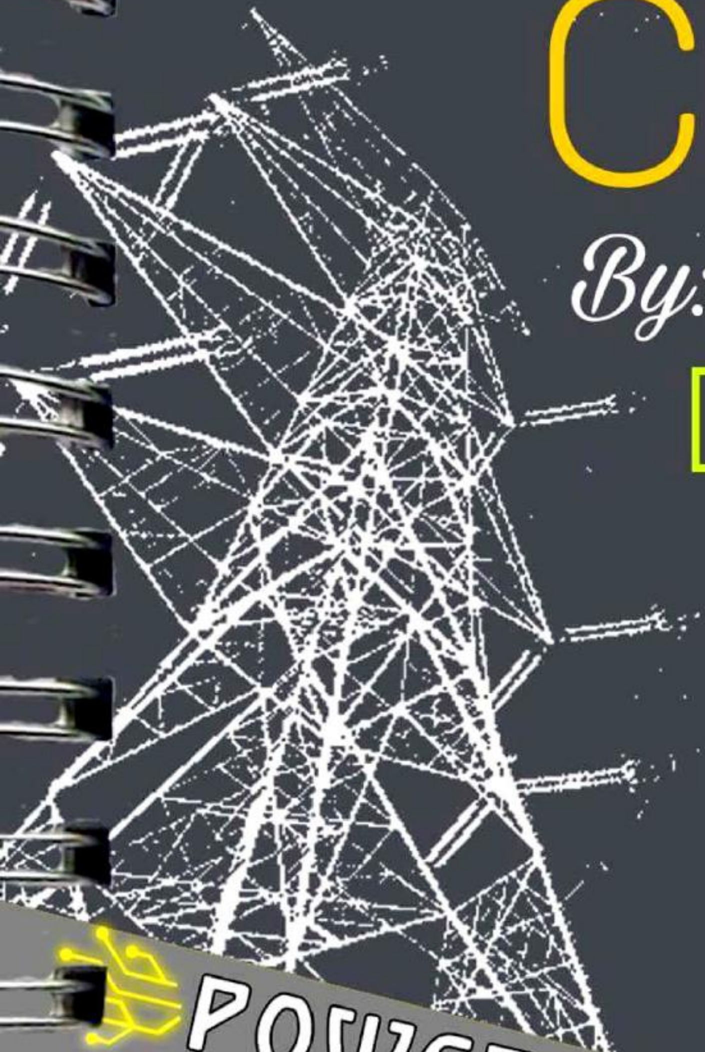
Circuits2

By: Mhmd Abuhashieh

Dr.Mhmd Alhajj



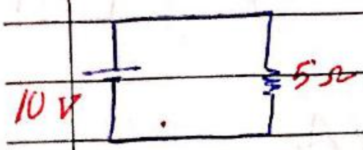
POWERUNIT

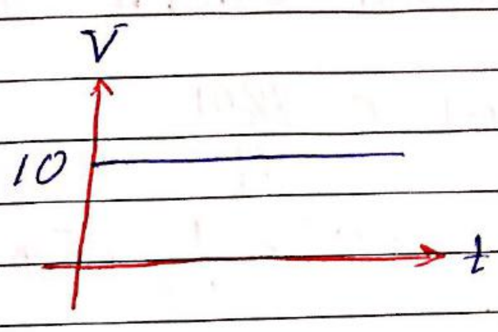
Fall016

No.

Introduction:

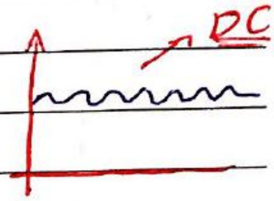
$P = I * V$

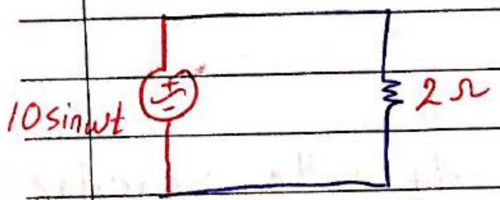




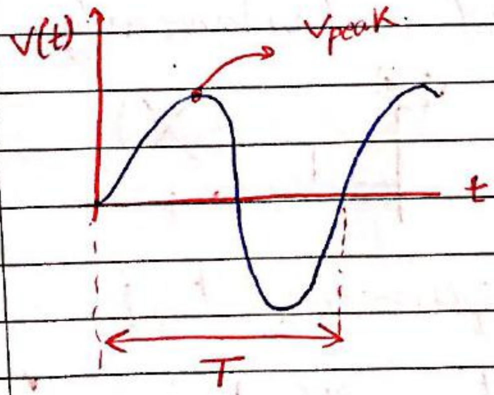
$\Rightarrow I = 2A$

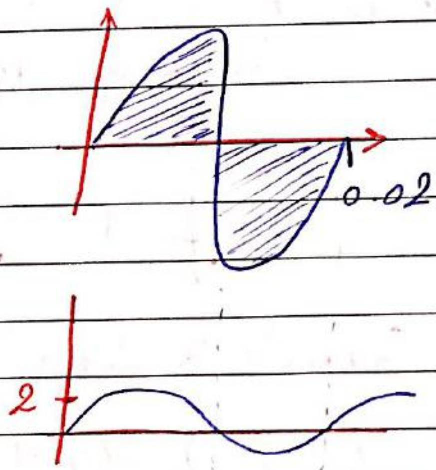
$P = (2)(10) = 20\ W$





$I = \frac{V}{R} = 5 \sin wt$

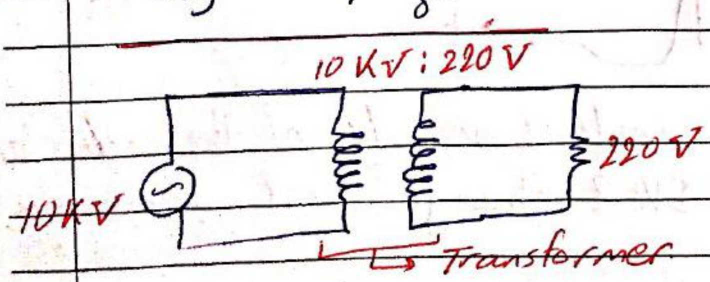


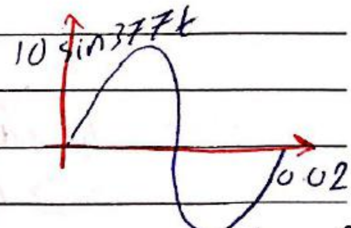


average $P = VI = 10 * 2 = 20$

$P(0.05) = 2 * 10 = 20$

$$V_{avg} = \frac{1}{T}\int_0^T 10 \sin wt \ dt = \frac{1}{0.02}\int_0^{\frac{1}{2}T} 10 \sin 377t \ dt = 0$$





$T = \frac{1}{f} = \frac{2\pi}{w}$

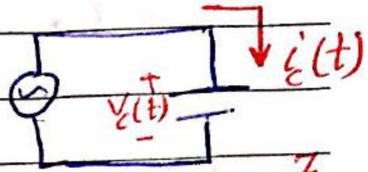
$Z = R + jX$

Impedence — Resistance — Reactance.

No.

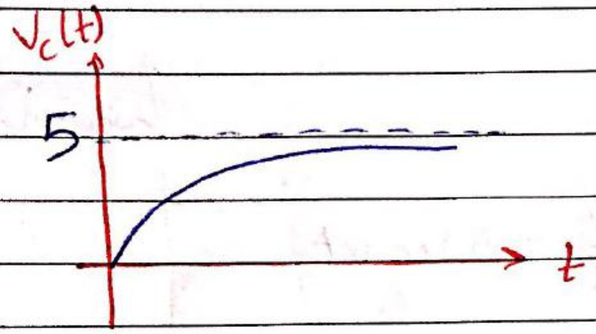
※ Calculate & sketch $i_c(t)$ when $V_c(t) = 5\left(1 - e^{-t/10^{-6}}\right)$ V

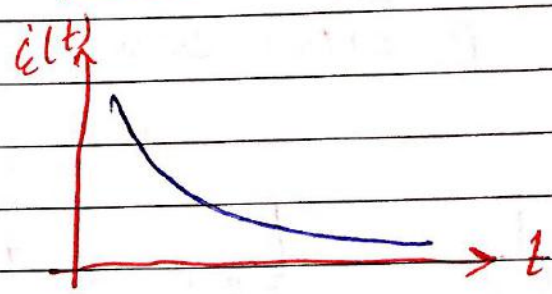
$$i_c(t) = C\,\frac{dV_c(t)}{dt}$$



$$= 10^{-7} * \frac{1}{10^{-6}} * 5 * e^{-t/10^{-6}} = \boxed{0.5\,e^{-t/10^{-6}}}\ \text{A.}$$

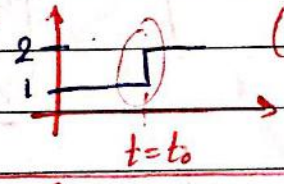
$C = 10^{-7}$



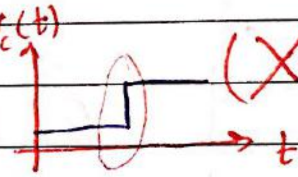


*Note: I_c can be changed instantaneously in the capacitor.
⟹ V_c can NOT be changed instantaneously.

i.e:



(correct)

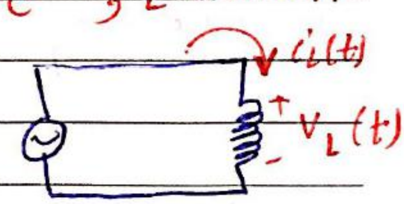


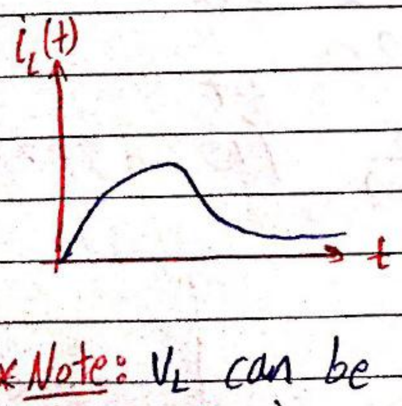
(X) can't be.

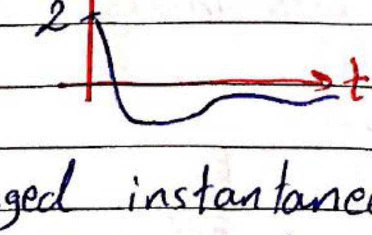
※ Calculate & sketch $V_L(t)$ when $i_L(t) = 20t\,e^{-2t}$, $L = 0.1$ H

$$V_L(t) = L\,\frac{di_L(t)}{dt} = 0.1 * \left[-40e^{-2t}\,t + 20e^{-2t}\right]$$

$$= \boxed{2e^{-2t}(1-2t)}\ \text{V.}$$



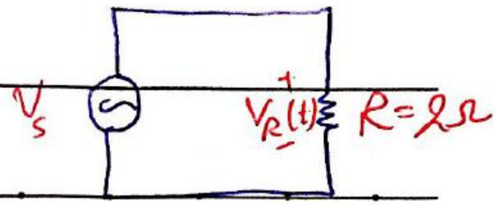




*Note: V_L can be changed instantaneously in the inductor.
⟹ i_L can NEVER EVER change instantaneously.

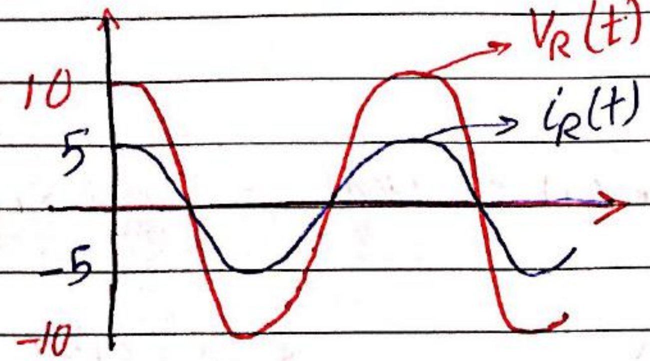
No.

Ex.

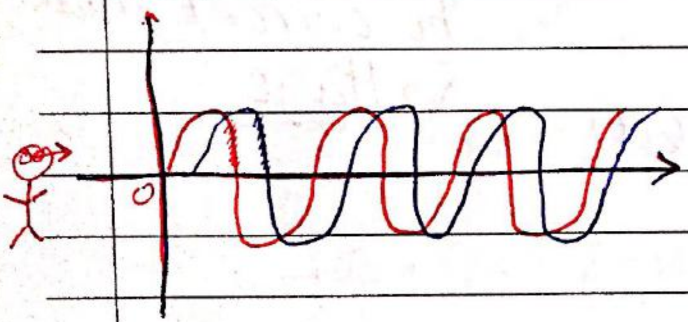


$i_R(t) = V_R(t)/R = \boxed{5\cos\omega t}$

$V_s(t) = 10\cos\omega t.$



* phase shift: is a compare between two waves form.



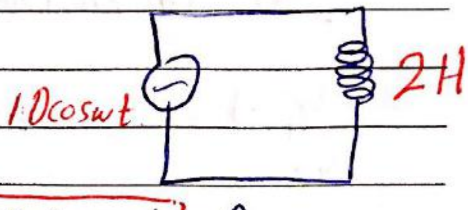
* The first wave interact with x-axis $\Rightarrow$ it will be the lead to the otherone.

$\Rightarrow$ The red wave form leads the blue wave form.

** Resistor voltage & current are "inphase".

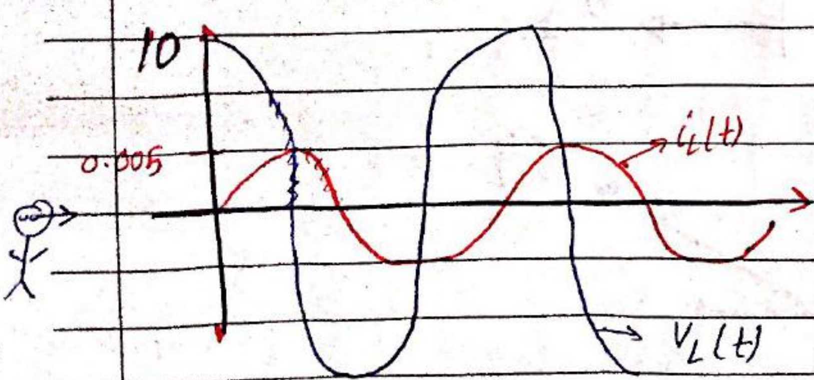
** Find the inductor current if $I_L(0^-) = $ Zero , $\omega = 1000$

$V_L(t) = L\dfrac{di_L}{dt}$



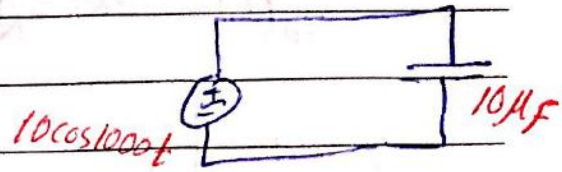
$\Rightarrow i_L(t) = \dfrac{1}{L}\int_0^t V_L(t')\,dt' + i_L(0^-)$

$= \dfrac{1}{2} * 10 * \dfrac{1}{1000}\sin 1000t = \boxed{0.005\sin 1000t}$ A.



* In inductor the voltage leads the current by $\left(\frac{\pi}{2}\right)$.

※ Find $i_c(t)$, $i_c(0^-) = 0$, $V_c(0) = 0$



$$i_c(t) = C\,\frac{dV_c(t)}{dt}$$

$$= 10^{-5} * 10 * 1000 * (-1)\sin 1000t = \boxed{-0.1\sin(100t)}\ A$$

* In capacitor the current leads voltage.

*Phasors:

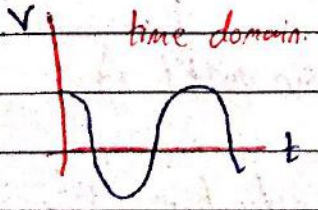
$V_1(t) = 340\cos(100\pi t) \Rightarrow V_1 = 340\angle 0^\circ$

$i(t) = 10\cos(100\pi t - 45^\circ) \Rightarrow I = 10\angle -45^\circ$

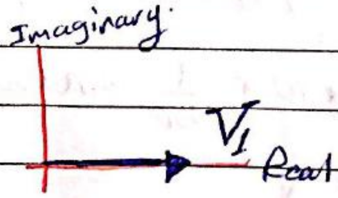
$i(t) = 20\cos(100\pi t - 10^\circ) \Rightarrow I = 20\angle -10^\circ$

$V_2(t) = 340\cos(100\pi t + 120^\circ) \Rightarrow V_2 = 340\angle +120^\circ$

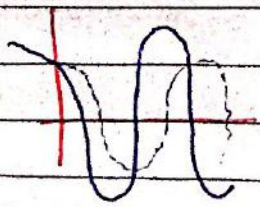
$V_1(t) = \cos\omega t = 1\cos(\omega t + 0)$



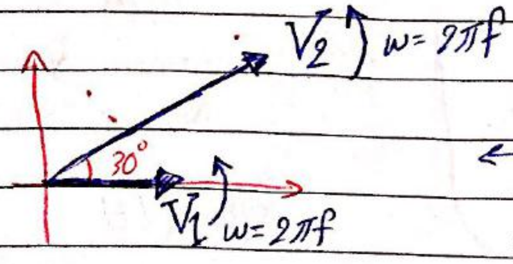
⇒



$v_2(t) = 3\cos(\omega t + 30^\circ)$



⇒



V_2 reach x-axis firstly.

V_2 leads V_1.

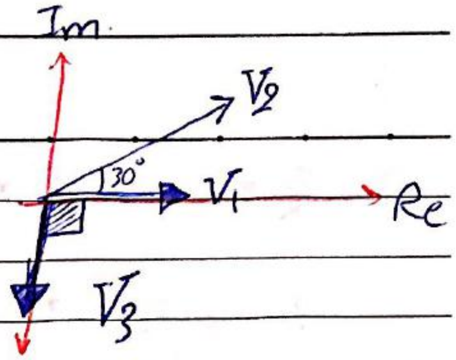
V_2 leading.

V_1 lagging.

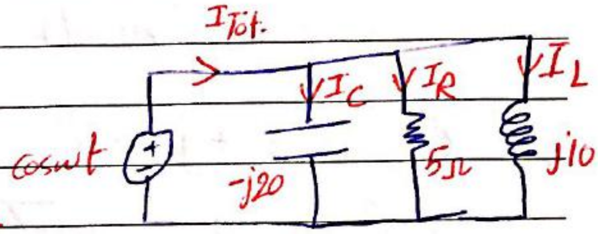
No.

$V_3(t) = 2\sin\omega t$

$= 2\cos\left(\frac{\pi}{2} - \omega t\right) = \underline{2}\cos\left(\omega t - \frac{\pi}{2}\right)$



Ex. Draw the currents phasors:



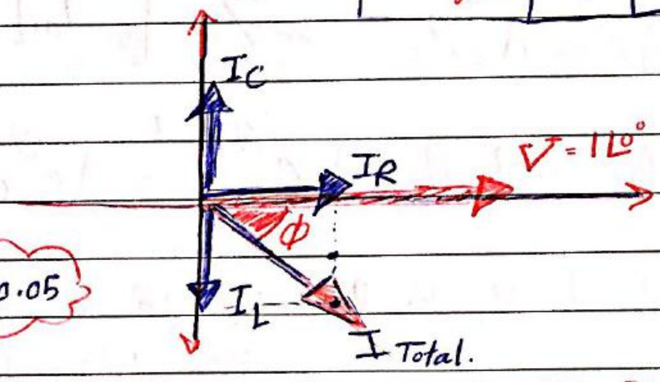
$v(t) = \cos\omega t \Rightarrow V = 1\angle 0°$

$I_R = \frac{1\angle 0°}{5} = \boxed{0.2\angle 0°}$

$I_C = \frac{1\angle 0°}{-j20} = \boxed{j0.05}$

$I_L = \frac{1\angle 0°}{j10} = \boxed{-j\,0.1}$

$I_{Total} = I_C + I_R + I_L = 0.2 - j0.05$



$\phi = \tan^{-1}\left(\frac{-0.05}{0.2}\right)$

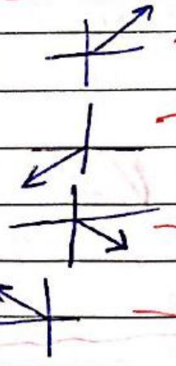
put in your mind in what quarter ϕ in.

$\tan^{-1}\left(\frac{1}{2}\right)$ → $\tan^{-1}\left(\frac{1}{2}\right)$

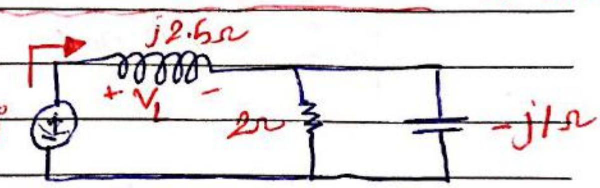
$\tan^{-1}\left(\frac{-1}{-2}\right)$ → $\pi \pm \tan^{-1}\left(\frac{1}{2}\right)$

$\tan^{-1}\left(\frac{-1}{2}\right)$ → $-\tan^{-1}\left(\frac{1}{2}\right)$

$\tan^{-1}\left(\frac{1}{-2}\right)$ → $-\pi \pm \tan^{-1}\left(\frac{1}{2}\right)$

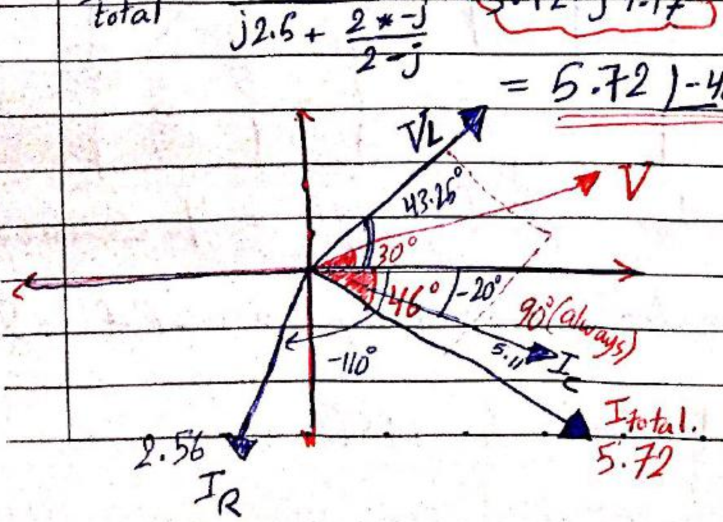


Ex. Draw the phasors:



$I_{total} = \frac{10\angle 30°}{j2.5 + \frac{2 * -j}{2 - j}} = 3.92 - j4.17$

$= \underline{5.72}\,\angle -46.75°$



* since V leads $I_{tot.}$ the main part in the circuit is the inductor.

for $V_L = I_L * j2.5$

$V_L = 14.3\angle 43.25°$ ⇒

* I_R is inphase just with V_R .

No.

$\Rightarrow I_R = 5.72\angle 46.75^\circ * \left[\frac{-j1}{2-j1}\right] = \boxed{2.56\angle -110^\circ}$

$I_C = 5.72\angle 46.75^\circ * \left[\frac{2}{2-j1}\right] = \boxed{5.11\angle -20^\circ}$

* Euler's Equation:

$\boxed{e^{j\theta} = \cos\theta + j\sin\theta}$

$A e^{j\theta} = A\angle\theta \Rightarrow \begin{cases} Re[A e^{j\theta}] = A\cos\theta \\ Im[A e^{j\theta}] = jA\sin\theta \end{cases}$

$\underline{A\cos(\omega t+\theta)} = A\, e^{j\theta + j\omega t} = A e^{j\theta} e^{j\omega t} = \underline{A e^{j\theta}}$

Ex * Find $V_1 + V_2$? $V_1 = 15\cos(\omega t + \frac{\pi}{4})$

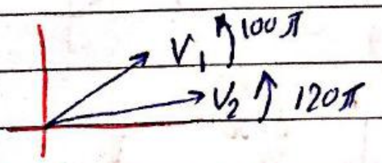
$V_2 = 10\cos(\omega t + \frac{\pi}{6})$

$\Rightarrow V_1 = 15\angle\frac{\pi}{4} = 15\cos\frac{\pi}{4} + j15\sin\frac{\pi}{4}$

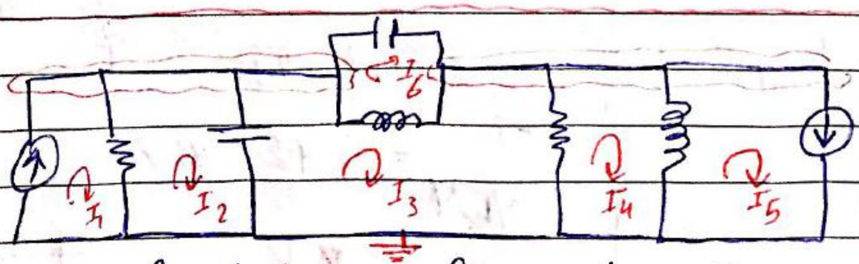
$V_2 = 10\angle\frac{\pi}{6} = 10\cos\frac{\pi}{6} + j10\sin\frac{\pi}{6}$

Ex: $V_1 = 15\cos(100\pi t + \frac{\pi}{4}) = 15\cos(100\pi) * \cos\frac{\pi}{4} - 15\sin 100\pi t \sin\frac{\pi}{4}$

$V_2 = 10\cos(120\pi t + \frac{\pi}{6})$



*we can't add them if ω different.



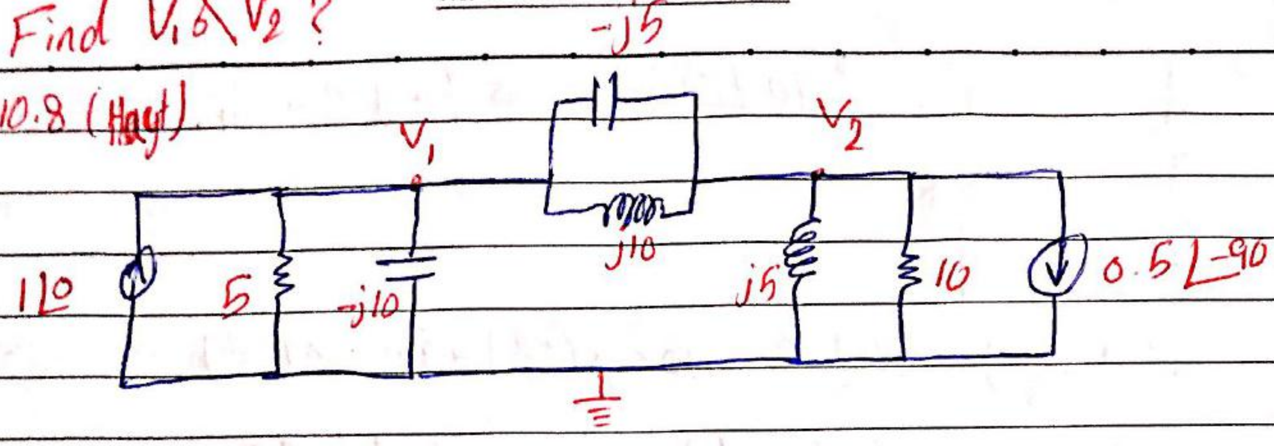
mesh analysis: # of meshes − # of current sources = 4 variables.

nodal analysis: # of nodes − # of voltage sources − Ref. = 2.

nodal best to use.

Find V_1 & V_2 ?

Ex 10.8 (Hayt)



⇒ Nodal analysis:

at V_1:

$$1\angle 0 + \frac{0-V_1}{5} + \frac{0-V_1}{-j10} + \frac{V_2-V_1}{-j5} + \frac{V_2-V_1}{j10} = 0$$

at V_2:

$$-0.5\angle -90 + \frac{0-V_2}{10} + \frac{0-V_2}{j5} + \frac{V_1-V_2}{-j5} + \frac{V_1-V_2}{j10} = 0$$

solving:

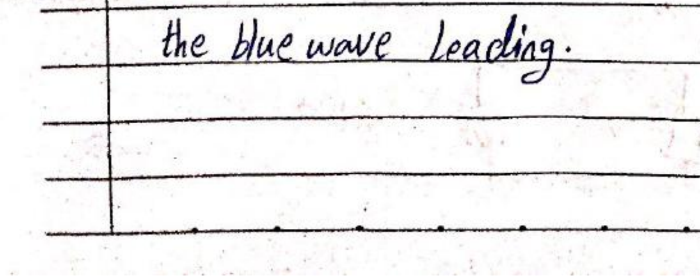
$V_1 = 2.24\angle -63.4$

$V_2 = 4.47\angle 116.6$

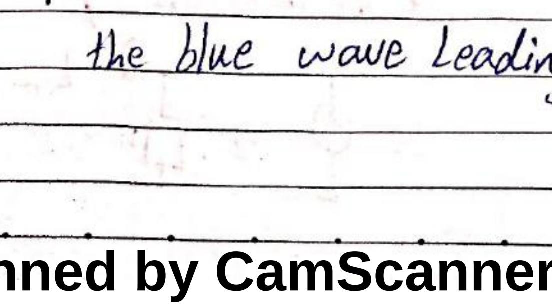
⇒ v_1 in time domain: $v_1(t) = 2.24\cos(\omega t - 63.4)$ volt.

Time domain	phasor domain
R	R
L	$j\omega L$
C	$1/j\omega C$
$V_L(t) = L\,\dfrac{di_L(t)}{dt}$	$V_L = j\omega L\, I$
$i_c(t) = C\,\dfrac{dV_c(t)}{dt}$	$I_C = j\omega C\, V_C$

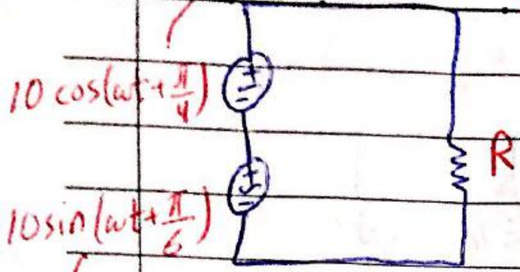
* phase shift:



the blue wave Leading.



the blue wave Leading.

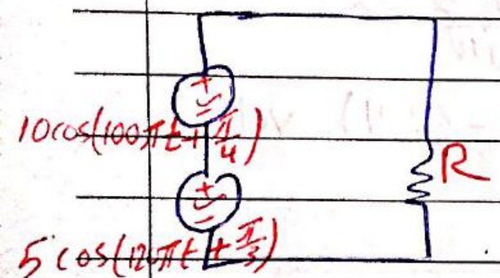


$= 10\angle\frac{\pi}{4} = 10\cos\frac{\pi}{4} + j\,10\sin\frac{\pi}{4}$

$\rightarrow 10\cos(\omega t - \frac{\pi}{3}) = 10\angle\frac{-\pi}{3} = 10\cos(\frac{-\pi}{3}) + j\,10\sin(\frac{-\pi}{3})$

$\Rightarrow i(t) = \dfrac{10\cos(\omega t+\frac{\pi}{4})}{R} + \dfrac{10\sin(\omega t+\frac{\pi}{6})}{R}$

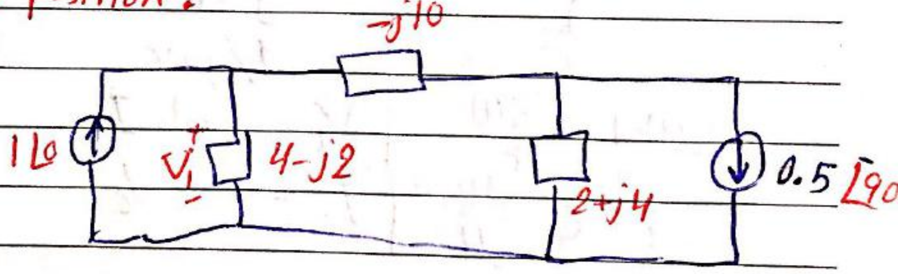
$\Rightarrow I = \dfrac{V}{R}$



$\Rightarrow$ "super position"

$I = I' + I''$, $I' = \dfrac{5\cos(120\pi t+\frac{\pi}{3})}{R}$

Ex. Find V_1 using super position?



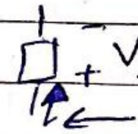
$V_1 = V_1' + V_1''$

Killing current source :

$V_1' = 1\angle0 * \dfrac{(4-j2)(2+j6)}{6-j8}$

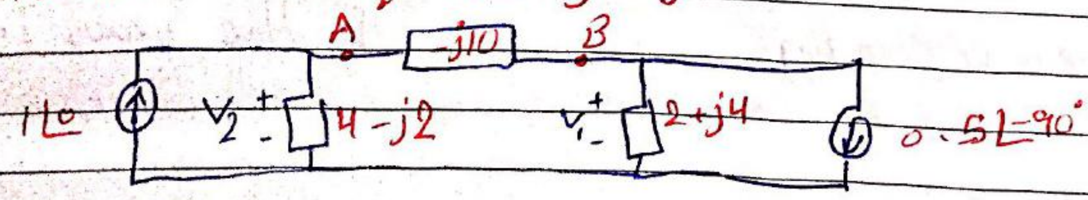
$= \boxed{2-j2}$ volt.

Killing voltage source: $\Rightarrow V_1'' = (0.5\angle-90^\circ)\ \dfrac{(2+j4)}{6-j8} = \boxed{1}$ volt.

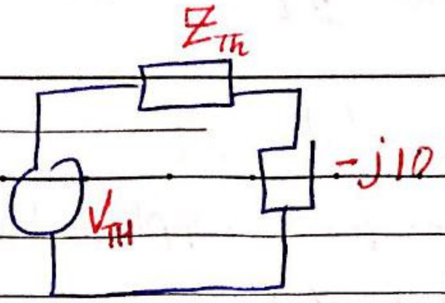


$\Rightarrow V_1 = (2-j2) - 1 = 1-j2$ volt.

Ex. Find the thevenin eq. seen by $-j10$:



$\Rightarrow$



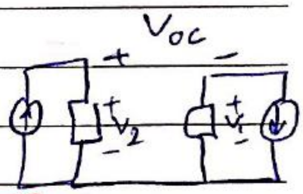
No.

$\Rightarrow$ The expected circuit:

~~since there is No dependent sources: (Kill the sources)~~

$Z_{TH} = \frac{V_{o.c}}{I_{SC}}$

$Z_{TH} = (4-j2) + (2+j4) = \boxed{6+j2}\ \Omega$



$\Rightarrow V_{oc} = V_2 - V_1$

$V_1 = -(2+j4)(0.5\angle{-90})$

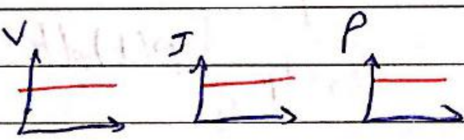
$V_2 = 1\angle 0 * (4-j2)$

$V_{oc} = (4-j2)(1\angle 0) + (2+j4)(0.5\angle{-90}) = \boxed{6-j3}$ volt.

New Subject:

※ Instantaneous Power:

* in DC systems: $P = VI$



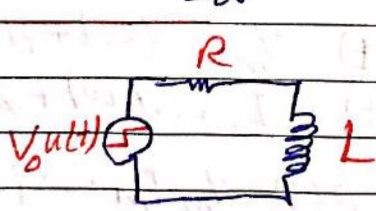
* in AC systems: R: $P(t) = v(t)\, i(t) = i^2(t) * R = \frac{V^2(t)}{R}$

L: $P(t) = i(t)\, v(t) = i(t)\, L \frac{di(t)}{dt} = v(t) * \frac{1}{L} \int_{-\infty}^{t} V(\tau)\, d\tau$

C: $P(t) = i(t)\, v(t) = v(t)\, C \frac{\partial V_c(t)}{\partial t} = i(t) * \frac{1}{C} \int_{-\infty}^{t} i(\tau)\, d\tau$

Ex. given: $v(t) = V_o\, u(t)$

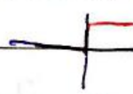
$i(t) = \frac{V_o}{R}\left[1 - e^{-\frac{Rt}{L}}\right] u(t)$



* inst. power supplied by the source:

$P(t) = \frac{V_o^2}{R}\left[1 - e^{-\frac{Rt}{L}}\right] u(t)$

Note: $u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$



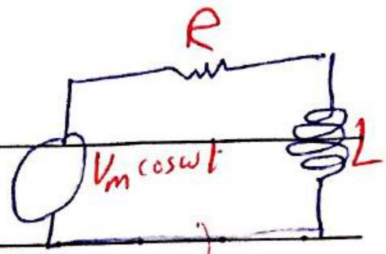
* inst. power absorbed by the resistor:

$P(t) = i^2(t)\, R = \frac{V_o^2}{R}\left[1 - e^{-Rt/L}\right]^2 u(t)$

~~* absorbed power by the inductor:~~

$P(t) = i(t) * L \frac{di(t)}{dt} = \frac{V_o}{R}\left[1 - e^{-Rt/L}\right] * L * -\frac{R}{L} * \frac{V_o}{R} * -e^{-\frac{Rt}{L}} u(t) = \frac{V_o^2}{R}\left(e^{-\frac{Rt}{L}}\right)\left(1 - e^{-\frac{Rt}{L}}\right) u(t)$

*Note: $\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$



Ex. Sinusoidal ckt:

Find the power supplied by the source?

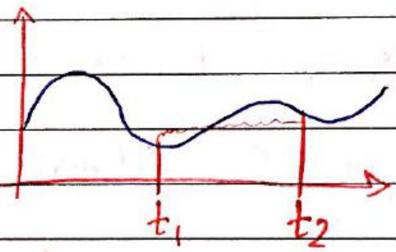
Lets suppose $i(t) = I_m \cos(\omega t + \phi)$

$$p(t) = i(t)\, v(t)$$
$$= V_m \cos\omega t * I_m \cos(\omega t + \phi)$$
$$= \frac{V_m I_m}{2}\left[\cos(2\omega t + \phi) + \cos\phi\right]$$ **

⇒ ** Average Power:

↳ more accurate & more descriptive.

$$\boxed{\underline{P} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} p(t)\,dt}$$ **



⇒ for a periodic signal:

$$\underline{P} = \frac{1}{T}\int_T p(t)\,dt$$ **

Let us assume for a sinusoidal ckt:

$v(t) = V_m \cos(\omega t + \theta)$
$i(t) = I_m \cos(\omega t + \phi)$
For any element.

$$\Rightarrow p(t) = V_m I_m \cos(\omega t + \theta)\cos(\omega t + \phi) = \frac{V_m I_m}{2}\left[\cos(2\omega t + \theta + \phi) + \cos(\theta - \phi)\right]$$

Average Power:

$$P = \frac{1}{T}\int_T p(t)\,dt = \frac{V_m I_m}{2}\cos(\theta - \phi)$$ **

let $T = 2\pi$

$$\hookrightarrow P = \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m I_m}{2}\cos(2\omega t + \theta + \phi)\,dt + \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m I_m}{2}\cos(\theta - \phi)\,dt$$

$$= \frac{1}{2\pi}\frac{V_m I_m}{2}\cos(\theta - \phi) * 2\pi = \frac{V_m I_m}{2}\cos(\theta - \phi)$$

voltage for Z Not V on R.

Ex. $v(t) = 4\cos\left(\frac{\pi}{6}t\right)$, $Z = 2\angle 60^\circ$

* find the average power absorbed by the imbedance?

$V = 4\angle 0^\circ$, $I = \frac{V}{Z} = \frac{4\angle 0}{2\angle 60} = 2\angle -60^\circ$

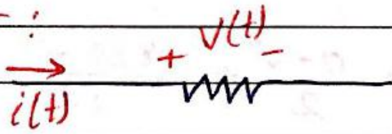
$\Rightarrow P = \frac{1}{2} V_m I_m \cos(\theta - \phi) = \frac{1}{2} \times 4 \times 2 \times \cos(0 - -60) = \boxed{2}\ W$

* find the instantaneous power?

$v(t) = 4\cos\left(\frac{\pi}{6}t\right)$ & $i(t) = 2\cos\left(\frac{\pi}{6}t - 60^\circ\right)$

$\Rightarrow p(t) = 8\cos\left(\frac{\pi}{6}t\right)\cos\left(\frac{\pi}{6}t - 60\right) = 2 + 4\cos\left(\frac{\pi}{3}t - 60\right)$

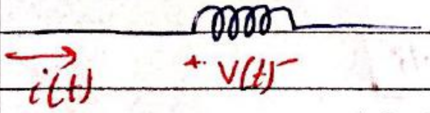
In a Resistor:

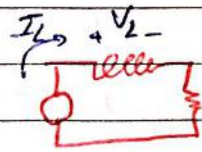


Average Power absorbed in R:

$P = \frac{V_m I_m}{2}\cos(\theta - \phi) = \boxed{\frac{V_m I_m}{2}}$ ** **

In inductor:





always between I_L & V_L here $\underline{\underline{90^\circ}}$

$P = \frac{V_m I_m}{2}\cos(\underbrace{\theta - \phi}_{90^\circ}) = \boxed{0}$ ** **

First 2 weeks

※ average power (R) $= \frac{1}{2} V_m I_m = \frac{1}{2}\frac{V_m^2}{R} = \frac{1}{2} I_m^2 R$

(ex.) Find the average power delivered to the impedance $Z_L = 8 - j11$ when $I = 3 + j4$

** $P_{average} = 0$ for any X.

$P_{avg} = \frac{1}{2} R I_m^2 = \frac{1}{2}(8)(5)^2 = \boxed{100}\ W$

for instantanuous power: $p(t) = i(t)\,v(t)$

$i(t) = 5\cos(\omega t + \phi)$; $\phi = \tan^{-1}\left(\frac{4}{3}\right)$

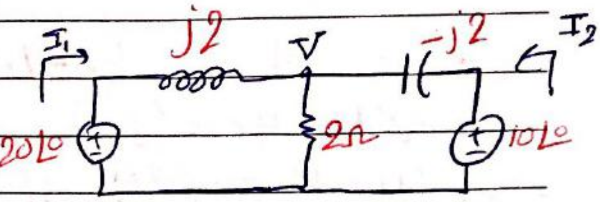
$\Rightarrow V = (8-j11)(3+j4) = 64 - j = 68\angle{-0.84}$

$v(t) = 68\cos(\omega t - 0.84)$

$\Rightarrow P = i(t) * v(t)$

$= 100 + 170\cos(\omega t + 52.26^\circ)$

ex: Find the average power absorbed by elements and generated by sources?



$P_{avg}\Big|_{2j\,\&\,-j2} = \text{Zero}.$

* Nodal analysis:

At V: $\dfrac{20\angle 0 - V}{j2} + \dfrac{0 - V}{2} + \dfrac{10\angle 0 - V}{-j2} = 0$

$\Rightarrow -j10 + \cancel{0.5j\,V} - 0.5V + j5 - \cancel{0.5j\,V} = 0$

$\Rightarrow \boxed{V = 10\angle{-90^\circ}}$

$P_{avg}\Big|_{(2\Omega)} = \dfrac{V^2}{2R} = \dfrac{100}{2*2} = \boxed{25}\ W$

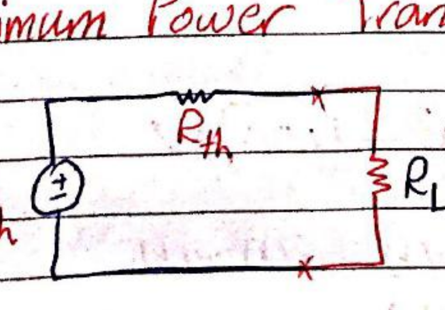
$I_1 = \dfrac{20\angle 0 - 10\angle{-90}}{j2} = \boxed{11.18\angle{-63.43^\circ}}$

$P_{avg}\Big|_{(20V)} = \dfrac{20*11.18}{2}\cos(0 + 63.43^\circ) = \boxed{50}\ W$

$I_2 = \dfrac{10\angle 0 - 10\angle{-90}}{-j2} = \boxed{7.07\angle{-45^\circ}} \Rightarrow P_{avg}\Big|_{(10V)} = \dfrac{7.07*10}{2}\cos(45)$

$= \boxed{25}\ W$

Maximum Power Transfer:



$\rightarrow$ occurs: $R_{th} = R_L$

$\Rightarrow$

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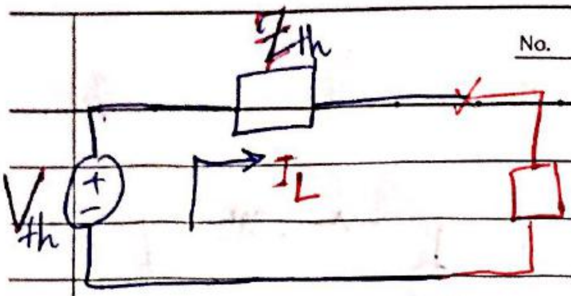
*maximum power transfer: (occurs)

$$Z_{th} = Z_L^*$$



No.

$\Rightarrow$ $Z_{th} = R_{th} + jX_{th}$

$Z_L = R_{th} - jX_{th}$

we know:

$$I_L = \frac{V_{th}}{Z_{th}+Z_L} = \frac{V_{th}}{R_{th}+jX_{th}+R_L+jX_L}$$

also: $$V_L = V_{th} * \frac{Z_L}{Z_L+Z_{th}} = \frac{V_{th}(R_L+jX_L)}{R_L+jX_L+R_{th}+jX_{th}}$$

so, $$\underset{(load)}{P_{avg}} = \frac{1}{2}|V_L||I_L|\cos(\theta-\phi)$$

$$|I_L| = \frac{|V_{th}|}{\sqrt{(R_L+R_{th})^2+(X_L+X_{th})^2}}, \quad \angle I_L = \angle V_{th} - \tan^{-1}\left(\frac{X_{th}+X_L}{R_{th}+R_L}\right) \rightarrow \phi$$

$$|V_L| = \frac{|V_{th}| * \sqrt{R_L^2+X_L^2}}{\sqrt{(R_L+R_{th})^2+(X_L+X_{th})^2}}, \quad \angle V_L = \angle V_{th} + \tan^{-1}\left(\frac{X_L}{R_L}\right) - \tan^{-1}\left(\frac{X_{th}+X_L}{R_{th}+R_L}\right) \rightarrow \theta$$

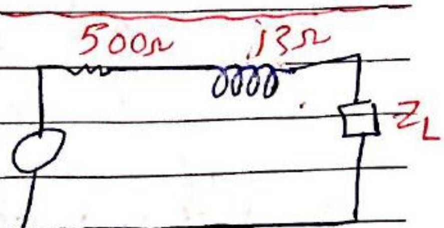
Now: $$\boxed{P = \frac{|V_{th}|^2\sqrt{R_L^2+X_L^2}}{2\left[(R_L+R_{th})^2+(X_L+X_{th})^2\right]} * \cos\left[\tan^{-1}\left(\frac{X_L}{R_L}\right)\right]}$$

$\Rightarrow$ Derive P w.r.t R_L $\Rightarrow$ $\boxed{R_L = R_{th}}$

& Derive P w.r.t X_L $\Rightarrow$ $\boxed{X_L = -X_{th}}$

so $\boxed{Z_{th} = Z_L^*}$ maximize the power.

ex) Find Z_L that maximize the power to the load?

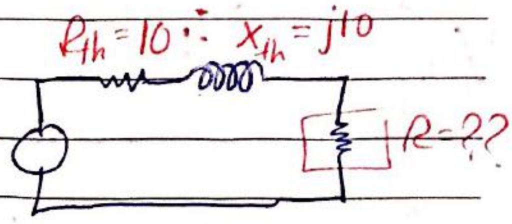


$Z_{th} = 500 + j3$

$Z_L = Z_{th}^* = 500 - j3$

No.

