}{2} L_2 i_2^2(t) + M i_1(t) i_2(t)$$

$$w(t) = 151 \text{ } \mu\text{J}$$

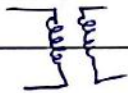
No \_\_\_\_\_

### Transformers:-

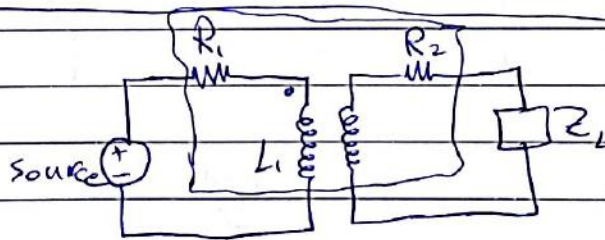
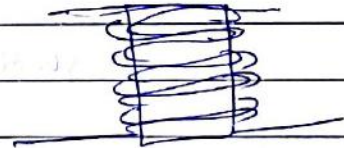


### transformers:-

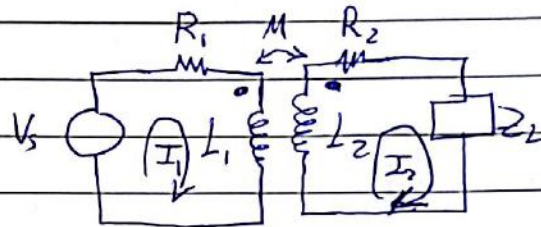
linear  
The coils are wound  
in Air



Ideal  
The coils are  
wound around  
a magnetic core



\* linear transformer :-

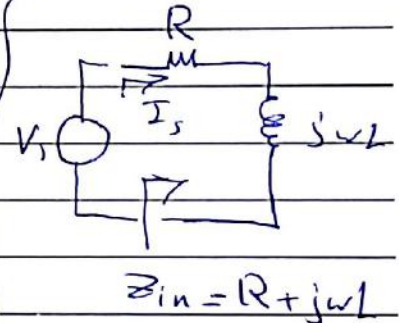


$$\begin{cases} V_s = (R_1 + j\omega L_1) I_1 - j\omega M I_2 \\ (R_2 + j\omega L_2 + Z_L) I_2 - j\omega M I_1 = 0 \end{cases}$$

$$\begin{aligned} Z_{11} &= R_1 + j\omega L_1 \\ Z_{22} &= R_2 + Z_L + j\omega L_2 \end{aligned}$$

$$\Rightarrow V_s = Z_{11} I_1 - j\omega M I_2 \quad \text{--- (1)}$$

$$0 = -j\omega M I_1 + Z_{22} I_2 \quad \text{--- (2)}$$



$$\begin{bmatrix} V_s \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{11} & -j\omega M \\ -j\omega M & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

From 2 :-  $I_2 = \frac{j\omega M}{Z_{22}} I_1$

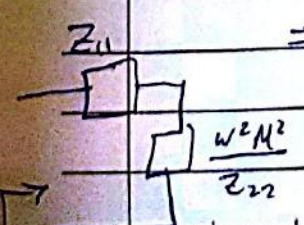
\*\* substitute in 1 :-

$$V_s = Z_{11} I_1 - j\omega M \left( \frac{j\omega M}{Z_{22}} \right) I_1$$

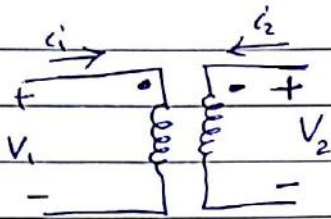
$$Z_{in} = \frac{V_s}{I_1} = Z_{11} + \frac{\omega^2 M^2}{Z_{22}} = \underbrace{Z_{11}}_{\text{Real}} + \underbrace{\frac{\omega^2 M^2}{Z_{22}}}_{\text{Imag}} \quad \left\{ \begin{aligned} Z_{22} &= R_{22} + jX_{22} \\ &= R_{22} + jX_{22} \end{aligned} \right.$$

$$= Z_{11} + \frac{\omega^2 M^2 R_{22}}{R_{22}^2 + X_{22}^2} - \frac{j\omega^2 M^2 X_{22}}{R_{22}^2 + X_{22}^2}$$

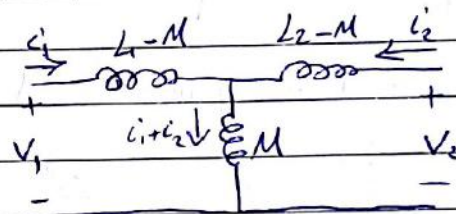
Reflected Impedance



→ The presence of the secondary coil, increases the input impedance seen by the source.



→ The T-equivalent CKT.



$$V_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

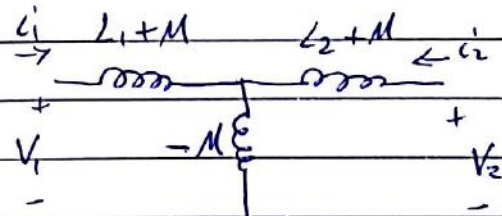
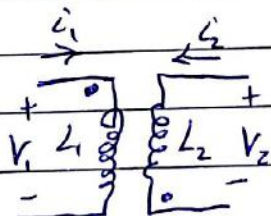
$$V_1 = (L_1 - M) \frac{di_1}{dt} + M \frac{d(i_1 + i_2)}{dt}$$

$$V_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

$$V_2 = L_1 \frac{di_1}{dt} - M \frac{di_1}{dt} + M \frac{di_2}{dt} + M \frac{di_2}{dt}$$

$$V_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$V_2 = (L_2 - M) \frac{di_2}{dt} + M \frac{d(i_1 + i_2)}{dt}$$

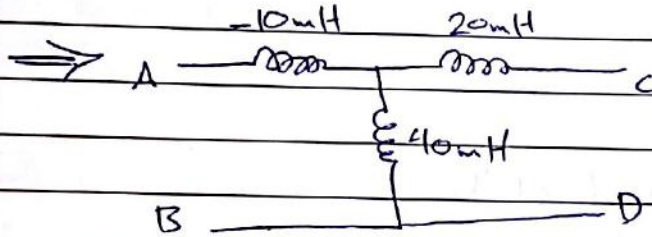
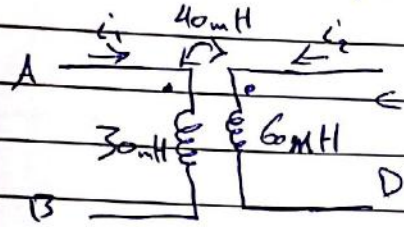


T-equivalent CKT

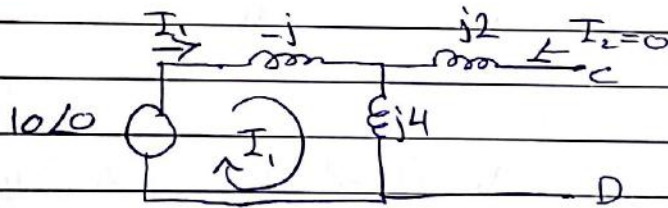
No \_\_\_\_\_

ⓧ Find the T-equivalent CKT

لو اکل  
تاران بنفرض  
اکسٹری  
لیا



if  $V_{AB} = 10\angle 0$  &  $\omega = 100$ , find  $I_1$  &  $V_{CD}$

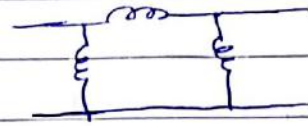
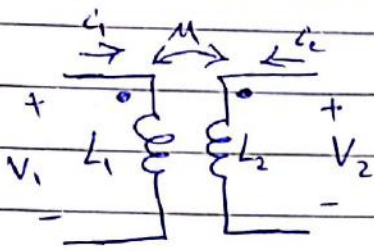


$$I_1 = \frac{10\angle 0}{-j + 4j} = 3.33 \angle -90^\circ = 3.33 \sin(100t) \text{ A}$$

$$V_{CD} = I_1 * j4 = 13.33 \angle 0 = 13.33 \cos 100t$$



\* The  $\pi$  equivalent ckt is-



$$V_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$V_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

$$\frac{di_2}{dt} = \frac{1}{L_2} V_2 - \frac{M}{L_2} \frac{di_1}{dt}$$

$$V_1 = L_1 \frac{di_1}{dt} + M \left[ \frac{1}{L_2} V_2 - \frac{M}{L_2} \frac{di_1}{dt} \right]$$

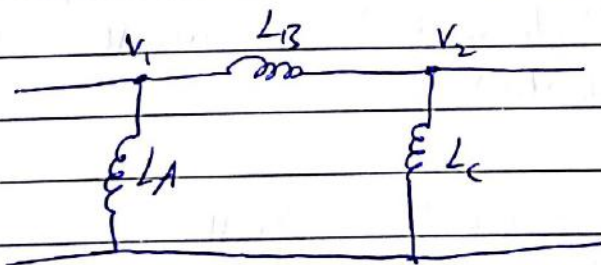
$$V_1 = L_1 \frac{di_1}{dt} + \frac{M}{L_2} V_2 - \frac{M^2}{L_2} \frac{di_1}{dt}$$

$$V_1 - \frac{M}{L_2} V_2 = \left( L_1 - \frac{M^2}{L_2} \right) \frac{di_1}{dt}$$

Like nodal analysis

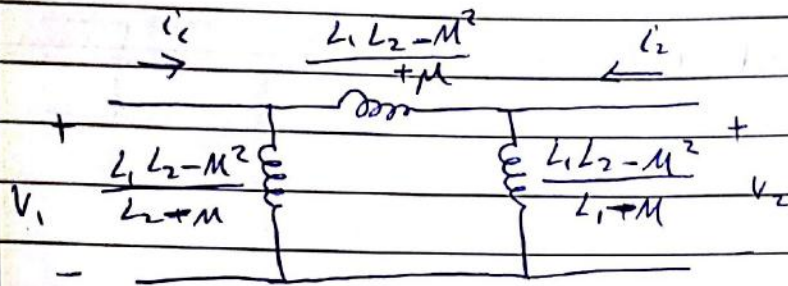
$$\frac{di_1}{dt} = \frac{L_2}{L_2 L_1 - M^2} V_1 - \frac{M}{L_2 L_1 - M^2} V_2$$

$$\frac{di_2}{dt} = \frac{-M}{L_2 L_1 - M^2} V_1 + \frac{L_1}{L_2 L_1 - M^2} V_2$$

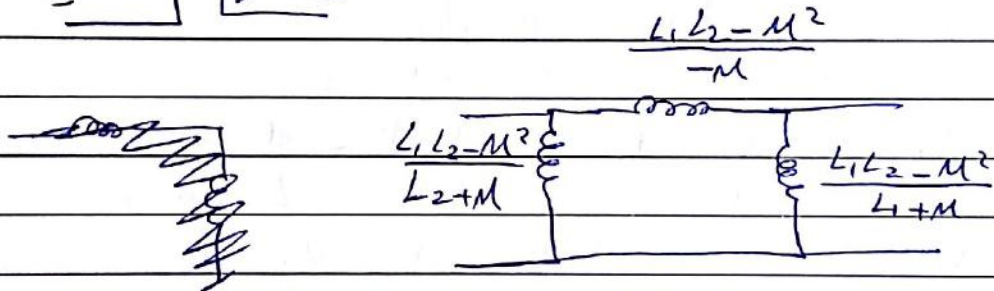
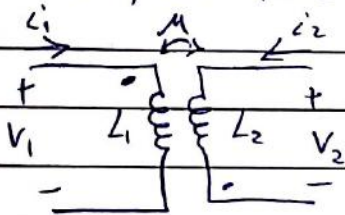


$$0 = \frac{V_1 - V_2}{( )} + \frac{V_1 - 0}{( )} + I_1$$

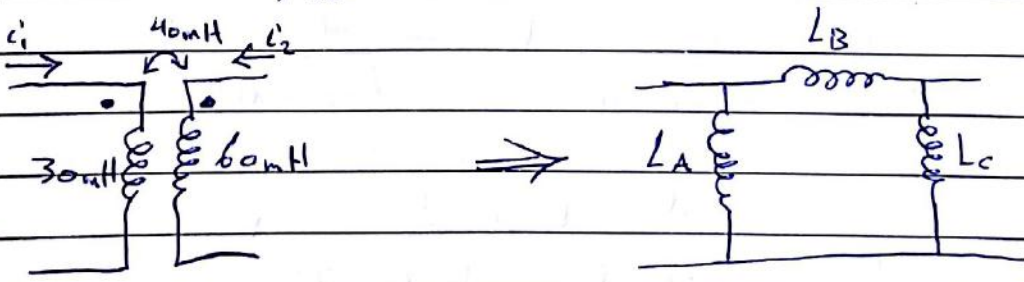
No



What happens if we change the dots



2913 ⇒ (ex) Find the  $\pi$  equivalent CKT for the linear transformer shown :-



$$L_A = \frac{30 \times 10^{-3} \times 60 \times 10^{-3} - 1600 \times 10^{-6}}{60 \times 10^{-3} - 40 \times 10^{-3}} = 10 \text{ mH}$$

$$L_B = \frac{30 \times 60 \times 10^{-6} - 1600 \times 10^{-6}}{40 \times 10^{-3}} = 5 \text{ mH}$$

$$L_C = \frac{30 \times 60 \times 10^{-6} - 1600 \times 10^{-6}}{30 \times 10^{-3} - 40 \times 10^{-3}} = -20 \text{ mH}$$

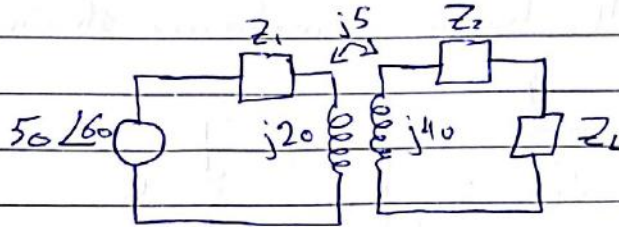
No \_\_\_\_\_

⊗ Calculate the input impedance & current  $I_1$

$$Z_1 = 60 - j100$$

$$Z_2 = 30 + j40$$

$$Z_L = 80 + j60$$



sd

قوة لا يتفق اتجاه التيار  $M^2$

$Z_{in}$ : impedance seen by the source ( $Z_{in}$ )

$$Z_{in} = Z_{11} + \frac{M^2}{Z_{22}} \Rightarrow \text{reflected impedance}$$

$$Z_{11} = R_1 + j\omega L_1$$

$$Z_{22} = R_2 + j\omega L_2 + Z_L$$

$$\begin{aligned} Z_{11} &= Z_1 + j\omega L_1 \\ &= 60 - j100 + j20 \\ &= 60 - j80 \Omega \end{aligned}$$

$$j\omega M = j5$$

$$\omega M = 5$$

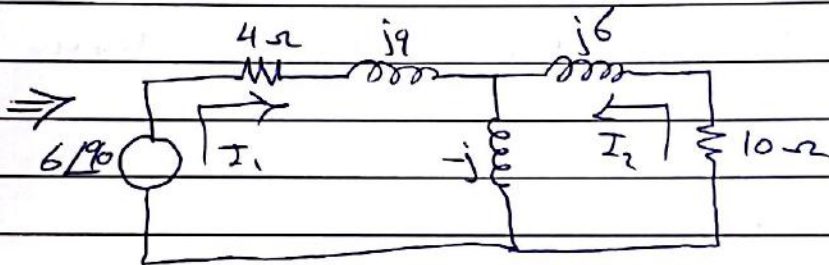
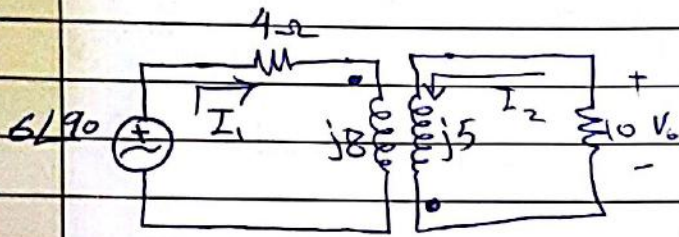
$$\begin{aligned} Z_{22} &= Z_2 + j\omega L_2 + Z_L \\ &= 30 + j40 + j40 + 80 + j60 \\ &= 110 + j140 \Omega \end{aligned}$$

$$Z_{in} = 60 - j80 + \frac{25}{110 + j140} = 100.14 \angle -53 \Omega$$

$$Z_{in} = \frac{V_s}{I_1} \Rightarrow I_1 = \frac{V_{in}}{Z_{in}} = \frac{50 \angle 60}{100.19 \angle -53} = 0.5 \angle 113.1$$



ex) Solve for  $I_1, I_2, V_o$  using T-equivalent CKT for the linear transformer shown:-



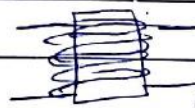
$$-6\angle 90^\circ + (4+j9)I_1 - j(I_1 + I_2) = 0$$

$$(10+j6)I_2 - j(I_2 + I_1) = 0$$

$$I_2 = 0.06\angle 90^\circ \quad I_1 = 0.6 + j0.3$$

$$V_o = -10I_2 = 0.6\angle 90^\circ$$

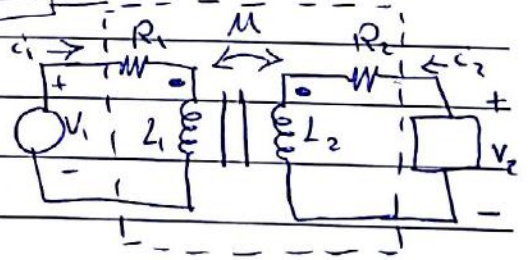
\* Ideal Transformer :-



→ the coils & mutual inductances large ( $L_1 = \infty, L_2 = \infty, M = \infty$ )

→  $k = 1$

→  $R_1 = 0 = R_2$



$$-V_1 = j\omega L_1 I_1 + j\omega M I_2 \quad \text{--- (1)}$$

$$V_2 = j\omega L_2 I_2 + j\omega M I_1 \quad \text{--- (2)}$$

$$\rightarrow I_1 = \frac{(V_1 - j\omega M I_2)}{j\omega L_1}$$

No \_\_\_\_\_

substitution in (2) :-

$$V_2 = j\omega L_2 I_2 + j\omega M \left[ \frac{V_1 - j\omega M I_2}{j\omega L_1} \right]$$

$$= j\omega L_2 I_2 + \frac{M}{L_1} V_1 - \frac{j\omega M^2 I_2}{L_1}, \quad M = \sqrt{L_1 L_2}$$

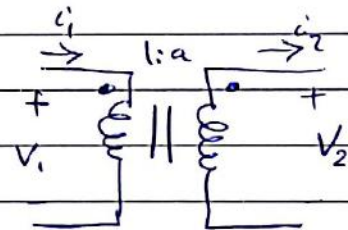
$$V_2 = j\omega L_1 I_2 + \frac{\sqrt{L_1 L_2}}{L_1} V_1 - j\omega L_1 I_2$$

$$V_2 = \sqrt{\frac{L_2}{L_1}} V_1 = n V_1 = a V_1 = \frac{N_2}{N_1} V_1$$

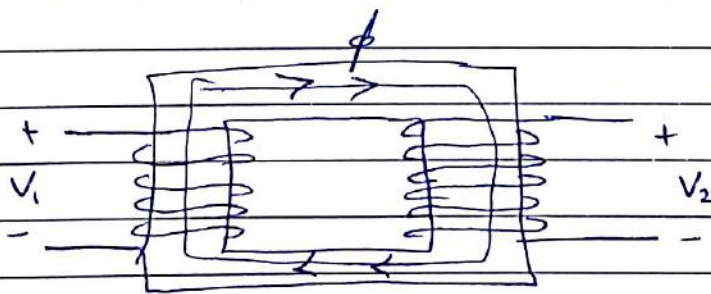
↳ turns ratio

$$L_1 = \frac{N_1^2}{X}$$

$$L_2 = \frac{N_2^2}{X}$$



$$\sqrt{\frac{L_2}{L_1}} = \frac{N_2}{N_1}$$



$$V_1 = -N_1 \frac{d\phi}{dt}, \quad V_2 = -N_2 \frac{d\phi}{dt}$$

$$\frac{d\phi}{dt} = \frac{V_1}{N_1} = \frac{V_2}{N_2} \Rightarrow \boxed{\frac{V_2}{V_1} = \frac{N_2}{N_1}}$$

No

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \Rightarrow N_2 > N_1 \rightarrow \text{step up transformer}$$

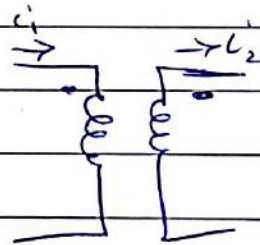
$$N_2 < N_1 \rightarrow \text{step down transformer}$$

