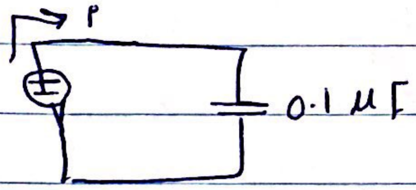
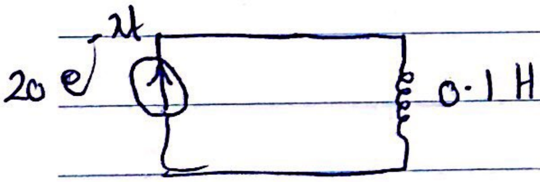
① Calculate $I_c(t)$ when $V_c(t) = 5(1-e^{t/10^{-6}})$ and $C = 0.1\mu F$



$$i_c(t) = \frac{C\,dv(t)}{d(t)} = 0.1 \times 10^{-6} \times \frac{d\ 5(1-e^{t/10^{-6}})}{dT}$$

$$= 0.1 \times 10^{-6} (5 + e^{-1/10^{-6}})$$

② Calculate the inductor voltage when $i_L(t) = wte^{-2t}$ and $L = 0.1\ H$



$$V_L(t) = L\frac{di(t)}{d(t)} = 0.1\left[20e^{-2t} - 40te^{-}\right.$$

$$= 2e^{-2t} - 4te^{-2t} \text{ Volt}$$

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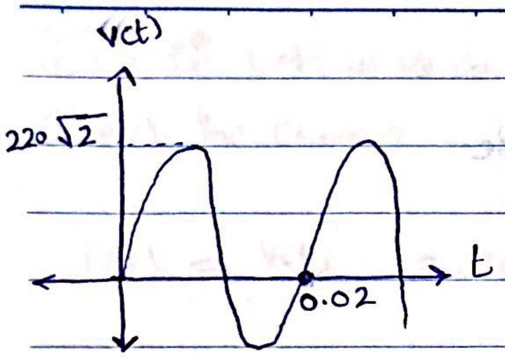


$f = 50\ Hz$

$T = \frac{1}{f} = \frac{1}{50}$

$f = 50 \Rightarrow 2\pi f$

$v(t) = 220\sqrt{2}\,\sin(\omega t + 0) \equiv 220\sqrt{2}\,\cos(\omega t + 0)$ $R = 5$

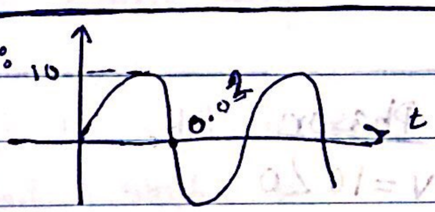
$i(t) = 44\sqrt{2}\,\sin(\omega t + 0)$ $R = 2$

$i(t) = 110\sqrt{2}\,\sin(\omega t + 0)$ $L = 0.1\ L$

$i(t) = 10\cos(\omega t \cancel{+0} - \frac{\pi}{2})$

$v(t) = 132 \times 10^{-3}\sqrt{2}\,\sin(\omega t + 0)$

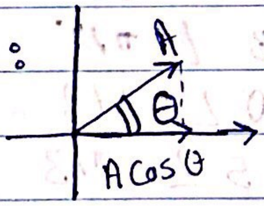
EX:



$t = 0 \to 0$ $t = 0.015 \to -10$

$t = 0.05 \to 10$ $t = 0.02 \to 0$

Phasor:



The real Value of a phasor is the Projection on x-axis

*We will be using [Cos].

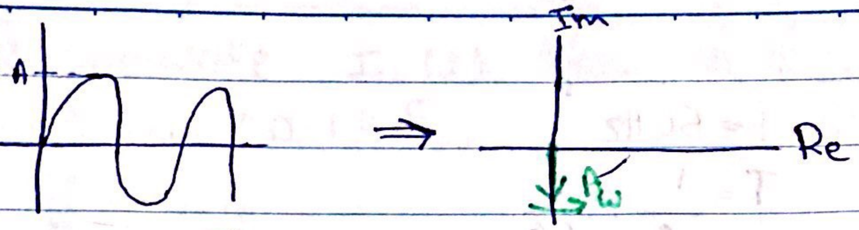
$e^{j\theta} = \cos\theta + j\sin\theta$

$Re\{e^{j\theta}\} = \cos\theta$

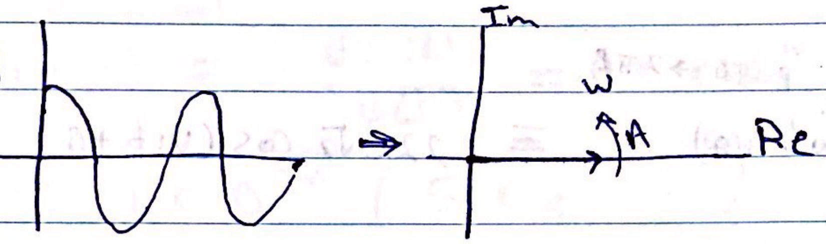
$Im\{e^{j\theta}\} = \sin\theta$

Scanned with CamScanner

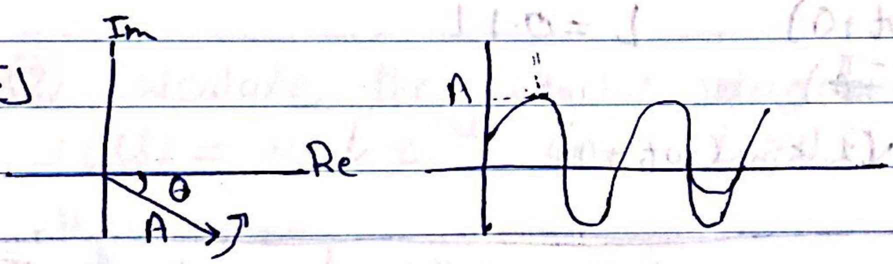
[a]



[b]



[c]



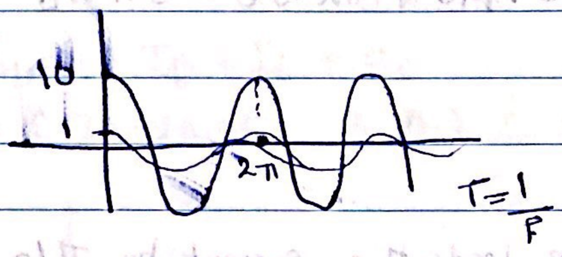
Time	Phasor
$v(t) = 10\cos(\omega t)$	$V = 10 \angle 0$
$v(t) = 5\cos(\omega t - \pi/3)$	$V = 5 \angle -\pi/3$
$i(t) = 3\cos(\omega t + \pi/4)$	$I = 3 \angle \pi/4$
$v(t) = 10 \sin(\omega t)$	$V = 10 \angle -\pi/2$
$i(t) = 5 \sin(\omega t + \pi/3)$	$I = 5 \angle -\pi/3$

Time	Phasor
R	R
L	$j\omega L$
$v_L(t) = L \dfrac{di(t)}{dt}$	$V_L = j\omega L I$
C	$1/j\omega C$
$i_c(t) = C \dfrac{d_c v(t)}{dt}$	$I = j\omega C V$

Ex: if $v(t) = 10\cos wt$, $w=1$, $R=10$, Find θ

Sketch the current

$$i(t) = \frac{v(t)}{R} = \cos wt \qquad f = \frac{w}{2\pi}$$



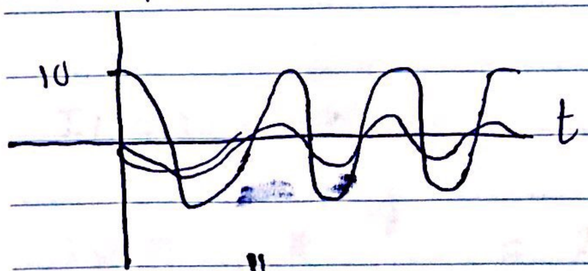
- How to differentiate between the leading and laging?

→ check 2 rising or falling edges and check which one reaches the x-axis first

Ex: if $V(t) = 10\cos(wt)$, $w=1$, $C=0.1$

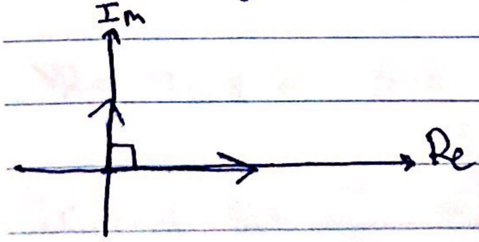
Find & sketch $V(t)$

$$i_c(t) = C\frac{dV_c(t)}{dt} = -0.1 * 10 * \sin wt = -\sin wt$$



in the capacitor, the current leads the voltage by $\pi/2$

⇓

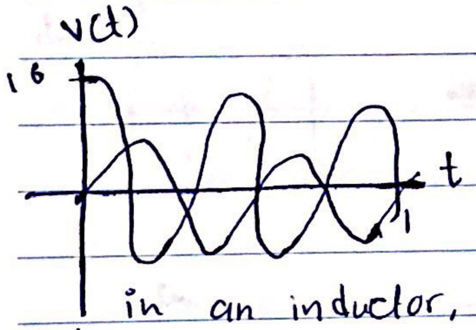


$\boxed{Ex}$ let $v(t) = 10\cos wt$, $w = 1$, $L = 10$

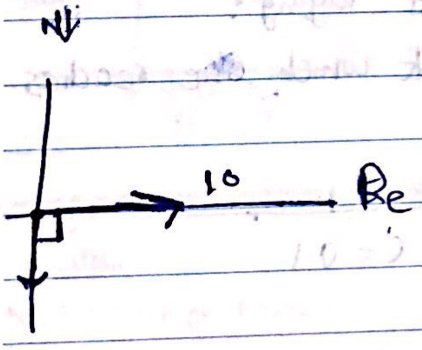
Find & sketch $i(t)$

$v(t) = L\dfrac{di(t)}{dt}$, $i(t) = \dfrac{1}{L}\int_0^t v(\tau)\,d\tau$

$= 0.1 * 10 * \sin wt = \sin wt$



in an inductor, the voltage leads the current by $\pi/2$



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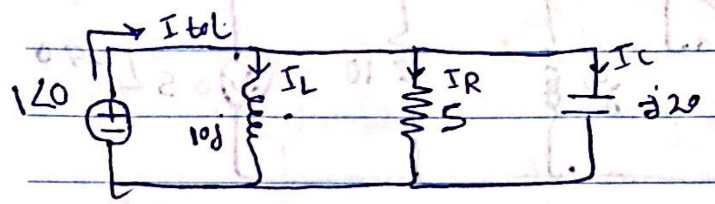
$\boxed{Ex}$ Draw all Phasors



