

Name: \_\_\_\_\_

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Section # 1

Serial # 50

Section 1 Dr. Mohammed Abuelhaj

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Questions 1 (6 points)

If the source voltage in the figure below is 220 V<sub>rms</sub>, determine the source currents and the current in the neutral line, I<sub>NN</sub>.

I <sub>a</sub>	I <sub>b</sub>	I <sub>c</sub>	I <sub>NN</sub>
7.64 / 20.3	10 / -120	16.92 / 97.38	10.77 / 90

$$V_{an} = 220 \angle 0^\circ$$

$$V_{bn} = 220 \angle -120^\circ$$

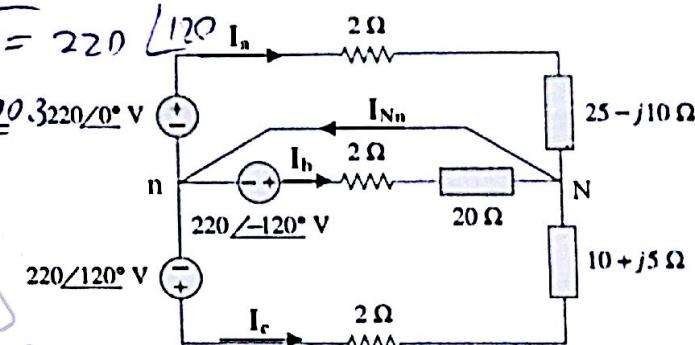
$$V_{cn} = 220 \angle 120^\circ$$

$$(6) I_a = \frac{220}{2 + 25 - 10j} = 7.64 / 20.3220.0^\circ \text{ V}$$

$$I_b = \frac{220 \angle -120^\circ}{22} = 10 \angle -120$$

$$I_c = \frac{220 \angle 120^\circ}{2 + 10 + 5j} = 16.92 / 97.38$$

$$I_{NN} = I_a + I_b + I_c = 10.77 / 90$$



Questions 2 (6 points)

In the balanced three-phase Y-Δ system shown in the figure, find

- the magnitude of the line current  $I_L$
- the total average power delivered to the load.

$ I_L $	$P_{tot}$
20.42	6256.7

b. balanced

$$\frac{Z_D}{3} = 2\Omega$$

$$\frac{9-j6}{3} = 3-2j$$

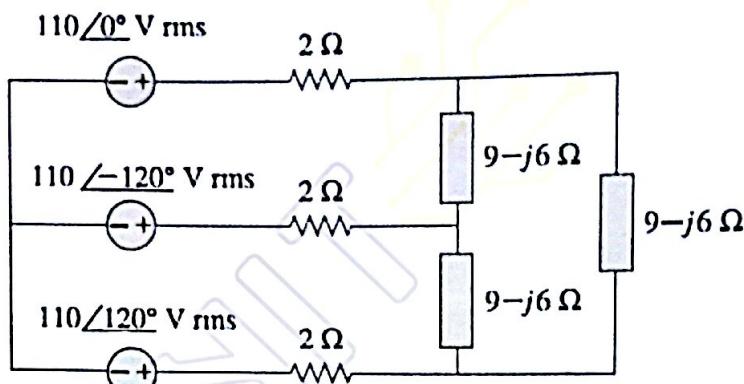
$$I_L = \frac{110}{2+3-2j} = 20.42 \angle 21.8^\circ$$

$$I_L = I_P = 20.42 \angle 21.8^\circ$$

$$P_{tot} = 3(V_P I_P \cos(\theta_P - \phi_i))$$

$$= 3(110 \times 20.42 \cos(0 - 21.8))$$

$$= 6256.7$$



Questions 3 (6 points)

(4) The circuit in the figure is excited by a balanced three-phase source with a line voltage of  $210 \text{ V}_{\text{rms}}$ . If  $Z_L = 1 + j1 \Omega$ ,  $Z_\Delta = 24 - j30 \Omega$ , and  $Z_Y = 12 + j5 \Omega$ , determine the magnitude of the line current of the combined loads, and the magnitude of the phase current in the delta load.

$I_L (\text{line})$	$I_P (\text{phase in } \Delta)$
<del>13.66</del>	<del>7.89</del>

$$V_L = 210 \angle 0^\circ \text{ (reference)}$$

$$Z_\ell = 1 + j$$

$$Z_\Delta = 24 - j30$$

$$Z_Y = 12 + j5$$

$\Delta \rightarrow Y$

$$\frac{Z_\Delta}{3} = 8 - 10j$$

$$Z_P = \frac{Z_\Delta}{3} || Z_Y$$

$$= (8 - 10j)(1(12 + 5j))$$

$$= \frac{(8 - 10j)(12 + 5j)}{8 - 10j + 12 + 5j}$$

$$= 7.81 - 2.04j$$

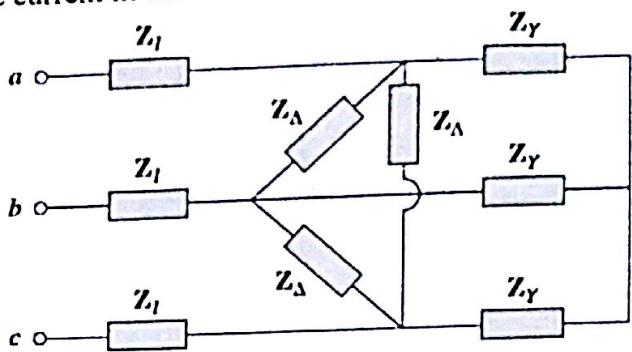
$$Z_T = Z_P + Z_\ell$$

$$= 7.81 - 2.04j + 1 + j$$

$$= 8.81 - 1.04j - 2$$

$$V_P = \frac{210}{\sqrt{3}} \angle -30^\circ$$

$$I_L = \frac{210 \angle -30^\circ}{\sqrt{3}(8.81 - 1.04j)} = 13.66 \angle -23.26^\circ$$



$$I_P = \frac{I_L}{\sqrt{3}}$$



Questions 4 (9 points)

In the figure below, two wattmeters are properly connected to the unbalanced load supplied by a balanced source such that  $V_{ab} = 208 \angle 0^\circ$  V<sub>rms</sub> with positive phase sequence.