* ~~what is the maximum power transfer for the shown cKt:~~



⇒ what is R that maximize the transferred power ?

$$R_L = \sqrt{R_{th}^2 + X_{th}^2} \Rightarrow R_L = 10\sqrt{2}\ \Omega$$

* Non-periodic signals:

Lets suppose:

$$i(t) = \sin t + \sin \pi t \ ,\ R = 1$$

find the average power?

$$\underline{P} = \frac{1}{T}\int p(t)\,dt \ ;\ p(t) = i^2(t) * R$$

$$\Rightarrow p(t) = (\sin^2 t + 2\sin t \sin \pi t + \sin^2 \pi t)\, R$$

$$\Rightarrow \underline{P} = \frac{R}{T}\int_T \sin^2 t\,dt + \frac{R}{T}\int_T 2\sin t \sin \pi t\,dt + \frac{R}{T}\int_T \sin^2 \pi t\,dt$$

$\sin^2 t = \frac{1}{2} - \underbrace{\frac{1}{2}\cos 2t}_{0}$; $2\sin t\sin\pi t$: using trigonometric rules it will give Zero ; $\sin^2\pi t = \frac{1}{2} - \underbrace{\frac{1}{2}\cos 2\pi t}_{0}$

$$\underline{P} = \frac{R}{T}\left(\frac{T}{2} + \frac{T}{2}\right) = \boxed{1}\ W.$$

* Suppose $i(t) = I_{m_1}\sin\omega_1 t + I_{m_2}\sin\omega_2 t + I_{m_3}\sin\omega_3 t + \dots$

*** $$P_{avg} = \frac{1}{2} I_{m_1}^2 R + \frac{1}{2} I_{m_2}^2 R + \frac{1}{2} I_{m_3}^2 R + \dots$$

$$P_{avg} = \frac{V_{m_1}^2}{2R} + \frac{V_{m_2}^2}{2R} + \dots$$

$i(t)$ R

(ex) find the average power: $i(t) = 3\cos 10t - 4\sin 20t$
$R = 5\Omega$?

$$P_{avg} = \frac{1}{2} I_{m_1}^2 R + \frac{1}{2} I_{m_2}^2 R = \frac{1}{2} * 9 * 5 + \frac{1}{2} * 16 * 5 = \boxed{62.5}\ W$$

→ it is Not (−) always the magnitude +ve.

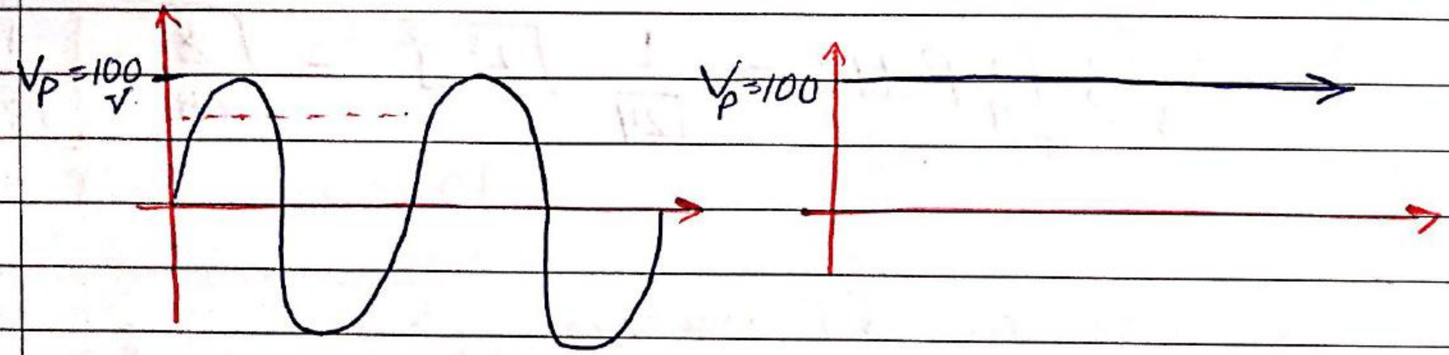
ex. Find the average power:

$i(t) = 2\cos 10t - 3\cos 10t$, $R = 4$

↳ $i(t) = -\cos 10t$ $\Rightarrow P_{avg} = \frac{1}{2}(1)^2 * 4 = \boxed{2}$ W.

ex. $i(t) = 2\cos 10t + 3\sin 10t$ find P_{avg} ?

※ Root Mean Square values [RMS]:

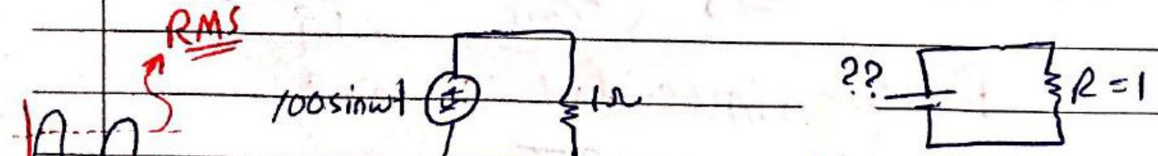


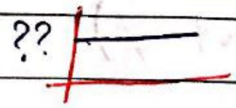
$V_p = ??$

$V_{avg} = \text{zero}$

* Compare between:

RMS





$P = \frac{1}{2}\frac{V_m^2}{R}$

$= 5000 W$

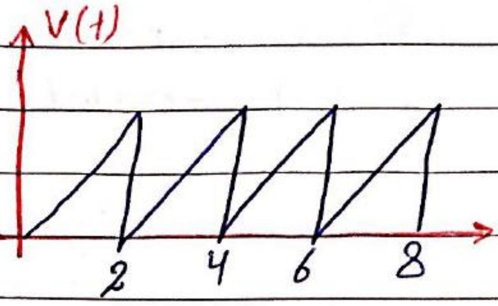
$5000 = \frac{V^2}{R}$

$\Rightarrow \boxed{V = 70.7 \text{ volt}}$

$P_{ac} = P_{dc} \Rightarrow \cancel{\frac{1}{R}} * \frac{1}{T}\int_T v^2(t)dt = \frac{V^2}{\cancel{R}}$

$$\Rightarrow V_{eff} = V_{DC} = V_{RMS} = \sqrt{\frac{1}{T}\int_T v^2(t)dt}$$

(ex.) Find the RMS value for the shown



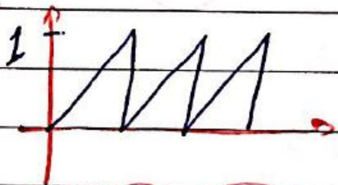
$\Rightarrow T = 2$ sec.

$$V_{rms} = \sqrt{\frac{1}{T}\int_T v^2(t)\,dt}$$

find $v(t)$ $\Rightarrow$ $v(t) = \frac{1}{2}t$ volt.

$$V_{rms} = \sqrt{\frac{1}{2}\int_T \underbrace{\frac{1}{4}t^2\,dt}_{\frac{t^3}{3}}} = \frac{1}{\sqrt{24}}\sqrt{t^3\Big]_0^2} = \sqrt{\frac{8}{24}} = \boxed{\frac{1}{\sqrt{3}}} \text{ volt}$$

* Rms value for ST waveform:



$$I_{rms} = \frac{1}{\sqrt{3}}$$

$\Rightarrow$ in general for ST

$I_m \to$ peak $\Rightarrow$ $I_{rms} = \frac{I_m}{\sqrt{3}}$

* Rms value for a sinusoidal signal:

$i(t) = I_m\cos(\omega t+\phi)$, $T = \frac{2\pi}{\omega}$

$$I_{rms}^2 = \frac{1}{T}\int_T i^2(t)\,dt = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} I_m^2\left[\frac{1}{2} + \overbrace{\frac{1}{2}\cos(2\omega t+2\phi)}^{0}\right]dt$$

$$I_{rms}^2 = \frac{\cancel{\omega}}{\cancel{2\pi}} \times \frac{I_m^2}{2} \times \frac{\cancel{2\pi}}{\cancel{\omega}} \Rightarrow \boxed{I_{rms} = \frac{I_m}{\sqrt{2}}}$$

** Note: Always if the quistion doesn't determine that the value peak or rms $\Rightarrow$ solve as rms.

$v(t) = 10\cos(\omega t+\theta)$

↳ always it is a peak value.

$V = 10$ volt → it is rms value.

No.

* For a sinusoidal signals: [for a Resistor]

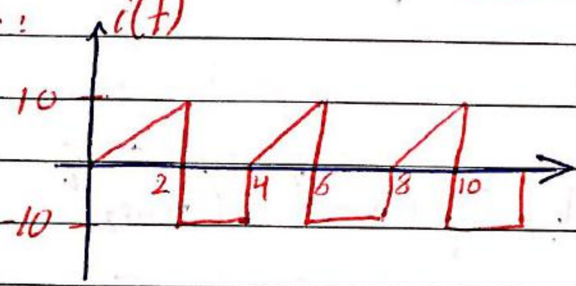
$$P_{avg} = \frac{1}{2} V_m I_m = \frac{1}{2}(\sqrt{2} V_{rms})(\sqrt{2} I_{rms}) = V_{rms} I_{rms}$$

$$\Rightarrow P_{avg} = V_{rms} I_{rms} = V_{eff} I_{eff} \Rightarrow P_{avg} = \frac{V_{rms}^2}{R} = I_{rms}^2 R$$

* In general:

$$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta - \phi) = V_{rms} I_{rms} \cos(\theta - \phi)$$

ex) Find the rms value for:

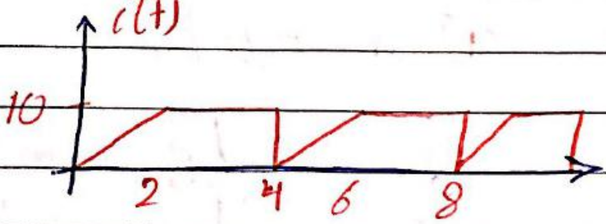


$$T = 4, \quad i(t) = \begin{cases} 5t, & 0 < t < 2 \\ -10, & 2 < t < 4 \end{cases}$$

$$I_{rms}^2 = \frac{1}{4}\left[\int_0^2 25t^2\, dt + \int_2^4 100\, dt\right] = \frac{1}{4}\left[\frac{25(8)}{3} + 200\right]$$

$$\Rightarrow I_{rms} = \frac{1}{2}\sqrt{\frac{25(8)}{3} + 200} = \boxed{8.165\ A}$$

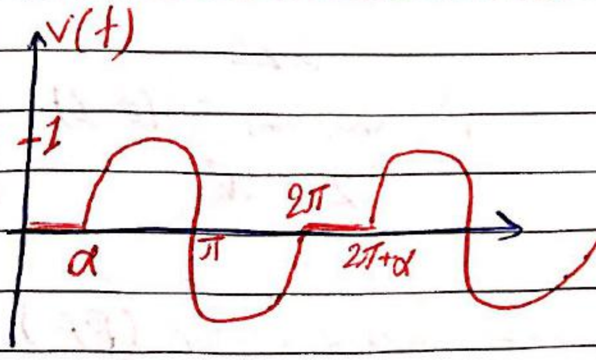
ex.) find rms value for:



$\Rightarrow$ the same as the previous example

$\Rightarrow \boxed{I_{rms} = 8.165\ A}$

ex.) find rms:



$$T = 2\pi, \quad v(t) = \begin{cases} 0, & 0 < t < \alpha \\ \sin t, & \alpha < t < 2\pi \end{cases}$$

$$I_{rms}^2 = \frac{1}{2\pi}\int_\alpha^{2\pi} \sin^2 t\, dt$$

$\hookrightarrow \frac{1}{2} - \frac{1}{2}\cos 2t$

※ Effective value for signals with multiple frequencies:

No.

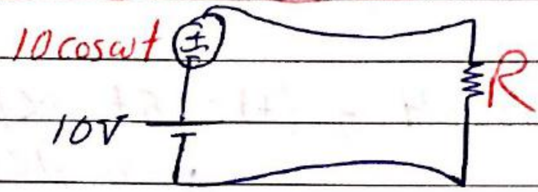
$$P_{avg} = \frac{1}{2}\left(I_{m_1}^2 + I_{m_2}^2 + I_{m_3}^2 + \cdots\right) R$$

$\hookrightarrow (\sqrt{2}\, I_{rms_1})$

$$\Rightarrow \boxed{P_{avg} = \left(I_{rms_1}^2 + I_{rms_2}^2 + \cdots\right) R} \quad ***$$

$$I_{rms} = \sqrt{I_{rms_1}^2 + I_{rms_2}^2 + I_{rms_3}^2 + \cdots} \quad ***$$

ex. Find rms value:



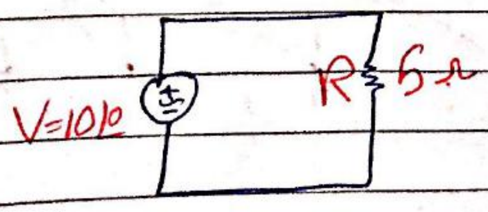
$$V_{rms} = \sqrt{(V_{rms_1})^2 + (V_{rms_2})^2}$$

$$= \sqrt{\left(\frac{10}{\sqrt{2}}\right)^2 + 100} = \boxed{\sqrt{150}} \text{ volt}$$

* if he asked about average power: $P_{avg} = V_{rms}^2 / R$

$$= \boxed{\frac{150}{R}} \text{ watt.}$$

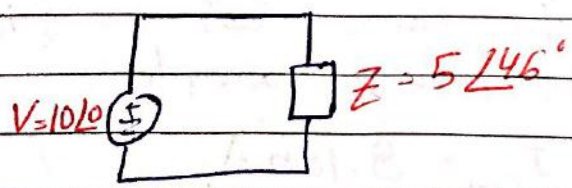
※ Power Factor:



$I = 2\angle 0^\circ$

$P = V_{rms} I_{rms} \cos(\theta - \phi)$

$= \boxed{20\ W}$



$I = 2\angle -45^\circ$

$P = V_{rms} I_{rms} \cos(\theta - \phi)$

$= 10 * 2 * \frac{1}{\sqrt{2}} = \boxed{\frac{20}{\sqrt{2}}\ W}$

$\Rightarrow$ Power factor $\boxed{(PF) = \frac{\text{Real Power}}{\text{Apparent power}}}$

$$= \frac{V_{rms} I_{rms} \cos(\theta - \phi)}{V_{rms} I_{rms}} = \cos(\theta - \phi) = PF$$

No.

* Power factor for a sinusoidal signal:

for $\underline{\underline{R}}$ Loads $\longrightarrow \cos(\theta - \phi) = 1$

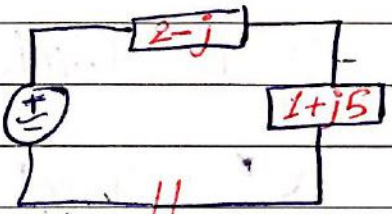
for $\underline{\underline{X}}$ Loads $\longrightarrow \cos(\theta - \phi) = 0$

for RX loads $\longrightarrow [0 - 1]$

ex. Find the source PF ?

$V = 60\angle 0$ (it is rms value)

$V = 60\angle 0°$

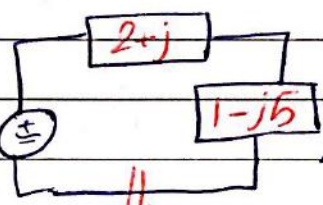


$I_{rms} = \dfrac{V_{rms}}{3+j4} = 12\angle -53.13°$

$\Downarrow$ inductive

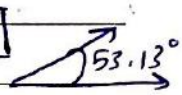
$PF_{source} = \cos(0 + 53.13°) = 0.6$ (lagging)

ex. $I_{rms} = \dfrac{V_{rms}}{3-j4} = 12\angle +53.13°$



$PF_{source} = \cos(0 - 53.13°) = 0.6$ (leading)

$60\angle 0°$



$\Downarrow$ Capacitive.

Another way

$PF = \dfrac{\text{Real Power}}{\text{Apparent Power}}$; Apparent power $= 60 \times 12 = 720$ W

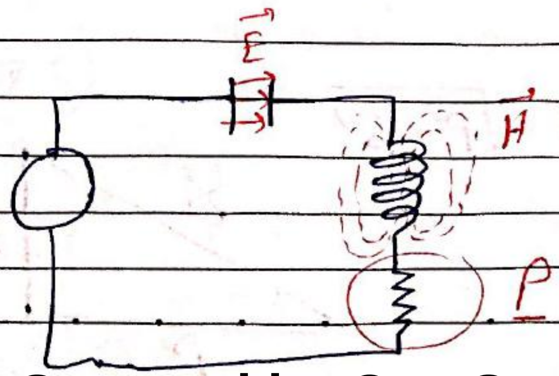
; Real Power $= (I_{rms})_1^2 + (I_{rms})_2^2 = 144 \times 1 + 288 = 432$ W.

$PF = {}^{432}/_{720} = \boxed{0.6}$

※ Complex Power: [some power is Lost]

complex Power $\rightarrow$ Real (W)

$\rightarrow$ Imajinary (Reactive) power $\Downarrow$ VAR



Complex power → Real (W)

→ imaginary (Reactive power) ⇒ VAR

→ Rate of energy transfer in L & C.

→ Rate of charge & discharge.

* Suppose $V = |V_{eff}| \angle \theta$, $I = |I_{eff}| \angle \phi$

$$\Rightarrow P = |V_{eff}|\,|I_{eff}| \cos(\theta-\phi) = |V_{eff}|\,|I_{eff}|\, Re\{e^{j(\theta-\phi)}\}$$

$e^{j(\theta-\phi)} = \cos(\theta-\phi)+j\sin(\theta-\phi)$

$$S = |V_{eff}|\,|I_{eff}|\, e^{j(\theta-\phi)}$$

$$= \underbrace{|V_{eff}|\, e^{j\theta}}_{V}\ \underbrace{|I_{eff}|\, e^{-j\phi}}_{I^*} = \boxed{V I^*} \equiv S$$

Complex Power = Real Power + Reactive Power

$$\boxed{S = P + jQ} \quad ***$$

$\Rightarrow S = V_{rms}\, I^*_{rms} = \frac{1}{2} V_m I^*_m$ VA.

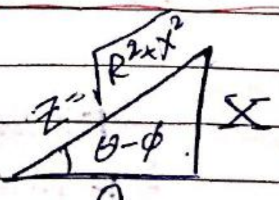
$P = V_{rms}\, I_{rms} \cos(\theta-\phi)$ W

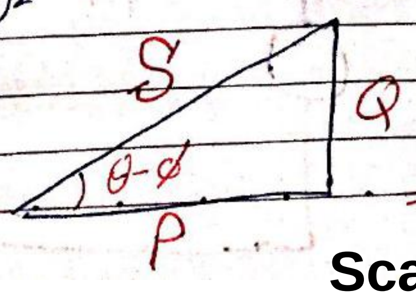
$Q = V_{rms}\, I_{rms} \sin(\theta-\phi)$ VAR.

$|S| = |V_{rms}| * |I_{rms}|$ VA

* Power Triangle:

$|S| = \sqrt{P^2+Q^2}$

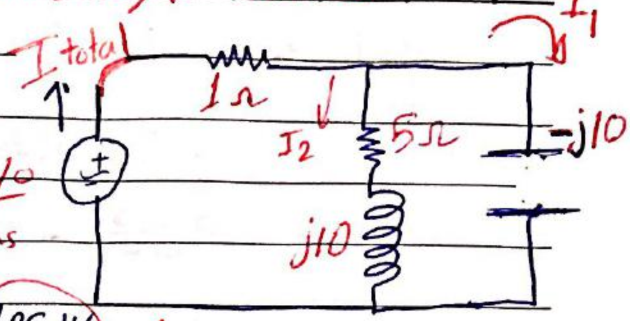




$\Rightarrow \angle S = \theta - \phi$

No.

ex.) find the Complex Power absorbed/produced By sources & elements:



assume I_{total} flowing from the voltage source:

$$I_{total} = \frac{\cancel{100}\angle 0}{1 + \frac{(5+j10)*(-j10)}{5}} = 5.16\angle 25.46\ A.$$

$\boxed{1\Omega}$

* the resistor always give a real power.

$$P_{avg(1\Omega)} = I_{rms}^2 R = (5.16)^2 * 1 = \boxed{26.6\ W}$$

$$S_{(1\Omega)} = V_{rms} * I_{rms}^* = 5.16\angle 25.46 + 5.16\angle -25.46$$
$$= \boxed{26.6\ VA}$$

$\boxed{-j10}$

$$S_{(-j10)} = V_{rms} I_{rms}^*$$

$$I_1 = I_{total} * \frac{5+j10}{5} = 11.53\angle 88.89°\ A$$

$$V_{rms(-j10)} = I_{rms}(-j10) = (11.53\angle 88.89°)(-j10) = 115.3\angle -1.11 \text{ volt}$$

$$S_{(-j10)} = (115.3\angle -1.11)(11.53\angle -88.89°) = 1331\angle -90$$
$$= \boxed{-j1331\ VA}$$

$$\Rightarrow Q = V_{rms} I_{rms} \sin(-1.11 - 88.89)$$
$$= -1331\ VAR$$

↳ VAR since it represent Q here.

$\boxed{5+j10}$

$$S_{(5+j10)} = V_{rms} I_{rms}^* \Rightarrow I_2 = I_{tot.} - I_1 = 10.31\angle -64.53\ A$$

$$V_{(5+j10)} = 115.3\angle -1.11\ V$$

$$= (115.3\angle -1.11)(10.31\angle +64.53)$$
$$= \boxed{531.48 + j1062.96\ VA}$$

$\boxed{120\angle 0}$

$$S_{(source)} = (120\angle 0)(5.16\angle -25.46) = \boxed{558.98 - j266.14\ VA}.$$

No.

* Find the average power across $4\,\Omega$ Resistor when:

$v(t) = 8\sin(200t) - 6\cos(200t - 45)$

$\hookrightarrow 8\cos(200t - \frac{\pi}{2})$

$\Rightarrow V = 8\angle\frac{-\pi}{2} - 6\angle{-45} = 8\cos(\frac{-\pi}{2}) + j\sin(\frac{-\pi}{2}) - 6\cos(-45) - j6\sin(-45)$

$V = -4.24 - j3.75$

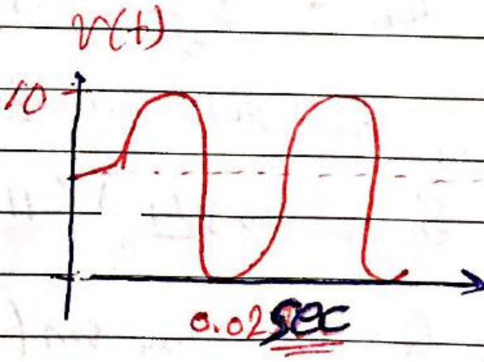
$|V| = 5.66 \Rightarrow P = \frac{V_m^2}{2R} = \boxed{4.01\ W}$

$\Rightarrow$ if we add (4) to v(t):

$$V(t) = \underbrace{8\sin 200t - 6\cos(200t - 45)}_{f_1} \underbrace{+4}_{f_2}$$

$$V_{rms} = \sqrt{V_{rms_1}^2 + V_{rms_2}^2} = \sqrt{\left(\frac{5.66}{\sqrt{2}}\right)^2 + (4)^2} = \boxed{5.66}\ \text{volt}$$

ex) Find the RMS value for:



$T = 0.02$ sec.

$v(t) = 5 + 5\sin(\frac{2\pi}{0.02}t)$ (ω)

$$V_{rms} = \sqrt{\frac{1}{0.02}\int_0^{0.02}(5 + 5\sin(100\pi t))^2 dt}$$

if we want $V_{avg} = \frac{1}{0.02}\int_0^{0.02}(5 + \underbrace{5\sin(100\pi t)}_{\text{gives zero}})dt$

$= \frac{1}{0.02} * 0.1 = \boxed{5\ \text{volt}}$

* Another way to find V_{rms}:

$$V_{rms} = \sqrt{V_{rms_1}^2 + V_{rms_2}^2} = \sqrt{(5)^2 + \left(\frac{5}{\sqrt{2}}\right)^2} = \boxed{6.124\ \text{volt}}$$

No.

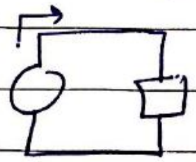
(ex) Find the PF for $i(t) = 4\cos(100\pi t + 10^\circ)$ A

$v(t) = 120\cos(100\pi t - 20^\circ)$ V

$PF = \cos(\theta - \phi) = \cos(-30) = \frac{\sqrt{3}}{2}$ Leading

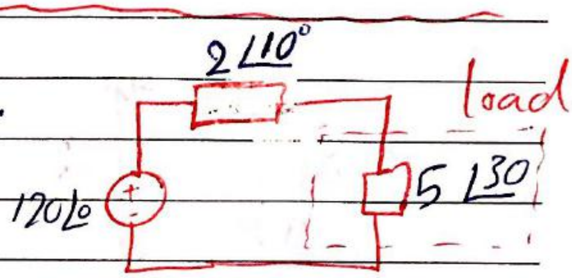
$Z = \frac{120\angle -20}{4\angle 10} = 30\angle -30$

$\rightarrow PF = \cos(-30) = \frac{\sqrt{3}}{2}$



(ex) Load power factor:

$PF = \cos(-30) = \frac{\sqrt{3}}{2} = 0.866$



Source PF:

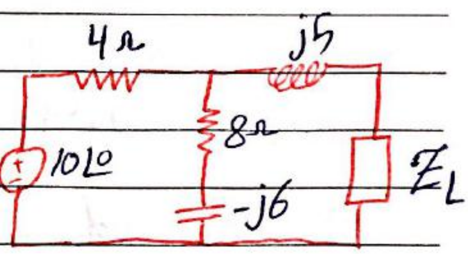
$PF = \cos(\theta - \phi)$

$I = \frac{120}{2\angle 10 + 5\angle 30} = 17.36\angle -24.32^\circ \Rightarrow PF = \cos(24.32)$

$= \boxed{0.911}$

(ex) Determine the load impedance Z_L that maximize the average power drawn from the ckt & what is that average max power?

Need Z_{th} at first:



Killing the source:

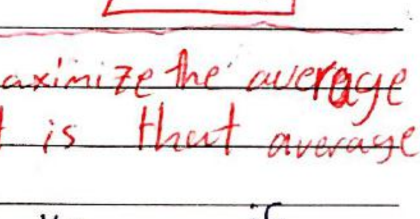
we have $(4\Omega) // (8 - j6) = Z_o$

then Z_o series with $(j5)$ gives Z_{th}

$Z_{th} = j5 + \frac{(4)(8-j6)}{12-j6} = \underline{2.933 + j4.467}\ \Omega$

So, $\boxed{Z_L = 2.933 - j4.467}\ \Omega$.

Thevenin Equivalent:



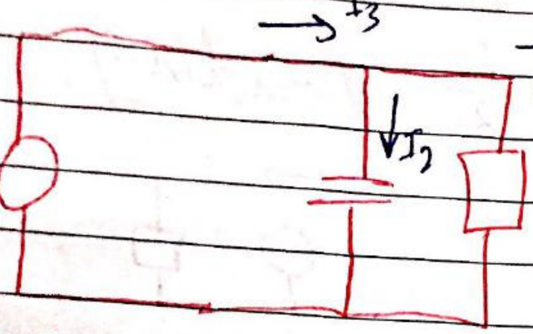
using voltage division to find V_{th}:

$V_{th} = 10\angle 0 \frac{(8-j6)}{12-j6} = \boxed{7.454\angle -10.3^\circ}$

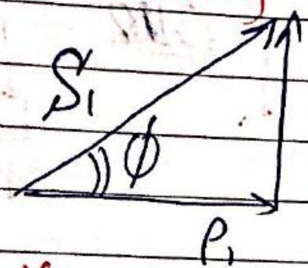
$\Rightarrow P_{max} = \frac{|V_{th}|^2}{8|Z_{th}|} = 1.3\text{ W}$, $P_{avg}\Big|_{load} = \frac{V_{th}^2}{8R} = \boxed{2.368\text{ W}}$

* PF Correction:

No.

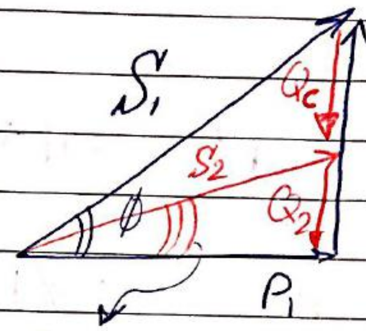


* Before adding the Capacitor:



PF = cosϕ

* After adding the Capacitor:



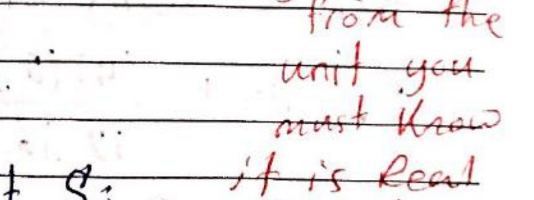
the angle $< \phi$ so PF will increase.

$$\boxed{\underset{\text{(total)}}{Q_2} = Q_1 + Q_C}$$

ex: Induction motor is operating @ 50KW & 0.8 Lagging PF & $V_s = 230 V_{rms}$ (given f=50Hz)

demand:

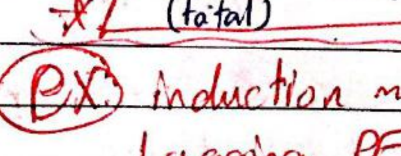
$\Rightarrow$ we need to make the PF = 0.95



from the unit you must know it is Real power here.

$\Rightarrow$ we need to add a capacitor in parallel with the motor.

Before:



if he want S:

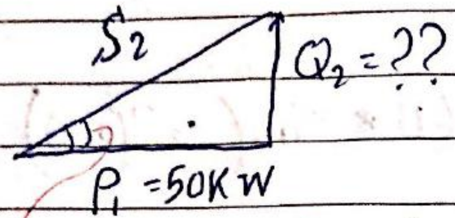
$S = \sqrt{(Q_1)^2 + (P_1)^2}$

$= 62.5 KVA.$

$\phi = \cos^{-1}(0.8) = 36.87°$

$\tan(36.87) = \frac{Q_1}{P_1} \Rightarrow Q_1 = 37.5\ KVAR$

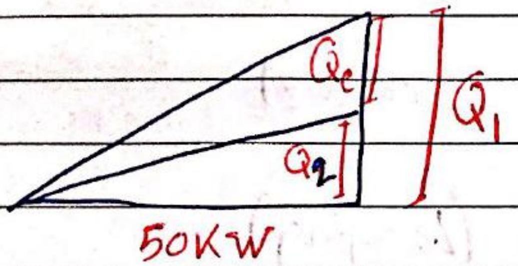
After:



$\cos^{-1}(0.95) = 18.19$

$\Rightarrow \tan 18.19 = \dfrac{Q_2}{50KW}$

$\underline{Q_2 = 16.43 \text{ KVAR}}$



$Q_c = 16.43 - 37.5 = -21.06 \text{ KVAR}$

$Q_2 - Q_1$

Now How to find the capacitance?

$Q = V_{rms} I_{rms} \sin(\theta_v - \theta_i) \Rightarrow |Q_c| = |V_{rms}||I_{rms}|$; $|I_{rms}| = \dfrac{|V_{rms}|}{|X_c|}$

$X_c = 1/j\omega C \Rightarrow |X_c| = \dfrac{1}{\omega C}$

so $Q_c = |V_{rms}|^2 \omega C \Rightarrow C = \dfrac{|Q_c|}{|V_{rms}|^2 \omega}$

$\Rightarrow C = \dfrac{21.67 \times 10^3}{(230)^2 \times 2\pi \times 50} = \boxed{1.3 mF}$

suggested problems: 10.7/10.11/10.26/10.38/10.43/10.49/10.53
10.56/10.60

CH11 (2, 5, 12, 13, 14, 18, 22, 24, 26, 28, 32, 34, 38, 41, 42, 46 → 54, 69, 72, 73, 74, 75) sixth edition of sadiku

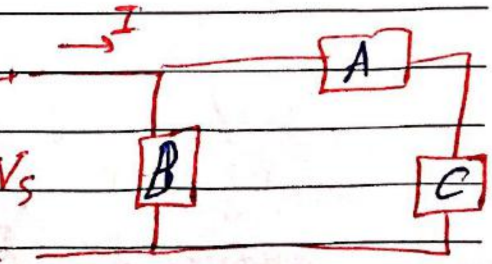
Problem 11.52 sadiku sixth edition:

given:

A: 2KW @ 0.8 pf Lag.

B: 3K VA @ 0.4 pf leading.

C: consumes 1KW & receives 500 VAR.



(1)

⇒ find pf of the source?

No.

since it is leading.

or $\sqrt{1-(0.4)^2}$

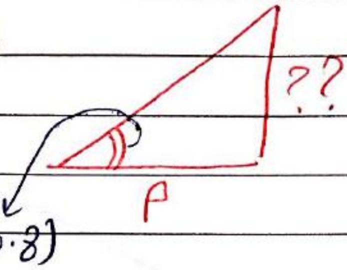
for B: $S = 3KVA$, $pf = 0.4$ lead

$P = |S|\, pf = 1.2\,KW$, $Q = |S| * (-1) * \sin(\cos^{-1}(0.4))$

for A: $P = 2KW$, $Q = P \tan(\cos^{-1}(0.8))$

for C: $P = 1KW$

$Q = 500\ VAR$



positive : always receiving or consuming means (Lagging)

* (generating ≡ Leading) *

$P_{total} = P_A + P_B + P_C$ & $Q_{total} = Q_A + Q_B + Q_C$

$\Rightarrow \phi = \tan^{-1}\dfrac{Q_{total}}{P_{total}} \Rightarrow pf = \cos\phi$

final answer: $\boxed{pf = 0.98443}$ Leading.

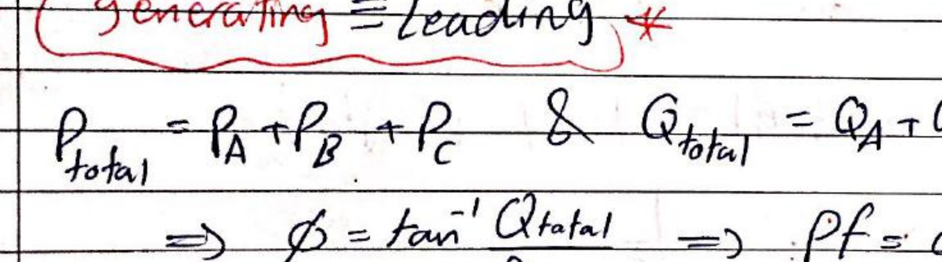
(2) Find I if $V_s = 120\angle 45°$?

نقوم فقط بعكس اشارة الزاوية لـ I^*

$S = V I^* \Rightarrow I^* = \dfrac{S_{total}}{V_s} \Rightarrow$ then we find I.

final answer: $\boxed{I = 35.554\angle 55.12°\ A_{rms}}$

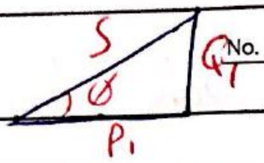
Ex.



Need value of (C) parallel to (10Ω) that will make pf on the source 0.95?

assume $f = 50Hz$

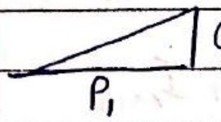
* Before:



$P_1 = I_1^2 * 20 + I_2^2 * 10$ $\Rightarrow \phi = \tan^{-1}\frac{Q}{P}$

$Q_1 = I_2^2 * j20 - I^2 * j10$ $\quad pf = \cos\phi$

* After:



$\Rightarrow Q_C = Q_T - Q_1$

$\phi = \cos^{-1} 0.95$

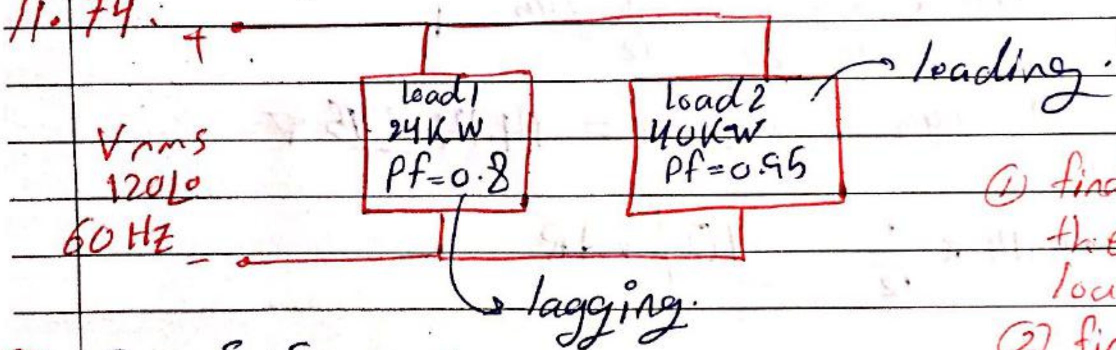
$Q_T = P_1 \tan 18.19°$

$C = \frac{Q_C}{\omega |V|^2_{rms}}$

$\omega \to 2\pi * 50$

$|V|_{rms} \to$ voltage on the 10Ω resistor.

11.74.



① find pf for the combined load.

② find (C) that make pf=1

① $S_{total} = S_1 + S_2$

$P_{total} = P_1 + P_2$

$Q_{total} = Q_1 + Q_2$

$P_1 = 24KW$

$Q_1 = P_1 \tan 36.81$

$= 18K\,VAR$

$P_2 = 40KW$

$Q_2 = P_2 \tan 18.19$

$= 14.73\,K\,VAR$

$P_{total} = 64KW$, $Q_{total} = 32.73\,K\,VAR$

$\phi = \tan^{-1}\frac{Q_{tot}}{P_{tot}} \Rightarrow pf = \cos\phi$

② $pf = 1 \Rightarrow \phi = 0 \Rightarrow Q_T = \text{zero}$

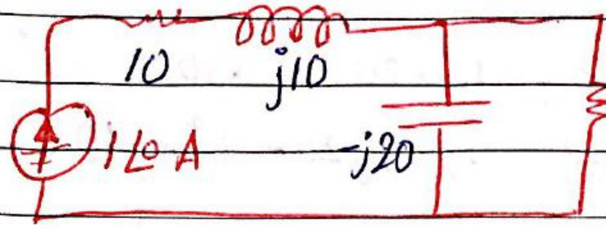
so $Q_C = Q_T - \underbrace{Q_1}_{Q_{total}\text{ before}} = -32.73K\,VAR$

Now: $C = \frac{|Q_c|}{|V_{rms}|^2\,\omega}$

$\omega \to 2\pi * 60$

No.

11.54: find the reactive power for each element in the following cKt:



directly Q on 10 & 20 Ω is equal to $\boxed{\text{Zero}}$

$Q = V_{rms} I_{rms} \sin(\theta_v - \theta_i)$

for pure C or L:

$Q_L = V_{rms} I_{rms}$

$Q_C = -V_{rms} I_{rms}$

comes from $\sin(-90)$

$\Rightarrow V_L = 1\angle 0 * j10$

$= 10\angle 90^\circ$

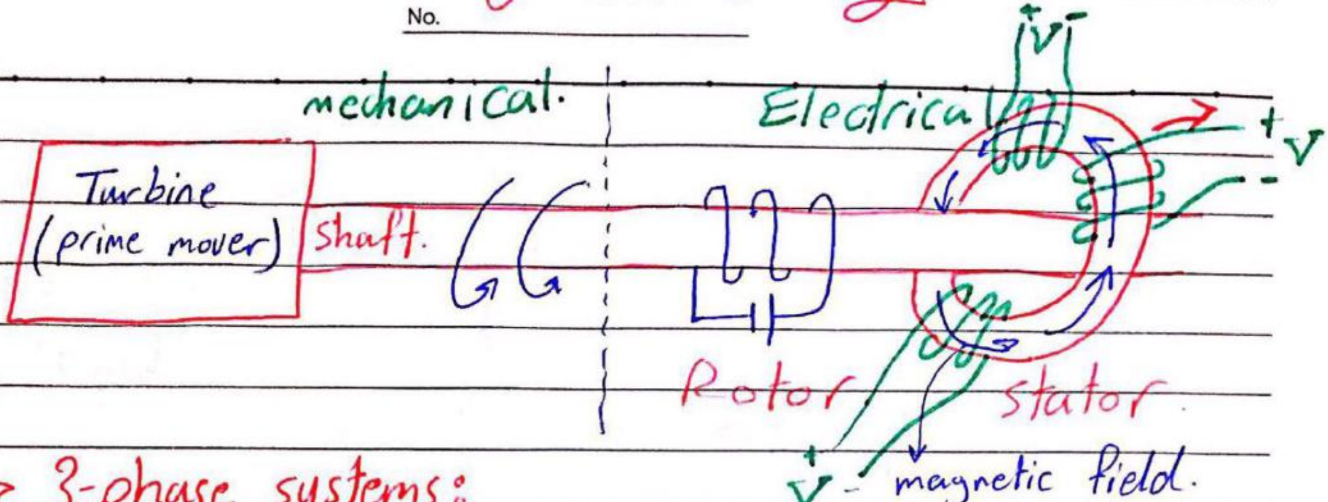
$Q_L = 10 * 1 = \boxed{10}$ VAR

$I_{(capacitor)} = 1\angle 0 * \dfrac{20}{20-j20} = \dfrac{1}{\sqrt{2}}\angle 45^\circ$ A

$V_{rms\,(capacitor)} = \dfrac{1}{\sqrt{2}}\angle 45 * 20\angle -90 = 14.14\angle -45$ V

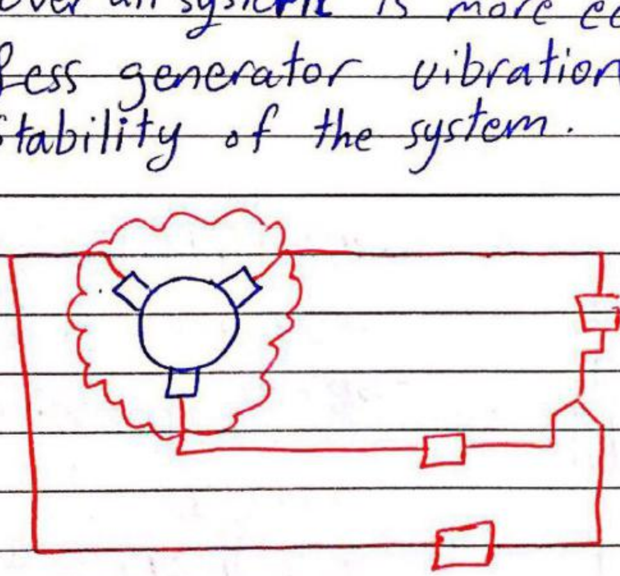
$Q_C = 14.14 * \dfrac{1}{\sqrt{2}} = \boxed{10}$ VAR

CHAPTER (12): Poly Phase Systems:

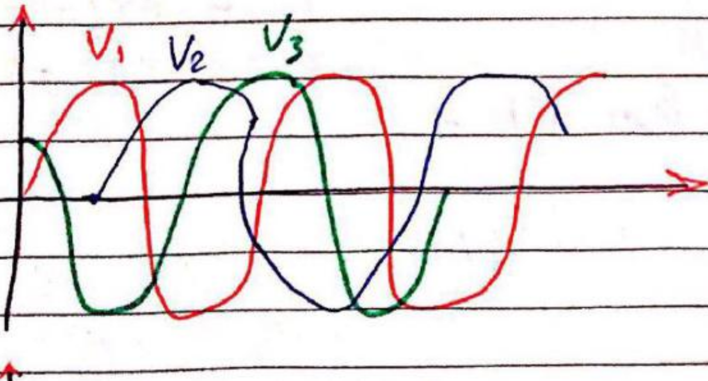


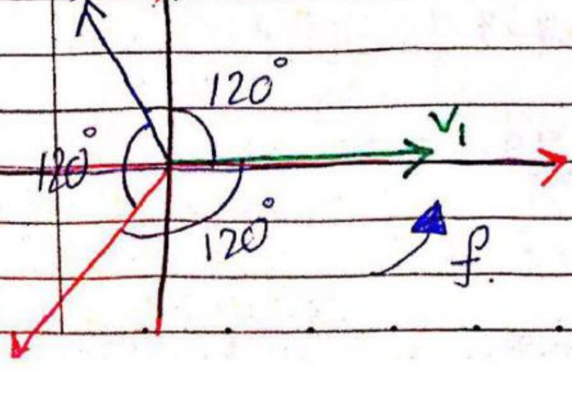
⇒ 3-phase systems:

1. over all system is more economical.
2. less generator vibration ⇒ since P_{avg} = constant.
3. Stability of the system.

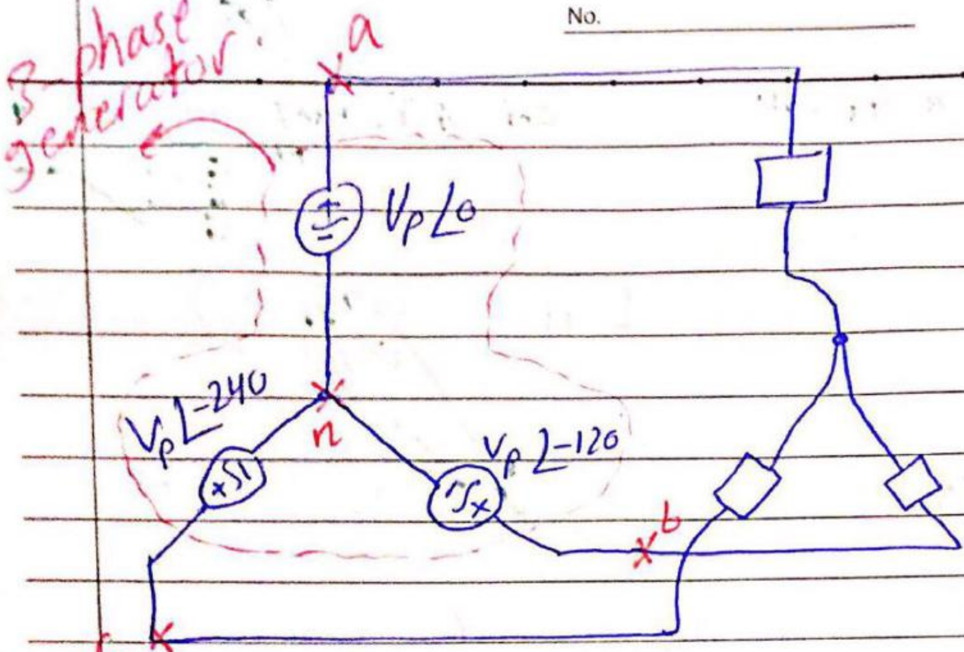


* all the three loads have same voltage but different in phase shift.





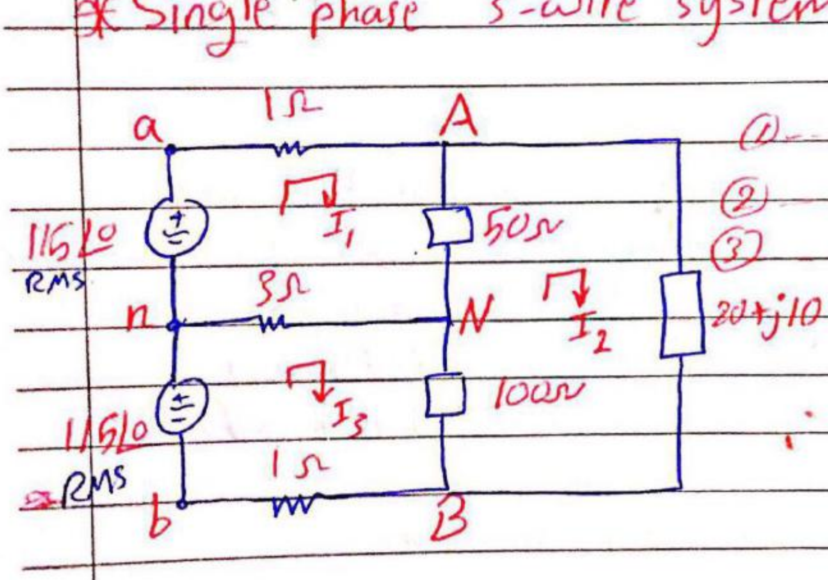
No.



* Instead of using: V_1, V_2, V_3

$\Rightarrow V_{an}, V_{bn}, V_{cn}$

* Single phase 3-wire systems



① $-115\angle 0 + I_1 + 50(I_1 - I_2) + 3(I_1 - I_3) = 0$

② $(20+j10) I_2 + 100(I_2 - I_3) + 50(I_2 - I_1) = 0$

③ $-115\angle 0 + 3(I_3 - I_1) + (I_3 - I_2)100 + I_3 = 0$

solving:

$I_1 = 11.24 \angle -19.83$

$I_2 = 9.38 \angle -24.47$

$I_3 = 10.37 \angle -21.8$

I_{aA}: the current flow from a to A.