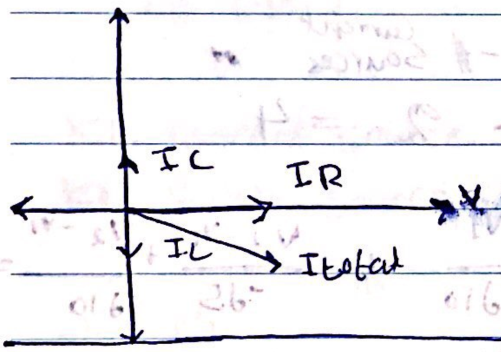
$- I_R = \frac{1\angle 0}{5} = 0.2\angle 0\ A$

$- I_L = \frac{1\angle 0}{j10} = -j0.1 = 0.1\angle -90$

$- I_C = \frac{1\angle 0}{-j20} = j0.05\ \angle$

- $I_{total} = I_R + I_L + I_C$

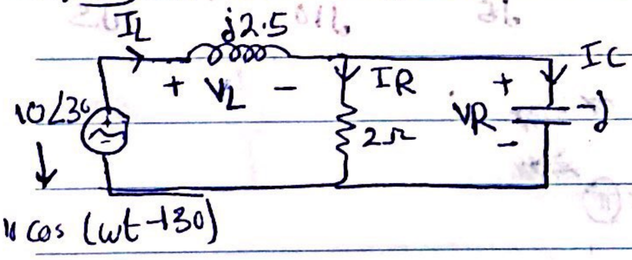
$= 0.2 - j0.05 \approx 0.2\angle -14$



$I = -0.2 + j0.05 =$

$= \sqrt{(0.2)^2 + 0.05^2}\ \angle \tan\left(\frac{0.05}{0.2}\right) + \pi$

$\boxed{Ex}$ Draw all Phasors



$I_L = \frac{10\angle 30}{j2.5 + (2 \parallel -j)}$

$= \frac{10\angle 30}{j2.5 + \left(\frac{j2}{2-j}\right)}$

$= 5.72\angle -46.75\ A$

$I_L = I_R + I_C$

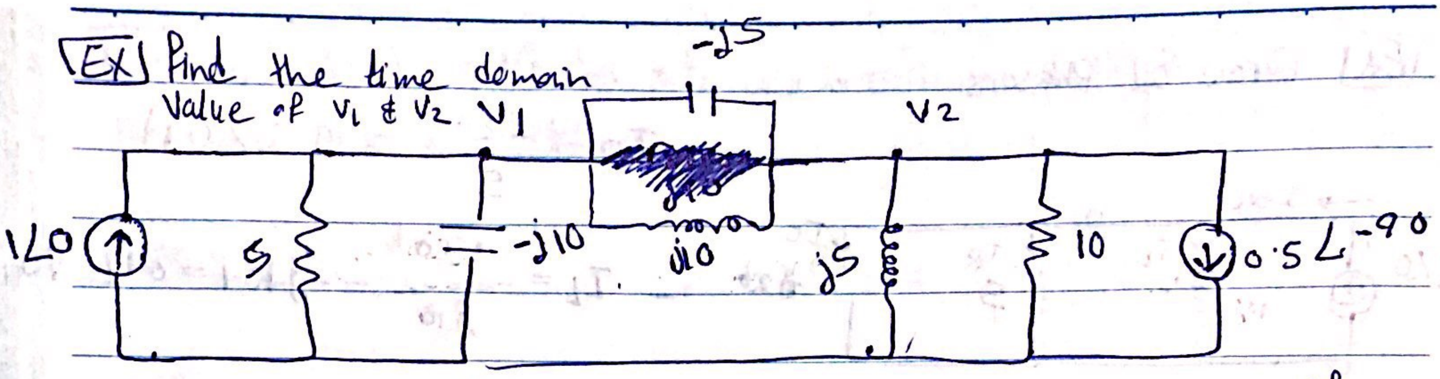
$I_R = I_L * \frac{-j}{2-j} = 2.56\angle -110$

$V_R = I_R * R = 5.2\ \angle -110$ $\{ V_C = V_R$

$I_C = I_L - I_R = \frac{V_C}{-j} = 5.12\angle -20$ $\{ V_L = j2.5 * I_L = V_s - V_R$

$= 14.3\angle 43.25$

[EX] Find the time domain value of V_1 & V_2



# of equations in nodal analysis = # nodes − # voltage sources − 1 (ref)

$= 3 - 0 - 1 = 2$

# of equations in mesh analysis = # loops − # current sources

$= 6 - 2 = 4$

• at node V_1: $1\angle 0 + \frac{0 - V_1}{5} + \frac{0 - V_1}{-j10} + \frac{V_2 - V_1}{-j5} + \frac{V_2 - V_1}{j10} = 0$

• at node V_2: $-0.5\angle -90 + \frac{0 - V_2}{10} + \frac{V_2}{j5} + \frac{V_1 - V_2}{j10} + \frac{V_1 - V_2}{-j5} = 0$

$V_1 = 1 - j2 = \sqrt{5} \angle \tan^{-1} \frac{-2}{1}$

$V_2 = -2 + j4 = \sqrt{20} \angle \tan^{-1} \frac{4}{2} + \pi$

$v_1(t) = 2.24 \cos(\omega t - 63.4)$

$v_2(t) = 4.47 \cos(\omega t + 116)$

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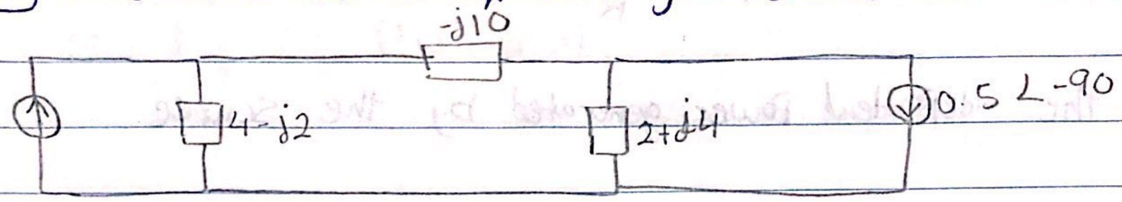
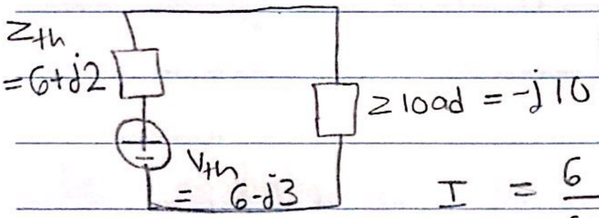
[Ex] Find the thevinin eq seen by the $(-j10)$ element

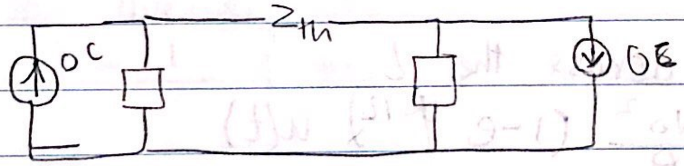




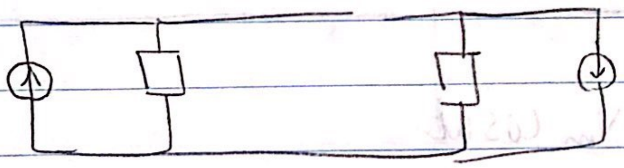
$Z_{th} = Z_{eq} = \frac{V_{oc}}{I_{sc}}$

$I = \frac{6-j2}{6+j2-j10}$

Kill current source by O.C
Kill Voltage source by S.C



$Z_{th} = 4-j2+2+j4 = (6+j2)\,\Omega$



$V_{AB} = (4-j2)\angle 90$
$V_{CD} = (0.5\angle 90)(2+j4)$

* Instantaneous P

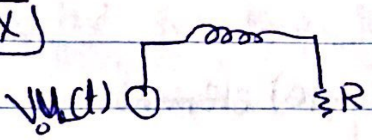
$P(t) = V(t)\,I(t) = \frac{V(t)^2}{R} = I(t)^2 R$

$L = v(t)$ and $i(t)$

$P(t) = v(t)\,I(t) = \frac{L\,di(t)}{dt} * i(t) = v(t) * \frac{1}{L}\int_0^t v(\tau)\,d\tau$

~~$L = v(t)$~~

Ex



$i(t) = \frac{V_o}{R}\left(1-e^{-tR}\right) u(t)$

① Find the independent Power generated by the source

$P(t) = v(t)\, i(t)$

$= V_o u(t) * \frac{V_o}{R}\left(1-e^{-tR/L}\right) u(t)$

$P(t) = \frac{V_o^2}{R}\left(1-e^{-R/L}\right) w$

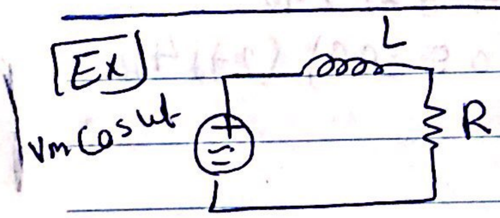
② Find instantenous p accross the resistor:

$P(t) = I(t)\, v(t) = I^2(t) R = \frac{V_o^2}{R}\left(1-e^{-tR/L}\right)^2 R\, u(t)$

