Circuits1

Summer017

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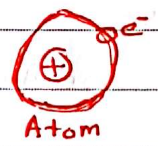
Circuits1 . Summer 017
Dr. Yanal

4/6 , lec1

*Ch1:

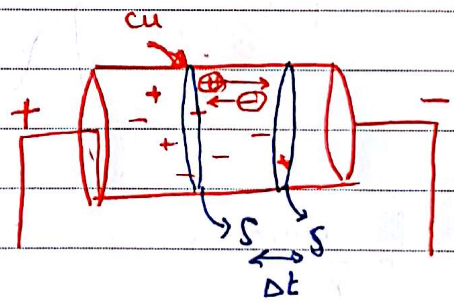
Charge : Q or q
↳ +ve charge
↳ -ve charge

$1e^- = -1.6 \times 10^{-19}$ C , Proton = 1.6×10^{-19} C



~~current~~

Current : I or i (A)
↳ charge in motion

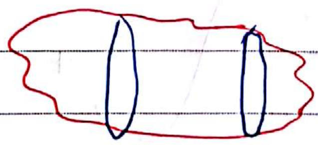


In Uniform surfaces ← $i = \frac{\Delta q}{\Delta t}$ ⇒ [C/s] [A]

A ≡ C/s
C → columb
s → Second
A → Ampere

In General ~~Surfaces~~ case

$i = \frac{dq}{dt}$



* I is in the direction of +ve Q.

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* Convention:

Capital letters : I, V, Q, P

for quantities that don't change with time

Example:

$Q = 4\,C$, $I = 5\,A$, $V = -2\,V$

Small letters : q or $q(t)$, i or $i(t)$

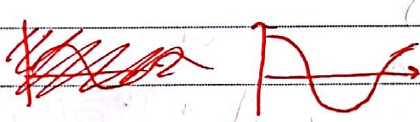
for general case

either change or not change with time

Example

$i = 4A$

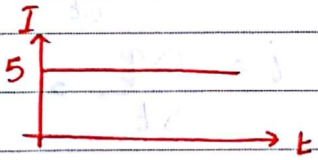
$i = 4\cos t$



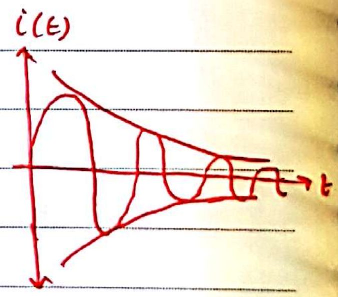
* Types of current:

$I = 5A$

1) DC

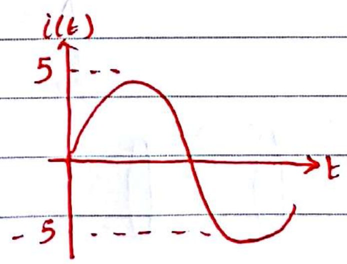


4) Damped sinusoidal current

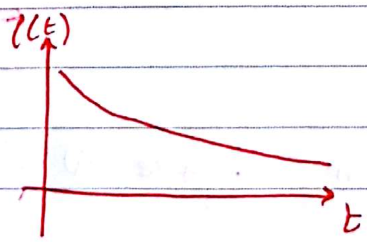


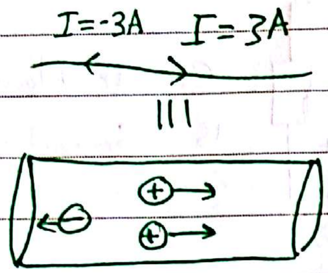
$i = 5\sin t$

2) AC



3) exponentialy decaying





Ex: $i(t) = \begin{cases} 5e^{4t}\ A, & t<0 \\ 5e^{-4t}\ A, & t>0 \end{cases}$

Find the total +ve charge crossed from left to right during $[-0.25, 0.25]$ s

$i = \frac{dq}{dt} \Rightarrow \int dq = \int i\,dt$

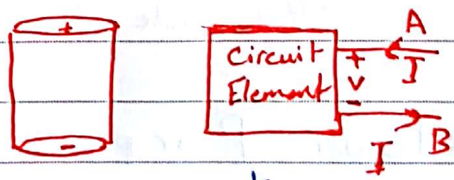
$$q = \int_{-0.25}^{0.25} i(t)\,dt = \int_{-0.25}^{0} 5e^{4t}\,dt + \int_{0}^{0.25} 5e^{-4t}\,dt \quad C$$

$= 1.58$ C

* Voltage : V or v

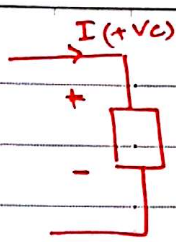
Unit is Volt (V)

→ Potential difference.



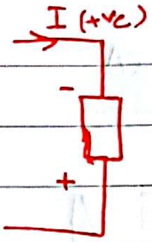
$\text{Voltage} = \frac{\text{work}}{\text{Charge}}$

1 Volt = 1 Joule / Coulomb

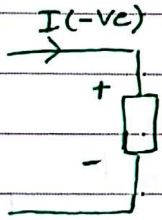


circuit Element is absorbing Power

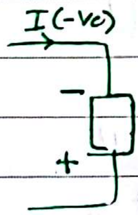
(IV = P > 0 absorb)
(IV = P < 0 Generate)



Circuit Element is Generating Power

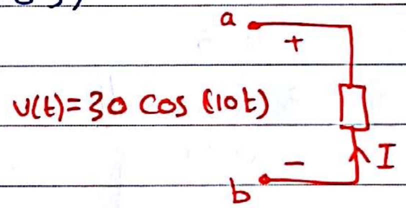


Supply / Generate Power



Absorb Power

<u>Ex</u>: what is the Energy required to move 3C from (a) to (b) during (t = 6 S)



$$w = V * Q$$

$$= 30\cos(10t)\Big|_{t=6} * 3$$

$$= 90\cos(60^\circ)$$

$$= 45 \text{ Joule}$$

* Power : P or p

$$P = \frac{work}{time} = \frac{w}{t} \qquad J/s \text{ or watt}$$

$$P = \frac{dw}{dt} * \frac{dq}{dq} = \frac{dw}{dq} \cdot \frac{dq}{dt} = VI$$

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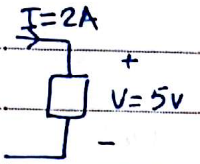
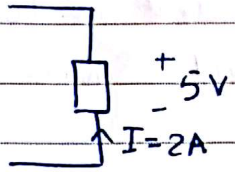
Ex:



$P = 10W$ (absorb)

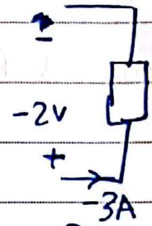
$P_{abs} = 10\,w$



$P = 10\,w$ (generate)

$P_{gen} = 10W$

$P_{abs} = -10\,W$

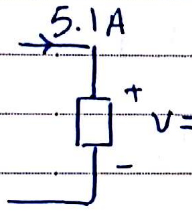


$P = 6\,W$ (absorb)

$P_{Gen} = -6W$

$P_{abs} = 6\,W$

* $\boxed{P_{Gen} = -P_{Abs.}}$



$v = 5(t^2 - 2)\ V$

at $t = 0.8s$

$P = 5.1 * 5(.8^2 - 2)\ w$ (Generate)

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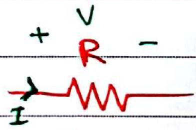
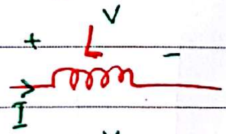
* Types of Circuit Elements:

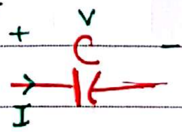
If $V \propto i \Rightarrow$ Resistance



If $V \propto \frac{di}{dt} \Rightarrow$ Inductance



If $V \propto \int i\,dt \Rightarrow$ Capacitance



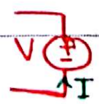
Passive Elements

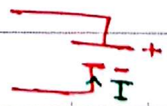
* these elements absorb Power. called Passive elements

C, L are also called Storing devices. ~~there~~

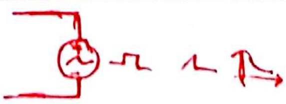
If V is independent of current

→ Independent Voltage Source.





DC



AC

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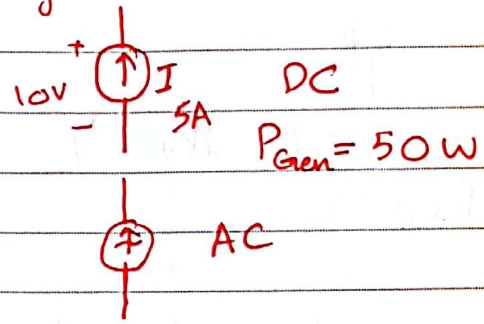
If i is independent of voltage

→ Independent current Source



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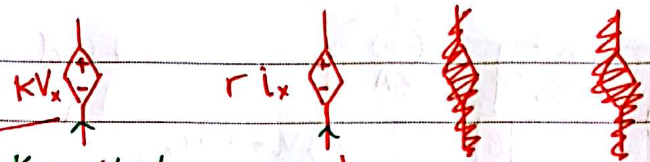
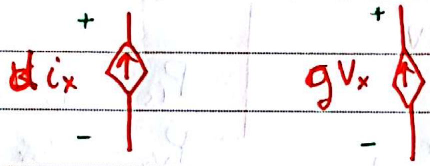
* Dependent Sources



K: constant
V_x: Voltage defined S.where in the circuit.

1) Voltage controlled voltage Source
2) current controlled Voltage Source

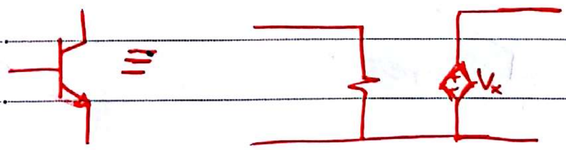


3) current controlled current Source
4) ~~current~~ Voltage controlled ~~voltage~~ current Source.

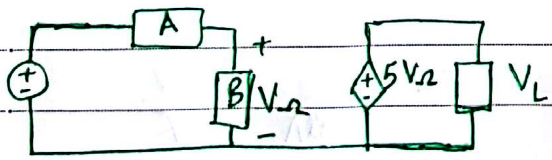
k, ~~r~~ d, g are all constants ~~[unit]~~

k, d [Unitless] / r [ohm Ω] / g [Seimens $S \equiv \Omega^{-1}$]

Note
* You'll see Dependent Sources in Electronics, such as Transistors



* Ex:



find V_L if $V_\Omega = 3V$??

$V_L = 5V_\Omega$ (Parallel)

$V_L = 15\ V$

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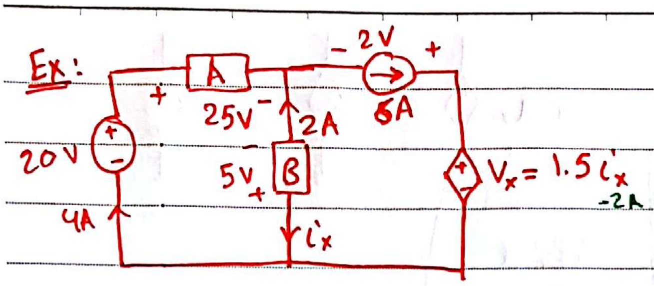
Ex:



Find the Power Generated by each element ??

$P_{V_s} = 80$ watt

$P_A = -100$ watt

$P_B = -10$ watt

$P_{i_s} = 12$ watt

$P_{V_x} = \cancel{36}$ 18 watt

$V_x = 1.5\, i_x$

$i_x = -2A$

$V_x = -6V$

this is for all circuit.

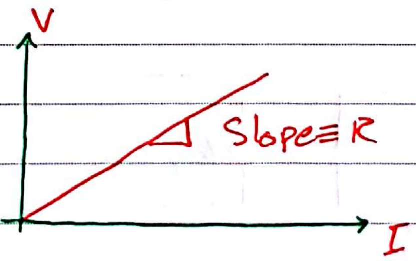
$\boxed{\Sigma P_{Gen} = 0}$

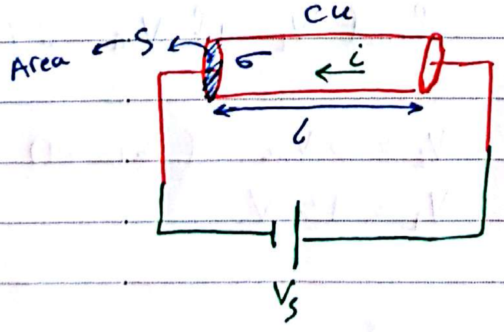
$\boxed{\Sigma P_{Gen} = -\Sigma P_{loss}}$

* Ohm's law

$V \propto i$, $V = iR$, $R \equiv$ Resistor , $R = \frac{V}{I}$

$\frac{V}{A} \equiv \Omega$





$R = \frac{\ell}{\sigma A} = \frac{V}{I}$

σ: conductivity $\equiv \frac{1}{\rho}$ [S/m] [℧/m]

ρ: Resistivity [Ω/m]

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$P = VI$, $V = IR$, $I = \dfrac{V}{R}$

$$\boxed{P = I^2 R = \frac{V^2}{R}}$$ Joule's law

$R = \dfrac{V}{I}$, $G = \dfrac{I}{V} = \dfrac{1}{R}$ $[\Omega^{-1}, S, \mho]$

$G \equiv$ Conductance.

$I = GV$

$$\boxed{P = GV^2 = I^2/G}$$

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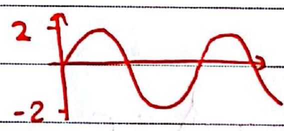
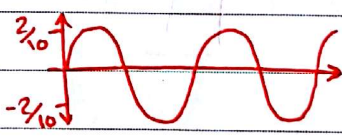
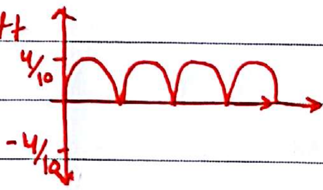
* Ex: if $V = 2\sin(100t)\ V$
$R = 10\ \Omega$



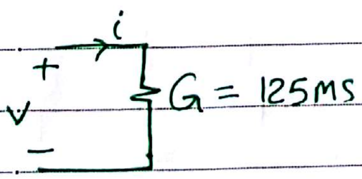
$i = \dfrac{2}{10}\sin(100t)\ A$



$P = \dfrac{4}{10}\sin^2(100t)$ watt



Ex: if $P_{abs} = 200\ mW$, find V ??



Sol: $V = \pm\sqrt{\dfrac{P}{G}} = \pm\sqrt{\dfrac{200}{125}}$

$V = \pm 1.26\ V$

~~Since~~ ~~P absor~~ according to current sign.

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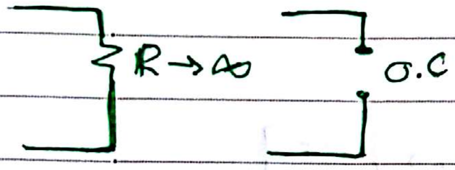
* Open & Short Circuit.

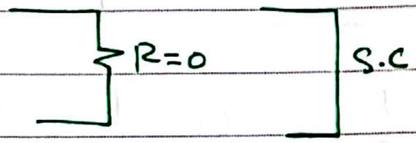
Open Circuit ($R=\infty$)



$$I = \frac{V}{R} = \frac{V}{\infty} = \text{zero A}$$

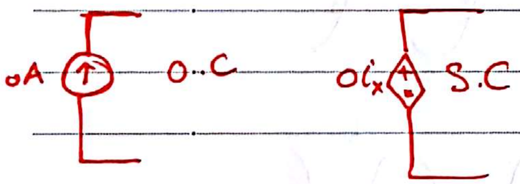
$I_{o.c} = \text{zero}$

Short Circuit ($R=0$)



$$V = IR = I(0) = \text{zero V}$$

$V_{o.c} = \text{zero}$



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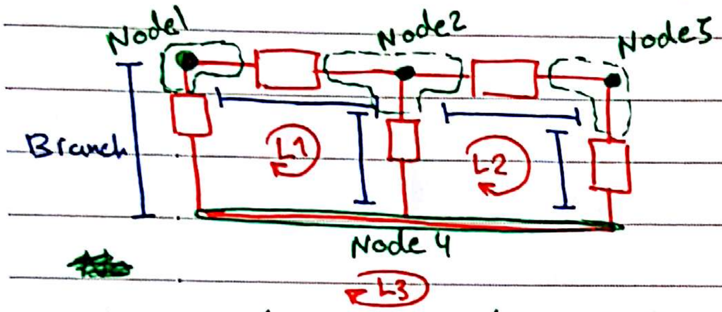
Ch2: Voltage & current laws.

* Node, Branch, Path, loop, Mesh



This circuit contains 4 Nodes, 5 branches, 3 loops, 2 Meshes

Node ≡ Connection between two Circuit Elements

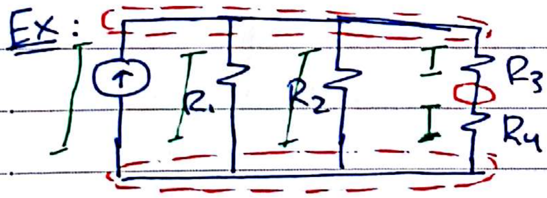
Branch ≡ Node - element - Node

Path ≡ Start from a Node and go to another Node.

loop ≡ Start from a Node and return to it.

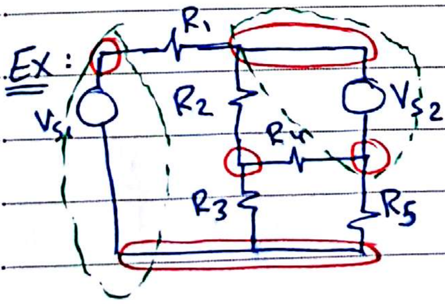
Mesh ≡ loop that does not contain another loop.

Ex:



3 Nodes, 5 branches, 6 loops, 3 Meshes

Ex:



5 Nodes, 7 Branches, 7 loops, 3 Meshes

Supernode ≡ V_s between 2 Nodes

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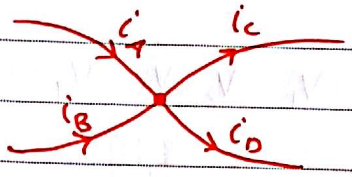
*Kirchoff 's laws

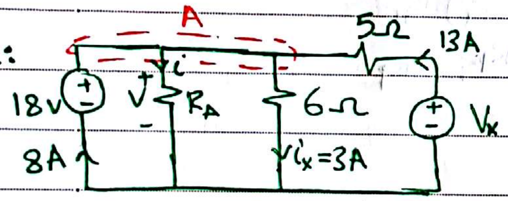
*Kirchoff 's current law (KCL)



$$\boxed{\sum I_{entering} = \sum I_{leaving}}$$

$i_A + i_B = i_c + i_D \Rightarrow i_A + i_B - i_c - i_D = \text{zero}$

Ex:



find R_A ??

$V = 18V$

Apply KCL @ Node A

$13 + 8 = i + 3$

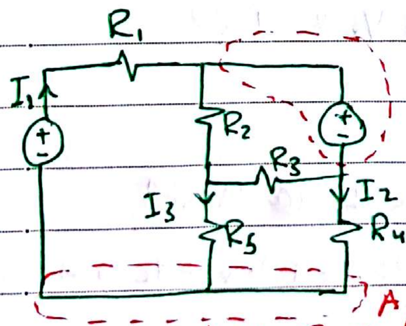
$\boxed{i = 18\ A}$

$R_A = \frac{V}{i}$

$= \frac{18}{18} = 1\,\Omega$

$\boxed{R_A = 1\,\Omega}$

Ex:



$I_3 = 2A$

$I_2 = 0A$

Apply KCL @ Node A

$I_1 = I_2 + I_3 \Rightarrow \boxed{I_1 = 2A}$

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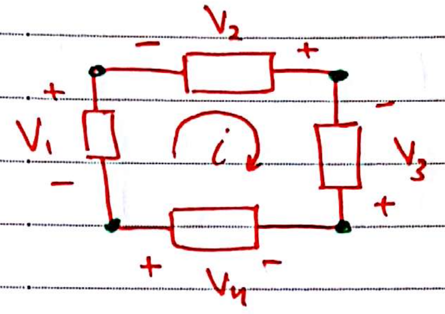
* Kirchoff's Voltage laws (KVL)

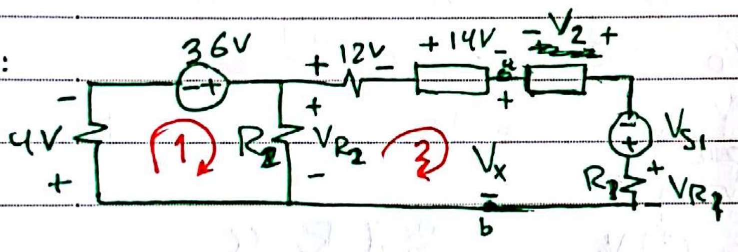
$$\sum V = 0$$



$-V_1 - V_2 - V_3 - V_4 = \text{zero}$

$\boxed{V_1 + V_2 + V_3 + V_4 = \text{Zero}}$

Ex:



Find V_x, V_{R2} ??

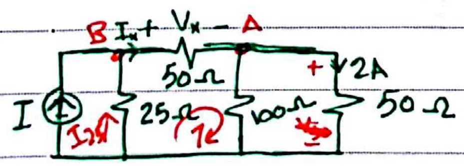
KVL @ Mesh 1

$+4 - 36 + V_{R2} = 0 \Rightarrow \boxed{V_{R2} = 32 \text{ Volt}}$

KVL @ Mesh2

$-32 + 12 + 14 + V_x = 0 \Rightarrow \boxed{V_x = 6 \text{ Volt}}$

Ex:



Find V_x & I ??

$V_{50\Omega} = 2 * 50 = 100V$

$V_{50\Omega} = V_{100\Omega} = 100V$

KCL @ Node A

$I_x = I_{100\Omega} + I_{50\Omega}$

$\boxed{I_x = 3A} \rightarrow \boxed{V_x = 150V}$

KVL @ Mesh1

$25 I_{25} + V_x + 100 = 0$

$I_{25\Omega} = \frac{-250}{25} = -10A$

KCL @ Node B

$I = I_x - I_{25}$

$\boxed{I = 13 A}$

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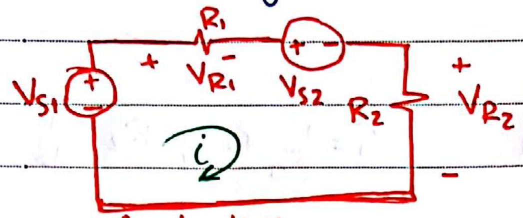
* The Single loop Circuit:



Apply KVL:

$$-V_{S_1} + V_{R_1} + V_{S_2} + V_{R_2} = 0$$

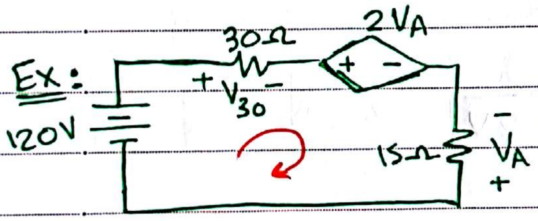
$V_{R_1} = i R_1$

$V_{R_2} = i R_2$

$$-V_{S_1} + iR_1 + V_{S_2} + iR_2 = 0$$

$$\boxed{i = \frac{V_{S_1} - V_{S_2}}{R_1 + R_2}}$$

Ex:



Find the Power Absorbed by each element ??