$\Rightarrow I_{aA} = I_1$

I_{bB}: the current flow from b to B

$\Rightarrow I_{bB} = -I_3$

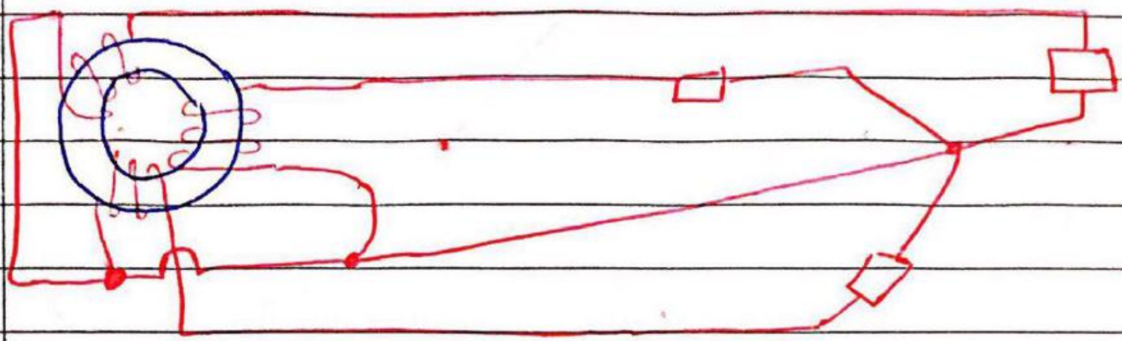
I_{nN}: $\Rightarrow I_{nN} = I_3 - I_1 = 0.9459 \angle -177.7^\circ$

$P_{(50\Omega)} = 50 * |I_1 - I_2|^2 = 206\ W$

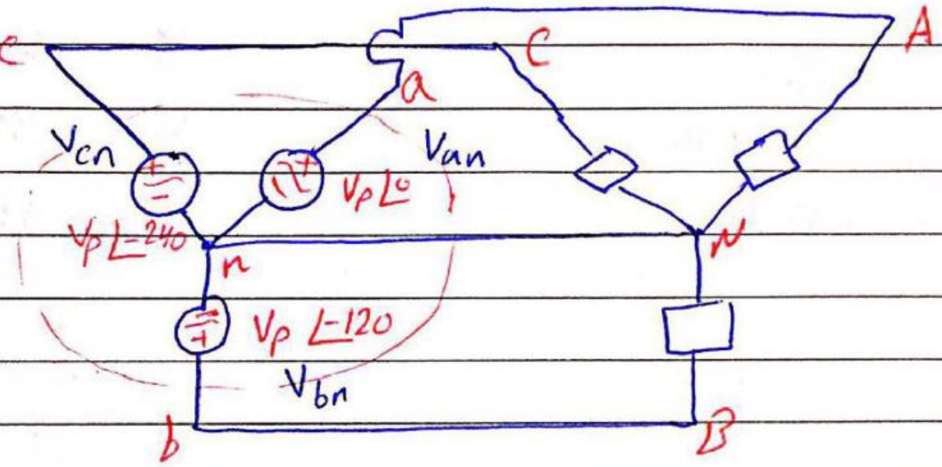
$P_{(100\Omega)} = 100 * |I_3 - I_2|^2 = 117\ W$

$P_{(20+j10)} = 20 * I_2^2 = 1763\ W$

※ A Balanced 3∅ system Connected Y-Y:



⇒



Y source: the neutral of the phase are connected together.

Y Load: the neutrals of th loads are connected together.

$$|V_{an}| = |V_{bn}| = |V_{cn}|$$

$$V_{an} = V_p\angle 0$$
$$V_{bn} = V_p\angle -120$$
$$V_{cn} = V_p\angle -240$$

same +120

⇒

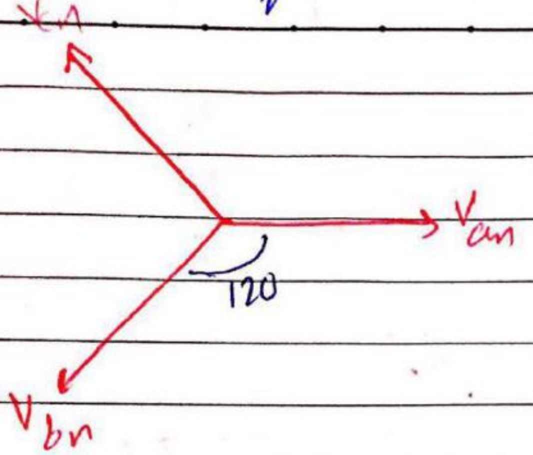
$$V_{an} + V_{bn} + V_{cn} = 0$$

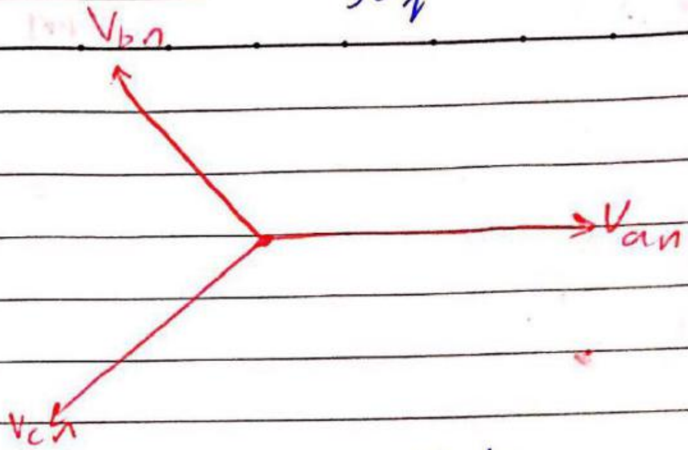
$$V_p\angle 0 + V_p\angle -120 + V_p\angle +120 = 0$$

$$\Rightarrow V_p + V_p\cos(-120) + jV_p\sin(-120) + V_p\cos(120) + jV_p\sin(-120) = 0$$

$$V_p - \frac{V_p}{2} - \frac{V_p}{2} = 0 \Rightarrow V_p - V_p = 0$$

positive sequance. | negative sequance.





$V_{an} = V_p \angle 0$
$V_{bn} = V_p \angle -120$
$V_{cn} = V_p \angle +120$

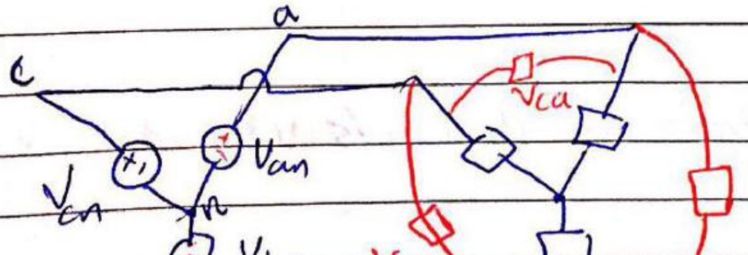
⇓

used in the rest of the word.

$V_{an} = V_p \angle 0$
$V_{cn} = V_p \angle -120$
$V_{bn} = V_p \angle +120$

⇓

used in California.



$V_{ab} = ??$

line-to-neutral voltage.
line-neutral voltage.

KVL:

$$\Rightarrow V_{ab} = V_{an} - V_{bn} = V_p \angle 0 - V_p \angle -120$$
$$= V_p - [V_p \cos(-120) + j V_p \sin(-120)]$$
$$= V_p + \frac{V_p}{2} + j V_p \frac{\sqrt{3}}{2}$$

$30°$

$$\Rightarrow = V_p\left(\frac{3}{2} + j\frac{\sqrt{3}}{2}\right) = \sqrt{3}\, V_p\left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right)$$

$$V_{ab} = \sqrt{3}\, V_p \angle 30°$$

⟹

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$V_{bc} = V_{bn} - V_{cn} = V_p \angle -120 - V_p \angle +120$

$= V_p \cos(-120) + jV_p \sin(-120) - V_p \cos(120) - jV_p \sin(120)$

$= -j\frac{\sqrt{3}}{2}V_p - j\frac{\sqrt{3}}{2}V_p = -j\sqrt{3}V_p = \sqrt{3}V_p \angle -90°$

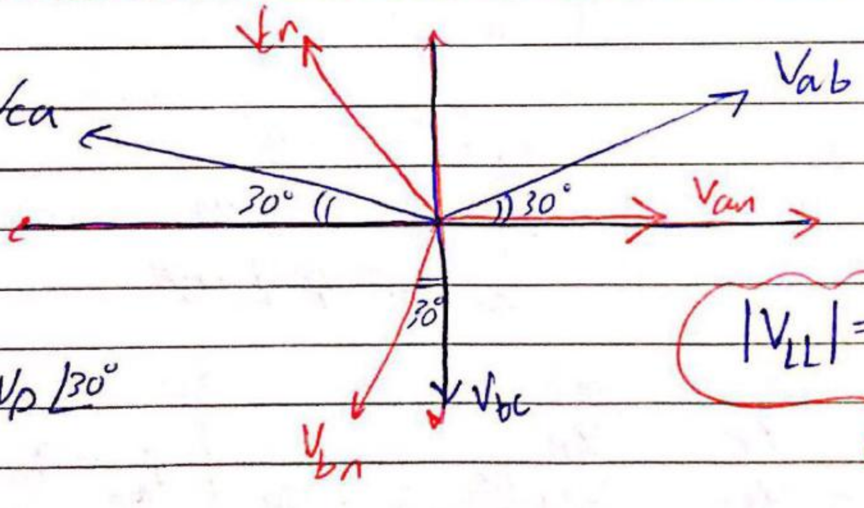
$V_{bc} = \sqrt{3}V_p \angle -90°$ "Line-to-Line Voltage"

$V_{ca} = V_{cn} - V_{an} = V_p \angle +120 - V_p = V_p \cos 120 + jV_p \sin 120 - V_p$

$= -\frac{V_p}{2} + jV_p\frac{\sqrt{3}}{2} - V_p = V_p\left(\frac{-3}{2} + j\frac{\sqrt{3}}{2}\right)$

$= \sqrt{3}V_p\left(\frac{-\sqrt{3}}{2} + j\frac{1}{2}\right) = \sqrt{3}V_p \angle 150$

$\Rightarrow V_{ca} = \sqrt{3}V_p \angle 150°$ "Line-Line Voltage"

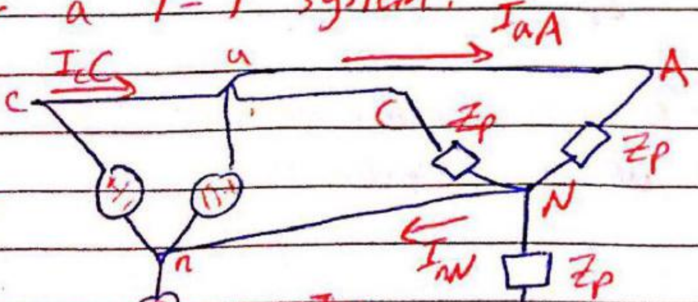


$V_{ab} = \sqrt{3}V_p \angle 30°$

$|V_{LL}| = \sqrt{3}\ V_{LN}$

in 3Ø balanced Y system

For a Y-Y system:



$I_{aA} = I_a = \frac{V_{an}}{Z_p} = \frac{V_p \angle 0}{Z_p}$

$I_{bB} = I_b = \frac{V_{bn}}{Z_p} = \frac{V_p \angle -120}{Z_p}$

$I_b = I_a \angle -120$

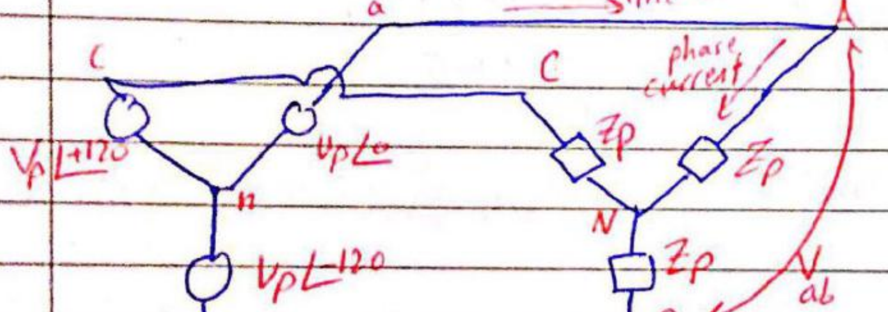
$I_{cC} = I_C = \frac{V_{CN}}{Z_p} = \frac{V_p \angle +120}{Z_p} = I_a \angle +120$

No.

$$I_{Nn} = I_a + I_b + I_c$$
$$= I_a + I_a \angle -120 + I_a \angle +120$$
$$= I_a + I_a \cos(-120) + I_a j \sin(-120) + I_a \cos(120) + I_a j \sin(120)$$
$$= I_a + \frac{-I_a}{2} - \frac{I_a}{2} = I_a - I_a = \text{Zero}$$

* التيار المار بالنيوترال صفر لذلك لو قمنا بإزالته لن يؤثر شيء.

Y-Y connected ckts:



line-neutral: V_{an}, V_{bn}, V_{cn} } phase voltages

line-line: V_{ab}, V_{bc}, V_{ca} } line voltages.

$$V_{an} = V_p \angle 0 \qquad V_{ab} = \sqrt{3}\, V_p \angle 30^\circ$$
$$V_{bn} = V_p \angle -120 \qquad V_{bc} = \sqrt{3}\, V_p \angle -90^\circ$$
$$V_{cn} = V_p \angle +120 \qquad V_{ca} = \sqrt{3}\, V_p \angle +150^\circ$$

$$I_{aA} = I_a = \frac{V_{an}}{Z_p} = \frac{V_p \angle 0}{Z_p}$$
$$I_{bB} = I_b = \frac{V_{bn}}{Z_p} = \frac{V_p \angle -120}{Z_p} = I_a \angle -120$$
$$I_{cC} = I_c = \frac{V_{cn}}{Z_p} = \frac{V_p \angle +120}{Z_p} = I_a \angle +120$$

$$I_{Nn} = I_a + I_b + I_c = 0$$

Example: for the Y-Y system shown find:

(a) the phase current and line currents.

(b) find the line voltages.

(c) total real power absorbed by the load.

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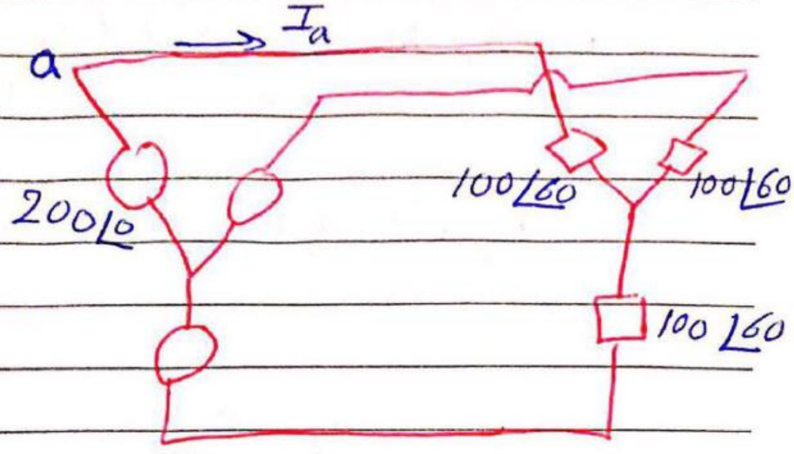
phase voltages:

$V_{an} = 200\angle 0$

$V_{bn} = 200\angle -120$

$V_{cn} = 200\angle +120$



(b) line voltages:

$V_{ab} = \sqrt{3}\ 200\angle 30° = 346\angle 30°$

$V_{bc} = 346\angle -90°$

$V_{ca} = 346\angle +150°$

(a) phase currents: & line currents:

$I_a = \frac{V_{an}}{Z_p} = \frac{200\angle 0}{100\angle 60} = \boxed{2\angle -60°}$ $\Rightarrow$ phase current = line current [Always in Y]

$I_b = \frac{V_{bn}}{Z_p} = \frac{200\angle -120}{100\angle 60} = 2\angle -180°$ or $I_b = I_a\angle -120° = 2\angle -180°$

$I_c = I_a\angle +120° = 2\angle +60°$

(b) power in phase A: $P_A = V_A I_A \cos(\theta) = 200 * 2 * \cos 60 = \boxed{200\,W}$

$P_B = V_B I_B \cos(\theta) = 200 * 2 * \cos(-120+180) = \boxed{200\,W}$

$P_C = 200 * 2 * \cos(120-60) = \boxed{200\,W}$

$P_{total} = P_A + P_B + P_C = 600\,W$

$\Rightarrow$ Always in Balanced systems:

$P_{3\phi} = 3P_{1\phi}$ **

* Instantaneous Power:

$v_a(t) = 200\cos\omega t * \sqrt{2} = \underbrace{200\sqrt{2}}_{\text{peak value}}\cos\omega t$

$i_a(t) = 2\sqrt{2}\cos(\omega t - 60)$

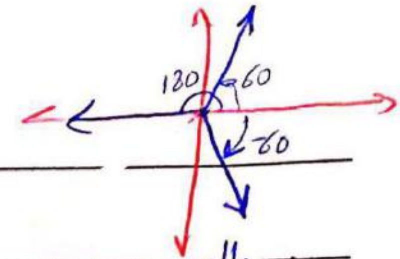
$P_a(t) = 800\cos(\omega t)\cos(\omega t - 60)$

$= 400[\cos(-60) + \cos(2\omega t - 60)]$

$= 200 + 400\cos(2\omega t - 60)$

remember:
$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$

$\Rightarrow$



gives Zero

No.

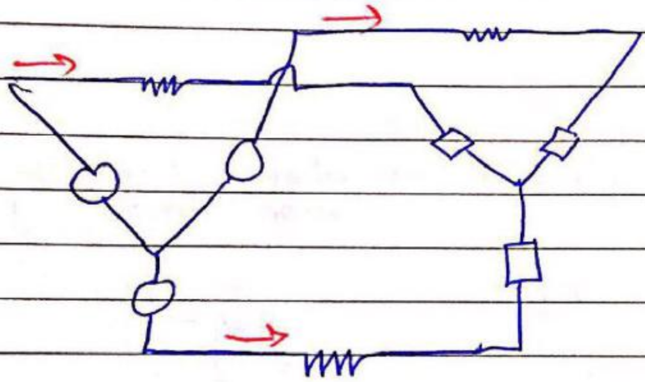
$P_a(t) = 200 + 400\cos(2\omega t - 60)$

$\Rightarrow P_b(t) = 200 + 400\cos(2\omega t + 60)$

$P_c(t) = 200 + 400\cos(2\omega t + 180)$

$\Rightarrow P_{3\phi}(t) = P_A(t) + P_B(t) + P_C(t) = 600 + 0 = 600\,W$

since also $V_a + V_b + V_c = 0$



* $Losses = I_a^2 R + I_b^2 R + I_c^2 R$
$= 4R + 4R + 4R$
$= \boxed{12R}$ W.

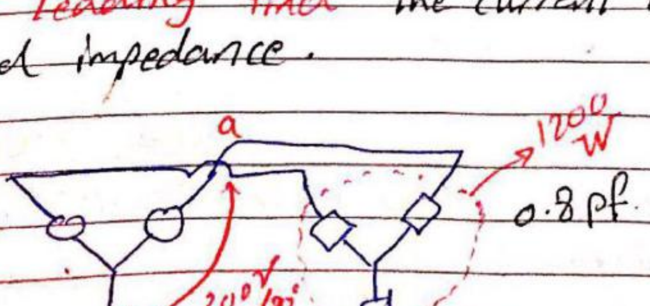
* What if single phase is used:

$P = V_{rms} I_{rms} \cos\theta \Rightarrow 600 = 200\, I_{rms} * \frac{1}{2}$

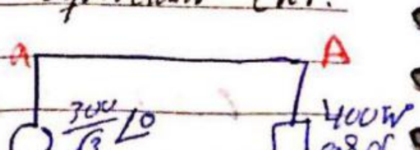
$I_{rms} = 6\,Amp$

$\Rightarrow Losses = |I_{rms}|^2 R$
$= \boxed{36R}$ W.

ex). A balanced 3ϕ system with $V_L = 300\,V$ & supplying a balanced Y-connected load with 1200W @ 0.8 pf leading find the current & the per-phase load impedance.



* single phase equivelant ckt:



No. ____

$\Rightarrow |V_{ab}| = 300V \Rightarrow V_{an} = \frac{300}{\sqrt{3}}$

* Take V_{an} as your reference

$P = V_{an} I_a \cos\theta \Rightarrow 400 = \frac{300}{\sqrt{3}} I_a * 0.8$

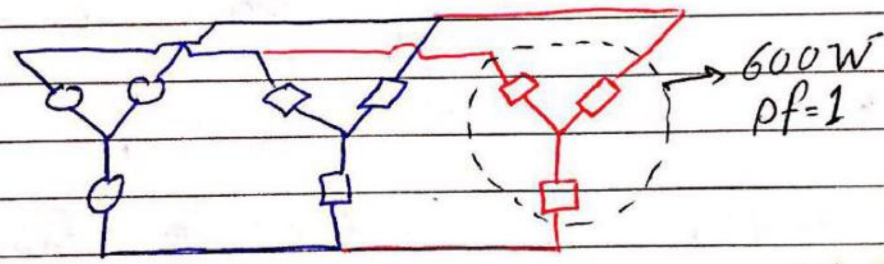
$\Rightarrow |I_a|_{rms} = 2.89 \angle \overset{+}{\cos^{-1}}(0.8) \Rightarrow \boxed{|I_a| = 2.89 \angle 36.87}$ A.

$\Rightarrow Z = \frac{300/\sqrt{3} \angle 0}{2.89 \angle 36.87} = 60 \angle -36.87 \ \Omega$

* if the quistion doesn't determine the pf.

$\Longrightarrow$ pure resistor (pf=1)

ex) a balanced 600 W is added to the system in parallel find the new line current?



$P = V_{an} I_a$

$\Rightarrow I_a = \frac{200}{300/\sqrt{3}} = \boxed{1.15 \angle 0}$ A

$\Rightarrow I_{Line} = 2.89 \angle +36.87 + 1.15$

$= 3.87 \angle 26.6^\circ$ A.

we also could find:

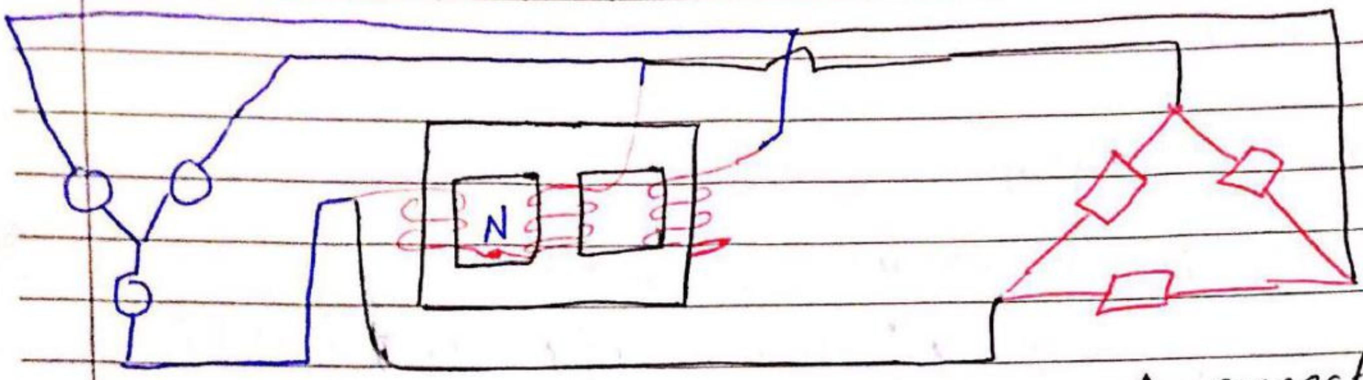
$V_{an} = \frac{300}{\sqrt{3}}$ $\quad V_{bn} = \frac{300}{\sqrt{3}} \angle -120^\circ$ $\quad V_{cn} = \frac{300}{\sqrt{3}} \angle +120$

$V_{ab} = 300 \angle +30$ $\quad V_{bc} = 300 \angle -90$ $\quad V_{ca} = 300 \angle +150$

case 1: $I_a = 2.83 \angle 36.87^\circ$ $\quad I_b = 2.83 \angle 36.87 - 120$ $\quad I_c = 2.83 \angle 36.87 + 120$

case 2: $I_a = 3.87 \angle 26.6^\circ$ $\quad I_b = 3.87 \angle 26.6 - 120$ $\quad I_c = 3.87 \angle 26.6 + 120$

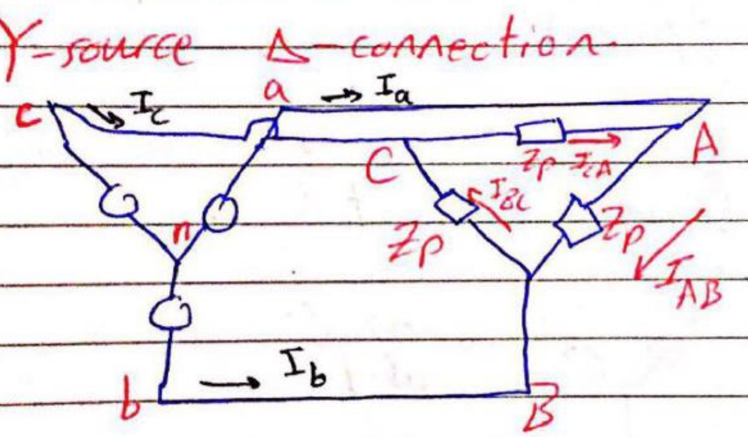
No.



Δ-connection.

we use it since we can't access the Neutral point.

Y-source Δ-connection.



In Δ-connection:

$$V_{phase} = V_{Line-line}$$

$$I_{AB} = \frac{V_{AB}}{Z_P}$$

$$I_{BC} = \frac{V_{BC}}{Z_P} = I_{AB}\angle -120^\circ$$

$$I_{CA} = \frac{V_{CB}}{Z_P} = I_{AB}\angle +120^\circ$$

⇒ in Δ-connection:

$$I_{phase} \neq I_{Line}$$

$$I_a + I_{CA} = I_{AB} \Rightarrow I_a = I_{AB} - I_{CA}$$

$$= I_{AB} - I_{AB}\angle 120^\circ$$

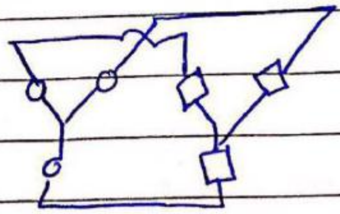
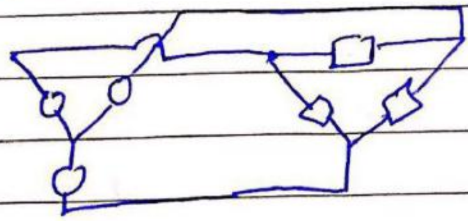
$$= I_{AB} - I_{AB}\cos 120 - j\, I_{AB}\sin 120 = I_{AB}\left(1 + 0.5 - j\frac{\sqrt{3}}{2}\right)$$

$$I_a = I_{AB}\sqrt{3}\angle -30$$

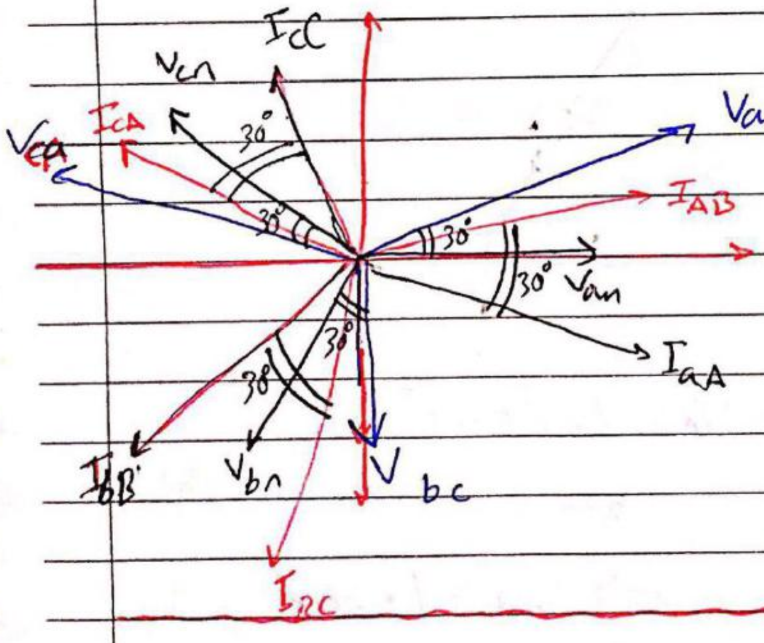
line current

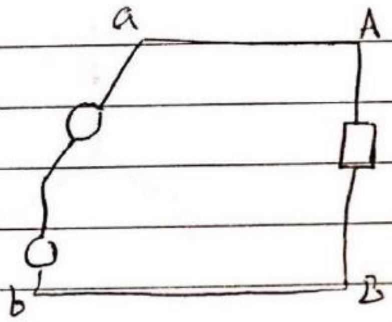
phase current.

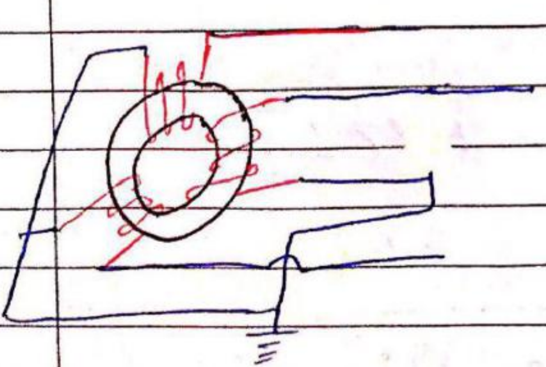
No.

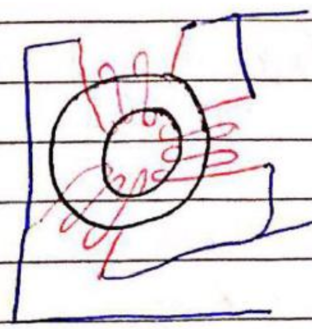
Y	Δ
$V_{LL} = \sqrt{3} V_{LN} \angle 30$	$V_{LL} = V_{phase}$
$I_L = I_p$	$I_L = \sqrt{3}\, I_p \angle -30$
	

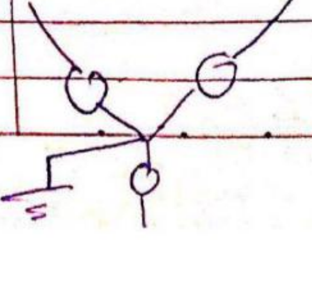
*Assume an inductive load:

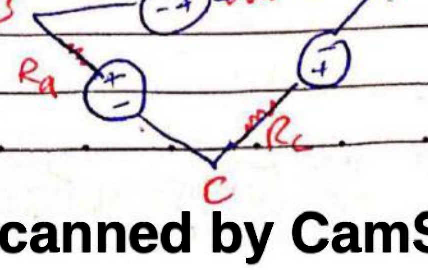










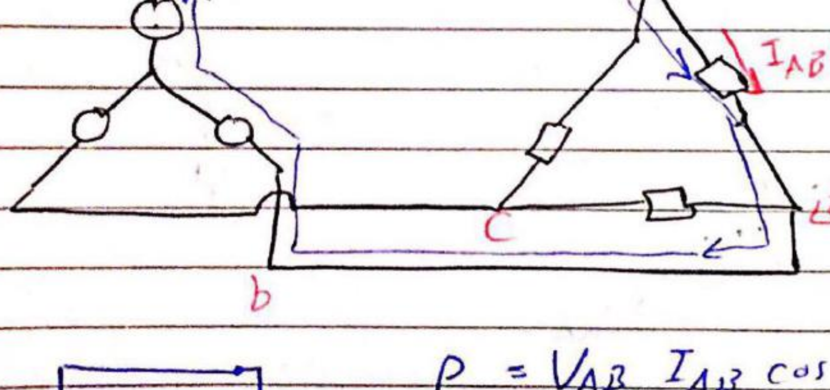


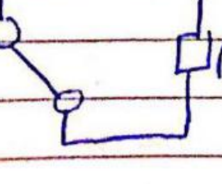
* 4 types of connection:

① Y_{source} Y_{load}. ② Y_{source} Δ_{load}.

③ Δ_{source} Y_{load}. ④ Δ_{source} Δ_{load}.

~~Ex~~ what are the line currents in a 3ϕ system with line voltage of 300 V & supplies 1200 W to Δ connection load at lagging pf = 0.8 & find the phase impedance.



=> 

$P = V_{AB} I_{AB} \cos\theta$

$400 = 300\, I_{AB}\,(0.8)$

$\Rightarrow I_{AB} = 1.667$

=> Take V_{AB} as your reference: $V_{AB} = 300\angle 0°$

so $I_{AB} = 1.667 \angle -\cos^{-1}(0.8) = 1.667 \angle -36.87°$

$\Rightarrow I_a = 1.667 \times \sqrt{3} \angle -36.87-30 = \boxed{2.89 \angle -66.87}$

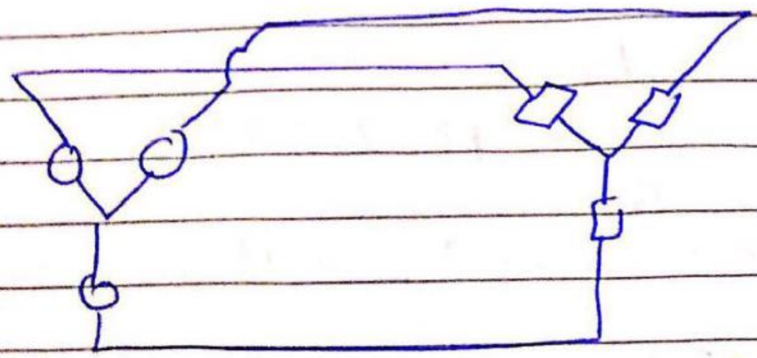
$\boxed{\begin{aligned} I_c &= 2.89 \angle -66.87+120° \\ I_b &= 2.89 \angle -66.87-120° \end{aligned}}$

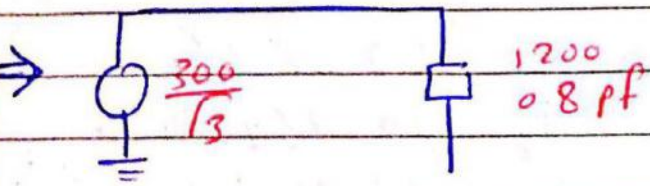
$\Rightarrow \boxed{\begin{aligned} I_{BC} &= 1.66 \angle -36-120 \\ I_{CA} &= 1.66 \angle -36+120. \end{aligned}}$

$Z_p = \dfrac{V_{AB}}{I_{AB}} = \dfrac{300\angle 0}{1.66 \angle -36.87}$

$= 180 \angle 36.87$

⇒ what if the load is Yconnection:





$$P = V_{an} I_a \cos\theta$$

$$400 = \frac{300}{\sqrt{3}} * I_a * 0.8$$

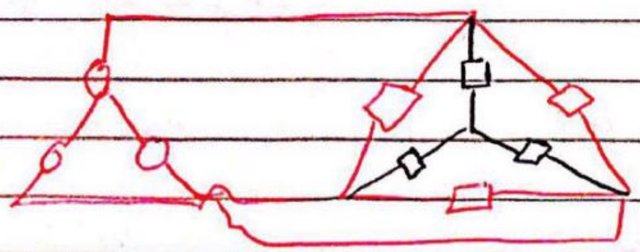
$$\Rightarrow \boxed{I_a = 2.89} \text{ A.}$$

Take V_{an} as your ref.

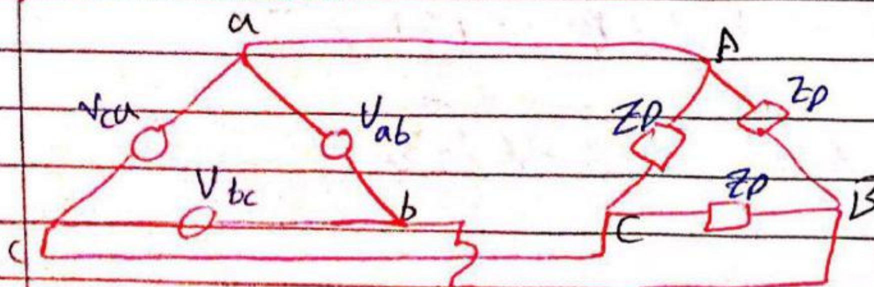
$$V_{an} = \frac{300}{\sqrt{3}} \angle 0 \quad \Rightarrow \quad I_a = 2.89 \angle -36.87$$

$$Z_p = \frac{V_{an}}{I_a} = 60 \angle 36.87$$

$$\Rightarrow \quad Z_\Delta = 3 Z_Y \qquad Z_Y = \frac{1}{3} Z_D$$



Δ - Δ connection:



$$I_{AB} = \frac{V_{ab}}{Z_p}$$

$$I_{BC} = \frac{V_{bc}}{Z_p}$$

$$I_{CA} = \frac{V_{ca}}{Z_p}$$

$$I_a = \sqrt{3}\, I_{AB} \angle -30^\circ$$

No.

ex: Δ-Δ connection system with load $20-j15$ is connected to Δ source with $V_{ab} = 330\angle 0$ find I_{phase} & I_{line}?

phase current

$$I_{AB} = \frac{V_{AB}}{Z_P} = \frac{330\angle 0}{20-j15} = 13.2\angle 36.87 \text{ A}$$

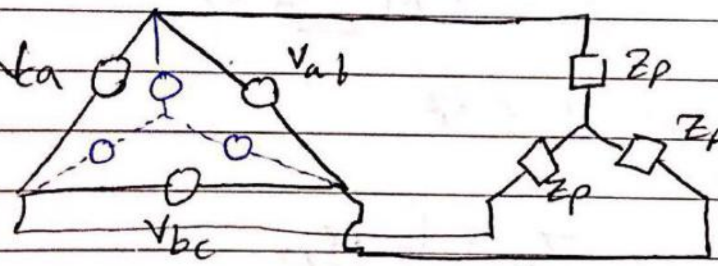
$$I_{BC} = 13.2\angle 36.87-120 = 13.2\angle -83.13 \text{ A}$$

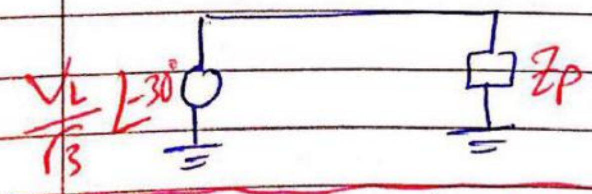
$$I_{CA} = 13.2\angle 156.87 \text{ A}$$

Line currents:

$$\Rightarrow I_a = \sqrt{3}\, I_{AB}\angle -30 \Rightarrow \begin{cases} I_a = 22.86\angle 6.87 \text{ A} \\ I_b = 22.86\angle -113.13 \text{ A} \\ I_c = 22.86\angle 126.87 \text{ A} \end{cases}$$

Δ-Y connection:





$$I_a = \frac{V_{an}}{Z_P} = \frac{V_{ab}\angle -30^\circ}{\sqrt{3}\, Z_P}$$

ex a balanced Y connected load with $Z_P = 40+j25$ is supplied by Δ source with line voltage $V_{ab} = 210\angle -20$ calculate the phase current?

$$V_{an} = \frac{V_{ab}}{\sqrt{3}}\angle -30 = \frac{210}{\sqrt{3}}\angle -50 = 121.2\angle -50$$

$$I_a = \frac{V_{an}}{Z_P} = \frac{121.2\angle -50}{40+j25} = 2.57\angle -82^\circ \text{ A}$$

$I_b = 2.57\angle -202^\circ$ A.

$I_c = 2.57\angle 38^\circ$ A.

No.

* Power in 3ϕ systems:

- for a Y connected load:

$V_{an} = \sqrt{2} V_p \cos\omega t$, $V_{bn} = \sqrt{2} V_p \cos(\omega t - 120)$, $V_{cn} = \sqrt{2} V_p \cos(\omega t + 120)$

↳ peak value since V_p (rms)

* suppose an inductive load ($Z\angle\theta$):

$i_{an} = \sqrt{2} I_p \cos(\omega t - \theta)$, $i_{bn} = \sqrt{2} I_p \cos(\omega t - 120 - \theta)$, $i_{cn} = \sqrt{2} I_p \cos(\omega t + 120 - \theta)$

$$\Rightarrow \begin{cases} P_A = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = V_p I_p \cos\theta \\ P_B = V_p I_p \cos\theta \\ P_C = V_p I_p \cos\theta \end{cases} \Rightarrow P_{3\phi} = 3 P_{1\phi} = 3 V_p I_p \cos\theta$$ **

$Q_{3\phi} = 3 Q_{1\phi} = 3 V_p I_p \sin\theta$ ** $|S_{3\phi}| = 3 |S_{1\phi}| = 3 |V_p| |I_p|$ **

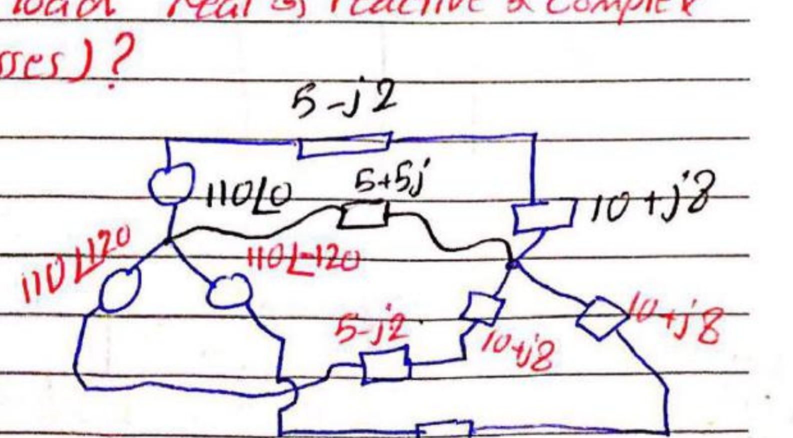
* $P_{Total} = 3 V_p I_p \cos\theta \Rightarrow$ in Y: $V_p = \frac{V_L}{\sqrt{3}}$ & $I_L = I_p$

in Δ: $V_p = V_L$ & $I_p = \frac{I_L}{\sqrt{3}}$

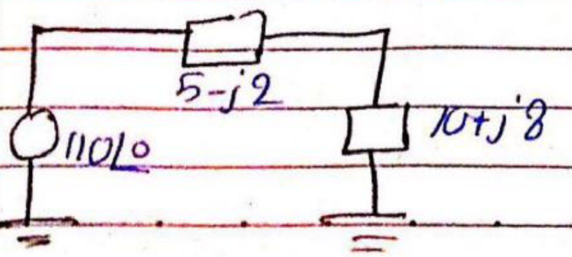
$\Rightarrow$ so in Y & Δ:

$$\boxed{P_{Total} = \sqrt{3} V_L I_L \cos\theta , \quad Q_{Total} = \sqrt{3} V_L I_L \sin\theta , \quad S_{total} = \sqrt{3} V_L I_L^*}$$

* Ex. Find the source & the load real & reactive & complex power? (find the losses)?



* since it is Balanced so (5+j5) ⇒ has No effect. since $I_N = 0$



$I_p = \frac{110\angle 0}{5 - j2 + 10 + j8} = \boxed{6.8\angle -21.8^\circ}$

No.

$$\Rightarrow \underset{(source)}{P_{3\phi}} = 3 V_p I_p^* = 3 * (110\angle 0) * (6.8 \angle 21.8^\circ)$$

$$= \underbrace{2247 \angle 21.8}_{S_{3\phi}} = \underbrace{2087}_{P_{3\phi}} + j \underbrace{834.6}_{Q_{3\phi}}$$

at load:

$$S_L = 3 V_p I_p^* = 3 (Z_p I_p) * I_p^* = 3 |I_p|^2 Z_p$$

$$= \underbrace{1782 \angle 38.66^\circ}_{S_{3\phi}} = \underbrace{1392}_{P_{3\phi}} + j \underbrace{1113}_{Q_{3\phi}}$$

Losses:

method(1): $S_{losses} = S_{source} - S_{load} = 695.6 - j\,278$

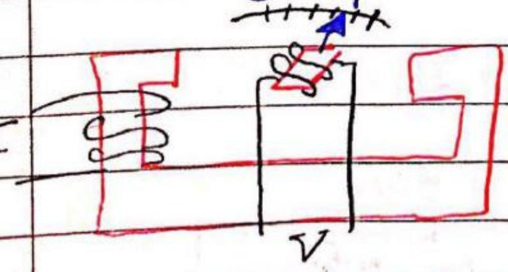
method(2): $S_{losses} = 3 |I_p|^2 Z_{T.L} = 3 * (6.8)^2 * (5 - j2)$

$$= 695.6 - j\,278$$

* Power Measurments :

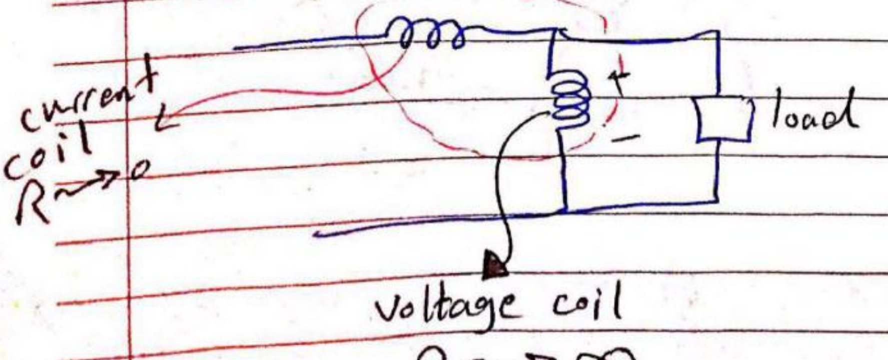
* in single phase :

- wattmeter: is an instrument used to measure the average power.



Deflection is proportional with V & I.

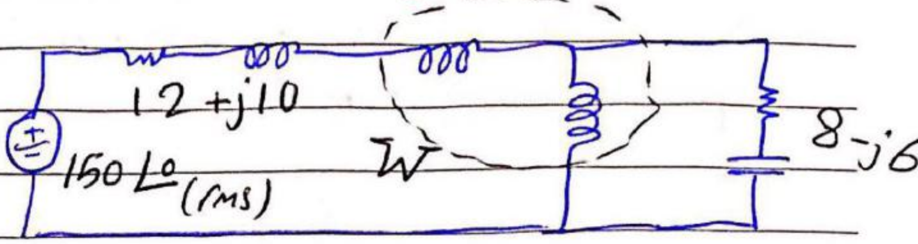
* structure of wattmeter:



$$P = |V_{rms}| |I_{rms}| \cos(\theta_v - \theta_i)$$

NO.