- Determine the reading of each wattmeter.
- Calculate the total apparent power absorbed by the load.

$W_1$	$W_2$	$ S_{\text{tot}}$
2586.25	4808.3	8338.16

$$V_{ab} = 208 \angle 0^\circ$$

$$V_{bc} = 208 \angle -120^\circ$$

$$V_{ca} = 208 \angle 120^\circ$$

$$I_{AB} = \frac{208 \angle 0^\circ}{20} = 10.4 A$$

$$I_{BC} = \frac{208 \angle -120^\circ}{10 - 10j} = 14.7 \angle -75^\circ$$

$$I_{CA} = \frac{208 \angle 120^\circ}{12 + 5j} = 16 \angle 97.3^\circ$$

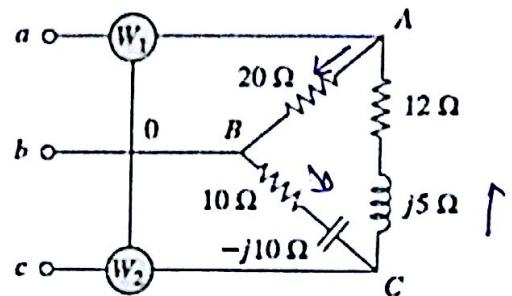
$$I_a, I_c$$

$$I_a = I_{AB} - I_{CA}$$

$$I_c = I_{CA} - I_{BC}$$

$$I_a = 20.16 \angle -51.92^\circ$$

$$I_c = 30.63 \angle 100.98^\circ$$



$$\begin{aligned} W_1 &= V_{ab} \cdot I_a \cos(\phi_2 - \phi_1) \\ &= 208 \cdot 20.16 \cos(0 + 51.92) \\ &= 2586.25 \text{ W} \end{aligned}$$

$$\begin{aligned} W_2 &= V_{bc} \cdot I_c \cos(\phi_2 - \phi_1) \\ &= 208 \cdot 30.63 \cos(60 - 101) \\ &= 4808.3 \text{ W} \end{aligned}$$

$$\left. \begin{aligned} V_{bc} &= 208 \angle 120^\circ \\ V_{cb} &= 208 \angle 60^\circ \end{aligned} \right\}$$

$$P_T = 7394.53$$

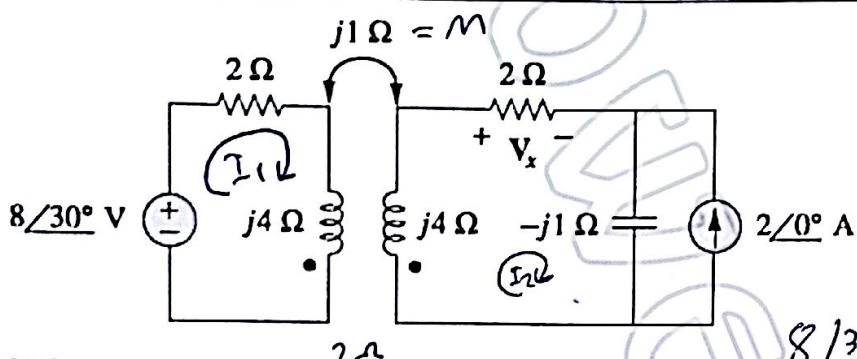
$$\begin{aligned} Q_T &= \sqrt{3} (4808.3 - 2586.25) \\ &\approx 3848.7 \text{ VAR} \end{aligned}$$

Questions 5 (7 points)

In the shown network,

- use source transformation to write down the two mesh equations that govern the current relations in the circuit in their simplest form.
- Find  $V_x$ .

Equation (1)	$I_1(2+j4) + I_2(-j) = 8\angle 30^\circ$	$v_x$
Equation (2)	$I_1(-j) + I_2(2+3j) = 2j$	$1.896 \angle 144.38^\circ$



$$8\angle 30^\circ = (2+j4)(3-2j)I_2 - 2$$

$$-jI_2$$

$$8\angle 30^\circ = (-2-17j)I_2$$

$$-4-8j$$

$$I_2 = 0.048 \angle 144.38^\circ$$

$$v_x = I_2 \times 2$$

$$= 1.896 \angle 144.38^\circ$$

mesh ①

$$8\angle 30^\circ = I_1(2+j4) - jI_2 \quad \text{--- (1)}$$

mesh ②

$$I_2(4j+2-j) - jI_1 + (-2j) = 0$$

$$I_1(-j) + I_2(2+3j) = 2j \quad \text{--- (2)}$$

$$I_1 = \frac{I_2(2+3j) - 2j}{j}$$

$$I_1 = (3-2j)I_2 - 2 \rightarrow \text{substitute} \quad \text{--- (1)}$$

### Question 6 (8 points)

For the circuit shown in the figure, find:

- the self-impedance of the primary circuit  $Z_{11}$ .
- the self-impedance of the secondary  $Z_{22}$ .
- the source and load currents  $I_1$  and  $I_2$ .