→ 2 rules to define the voltage polarities & the current directions:

(1) if  $V_1$  &  $V_2$  are both positive or both negative at the dotted terminals use  $+n$ , or otherwise use  $-n$

(2) if  $I_1$  &  $I_2$  both enter or both leaves the dotted terminals use  $-n$  for  $\frac{I_1}{I_2}$ , otherwise use  $+n$

$$\frac{I_1}{I_2} = -n$$



# Circuits 2

POWER UNIT

By: Maen Mahadeen

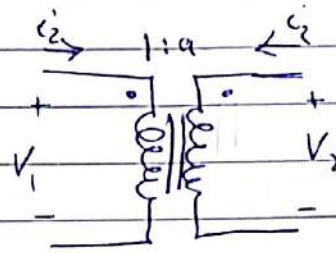
Dr. Mohd Al-haj

Spring 16

No \_\_\_\_\_

Ideal transformers:-

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = a = n$$

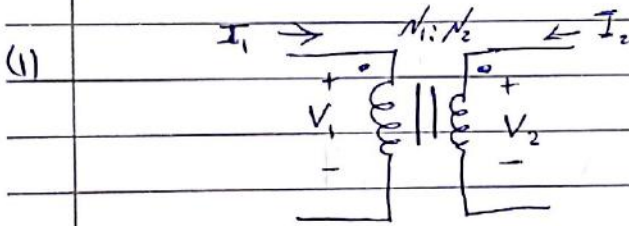


⇒ The ideal Tr. is lossless :-

$$V_1 I_1 = V_2 I_2$$

$$\frac{V_2}{V_1} = \frac{I_1}{I_2} \Rightarrow \frac{I_1}{I_2} = \frac{N_2}{N_1}$$

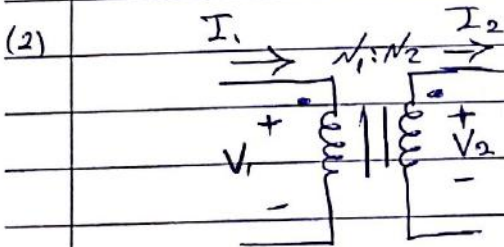
⇒ 2 rules :- (1) --- (2) ---



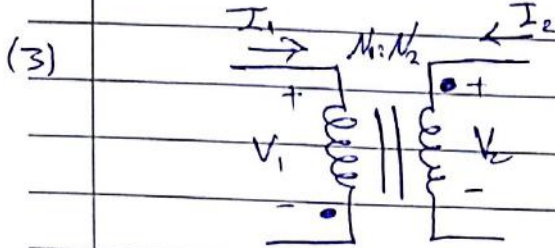
$$\frac{V_2}{V_1} = + \frac{N_2}{N_1} = +n$$

$$\frac{I_1}{I_2} = -n$$

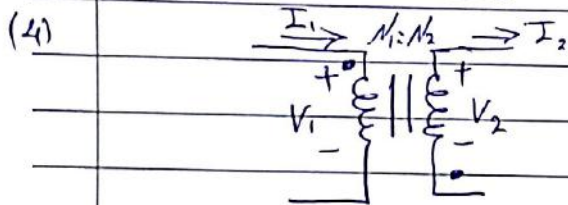
$$\left. \begin{array}{l} V_1 = 10, N_2 = 5, N_1 = 1 \\ \Rightarrow V_2 = +50 \end{array} \right\}$$



$$\frac{V_2}{V_1} = +n, \quad \frac{I_1}{I_2} = +n$$



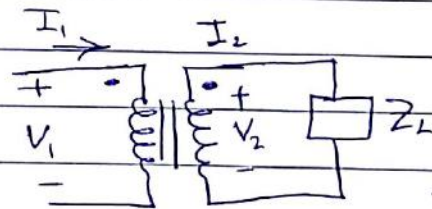
$$\frac{V_2}{V_1} = -\frac{N_2}{N_1} = -n \quad , \quad \frac{I_1}{I_2} = +\frac{N_2}{N_1} = +n$$



$$\frac{V_2}{V_1} = -\frac{N_2}{N_1} = -n \quad , \quad \frac{I_1}{I_2} = -\frac{N_2}{N_1} = -n$$

$$|S_1| = V_1 I_1 = \frac{V_2}{n} (n I_2) = V_2 I_2 = |S_2|$$

$$Z_{in} = \frac{V_1}{I_1}$$

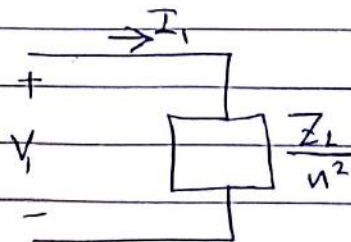


$$= \frac{V_2}{n I_2}$$

$$Z_L = \frac{V_2}{I_2}$$

$$= \frac{V_2}{I_2} * \frac{1}{n^2}$$

=  $\frac{Z_L}{n^2}$  ↗ Reflected Impedance



(ex) An ideal TR. is rated  $\frac{2400}{V_1} / \frac{120}{V_2} \text{ V}$ ,  $\frac{9.6}{S} \text{ kVA}$

and has 50 turns on the secondary side, Find:-

1) turns ratio:  $n = a = \frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{120}{2400} = 0.05$  (unitless)

2) turns on primary side:-

$$n = \frac{N_2}{N_1} \Rightarrow 0.05 = \frac{50}{N_1} \Rightarrow N_1 = 1000 \text{ turns}$$

3) The currents ratings for the primary & the secondary  
cKT:-

$$S = S_1 = S_2$$

$$S_1 = V_1 I_1 \Rightarrow I_1 = \frac{S_1}{V_1} = \frac{9.6 \times 10^3}{2400} = 4 \text{ A}$$

$$S_2 = V_2 I_2 \Rightarrow I_2 = \frac{S_2}{V_2} = \frac{9600}{120} = 80 \text{ A}$$

Current rating: maximum current flow through the load

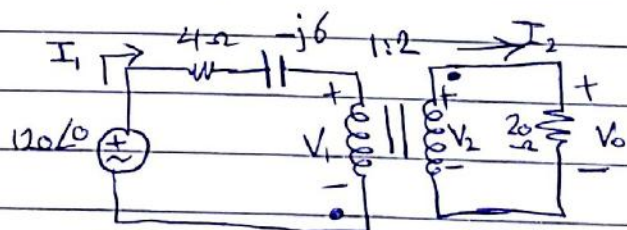
(ex) For the ideal transformer shown: Find  $\Rightarrow$

1) the source current  $I_1$ , 2) the output voltage  $[V_o]$

3) the complex power supplied by the source.

Sol:-

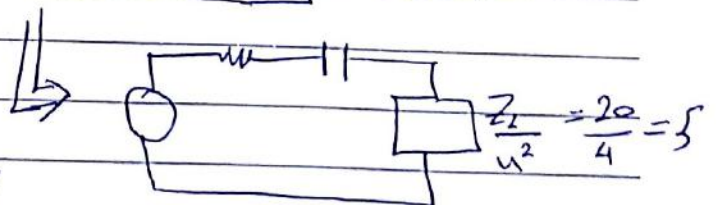
$$Z_{in} = V_s / I_1$$



$$Z_{in} = 4 - j6 + \frac{Z_L}{n^2}$$

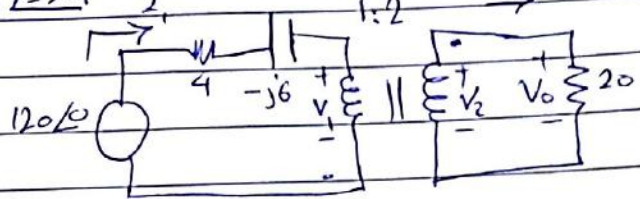
$$= 4 - j6 + 5$$

$$= 9 - j6 = 10.82 \angle -33.7^\circ$$



No \_\_\_\_\_

$$I_1 = \frac{120 \angle 0}{10.82 \angle -33.7} = 11.09 \angle 33.7$$



$$V_0 = R \times I_2$$

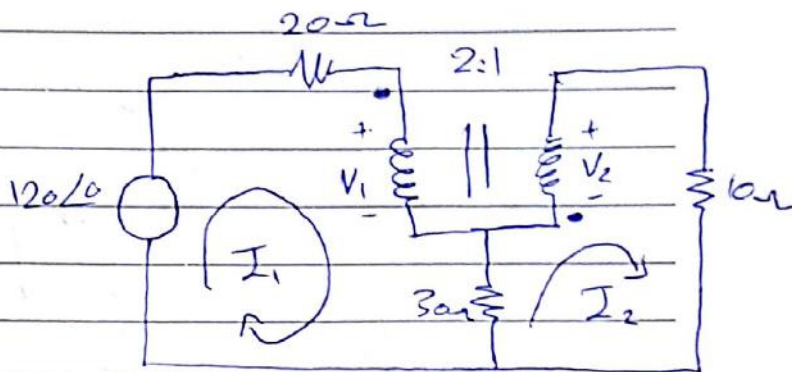
$$\frac{I_1}{I_2} = (-) \frac{n}{1}$$

$$\Rightarrow I_2 = \frac{-1}{n} I_1 = \frac{-1}{2} * 11.09 \angle 33.7 = -5.545 \angle 33.69 = 5.545 \angle 213.7$$

$$V_0 = 20 I_2 = 110.9 \angle 213.7$$



ex) calculate the power supplied to the  $10\ \Omega$  - resistor.



@ Mesh 1:  $-120 + 20I_1 + V_1 + 30(I_1 - I_2) = 0$

@ Mesh 2:  $10I_2 + 30(I_2 - I_1) - V_2 = 0$

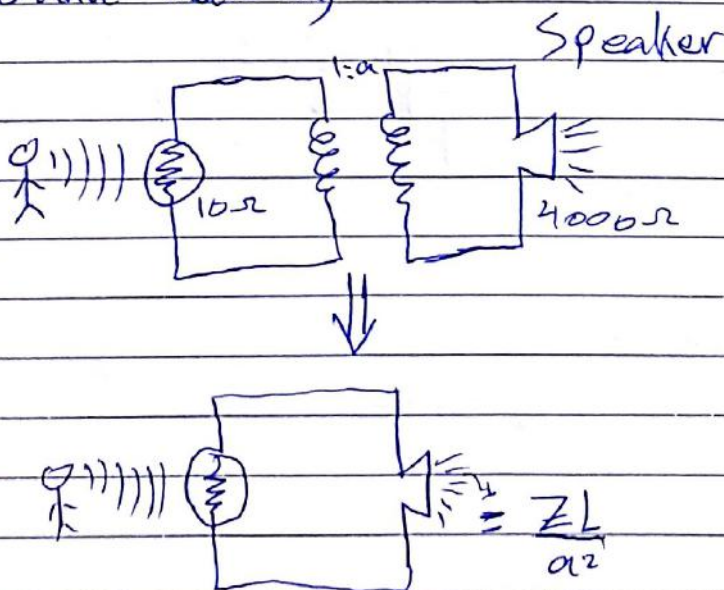
$$\frac{V_2}{V_1} = \frac{-1}{2}, \quad \frac{I_1}{I_2} = \frac{-1}{2}$$

$\Rightarrow I_2 = -0.7272\text{ A}$

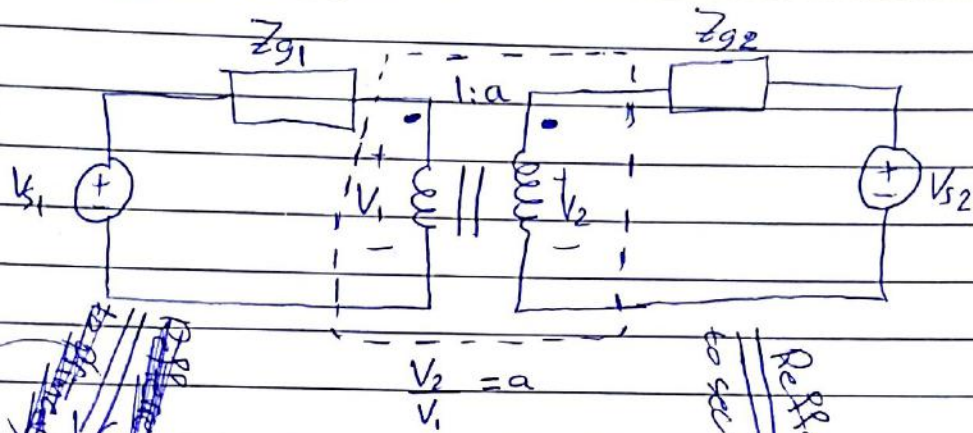
$\Rightarrow P = (-0.7272)^2 \times 10$   
 $= 5.3\text{ W}$

$\Rightarrow$  All equation are valid for both time & frequency domains

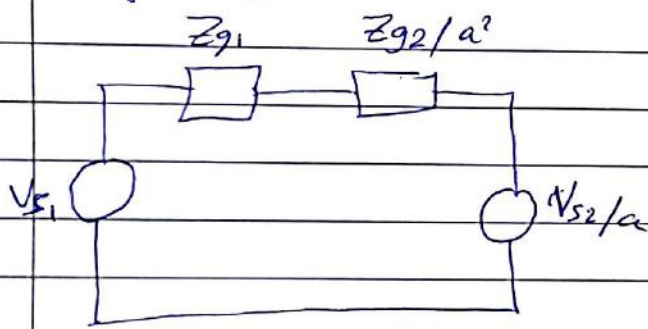
$\Rightarrow$  Impedance matching



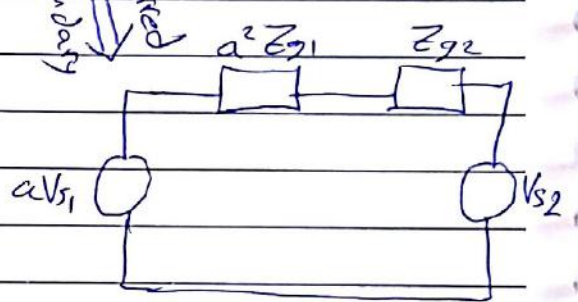
### # Equivalent CKT :-



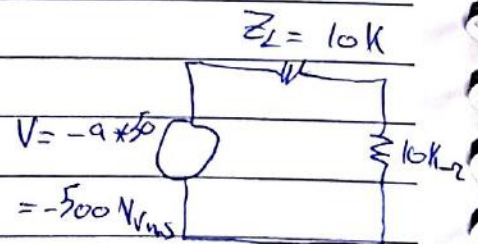
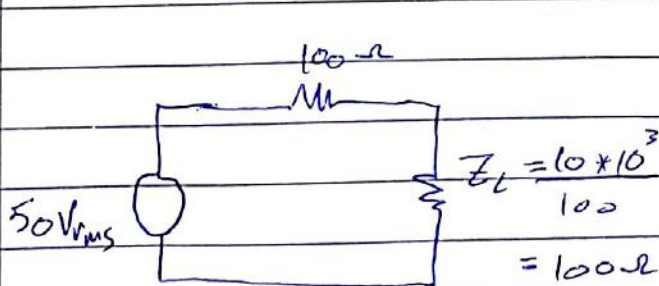
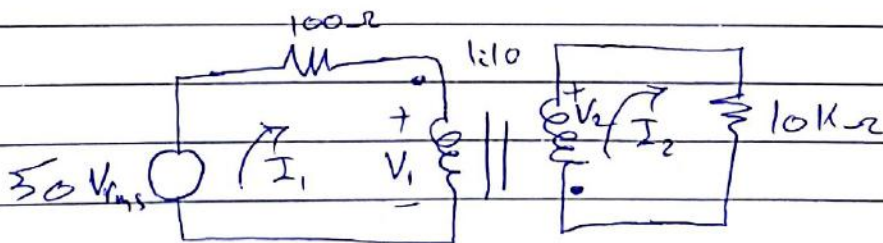
Referred to Primary



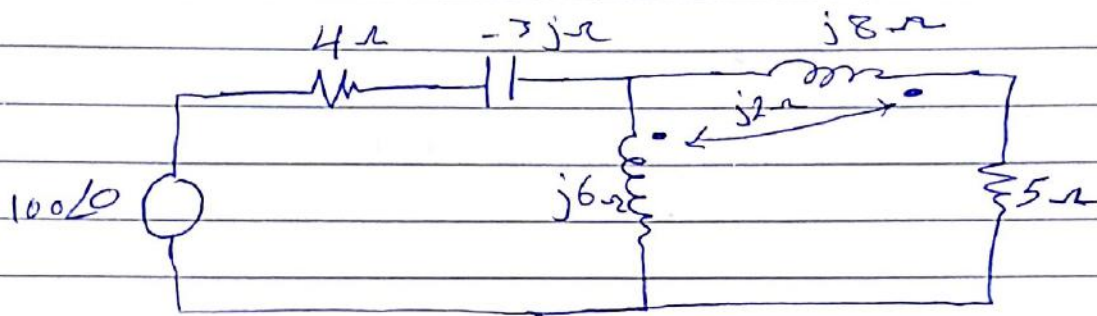
Referred to secondary



(ex) Draw the equivalent impedance referred to Primary & secondary.

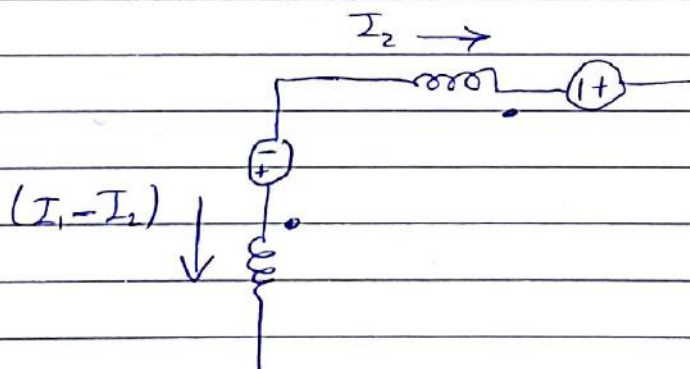


## # Review to second exam

(ex) Calculate the mesh currents in the CK T<sub>2</sub>-

sol - :

$$\text{@ Mesh 1: } -100\angle 0 + (4 - j3)I_1 + j6(I_1 - I_2) - j2I_2 = 0$$

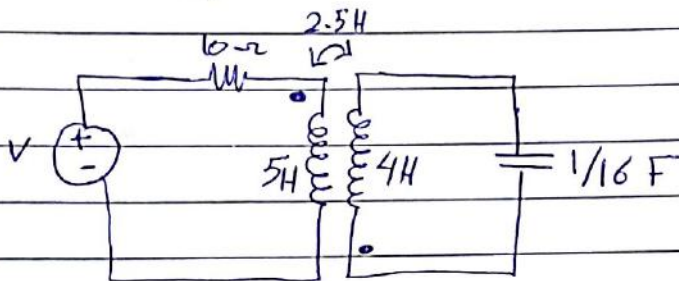


@ Mesh 2:

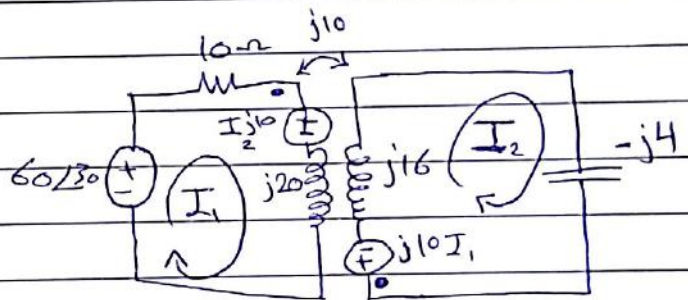
$$(5 + j8)I_2 + j6(I_2 - I_1) + j2I_2 - j2(I_1 - I_2) = 0$$

No \_\_\_\_\_

ex) For the CKT shown, Find the coupling coefficient & the total energy stored at  $t = 1 \text{ sec}$  if  $v = 60 \cos(4t + 30^\circ) \text{ V}$



$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{2.5}{\sqrt{5 \times 4}} = 0.56$$



