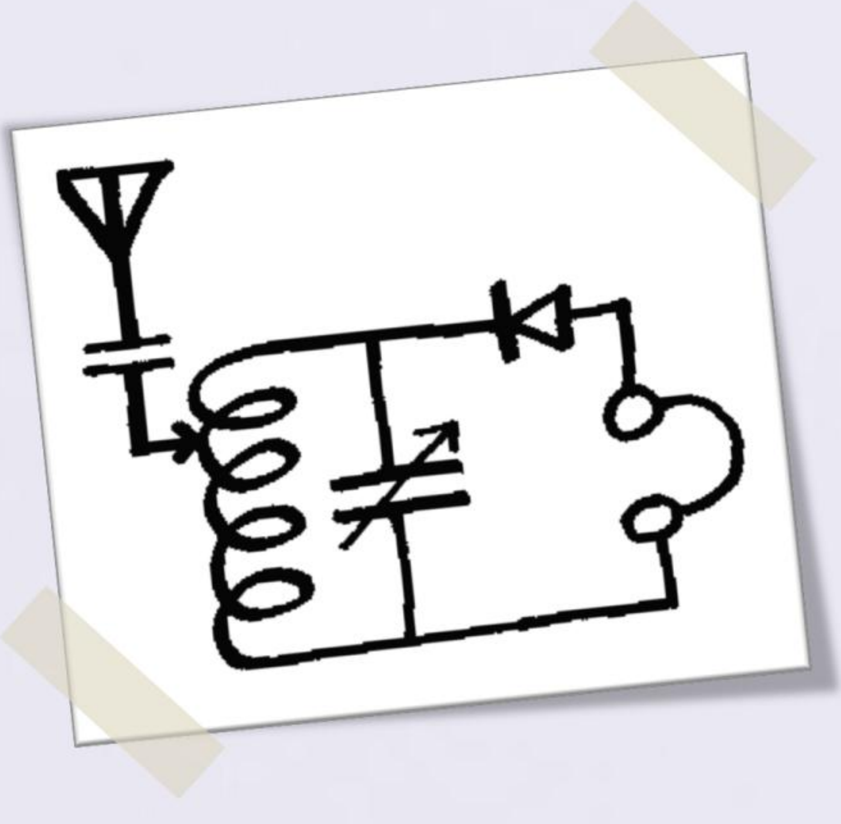
Circuits II Notebook

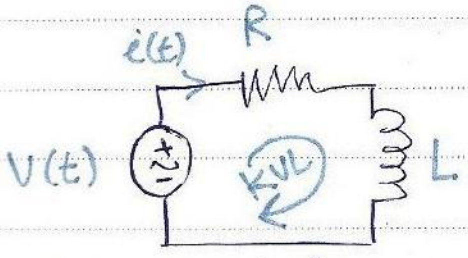
Dr.Ghazi Alsukkar

By: Asma' Hakouz

Spring-2013



Tuesday
5 / 2 / 2013



$v(t) = V_m \cos(\omega t)$

KVL: $-V(t) + i(t)R + l\frac{d\, i(t)}{dt} = 0$ ... ①

→ 1^{st} order differantial equ.

$v(t) = V_m e^{j(\omega t+\theta)}$

$V_r(t) = V_m \cos(\omega t+\theta)$

$i(t) = I_m e^{j(\omega t+\phi)}$

$i_r(t) = I_m \cos(\omega t+\phi)$

- linear CKT means that both input & output ~~current~~ variables have the same form!
- $V_r(t) = Re\{V(t)\} \triangleq$ Real part of $v(t) \equiv V_m \cos(\omega t+\theta)$

- Plug in 1:

$$R\, i(t) + l\frac{di}{dt} = V(t)$$

$$R I_m e^{j(\cancel{\omega t}+\phi)} + l I_m j\omega e^{j(\cancel{\omega t}+\phi)} = V_m e^{j(\cancel{\omega t}+\theta)} \quad \nwarrow e^{j\omega t} \neq 0$$

$$R\left(I_m e^{j\phi}\right) + j\omega L\left(I_m e^{j\phi}\right) = V_m e^{j\theta}$$

$$R I_m \angle\phi + j\omega L I_m \angle\phi = V_m \angle\theta$$

$$I_m \angle\phi \left(\underbrace{R + j\omega L}_{\text{Rectangular}}\right) = V_m \angle\theta$$

$$I_m \angle\phi \left(\sqrt{R^2+\omega^2L^2}\ \angle \tan^{-1}\left(\frac{\omega L}{R}\right)\right) = V_m \angle\theta$$

$$\therefore\ I_m = \frac{V_m}{\sqrt{R^2+\omega^2L^2}} \ldots ② \quad \& \quad \phi = \theta - \tan^{-1}\left(\frac{\omega L}{R}\right) \ldots ③$$

$\vec{I} = I_m \angle \phi$

-from ② & ③

$$\vec{I} = \frac{V_m}{\sqrt{R^2+\omega^2L^2}} \angle \theta - \tan^{-1}\left(\frac{\omega L}{R}\right) \rightleftharpoons i(t) = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\left(\cos(\omega t+\theta-\tan^{-1}(\frac{\omega L}{R})\right.$$

time domain

- Phasor:-

$$i(t) = I_m \cos(\omega t+\phi) \rightleftharpoons \vec{I} = I_m \angle \phi \equiv I_m e^{j\phi}$$

$$\equiv I_m \cos\phi + I_m \sin\phi$$

ex: a) $v_1(t) = 12\cos(200t-35^\circ)$

$v_2(t) = -12\cos(200t-35^\circ)$

find $\vec{V}_1$ & $\vec{V}_2$ → the (-) sign means a phase shift by 180°

$\vec{V}_1 = 12\angle -35^\circ$

$\vec{V}_2 = -12\angle -35^\circ \equiv 12\underset{(-35+180)}{\angle 145^\circ}$

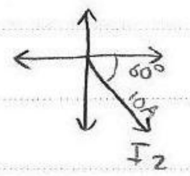
b) $i_2(t) = 10\sin(12t+30^\circ)$, find $\vec{I}_2$?

first we have to transform sine to cosine:

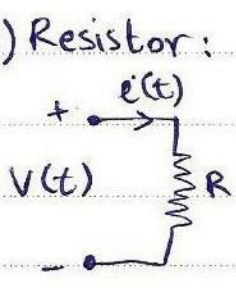
$i_2(t) = 10\cos(12t+30^\circ-90^\circ) = 10\cos(12t-60^\circ)$

$\therefore \vec{I}_2 = 10\angle -60^\circ$

* negative freq. means rotating clockwise!



1) Resistor:



$V(t) = i(t).R$

$\rightarrow V(t) = V_m\cos(\omega t+\theta)$

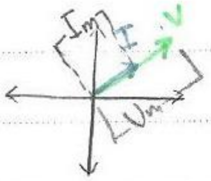
$\rightarrow i(t) = I_m\cos(\omega t+\phi)$

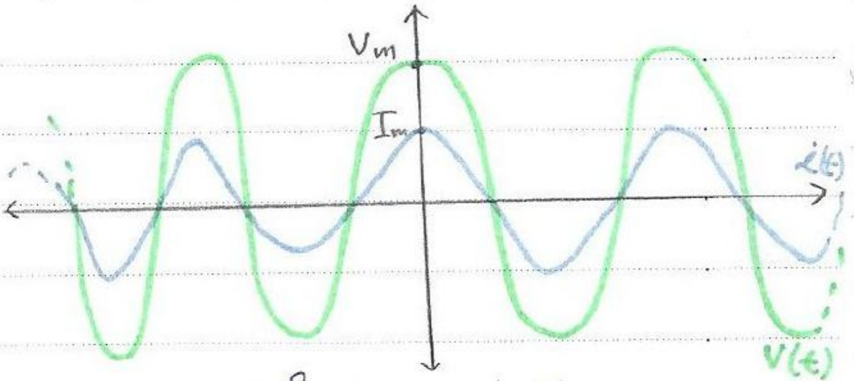
$\boxed{\vec{V} = \vec{I}R}$, $V_m\angle\theta = I_m R\angle\phi$ ...①

from ① :

$\phi = \theta$ &

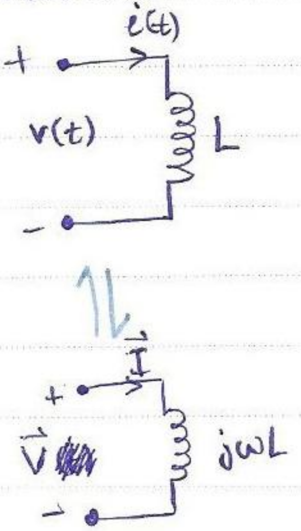
$V_m = I_m . R$





i & v are inphase

2) Inductor:



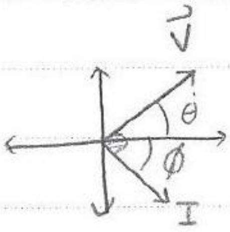
$v(t) = L\frac{di}{dt} \rightleftharpoons V_m e^{j(\omega t+\theta)} = l I_m j\omega e^{j(\omega t+\phi)}$

time domain

$V_m \angle\theta = \boxed{j}\omega L I_m \angle\phi$

phase shift = 90°

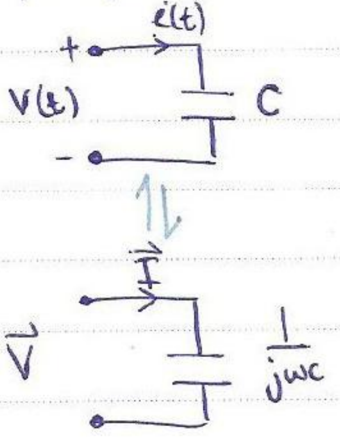
$$\boxed{\vec{V} = j\omega L \vec{I}}$$



$\therefore \phi = \theta - 90°$

$\boxed{\vec{V} \text{ leads } \vec{I} \text{ by } 90°}$

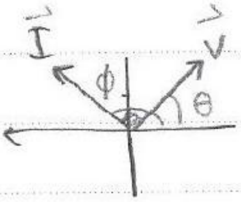
3) Capacitor:



$i(t) = C\frac{d\,v(t)}{dt} \rightleftharpoons I_m\angle\phi = j\omega C V_m \angle\theta$

$\therefore \phi = \theta + 90°$

& $I_m = \omega C V_m$



$\boxed{\vec{I} \text{ leads } \vec{V} \text{ by } 90°}$

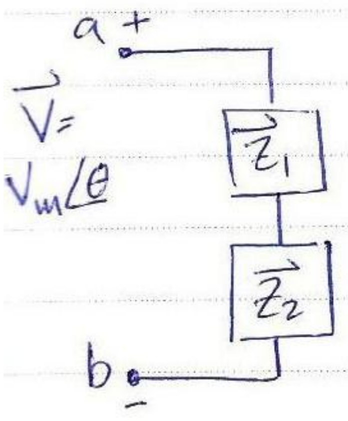
* Generalized Ohm's law:

$$\boxed{\vec{V} = \vec{I}\,\vec{Z}} \rightarrow \text{impedance } [\Omega]$$

- $\vec{Y} = \frac{1}{\vec{Z}}$ → admittance $\left[\frac{1}{\Omega} \equiv S \equiv \mho\right]$ ↳ mho!!

7/2/2013

$\vec{Z} =$ R $\quad j\omega L \quad \frac{1}{j\omega C}$



a) $\vec{V}_1 = \frac{\vec{V} \times \vec{Z}_1}{\vec{Z}_1 + \vec{Z}_2}$ ... voltage division

b) $\vec{Z}_{eq} = \vec{Z}_1 + \vec{Z}_2$ in series

$\vec{Z}_{eq} = \frac{\vec{Z}_1 \times \vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2}$ in parallel

in general: $\vec{Z}_{eq} = \frac{1}{\sum_{i=1}^{n} \frac{1}{\vec{Z}_i}}$ } in parallel

& $\vec{Y}_{eq} = \vec{Y}_1 + \vec{Y}_2 + \dots + \vec{Y}_n$

↳ better to use it in nodal analysis!

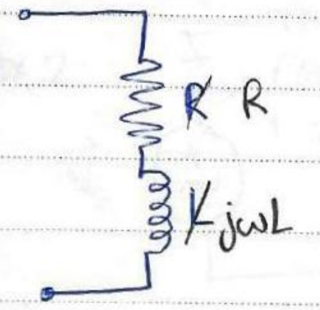
$$\boxed{\vec{Z} = R + jX \equiv \sqrt{R^2+X^2}\,\angle \tan^{-1}\left(\frac{X}{R}\right)}$$

R: Real part; the resistance of the impedance [Ω].

X: Imaginary part; the reactance → +ve inductive

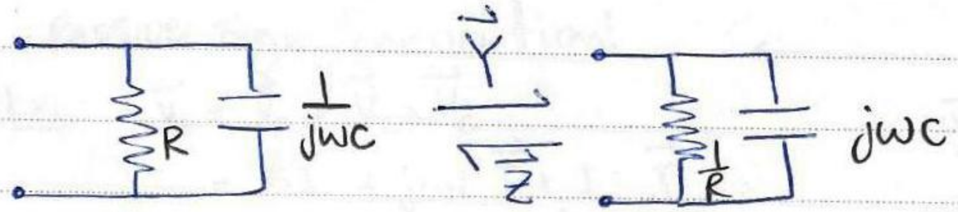
↳ -ve capacitive $\frac{1}{j\omega C} = -\frac{j}{\omega C}$

1)



$\vec{Z}_{eq} = R + j\omega L$

2)



$\vec{Y}_{eq} = \frac{1}{R} + j\omega C$

$\rightarrow \vec{Z}_{eq} = \frac{1}{\vec{Y}_{eq}} = \frac{1}{1/R + j\omega C} \frac{*(1/R - j\omega C)}{*(1/R - j\omega C)}$

$= \frac{1/R - j\omega C}{(1/R)^2 + (\omega C)^2}$

$= \frac{1/R}{(1/R)^2 + (\omega C)^2} - j \frac{\omega C}{(1/R)^2 + (\omega C)^2}$

$\rightsquigarrow \boxed{\vec{Y} = G + jB}$

G: conduct~~ivity~~ance

B: Suseptance

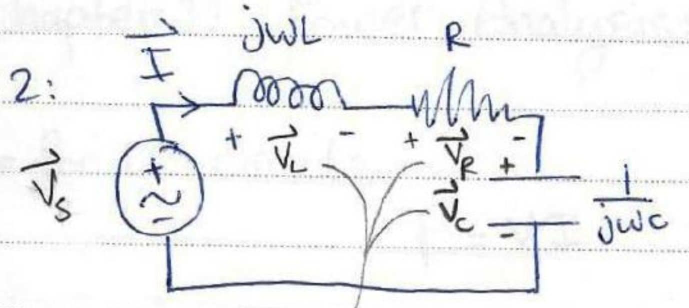
ex:

KVL: $-\vec{V} + R\vec{I} + j\omega L \vec{I} = 0$

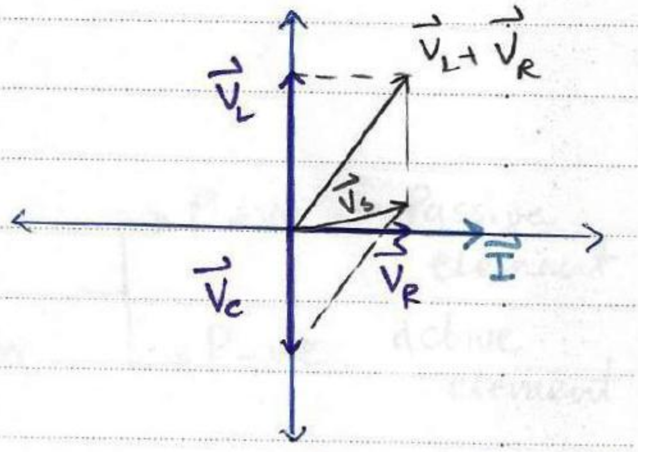
$(R + j\omega L)\vec{I} = \vec{V}$

$\vec{I} = \frac{V_m \angle\theta}{R + j\omega L}$

ex 2:



Passive sign convention!

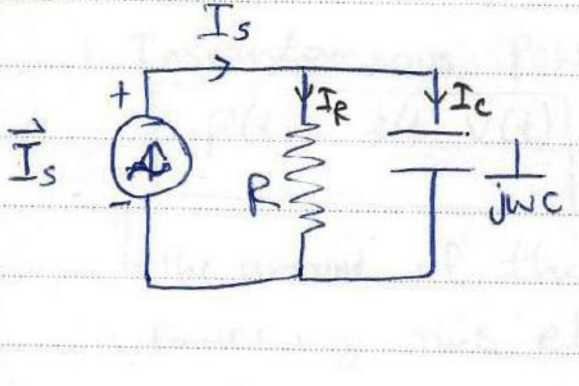


KVL: $\vec{V}_s = \vec{V}_R + \vec{V}_L + \vec{V}_C$

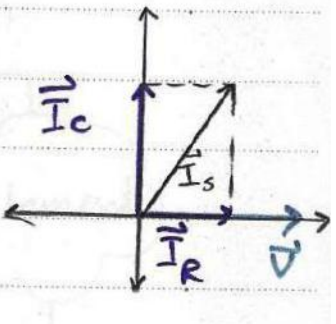
$= R\vec{I} + j\omega L\vec{I} + \frac{1}{j\omega C}\vec{I}$

** If $\vec{V}_s$ is inphase with $\vec{I}$ it's called a (resonant CKT) ; $\vec{V}_C$ & $\vec{V}_L$ cancels eachother.

ex3:



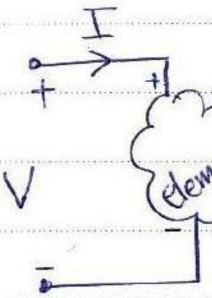
$\rightsquigarrow$ $\vec{I}_s = \vec{I}_R + \vec{I}_C$



Chapter 11: Power Analysis:-

- for DC circuits :

$$P = VI$$



; Passive-sign convention

→ P +ve Passive element

→ P −ve active element

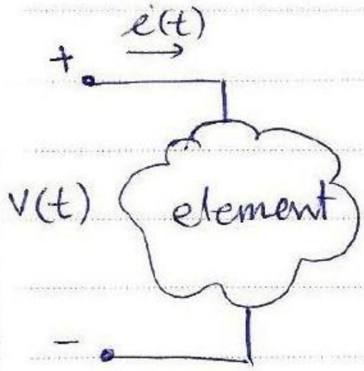
⤳ the element absorbs (P) amount of power ; then (P) is the amount of power <u>delivered to</u> this element.

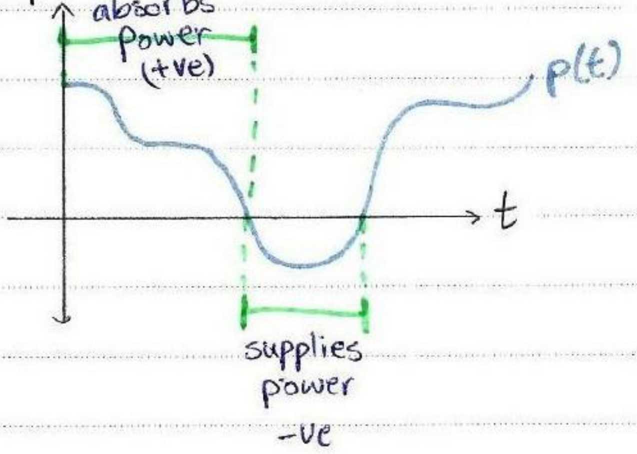
10/2/2013

1. Instantaneous Power:

$$\boxed{p(t) = i(t)\, v(t)}$$

↳ the amount of the absorbed power by this element.





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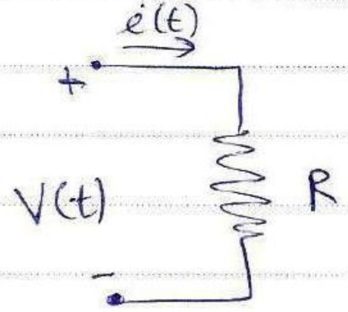
* P in Resistors:

$v(t) = R\, i(t) \ldots *$

$P = i(t)\, v(t)$

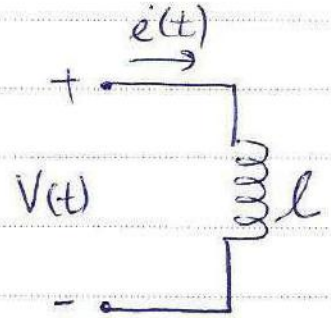
$$\boxed{\begin{aligned} P &= R\, i^2(t) \\ P &\equiv v^2(t)/R \end{aligned}}$$



* in Inductors:

$v(t) = \ell \dfrac{d\, i(t)}{dt} \ldots *$

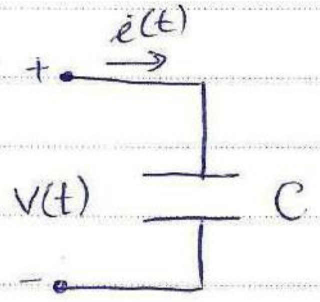
$$\boxed{\begin{aligned} P &= \ell\, i(t) \frac{d\, i(t)}{dt} \\ P &\equiv \frac{1}{L} v(t) \int_{-\infty}^{t} v(T)\, dT \end{aligned}}$$



* in Capacitors:

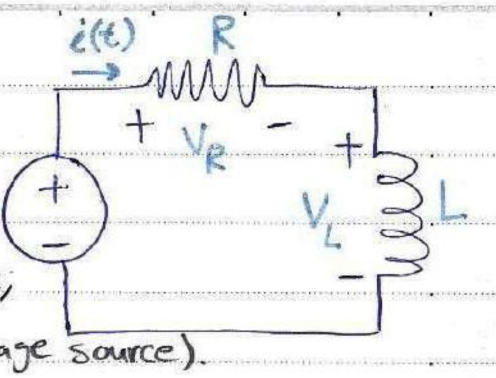
$i(t) = C \dfrac{d\, V(t)}{dt} \ldots *$

$$\boxed{\begin{aligned} P &= C\, v(t) \frac{d\, v(t)}{dt} \\ P &\equiv (1/C)\, i(t) \int_{-\infty}^{t} i(T)\, dT \end{aligned}}$$



ex:- in the following circuit:

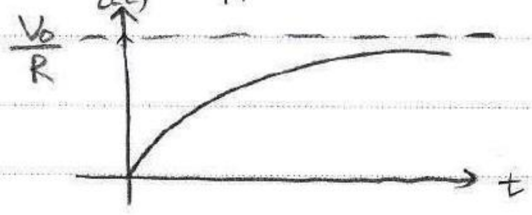
find $P_s(t)$ & prove that $P_s(t) = P_R(t) + P_L(t)$



Ac source;

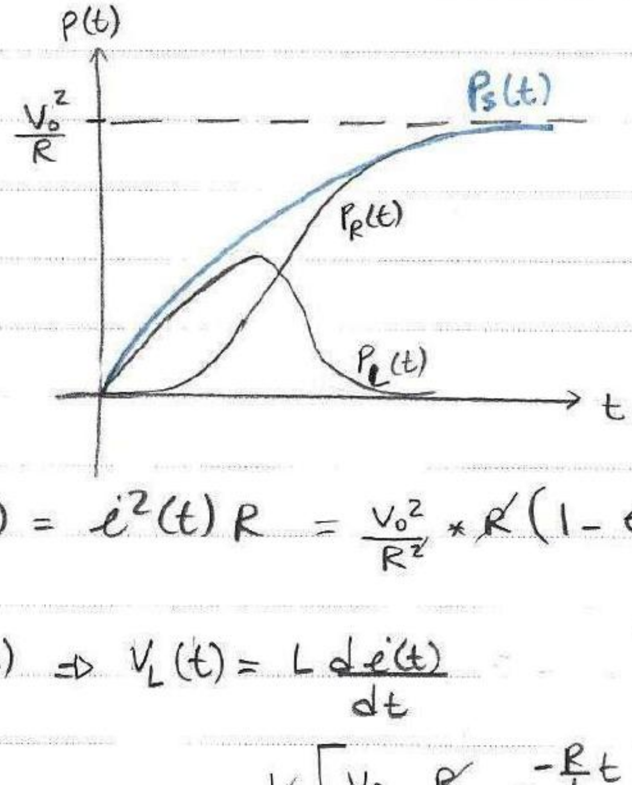
(step voltage source).

sol: $i(t) = \frac{V_0}{R}\left(1 - e^{-\frac{R}{L}t}\right) u(t)$



$$P_s(t) = v(t) \cdot i(t) = \left(V_0 u(t)\right) \cdot \left(\frac{V_0}{R}\left(1 - e^{-\frac{R}{L}t}\right)u(t)\right)$$

$$P_s(t) = \frac{V_0^2}{R}\left(1 - e^{-(R/L)t}\right)\boxed{u(t)} \longrightarrow \boxed{\begin{array}{l} u^2(t) \equiv u(t) \\ ;\ 0^2 = 0 \\ \ \ 1^2 = 1 \end{array}}$$



$\rightarrow P_R(t) = i^2(t) R = \frac{V_0^2}{R^{\cancel{2}}} * \cancel{R}\left(1 - e^{-\frac{R}{L}t}\right)^2 u(t) \ldots$ ①

** $\frac{du(t)}{dt} = 0$, $t > 0$

$\rightarrow P_L(t) \Rightarrow V_L(t) = L \frac{di(t)}{dt}$

$$= \cancel{L}\left[\frac{V_0}{\cancel{R}} \cdot \frac{\cancel{R}}{\cancel{L}} e^{-\frac{R}{L}t} \cdot u(t) + \overset{0}{\cancel{\frac{V_0}{R}\left(1 - e^{-(R/L)t}\right)\frac{du(t)}{dt}}}\right]$$

$$V_L(t) = V_0 e^{-\frac{R}{L}t} \cdot u(t)$$

cont→ $P_L(t) = V_L(t)\, i(t)$

$$P_L(t) = \frac{V_0^2}{R}\left(1-e^{-Rt/L}\right)e^{-Rt/L}\, u(t) \quad \text{--} ②$$

$$① + ② \Rightarrow P_R + P_L = \frac{V_0^2}{R}\left(1-e^{-Rt/L}\right)^2 u(t) + \frac{V_0^2}{R}\left(1-e^{-Rt/L}\right)e^{-Rt/L}u(t)$$

$$= \frac{V_0^2}{R}\left(1-e^{-Rt/L}\right)u(t)\left(1-\cancel{e^{-Rt/L}}+\cancel{e^{-Rt/L}}\right)$$

$$P_R(t)+P_L(t) = \frac{V_0^2}{R}\left(1-e^{-Rt/L}\right)u(t) \equiv P_s(t) \quad ※$$

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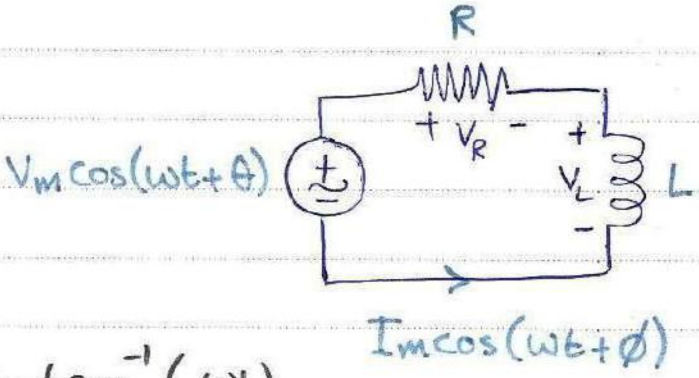
ex: $\vec{V} = V_m \angle\theta$

$\vec{I} = I_m \angle\phi$

find $p(t)$?!



sol: $I_m = \frac{V_m}{\sqrt{R^2+w^2L^2}}$, $\phi = \theta - \tan^{-1}\left(\frac{wL}{R}\right)$

$\rightarrow p(t) = v(t)\, i(t)$

$= V_m \cos(\omega t+\theta) \,.\, I_m \cos(\omega t+\phi)$

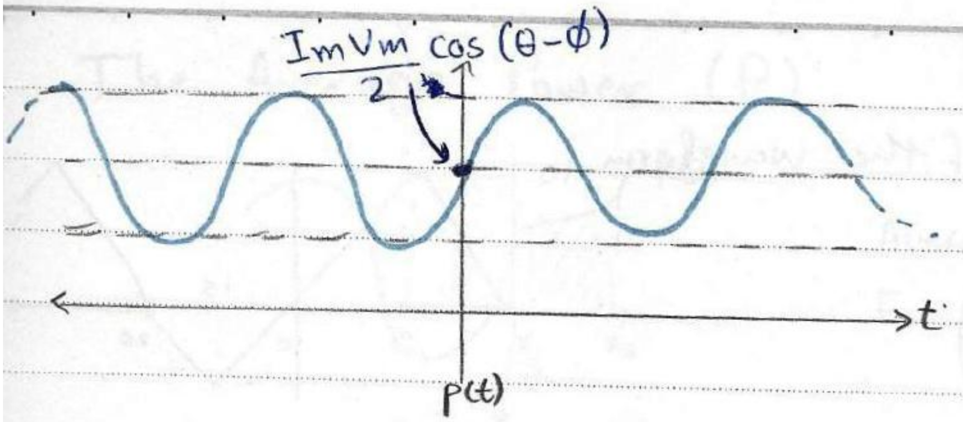
** $\boxed{\cos\alpha \,.\, \cos\beta = \frac{1}{2}\cos(\alpha-\beta) + \frac{1}{2}\cos(\alpha+\beta)}$

$$\therefore p(t) = \frac{V_m I_m}{2}\left(\cos(\theta-\phi) + \cos(2\omega t+\theta+\phi)\right)$$

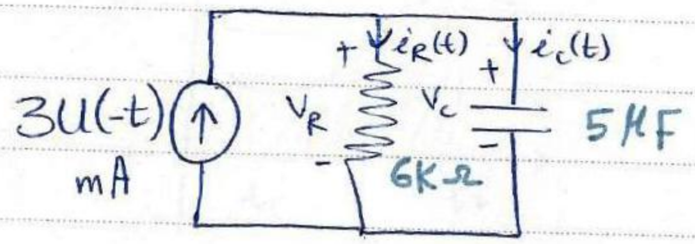
ضعف التردد

$$p(t) = \underbrace{\frac{1}{2}V_m I_m \cos(\theta-\phi)}_{\text{constant!}} + \underbrace{\frac{1}{2}V_m I_m \cos(\underline{2\omega t}+\theta+\phi)}_{\text{function of time}}$$

→ average Power
(DC Power)



Problem 7:



find $P_R(t)$?

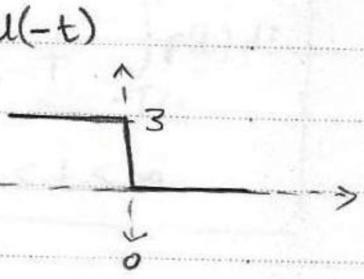
$P_R(0^+)$?

$P_R(30\,m\,sec)$?

$V_c(0^-) = V_c(0^+) = 3m \cdot 6K = 18V$

$i_R(0^-) = 3mA$, $\quad i_R(0^+) = \frac{18\,V}{6k} = 3m\,A$

↑ Just a coincedance



$\rightsquigarrow \; P_R(t) = i_R^2(t) \cdot R \;\ldots ①$

$i_R(t) = A e^{-t/\tau} \ldots ②$, $\quad \tau = RC = 5\mu * 6k = (0.03)sec$

but $i_R(0^+) = 3mA \rightarrow$ plug in ② $\rightarrow A = 3mA$

$\therefore i_R(t) = 3 \cdot e^{-t/\tau}\,mA$

plug in ① ---

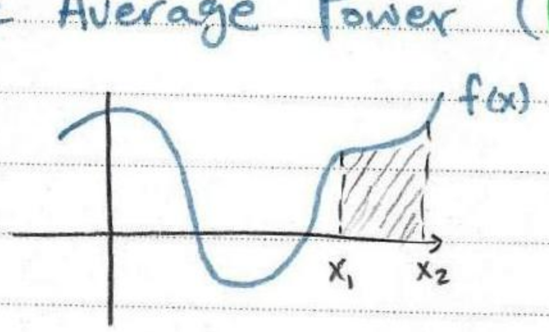
$* \; P_R(t) = (3e^{-t/\tau}mA)^2 \cdot 6K$

$* \; P(0^+) = (3*10^{-3})^2 * 6*10^3 = 54 m Watt$

$* \; P_R(30\,m\,sec) = (3*10^{-3})^2 * (e^{-30m/0.03})^2 * 6*10^3$

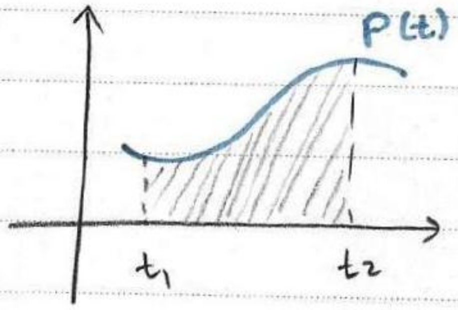
$= 7.308$ m Watt

The Average Power (P)



Average value of f(x):

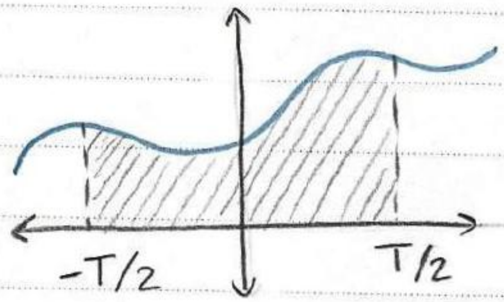
$$F = \left(\int_{x_1}^{x_2} f(x)\,dx\right) \Big/ (x_2 - x_1)$$



$$P = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} p(t)\,dt$$

for $-\infty < t < \infty$

في حال لم يتم
تحديد فترة !

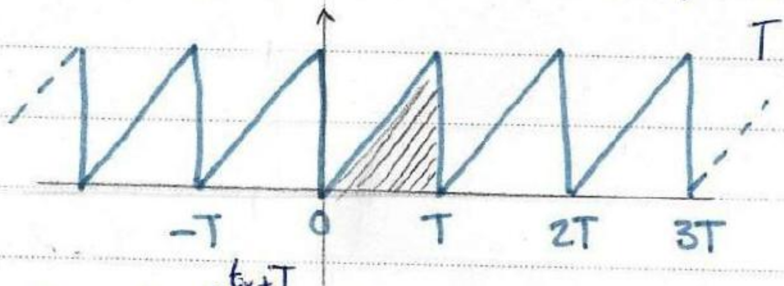


$$\boxed{P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} p(t)\,dt \qquad -\infty < t < \infty}$$

** For Periodic ~~s~~waveforms:-

$p(t) = p(t+T) = p(t+nT)$; $n \in \mathbb{Z}$ (integers)

T is the period of the wave --



$$\left.\begin{aligned} P &= \frac{1}{T} \int_{t_x}^{t_x+T} p(t)\,dt \\ &= \frac{1}{nT} \int_{t_x}^{t_x+nT} p(t)\,dt \\ &= \frac{1}{T} \int_{0}^{T} p(t)\,dt \end{aligned}\right\}$$

$$\boxed{P = \frac{1}{T} \int_{\textcircled{T}} p(t)\,dt}$$

integrate over T

$-T/2 \rightarrow T/2$
or $0 \rightarrow T$

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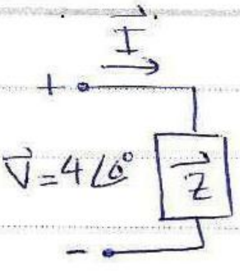
ex: $v(t) = 4\cos\left(\frac{\pi t}{6}\right)$

$\vec{Z} = 2\angle 60^\circ\ \Omega$

find $p(t)$?!



sol: $\vec{I} = \frac{\overset{2}{\cancel{4}}\angle 0^\circ}{\cancel{2}\angle 60^\circ} = 2\angle -60^\circ$ A ... $P = \frac{1}{2} I_m V_m \cos(\theta - \phi)$

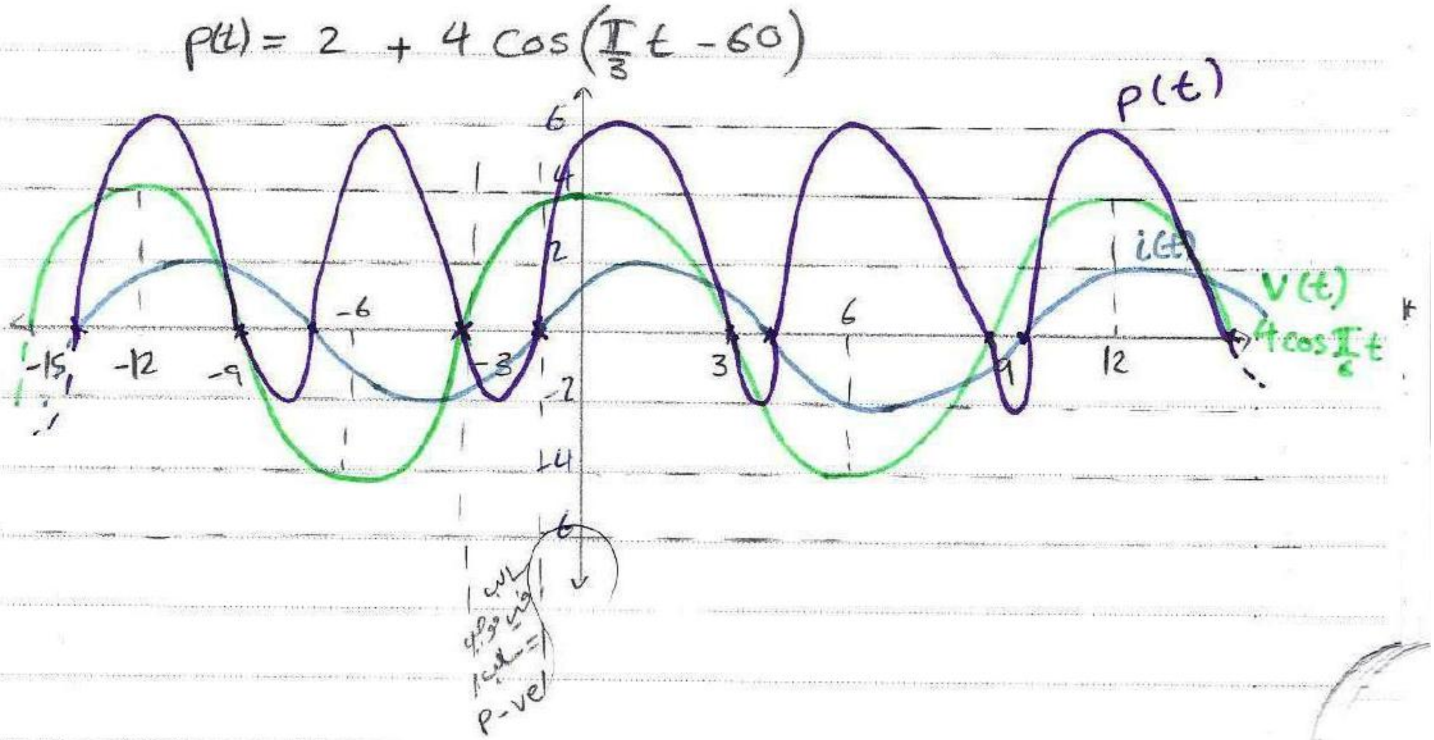
$P = 4 \frac{*2}{2} \cos 60^\circ = 2$ watt ... average power / DC P

$p(t) = v(t)\, i(t)$

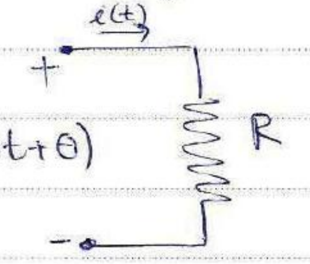
$i(t) \longrightarrow\ i = 2\cos\left(\frac{\pi}{6}t - 60^\circ\right)$

$$p(t) = \underbrace{\underline{2}}_{\text{avg } P} + \frac{1}{\cancel{2}} * \underset{V_m}{4} * \underset{I_m}{\cancel{2}} \cdot \cos\left(\underset{2\omega}{\frac{\pi}{3}}t - \underset{\theta+\phi}{60^\circ}\right)$$

$$p(t) = 2 + 4\cos\left(\frac{\pi}{3}t - 60\right)$$

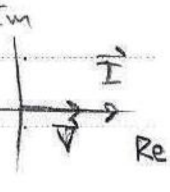


* Average power absorbed by an "Ideal Resistor":



$V(t) = V_m \cos(\omega t + \theta)$

$i(t) = \frac{V(t)}{R} = \frac{V_m}{R} \cos(\omega t + \theta)$; $\theta = \phi$ $\therefore \theta - \phi = 0^\circ$

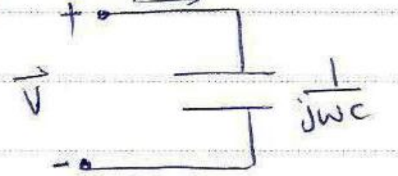


$$P = \frac{1}{2} V_m \overset{I_m}{\boxed{\frac{V_m}{R}}} \cos(\overset{\theta-\phi}{\boxed{0^\circ}})$$

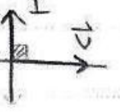
$$P_R = \frac{1}{2}\frac{V_m^2}{R} = \frac{1}{2} I_m^2 R$$

* Average power absorbed by purely reactive impedance:

a)



$\theta - \phi = -90^\circ$

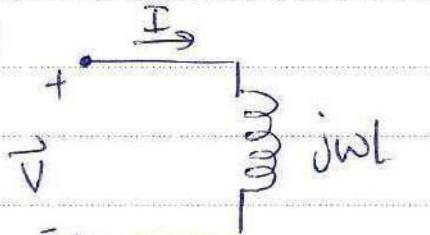


$P = \frac{1}{2} I_m V_m \cos(-90^\circ)$

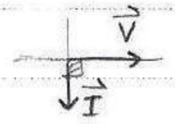
$P = 0$

capacitor doesn't consume average power (DC Power).

b)



$\theta - \phi = 90^\circ$



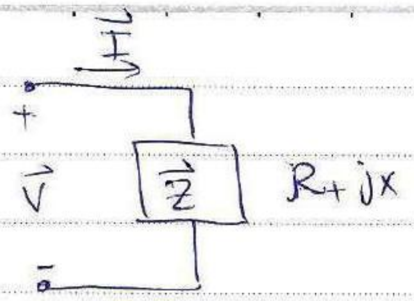
$P = \frac{1}{2} I_m V_m \cos(90^\circ) = 0$

** When $\vec{Z} = jX$ (pure reactance) : $\overset{\text{avg power.}}{P} = 0$

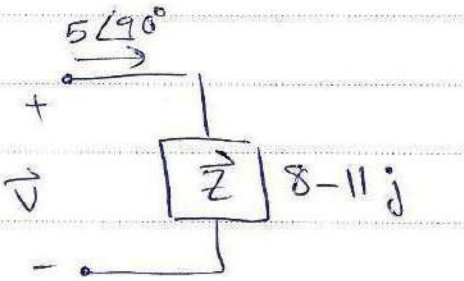
** 〃 $\vec{Z} = R$ (〃 resistance) : $P = \frac{1}{2}\frac{V_m^2}{R} = \frac{1}{2} I_m^2 R$

** 〃 $\vec{Z} = R + jX$ $\xrightarrow[\text{Page}]{\text{next}}$

$P = \frac{1}{2} I_m^2 R$



ex:-



, find p?

sol: $P = \frac{1}{2} * 5^2 * 8 = 100$ Watt

another way:

$$\vec{V} = 5\angle 90 * \left(\sqrt{8^2+11^2}\ \angle \overset{= \tan^{-1}\left(-\frac{11}{8}\right)}{-\tan^{-1}\left(\frac{11}{8}\right)}\right) \qquad (\vec{V} = \vec{I}\,\vec{Z})$$

$$P = \frac{1}{2} I_m V_m \cos(\theta - \phi)$$

$$= \frac{1}{2}\, 5 * \left(\sqrt{8^2+11^2}\right) * 5 \cos\left(\underbrace{-\tan^{-1}\left(\frac{11}{8}\right)+90^\circ}_{\theta} - \underbrace{90}_{\phi}\right)$$

$P = 100$ Watt

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ex:- find:

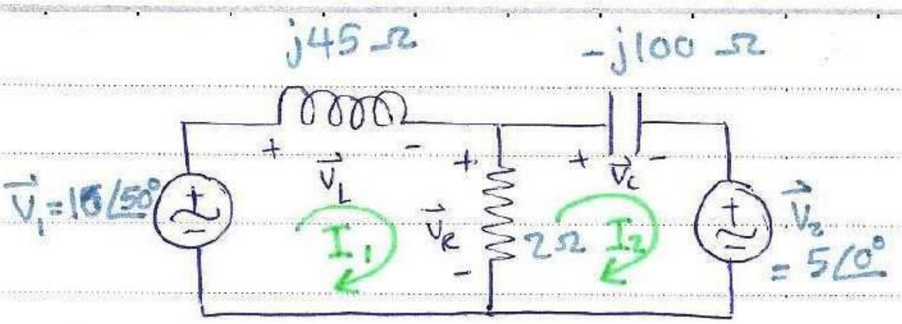
$P_R = ?$

$P_L = ?$

$P_C = ?$

$P_1 = ?$

$P_2 = ?$



Sol: $\boxed{* P_L = P_C = 0}$

Mesh analysis

@ loop 1: $-10\angle 50^\circ + j45\,\vec{I}_1 + (\vec{I}_1 - \vec{I}_2) * 2 = 0$

$$(2 + j45)\vec{I}_1 - 2\vec{I}_2 = 10\angle 50^\circ \quad \ldots ①$$

@ loop 2: $(\vec{I}_2 - \vec{I}_1) * 2 - j100\,\vec{I}_2 + 5\angle 0^\circ = 0$

$$-2\vec{I}_1 + (2 - j100)\vec{I}_2 = -5\angle 0^\circ \quad \ldots ②$$

if we do the math ; cramer, gaussian elemination, ...

$$\vec{I}_1 = 0.1724 - 0.1352\,j = 0.2205\angle -37.8^\circ$$

$$\vec{I}_2 = 0.0018 - 0.0466\,j = 0.0466\angle -87.78^\circ$$

$$P_R = \frac{1}{2}\left(|\vec{I}_1 - \vec{I}_2|\right)^2 * 2 \longrightarrow \vec{I}_1 - \vec{I}_2 = 0.1706 - 0.0886\,j$$

$$= 0.1922\angle -27.4^\circ$$

$$= \frac{1}{2}(0.1922)^2 * \cancel{2} = 0.0369$$

$\boxed{* P_R = (36.9)\,\text{m Watt}}$ (absorbed)

$$* \; P_1 = \frac{1}{2}\,|\overset{I_m}{\vec{I}_1}|\,|\overset{V_m}{\vec{V}_1}|\cos(\theta_1 - \phi_1)$$

$$P_1 = \frac{1}{2}(0.2205)(10)\cos(50 - (-37.8)) = 42.324 \text{ Watt}$$

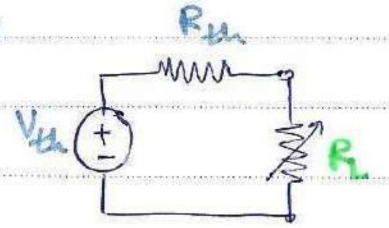
(supplied)

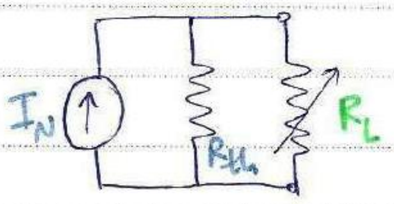
$$* P_2 = \frac{1}{2}(0.0466)(5)\cos(0 + 87.78^\circ) = 4.516 \text{ m Watt}$$

(absorbed)

Maximum Power Transfer Theorem:

DC:



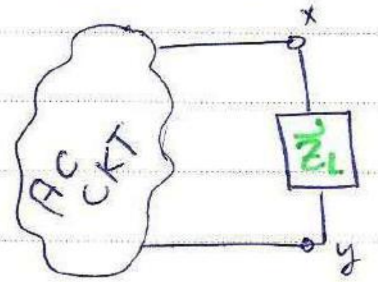


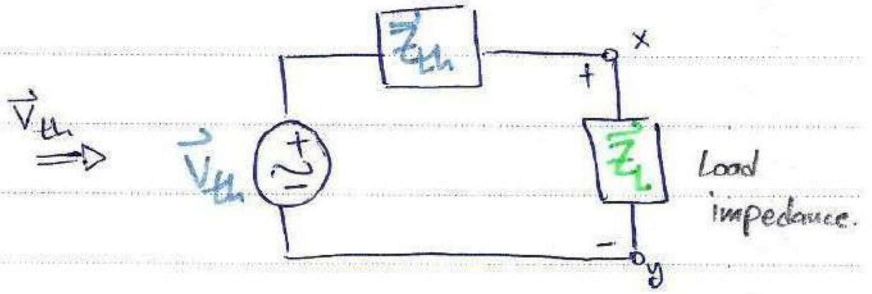
$I_N = \frac{V_{th}}{R_{th}}$

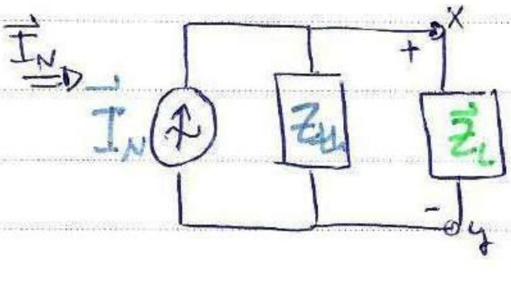
→ V_{th} or I_N delivers maximum power for R_L when

$$\boxed{R_L = R_{th}}$$

Sinusoidal Sources:





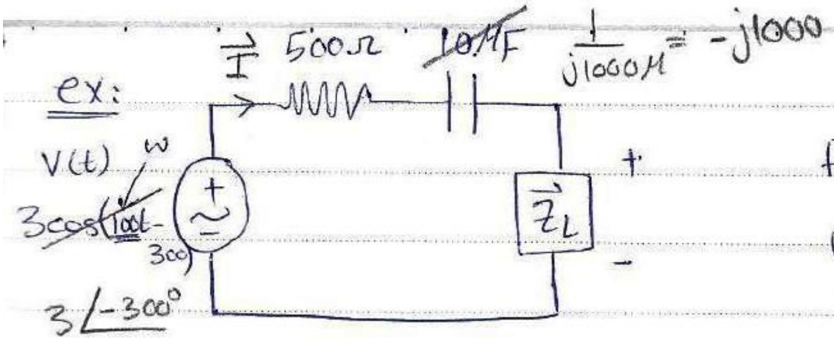


→ $\vec{V}_{th}$ or $\vec{I}_N$ delivers maximum average* power (P_L) to $\vec{Z}_L$ when:

$$\boxed{\vec{Z}_L = \vec{Z}_{th}^{*}}$$ ← the conjugate

↝ proof in the book!!

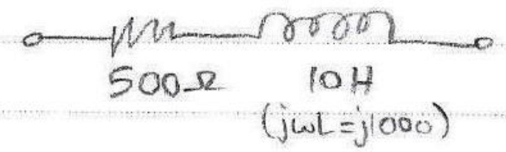
ex:



find $\vec{Z}_L$ that absorbs the max. power?
and find P_L max.?

sol: * $\vec{Z}_{th} = 500 - j1000$

$\vec{Z}_L = \vec{Z}^*_{th} = 500 + j1000 \rightarrow$



$$* \vec{I}_L = \frac{\vec{V}_{th}}{Z^*_{th} + \vec{Z}_{th}} = \frac{\vec{V}_{th}}{2R}$$

$$P_L = \frac{1}{2} |\vec{I}_L|^2 R$$

$$= \frac{1}{2} \left(\frac{|\vec{V}_{th}|}{2R}\right)^2 R = \frac{1}{2} \frac{|V_{th}|^2}{4R^{\cancel{2}}} \cancel{R}$$

$$= \frac{|V_{th}|^2}{8R}$$

$$P_L = \frac{3^2}{8 \times 500} \text{ watt}$$

* Non-periodic wave forms & average power:

let $i(t) = \underbrace{i_1(t)}_{\text{Periodic}} + \underbrace{i_2(t)}_{\text{periodic}}$

ex: $i(t) = \sin \underset{\omega_1 = 1}{t} + \sin \underset{\omega_2 = \pi}{\pi} t$

$T_1 = \frac{2\pi}{1}$ sec $\quad T_2 = \frac{2\pi}{\pi} = 2$ sec

$\frac{T_1}{T_2} = \frac{2\pi}{2} = \pi$ irrational ⇏ ∴ $i(t)$ is non-periodic!

↳: can't be written as $\frac{a}{b}$; a & b are integers.

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* A non-periodic function could be a sum of multi-periodic functions having differant periods, such that no greater common Period can be found.