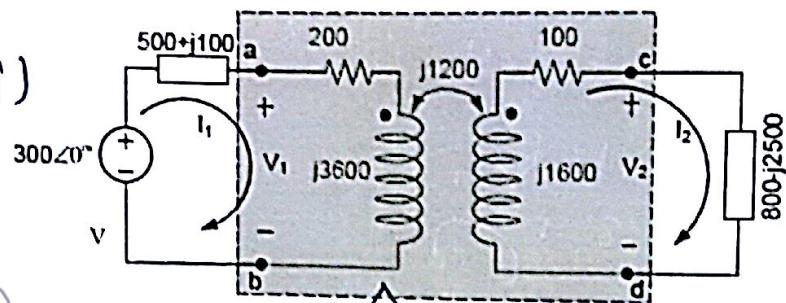
$Z_{11}$	$Z_{22}$	$I_1$	$I_2$
<del><math>200+3600j</math></del> $1000+4400j$	<del><math>100+1600j</math></del> $100+800j$	<del><math>0.063 \angle -71.56^\circ</math></del> $0.063 \angle -71.56^\circ$	<del><math>0.059 \angle 63.43^\circ</math></del> $0.059 \angle 63.43^\circ$

$m < 5h$  (1)

$$300 = I_1 (500 + 100j + 200 + 3600j) - I_2 (j1200)$$

mesh (2)

$$-(j1200)I_1 + I_2 (j1600 + 100 + 800j - 2500j) = 0$$



$$-(j1200)I_1 + I_2 (900 - 900j) = 0$$

$$I_1 = \frac{(900 - 900j)}{j1200} I_2$$

mesh (1)

$$300 = I_1 (700 + 3700j) - I_2 (1200j)$$

$$300 = (2250 - 3300j - 1200j) I_2$$

$$I_2 = 0.059 \angle 63.43^\circ$$

$$I_1 = 0.063 \angle -71.56^\circ$$

$$Z_{11} = 200 + 3600j + \frac{(1200)^2}{100 + 1600j + 800 - 2500j}$$

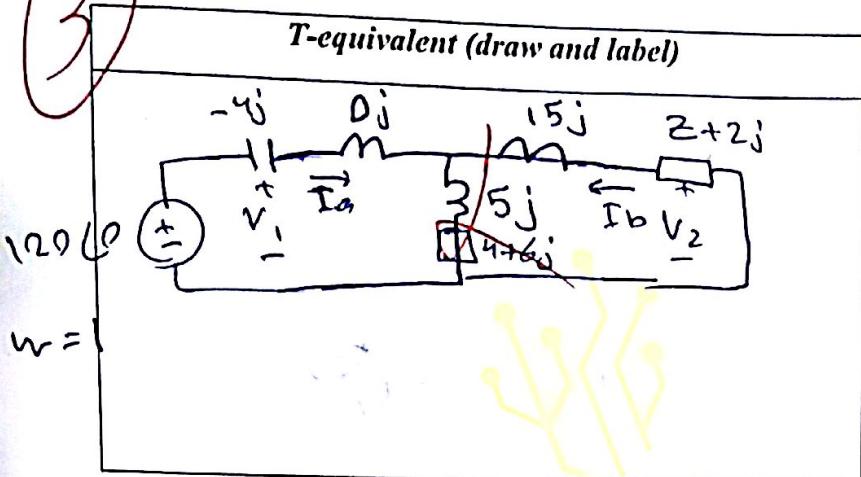
$$= 200 + 3600j + 800 + 800j$$

$$= 1000 + 4400j$$

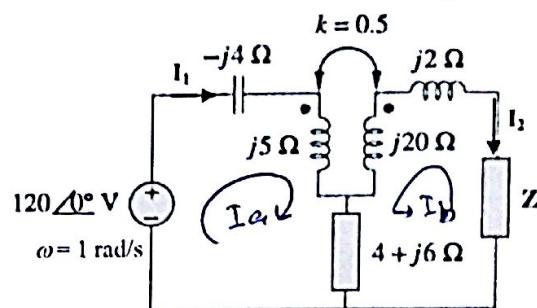
Question 7 (9 points)

For the circuit shown in the figure, replace the transformer by its T-equivalent model to find the Thevenin equivalent to the left of the load  $Z$ .

(3)



$Z_{TH}$	$V_{TH}$
$-0.285 - j1.27j$	



$$M = 0.5 \sqrt{L_1 L_2} \quad L_1 = 5j$$

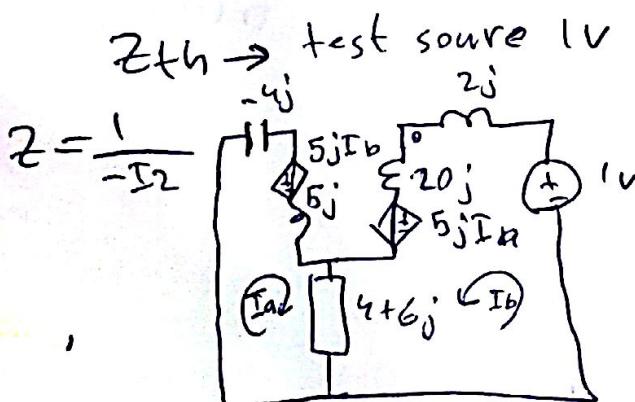
$$= 0.5 \sqrt{5 \times 20} \quad L_2 = 20j$$

$$= 5j$$

$$= L_1 - M = 5 - 5 = 0j$$

$$L_2 - M = 20 - 5 = 15j$$

$$m = 5j$$



mesh ①

$$I_a(-4j + 5j + 4 + 6j) + I_b(5j + 4 + 6j) = 0$$

$$I_a(-4 + 7j) + I_b(4 + 11j) = 0$$

$$I_a = \frac{(4 + 11j)}{-(4 + 7j)} I_b$$

mesh ②

$$I = I_b(2j + 20j + 4 + 6j) + I_a(5j + 4 + 6j)$$

$$I = I_b(4 + 28j) + I_a(4 + 11j)$$

$$I_b = 0.088 \angle -85^\circ$$

$$Z_{th} = -0.285 - j1.27j$$

mesh ①

$$120\angle0^\circ = I_a(-4 + 7j) + I_b(4 + 11j)$$

mesh ②

$$I_b(3.015 + 16.73j) + I_a(4 + 11j) = 0$$

$$I_a = \frac{(3.015 + 16.73j)}{-(4 + 11j)} I_b$$

### Question 8 (9 points)

For the circuit shown in the figure, if  $\omega = 1,000 \text{ rad/s}$ , determine

- the currents  $I_1$  and  $I_2$ .
- the energy stored in the coupled coils at  $t = 2 \text{ ms}$ ,  $W(t)$ .