③ Find the inst. Power across the L

$P(t) = v(t)\, i(t) = \frac{V_o^2}{R}\left(1-e^{-tR/L}\right) u(t)$

Ex



$v(t) = V_m \cos \omega t$

Find the ins. power by the source

$P(t) = v(t) * i(t) = V_m \cos(\omega t) * I_m \cos(\omega t + \theta)$

$= \frac{V_m I_m}{2}\left[\cos(2\omega t + \theta) + \cos(\theta)\right]$

$\cos x \cos y = \frac{1}{2}\left[\cos(x+y) + \cos(x-y)\right]$

• Average Power: ⓐ accurate ⓑ representation

$$P = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} P(t)\, d(t)$$

$$P_{avg} = \frac{1}{T} \int_0^{t} P(t)\, dt$$

_let suppose / any elements

$v(t) = V_m \cos(\omega t + \theta)$

$i(t) = I_m \cos(\omega t + \theta)$

The average power:

$$P = \frac{1}{T} \int_0^{T} v(t)\, d\Upsilon = \frac{1}{T} \int_0^{\pi} V_m I_m \cos(\omega t + \theta) + \cos(\omega t + \phi)$$

$$= \frac{1}{T} \int_0^{T} \frac{V_m - I_m}{2} \left[\cos(2\omega t + \theta + \phi) + \cos(\theta - \phi) \right] dt$$

$$= \frac{1}{T} \frac{V_m - I_m}{2} \int_0^{T} \cos(2\omega t + \theta + \phi) + \frac{1}{T} \left(\frac{V_m - I_m}{2} \cos(\theta - \phi) \right) dt$$

$$= \frac{1}{T} \int_0^{T} \frac{V_m - I_m}{2} \cos(\theta - \phi)\, dt$$

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* For asinosodal signals:

$$P_{avg} = \frac{V_m - I_m}{2} \cos(\theta - \phi)$$

$\boxed{EX}$ if $v(T) = 4\cos(\frac{\pi}{4} t)$. $Z = 2\angle -60$
find the average $v = 4\angle 0$?

$$I = \frac{V}{Z} = \frac{4\angle 0}{2\angle 60} = 2\angle -60^\circ = 2\cos(\frac{\pi}{4} - 60)$$

$$P_{avg} = \frac{4*2}{2} \cos(\theta + 60) = 2W$$

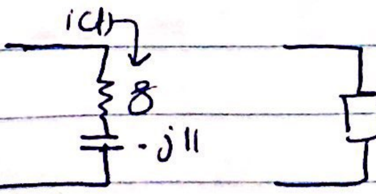
$$p(t) = v(t)\, i(t)$$
$$= 4\cos(\frac{\pi}{4} t) * 2\cos(\frac{\pi}{4} - 60) = 4\left[\cos(\frac{\pi}{2} t - 60 + \cos(60)\right]$$
$$= 2 + 4\cos(\frac{\pi}{2} t - 60)$$

* For a Pure resistive loads

$$P_{avg} = \frac{V_m - I_m}{2} = \frac{V_m^2}{2R} = \frac{1}{2} I_m^2 R$$ real average power شكل حراة

* For a L and C : $P_{avg} = 0$ مافي حرارة

$\boxed{EX}$ Find the average real power absorved by $Z = 8 - j11$ when Current $I = 3 + j4$ access through it

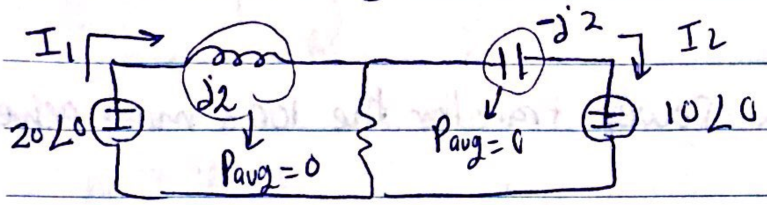


$i = (3 + j4)A = \sqrt{3^2 + 4^2} \angle \tan^{-1}\frac{4}{3}$

$Z = (8 - j11)\,\Omega$

$$P = \frac{1}{2} I_m^2 R = \frac{1}{2} \times 5^2 \times 8 = 100 \text{ watt}$$

EX Find the real avg. power consumed / generated from elements or sources



Nodal analysis:

$$\frac{20\angle 0 - V_0}{j2} + \frac{-V_0}{2} + \frac{10 - V_0}{-j2} = 0$$

$$V_0 = 10\angle -90 \rightarrow V_0 = -j10\ V$$

$$P_{avg}(L) = \frac{1}{2}\frac{V_m^2}{2} = \frac{1}{2} \times \frac{10^2}{2} = 25$$

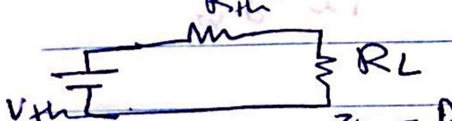
$$I_1 = 20\angle 0 - 10\angle -90 = 11.18\angle 63.43$$

$$P_{avg} = \frac{20 \times 11.18}{2} \cos(0 + 63.43) = 50$$

$$I_2 = \frac{10\angle -90 - 10\angle 0}{-j10} = 7.07\angle -45$$

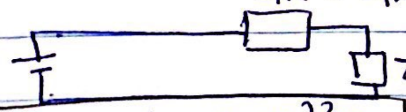
$$P_{avg} = \frac{1}{2} \times 10 \times 7.07 \cos(0 + 45) = 25 \text{ watt (absorbed)}$$

* maximum power transfer:



$\boxed{R_{th} = R_L}$

$Z_{th} = R_{th} + jX_{th}$



$$\boxed{P_{avg} = \frac{1}{2}(I)^2 R_L}$$

$$I = \frac{V_{th}}{R_{th} + Z_{th}}$$

$$\boxed{I = \frac{|V_{th}|}{\sqrt{(R_{th}+R_L)^2 + (X_{th}+X_L)^2}} \angle \theta_{th} - \tan^{-1}\left(\frac{X_{th} - X_L}{R_{th}+R_L}\right)}$$

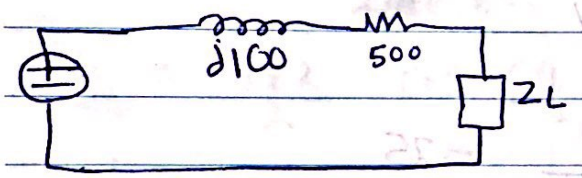
$\frac{\partial P_{avg}}{\partial R_L} \rightsquigarrow R_L = R_{th}$ $\qquad \frac{\partial P_{avg}}{\partial x_L} \Rightarrow x_L = -x_{th}$

*conclusion: for maximum power transfer the load must achieve

$Z_L = Z_{th}^*$

$j = -j$ $\qquad \angle + \to \angle -$

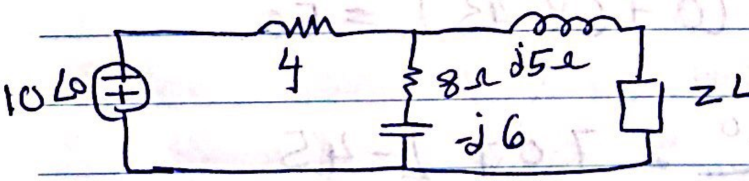
$\boxed{Ex}$ Find the value of ~~the~~ Z_L that maximized the maximum Power transfer



$Z_L = Z_{th}^*$

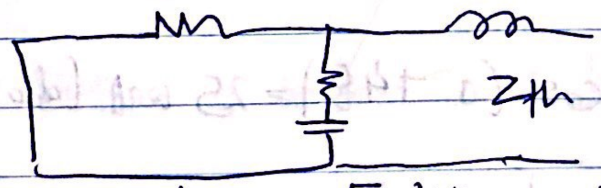
$Z_L = 500 - j100$

$\boxed{Ex}$ Determine the load impedance Z_L that maximized the average real power & find the power value



$Z_L = Z_{th}^*$

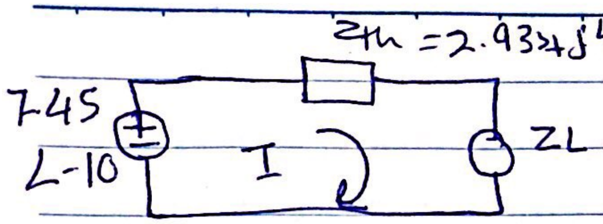
$Z_{th} = Z_{eq}$ after killing sources



$Z_{th} = j5 + [4 \parallel (8 - j6) = j5 + \frac{4(8-j6)}{12-j6}$

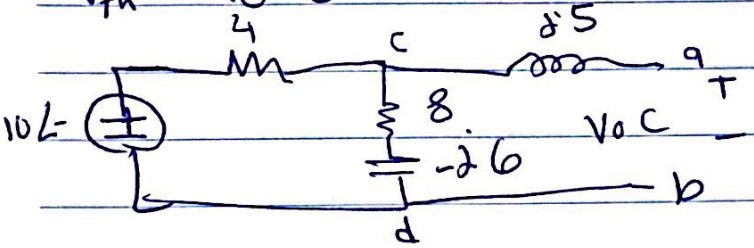
$= (2.933 + j4.467)\,\Omega$

$Z_L = (2.933 - j4.467)\,\Omega$



$2.933 - j4.467$

$V_{th} = V_{O.C}$



$V_{ab} = V_{cd} = \frac{8 - j6}{12 - j6} \times 10\angle 0$

$P_{avg} = \frac{1}{2} \times I^2 \times 2.933$

$I = \frac{7.45\angle -10.3}{2.933 \times 2} = 1.27 = 1.27\ \angle -10.3$

$P_{avg} = \frac{1}{2} (1.27)^2 \times 2.933 = 2.36$

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* In general:

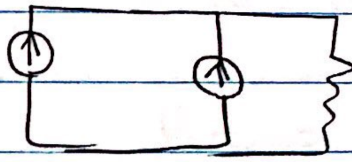
$-P_{avg} \rightarrow P(t) = \frac{1}{2} V_m I_m \cos(\theta - \phi)$

$$P = \frac{1}{T} \int_0^T P(t)\, dt$$

$\hookrightarrow = i(t) * v(t)$

$-P_{inst} \rightarrow P(t) = \frac{1}{2} V_m I_m \cos(\theta - \phi) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta + \phi)$

* Source with multiple Freq:



$\boxed{Ex}$ $i(t) = 3\sin 2t + 4 \sin 10t$

$R = 1$, find P_{avg}

$$P_{avg} = \frac{1}{T}\int_0^T P(t)\, d\tau - \frac{1}{T}\int_0^T i^2(t) * R = \frac{1}{T}\int_0^T (3\sin 2t + 4\sin 10t)^2 dt$$

$$= \frac{1}{T}\int_0^T 9\sin^2 2t + 24 \sin 2t \sin 10t + 16 \sin^2 10t$$

$$= \frac{1}{T}\int_0^T \left(9 * \frac{1}{2} + 16 * \frac{1}{2}\right) dt = \frac{1}{2}[9+16] = 12.5$$

* For sinoidal signals with multiple frequencies
if $i(t) = I_{m1} \cos \omega_1 t + I_{m2} \cos \omega_2 t + \cdots + I_{mn} \cos \omega_n t$

$$P_{avg} = \frac{1}{2} R \left[I_{m1}^2 + I_{m2}^2 + \cdots + I_{mn}^2 \right]$$

$$\text{OR} = \frac{1}{2} \frac{\left[V_{m1}^2 + V_{m2}^2 + \cdots V_{mn}^2 \right]}{R}$$

Ex ① $i(t) = 2\cos 10t - 3\cos 20t$, $R=4$, find P_{avg}

$P_{avg} = \frac{1}{2} * 4 * (4+9) = 26\,W$

② $v(t) = 3\cos w_1 t + 4\sin w_2 t$, $R=2$, find P_{avg}

$P_{avg} = \frac{1}{2} * \frac{(9+16)}{2} = \frac{25}{4} = 6.25\ W$

③ $i(t) = 3\cos 10t - 5\cos 10t$, $R=1$, find P_{avg}

$= -2\cos 10t$

$P_{avg} = \frac{1}{2} * (2)^2 * 1 = 2W$

④ $i(t) = 5 + 4\cos wt$, $R=1$, find P_{avg}

$P_{avg} = (5)^2 + \frac{1}{2} * 4^2 * 1 = 33\ W$

note : DC power $\rightarrow (i^2R)$ [no $\frac{1}{2}$ in DC]

AC power $\rightarrow \frac{1}{2} I^2 R$ or $\frac{1}{2} \frac{V^2}{R}$

⑤ $v(t) = 3\cos wt + 4\sin wt$, $R=2$, P_{avg}?

$= 3\angle 0 + 4\angle -90 = 3 - j4 \Rightarrow 5\angle -53$

$P_{avg} = \frac{1}{2} \frac{V^2}{R} = \frac{1}{2} \frac{(25)}{2} = 6.25\ W$

⑥ $v(t) = 3\cos wt + 4\cos(wt - \pi/3)$

$= 3\angle 0 + 4\angle -\pi/3 = 5 - j3.5 \Rightarrow 6.1\angle -35$

$P_{avg} = \frac{1}{2} * \frac{(6.1)^2}{R}$

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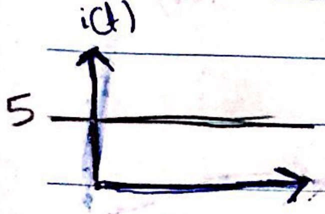
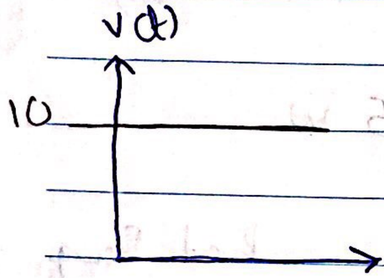
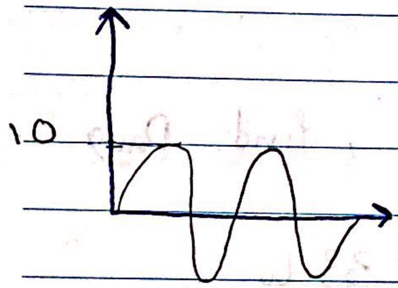
* Effective values:



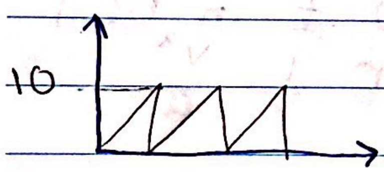
$P_{avg} = 5^2 R$

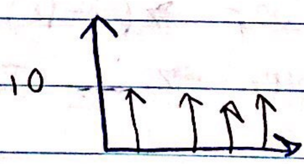


$P_{avg} = 10^2 / R$

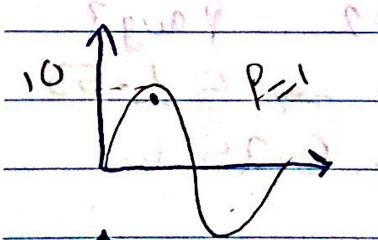


$P_{avg} = 10^2 / 2R$

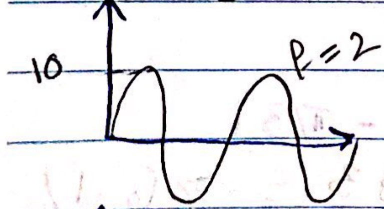




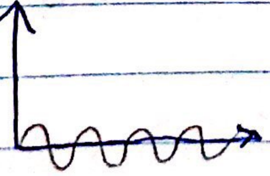
$P_{avg} \neq \frac{10^2}{2R}$



$P_{avg} = \frac{10^2}{2R}$



$P_{avg} = \frac{10^2}{2R}$



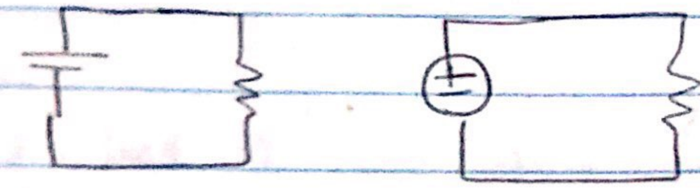
$P_{avg} = \frac{1}{2} R$

* Find the RMS (effective) value of:

$$i(t) = I_m \cos(\omega t + \phi)$$

$$I_{RMS} = I_{eff} = \sqrt{\frac{1}{T}\int_0^T I^2 \cos^2(\omega t + \phi)\, dt}$$

$$= \sqrt{\frac{1}{T}\int_0^T \frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}}$$

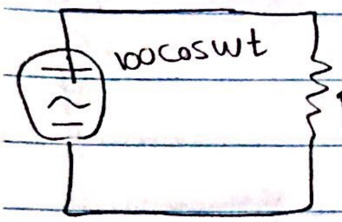


$$P = \frac{V_{DC}}{2}$$

$$P = \frac{1}{T}\int_0^T \frac{v^2(t)}{R}\, dt$$

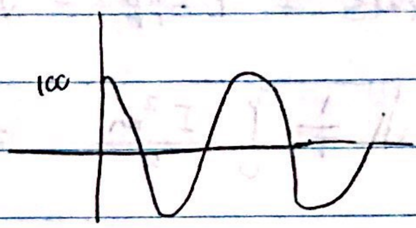
$$\frac{V_{DC}^2}{R} = \frac{1}{T}\int_0^T \frac{v^2(t)}{R}\, dt$$

$$V_{eff} = V_{DC} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\, dt}$$

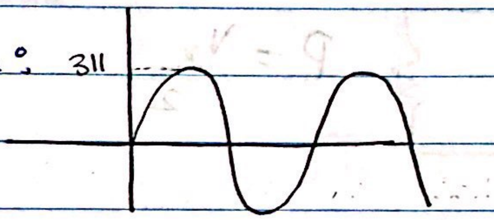


$P_{avg} = 5000$ W $\quad \frac{V_{DC}^2}{R=1} = 5000$

$V_{DC} = \sqrt{5000} = 70.2$



In Jordan:



- For asinosidal signal:

$$P_{avg} = \frac{I_m V_m}{2} \cos(\theta - \phi) = I_{rm} V_{rms} \cos(\theta - \phi)$$

- For a resistor:

$$P_{avg} = \frac{V_m I_m}{2} = \frac{V_m^2}{2R} = \frac{1}{2} I_m^2 R$$

$$= V_{rms} I_{rms} = \frac{V_{rms}^2}{2} = I_{rms}^2 R$$

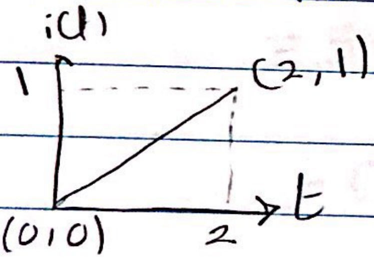
* Effective value: $V_{rms} = \sqrt{\frac{1}{T} \int_0^T v^2(t)\, dt}$

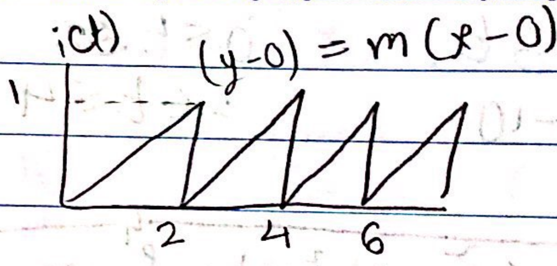
• For sinosodial:

$V_{rms} = V_m / 2$ $\qquad$ $I_{rms} = I_{Peak} / \sqrt{2}$

$P_{avg} = \frac{V_m I_m}{2} \cos(\theta - \phi) = V_{rms} I_{rms} \cos(\theta - \phi)$

Ex: Find the avg power deliverd to $(5\,\Omega)$ resistor





$P_{avg} = \frac{V_m I_m}{2} \cos(\theta - \phi) = I^2_{rms} R$

$I_{rms} = \sqrt{\frac{1}{T} \int_0^T i^2(t)\, dt}$ $\quad (T = 2)$

$i(t) = \frac{1}{2} t \qquad 0 \leq t \leq 2$

$I_{rms} = \sqrt{\frac{1}{2} \int_0^2 \left(\frac{1}{2} t\right)^2 dt} = \sqrt{\frac{1}{2} \int_0^T \frac{1}{4} t^2\, dt}$