* Ex. Find the wattmeter reading for the shown ckt:



$$I_{rms} = \frac{150\angle 0}{12+j10+8-j6}$$

$$= \boxed{7.35\angle -11.3^{\circ}}\ A$$

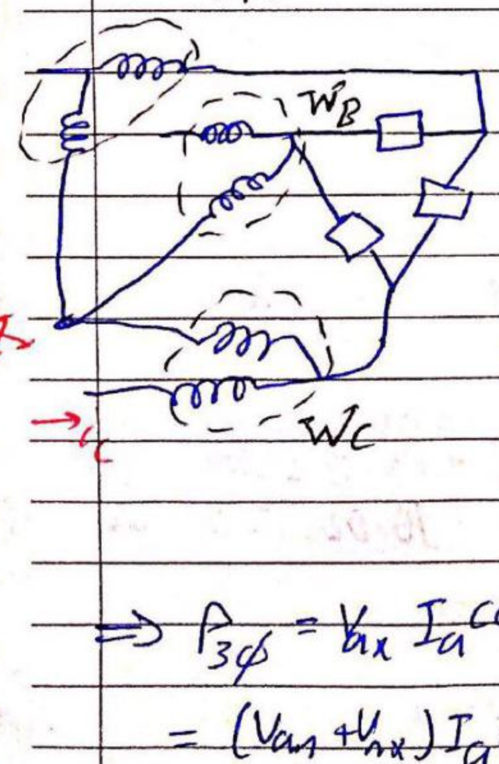
$$V_{rms\,(load)} = (8-j6)(7.35\angle -11.3^{\circ}) = \boxed{73.5\angle -48^{\circ}}\ V$$

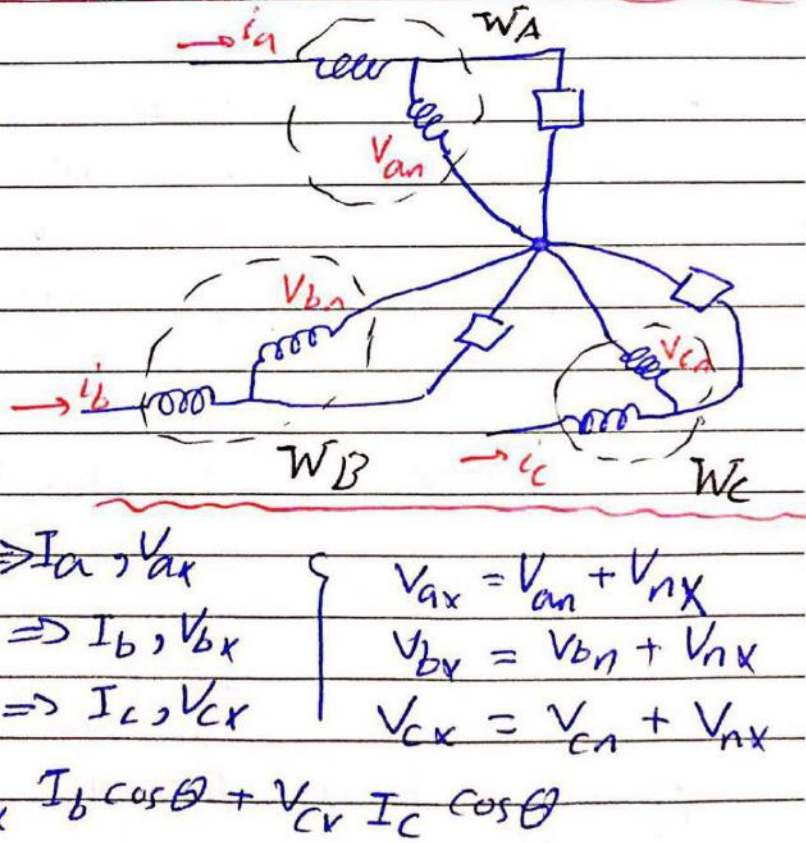
wattmeter reading $\equiv P = |V_{rms}||I_{rms}|\cos(\theta_v - \theta_i)$

$$= \boxed{432.1\ W}$$

※ in three phase:

using Δ





$\Rightarrow P_A \Rightarrow I_a, V_{ax}$ $V_{ax} = V_{an} + V_{nx}$

$P_B \Rightarrow I_b, V_{bx}$ $V_{bx} = V_{bn} + V_{nx}$

$P_C \Rightarrow I_c, V_{cx}$ $V_{cx} = V_{cn} + V_{nx}$

$$\Rightarrow P_{3\phi} = V_{ax} I_a \cos\theta + V_{bx} I_b \cos\theta + V_{cx} I_c \cos\theta$$

$$= (V_{an} + V_{nx}) I_a \cos\theta + (V_{bn} + V_{nx}) I_b \cos\theta + (V_{cn} + V_{nx}) I_c \cos\theta$$

$$= V_{an} I_a \cos\theta + V_{bn} I_b \cos\theta + V_{cn} I_c \cos\theta + V_{nx}\underbrace{(I_a + I_b + I_c)}_{0}\cos\theta$$

$$P_{3\phi} = V_{an} I_a \cos\theta + V_{bn} I_b \cos\theta + V_{cn} I_c \cos\theta$$

No.

Conclusion:

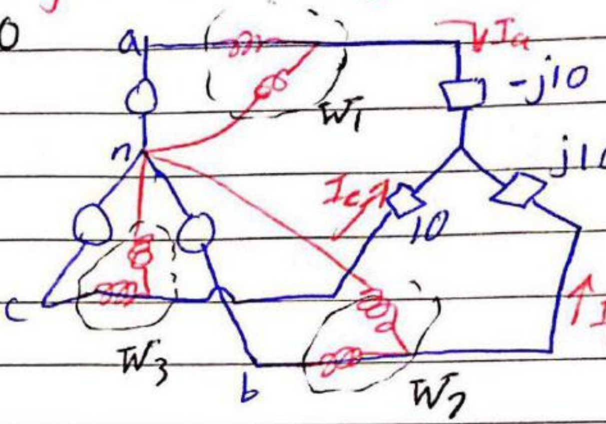
⇒ The choice of x does not change the total power measured.

ex) Find the 3-wattmeters reading when x @ n ?

given: $V_{ab} = 100 \angle 0$, $Z_a = -j10$, $Z_b = j10$, $Z_c = 10$

W_1 : V_{an}, I_a W_2 : V_{bn}, I_b

W_3 : V_{cn}, I_c



$V_{an} = \frac{100}{\sqrt{3}} \angle -30^\circ$

$V_{bn} = \frac{100}{\sqrt{3}} \angle -150^\circ$

$V_{cn} = \frac{100}{\sqrt{3}} \angle +90^\circ$

taking loops in two phases:

$-V_{an} - j10\, I_a - 10\, I_c + V_{cn} = 0$ --- ①

$-V_{cn} - j10\, I_a - j10\, I_b + V_{bn} = 0$ --- ②

$-V_{bn} + j10\, I_c - 10\, I_b + V_{cn} = 0$ --- ③

⇒ Hard to solve it will be very long.

⇒ A way to solve:

$I_a = \frac{V_{an} - V_N}{-j10}$, $I_b = \frac{V_{bn} - V_N}{j10}$, $I_c = \frac{V_{cn} - V_N}{10}$

use: ⇒ $I_a + I_b + I_c = 0$

Answers: $I_a = 19.32 \angle 15$ A.
$I_b = 19.32 \angle 165$ A.
$I_c = 10.00 \angle -90$ A.

so Now:

$P_1 = \frac{100}{\sqrt{3}} * 19.32 \cos(-30 - 15) = 788.7$ W

$P_2 = \frac{100}{\sqrt{3}} * 19.32 \cos(-150 - 165) = 788.7$ W

$P_3 = \frac{100}{\sqrt{3}} * 10 \cos(90 + 90) = -577.4$ W

so $P_{total} = P_1 + P_2 + P_3 = 1000$ W

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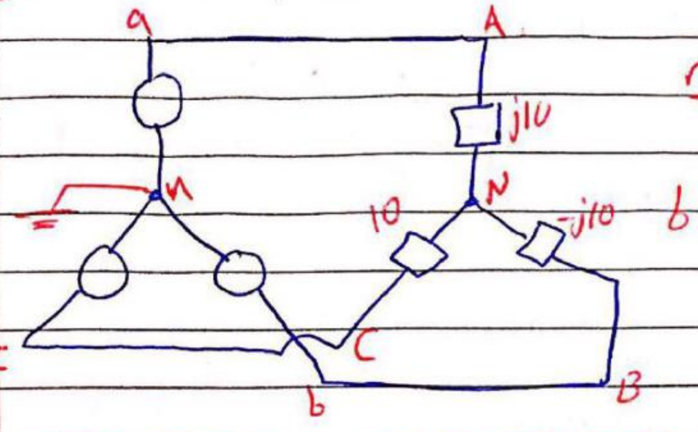
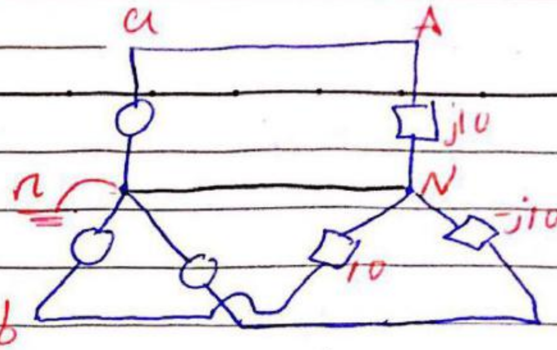
$I_a + I_b + I_c = 0$

$V_N \neq 0$

Note:





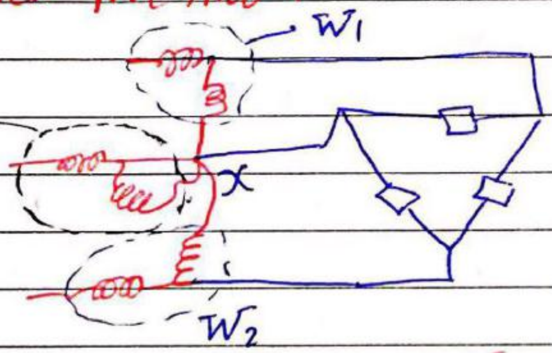
$I_{nN} = I_a + I_b + I_c$

$V_N = V_n = 0$

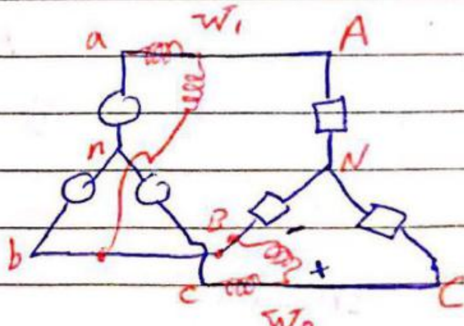
if there is $Z_n \Rightarrow V_N \neq 0$

※ The Two Wattmeter Method:

$$P_T = P_1 + P_2$$



for the last ex.:



$W_1 : V_{ab}, I_a$

$W_2 : V_{cb}, I_c$

$\Rightarrow V_{ab} = 100\angle 0$

$V_{cb} = -V_{bc} = -100\angle{-120} = 100\angle{+60^\circ}$

$I_a = 19.32\angle 15^\circ$

$I_c = 10\angle{-90^\circ}$

$\Rightarrow P_1 = 100 * 19.32 \cos(0-15) = 1866\ W$

$\theta_{V_{ab}}$ θ_{I_a}

$P_2 = 100 * 10 \cos(60+90)$

$= -866\ W$

$\Rightarrow P_T = P_1 + P_2$

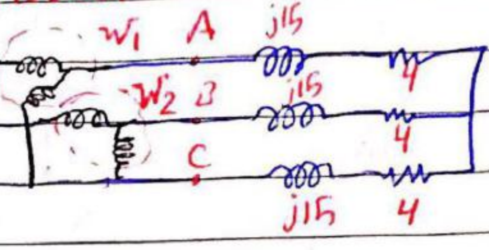
$= 1000\ W$

P_1 & P_2 $\Rightarrow$ they are just reading (so actually they have No meaning)

$P_T \longrightarrow$ The power supplied by the source

No.

Ex. given $V_{ab} = 230\angle 0^\circ$ find the power measured by each wattmeter & the total Power



$P_T = P_1 + P_2$

$P_1 \Rightarrow V_{ac}, I_a \qquad P_2 \Rightarrow V_{bc}, I_b$

$V_{ab} = 230\angle 0^\circ$

$\boxed{V_{bc} = 230\angle -120}$

$V_{ca} = 230\angle +120 \Rightarrow V_{ac} = 230\angle 120-180 = \boxed{230\angle -60^\circ}$

$\Rightarrow I_a = \dfrac{V_{an}}{Z_p} = \dfrac{230/\sqrt{3}\angle -30^\circ}{4+j15} = 8.554\angle -105^\circ$

$\Rightarrow I_b = 8.554\angle 225$

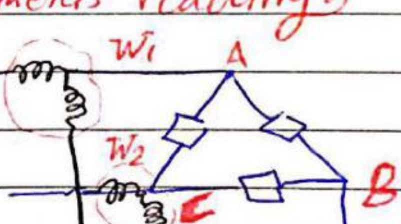
so $P_1 = V_{ac} I_a \cos(\theta_{V_{ac}} - \theta_{I_a}) = \boxed{1389\ W} \Rightarrow P_T = P_1 + P_2$

$P_2 = V_{BC} I_b \cos(\theta_{V_{bc}} - \theta_{I_b}) = \boxed{-512\ W}$

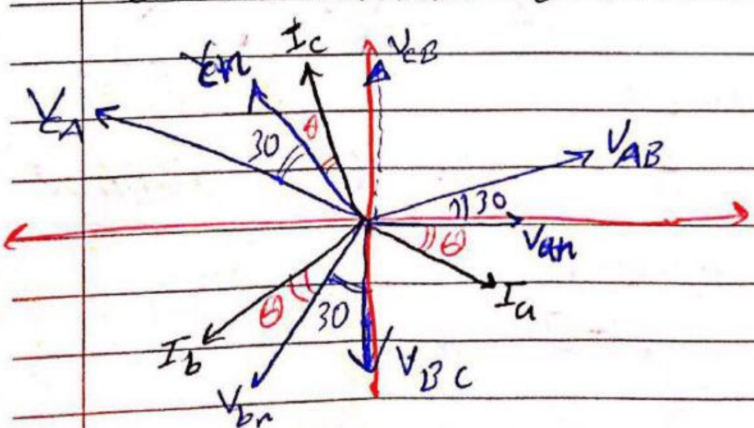
$P_T = 876.5\ W$

※ How to calculate PF using the wattmeters reading:

$W_1 : V_{AB}, I_a \quad , \quad W_2 : V_{CB}, I_C$



consider an inductive load:



$P_1 = V_{AB} I_a \cos(\theta_{V_{AB}} - \theta_{I_a})$

$= V_{AB} I_a \cos(30+\theta)$

$= V_{LL} I_L \cos(30+\theta)$

$P_2 = V_{CB} I_C \cos(\theta_{V_{CB}} - \theta_{I_C})$

$= V_{CB} I_C \cos(30-\theta)$

$= V_{LL} I_L \cos(30-\theta)$

Now: $\dfrac{P_1}{P_2} = \dfrac{V_{LL} I_L \cos(\theta+30)}{V_{LL} I_L \cos(30-\theta)} \Longrightarrow$

* All relations in this page work on Δ & Y *

No.

$$\Rightarrow \frac{P_1}{P_2} = \frac{\cos\theta\cos 30 - \sin\theta\sin 30}{\cos\theta\cos 30 + \sin\theta\sin 30} = \frac{\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta}{\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta}$$

$$= \frac{\sqrt{3} - \tan\theta}{\sqrt{3} + \tan\theta} \Rightarrow \sqrt{3}P_1 + P_1\tan\theta = \sqrt{3}P_2 - P_2\tan\theta$$

$$\Rightarrow \tan\theta(P_1 + P_2) = \sqrt{3}(P_2 - P_1)$$

$$\Rightarrow \tan\theta = \frac{\sqrt{3}(P_2 - P_1)}{P_1 + P_2} \Rightarrow pf = \cos\theta$$

$\rightarrow P_1 = P_2$ resistive load.

$\rightarrow P_2 > P_1$ inductive load.

capacitive load. $P_1 > P_2$

$P_1 = V_{LL} I_L \cos(30+\theta)$

$P_2 = V_{LL} I_L \cos(30-\theta)$

$$\Rightarrow P_T = V_{LL} I_{LL}\cos 30\cos\theta - V_{LL} I_{LL}\sin 30\sin\theta + V_{LL} I_L \cos 30\cos\theta + V_{LL} I_L \sin 30 \sin\theta$$

$$= 2V_{LL} I_L \frac{\sqrt{3}}{2}\cos\theta$$

$$\Rightarrow P_T = \sqrt{3} V_{LL} I_L \cos\theta$$

How to calculate Q!

$$P_1 - P_2 = V_{LL} I_L \cos(30+\theta) - V_{LL} I_L \cos(30-\theta) = -2V_L I_L \sin\theta\sin 30$$

$$= -V_L I_L \sin\theta$$

we know that $Q_T = \sqrt{3} V_{LL} I_L \sin\theta$

so, $Q_T = \sqrt{3}(P_2 - P_1)$ & $S_T = \sqrt{Q_T^2 + P_T^2}$

ex) The two wattmeters reading are $P_1 = 1560\,W$ & $P_2 = 2100\,W$ when connected to Δ-load.

Find:

(1) $P_{1\phi} = \frac{P_{3\phi}}{3} = \frac{P_1 + P_2}{3} = \boxed{1220\,W}$

(2) $Q_{1\phi} = \frac{Q_T}{3} = \frac{\sqrt{3}(P_2 - P_1)}{3} = \boxed{311.77\ VAR}$

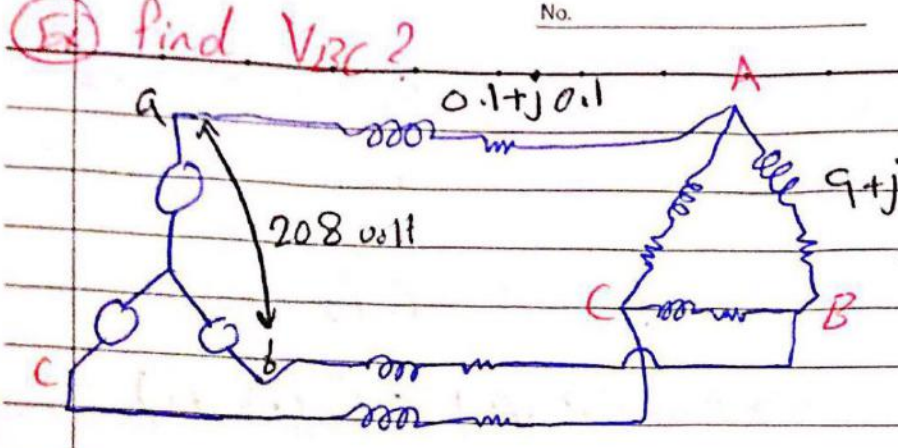
(3) $\theta = \tan^{-1}\left(\frac{Q_T}{P_T}\right) = \boxed{14.34^\circ}$

(4) $pf = \cos\theta = \boxed{0.9688}$ lagging.

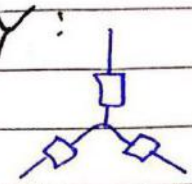
since $P_2 > P_1$

Ex Find V_{BC} ?

No.

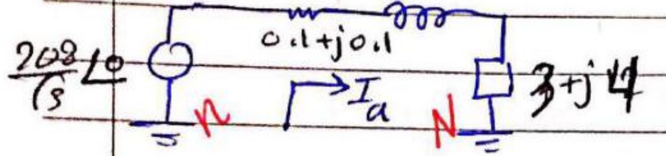


→ convert the load to Y:



$Z_Y = \frac{1}{3} Z_\Delta$

equivalent ckt: (single phase):



$\Rightarrow V_{an} = \frac{208}{\sqrt{3}} \angle 0$

$I_a = \dfrac{\frac{208}{\sqrt{3}} \angle 0}{0.1 + j0.1 + 3 + j4}$

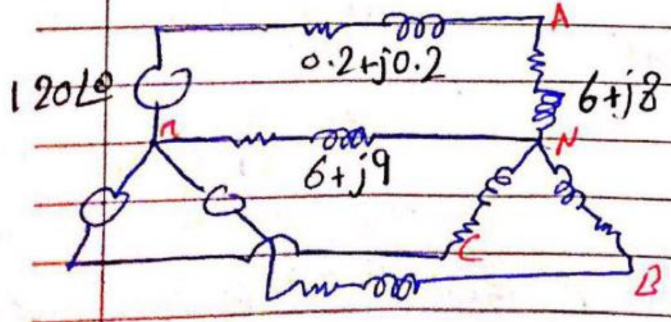
$\Rightarrow V_{AN} = I_a * Z$

$= I_a * (3+j4) = 117 \angle 0.22$ volt

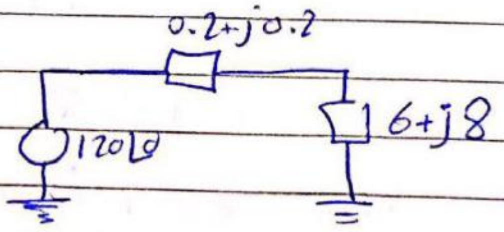
so $V_{AB} = \sqrt{3} * 117 \angle 30.22 = 203 \angle 30.22°$

so $V_{BC} = 203 \angle -89.78°$

* For the figure shown solve for line current, phase voltage and line voltage at load:



$\Rightarrow$



$I_a = \dfrac{120\angle 0}{6.2+j8.2} = \boxed{11.7 \angle -52°}$

$\boxed{\begin{array}{l} I_b = 11.7 \angle -52-120° \\ I_c = 11.7 \angle -52+120 \end{array}}$

$V_{AN} = (6+j8)(11.7\angle -52°)$

$= 117 \angle 0.23$

$V_{AB} = 117\sqrt{3} \angle 30.23°$

$= 202.6 \angle 30.23°$

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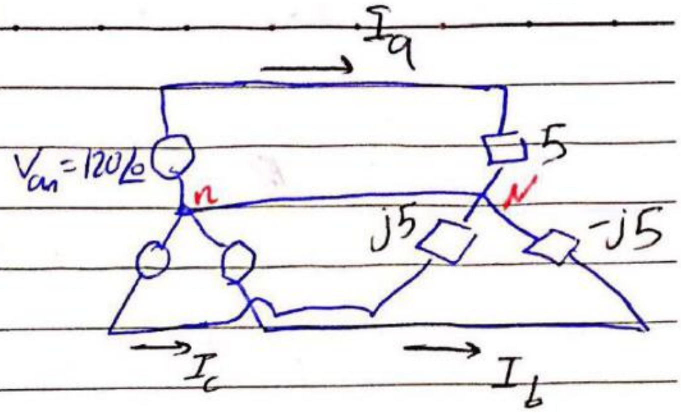
NO.

ex.) Solve for I_1, I_2, I_3, I_{Nn}?



since No load on the Neutral

$V_N = 0$

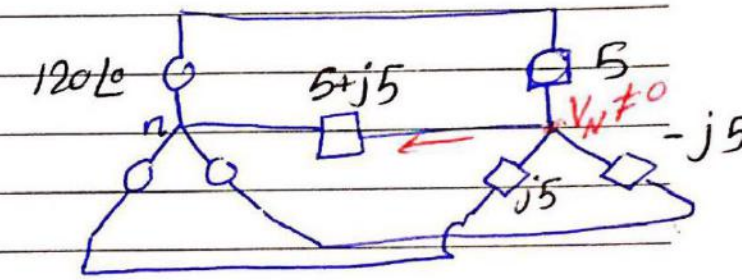
so $I_a = \frac{V_{an}}{5} \Rightarrow \boxed{I_a = 24\ A}$

$I_b = \frac{V_{bn}}{-j5} \Rightarrow \boxed{I_b = 24\angle -30^\circ\ A}$

$I_c = \frac{V_{cn}}{j5} \Rightarrow \boxed{I_c = 24\angle 30^\circ}$ $\Rightarrow$ $I_{nN} = -(I_a + I_b + I_c)$

$I_{Nn} = I_a + I_b + I_c \simeq \underline{65.6\ A}$

ex.) find I_a, I_b, I_c, I_{Nn}?



$I_a = \frac{V_{an} - V_N}{5}$, $I_b = \frac{V_{bn} - V_N}{-j5}$

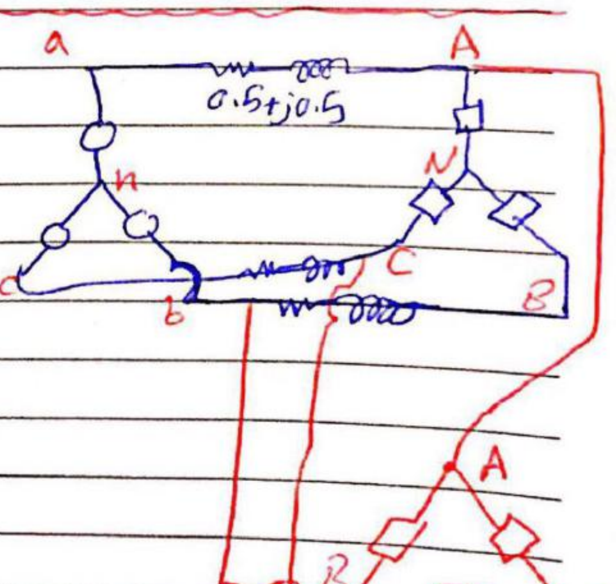
$I_c = \frac{V_{cn} - V_N}{j5}$

$I_a + I_b + I_c = I_{Nn} = \frac{V_N - \cancel{V_n}}{5+j5}$ Zero (Always V_n equal zero)

$\frac{120 - V_N}{5} + \frac{120\angle -120 - V_N}{-j5} + \frac{120\angle 120 - V_N}{j5} = \frac{V_N}{5+j5}$

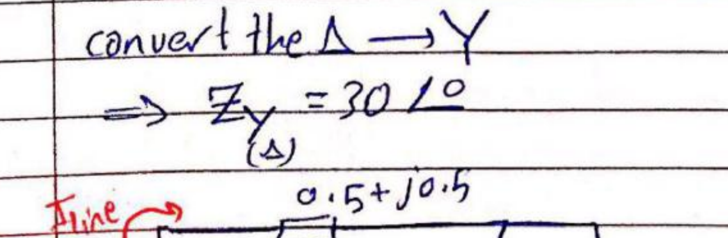
$\Rightarrow$ we find V_N then find the currents...

ex.) given voltage of generator 208 & $Z_Y = 20\angle 0$ & $Z_\Delta = 90\angle 0$



convert the $\Delta \longrightarrow Y$

$\Rightarrow Z_{Y(\Delta)} = 30\angle 0$



$I_{Line} = \frac{120\angle 0}{12.5 + j0.5} = \underline{9.6\angle -2.3}$

$\Longrightarrow$

⇒ Need $I_{(20\angle 0)}$ & $I_{(30\angle 0)}$

$I_{(30\angle 0)} = I_{line} \frac{20}{50}$ $I_{(20\angle 0)} = I_{line} \frac{30}{50} = \underline{5.76\angle -23}$ A

~~But for Δ we need the current to (Δ):~~

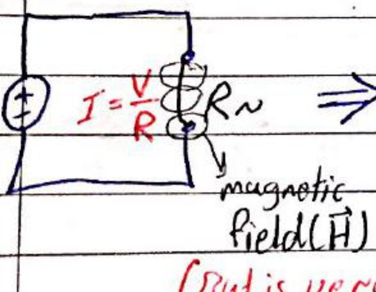
we have the Δ-line current ($I_{(30\angle 0)}$)

so $I_{line\,(\Delta)} = \frac{I_{30\angle 0}}{\sqrt{3}} \angle 30^\circ = \underline{2.22\angle 27.7}$ A

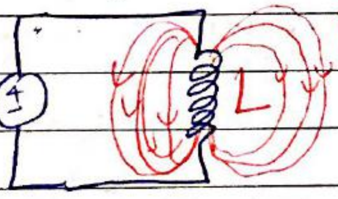
After first

End of CH12

CHAPTER(13): Magnetically coupled circuits:

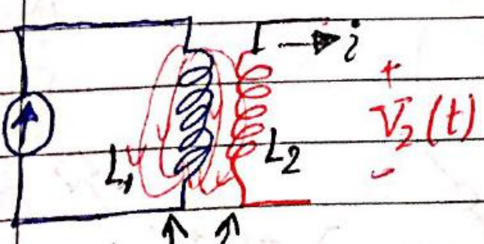


⇒



L Can't be neglected

so $V = L \frac{\partial i}{\partial t}$

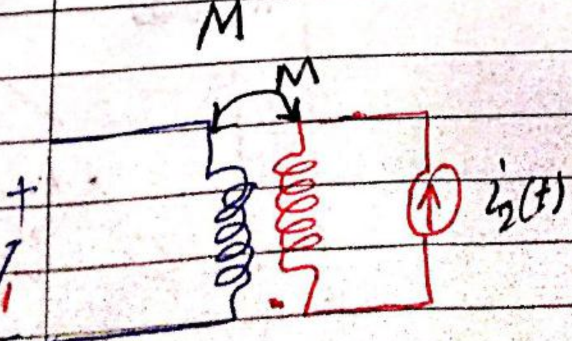


⇒ magnetically coupled circuit.

$$V_2(t) = M_{21} \frac{\partial i_1(t)}{\partial t}$$

effect source

M_{12} effect source



$$V_1 = M_{12} \frac{\partial i_2(t)}{\partial t}$$

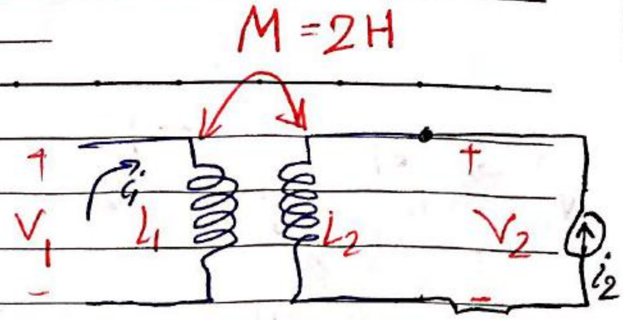
{A}

ex. Determine V_1 if $i_2 = 5\sin 45t$ A, $i_1 = 0$

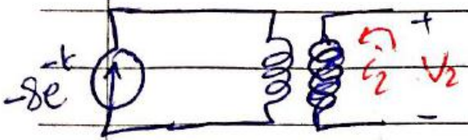
No.

$$V_1 = M\frac{\partial i_2}{\partial t} = (2)(5)(45)\cos 45t$$

$$V_1 = 450\cos 45t \text{ volt.}$$

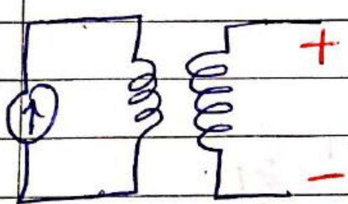


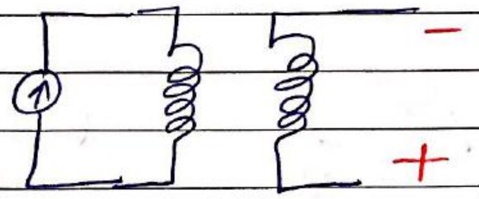
{B} Determine V_2 if $i_1 = -8e^{-t}$ and $i_2 = 0$



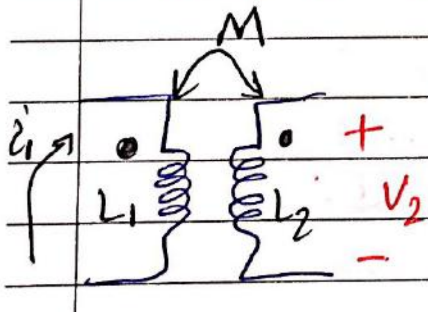
$$V_2 = M\frac{\partial i_1}{\partial t} = (2)(-8)(-1)e^{-t}$$

$$V_2 = 16e^{-t} \text{ volt.}$$

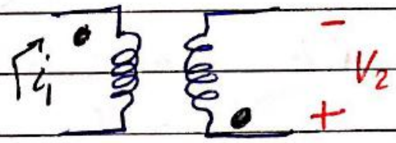




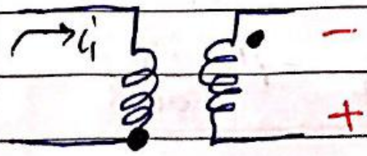
⇒ Dot Convention:

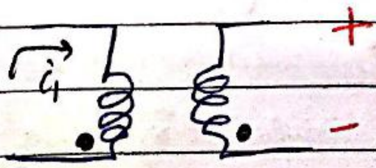


① if i enters the dot in one coil it makes the dotted terminal on the other coil positive.



② if i leaves the dot in one coil, it makes the dotted terminal on the other coil negative.

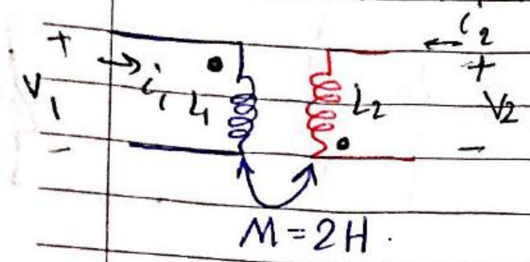




⇒

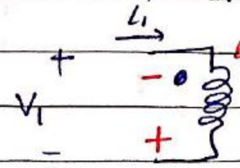
Now Back to the (ex) after we Know about Dot convention:

No.



A) $i_2 = 5\sin 45t$, $i_1 = 0$, find V_1?

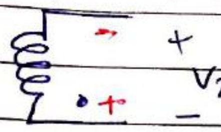
$$V_1 = -M\frac{\partial i_2}{\partial t} = -450\cos 45t \text{ volt.}$$



since i_2 leaving it make the other (+ve).

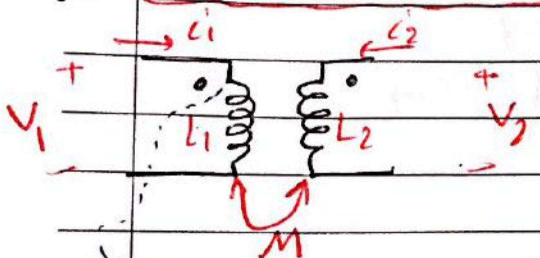
B) find V_2 if $i_1 = -8e^{-t}$, $i_2 = 0$?

since i_1 entering it make the other (+ve).



$$V_2 = -M\frac{\partial i_1}{\partial t} = -16e^{-t} \text{ volt.}$$

Ex.



$$V_1 = \underbrace{L_1\frac{\partial i_1}{\partial t}}_{\text{self inductance}} + \underbrace{M\frac{\partial i_2}{\partial t}}_{\text{Mutual inductance.}}$$

Assume here Like a voltage source $\pm M\frac{\partial i_2}{\partial t}$

* so you can reach this By KVL:

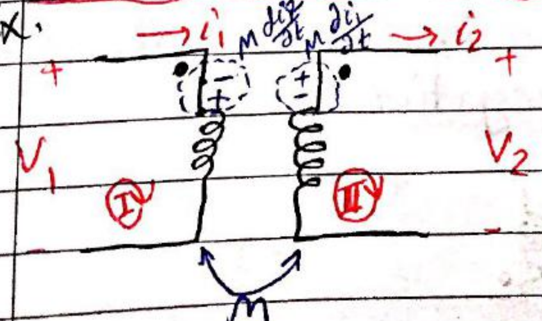
$$-V_1 + M\frac{\partial i_2}{\partial t} + L_1\frac{\partial i_1}{\partial t} = 0$$

$$\Rightarrow V_1 = L_1\frac{\partial i_1}{\partial t} + M\frac{\partial i_2}{\partial t}$$

* Do the same for the other one:

$$V_2 = L_2\frac{\partial i_2}{\partial t} + M\frac{\partial i_1}{\partial t}$$

Ex.



from loop(I):

$$-V_1 - M\frac{\partial i_2}{\partial t} + L_1\frac{\partial i_1}{\partial t} = 0$$

$$\Rightarrow \boxed{V_1 = L_1\frac{\partial i_1}{\partial t} - M\frac{\partial i_2}{\partial t}}$$

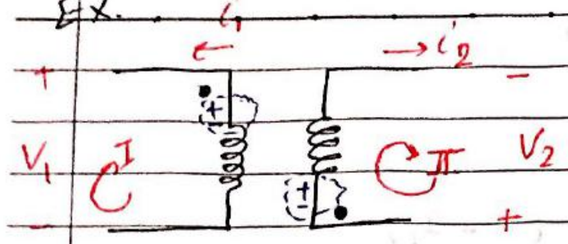
from loop(II):

$$V_2 + L_2\frac{\partial i_2}{\partial t} - M\frac{\partial i_1}{\partial t} = 0$$

$$\Rightarrow \boxed{V_2 = -L_2\frac{\partial i_2}{\partial t} + M\frac{\partial i_1}{\partial t}}$$

No.

Ex.



loop(I): $V_1 + L_1 \frac{\partial i_1}{\partial t} - M \frac{\partial i_2}{\partial t} = 0$

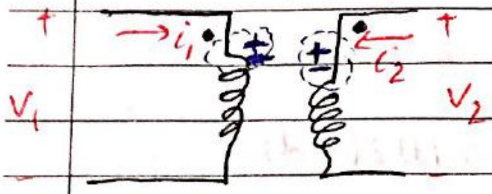
$\Rightarrow \boxed{V_1 = -L_1 \frac{\partial i_1}{\partial t} + M \frac{\partial i_2}{\partial t}}$

loop(II):

$-V_2 - M \frac{\partial i_1}{\partial t} + L_2 \frac{\partial i_2}{\partial t} = 0$

$\Rightarrow \boxed{V_2 = L_2 \frac{\partial i_2}{\partial t} - M \frac{\partial i_1}{\partial t}}$

※ In frequancy Domain:

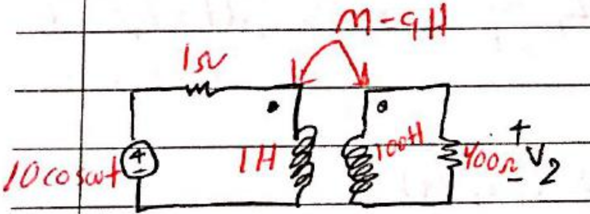


$v_1 = L_1 \frac{\partial i_1}{\partial t} + M \frac{\partial i_2}{\partial t}$

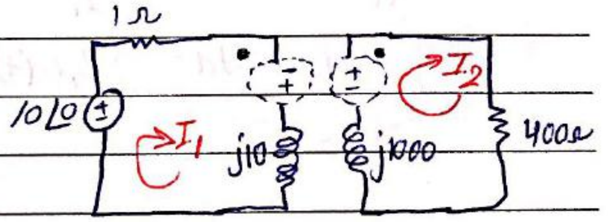
$\Rightarrow \boxed{\begin{aligned} V_1 &= j\omega L_1 I_1 + j\omega M I_2 \\ V_2 &= j\omega L_2 I_2 + j\omega M I_1 \end{aligned}}$

$\Rightarrow$ Do the same for all previous examples.

Ex. Find V_2 for the shawn cKt: ($\omega = 10$)



freq. $\Longrightarrow$ Domain



$L \rightarrow j\omega L$

$M \rightarrow j\omega M$

mesh analysis:

$-10\angle 0 + I_1 + j10 I_1 - j90 I_2 = 0$ --- (1)

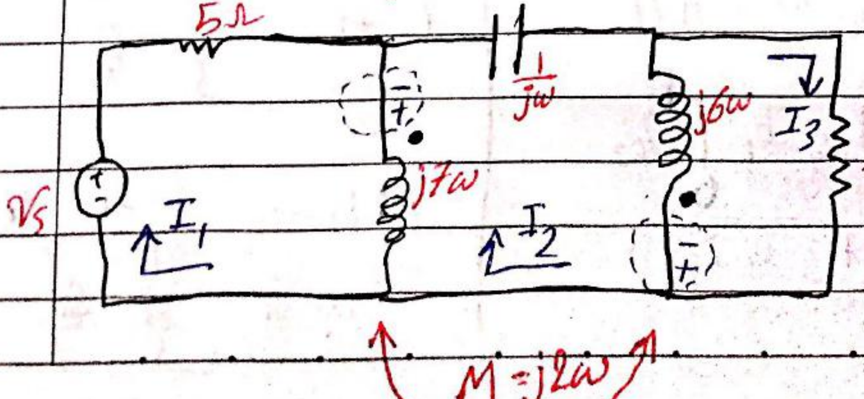
$I_2(400 + j1000) - j90 I_1 = 0$ --- (2)

$\Rightarrow$ solving for (1) & (2):

$I_2 = 0.172 \angle -16°$

so $V_2 = I_2 R \Rightarrow V_2 = 68.8 \angle -16°$ volt.

Ex. Write the equations for the meshes in the shawn cKt:



$I_2 - I_3$ (leaving)

$I_1 - I_2$ (entering)

$\longrightarrow$

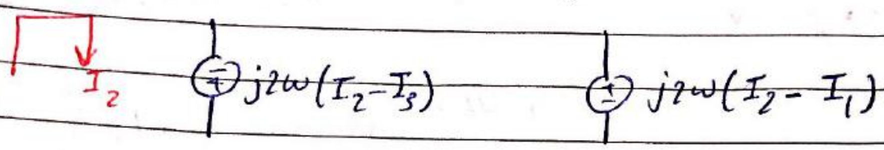
* mesh(1): (one virtual source).

$-V_s + 5I_1 + j7\omega(I_1 - I_2) - j2\omega(I_2 - I_3) = 0$

* mesh(3): (one virtual source)

$3I_3 + j6\omega(I_3 - I_2) + j2\omega(I_1 - I_2) = 0$

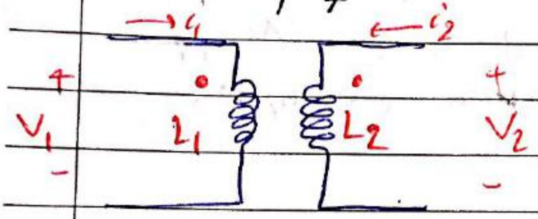
* mesh(2): (two virtual sources)



$\Rightarrow \frac{1}{j\omega} I_2 + j6\omega(I_2 - I_3) + j7\omega(I_2 - I_1) + j2\omega(I_2 - I_1) + j2\omega(I_2 - I_3) = 0$

* Energy stored in a ckt with mutual inductance:

$P = \frac{1}{T}\int_T v(t) \cdot i(t)\,dt$, $W = \int_T v(t)\, i(t)\, dt$.



** set $I_2 = 0$

lets make $i_1 : 0 \to I_1$

$t : 0 \to t_1$

$v_1 i_1 = L_1 \frac{\partial i_1}{\partial t} i_1 \Rightarrow v_1 i_1 dt = L_1 i_1 \partial i_1$

integrate...

$\Rightarrow \underbrace{\int_0^{t_1} v_1(t)\, i_1(t)\, dt}_{W} = \int_0^{I_1} L_1 i_1(t)\, di_1$

$\Rightarrow \boxed{W = \frac{1}{2} L_1 i_1^2(t)}$

**

set $i_1(t) = I_1$

let $i_2(t) : 0 \to I_2$

$t : t_1 \to t_2$

$\Rightarrow \underbrace{\int_{t_1}^{t_2} v_2 i_2\, dt}_{W} = \int_0^{I_2} L_2 i_2(t)\, di_2$

$\Rightarrow \boxed{W = \frac{1}{2} L_2 i_2^2(t)}$

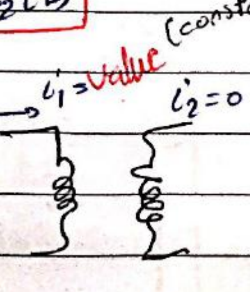
$v_1(t) = M_{12} \frac{\partial i_2}{\partial t}$

so $\int_{t_1}^{t_2} v_1(t)\, i_1(t)\, dt = \int_0^{I_2} M_{12} \frac{\partial i_2}{\partial t} i_1(t)\, dt$

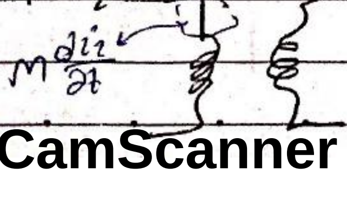
$= M_{12} I_1 \int_0^{I_2} \partial i_2$

$= \boxed{M_{12} I_1 I_2}$

$t = 0 \to t_1$



$t : t_1 \to t_2$



Total energy:

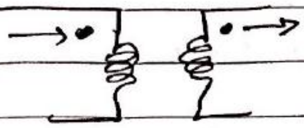
$$W_{total} = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M_{12} I_1 I_2$$

$$W = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M_{21} I_1 I_2$$

$\Rightarrow M_{12} = M_{21} = M$

instantaneous Energy:

$$W(t) = \frac{1}{2} L_1 i_1^2(t) + \frac{1}{2} L_2 i_2^2(t) + M i_1(t) i_2(t)$$



$$W = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 - M I_1 I_2$$

* if the two current leaving or entering the dot (M (+ve)).

* if one of the current leaving & the other entering (M (-ve)).

$\Rightarrow$ Set an upper limit for M:

$W > 0$, $i_1 > 0$, $i_2 > 0$

$$\Rightarrow \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 - M I_1 I_2 > 0$$

$$\Rightarrow \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + I_1 I_2 \sqrt{L_1 L_2} - I_1 I_2 \sqrt{L_1 L_2} - M I_1 I_2 > 0$$

$$\Rightarrow \frac{1}{2}\left(I_1 \sqrt{L_1} - I_2 \sqrt{L_2}\right)^2 + I_1 I_2 \left(\sqrt{L_1 L_2} - M\right) > 0$$

To guarantee that $W > 0 \rightarrow I_1 I_2 \left(\sqrt{L_1 L_2} - M\right) > 0$

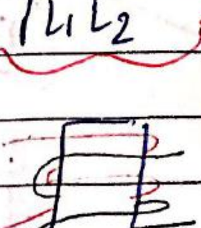
$$\Rightarrow \sqrt{L_1 L_2} - M > 0 \Rightarrow \sqrt{L_1 L_2} \geq M$$

this is the upper limit.

* Coupling Coefficient:

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

$$0 \leq K \leq 1$$



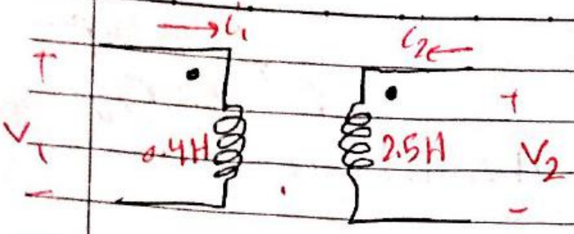
Ex. find $v_1(0)$, $w(0)$: given:

$K = 0.6$

$i_1 = 20\cos(500t - 20)\,mA$

$i_2 = 5\cos(500t - 20)\,mA$.

No.



first find M :

$$M = K\sqrt{L_1 L_2} = \underline{0.6\,H}$$

$$v_1(t) = L_1 \frac{\partial i_1(t)}{\partial t} + M \frac{\partial i_2(t)}{\partial t}$$

$$v_1(0) = 0.4 * [-20 * 500 \sin(-20)] + 0.6\,[-5 * 500 \sin(-20)]$$

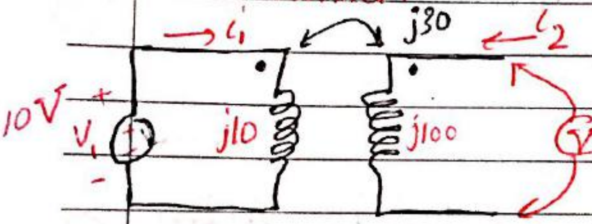
$$\boxed{v_1(0) = 1.881\ \text{volt}}.$$

$$w(0) = \frac{1}{2} i_1^2(0)\, L_1 + \frac{1}{2} L_2\, i_2^2(0) + M i_1(0) i_2(0)$$

$$= \frac{1}{2}[20\cos(-20)]^2 * 0.4 + \frac{1}{2}[5\cos(-20)]^2 * 2.5 + 0.6 * 20\cos(-20) * 5\cos(-20)$$

$$\boxed{w(0) = 161.2\,\mu J}$$

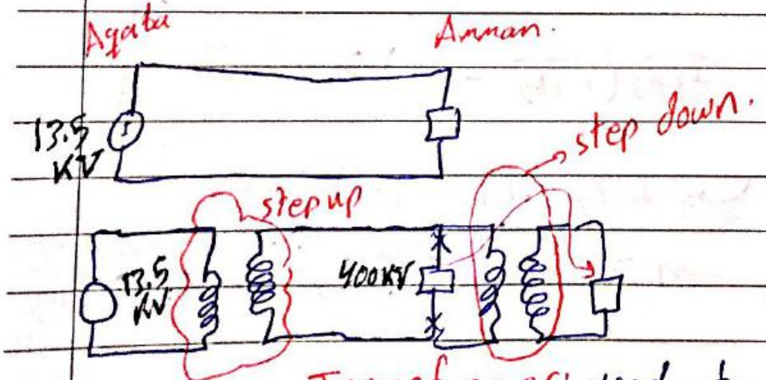
※ Transformer:



$10 = V_1 = j10\, I_1 + j30\, I_2$

$10 \overset{??}{<} V_2 = j100\, I_2 + j30\, I_1$

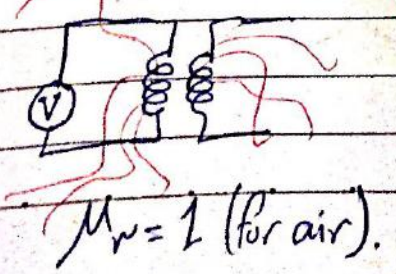
(acceptable)

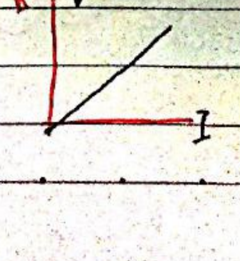


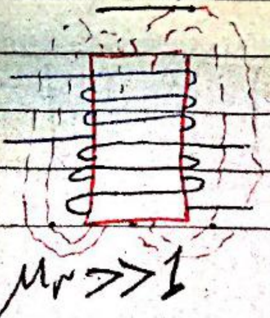
Transformer: used to step up & step down the voltage.