Mesh 1:  $-60 \angle 30^\circ + 10 I_1 + j20 I_1 + j10 I_2 = 0$

Mesh 2:  $(j16 - j4) I_2 + j10 I_1 = 0$

$$I_1 = 3.905 \angle -19.4^\circ$$

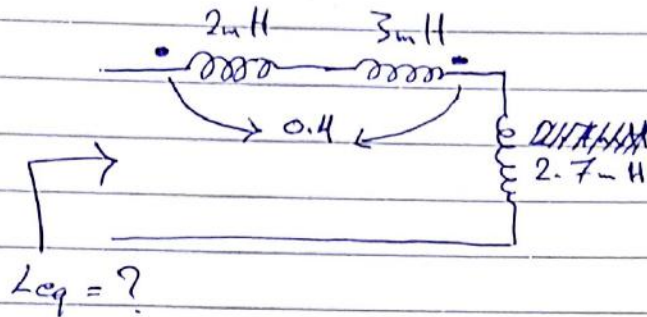
$$I_2 = 3.25 \angle 160.6^\circ$$

$$i_1(t) = 3.905 \cos(4t + 180^\circ - 19.4^\circ)$$

$$i_2(t) = 3.25 \cos(4t + 160.6^\circ)$$

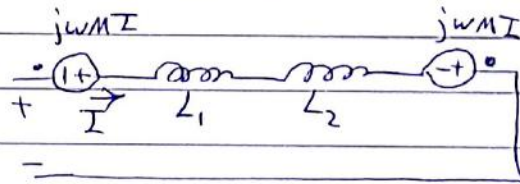
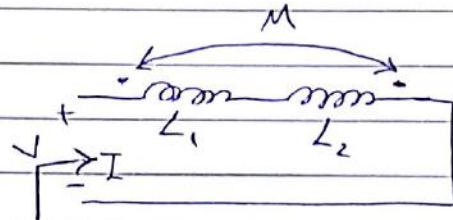
$$w(1) = \frac{1}{2} L_1 i_1^2(1) + \frac{1}{2} L_2 i_2^2(1) + M i_1(1) i_2(1) = 20.73 \text{ J}$$

ex) Find the total inductance at the CKT :-



حل المسألة

$$Z_{in} = \frac{V}{I}$$

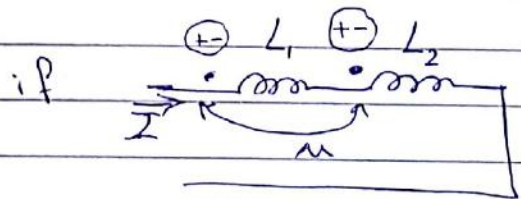


$$-V - j\omega M I + j\omega L_1 I + j\omega L_2 I - j\omega M I = 0$$

$$\frac{V}{I} = j\omega L_1 + j\omega L_2 - 2j\omega M$$

$$L_{eq} = L_1 + L_2 - 2M$$

إذا واتت دوائر في اتجاه واحد



$$L_{eq} = L_1 + L_2 + 2M$$

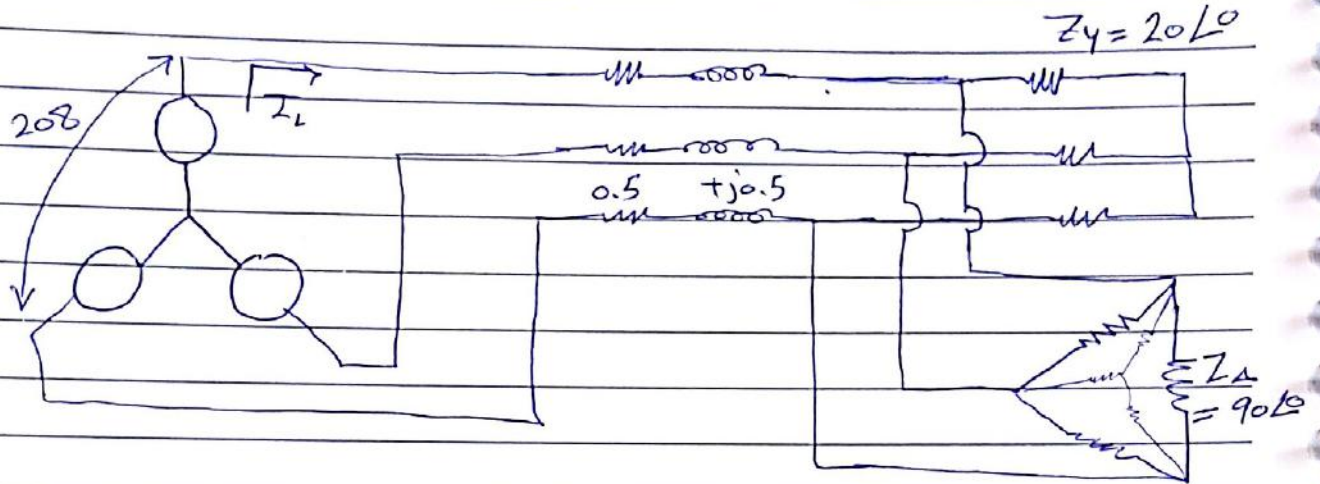
إذا واتت الدوائر في اتجاهين متعاكسين

⇒ in the ex. above :-

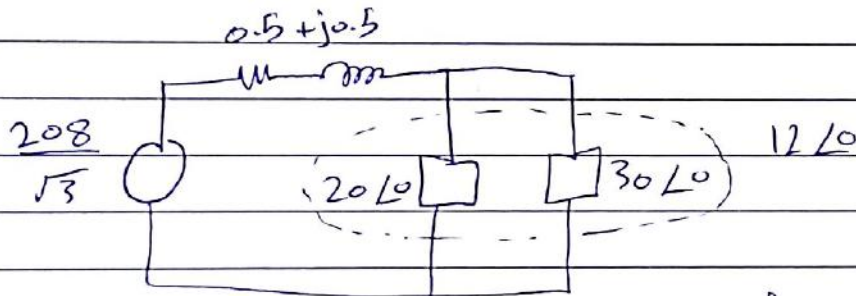
$$L_{eq} = 2m + 3m - 2 \times 0.4m + 2.7m$$

No \_\_\_\_\_

Voltage of generator = 208V



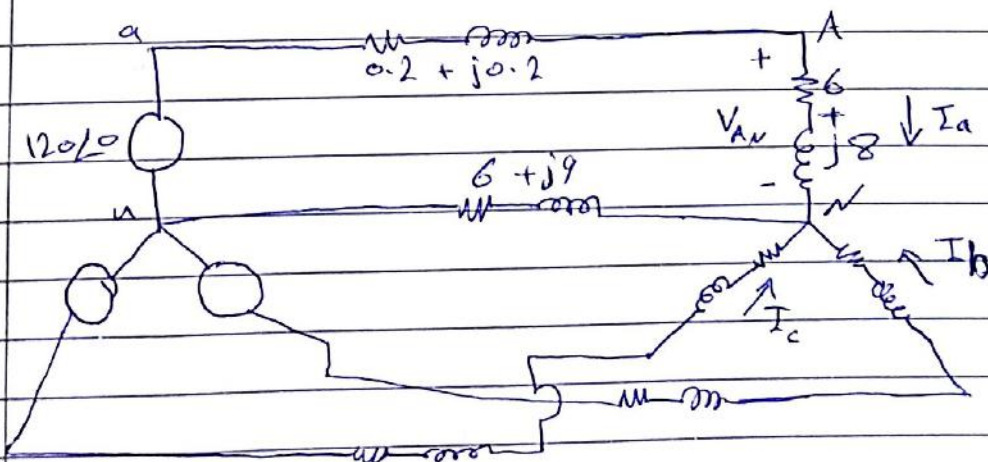
Sol:



$$I = \frac{208 / \sqrt{3} \angle 0^\circ}{12 \angle 0^\circ + 0.5 + j0.5}$$

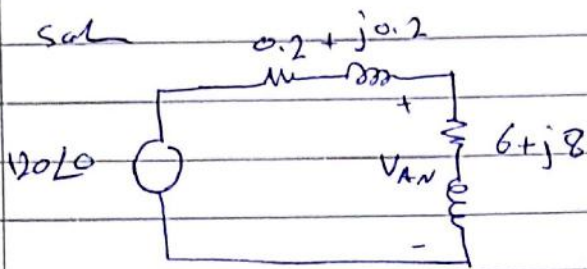
reference

Solve for line currents, phase voltages at load & line voltages.



Follow

No \_\_\_\_\_

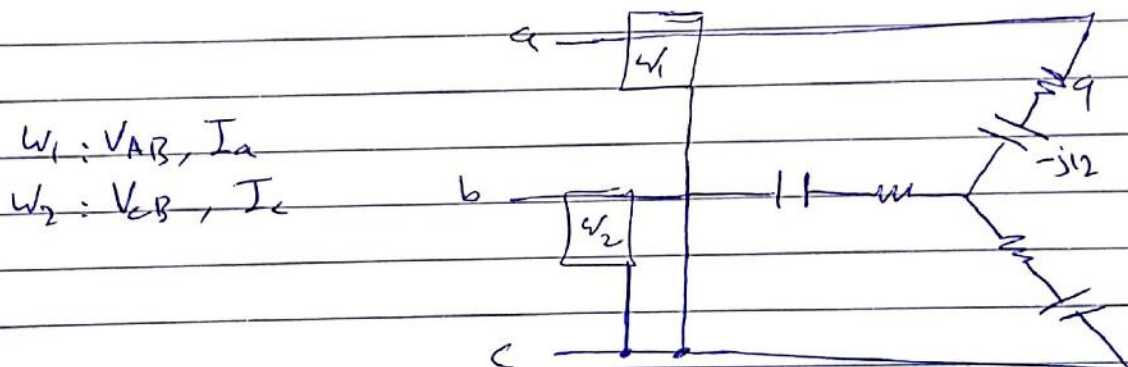


$$I_{aA} = \frac{120 \angle 0}{6.2 + j8.2} = 11.7 \angle -52.9$$

$$V_{AN} = I_{aA} \times (6 + j8) = 117 \angle 0.23$$

$$V_{AB} = \sqrt{3} \times 117 \angle 0.23 + 30, \quad V_{BC} = \sqrt{3} \times 117 \angle 0.23 + 30 + (-120)$$

$$PF = \cos(0 + 52.9)$$



$$W_1: V_{AB}, I_a$$

$$W_2: V_{CB}, I_c$$

$$V_{AB} = \sqrt{3} \times 120 \angle 30$$

$$V_{BC} = \sqrt{3} \times 120 \angle -90 = 208 \angle -90$$

$$V_{CB} = 208 \angle +90$$

$$I_a = \frac{V_{AN}}{9 - j12} = 8 \angle 53.13, \quad I_c = 8 \angle 53.13 \cdot 120$$

$$P_1 = 208 \times 8 \times \cos(30 - 53.13) = 1530 \text{ W}$$

$$P_2 = 208 \times 8 \times \cos(90 - 173.13) = 199 \text{ W}$$

Follow  $\Rightarrow$   
solution

No \_\_\_\_\_

$$V_{AB} = \sqrt{3} \times 117 \angle 0.23 + 30$$

$$V_{BC} = \sqrt{3} \times 117 \angle 0.23 + 30 - 120$$

$$\text{PF} = \cos(0 + 52.9)$$

$$P_T = P_1 + P_2 = 1729, \quad P_{\phi} = \frac{1729}{3}$$

$$Q_T = \sqrt{3} (P_2 - P_1) = -2305.36 \text{ VAR}$$

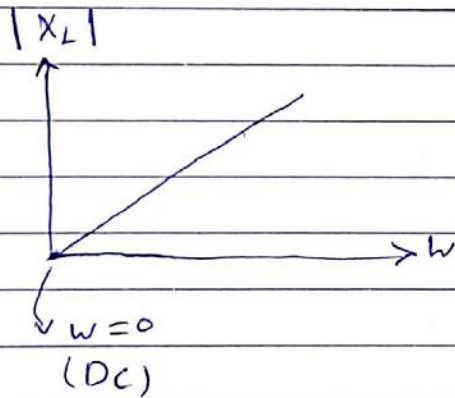
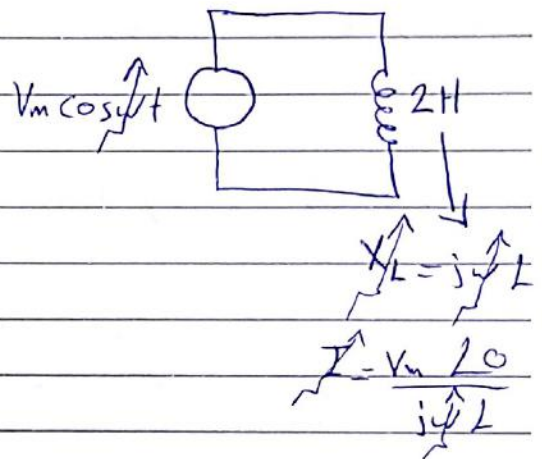
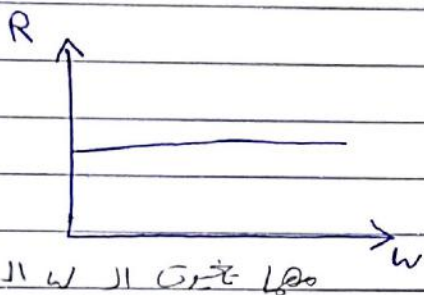


# # CH 14

No \_\_\_\_\_

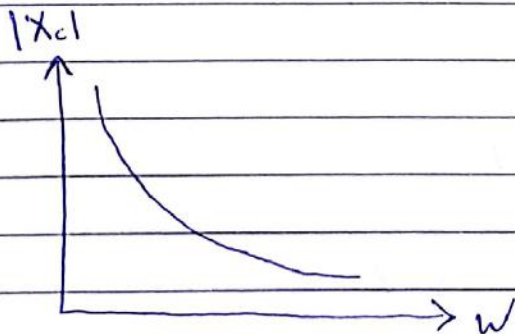
## # Frequency Response :-

→ what is the behaviour of the ckt if the frequency is changed.



$$|X_L| = |2\pi f L|$$

$$= |\omega L|$$

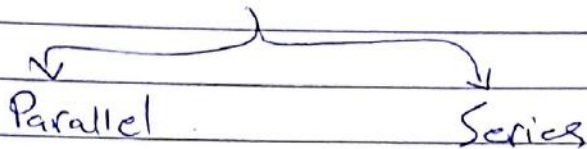


$$|X_C| = \frac{1}{|\omega C|}$$

$$\omega = 2\pi f$$

rad  $\rightarrow$  Hz

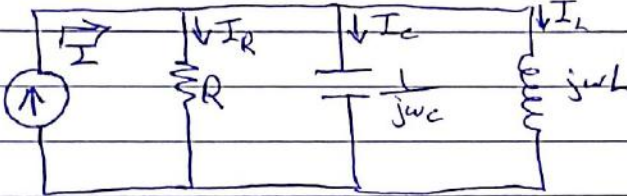
\* Resonance circuits :-



⇒ it is ~~contains~~ a circuit that contains an inductor & capacitor in a parallel or in series.

let us study this ckt :-

$$I = I_R + I_C + I_L$$



$$Z_{total} = R \parallel \frac{1}{j\omega C} \parallel j\omega L$$

$$Y_{total} = \frac{1}{R} + j\omega C + \frac{1}{j\omega L} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

$$|Y_{total}| = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

$$V = I Z_{total} = I * \frac{1}{Y_{total}}$$

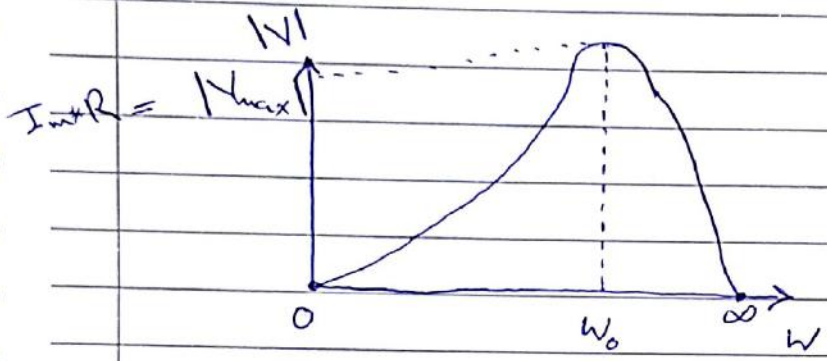
⇒  $|Y_{total}|$  is minimum &  $V$  is maximum when  
imaginary part  $\leftarrow \omega C - \frac{1}{\omega L} = 0$   
= Zero

we call it  
resonance frequency

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

No

$$\omega_0 = \frac{1}{\sqrt{LC}}, \quad P_0 = \frac{1}{2\pi\sqrt{LC}}$$

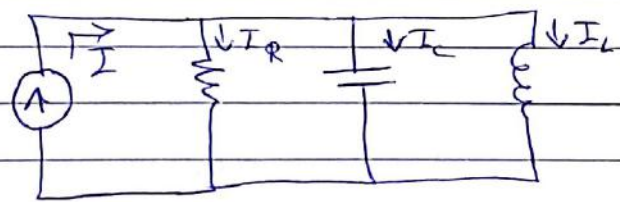


→ at resonance

→ V is maximum =  $I_m \times R$

→  $|Y_{total}|$  is minimum  $\Rightarrow Z$  is maximum

→ I & V are in phase.



$$I = I_R + I_C + I_L$$

at resonance  $I_m = I_R$  &  $V = I_m \times R$

$$I_C = \frac{V}{X_C}, \quad I_L = \frac{V}{X_L}$$

→ at resonance  $I_L$  &  $I_C$  does not equal zero

But  $I_C = -I_L \Rightarrow I_C + I_L = 0$

No \_\_\_\_\_

$$I_{L,0} = \frac{V_{L,0}}{j\omega L}$$

$I_L$  at resonance  
Voltage at resonance

$$= \frac{I_m R}{j\omega L}$$

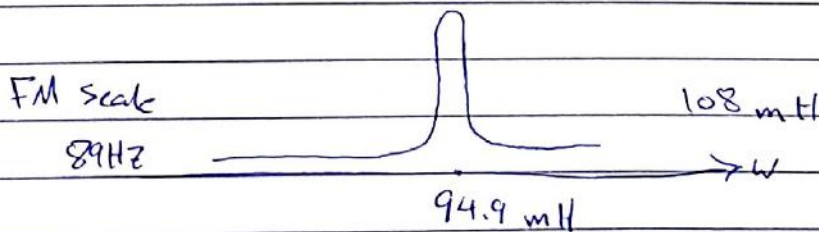
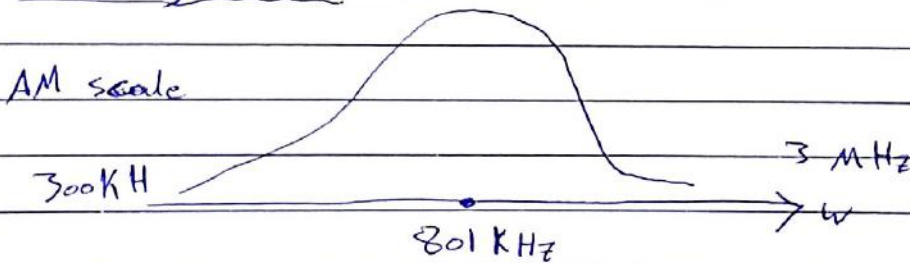
$$|I_{L,0}| = \frac{|V_{L,0}|}{|\omega L|}$$

capacitor current at resonance

$$I_{C,0} = \frac{V_{C,0}}{1/j\omega C} \rightarrow \text{capacitor voltage at resonance.}$$

$$= j\omega C V_{C,0} \quad | \quad |I_{C,0}| = \omega C |V_{C,0}|$$

Quality Factor: Q



No \_\_\_\_\_

Quality Factor = Q

$$Q = 2\pi \frac{\text{maximum energy stored}}{\text{total energy lost per period}}$$

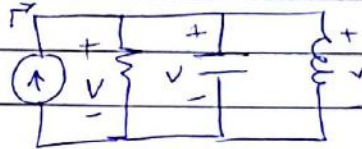
$$= 2\pi \frac{\text{Energy} [W_L(t) + W_C(t)]_{\max}}{P_R T}$$

$$P = \frac{1}{T} \int v(t)i(t)$$

$$P_T = \int v(t)i(t)$$

$$i(t) = I_m \cos \omega t$$

$$v(t) = R I_m \cos \omega t$$



$$W_C(t) = \frac{1}{2} C V^2(t) = \frac{C * R^2 I_m^2 \cos^2 \omega t}{2}$$

$$W_L(t) = \frac{1}{2} L i^2(t) = \frac{1}{2} L \left[ \frac{1}{L^2} R^2 I_m^2 \sin^2 \omega t \right] = \frac{C R^2 I_m^2 \sin^2 \omega t}{2}$$