Apply KVL

$$-120 + V_{30} + 2V_A - V_A = 0$$

$$-120 + V_{30} + V_A = 0$$

$V_{30} = 30i$, $V_A = -15i$

$$-120 + 30i - 15i = 0$$

$$\boxed{i = 8A}$$

Pabs

$P_{120V} = -960$ watt

$P_{30\Omega} = 30\,(8)^2$
$= 1920$ watt

$P = i^2 R$

$P_{2V_A} = 2(-15(8))(8)$
$= -1920$ watt

$P_{15\Omega} = (8)^2 * 15$
$= 960$ watt

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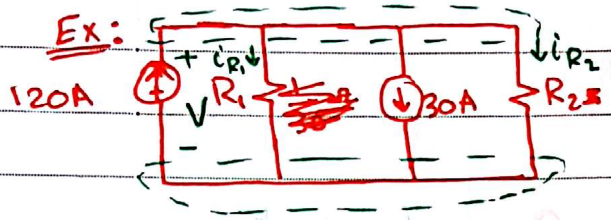
* The Single Node Pair circuit :-

Ex:



$R_1 = \frac{1}{30}\,\Omega$, $R_2 = \frac{1}{15}\,\Omega$

Find Power Absorbed by each element

KCL @ upper Node

~~$120A = \frac{V}{R_1} + 30A + \frac{V}{R_2}$~~

$i_{R_1} = \frac{V}{R_1}$

$i_{R_2} = \frac{V}{R_2}$

$120A = i_{R_1} + 30A + i_{R_2}$

$90 = \frac{V}{R_1} + \frac{V}{R_2}$

$90 = 30V + 15V \Rightarrow \boxed{V = 2V}$

Pabs

$P_{120A} = -120 * 2 = -240$ watt

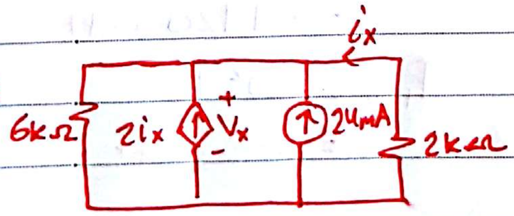
$P_{R_1} = \frac{(2)^2}{R_1} = \cancel{60}\ 120$ watt

$P = \frac{V^2}{R}$

$P_{R_2} = \frac{(2)^2}{R_2} = 60$ watt

$P_{30A} = 30 \cdot 2 = 60$ watt

Ex:



Find V & $P_{Gen\ 24mA}$??

Answers: $V = 14.4V$, $P_{Gen\ 24mA} = 345.6\ mW$

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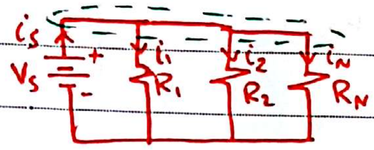
* Series & Parallel combinations:

→ Parallel



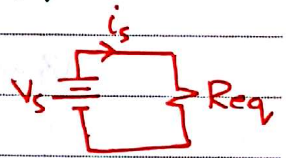
KCL @ upper Node

$$i_s = i_1 + i_2 + \dots + i_N$$

$$i_s = \frac{V_s}{R_1} + \frac{V_s}{R_2} + \dots + \frac{V_s}{R_N}$$

$$i_s = V_s\left(\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N}\right)$$

$$R_{eq} = \frac{V_s}{i_s} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots \frac{1}{R_N}}$$



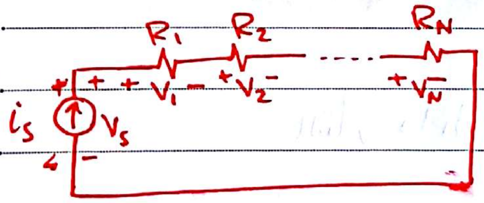
2 Resistors in Parallel

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

$$\boxed{R_{eq} = \frac{R_1 \cdot R_2}{R_1 + R_2}}$$

$$G_{eq} = \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N}$$

→ Series



KVL

$$-V_s + V_1 + V_2 + \dots + V_N = 0$$

$$V_s = V_1 + V_2 + \dots + V_N$$

$$= i_s\,(R_1 + R_2 + \dots + R_N)$$

$$\frac{V_s}{i_s} = R_1 + R_2 + \dots + R_N$$

$$\boxed{R_{eq} = R_1 + R_2 + \dots + R_N}$$

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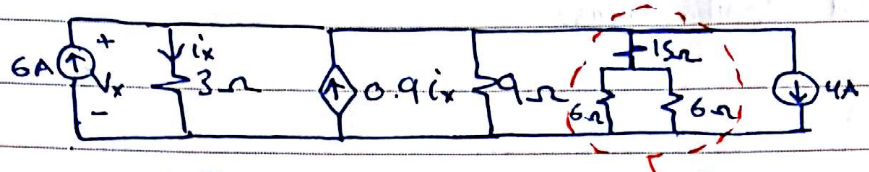
Ex:

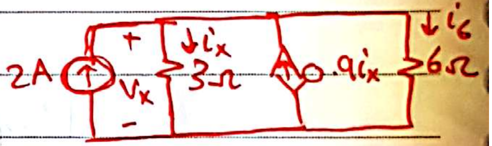


Find $P_{Gen_{6A}}$, $P_{Gen_{4A}}$, V_x, i_x

$6\Omega \,//\, 6\Omega \rightarrow R_{eq_1} = \frac{6 \cdot 6}{12} = 3\Omega$

$3\Omega \,//\, 15\Omega \rightarrow R_{eq_2} = 18\Omega$

$18\Omega \,//\, 9\Omega \rightarrow R_{eq_3} = \frac{18 \cdot 9}{27} = 6\Omega$



$i_x = \frac{V_x}{3}$

$i_6 = \frac{V_x}{6}$

KCL @ upper Node

$2 + 0.9 i_x = V_x \left(\frac{1}{3} + \frac{1}{6}\right)$

$2 + \frac{\cancel{9}^{3}}{10} \cdot \frac{V_x}{\cancel{3}_1} = V_x \left(\frac{1}{3} + \frac{1}{6}\right)$

$2 + 0.3 V_x = \frac{1}{2} V_x$

$0.2 V_x = 2 \Rightarrow \boxed{V_x = 10 \text{ Volt}} \rightarrow \boxed{i_x = \frac{10}{3} A}$

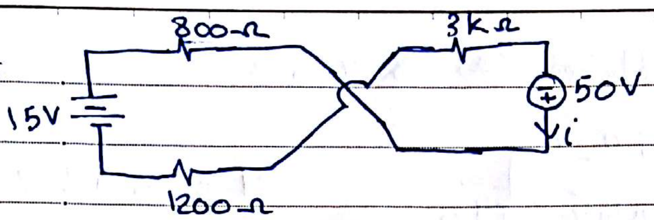
$P_{Gen_{6A}} = 60$ watt ~~Because current in the~~

التيار داخل في ~~الطرف~~ السالب

$P_{Gen_{4A}} = -40$ watt

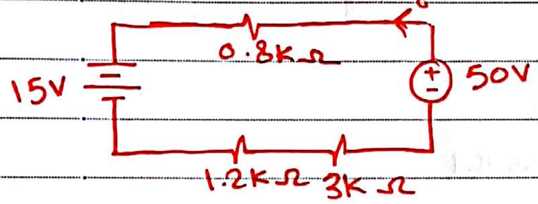
التيار داخل في الموجب

Ex:

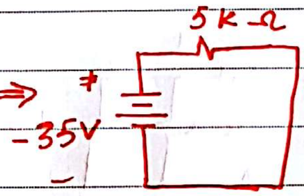


find i ??

Redraw the circuit



⇒

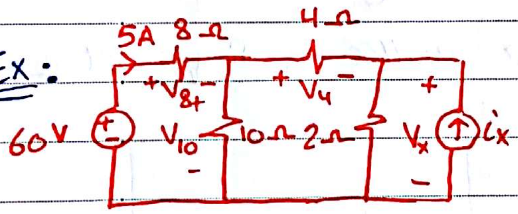


KVL

$-35 + 5i = 0$

$i = \frac{35}{5} = 7A$

Ex:



Find V_x, i_x ??

$V_8 = 5(8) = 40V$

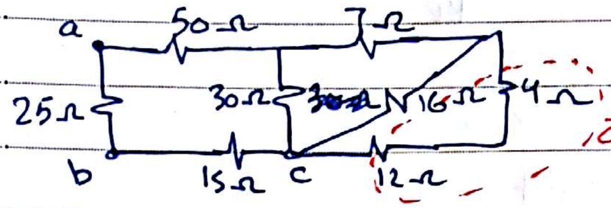
$V_{10} = V_8 + 60$

$V_{10} = 100V$

$V_x = 8V$, $i_x = 1A$

$100 = V_4 + V_x$

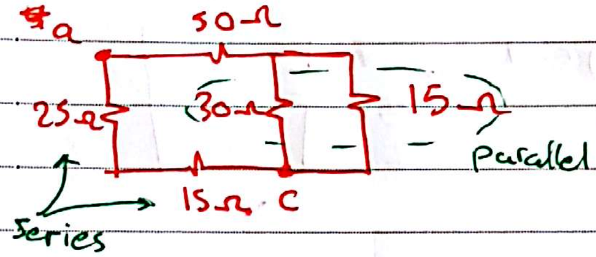
Ex: Find R_{eq} between (a) & (c)



4 series 12 ⇒ 16

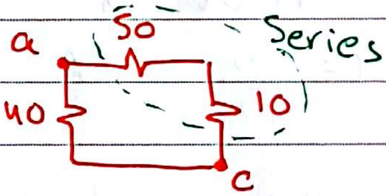
16 // 16 ⇒ 8

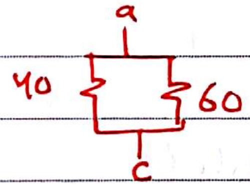
8 series 7 ⇒ 15



15//30 ⇒ 10 Ω

25 S 15 ⇒ 40 Ω





Parallel → 40//60 ⇒ R_{eq} = 24 Ω

~~Resolve~~

Ex: Find R_{eq} between (a) & (b)

R_{eq} = 18.75 Ω

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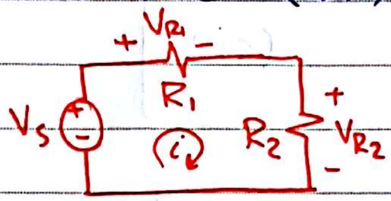
* Voltage & current division

⇒ Voltage divider rule (VDR) "only for Series Elements"



<u>KVL</u>

$$-V_s + V_{R_1} + V_{R_2} = 0$$

$$V_s = iR_1 + iR_2$$

$$V_s = i(R_1 + R_2)$$

$$\frac{V_s}{R_1+R_2} = i$$

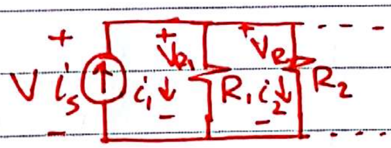
$$V_{R_1} = iR_1 \rightarrow i = \frac{V_{R_1}}{R_1}$$

$$V_{R_2} = iR_2 \rightarrow i = \frac{V_{R_2}}{R_2}$$

$$* \quad \frac{V_{R_1}}{R_1} = \frac{V_s}{R_1+R_2} \Rightarrow V_{R_1} = V_s \frac{R_1}{R_1+R_2}$$

$$* \quad \frac{V_{R_2}}{R_2} = \frac{V_s}{R_1+R_2} \Rightarrow V_{R_2} = V_s \frac{R_2}{R_1+R_2}$$

⇒ current divider rule ~~(CDR)~~ (CDR) "only for Parallel Elements"



<u>KCL @ upper Node</u>

$$i_s = i_1 + i_2 + \dots$$

$$i_s = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$V = i_s \frac{R_1 \cdot R_2}{R_1 + R_2}$$

$$i_1 = \frac{V_{R_1}}{R_1} \rightarrow V_{R_1} = i_1 R_1$$

$$i_2 = \frac{V_{R_2}}{R_2} \rightarrow V_{R_2} = i_2 R_2$$

$$V = V_{R_1} = V_{R_2}$$

$$i_1 \cancel{R_1} = i_s \frac{\cancel{R_1} R_2}{R_1+R_2} \Rightarrow i_1 = i_s \frac{R_2}{R_1+R_2} \quad , \quad i_2 = i_s \frac{R_1}{R_1+R_2}$$

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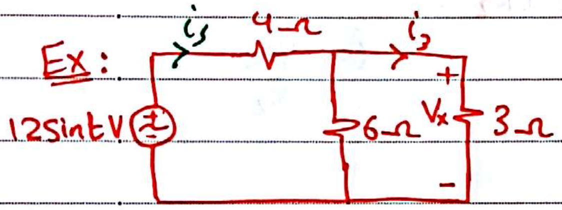
* for N elements

$$\boxed{i_K = \frac{R_{eq}}{R_K} * i_s \qquad \text{(CDR)}}$$

$$\boxed{V_K = \frac{R_K}{R_{eq}} * V_s \qquad \text{(VDR)}}$$

Ex:



Find V_x??

$$i_s = \frac{12 \sin t}{R_{eq}} = 2 \sin t \quad ; \quad R_{eq} = \frac{3 \cdot 6}{9} + 4 = 6\Omega$$

$$\cancel{R_{eq} = \frac{3 \cdot 6}{9} + 4 = 6}$$

$$\cancel{i_3 =} \quad i_3 = i_s \cdot \frac{6}{3+6}$$

$$i_3 =$$

$$V_x = 3 * i_3$$

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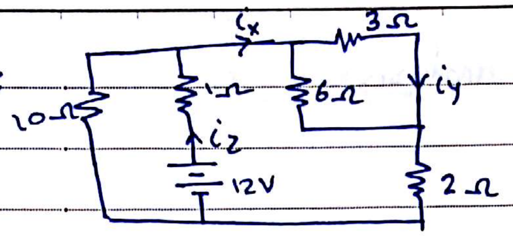
Ex:

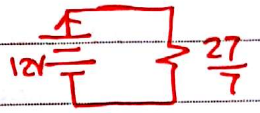


find i_x, i_y, i_z ??

Simplify the ckt

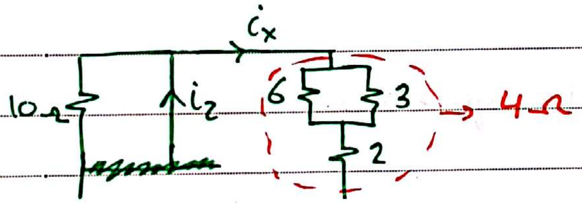
$$R_{eq} = [(\underbrace{3 /\!/ 6}_{2} + 2) /\!/ 10] + 1 \Rightarrow R_{eq} = \frac{27}{7}\ \Omega$$

$4 /\!/ 10$



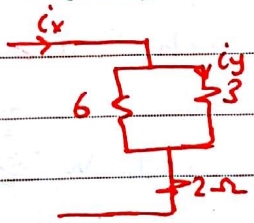
$$i_z = \frac{12}{27/7}$$

$$\boxed{i_z = \frac{28}{9}\ A}$$



$$i_x = i_z * \frac{10}{10+4} \rightarrow \boxed{i_x = \frac{20}{9}\ A}$$

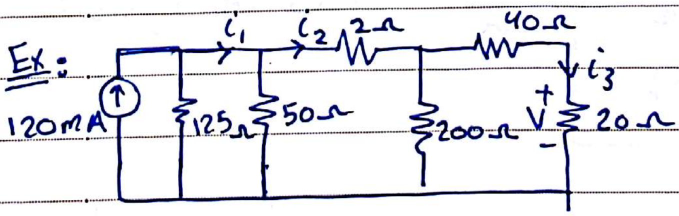
(CDR)



$$i_y = i_x \cdot \frac{6}{3+6} \Rightarrow \boxed{i_y = \frac{40}{27}\ A}$$

(CDR)

Ex:



find $i_1, i_2,$ ~~i_3~~ V ??

Solutions: $i_1 = 100\,mA$

$i_2 = 50\,mA$

$V = 0.8\,V$

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Ch 3: Nodal & Mesh Analysis.

* Nodal Analysis:

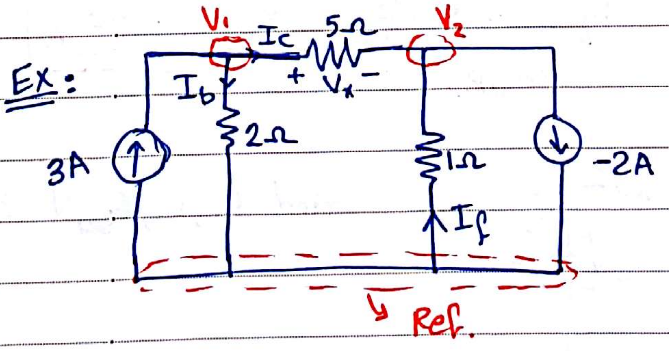
* Procedure:-

1) Assume a ckt with N-nodes
   choose one node as a reference.
   then write KCL equation for the remaining (N−1) nodes
   and rename the Nodes as $V_1, V_2, \dots, V_{N-1}$

2) Solve the N−1 equations.

3) If a Voltage Source exists in a circuit, You can form a Super Node, thus reducing the No. of eq.s by 1 for each Source, and a replacement eqn should be written for each Source.

Ex:



Find V_x, I_b, I_f using Nodal Analysis??

KCL eqn @ Node 1

$-3 + I_b + I_c = 0$

$$\boxed{\frac{V_1}{2} + \frac{V_1 - V_2}{5} - 3 = 0} \quad --- \; (1)$$

$$\boxed{V_1\left(\frac{1}{2} + \frac{1}{5}\right) + V_2\left(-\frac{1}{5}\right) - 3 = 0} \; - (1)$$

$$I_b = \frac{V_1 - 0}{2}$$

$$I_c = \frac{V_1 - V_2}{5}$$

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KCL @ Node 2

$$-2 + \frac{V_2}{1} + \frac{V_2 - V_1}{5} = 0$$

$$\boxed{-2 + V_2\left(1 + \frac{1}{5}\right) + V_1\left(-\frac{1}{5}\right) = 0} \quad (2)$$

You can Solve eqns (1) & (2) by الحزف والتعويض

$0.7V_1 - 0.2V_2 = 3$

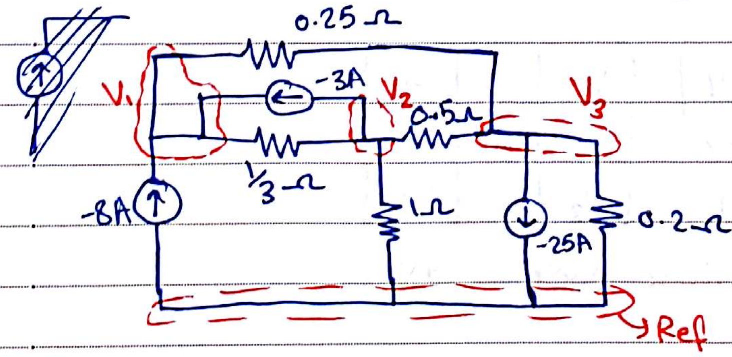
$-0.2V_1 + 1.2V_2 = 2$

$\Rightarrow \quad V_1 = \frac{3 + 0.2V_2}{0.7}$

$\boxed{V_1 = 5V}$, $\boxed{V_2 = 2.5V}$

$V_x = V_1 - V_2 = 2.5V$, $I_b = \frac{V_1}{2} = 2.5A$, $I_c = \frac{-V_2}{1} = -2.5A$

Ex:



Find the Nodal Voltages ??

* 3 KCL eqns.
* KCL @ Node 1

$$8 + 3 + \frac{V_1 - V_2}{1/3} + \frac{V_1 - V_3}{0.25}$$

$$\boxed{V_1(3+4) + V_2(-3) + V_3(-4) = -11} \quad (1)$$

* KCL @ Node 2

$$\frac{V_2}{1} + -3 + \frac{V_2 - V_1}{1/3} + \frac{V_2 - V_3}{0.5} = 0$$

$$\boxed{V_1(-3) + V_2(1+3+2) + V_3(-2) = 3} \quad \text{—②}$$

* KCL @ Node 3

$$-25 + \frac{V_3}{0.2} + \frac{V_3 - V_2}{0.5} + \frac{V_3 - V_1}{0.25} = 0$$

$$\boxed{V_1(-4) + V_2(-2) + V_3(5+2+4) = 25} \quad \text{—③}$$

Solve using matrix

$$\begin{bmatrix} 7 & -3 & -4 \\ -3 & 6 & -2 \\ -4 & -2 & 11 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -11 \\ 3 \\ 25 \end{bmatrix}$$

↳ these must be +ve.

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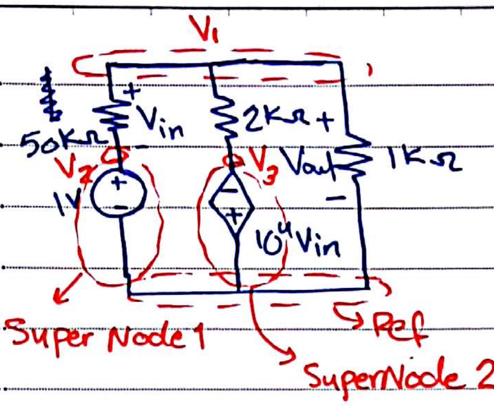
Ex:



Find the Node to ref Voltages

KCL @ Node 1

$\boxed{V_2 = 1V}$

$V_3 = -10^4 V_{in}$

$V_{in} = V_1 - V_2$

$$\frac{V_1 - V_2}{50} + \frac{V_1 - V_3}{2} + \frac{V_1}{1}$$

$$\frac{V_1 - 1}{50} + \frac{V_1 - (-10^4(V_1 - V_2))}{2} + \frac{V_1}{1} = 0$$

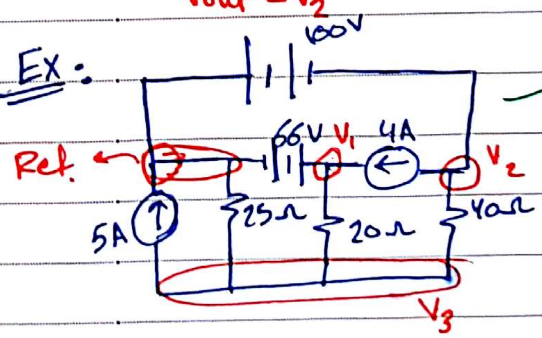
$$V_1\left(\frac{1}{50} + \frac{1+10^4}{2} + 1\right) = \frac{1}{50} + \frac{10^4}{2} \Rightarrow \boxed{V_1 = 0.9997V}$$

$V_3 = -10^4(V_1 - 1)$

$\boxed{V_3 = 2.999V}$

$V_{out} = V_3$

Ex:



Resolve with the Ref bottom Node

$V_1 = 102.52V$, $V_2 = -63.\ V$

Ref ← $V_3 = 36.52V$

$V_1 = 66V$

$V_2 = -100V$

KCL @ Node 3

$$5 + \frac{V_3}{25} + \frac{V_3 - V_1}{20} + \frac{V_3 - V_2}{40} = 0$$

$$\boxed{V_3 = -36.52V} \leftarrow V_3\left(\frac{1}{25} + \frac{1}{20} + \frac{1}{40}\right) = -5 + \frac{66}{20} - \frac{100}{40}$$

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11/6 , lec6

* Mesh Analysis

Only applied for a planner circuits

Procedure:

① Assume a circuit with M- meshes
Assign a Mesh current inside each mesh $i_1, i_2 \ldots i_M$.