Transformer.

Linear Ideal.



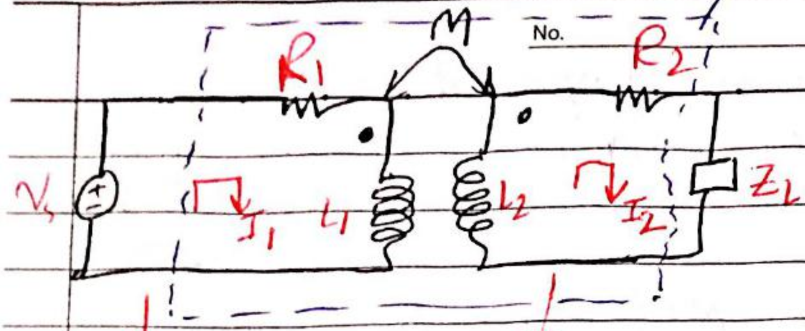




$\mu_r = 1$ (for air).

$\mu_r >> 1$

* Linear Transformer:



primary circuit.

secondary Circuit.

→ Linear Transformer.

from the two meshes:

$$V_s = (R_1 + j\omega L_1)I_1 - j\omega M I_2$$

$$0 = (j\omega L_2 + R_2 + Z_L)I_2 - j\omega M I_1$$

* Before M:

$$V_s = (R_1 + j\omega L_1)\, I_1$$

$$Z_{in} = \frac{V_s}{I_1} = Z_{11}$$

$$\Rightarrow Z_{11} = R_1 + j\omega L_1$$

$$Z_{22} = R_2 + Z_L + j\omega L_2$$

The mesh equ. becomes:

$$\Rightarrow V_s = Z_{11} I_1 - j\omega M I_2$$

$$0 = -j\omega M I_1 + Z_{22} I_2 \Rightarrow \begin{bmatrix} V_s \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{11} & -j\omega M \\ -j\omega M & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Rightarrow I_2 = \left(\frac{j\omega M}{Z_{22}}\right) I_1 \Rightarrow V_s = Z_{11} I_1 - j\omega M I_1 \left(\frac{j\omega M}{Z_{22}}\right)$$

$$\Rightarrow V_s = Z_{11} I_1 + \frac{\omega^2 M^2}{Z_{22}} I_1 \Rightarrow \frac{V_s}{I_1} = Z_{in}$$

$$\Rightarrow Z_{in} = Z_{11} + \frac{\omega^2 M^2}{Z_{22}} = Z_{11} + \frac{\omega^2 M^2}{R_{22} + jX_{22}} * \frac{(R_{22} - jX_{22})}{(R_{22} - jX_{22})}$$

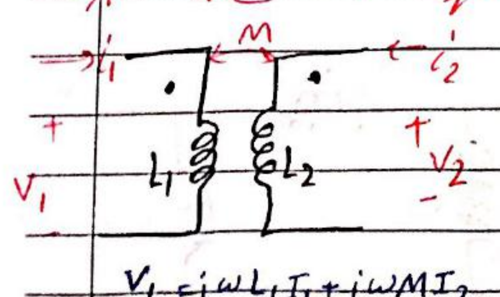
$R_{22} + jX_{22}$

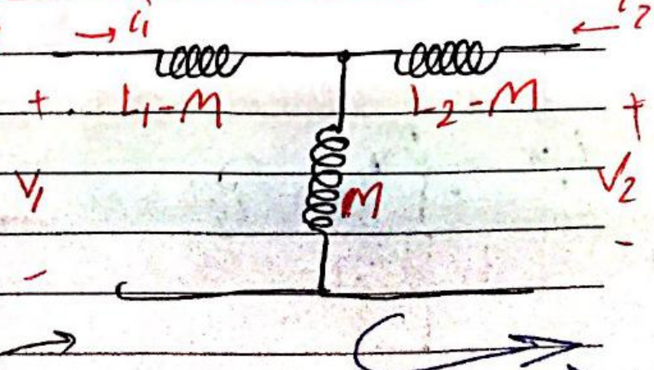
$$\Rightarrow Z_{in} = Z_{11} + \frac{\omega^2 M^2 R_{22}}{R_{22}^2 + X_{22}^2} - j\frac{\omega^2 M^2 X_{22}}{R_{22}^2 + X_{22}^2}$$

[Reflected Impedance]

* The presence of secondary increase losses in the primary.

* The T-equivalent circuit:





$$V_1 = j\omega L_1 I_1 + j\omega M I_2$$

$$V_2 = j\omega L_2 I_2 + j\omega M I_1$$

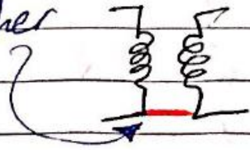
if the currents leaving or entering the dot.

⇒ left mesh: $V_1 = (j\omega L_1 - j\omega M) I_1 + j\omega M (I_1 + I_2)$

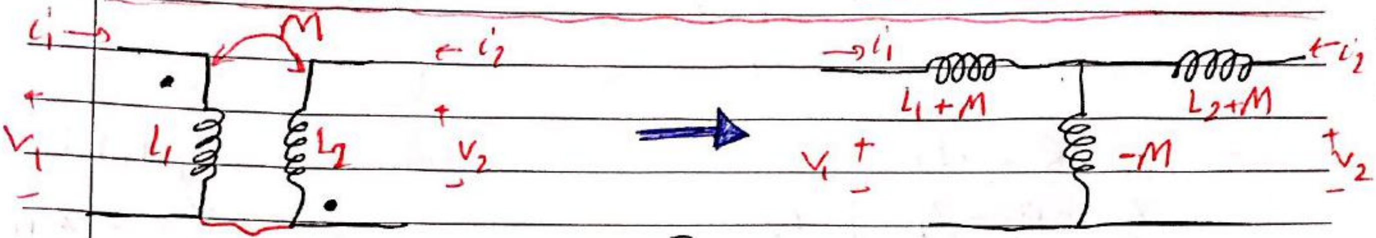
$= j\omega L_1 I_1 - \cancel{j\omega M I_1} + \cancel{j\omega M I_1} + j\omega M I_2 = j\omega L_1 I_1 + j\omega M I_2$

No.

* ~~There is an condition~~ To use T-equivalent ckt:
~~that the two coil must be connected together~~

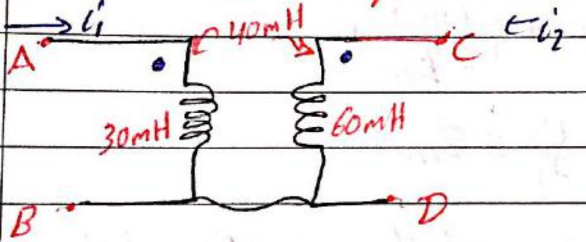


⇒ ~~right mesh:~~
~~$V_2 = (j\omega L_2 - j\omega M) I_1 + j\omega M (I_1 + I_2)$~~ $= j\omega L_2 I_2 + j\omega M I_1$

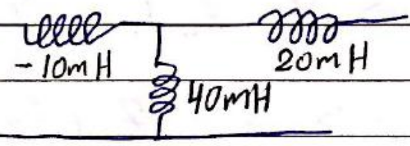


if one of the currents enter the dot
& the other leave.

ex) Find the T-equivalent ckt.

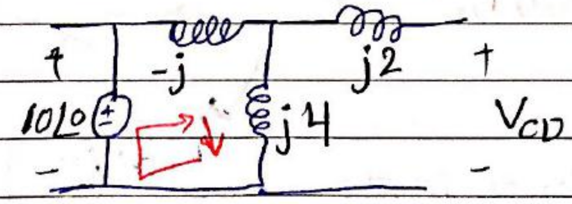


since the two currents entering it would be the first figure for T-equ. ckt.



* if $V_{AB} = 10\angle 0°$ & $\omega = 100$
find I_1 & V_{CD} ?

converting to freq. domain

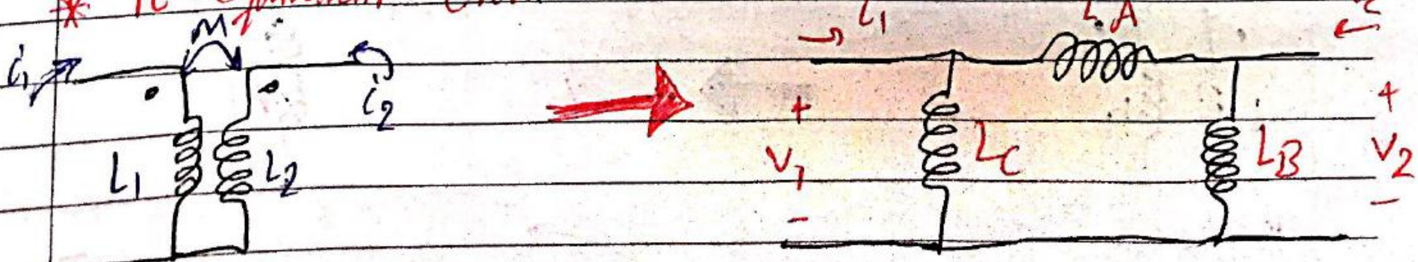


$I_1 = \frac{10\angle 0°}{j3} = 3.33\angle -90°$

$i_1(t) = 3.33 \cos(100t - 90)$ A

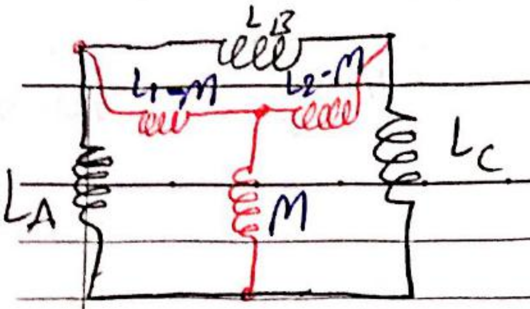
so $V_{CD} = j4 * I_1 = \boxed{13.33\angle 0°}$ volt.

* π-equivalent ckt:



Scanned by CamScanner

To Know the Values of L_A & L_B & L_C:

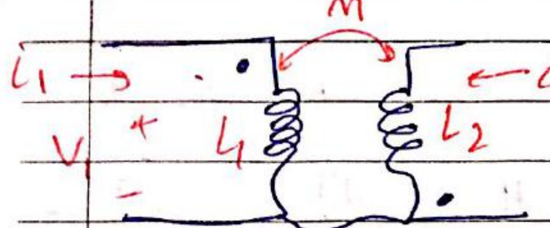


this situation if the two currents entering the dot or Leaving.

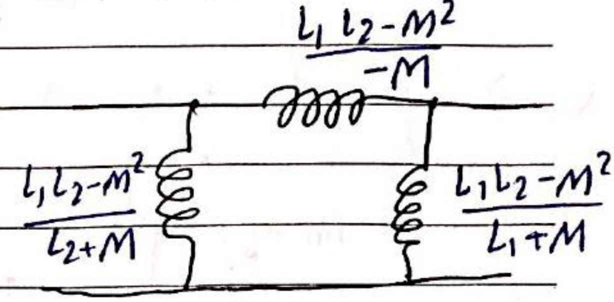
$$\Rightarrow L_B = \frac{(L_1-M)(L_2-M)+(L_1-M)M+(L_2-M)M}{M} = \boxed{\frac{L_1L_2-M^2}{M}}$$

$$\boxed{L_A = \frac{L_1L_2-M^2}{L_2-M}} \qquad \boxed{L_C = \frac{L_1L_2-M^2}{L_1-M}}$$

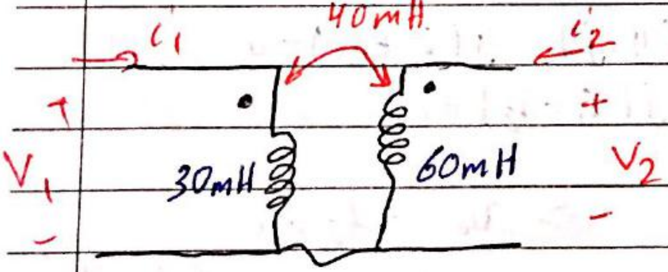
※ if one of the current leave & the other enter:

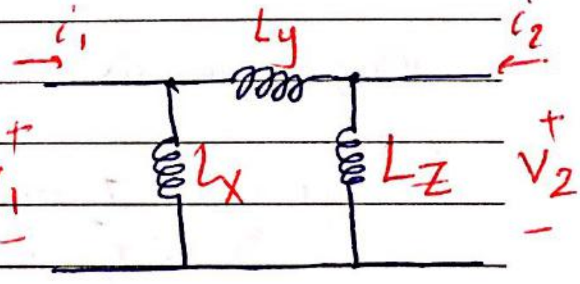


$\Rightarrow$



(Ex.) Find the π-equivalent ckt for:

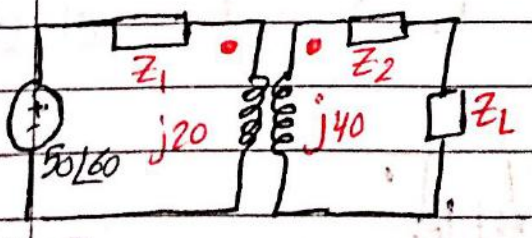




$$\Rightarrow L_x = \frac{L_1L_2-M^2}{L_2-M} = \boxed{10\text{mH}} \qquad \Rightarrow L_y = \frac{L_1L_2-M^2}{M} = \boxed{5\text{mH}}$$

$$\Rightarrow L_Z = \frac{L_1L_2-M^2}{L_1-M} = \boxed{-20\text{mH}}$$

(Ex.) Calculate the input impedance & current I_1 for:



given: $Z_1 = 60 - j100$

$Z_2 = 30 + j40$

$Z_L = 80 + j60$

$Z_{11} = Z_1 + j20 = \underline{60 - j80}$, $Z_{22} = Z_L + Z_2 + j40 = \underline{110 + j140}$

$Z_{in} = Z_{11} + \dfrac{\omega^2 M^2}{Z_{22}} \Longrightarrow$ we have $\omega M = 5$ $\Rightarrow$ $Z_{in} = 60 - j80 + \dfrac{25}{110 + j140}$

No.

$\Rightarrow$ so $Z_{in} = 100.14 \angle -53^\circ$

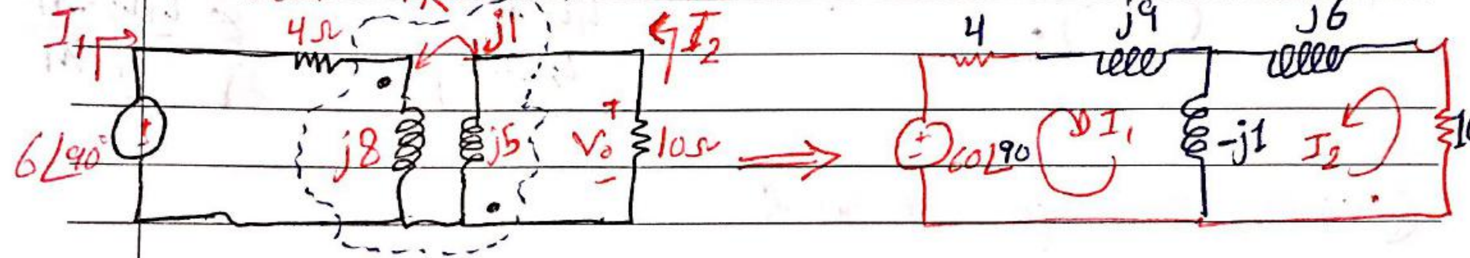
$Z_{in} = \dfrac{V_1}{I_1} \Rightarrow I_1 = \dfrac{V_1}{Z_{in}} = \dfrac{50 \angle 60}{100.14 \angle -53^\circ} \Rightarrow I_1 = 0.5 \angle 113^\circ$

if he asked about I_2:

we can take the right mesh ; But more easier:

$\Rightarrow I_2 = I_1 \left(\dfrac{j\omega M}{Z_{22}} \right) \Rightarrow$ Answer $= 0.014 \angle 151.2^\circ$

ex. Solve for I_1, I_2, V_o using T-equivalent ckt for the linear TR:



Using mesh analysis: $-(6\angle 90) + I_1(4 + j9) - j(I_1 + I_2) = 0$ ---(1)

$(10 + j6) I_2 - j(I_1 + I_2) = 0$ ---(2)

solving: $I_2 = 0.06 \angle 90^\circ$ A

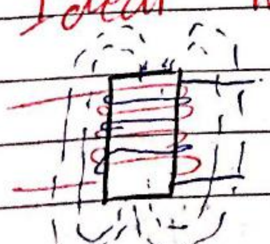
$I_1 = 0.6 + j0.3$ A

$\Rightarrow V_o = -I_2 \times 10$

$\Rightarrow V_o = 0.6 \angle -90^\circ$

* Second material *

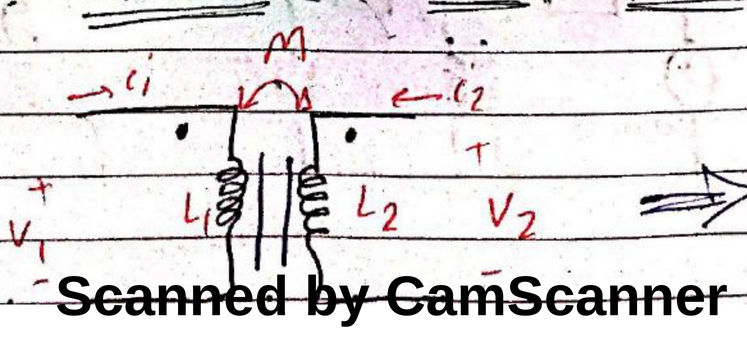
* Ideal Transformer:



$\mu_r \neq 1$

Assuming the two coils purly inductive $\Rightarrow R_1 = 0 = R_2$

Also $\Rightarrow$ $L_1 = \infty = L_2$ & $K = 1$ so $M = \infty$



$\Rightarrow V_1 = j\omega L_1 I_1 + j\omega M I_2 \Longrightarrow I_1 = \frac{V_1 - j\omega M I_2}{j\omega L_1}$

$V_2 = j\omega L_2 I_2 + j\omega M I_1 \rightsquigarrow V_2 = j\omega L_2 I_2 + j\omega M \left(\frac{V_1 - j\omega M I_2}{j\omega L_1}\right)$

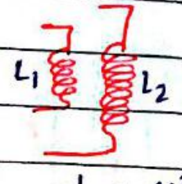
No.

$$V_2 = j\omega L_2 I_2 + \frac{M}{L_1} V_1 - j\omega \frac{M^2}{L_1} I_2 = \cancel{j\omega L_2 I_2} + \frac{\sqrt{L_1 L_2}}{L_1} V_1 - \cancel{j\omega \frac{(L_1 L_2)}{L_1} I_2}$$

$$\Rightarrow V_2 = \sqrt{\frac{L_2}{L_1}}\, V_1 \quad ***$$

$$\Rightarrow V_2 = \frac{N_2}{N_1} V_1 \quad ***$$

$\frac{N_2}{N_1}$ → turns ratio (a) → or (n)



$L_2 > L_1$

$L = \frac{N^2}{\mathcal{R}}$ → reluctance

$L \propto N^2$

$L_1 \propto N_1^2$

$L_2 \propto N_2^2$

$|S|_{primary} = |S|_{secondary}$

$|S_1| = |S_2| \Rightarrow V_1 I_1 = V_2 I_2$

$$\Rightarrow * \quad \frac{V_2}{V_1} = \frac{I_1}{I_2} = a = \frac{N_2}{N_1} \quad *$$

* if $N_2 > N_1 \Rightarrow$ step up transformer.

* if $N_1 > N_2 \Rightarrow$ step down transformer.

ex: An ideal transformer is rated 2400/120 V, 9.6 KVA and it has 50 turns on the secondary

find:

① turns ratio. $\frac{2400 \to V_1}{120 \to V_2} \Rightarrow n = a = \frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{120}{2400} = \boxed{0.05}$

② turns on primary side. $\frac{N_2}{N_1} = a = \frac{50}{N_1} = 0.05 \Rightarrow \boxed{N_1 = 1000}$ turns.

③ The rated currents on primary & secondary.

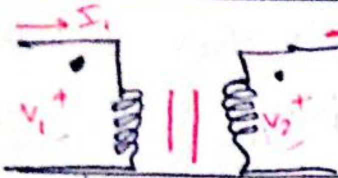
$|S_1| = |S_2| \quad S_1 = V_1 I_1 \Rightarrow I_1 = \frac{9.6 \times 10^3}{2400} = \boxed{4A}$

$S_2 = V_2 I_2 \Rightarrow I_2 = \frac{9.6 \times 10^3}{120} = \boxed{80A}$

* 2 Rules to define the voltage polarities & current directions:

① If V_1, V_2 are both (+ve) or both (−ve) at the dotted terminals use $+n$ for $\frac{V_2}{V_1}$ or ($-n$, otherwise).

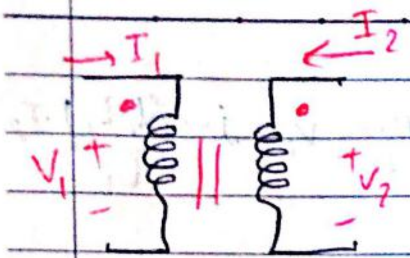
② If I_1 & I_2 both enter into or both leave the dotted terminals use $-n$ for I_1/I_2, otherwise use $+n$.



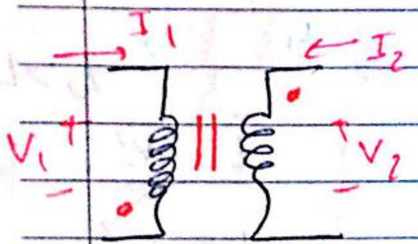
⇒ since V_1, V_2 both +ve ⇒ $\frac{V_2}{V_1} = a$

⇒ since one enter & one leave for currents

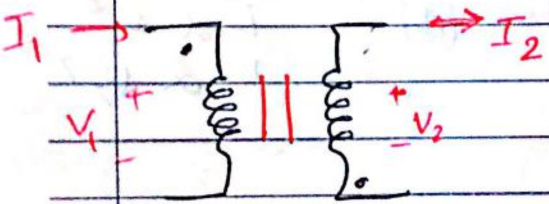
No. ⇒ $\frac{I_1}{I_2} = a$



⇒ $\frac{V_2}{V_1} = a$; $\frac{I_1}{I_2} = -a$.



$\frac{V_2}{V_1} = -a$; $\frac{I_1}{I_2} = a$



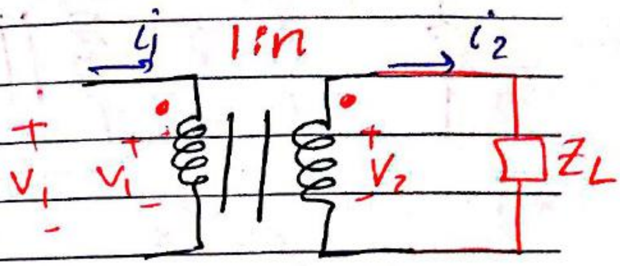
$\frac{V_2}{V_1} = -a$; $\frac{I_1}{I_2} = -a$

No.

Complex Power in the transformer: (Lossless)

$$S_1 = V_1 I_1^* = \left(\frac{V_2}{n}\right)(n I_2^*) = V_2 I_2^* = S_2$$

*Input Impedance: $\frac{N_2}{N_1} = n$



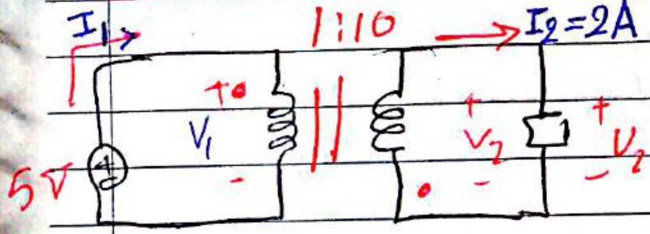
$$Z_{in} = \frac{V_1}{I_1} = \frac{V_2/n}{n I_2}$$

$$= \frac{1}{n^2}\frac{V_2}{I_2} = \frac{1}{n^2} Z_L$$

"Reflected Impedance"

$Z_2 = \frac{V_2}{I_2}$, $\frac{V_2}{V_1} = +n$

$\frac{I_1}{I_2} = +n$



find V_2 ? & I_1

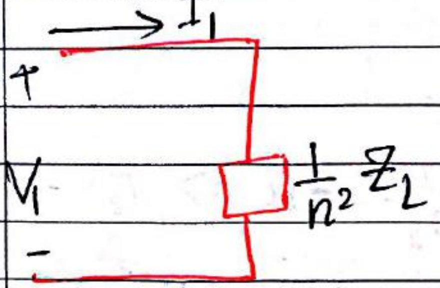
since the dots one (-ve) & one (+ve) $\frac{V_2}{V_1} = -n$

$\Rightarrow \boxed{V_2 = -50}$ volt.

since both currents entering

$\frac{I_1}{I_2} = -n \Rightarrow \boxed{I_1 = -20A}$

for the ckt in the top of this page;

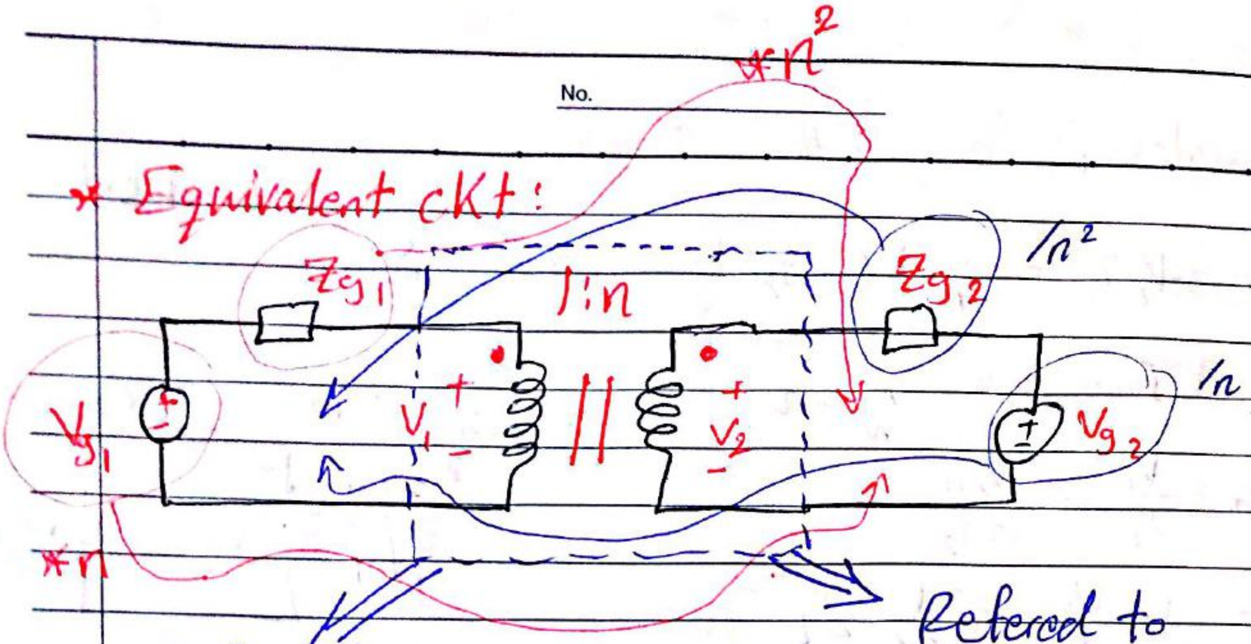


$\Rightarrow$ [Equivalent ckt refer to Primary.]

$\Rightarrow$

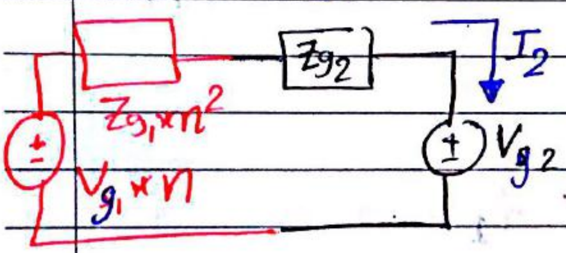
No.

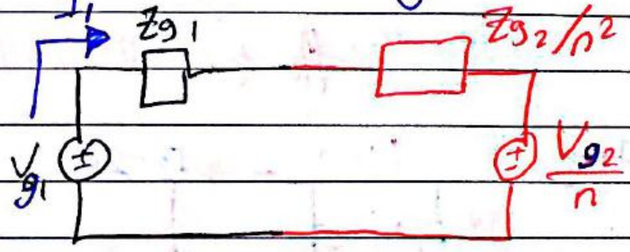
* Equivalent cKt:



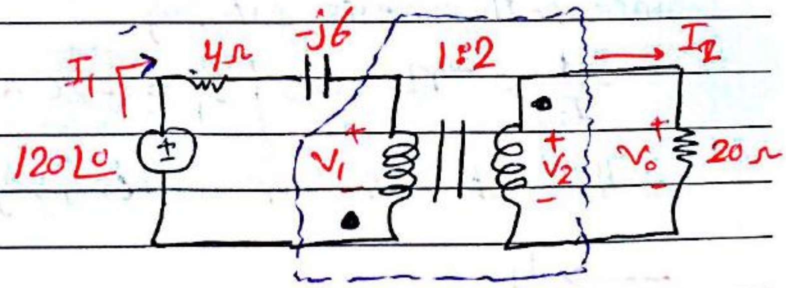
Refered to Secondary

Refered to Primary:

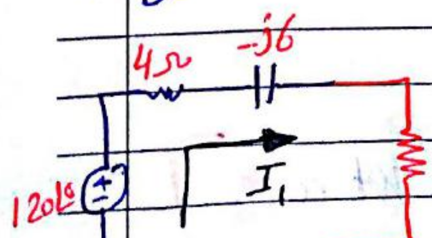




ex) For the shown ideal TR find the source current I_1, the ouput voltage V_o, the complex power supplied by the source



using Reflection:



$5\Omega \rightarrow \frac{20\Omega}{n^2} = \frac{20}{4} = 5\Omega$

$$I_1 = \frac{120\angle 0}{9-j6} = 11.1\angle 33.7°$$

$$\frac{I_1}{I_2} = -n \Rightarrow \boxed{I_2 = -5.545\angle 33.7°}$$

$$\Rightarrow V_o = 110.9\angle 213.7°$$

from the main cKt:

$V_o = I_2 * 20$

* Anothe method to find v_0:

$V_0\Big|_{\text{new CKt}} = 5 * (11.1 \angle 33.7°) \Rightarrow$ it is a reflected voltage $\equiv V_0'$

~~$\frac{V_2}{V_1} = -2 \Rightarrow V_2 = -2V_1 = -2 * 5 * I_1 = 110.9 \angle 213.7°$~~ V_0'

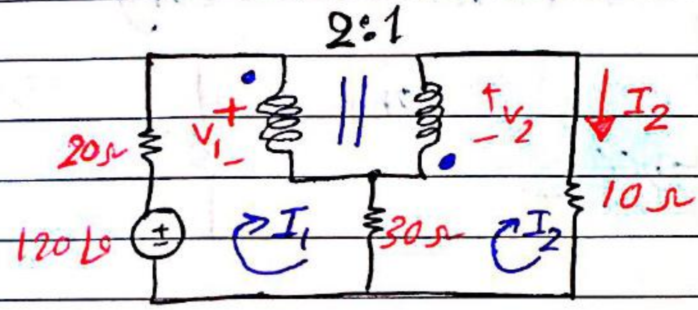
~~* complex power:~~

~~$S_1 = V_1(I_1^*) = 120\ (11.1 \angle -33.7°)$~~ $= \boxed{1330.8 \angle -33.7°}$ VA.

(ex) Calculate the power supplied to the 10 Ω Resistor:

We Can't do Reflection.

using (KVL):



mesh(1):

$-120\angle 0° + 20 I_1 + V_1 + 30(I_1 - I_2) = 0$ --- (1)

mesh(2):

$30(I_2 - I_1) - V_2 + 10 I_2 = 0$ --- (2) $\Rightarrow$ also $\boxed{\frac{V_2}{V_1} = \frac{-1}{2}}$

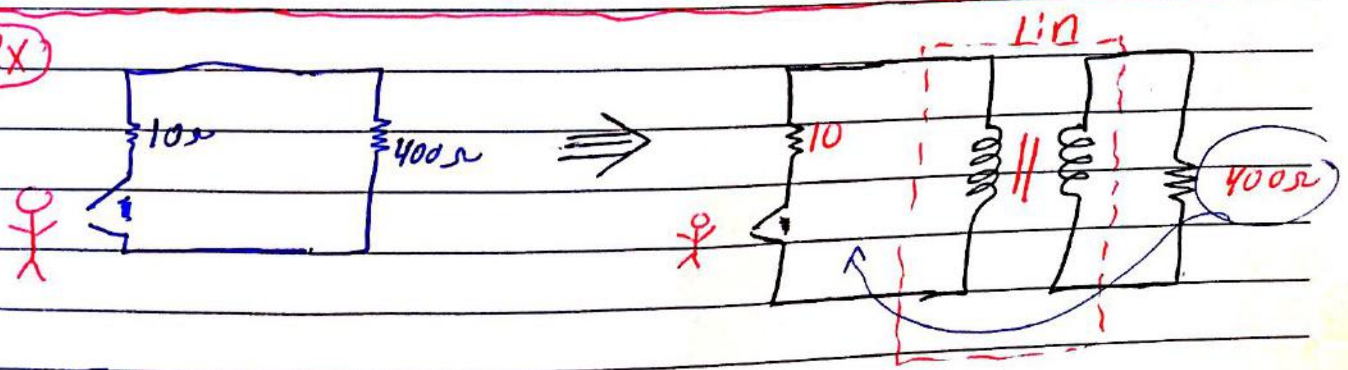
put them in (1) & (2) $\Rightarrow \boxed{\frac{I_1}{I_2} = -\frac{1}{2}}$

solving:

$\boxed{I_2 = -0.7272\ A}$

$\Rightarrow P = I_2^2 R \Rightarrow P_{10\Omega} = 5.3\ W$

(ex)



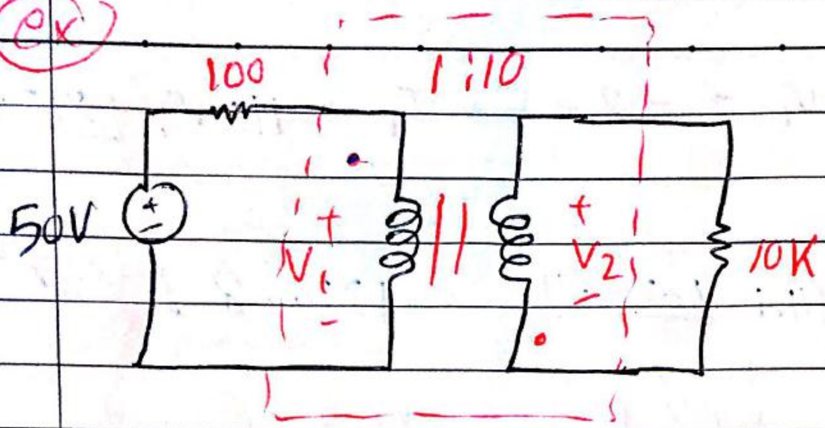
$Z_{L\,\text{reflected}} = 10 = \frac{1}{n^2} * 400$

$\boxed{n = \sqrt{40}}$

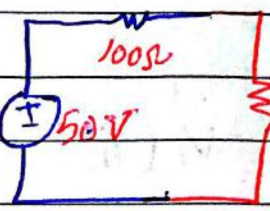
Reflect the following ckt to primary & to secondary:

No.

ex

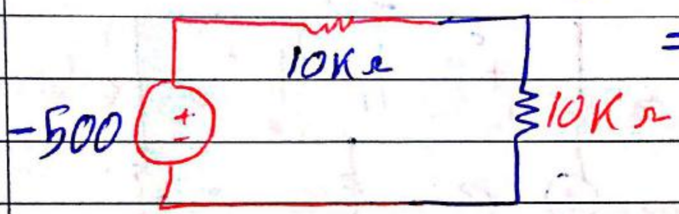


⟹ Refer to primary:



$\frac{10K}{n^2} = \underline{100\Omega}$

⟹ Refer to Secondary:



⟹ * Be aware for the sign of n.

$\frac{V_2}{V_1} = -10$

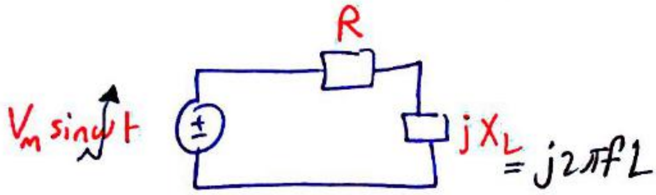
$\Rightarrow (50)(-10) = -500$ volt.

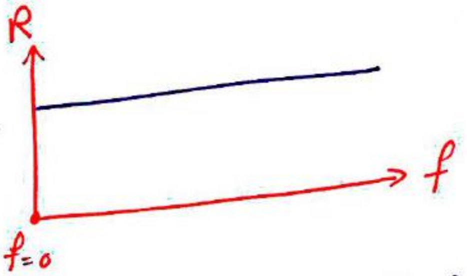
End of CH_{13}

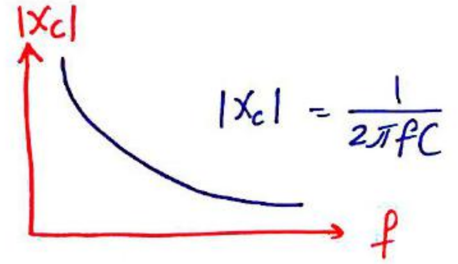
* * *

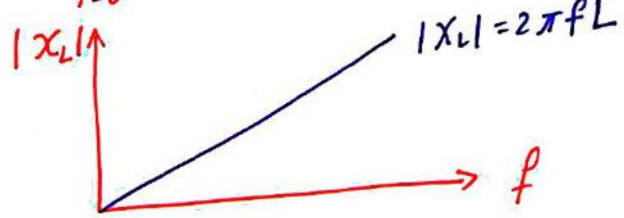
CHAPTER (14)

Frequancy Response.





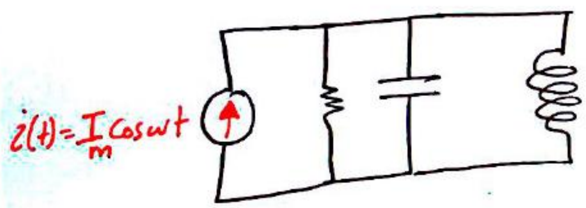




These 3-figures for $\underline{\underline{Z}}$.

※ Parallel Resonance:

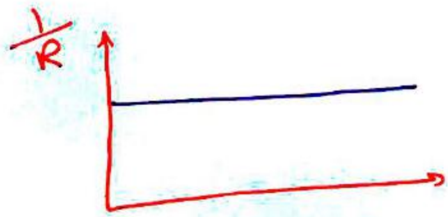
*What is Resonance CKt? it is a cKt that contains R, L, C when in Parallel or in series or combination.

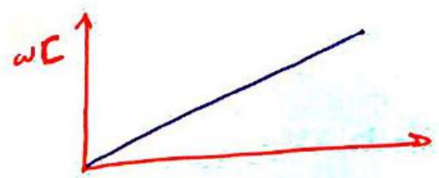


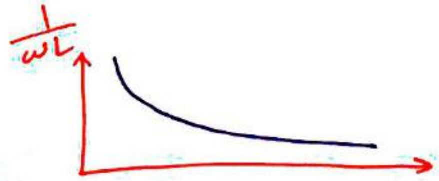
$$Z_{eq} = R /\!/ X_C /\!/ X_L$$

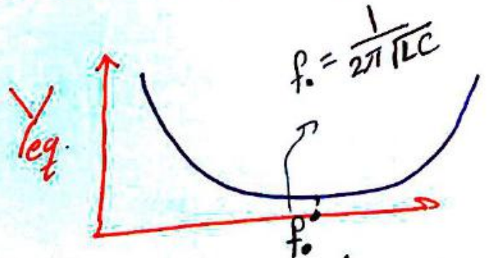
$$Y_{eq} = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

The following 3-figures for $\underline{\underline{Y}}$.









$$\boxed{Y_{eq} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}}$$

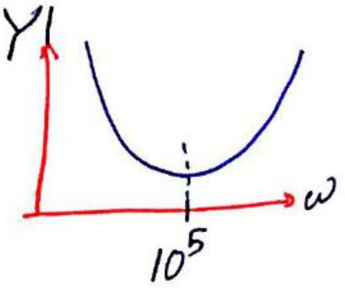
lets Assume $|\omega C| = \frac{1}{\omega L}$

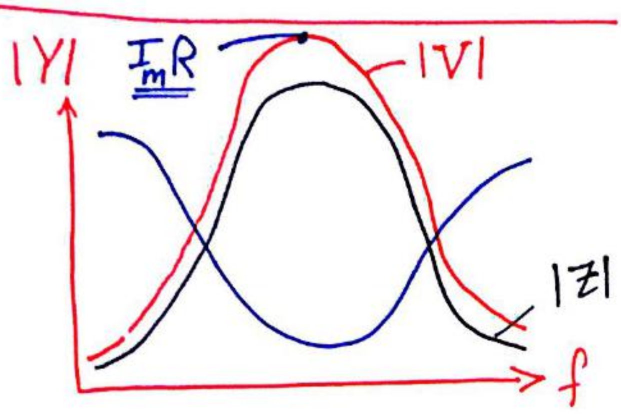
$\Rightarrow \omega C = \frac{1}{\omega L} \Rightarrow \omega^2 = \frac{1}{LC} \Rightarrow \omega_o = \frac{1}{\sqrt{LC}}$ "Resonance" Frequancy.

$\Rightarrow$ At Resonance : $\boxed{Y_{eq} = \frac{1}{R}}$

Ex. $L = 1\,mH$, $C = 0.1\,\mu F$

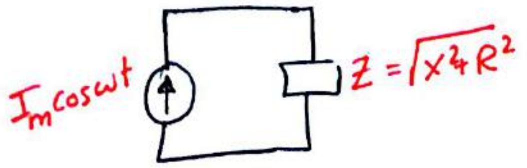
$$\omega = \frac{1}{\sqrt{LC}} = \underline{\underline{10^5}}\ rad/sec.$$





$V = IZ$

$|V| = |I_m|\,|Z|$

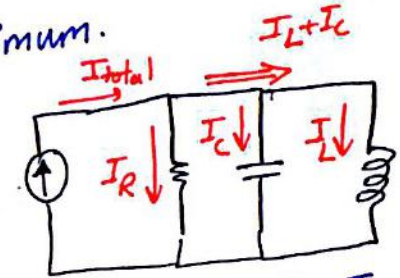


$\Rightarrow$ $\underline{\underline{|V| = I_m\,|Z|}}$

* At parallel Resonance: the voltage is maximum.

* There is a current in L, C at resonance.

, But these currents are equal and opposite, They may exceed source current.



$I_{total} = I_R + I_C + I_L$

@ resonance

$I_{total} = I_R$

pass through R.

$I_{L,o} + I_{C,o} = 0$ $\Rightarrow$ $|I_{C,o}| = |I_{L,o}|$

means @ resonance.

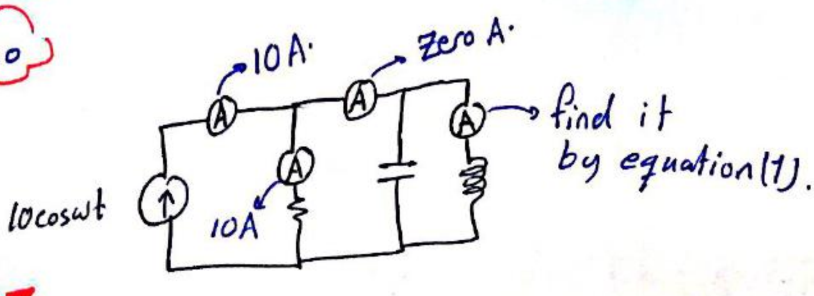
$$I_{L,o} = \frac{V_o}{j\omega_o L}$$

the voltage @ resonance: $= IR$

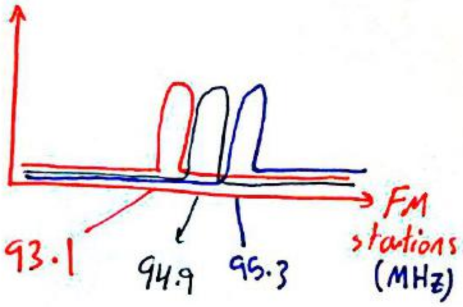
$$\boxed{I_{L,o} = \frac{IR}{j\omega_o L}} \cdots (1) \quad \Rightarrow \quad I_{C,o} = \frac{V_o}{1/j\omega_o C} = \boxed{j\,IR\,\omega_o C} \cdots (2)$$

$$I_{C,o} = -I_{L,o}$$

Ex. find the readings for each (A)?



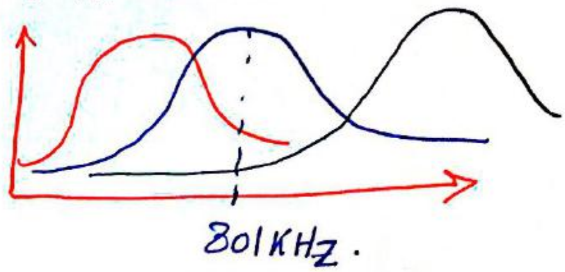
* FM station:



$\Downarrow$

High quality factor

* AM station:



$\Downarrow$

poor quality factor.

* Quality Factor:

it is a parameter that represents the Bandwidth of the signal.
↳ The region that contains the most of the power of the signal.

$$Q \propto \frac{1}{B}$$

Quality factor.
Bandwidth.

$$Q \equiv 2\pi \frac{\text{maximum energy stored}}{\text{Total energy lost per period}}$$

$$\boxed{Q = 2\pi \frac{[W_L + W_C]}{P_R T}}$$

$$W = \int_0^T v(t)i(t)\,dt$$

$$P = \frac{1}{T_0}\int_0^T v(t)i(t)\,dt$$

$$\Rightarrow W = PT$$

* suppose that $i(t) = I_m \cos\omega t$

$\Rightarrow$ @ resonance: $\boxed{i(t) = I_m \cos\omega_0 t}$, $\omega_0 = \frac{1}{\sqrt{LC}}$

$\Rightarrow \boxed{v(t) = R I_m \cos\omega_0 t}$ @ resonance I_m pass just through "R".