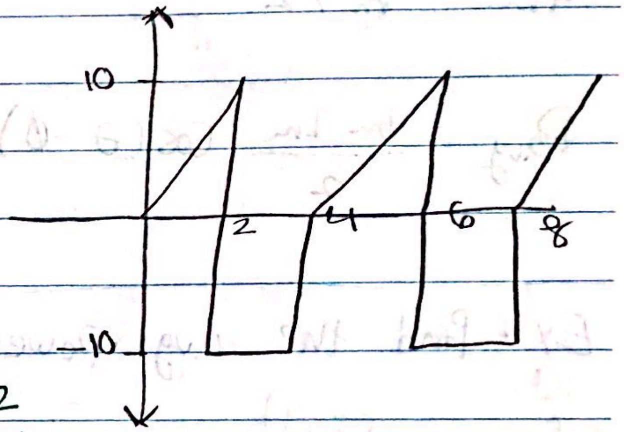
$= \sqrt{\frac{1}{2} \left(\frac{1}{12} t^3\right)} = \frac{1}{\sqrt{3}}\, A$

$P_{avg} = 5 * \frac{1}{3}\, W$

- For sawtooth functions :

$V_{rms} = V_m / \sqrt{3}$, $I_{rms} = I_m / \sqrt{3}$

$\boxed{Ex}$ Find the rms values:



$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\, dt}$$

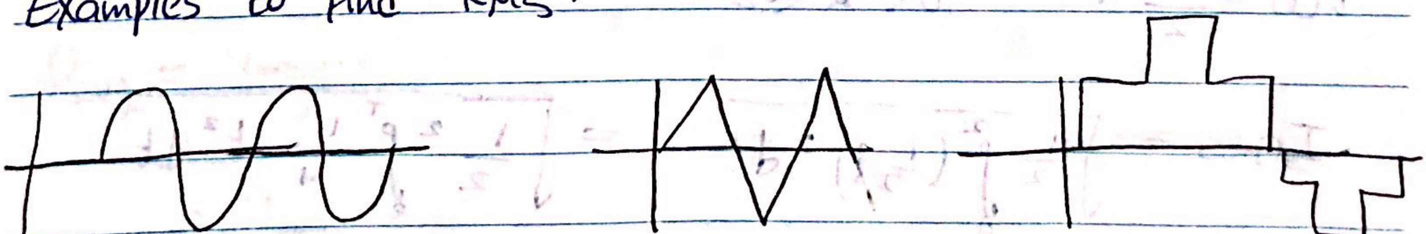
$T = 4$

$$v(t) = \begin{cases} 5t & 0 \le t \le 2 \\ -10 & 2 < t \le 4 \end{cases}$$

$$V_{rms} = \sqrt{\frac{1}{4}\left(\int_0^2 25t^2\,dt\right) + \int_2^4 100\, dt}$$

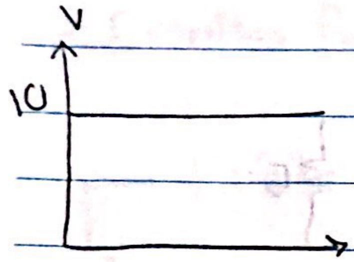
$$= \sqrt{\frac{1}{4}\left(\frac{25}{3}\, t^3\Big]_0^2 + 100(2)\right)} = 8.165\ V$$

Examples to find RMS:



* Source with multiple freq : $v(t) = 3\cos\omega_1 t + 4\cos\omega_2 t$, find rms

$$V_{rms} = \sqrt{(V_{rms1})^2 + (V_{rms2})^2 + \cdots} = \sqrt{\left(\frac{3}{\sqrt{2}}\right)^2 + \left(\frac{4}{\sqrt{2}}\right)^2}$$



$V_{rms} = 10$

$i(t) = 5 + 4\sin(\omega t - \pi/3)$, find I_{rms}

$v(t) = 5\cos \omega t + 3\cos(\omega t - \pi/3)$

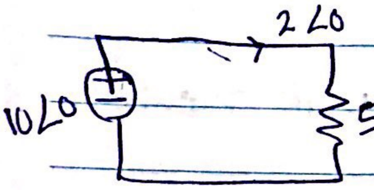
$= 5\angle 0 + 3\angle -\pi/3$

$= 5 + j0 + 3\cos(-\pi/3) + j3\sin(-\pi/3)$

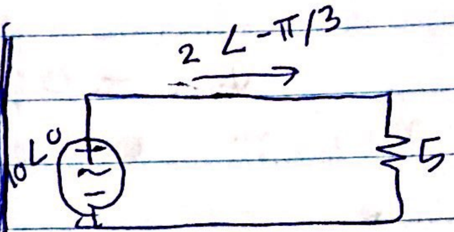
$= 6.5 - j\frac{3\sqrt{3}}{2} = \sqrt{(6.5)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2}\ \angle \tan^{-1}\left(\frac{\frac{3\sqrt{3} \times -1}{2}}{6.5}\right)$

$$\boxed{V_{rms} = \frac{V_{Peak}}{\sqrt{2}}}$$

* Power Factor



$$P_{avg} = \frac{V_m I_m}{2} = \frac{10*2}{2}$$
$$= 10W$$



$$P_{avg} = \frac{V_m I_m}{2} \cos(\theta - \phi)$$
$$= \frac{10*2}{2} \cos(0 - -\pi/3) = 5W$$

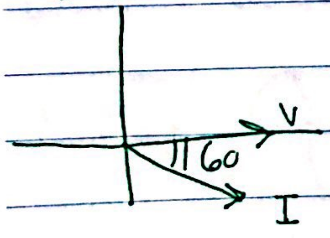
* The power factor represents the losses in the circuit

$$PF = \frac{\text{Real Power}}{\text{apparent Power}} = \frac{V_{rms} I_{rms} \cos(\theta - \phi)}{V_{rms} I_{rms}} = \cos(\theta - \phi)$$

$0 \leq PF = \cos(\theta - \phi) \leq 1$, [For R $\cos(\theta - \phi) = 1$, $PF = 1$]

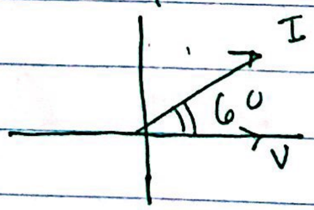
[For L, C $\cos(\theta - \phi) = 0$, $PF = 0$]

L & R



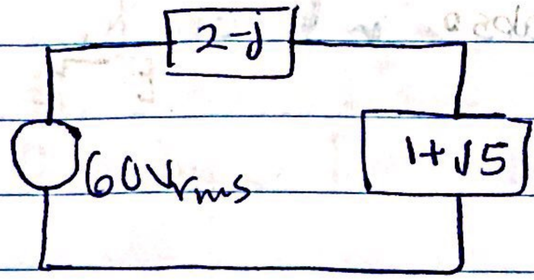
$PF = \cos(0 + 60) = 0.5$
lagging
(I lags V) → inductive load

C & R



$PF = \cos(0 - 60) = +0.5$
Loading
(I leads V) → capacitive load

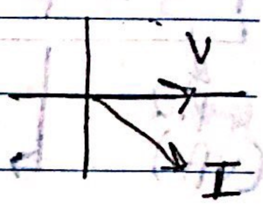
* Example: Find the source PF



$v(t) = \sqrt{2}\ 60 \cos(\omega t)$

- $PF_{source} = \cos(\theta - \phi) = \cos(0 - -53.13) = 0.6$ lagging (I lags V)

- $I = \dfrac{60\angle 0}{2 - j + 1 + j5} = 12\angle -53.13$ Arms



$P_{avg} = (2-j) = (12)^2 * 2$

$P_{avg} = (1+j5) = (12)^2 * 1$

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4/10/2018

*Complex power: This concept is introduced to represent the power consumed/stored in R or X (L, C)

Ⓢ → complex → Real Power (R) → Ⓟ
→ Reactive Power (L, C) → Ⓠ

$$\boxed{S = P + jQ}$$

S: complex (VA)
P: Power (real Power (W))
Q: Reactive Power (VAR) volt ampere reactive

$\Rightarrow x = 5\cos\theta$

$z = 5e^{j\theta} = 5(\cos\theta + j\sin\theta)$

$x = R_e\{z\}$

$e^{j(\theta-\phi)} = \cos(\theta-\phi) + j\sin(\theta-\phi)$

$\cos(\theta-\phi) = Re\{e^{j(\theta-\phi)}\}$

$P = R_e\{S\}$, $Q = I_m\{S\}$

$P = |V_{rms}||I_{rms}|\cos(\theta-\phi)$

$= |V_{rms}||I_{rms}| \, re\{e^{j(\theta-\phi)}\}$

$= R_e\{|V_{rms}||I_{rms}|e^{j(\theta-\phi)}\}$

$= Re|V_{rms}|e^{j\theta} * |I_{rms}|e^{-j\phi}$

$= Re\{V * I^*\}$

$S = \frac{1}{2} V_m I_m^*$

$S = V I^*$

$P = \frac{1}{2} V_m I_m \cos(\theta-\phi)$

• Suppose $V_{rms} = 10 \angle 30$
$I_{rms} = 5 \angle 20$

$S = V_{rms} I^*_{rms} = 10\angle 30 * -5 \angle -20 = 50 \angle 10$ VA
$P = V_{rms} I_{rms} \cos(\theta - \phi) = 10 * 5 \cos(30-20)$

• Suppose : Given Peak Values
$V = 10 \angle 30$, $I = 5 \angle 20$

$S = \frac{1}{2} V I^* = \frac{1}{2} * 10 \angle 30 * 5 \angle -20 = 25 \angle 10$ VA

$P = \frac{1}{2} V_m I_m \cos(\theta - \phi) = \frac{1}{2} * 10 * 5 \cos(10)$

• Suppose : $v(t) = 10 \cos(wt + 30)$
$i(t) = 5 \cos(wt + 20)$

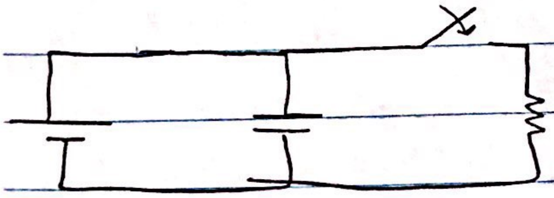
$P \rightarrow R_e\{S\}$, $Q \rightarrow I_m\{S\}$

$S = V I^*$
$= |V_{rms}| \angle \theta \quad I_{rms} \angle -\theta$
$= |V_{rms}| \, |I_{rms}| \angle \theta - \phi$