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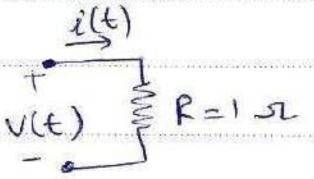
ex: $i(t) = \underline{\overset{i_1(t)}{\sin(t)}} + \underline{\overset{i_2(t)}{\sin(\pi t)}}$

find p?



sol: $P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \underline{\underline{p(t)}}\, dt \quad \ldots ①$

$p(t) = i^2(t) R = i^2(t)$; normalized Power $(R = 1\Omega)$

$i^2(t) = [\sin t + \sin \pi t]^2$

$i^2(t) = \sin^2 t + 2\sin(t)\sin(\pi t) + \sin^2(\pi t) = p(t)$

...plug in①

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \left(\sin^2 t + 2\sin(t)\sin(\pi t) + \sin^2(\pi t) \right) dt$$

$$= \underbrace{\lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \frac{1}{2} - \frac{1}{2}\cos 2t \, dt}_{A_{V_1} = \frac{1}{2} + 0} + \underbrace{\lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} (\cos(t - \pi t) - \cos(t + \pi t)) dt}_{A_{V_2} = 0} + \underbrace{\lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \frac{1}{2} - \frac{1}{2}\cos 2\pi t \, dt}_{A_{V_3} = \frac{1}{2}}$$

تكامل ال cos على دورة كاملة

$$P = \frac{1}{2} + 0 + \frac{1}{2} = 1 \text{ watt}$$

* but can we apply super position here? does ~~does~~ $P = P_1 + P_2$?

→ In general, super position can't be applyed with power because it's not linear, but sinusoidals sources are an exception!

So, in the previous example:

$$P = P_1 + P_2 = \frac{1}{2}(1^2)(1) + \frac{1}{2}(1^2)(1) = 1 \text{ watt } *$$

* In general:

a) if

$$i(t) = I_1\cos(w_1t+\phi_1) + I_2\cos(w_2t+\phi_2) + \cdots + I_N\cos(w_Nt+\phi_N)$$

$$= \sum_{L=1}^{N} I_L \cos(w_Lt+\phi_L) \quad \ldots\text{(sinusoidal)}.$$

& $w_1 \neq w_2 \neq w_3 \neq \cdots \neq w_N$; since $(I_1^2 + I_2^2) \neq (I_1+I_2)^2$

Then

$$\boxed{P = \frac{1}{2}\left(I_1^2 + I_2^2 + \cdots + I_N^2\right) \cdot R}$$

b) if

$$V(t) = \sum_{l=1}^{N} V_l \cos(w_lt+\theta_l) \qquad \& \ w_1 \neq w_2 \neq \cdots \neq w_N$$

Then

$$\boxed{P\cancel{t} = \frac{1}{2}\cdot\frac{1}{R}\left(V_1^2 + V_2^2 + \cdots + V_N^2\right)}$$

ex:- a) $R = 4\Omega$

$i_1(t) = \overset{I_1}{2}\cos(10t) \underline{\underline{\overset{I_2}{-3}}}\cos(20t)$ A , find P_1?

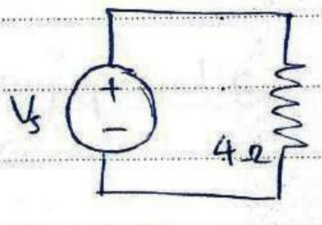
$$P_1 = \frac{1}{2} * 4\left((2)^2 + (-3)^2\right) = 26 \text{ watt}$$

b) $i_2(t) = 2\cos(10t) - 3\cos(10t)$, $w_1 = w_2$!

$$i_2(t) = -\cos(10t)$$

$$P = \frac{1}{2} * 4 * (-1)^2 = 2 \text{ watt}$$

<u>ex2</u>: $V_s(t) = 8\sin(200t) - 6\cos(200t-45°) - 5\sin(100t) + 4$



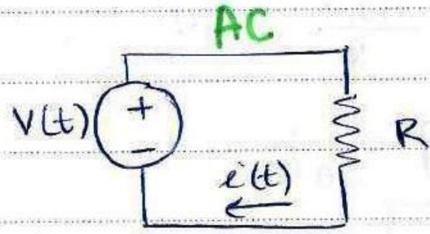
find P?

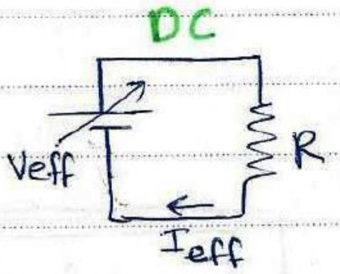
<u>Sol</u>: $V_s(t) = \underbrace{8(\cos(200t-90)) - 6\cos(200t-45°)}_{8\angle -90° - 6\angle -45°} - 5\sin(100t) + 4$

$= 5.667\angle\theta$ doesn't affect P $- 5\sin(100t) + \underline{\underline{4}}$

$$P = \frac{1}{2} * \frac{1}{4}\left((5.667)^2 + (-5)^2\right) + \overbrace{\frac{1}{4}(4^2)}^{\text{DC power}} = 11.139 \text{ watt}$$

Effective value of current & voltage: (r.m.s)





intuition * if we replaced the Ac source with Dc source, which delivers the same amount of power (P) to the same resistance (R) then the voltage of that DC source =

$V_{effective} \equiv V_{rms}$

in <u>AC</u>: $P = \frac{1}{T}\int_0^T i^2(t) R\, dt = \frac{R}{T}\int_0^T i^2(t)dt$ ...①

in <u>DC</u>: $P = R I^2_{eff}$ ...②

but from definition: $P_{AC} = P_{DC}$

$\therefore \frac{\cancel{R}}{T}\int_0^T i^2(t)\,dt = \cancel{R} I^2_{eff}$

$$\boxed{I_{rms} \equiv I_{eff} = \sqrt{\frac{1}{T}\int_T i^2(t)\,dt}}$$ for any periodic source

RMS: Root-Mean-Square

* to represent an RMS value we can say:-

$I = 5\,A_{rms}$ Ampere rms

$V = 6\,V_{rms}$

for V...

$$P_{AC} = \frac{1}{T}\int_T \frac{V^2(t)}{R}\,dt$$

$$P_{DC} = \frac{V^2_{eff}}{R}$$

$$\ldots P_{AC} = P_{DC}$$

then $$\boxed{V_{eff} = \sqrt{\frac{1}{T}\int_T V^2(t)\,dt}}$$ for any periodic function!

~> rms for sinusoidal waveforms:

let $i(t) = I_m \cos(\omega t + \phi)$

find I_{eff}?

$$I_{eff} = \sqrt{\frac{1}{T}\int_T (I_m \cos(\omega t+\phi))^2\,dt}$$

$$= I_m\sqrt{\frac{1}{T}\int_0^T \frac{1}{2} + \underbrace{\cancel{\frac{1}{2}\cos(2\omega t+2\phi)}}_{Av=0}\,dt}$$

$$I_{eff} = I_m\sqrt{\frac{1}{2}} = \frac{I_m}{\sqrt{2}}$$

$$\boxed{\therefore I_{eff} \cong 0.707\,I_m}$$ only for sinusoidals!

$V(t) = V_m\cos(\omega t+\theta)$

$V_{rms} = \frac{|\vec{V}|}{\sqrt{2}} = \frac{V_m}{\sqrt{2}}$

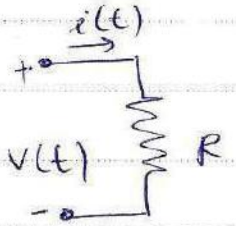
$$\begin{array}{llll} & & \underline{\text{Unit:}} & \\ ** & \vec{V} = (\underline{V_m\angle\theta}) & \to \underline{V} & ;\ \text{peak value} \\ & \quad\ \ \underline{V_{rms}} & \hookrightarrow \underline{V_{rms}} & ;\ \text{Rms value} \end{array}$$

ex: a) $\vec{V}_1 = 5\angle 30\ V \to V_m=5\ ,\ V_{rms}=\frac{5}{\sqrt{2}}$

b) $\vec{V}_2 = 5\angle 30^\circ\ V_{rms} \to V_{rms}=5\ ,\ V_m=5\sqrt{2}$

RMS value & average power!

$P_R = \frac{1}{2} I_m^2 R = \frac{1}{2}\frac{V_m^2}{R} = \frac{1}{2} V_m I_m$ ---①



but $I_m = I_{\underset{rms}{eff}} * \sqrt{2}$

$V_m = V_{eff/rms} * \sqrt{2}$

$$\left.\begin{array}{l} i(t) = I_m\cos(\omega t+\phi) \\ v(t) = V_m\cos(\omega t+\theta) \end{array}\right\} \theta=\phi \ \text{in-phase}$$

plug in ①

$P_R = I_{rms}^2 R = \frac{V_{rms}^2}{R} = I_{rms}V_{rms}$

in general:-

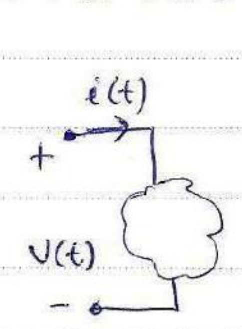
$v(t) = V_m\cos(\omega t+\theta) = \sqrt{2}\, V_{eff}\cos(\omega t+\theta)$

$i(t) = I_m\cos(\omega t+\phi) = \sqrt{2}\, I_{eff}\cos(\omega t+\phi)$

$P = \frac{1}{2} I_m V_m \cos(\theta-\phi)$

$= \frac{1}{\cancel{2}} I_{eff} V_{eff} (\cancel{2}) \cos(\theta-\phi)$

$P = I_{eff} V_{eff} \cos(\theta-\phi)$



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~> rms for multi frequency waveforms:-

$$i(t) = \sum_{L=1}^{N} I_L \cos(w_L t + \phi_L) \quad ; \quad w_1 \neq w_2 \neq \dots \neq w_N$$

I_{eff}?

$$P = \frac{1}{2}\left(I_1^2 + I_2^2 + \dots + I_N^2\right) R$$

$$= \left(\left(\frac{I_1}{\sqrt{2}}\right)^2 + \left(\frac{I_2}{\sqrt{2}}\right)^2 + \dots + \left(\frac{I_N}{\sqrt{2}}\right)^2 R\right.$$

$$= \left(I_{eff_1}^2 + I_{eff_2}^2 + \dots + I_{eff_N}^2\right) R$$

$$\boxed{P = I_{eff_{total}}^2 \cdot R}$$

where $I_{eff\ total} = \sqrt{I_{eff_1}^2 + I_{eff_2}^2 + \dots + I_{eff_N}^2}$

$$v(t) = \sum_{l=1}^{N} V_L \cos(w_L t + \theta_L) \quad ; \quad w_1 \neq w_2 \neq \dots \neq w_N$$

$$V_{eff} = \sqrt{V_{eff_1}^2 + V_{eff_2}^2 + \dots + V_{eff_N}}$$

$$\boxed{P = \frac{V_{eff}^2}{R}}$$

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ex: $i(t) = 6\cos(25t) + 5\cos^2(25t)$. A

find I_{eff}?

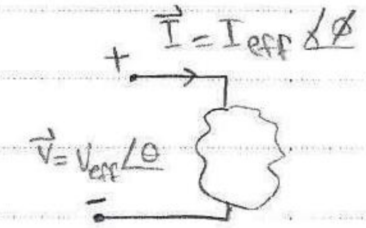
Sol: $i(t) = 6\cos(25t) + 5\left(\frac{1}{2} + \frac{1}{2}\cos(50t)\right)$

$$= \underbrace{6\cos(25t)}_{I_{eff_1} = \frac{6}{\sqrt{2}}} + \underbrace{\frac{5}{2}}_{I_{eff_2} = \frac{5}{2}} + \underbrace{\frac{5}{2}\cos(50t)}_{I_{eff_3} = \frac{5}{2\sqrt{2}}}$$

$$I_{eff} = \sqrt{\left(\frac{6}{\sqrt{2}}\right)^2 + \left(\frac{5}{2}\right)^2 + \left(\frac{5}{2\sqrt{2}}\right)^2} = 5.232\ A_{rms}$$

* Apparent Power & power factor:

- apparent power = $I_{eff} V_{eff}$
  *unit used is : volt-Ampere [VA]



- Power factor $P.F = \frac{\text{Average Power}}{\text{Apparent Power}} = \frac{P}{V_{eff} \cdot I_{eff}}$

↳ for sinusoidal waveforms:

$v(t) = V_m \cos(\omega t + \theta) \equiv \sqrt{2}\, V_{eff} \cos(\omega t + \theta)$

$i(t) = I_m \cos(\omega t + \phi) \equiv \sqrt{2}\, I_{eff} \cos(\omega t + \phi)$

$P = V_{eff} I_{eff} \cos(\theta - \phi)$

$$\therefore P.F = \frac{P}{V_{eff} I_{eff}} = \cos(\overset{\alpha}{\underbrace{\theta - \phi}_{\text{P.F angle!}}}) \quad \therefore \overset{\text{for sinusoidal}}{\boxed{0 \leq P.F \leq 1}}$$

$\alpha = \theta - \phi = \cos^{-1}(P.F)$

↳ can be ∓ve!

→ power factor ≤ apparent power

P.F = apparent power when $\cos(\theta - \phi) = 1 = P.F$

P.F=1 ; Unity power factor

↳ $\theta = \phi$; in-phase! ; pure resistive load.

⇝ for purely reactive load:

$\theta - \phi = \pm 90^\circ \rightarrow$ capacitive −ve

$\hookrightarrow$ inductive +ve

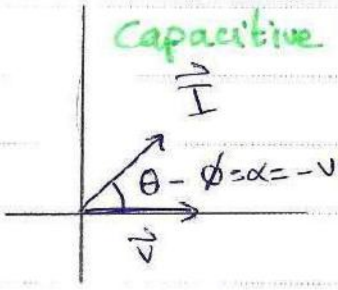
$\therefore$ $P.F = \cos(\theta - \phi) = \boxed{0}$

⇝ for $\vec{Z} = R + jX$ load:

$$* \quad \vec{Z} = \frac{\vec{V}}{I} = \frac{V_{eff}\angle\theta}{I_{eff}\angle\phi} \Rightarrow \frac{V_{eff}}{I_{eff}} = |\vec{Z}| \;,\; \theta - \phi = \measuredangle\vec{Z}$$

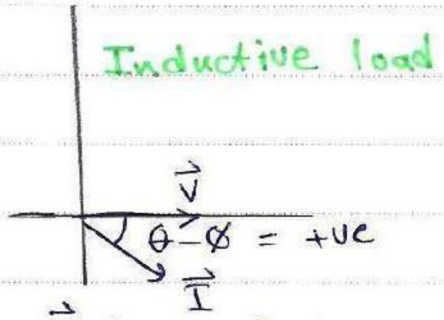
$\therefore$ phase differance between $\vec{V}$ & $\vec{I}$ is the phase of $\vec{Z}$!

$$\boxed{P.F = \cos(\measuredangle\vec{Z})}$$



$\vec{I}$ leads $\vec{V}$ by α

leading P.F



$\vec{I}$ lags $\vec{V}$ by α

lagging P.F

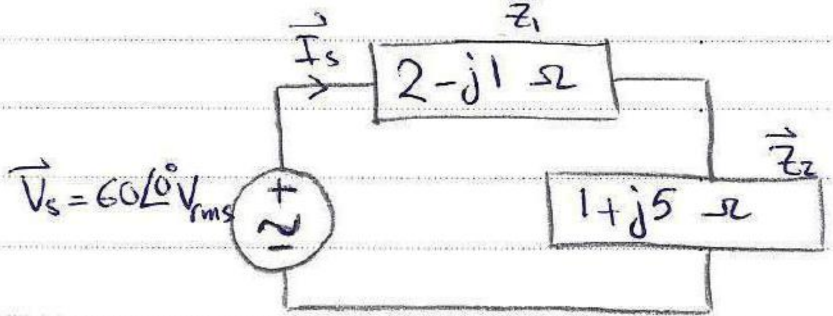
P.F
- → 0 ⟶ purely reactive load
- → 1 ⟶ Purely resistive load
- ↳ leading → capacitive load
- ↳ lagging → Inductive load

when we want to define P.F we say for example:

P.F = 0.5 leading !

ex:-

find the overall p.f of this CKT:



Sol: $P.F = \frac{P}{V_{eff} I_{eff}}$, $V_{eff} = 60\ V_{rms}$, $I_{eff} = ?$

$$\vec{I} = \frac{\vec{V}_s}{\vec{Z}_1 + \vec{Z}_2} = \frac{60\angle 0^\circ}{(2+1)+j(-1+5)} = \frac{60\angle 0^\circ}{3+j4} = 12\angle -53.1^\circ\ A_{rms}$$

$I_{eff} = 12\ A_{rms}$

$Z_{eq} = 3+j4$; inductive!

$$* \quad P.F = \cos(\theta - \phi) = \cos\left(0^\circ - (-53.1^\circ)\right) = 0.6 \ \underline{\text{lagging}}$$

another way:

$P_{\vec{Z}_1} = I_{eff}^2 \cdot (2) = 12^2 * 2 = 288$ watt

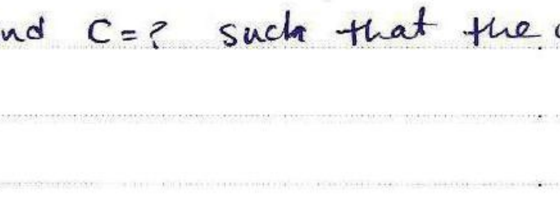
$P_{\vec{Z}_2} = I_{eff}^2\ (1) = 12^2 = 144$ watt

$P_{eq} = 432$ watt

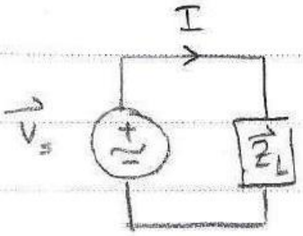
$$P.F = \frac{P}{V_{eff} I_{eff}} = \frac{432}{60 * 12} = 0.6 \text{ lagging} \ **$$

P.42: a) find overall p.F.

b) find the avg power supplied by this source.



c) if we add a capacitor parallel to the R/L , find C=? such that the overall P.F becomes unity.



$\vec{V}_s = 120\angle 0^\circ$

$\vec{Z}_L = 4 + \left(\frac{j16 * 12}{j16 + 12}\right) = 11.68 + j5.76 \ \Omega$

$= 13.023 \angle 26.25^\circ$ lagging!

$I = \frac{\vec{V}_s}{\vec{Z}_L} = 9.214 \angle -26.25^\circ \ A_{rms}$

a)

$P.F = \cos(\measuredangle \vec{Z}) = \cos(26.25^\circ) = 0.8968$ lagging!

b) $P = I_{eff} \cdot V_{eff} \cdot (P.F) = (9.214)(120)(0.8968) = 991.6$ watt

c) $P.F = 1$; resistive load

$\vec{Y}_{eq} = jwc + \frac{1}{11.68 + j5.76}$

$= jwc + \frac{11.68 - j5.76}{(11.68)^2 + (5.76)^2}$

$= jwc + \frac{11.68}{(11.68)^2 + (5.76)^2} - \frac{j5.76}{(11.68)^2 + (5.76)^2}$

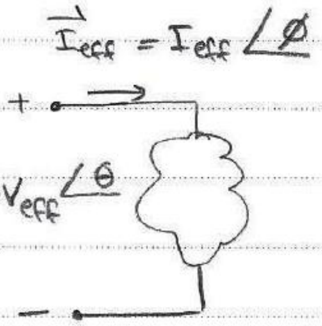
$Im\{Y_{eq}\} = 0$; because the load must be purely resistive.

$\therefore wc - \frac{5.76}{(11.68)^2 + (5.76)^2} = 0$, $w = 2\pi(60)$

$c = 90.09 \ \mu F$

21 / 2 / 2013

Complex Power :-



$\vec{I}_{eff} = I_{eff} \angle \phi$

$\vec{V}_{eff} = V_{eff} \angle \theta$

$$\vec{S} = \vec{I}_{eff}^{\,*}\ \vec{V}_{eff} \quad [VA]$$

$$\vec{I}^{\,*}_{eff} = I_{eff} \angle -\phi$$

$$P = Re\{\vec{S}\} = Re\{\vec{I}^{\,*}_{eff} \cdot V_{eff}\} \equiv V_{eff} I_{eff} \cos(\theta - \phi)$$

$$= Re\{V_{eff} \angle \theta \cdot I_{eff} \angle -\phi\}$$

$$= Re\{V_{eff} I_{eff}\, e^{j(\theta-\phi)}\}$$

$$= Re\{\underbrace{V_{eff} I_{eff} \cos(\theta-\phi)}_{\substack{\text{Average power} \\ \text{/ Dc Power} \\ \text{/ Real power}}} + j \underbrace{V_{eff} I_{eff} \sin(\theta-\phi)}_{\substack{\text{Reactive power} \\ \text{Quadrature Power (Q)}}}\}$$

$$\underset{\substack{\downarrow \\ [VA]}}{\vec{S}} = \underset{\substack{\downarrow \\ [Watt]}}{P} + j\, \underset{\substack{\downarrow \\ [VAR]}}{Q}$$

; Volt-Ampere ; Volt-Ampere-reactive!

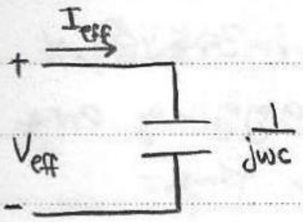
* Reactive Power : it's the time rate of energy flow back & forth between the $\overset{\text{Power supply}}{\text{source}}$ & the reactive component of the load ($\vec{Z}$)

Imaginary part of $\vec{Z}$

$\boxed{P = V_{eff} I_{eff} \cos(\theta - \phi)}$ [WAtt] ; measured using wattmeter.

$\boxed{Q = V_{eff} I_{eff} \sin(\theta - \phi)}$ [VAR] ; measured using VAR meter

- for inductive load (lagging p.F) : Q is +ve
- " Capacitive " (leading ") : Q is −ve



$$\vec{V}_{eff} = \vec{I}_{eff} \cdot \frac{1}{jwc}$$

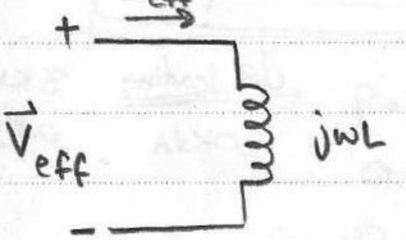
$$\vec{S} = \vec{V}_{eff} \cdot \vec{I}^{*}_{eff} = \vec{I}_{eff} \cdot \frac{1}{jwc} \cdot \vec{I}^{*}_{eff}$$

$$= I_{eff}^{2} \cdot \frac{1}{jwc}$$

$$= -j \frac{I_{eff}^{2}}{wc} = -jQ_c$$

$$\therefore \boxed{Q_c = -\frac{I_{eff}^{2}}{wc} \equiv -V_{eff}^{2}\, wc}$$

* only imaginary power Q apears here, because average pow
absorbed by a capacitor = 0 !



$$\vec{V}_{eff} = I_{eff}\, jwL$$

$$\vec{S} = \vec{I}_{eff} \cdot jwL \cdot \vec{I}^{*}_{eff}$$

$$= j I_{eff}^{2} (wL)$$

$$= j Q_L$$

$$\boxed{Q_L = wL\, I_{eff}^{2} \equiv \frac{V_{eff}^{2}}{wL}}$$

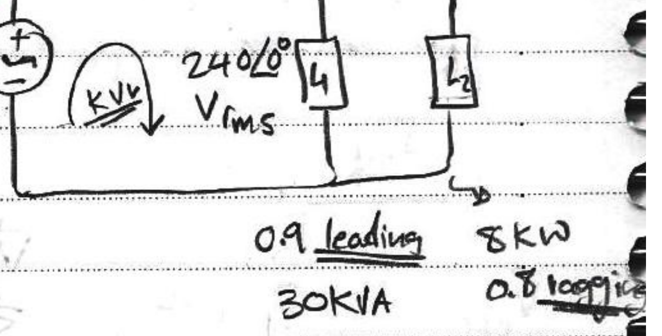
spring 2011

Given the network shown below. The load $L_1 = 30kVA$, 0.9 p.f leading & $L_2 = 80 kwatt$, 0.8 p.f lagging are connected in parallel & supplied from the source $V_s(t) = V_m \cos(100\pi t + \theta)$ through a line with an impedance of $0.1 + j0.1 \Omega$ as shown ~~in~~ below.

~~to~~ find :1) V_m, θ

2. complex power & p.f at the source.

3. what size capacitor should be installed in parallel with Vs to cause 0.95 $\overset{\text{lagging}}{\uparrow}$ p.f ?!



Sol :-

because it's leading!

① $\vec{S}_1 = \text{apparent } p \angle \boxed{-}\cos^{-1} p.f$

$= 30 \angle -25.84° kVA$

$\vec{S}_2 = \frac{\text{Average } p}{p.f} \angle +\cos^{-1}(p.f)$

$= 100 \angle 36.87 \; KVA$

$\vec{S}_{total} = \vec{S}_1 + \vec{S}_2 = 116.84 \angle 23.68° \; kVA$

$\vec{S}_{total} = \boxed{\vec{I}_s^*} \times \overset{240\angle 0}{\vec{V}_L}$

$\rightarrow I_s = 487 \angle -23.68 \; A$

KVL $-V_s + (0.1 + j0.1) I_s + 240 \angle 0° = 0$

$V_s = \underbrace{305.19}_{V_m} \underbrace{\angle 4.71°}_{\theta}$

② $S_s = \vec{V}_s \times \vec{I}_s^* = 148.6 \angle 28.4 \; KVA$

$P.F = \cos(\theta - \phi) = 0.88$ lagging (~~$\angle$~~ +ve)

③

$C = \frac{\tan\theta_{old} - \tan\theta_{new}}{\omega V_{rms}^2}$

$\theta_{old} = \cos^{-1}(0.88) = 28.36°$

$\theta_{new} = \cos^{-1}(0.95) = 18.19°$

$\omega = 100\pi$

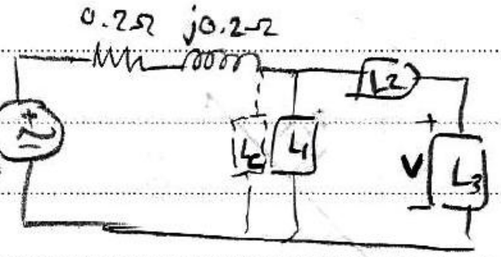
$V_{rms} = \frac{V_m}{\boxed{\sqrt{2}}} = \frac{305.19}{\sqrt{2}}$

$= 215.8$

$C = 1.45 nf$

Fall 2012

The particulars of the composite load shown in fig 1 are as follows :-



L_1 : 10kW @ 0.8 lagging p.f

L_2 : 2KVA @ 0.707 lagging p.f

L_3 : 4KVA @ 0.866 leading p.f

if **V** = 200 $\angle 0°$ Vrms, determine:

a) V_s, I_s, & p.f$_{source}$.

b) what value of C if connected ~~to~~ accross the Source will cause the source to operate @ 0.95 lagging p.f

Sol:- a) $\vec{S}_1 = \frac{10k}{0.8} \angle +\cos^{-1}0.8 = 12.5 \angle 36.87°$ KVA

$\vec{S}_2 = 2k \angle +\cos^{-1}0.707 = 2 \angle 45°$ KVA

$\vec{S}_3 = 4k \angle -\cos^{-1}0.866° = 4 \angle -30°$ KVA

$\vec{I}_3^* = \frac{\vec{S}_3}{\vec{V}_3} = 20 \angle -30°$

$\rightarrow \vec{I}_3 = 20 \angle 30°$

$\vec{I}_3 = \vec{I}_2$!

$\therefore \vec{V}_2 = \frac{\vec{S}_2}{\vec{I}_3^*} = 100 \angle 75°$ V

KVL $V_1 = V_2 + \overset{V}{V_3} = 245.67 \angle 23.15°$ V

$\therefore \vec{I}_1^* = \frac{\vec{S}_1}{\vec{V}_1} = 50.88 \angle 13.71°$

$\vec{I}_1 = 50.88 \angle -13.71°$

but $I_s = I_1 + I_3 = \boxed{66.78 \angle -1.77°}$

KVL $-V_s + (0.2 + j0.2) I_s + V_1 = 0$

$\boxed{\vec{V}_s = 263.49 \angle 24.56}$

p.f$_{source}$ = $\cos(24.56 + 1.77°) = 0.897$ lagging

b) $\theta_{old} = \cos^{-1}(pf_{old}) = 26.33°$

$\theta_{new} = \cos^{-1}(0.95) = 18.19°$

$\omega = 2\pi f = 100\pi$

$S_{total} = \frac{(10 + 2 \times 0.707 + 4 \times 0.866) \angle \cos^{-1} pf}{0.95}$

$= 15.66 \angle 18.19°$ k

$\vec{S}_c = S_{tot} - (S_1 + S_2 + S_3) = 2.03 \angle -90°$

$= -j\,2.03$ Ω k

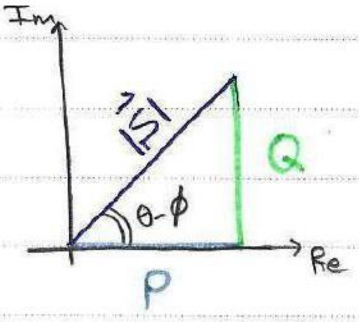
$\vec{I}_c^* = \frac{\vec{S}_c}{V_c = V_1} \Rightarrow \vec{I}_c = 8.22 \angle 113.15$ K

$\vec{Z}_c = \frac{V_c}{I_c} = 29878.09 \angle -90°$ m

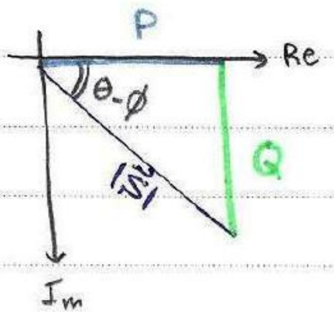
$\vec{Z}_c = -\frac{j}{\omega c}$

$C = \frac{-\cancel{j}}{\omega(-\cancel{j}\, 29878.09)} = 1.065$ nF

* Power triangle:-



inductive load



capacitive load

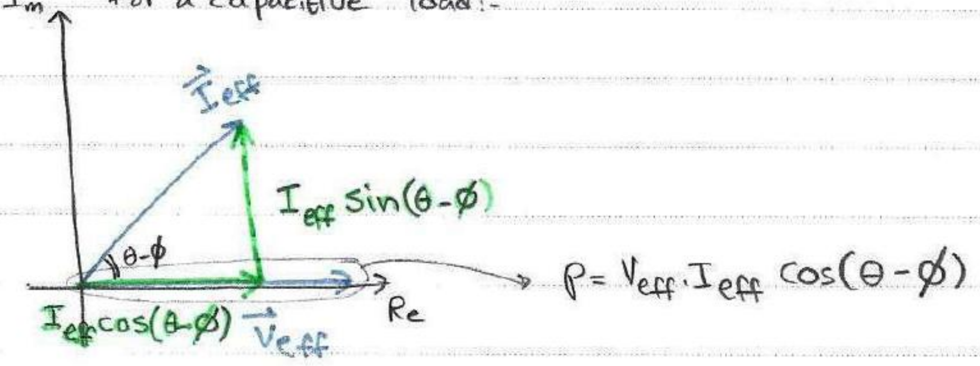
$$\boxed{|\vec{S}| = V_{eff}\, I_{eff}} \quad = \text{apparent power } [VA]$$

$$\& \quad \boxed{|\vec{S}| = \sqrt{P^2 + Q^2}}$$

↳ $P = |\vec{S}| \cos(\theta - \phi) = V_{eff}\, I_{eff} \cos(\theta-\phi)$

↳ $Q = |\vec{S}| \sin(\theta - \phi) = V_{eff}\, I_{eff} \sin(\theta-\phi)$

for a capacitive load:-

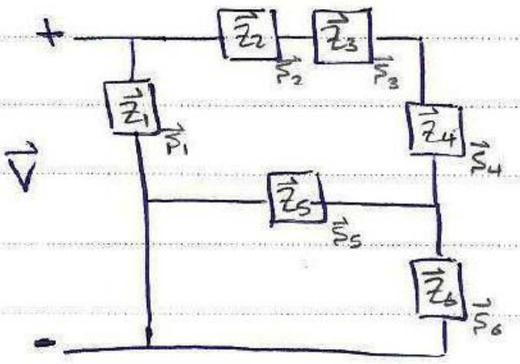


$$\vec{S} = P + j\, Q$$

↳ 0 ; pure reactive ↳ 0 ; Pure resistive

↳ ≠0 ; $\vec{z}$ is complex ↳ ≠0 ; $\vec{z}$ is complex

ex:-

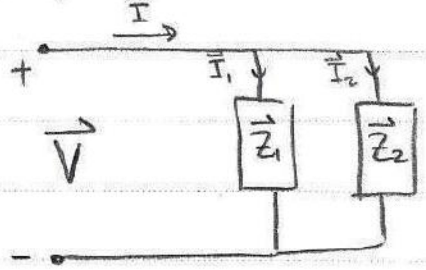


$$\underset{\sim\sim}{\vec{S}_{eq}} = \vec{S}_1 + \vec{S}_2 + \cdots + \vec{S}_6$$

power that $\vec{Z}_{eq}$ absorbs!

* complex power delivered to several interconnected loads/ impedances is "the sum of complex power delivered to each indivisual load!

Proof: a) in parallel:



دائماً مستخدم ح effective values!

$$\vec{S} = \vec{V}\,\vec{I}^*$$

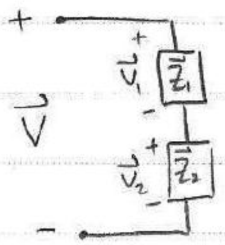
KCL $(\vec{I} = \vec{I}_1 + \vec{I}_2)$

$$= \vec{V}(\vec{I}_1 + \vec{I}_2)^* = \vec{V}(\vec{I}_1^* + \vec{I}_2^*)$$

$$= \vec{V}\vec{I}_1^* + \vec{V}\vec{I}_2^*$$

$$\therefore \vec{S} = \vec{S}_1 + \vec{S}_2 \quad *$$

b) in series:



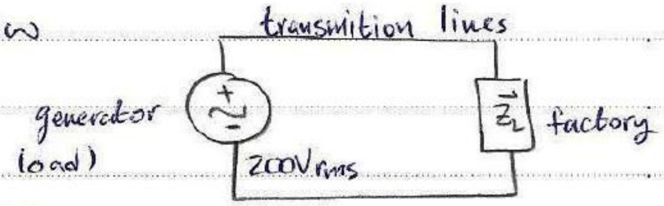
$$\vec{S} = \vec{V}_{eff}\,\vec{I}^*_{eff}$$

$$= (\vec{V}_1 + \vec{V}_2)\,\vec{I}^*$$

$$= \vec{V}_1\vec{I}^* + \vec{V}_2\vec{I}^*$$

$$\vec{S} = \vec{S}_1 + \vec{S}_2 \quad *$$

ex maximum power rating = 1kw



*if p.F = 1 (pure resistive load)

then: $I_{eff} = \frac{1k}{(1)(200)} = \underline{\underline{5}}\ A_{rms}$

* if the load has a lagging p.F of 0.5 :

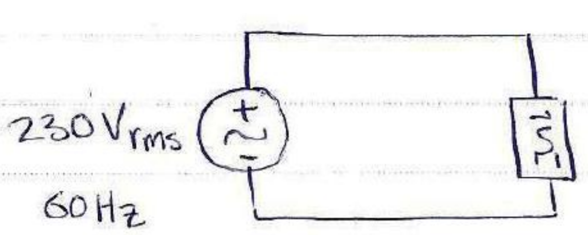
$|\vec{I}| = \frac{1k}{(0.5)(200)} = \underline{\underline{10}}\ A_{rms}$

... $I_1 (P.F=1) < I_2 (P.F=0.5)$

it's better when P.F approaches unity (=1) because power dropped in transmition lines is given by: $p = |I|^2 R$, so increasing of I increases the power lose!

24/12/2013

ex: An induction motor operates at 50Kw & lagging p.F of 0.8



$P = 50KW$

$P.F_1 = 0.8$ lagging

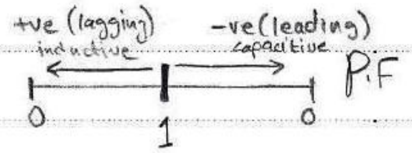
specify ~~a~~ the suitable solution to raise the p.f to 0.95 lagging.

Sol:- ∵ $P.F_2 = 0.95$

solution is to put a correction load in parallel with the motor.

- we chose to put it parallel in order to maintain the voltage (230Vrms) applied on $\vec{S}_1$.
- we add only an "imaginary /reactive part" load to maintain avg power (50kw) that's absorbed by the load.

+ in this case , to make p.f approaches 1 we need to add purely capacitive load!



- $\vec{S}_1 = P_1 + jQ_1$

↳ $P_1 = P = 50kW$

↳ $|\vec{S}_1| \cdot P.F_1 = P_1$

$|\vec{S}_1| = \frac{50k}{0.8}$

$P.F_1 = \cos(\theta - \phi) = \cos\alpha_1 \Rightarrow \alpha_1 = \cos^{-1}(P.F_1) = 36.9^\circ$ α +ve lagging p

S_1 can be also written as:-

$$\vec{S}_1 = |\vec{S}_1| \angle\alpha_1 = \frac{50k}{0.8}\angle 36.9^\circ \equiv (50 + j37.5)\ kVA$$

then, ↳ $Q_1 = (37.5)k\ VAR$

- $\vec{S}_{total} \equiv \vec{S} = \vec{S}_1 + \vec{S}_2$

$$|\vec{S}| = \frac{P}{P.F_2} = \frac{50k}{0.95}$$

$$\alpha = \angle\vec{S} = \cos^{-1}(P.F_2) = \cos^{-1}(0.95) \quad (+ve)$$

$$\Rightarrow \vec{S} = \frac{50k}{0.95}\angle\cos^{-1}(0.95) \equiv (50 + j16.43)\ kVA$$

$$\therefore Q_{tot} = (16.43)\ k\ VAR$$

but $\vec{S} = \vec{S}_1 + \vec{S}_2$

$(\cancel{50} + j16.43)k = (\cancel{50} + j37.5)k + \vec{S}_2$

$\vec{S}_2 = -j\,21.07\ kVA$

but $\vec{S}_2 = \vec{I}_2^{*}\vec{V}_s$

$I_2^{*} = \frac{-j21.07}{230} = -j91.6\ A_{rms}$

$\therefore \vec{I}_2 = j91.6\ A_{rms}$

$\& \ \vec{Z}_c = \dfrac{\vec{V}_s}{I_2} = \dfrac{230\angle 0^\circ}{j91.6} = \underbrace{-j\,2.51\,\Omega}_{-j/wc}$

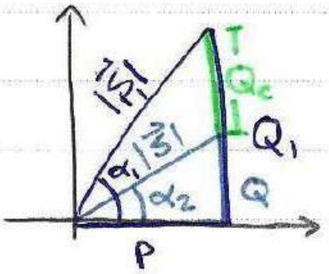
$-j/wc = -j\,2.51$

$wc = \dfrac{1}{2.51}$

$$\boxed{P.F \propto \frac{1}{\alpha}}$$

$C = \dfrac{1}{2.51(2\pi * 60)} = 1056\,\mu F \equiv 1.056\,mF$

another way: using power triangle:



$\vec{S}_1 = P + jQ_1$

$\vec{S} = P + jQ$

$Q_c = Q_1 - Q$

but... $Q_1 = P\tan(\alpha_1)$

$\& \ \alpha_1 = \cos^{-1}(P.F_1)$

$Q = P\tan(\alpha_2)$

$\& \ \alpha_2 = \cos^{-1}(P.F_2)$

$\Rightarrow Q_c = Q_1 - Q$

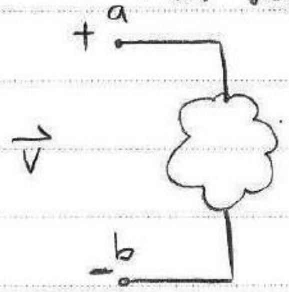
$Q_c = |V_s|^2 (wC)$

$\therefore C = \dfrac{Q_c}{|V_s|^2 \cdot w}$ ⋇

Chapter *12: Polyphase Circuits:

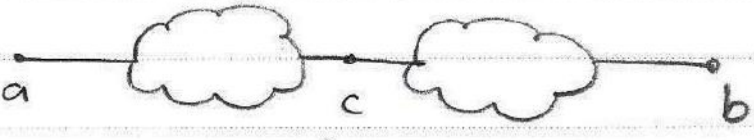
* revision: double subscript notation:-

→ for voltages:

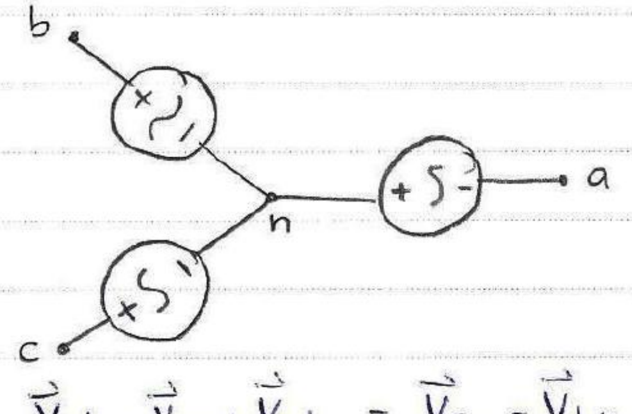


* instead of using ±ve notations, we describe voltage differance by:
$\vec{V}_{ab}$; terminal (a) is at hiegher potential (+) than terminal b (-).

$$\boxed{\vec{V}_{ab} = -\vec{V}_{ba}}$$

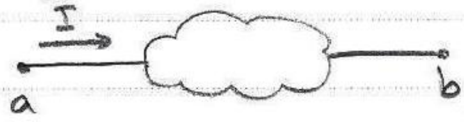


$$\vec{V}_{\underline{ab}} = \vec{V}_{\underline{a}c} + \vec{V}_{c\underline{b}} .$$

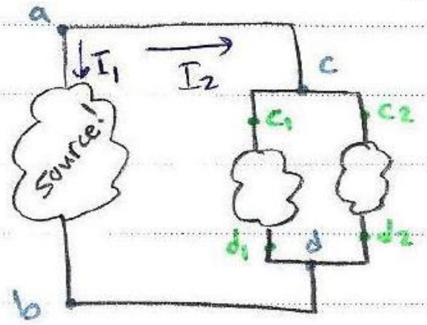


$$\vec{V}_{ab} = \vec{V}_{an} + \vec{V}_{nb} \equiv \vec{V}_{an} - \vec{V}_{bn}$$
$$\vec{V}_{ac} = \vec{V}_{an} + \vec{V}_{nc} = \vec{V}_{an} - \vec{V}_{cn}$$
$$\vec{V}_{bc} = \vec{V}_{bn} + \vec{V}_{nc} \equiv \vec{V}_{bn} - \vec{V}_{cn} \ldots$$

→ for currents:



$$\boxed{\vec{I}_{ab} \equiv -\vec{I}_{ba}}$$

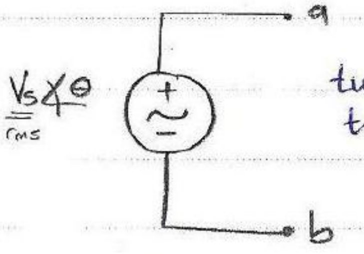


* $\vec{I}_{ab}$; does it mean I_1 or I_2 ?!!
  - we always follow the shortes / the most direct path. (I_1) فيه أقل عدد elements
* $\vec{I}_{cd}$; there's two currents which one to take?
  - here we have to say $\vec{I}_{c_1d_1}$ or $\vec{I}_{c_2d_2}$!

$$\underset{\sim\sim}{\vec{I}_{cd}} = \vec{I}_{c_1d_1} + \vec{I}_{c_2d_2}$$

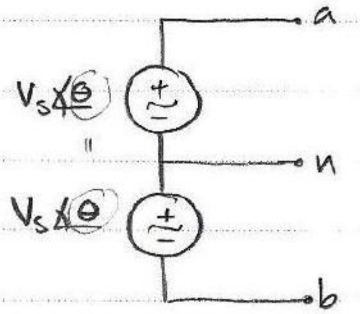
the overall currant

Poly-phase Systems:-



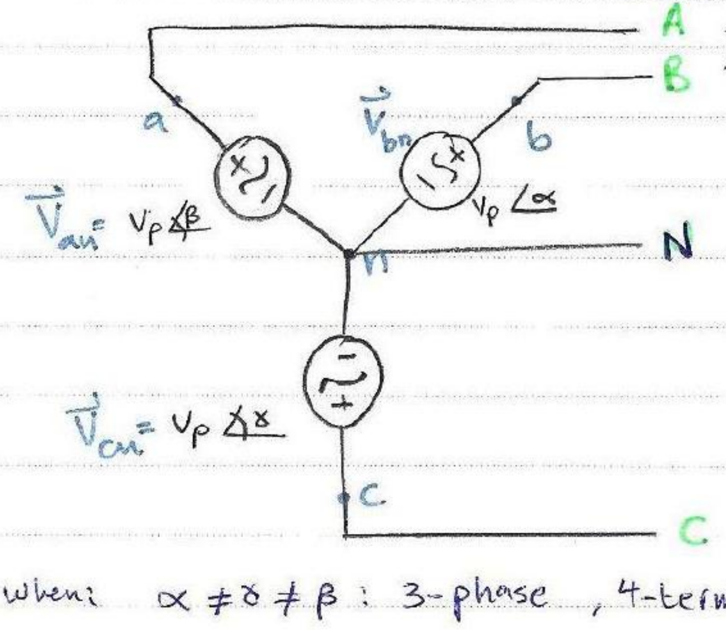
two terminals!

Single-phase



three terminals!

single - phase



A, B — line terminals

N neutral terminal

C live terminal

when: $\alpha \neq \gamma \neq \beta$: 3-phase , 4-terminals System

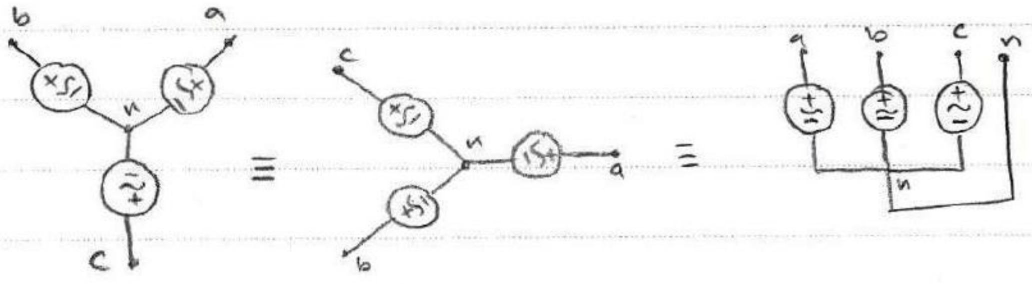
$\alpha = \beta \neq \gamma$: 2-phase , ~ ~ ~

$\alpha = \beta = \gamma$: 1- phase , ~ ~ ~

* In 3-phase systems, ~~was~~ the overall instantanuous power will be constant!

→ $\vec{V}_{an}$, $\vec{V}_{bn}$, $\vec{V}_{cn}$ are the 'phase Voltages' [V_{rms}]

→ $\vec{V}_{ab}$, $\vec{V}_{ac}$, $\vec{V}_{bc}$, $\vec{V}_{cb}$, $\vec{V}_{ac}$, ... are the 'line voltages' [V_{rms}]



it's called a Y-connected source!

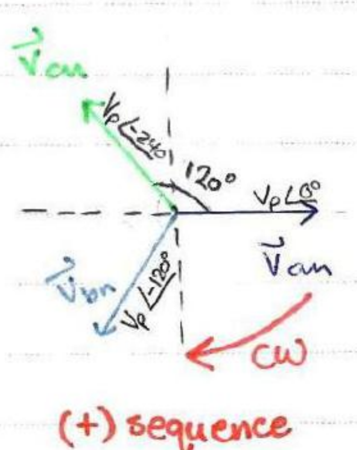
⇒ Balanced 3-phase source:

3-phase source is ~~p~~ balanced if:-

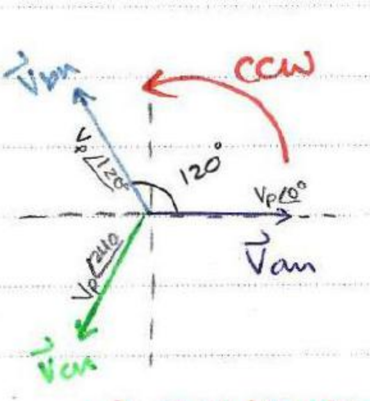
$$\boxed{\vec{V}_{an} + \vec{V}_{bn} + \vec{V}_{cn} = 0}$$

&

$$\boxed{|\vec{V}_{an}| = |\vec{V}_{bn}| = |\vec{V}_{cn}| = V_p} \quad \text{(rms)}$$



(+) sequence
(abc)

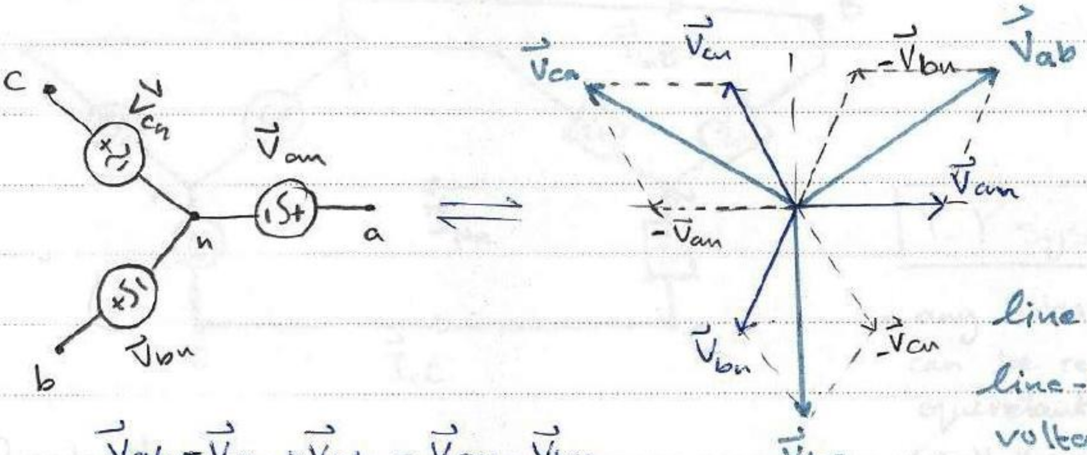


(−) sequence
(cba)

→ if $\vec{V}_{an}$ is the refferance , then $\vec{V}_{an} = V_p \angle 0°$ & :

① $\vec{V}_{bn} = V_p \angle -120° = V_p \angle +240°$
$\vec{V}_{cn} = V_p \angle -240° = V_p \angle +120°$ } (+ve-phase / abc) sequence

② $\vec{V}_{bn} = V_p \angle 240° = V_p \angle -120°$
$\vec{V}_{bn} = V_p \angle 120° = V_p \angle -240°$ } (-ve-phase/ cba) sequence



line voltages/ line-to-line voltages.

$$\vec{V}_{ab} = \vec{V}_{an} + \vec{V}_{nb} = \vec{V}_{an} - \vec{V}_{bn}$$
$$= V_p \angle 0° - V_p \angle -120°$$
$$= V_p - V_p \cos(-120) - j V_p \sin(-120°)$$
$$\vec{V}_{ab} = V_p \left(1 + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \sqrt{3} V_p \angle 30°$$

$$\boxed{V_L = \sqrt{3} V_p}$$

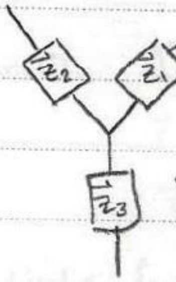
↳ V_p : phase voltage
↳ V_L : line voltage.

** line Voltage leads phase voltages by 30° (in +ve sequence)

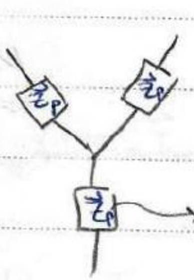
(abc) (+)
- $\vec{V}_{ab} = \sqrt{3} V_p \angle 30°$
- $\vec{V}_{bc} = \sqrt{3} V_p \angle -90° \equiv \sqrt{3} \angle 30° \, \vec{V}_{bn}$
- $\vec{V}_{ca} = \sqrt{3} V_p \angle 150° \equiv \sqrt{3} \angle 30° \, \vec{V}_{cn}$

(Cba) (-)
- $\vec{V}_{ab} = \sqrt{3} V_p \angle -30° \equiv \sqrt{3} \angle -30 \, \vec{V}_{an}$
- $\vec{V}_{bc} = \sqrt{3} V_p \angle 90° \equiv \sqrt{3} \angle -30 \, \vec{V}_{bn}$
- $\vec{V}_{ca} = \sqrt{3} V_p \angle 210° \equiv \sqrt{3} \angle -30 \, \vec{V}_{cn}$

default (if it isn't mintioned) ⇒ +ve-sequence.