$\frac{1}{L^2} \leftarrow \frac{1}{L^2}$

$$W_L(t) + W_C(t) = \frac{C R^2 I_m^2}{2} [\cos^2(\omega t) + \sin^2(\omega t)]$$

$$W_L(t) + W_C(t) = \frac{C R^2 I_m^2}{2}$$

$$P_R T = \frac{1}{2} I_m^2 R \frac{1}{f_0}$$

$$Q = 2\pi \frac{C R^2 I_m^2 / 2}{I_m^2 R / 2 f_0} \Rightarrow Q = \omega_0 R C, \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q = \frac{1}{\sqrt{LC}} R C = R \sqrt{\frac{C}{L}} = \frac{R}{X_{L0}} = \frac{R}{X_{C0}}$$

No

at resonance

$$I_{C,10} = -I_{C,10} = j\omega_0 C R I = j Q_0 I$$

$$|I| = 2 \text{ mA} \quad Q_0 = 50$$

$$|I_{C,10}| = 100 \text{ mA}$$

ex) Parallel RLC ckt  $L = 2 \text{ mH}$ ,  $Q_0 = 5$ ,  $C = 10 \text{ nF}$   
Find  $R$  & admittance at  $0.9\omega_0$ ,  $\omega_0$ ,  $1.1\omega_0$

sol:

$$R = Q_0 \sqrt{\frac{L}{C}} = 5 \sqrt{\frac{2 \times 10^{-3}}{10 \times 10^{-9}}} = 2.23 \text{ k}\Omega$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 223.6 \text{ k rad/s}$$

$$f_0 = \frac{\omega_0}{2\pi}$$

$$Y = \frac{1}{R} + j\omega C + \frac{1}{j\omega L}$$

$0.9\omega_0$   $j\omega_0$   $0.9$   $\omega_0$   $\omega_0$   $\omega_0$

$\times 0.9$

$$Y(\omega_0) = \frac{1}{223.6 \times 10^3} + j(223.6 \times 10^3) 10^{-9} + \frac{1}{j \times 223.6 \times 10^3 \times 2 \times 10^{-3}}$$

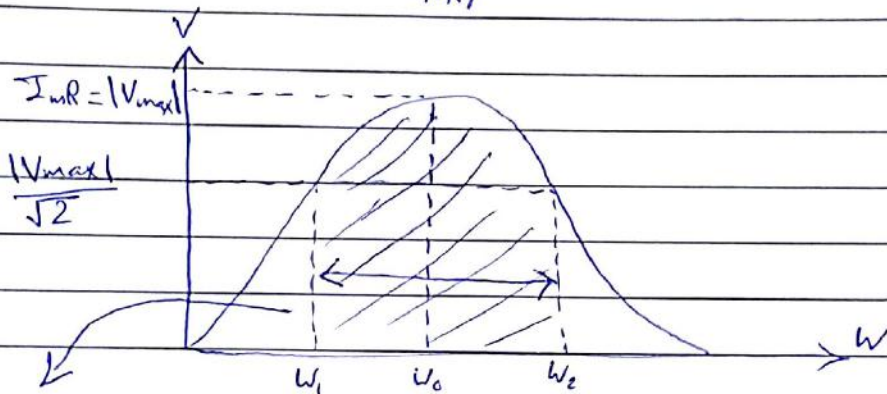
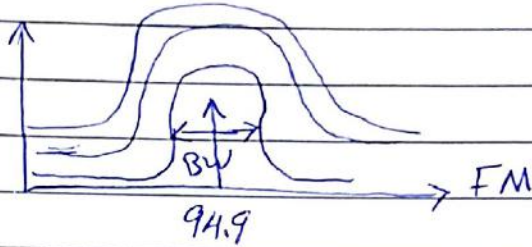
$\times 0.9$

$$|Y(\omega_0)| = 4.472 \times 10^{-4} \text{ S or } \Omega^{-1} \text{ Siemens}$$

$$|Y(0.9\omega_0)| = 6.504 \times 10^{-4} \text{ S}$$

$$|Y(1.1\omega_0)| = 6.182 \times 10^{-4} \text{ S}$$

Band width :-



Half-power Band width

↳ lower half-power frequency

↳ upper half-power frequency

$w_1$  &  $w_2$  are called two half power frequencies.

$$V(w_0) = \sqrt{2} V(w_1) = \sqrt{2} V(w_2)$$

$$V(w_0) = \frac{1}{\sqrt{2}} V(w_1) = \frac{1}{\sqrt{2}} V(w_2)$$

$B = w_2 - w_1$  → Band width

↳ half-power ~~BW~~

How to define BW :-

$$Y = \frac{1}{R} + j \left( wC - \frac{1}{wL} \right)$$

use  $Q_0 = w_0 RC$

⇒ Follow

No \_\_\_\_\_

$$Y = \frac{1}{R} + j \frac{1}{R} \left( \frac{W W_0 R C}{W_0} - \frac{W_0 R}{W W_0 L} \right)$$

$$Y = \frac{1}{R} \left[ 1 + j Q_0 \left( \frac{W}{W_0} - \frac{W_0}{W} \right) \right]$$

if we need  $Y(W_1) = \sqrt{2} Y(W_0)$  &  $Y(W_2) = \sqrt{2} Y(W_0)$

$$Q_0 \left( \frac{W_2}{W_0} - \frac{W_0}{W_2} \right) = 1 \quad \text{--- (1)}$$

&

$$Q_0 \left( \frac{W_1}{W_0} - \frac{W_0}{W_1} \right) = -1 \quad \text{--- (2)}$$

$$Y(W_0) = \frac{1}{R} \sqrt{(1)^2 + (0)^2} = \frac{1}{R}$$

$$Y(W_1) = \frac{1}{R} \sqrt{(1)^2 + (1)^2} = \frac{\sqrt{2}}{R}$$

$$\checkmark W_1 = W_0 \left[ \sqrt{1 + \left( \frac{1}{2Q_0} \right)^2} - \frac{1}{2Q_0} \right]$$

$$\checkmark W_2 = W_0 \left[ \sqrt{1 + \left( \frac{1}{2Q_0} \right)^2} + \frac{1}{2Q_0} \right]$$

$$\checkmark B = W_2 - W_1 = \frac{W_0}{Q_0}$$



No \_\_\_\_\_

$$\omega_1 = \omega_0 \left[ \sqrt{1 + \left(\frac{1}{2Q_0}\right)^2} - \frac{1}{2Q_0} \right]$$

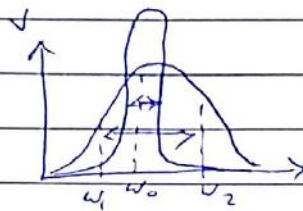
$$\omega_2 = \omega_0 \left[ \sqrt{1 + \left(\frac{1}{2Q_0}\right)^2} + \frac{1}{2Q_0} \right]$$

~~W<sub>1</sub> & W<sub>2</sub>~~

Half Power BW

$$\beta = \omega_2 - \omega_1$$

$$\beta = \frac{\omega_0}{Q_0}$$



$$\omega_0^2 = \omega_1 \omega_2 \Rightarrow \omega_0 = \sqrt{\omega_1 \omega_2}$$

\* Approximations for High Quality factor  $Q_0 > 5$

$$\omega_1 \approx \omega_0 \left[ 1 - \frac{1}{2Q_0} \right], \quad \omega_2 \approx \omega_0 \left[ 1 + \frac{1}{2Q_0} \right]$$

$$\omega_{1,2} \approx \omega_0 \mp \frac{1}{2} \beta$$

$$\frac{1}{2} \beta = \frac{\omega_0}{2Q_0}$$

(ex) Parallel RLC ckt with

$$R = 40 \text{ k}\Omega, \quad L = 1 \text{ H}, \quad C = \frac{1}{64} \mu\text{F}$$

Find the locations for the two half power BW & the approximate value for admittance at 8200 rad/s

Sol:

$$Q_0 = \omega_0 RC, \quad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 * \frac{1}{64} * 10^{-6}}} = 8 \text{ k rad/s}$$

$$Q_0 = 8 * 10^3 * 40 * 10^3 * \frac{1}{64} * 10^{-6} = 5$$

Follow

No \_\_\_\_\_

$$\Rightarrow \omega_1 = \omega_0 \left[ 1 - \frac{1}{10} \right] = 7200 \text{ rad/sec}$$

$$\omega_2 = \omega_0 \left[ 1 + \frac{1}{10} \right] = 8800 \text{ rad/s}$$

$$Y = \frac{1}{R} + j\omega C + 1/j\omega L$$

$$= \frac{1}{40 \times 10^3} + j(8200) \frac{1}{64} \times 10^{-6} + \frac{1}{j8200}$$

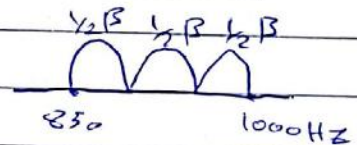
$$Y = \frac{1}{R} [1 + jN] \text{ approximate}$$

$$|Y| = \frac{1}{R} \sqrt{1 + N^2}, \quad \angle Y = \tan^{-1}(N)$$

$$N = \frac{\omega - \omega_0}{\frac{1}{2}B} \rightarrow \text{number of half BW OFF resonance}$$

ex) Find  $N$  at 850 Hz

$$N = \frac{850 - 1000}{50} = -3$$



@ 8200:-

$$N = \frac{8200 - 8000}{800} = 0.25$$

$$Y = \frac{1}{40 \times 10^3} [1 + j0.25]$$

$$|Y| = \frac{1}{40 \times 10^3} \sqrt{1 + (0.25)^2}, \quad \angle Y = \tan^{-1}(0.25) = 14.04^\circ$$

25.77  $\mu$ s

\* Series Resonance :-



$$Z = R + j\omega L + \frac{1}{j\omega C} = R + j\left[\omega L - \frac{1}{\omega C}\right]$$

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

when = 0

$$\omega L = \frac{1}{\omega C}$$

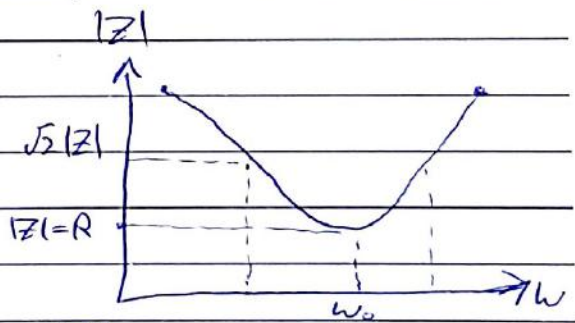
$$\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

Resonance frequency

\* For series resonance :-

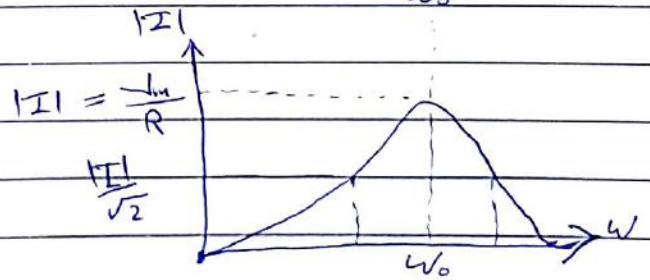
(a) Resonance frequency :-

- (1)  $|Z|$  is minimum
- (2)  $|I|$  is maximum
- (3)  $V$  &  $I$  are in phase  
Pure resistive load.

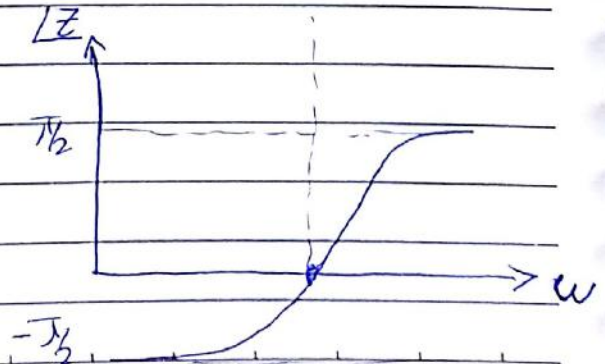


(4)  $V_{L0}$  &  $V_{C0} \neq 0$

$$V_{L0} = -V_{C0}$$



$$|V_{C0}| = |I| \frac{1}{\omega C} = \frac{V_m}{R\omega C}$$



$$\omega_0 = \frac{1}{\sqrt{LC}}, \quad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC} = \frac{\omega_0}{\beta}$$

$$\Rightarrow \beta = \frac{\omega_0}{Q_0}$$

$Q_0 = \frac{2\pi \times \text{maximum stored energy}}{\text{Total energy lost per period}}$

$$\beta = \omega_2 - \omega_1 = \frac{\omega_0}{Q_0}$$

$$\omega_1 = \omega_0 \left[ \sqrt{1 + \left(\frac{1}{2Q_0}\right)^2} - \frac{1}{2Q_0} \right]$$

$$\omega_2 = \omega_0 \left[ \sqrt{1 + \left(\frac{1}{2Q_0}\right)^2} + \frac{1}{2Q_0} \right]$$

For CKTs with high  $Q_0$  :-

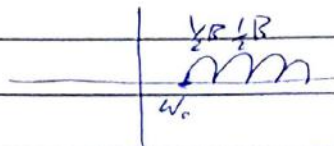
$$\omega_{1,2} = \omega_0 \left[ 1 \mp \frac{1}{2Q_0} \right]$$

281 4/17

No

$$Z_{eq} = R(1 + jN) \quad , \quad |Z| = R\sqrt{1+N^2} \quad , \quad \angle Z = \tan^{-1}(N)$$

$$N = \frac{W - W_0}{\frac{1}{2}\beta}$$



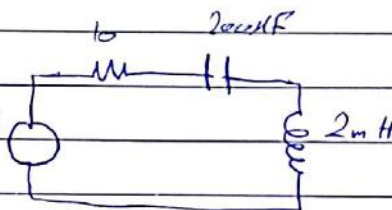
$N \equiv$  # of BW off resonance

ex)  $V_s = 100 \cos \omega t$  mV for a series RLC ckt with  $R = 10 \Omega$ ,  $C = 200 \text{ nF}$ ,  $L = 2 \text{ mH}$ , calculate  $|I|$  if  $W = 48 \text{ k rad/s}$

sol:

ob)  $V_s$  to Exact solutions

$$Z = R + j\omega L + \frac{1}{j\omega C} \quad [W = 48 \text{ k}] \quad 100 \cos \omega t$$



ob)  $V_s$  to sol:

$$W_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2 \times 10^{-3} \times 200 \times 10^{-9}}} = 50 \text{ k rad/sec}$$

$$Q_0 = \frac{1}{W_0 R C} = \frac{W_0 L}{R} = \frac{50 \times 10^3 \times 2 \times 10^{-3}}{10} = 10 > 5$$

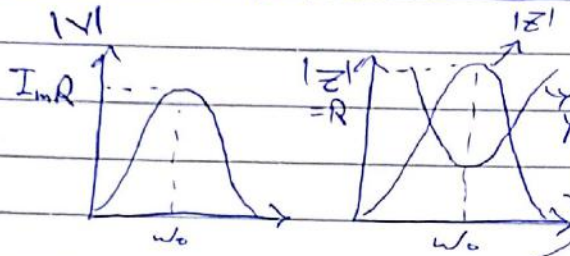
$$N = \frac{W - W_0}{\frac{1}{2}\beta} \quad , \quad \beta = \frac{W_0}{Q_0} = \frac{50}{10} = 5 \text{ k rad/s}$$

$$N = \frac{48 \text{ k} - 50 \text{ k}}{2.5 \text{ k}} = -0.8$$

$$Z_{eq} = [R\sqrt{1+N^2}] / \tan^{-1} N = 12.81 / -38.66 \Omega$$

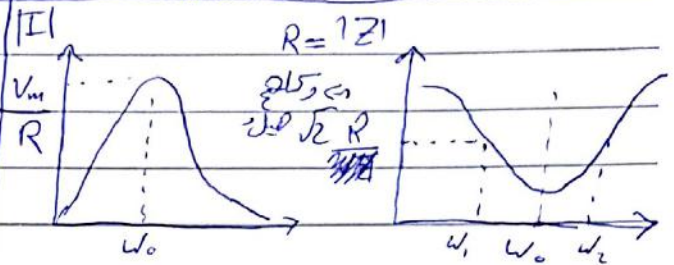
$$|I| = \frac{100 \text{ mV}}{12.81} = 7.806 \text{ mA}$$

Parallel



$$Q_0 = \omega_0 RC = \frac{R}{\omega_0 L} = \frac{|I|}{I} = \frac{1}{R}$$

Series



$$Q_0 = \frac{1}{\omega_0 RC} = \frac{\omega_0 L}{R}$$

$$|I_L(j\omega_0)| = |I_C(j\omega_0)| = Q_0 |I(j\omega_0)| \quad |V_L(j\omega_0)| = |V_C(j\omega_0)| = Q_0 |V(j\omega_0)|$$

Exact: [Parallel & series]

$$\omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{\omega_1 \omega_2}$$

$$\omega_{1,2} = \omega_0 \left[ \sqrt{1 + \frac{1}{4Q_0^2}} \mp \frac{1}{2Q_0} \right]$$

Approximate :- [Q₀ > 5]

$$\omega_{1,2} = \omega_0 \left[ 1 \mp \frac{1}{2Q_0} \right]$$

$$\omega_0 = \frac{1}{2} (\omega_1 + \omega_2)$$

$$\omega_{1,2} = \omega_0 \mp \frac{1}{2} \beta$$

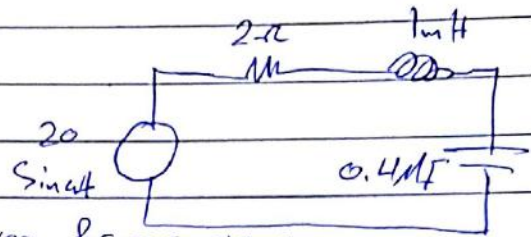
$$Z_s = R [1 + jN] \quad , \quad Y_p = \frac{1}{R} [1 + jN]$$

Series

Parallel

$$N = \frac{\omega - \omega_0}{\frac{1}{2} \beta}$$

(ex) For the series resonant circuit shown, find:-



(1) Resonance frequency, half power frequencies:-

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3} \times 0.4 \times 10^{-6}}} = 50 \text{ krad/sec}$$

$$Q_0 = \frac{\omega_0 L}{R} = \frac{50 \times 10^3 \times 10^{-3}}{2} = 25$$

$$\beta = \frac{\omega_0}{Q_0} = \frac{50 \text{ krad/sec}}{25} = 2 \text{ krad/sec}$$

$$\omega_1 = \omega_0 - \frac{1}{2}\beta = 49 \text{ krad/sec}$$

$$\omega_2 = \omega_0 + \frac{1}{2}\beta = 51 \text{ krad/sec}$$

(2) Find the amplitude of the current at  $\omega_0, \omega_1, \omega_2$ .  
at  $\omega_0$   $|I| = \frac{V_m}{R} = \frac{20}{2} = 10 \text{ A}$

$$\text{at } \omega_1, \omega_2 \Rightarrow |I| = \frac{|I_{\max}|}{\sqrt{2}} = \frac{10}{\sqrt{2}}$$

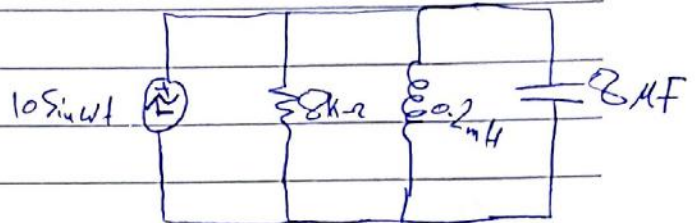
$$\text{loop vol} \Rightarrow n = \frac{\omega - \omega_0}{\frac{1}{2}\beta} = \frac{\omega_1 - \omega_0}{\frac{1}{2}\beta} = \frac{49 - 50}{\frac{1}{2} \times 2} = 1$$

(ex) For the Parallel RLC CKT shown

Find (1)  $\omega_0, Q_0, \beta, \omega_1, \omega_2$

(2) the Power dissipated @  $\omega_0, \omega_1, \omega_2$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 25 \text{ K rad/sec}$$



$$Q_0 = \omega_0 RC = 1600 \text{ [high selective ckt]}$$

$$\beta = \frac{\omega_0}{Q_0} = 15.625 \text{ rad/sec}$$

↑ impulse

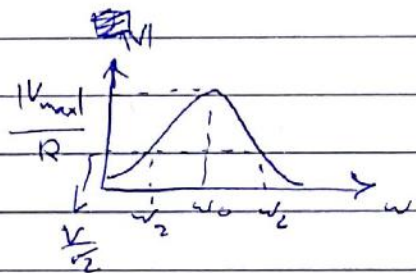
$$\omega_1 = \omega_0 - \frac{1}{2}\beta = 24,992 \text{ rad/sec}$$

$$\omega_2 = \omega_0 + \frac{1}{2}\beta = 25,008 \text{ rad/sec}$$

(2) Power at  $\omega_0$

$$P = \frac{1}{2} |I|^2 R$$

Peak  $\frac{V_m}{\sqrt{2}}$  ←



$$|I| = \frac{10 \angle -90}{8 \times 10^3} = 1.25 \angle -90 \mu\text{A}$$

↳ Peak

$$P = \frac{1}{2} \times (1.25 \times 10^{-3})^2 \times 8 \times 10^3 = 6.25 \text{ mW}$$

Power at  $\omega_1, \omega_2$  equal  $\frac{1}{2}$  Power at  $\omega_0$  (half power band width)

or  $P_1 = P_2 = \frac{1}{2} P = \frac{1}{2} \times 6.25 \text{ mW} = 3.125 \text{ mW}$

$$P_i = \frac{(V_m/\sqrt{2})^2}{2R} = 3.125 \text{ mW}$$

$$P @ \omega_0 = P @ \omega_1 = P @ \omega_2 = 6.25 \text{ mW}$$

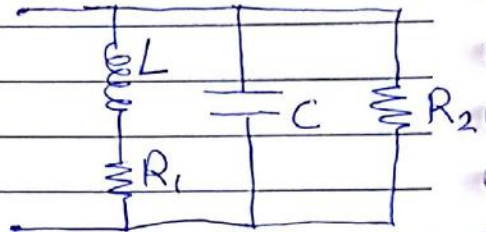
Voltage is 10 sin wt source



\* Other Resonance Forms -

→ Not Parallel, Not series.