$I_1$	$I_2$	$W(t)$
$1.87 \angle -141.34^\circ$	$2.846 \angle 132.2^\circ$	$0.0108 \text{ J}$

$$\omega = 1000 \text{ rad/s}$$

$$m = 0.5 \sqrt{100}$$

$$= 5 \text{ j}$$

mesh(1)

$$12 \angle 90^\circ + I_1 (4 + 10\text{j} - 5\text{j}) + I_2 (5\text{j} \cancel{- 5\text{j}}) = 0$$

$$I_1 (4 + 5\text{j}) = -12 \angle 90^\circ$$

$$I_1 = 1.87 \angle -141.34^\circ$$

mesh(2)

$$I_2 (-5\text{j} + 10\text{j} + 8) + 20 + 5\text{j} I_1 = 0$$

$$I_2 (5\text{j} + 8) + 20 + (5\text{j}) (1.87 \angle -141.34^\circ) = 0$$

$$I_2 = 2.846 \angle 132.2^\circ$$

$$i_1 = 1.87 \cos(1000t - 141.34)$$

$$i_2 = 2.846 \cos(1000t + 132.2)$$

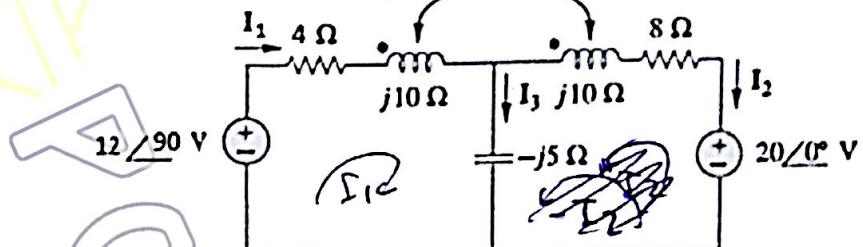
$$t = 2 \text{ ms} \quad \omega t = 2 \rightarrow 2 \times 57.3 \rightarrow 114.6^\circ$$

$$i_1 = 1.87 \cos(114.6 - 141.34) = 1.67$$

$$i_2 = 2.846 \cos(114.6 + 132.2) = -1.12$$

$$w = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M i_1 i_2$$

$$\begin{aligned} & \cancel{-5\text{j} I_2} + \cancel{5\text{j} I_1} \\ & \lambda = 0.5 \end{aligned}$$



$$\begin{aligned} m &= \\ L_1 &= 0.01 \text{ H} \\ L_2 &= 0.01 \text{ H} \end{aligned}$$

$$\begin{aligned} w &= \frac{1}{2} (0.01) (1.67)^2 \\ &+ \frac{1}{2} (0.01) (-1.12)^2 \\ &+ (0.005) (1.67)(-1.12) \\ &= 0.0108 \text{ J} \end{aligned}$$

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Section 4	Mohammed Abuelhaj		

**Questions 1 (6 points)**

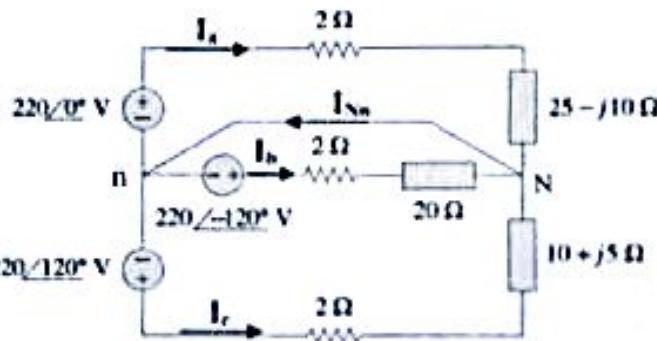
If the source voltage in the figure below is  $220 \angle 0^\circ$  V<sub>ms</sub>, determine the source currents and the current in the neutral line,  $I_n$ .

$I_a$ (A)	$I_b$ (A)	$I_c$ (A)	$I_{n_s}$ (A)
$7.6 \angle 20.3^\circ A$	$10 \angle -120^\circ A$	$16.9 \angle 97.4^\circ A$	$10.8 \angle -90^\circ A$

**Solution:**

For phase a.

$$I_a = \frac{V_{an}}{Z_A + 2} = \frac{220 \angle 0^\circ}{27 - j10} = \frac{220}{28} \angle 20.3^\circ A$$



For phase b,

$$I_b = \frac{V_{bn}}{Z_B + 2} = \frac{220 \angle -120^\circ}{12 + j5} = 10 \angle -120^\circ A$$

For phase c,

$$I_c = \frac{V_{cn}}{Z_C + 2} = \frac{220 \angle 120^\circ}{12 + j5} = \frac{220 \angle 120^\circ}{13 \angle 22.62^\circ} = 16.923 \angle 97.38^\circ A$$