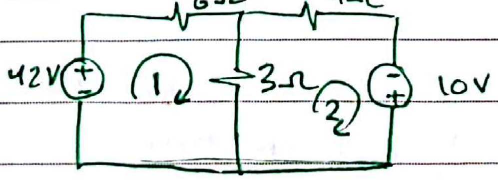
② write KVL eq.s for each Mesh (M-eq.s)

③ Solve M-eqs , to find M - unknown

④ if a ~~current~~ Circuit contains a current Sources, create a Super Mesh for each current (by applying KVL around the larger loop formed by the branches that are not common to the Meshes.

⑤ KVL eq. not needed if the current Source is On the Parameter of the circuit.

Ex: Find the Mesh currents.



KVL eq for Mesh-1

$-42 + 6i_1 + 3(i_1 - i_2) = 0$

$\boxed{9i_1 - 3i_2 = 42}$ — ①

KVL eq. for Mesh-2

$3(i_2 - i_1) + 4i_2 - 10 = 0$

$\boxed{7i_2 - 3i_1 = 10}$ — ②

$i_1 = 6A$
$i_2 = 4A$

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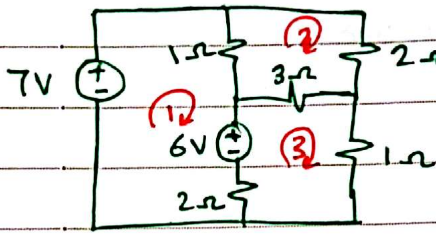
Ex: Use Mesh analysis to find the Mesh currents.



KVL eq. for Mesh1

$-7 + 1(i_1 - i_2) + 6 + 2(i_1 - i_3)$

① -- $\boxed{3i_1 - i_2 - 2i_3 = 1}$

KVL eq. for Mesh 2

$1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$

② -- $\boxed{-i_1 + 6i_2 - 3i_3 = 0}$

KVL eq. for Mesh3

$2(i_3 - i_1) - 6 + 3(i_3 - i_2) + i_3$

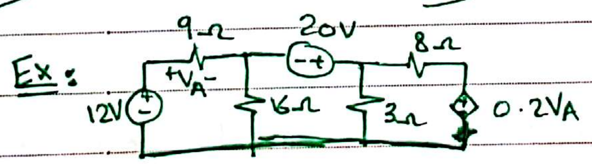
③ -- $\boxed{-2i_1 - 3i_2 + 6i_3 = 6}$

$i_1 = 3A$

$i_2 = 2A$

$i_3 = 3A$

Ex:

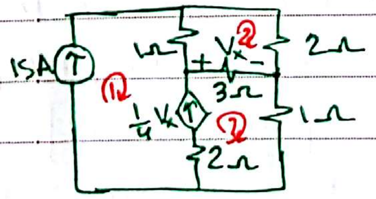


$i_1 = 2.621 A$

$i_2 = 3.265 A$

$i_3 = 0.462 A$

Ex: Find the three Mesh currents & V_x



$i_1 = 15A$ → Super Node

KVL eq. @ Mesh 2

$1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3)$

$-15 + 6i_2 - 3i_3 = 0$

① - $\boxed{6i_2 - 3i_3 = 15}$

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KVL eq. @ Mesh3

$3(i_3-i_2) + i_3 + 2(i_3-i_2)$

$\boxed{i_3 - i_1 = \frac{1}{9} V_x}$ $\boxed{V_x = 3(i_3-i_2)}$

$i_3 - 15 = \frac{1}{9} V_x$

$i_3 - 15 = \frac{3}{9}(i_3-i_2)$

$\boxed{\frac{2}{3} i_3 + \frac{1}{3} i_2 = 15}$ --- ②

by eq.s ① & ②

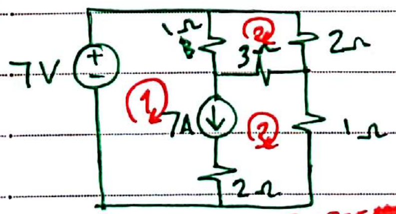
$i_1 = 15A$

$i_2 = 11A$

$i_3 = 17A$

$V_x = 18V$

Ex: Find the Mesh Currents.



Super Mesh

$\boxed{i_1 - i_3 = 7A}$ ①

~~SuperMesh~~ KVL eq. @ supermesh 1,3

$-7 + 1(i_1 - i_2) + 3(i_3 - i_2) + 3 i_3 = 0$

$\boxed{i_1 - 4i_2 + 3i_3 = 7}$ ②

KVL eq. @ Mesh2

$1(i_2 - i_1) + 2 i_2 + 3(i_2 - i_3) = 0$

$\boxed{-i_1 + 6i_2 - 3i_3 = 0}$ ③

$i_1 = 9A$

$i_2 = 2.5A$

$i_3 = 2A$

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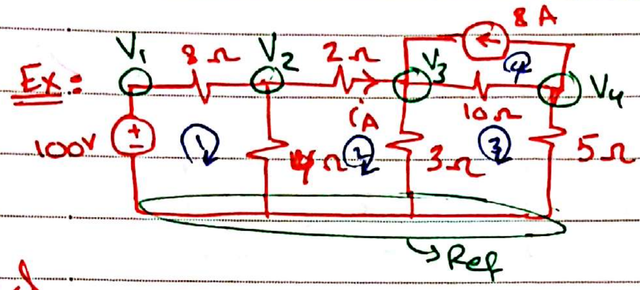
*Nodal V.S Mesh Analysis.

Ex:



Find i_A ??

Nodal

$V_1 = 100V$

Node 2

$$\frac{V_2 - 100}{8} + \frac{V_2}{4} + \frac{V_2 - V_3}{2} = 0$$

Node 3

$$\frac{V_3 - V_2}{2} + \frac{V_3}{3} + \frac{V_3 - V_4}{10} = 8$$

Node 4

$$\frac{V_4 - V_3}{10} + \frac{V_4}{5} + 8 = 0$$

$V_2 = 25.89V$

$V_3 = 20.31V$

$V_4 = 19.89V$

$$\boxed{i_A = \frac{V_2 - V_3}{2} = 2.79A}$$

Mesh

Mesh1

$$-100 + 8i_1 + 4(i_1 - i_2) = 0$$

Mesh2

$$4(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$$

Mesh3

$$3(i_3 - i_2) + 10(i_3 + 8) + 5i_3 = 0$$

$$\boxed{i_4 = 8A}$$

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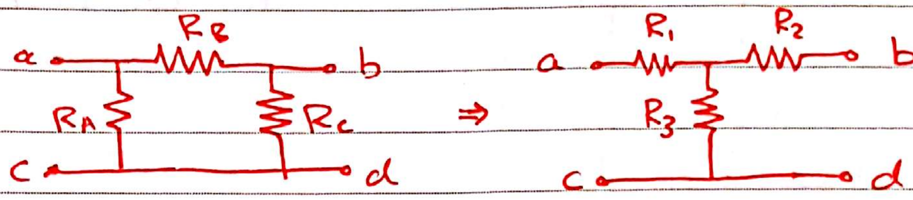
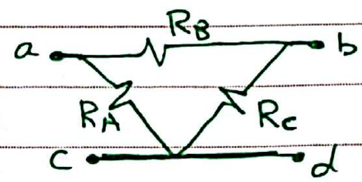
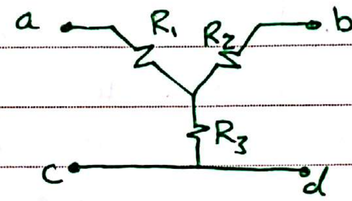
12/6, lec7

Delta to Wye Conversion:





Δ



Y

$Y \longrightarrow \Delta$	$\Delta \longrightarrow Y$
R_1, R_2, R_3 known	R_A, R_B, R_C known
$R_A = \dfrac{R_1R_2 + R_2R_3 + R_1R_3}{R_2}$	$\cancel{R} -$
$R_B = \dfrac{R_1R_2 + R_2R_3 + R_1R_3}{R_3}$	$R_1 = \dfrac{R_A R_B}{R_A + R_B + R_C}$
$R_C = \dfrac{R_1R_2 + R_2R_3 + R_1R_3}{R_1}$	$R_2 = \dfrac{R_B R_C}{R_A + R_B + R_C}$
	$R_3 = \dfrac{R_A R_C}{R_A + R_B + R_C}$

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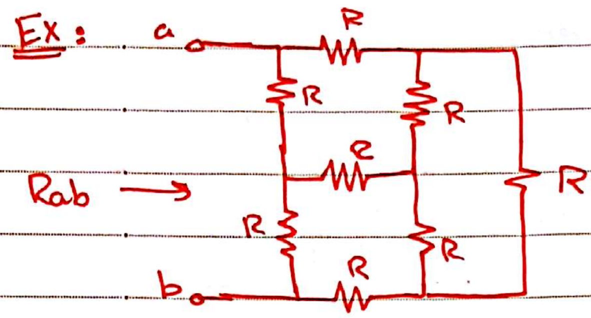
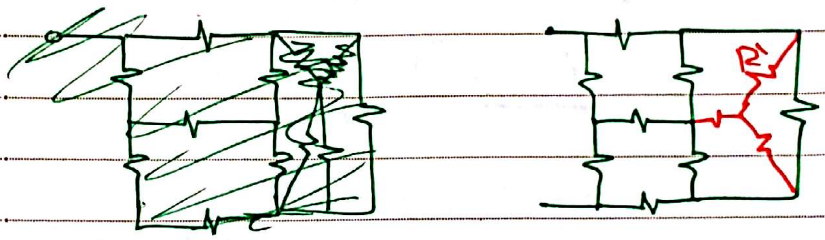
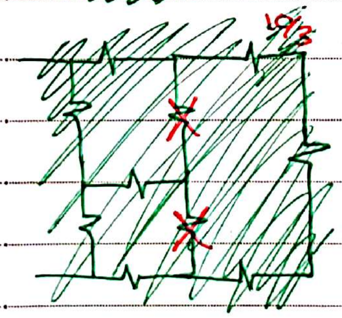
Ex: $R = 10\,\Omega$

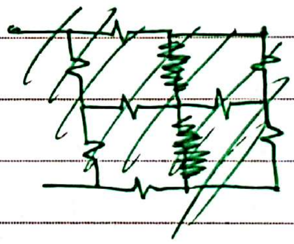
All R's are $10\,\Omega$



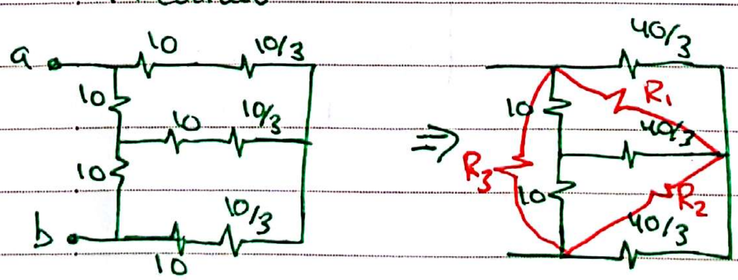


$$R' = \frac{10 \cdot 10}{10+10+10} = \frac{10}{3}$$





* Redraw



$$R_1 = \frac{10 \cdot \frac{40}{3} + 10 \cdot \frac{40}{3} + 10 \cdot 10}{10}, \quad R_3 = \frac{10 \cdot 40/3 + 10 \cdot 40/3 + 10 \cdot 10}{40/3}$$

$R_2 = R_1$ $\qquad$ $R_1 = R_2 =$

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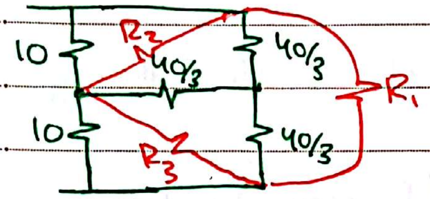
~~Ex Another Method of Solving~~

Another way to solve this ckt

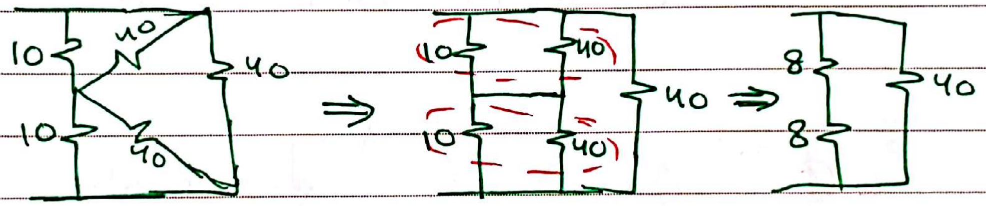
~~Ex~~



$R_1 = R_2 = R_3 = \frac{(\cancel{40/3} \cdot 40/3) + 3}{\cancel{40/3}}$

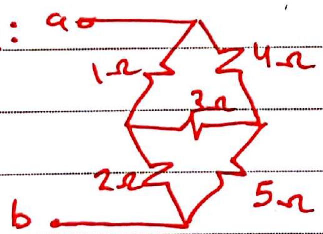
$R_1 = R_2 = R_3 = 40\,\Omega$

Redraw



$40 \parallel 16 \Rightarrow R_{ab} = 11.43\,\Omega$

Ex:



$\boxed{R_{ab} = 1.74\,\Omega}$

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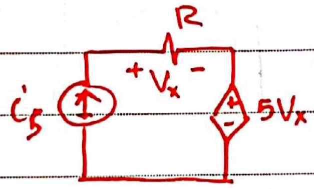
Ch.4 Useful Circuit Analysis techniques.

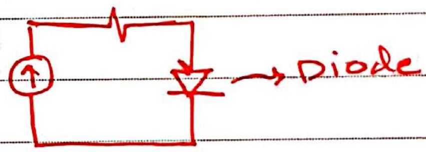
* linearity & Superposition

any linear Circuit consists of the following

1) Independent Sources. (Forcing function)
2) Linear dependent Sources. AV_x
3) linear Passive circuits elements.
   ↳ the loads must all be resistive.



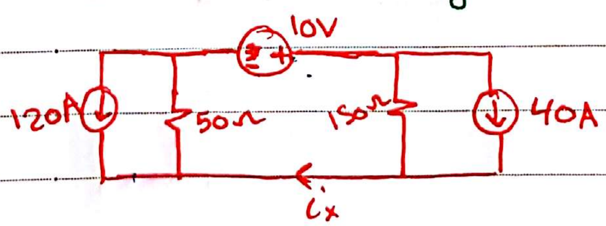
Linear



Non-Linear

* Superposition

Ex: Find i_x using S.P



S.P: only one Independent Source Should be active at a time.

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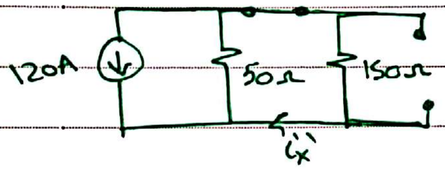
* To kill a Source
- If the Source is Voltage Source replace it with Short circuit.
- If the Source is current Source replace it with open circuit.

for 120A only



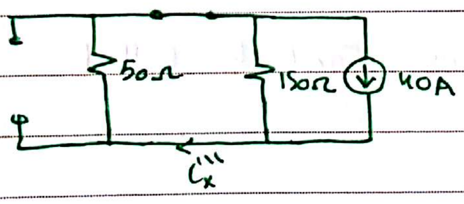
i_x' =

by current Divide rule

$$i_x' = -120 * \frac{50}{200}$$

$$\boxed{i_x' = -30A}$$

for 40A only



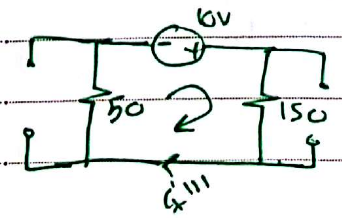
i_x''

by CDR

$$i_x'' = 40 * \frac{150}{200}$$

$$\boxed{i_x'' = 30A}$$

for 10V only



i_x'''

KVL

$$i_x''' = \frac{10}{200}$$

$$\boxed{i_x''' = 50mA}$$

$$i_x = i_x' + i_x'' + i_x''' = 50mA$$

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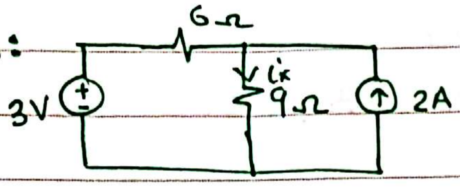
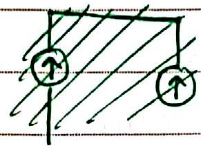
Ex:



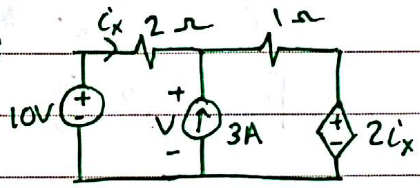
Find i_x using S.P

$\boxed{i_x = 1A}$

~~Ex:~~



Ex:

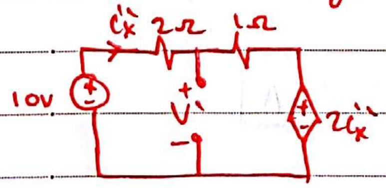


Find i_x & V using S.P.

~~* Independent~~

* Independent Sources can't be killed.

① for 10V only



KVL

$$i_x' = \frac{10 - 2i_x'}{3}$$

$\boxed{i_x' = 2A}$

$V' = 10 - 2i_x'$

$\boxed{V' = 6V}$

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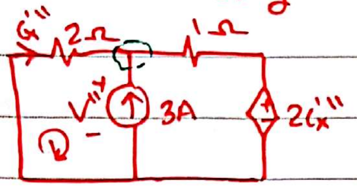
② for 3A only



KCL @ upper node

$$\frac{V''}{2} + \frac{V''-2i_x''}{1} = 3$$

$$\boxed{\frac{3}{2}V''-2i_x''=3} \text{ --- ①}$$

KVL @ Mesh 1

$$\boxed{V''-2i_x''=0} \text{ --- ②}$$

V''

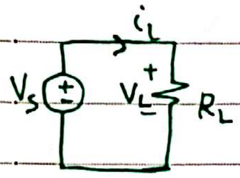
$V = V' + V''$

$i_x = i_x' + i_x''$

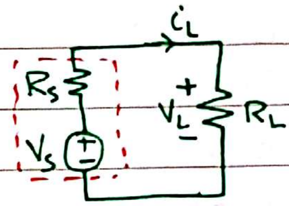
* S.P can't be applied for non-linear elements, sources or quantities such as Power.

* Source Transformation :

* Ideal Voltage Source



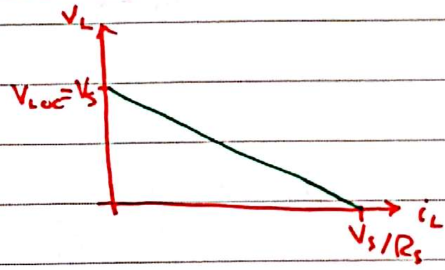
* Practical Voltage Source



KVL : $-V_s + i_L R_s + V_L = 0$

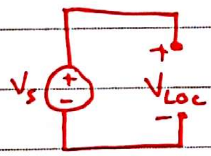
$$\boxed{V_L = V_s - i_L R_s}$$

$V_L = 0 \Rightarrow i_L R_s = V_s$



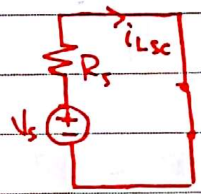
If $I_L = 0 \longrightarrow$ o.c

$V_L = V_s = V_{Loc}$



If $V_L = 0 \Rightarrow$ S.C

$i_L = \frac{V_s}{R_s} = i_{Loc}$

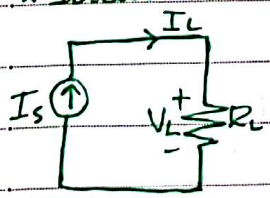


If $R_s = R_L$

$V_L = V_s \cdot \frac{R_L}{R_s + R_L}$

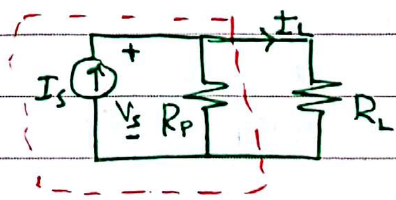
at $R_s = R_L \Rightarrow V_L = V_s / 2$

* Ideal current Source



$$\boxed{I_s = I_L}$$

* Practical current Source



$$\boxed{I_L = I_s \frac{R_P}{R_P + R_L}} \quad , V_L = V_s$$

$$I_L = I_s - \frac{V_s}{R_P}$$

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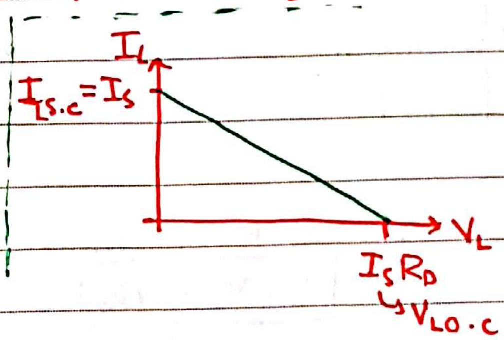
$$I_L = I_s - \frac{V_L}{R_p}$$

at $V_L = 0 \Rightarrow V_s = 0 \Rightarrow$ S.C $\rightarrow V_L = V_s$

$\hookrightarrow R_L = 0$

at $I_L = 0 \Rightarrow$ O.C

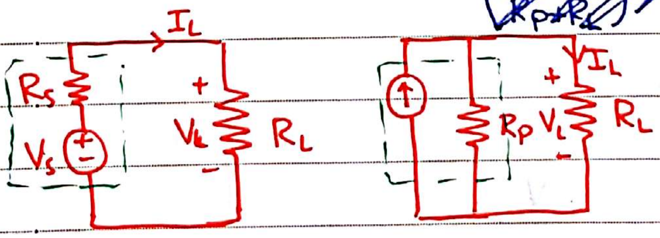
$\hookrightarrow R_L = \infty$



$$I_L = I_s \frac{R_p}{R_p + R_L}$$

$\gg$ if $R_p = R_L$

$$I_L = I_s \frac{R_p}{R_p + R_p} = I_s/2$$



(a) (b)

$V_L = I_L R_L$

(a) $V_L = \frac{V_s}{R_s + R_L} R_L \equiv$ (b) $V_L = I_s \left(\frac{R_p}{R_p + R_L}\right) \cdot R_L$

equal

* $R_s = R_p$

$\Rightarrow V_s = I_s R_s = I_s R_p$

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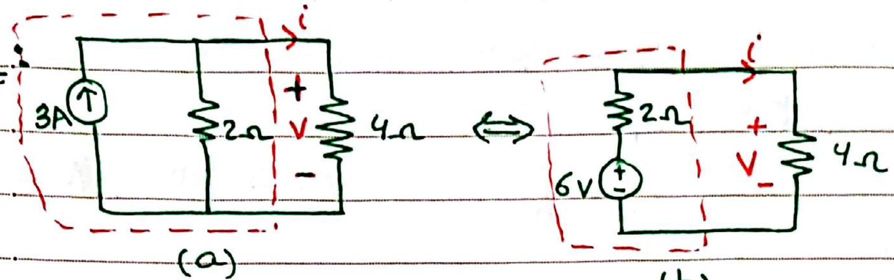
* **Ex:**



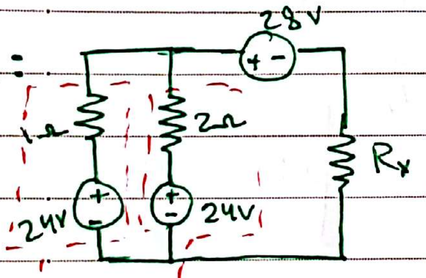
(a) ⟺ (b)