Key: 
$$\begin{aligned} \text{Im}[Z] &= 0 \\ \text{Im}[Y] &= 0 \end{aligned}$$



$$Y(\omega) = j\omega C + \frac{1}{R_2} + \frac{1}{j\omega L + R_1} \cdot \frac{(-j\omega L + R_1)}{j\omega L + R_1}$$

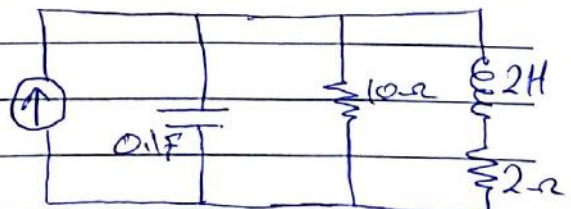
$$Y(\omega) = \frac{1}{R_2} + j\omega C + \frac{R_1 - j\omega L}{R_1^2 + (\omega L)^2}$$

$$\text{Im}[Y] = \omega C - \frac{\omega L}{R_1^2 + (\omega L)^2} = 0$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \left(\frac{R_1}{L}\right)^2} \Rightarrow \text{Resonance Frequency for this form}$$

(ex) Determine the resonance frequency for the ckt shown

$$Y = j\omega \cdot 0.1 + \frac{1}{10} + \frac{1 \cdot (2 - j\omega 2)}{(2 + j\omega 2) \cdot (2 - j\omega 2)}$$

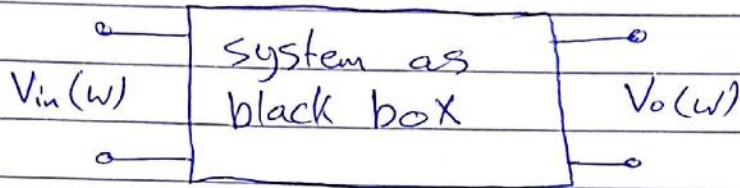


$$Y = j\omega \cdot 0.1 + 0.1 + \frac{2 - j\omega 2}{4 + 4\omega^2}$$

$$\text{Im}[Y] = 0.1\omega - \frac{2\omega}{4 + 4\omega^2} = 0 \Rightarrow \omega_0 = 2 \text{ rad/s}$$

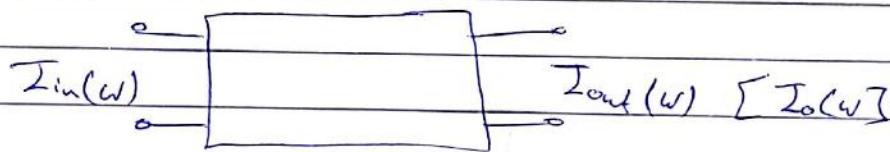
## Transfer Function :-

↳ How to relate the input to the output irrespective of the system



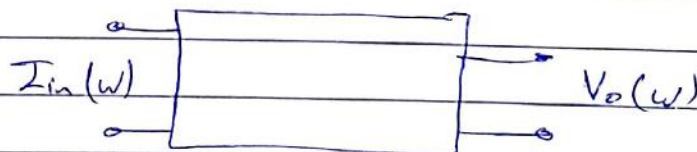
$H_V(\omega) \triangleq$  Voltage gain

$$H_V(\omega) = \frac{V_o(\omega)}{V_{in}(\omega)} \equiv \text{Voltage gain}$$



$H_I(\omega) =$  current gain

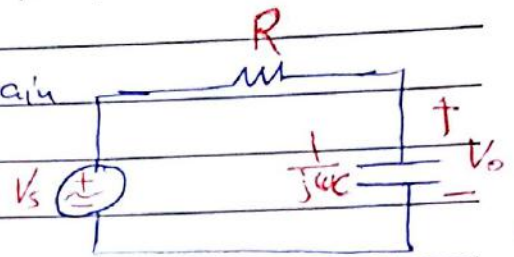
$$H_I(\omega) = \frac{I_o(\omega)}{I_{in}(\omega)}$$



$$H_Z(\omega) = \frac{V_o(\omega)}{I_{in}(\omega)} \triangleq \text{Transfer Impedance}$$

(ex) For the circuit shown obtain the transfer function  $\frac{V_o}{V_s}$

, where  $V_s = V_m \cos \omega t$



Sol —

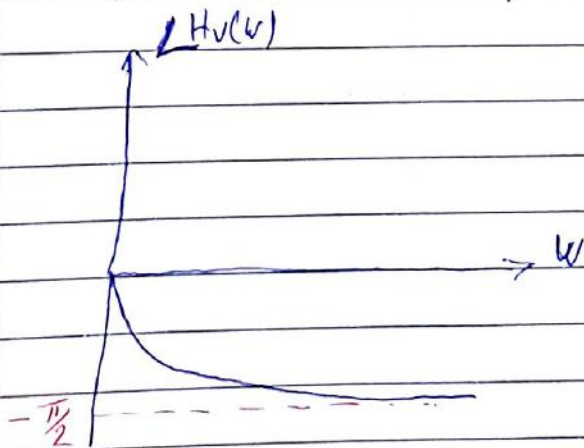
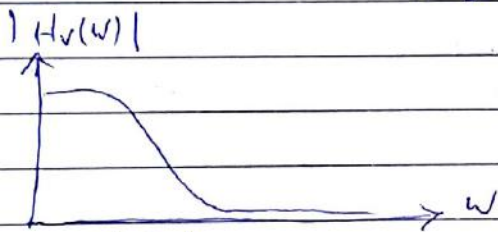
$$H_v(\omega) = \frac{V_o}{V_{in}} = \frac{V_o}{V_s} = \frac{I * \frac{1}{j\omega C}}{I * (R + \frac{1}{j\omega C})}$$

$$H_v(\omega) = \frac{\frac{1}{j\omega C} * j\omega L}{(\frac{1}{j\omega C} + R) * j\omega L} = \frac{1}{1 + j\omega RC}$$

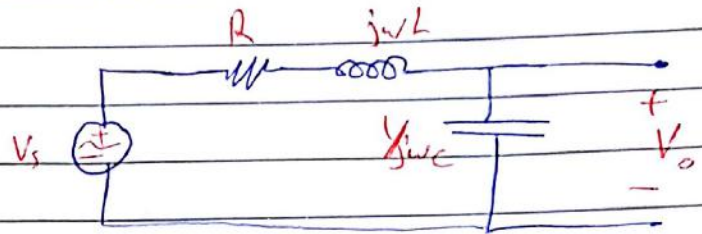
$$|H_v(\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

$$\angle (H_v(\omega)) = \angle \left( \frac{1}{1 + j\omega RC} * \frac{(1 - j\omega RC)}{(1 - j\omega RC)} \right) \rightarrow \tan^{-1}(-\omega RC)$$

Real Part is 1 ←



(ex) Find the Voltage transfer Function for these ckt.



$$H_v(\omega) = \frac{V_o(\omega)}{V_s(\omega)}$$

$$= \frac{I * 1/j\omega C}{I * (R + j\omega L + 1/j\omega C)}$$

$$= \frac{(1/j\omega C) * (j\omega C)}{(R + j\omega L + 1/j\omega C) * (j\omega C)} \quad j*j = -1$$

$$H_v(\omega) = \frac{1 * (1 - \omega^2 LC) - j\omega RC}{(j\omega RC - \omega^2 LC + 1) * (j\omega C)}$$

$$|H_v(\omega)| = \frac{1}{\sqrt{(1 - \omega^2 LC)^2 + (\omega RC)^2}}$$

$$\angle H_v(\omega) = \tan^{-1} \left( \frac{-\omega RC}{1 - \omega^2 LC} \right)$$

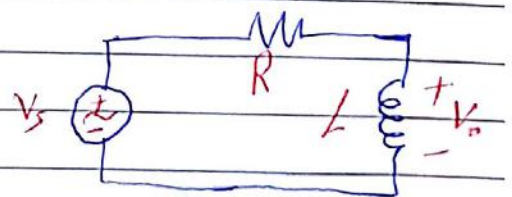
$$H(\omega) = \frac{\alpha - j\beta}{\alpha^2 + \beta^2}$$

$$\angle H(\omega) = \tan^{-1}(-\beta/\alpha)$$

(ex) obtain the transfer Function  $\frac{V_o}{V_s}$  at the RL ckt

$$H_v(\omega) = \frac{V_o(\omega)}{V_s(\omega)} = \frac{I * j\omega L}{I * (R + j\omega L)}$$

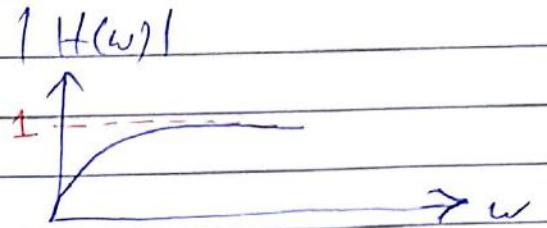
$$= \frac{(j\omega L) * (R - j\omega L)}{(R + j\omega L) * (R - j\omega L)}$$



Follow

$$H(\omega) = \frac{\omega^2 L^2 + j\omega RL}{R^2 + \omega^2 L^2}$$

$$|H(\omega)| = \frac{\omega L}{\sqrt{R^2 + (\omega L)^2}}$$



$$\angle H(\omega) = \tan^{-1} \left[ \frac{\omega RL}{\omega^2 L^2} \right] = \tan^{-1} \left[ \frac{R}{\omega L} \right]$$

↳ From  $H(\omega) = \frac{\omega^2 L^2 + j\omega RL}{R^2 + \omega^2 L^2}$



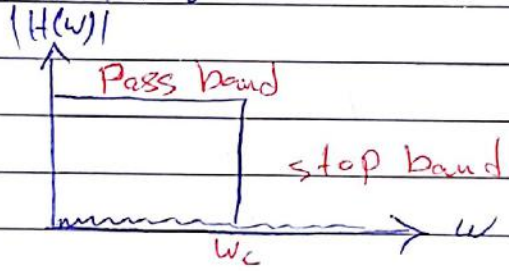
### # Filters :

→ a filter is a ckt that is defined to pass signals with disired frequencies & reject others.

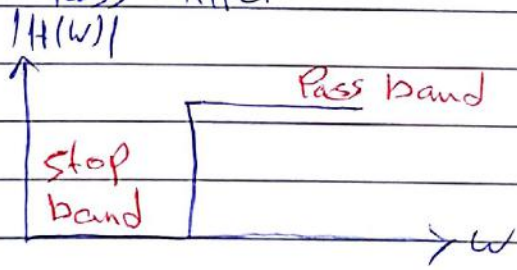
⇒ A filter is a frequency selective device.

#### Types :-

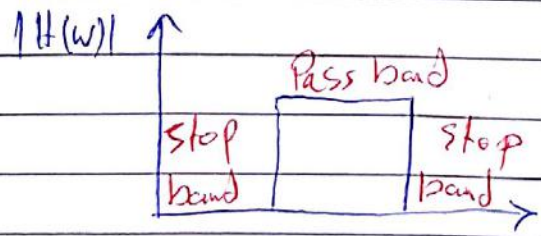
(1) Low Pass Filter :-



(2) High Pass Filter



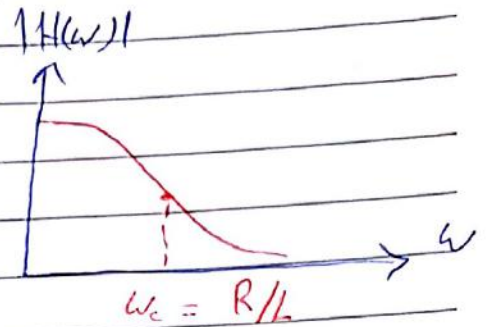
(3) Band Pass Filter :





$$H(\omega) = \frac{R}{j\omega L + R}$$

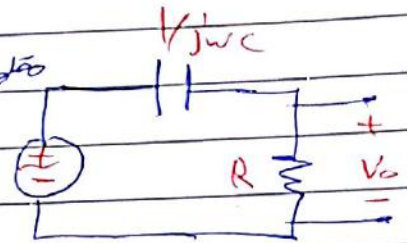
$$|H(\omega)| = \frac{R}{\sqrt{R^2 + (\omega L)^2}}$$



Low Pass \* High Pass filter :-

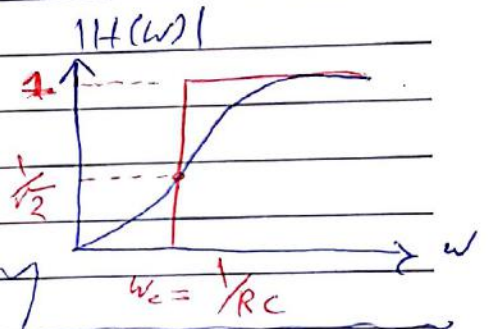
Low Pass :-

$$H(\omega) = \frac{V_o(\omega)}{V_{in}(\omega)}$$

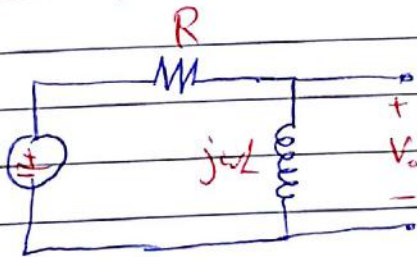


$$= \frac{R}{R + 1/j\omega C} = \frac{j\omega RC}{1 + j\omega RC}$$

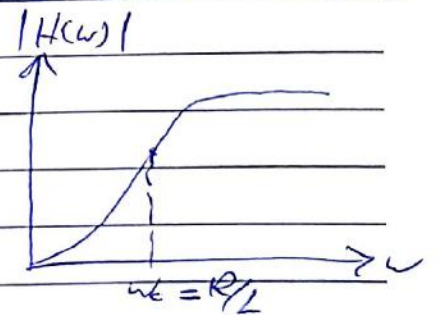
$$|H(\omega)| = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}}$$



another approach :-



$$H(\omega) = \frac{j\omega L}{R + j\omega L}$$



$$|H(\omega)| = \frac{\omega L}{\sqrt{R^2 + (\omega L)^2}}$$

$$H(0) = 0$$

$$H(\omega) = 1$$

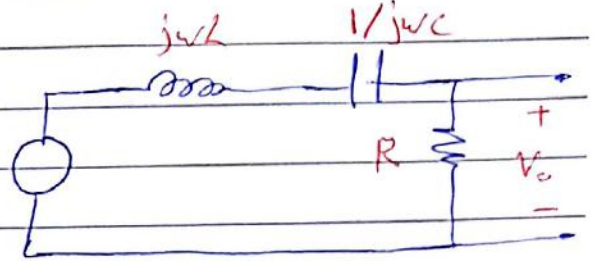


\* Band Pass Filter:-

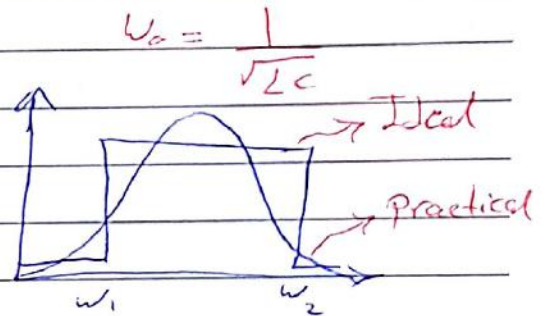
→ Series RLC Ckt without output taken at R

$$H(\omega) = \frac{R}{j\omega L + 1/j\omega C + R}$$

$$= \frac{j\omega RC}{(1 - \omega^2 LC) + j\omega RC}$$

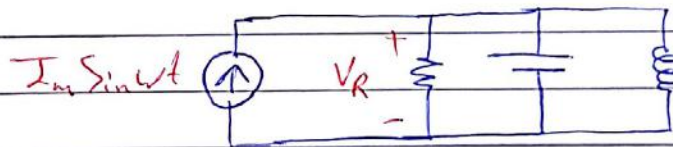


$$|H(\omega)| = \frac{\omega RC}{\sqrt{(1 - \omega^2 LC)^2 + (\omega RC)^2}}$$



$H(0) = 0, H(\infty) = 0$

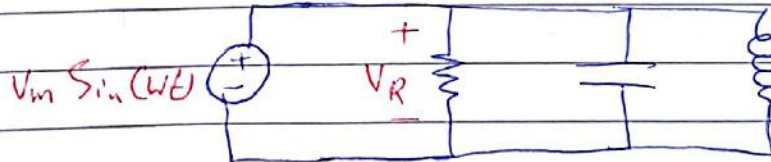
الطاقة الممتصة في المقاومة



~~P~~  $P(\omega_0) = \frac{V_m^2}{2R}, V_m = I_m R$

$P(\omega_1) = P(\omega_2) = \frac{1}{2} P(\omega_0) = \frac{V_m^2}{4R}$

14.8



$V_R = V_m$

$P = \frac{V_m^2}{2R} [\omega_0, \omega_1, \omega_2]$

\*

24/4 2021

No \_\_\_\_\_

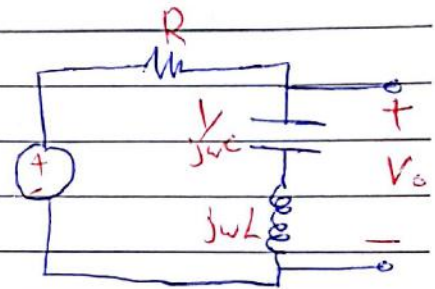
### 4) Band stop filter :-

↳ a filter that prevents a band of frequency and allow other frequency :-

⇒ RLC series ckt with output taken on LC

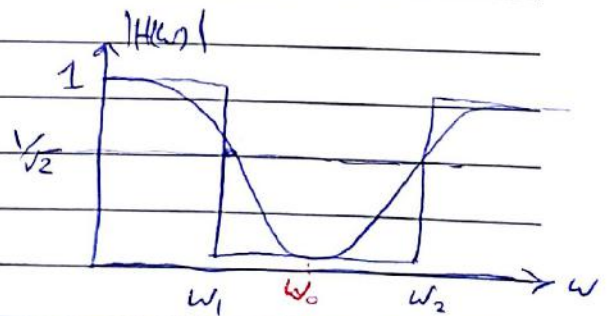
$$H(\omega) = \frac{V_o}{V_{in}} = \frac{j(\omega L - 1/\omega C)}{R + j(\omega L - 1/\omega C)}$$

$$= \frac{j(1 - \omega^2 LC)}{-\omega CR + j(1 - \omega^2 LC)}$$



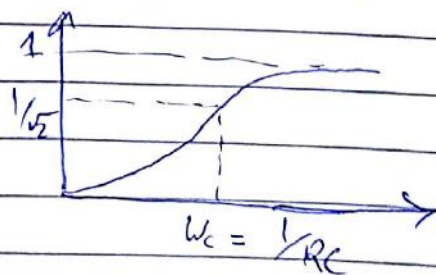
$$|H(\omega)| = \frac{(1 - \omega^2 LC)}{\sqrt{(\omega RC)^2 + (1 - \omega^2 LC)^2}}$$