The current in the neutral line is

$$I_n = -(I_a + I_b + I_c) \text{ or } -I_n = I_a + I_b + I_c$$

$$-I_n = (7.166 + j2.654) + (-5 - j8.667) + (-2.173 + j6.783)$$

$$I_n = 0.007 - j10.77 = \underline{\underline{10.77 \angle -90^\circ A}}$$

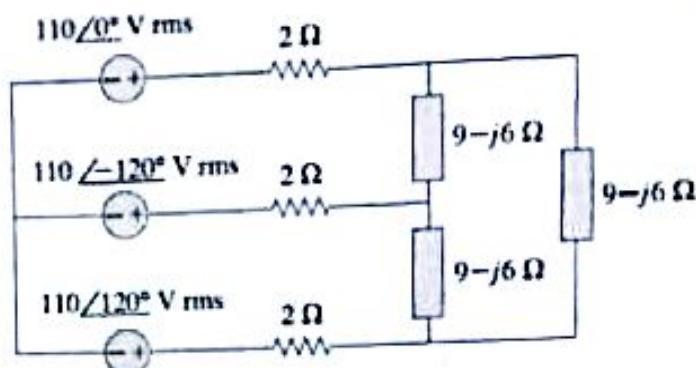
$$\underline{\underline{I_{n_s} = -I_n = 10.8 \angle -90^\circ A}}$$

**Questions 2 (6 points)**

In the balanced three-phase Y-Δ system shown in the figure, find

- the magnitude of the line current  $I_L$
- the total average power delivered to the load.

$ I_L $ (A)	$P_{tot}$ (W)
20.4 A	3744 W



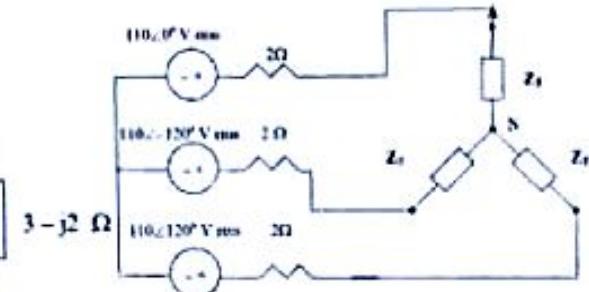
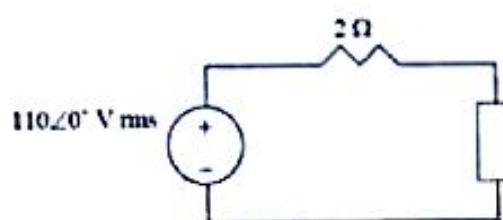
**Solution:**

Convert the delta load to Y as shown below.

$$Z_Y = \frac{Z_\Delta}{3}$$

$$Z_Y = \frac{9-j6}{3} = 3-j2 \Omega$$

We consider the single phase equivalent shown below.



$$I_a = \frac{110}{2+3-j2} = 20.4265 < 21.8^\circ$$

$$I_L = |I_a| = 20.43 \text{ A}$$

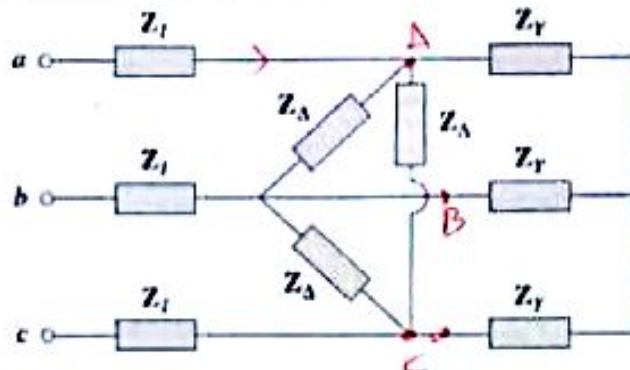
$$S = 3|I_a|^2 Z_Y = 3(20.43)^2 (3-j2) = 4514 \angle -33.96^\circ = 3744 - j2522$$

$$P = \operatorname{Re}(S) = 3744 \text{ W.}$$

Questions 3 (6 points)

The circuit in the figure is excited by a balanced three-phase source with a line voltage of  $210 \text{ V}_{\text{rms}}$ . If  $Z_L = 1 + j1 \Omega$ ,  $Z_A = 24 - j30 \Omega$ , and  $Z_Y = 12 + j5 \Omega$ , determine the magnitude of the line current of the combined loads, and the magnitude of the phase current in the delta load.

$I_{\text{line}} (\text{A})$	$I_p (\text{phase in } A) (\text{A})$
13.66 A	5.47 A 4.97



Solution:

Convert the delta load,  $Z_\Delta$ , to its equivalent wye load.

$$Z_{Y_\Delta} = \frac{Z_\Delta}{3} = 8 - j10$$

$$Z_p = Z_Y \parallel Z_{Y_\Delta} = \frac{(12 + j5)(8 - j10)}{20 - j5} = 8.076 \angle -14.68^\circ$$

$$Z_p = 7.812 - j2.047 = 8.07 \angle -14.68^\circ \Omega$$

$$Z_T = Z_p + Z_L = 8.812 - j1.047$$

$$Z_T = 8.874 \angle -6.78^\circ$$

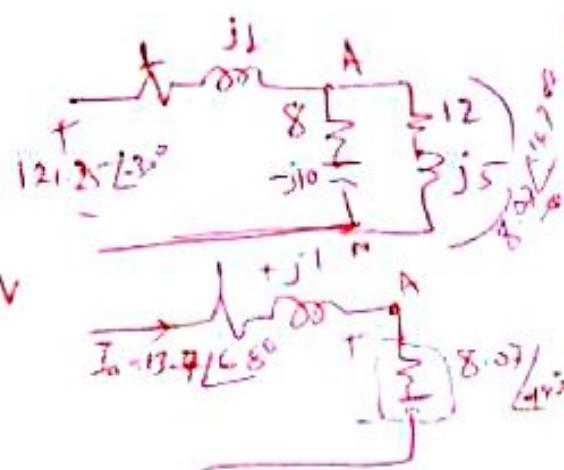
We now use the per-phase equivalent circuit.

$$I_p = \frac{V_p}{Z_p + Z_L}, \quad \text{where } V_p = \frac{210}{\sqrt{3}} = 121.25 \angle -30^\circ$$

$$I_p = \frac{210}{\sqrt{3}(8.874 \angle -6.78^\circ)} = 13.66 \angle 6.78^\circ$$

$$I_L = |I_p| = 13.66 \text{ A}$$

$$I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{210/\sqrt{3}}{\sqrt{3}} = \frac{210/\sqrt{3}}{\sqrt{8^2 + 10^2}} = \frac{121.24}{\sqrt{3}} = \frac{12.81}{\sqrt{3}} = \frac{9.45}{\sqrt{3}} = 5.46 \text{ A}$$



$$V_{AB} = 190.9 \angle 37.9^\circ$$

$$= 170.24 \angle 7.9^\circ \text{ V}$$

$$V_{AB} = 190.9 \angle 37.9^\circ$$

**Questions 4 (9 points)**

In the figure below, two wattmeters are properly connected to the unbalanced load supplied by a balanced source such that  $V_{ab} = 208\angle 0^\circ$  V<sub>ms</sub> with positive phase sequence.