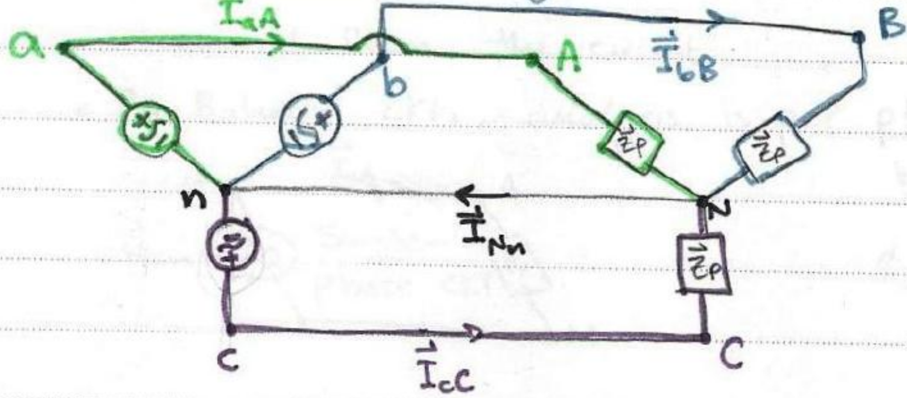
Y-load
3-phase!



Balanced Y-load

→ $\vec{Z}_P$: phase impedance
1-phase.

Balanced Y-connected System:-



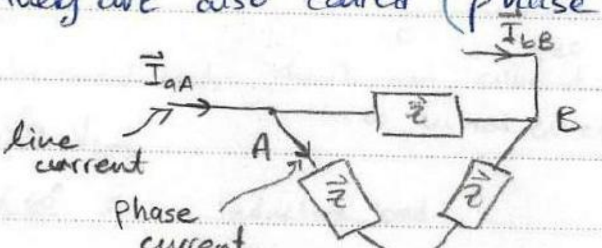
$\boxed{\text{Y-Y system}}$

* any Balanced 3-ϕ system can be reduced to an equevelant Y-Y system, so, Y-Y analysis is The key to solving other 3-ϕ systems!

@ node N:

$$\boxed{\vec{I}_{aA} + \vec{I}_{bB} + \vec{I}_{cC} = \vec{I}_{Nn}} \quad \cdots ①$$

- $\vec{I}_{aA}, \vec{I}_{bB}, \vec{I}_{cC}$ are called (line currents). & in this case (transition lines has no impedance) they are also called (phase carrents).



another case: delta-load!

$$\vec{I}_{aA} = \frac{\vec{V}_{AN}}{\vec{Z}_P} = \frac{\vec{V}_{an}}{\vec{Z}_P} = \frac{V_P \angle 0^\circ}{\vec{Z}_P}$$

$\vec{V}_{AN} = \vec{V}_{an}$ because it's a short ckT between a&A, n&N

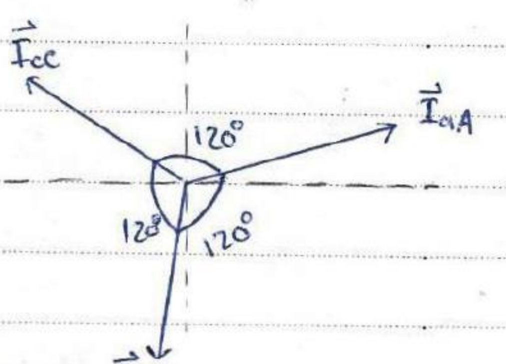
$$\vec{I}_{bB} = \frac{\vec{V}_{BN}}{\vec{Z}_P} = \frac{\vec{V}_{bn}}{\vec{Z}_P} = \frac{\vec{V}_{an} \angle -120}{\vec{Z}_P} = \frac{V_P \angle -120}{\vec{Z}_P} = \vec{I}_{aA} \angle -120^\circ$$

$$\vec{I}_{cC} = \frac{\vec{V}_{CN}}{\vec{Z}_P} = \frac{\vec{V}_{cn}}{\vec{Z}_P} = \frac{\vec{V}_{an} \angle -240^\circ}{\vec{Z}_P} = \frac{V_P \angle -240^\circ}{\vec{Z}_P} = \vec{I}_{aA} \angle -240^\circ$$

from ①: $\vec{I}_{aA}+\vec{I}_{bB}+\vec{I}_{cC}=\vec{I}_{Nn}$

$\vec{I}_{aA}\angle 0+\vec{I}_{aA}\angle -120+\vec{I}_{aA}\angle -240=\vec{I}_{Nn}$

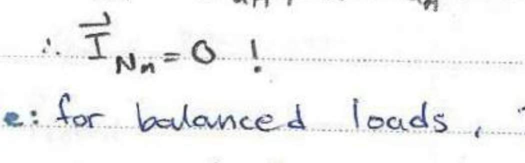
$\therefore \vec{I}_{Nn}=0$!

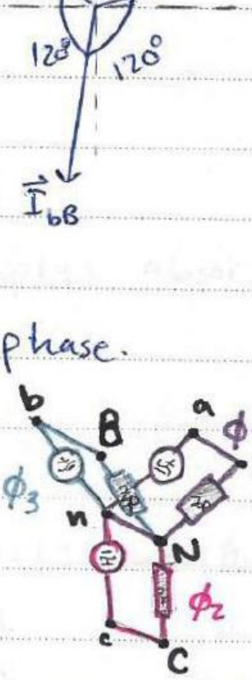


Note: for balanced loads, $I_{Nn}=0$,

∴ the neutral wire ~~cord~~ can be removed from the circuit.

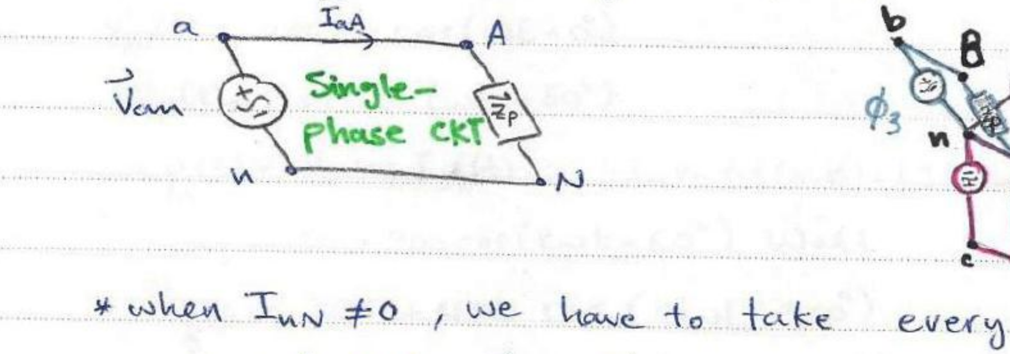
* for Balanced ckts, analysis is per phase.





I can solve it per phase

* when $I_{nN}\neq 0$, we have to take every CKT alone and calculate $\vec{I}_{aA}$, $\vec{I}_{bB}$, --- one by one



2 loops (mesh)

*$I_n(Y)$, if I have an unbalanced load, the I can solve it per phase only if I have a neutral line! else, I have to use other analysis techiques (nodal, mesh, ---).

ex:- $\vec{V}_{an}=200\angle 0\ V_{rms}$

$\vec{Z}_p=100\angle 60°\ \Omega$; inductive load.

for Balanced ckt, find:-

a) $\vec{I}_{aA}=\frac{\vec{V}_{an}}{\vec{Z}_p}=\frac{20\cancel{0}\angle 0}{10\cancel{0}\angle 60}=2\angle -60°$

* (+ve)-sequance (default)

$\vec{V}_{bn}=200\angle -120\ V_{rms}$

$\vec{V}_{cn}=200\angle -240\ V_{rms}$

b) $\vec{I}_{bB}=\vec{I}_{aA}\angle -120=2\angle -60-120=2\angle -180\ A_{rms}$

c) $\vec{I}_{cC}=\vec{I}_{aA}\angle -240°=2\angle -60-240=2\angle -300\ A_{rms}$

d) $\vec{V}_{ab}=\sqrt{3}\angle 30\,\vec{V}_{an}=200\sqrt{3}\angle 30°\ V_{rms}$

e) $\vec{V}_{bc}=\sqrt{3}\angle 30\,\vec{V}_{bn}=200\sqrt{3}\angle -90°\ V_{rms}$

f) $\vec{V}_{ca}=200\sqrt{3}\angle -210°\ V_{rms}$

g) $\vec{V}_{ac}=-200\sqrt{3}\angle -210=200\sqrt{3}\angle \overset{150}{-210-180}$

$=200\sqrt{3}\angle -30°$

/ 3 / 2013

$\xrightarrow{\text{cont.}}$ h) P_{AN} :

$P_{AN} = V_{rms} I_{rms} \cos(\theta - \phi)$

$= 200 * 2 \cos(0 - (-60))$

$P_{AN} = 200$ Watt

i) P_{total} :

$Ptotal = 3 P_{AN} = 3 * 200 = 600$ Watt

j) what's the instantaneous power $p(t)$ absorbed by the load?

$V_{AN}(t) = 200\sqrt{2} \cos(\omega t + 0^\circ)$

$I_{AN}(t) = 2\sqrt{2} \cos(\omega t - 60^\circ)$

$\rightarrow P_A(t) = V_{AN}(t) I_{AN}(t) = \frac{1}{2} I_m V_m \cos(\theta - \phi) + \frac{1}{2} I_m V_m \cos(2\omega t + \theta + \phi)$

$= 200 + 400 \cos(2\omega t - 60^\circ)$ Watt

$P_B(t) = 200 + 400 \cos(2\omega t - 300^\circ)$

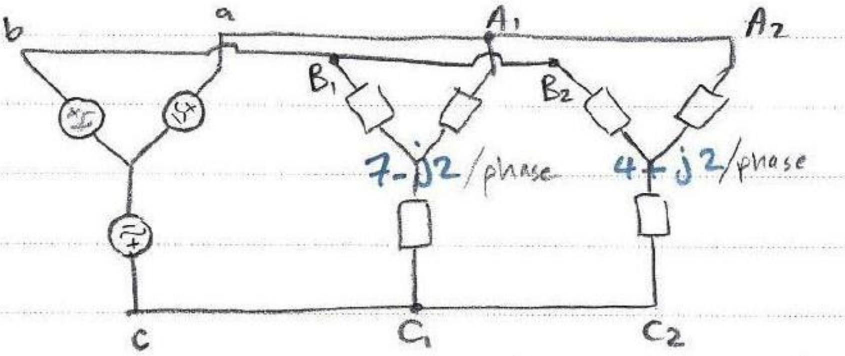
$P_c(t) = 200 + 400 \cos(2\omega t - 180^\circ)$

$Ptotal(t) = P_a(t) + P_B(t) + P_c(t) = 600$ Watt

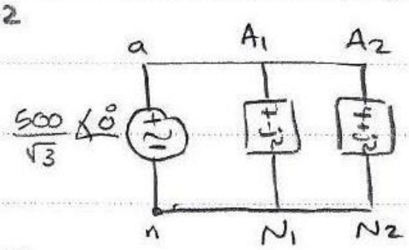
3/3 /2013

ex:- A Balanced 3-phase, 3-wires system has a line voltage of 500 Vrms, two balanced Y-connected loads are present, one is a capacitive load with impedance 7-j2 Ω per phase, & the other one is an inductive load with 4+j2 per phase, find:-

1. the phase voltage.
2. the line current
3. p.F of the load.
4. the total power drawn by the load.
5. the p.F at which the source is operating.



loads are parallel (can't be inseries).



$V_L = 500$ Vrms

① $V_P = V_L/\sqrt{3} = (500/\sqrt{3})$ Vrms

$V_{an} = V_P \angle 0 = (500/\sqrt{3})\angle 0°$ Vrms

↳ chose it to be a refferance.

② $\vec{Z}_{eq} = \vec{Z}_p = (7-j2) // (4+j2) = \left(\frac{32}{11} + j\frac{6}{11}\right)\Omega$

$|\vec{Z}_p| = \frac{1}{11}\sqrt{32^2+6^2} = 2.69\ \Omega$

$\to |\vec{I}_{aA_1}| = \frac{(500/\sqrt{3})}{2.96} = 97.53$ Arms

inductive load

OR $\vec{I}_{aA_1} = \frac{\vec{V}_{an}}{\vec{Z}_p} = \frac{(500/\sqrt{3})\angle 0°}{\frac{32}{11} + j\frac{6}{11}} = 97.53 \angle -10.62$ Arms

③ $p.F_L = \cos(0-(-10.62)) = 0.9828$ lagging.

④ $P_{Total} = 3 * \underline{\underline{P_{phase}}}$ ...①

$P_p = (97.53)^2 \left(\frac{32}{11}\right)$ ← real part of $\vec{Z}$

Plug in ① ... $P_T = 83.018$ kWatt.

* we take the current I because Vp is n't applied only on $Re\{\vec{Z}\}$!

⑤ $P.F_S = P.F_L$ "because there's zero impedance through transmition lines"

$\therefore P.F_S = \cos(0 + 10.62°) \equiv \cos\left(\tan^{-1}\left(\frac{6}{32}\right)\right)$ lagging

$= 0.98285$ lagging.

<u>OR</u> $P.F = \frac{P_p}{|V||I|} = \frac{27.67k}{\left(\frac{500}{\sqrt{3}}\right)(97.53)} = 0.9828$ ※ lagging.

<u>EX2</u>:- 3 Balanced Y-connected loads are installed on a balanced $\underline{\overset{\text{3-phase}}{3\text{-}\emptyset}}$, 4-wires system, $\boxed{\text{load 1}}$ draws a total power of $\underline{\overset{\text{Real power (avg)}}{6\,KW}}$ @ $\underline{\overset{\text{Pure resistive}}{\text{unity P.F}}}$, $\boxed{\text{load 2}}$ requires $\underline{\overset{\text{apparent Power}}{10\,KVA}}$ @ P.F = 0.96 lagging, & $\boxed{\text{load 3}}$ needs 7 Kw @ P.F = 0.85 lagging, if the phase voltage @ loads is 135V, & each line has a resistance of 0.1Ω, while the neutral line has a resistance = 1Ω, find:-

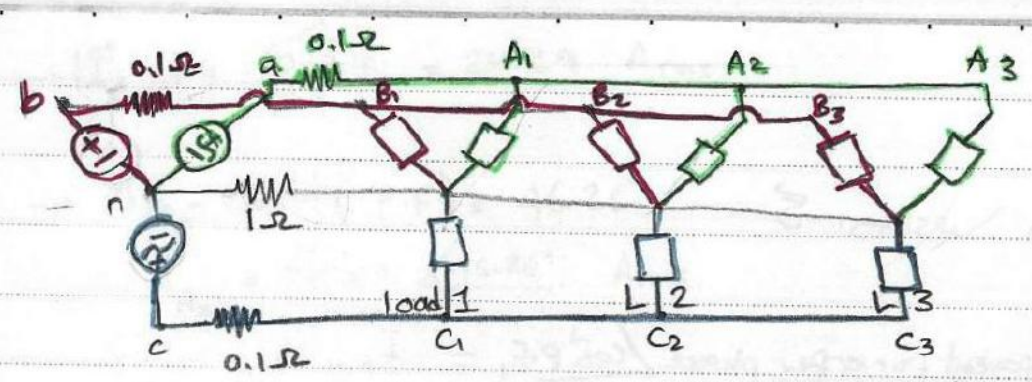
① the total power drawn by the loads.

② the combined P.F of the loads.

③ the total power lost in the 4 lines.

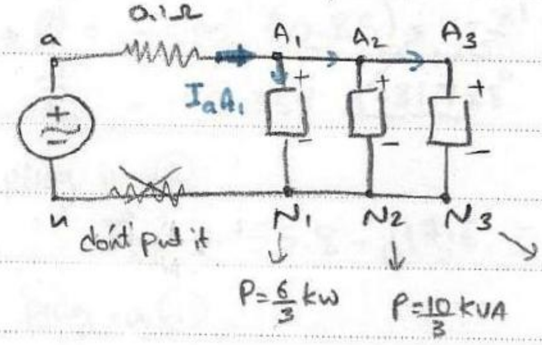
④ the phase voltage

⑤ the P.F @ which the source is operating



Pure resisteve	inductive	inductive
6KW	10KVA	7KW
P.F=1	P.f=0.96	P.f=0.85

Sol:-



فرضناها صفر!

$V_{Load} = 135\angle 0°$ Vrms

① $P_{total} = 6KW + \underbrace{10KVA * 0.96}_{\text{apparent } P * P.F = \text{Real } P} + 7kW = 22.6KW$

② $P_{phase} \equiv P_P = \frac{22.6}{3}$ kw

also $P_P = |\vec{V}_{Load}|\ |I_{aA_1}| * P.F_\ell$

$\therefore P.F_\ell = \frac{P_P}{|\vec{V}_{Load}||\vec{I}_{aA_1}|} = \frac{22.6/3\,K}{135 * \underline{\underline{|\vec{I}_{aA_1}|}}}$ ...①

KCL $\vec{I}_{aA_1} = \vec{I}_{A_1N} + \vec{I}_{A_2N} + \vec{I}_{A_3N}$ --②

apparent power ≡ VI ! ≡ Real power/P.F

$\Rightarrow$ 1) $|\vec{I}_{A_1N}| = \boxed{\frac{6/3\,K}{V_\ell * P.F}} = 14.8148 A_{rms}$

2) $\phi = -\cos^{-1}(P.F)$, since $\theta = 0$

$= 0°$

$\therefore \vec{I}_{A_1N} = 14.8148\angle 0°$ A_{rms}

OR we solve it using complex power $S_1 = \vec{V}_\ell * \vec{I}^*_{A_1}$

already apparent

$\rightarrow |\vec{I}_{A_2N}| = \frac{\boxed{10/3}\ k}{135} = 24.69$ Arms

$\rightarrow \phi = -\cos^{-1}(P.F) = -16.26^\circ$

$\therefore \vec{I}_{A_2N} = 24.69 \angle -16.26^\circ$ Arms.

$\rightarrow |\vec{I}_{A_3N}| = \frac{7/3\ k}{135 * 0.85} = 20.334$ Arms

$\rightarrow \phi = -\cos^{-1}(0.85) = -31.788^\circ$

$\therefore \vec{I}_{A_3N} = 20.334 \angle -31.788^\circ$

plug in ②

$\vec{I}_{aA_1} = 55.8 - j17.6 = 58.5 \angle -17.5287^\circ$

plug in ①

$P.F_\ell = \frac{22.6/3}{135 * 58.5} = 0.9535$ lagging.

③ $P_{Lost} = (|\vec{I}_{aA_1}|)^2 * 3 * 0.1 = 1027.317$ Watt.

④ $\vec{V}_{bn} = \vec{V}_s + \vec{V}_\ell$

$= 0.1 * \vec{I}_{aA_1} + 135 \angle 0^\circ$

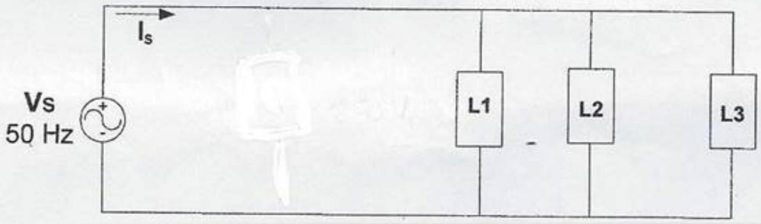
$= 140.59 \angle -0.718^\circ$

⑤ $P.F_s = \frac{1027.317/3 + (22.6/3)\ k}{\underbrace{140.5912}_{V_P} * \underbrace{58.5183}_{I_{aA_1}}} = 0.9573$ lagging.

first exam

(8 marks) Q(1): Three loads are connected in parallel across 250V (rms) as shown in Fig.1. Load 1 absorbs 16kW and 28 kVA. Load 2 absorbs 10kVA at 0.6 leading PF. Load 3 absorbs 8KW at unity PF.

1- Find the Impedance that is equivalent to the three parallel loads.
2- Find the power factor of the total load.
3- If the source operates at 50 Hz, find the value of the capacitor to be connected in order to improve the power factor to 0.96.



$S_1 = 28 \angle 55.5$ KVA

$S_2 = 10 \angle -53$ KVA

$S_3 = 8 \angle 0$ KVA

$S_T = S_1 + S_2 + S_3 = 33.65 \angle 26.54$ k

$S_T = 250 \angle 0 \, I_T^* = \underline{30.15} + j\,\underline{14.7}$ k

$I_T = 134.2 \angle -26.5$ A

① $Z_T = \frac{V_T}{I_T} = \boxed{1.86 \angle 26.54} \ \Omega$

② $\boxed{P.F = \cos +26.54 = 0.89 \text{ lag}}$

theta

$\Phi_{New} = \cos^{-1}(0.96) = 16.26.$

$Q_{New} = 8.8$ KVA.

$Q_c = 14.7 - 8.8 = 5.2$ KVA

$Q_c = \frac{V^2}{Z_c} \Rightarrow Z_c = 12\Omega$

$Z_c = \frac{1}{2\pi f c} \Rightarrow \boxed{C = 3.14 \mu F}$

Section:................. ....الاسم:

(4 marks) Q(2): Fig.2 shows the instantaneous power $p(t)$ consumed by the load terminals, where the terminal voltage is given by $v(t) = V_m \cos(\omega t)$, then find the followings:

1- PF.
2- The reactive power Q.
3- The Complex power $\boldsymbol{S}$.

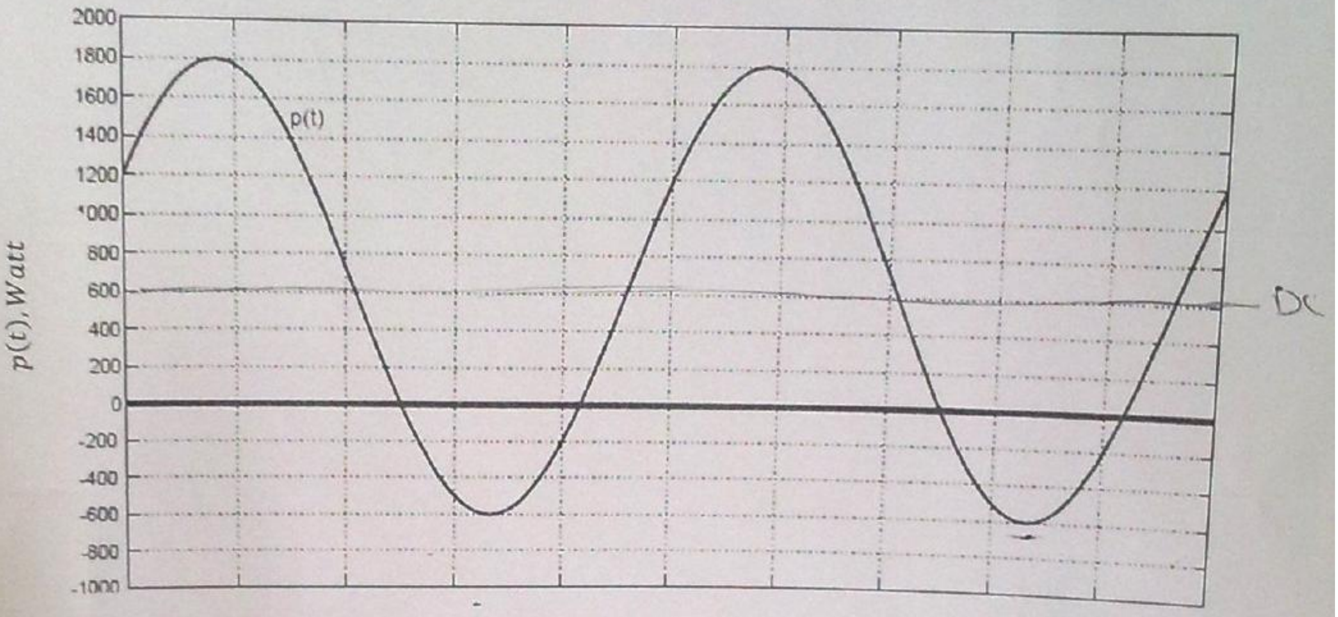


Fig. 2

$v(t) = V_m \cos(\omega t + \theta) \Rightarrow \theta = 0$

$i(t) = I_m \cos(\omega t + \phi)$

$$\Rightarrow p(t) = \frac{V_m I_m}{2} \cos(\theta - \phi) + \frac{V_m I_m}{2} \cos(2\omega t + \theta + \phi)$$

$$P = \frac{V_m I_m}{2} \cos(\theta - \phi) = 600$$

$$Q = \frac{V_m I_m}{2} \sin(\theta - \phi)$$

$\boxed{PF = 1/2 \text{ lagging}}$

$\Rightarrow p(t) = 600 + 1200 \cos(2\omega t + \theta + \phi)$

$600 = 1200 \cos(\theta - \phi)$

$\Rightarrow \cos^{-1}(0.5) = \pm 60° \Rightarrow$ from graph $\Rightarrow \theta + \phi = -60°$

$\phi = -60° \Rightarrow \theta - \phi = 60°$

$Q = 1200 \sin(+60°) = 1039.23$ VAR

$\boxed{Q = 1039.23 \text{VAR}}$

$S' = (600 + j1039.23)$ VA

$\boxed{S = 600 + j1039.23}$ VA

(8 marks) Q(3): In the system shown in Fig.3, given $Z_Y = 20 + j8\ \Omega$, and (+) phase sequence is assumed. If $\boldsymbol{I}_{aA} = 20\measuredangle{-46^\circ}\ A\ rms$, and the source is operating with $PF = 0.94$ lagging, find the following:

1- R_w
2- The total complex power supplied by the source
3- $\boldsymbol{V}_{an}$.
4- $\boldsymbol{V}_{AB}$.

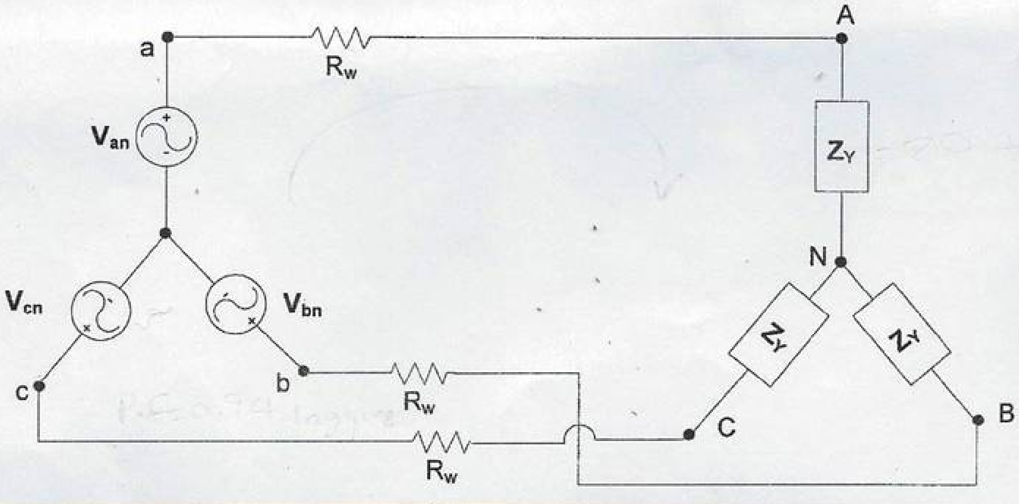


Fig. 3

① $Z_T = R_w + 20 + j8$

$PF = 0.94$

$\theta - \emptyset = \cos^{-1}(0.94) = 19.94$

$Z = a + jb = (R_w + 20) + j8$

$\therefore \theta - \emptyset \overset{\tan}{=} (19.94 = \tan^{-1}(\frac{8}{R_w + 20})$

$\frac{8}{R_w + 20} = \tan(19.94) = 0.362$

$\Rightarrow R_w = \frac{8 - 20(0.362)}{0.362}$

$$\boxed{R_w = 2.09}$$

③ $V_{an}{}^{Y} = V_{aN}{}^{Y} = Z\,I$

$= (2.09 + 20 + j8)\,20\angle{-46^\circ}$

$$\boxed{V_{an}{}^{Y} = 469.88\angle{-26.09}}$$

② $S_{T_s} = 3\left[I^2 \cdot Z_T\right] = S_{T_{load}}$

$= 3\,(20)^2\,(22.09 + j8)$

$$\boxed{\begin{aligned} &= (26.508 + j9.6)\ KVA \\ &= 28.19\angle{19.908^\circ} \end{aligned}}$$

④ $V_{AN} = Z\,I = (20 + j8)(20\angle{-46}$

$= \boxed{470.8\angle{-24^\circ}}$

$\therefore V_{AB} = \sqrt{3}\,(470.8)\angle{-24 + 30^\circ}$

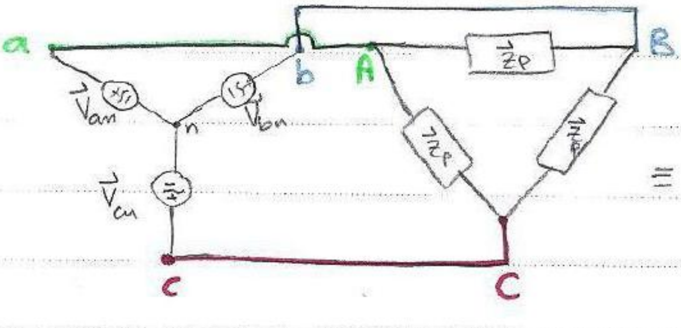
$$\boxed{V_{AB} = 746.18\angle{5.8^\circ}}$$

$R_w =$

5/3/2013

* Balanced Δ-connected load:

→ Y-Δ connected system:



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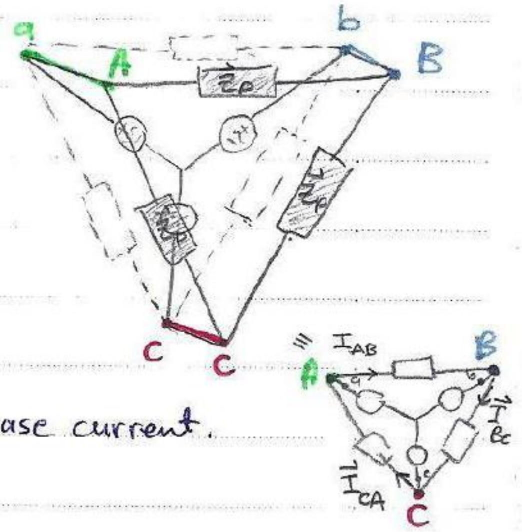


* In Δ-connected load, line current ≠ phase current.
* In a balanced source:

$V_p = |\vec{V}_{cn}| = |\vec{V}_{bn}| = |\vec{V}_{an}|$ ... Phase voltage

$V_L = |\vec{V}_{ab}| = |\vec{V}_{ca}| = |\vec{V}_{bc}|$ ... line voltage

$V_L = \sqrt{3}\ V_p$

$\vec{V}_{an} = V_p \measuredangle 0°$ $\qquad$ $\vec{V}_{ab} = V_L \measuredangle \underline{30°}$

$\vec{V}_{bn} = V_p \measuredangle \underline{-120°}$ $\qquad$ $\vec{V}_{bc} = V_L \measuredangle \underline{-90°}$

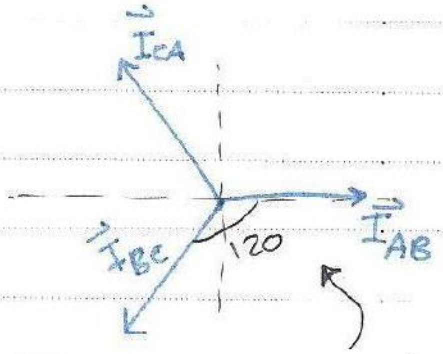
$\vec{V}_{cn} = V_p \measuredangle \underline{120°}$ $\qquad$ $\vec{V}_{ca} = V_L \measuredangle \underline{-210°}$

→ Phase currents: (since transmition lines has zero impedance).

$$\vec{I}_{AB} = \frac{\vec{V}_{AB}}{\vec{Z}_P} = \frac{\vec{V}_{ab}}{\vec{Z}_p} = \frac{V_L \measuredangle 30°}{|\vec{Z}_P| \measuredangle \vec{Z}_P}$$

$$\vec{I}_{BC} = \frac{\vec{V}_{BC}}{\vec{Z}_P} = \frac{\vec{V}_{bc}}{\vec{Z}_P} = \frac{V_L \measuredangle -90°}{|\vec{Z}_P| \measuredangle \vec{Z}_P}$$

$$\vec{I}_{CA} = \frac{\vec{V}_{CA}}{\vec{Z}_P} = \frac{\vec{V}_{ca}}{\vec{Z}_P} = \frac{V_L \measuredangle 210°}{|\vec{Z}_P| \measuredangle \vec{Z}_P}$$



we assumed $\vec{I}_{AB}$ to be the referance but that doesn't mean that $\measuredangle \vec{I}_{AB} = 0$

* line currents:-

$\vec{I}_{aA} = \vec{I}_{AB} + \vec{I}_{AC} = \vec{I}_{AB} - \vec{I}_{CA}$

* let $I_p = |\vec{I}_{AB}| = |\vec{I}_{BC}| = |\vec{I}_{CA}|$

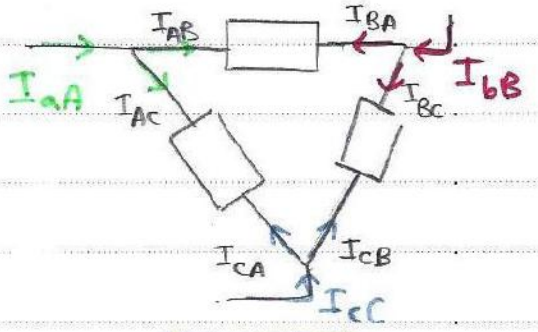
$\therefore \vec{I}_{aA} = \overset{I_L}{\underline{\sqrt{3}\ I_p}} \angle{-30} = I_L \angle{-30}$

$\vec{I}_{bB} = \vec{I}_{BC} + \vec{I}_{BA} = \vec{I}_{BC} - \vec{I}_{AB}$

$= I_L \angle{-150}$

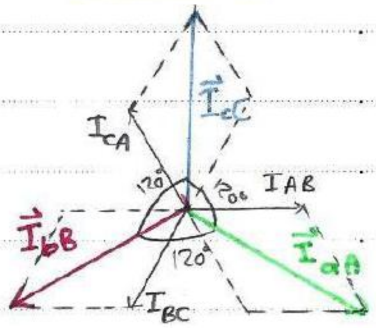
$\vec{I}_{cC} = \vec{I}_{CA} + \vec{I}_{CB} = \vec{I}_{CA} - \vec{I}_{BC}$

$= I_L \angle{-270}$



$$\boxed{I_L = \sqrt{3} I_p}$$

then $\Rightarrow$
$$\boxed{\begin{aligned} \vec{I}_{aA} &= \sqrt{3} \angle{-30^\circ}\ \vec{I}_{AB} \\ \vec{I}_{bB} &= \sqrt{3} \angle{-30^\circ}\ \vec{I}_{BC} \\ \vec{I}_{cC} &= \sqrt{3} \angle{-30^\circ}\ \vec{I}_{CA} \end{aligned}}$$



line currents.

ex: Determine the amplitude of the line currents in a 3-ϕ system with a line voltage (بقصدها الـ load إذا ذكر العكس) of 300 V_{rms} that supplies 1200 W to a Δ-connected load at a lagging p.f of 0.8, then find the phase impedance.

sol: I_L? , $\vec{Z}_p$?

$|\vec{V}_{AB}| = |\vec{V}_{BC}| = |\vec{V}_{CA}| = 300\ V_{rms}$

→ the whole load absorbs 1200 W , ∴ one phase absorbs $\frac{1200}{3} = 400W$

$P_p = V_L \cdot I_p \cdot P.F$

$400 = 300 \times I_p \times 0.8$

$I_p = \frac{5}{3}\ A_{rms} = 1.667\ A_{rms}$

* $I_L = \sqrt{3}\ I_p = \boxed{2.89\ A_{rms}}$

$\vec{Z}_p = \underbrace{\frac{V_{AB}}{I_{AB}}}_{\hookrightarrow} = \frac{300\angle 0^\circ}{2.89 \angle I_{AB}} = 180 \angle{-I_{AB}} = 180 \angle 36.9^\circ$

$\angle\vec{Z} = +\cos^{-1}(p.F) = \cos^{-1}(0.8) = \angle 36.9^\circ$

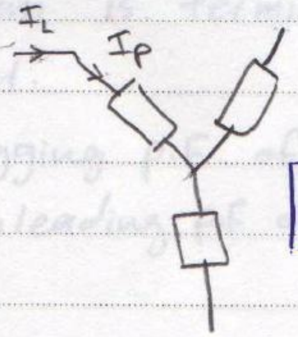
$\angle \vec{I}_{AB} = -\angle\vec{Z} = \angle{-36.9^\circ}$ lag

* Power in Y-load :

let Ψ be the P.F angle

$$\therefore P_p = V_p I_p \cos(\Psi)$$

$$= \left(\frac{V_L}{\sqrt{3}}\right) I_L \cos(\Psi)$$



$$\boxed{I_L = I_p}$$

but... $P_{total} = 3 \times P_p$ ... in a Balanced load

$$= \sqrt{3}\, V_L I_L \cos(\Psi)$$

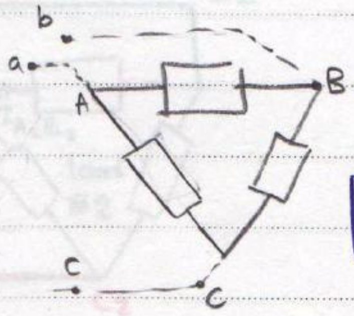
* Power in Δ- load:

$$P_p = V_p I_p \cos(\Psi)$$

$$\rightarrow P_p = V_L \left(\frac{I_L}{\sqrt{3}}\right) \cos(\Psi)$$

$$P_{total} = 3 \times P_p$$

$$\rightarrow P_{total} = \sqrt{3}\, V_L I_L \cos(\Psi)$$



$$\boxed{V_L = V_p}$$

for $\boxed{\underline{\underline{\text{load}}}}$	phase Voltage	Line Voltages	phase current	Line current
Y	$\vec{V}_{AN} = V_p \angle 0^\circ$ $\vec{V}_{BN} = V_p \angle -120^\circ$ $\vec{V}_{CN} = V_p \angle -240^\circ$	$\vec{V}_{AB} = (\sqrt{3} \angle 30)\, \vec{V}_{AN}$ $= \sqrt{3}\, V_p \angle 30^\circ$ $\vec{V}_{BC} = (\sqrt{3} \angle 30^\circ)\, \vec{V}_{BN}$ $= \sqrt{3} V_p \angle -90^\circ$ $\vec{V}_{cA} = (\sqrt{3} \angle 30^\circ) \vec{V}_{CN}$ $= \sqrt{3} V_p \angle -210^\circ$	$I_{aA} = I_{AN} = \frac{V_{AN}}{Z_p} \longrightarrow$ ″ $I_{bB} = I_{BN} = \frac{V_{BN}}{Z_p} \longrightarrow$ ″ $I_{cC} = I_{CN} = \frac{V_{CN}}{Z_p} \longrightarrow$ ″	
Δ	$\vec{V}_{AB} = \sqrt{3} V_p \angle 30 \xrightarrow{\equiv} \vec{V}_{AB}$ $\equiv V_{ab}$ (if there's no $\vec{Z}$ in transmition lines) $\vec{V}_{BC} = \sqrt{3} V_p \angle -90 \xrightarrow{\equiv} \vec{V}_{BC}$ $\vec{V}_{CA} = \sqrt{3} V_p \angle -210^\circ \xrightarrow{\equiv} \vec{V}_{CA}$		$I_{AB} = \frac{V_{AB}}{Z_p}$ $I_{BC} = \frac{V_{BC}}{Z_p}$ $I_{CA} = \frac{V_{CA}}{Z_p}$	$I_{aA} = (\sqrt{3} \angle -30^\circ) \overset{I_{AB}}{\boxed{\frac{V_{AB}}{Z_p}}}$ $= (\sqrt{3} \angle -30^\circ)\, I_{AB}$ $I_{bB} = (\sqrt{3} \angle -30^\circ)\, I_{BC}$ $I_{cC} = \sqrt{3} \angle -30\, I_{cA}$

for both Δ & Y

$$P_{phase} = V_L I_L \underline{\cos(\theta - \overset{\swarrow P.F}{\emptyset})}$$

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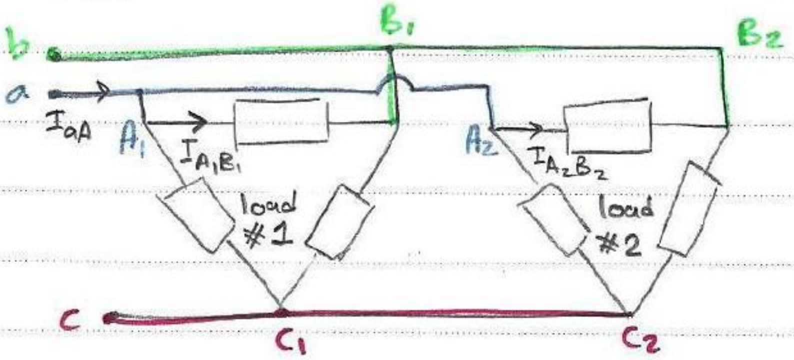
ex: A Balanced 3-ϕ, 3-wires system is terminated with 2 Δ-connected loads in parallel.

→ load #1 draws 40K VA at lagging p.F of 0.8.

→ load #2 absorbs 24k Watt at leading p.f of 0.9

assume $\vec{V}_{ab} = 440\angle 30^\circ$ V, find:

1. Total $\overset{\text{average P}}{\underline{\text{power}}}$ drawn by the 2 loads.
2. The phase current $\vec{I}_{A_1B_1}$ for the lagging load
3. $\vec{I}_{A_2B_2}$
4. $\vec{I}_{aA}$



Sol:-

$$1)\ P_{total} = \overset{\text{apparent P} \times \text{p.F}}{\underline{40k\ VA} \times 0.8} + 24\ k = 56k\ \text{Watt}$$

$$2)\ \vec{I}_{A_1B_1} = \frac{\vec{V}_{A_1B_1}}{\vec{Z}_1} \equiv \frac{\vec{V}_{ab}}{\vec{Z}_1} = \frac{440\angle 30^\circ}{|\vec{Z}|\angle\cos^{-1}(P.F_1)} = \frac{440}{|Z_1|}\angle 30-\cos^{-1}(0.8)$$

$|\vec{I}_{A_1B_1}| \longrightarrow$

$$P_{P_1} \Rightarrow \overset{V_{eff}}{\underline{440}}\ \overset{I_{eff}}{\underline{|I_{AB}|}} \times \overset{\cos(\theta-\phi)}{\underline{P.F}} = 40 \times 0.8/3$$

$$|I_{A_1B_1}| = \frac{(40/3)\ k}{440} = 30.303\ A_{rms}$$

$$\therefore \vec{I}_{A_1B_1} = 30.303\ \underline{\angle -6.87^\circ}$$

⟶

3) $\vec{I}_{A_2B_2} = \dfrac{\vec{V}_{ab}}{|\vec{Z}_2| \angle \underline{-\cos^{-1}(P.F)}} = \dfrac{440}{|\vec{Z}_2|} \angle \underline{30+\cos^{-1}(0.9)}$ --- ①

$\longrightarrow P_{P_2} \Rightarrow \frac{24}{3}k = 440 \times |\vec{I}_{A_2B_2}| \times P.F$

$$|\vec{I}_{A_2B_2}| = 20.202 \text{ --- ②}$$

from ① & ② : $\vec{I}_{A_2B_2} = 20.202 \angle \underline{55.84^\circ}$

4) $\vec{I}_{aA} = \sqrt{3} \angle \underline{-30} \; \underline{\underline{\vec{I}_{AB}}}$

$\hookrightarrow \vec{I}_{AB} = \vec{I}_{A_1B_1} + \vec{I}_{A_2B_2}$

$$\vec{I}_{aA} = \sqrt{3} \angle \underline{-30} \left(\vec{I}_{A_1B_1} + \vec{I}_{A_2B_2} \right) = 75.26 \angle \underline{-12.477^\circ} A$$

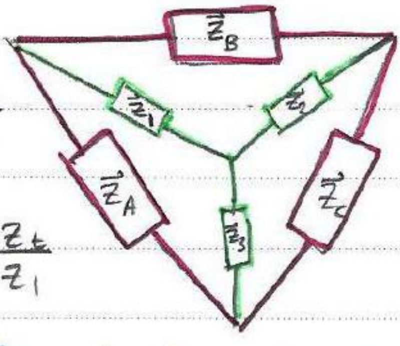
Y ↔ Δ transformation

Y → Δ :

$$Z_A = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_2} = \frac{Z_t}{Z_2}$$

$$Z_B = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_3} = \frac{Z_t}{Z_3}$$

$$Z_C = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_1} = \frac{Z_t}{Z_1}$$



Δ → Y :

$$Z_1 = \frac{Z_A \times Z_B}{Z_A + Z_B + Z_C}$$

$$Z_2 = \frac{Z_B \times Z_C}{Z_A + Z_B + Z_C}$$

$$Z_3 = \frac{Z_A \times Z_C}{Z_A + Z_B + Z_C}$$

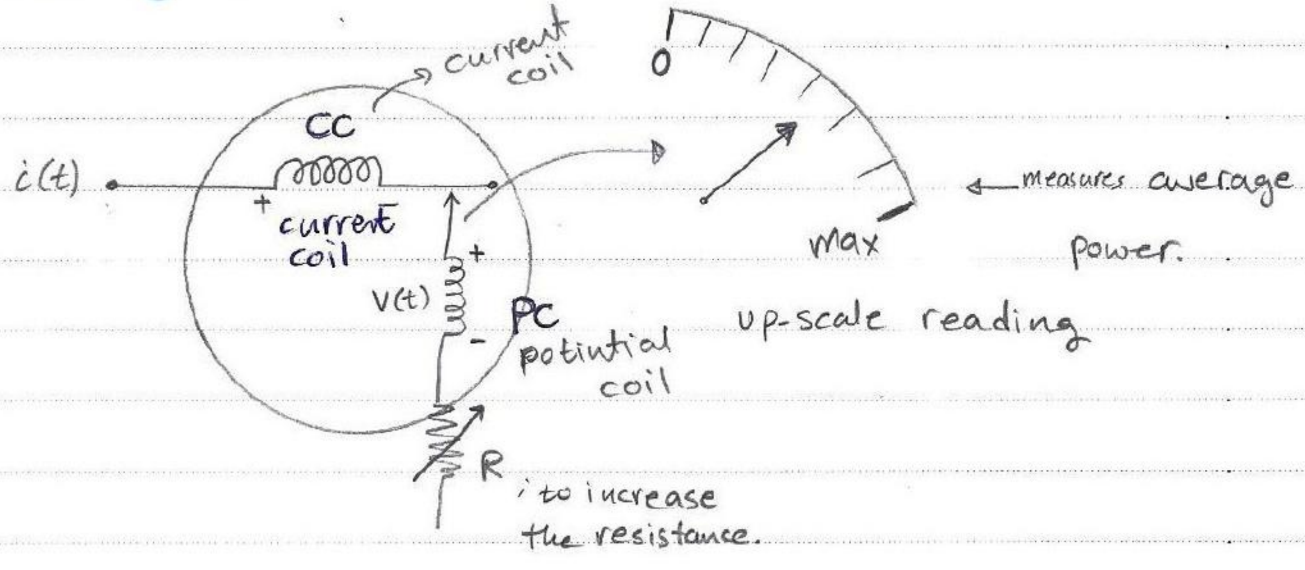
for a balanced source:

$Z_1 = Z_2 = Z_3 = Z_Y$

$Z_A = Z_B = Z_C = Z_\Delta$

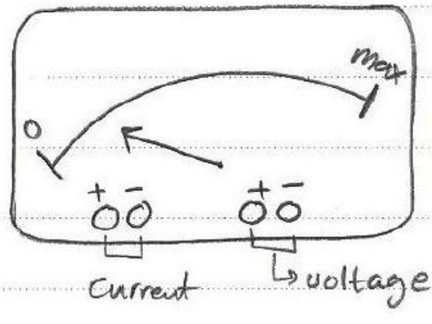
where, $Z_Y = \vec{Z}_\Delta / 3$

* Analog Wattmeter:



* Current coil (cc) is made of heavy wire

* PC is made of a fine wire, relatively high resistance.
* Pc has ~~a~~ greater number of turns.



* How to connect it with load?!

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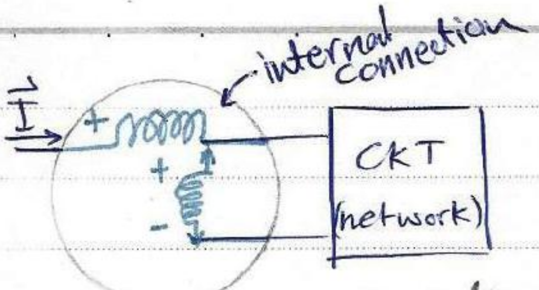
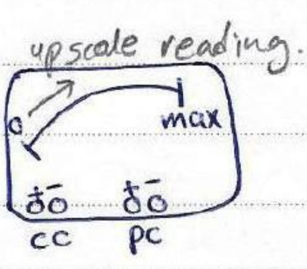
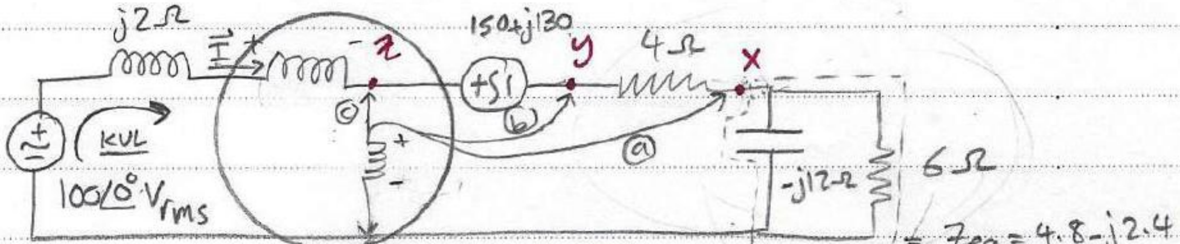


passive sign convention



p: +ve quanitiy → absorbs
-ve 〃 → delivers ; there will be no reading (because it's an upscale).
(we reverse the order of Pc to have a reading).

ex:- Determine the wattmeter reading state weither or not the potential coil had to be reversed in order to obtain an upscale reading, & identify the device (s) absorbing or generating this power.
The +ve terminal of the wattmeter is connected to: a) x. b) y. c) z.



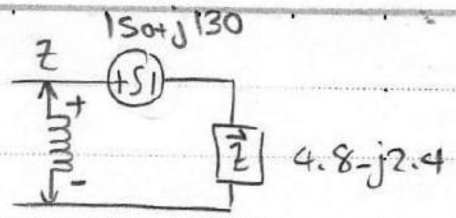
sol:- $\vec{I}$? KVL $-100\angle 0^\circ + j2\vec{I} + (150+j130) + 4\vec{I} + (4.8-j2.4)\vec{I} = 0$

$$\vec{I} = \frac{-(150+j130)}{4+j2+4.8-j2.4} = -5-j15 = 15.81 \measuredangle -108.43^\circ$$

a) $P_x = |\vec{I}|^2 \, Re\{Z\} = (15.81)^2 \times 4.8 = 1200$ Watt absorbs, ∴ no change to pc.

b) $P_y = |\vec{I}|^2 \times Re\{4.8 - 2.4j + 4\} = (15.81)^2 \times (6.8) = 2200$ Watt absorbs.

c) $P_Z = P_Y + P_{source}$



$P_{source} = |\vec{V}_{zy}||\vec{I}|\cos(\theta-\phi)$

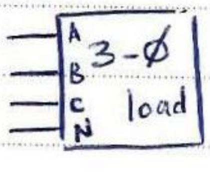
$= 198.49 \times 15.81 \times \cos(40.914° + 108.43°)$

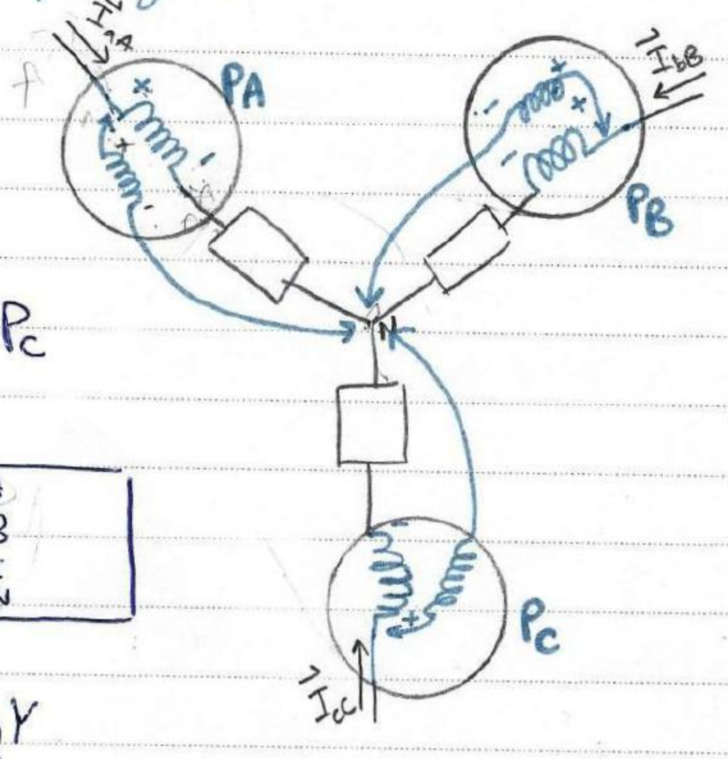
$= -2700$ Watt ; delivers Power

$P_Z = 2200 - 2700 = -500$ watt ; also delivers power

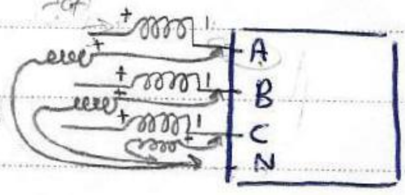
∴ PC connection has to be reversed.