~~at~~  $H(0) = 1$        $\omega_0 = \frac{1}{\sqrt{2LC}}$   
 $H(\omega) = 1$



ex) Design a high-pass filter with a corner frequency of 3 kHz

Follow  
⇒



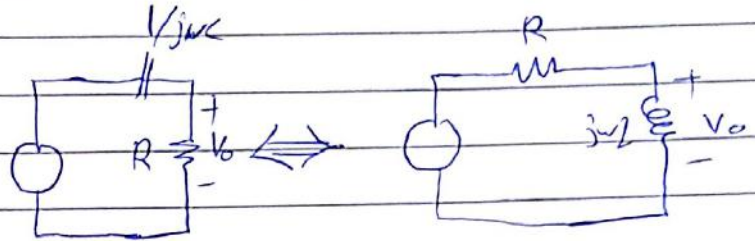
sol :-

choose an RC CKT with output on R

$$\omega_c = 2\pi(3000) = \frac{1}{RC}$$

$$R = 4.7 \text{ k}\Omega$$

$$C = 11.29 \text{ nF}$$



C لیسٹ اور R لیاؤ \*  
 جس میں آؤٹ پٹ لے لیں

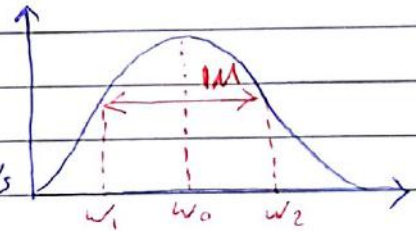
(ex) Design a band pass filter characterized by  
 BW = 1 MHz & high cutoff frequency of 1.1 MHz

لو لیاؤ

$$\omega_2 = 1.1 \times 2\pi = 6.912 \text{ M rad/s}$$

$$\omega_1 = \omega_2 - \beta$$

$$= (1.1 \text{ M} - 1 \text{ M}) \times 2\pi = 628.3 \text{ K rad/s}$$



$$\beta = \frac{\omega_0}{Q_0} = \frac{1}{RC} = 6.283 \text{ M rad/s} \quad \text{--- (1)}$$

$$\omega_0 = \sqrt{\omega_1 \omega_2} = \sqrt{(6.912) \times 628.3 \times 10^9}$$

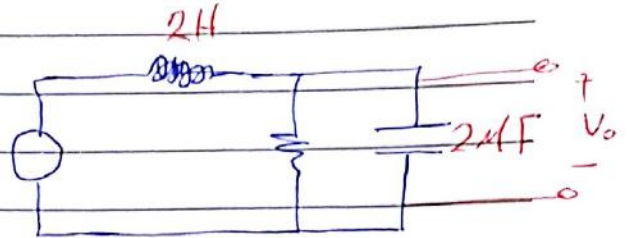
$$\omega_0 = \frac{1}{\sqrt{LC}} = 2.084 \times 10^6 \quad \text{--- (2)}$$

$$\Rightarrow R = 314 \text{ k}\Omega, L = 50 \text{ mH}, C = 46 \text{ pF}$$

(ex) Determine what type of filter is shown in the figure, what is the corner frequency, cutoff frequency

$$H(s) = \frac{R \parallel \frac{1}{sC}}{sL + R \parallel \frac{1}{sC}}$$

$$s = \sigma + j\omega$$



دائرة فلتر من النوع RL

$$H(\omega) = \frac{R \parallel \frac{1}{j\omega C}}{j\omega L + R \parallel \frac{1}{j\omega C}}$$

$$H(s) = \frac{R}{s^2 RLC + sL + R}$$

$$H(\omega) = \frac{R}{R - \omega^2 LCR + j\omega L}, \quad |H(\omega)| = \frac{R}{\sqrt{(R - \omega^2 LCR)^2 + (\omega L)^2}}$$

$$H(\omega_c) = \frac{1}{\sqrt{2}}$$

$$H(\infty) = 0, \quad H(0) = 1$$

$$\frac{1}{\sqrt{2}} = \frac{R}{\sqrt{(R - \omega^2 LCR)^2 + (\omega L)^2}}$$

$$\frac{1}{2} = \frac{R^2}{(R - \omega^2 LCR)^2 + (\omega L)^2}$$

$$2R^2 = (R - \omega^2 LCR)^2 + (\omega L)^2$$

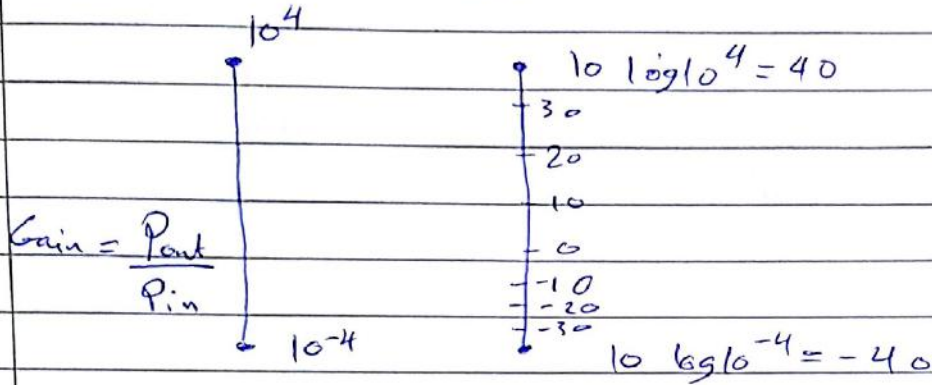
$$\omega_c^2 = 0.5509 \text{ M rad/s}^2 \text{ OR } \omega_c^2 = 0.1154 \text{ X}$$

$$2R^2 = R^2 - 2\omega^2 LCR + \omega^4 (LCR)^2 + \omega^2 L^2 \quad \omega_c = 0.742 \text{ k rad/s}$$

$$\omega^4 (LCR)^2 + (L^2 - 2R^2 LC) \omega^2 - R^2 = 0$$

$$\omega^2 = \frac{-(L^2 - 2R^2 LC) \pm \sqrt{(L^2 - 2R^2 LC)^2 - 4(LCR)^2 R^2}}{2(LCR)^2}$$

## # The Decible scale :-



$$\text{Gain}_{dB} = 10 \log \frac{P_{out}}{P_{in}}$$

$$\text{Gain}_{dB} = 10 \log \frac{V_{out}^2 / R_{out}}{V_{in}^2 / R_{in}}$$

if  $R_{in} = R_{out}$

$$\Rightarrow \text{Gain}_{dB} = 10 \log \frac{V_{out}^2}{V_{in}^2} = 10 \log \left( \frac{V_{out}}{V_{in}} \right)^2 = 20 \log \left( \frac{V_{out}}{V_{in}} \right)$$

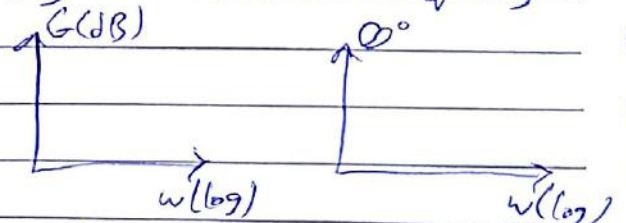
$$\text{Gain}_{dB} = 20 \log \frac{I_{out}}{I_{in}}$$

Bode plots :- plots of the magnitude in decibels  
OR the phase shift in degrees versus frequency

in logarithmic scale

→ easy to construct

→ Approximate plots



⇒ write transfer function (T.F) in standard form

$$H(\omega) = \frac{N(\omega)}{D(\omega)} = \frac{k (j\omega)^{F_N} \left[ 1 + \frac{j\omega}{z_1} \right]}{\left[ 1 + \frac{j\omega}{p_1} \right]}$$

$k =$  constant gain

Zero  $\equiv \Delta$   $\frac{1}{s}$   $\frac{1}{s^2}$

$j\omega$ : Zero or Pole at Zero

Pole  $\equiv \nabla$   $s$   $s^2$

$\left[1 + \frac{j\omega}{Z_1}\right]$ : Zero at  $Z_1$

$\left[1 + \frac{j\omega}{P_1}\right]$ : Pole at  $P_1$

$$H(\omega) = \frac{200 j\omega}{(j\omega + 2)(j\omega + 10)}$$

$$H(\omega) = \frac{200 * (j\omega)}{2 \left(1 + \frac{j\omega}{2}\right) * 10 \left(1 + \frac{j\omega}{10}\right)}$$

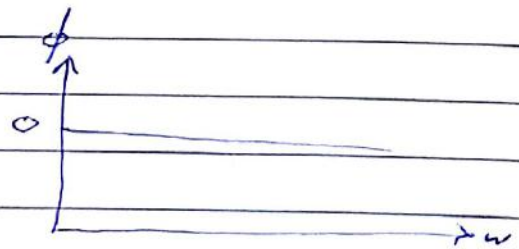
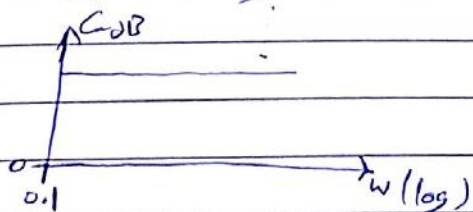
$$= \frac{10 (j\omega)}{\left(1 + \frac{j\omega}{2}\right) \left(1 + \frac{j\omega}{10}\right)}$$

$$20 \log_{10} H(\omega) = 20 \log_{10} \left[ \frac{10 (j\omega)}{\left(1 + \frac{j\omega}{2}\right) \left(1 + \frac{j\omega}{10}\right)} \right]$$

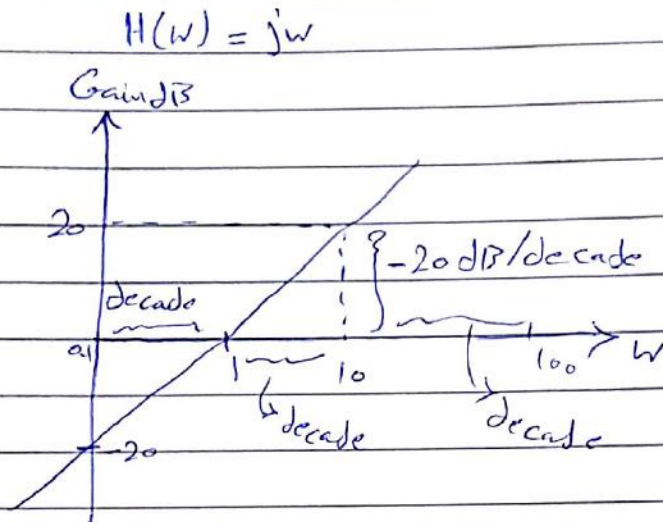
$$20 \log_{10} H(\omega) = H(\omega)_{dB} = 20 \log_{10} 10 + 20 \log_{10} |j\omega| - 20 \log_{10} \left|1 + \frac{j\omega}{2}\right| - 20 \log_{10} \left|1 + \frac{j\omega}{10}\right|$$

plot of a constant  $K$ :

$$G_{dB} = 20 \log K$$



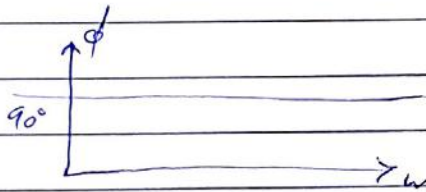
Plot of a Zero or Pole at origin :-



$$H(1) = 20 \log(1) = 0$$

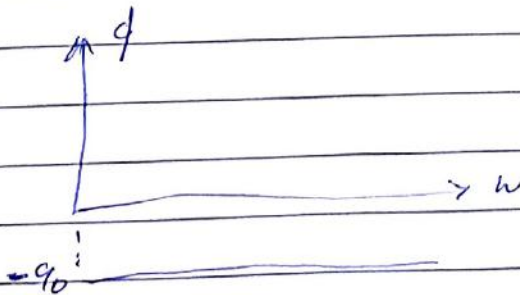
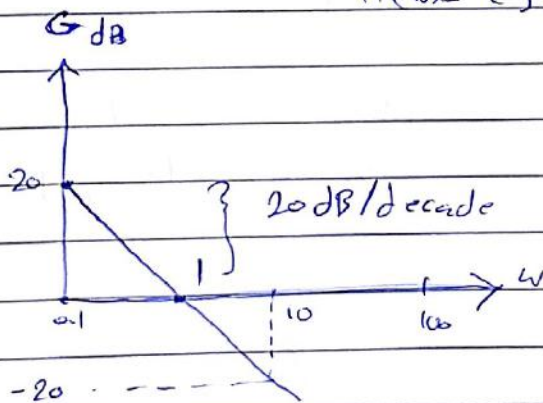
$$H(10) = 20$$

$$H(0.1) = -20$$



Plot of a Pole at origin

$$H(\omega) = (j\omega)^{-1} = \frac{1}{j\omega}$$

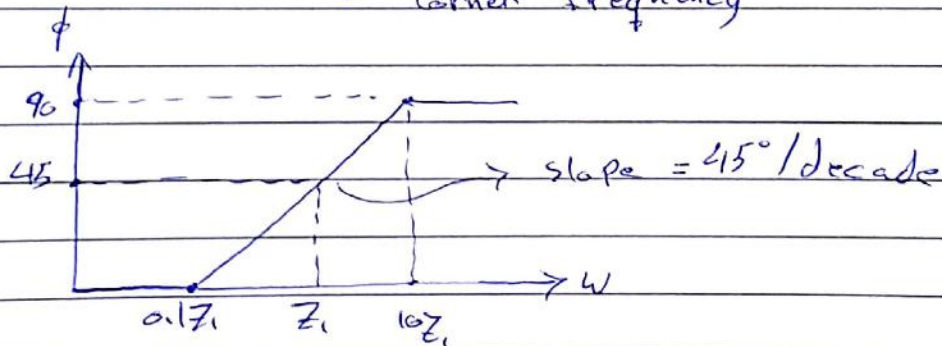
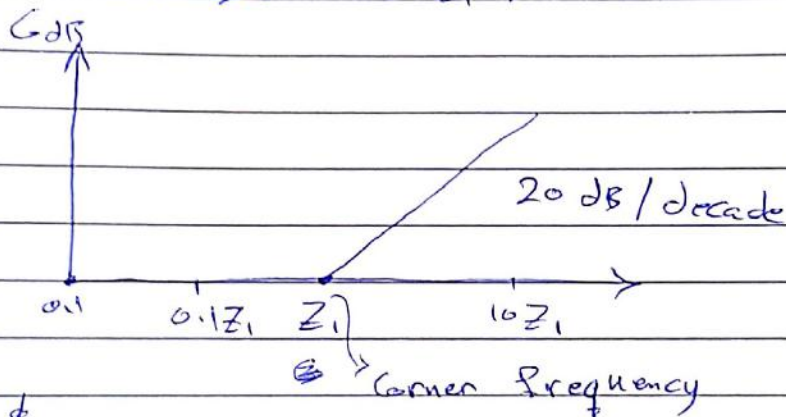


$$\text{Simple Zero} = \left[ 1 + \frac{j\omega}{z_1} \right]$$

$$G_{dB} = 20 \log \left[ 1 + \frac{j\omega}{z_1} \right]$$

$$= \begin{cases} 0 \text{ dB} & , \omega \approx 0 \end{cases}$$

$$\approx 20 \log \frac{\omega}{z_1} & , \omega \approx \infty$$



$$H(\omega) = 1 + \frac{j\omega}{100}$$

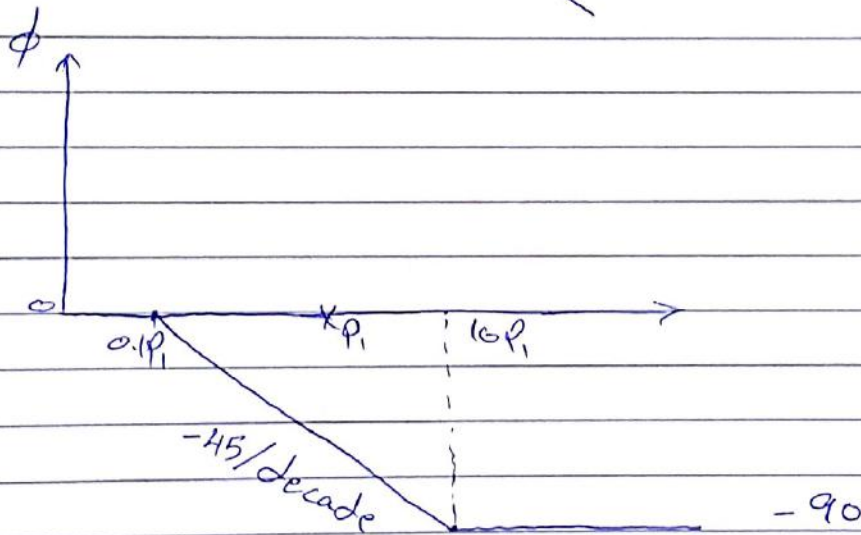
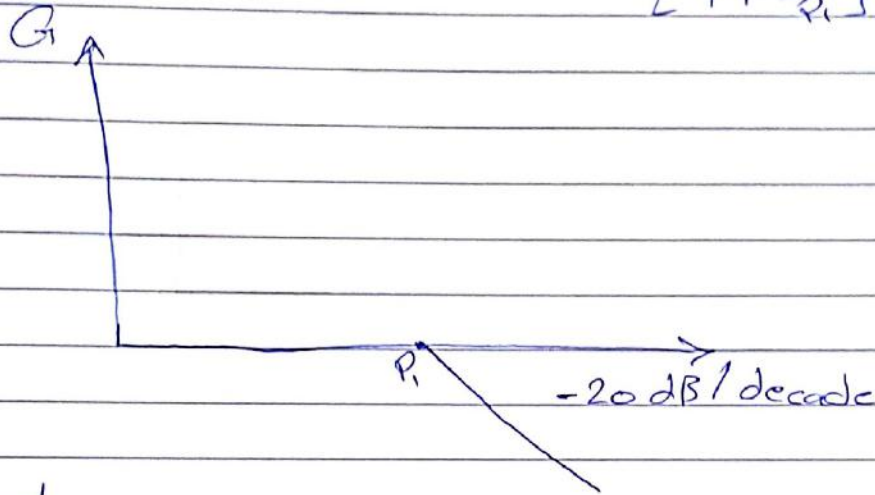
$$\underline{H}(10) = 1 + \frac{j10}{100} = 1 + j0.1 \approx 0.1$$

$$\underline{H}(100) = 1 + j10$$

$$\angle H(100) = 45^\circ = 90$$



Simple ~~Pole~~ Pole :  $20 \log \frac{1}{[1 + \frac{\omega}{P_1}]} = -20 \log [1 + \frac{\omega}{P_1}]$