$= |V_{rms}| \, |I_{rms}| \, e^{j\theta - \phi}$
$= |V_{rms}| \, |I_{rms}| \, (\cos(\theta - \phi) + j \sin(\theta - \phi))$
$= |V_{rms}| \, |I_{rms}| \, (\cos\theta - \phi) + j \, |V_{rms}| \, |I_{rms}| \sin(\theta - \phi)$
$S = P + jQ$

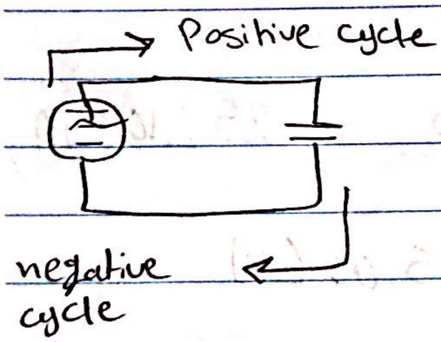
$$P = |V_{rms}|\,|I_{rms}| \cos(\theta - \phi) \quad W$$

$$Q = |V_{rms}|\,|I_{rms}| \sin(\theta \ \phi) \quad VAR$$



P : real Power (heat in R)

Q : Rate of change in stored energy caused for produce em waves



$$Z = R + jX$$

$$Z = \sqrt{R^2 + X^2} \ \angle \tan^{-1} \frac{X}{R}$$

$X = |Z| \sin\theta$, $R = |Z| \cos\theta$

$$S = P + jQ$$

$$|S| = \sqrt{P^2 + Q^2} \ \angle \tan^{-1} Q/P$$

$P = |S| \cos\theta$

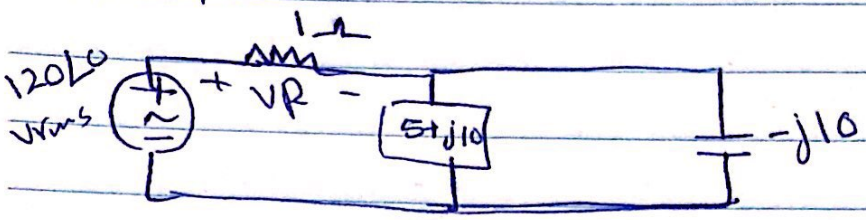
$Q = |S| \sin\theta$

Complex power is a phasor (mag + angle)

apparent Power : $|S| \rightarrow VA$

$|S| = |V_{rms}|\,|I_{rms}|$

Example :



$S = I^* V$

$I = \dfrac{120 \angle 0}{1 + ((5+j10) \,||\, (-j\,10))}$

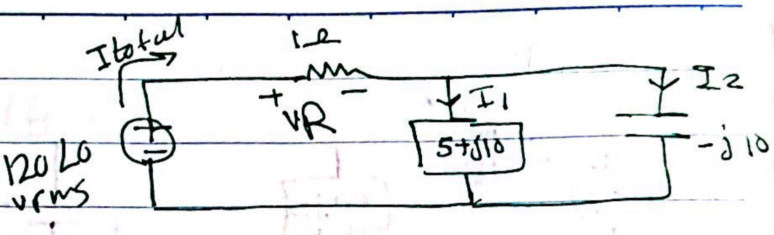
$I = \dfrac{120 \angle 0}{1 + \dfrac{100 - j5}{5}} = 5.16 \angle 25.46 \text{ A}$

$V_R = IR = (5.16 \angle 25.46) * 1 \text{ volt}$

$S(1\Omega) = V I^* = 5.16 \angle 25.46 * 5.16 \angle -25.46$

$= 26.6 \angle 0 = 26.6 + j0 \text{ VA}$

• Example:



$S = V I^k$

$$I = \frac{120\angle 0}{1 + ((5+j10) \parallel (-j10))}$$

$V_R = I\,R = (5.16 \angle 25.46) * I$ Volt

$S(1\Omega) = V I = 5.16 \angle 25.46 * 5.16 \angle 25.46$

$= 26.2\angle 0 = 26.6 + j0$ VA

For capacitor

$$I_2 = \frac{5+j10}{5} * I_{total} = 11.53 \angle 88.9 \text{ A}$$

$V(-j10) = -j10 * I_2 = 115.3 \angle -1.1$ V

$S = 115.3 \angle -1.1 * 11.53 \angle 88.9 = 1333.1 \angle -90$ A

$= 0 - j\,1331$ VA $= -j\,1331$ VA

For inductor

$I_2 = I_{tot} - I_2 = 10.31 \angle -64.53$ A

$V = V_{capacitor} = 115.3 \angle -1.1$ v

$S = 115.3 \angle -1.1 * 10.31 \angle 64.53 = 531.5 + j1063$ VA

$S'(source) = VI^* = 120 \angle 0 * 5.16 \angle -25.46 = 558 - j226$ VA

- Capacitor is generated for reactive ~~relative~~ power
- Inductor consumed for reactive power
- Source → Generated for real power
  → ~~Consumed~~ for reactive power
- Resistor consumed real power

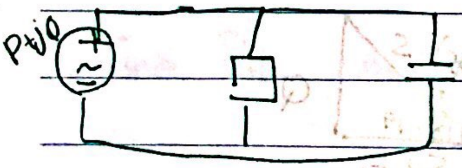
⊖ Generated ⊕ Consumed

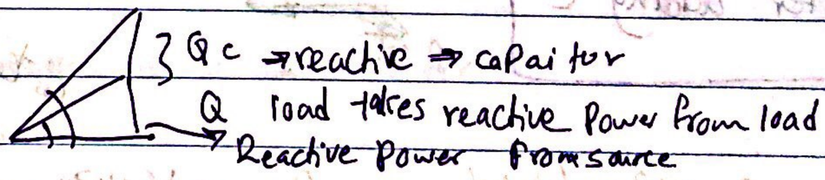
$-j1331$ generated reactive power (For capacitor)
$+j1063$ Consumed reactive power (For inductor)

---

* PF correction:

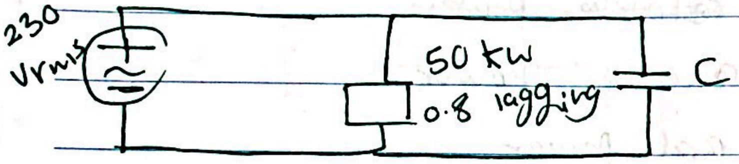
Increasing the PF by installing a capacitive element in the circuit
⇒ should not affect the load voltage or current ($PF = \cos(\theta - \phi)$)





A Load of 50 kW with 0.8 lagging PF is supplied by a 230 Vrms source

it is required to increase the PF = 0.95 lagging



Add capacitor in parallel

Before adding C

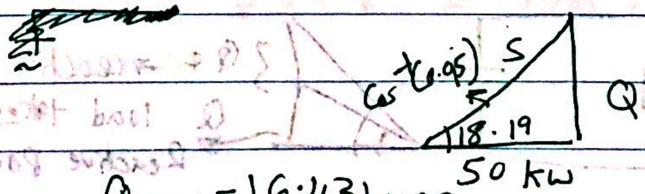
$P = S \cos 36.9$, $\tan 36.9 = Q / P$ $\rightarrow$ $Q = 37.5$ kVAR, $S = 50$

$I_s = I_L \Rightarrow P = V_{rms} I_{rms} PF$

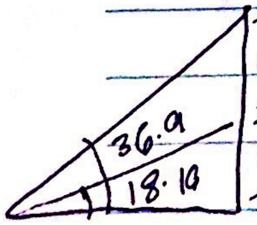
$50 \times 10^3 = 230 \times I \times 0.8$

$I = 271.74$

After adding C



$\tan 18.199 = \frac{Q_{new}}{50 \times 10^3}$, $Q_{new} = 16.43$ kVAR



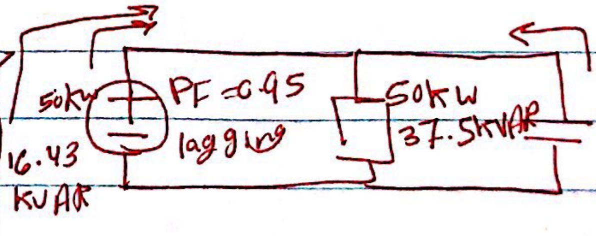
$\rightarrow Q_C = Q_{new} - Q_{old}$

$Q_{old} = 37.5$ kVAR

$Q_{new} = 16.43$ kVAR

$Q_C = -21.07$ kVAR

$|Q_C| = 21.07$ kVAR



$I_s \neq I_L$

$I_s = I_L + I_C$

$Q = |V_{Rms}|\,|I_{rms}| \sin(\cancel{\theta - \phi})$ 90

$|I_C| = \dfrac{Q_C}{V_{rms}} = \dfrac{21.07 \times 10^3}{230}$

$|I_C| = 91.6 A$, $I_C = 91.6 \angle 90$ A

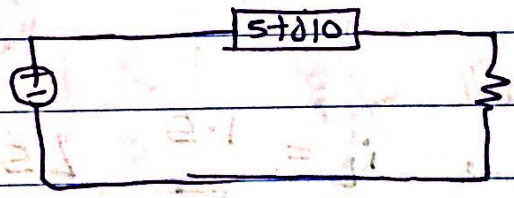
$Q_C = V_C I_C \quad \dfrac{|V_C^2|}{|X_C|} = I_C^2 X_C$

$|Q_C| = \dfrac{V_C^2}{X_C} = w_C V_C^2$

$C = |Q_C| / w V^2 C = 21.07 \times 10^3 / (2\pi F) \times (230)^2 = 1.26 uf$

$I_s = 271.4 \angle -36.9 + 91.6 \angle 90 = 228.78 \angle -18.19$

- Example :