$i = \frac{3\,(2)}{6} = 1A$ (CDR) | $i = \frac{6}{6} = 1A$ (KVL)

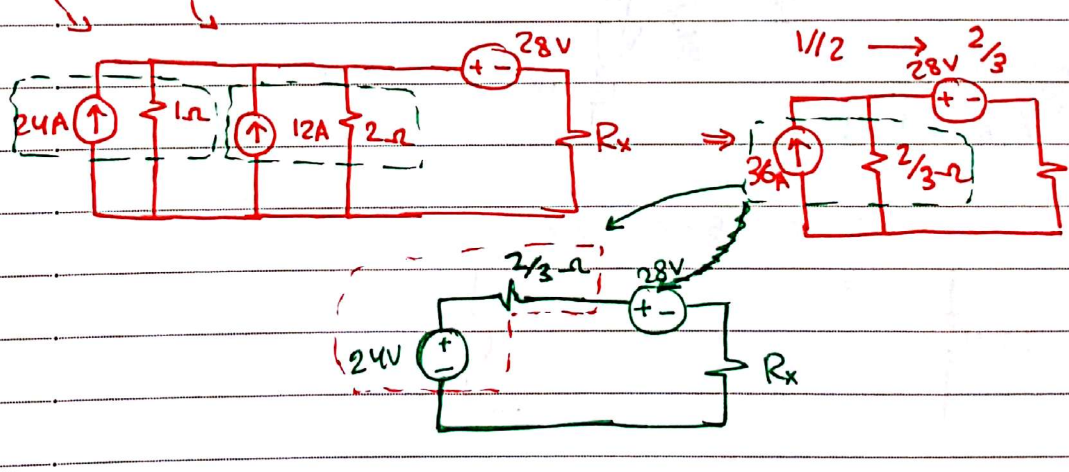
$V = 4V$ | $V = 4V$

$P = 4w$ | $P = 4w$

***Ex:**



Find R_x that absorb 5w ??
"using S.T"



KVL eq

$$-24 + \frac{2}{3}I + 28 + IR_x = 0$$

$$\boxed{\left(\frac{2}{3} + R_x\right) I = -4} \quad \text{--- (1)}$$

$$\boxed{I^2 R_x = 5\,w} \quad \text{--- (2)} \rightarrow \text{from } R_x \text{ absorb } 5w$$

from eqs. (1), (2) we find

$R_x = 1.587\,\Omega$

$R_x = 0.28\,\Omega$ ⇒ both are the solution, if one is negative ⇒ neglected

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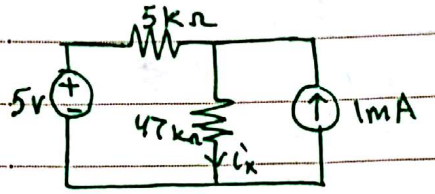
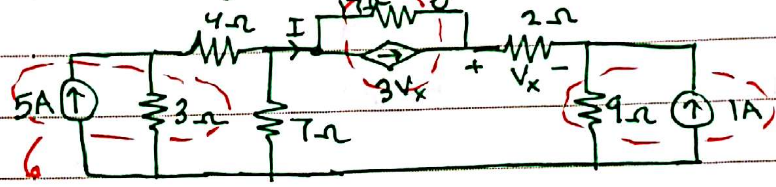
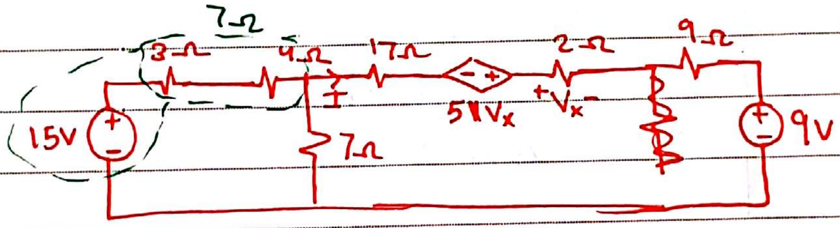
***Ex:** Find I_x using S.T

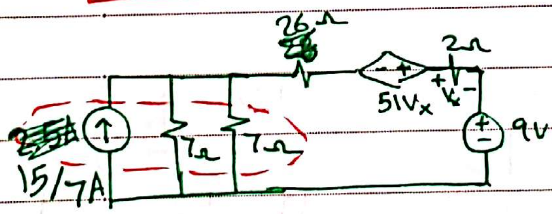


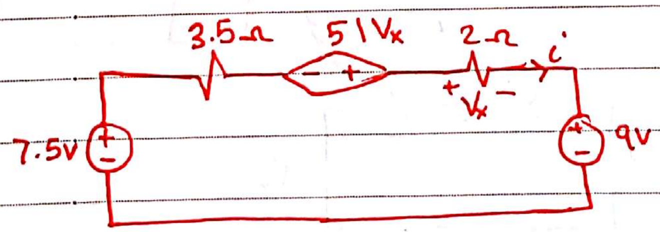
$i_x = 0.192\ mA$

Ex: Find I, V_x using S.T









KVL

$-7.5 + 3.5i - 51V_x + 2i + 9 = 0 \Rightarrow \boxed{5.5i - 51V_x = -1.5}$ --- ①

$\boxed{V_x = 2i}$ --- ②

$V_x = 42.56\ mV$

$i = 21.28\ mA$

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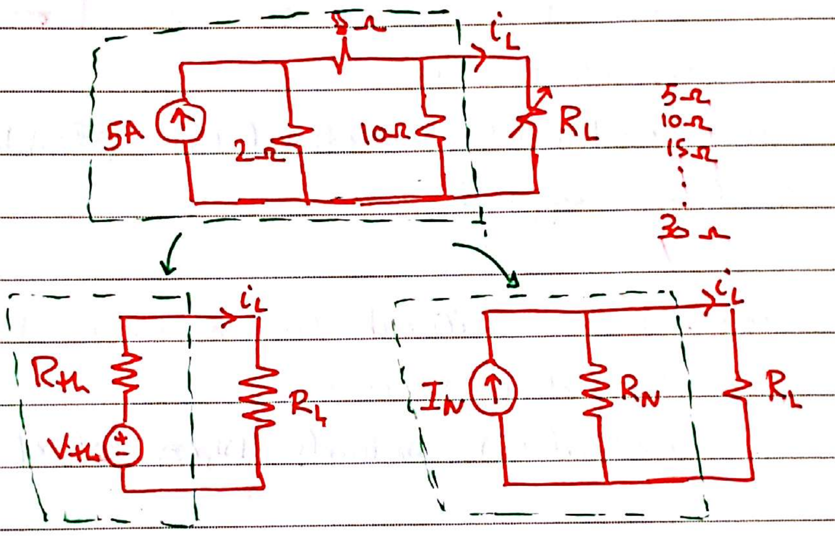
* Thevenin & Norton's equivalent:

» If a Variable load exists in the Circuit

» If you are intrested in a partial of the circuit

→ Applied for linear Circuits.



$R_{th} = R_N$

$V_{th} = I_N R_N$

$I_N = \dfrac{V_{th}}{R_{th}}$

* methods to find thevenin's or Norton's equivalent:

1) R_{th} & V_{th} or R_N & I_N

2) V_{oc} & I_{sc}

3) $V_{test} = 1V$ or $I_{test} = 1A$

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* When to use these methods??

Case 1: If a circuit contains only Resistors and V,I Sources.

Use method (1), but methods (2) & (3) are Possible.

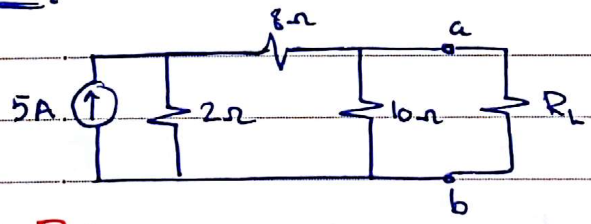
Case 2: If a circuit contains Resistors, V,I Sources & V,I dependent Sources.

Use method (2), method (3) possible, method (1) can't be applied.

Case 3: If a circuit contains only Resistors & V,I dependent sources.

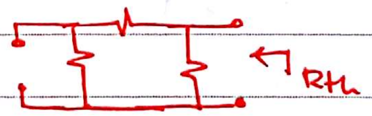
Use method (3), methods (1),(2) can't be applied.

***Ex:** Find the thevenin's



* R_{th}

Disconnect R_L, then kill the sources.



$R_{th} = 10 \,//\, (8+2)$

$R_{th} = 5\Omega$

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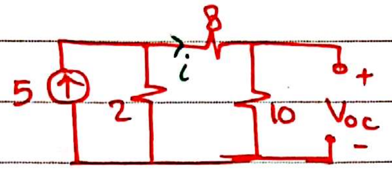
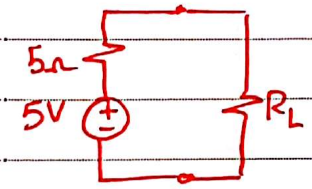
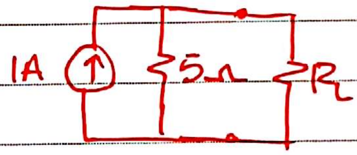
* V_{th} Disconnect R_L, then find V_{oc}.

$V_{th} = V_{oc}$



$i = 5 * \frac{2}{20} = \frac{1}{2} A$, $V_{oc} = 10 * \frac{1}{2} = 5V$





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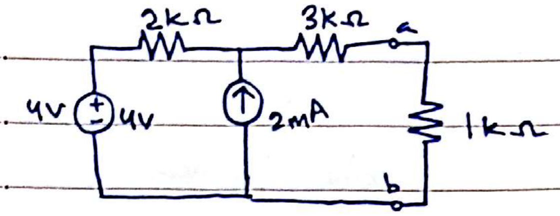
Ex:

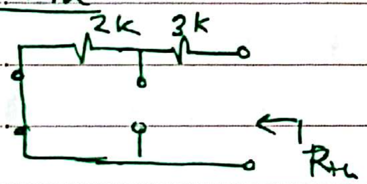


Find thevenin's (on method (1)) ~~Rload~~ between a & b

1) Remove R_L
2) kill the Sources to find R_{th}
3) Place O.C if thevenin's eq. (find V_{oc})
 " S.C if Norton's eq. (find i_{sc})

Solve by thevenin

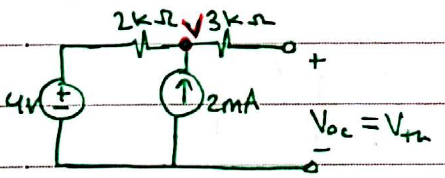
R_{th}



$R_{th} = 2k + 3k = 5k\Omega$

$\boxed{R_{th} = 5\ k\Omega}$

V_{th}



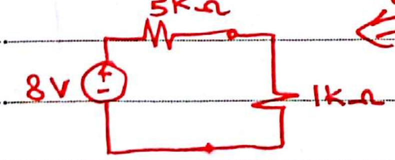
$V = V_{oc} = V_{th}$

*Solve by Nodal

$$\frac{V-4}{2k} = 2 \Rightarrow \boxed{V = 8V}$$

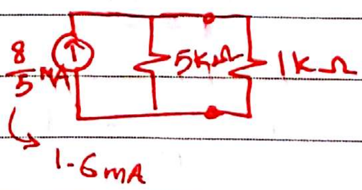
$\boxed{V_{th} = 8V}$

thevenin eq.



S.T

Norton's eq.



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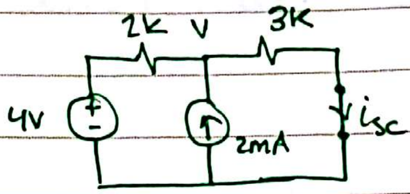
Resolve by Norton



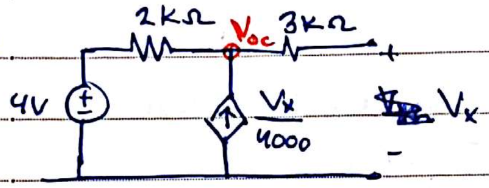
$$i_{sc} = \frac{V}{3k}$$

Solve by Nodal

$$\frac{V-4}{2} + \frac{V}{3} = 2 \Rightarrow V =$$

$$\boxed{i_{sc} = \frac{V}{3k} = 1.6\,mA}$$

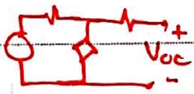
Ex:



find thevenin's eq. between the terminals.
(on method(2))

$$R_{th} = \frac{V_{oc}}{i_{sc}} = \frac{V_{th}}{i_N}$$

V_{oc}



$V_{oc} = V_x$

by Nodal

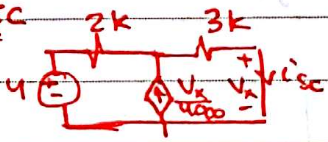
$$\frac{V_x - 4}{2000} = \frac{V_x}{\cancel{4000}\ 2}$$

$$2V_x - 8 = V_x$$

$$\boxed{V_x = 8\,V}$$

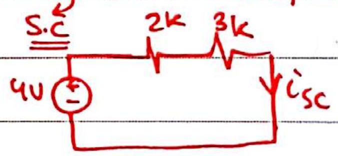
$$\boxed{V_{th} = V_{o.c} = V_x = 8V}$$

i_{sc}



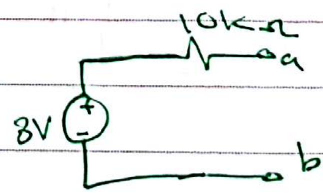
$V_x = 0$, so dependent i_s is o.c

S.C



$$\boxed{i_{sc} = \frac{4}{5} = 0.8\,mA}$$

$$\boxed{R_{th} = R_N = \frac{V_{th}}{i_N} = 10\,k\Omega}$$



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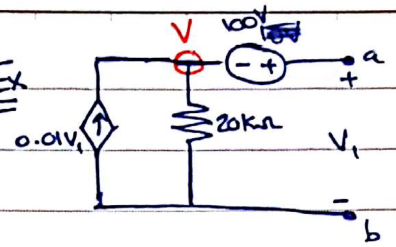
Ex

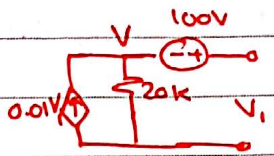


Find thevenin's eq. between a & b.

Voc, isc

Voc

$V_1 = V_{oc}$



~~Nodal~~

Nodal

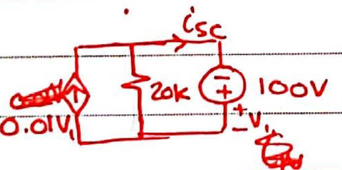
$\boxed{V = V_1 - 100}$ --- (1)

$\boxed{\frac{V}{20k} = 0.01V_1}$ --- (2)

V_{th}

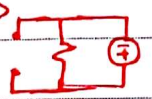
$\longrightarrow \boxed{V_1 = V_{oc} = \frac{-100}{199} V = -502.5\,mV}$

isc



S.C

$V_1 = 0 \Rightarrow \boxed{i_{sc} = \frac{100}{20k} = 5\,mA}$ $\leftarrow i_N$



$R_{th} = R_N = \frac{V_{th}}{i_N} = \frac{-502.5}{5} = -100.5\ \Omega$

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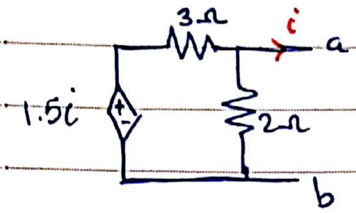
Ex on method (3)

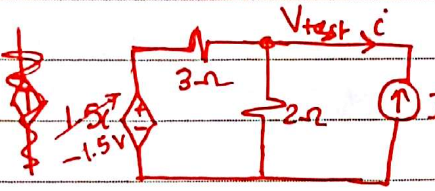


Find thevenin's eq.

① Assume ~~Vtest~~ $V_{test} = 1V$ or $i_{test} = 1A$

② find i_{test} or V_{test}

③ $R_{th} = \frac{V_{test}}{i_{test}}$

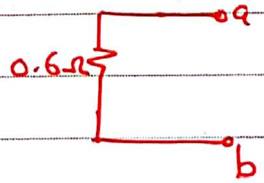


$i = -1A$, Vtest by Nodal

$$\frac{V-(-1.5)}{3} + \frac{V}{2} = 1 \Rightarrow \boxed{V_{test} = 0.6V}$$

$\boxed{R_{th} = 0.6\Omega} \rightarrow \frac{V_{test}}{i_{test}}$

thevenin eq.



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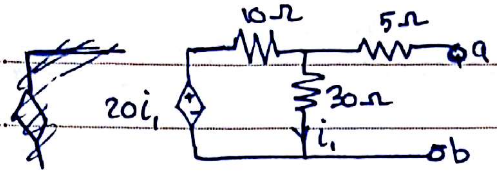
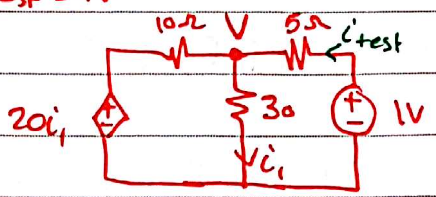
Ex: Find thevenin's eq.



assume $V_{test} = 1V$



by Nodal

$$\boxed{\frac{V-1}{5} + \frac{V-20i_1}{10} + \frac{V}{30} = 0} \quad \text{--- } (1)$$

$$\boxed{i_1 = \frac{V}{30}} \quad \text{--- } (2)$$

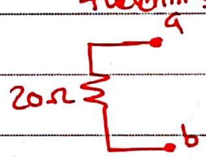
$V = 0.75V$

we find i_{test}

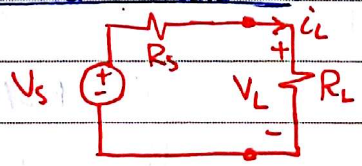
$$\Rightarrow i_{test} = \frac{1-V}{5} = 50mA$$

$$R_{th} = \frac{0.75}{50mA} = 20\,\Omega$$

thevenin's eq.



* Maximum Power Transfer:



$$P_L = V_L * i_L = i_L^2 * R_L \quad , \quad i_L = \frac{V_s}{R_s + R_L}$$

$$P_L = \frac{V_s^2}{(R_s+R_L)^2} * R_L$$

P_L max ?? $\Rightarrow \dfrac{dP_L}{dR_L} = 0$

$\cancel{\dfrac{dP_L}{dR_L} = \dfrac{V_s^2 R_L}{(R_s+R_L)^2}}$

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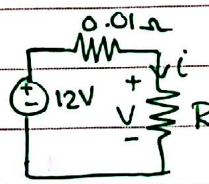
$$\frac{dP_L}{dR_L} = \frac{(R_S+R_L)^2 V_S^2 - V_S^2 R_L \left(2(R_S+R_L)\right)}{(R_S+R_L)^4} = 0$$

$$\Rightarrow (R_S+R_L)^2 \cancel{V_S^2} = \cancel{V_S^2}\, 2R_L(R_S+R_L)$$

$$R_S^2 + \cancel{2R_L R_S} + R_L^2 = \cancel{2R_L R_S} + 2R_L^2$$

* $R_S = R_L$ for max Power Transfer

* i.e



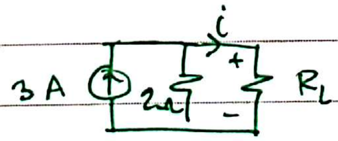
$R_L = 0.01$

$i = \frac{12}{2(0.01)} \Rightarrow i = 600$

$V = 6V$ (VDR)

$$P_{Lmax} = \frac{V_L^2}{R_L} = \frac{36}{0.01} = 3.6\,kW$$

* i.e



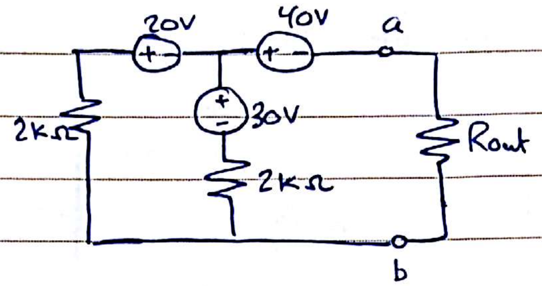
$i = 3 * \frac{2}{2+R_L}$ $R_L = 2\Omega$

$i = 1.5A$

$P_{Lmax} = i^2 R = 4.5\,kW$

15/6 , lec 10

Ex:



a) Find Pout if $R_{out} = 3k\Omega$

b) Find Rout that will absorb Maximum Power Transfer

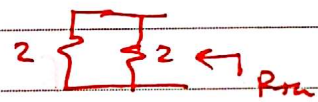
c) Find Rout if Pout = 20mW

we Solve by thevenin (Method 1)

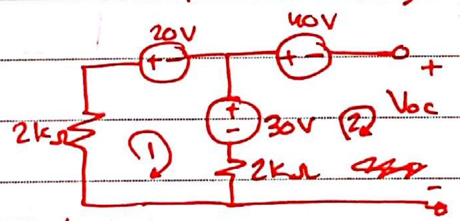
~~b) find Vth~~

R_{th} (Remove Rout, kill the Sources)

$R_{th} = 2 // 2 = 1k\Omega$



V_{th} (Remove Rout, find Voc)



KVL @ mesh 1

$+30 + 2i + 2i + 20 = 0$

$\boxed{i = -\frac{50}{4} mA}$

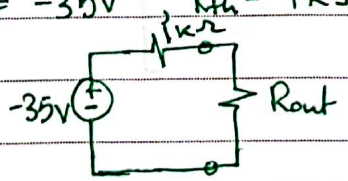
KVL @ mesh 2

$+40 + V_{oc} - 2i - 30 = 0$

$V_{oc} = -10 - \frac{100}{4}$

$\boxed{V_{oc} = -35V}$

$V_{th} = -35V \quad R_{th} = 1k\Omega$



a) $P_{out}\big|_{R_{out} = 3k\Omega} = i^2 R_{out} = \frac{V^2}{R_{out}} = \left(\frac{-35}{1+3}\right)^2 * 3 = \frac{\left(-35 * \frac{3}{1+3}\right)^2}{3}$

$\boxed{P_{out} = 239.69mW}$

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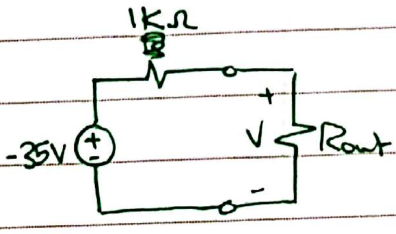
b) R_{out} (max Power) $= R_{th} = 1k\Omega$

c) $R_{out} = \frac{V^2}{P_{out}}$

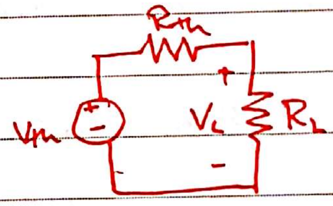


$$V = -35 * \frac{R_{out}}{R_{out}+1}$$

$$R_{out} = \frac{(-35)^2 \left(\frac{R_{out}}{R_{out}+1}\right)^2}{20m} \Rightarrow \boxed{\begin{array}{l} R_{out} = 59.2k\Omega \\ R_{out} = 16.88\ k\Omega \end{array}}$$

$$\frac{R_{out}(R_{out}+1)^2}{(R_{out})^2} = \frac{(-35)^2}{20m}$$

* $P_{max} = \frac{V_L^2}{R_L}$, $R_L = R_{th}$



$V_L = V_{th} * \frac{R_L}{R_L+R_{th}}$, $P_{Lmax} = \frac{\left(V_{th} * \frac{R_L}{R_L+R_{th}}\right)^2}{R_L}$

$$P_{Lmax} = \frac{V_{th}^2\ \cancel{R_L^2}}{4\cancel{R_L^2}\ R_L}$$

$$\boxed{P_{Lmax} = \frac{V_{th}^2}{4R_{th}}}$$

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Ch6 : Capacitors & Inductors:

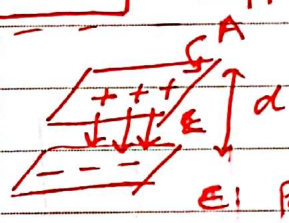
* They are storing elements.