No \_\_\_\_\_

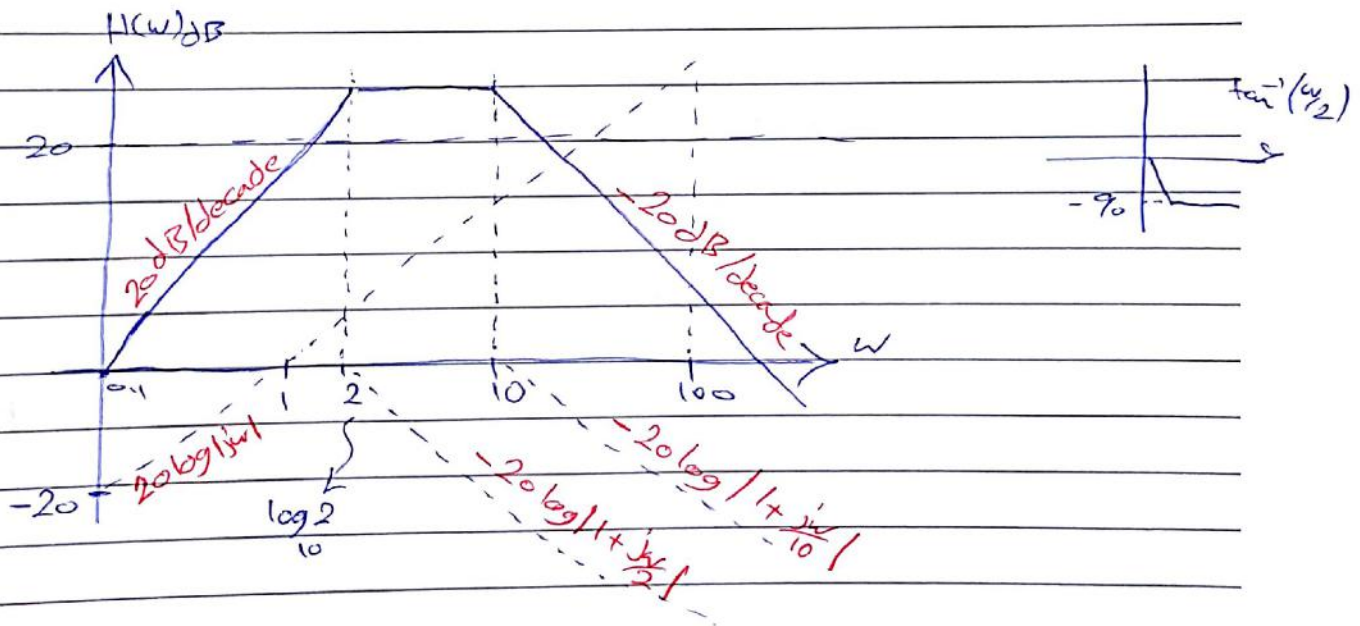
Ex) Construct the bode plots for:-

$$H(\omega) = \frac{200j\omega}{(j\omega+2)(j\omega+10)}$$

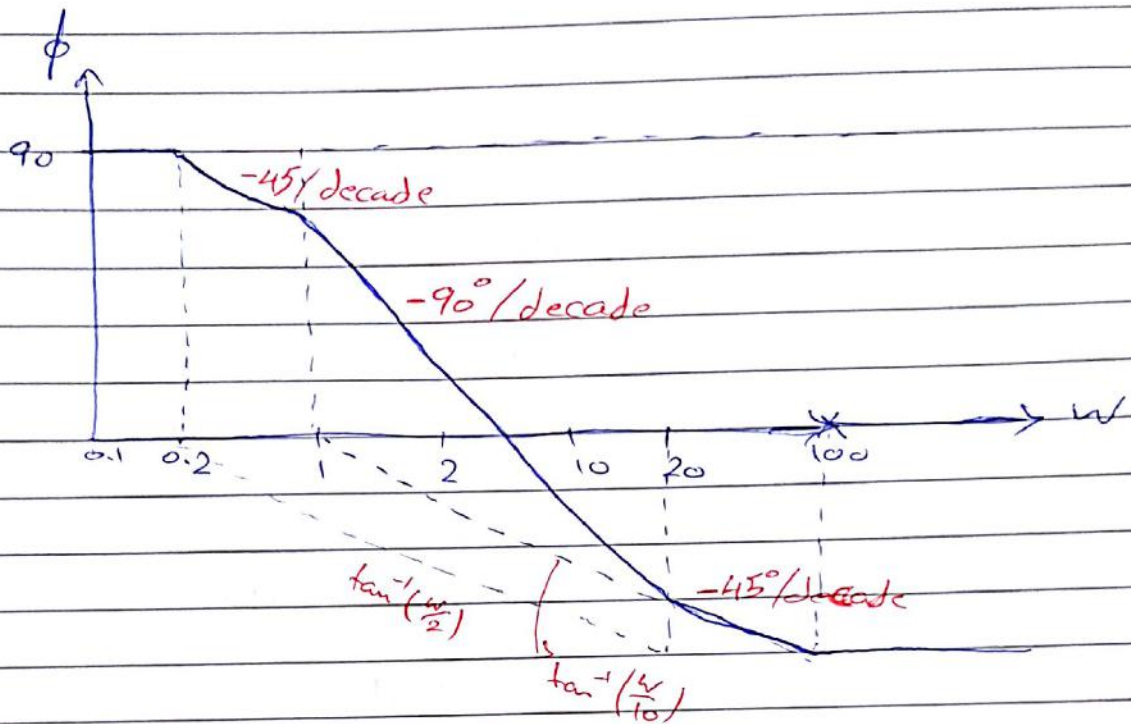
in standard form  $\Rightarrow H(\omega) = \frac{200(j\omega)}{2(j\frac{\omega}{2}+1) \times 10(j\frac{\omega}{10}+1)}$

$$H(\omega) = \frac{10(j\omega)}{(j\frac{\omega}{2}+1) \times (j\frac{\omega}{10}+1)}$$

$$H(\omega)_{dB} = 20 \log 10 + 20 \log |j\omega| - 20 \log |1 + j\frac{\omega}{2}| - 20 \log |1 + j\frac{\omega}{10}|$$



No \_\_\_\_\_

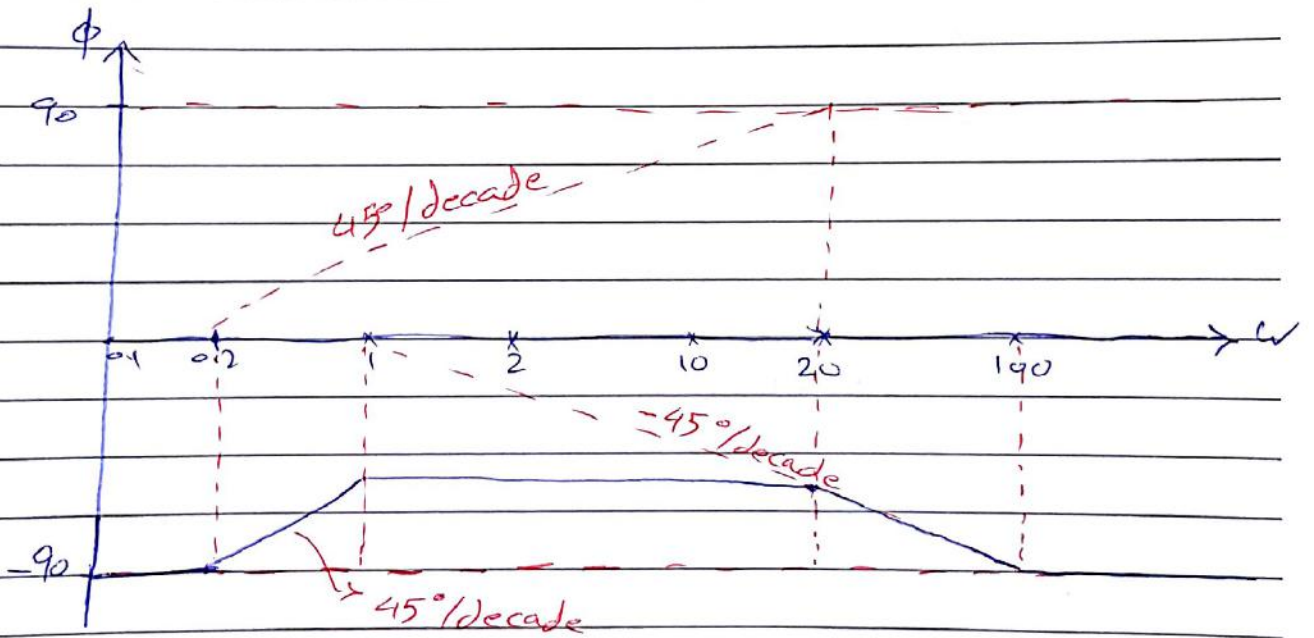
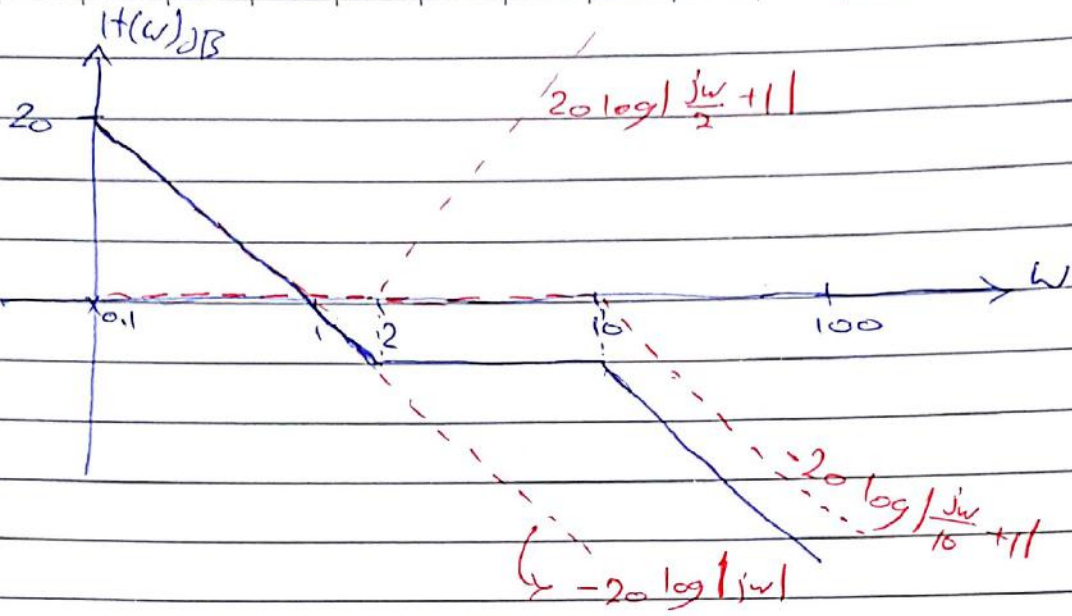


$$H(\omega) = \frac{5(j\omega + 2)}{j\omega(j\omega + 10)} = \frac{5 \times 2(j\omega/2 + 1)}{j\omega \times 10(j\omega/10 + 1)}$$

$$= \frac{(j\omega/2 + 1)}{j\omega \times (j\omega/10 + 1)}$$

$$H(\omega)_{dB} = 20 \log |j\omega/2 + 1| - 20 \log |j\omega| - 20 \log |j\omega/10 + 1|$$

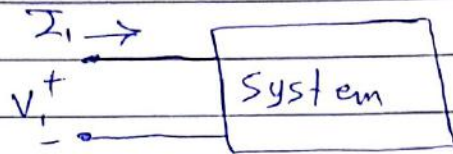
$$\angle H(\omega) = \angle \tan^{-1} \frac{\omega}{2} - 90 - \angle \tan^{-1} \frac{\omega}{10}$$



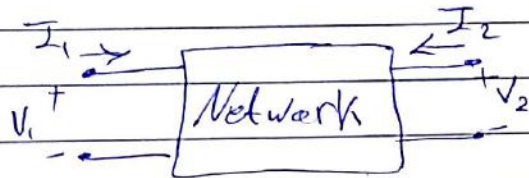
No \_\_\_\_\_

## # 2-port network :-

⇒ 1 port network :-



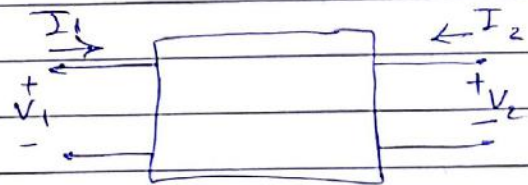
⇒ 2 port network :-



## Impedance Parameter [Z-Parameters]

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{22} I_2 + Z_{21} I_1$$



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \underbrace{\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}}_{\text{System}} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$Z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0}$$

$$Z_{12} = \frac{V_1}{I_2} \Big|_{I_1=0}$$

$$Z_{21} = \frac{V_2}{I_1} \Big|_{I_2=0}$$

$$Z_{22} = \frac{V_2}{I_2} \Big|_{I_1=0}$$

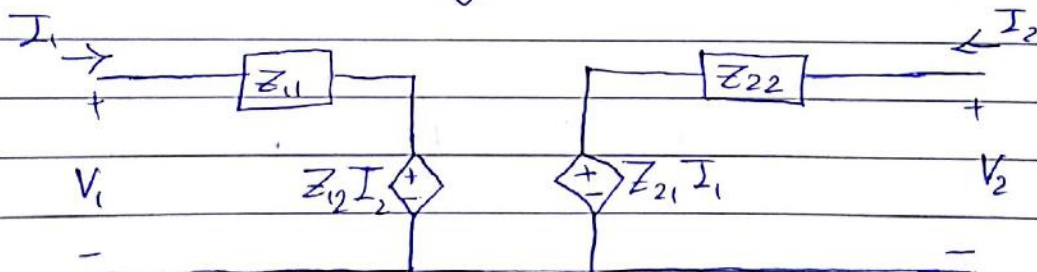
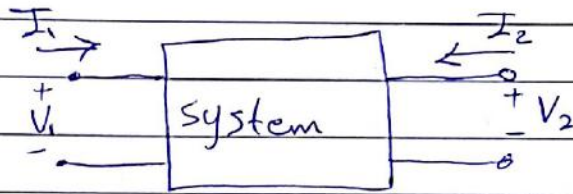
$Z_{11} \triangleq$  Open-circuit input impedance.

$Z_{12} \triangleq$  Open-circuit transfer impedance from Port 1 to Port 2.

$Z_{21} \triangleq$  Open-circuit transfer impedance from Port 2 to Port 1.

$Z_{22} \triangleq$  Open-circuit output impedance.

\* Equivalent ckt for Z Parameters :

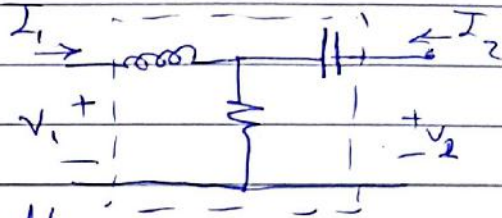


$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

⇒ If the network has no dependent sources.

⇒ This system is called reciprocal system.

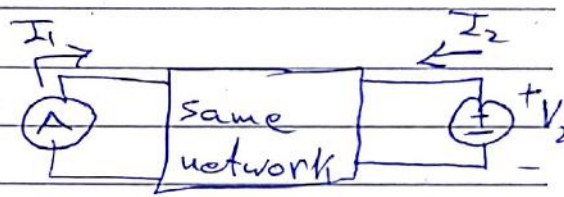


⇒ Source & effect are interchangeable.

⇒  $Z_{21} = Z_{12}$



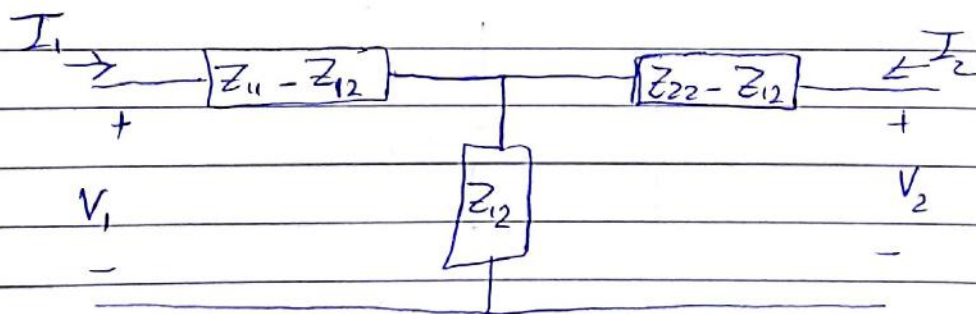
$V_1$  causes  $I_2$



$V_2$  causes  $I_1$

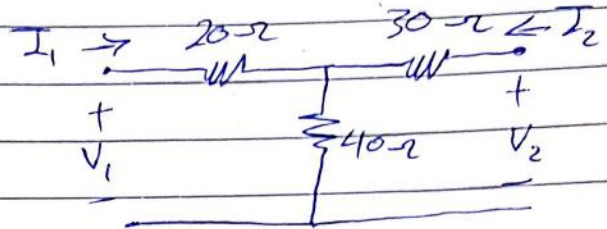
if  $V_1 = V_2$  then  $I_1 = I_2$

For a reciprocal network :-



ex) Determine the Z parameters for the CKT shown:-

$$\text{Find } Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$



Approach 1:-

$$Z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0} = \frac{I_1 \times (20+40)}{I_1} = 60 \Omega$$

$$Z_{21} = \frac{V_2}{I_1} \Big|_{I_2=0} = \frac{40 \times I_1}{I_1} = 40 \Omega$$

$$Z_{12} = \frac{V_1}{I_2} \Big|_{I_1=0} = \frac{(30+40) I_2}{I_2} = 70 \Omega$$

$$Z_{22} = \frac{V_2}{I_2} \Big|_{I_1=0} = \frac{(30+40) I_2}{I_2} = 70 \Omega$$

$$Z = \begin{bmatrix} 60 & 40 \\ 40 & 70 \end{bmatrix}$$

Approach 2:-

$$40 = Z_{22} = Z_{21} \text{ [reciprocal]}$$

$$20 = Z_{11} - Z_{12} \Rightarrow Z_{11} = 60 \Omega$$

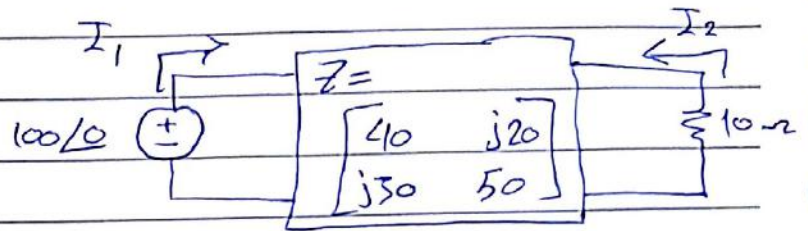
$$30 = Z_{22} - Z_{21} \Rightarrow Z_{22} = 70 \Omega$$



(ex) Find  $I_1$  &  $I_2$  in the shown network

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$



$$V_1 = 100 \angle 0$$

$$V_2 = -10 I_2$$

$$\rightarrow 100 \angle 0 = 40 I_1 + j20 I_2 \quad \Rightarrow \quad I_1 = 2 \angle 0 \text{ A}$$

$$\rightarrow -10 I_2 = j30 I_1 + 50 I_2 \quad \Rightarrow \quad I_2 = 1 \angle -90 \text{ A}$$

Y-Parameters [~~and~~ admittance Parameters].

→ Sometimes Z-Parameters are not easy to find.

$$Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

$$Y^{-1} = Z$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

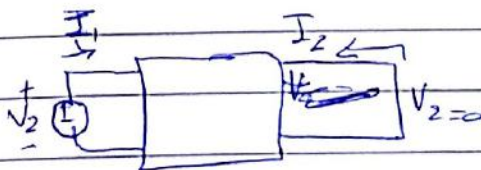
system Y-Parameters.

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$Y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0}$$

$$Y_{21} = \frac{I_2}{V_1} \Big|_{V_2=0}$$



No

$$Y_{12} = \frac{I_1}{V_2} \Big|_{V_1=0}$$

$$Y_{21} = \frac{I_2}{V_1} \Big|_{V_2=0}$$

$Y_{11} \triangleq$  Short ckt input admittance.

$Y_{12} \triangleq$  short ckt transfer admittance from Port 1 to Port 2.

$Y_{21} \triangleq$  short ckt transfer admittance from Port 2 to Port 1.

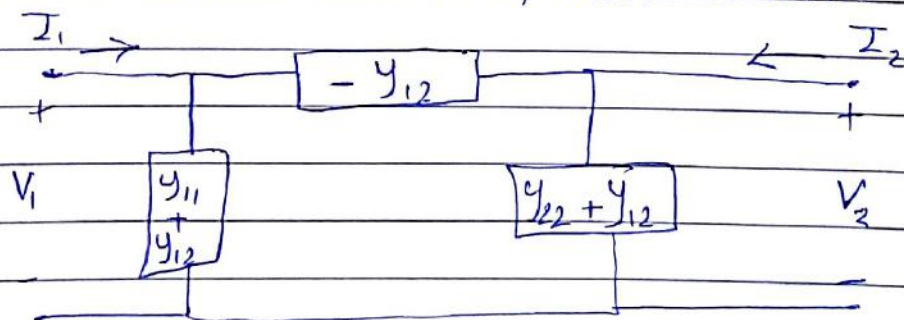
$Y_{22} \triangleq$  short - ckt output admittance.

$\Rightarrow$  if the network has no dependent sources.

$\Rightarrow$  it is call reciprocal network.

$$\Rightarrow \boxed{Y_{12} = Y_{21}}$$

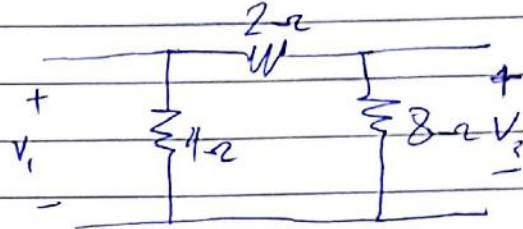
$\Rightarrow$  Equivalent ckt for a reciprocal network.