- Determine the reading of each wattmeter.
- Calculate the total apparent power absorbed by the load.

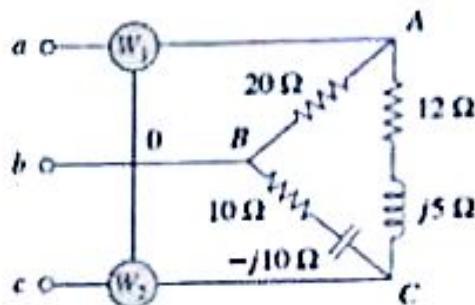
W <sub>1</sub> (W)	W <sub>2</sub> (W)	S <sub>tot</sub>   (VA)
2589 W	4808 W	8335 VA

(X)

(X)

(X)

**Solution**



If  $V_{ab} = 208\angle 0^\circ$ ,  $V_{bc} = 208\angle -120^\circ$ ,  $V_{ca} = 208\angle 120^\circ$ .

$$I_{AB} = \frac{V_{ab}}{Z_{AB}} = \frac{208\angle 0^\circ}{20} = 10.4\angle 0^\circ$$

$$I_{BC} = \frac{V_{bc}}{Z_{BC}} = \frac{208\angle -120^\circ}{10\sqrt{2}\angle -45^\circ} = 14.708\angle -75^\circ$$

$$I_{CA} = \frac{V_{ca}}{Z_{CA}} = \frac{208\angle 120^\circ}{13\angle 22.62^\circ} = 16\angle 97.38^\circ$$

$$I_{AA} = I_{AB} - I_{CA} = 10.4\angle 0^\circ - 16\angle 97.38^\circ$$

$$I_{AA} = 10.4 + 2.055 - j15.867$$

$$I_{AA} = 20.171\angle -51.87^\circ$$

(X)

(X)

$$I_{AC} = I_{CA} - I_{AB} = 16\angle 97.38^\circ - 14.708\angle -75^\circ$$

$$I_{AC} = 30.64\angle 101.03^\circ$$

$$P_1 = |V_{ab}| |I_{AA}| \cos(\theta_{V_{ab}} - \theta_{I_{AA}})$$

$$P_1 = (208)(20.171) \cos(0^\circ + 51.87^\circ) = 2590 \text{ W}$$

(X)

$$P_2 = |V_{cb}| |I_{AC}| \cos(\theta_{V_{cb}} - \theta_{I_{AC}})$$

$$\text{But } V_{cb} = -V_{bc} = 208\angle 60^\circ$$

(X)

$$P_2 = (208)(30.64) \cos(60^\circ - 101.03^\circ) = 4808 \text{ W}$$

$$P_T = P_1 + P_2$$

$$= 7398$$

(X)

$$Q_T = \sqrt{(4808 - 2590)^2} = 3841$$

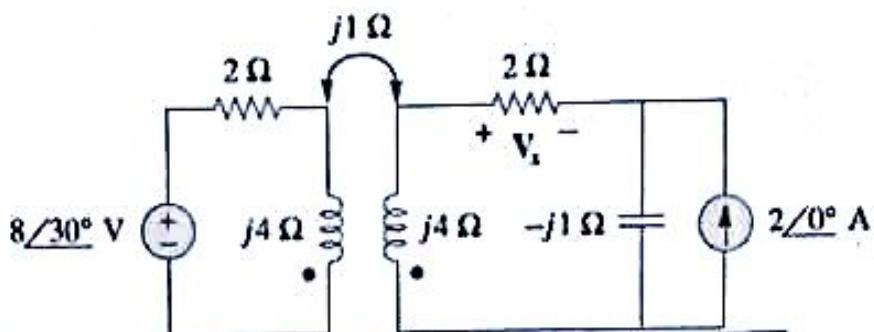
(X)

$$S_T = \sqrt{P_T^2 + Q_T^2}$$

**Questions 5 (7 points)**

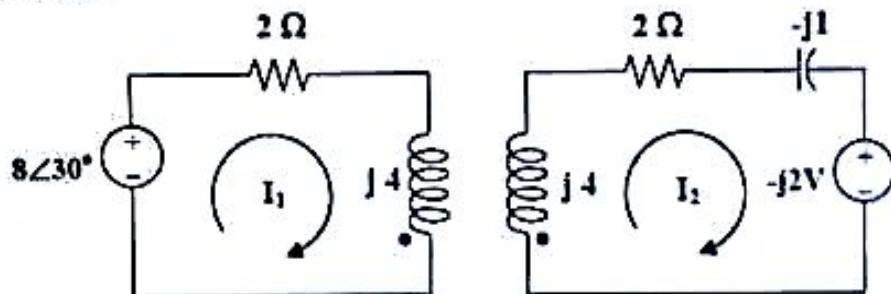
In the shown network,

- use source transformation to write down the two mesh equations that govern the current relations in the circuit in their simplest form.
- Find  $V_x$ .