* Capacitors:

$v \propto \int i\,dt \Rightarrow \boxed{V = \frac{1}{C}\int_t i\,dt}$

C: capacitor

$-||-$



EM

ε: Permittivity.

$\varepsilon = \varepsilon_o \varepsilon_r$

ε_o : Permitivity for Air

ε_r : Relative Permittivity.

$C = \frac{\varepsilon A}{d}$ [F]

1F is a very big capacitor.

C → mF, μF, nF, PF

$V = \frac{1}{C}\int i\,dt$

$dV = \frac{1}{C}\, i\,dt$

$\boxed{i = \frac{C\,dv}{dt}}$, $i = \frac{dq}{dt}$

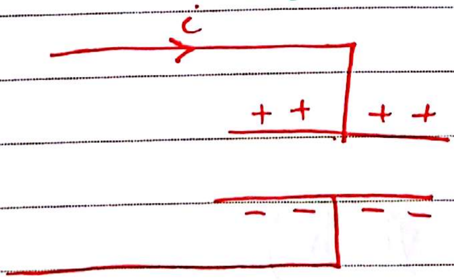
$\frac{dq}{dt} = \frac{C\,dv}{dt}$.

$\boxed{C = \frac{Q}{V}}$

Unit is Farad [F] or $\left[\frac{C}{V}\right]$ (C → colomb, V → Volt)

Symbol: $\xrightarrow{i_c} -||-$ with $+\ V_c\ -$, $\xrightarrow{i} -|(-$ with $+\ -$

In Circuit lab



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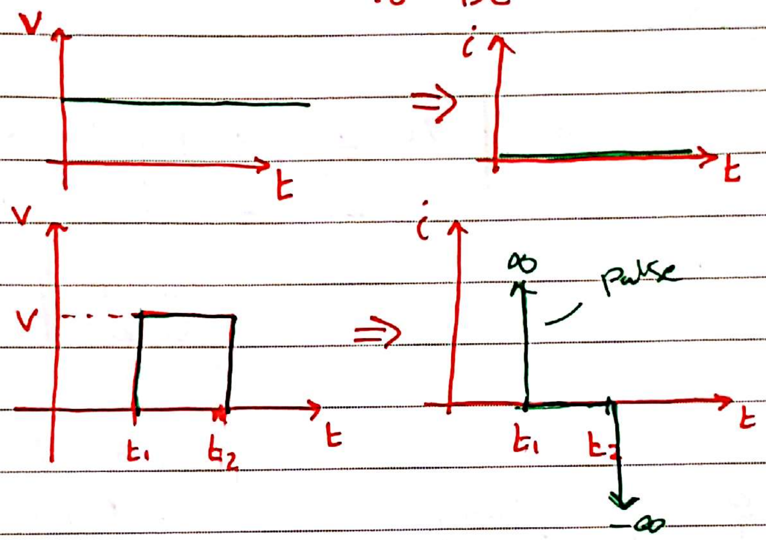
$$i = C\frac{dv}{dt}$$

if V = constant or $V = 0$

i = zero (open circuit to DC)



* Energy Stored in a Capacitor :

$w_c(t)$ or w_c

* $W_c(t)$ = Power * time.

$$P_c(t) = V \cdot i = V\, C\frac{dv}{dt}$$

$$W_c(t) = \int_{-\infty}^{t_o} P_c(t)\,dt$$

$$\boxed{W_c = \frac{1}{2} C V^2}$$

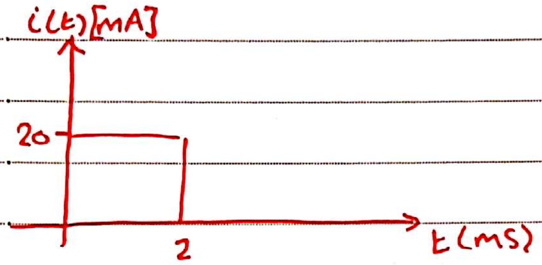
$$W_c(t) = \int_{-\infty}^{t_o} VC\frac{dv}{\cancel{dt}}\cancel{dt}$$

$$W_c(t) = \frac{1}{2} C V^2_{(t)} \Big|_{-\infty}^{t_o}$$

$$= \frac{1}{2} C \left(V^2_{(t_o)} - \underbrace{V^2_{(-\infty)}}_{\text{zero}}\right)$$

$$\boxed{W_c(t) = \frac{1}{2} C V^2_{(t_o)}}$$

***Ex:** Find $V_c(t)$ & $W_c(t)$ if $C = 5\mu f$ & $i(t)$ is shown



$$V_c(t) = \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt$$

***Vc(t)**

for $t \leq 0$

$$V_c(t) = \frac{1}{C}\int_{-\infty}^{0} i(t)\,dt = \text{zero}$$

for $0 < t < 2$

$$V_c(t) = \frac{1}{C}\Big(\underbrace{\cancel{\int_{-\infty}^{0}}}_{\text{zero}} + \int_{0}^{t} i(t)\,dt\Big)$$

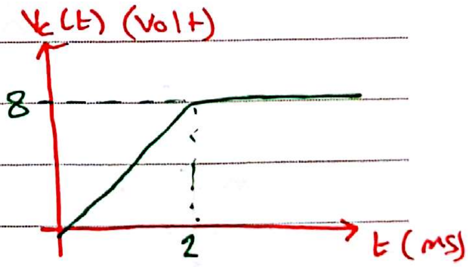
$$= \frac{1}{5\mu}\int_{0}^{t} 20m\,dt \Rightarrow V_c(t) = 4000t,\ 0<t<2ms$$

for $2 \leq t$

$$V_c(t) = \frac{1}{C}\Big(\int_{-\infty}^{0} + \int_{0}^{2} + \int_{2}^{\infty} i(t)\,dt\Big)$$

$$V_c(t) = 8\text{ Volt}, \quad 2 \leq t$$

$$V_c(t) = \begin{cases} \text{zero}, & t \leq 0 \\ 4000t, & 0 < t < 2ms \\ 8, & 2ms \leq t \end{cases}$$



* $W_c(t)$

$$W_c(t) = \frac{1}{2} C V_c^2(t)$$

$$\cancel{= \frac{1}{2} C (}$$

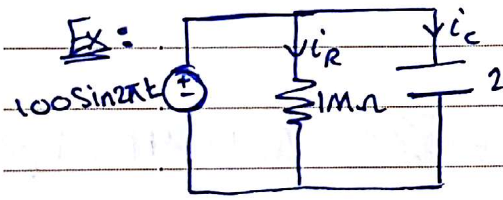
$$W_c(t) = \begin{cases} \text{zero}, & t \le 0 \\ 40t^2, & 0 < t < 2ms \\ \cancel{64 \ldots} \\ 160\mu, & 2ms < t \end{cases} \quad [\text{Joule}]$$

$\frac{1}{2} C (\text{zero})^2$

$\frac{1}{2} C \ (4000\, t)^2$

$\frac{1}{2} C \ (8)^2$

Ex:



Find $W_c(t)_{max}$, $W_R(t)$

In $0 < t \le 0.5S$

$$W_c(t) = \frac{1}{2} C V_c^2(t)$$

$$= \frac{1}{\cancel{2}} * \overset{10}{\cancel{20}} * 10^{-6} * 10^4 * \sin^2 2\pi t$$

$$= 0.1 \sin^2 2\pi t \ \text{Joule}$$

max @ $\sin^2 2\pi t = 1$

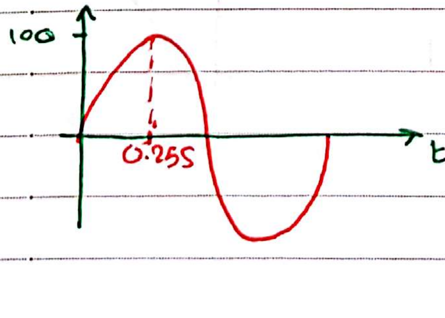
this is max value for Sin

$$\boxed{W_c(t)_{max} = 0.1\, J = 100\, mJ}$$

$\sin 2\pi t = 1$

$2\pi t = \frac{\pi}{2}$

$$\boxed{t = \cancel{\#}\ 1/4\ S}$$



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$W_R(t) = P_R\, t$ | $P_R(t) = \frac{V^2(t)}{R} = \frac{10^4 \sin^2(2\pi t)}{10^6}$

$P_R(t) = 10^{-2} \sin^2 2\pi t$ Watt

$W_R(t) = \int P_R(t)\,dt$

$$= \int_0^{0.5} P_R(t)\,dt = \int_0^{0.5} 10^{-2} \left(\tfrac{1}{2}(1 - \cos 4\pi t)\right) dt$$

$$= \frac{10^{-2}}{2}\left(t - \underbrace{\frac{\sin 4\pi t}{4\pi}}_{\text{zero}} \Big|_0^{0.5} \right)$$

$$= \frac{10^{-2}}{2}\left(\frac{1}{2} - 0\right)$$

$$= \frac{10^{-2}}{4} = \frac{1}{400}\,J = 2.5\,mJ$$ ✓ loss in the charging capacitor

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* The Inductor:

$V \propto \frac{di}{dt}$

$\boxed{V = L\frac{di}{dt}}$, $i = C\frac{dv}{dt}$

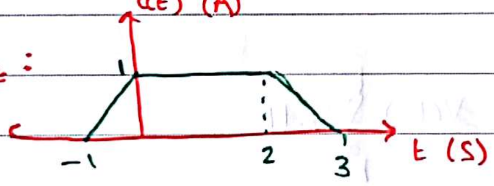
L: Inductance (H).

if i is Constant or Zero

$\Rightarrow V = 0$ (S.C)

Inductor is Short Circuit to DC

*Ex :



if L = 3H , find $V_L(t)$??

$V_L(t) = L\frac{di}{dt}$

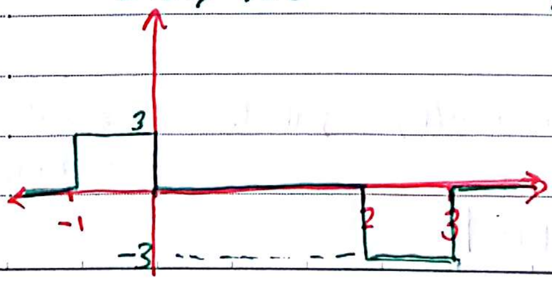
$t < -1 \Rightarrow i = 0$, $V = 0$

$-1 < t < 0 \Rightarrow V = 3\frac{(1-0)}{(0--1)} = 3$

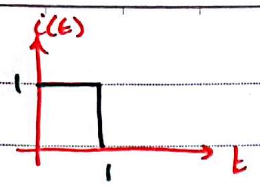
$0 < t < 2 \Rightarrow V = 0$

$2 < t < 3 \Rightarrow V = 3\frac{(0-1)}{3-2}$, $V = -3$

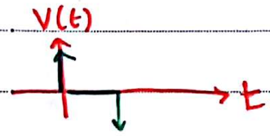
$3 < t \Rightarrow V = 0$



Ex :



find $V_L(t)$??



$$V = L\frac{di}{dt}$$

$$\int_{t_0}^{t} V\,dt = \int_{i(t_0)}^{i(t)} L\,di \qquad \text{at} \quad t = t_0 \Rightarrow i = i(t_0)$$

$$L\left(i(t) - i(t_0)\right) = \int_{t_0}^{t} V\,dt$$

$$\boxed{i(t) = \frac{1}{L}\int_{t_0}^{t} V\,dt + i(t_0)} \quad ; \text{ if } t_0 \to \infty \Rightarrow i(t_0) = 0$$

* Energy Stored

$$W_L(t) = P_L * t$$

$$P_L = V * i = i\left(L\frac{di}{dt}\right) \qquad \boxed{W_L(t) = \frac{1}{2}L\,i(t)^2}$$

$$W_L(t) = \int_{t} P_L(t)\,dt$$

$$= \int_{t_0}^{t} i\,L\frac{di}{dt}\,dt + w(t_0) \quad ; \text{ if } t_0 \to \infty \Rightarrow w(t_0) = 0$$

$$\Rightarrow i(t_0) = 0$$

$$= \frac{1}{2}L\,i^2 \Big|_{t_0}^{t}$$

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Ex: if $V_L(t) = 6\cos 5t$ V, $L = 2H$ & $i(t=-\frac{\pi}{2}) = 1A$
find $i_L(t)$??

$$i_L(t) = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0)$$

$$i_L(t) = \left(\frac{1}{2}\int_{-\frac{\pi}{2}}^{t} 6\cos 5t\,dt\right) + 1$$

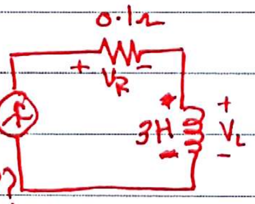
$$= \frac{1}{2}\left(\frac{6}{5}\sin 5t \Big|_{-\frac{\pi}{2}}^{t}\right) + 1$$

$$= \frac{6}{10}(\sin 5t + 1) + 1$$

$$i_L(t) = 0.6\sin 5t + 1.6 \quad A$$

$$W_L(t) = \cancel{\frac{1}{2}}\cancel{(2)}\,(0.6\sin 5t + 1.6)^2$$

Ex: Find $W_L(t)_{max}$ & $W_R(t)$
$W_R(t)$ during charging & Discharging of the inductor??

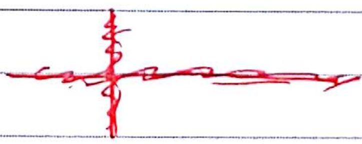


$$W_L(t) = \frac{1}{2}L\,i^2(t)$$

$$= \frac{1}{2}(3)\left(12\sin\frac{\pi}{6}t\right)^2$$

$$= 216\sin^2\frac{\pi}{6}t \rightarrow \text{for max } \frac{\pi}{6}t = \frac{\pi}{2} \Rightarrow \boxed{t = 3s}$$

* $\boxed{W_L(t)_{max} = 216\,J \text{ at } t = 3s}$



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$W_R(t)$ during charging & Discharging of the Inductor

~~$W_R(t)=$~~ $0 < t < 6 \rightsquigarrow$ 3S Charge, 3S Discharge.

$W_R(t) = \int_0^6 P_R(t)\,dt$

$P_R(t) = V \cdot i = \frac{V^2}{R} = i^2 . R$

$= 14.4 \sin^2 \frac{\pi}{6} t$ w

$W_R(t) = \int_0^6 14.4 \sin^2 \pi/_6 t\,dt$

$= 14.4 \int_0^6 \frac{1}{2}(1 - \cos \pi/_3 t)\,dt$

$= 7.2\,(6 - \underbrace{\cancel{\cos \pi/_3 t \big|_0^6}}_{\text{zero}})$

$\boxed{W_R(t) = 43.2 \text{ Joule}} \rightarrow$ losses

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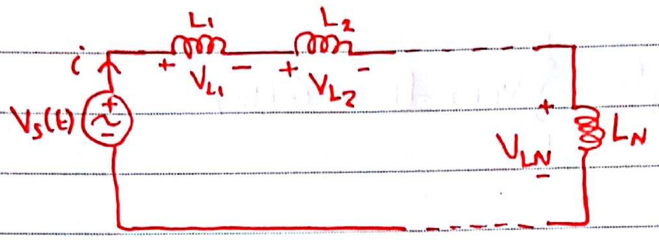
* Inductance & Capacitance Combinations :

→ Series Inductance :



KVL

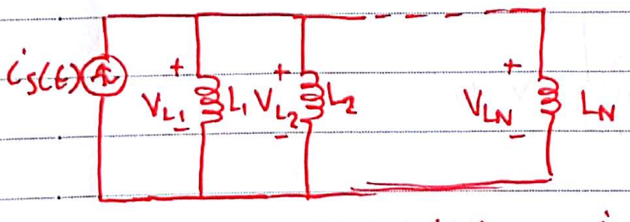
$$\sum_{k=1}^{N} V_k \Rightarrow -V_s + V_{L_1} + V_{L_2} + \dots + V_{L_N} = 0$$

$$0 = -V_s + L_1\frac{di}{dt} + L_2\frac{di}{dt} + \dots + L_N\frac{di}{dt}$$

$$V_s = (L_1 + L_2 + \dots + L_N)\frac{di}{dt}$$

$\boxed{L_{eq} = L_1 + L_2 + \dots + L_N}$ in Series

→ Parallel Inductance :



KCL $\Rightarrow i_s(t) = i_1 + i_2 + \dots + i_N$

$$i_s(t) = \frac{1}{L_1}\int_{t_0}^{t} V(t)\,dt + i(t_0)$$

$$+ \frac{1}{L_2}\int_{t_0}^{t} V(t)\,dt + i(t_0)$$

$$\dots + \frac{1}{L_N}\int_{t_0}^{t} V(t)\,dt + i(t_0)$$

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$$i_s(t) = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \dots \frac{1}{L_N}\right) \int_{t_0}^{t} v(t)\,dt + i_s(t_0)$$

$$i_s(t) = \frac{1}{L_{eq}} \int_{t_0}^{t} v(t)\,dt + i_s(t_0)$$

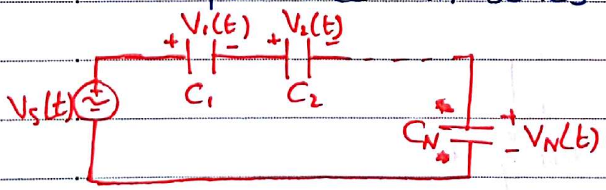
$$\cancel{L_{eq}}\ \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_N}$$

$$\boxed{L_{eq} = \frac{1}{1/L_1 + 1/L_2 + \dots 1/L_N}}$$ In parallel

for 2 inductances

$$\boxed{L_{eq} = \frac{L_1 \cdot L_2}{L_1 + L_2}}$$

→ Capacitance In Series



<u>KVL</u>

$$V_s(t) = V_1(t) + V_2(t) + \dots + V_N(t)$$

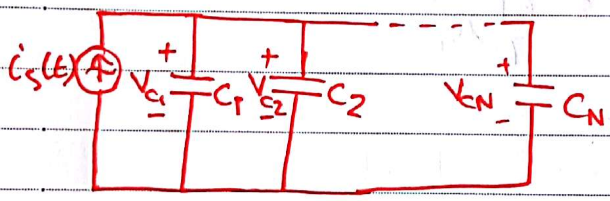
$$V_s(t) = \frac{1}{C_1}\int i(t)\,dt + \cancel{V(t_0)}\ V(t_0)$$
$$+ \frac{1}{C_2}\int i(t)\,dt + V(t_0)$$
$$\vdots$$
$$+ \frac{1}{C_N}\int i(t)\,dt + V(t_0)$$

$$V_s(t) = \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots \frac{1}{C_N}\right) \int i(t)\,dt + i_s(t_0)$$

$$V_s(t) = \frac{1}{C_{eq}} \int i(t)\,dt + i_s(t_0)$$

$$\boxed{C_{eq} = \frac{1}{1/C_1 + 1/C_2 + \cdots + 1/C_N}}$$ In Series

→ Capacitance In Parallel



KCL ⇒ $i_s(t) = i_1 + i_2 + \cdots i_N$

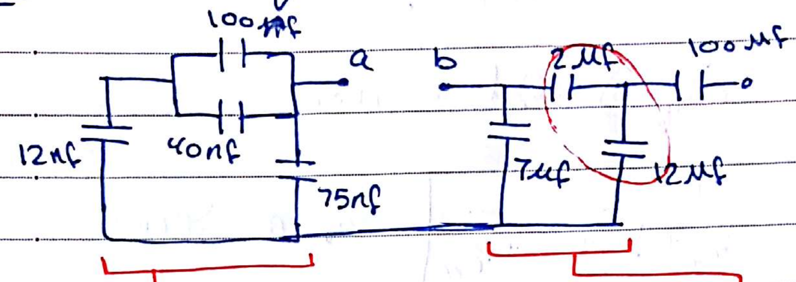
$$i_s(t) = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + \cdots C_N\frac{dv}{dt}$$

$$i_s(t) = (C_1 + C_2 + \cdots + C_N)\frac{dv}{dt}$$

$$i_s(t) = C_{eq}\frac{dv}{dt}$$

$$\boxed{C_{eq} = C_1 + C_2 + \cdots + C_N}$$ In Parallel

Ex: Find C_{eq}. between a & b ??



$*$ $2\mu f + 12\mu f \longrightarrow \frac{2.12}{14} = \frac{12}{7}$

$7 \,//\, \frac{12}{7} \longrightarrow \frac{61}{7}\mu f$

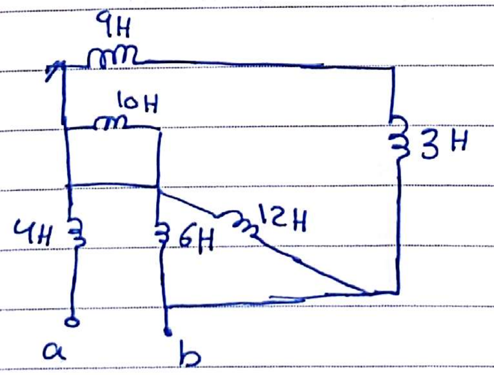
$100 \,//\, 40 \rightarrow 140 nf$

$140 + 12 \rightarrow \frac{140.12}{152} = 11 nf$

$11 \,//\, 75 \rightarrow 86 nf$

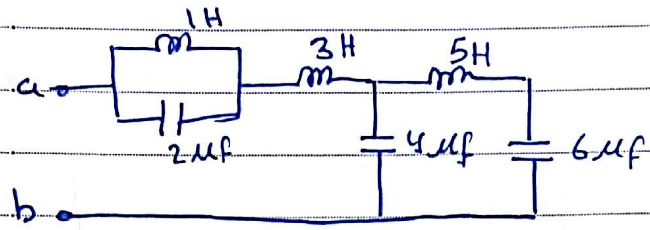
$\frac{61}{7}\mu + 86n \longrightarrow \dfrac{\frac{61}{7} * 86\, \mu.n}{\frac{61}{7}\mu + 86n} \Rightarrow \boxed{C_{eq} = 35.2 nf}$

Ex:



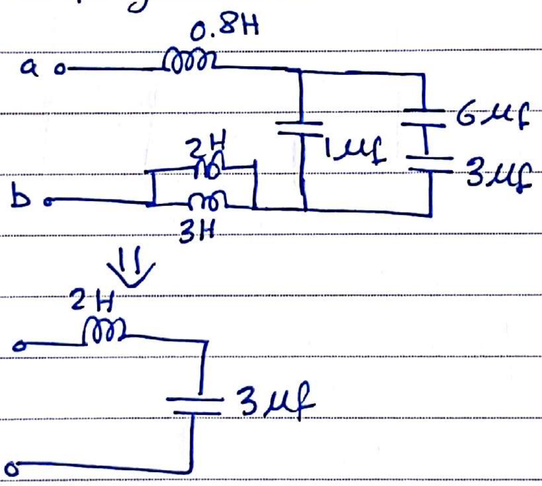
$L_{eq} = 7H$

*Simplify :



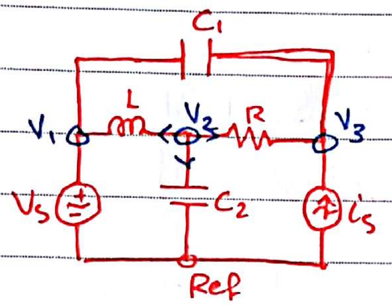
⇒ This Can not be Simplified.