(ex) obtain the Y-Parameters for the system network.

$$y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0} = \frac{I_1}{\frac{8}{6} I_1}$$

$$= \frac{3}{4} \text{ S (or)}$$



$$V_1 = I_1 \times \frac{4 \times 2}{4+2} = \frac{8}{6} I_1$$

$$y_{21} = \frac{I_2}{V_1} \Big|_{V_2=0} = \frac{-\frac{4}{6} I_1}{\frac{8}{6} I_1} = -0.5 \text{ S}$$

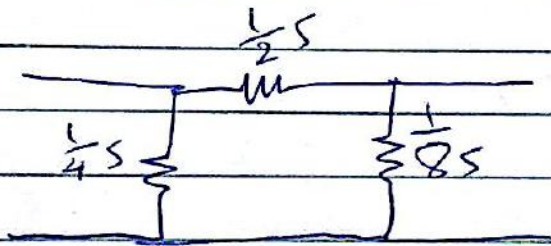
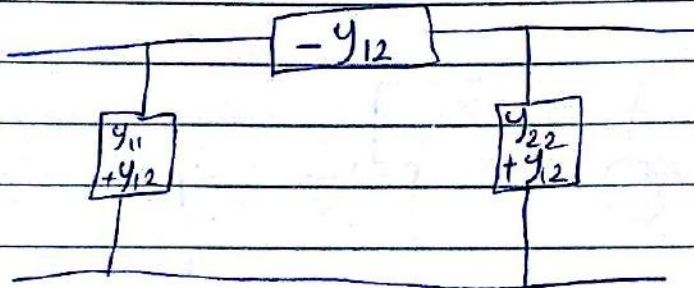
$$-I_2 = \frac{4}{2+4} I_1 = \frac{4}{6} I_1$$

No \_\_\_\_\_

$$Y = \begin{bmatrix} 0.75 & -0.5 \\ -0.5 & 0.625 \end{bmatrix}$$

Current source, short ckt  $\Delta V_i \bar{I}_i \Delta V_j \bar{I}_j$   
Voltage source, open ckt " "  $\bar{I}_i \bar{I}_j$

approach 2 :-



$$-y_{12} = \frac{1}{2}$$

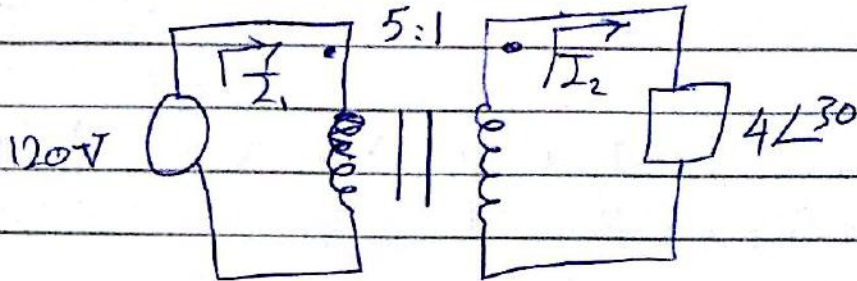
$$, y_{12} = -0.5 = y_{21}$$

$$, \frac{1}{4} = y_{11} + y_{12} \Rightarrow y_{11} = 0.75$$

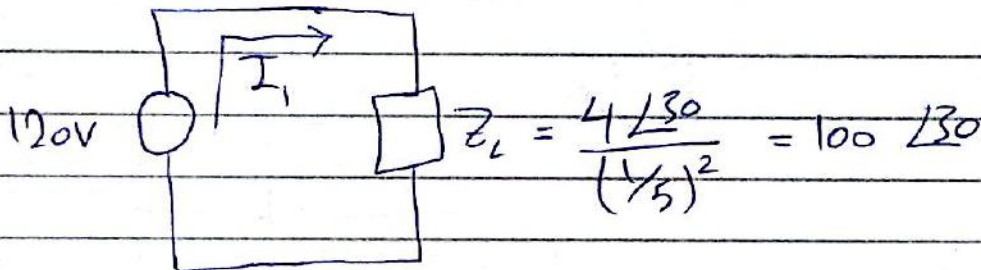
$$\frac{1}{8} = y_{22} + y_{12} \Rightarrow y_{22} = 0.625$$

انتهت الكادة  $\equiv \equiv$

ex Find  $I_1$  &  $I_2$

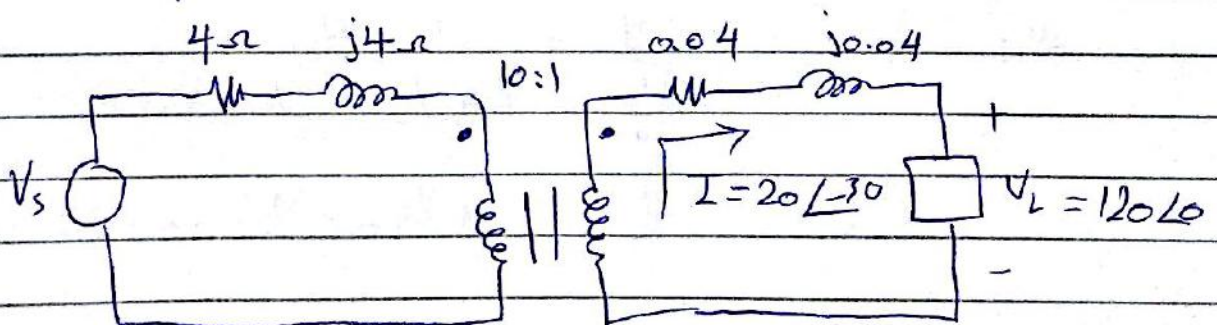


Sol :-



$$I_1 = \frac{120 \angle 0}{100 \angle 30} = 1.2 \angle -30$$

ex For the ckt shown find  $V_s$  :-

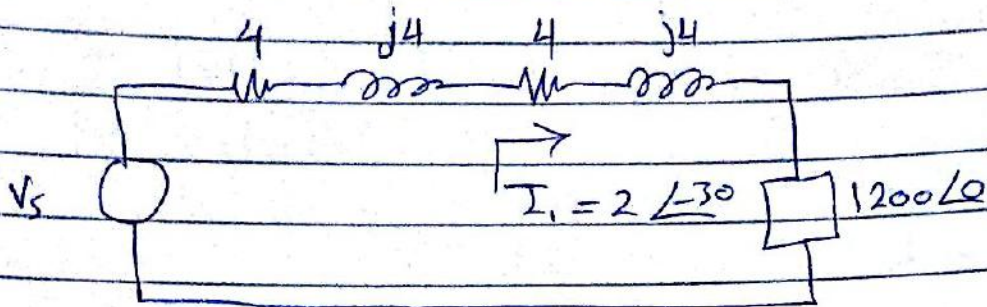


$$R_{ref} = \frac{0.04}{(1/10)^2} = 4 \Omega$$

$$V_{L ref} = 120 \times 10 = 1200$$

$$X_{ref} = \frac{j0.04}{(1/10)^2} = j4$$

$$I_1 = I_2 / 10 = 2 \angle -30$$



$$V_s = (8 + j8)I_1 + 1200 \angle 0 = 1222 \angle 0.275$$

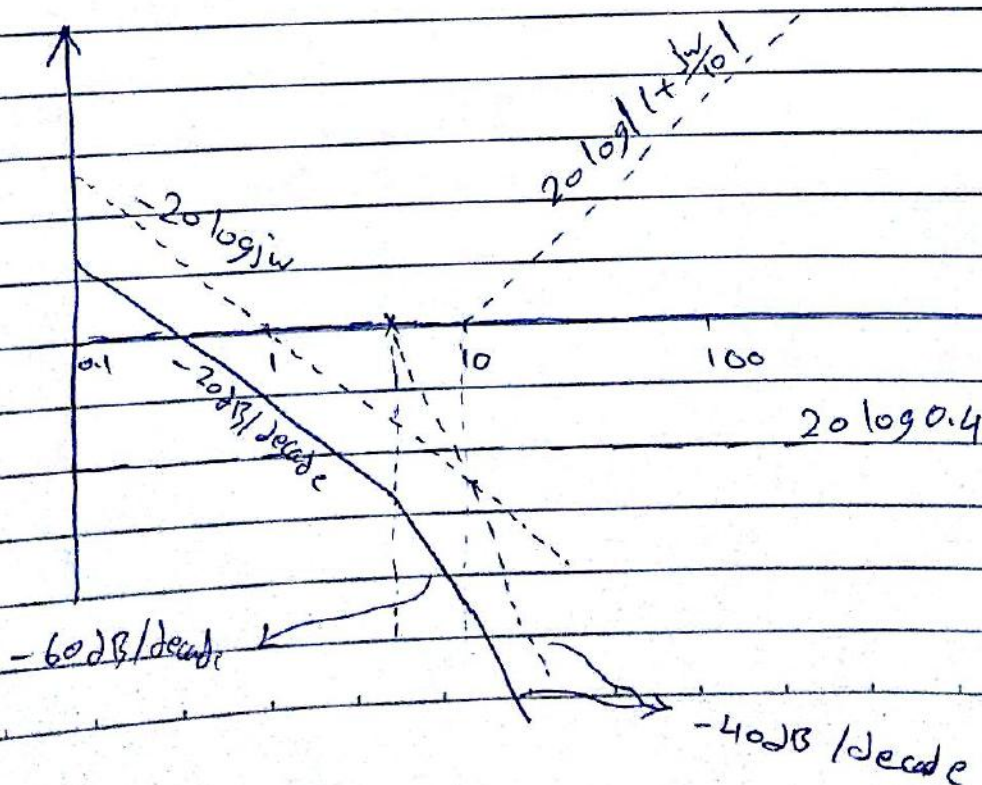
(ex) obtain the Bode plots for :-

$$H(\omega) = \frac{j\omega + 10}{j\omega(j\omega + 5)^2}$$

$$\Rightarrow H(\omega) = \frac{10 \left(1 + \frac{j\omega}{10}\right)}{j\omega \times 25 \left(1 + \frac{j\omega}{5}\right)^2} = \frac{0.4 \left(1 + \frac{j\omega}{10}\right)}{j\omega \left(1 + \frac{j\omega}{5}\right)^2}$$

$$H(\omega)_{dB} = 20 \log 0.4 + 20 \log \left(1 + \frac{j\omega}{10}\right) - 20 \log j\omega$$

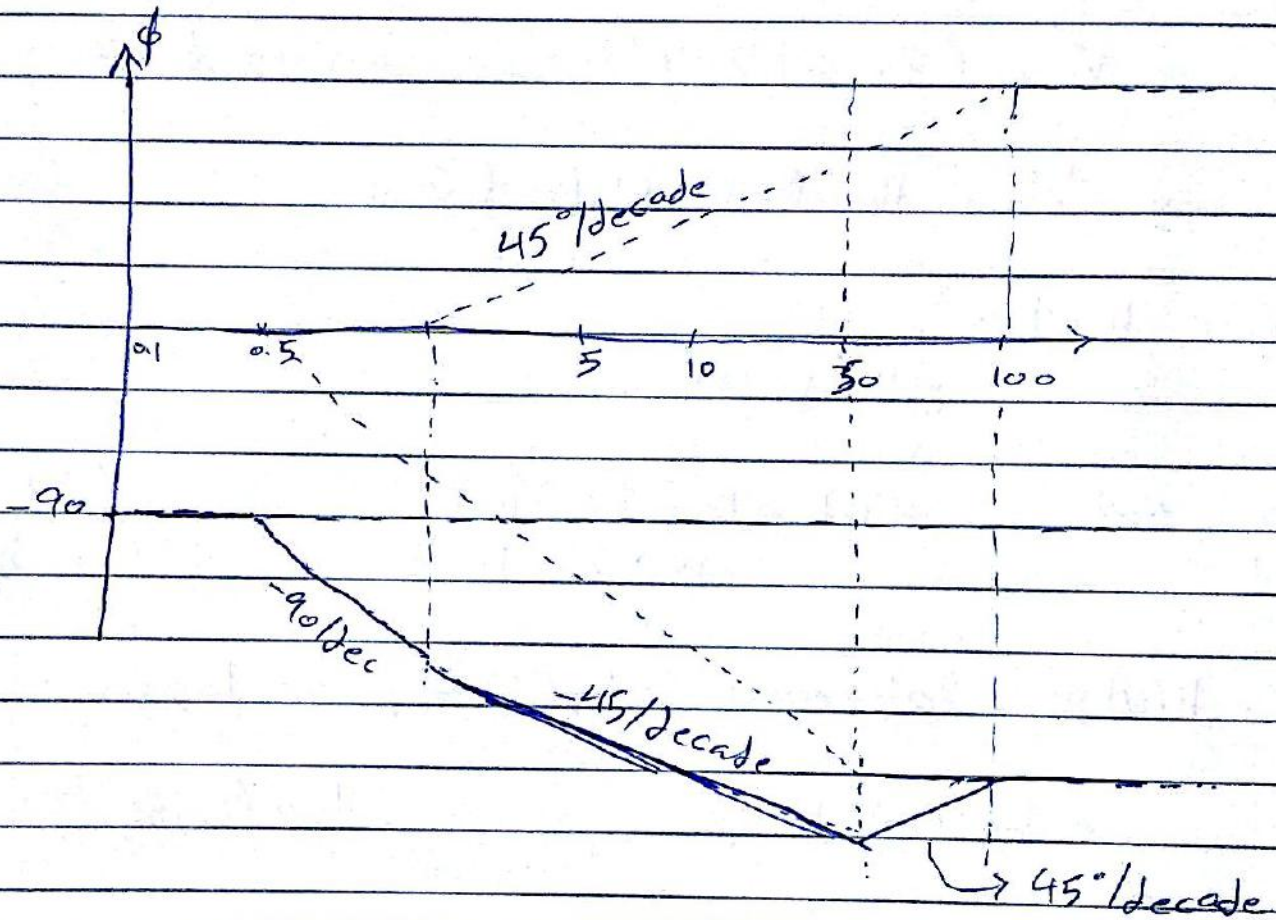
$$-40 \log \left(1 + \frac{j\omega}{5}\right) \leftarrow -20 \log \left(1 + \frac{j\omega}{5}\right)^2$$



No \_\_\_\_\_

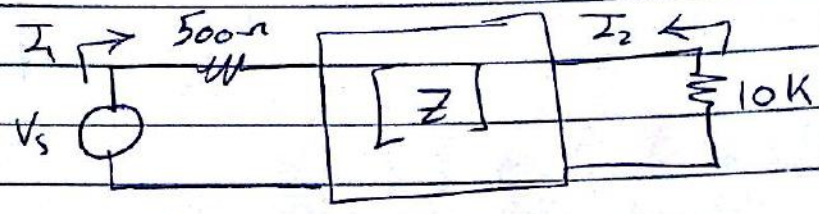
$$H(\omega) = 20 \log(0.4) + 20 \log \left| 1 + j \frac{\omega}{10} \right| - 20 \log |\omega| - 40 \log \left| 1 + j \frac{\omega}{5} \right|$$

$$\phi = 0 + \tan^{-1} \frac{\omega}{10} - 90 - 2 \tan^{-1} \frac{\omega}{5}$$



(ex)  $Z$  is given as  $\begin{bmatrix} 10^3 & 10 \\ -10^6 & 10^4 \end{bmatrix} \Omega$

Find Voltage, current & Power gains then find the input impedance for the Ckt shown.



Sol - :

$$G_I = \frac{I_2}{I_1}$$

$$G_V = \frac{V_2}{V_1}$$

$$G_P = G_I G_V$$

\* الفكرة من السؤال  
في اجزاء العلاقات

$$V_1 = 10^3 I_1 + 10 I_2$$

$$V_2 = -10^6 I_1 + 10^4 I_2$$

$$V_s = 500 I_2 + V_1$$

$$V_2 = -10^4 I_2$$

$$-10^4 I_2 = -10^6 I_1 + 10^4 I_2$$

$$I_2 = \frac{10^6}{2 \times 10^4} I_1 = 50 I_1$$

$$G_I = \frac{50 I_1}{I_1} = 50$$

$$V_1 = 10^3 I_1 + 10(50 I_1) = 1500 I_1$$

$$V_2 = -10^4 \times 50 I_1$$

$$G_V = \frac{V_2}{V_1} = \frac{-10^4 \times 50 I_1}{1500 I_1} = -333$$

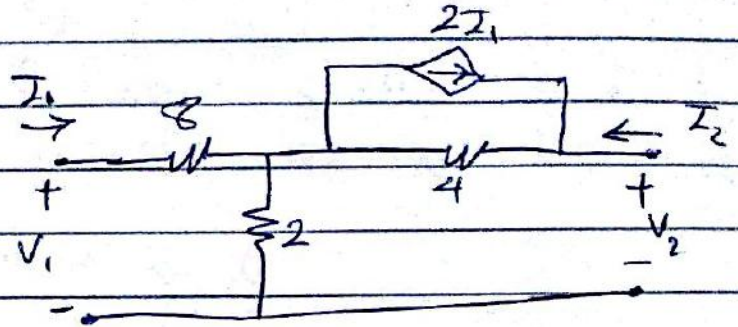
$$G_P = 333 \times 50 = 16650$$

$$Z_{in} = \frac{V_1}{I_1} = \frac{1500 I_1}{I_1}$$

$$= 1500 \Omega$$

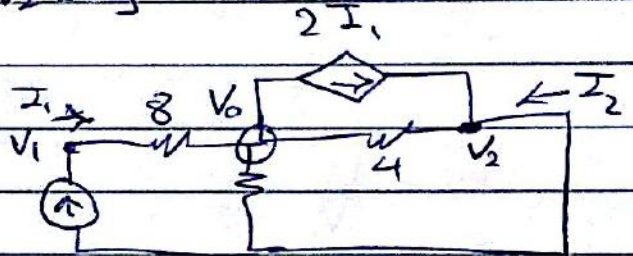


ex) Determine the Y-parameters for the 2-port network shown:-



① to find  $y_{11}$  &  $y_{21} \Rightarrow$   
short CKT the output [ $V_2 = 0$ ]

$$y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0}$$



$$y_{21} = \frac{I_2}{V_1} \Big|_{V_2=0}$$

at node  $V_0$ :-

$$\frac{V_1 - V_0}{8} + \frac{-V_0}{2} + \frac{V_2 - V_0}{4} - 2I_1 = 0$$

$$V_1 = -5V_0$$

$$I_1 = \frac{-5V_0 - V_0}{8} = \frac{-3}{4} V_0$$

$$y_{11} = \frac{\frac{-3}{4} V_0}{-5V_0} = 0.15 \text{ S}$$

at node  $V_2$ :-

$$\frac{V_0 - V_2}{4} + I_2 + 2I_1 = 0$$

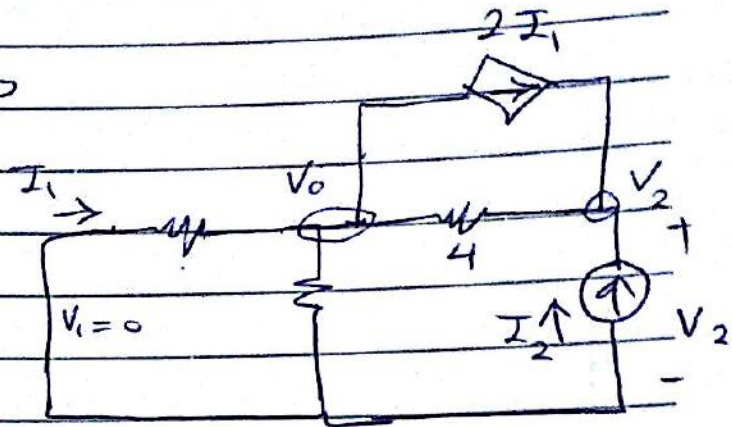
$$I_2 = 1.25 V_0$$

$$y_{21} = \frac{I_2}{V_1} \Big|_{V_2=0} = \frac{1.25 V_0}{-5 V_0} = -0.25 S$$

$y_{12} \neq y_{21}$  because the dependent source

to find  $y_{12}$  &  $y_{22}$  set  $V_1 = 0$

$$y_{12} = \frac{I_1}{V_2} \Big|_{V_1=0}$$



$$y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0}$$

at node  $V_0 \Rightarrow$

$$\frac{V_1 - V_0}{8} - \frac{V_0}{2} + \frac{V_2 - V_0}{4} - 2(I_1) = 0$$

$$V_2 = 2.5 V_0$$

$$y_{12} = \frac{I_1}{V_2} = \frac{-V_0/8}{2.5 V_0} = -0.05 S$$

@ node  $V_2$  :-  $\frac{V_0 - V_2}{4} + 2(I_1) + I_2 = 0$

$$I_2 = 0.625 V_0$$

$$y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0} = \frac{0.625 V_0}{2.5 V_0} = 0.25 S$$

$$Y = \begin{bmatrix} 0.15 & 0.05 \\ -0.25 & 0.25 \end{bmatrix}$$