**

$$W_C(t) = \frac{1}{2} C V^2 = \frac{1}{2} C * R^2 I_m^2 \cos^2\omega_0 t$$

$$W_L(t) = \frac{1}{2} L i^2 = \frac{1}{2} L \left[\frac{1}{L}\int v_L\,dt\right]^2$$

remember: $i_L = \frac{1}{L}\int v_L\,dt$

$$\int v_L\,dt = \frac{R I_m \sin\omega_0 t}{\omega_0}$$

$$\hookrightarrow = \frac{1}{2}\,\frac{R^2 I_m^2 \sin^2\omega_0 t}{L\,\omega_0^2}$$

$\Rightarrow$ where $\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \omega^2 = \frac{1}{LC}$

so $W_L(t) = \frac{1}{2}\frac{CR^2 I_m^2 \sin^2\omega_0 t}{1}$ $\nearrow$ 1

$$\Rightarrow W_C(t) + W_L(t) = \frac{1}{2} CR^2 I_m^2 [\sin^2\omega_0 t + \cos^2\omega_0 t]$$
$$= \frac{1}{2} CR^2 I_m^2$$

also $P_R = \frac{1}{2} I_m^2 R \Rightarrow P_R T = \frac{1}{2} I_m^2 R T_0 = \frac{1}{2}\frac{I_m^2 R}{f_0}$

Now:

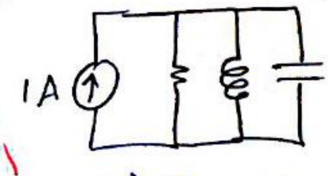
$$Q = 2\pi \frac{\frac{1}{2} CR^2 I_m^2}{\frac{1}{2} I_m^2 R \frac{1}{f_0}} = \underline{CR\omega_0} = 2\pi f_0 RC$$
$$= \frac{1}{\sqrt{LC}} RC = \underline{R\sqrt{\frac{C}{L}}}$$
$$= \frac{R}{X_{C,0}} = \frac{R}{X_{L,0}}$$

$$\boxed{\begin{aligned} Q &= \omega_0 CR \\ &= R\sqrt{C/L} \\ &= R/X_{C,0} \\ &= R/X_{L,0} \end{aligned}}$$ ***

---

* $I_{C,0} = -I_{L,0} = j\underbrace{\omega_0 CR}_{Q} I = j Q_0 I$

Q = 1000



$\Rightarrow I_{C,0} \neq 0$

$I_{C,0} = j1000$ Amp.

---

ex. Parallel RLC cKt with $L = 2mH$, $Q_0 = 5$, $C = 10nF$ find R & admittance at $\omega_0, 0.9\omega_0, 1.1\omega_0$?

Solution:

$Q = R\sqrt{\frac{C}{L}}$

$\Rightarrow R = Q\sqrt{\frac{L}{C}}$

$= 5\sqrt{\frac{2 * 10^{-3}}{10 * 10^{-9}}}$

so $\boxed{R = 2.236}$ $K\Omega$

$Y(\omega) = \frac{1}{R} + j\omega C + \frac{1}{j\omega L}$

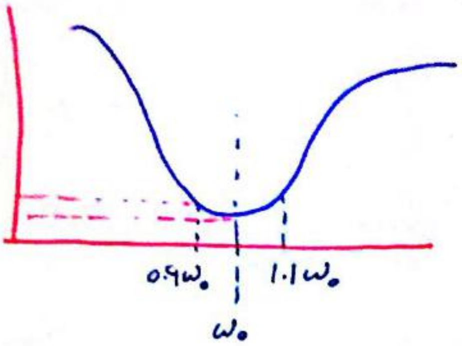
$\omega_0 = \frac{1}{\sqrt{LC}} = \underline{223.6K}$ rad/s.

@$0.9\omega_0$: $0.9\omega_0 = 0.9 \times 223.6K$

$Y(0.9\omega_0) = \frac{1}{2.236 * 10^3} + j * 0.9 \times 223.6 * 10^3 * 10^{-8} + \frac{1}{j * 0.9 * \omega_0 * 2 * 10^{-3}}$

$\boxed{|Y(0.9\omega_0)| = 6.504 * 10^{-4}}$ ℧.

$\Rightarrow$

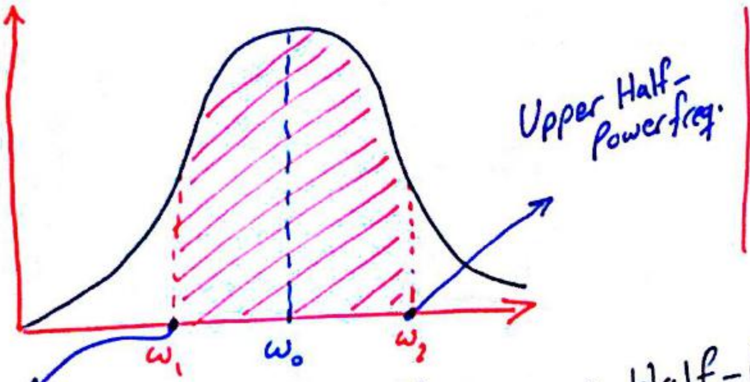


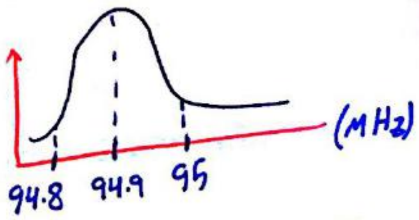
@ ω_o: $Y(\omega_o) = \frac{1}{R}$

$$\boxed{|Y(\omega_o)| = 4.472 * 10^{-4}} \ \mho.$$

@ $1.1\omega_o$:

$$\boxed{|Y(1.1\omega_o)| = 6.182 * 10^{-4}} \ \mho.$$





$\Rightarrow$ BW = 0.2 MHz

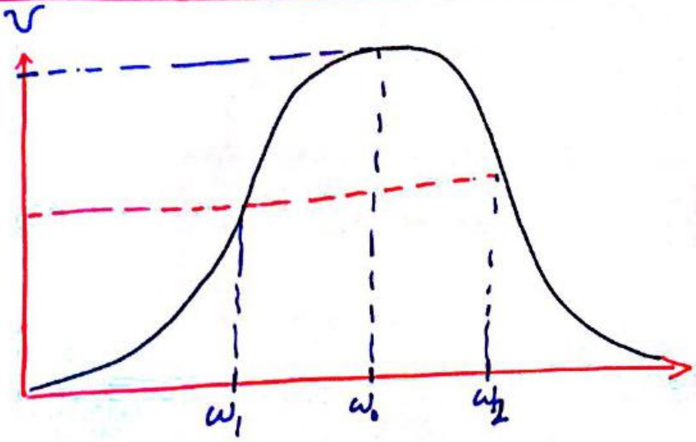
Lower Half-Power frequency.

Bandwidth (BW) $\rightarrow$ Half-Power Bandwidth.

$$* \ BW = \omega_2 - \omega_1 \ *$$

$V_{max} = I_m R$

$\frac{V_{max}}{\sqrt{2}}$



$Y = \frac{1}{R} + j\omega C + \frac{1}{j\omega L}$

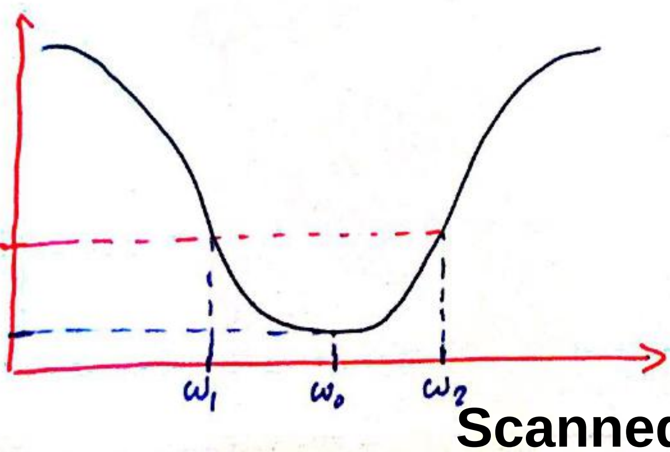
$Y(\omega_o) = \frac{1}{R}$

$Y(\omega_1) = \sqrt{2}\left(\frac{1}{R}\right)$

$\Longrightarrow$

$\sqrt{2}|Y|$

$|Y|$



$Y(\omega_1) = \sqrt{2}\left(\frac{1}{R}\right) = \frac{1}{R}\sqrt{1+1}$

$Y = \frac{1}{R} + j\omega C + \frac{1}{j\omega L}$ use $Q_0 = \omega_0 RC$

$\Rightarrow Y = \frac{1}{R} + j\frac{1}{R}\left[\frac{\omega \overbrace{\omega_0 CR}^{Q_0}}{\omega_0} - \frac{\omega_0 \overbrace{R}^{Q_0}}{\omega \omega_0 L}\right]$

$\Rightarrow Y = \frac{1}{R} + j\frac{Q_0}{R}\left[\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right]$

where: $|Y(\omega_1)| = \sqrt{2}\,|Y(\omega_0)| = \sqrt{2}\,\frac{1}{R}$

$|Y(\omega_1)| = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{Q_0}{R}\right)^2\left[\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right]^2} = \sqrt{2}\,\frac{1}{R}$

$\Rightarrow \cancel{\frac{1}{R}}\sqrt{1 + Q_0^2\left[\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right]^2} = \sqrt{2}\,\cancel{\frac{1}{R}}$

so $Q_0^2\left[\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right]^2 = 1 \Rightarrow$ we have: $\rightarrow Q_0\left(\frac{\omega_1}{\omega_0} - \frac{\omega_0}{\omega_1}\right) = 1$

OR $\rightarrow Q_0\left(\frac{\omega_2}{\omega_0} - \frac{\omega_0}{\omega_2}\right) = -1$

$\Rightarrow$ After simplify:

$$\omega_1 = \omega_0\left[\sqrt{1 + \left(\frac{1}{2Q_0}\right)^2} - \frac{1}{2Q_0}\right]$$

$$\omega_2 = \omega_0\left[\sqrt{1 + \left(\frac{1}{2Q_0}\right)^2} + \frac{1}{2Q_0}\right]$$

** These two expressions could take very long time to calculate ω_1 & ω_2 so be sure that $Q_0 < 5$ before use them.

$\Rightarrow$ from equations of ω_1 & ω_2:

Bandwidth: $\beta = \omega_2 - \omega_1 = 2\omega_0 * \frac{1}{2Q_0} = \frac{\omega_0}{Q_0} \Rightarrow \boxed{\beta = \frac{\omega_0}{Q_0}}$

also we could derive that:

$$\boxed{\omega_0 = \sqrt{\omega_1 \omega_2}}$$

※ Approximations for High Q_o cKts $[Q_o \geq 5]$:

for example take $Q_o = 5$

$$\sqrt{1+\left(\frac{1}{10}\right)^2} = \sqrt{1+0.01} \simeq 1$$

so: $\omega_1 = \omega_o \left[1 - \frac{1}{2Q_o}\right]$

$\omega_2 = \omega_o \left[1 + \frac{1}{2Q_o}\right]$

$$\Rightarrow \omega_1 = \omega_o - \frac{\omega_o}{2 Q_o} \rightarrow \beta$$

$$\Rightarrow \boxed{\omega_1 = \omega_o - \frac{1}{2}\beta \; , \; \omega_2 = \omega_o + \frac{1}{2}\beta \; , \; \omega_o = \frac{1}{2}[\omega_1 + \omega_2]}$$

*Approximate Admittance: $Y = \frac{1}{R}[1 + jN]$

N: is the number of Half-Band width off response.

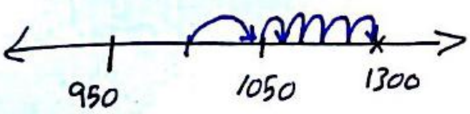
$$N = \frac{\omega - \omega_o}{\frac{1}{2}\beta}$$

ex.

if $\omega_1 = 950$ rad/s

$\omega_2 = 1050$ rad/s

find N @ $\omega = 1300$ rad/sec?



$\Rightarrow \beta = 100$ rad/sec.

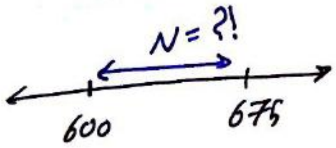
$\omega_o = 1000$ rad/sec

$$N = \frac{1300 - 1000}{\frac{1}{2} * 100} = 6 \Rightarrow \boxed{N = 6}$$

ex. ; if $\omega_o = 600$ rad/sec.

$\beta = 50$ rad/sec.

find N @ $\omega = 675$ rad/sec?



$$N = \frac{\omega - \omega_o}{\frac{1}{2}\beta} = \frac{675 - 600}{25}$$

$$= \frac{75}{25} \Rightarrow \boxed{N = 3}$$

also N @ $\omega = 660$ rad/sec:

$$N = \frac{660 - 600}{25} = \frac{12}{5} \Rightarrow \boxed{N = 2.4}$$

ex. Parallel RLC ckt with $R = 40K\Omega$, $L = 1\,H$, $C = \frac{1}{64}\mu F$

① Find the locations of the two Half power frequancies.

② Find the approximate value for admittance @ $\omega = 8200$ rad/sec.

Solution:

① ※Always the first step find Q_o ⟹ To Know which expressions of ω_1 & ω_2 you will use.

$$Q_o = \omega_o RC$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \boxed{8K \text{ rad/sec.}} \Rightarrow Q_o = 8 \times \cancel{10^3} \times 40 \times \cancel{10^3} \times \frac{1}{64} \times \cancel{10^{-6}}$$

$$\Rightarrow Q_o = 5$$ since $Q_o \geq 5$

use approximation @ resonance [Approximate relations]

method(1):

$$\omega_1 = \omega_o\left(1 - \frac{1}{2Q_o}\right) = \boxed{7200 \text{ rad/sec}}$$

$$\omega_2 = \omega_o\left(1 + \frac{1}{2Q_o}\right) = \boxed{8800 \text{ rad/sec.}}$$

method (2):

$$\beta = \frac{\omega_o}{Q_o} = 1600 \text{ rad/sec.} \Rightarrow \begin{aligned}\omega_1 &= \omega_o - \tfrac{1}{2}\beta \\ \omega_2 &= \omega_o + \tfrac{1}{2}\beta\end{aligned} \Rightarrow \boxed{\begin{aligned}\omega_1 &= 7200 \text{ rad/sec.} \\ \omega_2 &= 8800 \text{ rad/sec.}\end{aligned}}$$

---

②
$$Y(\omega) = \frac{1}{R} + j\omega C + \frac{1}{j\omega L}$$

$$Y(8200) = 25.75 \angle 13.9^\circ \ \mu S$$

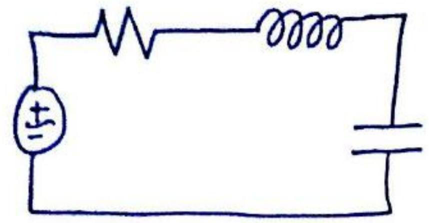
* Approximate Y : $Y = \frac{1}{R}[1 + jN]$.

we need N:

$$N = \frac{\omega - \omega_o}{\frac{1}{2}\beta} = \frac{8200 - 8000}{800} = \underline{\underline{0.25}}$$

$$\Rightarrow Y = \frac{1}{40K}[1 + j(0.25)] = 25.77 \angle 14.04^\circ \ \mu S$$

※ Series RLC Resonance:



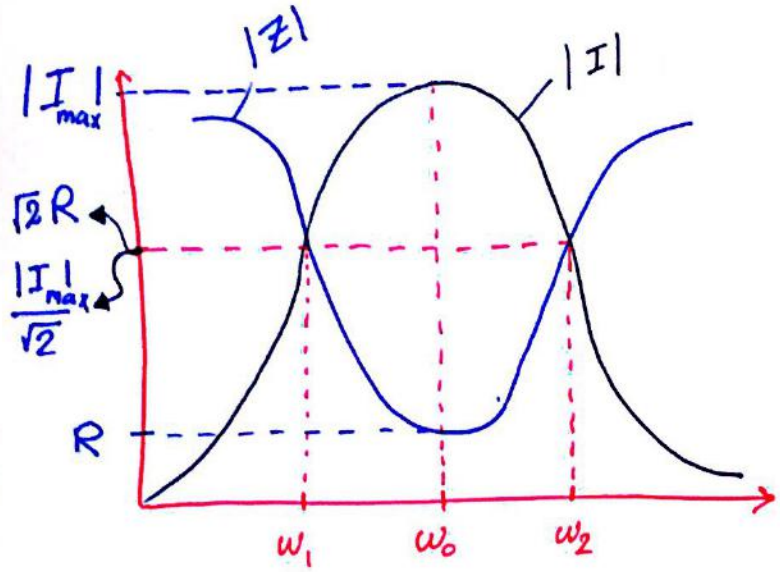
$$\Rightarrow \boxed{Z_{eq.} = R + j\omega L + \frac{1}{j\omega C}}$$

@ resonance $Z_{eq} = R$:

so $\quad 0 = j\omega L + \frac{1}{j\omega C}$

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$



$$\boxed{\beta = \omega_2 - \omega_1 = \frac{\omega_0}{Q_0}}$$

$$\Rightarrow |Z(\omega_0)| = R$$

$$|Z(\omega_1)| = \sqrt{2}\,R$$

$$|Z(\omega_2)| = \sqrt{2}\,R$$

* As the same we did to parallel resonance
Do the same you can reach that:

$$\boxed{Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC} = \frac{\omega_0}{\beta}}$$

ALSO:

$$w_1 = \omega_0\left[\sqrt{1+\left(\frac{1}{2Q_0}\right)^2} - \frac{1}{2Q_0}\right] \quad , \quad \omega_2 = \omega_0\left[\sqrt{1+\left(\frac{1}{2Q_0}\right)^2} + \frac{1}{2Q_0}\right]$$

$\Rightarrow$ For cKts with High Q_0 [$Q_0 \geq 5$]:

$$\boxed{\begin{aligned}\omega_1 &\simeq \omega_0 - \tfrac{1}{2}\beta \\ \omega_2 &\simeq \omega_0 + \tfrac{1}{2}\beta\end{aligned}}$$

$$Z_{eq} = R(1 + jN)$$

where: $N = \frac{\omega - \omega_0}{\frac{1}{2}\beta}$

Approximate impedance.

*Note: (1) Always if ω is unknown it means that the freq. is changing (resonance subject).

(2) Always the Ammeter reads RMS value.

ex. Given $V_s = 100\cos\omega t$ mvolt, for series RLC ckt with $R = 10\Omega$, $C = 200nF$, $L = 2mH$.

1. Calculate $|I|$ if $\omega = 48K$ rad/sec.
2. Calculate the approximate value of $|I|$ @ $\omega = 48K$ rad/sec.

Solution:

1. $|I| = \frac{|V|}{|Z_{eq}|}$ ⇒ we need Z_{eq}:

$$Z_{eq} = R + j\omega L + \frac{1}{j\omega C}$$

$$= 10 + j * 48K * 2m + \frac{1}{j * 48K * 200n}$$

$$\boxed{Z_{eq} = 12.91\angle{-34.24^\circ}}$$

$$|I| = \frac{100m}{12.91}$$

$$\Rightarrow \boxed{|I| = 7.746\,mA}$$

---

2. $\omega_0 = \frac{1}{\sqrt{LC}} = 50K$ rad/sec.

find Q_0 to know if it is allow to use approximation:

$Q_0 = \frac{\omega_0 L}{R} = 10$ (≥ 5) ⇒ use approximation.

$\Rightarrow \beta = \frac{\omega_0}{Q_0} = 5K$ rad/sec.

⇒ find N: $N = \frac{\omega - \omega_0}{\frac{1}{2}\beta} = -0.8$

$Z_{eq} = R(1 + jN)$

$$\Rightarrow \boxed{Z_{eq} = 12.81\angle{-38.66^\circ}}$$

$\Rightarrow |I| = \frac{|V|}{|Z_{eq}|} \Rightarrow |I| = 7.806\,mA$

ex. Find the Ammeters reading in the circuit shown in figure (1) & the Voltmeters reading in the circuit shown in figure (2):

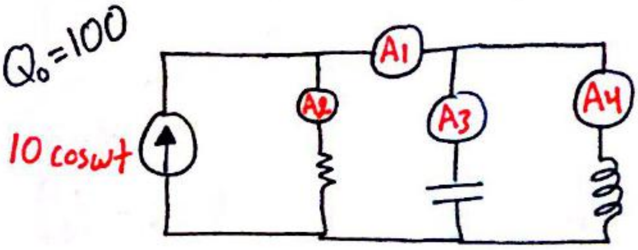


figure (1).

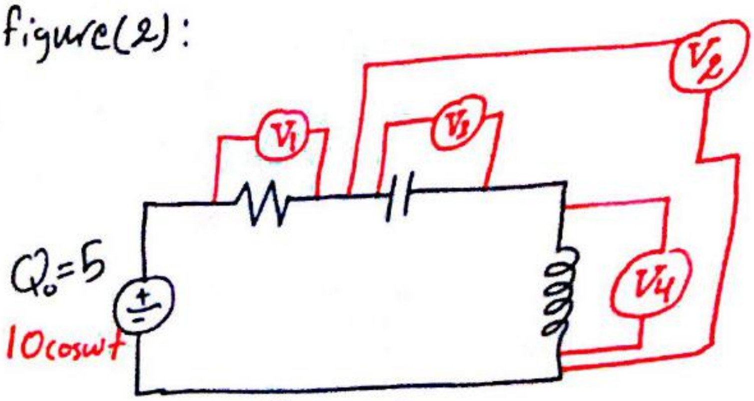


figure (2).

@ resonance:

$A_1 = \underline{0}$

$A_2 = \frac{10}{\sqrt{2}}\ A_{rms}$ or $\underset{(peak)}{A_2} = \underline{\underline{10\,A}}$.

$A_3 = j\,I Q_0 = j1000\,A \Rightarrow |A_3| = \underline{\underline{1000\,A}}$.

$A_4 = \frac{I Q_0}{j} = -j1000\,A \Rightarrow |A_4| = \underline{\underline{1000\,A}}$

@ resonance:

$\underset{(peak)}{V_1} = \underline{\underline{10 \text{ volt}}}$.

$V_2 = \underline{\underline{Zero}}$.

$V_3 = jQ_0 V = j\,50 \text{ volt}$.

$\Rightarrow |V_3| = \underline{\underline{50 \text{ volt}}}$.

$V_4 = \frac{Q_0 V}{j} = -j50 \text{ volt}$.

$\Rightarrow |V_4| = \underline{\underline{50 \text{ volt}}}$.

---

* A summary for series RLC & Parallel RLC resonance:

$\Rightarrow$ The $\underline{\underline{\text{common}}}$ relations for both:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

$$\omega_{1,2} = \omega_0\left[\sqrt{1 + \left(\frac{1}{2Q_0}\right)^2} \mp \frac{1}{2Q_0}\right]$$

$$\beta = \frac{\omega_0}{Q_0}$$

* Approximate relations:

$$\omega_{1,2} = \omega_0 \mp \frac{1}{2}\beta$$

$$\omega_0 = \frac{1}{2}(\omega_1 + \omega_2)$$

$$N = \frac{\omega - \omega_0}{\frac{1}{2}\beta}$$

Parallel RLC Resonance

$$Q_0 = \omega_0 RC = \frac{R}{\omega_0 L}$$

$$|I_L(\omega_0)| = |I_C(\omega_0)| = Q_0 I$$

$$Y = \frac{1}{R}\left[1 + jQ_0\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]$$

$$Y = \frac{1}{R} + j\omega C + \frac{1}{j\omega L}$$

* Approximate:

$$Y = \frac{1}{R}(1 + jN)$$

@ Resonance:

* V has maximum.
* Z has maximum.
* Y has minimum.

Series RLC Resonance

$$Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$$

$$|V_L(\omega_0)| = |V_C(\omega_0)| = Q_0 V$$

$$Z = R\left[1 + jQ_0\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]$$

$$Z = R + j\omega L + \frac{1}{j\omega C}$$

* Approximate:

$$Z = R(1 + jN)$$

@ Resonance:

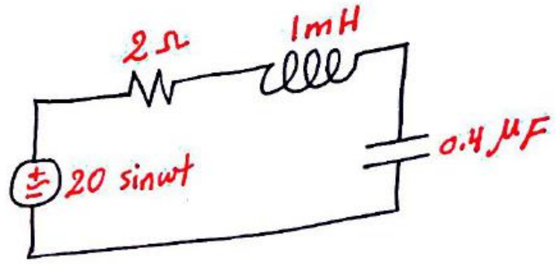
* I has maximum.
* Y has maximum.
* Z has minimum.

ex. For the series resonance ckt shown:

① Find the resonance frequancy & Half-power frequancies.

② Find Q_o & β.

③ Find the Amplitude of the current @ $\omega_0, \omega_1, \omega_2$.



Solution:

① $\omega_o = \frac{1}{\sqrt{LC}} = \boxed{50K\ rad/sec.}$

⇒ As always find Q_o at first:

$Q_o = \frac{1}{\omega_o RC} = \frac{\omega_o L}{R} = 25 \geq 5$ ⇒ so use approximations.

⇒ $\beta = \frac{\omega_o}{Q_o} = 2K\ rad/sec.$ ⇒ $\omega_1 = \omega_0 - \frac{1}{2}\beta \rightarrow \boxed{\omega_1 = 49K\ rad/sec.}$

$\omega_2 = \omega_0 + \frac{1}{2}\beta \rightarrow \boxed{\omega_2 = 51K\ rad/sec.}$

---

② As found in part (1)

$$\boxed{\begin{array}{l} Q_o = 25 \\ \beta = 2K\ rad/sec. \end{array}}$$

---

③ $\boxed{|I(\omega_0)| = \frac{20}{2} = 10\ Amp}$. ⟹ If he asked to find the angle: it would be the same angle of voltage since @ resonance I & V are inphase. so $\theta_I = \theta_V = 0$

$\boxed{|I(\omega_1)| = |I(\omega_2)| = \frac{10}{\sqrt{2}}\ Amp.}$

⇓

If he asked about the angles:

⇒ you have to find Z:

$Z = R(1+jN)$ ⇒ $N = \frac{51-50}{1} = 1$

$N = \frac{49-50}{1} = -1$

so $Z = 2 \pm j2$

⇒ $\theta = \tan^{-1}(\pm 1)$ → $+45°$ @ ω_2 ; $-45°$ @ ω_1

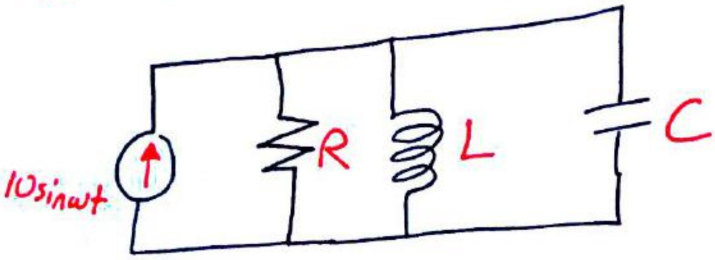
Warning: $|I(\omega_1)| = \frac{10}{\sqrt{2}}$ it is **Not** a RMS value

$\Rightarrow$ it is peak value.

ex. For the shown cKt, Given that: $R = 8K\Omega$. $L = 0.2mH$. $C = 8\mu F$.

① Find $\omega_0, Q, \beta, \omega_1, \omega_2$.

② Find the power dissapated @ $\omega_0, \omega_1, \omega_2$.



Solution:

① $\omega_0 = \frac{1}{\sqrt{LC}} = \boxed{25\,Krad/sec}$.

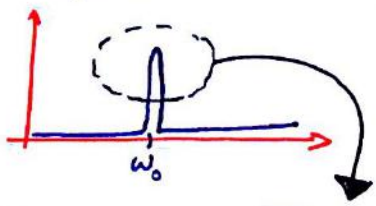
$Q_0 = \omega_0 RC = \boxed{1600}$

$\beta = \frac{\omega_0}{Q_0} = \boxed{15.625\,rad/sec}$.

$\omega_1 = \omega_0 - \frac{\beta}{2} \longrightarrow \omega_1 = 24.992\,Krad/sec.$

$\omega_2 = \omega_0 + \frac{\beta}{2} \longrightarrow \omega_2 = 25.008\,Krad/sec.$

*Note that: 15.625 very small value with respect to 25K so the figure is:



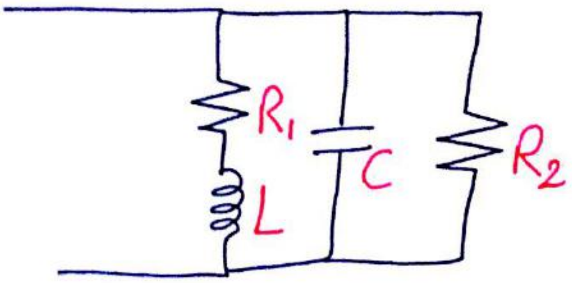
This is called: Highly Selective Circuit.

② $P_R(\omega_0) = \frac{1}{2} I^2 R = \boxed{400KW}$.

$P_R(\omega_1)$, $P_R(\omega_2)$ They will be half of $P_R(\omega_0)$

$\Rightarrow \boxed{P_R(\omega_1) = P_R(\omega_2) = 200KW.}$

※ Other Resonance Forms [Not series, Not parallel]:



*** Key:

$Imag\{Y\} = 0$

OR $Imag\{Z\} = 0$

=> Here it is easier to deal with $\underline{Y}$.

$$\Rightarrow Y = \frac{1}{j\omega L + R_1} + j\omega C + \frac{1}{R_2}$$

$$* \frac{(R_1 - j\omega L)}{(R_1 - j\omega L)}$$

equal zero @ resonance.

$$Y = \frac{1}{R_2} + j\omega C + \frac{R_1 - j\omega L}{R_1^2 + (\omega L)^2}$$

$$Imag\{Y\} = 0 \text{ @ } \omega_0 \Rightarrow \cancel{j}\omega_0 C - \frac{\cancel{j}\omega_0 L}{R_1^2 + (\omega_0 L)^2} = 0$$

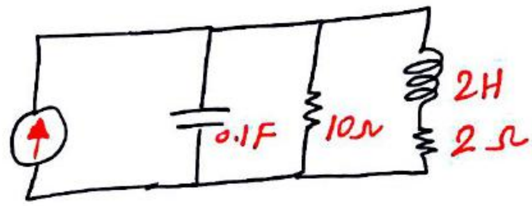
so $C = \dfrac{L}{R_1^2 + (\omega_0 L)^2} \Rightarrow (\omega_0 L)^2 = \dfrac{L}{C} - R_1^2$

$$\Rightarrow \omega_0^2 = \frac{1}{LC} - \left(\frac{R_1}{L}\right)^2$$

$$\Rightarrow \omega_0 = \sqrt{\frac{1}{LC} - \left(\frac{R_1}{L}\right)^2}$$

*** Now you can derive ω_0 to any ckt: All the idea build on the $\underline{KEY}$ and you just choose what is the best using Y or Z depending on the shape of the ckt.

ex.) Determine the resonance frequancy for the given cKt:



solution:

As derived before

$$\omega_0 = \sqrt{\frac{1}{LC} - \left(\frac{R_1}{L}\right)^2}$$

$$= \sqrt{\frac{1}{2*0.1} - \left(\frac{2}{2}\right)^2} \Rightarrow \boxed{\omega_0 = 2 \text{ rad/sec.}}$$

※ Frequancy Response:

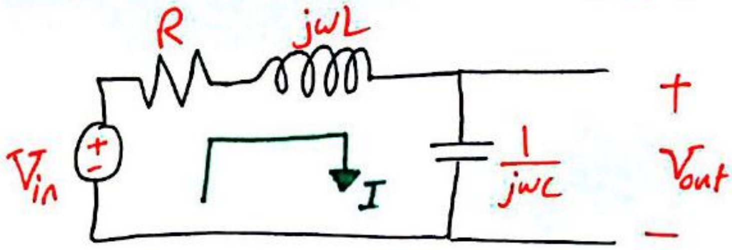
⟹ It is a relation between input & output at different frequancies.

$V_{in} \Rightarrow$ [ ] $\Rightarrow V_{out}$ OR $I_{in} \Rightarrow$ [ ] $\Rightarrow I_{out}$

OR * any relation between in & out.

* Transfer function is $\frac{V_{out}}{V_{in}}$.

ex.) Find the voltage transfer function for the shown cKt:



$$V_{out} = I * \frac{1}{j\omega C}$$

$$V_{in} = I\left(R + j\omega L + \frac{1}{j\omega C}\right)$$

$$H_v(\omega) = \frac{V_{out}}{V_{in}} = \frac{\cancel{I} * \frac{1}{j\omega C}}{\cancel{I} * \left(R + j\omega L + \frac{1}{j\omega C}\right)} * \frac{j\omega C}{j\omega C}$$

$$= \frac{1}{j\omega C R + (1 - CL\omega^2)} \Longrightarrow$$

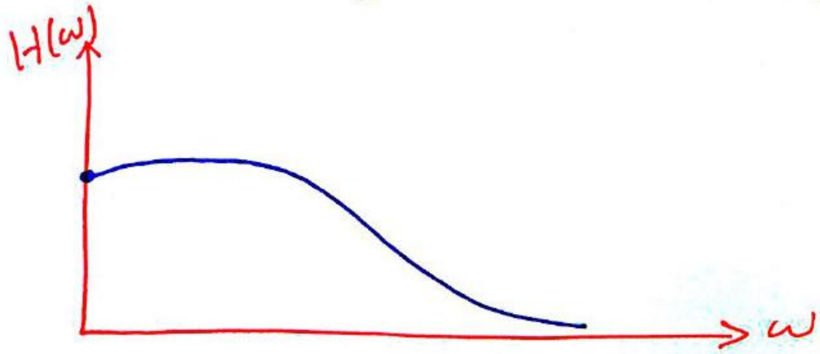
Now:

$$|H_v(\omega)| = \frac{1}{\sqrt{(\omega CR)^2 + (1 - CL\omega^2)^2}}$$

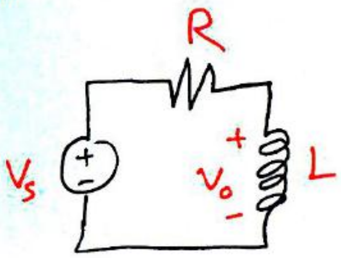
→ we want to draw this function

it is similar to $f(x) = \dfrac{1}{\sqrt{x^2 + (1 - x^2)^2}}$

By substituting some values take @ $x = 0$ & $x = \infty$



ex. Obtain the T.F V_o/V_s of the RL ckt:



solution:

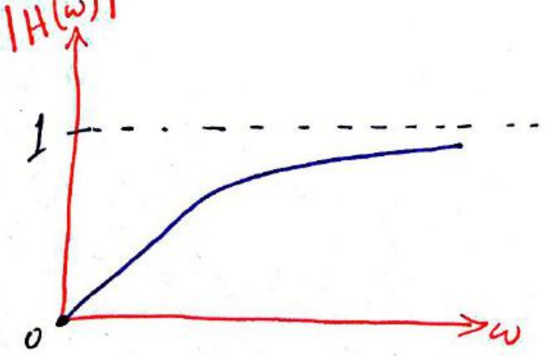
$$H_v(\omega) = \frac{V_o}{V_s} \qquad V_o = I * j\omega L \qquad V_s = I * (R + j\omega L)$$

$$\to = \frac{\cancel{I}\, j\omega L}{\cancel{I}\,(R + j\omega L)}$$

$$\boxed{|H_v(\omega)| = \frac{\omega L}{\sqrt{R^2 + (\omega L)^2}}}$$

To Draw it: $H(0) = \text{Zero}$

$H(\infty) = 1$



To find the angle:

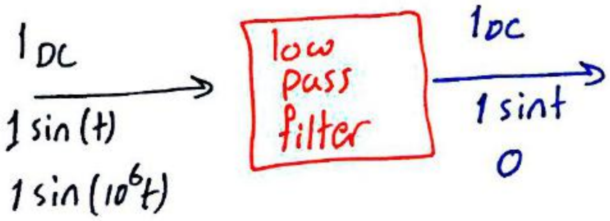
$$H_v(\omega) = \frac{j\omega L}{R + j\omega L} * \frac{R - j\omega L}{R - j\omega L}$$

$$= \frac{(\omega L)^2 + jR\omega L}{R^2 + (\omega L)^2}$$

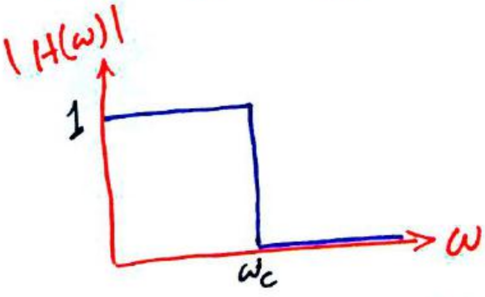
$$\angle H(\omega) = \tan^{-1}\left(\frac{Im}{Re}\right) = \tan^{-1}\left(\frac{R}{\omega L}\right)$$

※ Filters: [frequancy selective circuit]

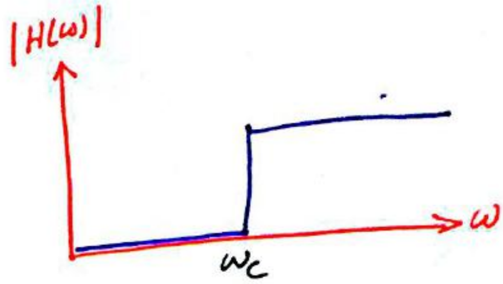
↳ It is a cKt/device that is designed to pass Signals with desired frequancy & reject others.



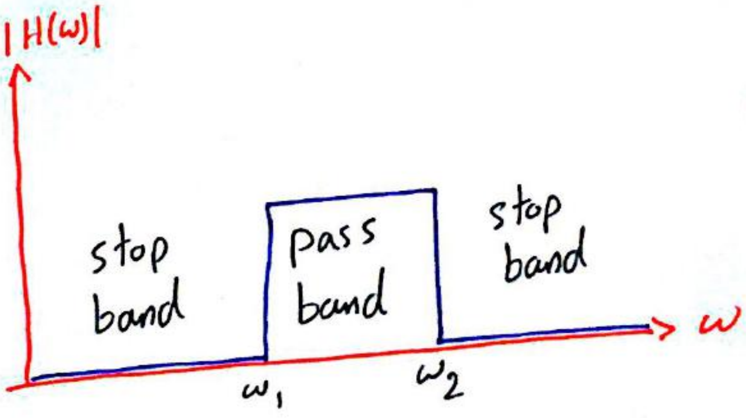
** Types: [Passive Filters]



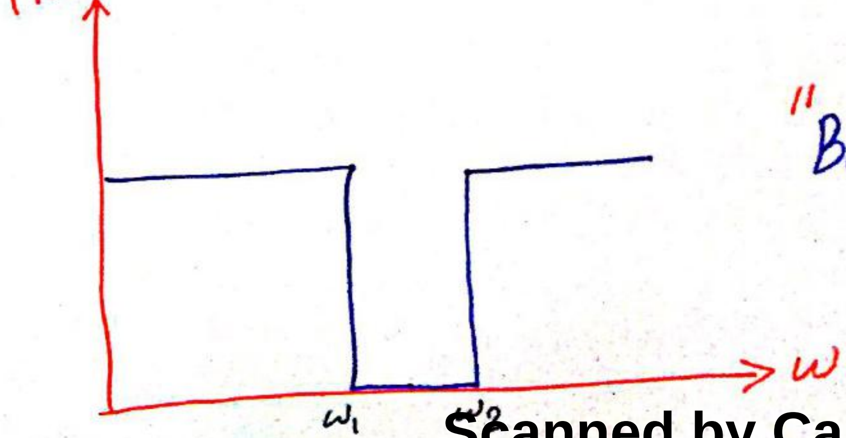
"Low pass Filter"



"High pass filter"

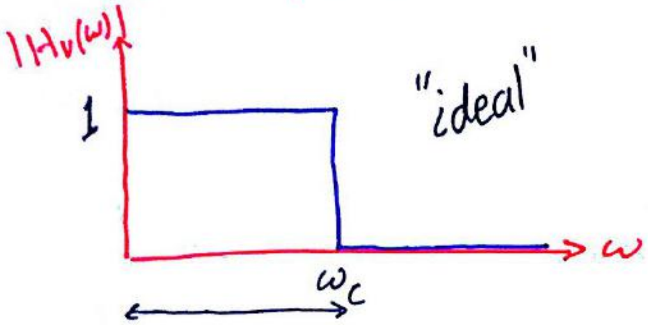


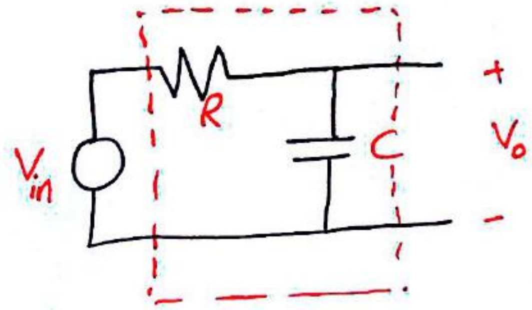
"Band pass filter"



"Band stop filter"

※ Low Pass Filter:



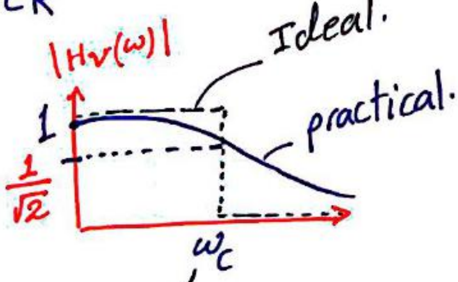


low pass.

⇒ The output of an RC ckt is taken off the capacitor.

$$H_v(\omega) = \frac{V_{out}}{V_{in}} = \frac{\cancel{I} * \frac{1}{j\omega C}}{\cancel{I}\left(\frac{1}{j\omega C} + R\right)} = \frac{1}{1 + j\omega C R}$$

$$\Rightarrow |H_v(\omega)| = \frac{1}{\sqrt{1 + (\omega C R)^2}} \rightarrow \begin{array}{l} H(0) = 1 \\ H(\infty) = 0 \end{array}$$



called: "Cutoff frequancy"

we know:

$$|H_v(\omega_c)| = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + (\omega_c RC)^2}}$$

solving:

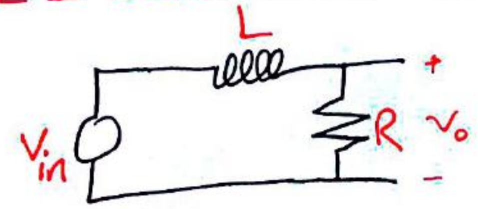
$$\Rightarrow \omega_c = \frac{1}{RC} \text{ rad/sec.}$$

if we want $\omega_c = 1000$ rad/sec

we choose: (for ex.)

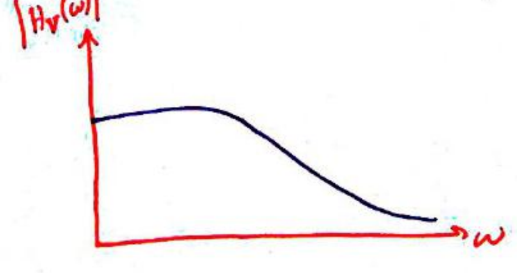
$R = 4.7 K\Omega$

$C = 0.2 \mu F$



$$H_v(\omega) = \frac{R}{R + j\omega L}$$

$$|H_v(\omega)| = \frac{R}{\sqrt{R^2 + (\omega L)^2}} \qquad \begin{array}{l} H(0) = 1 \\ H(\infty) = 0 \end{array}$$



⇒ it is also a low pass filter.

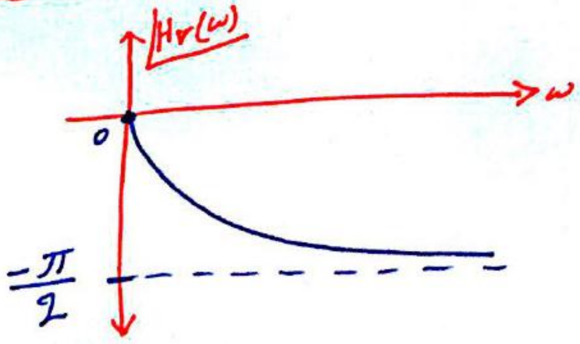
⇒ Now for the angles:

in the previous RC cKt:

$$H_v(\omega) = \frac{1}{1+j\omega RC} \Rightarrow \angle H_v(\omega) = \tan^{-1}(-\omega RC)$$

$\angle H_v(0) = \text{Zero}$

$\angle H_v(\infty) = \frac{-\pi}{2}$



in the previous RL cKt:

$$H_v(\omega) = \frac{R}{R+j\omega L} \Rightarrow \angle H_v(\omega) = \tan^{-1}\left(\frac{-\omega L}{R}\right)$$

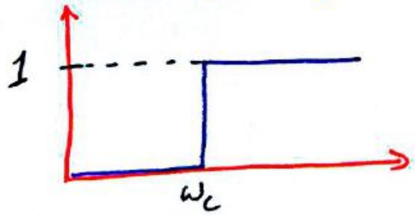
$\angle H_v(0) = 0$

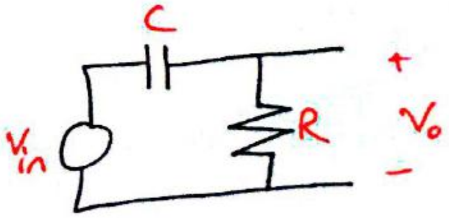
$\angle H_v(\infty) = \frac{-\pi}{2}$

⇒ it will be like the previous graph.

※ High Pass Filter:

⇒ is one that passes the frequancies above the cutoff frequancy.





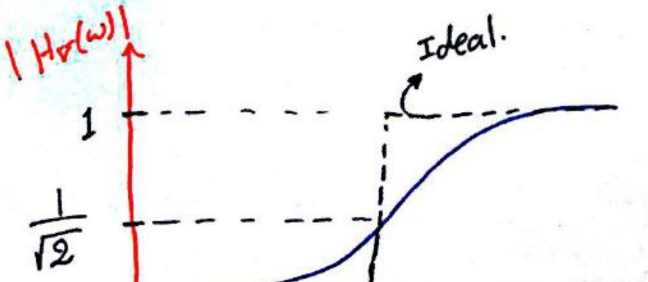
⇒ RC cKt with output at R.

$$H_v(\omega) = \frac{R}{R+\frac{1}{j\omega C}} = \frac{j\omega RC}{1+j\omega RC}$$

$$|H_v(\omega)| = \frac{\omega RC}{\sqrt{1+(\omega RC)^2}}$$

$|H_v(0)| = 0$

$|H_v(\infty)| = 1$



⇒

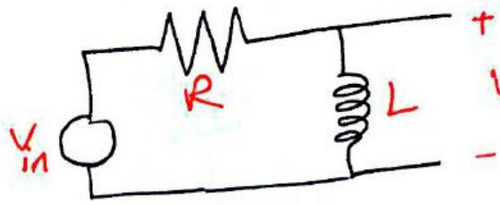
we have: $|H(\omega_c)| = \frac{1}{\sqrt{2}} \Rightarrow \boxed{\omega_c = \frac{1}{RC}}$ rad/sec.
"cutoff frequancy."

---

*Another ckt to build High Pass Filter: (RL ckt).

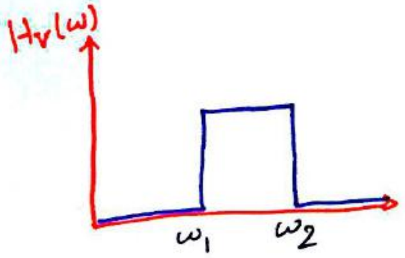
$$H_v(\omega) = \frac{j\omega L}{R + j\omega L}$$

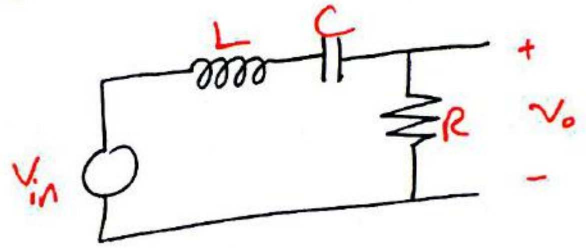
$$|H_v(\omega)| = \frac{\omega L}{\sqrt{R^2 + (\omega L)^2}}$$



※ Band Pass Filter:

$\Rightarrow$ Series RLC ckt with the output is taken off the resistor.