* Ex : Simplify



* Consequences of Linearity:

Ex: write the Nodal equations:



$i_L(t_0)$, $V_{c_1}(t_0)$, $V_{c_2}(t_0)$ $\Rightarrow$ Should be give

$\boxed{V_1 = V_s}$

KCL @ Node 2

$$\frac{V_2 - V_3}{R} + \left(\frac{1}{L}\int_{t_0}^{t}(V_2 - V_1)\,dt + i_L(t_0)\right) + C_2\frac{dV_2}{dt} = 0$$

KCL @ Node 3

$$-i_s + \frac{V_3 - V_2}{R} + C_1\frac{d(V_3 - V_1)}{dt}$$

Linear Integro differential equations

$V_L = L\frac{di}{dt}$

So linear

$KV_L = KL\frac{di}{dt}$

$i = \frac{1}{L}\int v(t)\,dt + i(t_0)$

So linear

$ki = \frac{k}{L}\int v(t)\,dt + ki(t_0)$

Simplify

$$\boxed{-\frac{1}{L}\int_{t_0}^{t} V_1\,dt + \frac{V_2}{R} + \frac{1}{L}\int_{t_0}^{t} V_2\,dt + C_2\frac{dV_2}{dt} - \frac{V_3}{R} = -i_L(t_0)} \quad (1)$$

$$\boxed{-C_1\frac{dV_1}{dt} + \frac{-V_2}{R} + \frac{V_3}{R} + C_1\frac{dV_3}{dt} = i_s} \quad (2)$$

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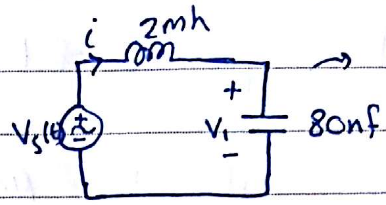
***Ex:** If $V_c(t) = 4\cos 10^5 t$ V
find $V_s(t)$??



→ LC ckt
lossless ckt

$$i = C\frac{dV_c(t)}{dt}$$

$$= 80\times10^{-9}\left(-4\times10^5 \sin 10^5 t\right)$$

$$\boxed{i = -32 \sin 10^5 t \text{ mA}}$$

$$V_L = L\frac{di}{dt}$$

$$= 2\times10^{-3}\left(-32\times10^5 \cos 10^5 t \times 10^{-3}\right)$$

$$\boxed{V_L(t) = -6.4 \cos 10^5 t \text{ V}}$$

KVL :

$$-V_s(t) + V_L(t) + V_c(t) = 0$$

$$V_s(t) = V_L(t) + V_c(t)$$

$$\boxed{V_s(t) = -2.4 \cos(10^5 t) \text{ V}}$$

***Duality:**

two Circuits are dual if Nodal eqs for one circuit is Similar to Mesh eqs. of the other circuit.

Dual

$V \leftrightarrow I$

$R \leftrightarrow \frac{1}{R}$

$L \leftrightarrow C$

Series $\leftrightarrow$ Parallel

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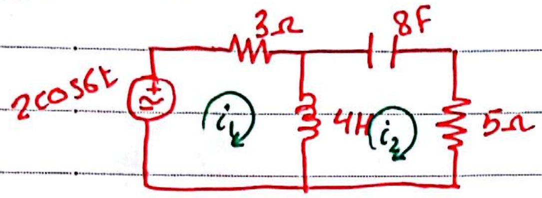
Ex: write the Mesh eq.s



$V_C(t_0) = 10V$

$i_L(t_0) = 0A$

(1) $-2\cos 6t + 3i_1 + \frac{4d(i_1 - i_2)}{dt} = 0$

(2) $\frac{4d(i_2 - i_1)}{dt} + \frac{1}{C}\int_{t_0}^{t} i_2(t)dt + V_C(t_0) + 5i_2 = 0$

$$\boxed{3i_1 + \frac{4di_1}{dt} - \frac{4di_2}{dt} = 2\cos 6t} \text{ — } ①$$

$$\boxed{-\frac{4di_1}{dt} + 5i_2 + \frac{4di_2}{dt} + \frac{1}{8}\int_{t_0}^{t} i_2(t)dt = -10} \text{ — } ②$$

Find the Dual Circuit

$$\boxed{3V_1 + \frac{4dV_1}{dt} - \frac{4dV_2}{dt} = 2\cos 6t} \text{ — } ①$$

$$\boxed{-\frac{4dV_1}{dt} + 5V_2 + \frac{4dV_2}{dt} + \frac{1}{8}\int_{t_0}^{t} V_2(t)dt = -10} \text{ — } ②$$

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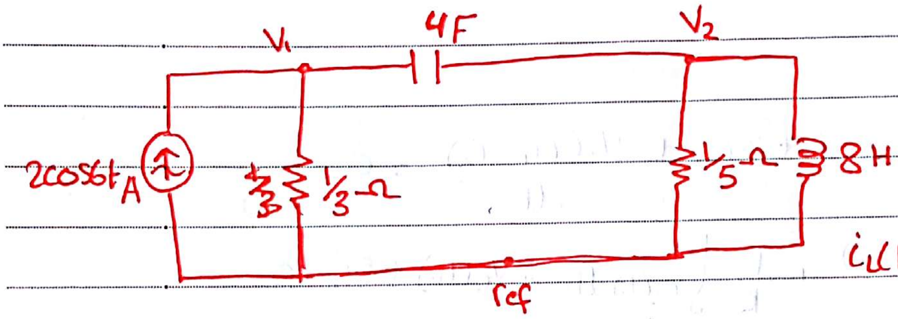
Draw the Dual Circuit
two methods
method (1) : from eqs.



$i_L(t_0) = 10A$

$\Omega \leftrightarrow \mho$
$L \leftrightarrow C$
$V \leftrightarrow I$
Series ↔ Parallel

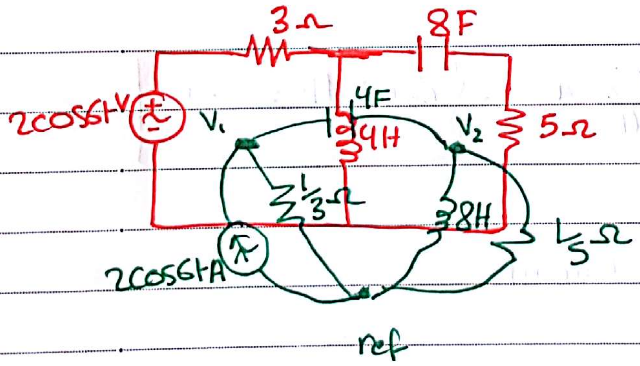
$\frac{1}{L}\int v\,dt$

$C\frac{dv}{dt}$

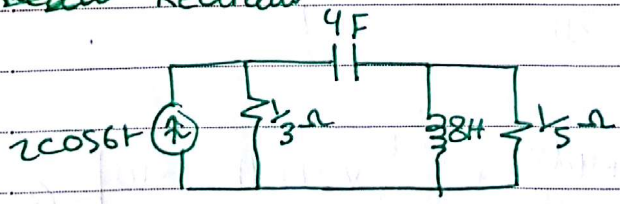
$L\frac{di}{dt}$

$\frac{1}{C}\int i\,dt$

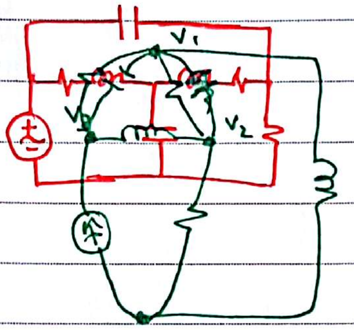
method(2): from circuit

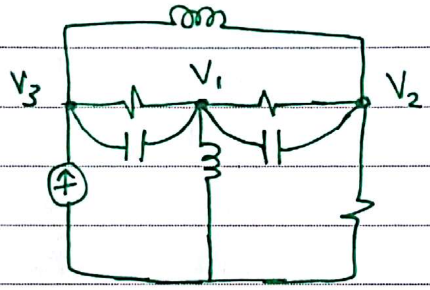


~~Redw~~ Redraw

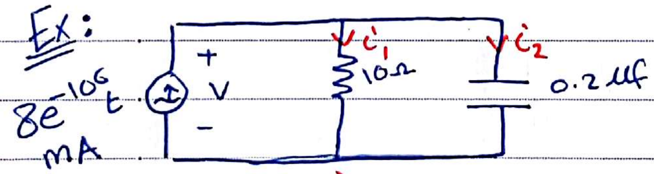


Ex: Draw the Dual Circuits

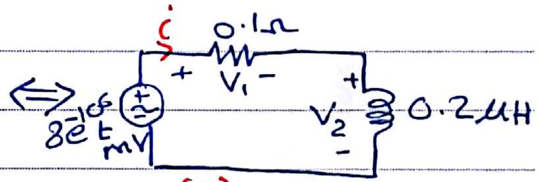




Ex:



(1)



(2)

Find V_1, V_2, and i if 2 circuits are dual ??
and $V = -80e^{-10^6 t}$ mV

$i_1\big|_{(2)} = V_1\big|_{(1)} \Rightarrow i_1\big|_{(1)} = V_1\big|_{(2)}$, $i_2\big|_{(1)} = V_2\big|_{(2)}$

$i = -80e^{-10^6 t}$ mA

$i_1 = -8e^{-10^6 t}$ mA

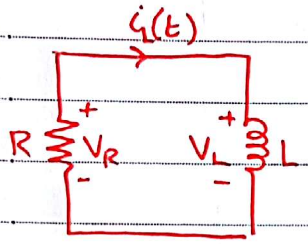
$i_2 = 16e^{-10^6 t}$ mA

$V_1 = -8e^{-10^6 t}$ mV

$V_2 = 16e^{-10^6 t}$ mV

CH.7: Basic RL & RC Circuits

- RL Circuits
  - Series
    - Source free
    - Driven
  - Parallel
    - Source free
    - Driven



$i_L(t_0) = I_0$

KVL:

$-V_R + V_L = 0$

$+iR + L\frac{di}{dt} = 0 \rightarrow$ linear homogenuos first order Differential equation

$$\frac{di(t)}{dt} + \frac{R}{L} i(t) = 0$$

$$\int_{I_0}^{i(t)} \frac{di(t)}{i(t)} = \int_{t_0}^{t} -\frac{R}{L}\, dt$$

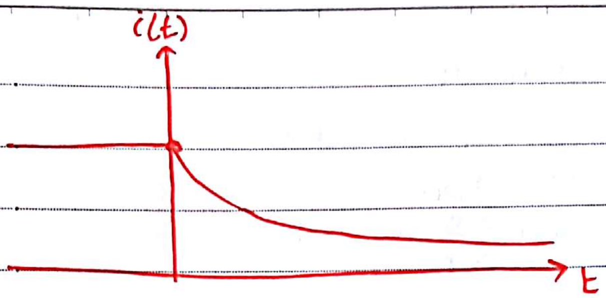
$$\ln\left(\frac{i(t)}{I_0}\right) = -\frac{R}{L}(t - t_0)$$

$$\boxed{i(t) = I_0 e^{-\frac{R}{L}(t-t_0)}}$$

Decaying exponential

If $t_0 = 0$

$$\boxed{i(t) = I_0 e^{-\frac{R}{L}t}}$$



discharging of the inductor (Dissipated by the series resistance).

* Power Dissipated:

$$P_R(t) = i^2(t)\,R$$
$$= I_o^2\, e^{-\frac{2R}{L}t} \cdot R \quad \text{watt}$$

$$w_R(t) = \int_0^\infty P_R(t)\,dt \qquad (t=0)$$

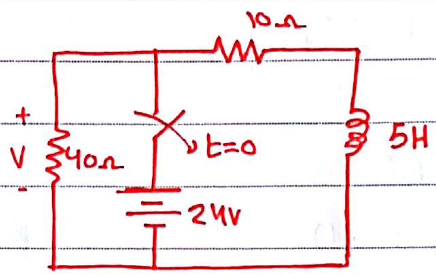
$$= \int_0^\infty I_o^2 R\, e^{-\frac{2R}{L}t}\,dt$$

$$= I_o^2 R \left(\frac{-L}{2R}\, e^{-\frac{2R}{L}t}\Big|_0^\infty\right)$$

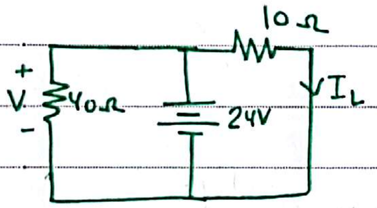
$$= \frac{-I_o^2 L}{2}\,(0-1)$$

$$\boxed{w_R(t) = \frac{1}{2} L I_o^2 = -w_L(t)}$$

*Ex: Find the current through the 5H at t = 200ms



① for $t<0$, $t=0^-$



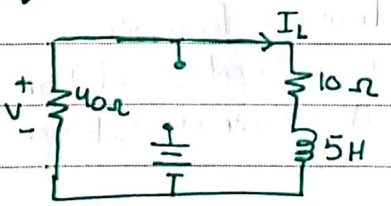
$$i_L(0^-) = \frac{24}{10} = 2.4A$$

$$i_L(0^+) = i_L(0^-) = i_L(0) = 2.4A$$

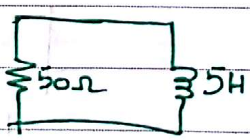
* Because current is not allowed to change suddenly across the Inductor.

$$\boxed{i_L(0) = 2.4A}$$

② for $t>0$



⇒



Series RL Circuit

$$i_L(t) = I_0 Re^{-\frac{R}{L}t}$$

$$I_0 = i_L(0) = 2.4A$$

$$\boxed{i_L(t) = 2.4\, e^{-10t}\ A} \longrightarrow i_L(200ms) = 2.4e^{-10(200m)}$$

$$\boxed{i_L(200ms) = 2.4e^{-2}\ A}$$

$$V = -40I = -40\,(2.4)e^{-10t}\ V$$

$$\boxed{V = -96\, e^{-10t}\ V}$$

$$\boxed{V = -96\, e^{-2}\ V}$$

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$P_R(t) = I_L^2 R$

Req

$= (2.4)^2 e^{-20t} (50)$

$$\boxed{P_R(t) = 288\, e^{-20t} \text{ watt}}$$

$W_L(t) = \frac{1}{2} L I_0^2$

$= \frac{1}{2} (5) (2.4)^2$

$$\boxed{W_L(t) = 14.4 J}$$

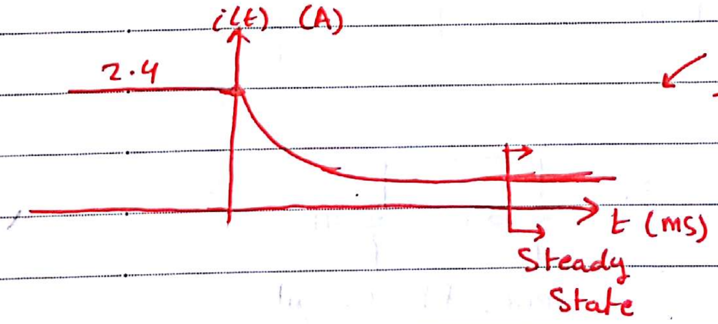
$W_R(t) = \int_0^\infty P_R(t)\, dt$

Switch

$= \int_0^\infty 288 e^{-20t} dt = \frac{288}{-20} e^{-20t} \Big|_0^\infty$

$$\boxed{W_R(t) = -14.4 J}$$

$$\boxed{W_L(t) = -W_R(t)}$$



Natural Response.
Transient Response.
Complementary Solution
homogeneous Solution

Define $\tau \equiv$ time Constant

$$i(t) = I_o e^{-\frac{R}{L}t} = I_o e^{-t/\tau}$$

$\boxed{\tau = \frac{L}{R}}$ for RL Circuits in Seconds.

at $t = \tau$ $\qquad\qquad \tau = \frac{5}{50} = 0.1$s

$i(\tau) = I_o e^{-t/\tau} = I_o e^{-1}$, $(e^{-1} = 0.368)$

$\boxed{i(\tau) = 0.368\, I_o}$

at $t = \tau \rightarrow$ current drop from I_o to $0.368 I_o$

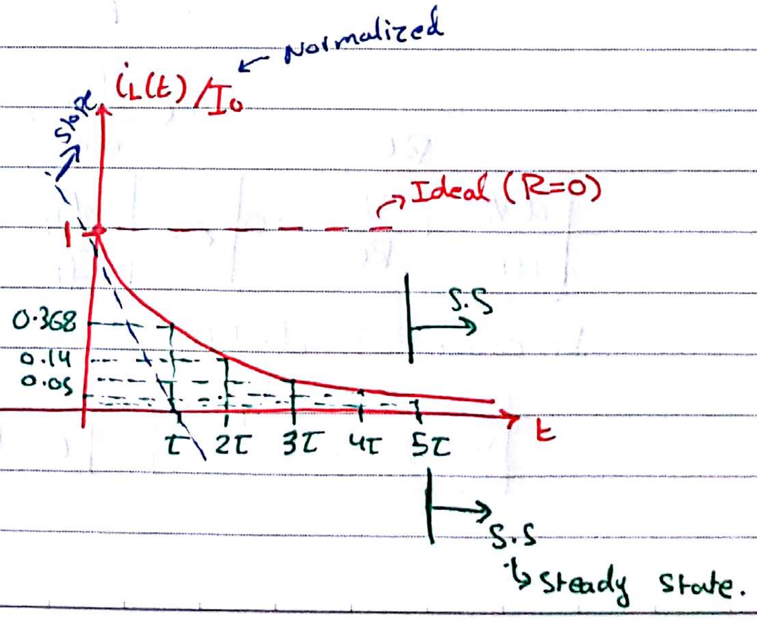
Reduction by 36.8%

$e^{-1} \approx 36.8\%$
$e^{-2} \approx 14\%$
$e^{-3} \approx 5\%$
$e^{-4} \approx 1.8\%$
$e^{-5} \approx 0.67\%$

Steady State begins after (5τ) because the current drops to less than 1% from it's initial condition.

$i_L(t) = I_o e^{-t/\tau}$

$\frac{i_L(t)}{I_o} = e^{-t/\tau}$



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$$d\left(\frac{i_L(t)}{I_0}\right) = e^{-t/\tau}\, dt$$

$$= -\frac{1}{\tau}\, e^{-t/\tau}$$

at $t=0$

$$-\frac{1}{\tau} = -\frac{R}{L}$$ derivative $\equiv$ Slope

$\tau\uparrow$, $\frac{L}{R}\uparrow$, $L\uparrow$, $R\downarrow$

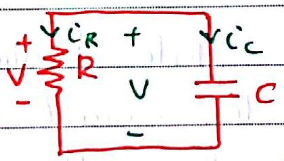
$\tau\uparrow$ better

$\tau \equiv$ تمثيل سرعة ال Discharging

---

* Source free ~~Series~~ Parallel RC Circuits.

*find $V_c(t)$



KCL @ upper Node

$$i_R + i_c = 0$$

$$\frac{V_c}{R} + C\frac{dV_c}{dt} = 0$$

$$C\frac{dV_c}{dt} = \frac{-V_c}{R}$$

RC $\boxed{\frac{dV_c}{dt} + \frac{V_c}{RC} = 0} \Leftrightarrow$ RL $\boxed{\frac{di_L}{dt} + \frac{R}{L} i_L(t) = 0}$

be cause of duality

$$\boxed{V_c(t) = V_0 e^{-t/\tau}} \qquad \boxed{\tau = RC}$$

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KVL

$-V_R + V_C = 0$

$$iR + \frac{1}{C}\int_{t_0}^{t} i(t)\,dt + V(t_0) = 0$$

$$R\frac{di}{dt} + \frac{1}{C}\,i(t)\,dt = 0$$

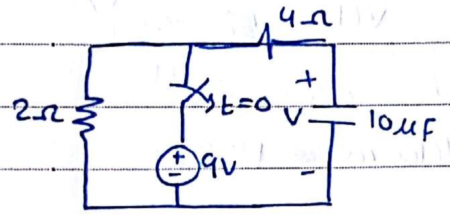
$$\frac{di}{dt} + \frac{1}{RC}\,i(t) = 0 \Rightarrow \boxed{i(t) = I_0 e^{-t/\tau}}, \boxed{\tau = RC}$$

Ex

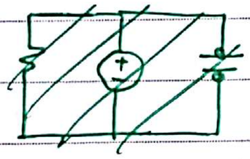
* Parallel RC circuits

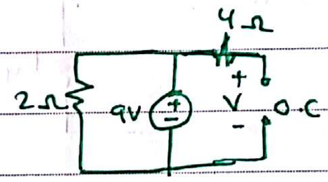
$$V_c(t) = V_o e^{-t/\tau} \quad , \quad \tau = RC$$

Ex: Find V at t = 200 μs



for $t<0$, $t=0^-$

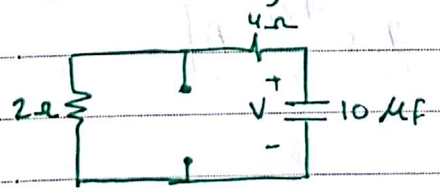




~~V(0^-)~~

$V(0^-) = 9V = V(0^+)$

for $t>0$, $t=0^+$



$$V(t) = V_o e^{-t/\tau}$$
$$= 9e^{-t/60}$$

$\tau = RC = 60\ \mu s$

$V(200\ \mu s) = 9e^{-200/60}$

$= 321.1\ mV$

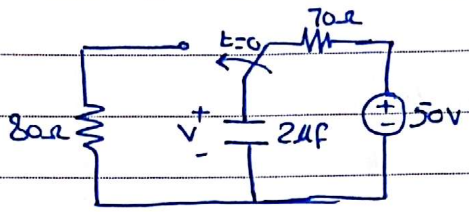
$\frac{0.321}{9} * 100\% = 3.57\%$

$> 3\tau$

$3\tau = 180\ \mu s$

Ex: Find $V(0)$, $V(0^-) = V(0^+) = V(0)$

& $V(160\mu s)$??