Find R_L that maximize power transfer

$R_L = |Z_{th}| = \sqrt{5^2 + 10^2} = \sqrt{125}$

• Example: Find the PF and P for the load

$i(t) = 4 \cos(100\pi t + 10)$

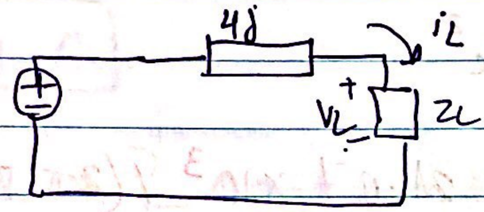
$v(t) = 120 \cos(100\pi t - 20)$

$PF = (-20 - 10) = 0.866$ leading

$P = \frac{120 \times 4}{2} \cos(-30)$

---

* Example



$V_L(t) = 60 \cos(\omega t - 10)$

$i_L(t) = 1.5 \cos(\omega t + 50)$

Find S_L & load PF

$V_L = \frac{60}{\sqrt{2}} \angle -10$, $i_L = \frac{1.5}{\sqrt{2}} \angle 50$

$S_L = V I^* = 45 \angle -60$ VA

Load $PF = \cos(\theta_{VL} - \phi_{iL}) = \cos(-10 - 50) = 0.5$ leading

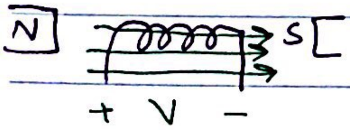
$V_S = V_L + (i_L)(1 + j) = 42.0 \angle -8$ V

$PF(\text{source}) = \cos(-8 - 50) = 0.53$ leading

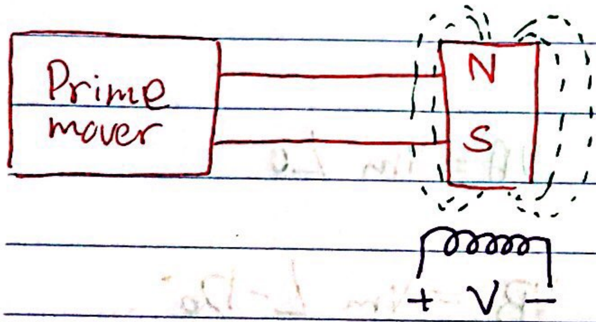
CH 12 Poly Phase system

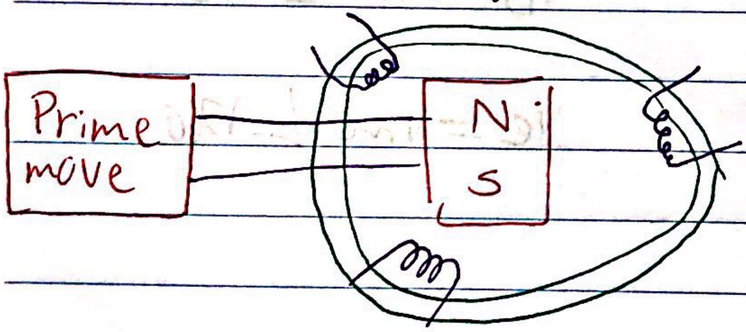
- How do we generate electricity

- Faraday : $V = -N \frac{d\phi}{dt}$



mechanical energy source
magnatic field

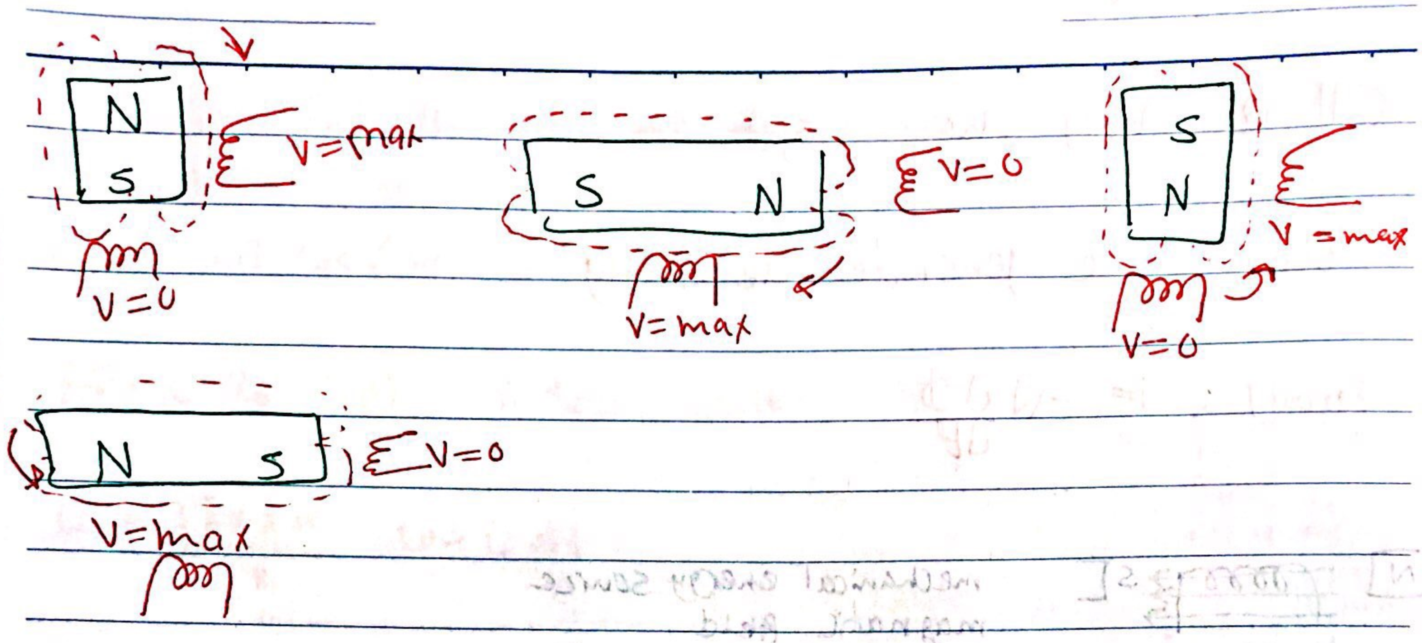




Three Phase system

~~2 loads (efficiency)~~

$|V_A| = |V_B| = |V_C| = N\phi\, V_A \sin \omega t$
different Phase

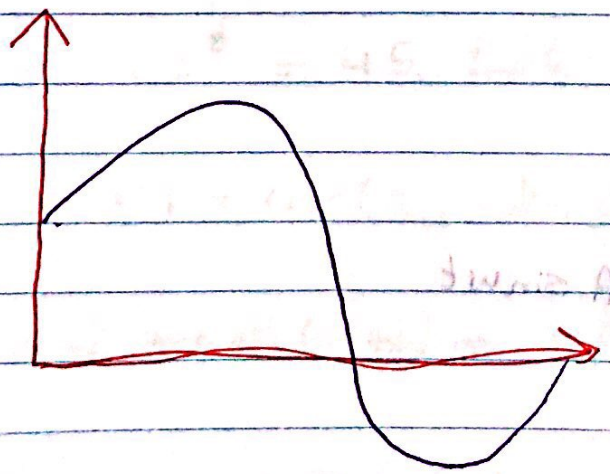


* In General :

$V_A = V_m \cos(\omega t)$ $\qquad$ $V_A = V_m \angle 0$

$V_B = V_m \cos(\omega t - 120)$ $\qquad$ $V_B = V_m \angle -120$

$V_C = V_m \cos(\omega t - 240)$ $\qquad$ $V_C = V_m \angle 120$



14/10/2018

Y - Y connected balanced systems

* The voltage across the load in Y-connected load is line-neutral voltage, also called phase to connect neutral voltages.

* A load can be connected between two lines (phases) & the voltage across this load is then (line - line voltage) or (phase - phase) voltage & can be calculated as follows

$\boxed{V_{AB} = V_{AN} - V_{BN}} = V_m \angle 0 - V_m \angle -120$

$= V_m - V_m \cos(-120) - jV_m \sin(-120)$

$= V_m + \frac{1}{2} V_m + j \frac{\sqrt{3}}{2} V_m = \frac{3}{2} V_m + j \frac{\sqrt{3}}{2} V_m$

$= \sqrt{3} V_m \left[\frac{\sqrt{3}}{2} + j \frac{1}{2} \right] = \boxed{\sqrt{3} V_m \left[\cos 30 + j \sin 30 \right]}$

$= \boxed{\sqrt{3} V_m \angle 30 \text{ V}}$

$V_{BC} = V_{BN} - V_{CN} = V_m \angle -120 - V_m \angle 120$

$= V_m \cos(-120) + j V_m \sin(-120) - V_m \cos(120) - j V_m \sin(120)$

$= -j\sqrt{3} V_m = \sqrt{3} V_m \angle -90$

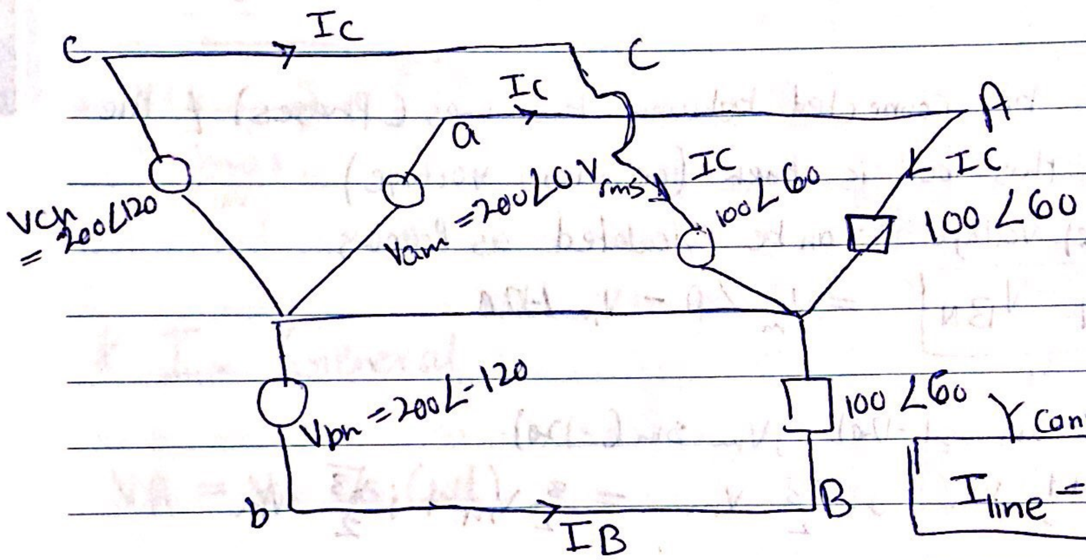
$V_{CA} = V_{CN} - V_{AN} = V_m \angle 120 - V_m \angle 0$