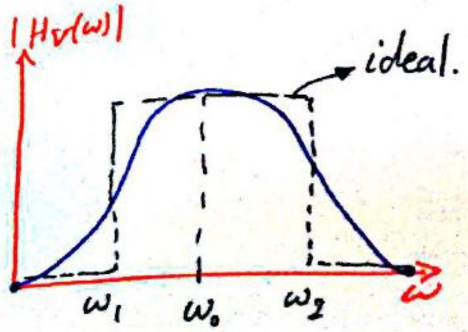




$$H_v(\omega) = \frac{v_o}{V_{in}} = \frac{R}{R + j\omega L + \frac{1}{j\omega C}} = \frac{j\omega RC}{j\omega RC - \omega^2 LC + 1}$$

$$|H_v(\omega)| = \frac{\omega RC}{\sqrt{(1-\omega^2 LC)^2 + (\omega RC)^2}}$$

$H(0) = 0$
$H(\infty) = 0$



$\Rightarrow \omega_1, \omega_2, \omega_0$: as defined in series resonance:

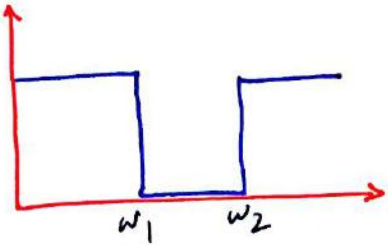
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

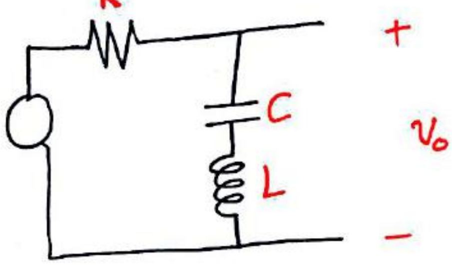
$$\omega_{1,2} = \omega_0 \left[\sqrt{1 + \left(\frac{1}{2Q_0}\right)^2} \mp \frac{1}{2Q_0} \right]$$

※ Band Stop Filter:

$\Rightarrow$ a filter that prevents a band of frequancies between two values (ω_1, ω_2)

$\Longrightarrow$ RLC series cKt with output at LC.

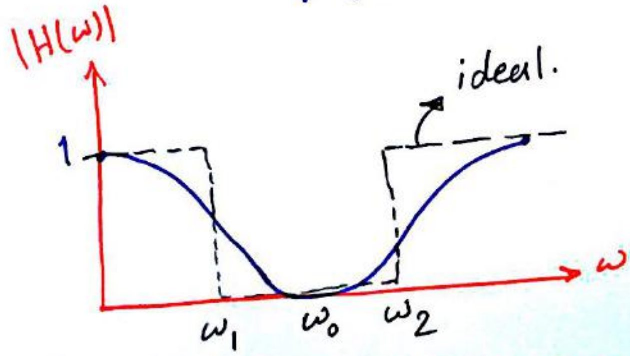




$$H(\omega) = \frac{V_o}{V_{in}} = \frac{j(\omega L - \frac{1}{\omega C})}{R + j(\omega L - \frac{1}{\omega C})}$$

$$|H(\omega)| = \frac{(1 - \omega^2 LC)}{\sqrt{(1-\omega^2 LC)^2 + (\omega RC)^2}}$$

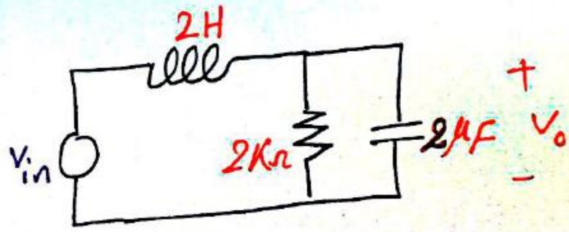
$H(0) = 1$
$H(\infty) = 1$



$$\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

---

ex. Determine what type of filter is shown in the figure, what is the corner frequancy:



solution: $H(\omega) = \frac{V_o}{V_{in}} = \frac{R // \frac{1}{j\omega C}}{R // \frac{1}{j\omega C} + j\omega L}$

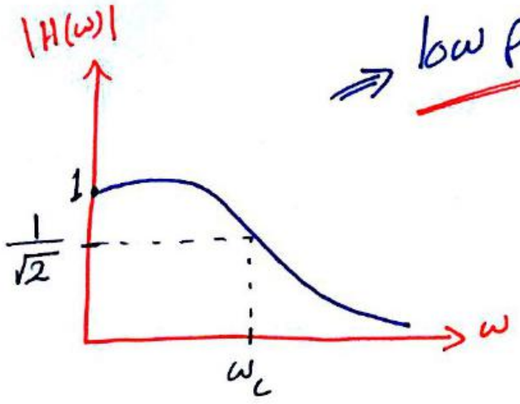
$$= \frac{\frac{R * \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}}{\frac{R * \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} + j\omega L} = \frac{R/(1+j\omega RC)}{R/(1+j\omega RC) + j\omega L} = \frac{R}{-\omega^2 RLC + j\omega L + R}$$

$$|H(\omega)| = \frac{R}{\sqrt{(R - \omega^2 RLC)^2 + (\omega L)^2}}$$

$H(0) = 1$
$H(\infty) = 0$

$\Longrightarrow$

$\Rightarrow$



$\Rightarrow$ low pass filter.

$\Rightarrow$ Now finding the cutoff freq:

$$\frac{1}{\sqrt{2}} = \frac{R}{\sqrt{(R-\omega_c^2 RLC)^2 + (\omega_c L)^2}}$$

solve this equation for ω_c:

$\Rightarrow \quad \omega_c^2 = 0.5509 \quad$ or $\quad -0.1134$

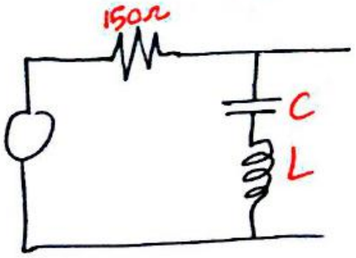
in this course we won't take the negative ω^2 that will give us complex freq.

$\Rightarrow \quad \omega_c = 0.792$ Krad/sec.

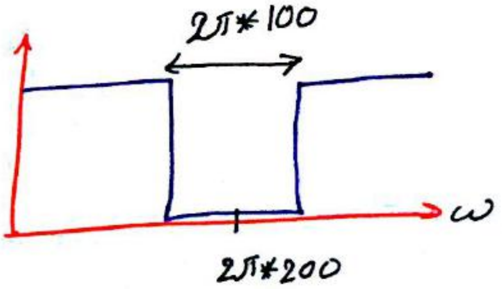
---

ex. A Band-Stop-Filter is designed to reject a 200 HZ frequancy while passing other frequancies. Calculate L & C ?
Take $R = 150\,\Omega$ & $BW = 100$ HZ.

solution:



$\Rightarrow$



$$\beta = \frac{\omega_0}{Q_0} = \frac{\omega_0}{\frac{\omega_0 L}{R}} = \frac{R}{L}$$

$\Rightarrow L = \frac{150}{200\pi} \Rightarrow \boxed{L = 0.238\,H}$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 400\pi$$

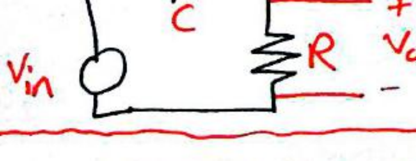
so $\boxed{C = 2.65\,\mu F}$

---

ex. Design a high-pass-filter with corner frequancy 3KHz ?

solution:

Assume it is RC cKt:



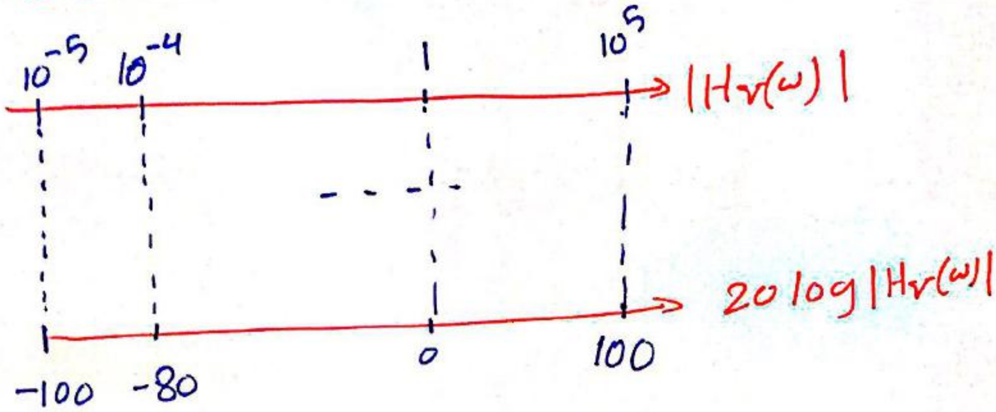
$\Rightarrow \omega_c = \frac{1}{RC} = 3K * 2\pi$

we can choose any value and find the other from the equation.

Take $\boxed{R = 4.7\,K\Omega}$

$\Rightarrow \boxed{C = 11.29\,nF}$

※ The Decibel Scale:



※ Bode Plots:

* To Draw $|H_v(\omega)|$

⇒ Approximate, Yet accurate, drawing of the logaritheme gain & phase as functions of frequancy.

$$H(\omega) = \frac{N(\omega)}{D(\omega)} = \frac{K\,(j\omega)^{\pm 1}\left[1+\frac{j\omega}{Z_1}\right]}{\left[1+\frac{j\omega}{P_1}\right]}$$

⇒ general form. (standard form).

where:

K: constant.

$(j\omega)^{\pm 1}$: pole or zero.

$\left[1+\frac{j\omega}{Z_1}\right]$: zero at Z_1.

$\left[1+\frac{j\omega}{P_1}\right]$: pole at P_1.

ex. write $H(\omega)$ in the standard form:

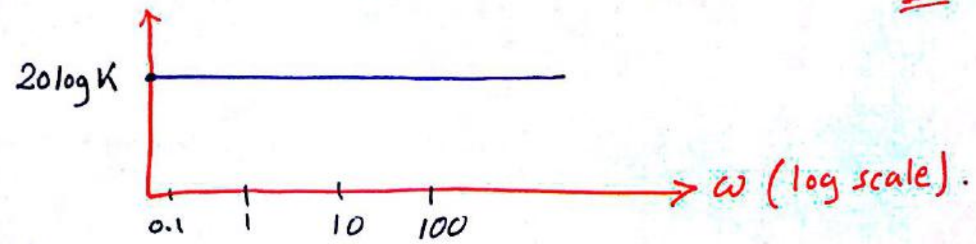
$$H(\omega) = \frac{200\,j\omega\,(j\omega+50)}{(j\omega+2)\,(j\omega+10)}$$

$$\Rightarrow H(\omega) = \frac{200 * j\omega * 50\left[1+\frac{j\omega}{50}\right]}{2*\left[\frac{j\omega}{2}+1\right]*10*\left[\frac{j\omega}{10}+1\right]} = \frac{500 * (j\omega) * \left[\frac{j\omega}{50}+1\right]}{\left[\frac{j\omega}{2}+1\right]*\left[\frac{j\omega}{10}+1\right]}$$

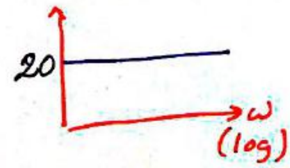
** $H(\omega) = \dfrac{K(j\omega)\left[1+\frac{j\omega}{Z_1}\right]}{\left[1+\frac{j\omega}{P_1}\right]}$

$\Rightarrow$ $20\log|H(\omega)| = 20\log|K| + 20\log|j\omega| + 20\log\left|\left(1+\frac{j\omega}{Z_1}\right)\right| - 20\log\left|\left(1+\frac{j\omega}{P_1}\right)\right|$

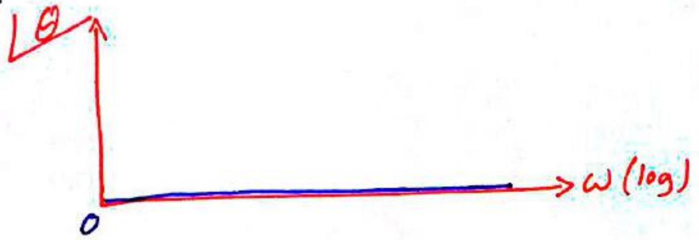
* Constant term K:



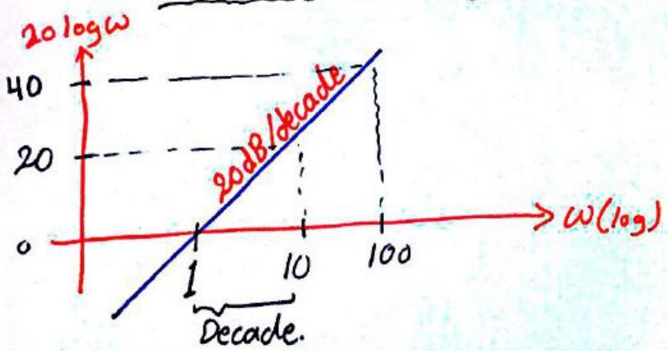
ex. if $H(\omega)=10$



$\Rightarrow$ Angle:

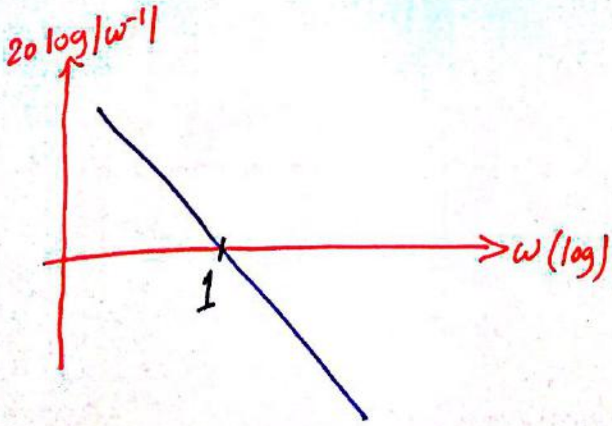


* Zero at Zero $\quad H(\omega)=j\omega$

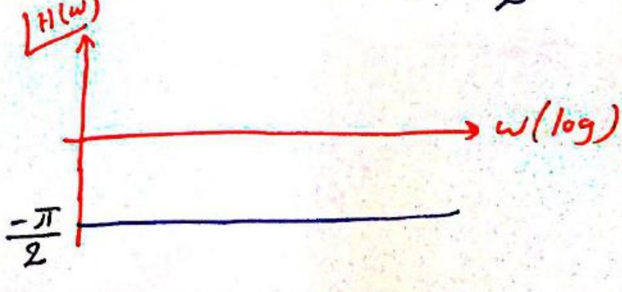


$\Rightarrow$ Angle: $\theta = \tan^{-1}\left(\frac{\omega}{0}\right) = \tan^{-1}(\infty)$
$= \frac{\pi}{2}$

* Pole at Zero $\quad H(\omega) = \frac{1}{j\omega}$

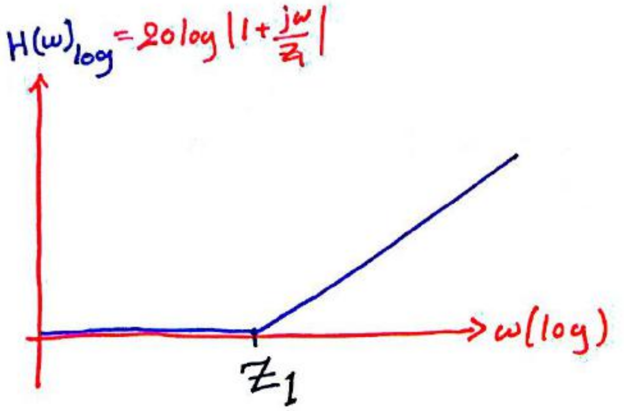


$\Rightarrow$ Angle: $\theta = \tan^{-1}\left(\frac{\frac{-1}{\omega}}{0}\right) = \frac{-\pi}{2}$



* Simple Zero: $H(\omega) = \left[1 + \frac{j\omega}{Z_1}\right]$

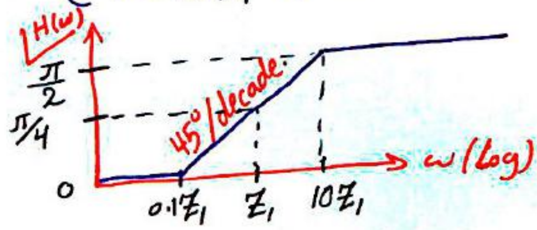
$$H(\omega)_{log} = 20\log\left|\left(1+\frac{j\omega}{Z_1}\right)\right| = \begin{cases} 0 & , Z_1 \gg \omega \rightarrow \text{imaginary part will be neglected.} \\ 20\log\frac{\omega}{Z_1} & , \omega \gg Z_1 \rightarrow \text{real part will be neglected.} \end{cases}$$



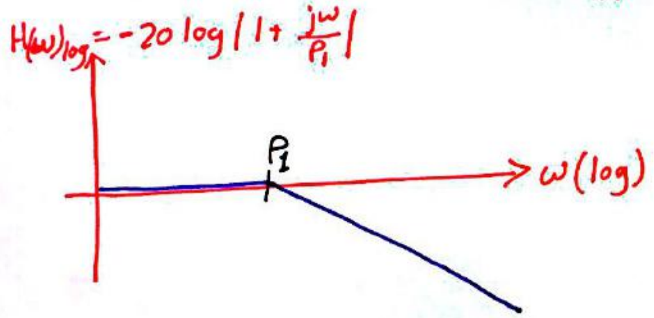
$\Rightarrow$ Angle: @ $\omega = 0.1Z_1 \Rightarrow \theta = 0$

@ $\omega = Z_1 \Rightarrow \theta = 45°$

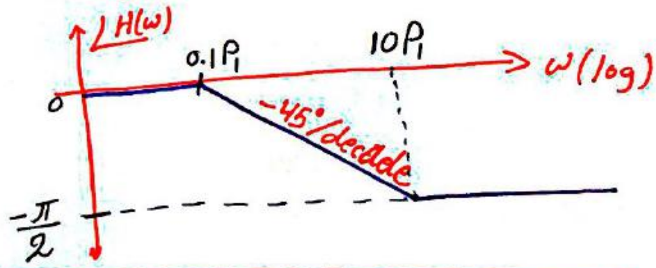
@ $\omega = 10Z_1 \Rightarrow \theta = 90°$



* Simple Pole: $H(\omega) = \frac{1}{\left[1+\frac{j\omega}{P_1}\right]}$



$\Rightarrow$ Angle:



---

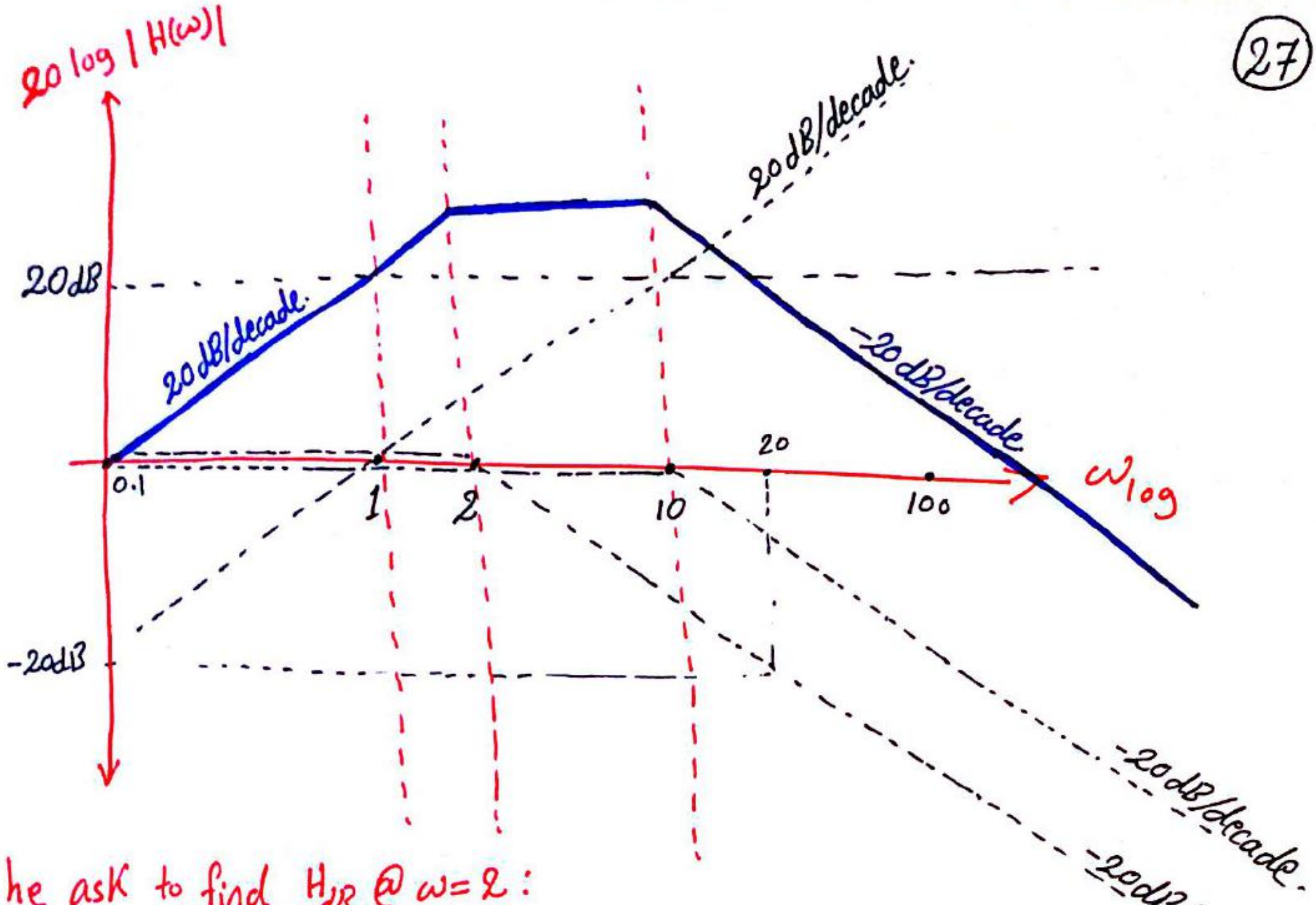
ex. Construct the Bode Plot for $H(\omega) = \frac{200(j\omega)}{(j\omega+2)(j\omega+10)}$

solution:

$$H(\omega) = \frac{200\,(j\omega)}{2\left(1+\frac{j\omega}{2}\right) * 10\left(\frac{j\omega}{10}+1\right)}$$

$$= \frac{10\,(j\omega)}{\left(1+\frac{j\omega}{2}\right)\left(1+\frac{j\omega}{10}\right)}$$

$\Rightarrow$ See the drawing on the next page. $\Longrightarrow$



if he ask to find H_{dB} @ $\omega = 2$:

$$H_{dB} = 20\log 10 + 20\log|j\omega| - 20\log\left|\frac{j\omega}{2}+1\right| - 20\log\left|\frac{j\omega}{10}+1\right|$$

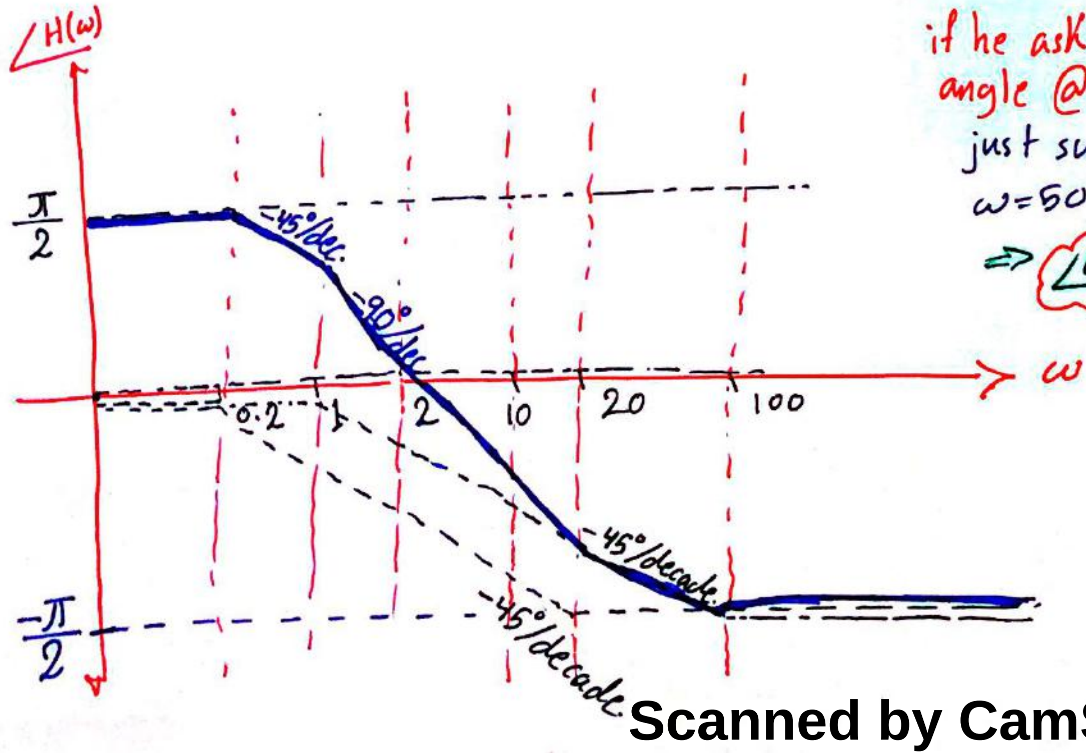
$$= 20\log 10 + 20\log 2 - 20\log\sqrt{2} - 20\log\sqrt{1.04} = \underline{\underline{22.84}}$$

Now for the angle:

$$\angle H(\omega) = 0 + 90 - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{10}$$

if he ask about the angle @ $\omega = 50$: just substitute $\omega = 50$ in the equation.

$\Rightarrow \angle H(50) = -76.4°$

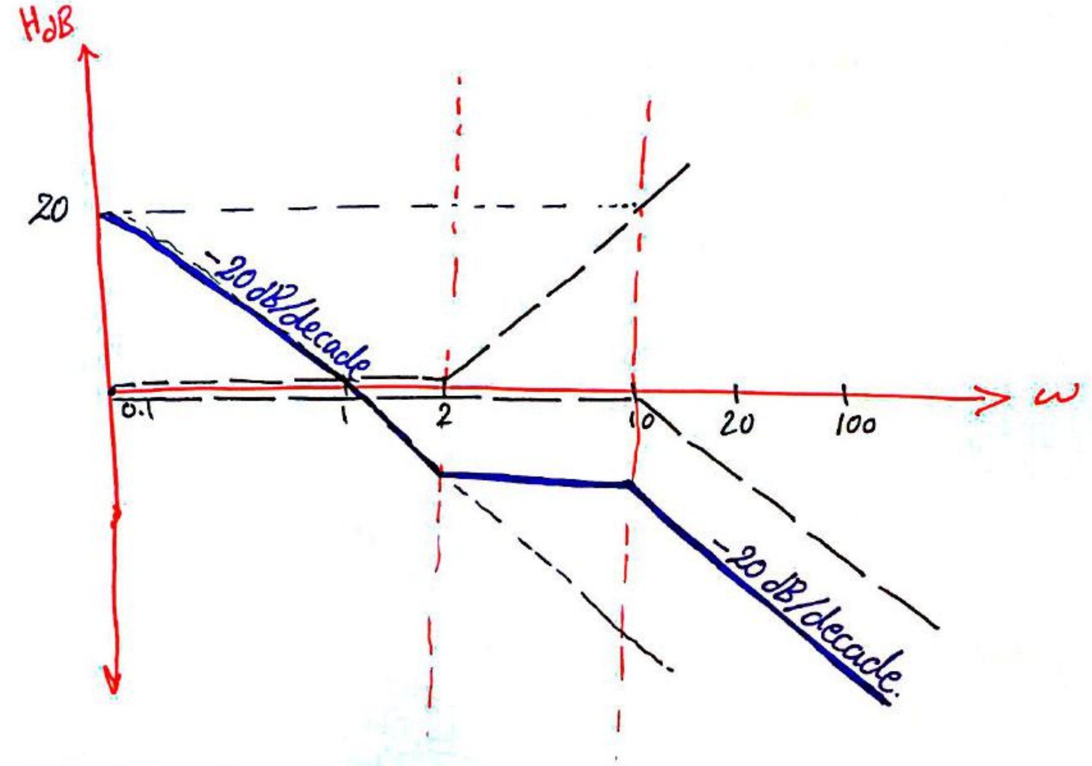


ex. $H(\omega) = \dfrac{5\,(j\omega+2)}{j\omega\,(j\omega+10)}$

solution:

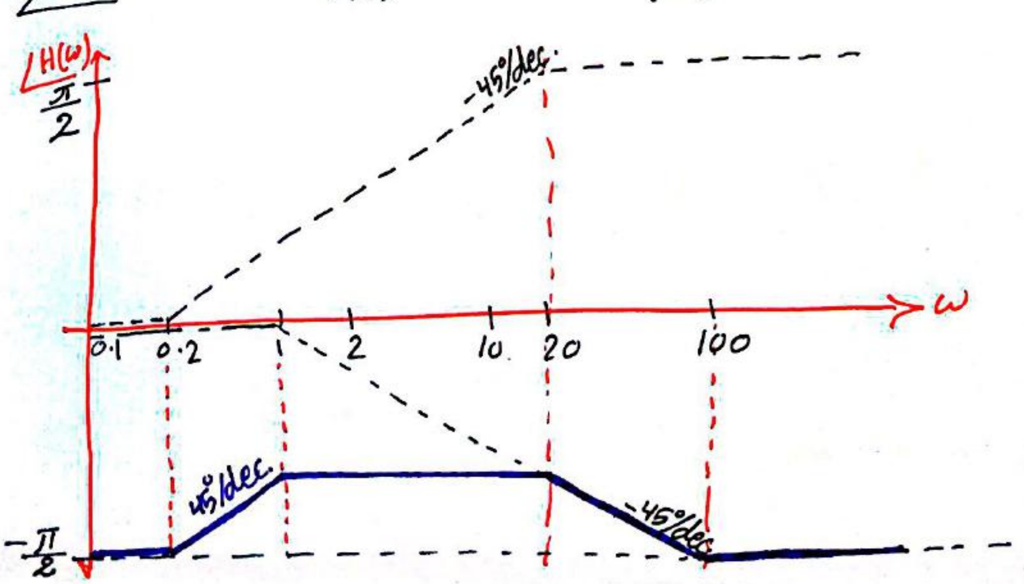
$$\Longrightarrow H(\omega) = \frac{5 * 2\left[1+\frac{j\omega}{2}\right]}{j\omega * 10\left[1+\frac{j\omega}{10}\right]} = \frac{\left(1+\frac{j\omega}{2}\right)}{j\omega\left(1+\frac{j\omega}{10}\right)}$$

$$H_{dB} = 20\log\left|1+\frac{j\omega}{2}\right| - 20\log|j\omega| - 20\log\left|1+\frac{j\omega}{10}\right|$$



Now for the angle:

$$\angle H(\omega) = \tan^{-1}\left(\frac{\omega}{2}\right) - 90 - \tan^{-1}\left(\frac{\omega}{10}\right)$$

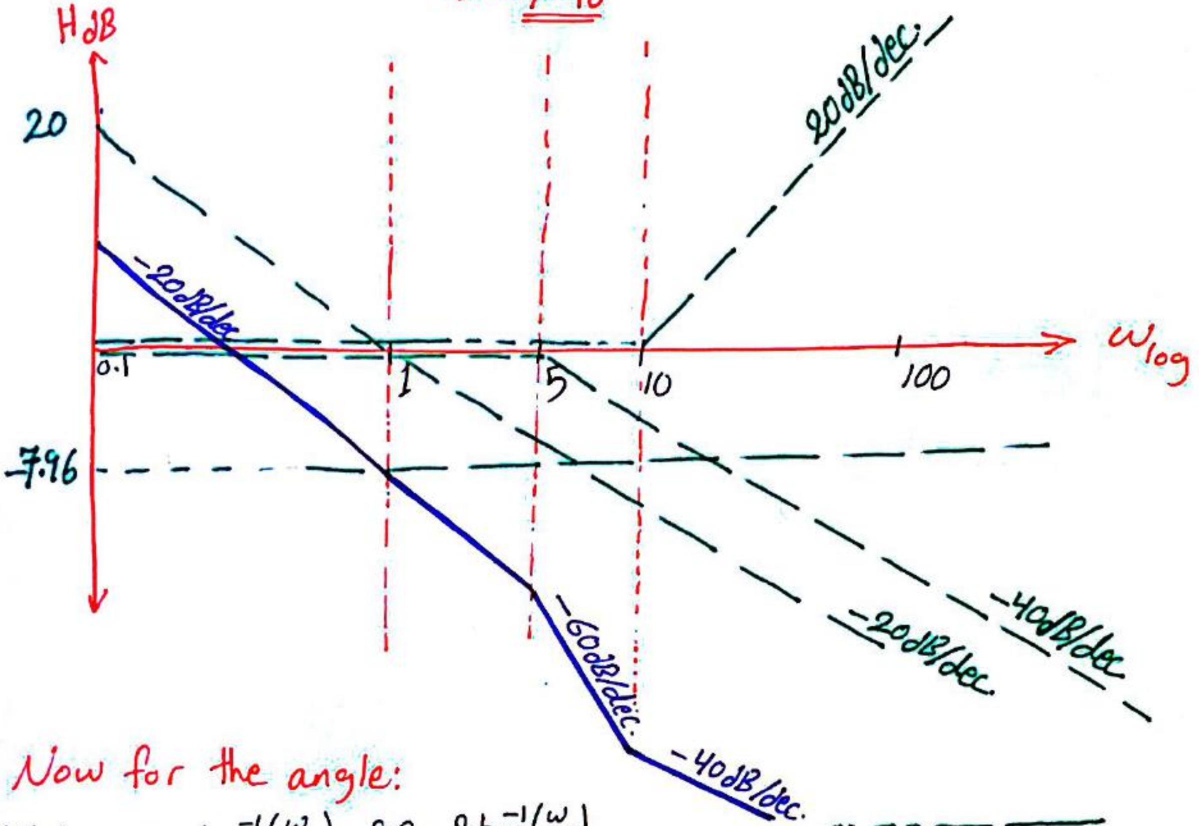


ex. $H(\omega) = \dfrac{j\omega + 10}{j\omega (j\omega+5)^2}$

Solution:

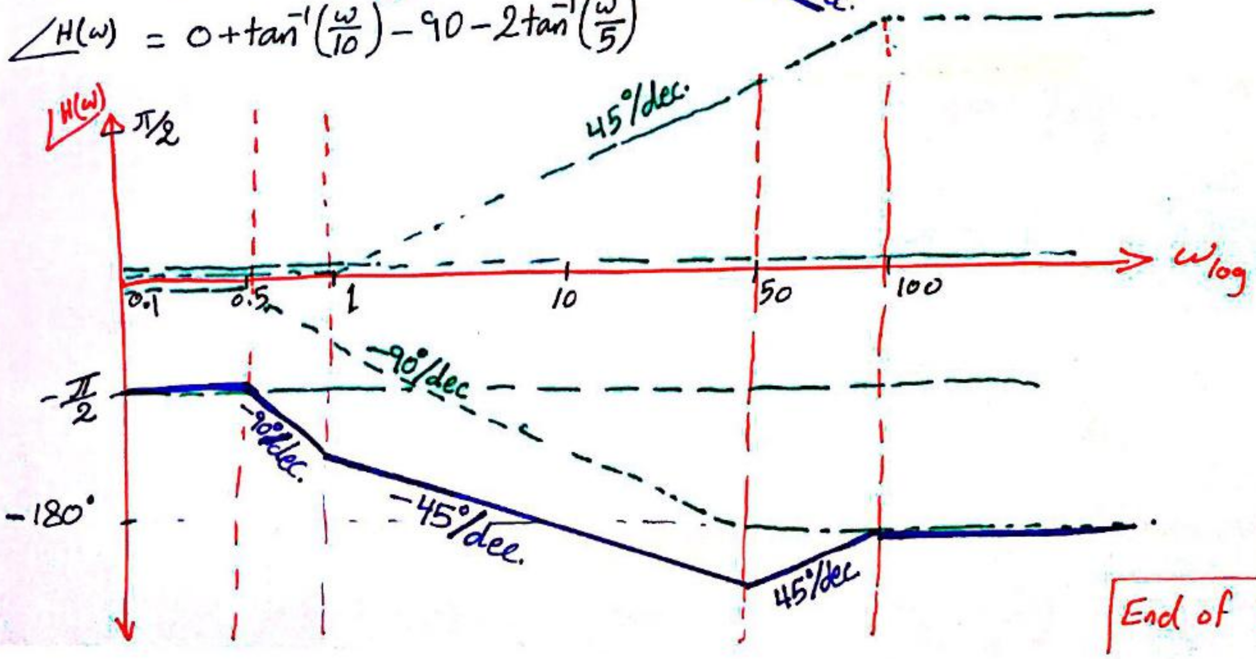
$$\Rightarrow H(\omega) = \frac{10\left(1+\frac{j\omega}{10}\right)}{j\omega * 25\left(\frac{j\omega}{5}+1\right)^2} = \frac{0.4\left(1+\frac{j\omega}{10}\right)}{j\omega\left(1+\frac{j\omega}{5}\right)^2}$$

$$\Rightarrow H_{dB} = \underbrace{20\log 0.4}_{\Rightarrow\ -7.96} + 20\log\left|1+\frac{j\omega}{10}\right| - 20\log|j\omega| - 40\log\left|1+\frac{j\omega}{5}\right|$$



Now for the angle:

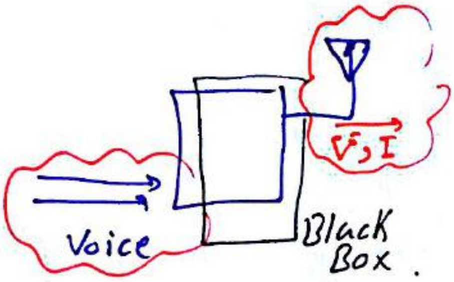
$\angle H(\omega) = 0 + \tan^{-1}\left(\frac{\omega}{10}\right) - 90 - 2\tan^{-1}\left(\frac{\omega}{5}\right)$

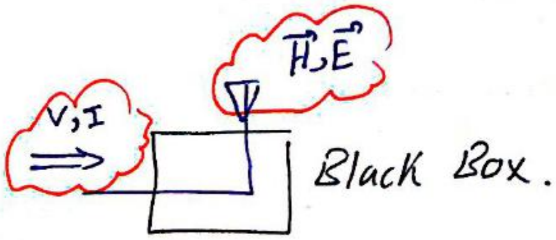


End of CH_{14}

CHAPTER (19)

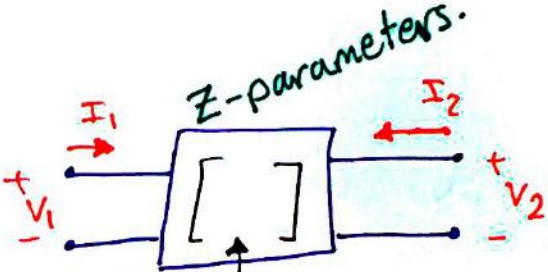
Two-Port Networks





⇒ What happens on one of the ports if we change the other one ?!

※ Z-Parameters:



$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

In matrix form:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \Rightarrow [V] = [Z][I]$$

⇒ Lets put $I_2 = 0$: (open cKt to the right side)

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$$

$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$$

⇒ lets put $I_1 = 0$: (open cKt to the left side)

$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$$

$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$$

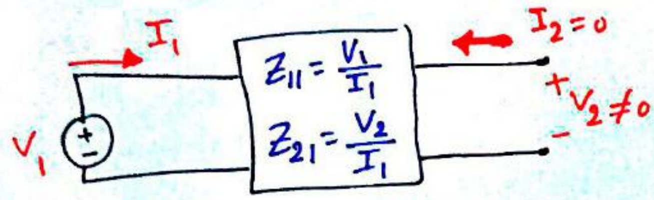
Z_{11}: open cKt input impedance.

Z_{22}: open cKt output impedance.

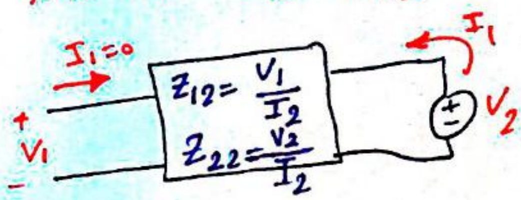
Z_{21}: open cKt transfer impedance. from port 2 to port 1.

Z_{12}: open cKt transfer impedance from port 1 to port 2.

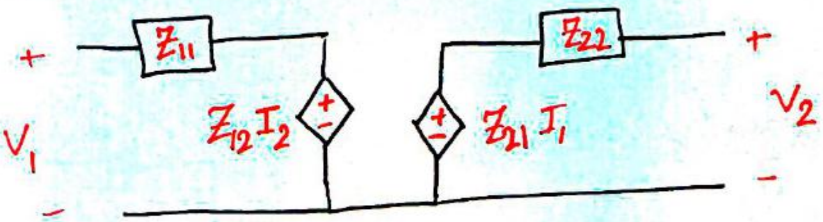
* To obtain Z_{11} & Z_{21}:



* To obtain Z_{12} & Z_{22}:



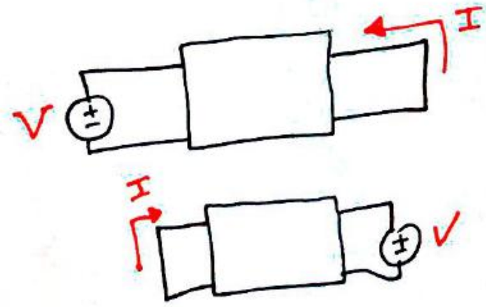
* Equivelant ckt for a 2-port network (with dependent power sources):



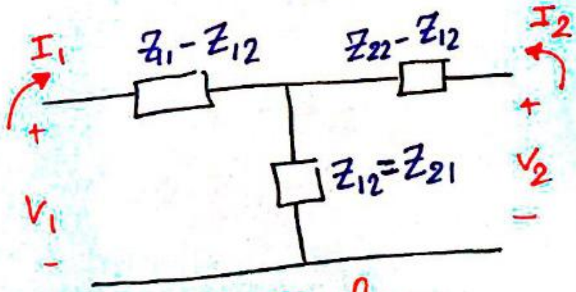
$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$\Rightarrow$ The same two equations that were obtained before.

* Special Case: if the networks have NO dependent power sources, we say that the system is "reciprocal".

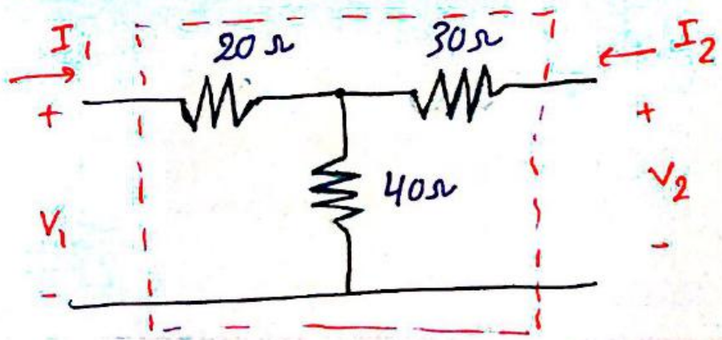


$\Rightarrow$



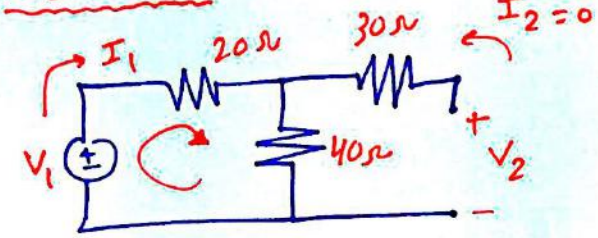
Equivelant ckt for a reciprocal network.

(ex.) Determine the Z-parameters for the ckt:



⇒ Solution:

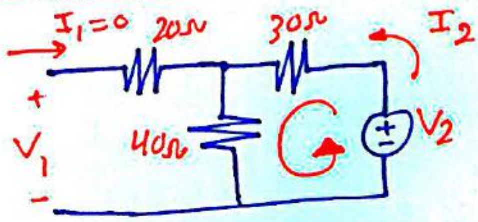
method (1):



$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0}$$

$$Z_{11} = \frac{I_1\,(20+40)}{I_1} = \boxed{60}\,\Omega$$

$$Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0} = \frac{40\,I_1}{I_1} = \boxed{40}\,\Omega$$



$$Z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0} = \frac{40\,I_2}{I_2} = \boxed{40}\,\Omega$$

$$Z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0} = \frac{(40+30)\,I_2}{I_2} = \boxed{70}\,\Omega$$

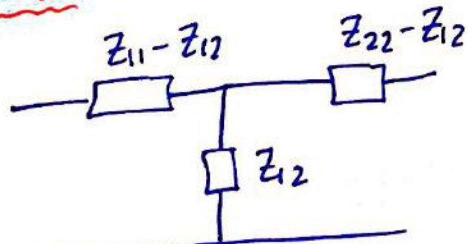
* What is the voltage where $V_1 = 10$ volt & $I_1 = 5A$.

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$
$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

⇒ solve for V_2 & I_2

$$\boxed{I_2 = -7.25A.\;,\; V_2 = -307.5\,\text{volt.}}$$

method (2):



⇒ $\boxed{Z_{12} = Z_{21} = 40\,\Omega}$

$20 = Z_{11} - Z_{12}$

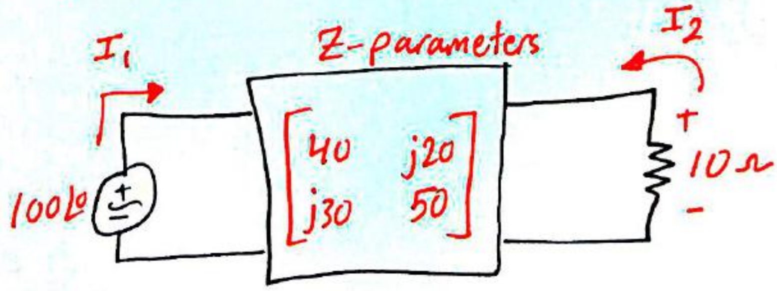
⇒ $\boxed{Z_{11} = 60\,\Omega}$

$30 = Z_{22} - Z_{12}$

⇒ $\boxed{Z_{22} = 70\,\Omega}$

This method just used if the cKt was in the shape of the special case "reciprocal network"

ex. Find I_1 & I_2 in the cKt shown:



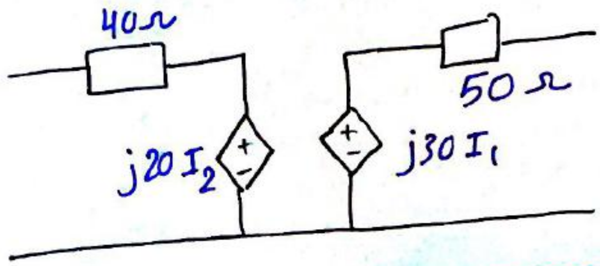
Solution:

$V_1 = Z_{11} I_1 + Z_{12} I_2 \Rightarrow 100\angle 0 = 40 I_1 + j20 I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2 \Rightarrow -10 I_2 = j30 I_1 + 50 I_2$

solving: $\boxed{I_1 = 2\angle 0\ A. \quad I_2 = 1\angle -90^\circ\ A}$

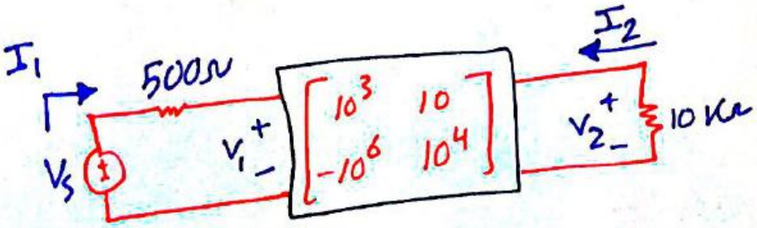
if he asked about the equivelant cKt for this example:



ex. Z is given by $\begin{bmatrix} 10^3 & 10 \\ -10^6 & 10^4 \end{bmatrix}$ Ω, find the voltage, current & power gain if the 2-port network is driven by V_s in series with 500Ω & terminated by 10KΩ resistor.

Solution:

* Driven ⇒ it means input.