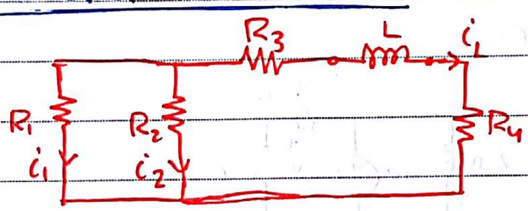
$V(0^-) = V(0^+) = V(0) = 50V$

$V(t) = \cancel{5e}\ 50e^{-t/(160\times10^{-6})}$

$V(160\mu s) = 50e^{-1}$

$V(160\mu s) = 18.39V$

* General RL Circuits



$$\tau = \frac{L_{eq}}{R_{eq}} = \frac{L_{eq}}{R_{th}}$$

$$R_{eq} = R_1 // R_2 + R_3 + R_4$$

$$\tau = \frac{L}{R_{eq}} \quad , \quad \boxed{i_L(t) = I_0 e^{-t/\tau} \ A}$$

$$i_L(0^{-1}) = i_L(0^+)$$

$$i_1(t) = i_1(0^+)\, e^{-t/\tau}$$

$i_1(0^+) \neq i_1(0^{-1})$ across a Resistor

at $t=0$ $\quad i_1(0^+) = -I_0 \left(\frac{R_2}{R_1+R_2}\right)$

$$i_1(t) = -I_0 \left(\frac{R_2}{R_1+R_2}\right) e^{-t/\tau}$$

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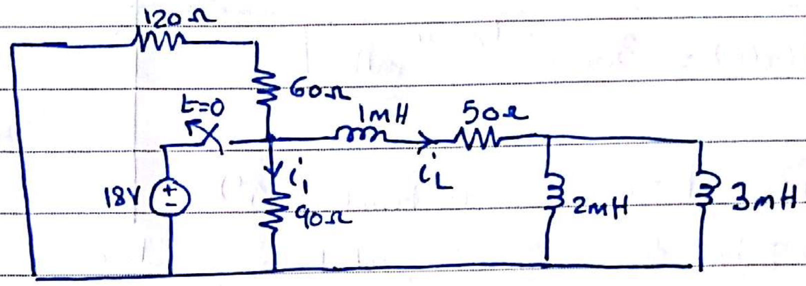
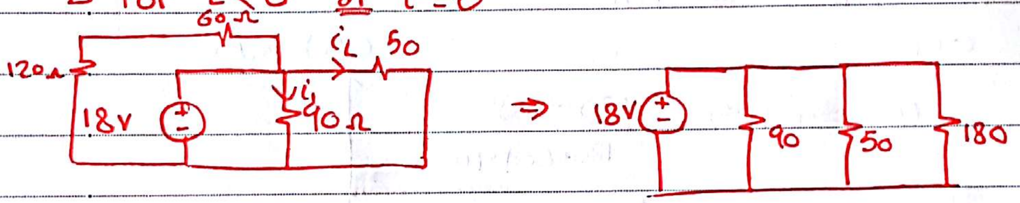
***Ex:** Find i_1 & i_L



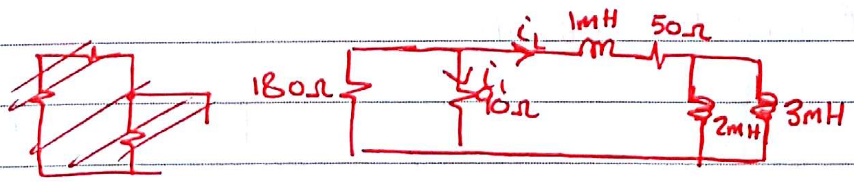
① for $t<0$ or $t=0^-$



$$i_1(0^-) = \frac{18}{90} = 200\,mA \neq i_1(0^+)$$

$$i_L(0^-) = \frac{18}{50} = 360\,mA = i_L(0^+)$$

② for $t>0$ or $t=0^+$



→ Source free circuit

$$R_{eq} = (180 // 90) + 50 = 110\,\Omega$$

$$L_{eq} = (2m // 3m) + 1m = 2.2\,mH$$

$$\tau = \frac{L_{eq}}{R_{eq}} = 20\,\mu s$$

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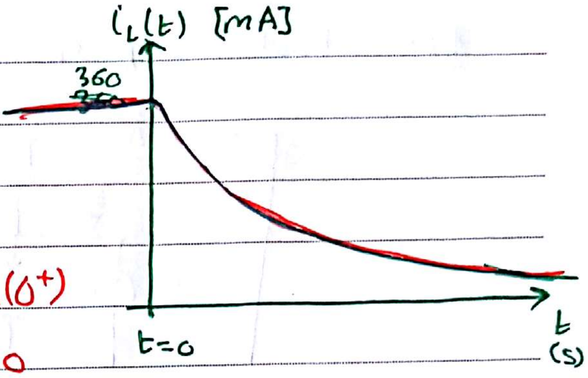
$i_L(t) = I_o e^{-t/\tau}$

$$\boxed{i_L(t) = 360 e^{-t/20\mu} \text{ mA}}$$



for $t=0^+$ to find $i_1(0^+)$

→ Solve the circuit at $t=0$

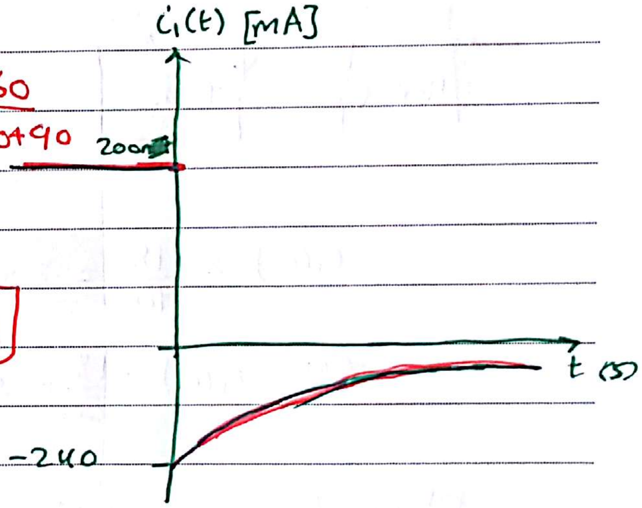
$i_L(0^+) = 360$ mA

CDR:

$$i_1(0^+) = -i_L(0^+) \times \frac{120+60}{120+60+90}$$

$$\boxed{i_1(0^+) = -240 \text{ mA}}$$

$$\boxed{i_1(t) = -240 e^{-t/20\mu} \text{ mA}}$$

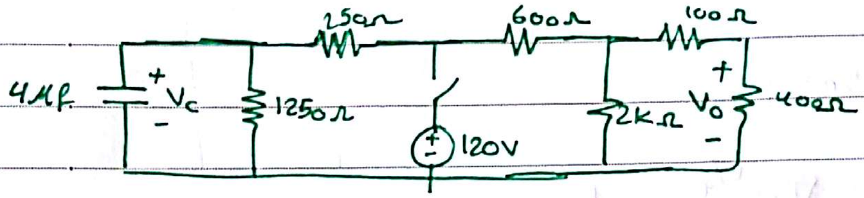


~~* General RC circuits~~

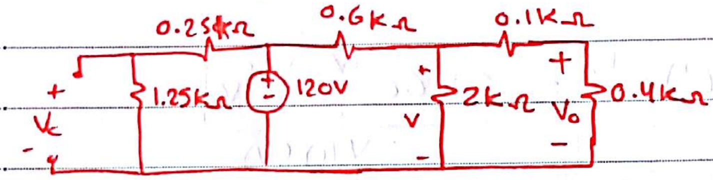
Ex

* General RC Circuits

Ex: find V_c & V_o at $t=0^-$, $t=0^+$, $t=1.3ms$



for $t<0$, $t=0^-$



$V_c = 120 \times \frac{1.25k}{1.5k}$, $\boxed{V_c(0^-) = 100V = V_c(0^+)}$

~~$V_o(0^-)$~~

$V = 120 \times \frac{2//0.5}{2//0.5+0.6} = 120 \times \frac{400}{1000}$

$V_o = V \times \frac{0.4}{0.5}$, $\boxed{V_o(0^-) = 38.4V \neq V_o(0^+)}$

for $t>0$, $t=0^+$

$V_c(0^+) = 100V = V_c(0^-)$

$V_o(0^+)$, Solve at $t=0$

$V_o(0^+) + V_{0.1k\Omega} = 100 * \frac{2//0.5}{2//0.5+0.85} = V_x$

$V_o(0^+) = V_x * \frac{0.4}{0.5}$ $\boxed{V_o(0^+) = 25.6V}$

$\tau = R_{eq} \cdot C$

$R_{eq} = 1250 // 1250 = 625\,\Omega$

$\boxed{\tau = 2.5ms}$

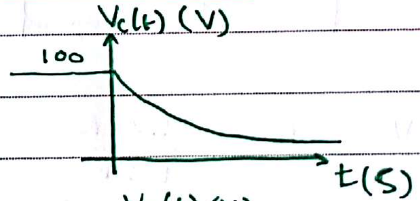
~~$V_c(t) = V_c e^{-t/\tau} = 100e$~~

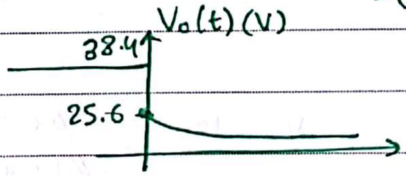
$\boxed{V_c(t) = V_c(0^+)\, e^{-t/\tau} = 100 e^{-t/2.5m} \text{ V}}$

$\boxed{V_o(t) = V_o(0^+)\, e^{-t/\tau} = 25.6 e^{-t/2.5ms} \text{ V}}$

$\boxed{V_c(1.3ms) = 59.45V}$

$\boxed{V_o(1.3ms = 15.22V}$





*Ex.

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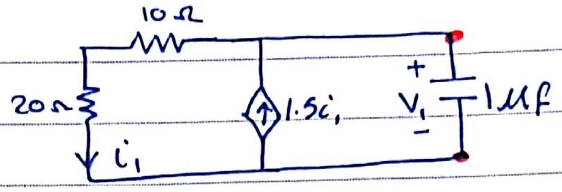
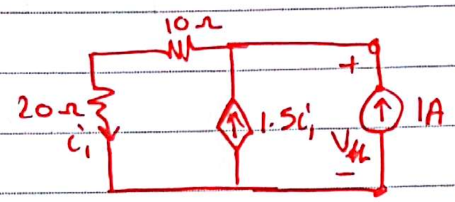
Ex:



Find $V_c(t)$ if $V_c(0^-) = 2V$

* Using thevenins method (3) [Remove C & add $I_s = 1$ A test]



~~$R_{th} = V_{test}$~~ , $R_{th} = \frac{V_{test}}{1}$

using Nod

$i_1 = V_{th}/30$

$1 + 1.5\left(\frac{V_{th}}{30}\right) = \frac{V_{th}}{30} \Rightarrow V_{th} = -60V$

$R_{th} = \frac{-60}{1} = -60\ \Omega$

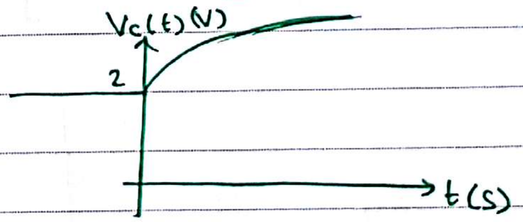
$\tau = R_{th} * C = (-60)(1\mu)$

$\boxed{\tau = -60\mu s}$

$V_c(t) = V_0\, e^{-t/\tau}$

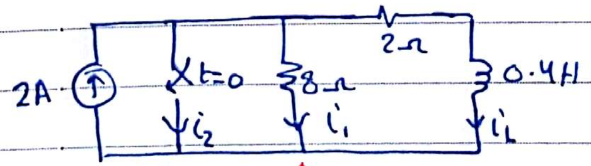
$= 2 e^{-t/-60\mu}$

$\boxed{V_c(t) = 2e^{t/60\mu}\ V}$



*This is an Unstable Circuit, circuit Increase without limit.

Ex: Find i_L, i_1, i_2 for all t , $t = 0^{-1}, 0^+$, any t

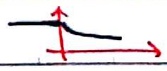


$i_L(0^{-1}) = i_L(0^+) = 1.6$ A

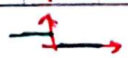
$i_1(0^{-1}) = 0.4A$, $i_1(0^+) = 0A$

$i_2(0^{-1}) = 0$, $i_2(0^+) = 0.4A$

$i_L(t) = 1.6 e^{-t/0.2}$ A



$i_1(t) = 0$ A



$i_2(t) = 2 - 1.6 e^{-t/0.2}$ A

$\tau = 0.2s$

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* Unit Step functions:

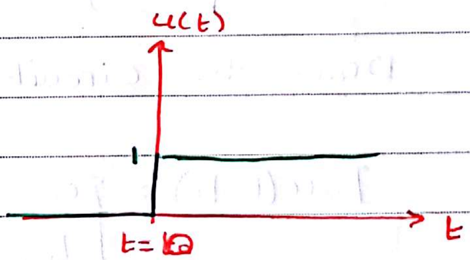
If a Source is Suddenly Inserted in a Circuit at $t=t_0$

$$\left.\begin{array}{ll} V=0 & , t<t_0 \\ V=V_0 & , t>t_0 \end{array}\right\}$$

Sudden change
⇓
Discontinuity
⇓
Singular Function
↙ Unit Step function $u(t)$
↘ Unit Impulse function $\delta(t)$

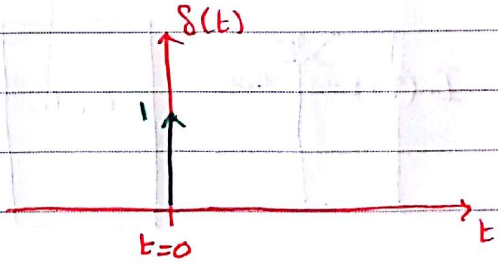
* $u(t)$

$$u(t)=\begin{cases} 0 & , t<\cancel{t_0}\,0 \\ 1 & , t>\cancel{t_0}\,0 \end{cases}$$



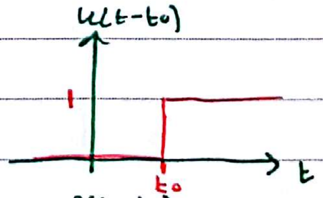
* $\delta(t)$

$$\delta(t)=\begin{cases} 0 & , t<0 \\ 1 & , t=0 \\ 0 & , t>0 \end{cases}$$



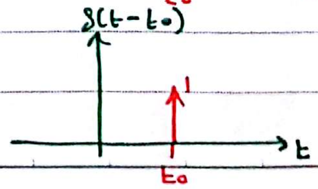
* $u(t-t_0)$

$$u(t-t_0)=\begin{cases} 0 & , t<t_0 \\ 1 & , t>t_0 \end{cases}$$



* $\delta(t-t_0)$

$$\delta(t-t_0)=\begin{cases} 0 & , t<t_0 \\ 1 & , t=t_0 \\ 0 & , t>t_0 \end{cases}$$



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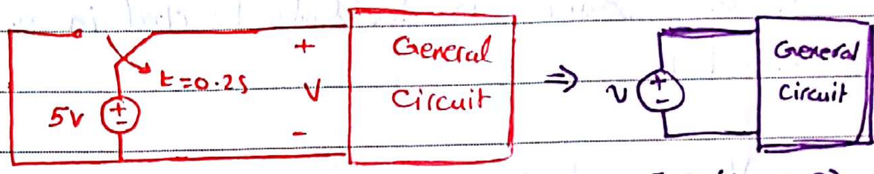
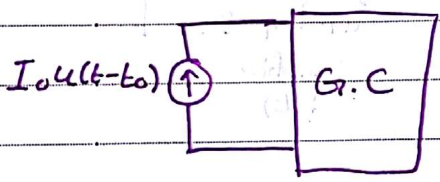
* Physical Sources & u(t)



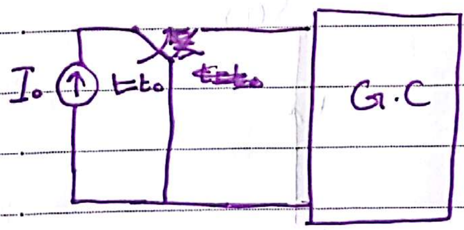
$v = 5\,u(t-0.2)$
↑
Switch

$$v = \begin{cases} 0, & t < 0.2 \\ 5, & t > 0.2 \end{cases}$$



Draw the circuit with a Switch.

$$I_o u(t-t_o) = \begin{cases} 0, & t < t_o \\ I_o, & t > t_o \end{cases}$$



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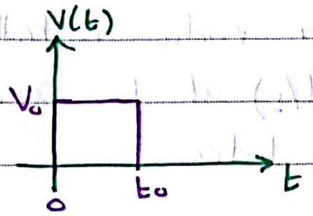
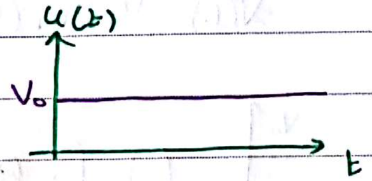
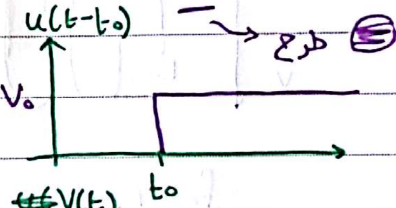
* The Rectangular Pulse Function:



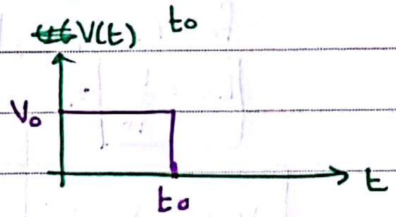
$$V(t) = \begin{cases} 0 & , t < 0 \\ V_o & , 0 < t < t_o \\ 0 & , t > t_o \end{cases}$$

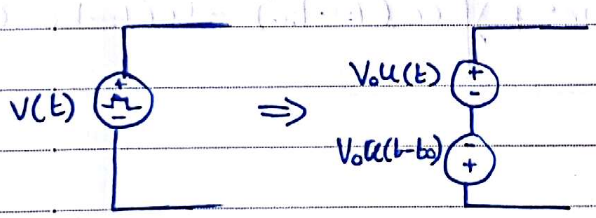




$\cancel{V(t) = u(t-t_o) - u(t)}$

$$\boxed{V(t) = V_o u(t) - V_o u(t-t_o)}$$



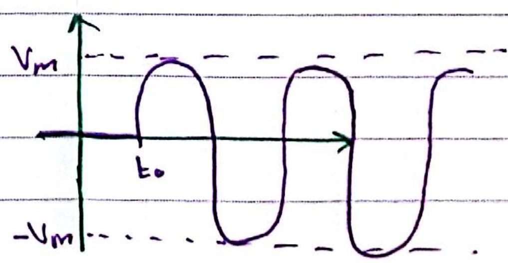


If a Sinusoidal Source ($V_m \sin \omega t$) is connected Suddenly to a network at $t = t_o$, then

~~V(t) = 0~~

$$V(t) = \begin{cases} 0 & , t < t_o \\ V_m \sin \omega t & , t > t_o \end{cases}$$

$$V(t) = V_m \sin \omega t \; u(t-t_o)$$



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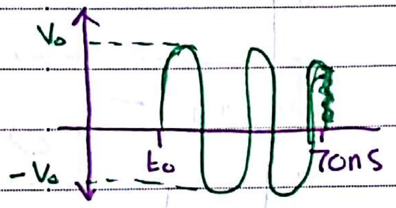
Ex: If a burst of energy need to be applied from the transmitter to a radio - controlled car operating at 47 MHz (295 Mrad/s) at t_0 and turn off the sourse 70 ns later

~~$V(t) = V_0 \sin(295Mt)$~~

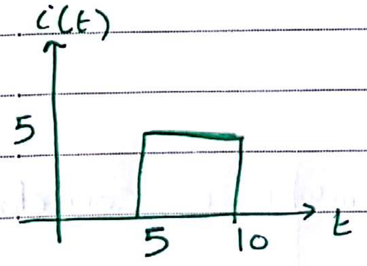


$f_0 = 47 MH_z$, $\boxed{\omega = 2\pi f}$ $\omega =$

$\boxed{t_0 = \frac{1}{f_0}}$

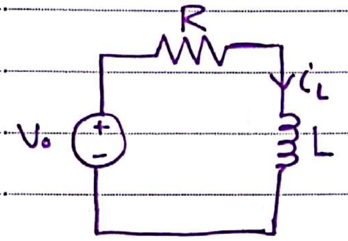
$$\boxed{V(t) = V_0 \sin(295\times10^6\, t)\left[u(t-t_0) - u(t-t_0-70\times10^{-9})\right]}$$

Ex:

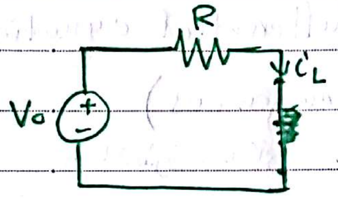


$$\boxed{i(t) = 5u(t-5) - 5u(t-10)}$$

* Driven, series RL Circuit

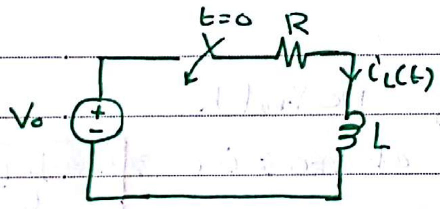


the Source exists in ~~the~~ the circuit
for $t > 5\tau$
Inductor will act as a Short Circuit

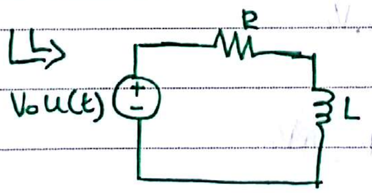


$i_L = \frac{V_0}{R}$ for $t > 5\tau$

$i_\infty = \frac{V_0}{R} \Rightarrow \infty \rightarrow t > 5\tau$



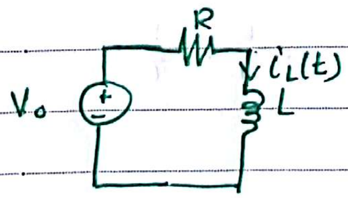
$V = \begin{cases} 0 & , t < 0 \\ V_0 & , t > 0 \end{cases} = V_0 u(t)$



$i_L(0^-) = i_L(0^+) = i_L(0)$

for $t < 0 \Rightarrow V = 0 \Rightarrow i_L(0^-) = 0 = i_L(0^+) = i_L(0)$

for $t > 0 \rightarrow i_L(0) = 0$



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<u>KVL</u>:

$$-V_o + IR + L\frac{di}{dt} = 0$$

$$L\frac{di}{dt} + iR = V_o$$

$$L\frac{di}{dt} = V_o - iR$$ → Linear differential equation (Not Homogenous)

* الطرف الثاني لا يساوي صفر *

$$\int \frac{L\,di}{V_o - iR} = \int dt$$

$$\frac{L\ln(V_o - iR)}{-R} = t + k$$, we find k

at $t=0 \Rightarrow i=0 \Rightarrow \boxed{k = -\frac{L}{R}\ln V_o}$

$$\frac{L}{-R}\ln(V_o - iR) = t - \frac{L}{R}\ln V_o$$

~~$\ln(V_o - iR) = t \ldots + \ln V_o$~~

~~$\ln(V_o - iR) = \ldots + \ln V_o$~~

$$-\frac{L}{R}\left(\ln(V_o - iR) - \ln V_o\right) = t$$

$$\ln\left(\frac{V_o - iR}{V_o}\right) = -\frac{R}{L}t \quad , \frac{R}{L} = \frac{1}{\tau}$$

$$\frac{V_o - iR}{V_o} = e^{-t/\tau}$$

$$V_o - iR = V_o e^{-t/\tau}$$

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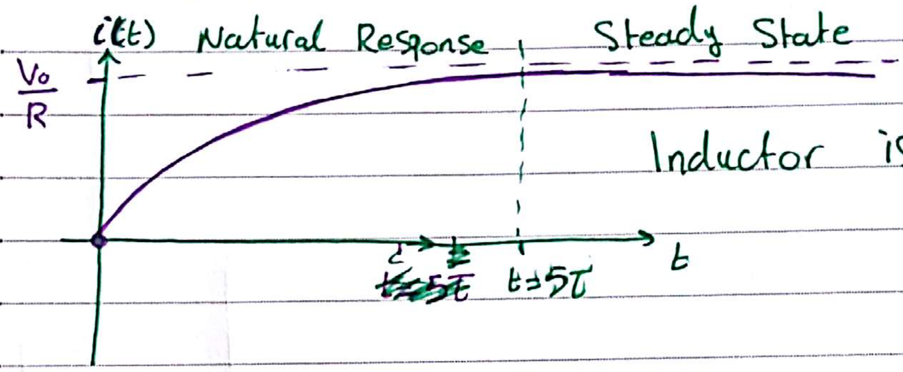
$$\boxed{i(t) = \frac{V_0}{R} - \frac{V_0}{R} e^{-t/\tau}} \quad , \tau = \frac{L}{R}$$