$= V_m \cos 120 + j V_m \sin 120 - V_m = \frac{1}{2} V_m + j \frac{\sqrt{3}}{2} V_m - V_m$

$= \frac{-3}{2} V_m + j \frac{\sqrt{3}}{2} V_m = \sqrt{3} V_m \left[\frac{-\sqrt{3}}{2} + j \frac{1}{2} \right]$

$= \sqrt{3} V_m \left[\cos 150 + j \sin 150 \right] = \sqrt{3} V_m \angle 150 \text{ V}$

Ex) For a Y-Y connected balanced system in the figure Find the phase & line currents & the total active power absorbed by the load (3ϕ)



Y connected loads

$$\boxed{I_{line} = I_{phase}}$$

Find the Phase-Phase voltages

$V_{AB} = \sqrt{3} V_{an} \angle 30 = 200\sqrt{3} \angle 30$

$V_{BC} = \sqrt{3} V_{bn} \angle 30 = 200\sqrt{3} \angle -90$

$V_{CA} = \sqrt{3} V_{cn} \angle 30 = 200\sqrt{3} \angle 150$

Phase Current = Line Current

$I_a = \frac{V_{an}}{Z_Y} = \frac{200\angle 0}{100\angle 60} = 2\angle -60 \ A_{rms}$

$I_b = 2\angle -180 \ A_{rms}$

$I_c = 2\angle 60 \ A_{rms}$

~~P~~ *

$*P_{total} = P_A + P_B + P_C$

$\cdot P_A = V_{AN} \, I_a \cos(\theta)$ $\quad \theta_{vm} - \phi_I$

$= 200 * 2 * \cos(0 + 60) = 200 W$

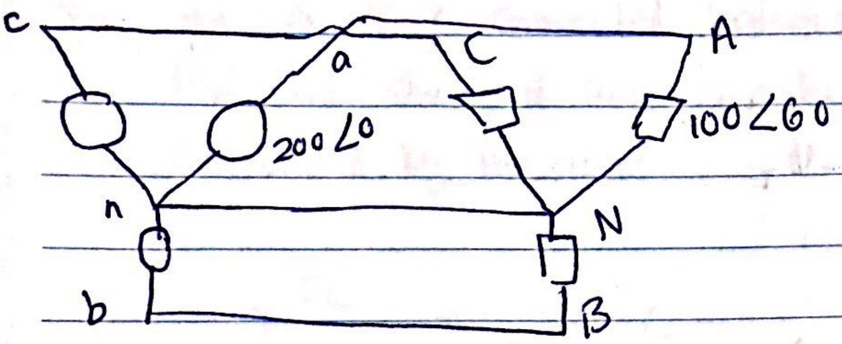
$\cdot P_B = V_{BN} I_B \cos(\theta)$ $\quad \theta_{vm} - \phi_I$

$= 200 \times 2 \times \cos(-120 + 180) = 200 W$

$\cdot P_C = V_{CN} \times I_C = 200 W$

$P_{total} = 600 W$

16/10/2018



$P = P_A + P_B + P_C = 600\,W$

$\underset{tot}{P(t)} = P_A(t) + P_B(t) + P_C(t)$

$V_{AN} = 200\angle 0 \rightarrow V_A(t) = 200\sqrt{2}\cos(\omega t)$

$V_{BN} = 200\angle -120 \Rightarrow V_{BN} = 200\sqrt{2}\cos(\omega t - 120)$

$V_{CN} = 200\sqrt{2}\cos(\omega t + 120)$

$i_A(t) = 2\sqrt{2}\cos(\omega t - 60)$

$i_B(t) = 2\sqrt{2}\cos(\omega t - 180)$

$i_C(t) = 2\sqrt{2}\cos(\omega t + 60)$

$P_A(t) = V_{AN}(t)\ i_a(t) = 800[\cos(\omega t)\cos(\omega t - 60)]$

$= \frac{800}{2}[\cos(2\omega t - 60) + \cos(60)]$

$= 200 + 400\cos(2\omega t - 60)\ W$

$P_B(t) = V_{BN}(t) + i_B(t) = 800\cos(\omega t - 120)\cos(\omega t - 180)$

$= 200 + 400\cos(2\omega t + 60)\ W$

$P_C(t) = V_{cn}(t)\ i_c(t) = 800\cos(\omega t + 120)\cos(\omega t + 60)$

$= \frac{800}{2}[\cos(2\omega t + 180) + \cos(60)] = 200 + 400\cos(2\omega t + 180)\ W$

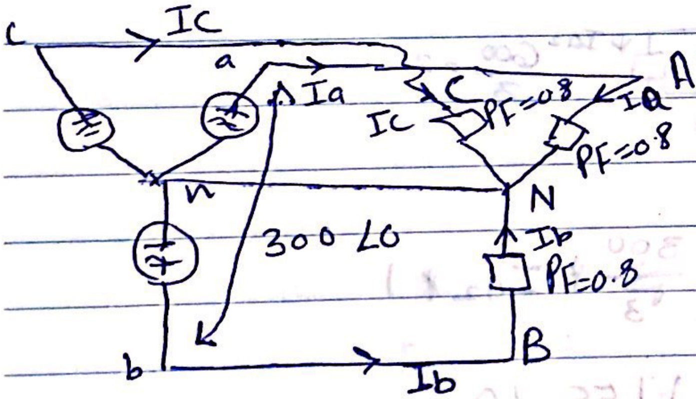
$P_{tot} = P_A(t) + P_B(t) + P_C(t) = \cancel{600 + 400\cos(2\omega t - 60) + 400\cos(\omega t + 60)}$

$= 600 + 400\cos(2\omega t - 60) + 400\cos(\omega t + 60) + 400\cos(2\omega t + 180)$ W

$W_Y = VI = P$

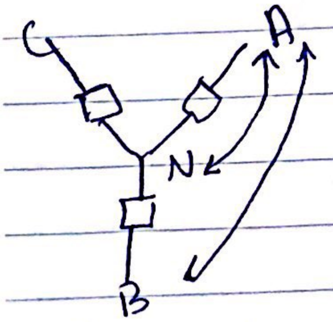
$P_{total} = 600\,W$

Ex A balanced 3ϕ system with $V_{ab} = 300\angle 30$ V & ~~applied~~ supplying a balanced Y-connected load with 1200 W @ & a leading PF of 0.8, find the line currents & the Pre-phase load independance



$V_{ab} = 300\angle 30$

$V_{an} = \frac{300}{\sqrt{3}} \angle -30\ V_{rms}$



$V_{ab} = \sqrt{3}\, V_{an} \angle 30$ (Y connected) $V_{LL} = \sqrt{3}\, V_{LN} \angle 30$

$P_A = V_{AN} I_a \cos(\theta_{V_{AN}} - \phi_{I_a}) = V_{AN} I_a PF$

$400 = \frac{300}{\sqrt{3}} * |I_a| * 0.8$

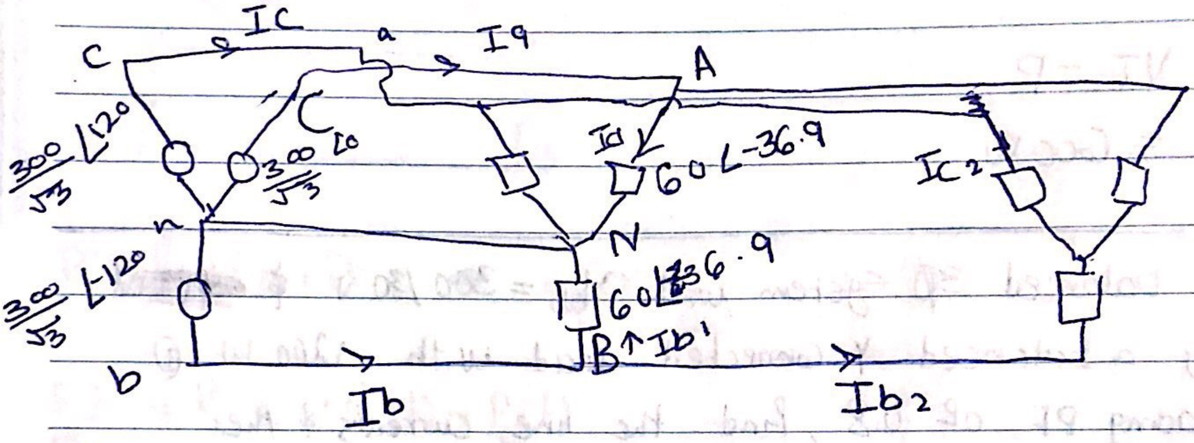
$|I_a| = 2.89\ A_{rms}$, $I_a = 2.89 \angle +\cos^{-1}(0.8) = 2.89 \angle 36.9$ A

I leading V

$I_{line\ cal} = 2.89 \angle 36.9$

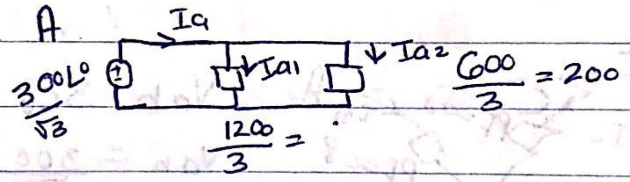
$Z_y = \frac{V_{AN}}{I_A} = \frac{\frac{300}{\sqrt{3}} \angle 0}{2.89\angle 36.9} = 60 \angle -36.9\ \Omega$

Ex) A balanced 600 W Y-connected load is added in Parallel to the Previous system, find the new line current



* Single Phase equivalent circuit

→ Study Phase A



$P_{A2} = V_{AN} * I_{a2}$ PE $\Rightarrow 200 = \frac{300}{\sqrt{3}} * I_{a2} * 1$

$|I_{a2}| = 1.155$ A $\Rightarrow I_{a2} = 1.155 \angle 0$

$I_a = I_{a1} + I_{a2}$

$= 2.89 \angle 36.9 + 1.155 \angle 0$

$= 3.87 \angle 26.6$ A

The real power supplied by the source is

$P_A = V_s I_s \cos(\theta - \phi_{I_s}) = \frac{300}{\sqrt{3}} * 3.87 \cos(0 - 26.6) = 600$ W

total Source Power = 1800 W

$\boxed{P_{3\phi} = P_{1\phi}}$