Wattmeter in 3-Ø system

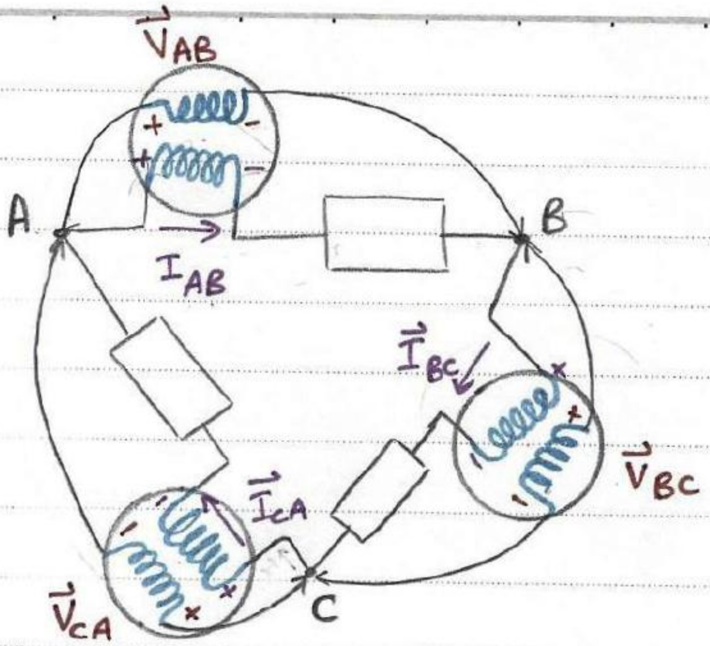


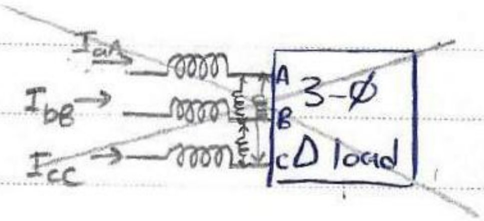


$P_{total} = P_A + P_B + P_C$



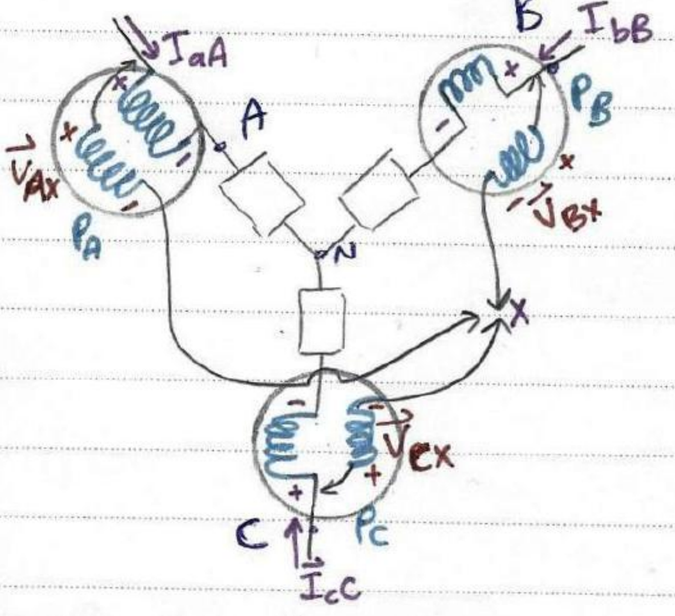
the problem in Y is that neutral label isn't always accessible





doesn't read the right power, since $I_{aA} \neq I_{AB}$!

problem in Δ that the phase load is not accessable.



$$P_A = \frac{1}{T}\int_T V_{Ax}(t)\, i_{aA}(t)\,dt$$

$$P_B = \frac{1}{T}\int_T V_{Bx}(t)\, i_{bB}(t)\,dt$$

$$P_C = \frac{1}{T}\int_T V_{cx}(t)\, i_{cc}(t)\,dt$$

$$P_{total} = P_A + P_B + P_C \quad \dots ①$$

$$V_{AX}(t) = V_{AN}(t) + V_{NX}(t)$$

$$V_{BX}(t) = V_{BN}(t) + V_{NX}(t)$$

$$V_{CX}(t) = V_{CN}(t) + V_{NX}(t)$$

in this case $P_{3\phi} \neq 3P_{\phi}$

$$P_{total} = \frac{1}{T}\int_T \left(V_{AX}(t)\, i_{aA}(t) + V_{BX}(t)\, i_{bB}(t) + V_{CX}(t)\, i_{cC}(t)\right) dt$$

$$P_{total} = \frac{1}{T}\int_T \left(V_{AN}(t)\, i_{aA}(t) + V_{BN}(t)\, i_{bB}(t) + V_{CN}(t)\, i_{cC}(t)\right)dt + \cancelto{0}{\frac{1}{T}\int_T V_{Nx}\underset{\substack{\parallel \\ 0 \text{ by } \underline{\underline{\text{KCl}}}}}{\underline{\underline{\left(i_{aA}(t)+i_{bB}(t)+i_{cC}(t)\right)}}}\,dt}$$

$$P_t = \frac{1}{T}\int_T \left(V_{AN}(t)\, i_{aA}(t) + V_{BN}(t)\, i_{bB}(t) + V_{CN}(t)\, i_{cC}(t)\right)dt$$

$$= \frac{1}{T}\int_T V_{AN}(t)\, i_{aA}(t)\,dt + \frac{1}{T}\int_T V_{BN}(t)\, i_{bB}(t)\,dt + \frac{1}{T}\int_T V_{CN}(t)\, i_{cC}(t)\,dt$$

$P_t = P_A + P_B + P_C$ same as before, so we can connect PC with x $\binom{\text{any}}{\text{point}}$ instead of N and still get the same readings:)

12/3/2013

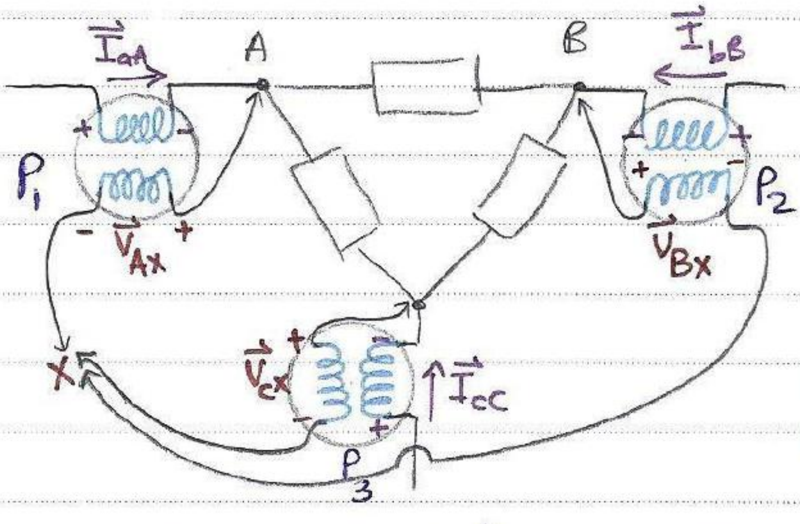


Fig.1

$$P_{total} = P_1 + P_2 + P_3 = \frac{1}{T}\int_T i_{aA}(t)V_{Ax}(t)dt + \frac{1}{T}\int_T i_{bB}(t)V_{Bx}(t)dt \quad \text{①}$$

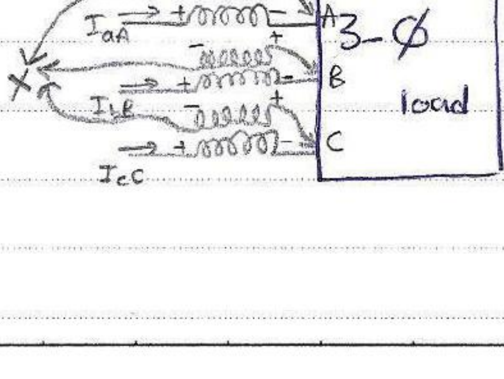
$$+\frac{1}{T}\int_T i_{cC}(t)V_{Cx}(t)dt$$

$$\rightarrow \left.\begin{aligned} i_{aA}(t) &= i_{AB}(t) - i_{CA}(t) \\ i_{cC}(t) &= i_{CA}(t) - i_{BC}(t) \\ i_{bB}(t) &= i_{BC}(t) - i_{AB}(t) \end{aligned}\right\} \text{plug them in ①}$$

$$P_t = \frac{1}{T}\int_T i_{AB}(V_{Ax}-V_{Bx}) + i_{CA}(V_{Cx}-V_{Ax}) + i_{BC}(V_{Bx}-V_{Cx})\, dt$$

$$= \frac{1}{T}\int_T i_{AB}V_{AB}\,dt + \frac{1}{T}\int_T i_{CA}V_{CA}\,dt + \frac{1}{T}\int_T i_{BC}V_{BC}\,dt$$

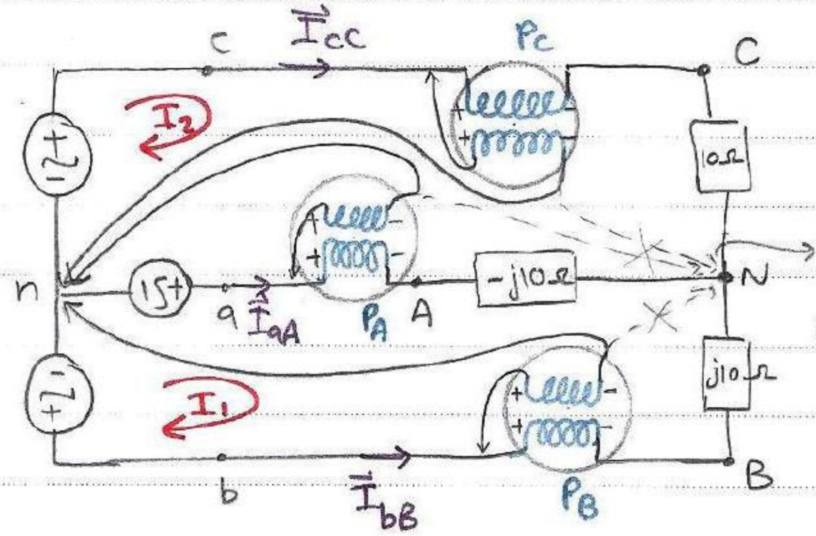
$$= P_A + P_C + P_B \text{ as before}$$



ex 1: A balanced Y-connected source is supplying an unbalanced Y-connected load.

Given $V_{ab} = 100 \angle 0^\circ$ V rms إذا اعطاها ولم يذكر ← نعتبر rms

- calculate the avg power absorbed by each load.



$V_{AN} \neq V_{an}$ (there's no neutral line here).

Sol: $V_L = 100$ V

$V_{ab} = 100 \angle 0^\circ$ V

$V_{bc} = 100 \angle -120^\circ$ V

$V_{ca} = 100 \angle 120^\circ$ V

$V_{an} = \frac{100}{\sqrt{3}} \angle -30^\circ$ V

$V_{bn} = 100/\sqrt{3} \angle -150^\circ$ V

$V_{cn} = 100/\sqrt{3} \angle -270^\circ$ V

mesh analysis:-

@ loop 1:-

$-V_{ab} \cancel{-j10(I_1 - I_2)} + \cancel{j10 I_1} = 0$

$\vec{I}_2 = \frac{100}{j10} \angle 0^\circ = 10 \angle -90^\circ$ Arms

@ loop 2:-

$-V_{ca} + 10 I_2 - j10(I_2 - I_1) = 0$

$\vec{I}_1 = 18.66 - j5$ Arms

$\vec{I}_{cC} = \vec{I}_2 = 10 \angle -90^\circ$ Arms

$\vec{I}_{bB} = -\vec{I}_1 = -18.66 + j5$ Arms

$I_{aA} = \vec{I}_1 - \vec{I}_2 = 19.318 \angle 15$ Arms

$P_A = |\vec{V}_{an}| \, |\vec{I}_{aA}| \cos(\text{ang } \vec{V}_{an} - \text{ang } \vec{I}_{aA})$

$= \frac{100}{\sqrt{3}} \times 19.318 \times \cos(-30 - 15^\circ) = 788.7$ Watt delivered power (we used active sign convention).

$P_B = |\vec{V}_{bn}|\,|\vec{I}_{bB}| \cos(\text{ang}\,\vec{V}_{bn} - \text{ang}\,\vec{I}_{bB}) = 788.7\,W$ delivered by the source.

$P_C = |\vec{V}_{an}|\,|\vec{I}_{cC}| \cos(\text{ang}\,\vec{V}_{an} - \text{ang}\,\vec{I}_{cC})$
$= -577.35$ Watt ; absorbed power

$P_{total} = P_A + P_B + P_C = 1000$ Watt ; delivered by the source

* If a system isn't balanced we solve 3 CKTs per phase only if there's a neutral line.

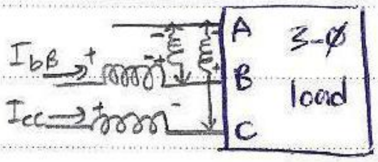
* If it's balanced I solve 1 phase and apply ±120° shift to the others. وجود N-line مابفرق ←

→ <u>OR</u> $P_{total} = |I_{cC}|^2 \times 10 = 1000\,W$ (because $\overset{c}{-j10}$ & $\overset{l}{j10}$ doesn't absorb avg powe).

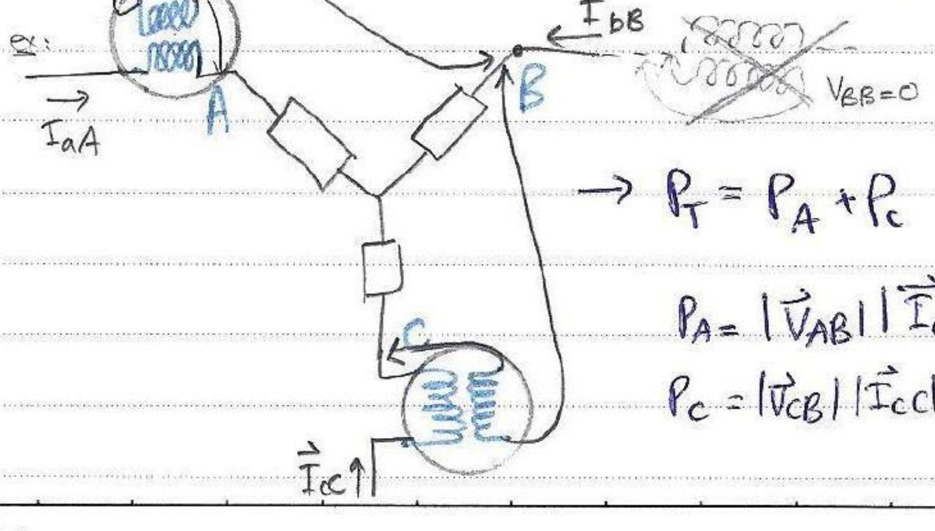
* The Two Wattmeter method:

in Fig 1, let x be A $\Rightarrow \vec{V}_{Ax} = \vec{V}_{AA} = 0$

$\longrightarrow P_T = P_2 + P_3$



ex:



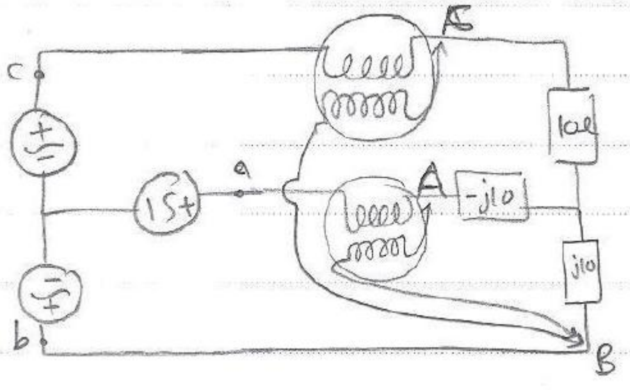
$\rightarrow P_T = P_A + P_c$

$P_A = |\vec{V}_{AB}|\,|\vec{I}_{aA}| \cos(\text{ang}\vec{V}_{AB} - \text{ang}\vec{I}_{aA})$

$P_c = |\vec{V}_{CB}|\,|\vec{I}_{cC}| \cos(\text{ang}\,\vec{V}_{CB} - \text{ang}\,\vec{I}_{cC})$

Return to ex1:

Using 2 wattmeters:-



$P_A = (19.318)(100)\cos(0-15)$

$= 1868$ Watt

$P_C = (100)(10)\cos(60+90^\circ)$

$= -866$ Watt

$P_T = 1000$ Watt #

$V_{cb} = 100\angle 60^\circ$ V

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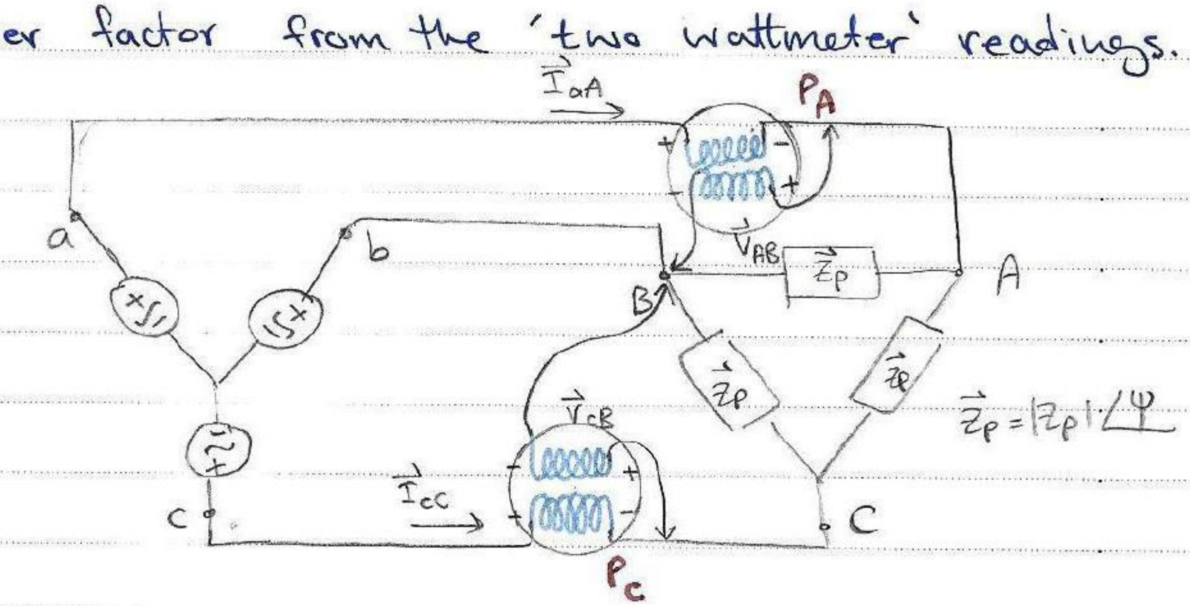
* power factor from the 'two wattmeter' readings.



$\vec{Z}_P = |Z_P| \angle \Psi$

$P_{total} = P_A + P_C$

P.F ? → Ψ ?

$$P_A = |\vec{V}_{AB}|\,|\underline{\vec{I}_{aA}}| \cos(\text{ang } \vec{V}_{AB} - \text{ang } \vec{I}_{aA}) \quad \ldots ①$$

$\hookrightarrow \quad \vec{I}_{aA} = \sqrt{3} \measuredangle{-30}\ \underline{\vec{I}_{AB}}$

$= \sqrt{3} \measuredangle{-30} (\vec{V}_{AB} / \vec{Z}_P)$

$\therefore |\vec{I}_{aA}| \measuredangle \vec{I}_{aA} = \dfrac{\sqrt{3} \measuredangle{-30°}\ |\vec{V}_{AB}| \ \measuredangle \vec{V}_{AB}}{|Z_P| \ \measuredangle \Psi}$

$\text{ang } \vec{I}_{aA} = -30 + \text{ang}(\vec{V}_{AB}) - \Psi$

$\therefore \text{ang}(\vec{V}_{AB}) - \text{ang}(\vec{I}_{aA}) = \Psi + 30$

plug in ①

$$\boxed{P_A = V_L I_L \cos(\Psi + 30)}$$

$$P_C = |\vec{V}_{CB}|\,|\vec{I}_{cC}| \cos(\text{ang } \vec{V}_{CB} - \text{ang } \vec{I}_{cC}) \quad \ldots ②$$

$\hookrightarrow \quad \vec{I}_{cC} = \sqrt{30} \measuredangle{-30}\ \vec{I}_{CA}$

$= \sqrt{30} \measuredangle{-30} (\vec{I}_{\underline{BC}} \measuredangle{-120})$

$= \sqrt{30} \measuredangle{-30} (\vec{I}_{CB} \angle{-120+180°}) = \sqrt{3}\ \vec{I}_{CB} \measuredangle{30°}$

/ /

$$\longrightarrow \quad |\vec{I}_{cc}| \measuredangle I_{cc} = \frac{\sqrt{3} \measuredangle 30 \; \vec{V}_{CB}}{\vec{Z}_p} = \frac{\sqrt{3} \measuredangle 30 \; |\vec{V}_{CB}| \measuredangle V_{CB}}{|\vec{Z}_p| \measuredangle \psi}$$

plug in ②:-

$$\boxed{P_c = V_L I_L \cos(\psi - 30^\circ)}$$

$$\therefore \frac{P_A}{P_c} = \frac{\cos(\psi + 30)}{\cos(\psi - 30^\circ)}$$

$$= \frac{\cos\psi\cos 30^\circ - \sin\psi \sin 30^\circ}{\cos\psi \cos 30^\circ + \sin\psi \sin 30^\circ} \times \left(\frac{\cos\psi\cos 30^\circ}{\cos\psi\cos 30^\circ}\right)$$

$$\frac{P_A}{P_c} = \frac{1 - \tan\psi \tan 30^\circ}{1 + \tan\psi \tan 30^\circ}$$

$$P_A + \frac{P_A}{\sqrt{3}} \tan\psi = P_c - \frac{P_c}{\sqrt{3}} \tan\psi$$

$$\frac{\tan\psi}{\sqrt{3}} (P_A + P_c) = P_c - P_A$$

$$\therefore \tan\psi = \sqrt{3} \left(\frac{P_c - P_A}{P_c + P_A}\right) \quad \text{... 4 cases!}$$

1. if $P_c = P_A \Rightarrow \tan\psi = 0$, $\psi = 0$ } P.F = 1 (pure resistive load).
2. if $P_c = -P_A \Rightarrow \tan\psi = \infty$, $\psi = \pm 90^\circ$ } P.f = 0 (pure reactive load).
3. if $P_c > P_A \Rightarrow \tan\psi$, ψ +ve $\rightarrow$ P.F lagging (inductive load).
4. if $P_A > P_c \Rightarrow \tan\psi \rightarrow \psi$ -ve $\rightarrow$ P.F leading (capacitive load).

/ /

$$\boxed{\tan \psi = \frac{\sqrt{3}\,(P_1 - P_2)}{P_1 + P_2}}$$

** If we don't know which terminal is A and which is C , or the common terminal was A or C instead of B , then:

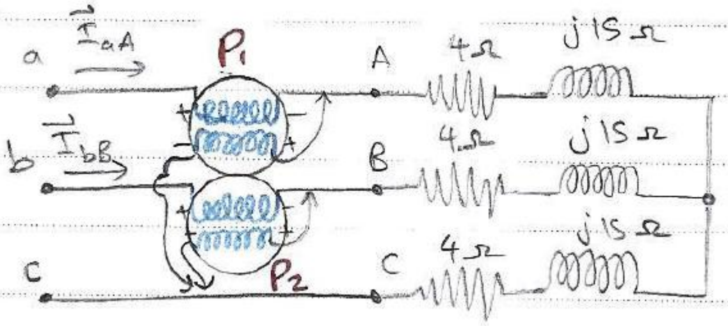
$\tan \psi = \frac{\sqrt{3}\,(P_1 - P_2)}{P_1 + P_2}$, can be used to find the magnitude of ψ , but not its sign then we can only determine the magnitude of the power factor without telling if it's leading or lagging.

⇒ to find the sign of ψ (if p.f is leading or lagging):-

→ Add a high reactive load in parallel with the original load.

- If the reactive load is 3-ϕ capacitive •
  - if the magnitude of ψ decreases , then the original load is inductive (lagging p.f).
  - if the magnitude of ψ increases , then the original load is capacitive (leading p.f)

<u>ex</u>: A Balanced load is fed by a balanced source with $\vec{V}_{ab} = 230\angle 0^\circ\ V_{rms}$, find: P_1, P_2 & P_{total} ?



• $P_1 = |\vec{V}_{AC}||\vec{I}_{aA}| \cos(\text{ang } \vec{V}_{AC} - \text{ang } \vec{I}_{aA})$ ...①

but $\vec{V}_{bc} = 230\angle -120^\circ\ V_{rms}$

$\vec{V}_{ca} = 230\angle -240^\circ\ V_{rms}$

$\vec{V}_{cA} = \vec{V}_{ca}$; since there's no impedance through transmition lines.

↳ $\therefore \vec{V}_{AC} = 230\angle -240+180 = 230\angle -60^\circ\ V_{rms}$

↳ $\vec{I}_{aA} = \dfrac{\vec{V}_{an}}{Z_p} = \dfrac{V_{AN}}{4+j15} = \dfrac{\frac{230}{\sqrt{3}}\angle -30^\circ}{4+j15} = 8.554\angle -105.1^\circ\ A_{rms}$

plug in ①

$P_1 = (230)(8.554)\cos(-60+105.1)$

$= 1389$ Watt

• $P_2 = |\vec{V}_{BC}|\underset{\sim\sim}{|\vec{I}_{bB}|}\cos(\text{ang } \vec{V}_{BC} - \text{ang } \vec{I}_{bB})$ ...②

↳ $\vec{I}_{bB} = \vec{I}_{aA}\angle -120$

$= 8.554\angle -225.1^\circ\ A_{rms}$

↳ $\vec{V}_{BC} = \vec{V}_{bc} = 230\angle -120^\circ\ V_{rms}$

plug in ② $P_2 = (230)(8.554)\cos(-120+225.1) = -512.5$ Watt

$\therefore P_{total} = P_1 + P_2 = 876.5$ Watt

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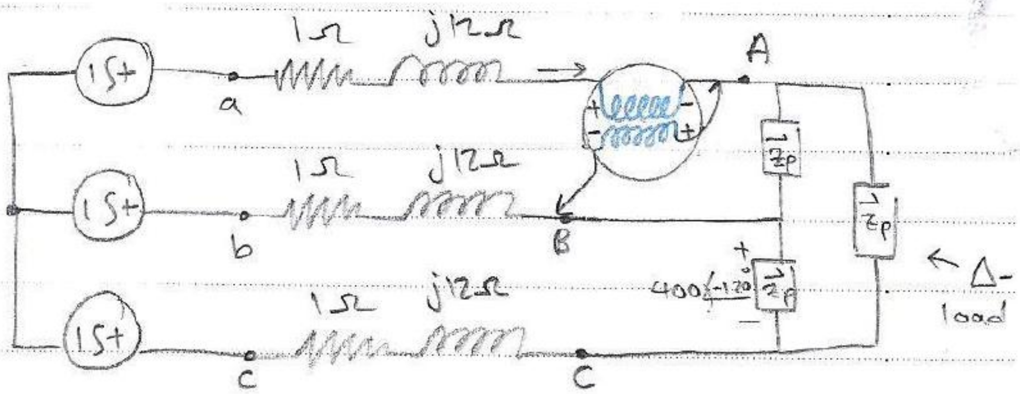
example 1: The wattmeter reading of 3-ϕ balanced load is 2400 Watt & has a p.f of 0.9196 lagging. Assume +ve phase sequence, find:

a) the power consumed by the load.
b) the line current $\vec{I}_{bB}$ & phase current $\vec{I}_{BC}$ of the load.
c) The voltage $\vec{V}_{bn}$ & $\vec{V}_{an}$
d) Calculate $\vec{Z}_p$ & total power loss of the line.
e) the complex power supplied by the source.



Sol:- $\vec{V}_{AB} = 400\angle 0°$ in ⓐ $P \neq 2400\times 3$ — Be careful, it's not the same current! it's I_{aA} not I_{AB}

$\vec{V}_{CA} = 400\angle 120$

$-180°$ $\vec{V}_{AC} = 400\angle -60°$

$$2400 = |\vec{V}_{AB}|\,|\vec{I}_{aA}|\,\underbrace{\cos(\text{ang}\,\vec{V}_{AB} - \text{ang}\,\vec{I}_{aA})}_{\equiv p.f}$$

$$2400 = 400\,|\vec{I}_{aA}| \times 0.9196$$

$$|\vec{I}_{aA}| = 6.525\ A_{rms}$$

but... $\cos(\text{ang}\,\vec{V}_{AB} - \text{ang}\,\vec{I}_{aA}) = p.f$

$$-\text{ang}\,I_{aA} = \cos^{-1} 0.9196$$

$$\text{ang}\,\vec{I}_{aA} = -23.132°.$$

$$\vec{I}_{aA} = 6.525\angle -23.132°\ A_{rms}.$$

& $\to \vec{I}_{aA} = \sqrt{3} \angle -30^\circ \, \vec{I}_{AB}$

$|I_{AB}| = |I_{aA}|/\sqrt{3} = 3.767 \; A_{rms}$

$\text{ang } \vec{I}_{AB} = (\text{ang } I_{aA} + 30) = 6.867^\circ$

$\Rightarrow \vec{I}_{AB} = 3.767 \angle 6.867^\circ$

a) $P_A = |\vec{V}_{AB}||\vec{I}_{AB}| \cos(\text{ang } \vec{V}_{AB} - \text{ang } \vec{I}_{AB})$

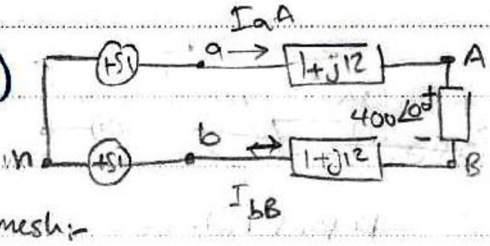
$= 1495.99$ Watt

$P_{total} = P_A \times 3 = 4487.966$ watt ; since it's balanced

b) $\vec{I}_{bB} = 6.525 \angle -23.132 - 120^\circ = 6.525 \angle -143.132^\circ$

$\vec{I}_{BC} = 3.767 \angle -113.132^\circ \; A_{rms} \to (I_{AB} \angle -120)$

c)



or you can transform Δ→Y

$\boxed{V_{an} = 241.19 \angle -11.0^\circ \; V_{rms}}$

mesh:-

$-V_{an} + (1+j12)I_{aA} + 400\angle 0^\circ - (1+j12)I_{bB} + V_{bn} = 0$

$V_{bn} - V_{an} = -417.75 \angle 18.998 \longrightarrow V_{an} \angle -120^\circ - V_{an} = -417.75 \angle 18.998^\circ$

d) $\vec{Z}_P = \vec{V}_{AB} / \vec{I}_{AB}$

$= 400\angle 0^\circ / 3.767 \angle 6.867 = 106.185 \angle -6.867^\circ \, \Omega$

$P_L = 3 \times |\vec{I}_{aA}|^2 (1) = 127.71$ Watt

e) $\vec{S}_P = \vec{V}_{an} \cdot \vec{I}_{aA}^* = 1573.76 \angle -17.867^\circ$

$\vec{S}_{total} = 3 \times \vec{S}_P .$

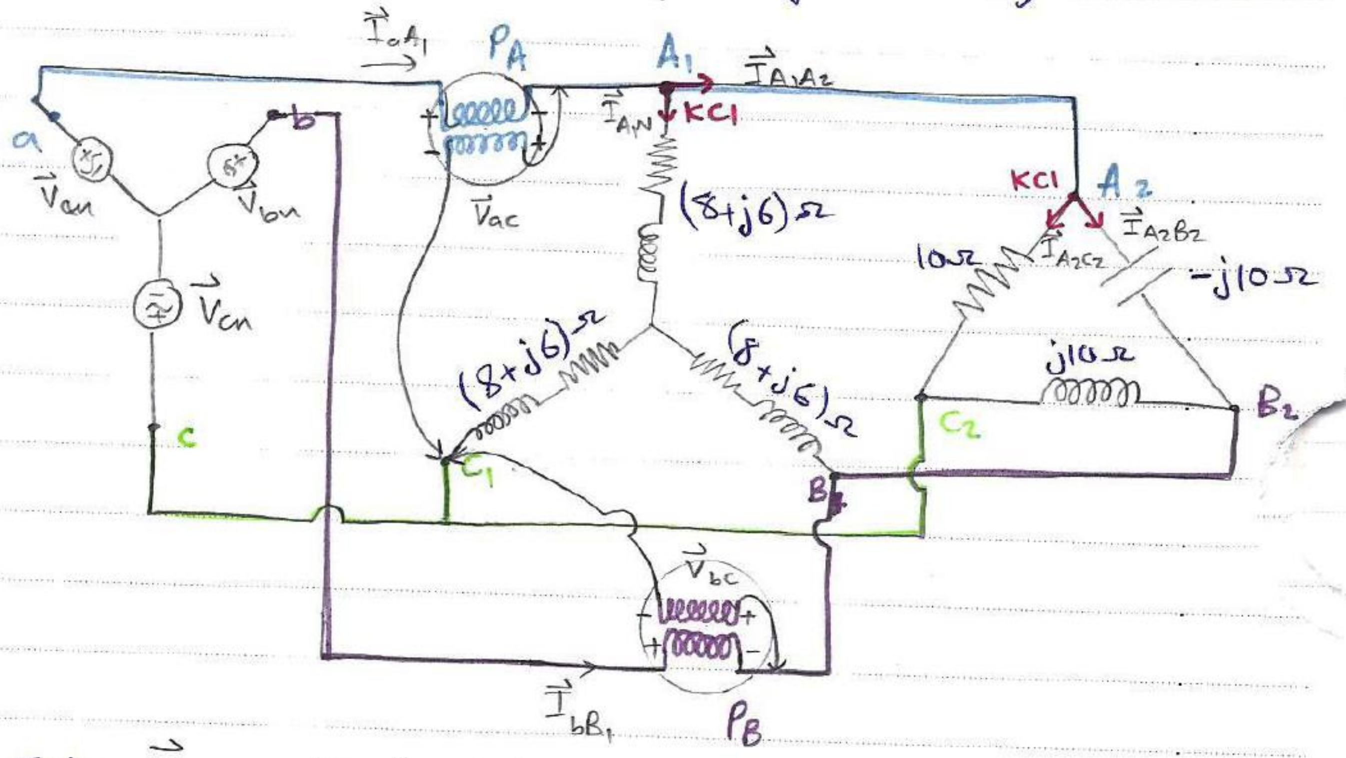
/ /

example 2:- $\vec{V}_{ca} = 400\angle{-270°}$ V_{rms}, determine:-

a) the reading of the two wattmeters P_A & P_B

b) if Both wattmeters are modified to read the reactive power, Q_A, Q_B, determine their readings!

c) determine the complex power by the sonre



Sol:- $\vec{V}_{ca} = 400\angle{-270°}$ V_{rms}

$\vec{V}_{ab} = 400\angle{-30°}$ V_{rms}

$\vec{V}_{bc} = 400\angle{-150°}$ V_{rms}

a) $\vec{I}_{aA_1} = \vec{I}_{A_1N} + \underbrace{\vec{I}_{A_1A_2}}$ KCl

$\hookrightarrow = \vec{I}_{A_2B_2} + \vec{I}_{A_2C_2}$

$\hookrightarrow \vec{I}_{A_2B_2} = \frac{\vec{V}_{A_2B_2}}{-j10} = \frac{\vec{V}_{ab}}{-j10} = \frac{400\angle{-30°}}{10\angle{-90°}} = 40\angle{60°}$ A_{rms}

$\vec{I}_{B_2C_2} = \frac{\vec{V}_{bc}}{j10} = \frac{400\angle{-150°}}{10\angle{90°}} = 40\angle{240°}$ A_{rms}

$\vec{I}_{C_2A_2} = \frac{\vec{V}_{ca}}{10} = 40\angle{-270°}$ Arms.

$\therefore \vec{I}_{A_1A_2} = I_{A_2C_2} + I_{A_2B_2} = 40 \angle -270° + 180° + 40 \angle 60°$
$= 20.705 \angle -150$ Arms

$\vec{I}_{B_1B_2} = \vec{I}_{B_2C_2} + \vec{I}_{B_2A_2} = -40$ Arms

$\vec{I}_{C_1C_2} = \vec{I}_{C_2A_2} + \vec{I}_{C_2B_2} = 20.705 \angle 15°$ Arms

$\hookrightarrow I_{A_1N}$?

$\vec{V}_{A_1N} = \vec{V}_{an} = \frac{\vec{V}_{ab}}{\sqrt{3}} \angle -30° = \frac{400}{\sqrt{3}} \angle -30 - 30° = 23094 \angle -60°$ V_{rms}

$\vec{I}_{A_1N} = \frac{\vec{V}_{A_1N}}{8+j6} = 23.094 \angle -96.87°$ Arms

Since it's a balanced load:-

$\vec{I}_{B_1N} = \vec{I}_{A_1N} \angle -120° = 23.094 \angle -216.87°$ Arms

$\vec{I}_{C_1N} = \vec{I}_{B_1N} \angle -120° = 23.094 \angle -336.87°$ Arms.

$\Rightarrow I_{aA_1} = I_{A_1N} + I_{A_1A_2} = 33.125 \angle -58.64°$ Arms

$I_{bB_1} = I_{B_1N} + I_{B_1B_2} = 60.094 \angle 165.67°$ Arms

$I_{cC_1} = I_{C_1N} + I_{C_1C_2} = 43.69 \angle 19.287°$ Arms

a) $P_A = |\vec{V}_{ac}| \, |\vec{I}_{aA_1}| \cos(\text{ang}\,\vec{V}_{ac} - \text{ang}\,\vec{I}_{aA_1})$
$= 11314.75$ watt

$P_B = |\vec{V}_{bc}| \, |\vec{I}_{bB_1}| \cos(\text{ang}\,\vec{V}_{bc} - \text{ang}\,\vec{I}_{bB_1})$
$= 17485.28$ watt

b) $Q_A = |\vec{V}_{ac}| \, |\vec{I}_{aA_1}| \sin(\text{ang}\,\vec{V}_{ac} - \text{ang}\,\vec{I}_{aA_1})$
$= -68$ VAR

$Q_B = |\vec{V}_{bc}| \, |\vec{I}_{bB_1}| \sin(\text{ang}\,\vec{V}_{bc} - \text{ang}\,I_{bB_1})$
$= 16494.6$ VAR.

c) $\vec{S}_{Source} = \boxed{(P_A + P_B) + j(Q_A + Q_B).}$

Chapter #13:

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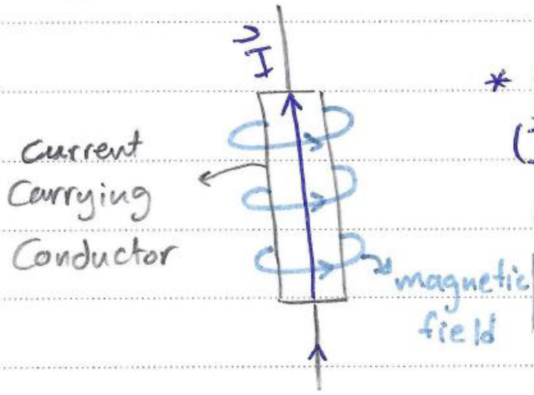
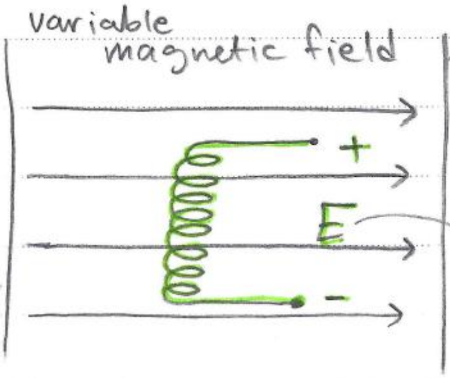
Magnetically Coupled Circuits:



* Orested has discovered that when $(\vec{I})$ passes through a current carrying conducter, a magnetic field is created.

*to know the direction of the magnetic field (right hand rule); thumb is in the direction of $\vec{I}$, other fingers with the magnetic field.

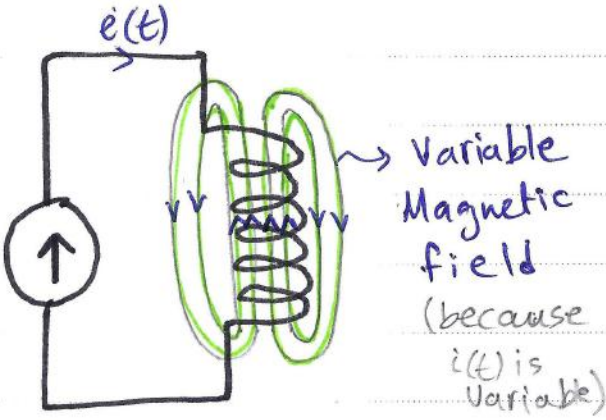
- Faraday's law:



المفروض المجال المغناطيسي يكون عبارات مغلقة لكن هنا فرضناه بهذا الشكل فقط للتوضيح.

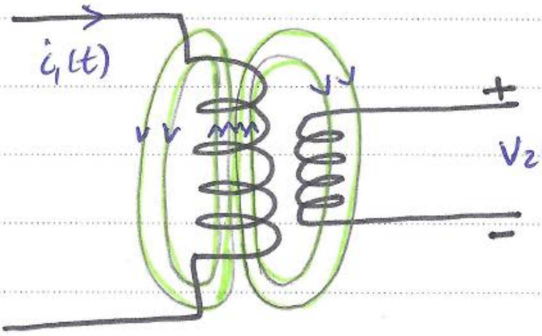
E: induced voltage.

→ Creating a variable Magnetic field:-



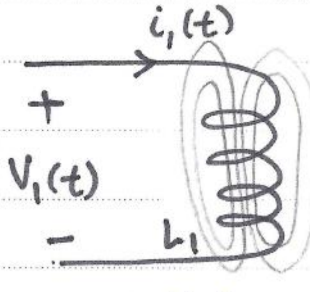
* to determine M.field direction we use the right hand rule; Thumb with M.field while other finger with the current.

/ /



$V_2(t)$; Induced Voltage

يجب أن يكون داخل مجال الملف الأول المغناطيسي
لينتج $V_2(t)$!



Coil 1

$$V_1(t) = L_1 \frac{d\, i_1(t)}{dt}$$

L_1 is the self inductance [H]

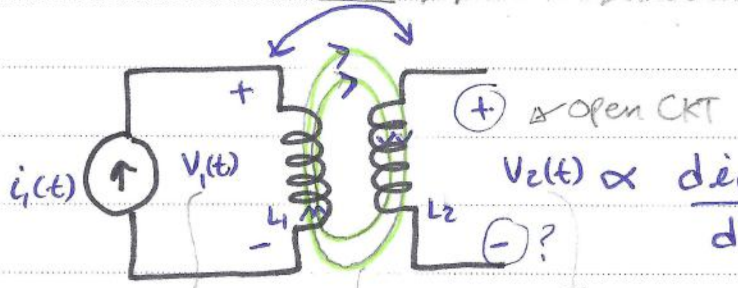
if i_1 was a DC current , V_1 & V_2 = 0

& the Magnetic field will be constant!

*

الاسم الرسمي:

$\underline{M_{21}}$; Mutual Inductance [H] / coeficient of Mutual inductance



(+) ← open CKT

(−) ?

$$V_2(t) \propto \frac{d i_1(t)}{dt}$$ (directly proportional).

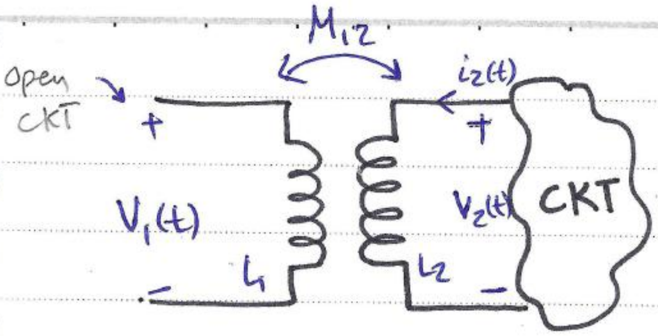
ناتجة عن ال self-inductance

induced Voltage

Variable flux ∅

$$V_1(t) = L_1 \frac{d i_1(t)}{dt} \quad , \quad V_2(t) = M_{21} \frac{d i_1(t)}{dt}$$

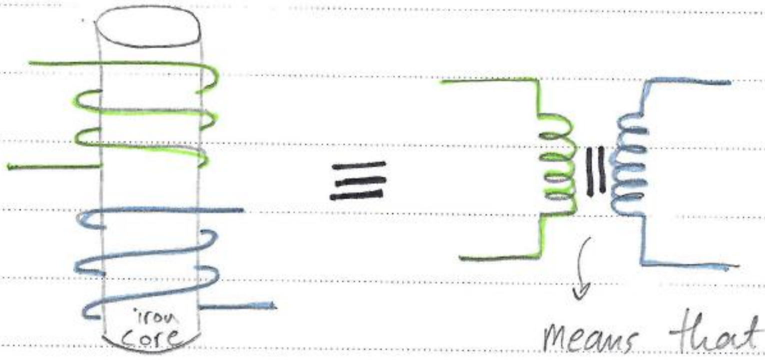
Self-induced Voltage　　　Mutual induced Voltage



$$V_2(t) = \underset{\text{self inductance}}{\underline{L_2}} \frac{d\, i_2(t)}{dt} \quad , \quad V_1(t) = \underset{\text{Mutual inductance}}{\underline{M_{12}}} \frac{d\, i_2(t)}{dt}$$

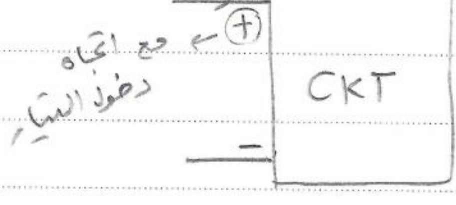
اثباتها بعدين! ← $\boxed{M_{21} = M_{12}} = M \quad [H]$

* M is measured in a magnetically coupled circuit & it's not a constant for an inductor

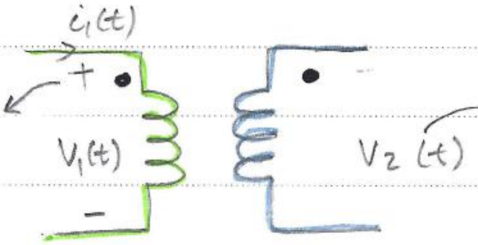


means that they're on the same core

* determining the induced Voltage's polarity:- (Dot convention or Mark convention).



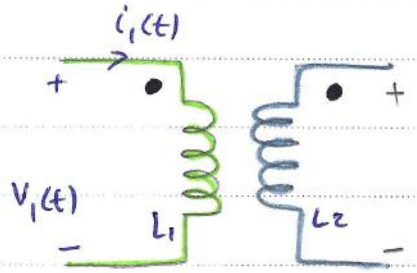
to determine Voltage's polarity in a CKT we use "passive sign convention" but, in a Magnetically Coupled CKT, we use "Dot/Mark convention".



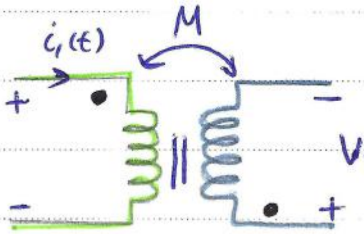
self-induced Voltage ($V_1(t)$) is determined using passive sign convention

الموجب مع اتجاه دخول التيار،

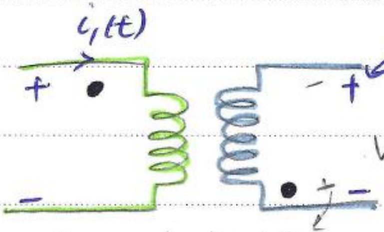
to determine V_2's (Mutual induced) polarity we use the Dot convention; we have 4 different combinations of Dot's locations.