Equation (1)	$(2+j4)I_1 - jI_2 = 8\angle 30^\circ$	$V_x (V)$
Equation (2)	$-jI_1 + (2+j3)I_2 = 2\angle 90^\circ$	$2.07\angle 21.1^\circ V$

**Solution**



For loop 1,

$$8\angle 30^\circ = (2+j4)I_1 - jI_2 \quad (1)$$

For loop 2,

$$(j4 + 2 - j)I_2 - jI_1 + (-j2) = 0$$

$$\text{or } I_1 = (3-j2)I_2 - 2 \quad (2)$$

Substituting (2) into (1),

$$8\angle 30^\circ + (2+j4)2 = (14+j7)I_2$$

$$I_2 = (10.928 + j12)/(14 + j7) = 1.037\angle 21.12^\circ$$

$$V_x = 2I_2 = 2.074\angle 21.12^\circ$$

**Question 6 (8 points)**

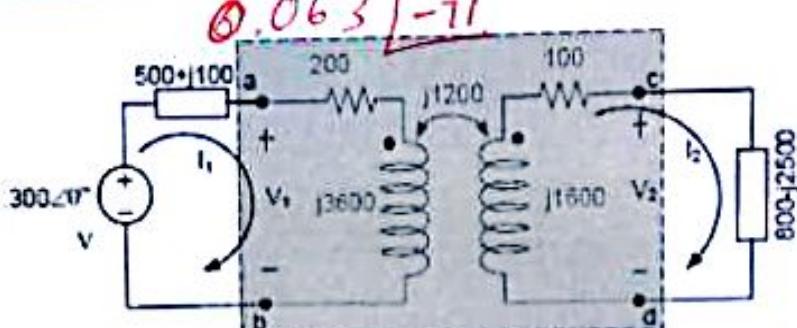
For the circuit shown in the figure, find:

- the self-impedance of the primary circuit  $Z_{11}$ .
- the self-impedance of the secondary  $Z_{22}$ .
- the reflected impedance as seen by the primary coil  $Z_r$ .
- the source and load currents  $I_1$  and  $I_2$ .

$$(0.02 - j0.06) \quad (0.026 + j0.053)$$

$Z_{11} (\Omega)$	$Z_{22} (\Omega)$	$Z_r (\Omega)$	$I_1 (\text{mA})$	$I_2 (\text{mA})$
$700 + j3700 \Omega$	$900 - j900 \Omega$	$800 + j800 \Omega$	$263.25 \angle -71.6^\circ \text{ mA}$	$59.63 \angle +63.4^\circ \text{ mA}$

$$3765 [79 \ 1272] L-45$$



**Solution:**

$$Z_{22} = R_2 + j\omega L_2 + Z_l = (R_2 + R_2) + j(X_2 + X_2)$$

$$Z_r = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_l} = \frac{\omega^2 M^2}{|Z_{22}|^2} [(R_2 + R_2) \cdot j(\omega L_2 + X_2)]$$

$$\text{Mesh 1} \quad V_s = (Z_1 + R_1 + j\omega L_1)I_1 - j\omega M I_2$$

$$\text{Mesh 2} \quad 0 = -j\omega M I_1 + (Z_2 + R_2 + j\omega L_2)I_2$$

$$Z_{11} = R_1 + j\omega L_1 + Z_1 \quad \text{Self impedance of the primary}$$

$$Z_{22} = R_2 + j\omega L_2 + Z_2 \quad \text{Self impedance of the secondary}$$

$$V_s = Z_1 I_1 - j\omega M I_2$$

$$0 = -j\omega M I_1 + Z_{22} I_2$$

$$I_2 = \frac{j\omega M}{Z_{22}} I_1$$

$$V_s = \frac{Z_1 Z_{22} + \omega^2 M^2}{Z_{22}} I_1 - \left( Z_{11} - \frac{\omega^2 M^2}{Z_{22}} \right) I_1$$

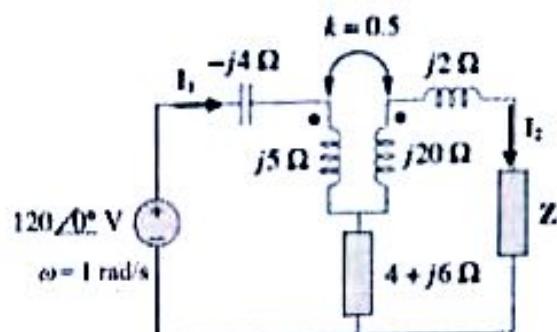
$$I_1 = \frac{Z_{22}}{Z_1 Z_{22} + \omega^2 M^2} V_s \quad \text{Primary Current}$$

$$I_2 = \frac{j\omega M}{Z_1 Z_{22} + \omega^2 M^2} V_s - \frac{j\omega M}{Z_{22}} I_1 \quad \text{Secondary Current}$$