$V_1 = Z_{11} I_1 + Z_{12} I_2$ ---(0)

$\Rightarrow \boxed{\underbrace{V_s - 500 I_1}_{V_1} = 10^3 I_1 + 10 I_2}$ ---(1)

$V_2 = Z_{21} I_1 + Z_{22} I_2$

$\boxed{-10^4 I_2 = -10^6 I_1 + 10^4 I_2}$ ---(2)

from (2): $\Rightarrow \boxed{I_2 = 50 I_1}$ ---(3)

substitute (3) in (0):

$V_1 = 1500 I_1$

also (3) in (1):

$V_s = 2000 I_1$

$\Longrightarrow$

$\Rightarrow \quad I_1 = \frac{V_s}{2000} \quad , \quad I_1 = \frac{V_1}{1500}$

$V_2 = -10^6 I_1 + 10^4 [50 I_1] = \underline{\underline{-0.5 * 10^6 I_1}}$

$V_{gain} = \frac{V_2}{V_1} = \frac{-0.5 * 10^6 \cancel{I_1}}{1500 \cancel{I_1}} = \boxed{-333.33}$

$Gain_I = \frac{I_2}{I_1} = \boxed{50}$

$Gain_P = \frac{-\frac{1}{2} V_2 I_2}{\frac{1}{2} V_1 I_1} = \frac{-\frac{1}{2} * -\frac{1}{2} * 10^6 I_1 * 50 I_1}{\frac{1}{2} * 1500 I_1 * I_1} = \boxed{16670}$

$\Rightarrow$ if he asked to find Z_{in} & Z_{out} ?!

$Z_{in} = \frac{V_1}{I_1}$ & $Z_{out} = Z_{22} - \frac{Z_{12} Z_{21}}{Z_{11} + Z_g}$

Z_g → generator impedance or source impedance.

for the last example:

$$\boxed{\begin{aligned} Z_{in} &= 1500\,\Omega. \\ Z_{out} &= 16.67\ K\Omega. \end{aligned}}$$

※ Y-parameters:

$\Rightarrow$ Some times Z parameters do not exist

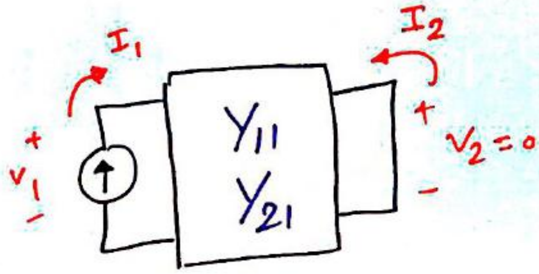
$\Rightarrow$ Use the Y-parameters.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \underbrace{\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}}_{\text{Y-parameters.}} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \Rightarrow [I] = [Y][V]$$

$\Rightarrow$

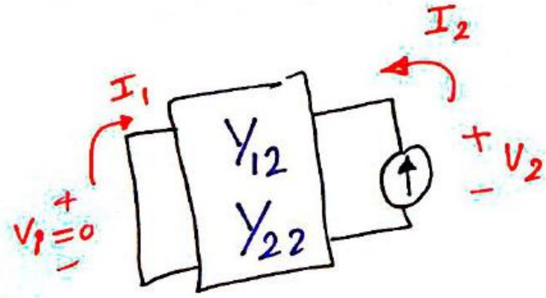
Equations:
$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$\Rightarrow$ set $V_2 = 0$ (short cKt on the right side)



$$\boxed{\begin{aligned} Y_{11} &= \frac{I_1}{V_1} \\ Y_{21} &= \frac{I_2}{V_1} \end{aligned}}$$ when $\underline{\underline{V_2 = 0}}$.

$\Rightarrow$ set $V_1 = 0$ (short cKt on the left side)



$$\boxed{\begin{aligned} Y_{12} &= \frac{I_1}{V_2} \\ Y_{22} &= \frac{I_2}{V_2} \end{aligned}}$$ when $\underline{\underline{V_1 = 0}}$.

Y_{11}: short cKt input admittance.

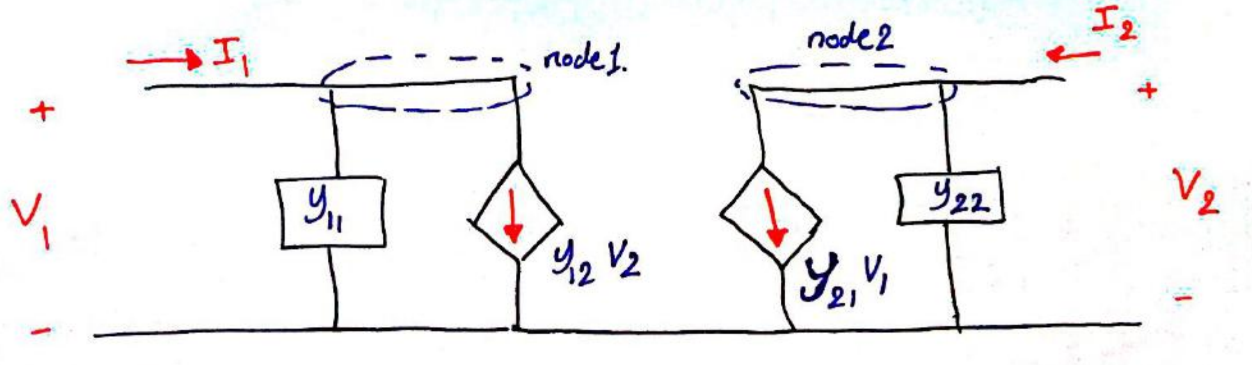
Y_{12}: short cKt transfer admittance from port 1 to port 2.

Y_{21}: short cKt transfer admittance from port 2 to port 1.

Y_{22}: short cKt output admittance.

---

*General equivalent cKt for 2-port network:



⇒ You can reach for the main two equations:

By nodal analysis:

@ node 1: $I_1 + (0 - V_1)\, y_{11} - y_{12} V_2 = 0$

$$\Rightarrow \begin{cases} I_1 = V_1 y_{11} + y_{12} V_2 \\ I_2 = V_2 y_{22} + y_{21} V_1 \end{cases}$$

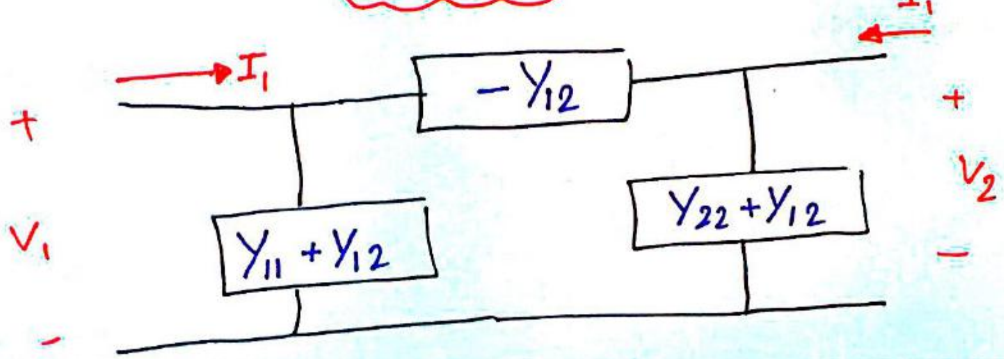
@ node 2: ⇒

as obtained before.

---

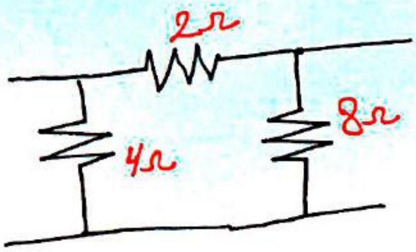
* Reciprocal cKt: [NO dependent power sources]

$\Rightarrow \boxed{Y_{12} = Y_{21}}$



---

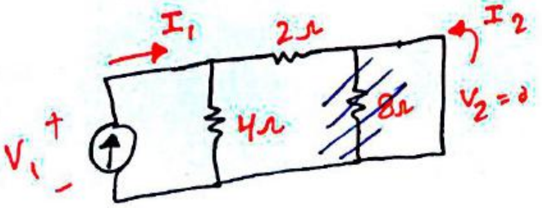
ex. Obtain th Y-parameters for the network shown:



Solution:

method (1):

for y_{11} & y_{21}



$4 // 2 \Rightarrow \frac{8}{6}\Omega$

so $V_1 = I_1 * \frac{8}{6}$

$y_{11} = \frac{I_1}{V_1} \Rightarrow \boxed{y_{11} = \frac{6}{8}\ \mho}$

$y_{21} = \frac{I_2}{V_1}$

current division:

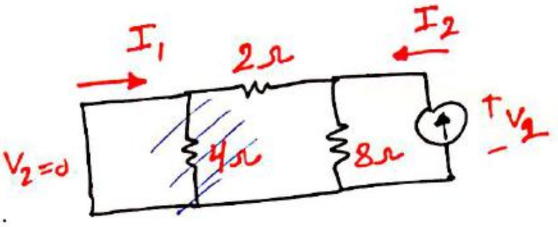
$I_2 = -I_1 \frac{4}{6}$

$= \dfrac{-\frac{4}{6} I_1}{\frac{8}{6} I_1}$

$\Rightarrow \boxed{y_{21} = -0.5\ \mho}$

⇒

Now for y_{22} & y_{12} :



$$y_{22} = \frac{I_2}{V_2}$$

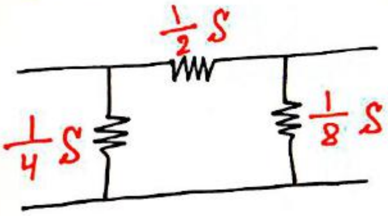
$2 // 8 \Rightarrow \frac{8}{5}\Omega$ $\Rightarrow y_{22} = \frac{I_2}{\frac{8}{5} I_2} \Rightarrow \boxed{y_{22} = \frac{5}{8} S}$

so $V_2 = \frac{8}{5} I_2$

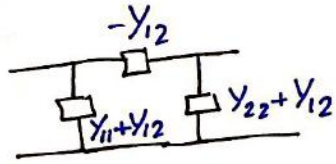
$$y_{12} = \frac{I_1}{V_2} = \frac{\frac{-8}{10} I_2}{\frac{8}{5} I_2} \Longrightarrow \boxed{y_{12} = -0.5 S}$$

Y parameters $= \begin{bmatrix} 0.75 & -0.5 \\ -0.5 & 0.625 \end{bmatrix}$

method(2):



$\equiv$



$\Rightarrow -y_{12} = \frac{1}{2} \Rightarrow \boxed{y_{12} = -0.5 = y_{21}}$

$\frac{1}{8} = y_{22} + y_{12} \Longrightarrow y_{22} = \frac{1}{8} - (-0.5)$

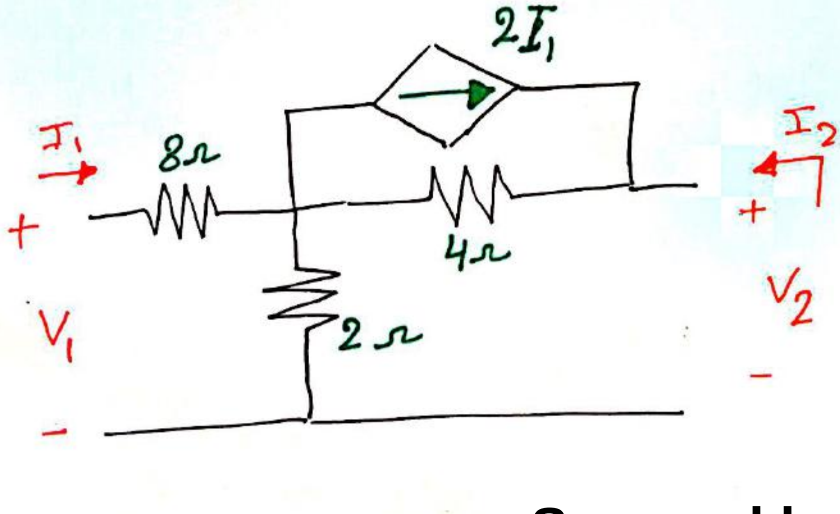
$\Rightarrow \boxed{y_{22} = 0.625 S}$

$\frac{1}{4} = y_{11} + y_{12}$

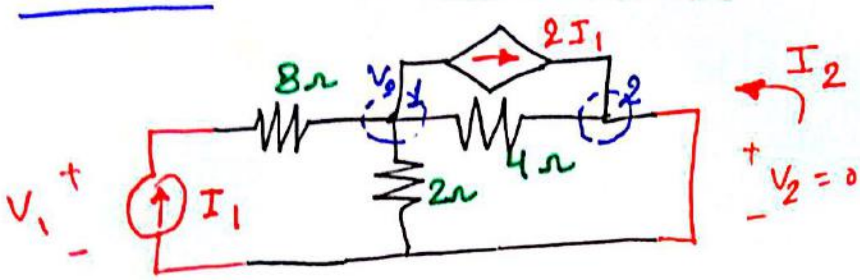
$\Rightarrow y_{11} = \frac{1}{4} - (-0.5)$

$\Rightarrow \boxed{y_{11} = 0.75}$

ex. Determine the Y-parameters for the 2-Port network:



Solution: To find y_{11}, y_{21}:



$$y_{11} = \frac{I_1}{V_1}$$

$$y_{21} = \frac{I_2}{V_1}$$

@ node(1): $\frac{V_1 - V_o}{8} + \frac{-V_o}{2} + \frac{-V_o}{4} - 2I_1 = 0$

$\longrightarrow I_1 = \frac{V_1 - V_o}{8}$ ---(*)

$$\Rightarrow \frac{V_1}{8} - \frac{V_o}{8} - \frac{V_o}{2} - \frac{V_o}{4} - \frac{V_1}{4} + \frac{V_o}{4} = 0$$

solving $\longrightarrow$ $V_1 = -5V_o$

for equ. (*):

$$I_1 = \frac{V_1 - V_o}{8} = \frac{-5V_o - V_o}{8} = -0.75V_o$$

$I_1 = -0.75V_o$

$\Rightarrow y_{11} = \frac{I_1}{V_1} = \frac{-0.75V_o}{-5V_o}$

$\Rightarrow \boxed{y_{11} = 0.15\,S}$

@ node(2):

$$2I_1 + I_2 + \frac{V_o - 0}{4} = 0 \Rightarrow 2I_1 + I_2 + \frac{V_o}{4} = 0$$

$I_1 = -0.75V_o$

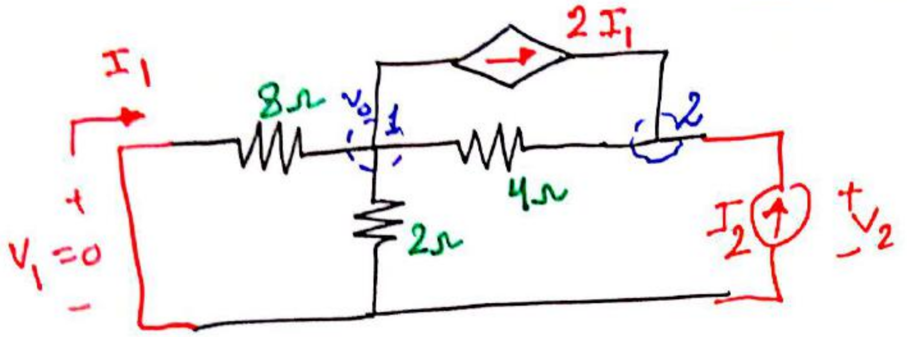
$$-1.5V_o + I_2 + \frac{V_o}{4} = 0$$

$\Rightarrow I_2 = 1.25V_o$

$$y_{21} = \frac{I_2}{V_1}$$

$$y_{21} = \frac{1.25V_o}{-5V_o} \Rightarrow \boxed{y_{21} = -0.25\,S}$$

$\Longrightarrow$ continue.



$y_{12} = \frac{I_1}{V_2}$

$y_{22} = \frac{I_2}{V_2}$

@ node(1):

$$\frac{0-V_0}{8} + \frac{0-V_0}{2} + \frac{V_2-V_0}{4} - 2I_1 = 0 \qquad I_1 = \frac{-V_0}{8}$$

$\xrightarrow{\text{solving}}$ $V_2 = 2.5V_0$

$$y_{12} = \frac{I_1}{V_2} \Rightarrow y_{12} = \frac{-V_0/8}{2.5V_0} \Rightarrow \boxed{y_{12} = -0.05S}$$

@ node(2): $I_1 = \frac{-V_0}{8}$ $\quad V_2 = 2.5V_0$

$$I_2 + 2I_1 + \frac{V_0 - V_2}{4} = 0 \xrightarrow{\text{solving}} I_2 = 0.625V_0$$

$$y_{22} = \frac{I_2}{V_2} \Rightarrow y_{22} = \frac{0.625V_0}{2.5V_0} \Rightarrow \boxed{y_{22} = 0.25S}$$

so Y-parameters is:

$$Y = \begin{bmatrix} 0.15 & 0.05 \\ -0.25 & 0.25 \end{bmatrix}$$

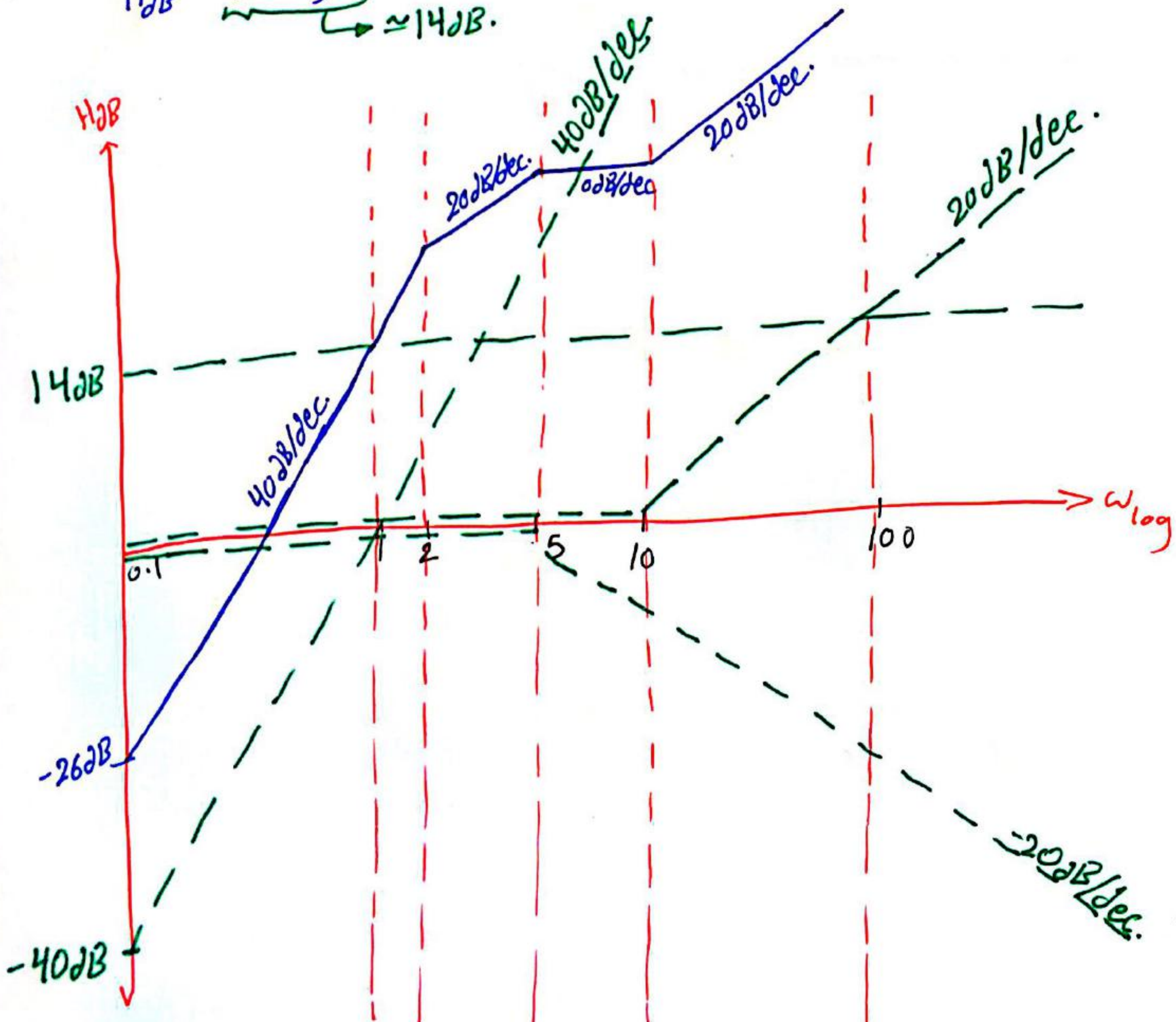
ex. Construct The Bode Plot for the following:

$$H(\omega) = \frac{5\,(j\omega)^2\,(j\omega+10)}{(j\omega+5)\,(j\omega+2)}$$

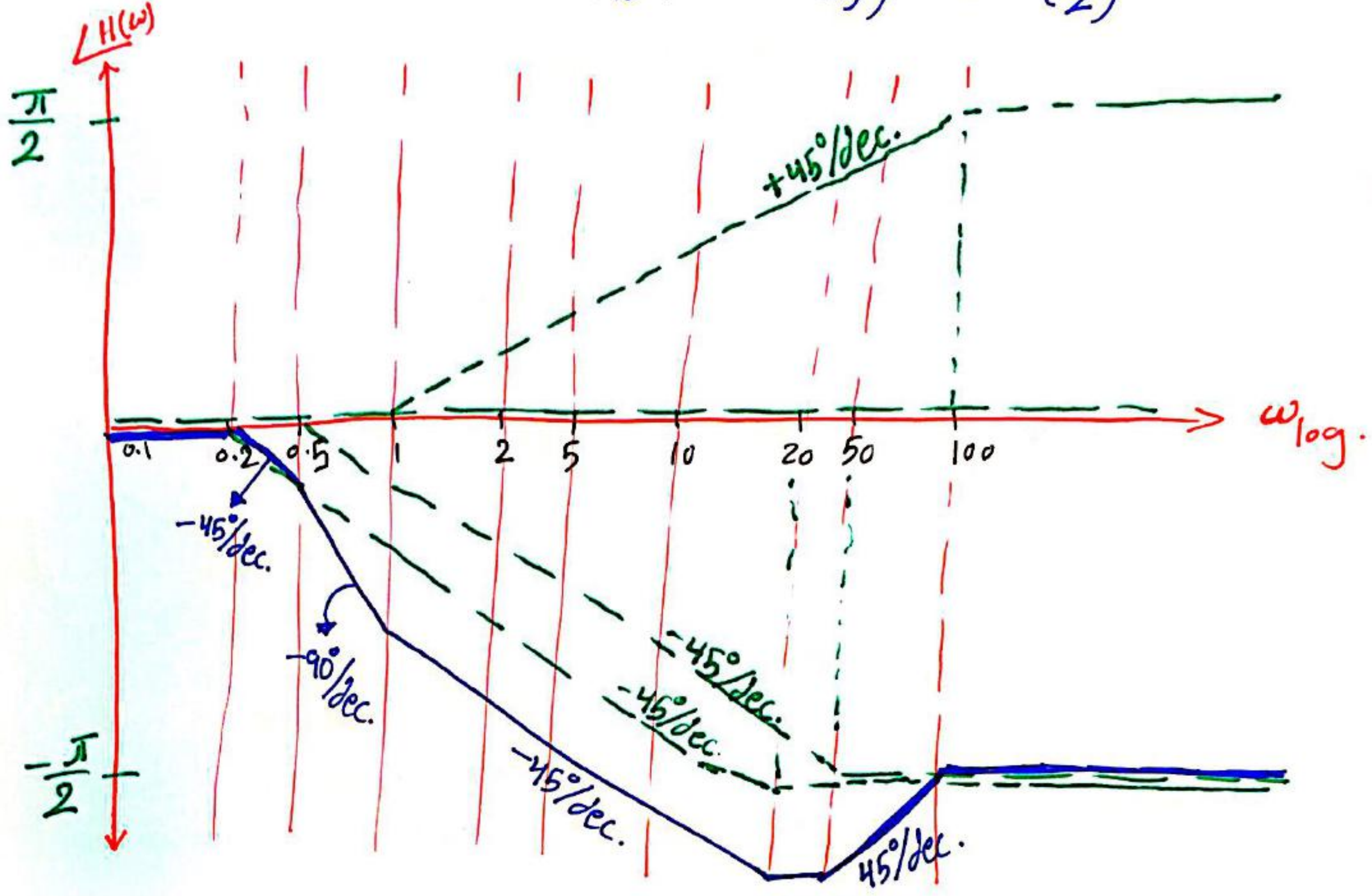
writing $H(\omega)$ in the standard form:

$$H(\omega) = \frac{5\,(j\omega)^2\left(1+\frac{j\omega}{10}\right)}{\left(1+\frac{j\omega}{5}\right)\left(1+\frac{j\omega}{2}\right)}$$

$$H_{dB} = \underbrace{20\log 5}_{\simeq 14\,dB} + 40\log(j\omega) + 20\log\left(1+\frac{j\omega}{10}\right) - 20\log\left(1+\frac{j\omega}{5}\right) - 20\log\left(1+\frac{j\omega}{2}\right)$$



$\angle H(\omega) = 0 + 0 + \tan^{-1}\left(\frac{\omega}{10}\right) - \tan^{-1}\left(\frac{\omega}{5}\right) - \tan^{-1}\left(\frac{\omega}{2}\right)$



* * * * * *

End of Material.

Best of Luck.

Electrical Circuits II

* Summary *

※ CHAPTER(10):

* for the capacitor:

$$i_c(t) = C\,\frac{\partial v_c(t)}{\partial t}$$

$$v_c(t) = \frac{1}{C}\int_0^t i_c(t')\,dt' + v_c(0^-)$$

* the current leads the voltage.

* for the inductor:

$$v_L(t) = L\,\frac{\partial i_L(t)}{\partial t}$$

$$i_L(t) = \frac{1}{L}\int_0^t v_L(t')\,dt' + i_L(0^-)$$

* the voltage leads the current.

* Converting from phasor to time domain:
  * the amplitude must be positive and (peak value).
* Converting from time domain to phasor:
  1. Amplitude mut be (+ve).
  2. it must be a cosine function.

※ CHAPTER (11):

* instantaneous power:

$$P(t) = v(t)\cdot i(t)$$

$$P(t) = \frac{V_m I_m}{2}\left[\cos(2\omega t+\theta_v+\theta_I) + \cos(\theta_v-\theta_I)\right]$$

* Average Power:

$$P = \frac{1}{T}\int_0^T P(t)\,dt$$

* just the resistance has an average power.

$$P_{avg} = \frac{V_m I_m}{2} \text{ (in R)}$$

$$P_{avg} = \text{Zero (in L \& C)}$$

$$P_{avg} = \frac{V_m I_m}{2}\cos(\theta_v-\theta_I) \text{ (in any element)}$$

$$\underset{(R)}{P_{avg}} = \frac{1}{2}V_m I_m = \frac{V_m^2}{2R} = \frac{1}{2}I_m^2 R$$

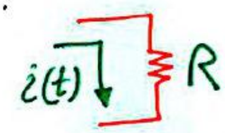
* Maximum Power Transfer:

happens when: $Z_{th} = Z_L^*$

if power is maximaized: $P_{avg}\Big|_{load} = \dfrac{V_{th}^2}{8R}$

* for multiple signals & multiple frequancies:

$i(t) = I_{m_1}\sin\omega_1 t + I_{m_2}\cos\omega_2 t + I_{m_3}\sin\omega_3 t + \cdots\cdots$

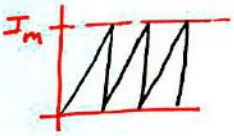


$$P_{avg} = \frac{1}{2}I_{m_1}^2 R + \frac{1}{2}I_{m_2}^2 R + \frac{1}{2}I_{m_3}^2 R + \cdots$$

$$P_{avg} = \frac{V_{m_1}^2}{2R} + \frac{V_{m_2}^2}{2R} + \frac{V_{m_3}^2}{2R} + \cdots\cdots$$

* RMS:

$$\boxed{V_{eff} = V_{rms} = \sqrt{\frac{1}{T}\int_0^T V^2(t)\,dt}}$$ $\Rightarrow$ in general.



$$\boxed{I_{rms} = \frac{I_m}{\sqrt{3}}}$$ $\Rightarrow$ for ST waveform.

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$ $\Rightarrow$ for a sinusoidal Signal.

$\Rightarrow$ for a resistor:

$$P_{avg} = V_{rms} I_{rms} = V_{rms}^2/R = I_{rms}^2 R$$

if there is multiple frequancies:

$$P_{avg} = \left(I_{rms_1}^2 + I_{rms_2}^2 + I_{rms_3}^2 + \cdots\right)R$$

* Power factor:

$Pf = \cos(\theta_V - \theta_I)$

→ Leading if $\theta_V - \theta_I \Rightarrow (-ve)$.

→ Lagging if $\theta_V - \theta_I \Rightarrow (+ve)$.

equal 1 — for resistance.

equal zero — for pure inductance or capacitance.

*** Complex Power:** (Assume RMS values)

$$S = V I^* = \frac{V^2}{Z^*} = I^2 Z \Longrightarrow \text{Complex Power (VA)}$$
$$|S| = |V|\,|I| \Longrightarrow \text{Apparent Power (VA)}$$
$$P = |V|\,|I| \cos(\theta_V - \theta_I) \Longrightarrow \text{Real/Average Power (W)}$$
$$Q = |V|\,|I| \sin(\theta_V - \theta_I) \Longrightarrow \text{Reactive Power (VAR)}$$

*** Adding capacitor to a cKt to give us new pf:**

we find P from the old pf since it will be always constant then find Q_1 Then Q_2 from $Q_2 = P * \tan(\cos^{-1}(pf_{new}))$

$\Rightarrow Q_c = Q_2 - Q_1$

$$\Rightarrow C = \frac{|Q_c|}{|V_{rms}|^2 \omega}$$

*** RMS value for multiple signals:**

$$V_{rms} = \sqrt{(V_{rms_1})^2 + (V_{rms_2})^2 + \cdots}$$

※ CHAPTER (12):

*** in Balanced systems:**

if the load is Δ-connected:

$$V_{LL} = V_{phase}$$
$$I_L = \sqrt{3}\, I_p \angle -30^\circ \quad \text{---(1)}$$

if the load is Y-connected:

$$V_{LL} = \sqrt{3}\, V_{LN} \angle +30^\circ$$
$$I_L = I_p \quad \text{---(2)}$$

$$Z_\Delta = 3 Z_Y \quad \text{---(3)}$$

*** Always in Balanced-systems:** convert to Y-Y and use the single phase cKt $\Rightarrow$ will be the easiest way to solve.

*** In UnBalanced systems:** DON'T use the relations in Box (1) & (2) & (3)

* Always the unbalanced system: ⇒ unbalanced in the load
(source is always balanced)
↳ $V_{ab} = \sqrt{3}\, V_{an} \angle +30^\circ$ (use it always)

* in Balanced system:
if there is a line (with or without impedance) ⇒ connected with n & N
⇒ we don't care to it, it has NO effect.
$I_a + I_b + I_c = 0$ ⇒ always in balanced-system.

* in Unbalanced system: (Three Cases)

1. if just only a line (without impedance) connected to n & N
⇒ $\boxed{I_a + I_b + I_c = I_{Nn}}$ & $\boxed{V_n = V_N = 0}$

2. if it is line (with impedance) ⇒ $\boxed{\begin{array}{l} I_a + I_b + I_c = I_{Nn} \\ V_n = 0 \quad V_N \neq 0 \end{array}}$
⇒ $V_N = Z_{line} * I_{Nn}$

3. if No line between n & N ⇒ $\boxed{V_N \neq 0 \,,\; I_a + I_b + I_c = 0}$

* Power in 3phase system:

$P_\Delta = P_Y = 3 V_p I_p \cos(\theta_v - \theta_i)$ $Q_\Delta = Q_Y = 3 V_p I_p \sin(\theta_v - \theta_i)$	$S_\Delta = S_Y = 3 V_p I_p^*$ $\lvert S_\Delta \rvert = \lvert S_Y \rvert = \sqrt{3}\, \lvert V_L \rvert \lvert I_L \rvert$

* Losses in the 3phase system:
→ $S_{losses} = S_{source} - S_{load}$.
→ $S_{losses} = 3 \lvert I_L \rvert^2 Z_{T.L}$

* Wattmeter mesurments:
⇒ Determine the voltage (V) & the current (I) that measured by this wattmeter then: $P = \lvert V \rvert \lvert I \rvert \cos(\theta_v - \theta_i)$

* if 3-wattmeters used: $\boxed{P_{Total} = P_1 + P_2 + P_3}$

* In Two-Wattmeters method:

$\boxed{P_T = P_1 + P_2}$ $\boxed{Q_T = \sqrt{3}(P_2 - P_1)}$ $\boxed{S_T = P_T + jQ_T}$

$\boxed{pf = \cos\theta}$ $\rightarrow$ $\boxed{\theta = \tan^{-1}\left(\frac{\sqrt{3}(P_2 - P_1)}{P_1 + P_2}\right)}$

※ CHAPTER (13):

※ Linear Transformer:

=> Virtual source: we put it between the dot & the coil

* choosing the polarities:

if the current enter the dot => take it (+ve) for the other dot.
if " " leaves " " => take it (-ve) " " " ".

* Energy stored:

$$W = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 \pm M I_1 I_2$$

(−) if one current leaves & the other enters.

(+) if both currents leaving or entering the dots.

* Coupling Coefficient: $\boxed{K = \frac{M}{\sqrt{L_1 L_2}}}$

* For Linear transformer with current entering in primary & leaving in secondary:

$\boxed{Z_{11} = R_1 + j\omega L_1}$ + any other impedances in primary.

$\boxed{Z_{22} = R_2 + j\omega L_2 + Z_L}$ + any other impedances in secondary.

$$\boxed{Z_{in} = Z_{11} + \frac{(\omega M)^2}{Z_{22}}}$$

$$\boxed{\text{Reflected Impedance} = \frac{(\omega M)^2}{Z_{22}}}$$

$\Longrightarrow$

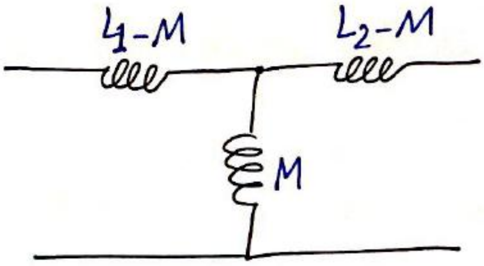
$\Rightarrow$ you can find I_1 & I_2 easly from:

$\boxed{I_1 = V_s / Z_{in}}$ for I_2: $\Rightarrow$ $\boxed{I_2 = \frac{j\omega M}{Z_{22}} I_1}$

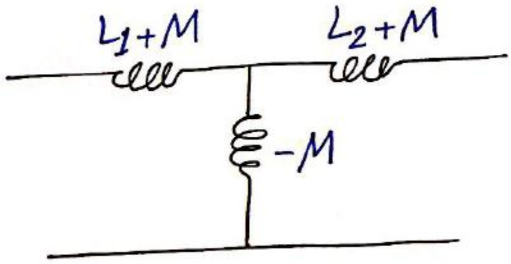
if both currents entering the dot here you have to multiply I_2 by (-1) if you use this way.

* T-equivalent cKt:

(Both currents entering or leaving)

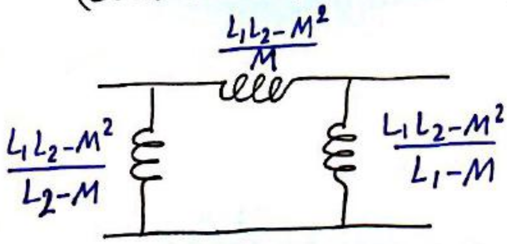


(one current enter and the other leave.)

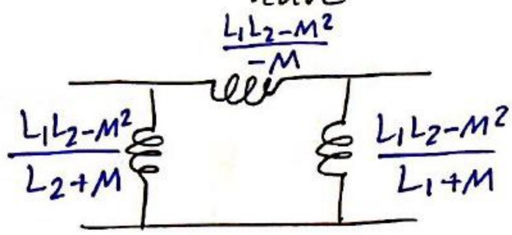


* π-equivalent cKt:

(Both currents enter or Leave)



(one current enter & other leave)



*We use T & π just if there is a connection between the primary & the secondary

※ Ideal Transformer:

$$\boxed{a \text{ or } n = \frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{I_2}{I_1}}$$

→ for currents: if the currents entering or Leaving the dots together (-ve) n otherwise (+ve) n

→ always n (+ve) for $\frac{N_2}{N_1}$

→ for voltages: if V_1, V_2 both (+ve) or both (-ve) $\Rightarrow$ (+ve) n otherwise (-ve) n.

*Complex power:

$$S_{primary} = S_{secondary} = V_1 I_1^* = V_2 I_2^*$$

*Reflection:

⇒ To reflect with respect to primary:
- Z/n^2
- V/n
- $I*n$

⇒ To reflect with respect to secondary:
- $Z*n^2$
- $V*n$
- I/n

Be aware for the sign.

*we just do the reflection if there is NO connection between primary & secondary. Otherwise, use mesh & nodal.

※ CHAPTER (14):

*Common Relations for series & Parallel Resonance:

$$\beta = \frac{\omega_0}{Q_0}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

$$\omega_{1,2} = \omega_0\left[\sqrt{1+\left(\frac{1}{2Q_0}\right)^2} \mp \frac{1}{2Q_0}\right]$$

*Approximate:

$$\omega_{1,2} = \omega_0 \mp \frac{1}{2}\beta$$

$$\omega_0 = \frac{1}{2}(\omega_1 + \omega_2)$$

$$N = \frac{\omega - \omega_0}{\frac{1}{2}\beta}$$

*Series Resonance:

$$Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$$

$$|V_L(\omega_0)| = |V_C(\omega_0)| = VQ_0$$

$$Z = R + j\omega L + \frac{1}{j\omega C}$$

$$Z_{app} = R(1+jN)$$

Series		Parallel
(max)	Y	(min)
(max) I		V (max)
(min)	Z	(max)

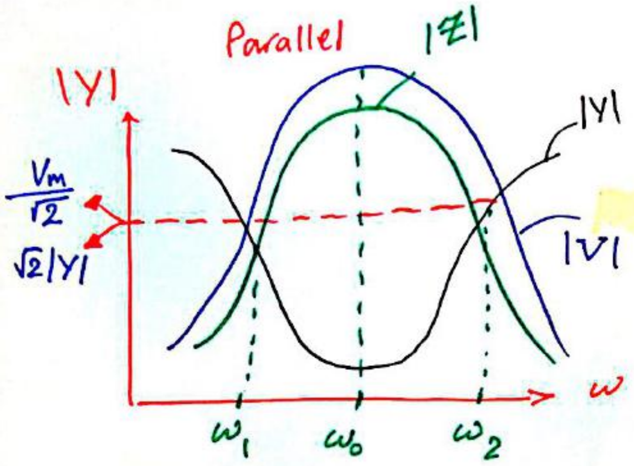
*Parallel Resonance:

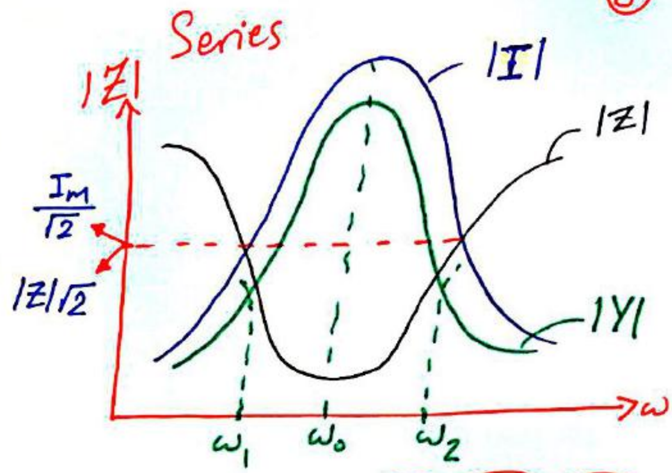
$$Q_0 = \omega_0 RC = \frac{R}{\omega_0 L}$$

$$|I_L(\omega_0)| = |I_C(\omega_0)| = I Q_0$$

$$Y = \frac{1}{R} + j\omega C + \frac{1}{j\omega L}$$

$$Y_{app} = \frac{1}{R}(1+jN)$$





**if you find $P(\omega_0)$ then $P(\omega_1) = P(\omega_2) = \frac{1}{2} P(\omega_0)$

*Other Resonance Forms:

<u>KEY</u>: To find ω_0 just choose what is easy to deal with Z or Y then find the equivelant & put the imaginary part $= 0$

* Transfer function : $\frac{\text{output}}{\text{input}}$.

* Filters:

* Low pass filter: RC cKt ⇒ output on C. → $\omega_c = \frac{1}{RC}$
RL cKt ⇒ output on R. → $\omega_c = \frac{R}{L}$
other cKt ⇒ make $|H_v(\omega_c)| = \frac{1}{\sqrt{2}}$
Then find ω_c.

* High pass filter: RL cKt ⇒ output on L. → $\omega_c = \frac{R}{L}$
RC cKt ⇒ output on R. → $\omega_c = \frac{1}{RC}$
other cKt ⇒ put $|H_v(\omega_c)| = \frac{1}{\sqrt{2}}$
Then find ω_c

* Band pass filter: RLC cKt ⇒ output on R. ⇒ ω_0 & ω_1 & ω_2 as in resonance.

* Band stop filter: RLC cKt ⇒ output on (L&C) ⇒ ω_0 & ω_1 & ω_2 as in resonance.

* Bode Plot:

$\Rightarrow$ standard form:

$$H(\omega) = \frac{K\,(j\omega)^{\pm 1}\left[1 + \frac{j\omega}{Z_1}\right]}{\left[1 + \frac{j\omega}{P_1}\right]}$$

could be zero or pole.

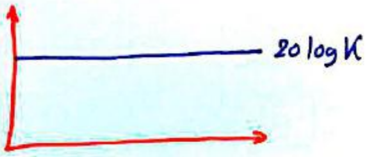
Zero @ Z_1

Pole @ P_1

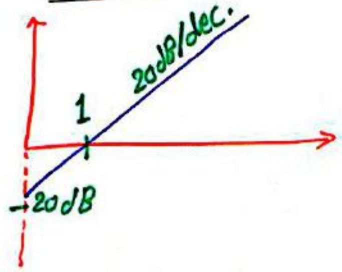
constant.

$\Rightarrow$ Drawing $H_v(\omega)$:

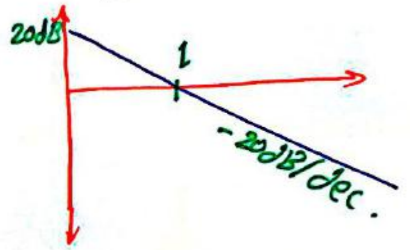
$\underline{20 \log K}$



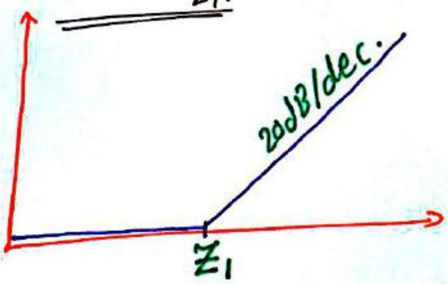
$\underline{20 \log |j\omega|}$



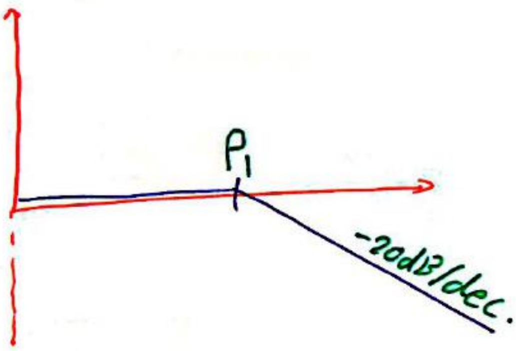
$\underline{-20 \log |j\omega|}$



$\underline{20 \log \left|1 + \frac{j\omega}{Z_1}\right|}$

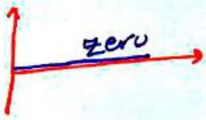


$\underline{-20 \log \left|1 + \frac{j\omega}{P_1}\right|}$

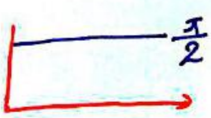


$\Rightarrow$ Drawing $\angle H_v(\omega)$:

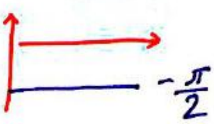
constant



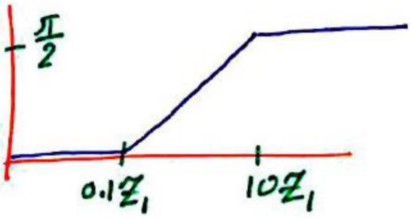
$20 \log |j\omega|$



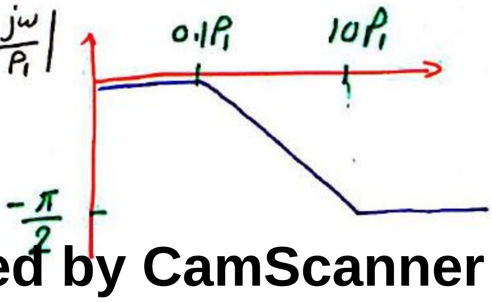
$-20 \log |j\omega|$



$20 \log \left|1 + \frac{j\omega}{Z_1}\right|$



$-20 \log \left|1 + \frac{j\omega}{P_1}\right|$



※ CHAPTER (19):

* Z-parameters:

* $\boxed{I_2 = 0}$:

$$Z_{11} = \frac{V_1}{I_1}$$

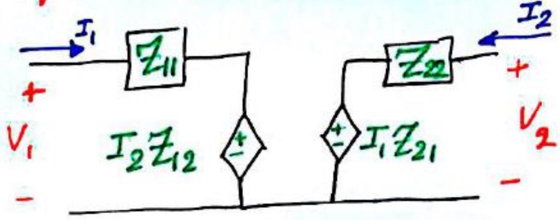
$$Z_{21} = \frac{V_2}{I_1}$$

* $\boxed{I_1 = 0}$:

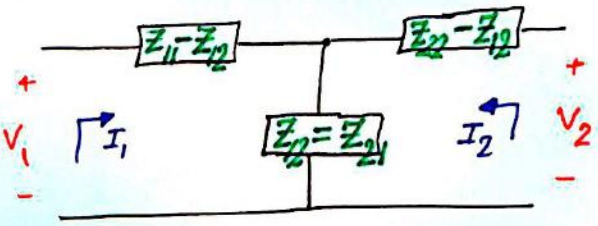
$$Z_{22} = \frac{V_2}{I_2}$$

$$Z_{12} = \frac{V_1}{I_2}$$

* equivalent cKt for 2-port network:



* Special case: (reciprocal)



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* Y-parameters:

* $\boxed{V_2 = 0}$:

$$Y_{11} = \frac{I_1}{V_1}$$

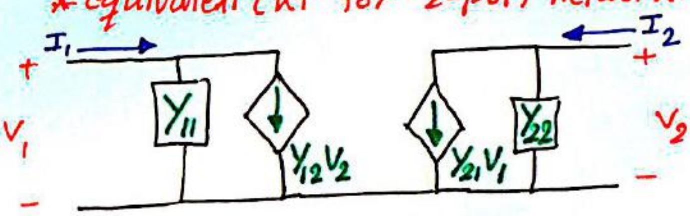
$$Y_{21} = \frac{I_2}{V_1}$$

* $\boxed{V_1 = 0}$:

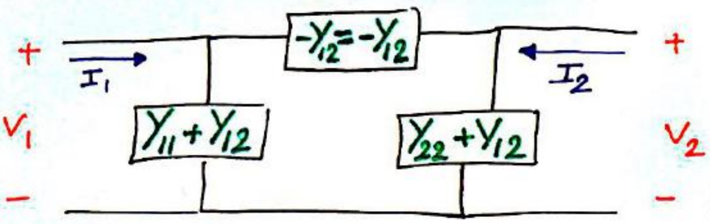
$$Y_{12} = \frac{I_1}{V_2}$$

$$Y_{22} = \frac{I_2}{V_2}$$

* equivalent cKt for 2-port network:



* special case: (reciprocal)



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* Z_{in} & Z_{out}:

$$Z_{in} = \frac{V_1}{I_1}$$

$$Z_{out} = Z_{22} - \frac{Z_{12} Z_{21}}{Z_{11} + Z_g}$$

GOOD LUCK