Solution for Driven Series RL circuit

$\Rightarrow i(0) = 0$



Inductor is charging

Steady State ≡ Forcing function ≡ Particular Solution

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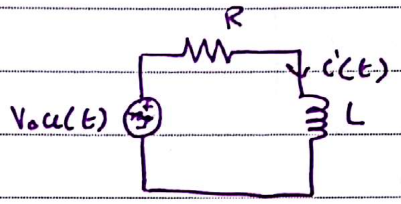
* Driven RL Circuits
series



$$V_o u(t) = \begin{cases} 0 & , t<0 \\ V_o & , t>0 \end{cases}$$

$$i(0^-) = I_o = i(0^+) = i(0)$$

KVL, $t>0$

$$-V_o + Ri + L\frac{di}{dt} = 0$$

$$L\frac{di}{dt} = V_o - Ri$$

$$\int \frac{L\,di}{V_o - Ri} = \int dt$$

$$\boxed{\frac{-L}{R}\ln(V_o - Ri) = t + k}$$

To find k $\quad i(0) = I_o \Rightarrow t=0 \Rightarrow i = I_o$

$$\boxed{K = -\frac{L}{R}\ln(V_o - RI_o)}$$

$$\frac{-L}{R}\ln(V_o - Ri) = t - \frac{L}{R}\ln(V_o - RI_o)$$

$$\frac{-L}{R}\left(\ln(V_o - Ri) - \ln(V_o - RI_o)\right) = t$$

$$\ln\left(\frac{V_o - Ri}{V_o - RI_o}\right) = -\frac{tR}{L}$$

$$V_o - Ri = (V_o - RI_o)\, e^{-t\frac{R}{L}}$$

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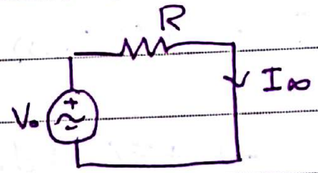
$$R\, i(t) = \frac{V_0}{R} - (V_0 - R I_0)\, e^{-t/\tau} \quad , \quad \tau = \frac{L}{R}$$

$$\boxed{i(t) = \frac{V_0}{R} - \left(\frac{V_0}{R} - I_0\right) e^{-t/\tau}}$$

at $t = \infty \Rightarrow (t > 5\tau)$

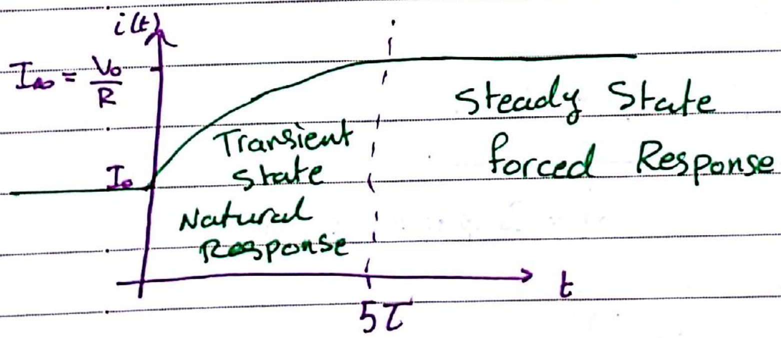
$L \rightarrow$ S.C



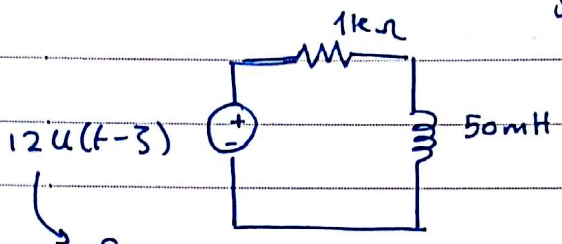
$$I_\infty = \frac{V_0}{R}$$

$$i(t) = I_\infty - (I_\infty - I_0)\, e^{-t/\tau}$$

$$i(t) = I_\infty + (I_0 - I_\infty)\, e^{-t/\tau}$$



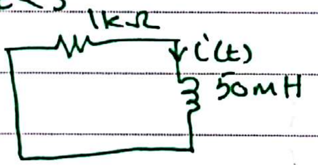
Ex: find $i(t)$ for $t=\infty$, 3^-, 3^+ and $100\mu s$ after the source changes value.



$$\begin{cases} 0, & t<3 \\ 12, & t>3 \end{cases}$$

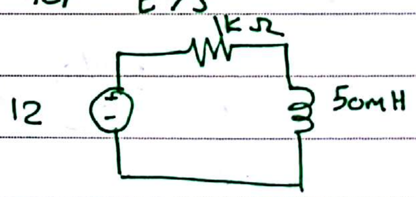
~~for t>3~~

for $t<3$



$i(3^-) = 0 = i(3^+) = i(3)$

for $t>3$



$\boxed{I_\infty = 12mA}$

$$\tau = \frac{L}{R} = \frac{50\times10^{-3}}{1\times10^{3}} = 50\mu s$$

$$i(t) = I_\infty + (I_0 - I_\infty)\,e^{-t/\tau}$$

$$i(t) = 12 + (0 - 12\,e^{-t/50\mu})\ mA$$

$$\boxed{i(t) = 12(1-e^{-t/50\mu})\ mA \quad , \quad t>3}$$

~~$i(3.0001) = 12(1-e$~~

$$\boxed{i(3.0001) = 10.38\ mA}$$

$$i(t) = \begin{cases} 0 & , \quad t<3 \\ 12(1-e^{-t/50\mu})\ mA & , \quad t>3 \\ 12\,mA & , \quad t=\infty \end{cases}$$

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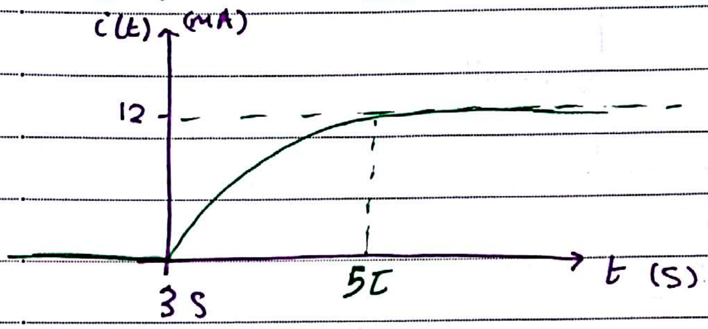
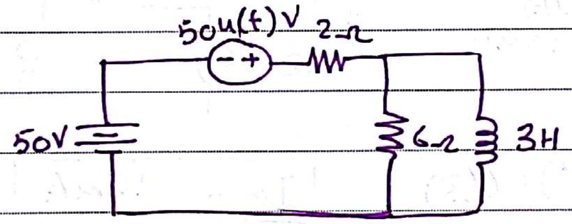
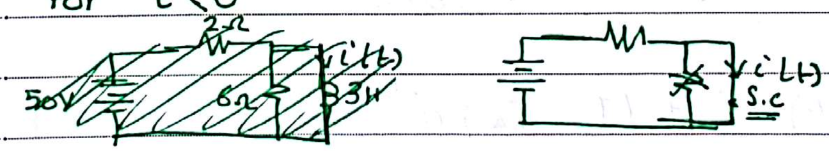
$\Rightarrow i(t) = 12\left(1 - e^{-t/50\mu}\right) u(t-3)\ mA$
$\Rightarrow$ for all t



Ex: find $i(t)$ for all t



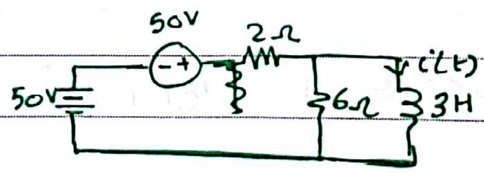
for $t<0$



L → Short circuit

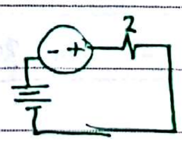
$I_o = i(0^-) = i(0^+) = \frac{50}{2} = 25A$

for $t>0$



$~~I_\infty =~~$

$I_\infty = \frac{100}{2} = 50A$



at ∞

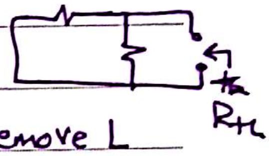
$I_0 = 25A$, $I_\infty = 50A$, now we find $i(t)$ for $t > 0$

$$i(t) = I_\infty + (I_0 - I_\infty)e^{-t/\tau}$$

find $\tau = \dfrac{L}{R_{th}}$ ⇝ seen from Inductor

$$= \frac{3}{6 // 2} = 2 \text{ Sec}$$

Thevenin's eq.



① Remove L

② kill Sources

$$\boxed{\tau = 2 \text{Sec}}$$

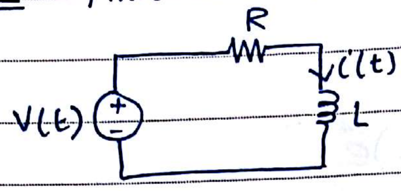
$$i(t) = 50 + (25 - 50)e^{-t/2} \ A$$

$$\boxed{i(t) = 50 - 25e^{-t/2} \ A} \ , \ t > 0$$

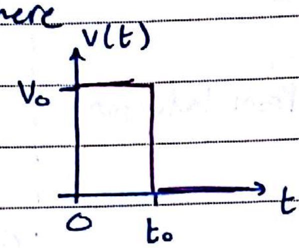
for all t

$$\boxed{i(t) = 25 + \left(25 - (50 - 25)e^{-t/2}\right)u(t)}$$

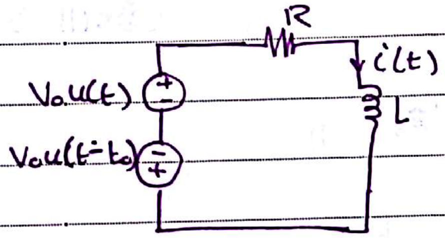
Ex: find $i(t)$ for all t



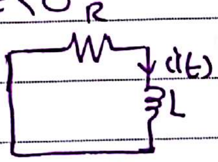
where



$$V(t) = V_0 u(t) - V_0 u(t - t_0)$$

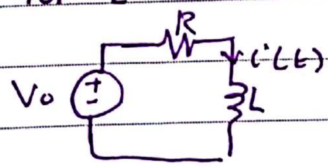


for $t < 0$



$$i(0^-) = i(0^+) = I_0 = 0 \text{ A}$$

for $0 < t < t_0$



$$I_\infty = \frac{V_0}{R}, \quad \tau = \frac{L}{R}$$

$$i(t) = \frac{V_0}{R} - \left(\frac{V_0}{R} - 0\right) e^{-t/\tau}$$

$$\boxed{i(t) = \frac{V_0}{R}\left(1 - e^{-t/\tau}\right) \text{ A}, \quad \text{for } 0 < t < t_0}$$

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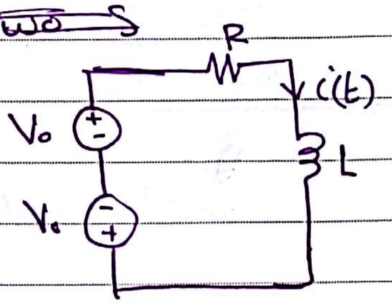
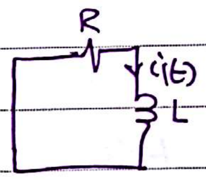
for $t > t_0$ → we need $i(t_0^-) = i(t_0^+)$

$$\boxed{i(t_0) = \frac{V_0}{R}\left(1 - e^{-t_0/\tau}\right) = i(t_0^-) = i(t_0^+)}$$

~~two S~~



⇒



$I_\infty = 0\ A$

$\cancel{i(t) = 0 - (0 - I_{t_0})e^{-t/\tau}}$, $\cancel{t > t_0}$

$$\boxed{\cancel{i(t) = i(t_0)\, e^{-t/\tau}, \quad t > t_0}}$$

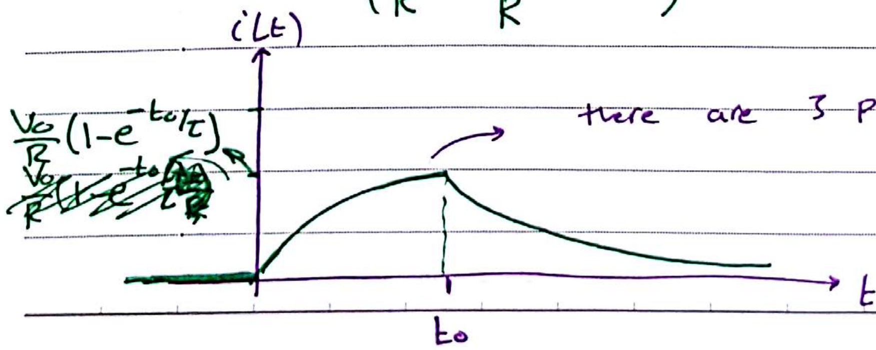
$$i(t) = 0 - (0 - i(t_0))\, e^{-(t-t_0)/\tau}$$

$$\boxed{i(t) = +\left(\frac{V_0}{R} - \frac{V_0}{R} e^{-(t_0/\tau)}\right) e^{-(t-t_0)/\tau}, \quad t > t_0}$$

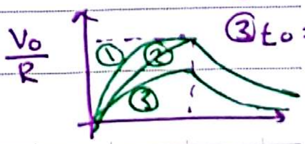
$$\cancel{i(t) = \frac{V_0}{R} - \frac{V_0}{R} e^{(t_0/\tau)} e^{-t t_0}}$$

$$i(t) = \left(\frac{V_0}{R} - \frac{V_0}{R} e^{-t_0/\tau}\right) e^{-(t-t_0)/\tau} \cdot u(t - t_0)$$

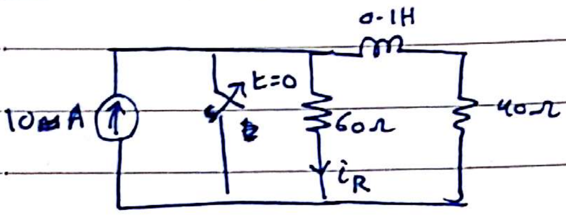
$$+\left(\frac{V_0}{R} - \frac{V_0}{R} e^{-t/\tau}\right) u(t) - \left(\frac{V_0}{R} - \frac{V_0}{R} e^{-t/\tau}\right) u(t - t_0)$$



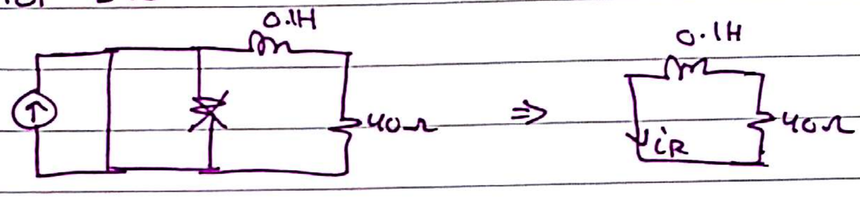
there are 3 possibilities either ① $t_0 > 5\tau$
② $t_0 < 5\tau$
③ $t_0 = 5\tau$



Ex: find $i_R(t)$ at $t = 0^-, 0^+, \infty, 1.5ms$



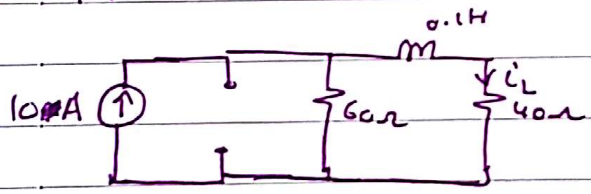
for $t<0$



$$i_L(0^-) = i_L(0^+) = 0$$

$$i_R(0^-) = -i_L(0^-) = 0 \neq i_R(0^+)$$

for $t>0$



$$R_{th} = 60 + 40 = 100\Omega$$

$$\tau = \frac{0.1}{100} = 1ms$$

$$i_{L\infty} = 10 \cdot \frac{60}{100} = 6 \text{ A}$$

$\cancel{I_\infty = 10 \cdot \frac{60}{100}} = 6$ A

$$\boxed{i_L(t) = 6 - (6e^{-t/1m}) \text{ A}}, \quad t>0$$

$$i_R = 10 - i_L(t)$$
$$= 10 - (6 - 6e^{-t/1m}) \text{ A}$$

$$\boxed{i_R(t) = 4 + 6e^{-t/1m} \text{ A}}, \quad t>0$$

$$\boxed{i_{R\infty} = 10 \cdot \frac{40}{100} = 4A}$$

$$\boxed{i_R(1.5ms) = 5.34 \text{ mA}}$$

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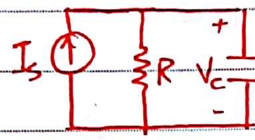
* Driven, RC Circuits
Parallel



$C \rightsquigarrow$ ~~o.c to DC~~

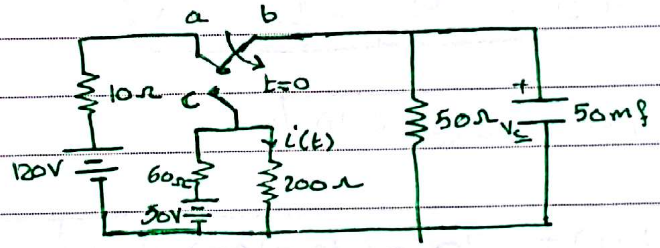
by Duality

$V_c(t) = V_\infty - (V_\infty - V_0)e^{-t/\tau}$ V

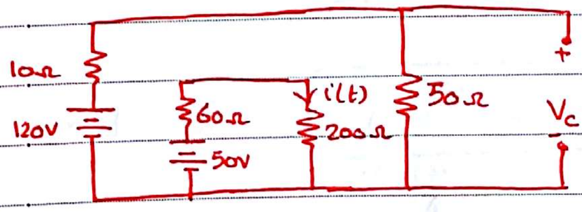
$\tau = RC$

$V_\infty = I_s R$

Ex: find $V_c(t)$ & $i(t)$



for $t<0$

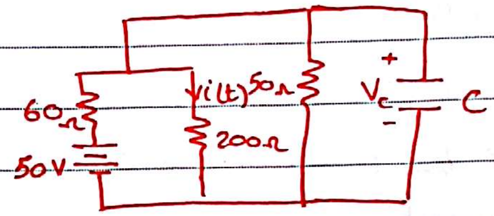


$V_c(0^-) = 120 \times \frac{50}{60} = 100V = V_c(0^+) = V_c(0)$

$i_c(0^-) = \frac{50}{260} = 0.1923A \neq i_c(0^+)$

for $t > 0$

$V_c(0) = 100V$



~~$V_{c\infty} = 50 \cdot 200$~~

$$\frac{200 \cdot 50}{250} \qquad 200 /\!/ 50 \rightarrow 40\ \Omega$$

$$V_{c\infty} = 50 * \frac{200/\!/50}{200/\!/50 + 60}$$

$$\frac{40 \cdot 60}{100} \qquad 40/\!/60 \rightarrow 24$$

$$= 50 \cdot \frac{40}{40+60}$$

$$\boxed{V_{c\infty} = 20V}$$

$$\boxed{i(0^+) = \frac{100}{200} = \frac{1}{2} A = 0.5A}$$

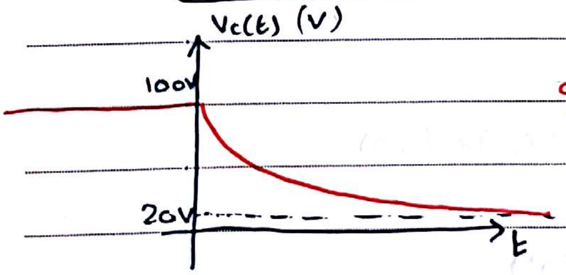
$$\boxed{i_\infty = \frac{20}{200} = 0.1A}$$

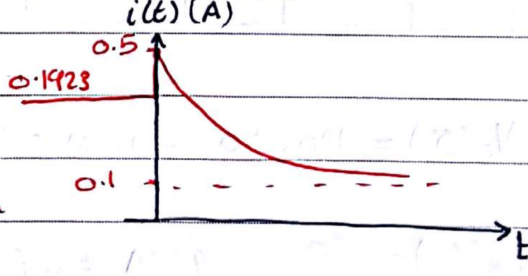
$$\boxed{V_c(t) = 20 - (20 - 100)\, e^{-t/1.2}\ V} \quad \text{for } t > 0$$

$$\tau = RC = 24 \times 50 \times 10^{-3} = 1.2$$

$$R_{eq} = 200/\!/50/\!/60 = 24\ \Omega$$

$$\boxed{i(t) = 0.1 - (0.1 - 0.5)\, e^{-t/1.2}\ A} \quad \text{for } t > 0$$





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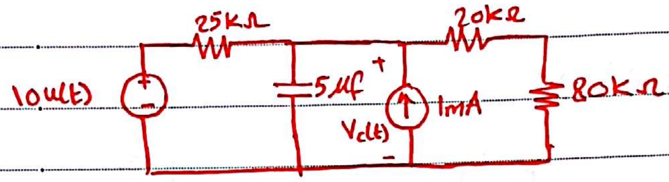
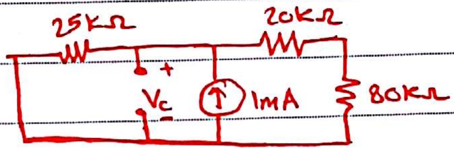
Ex: Find $V_c(t)$, for all t and at $t = 0.08S$



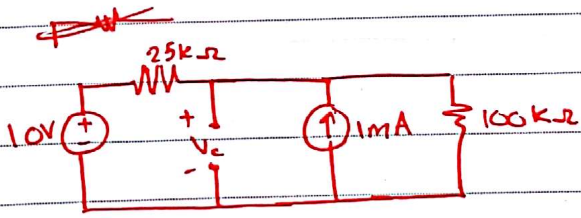
For $t<0$



$$V_c(0^-) = \left(1\times10^{-3} \times \frac{100\times10^3}{125\times10^3}\right) \times 25\times10^3$$

$$\boxed{V_c(0^-) = 20V = V_c(0^+) = V_c(0)}$$

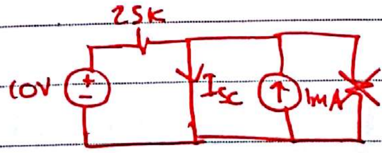
for $t>0$



Norton's eq.

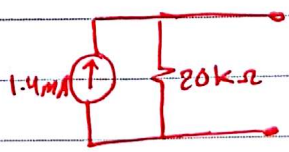
$R_N = 25 // 100 = 20k\Omega$

$\underline{I_{sc}}$



$$I_{sc} = 1m + \frac{10}{25k}$$

$$\boxed{I_{sc} = 1.4mA}$$



$\boxed{V_{th} = 28V} = V_\infty$

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$V_c(t) = 28 - (28 - 20)e^{-t/0.1}$ V

$\tau = R_N * C = 0.1S$

$V_c(0.08) = 24.4V$

لهون السكشن

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