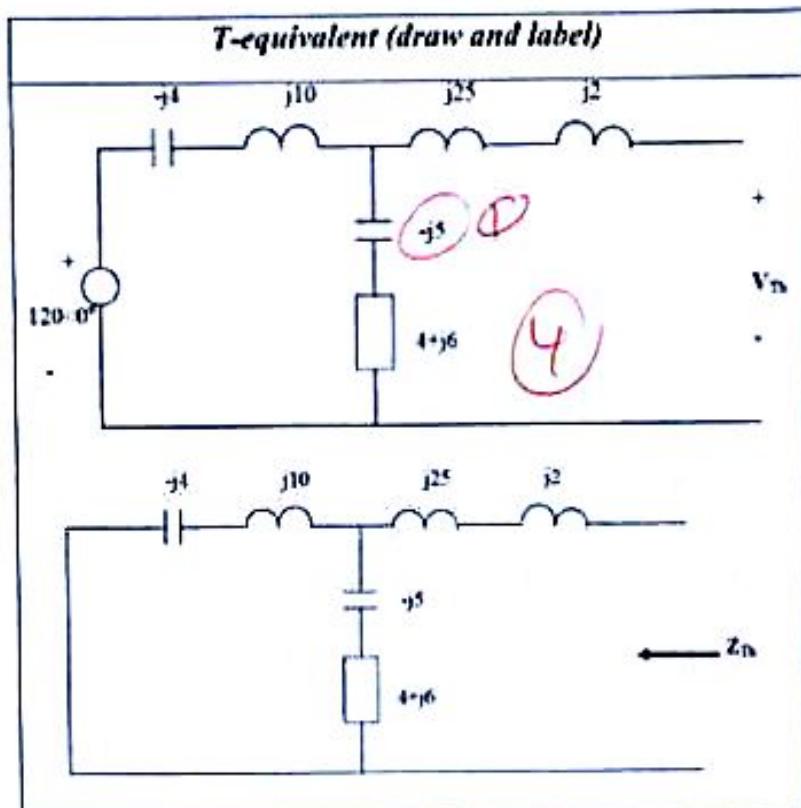
**Question 7 (9 points)**

For the circuit shown in the figure, replace the transformer by its T-equivalent model to find the Thevenin equivalent to the left of the load Z.

$V_{TH}$ (V)	$Z_{TH}$ ( $\Omega$ )
$61.37 \angle -46.2^\circ V$	$2.2 + j29.1 \Omega$
	$29.2 \angle 85.65^\circ \Omega$



Solution



$$V_{Th} = \frac{4+j}{4+j+j6} (120) = 61.37 \angle -46.22^\circ V \quad (2)$$

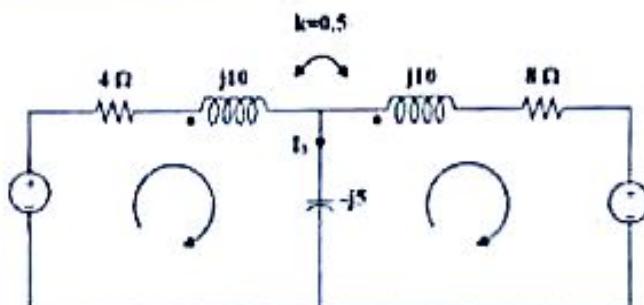
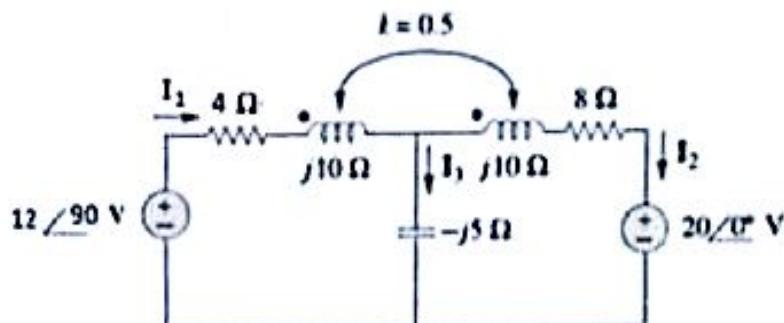
$$Z_{Th} = j27 + (4+j)/(j6) = j27 + \frac{j6(4+j)}{4+j7} = 2.215 + j29.12 \Omega \quad (2)$$

**Question 8 (9 points)**

For the circuit shown in the figure, if  $\omega = 1,000 \text{ rad/s}$ , determine

- the currents  $I_1$  and  $I_2$ .
- the energy stored in the coupled coils at  $t = 2 \text{ ms}$ ,  $W(t)$ .

$I_1 (\text{A})$	$I_2 (\text{A})$	$W(t) (\text{mJ})$
$2.46\angle 72.2^\circ \text{ A}$	$0.88\angle -97.5^\circ \text{ A}$	$43.7$ $23.2$



**Solution**

$$\text{For mesh 1. } jI_2 = (4 + j10 - j5)I_1 + j5I_2 + j5I_2 = (4 + j5)I_1 + j10I_2 \quad (1)$$

$$\text{For mesh 2. } 0 = 20 + (8 + j10 - j5)I_2 + j5I_1 + j5I_1$$

$$-20 = +j10I_1 + (8 + j5)I_2 \quad (2)$$

$$\begin{bmatrix} jI_2 \\ 20 \end{bmatrix} = \begin{bmatrix} 4 + j5 & +j10 \\ +j10 & 8 + j5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}, \Delta = 107 + j60, \Delta_1 = -60 - j296, \Delta_2 = 40 - j100$$

$$I_1 = \Delta_1 / \Delta = 2.462\angle 72.18^\circ \text{ A}$$

$$i_1 = 2.462 \cos(1000t + 72.18^\circ) \text{ A}$$

$$I_2 = \Delta_2 / \Delta = 0.878\angle -97.48^\circ \text{ A}$$

$$i_2 = 0.878 \cos(1000t - 97.48^\circ) \text{ A}$$

$$I_3 = I_1 - I_2 = 3.329\angle 74.89^\circ \text{ A}$$

$$i_1 = 0.9736 \cos(114.6^\circ + 143.09^\circ) = -2.445$$

$$\text{At } t = 2 \text{ ms, } 1000t = 2 \text{ rad} = 114.6^\circ, i_2 = 2.53 \cos(114.6^\circ + 153.61^\circ) = +0.8391$$

The total energy stored in the coupled coils is

$$w = 0.5L_1i_1^2 + 0.5L_2i_2^2 + Mi_1i_2$$

Since  $\omega L_1 = 10$  and  $\omega = 1000$ ,  $L_1 = L_2 = 10 \text{ mH}$ ,  $M = 0.5L_1 = 5 \text{ mH}$

$$w = 0.5(10)(-2.445)^2 + 0.5(10)(+0.8391)^2 + 5(-2.445)(+0.8391) \Rightarrow w = 43.67 \text{ mJ}$$