- if the current enters the dotted terminal of one coil, then it produces a +ve refferance at the dotted terminal of the 2nd coil.



$$V_2(t) = +M\frac{d\,i_1(t)}{dt}$$



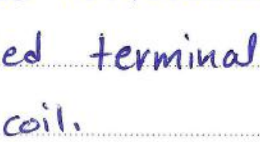
محدد من السؤال

$$V_2(t) = -M\frac{d\ i_1(t)}{dt}$$

المفروض بناءً على الـ Dot conv

يكون موجب لذا نضع (−) في القانون

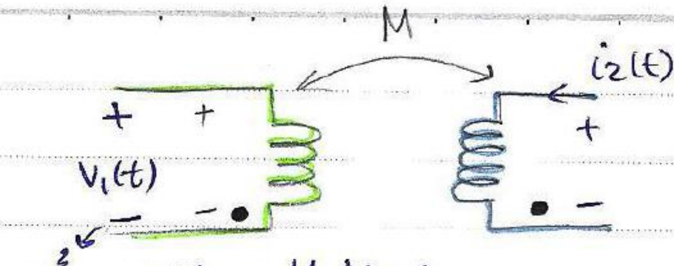
- if the current enters the undotted terminal of one coil, the the Voltage is +vely sensed at the undotted terminal of the ~~ϕ~~ 2nd coil.



$$V_2 = M\frac{d\,i_1(t)}{dt}$$

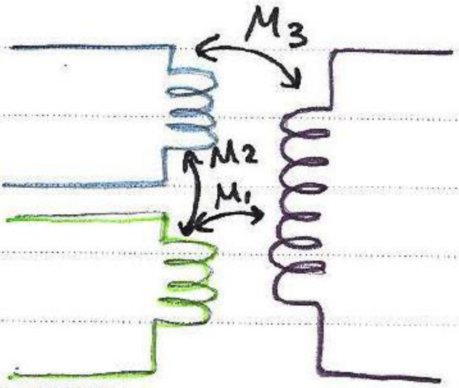
$$V_2 = -M\frac{d\ i_1(t)}{dt}$$

من السؤال



* if $M=0$, mutual induced voltage $=0$ بيكونوا بعاد عن بعض

$$V_1(t) = +M\frac{d\,i_2(t)}{dt}$$



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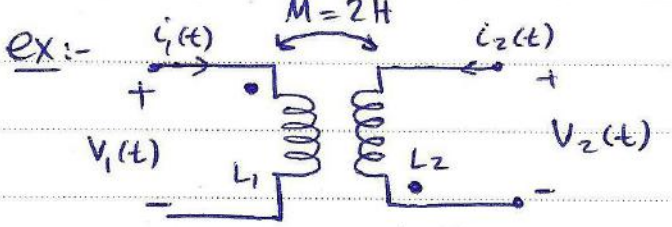
ex:-



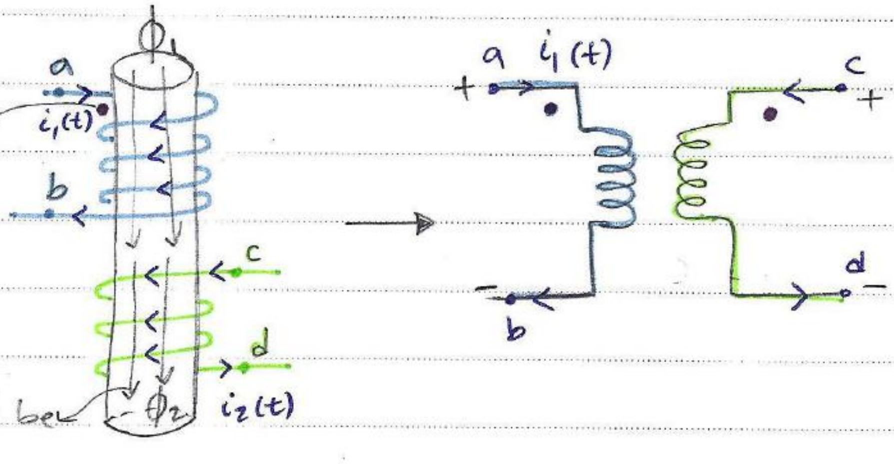
a) find $V_1(t)$ if $i_2 = 5\sin(45t)$ A , $i_1 = 0$

$$V_1(t) = -M\frac{d\,i_2(t)}{dt} = -2\left(5\times 45\cos(45t)\right) = -450\cos(45t)V$$

b) find $V_2(t)$ when $i_1(t) = -8e^{-t}$ A, $i_2 = 0$

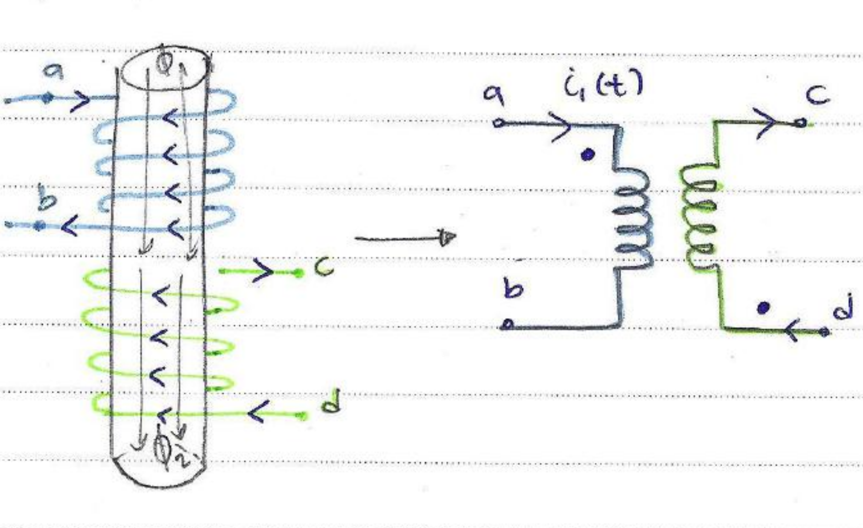
$$V_2(t) = -M\frac{d\,i_1(t)}{dt} = -2(-8(-1)e^{-t}) = -16\,e^{-t}\ A$$

* Physical Basics of the Dot convention:-

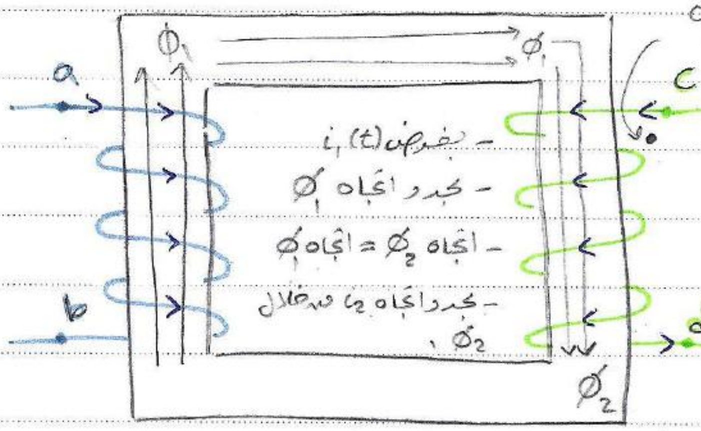


نضع • مع اتجاه دخول التيار الذي فرضناه

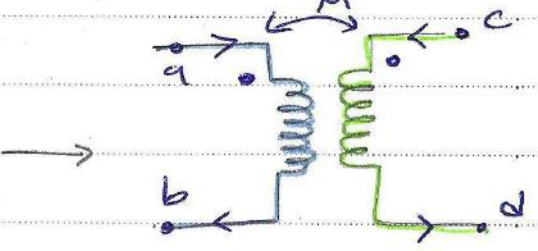
ϕ_2 must be in the same direction of ϕ_1



determined from the direction of Ø2



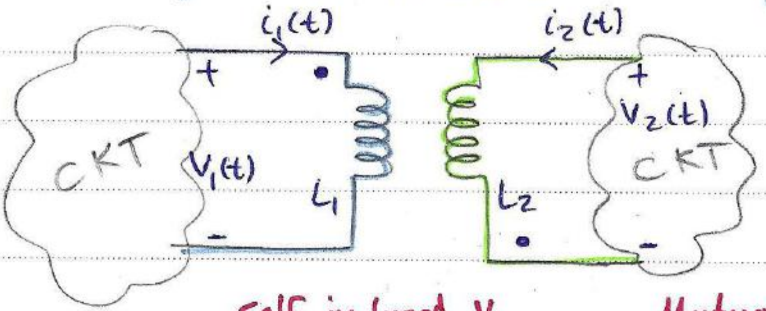
- بفرض $i_1(t)$
- نحدد اتجاه ϕ_1
- اتجاه ϕ_2 = اتجاه ϕ_1
- نحدد اتجاه i2 من خلال ϕ_2 ،



لو عكسنا الملف نعكس الـ dot

* Mutual & Self induced Voltage:-

ex1:



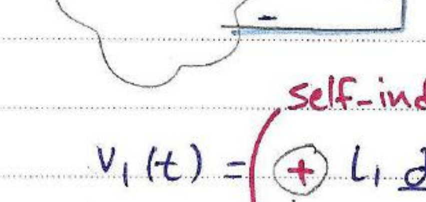
$$V_1(t) = \overbrace{\left(\textcircled{+}\ L_1 \frac{d i_1(t)}{dt}\right)}^{\text{Self-induced V}} + \overbrace{\left(\textcircled{-}\ M \frac{d i_2(t)}{dt}\right)}^{\text{Mutual induced V}}$$

determined using passive sign convention

determined using dot convention.

$$V_2(t) = + L_2 \frac{d i_2(t)}{dt} + - M \frac{d i_1(t)}{dt}$$

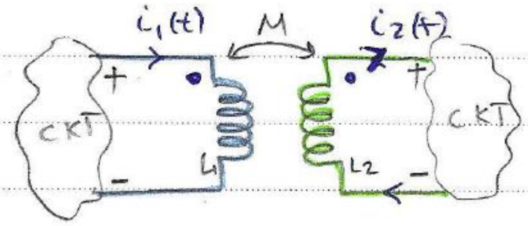
example In phasor domain:



$$\vec{V}_1 = j\omega L_1 \vec{I}_1 + j\omega M \vec{I}_2$$
$$\vec{V}_2 = j\omega L_2 \vec{I}_2 + j\omega M \vec{I}_1$$

/ /

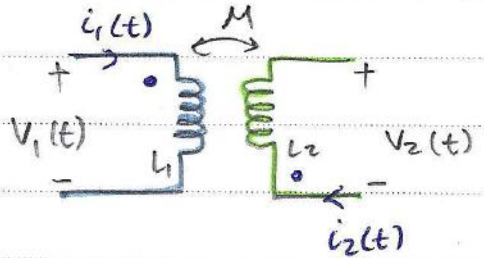
ex2:-



$$V_1(t) = +L_1 \frac{d i_1(t)}{dt} - M \frac{d\, i_2(t)}{dt}$$

$$V_2(t) = -L_2 \frac{d i_2(t)}{dt} + M \frac{d\, i_1(t)}{dt}$$

ex3:-



time-domain $\rightarrow V_1(t) = -L_1 \frac{d\, i_1(t)}{dt} - M \frac{d\, i_2(t)}{dt}$

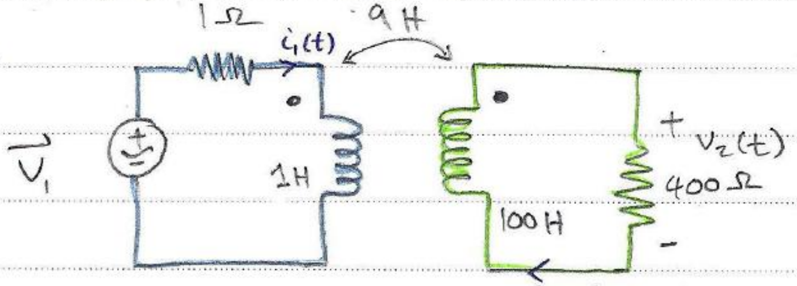
$\hookrightarrow V_2(t) = -L_2 \frac{d\, i_2(t)}{dt} - M \frac{d\, i_1(t)}{dt}$

Phasor-domain $\rightarrow \vec{V}_1 = -j\omega L_1 \vec{I}_1 - j\omega M \vec{I}_2$

على فرض أنها sinusoidal sources $\hookrightarrow \vec{V}_2 = -j\omega L_2 \vec{I}_2 - j\omega M \vec{I}_1$

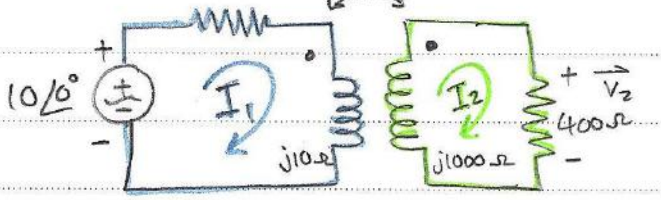
/ /

ex4:-



$i_2(t) \rightarrow$ from passive sign convention.

$V_1(t) = 10\cos(10t)\ V \Rightarrow \vec{V}_1 = 10\angle 0^\circ$, $\omega = 10$ rad/sec.



@ loop 1:-

$$-10\angle 0^\circ + (1)\vec{I}_1 + j10\,\vec{I}_1 - (j90)\vec{I}_2 = 0$$

$$(1+j10)\vec{I}_1 - j90\,\vec{I}_2 = 10\angle 0^\circ \ \ldots ①$$

@loop 2:- $\underbrace{j1000\,\vec{I}_2}_{\text{Self-induced}} \underbrace{- j90\,\vec{I}_1}_{\text{Mutual-induced}} + 400\,\vec{I}_2 = 0$

$$-j90\,\vec{I}_1 + (400+j1000)\vec{I}_2 = 0 \ \ldots ②$$

***Note:-** ** بـ loop2 لما افترضنا I_2 لحساب الـ self-induced V فرضناه بناءً على الـ passive sign convention وبالتالي الـ terminal السفلي أصبح موجب والعلوي سالب! وعندما أحدد الإشارة بناءً على الـ Mutual يكون السفلي سالباً ، لذا نضع إشارة سالب!!

solving ① & ②

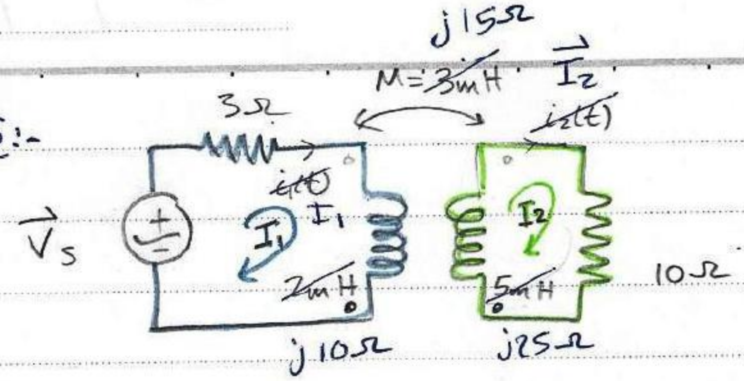
$\vec{I}_1 = (1.614 - j1.284)$ A not in rms

$\vec{I}_2 = (0.165 + j0.0495)$ A

$\vec{V}_2 = 400\,\vec{I}_2 = (66.055 - j19.812)$ V

ex5:

ex5:-



ω = 5krad/sec.

@ loop 1: $-\vec{V}_s + 3\vec{I}_1 + j10\vec{I}_1 - j15\vec{I}_2 = 0$

$(3+j10)\vec{I}_1 - j15\vec{I}_2 = \vec{V}_s$ --- ①

@ loop2: $j25\vec{I}_2 - j15\vec{I}_1 + 10\vec{I}_2 = 0$

$-j15\vec{I}_1 + (10+j25)\vec{I}_2 = 0$ --- ②

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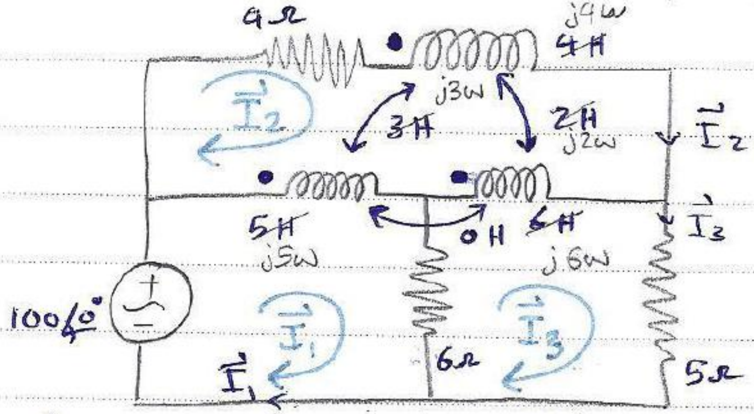
example :-

Find $\vec{I}_1, \vec{I}_2, \vec{I}_3$:



Sol:- @ loop I:-

$$-100\angle 0^\circ + \underbrace{j5\omega(\vec{I}_1-\vec{I}_2)}_{\text{Self-induced}} + \underbrace{\left(j3\omega\vec{I}_2 + j0\omega(\vec{I}_3-\vec{I}_2)\right)}_{\text{Mutual}} + 6(\vec{I}_1-\vec{I}_3) = 0$$

$$\vec{I}_1(6+j5\omega) - \vec{I}_2(j2\omega) - 6\vec{I}_3 = 100\angle 0^\circ \quad \dots ①$$

@ loop 2:

self. Mutual S.

$$4\vec{I}_2 + \underline{j4\omega\vec{I}_2} + \underline{j3\omega(\vec{I}_1-\vec{I}_2)} + \overset{s}{j2\omega}(\vec{I}_3-\vec{I}_2) + \underline{j6\omega(\vec{I}_2-\vec{I}_3)}$$
$$-\underline{j2\omega\vec{I}_2} \overset{M}{-} \overset{o}{0j\omega}(\vec{I}_1-\vec{I}_2) + \underline{j5\omega(\vec{I}_2-\vec{I}_1)} - \underline{j3\omega\vec{I}_2^{M}}$$
$$-\underline{0j\omega(\vec{J}_3-\vec{I}_2) = 0}$$

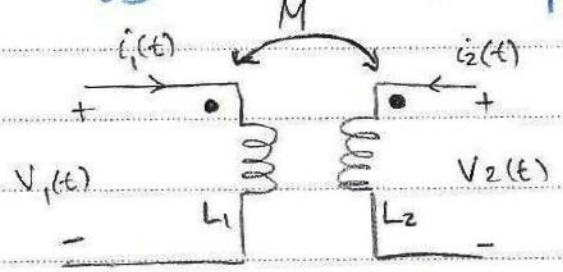
$$(-j2\omega)\vec{I}_1 + (4+j5\omega)\vec{I}_2 - (j4\omega)\vec{I}_3 = 0 \quad \dots ②$$

@ loop 3:-

$$6(\vec{I}_3-\vec{I}_1) + j6\omega(\vec{I}_3-\vec{I}_2) + j2\omega\vec{I}_2 + j0\omega(\vec{I}_1-\vec{I}_2)$$
$$+5\vec{I}_3 = 0$$

$$-6\vec{I}_1 - (j4\omega)\vec{I}_2 + (11+j6\omega)\vec{I}_3 = 0 \dots ③$$

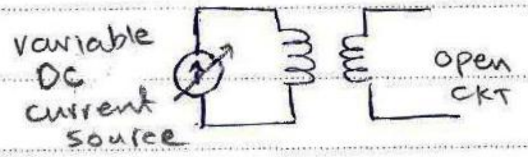
Energy stored & equality of M_{12} & M_{21} :-



@ zero initial conditions.

experiment 1:-

* let $i_2=0$; open circuit.



a) * increase i_1 from $0 \rightarrow I_1$
from $t=t_0 \rightarrow t_1$

instantanuous power → $p_1(t) = v(t)\, i(t) = L_1 \frac{d\, i_1(t)}{dt} \cdot i_1(t)$, $P_2(t) = 0$

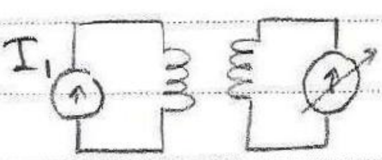
$$\text{Energy} = \int \text{power} \cdot dt$$

$$\underset{\text{coil1e}}{E}_{1,\underset{\text{exp1}}{1}} = \int_{t_0}^{t_1} L_1 \frac{d\, i_1(t)}{\cancel{dt}}\, i_1(t)\, \cancel{dt}$$

$$= L_1 \int_0^{I_1} i_1\, d i_1 = \frac{1}{2} L_1 I_1^2$$

$$E_{2,1} = 0$$

b) now, hold $i_1(t)$ @ I_1 & Increase i_2 from $0 \rightarrow I_2$
from $t=t_1 \rightarrow t_2$



$$E'_{2,1} = \int_{t_1}^{t_2} V_2\, i_2\, dt = L_2 \int_0^{I_2} i_2 d i_2 = \frac{1}{2} L_2 I_2^2 \leftarrow \text{self-induced}$$

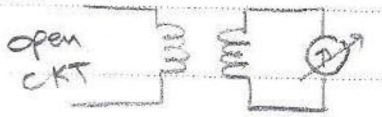
$$E'_{1,1} = \int_{t_1}^{t_2} M_{12} \frac{d i_2(t)}{dt}\, i_1\, dt$$

$$= M_{12} I_1 \int_0^{I_2} d i_2 = M_{12} I_1 I_2$$

$$\Rightarrow \quad E_{T,1} = E_{1,1} + E_{2,1} + E'_{1,1} + E'_{2,1}$$

total E stored in the system $\rightarrow$
$$E_{T,1} = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M_{12} I_1 I_2 \quad \ldots ①$$

experiment 2:



* @ $t=0$, $i_1 = i_2 = 0$ (zero initial conditions)

* increase i_2 from $0 \longrightarrow I_2$ from $t = t_0 \rightarrow t_1$

$$E_{2,2} = \int_{t_0}^{t_1} v_2 i_2 dt = \frac{1}{2} L_2 I_2^2$$

$$E_{1,2} = 0$$

* now, hold i_2 @ I_2 & increase i_1 from $0 \longrightarrow I_1$

from $t = t_1 \rightarrow t_2$

$$E'_{1,2} = \int_{t_1}^{t_2} v_1 i_1 dt = \frac{1}{2} L_1 I_1^2$$

$$E'_{2,2} = \int_{t_1}^{t_2} \underset{\text{Mutualy-induced}}{v_2'} i_2 dt = M_{21} I_2 \int_{t_1}^{t_2} \frac{di_1}{\cancel{dt}} \cancel{dt}$$

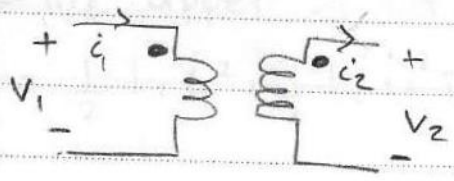
$$= M_{21} I_2 \int_0^{I_1} di_1 = M_{21} I_1 I_2$$

$$E_{T,2} = E_{1,2} + E_{2,2} + E'_{1,2} + E'_{2,2}$$

$$= 0 + \frac{1}{2} I_1^2 L_1 + \frac{1}{2} L_2 I_2^2 + M_{21} I_1 I_2 \quad \ldots ②$$

from ① & ②

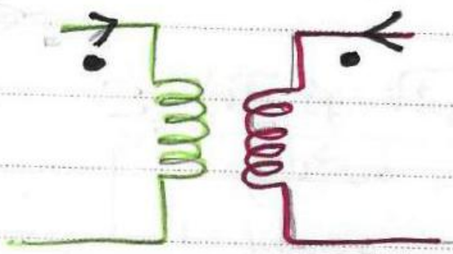
$$E_{T,1} = E_{T,2} \rightarrow \therefore \boxed{M_{21} = M_{12}} \quad \#$$



$$E = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2 - M I_1 I_2$$

in general:-

$$E_T(t) = \frac{1}{2} L_1 i_1^2(t) + \frac{1}{2} L_2 i_2^2(t) \mp M\, i_1(t)\, i_2(t)$$



$\boxed{+MI_1I_2}$

changing the direction of one of the currents <u>or</u> the location of one of the dots changes $M \rightarrow -M$

* عدد فردي من التغييرات ← $-MI_1I_2$ / عدد زوجي من التغييرات ← $+MI_1I_2$

In General

* "if one current enters a dotted terminal while the other leaves a dotted " then the sign of $+\cancel{M}$ MI_1I_2 is reversed"

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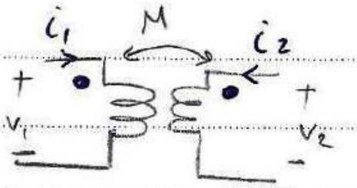
* The upper limit of M:



$$E_T = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 \mp M\, i_1 i_2$$

- if $i_1 \geqslant 0$ & $i_2 \geqslant 0 \Rightarrow E_T$ +ve
- if $i_1 \leq 0$ & $i_2 \leq 0 \Rightarrow$ +ve

E_T could be -ve only if

$$E_T = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 - M\, i_1 i_2$$

اكمال مربع

$$= \underbrace{\frac{1}{2}\left(\sqrt{L_1}\, i_1 - \sqrt{L_2}\, i_2\right)^2}_{\text{could be small as 0}} + \underline{\sqrt{L_1 L_2}\, i_1 i_2 - M i_1 i_2}$$

⇒ to inforce it to be +ve

$$\sqrt{L_1 L_2}\, i_1 i_2 - M i_1 i_2 \geqslant 0$$

$$\boxed{\sqrt{L_1 L_2} \geqslant M}$$

or

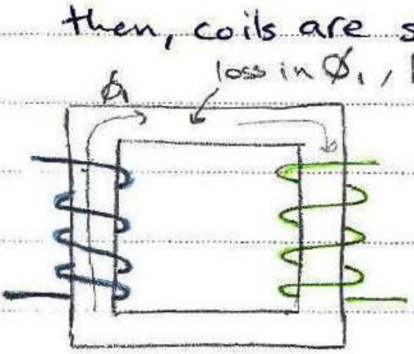
$$k = \frac{M}{\sqrt{L_1 L_2}} \leqslant 1 \quad \ldots$$ coupling coeficient.

$$0 \leq \frac{M}{\sqrt{L_1 L_2}} \leqslant 1$$

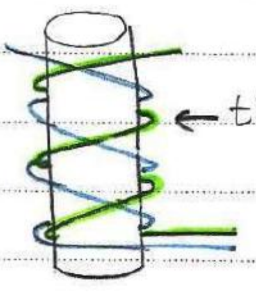
* if $k = 1 \Rightarrow \therefore M = \sqrt{L_2 L_1}$

then, coils are said to be tightly coupled.

loss in $\emptyset_1$, because of the reluctance (R) ; $R\emptyset$ = emf

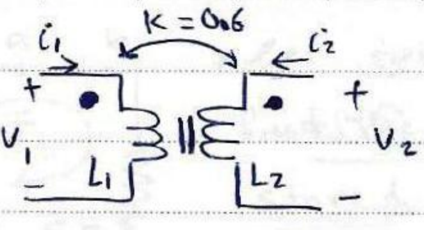


k<1



← tightly coupled

k ≅ 1



ex:- $L_1 = 0.4\ H$

$L_2 = 2.5\ H$

$k = 0.6$

$i_1 = 20\cos(500t - 20^\circ)\ mA$

$i_1 = 4\, i_2$, find :

1) i_2 at $t = 0$

2) V_1 at $t = 0$

3) $W(0)$; total stored energy in the two coils.

Sol:-

1) $i_2(t) = 5\cos(500t - 20^\circ)\ mA$ since: $(i_1 = 4\, i_2)$

$i_2(0) = 4.689\ mA$

2) $V_1(t) = L_1 \frac{d\,i_1(t)}{dt} + \underline{\underline{M}}\, \frac{d\,i_2(t)}{dt}$ $\boxed{M = k\sqrt{L_1 L_2} = 0.6\ H}$

radians → degrees → ** take care **

$\hookrightarrow \frac{d\,i_1(t)}{dt} = -20 \times 500 \sin(\underline{\underline{500t}} - 20^\circ)$

at $0 = -10\sin(-20^\circ)$

$\hookrightarrow \frac{d\,i_2(t)}{dt} = -5 \times 500 \sin(500t - 20^\circ)\ mA$

at $0 = -2.5\sin(-20^\circ)$

$V_1(0) = 0.4\,[-10\sin(-20^\circ)] + 0.6\,[-2.5\sin(-20^\circ)]$

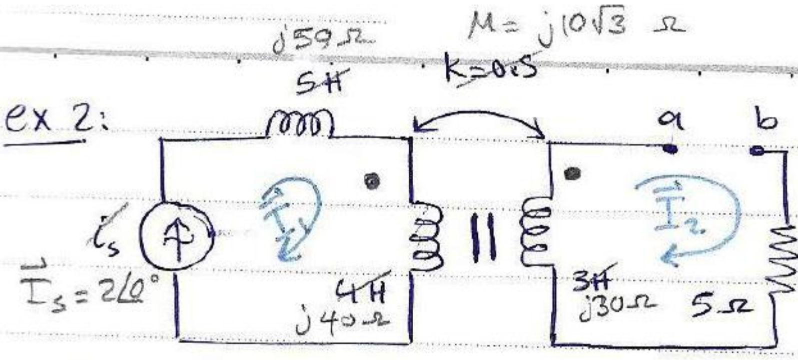
$= 1.881\ V$

3) $W(0)$?

$W(t) = \frac{1}{2} L_1\, i_1^2(t) + \frac{1}{2} L_2\, i_2^2(t) + M\, i_1(t)\, i_2(t)$

$W(0) = (151.2)\,\mu$ Joule

ex 2:



* let $i_s = 2\cos(10t)$ A, find the total energy stored at $t=0$, if:

a) a-b is open-circuited.

b) a-b is short-circuited.

$$M = K\sqrt{L_1 L_2} = \frac{1}{2}\sqrt{3\times4} = \sqrt{3}$$

sol: a) $w(t) = W_{5H} + W_{4H}$ (mutual + self induced).

$$w(t) = \frac{1}{2}(5)\, i_s^2(t) + \frac{1}{2}(4)\, i_s^2(t) + 0 \quad \swarrow ;\ i_2(t)=0 \text{ (open CKT)}$$

$$w(0) = \frac{1}{2}(5)(2)^2 + \frac{1}{2}(4)(2)^2 = 18 \text{ J}$$

b) 1st, transform the CKT to phasor domain.

KCL:-

$$j30\vec{I}_2 - j10\sqrt{3}\,\vec{I}_s + 5\vec{I}_2 = 0$$

$$\Rightarrow (5+j30)\,\vec{I}_2 = j\,10\sqrt{3}\,(2)$$

$$\vec{I}_2 = 1.139 \angle 9.462^\circ \text{ A}$$

back to time domain

$$i_2(t) = 1.139\cos(10t + 9.462^\circ)\text{ A}$$

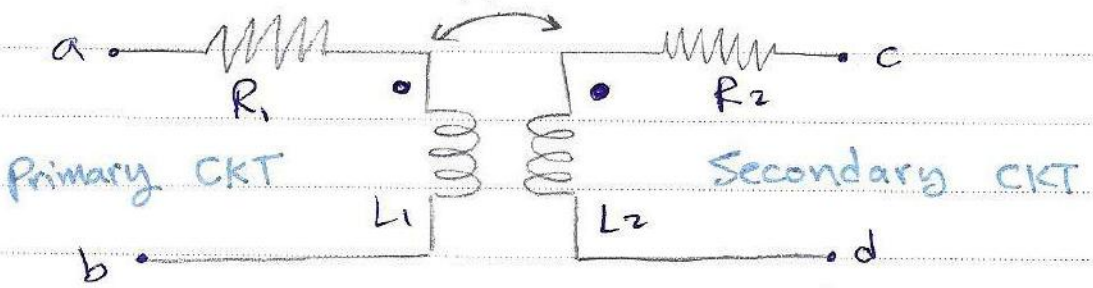
$$i_2(0) = 1.1235\text{ A}$$

$$W(0) = \frac{1}{2}\, 5\, i_s^2(0) + \frac{1}{2}(4)\,(i_s(0))^2 + \frac{1}{2}(3)\,(i_2(0))^2 - \sqrt{3}(2)(i_2(0))$$

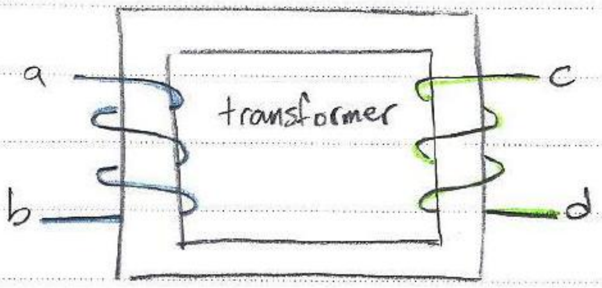
$$= \frac{1}{2}\times5\times2^2 + \frac{1}{2}\times4\times2^2 + \frac{1}{2}\times3\times1.1235^2 - \sqrt{3}\times2\times1.1235$$

$$= 16.001 \text{ J}$$

The linear Transformer:-

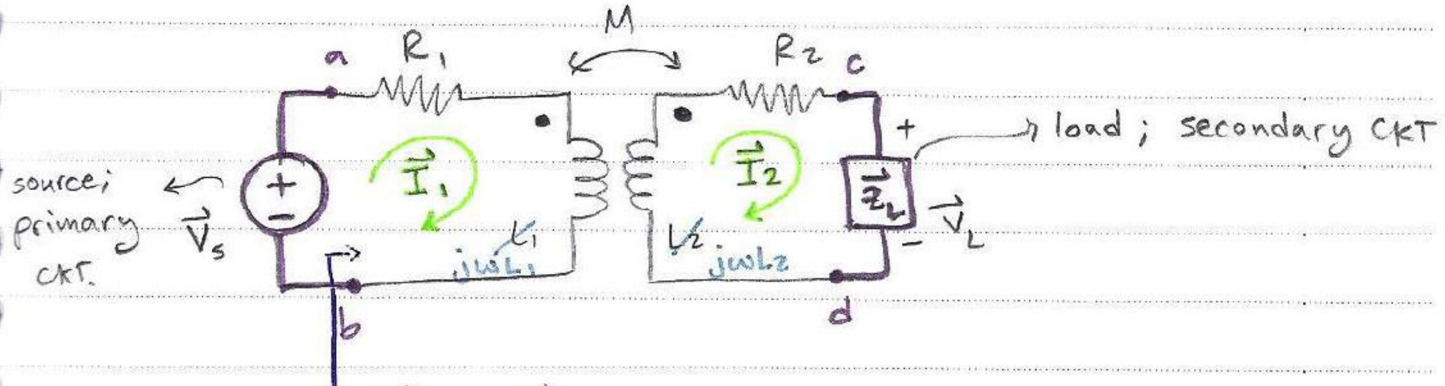


||| equivelant CKT for the transformer



* R_1 & R_2 are the resistances of wires.

* Transformers are used as isolaters.



$\vec{Z}_{in} = \dfrac{\vec{V}_s}{\vec{I}_1}$ → if the secondary CKT was open-circuited, $Z_{in} = R_1 + j\omega L_1$

@ loop 1: $-\vec{V}_s + (R_1 + j\omega L_1)\vec{I}_1 - j\omega M \vec{I}_2 = 0$

$V_s = (R_1 + j\omega L_1)\vec{I}_1 - j\omega M \vec{I}_2$ ... ①

@ loop 2: $(R_2 + j\omega L_2 + \vec{Z}_L)\vec{I}_2 - j\omega M \vec{I}_1 = 0$

<u>Def.</u> let $\vec{Z}_{11} = R_1 + j\omega L_1$

$\vec{Z}_{22} = R_2 + j\omega L_2 + \vec{Z}_L = R_{22} + jX_{22}$

$$\therefore \quad R_{22} = R_2 + Re\{\vec{Z}_L\}$$

$$X_{22} = WL_2 + Im\{\vec{Z}_L\}$$

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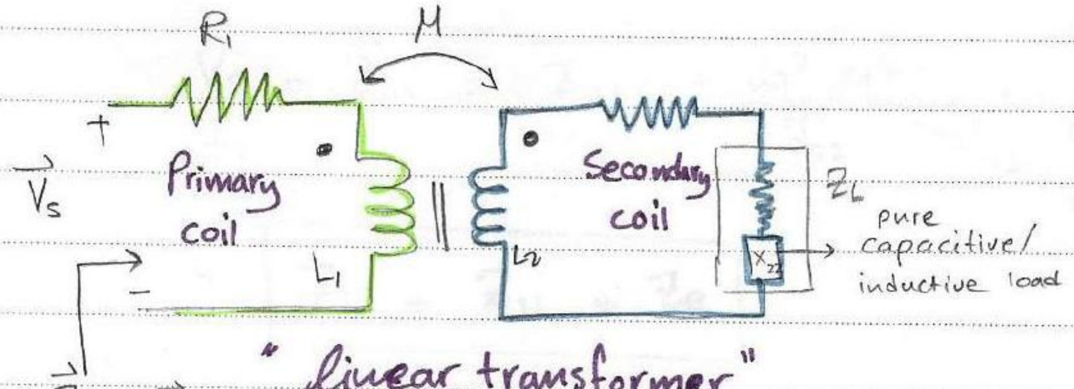


$\vec{Z}_{in} = \frac{\vec{V}_s}{\vec{I}_1}$

" linear transformer"

from the previous lec.

$$\vec{V}_s = (R_1 + j\omega L_1)\,\vec{I}_1 - j\omega M\,\vec{I}_2 \quad \text{...①}$$

$$0 = (R_2 + j\omega L_2 + \vec{Z}_L)\vec{I}_2 - j\omega M\,\vec{I}_1 \quad \text{...②}$$

let:

$$\boxed{\begin{aligned} \vec{Z}_{11} &= R_1 + j\omega L_1 \\ \vec{Z}_{22} &= R_2 + j\omega L_2 + \vec{Z}_L = R_{22} + j X_{22} \\ &\hookrightarrow R_{22} = R_2 + \mathrm{Re}\{\vec{Z}_L\} \\ &\quad\; X_{22} = \omega L_2 + \mathrm{Im}\{\vec{Z}_L\} \end{aligned}}$$

Plug in ① & ②

$$\vec{V}_S = \vec{Z}_{11}\,\vec{I}_1 - j\omega M\,\vec{I}_2 \quad \text{---③}$$

$$0 = \vec{Z}_{22}\,\vec{I}_2 - j\omega M I_1 \quad \text{---④}$$

from ④ $I_2 = \frac{j\omega M I_1}{\vec{Z}_{22}}$

Plug in ③.. $\vec{V}_S = \vec{Z}_{11}\vec{I}_1 + j\omega M\left(\frac{j\omega M\, I_1}{\vec{Z}_{22}}\right)$

$$\frac{\vec{V}_s}{I_1} = \frac{\cancel{I_1}}{\cancel{I_1}}\left(Z_{11} - \frac{(j\omega M)^2}{\vec{Z}_{22}}\right) \longrightarrow$$

$$\frac{V_s}{I_1} = \vec{Z}_{in} = Z_{11} + \frac{\omega^2 M^2}{\vec{Z}_{22}} \ldots$$

this equation doesn't depend on dots location
↳ $(\pm M)^2 = M^2$

$$\boxed{Z_{in} = Z_{11} + Z_R}$$

↳ • $Z_{11} = R_1 + j\omega L_1$

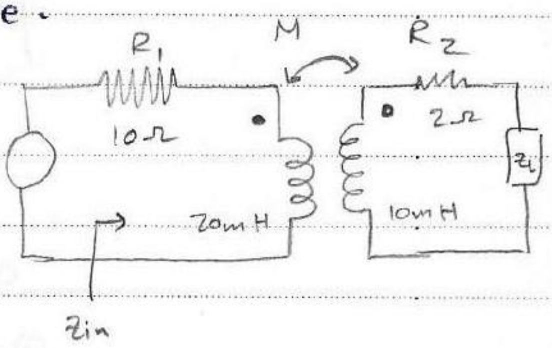
• $\boxed{Z_R = \frac{\omega^2 M^2}{Z_{22}}}$; the reflected impedance

• $Z_{22} = R_2 + j\omega L_2 + \vec{Z}_L$

also $Z_{in} = \vec{Z}_{11} + \frac{\omega^2 M^2 R_{22}}{R_{22}^2 + X_{22}^2} - \frac{j\,\omega^2 M^2 X_{22}}{R_{22}^2 + X_{22}^2}$

ex:- let $\vec{Z}_L = 7\angle 32^\circ$, $f = 50\ Hz$, $M = 800\ mH$, find:

1) the reflected impedance.



$$Z_R = \frac{\omega^2 M^2}{Z_{22}}$$

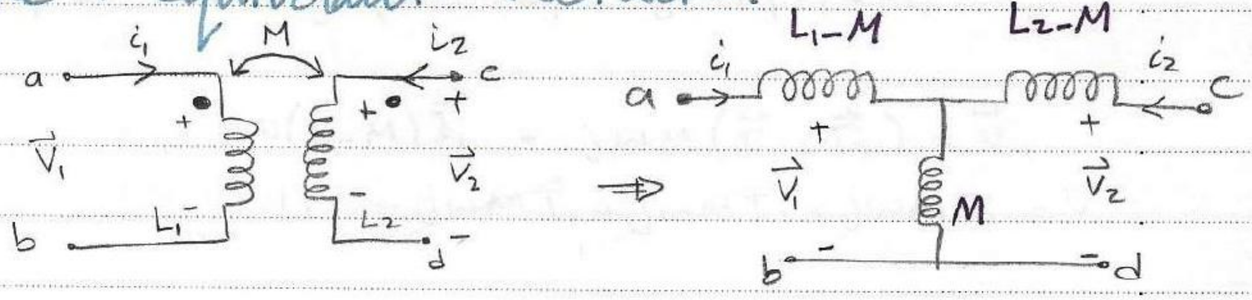
$$= \frac{(2\pi f)^2 M^2}{(R_2 + j\omega L_2 + Z_L)}$$

$$= 4.56 - j\,3.94\ k\Omega$$

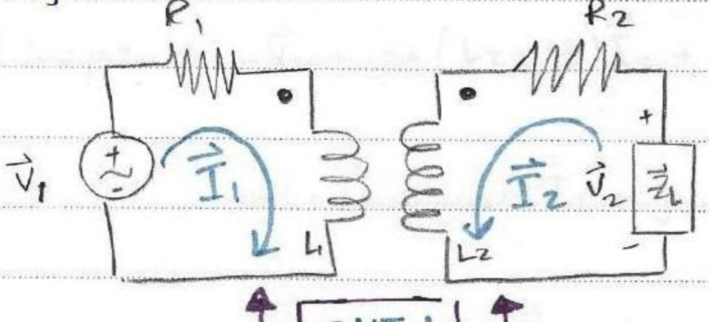
2) Zin?!

$$Z_{in} = Z_{11} + Z_R = 10 + j\,62.84\ \Omega + \vec{Z}_R$$

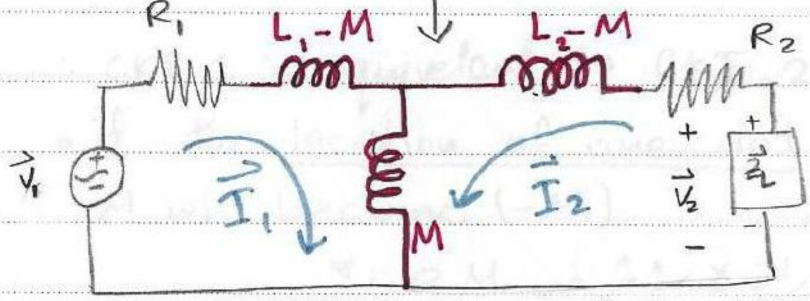
* The T equivalent Network:



→ The input voltages & currents = the output voltages & currents.



CKT 1



CKT 2

Proving that ckt 1 is equal to CKT2!

for CKT 1: let R_1, R_2 be zero

loop1: $-\vec{V}_1 + j\omega L_1 \vec{I}_1 + j\omega M \vec{I}_2 = 0$

$$j\omega L_1 \vec{I}_1 + j\omega M \vec{I}_2 = \vec{V}_1 \quad \cdots ①$$

loop2: $-V_2 + j\omega L_2 \vec{I}_2 + j\omega M \vec{I}_1 = 0$

$$j\omega L_2 \vec{I}_2 + j\omega M \vec{I}_1 = \vec{V}_2 \quad \cdots ②$$

for CKT 2 :-

@ loop 1 :- $-\vec{V}_1 + j\omega(L_1 - M) I_1 + j\omega M(\vec{I}_1 + \vec{I}_2) = 0$

$$j\omega(L_1 - M)\vec{I}_1 + j\omega M(\vec{I}_1 + \vec{I}_2) = \vec{V}_1$$

$$j\omega L_1 I_1 \cancel{- j\omega M I_1} \cancel{+ j\omega M I_1} + j\omega M I_2 = \vec{V}_1$$

$\therefore \vec{V}_1 = j\omega L_1 \vec{I}_1 + j\omega M I_2$ equal to eq ①

@ loop 2 :- $-\vec{V}_2 + j\omega(L_2 - M)\vec{I}_2 + j\omega M(\vec{I}_1 + \vec{I}_2) = 0$

$$j\omega L_2 \vec{I}_2 \cancel{- j\omega M \vec{I}_2} + j\omega M \vec{I}_1 \cancel{+ j\omega M \vec{I}_2} = \vec{V}_2$$

$\therefore \vec{V}_2 = j\omega L_2 \vec{I}_2 + j\omega M \vec{I}_1$ equal to eq ② *

$\therefore$ CKT 1 is equivelant to CKT 2 !

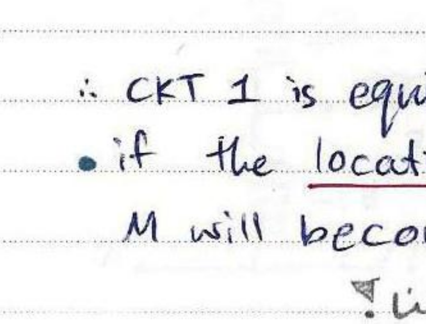
- if the location of one dot is changed M will become (−M)

تغيير اتجاه التيارات لا يؤثر في M هنا!

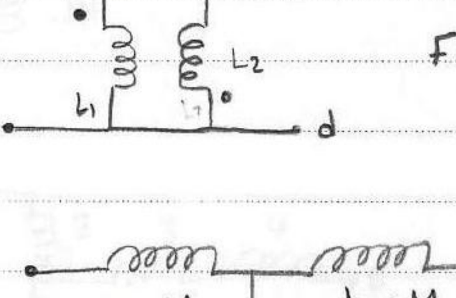
بينما في السابق عند حساب E_{total} كان تغيير التيار يؤثر في اشارة

الحد (MI_1I_2) كاملاً وليس M لوحدها!

ex:



Find the equivelant



it's impossible to build this CKT practically, but we can deal with it mathematically

the Same node

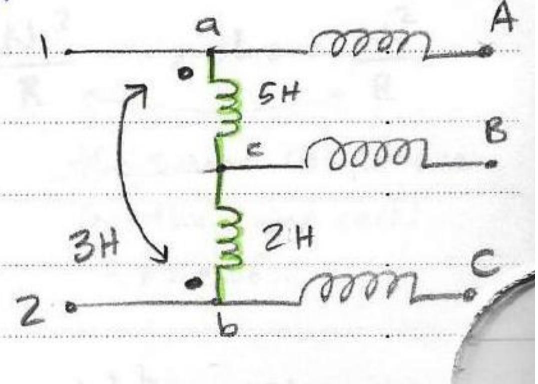
ex:- Find L_{eq} seen between terminals 1 & 2 if:

① all terminals (A, B, C) are open.

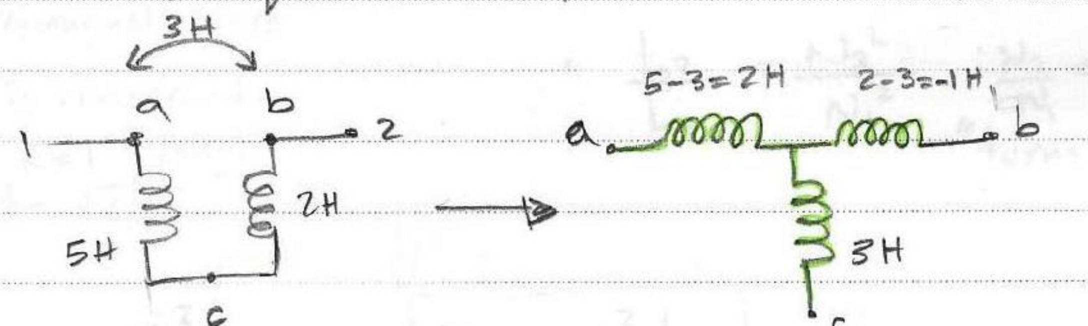
② A & B are connected.

③ B & C are connected.

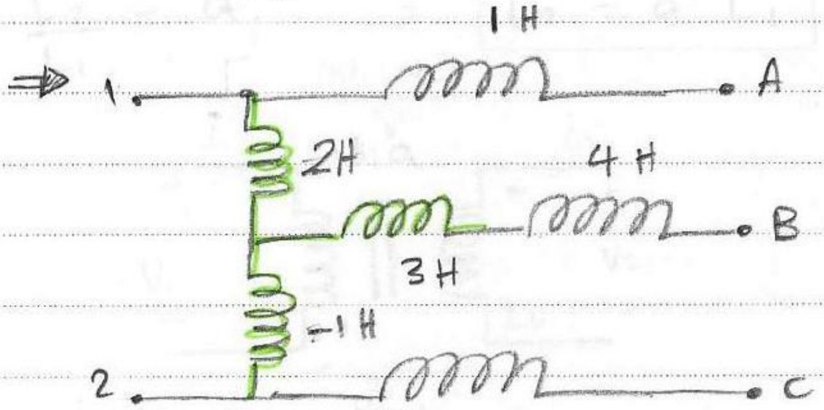
④ A & C are connected.



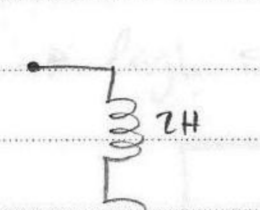
→ find the T equivelant CKT



⇒

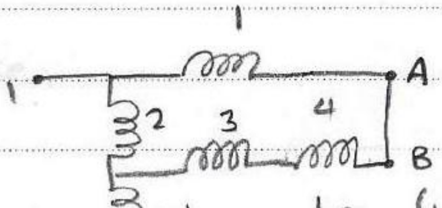


1)



$L_{eq} = 2-1 = 1H$

2)

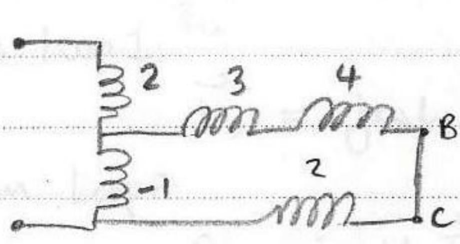


$L_{eq} = (1+4+3) // 2-1$

$= 8 // 2 \ -1$

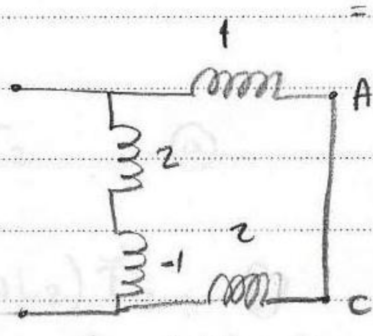
$= 0.6 \ H$

3)



$L_{eq} = (3+4+2) // -1 + 2 = 0.875$

4)



$L_{eq} = (2+-1) // (2+1) = 0.75 \ H$

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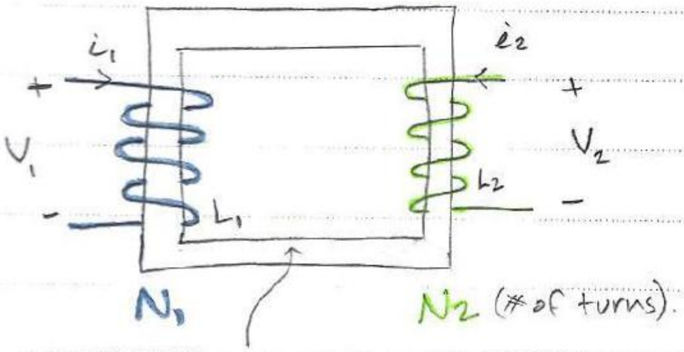
The Ideal Transformer:



Ferromagnetic core

ϕ is conserved

$k = 1$ (unity)

$\therefore M = \sqrt{L_1 L_2}$

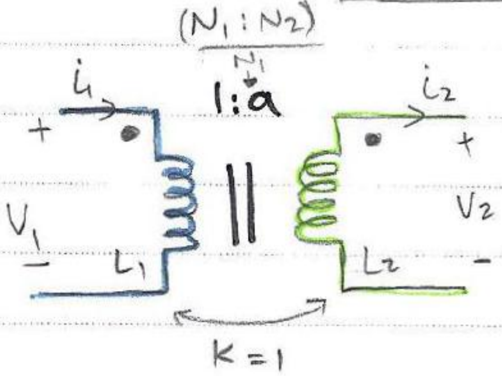
* $L_1 = \frac{N_1^2}{R}$, $L_2 = \frac{N_2^2}{R}$

the same reluctance (on the same core).

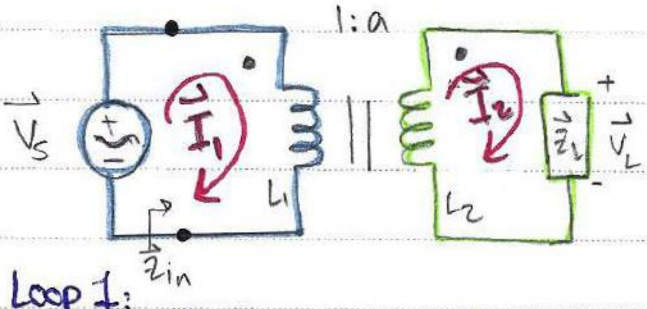
$R\phi = emf ...$

* $\frac{L_2}{L_1} = \frac{N_2^2}{N_1^2}$; $\frac{N_2}{N_1} = a$ "turns ratio"

* $\frac{L_2}{L_1} = a^2 \equiv \boxed{L_2 = a^2 L_1}$



→ high secondary impedance:



- from Loop 1:

$$\vec{V}_s = j\omega L_1 \vec{I}_1 - j\omega M I_2 \quad ... ⓐ$$

- from Loop 2:

$$0 = -j\omega M \vec{I}_1 + (\vec{Z}_L + j\omega L_2)\vec{I}_2 \quad ... ⓑ$$

from ⓑ

$$\vec{I}_2 = \frac{j\omega M \vec{I}_1}{\vec{Z}_L + j\omega L_2} \quad \ldots ①$$

-plug ① in ⓐ

$$\vec{V}_s = j\omega L_1 \vec{I}_1 + \frac{\omega^2 M^2 \vec{I}_1}{\vec{Z}_L + j\omega L_2} \quad \ldots ②$$

$$-\vec{Z}_{in} = \frac{\vec{V}_s}{\vec{I}_1} = j\omega L_1 + \frac{\omega^2 M^2}{\vec{Z}_L + j\omega L_2} \Rightarrow \boxed{M^2 = L_1 L_2}$$

(input impedance)

$$= j\omega L_1 + \frac{\omega^2 L_1 L_2}{\vec{Z}_L + j\omega L_2} \Rightarrow \boxed{L_2 = a^2 L_1}$$

$$= j\omega L_1 + \frac{\omega^2 a^2 L_1^2}{\vec{Z}_L + j\omega a^2 L_1} \quad \ldots ③$$

$$= \frac{j\omega L_1 \vec{Z}_L \cancel{- \omega^2 a^2 L_1^2} \cancel{+ \omega^2 a^2 L_1^2}}{\vec{Z}_L + j\omega a^2 L_1}$$

$$= \left(\frac{j\omega L_1 \vec{Z}_L}{\vec{Z}_L + j\omega a^2 L_1} \right) \times \frac{j\omega L_1}{j\omega L_1}$$

$$\vec{Z}_{in} = \frac{\vec{Z}_L}{\frac{\vec{Z}_L}{j\omega L_1} + a^2}$$

as $L_1 \longrightarrow \infty$

#1: $$\boxed{\vec{Z}_{in} = \frac{\vec{Z}_L}{a^2}} \quad \ldots ④$$

/ /

- from ①

$$\frac{\vec{I}_2}{\vec{I}_1} = \frac{j\omega M / j\omega L_2}{\vec{Z}_L + j\omega L_2 / j\omega L_2}$$

$$= \frac{j\omega M / j\omega L_2}{\frac{Z_L}{j\omega L_2} + 1}$$

as $L_2 \longrightarrow \infty$

$$\frac{\vec{I}_2}{\vec{I}_1} = \frac{j\omega M}{j\omega L_2} = \frac{\sqrt{L_1 L_2}}{L_2} = \sqrt{\frac{L_1}{L_2}} = \frac{1}{a}$$

#2 $\boxed{\frac{I_2}{I_1} = \frac{1}{a}} \longrightarrow \frac{\vec{I}_2}{\vec{I}_1} = \frac{N_1}{N_2} \rightarrow \boxed{\vec{I}_2 N_2 = \vec{I}_1 N_1}$

also $\frac{i_2(t)}{i_1(t)} = \frac{1}{a}$

- if the dot location of one coil is changed then replase (a) with (−a) $\longrightarrow \frac{I_2}{I_1} = -\frac{1}{a}$
- Voltage level adjustment:

$$\vec{V}_2 = I_2 \vec{Z}_L \rightarrow Z_L = \frac{\vec{V}_2}{I_2}$$

$$\vec{V}_S = \vec{V}_1$$

$$\vec{Z}_{in} = \frac{\vec{V}_S}{I_1} \Rightarrow \vec{V}_1 = \vec{Z}_{in} \vec{I}_1$$

$$\therefore \vec{V}_1 = \left(\frac{\vec{Z}_L}{a^2}\right) \cdot \vec{I}_1 = \frac{\left(\frac{V_2}{I_2}\right)}{a^2} \cdot \vec{I}_1$$

$$\vec{V}_1 = \frac{\vec{V}_2}{a^2} \cdot \cancel{a}$$

#3: → $\boxed{\dfrac{\vec{V}_1}{\vec{V}_2} = \dfrac{1}{a}} \longrightarrow \boxed{\vec{V}_2 N_1 = \vec{V}_1 N_2}$

also:- $\dfrac{V_2(t)}{V_1(t)} = a$

\# of turns for the secondary coil.

- if: $a>1 \equiv N_2>N_1$
  → $V_2>V_1$: Step up transformer

- if $a<1 \equiv N_1>N_2$
  → $V_2<V_1$: Step down transformer

* $\dfrac{\vec{V}_2}{\vec{V}_1} = a = \dfrac{\vec{I}_1}{\vec{I}_2}$

$\therefore \boxed{\vec{V}_2 \vec{I}_2 = \vec{V}_1 \vec{I}_1}$

-if $\vec{Z}_L = |\vec{Z}_L| \angle\theta \Rightarrow \vec{V}_2$ leads (assume θ +ve) $\vec{I}_2$ by θ.
& $\vec{V}_1$ leads $\vec{I}_1$ by θ.

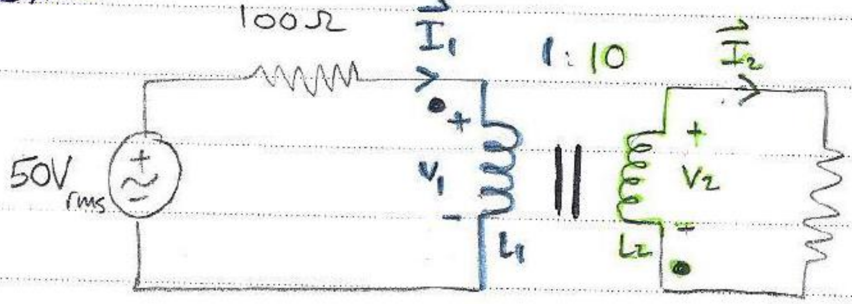
- Since $\vec{Z}_{in} = \dfrac{\vec{Z}_L}{\boxed{a^2}}$ $\therefore$ the angle fo $\vec{Z}_L = \text{ang}(\vec{Z}_{in}) = \theta$

ثابت من دون phase

$-P_2 = P_1$ for an ideal transformer.

$|\vec{V}_2||\vec{I}_2|\cos(\theta) = |\vec{V}_1||\vec{I}_1|\cos\theta$ from $\vec{V}_2\vec{I}_2 = \vec{V}_1\vec{I}_1$

ex:-



$\rightarrow \vec{V}_2 = a\vec{V}_1$

10K Ω $\rightarrow Z_{in} = \frac{10k}{(-10)^2}$

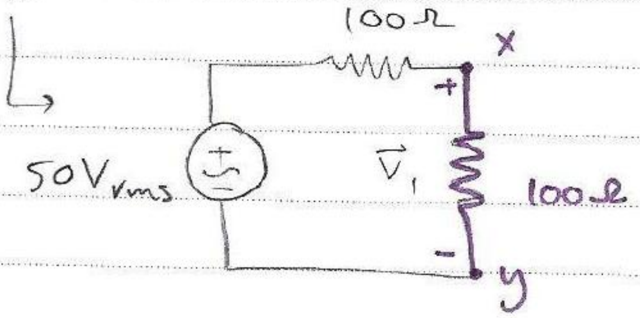
$= 100\ \Omega$

$a = -10$ (one of the dot's locations has changed).

$P = |\vec{I}_2|^2 (10k)$

or $P = \frac{|\vec{V}_2|^2}{10k}$

$Z_{in} = 100\ \Omega$



$\vec{V}_1 = 25\ V_{rms}$ (voltage division).

$\vec{V}_2 = a\vec{V}_1 = -10 \times 25 = -250\ V_{rms}$

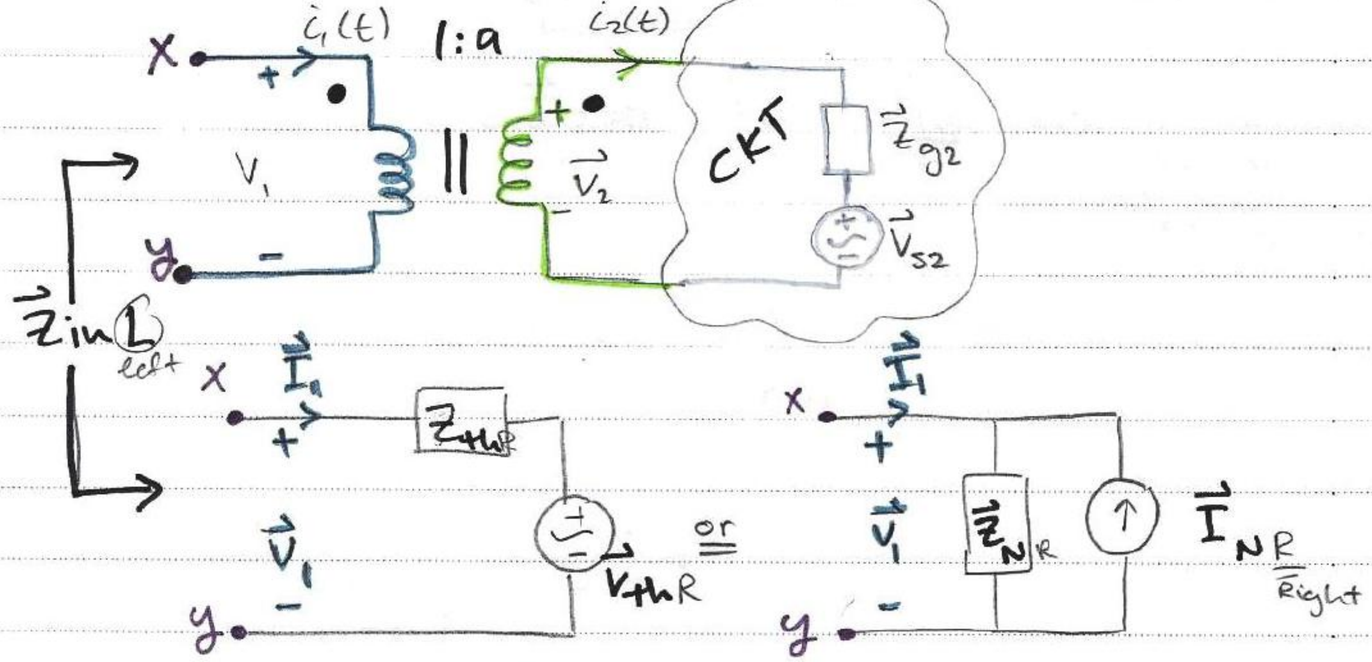
$P = \frac{(-250)^2}{10K} = 6.25$ Watt $\longrightarrow$ remember: if V isn't in rms → power is multiplied by (1/2)

$\vec{I}_1 = \frac{5\cancel{0}}{20\cancel{0}} = \frac{1}{4} A_{rms}$

$\vec{I}_2 = \frac{\vec{I}_1}{a} = \frac{1}{-40} A_{rms}$

2/4/2013

- Thevinin's & Notton's equivelant CKTs for the ideal transformer:



thevinin eq. CKT

Norton eq. CKT

$$* \; Z_{thR} = \vec{Z}_{g2} / a^2$$

$$* \; \vec{V}_{thR} = \vec{V}_{1\,\underset{\text{open CKT}}{oc}} \Rightarrow \vec{I}_1 = 0$$

$$\Rightarrow \frac{\vec{I}_1}{\vec{I}_2} = a \quad \therefore a\vec{I}_2 = 0 \rightarrow \vec{I}_2 = 0$$

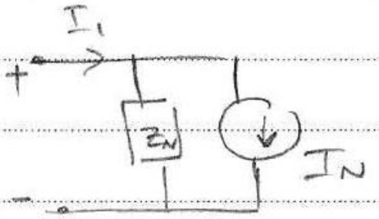
$$\hookrightarrow \vec{V}_{1oc} = \frac{\vec{V}_2}{a} = \frac{\vec{V}_{s2}}{a} = \vec{V}_{th}$$

$$* \; \vec{I}_{NR} = \frac{\vec{V}_{thR}}{Z_{thR}} = + \frac{\vec{V}_{s2}/a}{\vec{Z}_{g2}/a^2} = + \frac{a\vec{V}_{s2}}{\vec{Z}_{g2}}$$

⟶

/ /

another way:



$\vec{I}_N = I_{1\,S.C} = ?!$

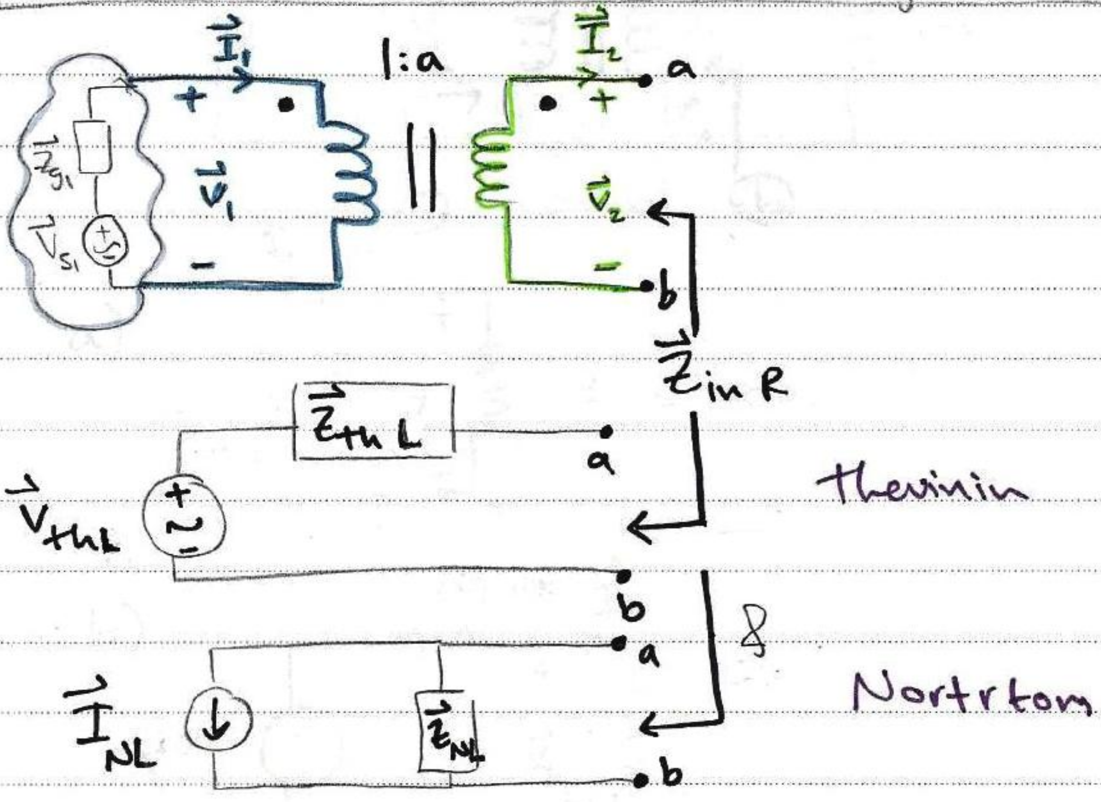
Short ckT

$\vec{I}_{1sc} = a\vec{I}_2 \Rightarrow \vec{I}_2 = \dfrac{\vec{I}_{1.sc}}{a} \Rightarrow V_1 = 0$

$V2 = 0$

$(\vec{I}_2 \times \vec{Z}_{g2}) + V_{S2} = 0$

$\therefore I_N = I_{1SC} = -a\dfrac{\vec{V}_{S2}}{\vec{Z}_{g2}}$ # على فرض ان اتجاه التيار للأسفل



$* \boxed{\vec{Z}_{th} = a^2\vec{Z}_{g1}}$

$* \ \vec{V}_{thL} = \vec{V}_{2\,o.c} = ? \quad , \ \vec{I}_2 = 0 = \vec{I}_1$

$\vec{V}_1 = \vec{V}_{S1} \Rightarrow \therefore \vec{V}_{2\,o.c} = a\vec{V}_1 = a\vec{V}_{S1}$

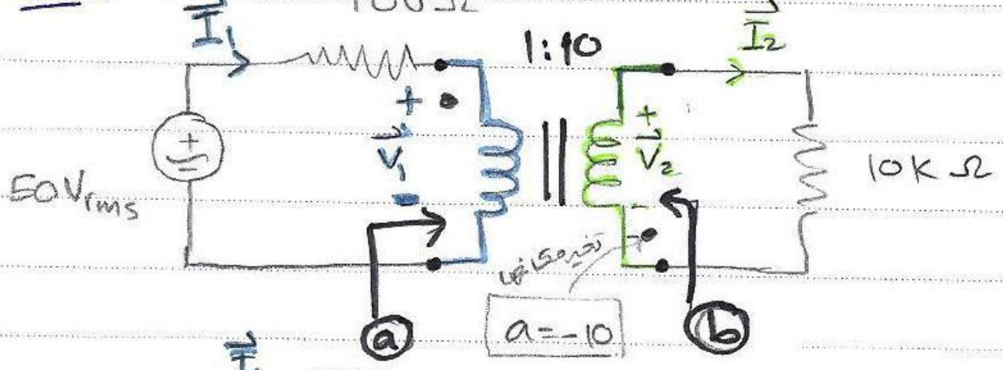
$\boxed{V_{thL} = a\vec{V}_{S1}}$

$$\vec{I}_N = \frac{-a\vec{V}_{S1}}{a^2\vec{Z}_{g1}} = \frac{-\vec{V}_{S1}}{a\vec{Z}_{g1}}$$

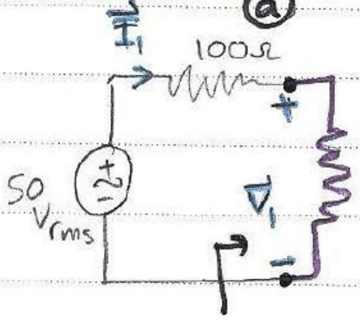
* Note:

- if the dot position of one of the coils is changed, then replace (a) with (-a)

ex 1:

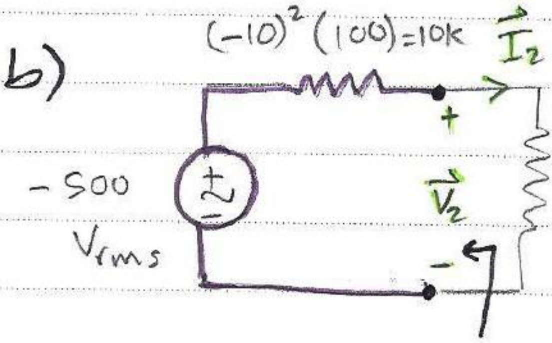


a)

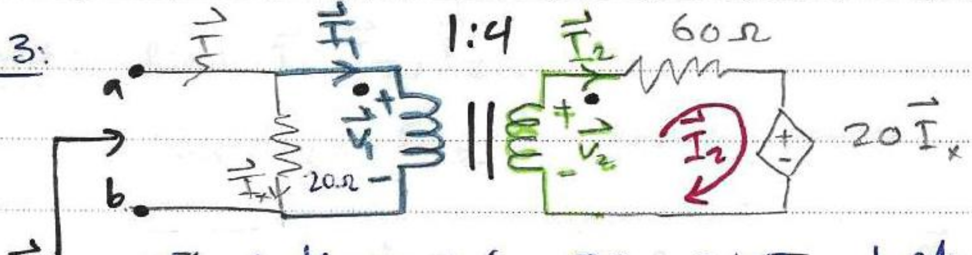


$\frac{10}{(-10)^2} = 100\Omega$

b)

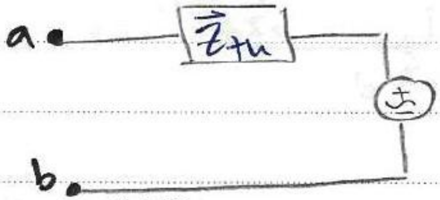


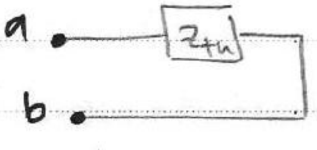
ex 3:



$\vec{Z}_{in}$ -Find thevenin's eq. CKT betwen terminals a & b?

Sol:-





$V_{th}=0$

$*\vec{V}_{th}=0$ (the source is dependant). cause

test voltage (نقوم بفرضها)

$$*\vec{Z}_{in} = Z_{th} = \frac{\textcircled{1}\,V}{\vec{I}} = \frac{1}{\vec{I}}$$

$$\hookrightarrow I_x = \frac{1}{20} A = 0.05 A$$

$$\vec{V}_1 = (1)\ V$$

$$\vec{V}_2 = a\vec{V}_1 = 4\ V \qquad (a=4)$$

@loop 2: $-V_2 + 60\vec{I}_2 + 20\vec{I}_x = 0$

$$\vec{I}_2 = \frac{V_2 - 20I_x}{60} = \frac{1}{20} = 0.05\ A$$

$V_2 = aV_1 = 4$

$$\frac{I_2}{I_1} = \frac{1}{a} \rightarrow \therefore \vec{I}_1 = I_2 \times 4 = 0.2\ A$$

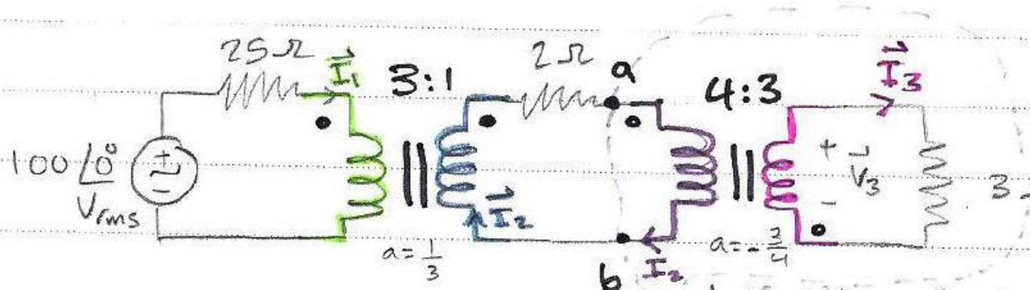
KCL:

$$\vec{I} = \vec{I}_x + \vec{I}_1 = 0.05 + 0.2 = 0.25\ A$$

$$\therefore Z_{th} = R_{th} = \frac{1}{0.25}\ \Omega = 4\ \Omega$$

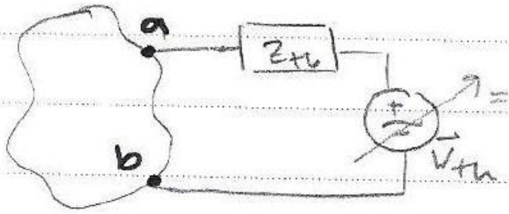
ex4: find $\vec{I}_1, \vec{I}_2, \vec{I}_3$

$P_{25\Omega}, P_{2\Omega}, P_{3\Omega}$? $P \Rightarrow$ avg power



كبي اشارة a بنعوض عن اشارتها!

①

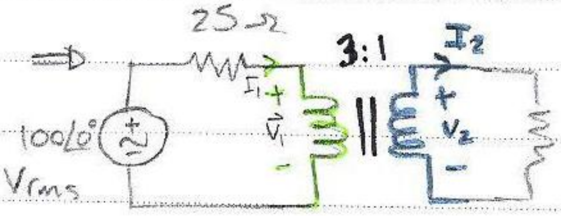


$= 0$ (there's no V source).

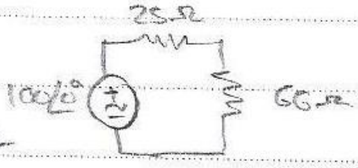
$$\vec{Z}_{th} = \frac{3}{(-\frac{3}{4})^2} = \frac{16}{3}\ \Omega$$

②

$\Rightarrow$



$\left(2 + \frac{16}{3}\right)$ in series



$$Z_{th} = \frac{\left(2 + \frac{16}{3}\right)}{(1/3)^2} = 66\ \Omega$$

$$\vec{I}_1 = \frac{100\angle 0°}{25 + 66} = 1.0989 \angle 0°\ A_{rms}$$

$$\vec{I}_2 = \frac{\vec{I}_1}{(1/3)} \Rightarrow I_2 = 3\vec{I}_1 = 3.297 \angle 0°\ A_{rms}$$

$$\vec{I}_3 = \frac{I_2}{-3/4} = 4.396 \angle 180$$

$P_{25\Omega} = I_1^2 R_{25} = (1.0989 \angle 0°)^2 \times 25$ Watt

$P_{2\Omega} = \vec{I}_2^2 \cdot R_{2\Omega} = (3.297 \angle 0°)^2 \times 2$ Watt

$P_{3\Omega} = \vec{I}_3^2\ R_{3\Omega} = (4.396 \angle 180)^2 \times 3$ Watt

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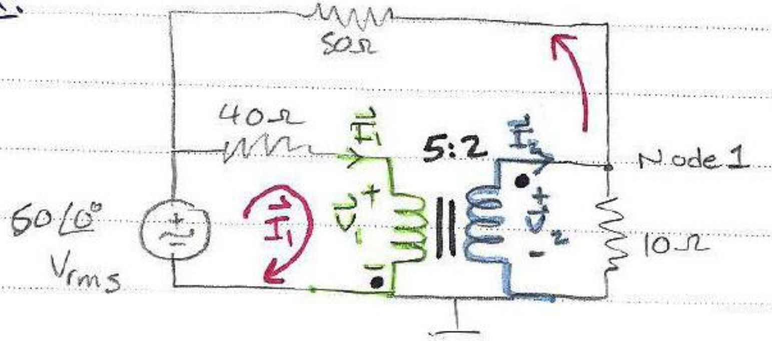
ex:



$V_2 = ?$

Sol: @ Node 1: $\frac{\vec{V_2} - 60}{50} + \frac{\vec{V_2}}{10} - \vec{I_2} = 0$

$6\vec{V}_2 - 50\vec{I}_2 = 66$ ...①

@ loop 1 : $-60\angle 0° + 40\vec{I}_1 + \vec{V}_1 = 0$

$\vec{I_2} = \frac{\vec{I_1}}{-2/5}$, $\vec{V_2} = -\frac{2}{5}\vec{V}_1$ ... ⓐ

$\therefore$ $-2.5\vec{V_2} - 16\vec{I_2} = 60\angle 0°$ ... ②

from ① & ② & ⓐ

→ $\vec{V_2} = -9.235\, V_{rms}$, $\vec{I_2} = -2.307\, A_{rms}$

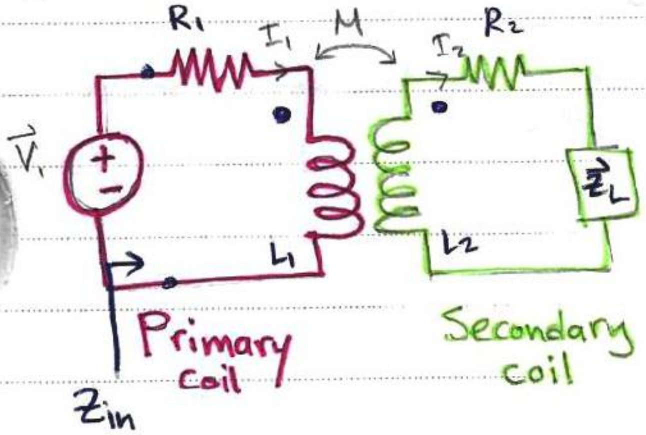
→ $\vec{V}_1 = 23.089\, V_{rms}$, $\vec{I}_1 = 0.92\, A_{rms}$

Summary:-

linear transformer

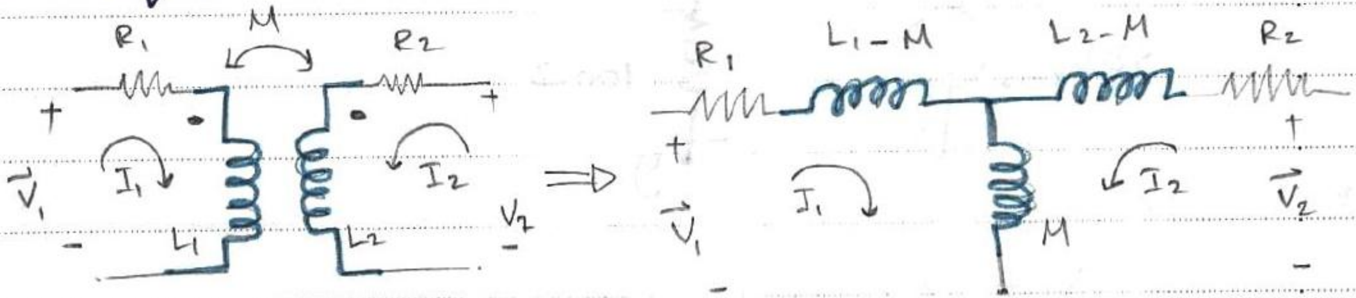
$$\boxed{\vec{Z}_{in} = Z_{11} + Z_R}$$

the Reflected impedance has suffered a sign changed in it's reactive part, so a capcitive load will lead to an inductive contribution to the input impedance.



$$\boxed{\vec{Z}_R = \frac{\omega^2 M^2}{\vec{Z}_{22}}}$$ the reflected impedance.

* T equivelant Network:-

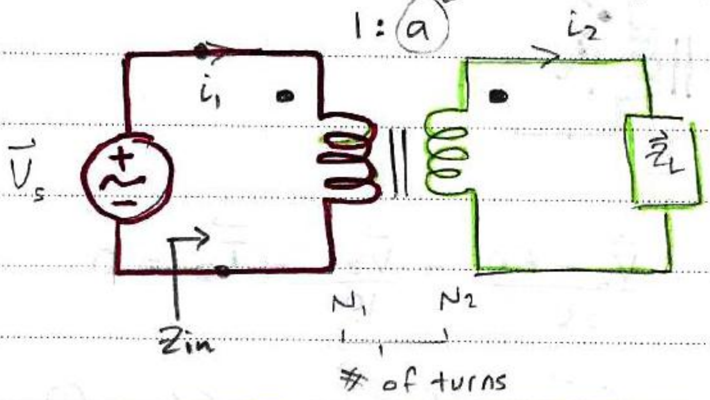


- changing one dot locations changes the sign of M.
- changing the Direction of current does n't affect the Values (M, L2 - M, L1 - M)

Summary:-

The Ideal Transformers

انتبه بالكتاب a ⟶ لو كانت النسبة $a:1 \Rightarrow 1:\boxed{\frac{1}{a}}$ a'



$k=1 \longrightarrow M=\sqrt{L_1L_2}$

turns ratio ⟵ $\boxed{\underline{\underline{a}} = \frac{N_2}{N_1}}$ $\boxed{\frac{L_2}{L_1} = a^2}$ $\boxed{\vec{V}_1\vec{I}_1 = \vec{V}_2\vec{I}_2}$

$\boxed{\vec{Z}_{in} = \frac{\vec{Z}_L}{a^2}}$ $\boxed{\frac{I_2}{I_1} = \frac{1}{a}}$ $\boxed{\frac{V_2}{V_1} = a}$

↳ $I_1N_1 = I_2N_2$ ↳ $V_1N_2 = V_2N_1$

↳ from this relation, if the load impedance (Z_L) is capacitive, the input impedance (Z_{in}) will also be capacitive! * عكس ال Linear transformer *

	$I_1N_1 = I_2N_2$	$V_1N_2 = V_2N_1$
a) change the location of one dot	$I_1N_1 = -I_2N_2$	$V_1N_2 = -V_2N_1$
b) chang the direction of one <u>current</u>	$I_1N_1 = -I_2N_2$	$V_1N_2 = V_2N_1$
c) change the polarity of one <u>voltage</u>	$I_1N_1 = I_2N_2$	$V_1N_2 = -V_2N_1$

Summary: (cont.)

$a>1$, $\therefore N_2 > N_1$

$\therefore V_2 > V_1$ ⟶ Step-up transformer

$a<1$, $N_1 > N_2$

$V_1 > V_2$ → Step-down transformer

$$* \ \text{ang}(\vec{Z}_L) = \text{ang}(\vec{Z}_{in}) = \theta$$

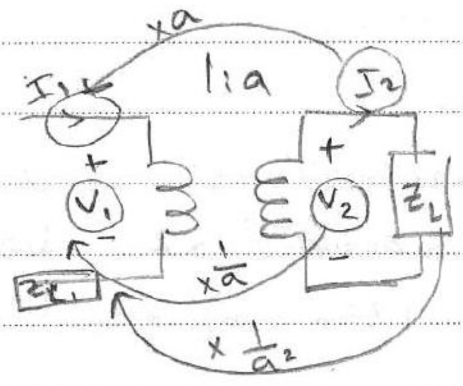
$$* \ P_2 = P_1 = |\vec{V}_2||\vec{I}_2| \cos(\theta) = |\vec{V}_1||\vec{I}_1| \cos(\theta)$$

$$* \ \text{apparent } P_1 = \text{apparent } P_2$$

* Thevinin & Norton's equivelant CKTs :-

$V_{th} = \underset{\text{open CKT}}{\underline{V_{o.c}}}$! → when I have only dependant sources then $V_{th} = I_N = 0$ & R_{th} is calcuted by applying a known I/V source the $R_{th} = \frac{V_{know}}{I_{calculated}}$ OR $\frac{V_{calcuted}}{I_{knon}}$

$I_N = \underset{\text{Short CKT}}{\underline{I_{S.C}}}$ → $I_N = \frac{V_{th}}{R_{th}}$ → $\underline{R_{th} = R_N}$

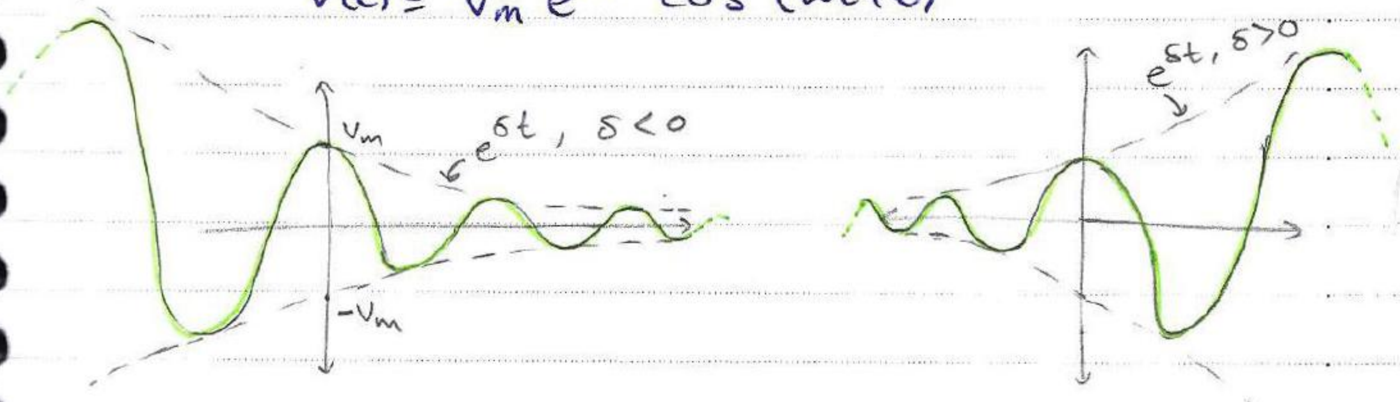


CH #14: Complex frequency

(this section is taken from 5th edition (without laplace).

* Exponentialy damped Sinusoidal Signal:

$$V(t) = V_m e^{\delta t} \cos(\omega t + \theta)$$



$$\boxed{S = \delta + j\omega}$$; complex frequency.

↳ δ: Nep er frequency (neper/sec = [Np/sec])
↳ ω: Angular ~ (rad/sec) ... real freq.

[S] = complex radian per sec.
~ nep er ~ ~.

→ $V(t) = V_m e^{\delta t} \cos(\omega t + \theta)$ ... time domain
→ $\vec{V}(s) = V_m \angle \theta$, $\vec{s} = \delta + j\omega$ ... freq. domain

$$\vec{V}(t) \xleftrightarrow{\mathcal{L}} \vec{V}(\vec{s})$$

t. domain laplace transform freq. domain

- Some special cases for $(\omega \,\&\, \delta)$:-

1. let $\underline{\omega = 0 \;,\; \delta = 0}$

$\therefore S = 0$

$\rightarrow V(t) = V_m \cos(\theta) = V_o$ constant DC Analysis

2. let $\underline{\delta = 0}$

$S = j\omega$

$\rightarrow V(t) = V_m \cos(\omega t + \theta)$ AC Analysis

$\vec{V} = V_m \measuredangle \theta$ (Sinusoidal signal)

3. let $\underline{\omega = 0}$

$S = \delta$

$\rightarrow V(t) = V_m e^{\delta t}$ transient Analysis

4. when $\delta \neq 0 \,\&\, \omega \neq 0 \rightarrow$ it's an exponentialy damping sinusoidal!

- How to find Complex frequency $(\vec{S})$?

$$f(t) = k e^{S t} \quad \overset{or}{\Rightarrow} \quad f(t) = k_1 e^{S_1 t} + k_2 e^{S_2 t}$$

$k \,\&\, S$ are complex #s.

- DC $(\omega = \delta = 0)$

$V(t) = V_o \Rightarrow V(t) = V_o e^{0 t}$ $\therefore \boxed{S = 0}$

- Exponential $(\omega = 0)$

$V(t) = V_o e^{\delta t} \Rightarrow \boxed{S = \delta}$

- Sinusoidal $(\delta = 0)$

$V(t) = V_m \cos(\omega t + \theta)$

→

from Euler's identity:

$$\cos(\alpha) = \frac{1}{2} e^{j\alpha} + \frac{1}{2} e^{-j\alpha}$$

$$\therefore\ v(t) = \frac{1}{2} V_m\, e^{j(\omega t+\theta)} + \frac{1}{2} V_m\, e^{-j(\omega t+\theta)}$$

$$= \underbrace{\frac{1}{2} V_m e^{j\theta}}_{k_1}\ e^{\underbrace{j\omega}_{\vec{S}_1} t} + \underbrace{\frac{1}{2} V_m e^{-j\theta}}_{k_2}\ e^{\underbrace{-j\omega}_{\vec{S}_2} t}$$

$$\vec{S}_1 = j\omega$$
$$\vec{S}_2 = -j\omega = \vec{S}_1^*$$
$$k_1 = k_2^*$$

$$\therefore\ V(t) = k_1 e^{S_1 t} + k_2 e^{S_2 t}$$

where

$$S_1 = S_2^*$$
$$k_1 = k_2^*$$

$$\boxed{V(t) = k e^{St} + k^* e^{S^* t}} \quad ;\ S = j\omega,\ k = \frac{1}{2} V_m e^{j\theta}$$

- **Exponentially damped sinusoidals:** $(\omega \neq 0,\ \delta \neq 0)$

$$V(t) = V_m e^{\delta t} \cos(\omega t + \theta)$$

$$= \frac{1}{2} V_m e^{\delta t} \left[e^{j(\omega t+\theta)} + e^{-j(\omega t+\theta)} \right]$$

$$e^{(\delta - j\omega)t}$$

$$= \underbrace{\frac{1}{2} V_m e^{j\theta}}_{k_1}\ e^{\underbrace{(\delta + j\omega)}_{S_1} t} + \underbrace{\frac{1}{2} V_m e^{-j\theta}}_{k_2}\ e^{-\underbrace{\cancel{(-\delta + j\omega)}}_{S_2} t}$$

$$= k_1 e^{S_1 t} + k_2 e^{S_2 t}$$

$$\boxed{V(t) = K_1 e^{S_1 t} + K_2 e^{S_2 t}}$$

; $\vec{S}_1 = \delta + j\omega = \vec{S}_2^*$

$\vec{K}_1 = \frac{1}{2} V_m e^{j\theta} = \vec{K}_2^*$

ex: find the complex frequency (S) for:-

① $i(t) = 160$

$S = 0$ (DC)

② $V(t) = 5e^{-2t}$

$S = -2$ (exponential)

③ $i(t) = 2\sin(500t)$

$S_1 = j500$, $S_2 = -j500$

$S = j500$

④ $V(t) = 4e^{-3t}\sin(6t + 10^\circ)$

$$\left.\begin{aligned} \vec{S}_1 &= -3 + j6 \\ S_2 &= -3 - j6 \end{aligned}\right] - S = -3 + j6$$

ex2: if $S_1 = j10$, $K_1 = 6 - j8$, $V(t) = K_1 e^{S_1 t} + K_2 e^{S_2 t}$
find $V(t) = ?$

$K_2 = 6 + j8$, $S_2 = -j10$, $\delta = 0$

$V(t) = \frac{1}{2} V_m e^{j\theta} e^{j\omega t} + \frac{1}{2} e^{-j\theta} e^{-j\omega t}$

$V(t) = V_m \cos(\omega t + \theta)$, $\omega = 10$

$$\underbrace{K_1}_{6 - j8} = \frac{1}{2} V_m e^{j\theta}$$

$\downarrow$

$10 e^{-j53.13^\circ} = \frac{1}{2} V_m e^{j\theta}$; $\theta = -53.13^\circ = -\tan^{-1}\left(\frac{8}{6}\right)$

$V_m = 10 \times 2 = 20V$

$\therefore V(t) = 20\cos(10t - 53.13^\circ)$

/ /

ex8: find S?

$$i(t) = (2e^{-100t} + e^{-200t}) \sin(200t)$$

Sol: $i(t) = 2e^{-100t} \sin(200t + e^{-200t} \sin(200t)$

$S_1 = ?$ $S_2 = ?$

$S_1 = -100 + j200$ $S_2 = -200 + j200$

محاضرة تعويضية
6/4/2013

Frequency response representation:

$$V(t) = V_m \, e^{\delta t} \cos(\omega t + \theta)$$ exponentialy damped.

$$\boxed{s = \delta + j\omega}$$

$$\hookrightarrow V(t) = \mathrm{Re}\{V_m e^{\delta t} \, e^{j(\omega t + \theta)}\}$$

$$= \mathrm{Re}\{V_m e^{\delta t} [\cos(\omega t + \theta) + j\sin(\omega t + \theta)]\}$$

$$= \mathrm{Re}\{V_m e^{j\theta} \, e^{\underline{(\sigma + j\omega)}t}\}$$

$$= \mathrm{Re}\{\underbrace{\boxed{V_m e^{j\theta}}}_{\text{phasor}} e^{st}\}$$

$$= \mathrm{Re}\{\underline{V_m \angle\theta} \; e^{st}\} \rightarrow \vec{V} = V_m e^{j\theta} = V_m \angle\theta$$

$$V(t) = \mathrm{Re}\{\vec{V} e^{st}\}$$

تعريف ال phasor بدون δ !

* when $i(t) = I_m \cos(\omega t + \phi)$

$$\vec{I} = I_m \angle\phi$$

$$i(t) = \mathrm{Re}\{I_m e^{j\theta} \, e^{j\omega t}\}$$

$$= \mathrm{Re}\{\vec{I} e^{j\omega t}\}$$

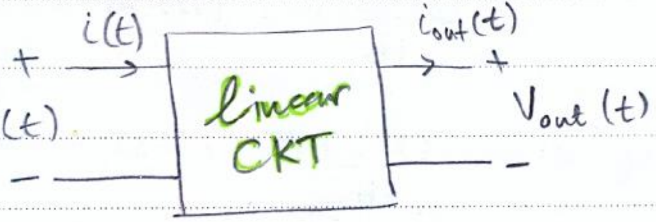
/ /

* Since we are dealing with linear circuits,

* input & output must take the same form.

e.g: exponentially damped input gives exponentially damped output!

∴ forced $\overset{\text{output}}{\text{response}}$ has the same form of the forcing function.. input



$$V(t) = \underline{\underline{V_m}}\, e^{\delta t} \cos(\omega t + \underline{\underline{\theta}})$$

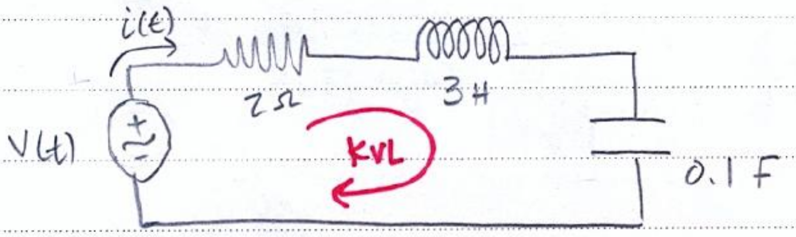
$$\rightarrow V_{out}(t) = V_o\, e^{\delta t} \cos(\omega t \theta_o)$$

only V_m, θ may change
ω & δ must stay the same.

$$i(t) = I_m e^{\delta t} \cos(\omega t + \phi)$$

$$\rightarrow i_{out}(t) = I_o\, e^{\delta t} \cos(\omega t + \phi_o)$$

ex1:- $V(t) = 60\, e^{-2t} \cos(4t + 10^\circ)$ V



find i(t)?

Sol:- Since it's a linear CKT...

$i(t) = I_m e^{-2t} \cos(4t + \phi)$ ...ⓐ, $S = -2 + j4$

I need I_m & ϕ

$$\Rightarrow V(t) = \mathrm{Re}\{60\, e^{j10^\circ} e^{(-2+j4)t}\}$$

$$= \mathrm{Re}\{\vec{V} e^{st}\} \; ...① \; ; \quad \vec{V} = 60\angle 10^\circ \, , \; s = -2+j4$$

it's the same for $i(t)$

$$\Rightarrow i(t) = \mathrm{Re}\{I_m e^{j\phi} e^{st}\}$$

$$= \mathrm{Re}\{\vec{I} e^{st}\} \; ...② \; ; \quad \vec{I} = I_m \angle \phi \, , \; s = -2+j4$$

KVL

$$V(t) = R\, i(t) + L\frac{d\, i(t)}{dt} + \frac{1}{c}\int i\, dt$$

$$V(t) = 2i(t) + 3\frac{d\, i(t)}{dt} + 10\int i(t).dt$$

form ①

$$\mathrm{Re}\{\vec{V} e^{st}\} = 2\mathrm{Re}\{\vec{I} e^{st}\} + 3\,\mathrm{Re}\{s\vec{I} e^{st}\} + 10\,\mathrm{Re}\left\{\frac{\vec{I}}{s} e^{st}\right\}$$

$$\cancel{\mathrm{Re}}\{60\angle 10^\circ \cancel{e^{st}}\} = \cancel{\mathrm{Re}}\left\{2\vec{I}\cancel{e^{st}} + 3s\vec{I}\cancel{e^{st}} + \frac{10}{s}\vec{I}\cancel{e^{st}}\right\}$$

since $e^{st} \neq 0$

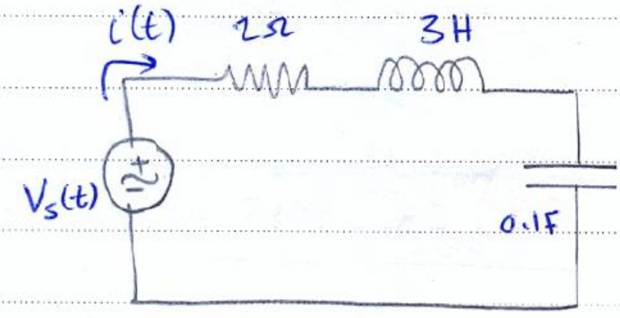
$$60\angle 10^\circ = 2\vec{I} + 3s\vec{I} + \frac{10}{s}\vec{I}$$

$$60\angle 10^\circ = \vec{I}\left(2 + 3s + \frac{10}{s}\right)$$

$$\therefore \vec{I} = \frac{60\angle 10^\circ}{2 + 3(-2+j4) + \frac{10}{(-2+j4)}} = \underset{I_m}{\underline{5.37}} \; \overset{\phi}{\underline{\angle -106.6^\circ}}$$

plug in ②

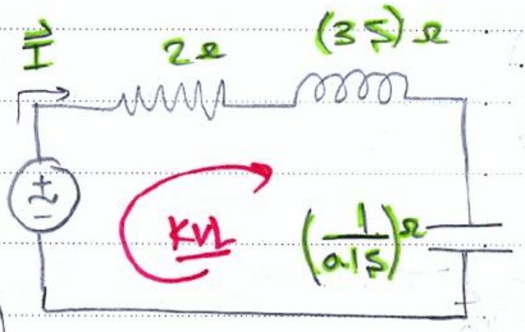
$$i(t) = 5.37\, e^{-2t} \cos(4t - 106.6^\circ)$$



time - domain

⇌

R ⇔ R
L ⇔ Ls
C ⇔ $\frac{1}{Cs}$



s - domain
(complex freq. domain).

KVL: $-\vec{V} + 2\vec{I} + 3s\vec{I} + \frac{1}{0.1s}\vec{I} = 0$

- impedance & admitance with respect to s!
  $\vec{Z}(s)$, $\vec{Y}(s)$ (they aren't phasors) the don't have time domain representation

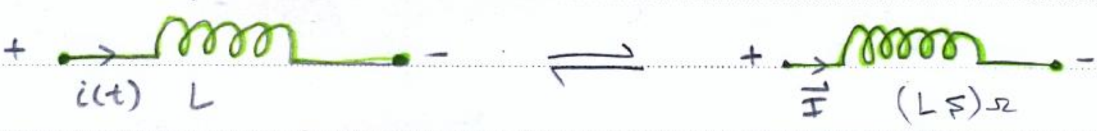
- Resistor:

$\boxed{V(t) = R\,i(t)}$; $\vec{V} = V_m e^{j\theta} \equiv V_m\angle\theta$

$\cancel{Re}\{\vec{V}\cancel{e^{st}}\} = R.\,\cancel{Re}\{\vec{I}\cancel{e^{st}}\}$ $\vec{I} = I_m e^{j\phi} \equiv I_m\angle\phi$

$\boxed{\vec{V} = \underbrace{R}_{\vec{Z}_R}\vec{I}}$ $\boxed{\vec{Z}_R = R}$

- Inductor:

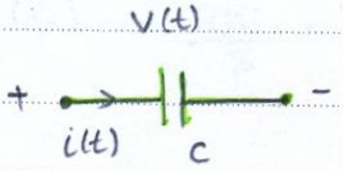


$V(t) = L\,\frac{d\,i(t)}{dt}$ ⇒ $\boxed{\vec{Z}_L = sL}$

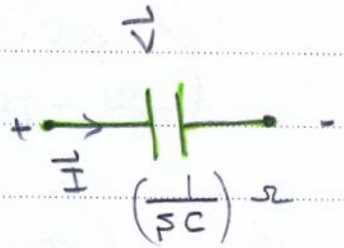
$\cancel{Re}\{\vec{V}\cancel{e^{st}}\} = L\,\cancel{Re}\{s\vec{I}\cancel{e^{st}}\}$

$\vec{V} = (sL)\vec{I}$

- Capacitor:



$\rightleftharpoons$



$$i(t) = C\,\frac{d\,v(t)}{dt}$$

$$\cancel{Re}\left\{\vec{I}\,\cancel{e^{st}}\right\} = C.\cancel{Re}\left\{s\vec{V}\,\cancel{e^{st}}\right\}$$

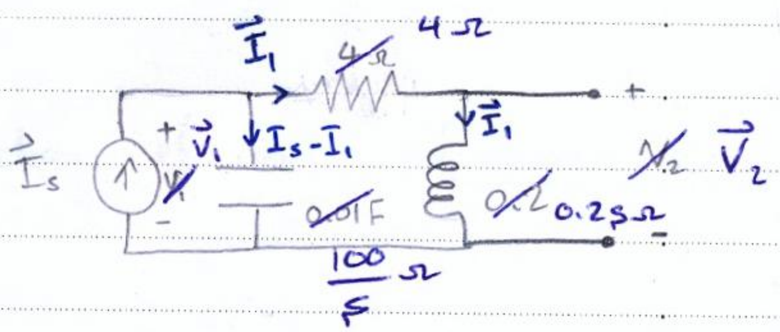
$$\vec{I} = sC\;\vec{V}$$

$$\vec{V} = \left(\frac{1}{sC}\right)\vec{I} \qquad \therefore \boxed{\vec{Z}_c = \frac{1}{sC}}$$

- Now, we can apply anytype of analysis (mesh, Nodal, ...) over these CKTs.

<u>ex</u>:-

Find $\vec{V}_1(s), \vec{V}_2(s)$:
(we'll solve with respect to s).



<u>Sol</u>:- $V_2(s) = \underline{\underline{\vec{I}_1}}\,(0.2s)$

current division $\longrightarrow$ $\vec{I}_1 = \dfrac{\vec{I}_s \cdot \frac{100}{s}}{\frac{100}{s}+4+0.2s}$

$$\rightarrow \vec{V}_2(s) = \frac{100\,s\,\vec{I}_s}{s^2+20s+500} \quad ...①$$

$$\vec{V}_1(s) = \frac{100}{s} \cdot \vec{I}_s \left(\frac{4+0.2s}{4+0.2s+\frac{100}{s}} \right)$$

$$\rightarrow \vec{V}_1(s) = \frac{100\,\vec{I}_s\,(s+20)}{s^2+20s+500} \quad \ldots ②$$

→ now, let $i_s(t) = 3e^{-5t}\cos(10t+18)$ A

& $s = -5+j10$

$\vec{I}_s = 3\angle 18^\circ$ A

from ① & ②

$\rightarrow \vec{V}_1(-5+j10) = 13.094 + j9.03 = 15.9 \angle 34.59^\circ$ V

$v_1(t) = 15.9e^{-5t}\cos(10t+34.59^\circ)$ V

$\rightarrow \vec{V}_2(-5+j10) = -4.55 + j8.75 = 9.86 \angle 117.5^\circ$ V

$v_2(t) = 9.86\,e^{-5t}\cos(10t+117.5^\circ)$ V

- **Sketch!**

step 1. Response as a function of ω: $\boxed{\text{Section 9.5}}$

Response function

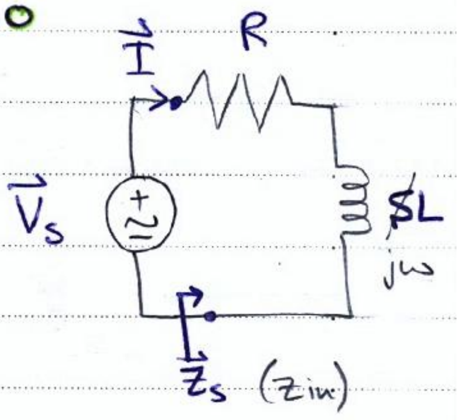
e.g. $\vec{Z}(s) = \frac{\vec{V}(s)}{\vec{I}(s)}$ $\frac{\text{Input}}{\text{Output}}$

$$\vec{Y}(s) = \frac{\vec{I}(s)}{\vec{V}(s)}$$

or $\frac{\vec{V}_i(s)}{\vec{V}_o(s)} = \vec{H}(s)$ "transfer function"

$$\left.\begin{aligned} H_v(S) &= \frac{\vec{V}_i(S)}{\vec{V}_o(S)} \\ H_i(S) &= \frac{\vec{I}_i(S)}{\vec{I}_o(S)} \end{aligned}\right] \text{transfer functions.}$$

* Sketch $H(j\omega)$, assume $\sigma = 0$



let $\vec{H}(S) = \frac{\vec{I}}{\vec{V}_s}$ ...①, transfer fnc.

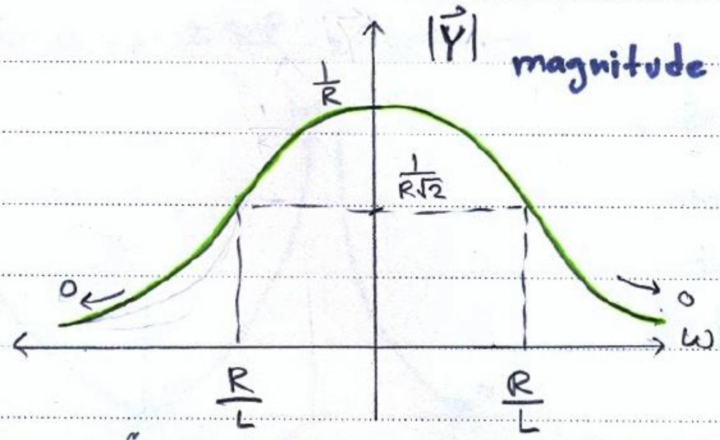
$\vec{I} = \frac{\vec{V}}{\vec{Z}_S} = \frac{\vec{V}_s}{R+SL}$

Plug in ①

$\vec{H}(S) = \frac{1}{R+SL}$ ← Notice, it's the input admittance! $\vec{Y}_{in}$.

$$\boxed{\overset{\vec{H}}{\vec{Y}}(j\omega) = \frac{1}{R+j\omega L}}$$

↳ $|\vec{Y}(j\omega)| = \frac{1}{\sqrt{R^2+\omega^2L^2}}$, $Ang(\vec{Y}(j\omega)) = -\tan^{-1}\left(\frac{\omega L}{R}\right)$



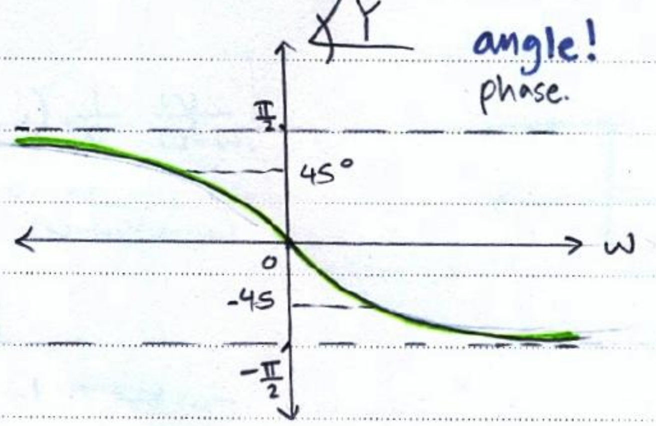
when $\omega = 0 \rightarrow \frac{1}{\sqrt{R^2}} = \frac{1}{R}$

As $\omega \rightarrow \pm\infty \rightarrow Y(j\omega) \rightarrow 0$

"Lower half power..."

"upper half power frequency"

$\frac{1}{2}\times$ maximum power!



angle!
phase.

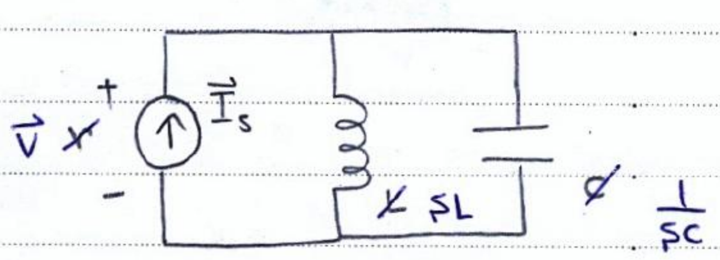
@ $\omega=0 \rightarrow \measuredangle\vec{Y} = 0°$

@ $\omega=\infty \rightarrow \measuredangle\vec{Y} = \frac{\pi}{2}$

$\omega = -\infty \ \measuredangle\vec{Y} = -\frac{\pi}{2}$

ex:

Sketch $\frac{V}{I_s}$!



Sol:

$$\vec{V} = \vec{I}_s \frac{(j\omega L)\left(\frac{1}{j\omega C}\right)}{j\omega L - j\frac{1}{\omega C}}$$

$$\vec{Z}_{in} = \frac{\vec{V}}{\vec{I}_s} = \frac{L/C}{j\left(\omega L - \frac{1}{\omega C}\right)} = -j\frac{1}{C}\frac{\omega}{\omega^2 - \frac{1}{LC}}$$

$$= -j\frac{1}{C}\frac{\omega}{\omega^2 - \omega_0^2}$$

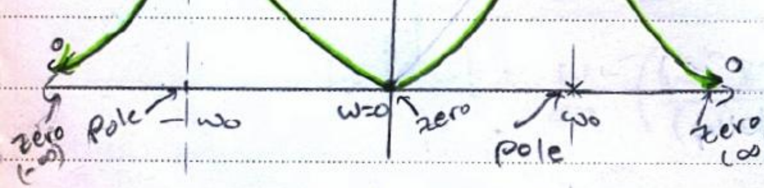
$\hookrightarrow \omega_0^2 = \frac{1}{LC}$

$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$

resonance freq.

mag. $\rightarrow |\vec{Z}| = \sqrt{\left(\frac{1}{C}\frac{\omega}{\omega^2-\omega_0^2}\right)^{\cancel{2}}}$ ← since the Real part=0.

Phase → ±90° → (ω)حسب



- Zeros: values of ω, which make the value of the transfer func. =0.
- Poles: values of ω, which make the value of the transfer func.= ±∞

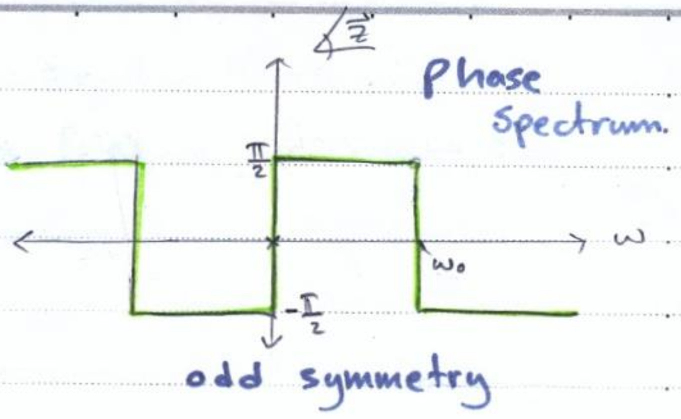
- magnitude spectrum is alway an even func. of ω.

& phase ″ ″ ″ ″ odd ″ ″ ″

/ /

$$\vec{Z} = -j\frac{1}{c}\frac{w}{w^2-w_0^2}$$

$$= -j\frac{1}{c}\frac{w}{(w-w_0)(w+w_0)}$$

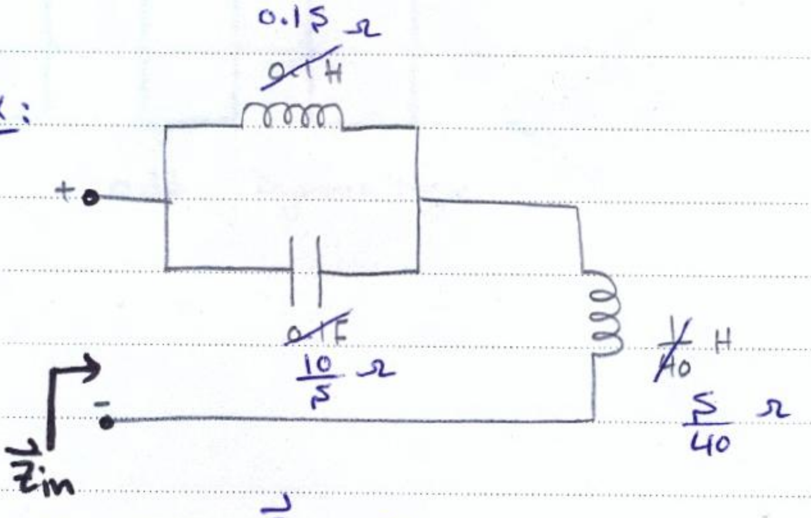


* critical points:-

@ $w = 0$ & $\to \pm\infty$ (Zeros)
@ $w = w_0, -w_0$ (Poles)

أول خطوة بالـsketch
إني أحددهم!

ex:



Sketch $\vec{Z}_{in}$?

Sol:-

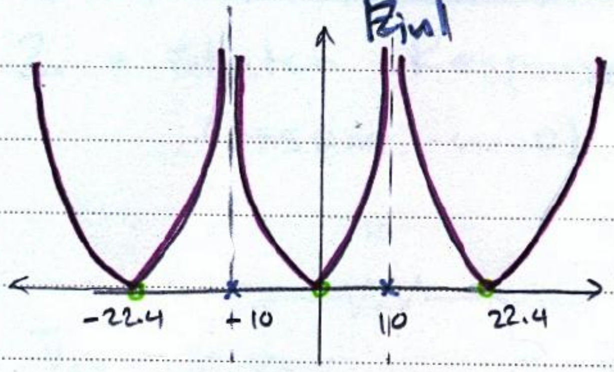
$$\vec{Z}_{in} = \frac{jw}{40} + \frac{(jw/10)\times(10/jw)}{jw/10 + 10/jw} \quad \cdots ⊛$$

$$= jw\left[\frac{100 - w^2 + 400}{40(100 - w^2)}\right]$$

$$\vec{Z}_{in} = \frac{jw}{40}\left[\frac{w^2 - 500}{w^2 - 100}\right] = \frac{jw}{40}\left[\frac{(w-\sqrt{500})(w+\sqrt{500})}{(w-10)(w+10)}\right]$$

$$\hookrightarrow |\vec{Z}_{in}| = \frac{w}{40}\left(\frac{(w-22.4)(w+22.4)}{(w-10)(w+10)}\right.$$

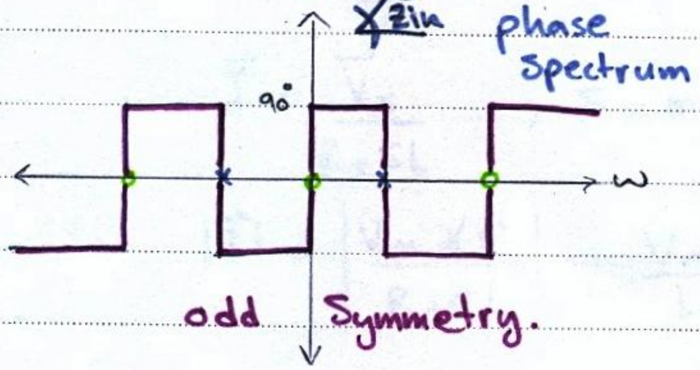
mag. Spectrum



* zeros → $\omega = 0$, $\omega = \pm 22.4$

* Poles → ± 10 , $\omega \to \pm\infty$

even symmetry.

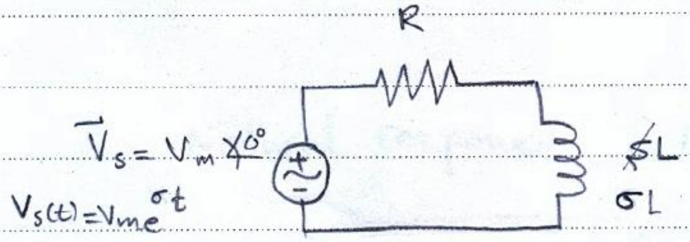


odd Symmetry.

نقوم بتحديد الاشارة بالتعويض ⟵

في ⊛ ← $\pm 90°$ ≡ $(\pm j)$

step 2. • Sketch Response as a function of σ:
(assume $\omega=0$)



$$\vec{I} = \frac{\vec{V}_s}{R + sL} \quad , \; s = \sigma$$

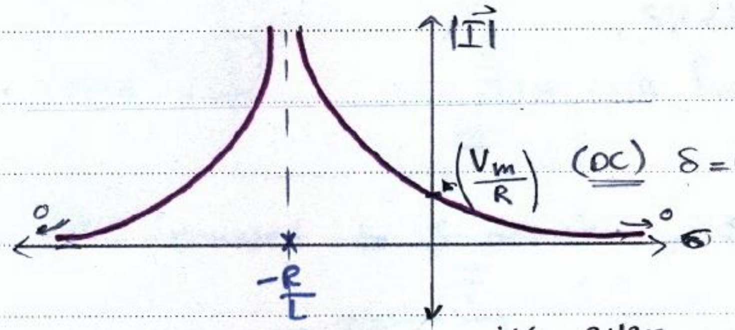
$$|\vec{I}| = \left| \frac{V_m \angle 0^\circ}{R + \sigma L} \right| = \frac{V_m}{L} \cdot \left| \frac{1}{\sigma + R/L} \right|$$

* Zeros of Denomenator
→ Poles of $\vec{I}$

* Zeros of Numenator
→ Zeros of $\vec{I}$

*Zeros → @ $\delta \to \pm\infty$

*pole → @ $\delta = -\frac{R}{L}$



(DC) $\delta = 0 \to \therefore \vec{V} = V_m$ (inducto will become short circuit).

it's even
not even (only with $\overset{s=}{\omega}$)

$$i(t) = \frac{V_m}{R + \sigma L} e^{\sigma t}$$

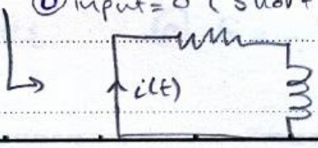
* pole at $-R/L$

means⇒ if $\vec{V}_s = V_m e^{\sigma t}$ & $\sigma = -R/L$

then $I \longrightarrow \infty$

$$\therefore \frac{\vec{I}}{\vec{V}_s} = \infty = \frac{1}{⓪} \; ; \text{when } \delta = -\frac{R}{L}$$

⓪ input = 0 (short ckT)



$$\boxed{i(t) = I_m e^{-(R/L)t}}$$

← Natural response from the inductor!

So, we can find the Natural res. from Poles.

$$\boxed{i(t) = I_m e^{-R/L\, t}} \Rightarrow i(t) = I_m e^{-t/\tau}$$

$; \tau = \frac{L}{R}$; 'time constant'

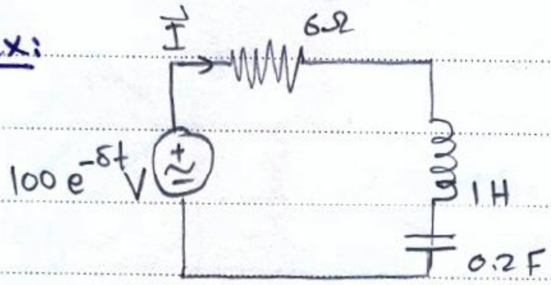
∴ Natural response $i_n(t)$ equals:-

$$\boxed{i_n(t) = I_m e^{-Rt/L}}$$

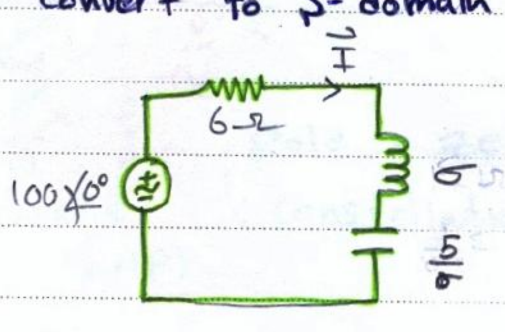
; natural response : without any forcing function.

⇒ applying a zero-Amplitude forcing function produces a non-zero forced response

ex: , sketch the response $(\vec{I})$ as a function of σ



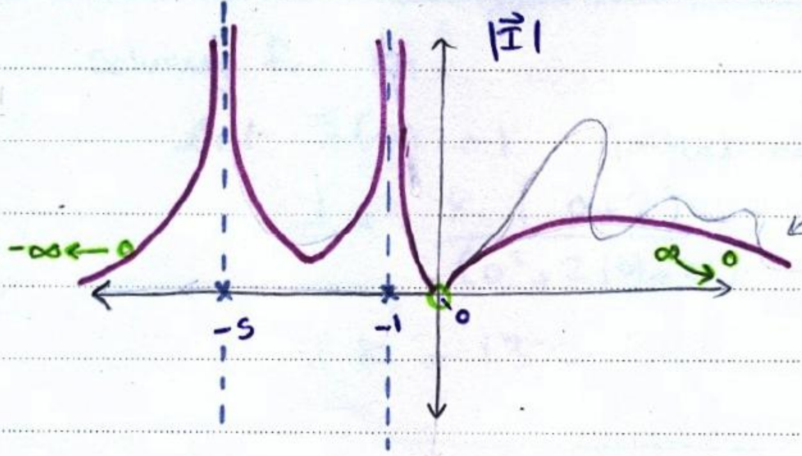
Sol:- convert to S-domain , S = σ :-



$$\vec{I} = \frac{100}{6+\sigma+\frac{5}{\sigma}} = \frac{100\,\sigma}{6\sigma+\sigma^2+5} = \frac{100\,\sigma}{(\sigma+1)(\sigma+5)}$$

* Critical points:-

- zeros @ σ = 0
- poles @ σ = -1, -5



@ $\sigma = \infty$

$|\vec{I}| = 0$

it's only a sketch! could be something else!

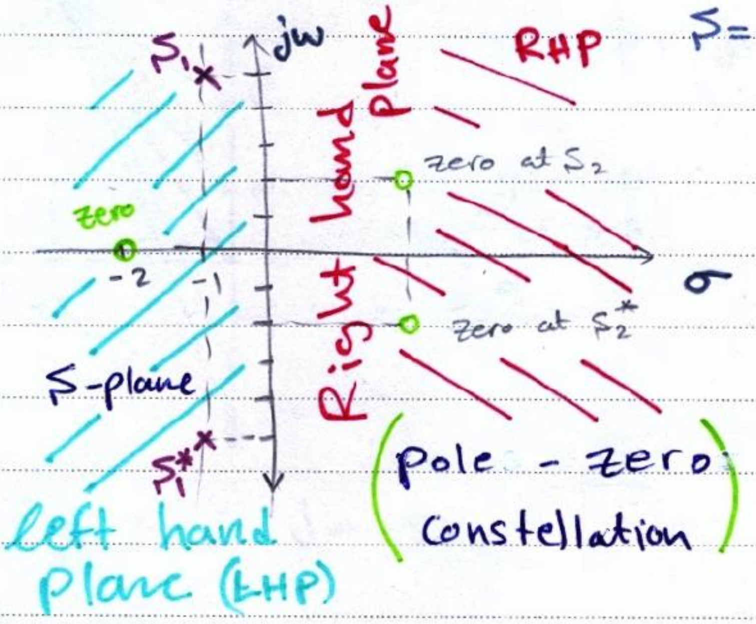
Natural response :-

$$i_n(t) = A\, e^{\overset{1^{st}\text{ pole}}{\boxed{-1}}\times t} + B\, e^{\overset{2^{nd}\text{ pole}}{\boxed{-5}}\times t}$$

Step 3 • sketch with respect to $s = \sigma + j\omega$ (3D)!

* introducing the s-plane:

$s = \sigma + j\omega$



$S_1 = -1 + j5$ (pole)

if I have a pole at S_1, then anothe pole must appear at S_1^*

if these poles & zeros were for some impedance $\vec{Z}(s)$, then:-

$$\vec{Z}(s) = K \frac{(s+2) \Rightarrow \text{zeros}}{(s+1-j5)\,(s+1+j5) \Rightarrow \text{poles}} \text{; K is a constant}$$

$$= K\, \frac{(s+2)}{s^2+2s+26}$$

solving for k:

let $\vec{Z}(0) = 1$ (initial condition).

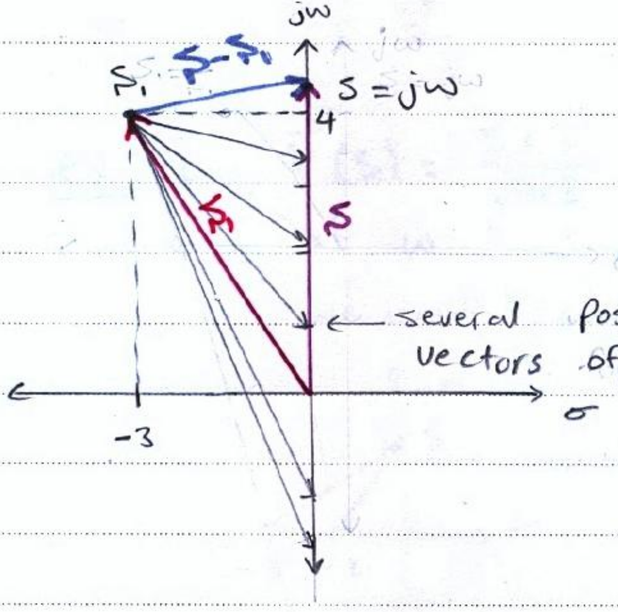
$$1 = \frac{k\,(0+2)}{(0^2+2(0)+26)} = \frac{k\times 2}{26}$$

$$\therefore k = 13$$

$$\rightarrow \boxed{\vec{Z}(s) = 13\,\frac{(s+2)}{s^2+2s+26}}$$

so, we found this equ. from the S-plane!

Sketch with respect to w using S-plane:



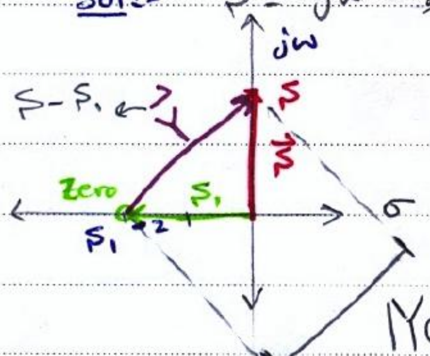
$s_1 = -3 + j4$

$s = jw$ ($\sigma = 0$, تتحرك على الـ Y-axis)

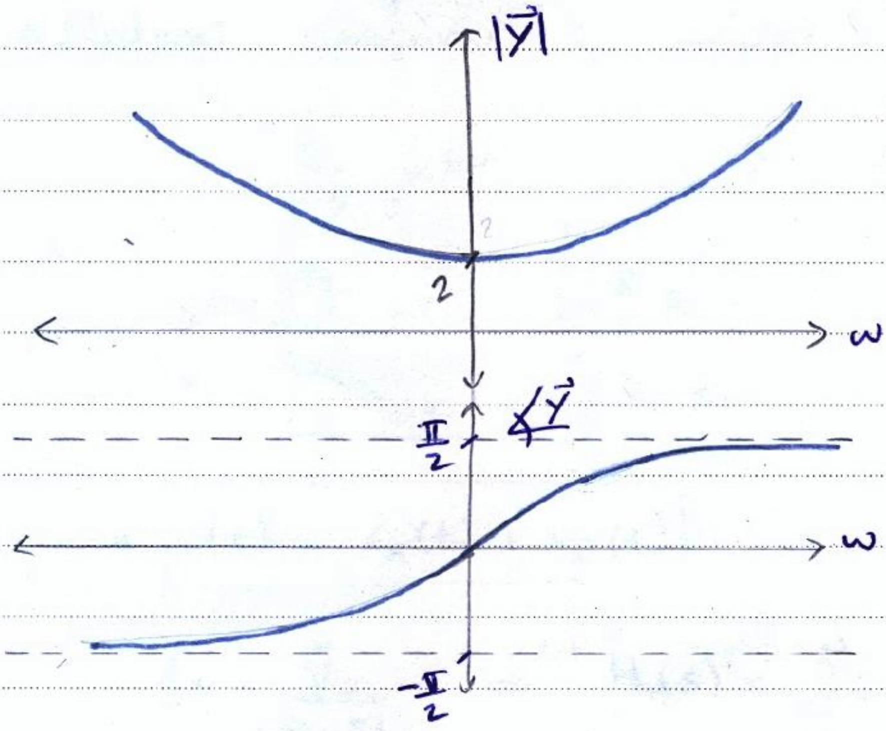
$$s - s_1 = jw - s_1$$

ex:- $\vec{Y}(s) = \frac{s+2}{s-s_1}$ $\Rightarrow$ Sketch Y with respect to w

sol:- $s = jw$, $s_1 = -2$



$$|Y(jw)| = |s+2| = |jw+2| = \sqrt{2^2+w^2}$$



* when $w=0$, $s=0$
$|\vec{Y}|=2$

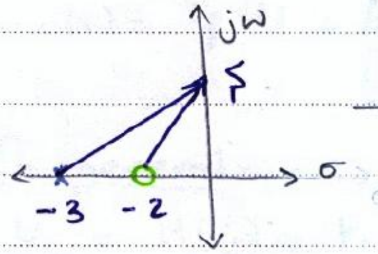
* when $w \to \pm\infty$
$|\vec{Y}| \Rightarrow \infty$

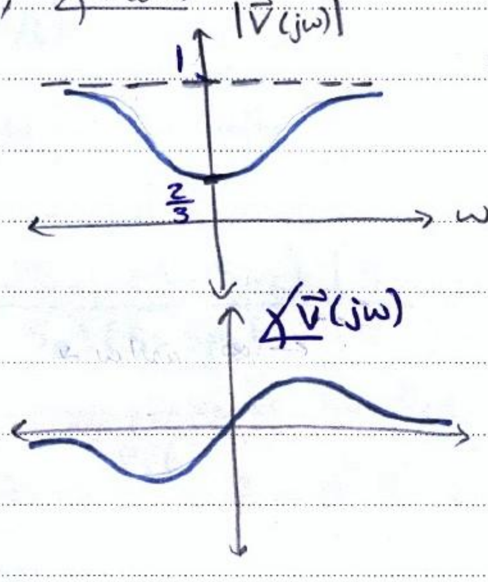
*when $w=0$, $s=0$
$\angle\vec{Y}=0°$

* when $w \to \pm\infty$.
$\angle\vec{Y} = \pm 90°$

ex2 :- $\vec{V}(s) = \frac{s+2}{s+3}$, Sketch $\vec{V}_s$ as a function of ω.

we need $|V(j\omega)|$, $\angle V(j\omega)$

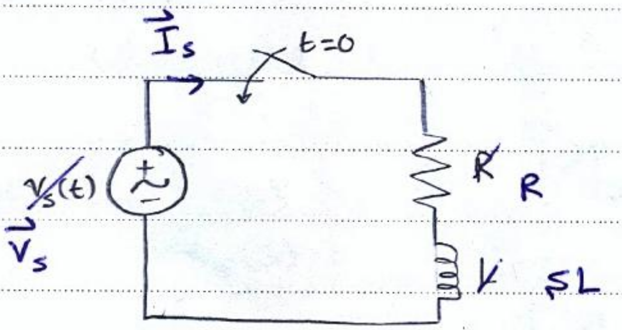




*when $w=0=s$
length of $\vec{V} = \frac{2}{3}$
; $|\vec{V}| = \frac{2}{3}$

7/4 /2013

* Natural Response & complex frequency:



$$* \boxed{i(t) = i_n(t) + i_f(t)}$$

$$\vec{I}_s = \frac{\vec{V}_s}{R+sL} \xrightarrow{let} H(s) = \frac{\vec{I}_s}{\vec{V}_s} = \frac{1}{R+sL}$$

$$H(s) = \frac{1}{L}\cdot\frac{1}{s+R/L}$$

$\rightarrow S_p = -R/L$

- $i_n(t) = Ae^{-\frac{R}{L}t}$

$$\therefore i(t) = i_f(t) + \underline{A}e^{-(R/L)t}$$

from the initial condition.

<u>In general:-</u>

if $\frac{V_2}{V_s} = H(s) = K\frac{(s-s_{o1})(s-s_{o2})(s-s_{o3})\cdots}{(s-s_{p1})(s-s_{p2})(s-s_{p3})\cdots}$ Zeros / Poles.

poles of the transfer function.

then $\rightarrow V_{2n}(t) = \underline{\underline{A_1}}e^{s_{p_1}t} + \underline{\underline{A_2}}e^{s_{p_2}t} + \underline{A_3e^{s_{p_3}t}} + \cdots$

from initial conditions.

$* \text{ if } \vec{s}_{p_3} = \vec{s}^{*}_{p_4} = \gamma + j\beta$

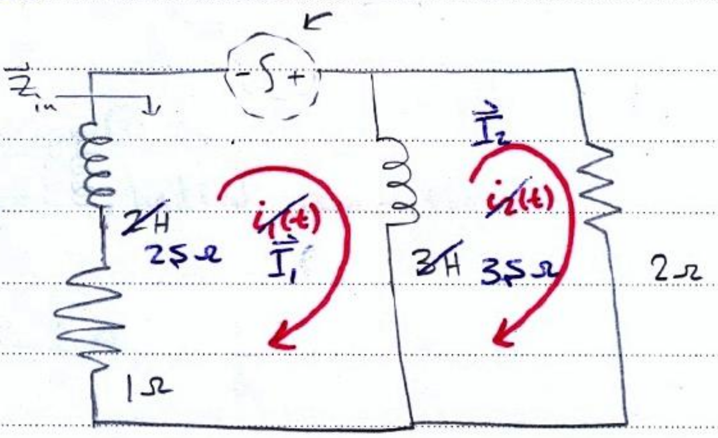
$$V_{2n}(t) = A_1e^{s_{p_1}t} + A_2e^{s_{p_2}t} + e^{\gamma t}\left[\underline{a}\cos(\beta t) + \underline{b}\sin(\beta t)\right] + \cdots$$

from initial cond.s

ex:- find $i_1(t)$ & $i_2(t)$, $t>0$

if $i_1(0) = 11\,A$

$i_2(0) = 11\,A$



Sol:

$$\boxed{H(s) = \frac{\vec{I}_1(s)}{\vec{V}_s(s)}}$$

* find $H(s)$

$$I_1(s) = \frac{\vec{V}_s}{\left(2s+1+\frac{6s}{3s+2}\right)} \leftarrow \vec{Z}_{in}$$

* نفرض وجود independant Vsource في الدارة

(لأكون بدي لاخفي بفرض Vsource

.(I source 〃 V 〃 〃 〃 〃

$$\boxed{\begin{aligned} i_1(t) &= i_{1n}(t) \\ i_2(t) &= i_{2n}(t) \end{aligned}}$$

$$H(s) = \frac{I_1(s)}{\vec{V}_s} = \frac{1}{2s+1+\frac{6s}{3s+2}}$$

$$= \frac{3s+2}{6s^2+13s+2}$$

$$H(s) = \frac{1}{2}\,\frac{s+\frac{2}{3}}{(s+2)(s+1/6)}$$

poles of $H(s)$; أصفار المقام

- $i_1(t) = A_1 e^{-2t} + B_1 e^{-\frac{1}{6}t}$ ....ⓐ

- $i_2(t) = A_2 e^{-2t} + B_2 e^{-\frac{1}{6}t}$ ....ⓑ

* finding initial conditions:

from the question

$i_1(0) = 11A$

$i_2(0) = 11A$

plugging ⓐ & ⓑ [illegible]

$i_1(0) = A_1 + B_1$ ....①

/ /

- for ⓐ:

$$i_1(0) = \boxed{A_1 + B_1 = 11} \quad \dots ①$$

we need another initial condition:

$$\underbrace{\left.\frac{di_1(t)}{dt}\right|_{t=0}} = -2A_1 - \frac{1}{6}B_1 \quad \dots ⊛$$

KVL (the outer loop)

$$1\,i_1(t) + 2\frac{di_1(t)}{dt} + 2i_2(t) = 0$$

$$\frac{di_1(t)}{dt} = -\frac{1}{2}\left[i_1(t) + 2i_2(t)\right]$$

$$\left.\frac{di_1(t)}{dt}\right|_{t=0} = -\frac{1}{2}\left[i_1(0) + 2i_2(0)\right] = -\frac{33}{2}$$

plug in ⊛

$$-\frac{33}{2} = -2A_1 - \frac{1}{6}B_1 \quad \dots ②$$

solving ① & ② $\longrightarrow \boxed{\begin{matrix} A_1 = 8 \\ B_1 = 3 \end{matrix}}$

$$\therefore \boxed{i_1(t) = 8e^{-2t} + 3e^{-t/6}}$$

- for equ. ⓑ :

$i_2(0) = \boxed{A_2 + B_2 = 11}$ ... ③

we need another initial condition:

$$\left.\frac{di_2(t)}{dt}\right|_{t=0} = -2A_2 - \frac{1}{6}B_2 \quad \ldots \; (**)$$

KVL (the 2nd loop)

$$2\,i_2(t) + 3\,\frac{d\left(i_2(t) - i_1(t)\right)}{dt} = 0$$

$$\frac{d\,i_2(t)}{dt} = \frac{d\,i_1(t)}{dt} - \frac{2}{3}\,i_2(t)$$

$$\left.\frac{d\,i_2(t)}{dt}\right|_{t=0} = \frac{-33}{2} - \frac{2}{3}\times 11 = -\frac{143}{6}$$

Plug in (**)

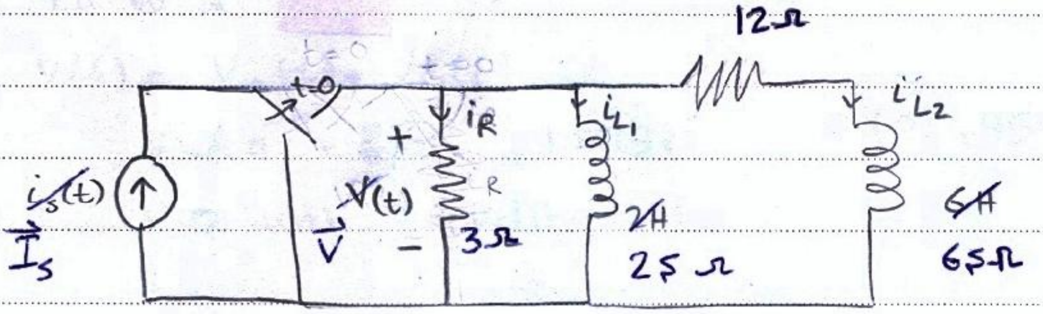
$$\frac{-143}{6} = -2A_2 - \frac{1}{6}B_2 \quad \ldots ④$$

Solve ③ & ④ $\longrightarrow A_2 = 12 \;, B_1 = -1$

$$\boxed{i_2(t) = 12\,e^{-2t} - e^{-t/6}}$$

/ /

ex:-



$i_s(t) = e^{-t}\cos(2t)$ A $\longrightarrow \vec{I}_s = 1\angle 0$

$s = -1+j2$, find V(t)?

Sol:-

- $\boxed{V(t) = V_n(t) + V_f(t)}$ ... ⊛

$$\vec{V} = \frac{\vec{I}_s}{\text{admittance}} = \frac{\vec{I}_s}{\frac{1}{3}+\frac{1}{2s}+\frac{1}{12+6s}} = \frac{3s(s+2)\vec{I}_s}{(s+1)(s+3)}$$

- $H(s) = \frac{\vec{V}}{\vec{I}_s} = \frac{3s\,(s+2)}{(s+1)(s+3)}$ $\Rightarrow$ poles $= -1, -3$

$\rightarrow V_n(t) = Ae^{-t} + Be^{-3t}$ natural response.

$$\rightarrow V_f(s) = \vec{I}_s * H(s)$$
$$= \vec{I}_s \times \frac{3s(s+2)}{(s+1)(s+3)}$$

$$V_f(-1+j2) = 1\angle 0^\circ \cdot \frac{3(-1+j2)(-1+j2+2)}{(-1+j2+1)(-1+j2+3)}$$

$$V_f(s)\Big|_{s=-1+j2} = 1.875\sqrt{2}\angle 45^\circ$$

$$V_f(t) = 1.875\sqrt{2}\,e^{-t}\cos(2t+45^\circ)$$

back to ⊛ :-

$$V(t) = V_n(t) + V_f(t)$$

$$= Ae^{-t} + Be^{-3t} + 1.875\sqrt{2}\, e^{-t}\cos(2t+45^\circ) \quad \ldots Ⓐ$$

* I need 2 initial conditions to find A & B

1^{st}) $\underline{\underline{V(0)}} = A + B + 1.875\cancel{\sqrt{2}}\ \cancel{\cos 45} \quad \ldots Ⓐ$

$$\hookrightarrow \quad i_{L_1}(0^+) = i_{L_2}(0^-) = 0$$

$$\therefore i_{L_1}(0^+) = i_{L_2}(0^+) = 0$$

$$V(0) = I_s(0) \times 3$$

$$= 1 \times 3 = 3\ V$$

plug in Ⓐ

$$\boxed{3 = A + B + 1.875} \quad \ldots ①$$

2^{nd}) $\frac{dv}{dt} = -Ae^{-t} - 3Be^{-3t} + 1.875\sqrt{2}\left[-2e^{-t}\sin(2t+45^\circ) - e^{-t}\cos(2t+45^\circ)\right]$

$$\underline{\underline{\left.\frac{dv}{dt}\right|_{t=0}}} = -A - 3B - 5.625 \quad \ldots ②'$$

$$\hookrightarrow \quad V(t) = 3\, i_R(t)$$

$$= 3\left(i_s(t) - i_{L_1}(t) - i_{L_2}(t)\right)$$

$$\frac{dv(t)}{dt} = 3\left(\underline{\underline{\frac{d\,i_s(t)}{dt}}} - \frac{d\,i_{L_1}(t)}{dt} - \frac{d\,i_{L_2}(t)}{dt}\right) \ldots Ⓑ$$

معطاة في السؤال

$$\hookrightarrow \frac{d\,i_s(t)}{dt} = -2e^{-t}\sin(2t) - e^{-t}\cos(2t)$$

$$\boxed{\left.\frac{d\,i_s(t)}{dt}\right|_{t=0} = -1}$$

$$\rightarrow L_1 \frac{di_{L_1}}{dt}\bigg|_{t=0} = \overset{\nearrow V(0)=3}{V(t)}\bigg|_{t=0}$$

$$\frac{\cancel{2}}{\cancel{2}}\ \boxed{\frac{di_{L_1}}{dt}\bigg|_{t=0} = \frac{3}{2}}$$

$$\rightarrow L_2 \frac{di_{L_2}}{dt}\bigg|_{t=0} = V(0)$$

$$\frac{\cancel{6}}{\cancel{6}}\ \boxed{\frac{di_{L_2}}{dt}\bigg|_{t=0} = \frac{3}{6}}$$

plug in Ⓑ'

$$\frac{dv}{dt}\bigg|_{t=0} = 3\left(-1 - \frac{3}{2} - \frac{3}{6}\right) = -9$$

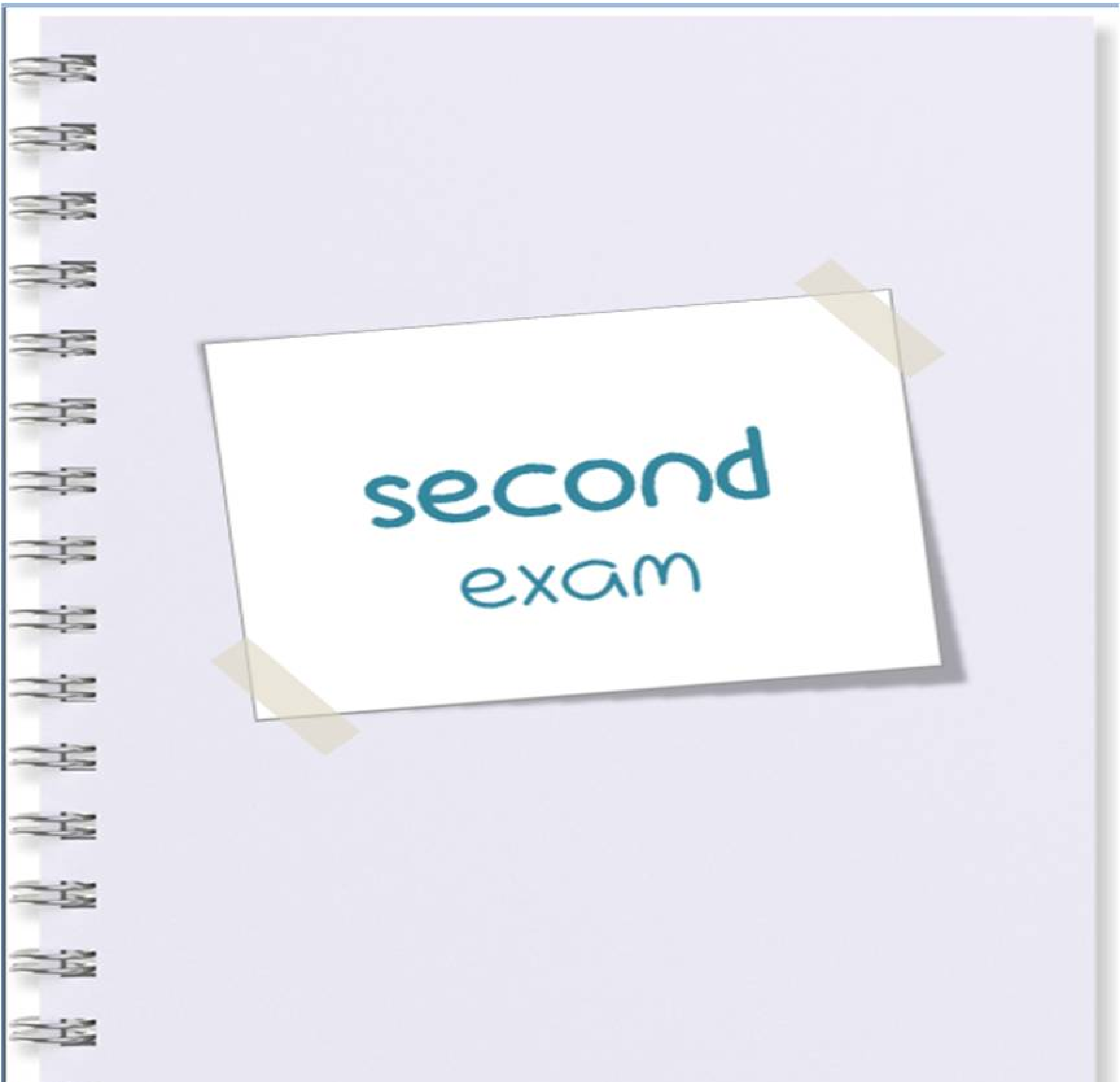
← the 2^{nd} initial condition $\left(\frac{dv}{dt}\big|_{t=0} = -9\right)$

plug in ②'

$$\boxed{-A - 3B - 5.625 = -9} \quad \ldots ②$$

solving ① & ② $\longrightarrow$ $A = 0$, $B = 1.125$

$$v(t) = \underbrace{1.125e^{-3t}}_{V_n(t)} + \underbrace{1.875\sqrt{2}\, e^{-t}\cos(2t + 45^\circ)\ V}_{V_f(t)}$$


second
exam

In fig1 below, the 3-phase source is balanced, P_1 reads upscale 4411 Watt, whele P_2. reads downscale 1000 Watt, & $V_{AB}=440\angle 0^\circ\ V_{rms}$

Determine:-

(2marks) The value of P.F for the load (is it leading or lagging?)

(4marks) I_{aA} and Z_p

(2marks) Van

(2marks) the P.f at the source (is it leading or lagging?) & the total ~~p~~ complex power of the source.

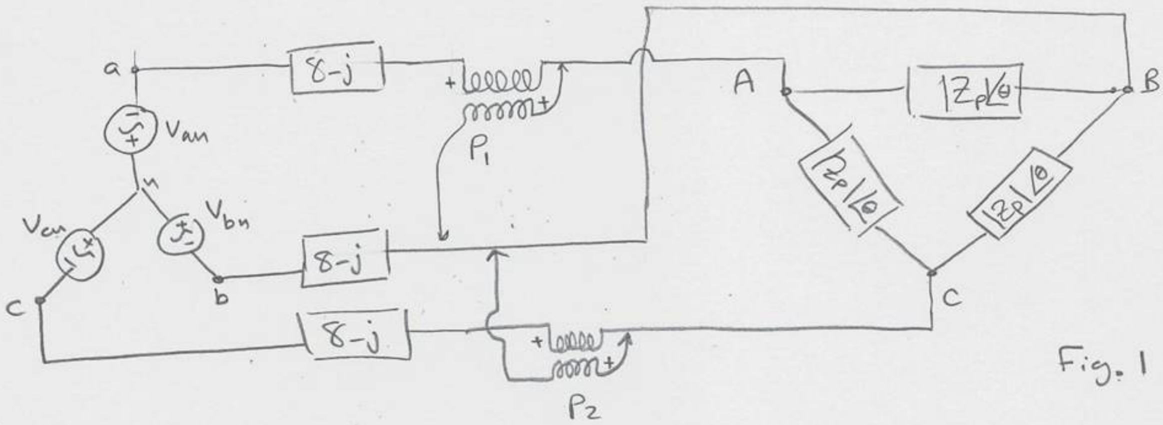


Fig. 1

$$\tan\theta = \sqrt{3}\,\frac{(P_2 - P_1)}{(P_2 + P_1)} \quad , P_1 = 4411 \ , P_2 = -1000$$

$$\tan\theta = \sqrt{3}\,\frac{(-1000-4411)}{(-1000+4411)} = -2.742$$

$$\Rightarrow \theta = \tan^{-1}(-2.742) = -70^\circ$$

$$\Rightarrow \boxed{P.F_L = \cos(-70^\circ) = 0.342 \text{ leading.}}$$

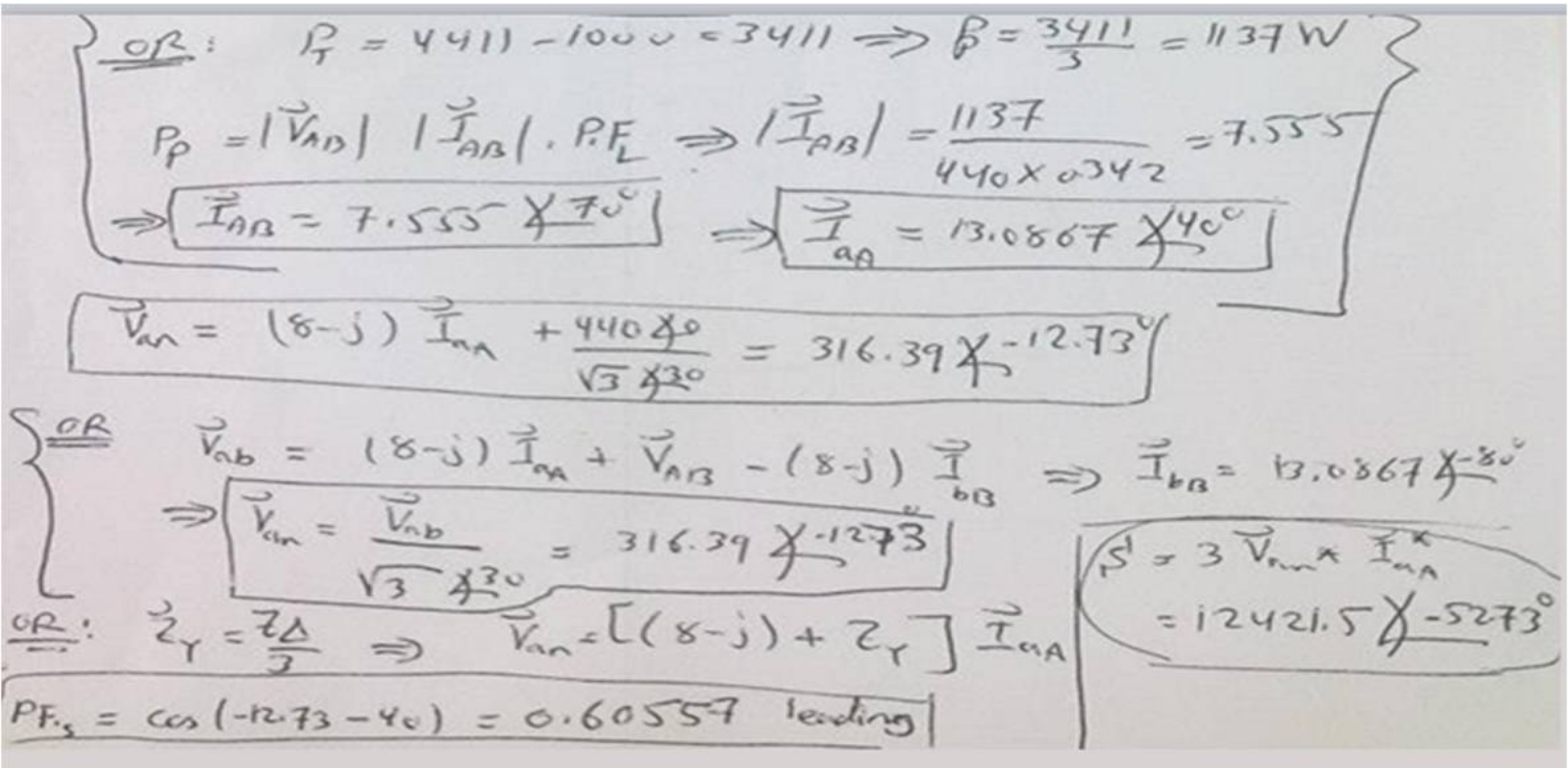
$$P_1 = |\vec{V}_{AB}|\,|\vec{I}_{aA}|\cos(30^\circ+\theta) \Rightarrow \frac{4411}{440 \times \cos(40^\circ)} = 13.0867 = |\vec{I}_{aA}|$$

$$\Rightarrow \vec{I}_{aA} = \sqrt{3}\,\angle{-30^\circ}\,\vec{I}_{AB} \Rightarrow |\vec{I}_{AB}| = \frac{13.0867}{\sqrt{3}} = 7.5556$$

$$|\vec{Z}_p| = \frac{440}{7.5556} = 58.23484 \Rightarrow \boxed{\vec{Z}_p = 58.23484\,\angle{-70^\circ}}$$

$$\Rightarrow \vec{I}_{AB} = \frac{\vec{V}_{AB}}{\vec{Z}_p} = \frac{440\angle 0^\circ}{58.23484\,\angle{-40}} = 7.5556\,\angle 70^\circ$$

$$\Rightarrow \boxed{\vec{I}_{aA} = 13.0867\,\angle 40^\circ}$$



OR: $P_T = 4411 - 1000 = 3411 \Rightarrow P = \frac{3411}{3} = 1137\ W$

$P_P = |\vec{V}_{AB}|\ |\vec{I}_{AB}| \cdot P.F_L \Rightarrow |\vec{I}_{AB}| = \frac{1137}{440 \times 0.342} = 7.555$

$\Rightarrow \boxed{\vec{I}_{AB} = 7.555 \angle 70^\circ} \Rightarrow \boxed{\vec{I}_{aA} = 13.0867 \angle 40^\circ}$

$$\boxed{\vec{V}_{an} = (8-j)\ \vec{I}_{aA} + \frac{440\angle 0}{\sqrt{3}\angle 30} = 316.39 \angle -12.73^\circ}$$

OR $\vec{V}_{ab} = (8-j)\ \vec{I}_{aA} + \vec{V}_{AB} - (8-j)\ \vec{I}_{bB} \Rightarrow \vec{I}_{bB} = 13.0867 \angle -80^\circ$

$\Rightarrow \boxed{\vec{V}_{an} = \frac{\vec{V}_{ab}}{\sqrt{3}\angle 30} = 316.39 \angle -12.73^\circ}$

OR: $\vec{Z}_Y = \frac{Z_\Delta}{3} \Rightarrow \vec{V}_{an} = [(8-j) + Z_Y]\ \vec{I}_{aA}$

$\boxed{PF_s = \cos(-12.73 - 40) = 0.60557 \text{ leading}}$

$$\boxed{S = 3\ \vec{V}_{an} \times \vec{I}_{aA}^{*} = 12421.5 \angle -52.73^\circ}$$

Section:…………………… الاسم:……………………………………

Q(2): (a) (6 marks) For the circuit shown in Fig.2 find: $\boldsymbol{V_2}$ and $\boldsymbol{I_x}$.

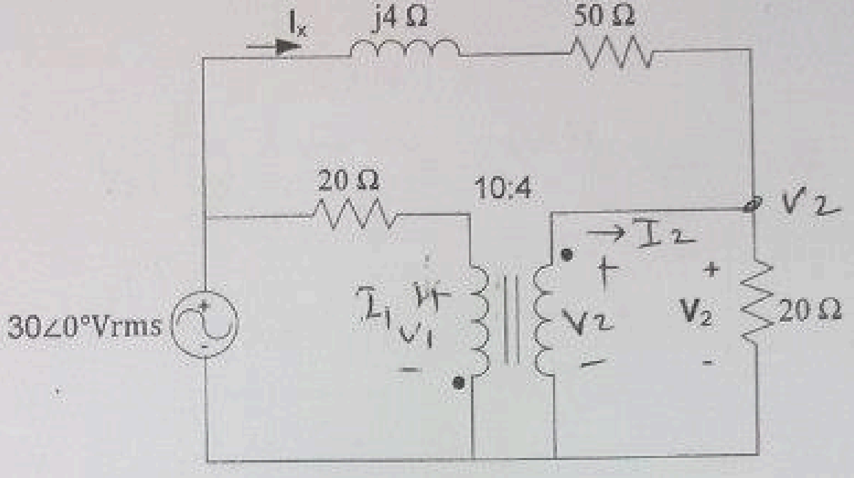


Fig. 2

Mesh1 $-30\angle 0 + 20I_1 + V_1 = 0$

$V_1 = 30\angle 0 - 20I_1$ --①

$\frac{I_1}{I_2} = 0.4$ --②

$\frac{V_2}{V_1} = -0.4.$ --③

Nodal. $\frac{30\angle 0 - V_2}{50+j4} + V_2 = \frac{V_2}{20}$ --④

Solve for V_2, I_x.

$$\boxed{\begin{array}{l} I_x = 0.76 \angle -4.6\,A. \\ V_2 = 8.9 \angle -178 \end{array}}$$

Section:........................ ....:الاسم

(b) (4 marks) For the circuit shown in Fig.3 given $v_s(t) = 20\cos 10t\ V$, find $v(t)$.

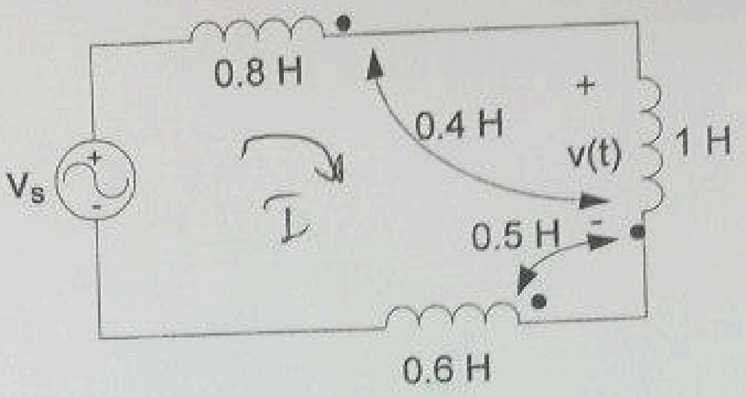


Fig. 3

$$-20 + j8I + j10I + j6I + j4I_1 + j4I$$

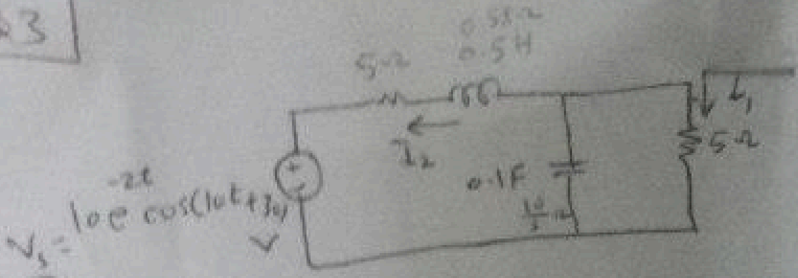
$$-j5I - j5I = 0$$

$$I = \frac{20\angle 0}{j22}$$

$$V = I\,[j10 + j4 - j5)$$

$$\boxed{v(t) = 8.18\cos 10t}$$

Q3



⑥ ① $i_1(t) = ?$

② ② the natural respong for $i_2(t)$ in the s-domain

② ③ Constelate all poles / Zeros for i_2

---

① $s = -2 + j10.$ ①

Given $V_s = 10e^{-2t}e^{j(10t+30)}$ ①

$\therefore V_{sp} = 10\angle 30$

Using V.D.:

$$V_{5\Omega} = V_p \frac{5(\frac{10}{s})/(5+\frac{10}{s})}{[5(\frac{10}{s})/(5+\frac{10}{s})] + 0.5s + 5}$$ ②

$$= \boxed{10\angle 30 \frac{50}{2.5s^2+30s+100}}\Bigg|_{s=-2+j10}$$

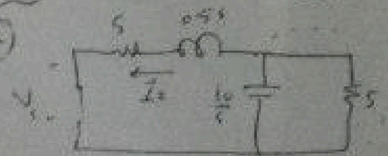
$$= \frac{500\angle 30}{(-240 - j100 - 60 + j300 + 100}$$

$$= \frac{500\angle 30}{-200+j20} = 1.76\angle -105$$

$\therefore i_1 = \frac{1.76\angle -105}{5} = 0.352\angle -105$

$\therefore i_1(t) = 0.352 e^{-2t}\cos(10t - 105)$ ①

② ①



$I_2 = ?$

$V_s = (\frac{50}{10+5s} + 0.5s + 5) I_2$ ① $\frac{5(\frac{10}{s})}{5+\frac{10}{s}}$ $\frac{50}{10+5s}$

$$\therefore \frac{I_2}{V_s} = \frac{1}{\frac{50}{10+5s}+0.5s+5}\left(\frac{10+5s}{10+5s}\right)$$

$$= \frac{10+5s}{50+25s+5s+2.5s^2+50}$$

$$\therefore \boxed{\frac{I_2}{V_s} = \frac{10+5s}{2.5s^2+30s+100}}$$

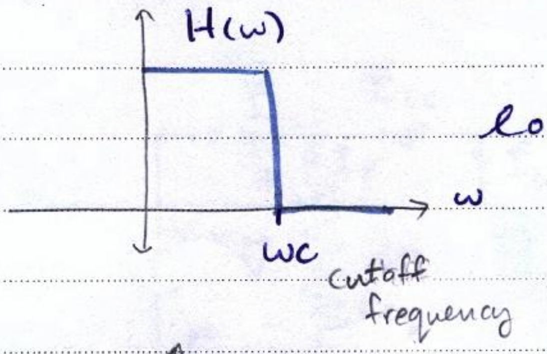
$Z_1 = -\frac{10}{5} = -2$ ①

$P_1 =$ ① ①

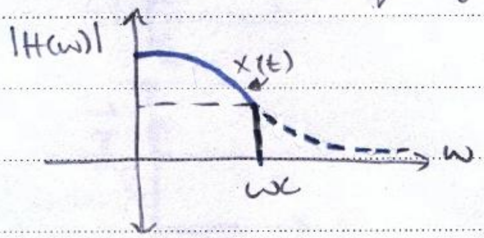
$P_2 =$

9/4 2013

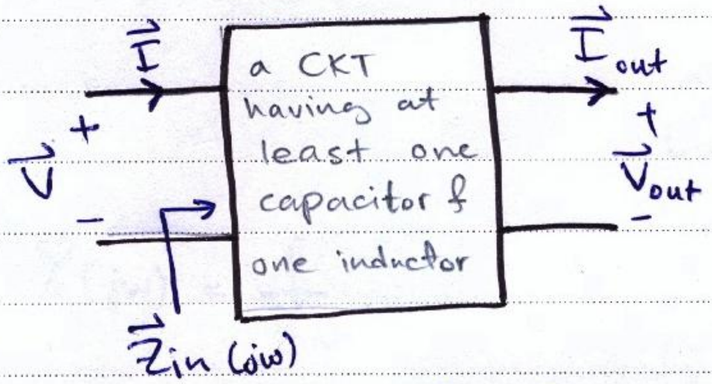
Chapter # 16: Frequency Response:-



low-pass Filter



Resonance:-

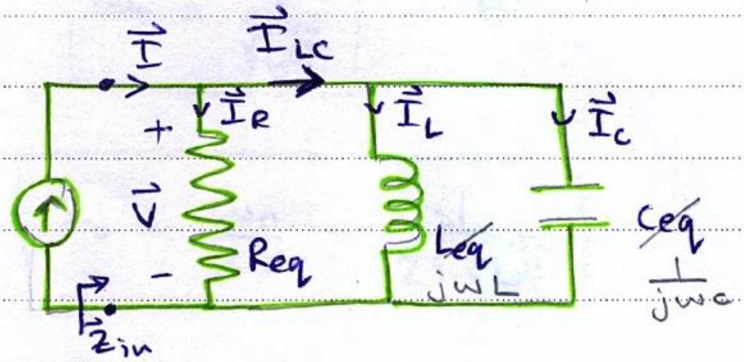


I need at least 1 (C) & 1 (L) to reach resonance!

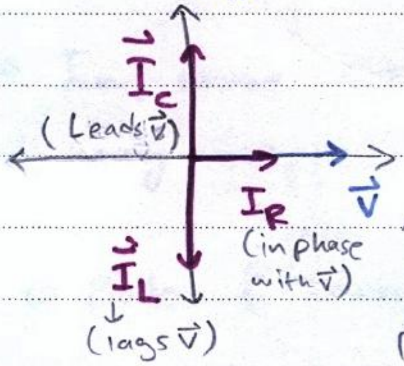
@ resonance:- $\vec{I}$ & $\vec{V}$ are in Phase (pure resistive $\vec{Z}_{in}$).
$\downarrow$ $w = w_0$ $\quad$ ↳ $Z_{in} = R$

@ resonance:- the amplitude of the response is either max or min ($\vec{V}$ أو $\vec{I}$ حسب)

→ Parallel resonant CKT:



$\sigma = 0, \ s = jw$



, take $\vec{V}$ as a refferance
لأنه مشترك بينهم

when $|\vec{I}_L| = |\vec{I}_C|$
they cancel each other
& $\vec{I} = \vec{I}_R$
$\therefore \vec{Z}_{in} = R$
"Resonant CKT"
where I_C & $I_L \neq 0$
but they cancel each other

$$\vec{Y}(jw) = \frac{1}{\vec{Z}_{in}}$$

$$= \frac{1}{R} + \frac{1}{jwL} + jwc$$

it's easer to deal with admittance

$$\vec{Y}(jw) = \frac{1}{R} + j\left(wc - \frac{1}{wL}\right) \dots \circledast$$

since it's a parallel RLC CKT.

@ resonance

$$\vec{Y}(jw) = \frac{1}{R} \dots ** ;$$

since $\vec{Z}_{in} = R$
*maximum impedance @ resonance.

$\therefore 1^{st}$ condition of resonance from $\circledast$, **

$$w_o C - 1/w_o L = 0$$

$$\Rightarrow w_0 C = \frac{1}{w_0 L}$$

$$\boxed{w_0 = \frac{1}{\sqrt{LC}}} \quad \cdots Ⓐ \quad [\text{rad/sec}]$$

$$\boxed{f_0 = \frac{w_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}} \quad [\text{Hz}]$$

- In some CKTs, multi resonance freq. may appear!

⟹ Poles & zeros of $\vec{Y}(\overset{s}{\cancel{jw}})$

$$\vec{Y}(s) = \frac{1}{R} + \frac{1}{sL} + sC$$

$$= \frac{C(s^2 + s/RC + 1/LC)}{s}$$

$$\vec{Y}(s) = \frac{C}{s}(s + \alpha - jw_d)(s + \alpha + jw_d)$$

$$\boxed{\alpha = \frac{1}{2RC}} \cdots ①$$

"exponential damping coeficient [Np/sec]"

$$\boxed{w_d = \sqrt{w_0^2 - \alpha^2}} \cdots ②$$

"Natural resonant freq. [rad/sec]"

$$\therefore \vec{Y}(s) = C\,\frac{s^2 + 2\alpha s + w_0^2}{s}$$

$$s = \frac{-2\alpha \pm \sqrt{4\alpha^2 - 4w_0^2}}{2}$$

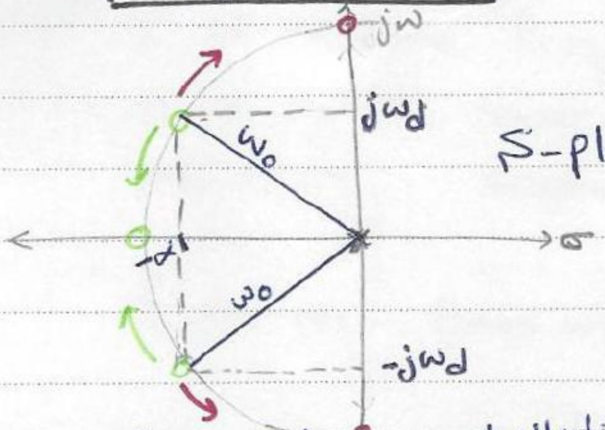
$$\rightarrow s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$= -\alpha \pm j\overset{\omega_d}{\left(\sqrt{\omega_0^2 - \alpha^2}\right)} \quad ; \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2} \; \ldots Ⓐ$$

$$= -\alpha \pm j\omega_d$$

from Ⓐ

$$\boxed{\omega_0 = \sqrt{\omega_d^2 + \alpha^2}} \ldots ③$$; ω_0 : resonance freq. for parallel RLC.



S-plane.

zeros of Y at $s \begin{cases} -\alpha + j\omega_d \\ -\alpha - j\omega_d \end{cases}$

** zeros of $\vec{Y}$ = Poles of $\vec{Z}$

zero-pole ○ constellation

if ω_0 is constant & :

1. $\sigma = 0$, $\alpha = 0$

$\therefore j\omega_0 = j\omega_d$

2. $j\omega_d \Rightarrow 0$

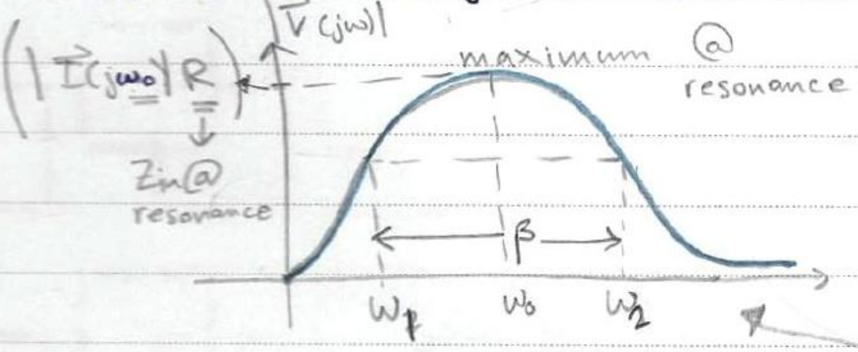
$\omega_0 = \alpha$

* Voltage response :-

$$\vec{V}(s) = \vec{I}(s)\,\vec{Z}(s)$$

$$\hookrightarrow \vec{Z}(s) = \frac{s/C}{(s+\alpha-j\omega_d)(s+\alpha+j\omega_d)} \leftarrow \frac{1}{\vec{Y}(s)}$$

$* \ |\vec{V}(j\omega)| = |\vec{I}(j\omega)| \ |\vec{Z}(j\omega)|$



Sketch for +ve frequency, in the -ve part it must satisfy the even symmetry.

this CKT is a "Band pass filter", & w0 is called "Center freq.".

$*$ calculate power @ $\omega = \omega_0$?

$$P(\omega_0) = |\vec{I}(j\omega_0)|^2 R$$

if P at certain $\underset{\uparrow}{\text{freq.}}^{(\omega_1),(\omega_2)} = \frac{1}{2}$ maximum P (at resonance)

∴ w_1 is called lower half power frequency.

w_2 " " ~~high~~upper " " "

& $w_2 - w_1$ = Band width (β)

@ Resonance

$$|\vec{V}(j\omega_0)| = |\vec{I}(j\omega_0)| \cdot R$$

$$\vec{I}_L(j\omega_0) = \frac{\vec{V}(j\omega_0)}{j\omega_0 L} \leftarrow \vec{V}_L = \vec{V}_R = \vec{V}_C$$

$$I_L(j\omega_0) = \frac{\vec{I}(j\omega_0) R}{j\omega_0 L} \quad \ldots ⓐ$$

$$I_C(j\omega_0) = \vec{V}(j\omega_0)(j\omega_0 C)$$
$$= j\omega_0 RC \ \vec{I}(j\omega_0) \quad \ldots ⓑ$$

→ but, we know that $\omega_0 L = \frac{1}{\omega_0 C}$ ... from Ⓐ

from ⓐ

$$\vec{I}_L\,(j\omega_0) = -j\vec{I}\,(j\omega_0)\ R\ (\omega_0 C) \quad ...ⓐ'$$

from ⓑ

$$\vec{I}_c\,(j\omega_0) = j\vec{I}\,(j\omega_0)\ RC\ \omega_0 \quad ...ⓑ'$$

$$\therefore I_c(j\omega_0) = -\vec{I}_L(j\omega_0) \quad \#$$

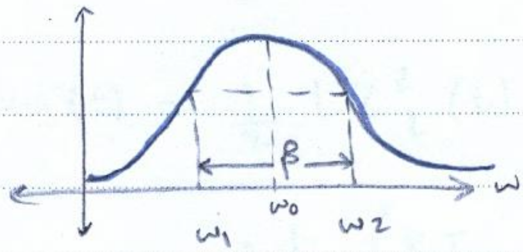
$$= j\omega_0 RC\ \vec{I}(j\omega_0)$$

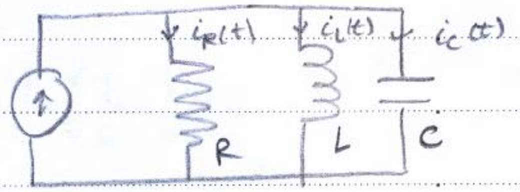
$$\therefore \vec{I}_{LC} = \vec{I}_L + \vec{I}_c = 0 \quad \text{@ resonance}$$

$$\boxed{|\vec{I}_L(j\omega_0)| = |\vec{I}_C(j\omega_0)| = \omega_0 RC\,|\vec{I}(j\omega_0)|}\ *$$

11 / 4 /2013

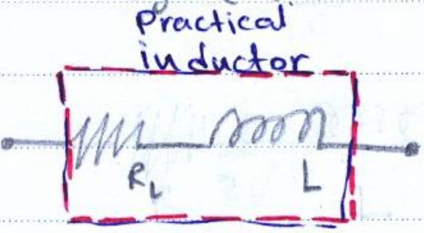
Quality Factor:-





as $\underline{Q}$ increases : Sharpness ↑ , Bandwidth ↓ ⇒ greater freq. selectivity

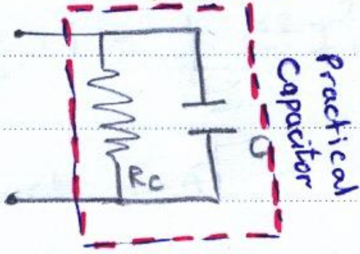
Quality (الجودة).



$Q \to \infty$

$R_L \to 0$ (pure inductance)

تزداد الكفاءة!



$Q \to \infty$

$R_C \to \infty$ (pure capacitance)

In general:

$$Q = 2\pi \times \frac{\text{maximum Energy stored in L\&C}}{\underbrace{\text{total Energy lost per period (in R)}}_{\text{power}}}$$

$$= 2\pi \frac{\left[W_L(t) + W_C(t) \right]_{max}}{P_R \underset{\text{period}}{\underline{T}}} \quad \text{... @ any freq.} \quad \text{...} ①$$

Q_0 : Quality factor at $\omega = \omega_0$ (resonance).

let $i(t) = I_m \cos(\omega_0 t)$ @resonance

$v(t) = R\, i(t)$ ⇒ (pure R)

$= R I_m \cos(\omega_0 t)$ $\vec{I}$ & $\vec{V}$ are in-phase.

$$W_c(t) = \frac{1}{2} C v^2(t) = \frac{I_m^2 R^2 C}{2} \cos^2(w_0 t)$$

$$W_L(t) = \frac{1}{2} L i_L^2(t) = \frac{1}{2} L\left(\frac{1}{L}\int v(t)\,dt\right)^2$$

$$= \frac{1}{2L}\left(\frac{R I_m}{w_0} \sin(w_0 t)\right)^2$$

but $w_0^2 = \frac{1}{LC}$

$$= \frac{1}{2\cancel{L}} R^2 I_m^2 \cancel{L} C \sin^2(w_0 t)$$

$$W_L(t) = \frac{1}{2} R^2 I_m^2 C \sin^2(w_0 t)$$

back to ①

$$\left[W_c(t) + W_L(t)\right]_{max}$$

$$= \frac{I_m^2 R^2 C}{2}\left(\cancelto{1}{\cos^2(w_0 t) + \sin^2(w_0 t)}\right)$$

$$= \frac{I_m^2 R^2 C}{2}$$ constant! ... $\boxed{a}$

$$P_R = \frac{1}{2} I_m^2 R$$

$$P_R T = \frac{1}{2} I_m^2 R \cdot \frac{1}{f_0} \quad \boxed{b}$$

∴ plug $\boxed{a}$ & $\boxed{b}$ in ①

$$Q_0 = 2\pi \cdot \frac{I_m^2 R^2 C/2}{I_m^2 R/2f_0} = 2\pi f_0 RC = w_0 RC$$

$$\therefore \boxed{Q_o = \omega_o RC}$$

$$\boxed{\omega_o = \frac{1}{\sqrt{LC}}}$$

$$Q_o = \omega_o RC = \frac{1}{\sqrt{LC}} . RC = R\sqrt{\frac{C}{L}}$$

$$\text{or} \rightarrow Q_o = \frac{R}{\underset{\underset{\underset{R+jX}{\underline{\underline{Im\{\vec{Z}\}}}}}{\downarrow}}{\underline{|X_{c(\omega_o)}|}}} = \frac{R}{\underset{\underset{R\omega_o C}{\downarrow}}{1/\omega C}} \underline{\underline{\text{or}}} \frac{R}{\omega_o L} = \frac{R}{|X_L(\omega_o)|}$$

$$= \frac{1}{\underset{\omega_o}{\underline{\underline{\omega C}}}} = R\omega_o C$$

ω_o @ resonance.

$$\boxed{\vec{I}_c(\omega_o) = -\vec{I}_L(\omega_o) = j\omega_o RC\ \vec{I}(\omega_o)}$$ from before

$$\hookrightarrow \boxed{= j\ Q_o\ \vec{I}(\omega_o)}$$

* if $Q_o > 1$ ⇒ it's a current amplifier

$$\boxed{\alpha = \frac{1}{2\underline{\underline{R}}C}}$$; exponential damping coeficient.

$$= \frac{1}{2(Q_o/\omega_o\cancel{C})\cancel{C}}$$

$$\boxed{\alpha = \frac{\omega_o}{2\ Q_o}}$$

$$\boxed{\omega_d = \sqrt{\omega^2 - \alpha^2}}$$

$$= \sqrt{\omega_0^2 - \frac{\omega_0^2}{4Q_0^2}}$$

$$\boxed{\omega_d = \omega_0\sqrt{1-\left(\frac{1}{2Q_0}\right)^2}}$$

* Damping factor :-

$$\vec{Y}(s) = C\,\frac{s^2 + s/RC + 1/LC}{s}$$

$$= C\,\frac{(s+\alpha-j\omega_d)(s+\alpha+j\omega_d)}{s}$$

also

$$= C\,\frac{(s^2+2\alpha s+\omega_0^2)}{s}$$

$$s^2+2\alpha s+\omega_0^2$$

$$\hookrightarrow s^2+2\underset{\substack{\uparrow\\ \text{zeta}}}{\zeta}\,\omega_0 s+\omega_0^2$$

$\overset{\swarrow \text{zeta}}{\zeta}$: Damping factor [dimension less]

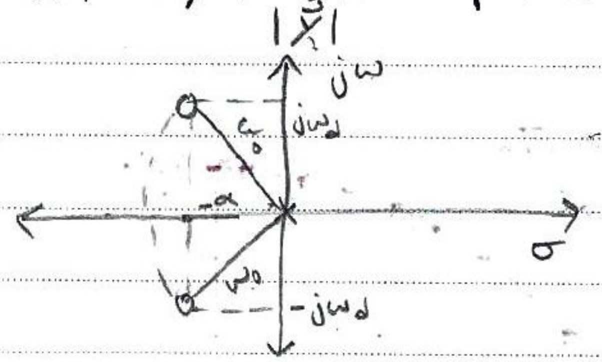
$$\boxed{\zeta = \frac{\alpha}{\omega_0} = \frac{1}{2Q_0}} \leftarrow \quad \frac{\alpha}{\omega_0} = \frac{\cancel{\omega_0}/2Q_0}{\cancel{\omega_0}} = 1/2Q_0 \,*$$

back to $\omega_d = \omega_0\sqrt{1-\left(\frac{1}{2Q_0}\right)^2}$

$$\boxed{\omega_d = \omega_0\sqrt{1-(\zeta)^2}}$$

/ /

Q_o, α, ζ & complex frequeney:-



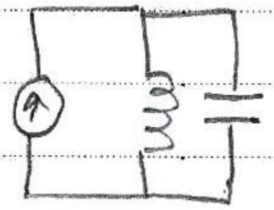
$\alpha = 0$

$\omega_d = \omega_o$

$Q_o = \infty$

$R = \infty$

parallel



مافي ضياع
في الطاقة !

when $\omega_d = 0$

$(\alpha) = \omega_o$

$\rightarrow \left(\frac{\omega_o}{2Q_o}\right) = \omega_o \quad \xrightarrow{also} \quad \alpha = \omega_o$

$\therefore Q_o = \frac{1}{2}$

$\frac{1}{2RC} = \frac{1}{\sqrt{LC}}$

$R = \frac{\sqrt{LC}}{2C}$

$\boxed{R = \frac{1}{2}\sqrt{\frac{L}{C}}}$

ex:- A parallel resonent CKT is composed of elements

$R = 8k\Omega$, $L = 50mH$, $C = 80nF$

find:-

1) $\omega_o = \frac{1}{\sqrt{LC}} = 15.811k$ rad/sec.

2) $Q_o = \omega_o RC = 10.119$; very high Q_o

3) $\omega_d = \omega_o\sqrt{1-\left(\frac{1}{2Q_o}\right)^2} = \omega_o\sqrt{1-\zeta^2}$

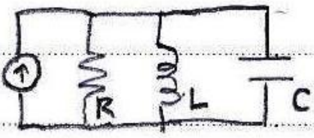
$= (15.791)$ k rad/sec.

4) $\alpha = \frac{\omega_o}{2Q_o} \approx 781$ Np/sec.

5) $\zeta = \frac{\alpha}{\omega^o} = 1/(2Q_o) \approx 0.0494$

ex2 :- find the components R , L , C for a parallel resonant CKT ; wo = 1000 rad/sec. ω_d = 998 rad/sec. $\vec{Y}(w_o) = 1$ m℧
@ resonance

Sol :-



$\vec{Y}_{in} = \frac{1}{R}$ at resonance

$(1 \times 10^{-3})℧ = \frac{1}{R}$

$\rightarrow \boxed{R = (1) k\Omega}$

$w_d = w_o \sqrt{1 - \left(\frac{1}{2Q_o}\right)^2} \Rightarrow \left(\frac{w_d}{w_o}\right)^2 = 1 - \left(\frac{1}{2Q_o}\right)^2$

$\rightarrow Q_o = \frac{1}{2\sqrt{1 - \left(\frac{w_d}{w_o}\right)^2}} = 7.909$

$Q_o = w_o RC$

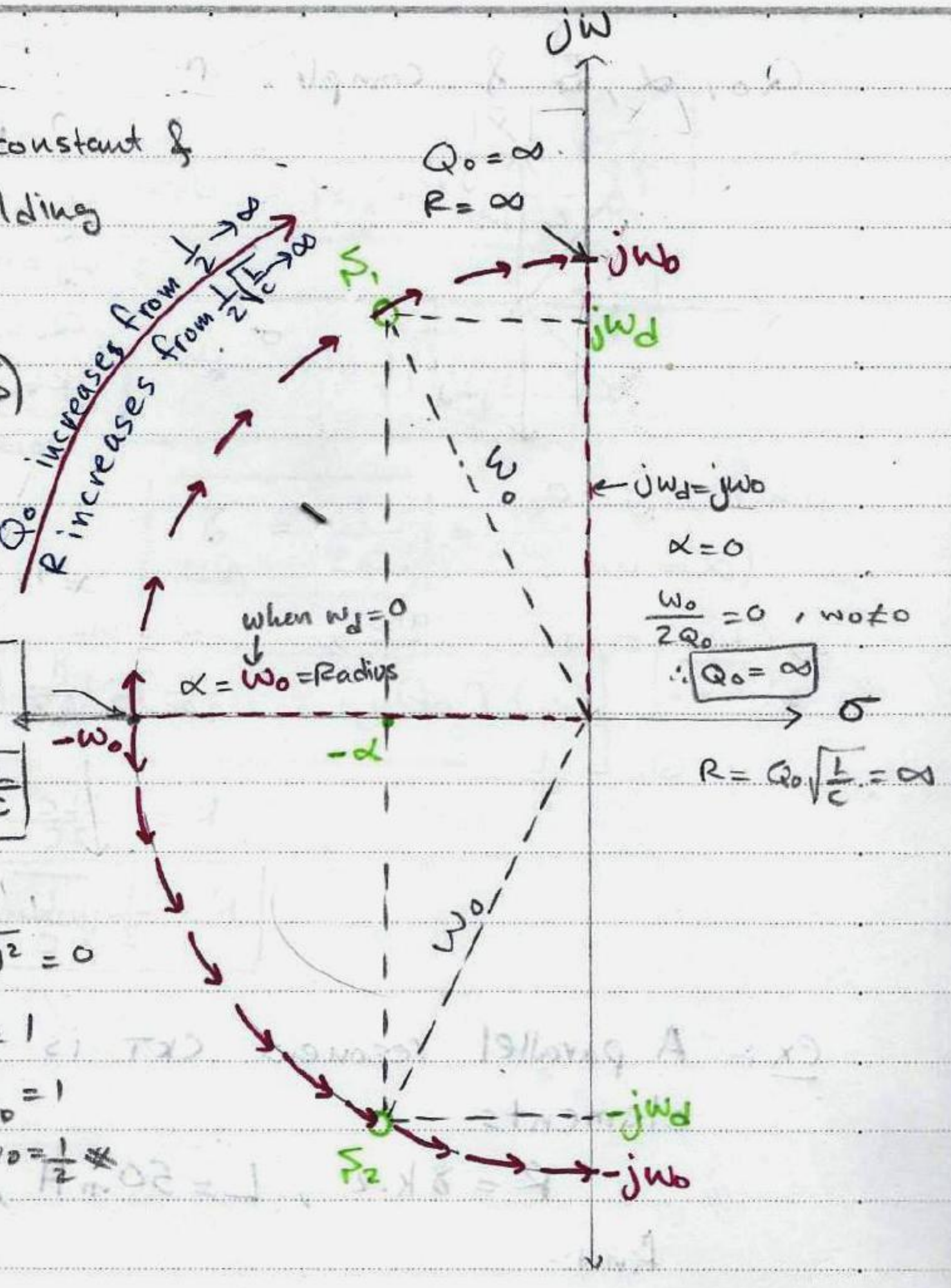
$\rightarrow C = \frac{Q_o}{w_o R} = \boxed{7.91 \mu F}$

$w_o^2 = \frac{1}{LC}$

$\hookrightarrow L = \frac{1}{w_o^2 C} = \boxed{126.4 \text{ mH}}$ ≉

we'll consider w_o constant & change R while holding L, C constant!

$\overrightarrow{Y}(S)$



$\alpha=0$

$\frac{w_o}{2Q_o}=0$, $w_o\neq0$

$\therefore \boxed{Q_o=\infty}$

$R = Q_o\sqrt{\frac{L}{C}}=\infty$

$w_d=0$

$-\alpha=-w_o$

$\frac{w_o}{2Q_o}=w_o$

$\boxed{Q_o=\frac{1}{2}}$ → or

$R = Q_o\sqrt{\frac{L}{C}}$

$\boxed{R=\frac{1}{2}\sqrt{\frac{L}{C}}}$

$\Leftarrow \boxed{\begin{matrix}Q=\frac{1}{2}\\ R=\frac{1}{2}\sqrt{\frac{L}{C}}\end{matrix}}$

$w_d=0$

$w_o\sqrt{1-(\zeta)^2}=0$

$\therefore \zeta=1$

$\frac{1}{2Q_o}=1$

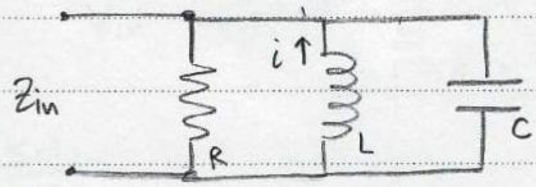
$Q_o=\frac{1}{2}$ *

$$Y(s)=\frac{C}{s}\,(s+\alpha-jw_d)\,(s+\alpha+jw_d)$$

Zeros at :- $s=-\alpha\pm jw_d$

Pole-Zero constellation for parallel resonant CKTs

+Problem 15 page 683:



$L = (1m)H$

$\alpha = (50)\ s^{-1}$

$w_d = (5000)$ rad/sec

Find w_o & Z_{in} at w_o

Sol:-

$* \quad w_d = \sqrt{w_o^2 - \alpha^2}$

$5000 = \sqrt{w_o^2 - 2500}$

$25 * 10^6 = w_o^2 - 2500$

$\boxed{w_o = (5000) rad/s}$

$* \quad \alpha = \frac{w_o}{2Q_o} \rightarrow Q_o = \frac{w_o}{2\alpha}$

$\therefore Q_o = \frac{5000}{2\times(50)} = 50$

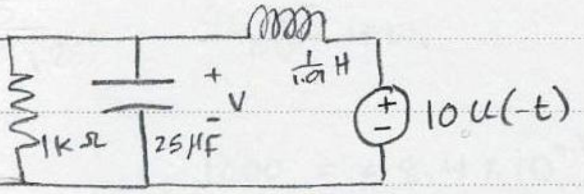
$* \quad Q_o = \frac{R}{w_o L} \rightarrow R = Q_o w_o L$

$\therefore R = 50(5000)(1\times10^{-3}) = 250\Omega$

$\boxed{Z_{in}(w_o) = R = 250\Omega}$

$++ \quad Q_o = w_o RC \rightarrow C = \frac{Q_o}{w_o R} = \frac{50}{5000(250)} = 40\mu F$

Problem 13:



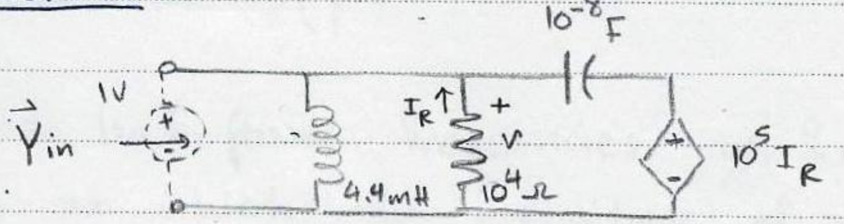
determine the resonant freq. at t>0.

Sol:-

at t>0, source is short circuited.

$\therefore w_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(\frac{1}{1.01})(25\times10^{-6})}} = 199.007$ rad/s

++ Problem 11 :- find Yin & wo , Zin(jwo)



Sol :-

$I_R = \frac{V}{R}$ ⇒ we add a voltage source of (1V)

$\therefore I_R = \frac{-1}{10^4} = -10^{-4}$ A

$$Y_{in} = \frac{I_{in}}{V} = \frac{I_{in}}{1} = \overset{I_L}{\left(\frac{1}{4.4\times10^{-3}s}\right)} + \overset{I_R}{\left(\frac{-1}{10^4}\right)} + \overset{I_C}{\left(\left(1+10^5 \times \underset{I_R}{\frac{-1}{10^4}}\right) 10^{-8} s\right)}$$

$$= \frac{10^4 + 4.4\times10^{-3}s + 484\times10^{-8}s^2}{.44s}$$

$$Y_{in}(j\omega) = \frac{1000 - 48.4\times10^{-8}\omega^2 + j4.4\times10^{-4}\omega}{j4.4\omega}$$

at $\omega = \omega_0$, imaginary part = 0

$$\therefore \frac{1000 - 48.4\times10^{-8}\omega_0^2}{j4.4\omega_0} = 0$$

$$1000 = 48.4\times10^{-8}\omega_0^2$$

$$\omega_0 = 45.45 \text{ rad/s}$$

$Z_{in}(j\omega_0) = \frac{1}{Y(j\omega_0)}$ ...

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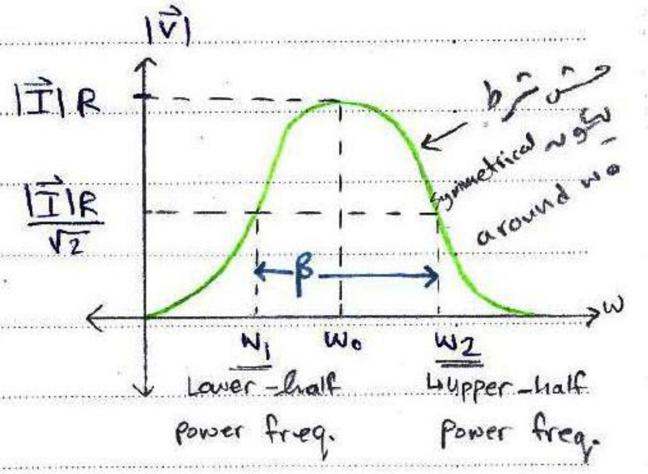
Band width (β)

→ half power frequencies ω_1 & ω_2 are frequencies at which the magnitude of the input admittance of a parallel resonant ckT is greater than the magnitude at resonance by a factor $= \sqrt{2}$.
also the voltage response & the impedance at ω_1, ω_2 are ($\frac{1}{\sqrt{2}}$ / 0.707) times their maximum value

(@ resonance).

- $\boxed{\beta = \omega_2 - \omega_1}$
  └ $\frac{1}{2}$-power
  └→ Band width



* Exact values of ω_1 & ω_2 :-

$$\vec{Y}(j\omega) = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right) \qquad \text{for parallel } \overset{RLC}{\uparrow}\text{ CKT}$$

$$= \frac{1}{R} + j\,\frac{1}{R}\left(\frac{\omega \overset{\leftarrow Q_0}{\boxed{C R\omega_0}}}{\omega_0} - \overset{\rightarrow}{\boxed{\frac{R\omega_0}{\omega_0\,\omega L}}}\right)$$

$$= \frac{1}{R} + j\,\frac{1}{R}\left(\frac{\omega Q_0}{\omega_0} - \frac{\omega_0 Q_0}{\omega}\right)$$

$$\boxed{\vec{Y}(j\omega) = \frac{1}{R}\left[1 + j\,Q_0\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]} \;\cdots \circledast$$

@ ω_1 , ω_2 :-

$$\left|\vec{Y}(j\omega_1)\right| = \left|\vec{Y}(j\omega_2)\right| = \sqrt{2}\,\overset{|\vec{Y}(j\omega_0)|}{\left(\frac{1}{R}\right)}$$

from ⊛ :-

$$|\vec{Y}(j\omega)| = \frac{1}{R}\sqrt{1+\left[Q_0\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)\right]^2}$$

$$|\vec{Y}(j\omega_1)| = \frac{\sqrt{2}}{R} \qquad\qquad |\vec{Y}(j\omega_2)| = \frac{\sqrt{2}}{R}$$

$$\frac{1}{\cancel{R}}\sqrt{1+\left[Q_0\left(\frac{\omega_1}{\omega_0}-\frac{\omega_0}{\omega_1}\right)\right]^2} = \frac{\sqrt{2}}{\cancel{R}} \qquad \frac{1}{\cancel{R}}\sqrt{1+\left[Q_0\left(\frac{\omega_2}{\omega_0}-\frac{\omega_0}{\omega_2}\right)\right]^2} = \frac{\sqrt{2}}{\cancel{R}}$$

$$\boxed{Q_0\left(\frac{\omega_1}{\omega_0}-\frac{\omega_0}{\omega_1}\right) = -1} \qquad \boxed{Q_0\left(\frac{\omega_2}{\omega_0}-\frac{\omega_0}{\omega_2}\right) = 1}$$

$$\boxed{\omega_2 > \omega_1}$$

$$\boxed{\begin{aligned}\omega_{1,2} &= \omega_0\left[\sqrt{1+\left(\frac{1}{2Q_0}\right)^2} \pm \frac{1}{2Q_0}\right]\\ &= \omega_0\left[\sqrt{1+\zeta^2} \pm \zeta\right]\end{aligned}}$$

$$\beta = \omega_2-\omega_1 = \frac{\omega_0}{Q_0} = \frac{\omega_0}{\omega_0 RC} = \boxed{\overset{\beta}{\frac{1}{RC}}} = \boxed{2\alpha}$$

$$\omega_0^2 = \omega_1\omega_2$$

$$\boxed{\omega_0 = \sqrt{\omega_1\omega_2}}$$

as Q_0 increases,
β decreases.

* Approximations for high Q CKTs:-

الشرط الأول → $\boxed{Q_0 \geq 5}$ higher Q leads to narrower bandwidth & sharper response curve!

zeros:- $S = -\alpha \pm j\omega_d$

$$= -\frac{1}{2Q_0} \pm j\sqrt{1-\zeta^2}$$

$$= -0.1\,\omega_0 \pm j\,0.995\,\omega_0 \quad \text{at } Q_0 = 5$$

الشرط الثاني → $\boxed{0.9\omega_0 \leq \omega \leq 1.1\,\omega_0}$ التقريب يكون فقط لقيم w القريبة من w0 ،

$$\left.\begin{aligned} \alpha &= \frac{\omega_0}{2Q_0} \\ \beta &= 2\alpha \end{aligned}\right\} \rightarrow \text{no change}$$

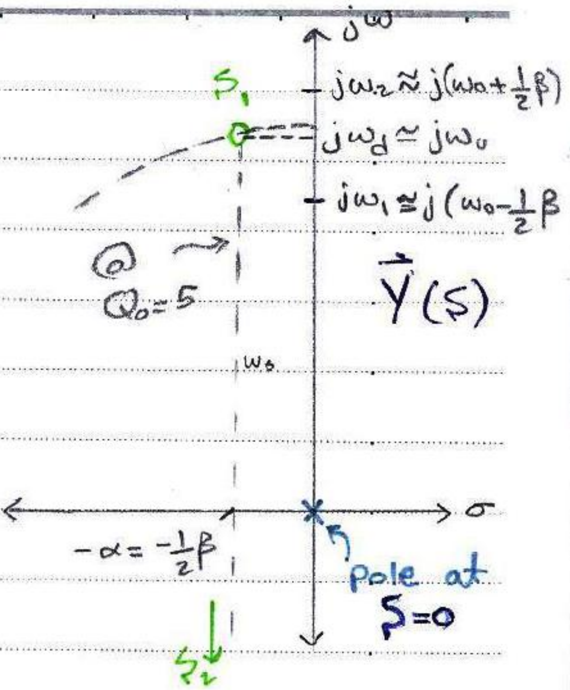
$$\omega_{1,2} = \omega_0 \left[\sqrt{1+\underbrace{\cancel{\left(\frac{1}{2Q_0}\right)^2}}_{}} \mp \frac{1}{2Q_0}\right] \qquad \overset{\approx 0 \rightarrow (2Q_0)^2 \rightarrow \infty}{} \text{ نسبياً}$$

$$\approx \omega_0 \left(1 \mp \frac{1}{2Q_0}\right)$$

$\boxed{\omega_{1,2} \approx \omega_0 \pm \frac{1}{2}\beta}$ → means that the response is symmetrical arbund $\boxed{\omega = \omega_0}$ .... ①

$\boxed{\omega_0 \approx \frac{1}{2}(\omega_1 + \omega_2)}$ the arithemetic mean of ω_1, ω_2 (avg المعدل)

the locations of S_1 & S_2 can be approximated :-



$$S_{1,2} = -\alpha \pm j\omega_d$$

$$\boxed{S_{1,2} \simeq -\frac{1}{2}\beta \pm j\omega_0}$$

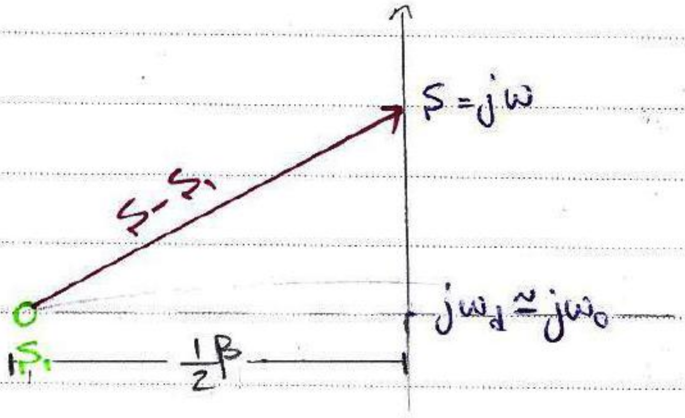
& from ①:-

$$\omega_{1,2} \simeq \omega_0 \mp \frac{1}{2}\beta$$

now, $\vec{Y}(S) \simeq$? where (ω) is in the neighborhood of ω_0.

→ take a test point $S = j\overset{\uparrow}{\omega}$ slightly above $j\omega_0$ to determine the admittance offered by the parallel RLC network at ω.

we have 3 critical pts S_1, S_2 (zeros), S (pole)

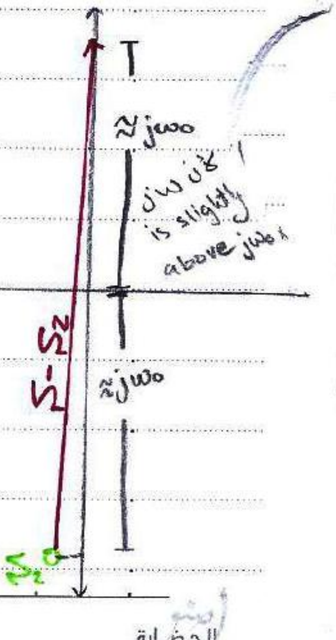


$$\vec{Y} = C\frac{(S-S_1)(S-S_2)}{(S-0)} \quad \dots ⊛$$

$$\rightarrow S-S_1 = \frac{1}{2}\beta + j(\omega-\omega_0)$$

$$\rightarrow S-S_2 \simeq j2\omega_0$$

$$\rightarrow S-0 = j\omega$$



/ /

Plug in ⊛

$$\vec{Y}(s) \approx C\,\frac{(\cancel{j}2\cancel{\omega_0})(s-s_1)}{\cancel{j\omega_0}} = 2C(s-s_1)$$

$$\vec{Y}(s) \approx 2C\left[\tfrac{1}{2}\beta + j(\omega-\omega_0)\right]$$

$$\approx \cancel{2}C\cdot\frac{1}{\cancel{2}}\beta\left[1 + j\overset{N}{\boxed{\frac{(\omega-\omega_0)}{\frac{1}{2}\beta}}}\right]$$

$$\beta = \frac{1}{RC}$$

$$\boxed{\begin{aligned}\vec{Y}(s) &= \frac{1}{R}(1+jN)\\ \vec{Y}(j\omega) &= \frac{1}{R}\sqrt{1+N^2}\;\angle\tan^{-1}(N)\end{aligned}}$$

$N = \dfrac{\omega-\omega_0}{\frac{1}{2}\beta}$; number of half-Bandwidths off resonance.

- Values of N :-

@ w_2: $\quad w_2 = w_0 + \frac{1}{2}\beta$

$$N = \frac{w_0 + \frac{1}{2}\beta - w_0}{\frac{1}{2}\beta} = 1$$

@ w_1: $\quad \omega_1 = w_0 - \frac{1}{2}\beta$

$$N = \frac{w_0 - \frac{1}{2}\beta - w_0}{\frac{1}{2}\beta} = -1$$

Summary:-

Parallel resonant CKTs:-

- Resonance is the condition existing in any physical system, when a response of a max. amplitude is produced
- There must be at least one capacitor & one inductor in a network in oder to be able to reach resonance.
- Resonance condition can be achieved by adjusting $w \overset{or}{/} L \overset{or}{/} C$, here we'll focus ~~of~~ on w as a variable, where w_o is the resonant frequency.

- At Resonance:-

$$\boxed{\begin{matrix} Z_{in} = R \\ \vec{Y}_{in} = 1/R \end{matrix}}$$

max. value of impedance

minimum " " admittance

$\Big]$ $\vec{I}$ & $\vec{V}$ are in-phase at w_o

$$\boxed{\omega_o = \frac{1}{\sqrt{LC}}}$$; resonant freq. [rad/s]

$$\boxed{f_o = \frac{1}{2\pi\sqrt{LC}}}$$; [Hz]

$$\boxed{\omega_d = \sqrt{\omega_o^2 - \alpha^2}}$$; Natural resonant frequency [rad/s]

$$\boxed{\alpha = \frac{1}{2RC}}$$; exponential damping coeficient $[\overset{\text{Neper}}{\downarrow}\text{Np/s}]$

$$\boxed{|I_L(j\omega_o)| = |I_C(j\omega_o)| = \omega_o RC\,|I(j\omega_o)|}$$

- Quality factor

$$\text{Quality factor} = \frac{2\pi \text{ max. E stored in } L/C}{\text{max. E lost in } R \text{ per period}}$$

$$\boxed{Q_o = \omega_o RC = \frac{R}{\omega_o L} = \frac{R}{|X_{L/C}^{or}(j\omega_o)|} = R\sqrt{\frac{C}{L}}}$$

* معادلات ال Resonance بدلالة Q_o :-

$$\boxed{\omega_d = \omega_o\sqrt{1-\left(\frac{1}{2Q_o}\right)^2} = \omega_o\sqrt{1-(\zeta)^2}}$$

$$\boxed{\zeta = \frac{1}{2Q_o} = \frac{\alpha}{\omega_o}}$$; damping factor [dimensionless]

$$\boxed{\alpha = \frac{\omega_o}{2Q_o}}$$

$$\boxed{|I_c(j\omega_o)| = |I_L(j\omega_o)| = jQ_o|\vec{I}(j\omega_o)|}$$

- Bandwidth (β) & half-power frequencies (ω_1, ω_2)

$$\beta = 2\alpha = \frac{1}{RC} = \omega_2 - \omega_1 = \frac{\omega_o}{Q_o}$$

exact values of ω_1, ω_2

$$\boxed{\omega_{1,2} = \omega_o\left[\sqrt{1+(\zeta)^2} \pm \zeta\right]}$$

$$\boxed{\omega_o = \sqrt{\omega_1\omega_2}}$$

approximations:-

when $Q_o \geq 5$

& $0.9\omega_o \leq \omega \leq 1.1\omega_o$

$$\boxed{\omega_{1,2} \simeq \omega_o \pm \frac{1}{2}\beta}$$

$$\boxed{\omega_o \simeq \frac{\omega_1 + \omega_2}{2}}$$

exact :

$$\boxed{\vec{Y}(j\omega) = \frac{1}{R}\left[1 + jQ_o\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)\right]}$$

approximation:-

$$\boxed{\vec{Y}(j\omega) = \frac{1}{R}(1 + jN)} = \frac{1}{R}\sqrt{1+N^2}\angle\tan^{-1}N$$

$$\boxed{N = \frac{\omega - \omega_o}{\frac{1}{2}\beta}}$$; # of half-Band widths off resonance

---

ex1: determine the approximate value of $\vec{Y}(s)$ for parallel RLC CKT, $R = 40K\Omega$, $L = 1H$, $C = \frac{1}{64}\mu F$ at $\omega = 8.2\ K$ rad/s

Sol:- First we find ω_o & Q_o to check approximation conditions

$$\omega_o = \frac{1}{\sqrt{LC}} = 8\ k\ \text{rad/s}$$

$$\left.\begin{array}{l} Q_o = \omega_o RC = 5 \\ \omega = 1.025\ \omega_o \end{array}\right] \rightarrow \begin{array}{l} Q_o \geq 5\ \checkmark \\ 0.9\omega_o \leq \omega \leq 1.1\ \omega_o\ \checkmark \end{array}$$ we can use approximations

$$\beta = \frac{\omega_o}{Q_o} = 1.6\ k\ \text{rad/s}$$

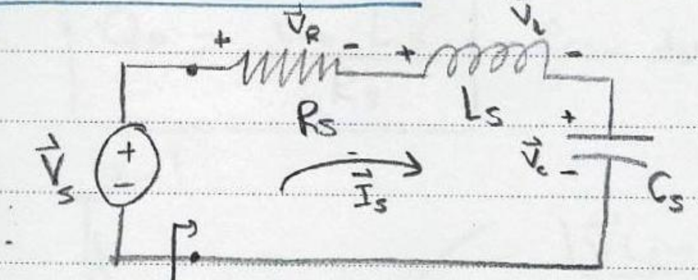
$$N = \frac{\omega - \omega_o}{\frac{1}{2}\beta} = \frac{(8.2 - 8)k}{\frac{1}{2}(1.6)\ k} = 0.25$$

$$\therefore |\vec{Y}| = \frac{1}{R}\sqrt{1+N^2} = 25\mu\sqrt{1+(0.25)^2} = 25.77\ \mu S$$

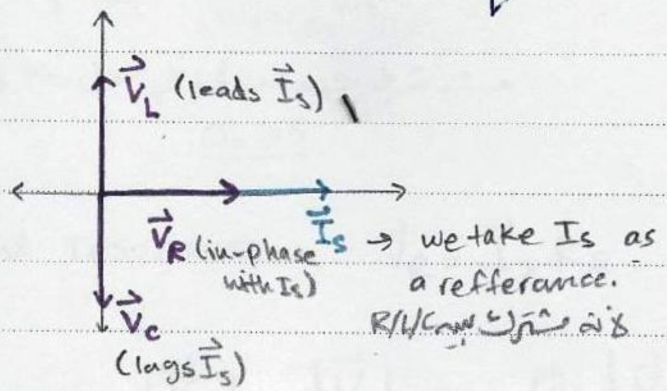
$$\angle\vec{Y} = \tan^{-1}(N) = 14.03^\circ$$

$$\therefore \vec{Y} = 25.77\ \angle 14.03^\circ\ \mu S$$

- Series Resonance :-



Z_{in} $(j\omega)$; dealing with Z in series is much easier than $\vec{Y}$ Since Parallel & series RLC CKTs are dual circuits then, we can apply duality concepts to find Series resonance equations.



at resonance :-

$$\boxed{\begin{aligned}\vec{V}_s &= \vec{V}_R \\ \vec{V}_c &= -\vec{V}_L\end{aligned}}$$

$$\vec{Z}_{in}(j\omega) = R_s + j\left(\omega L_s - \frac{1}{\omega C_s}\right) \quad \ldots ①$$

at resonance $\quad Im\{\vec{Z}_{in}\} = 0 \quad \ldots ②$

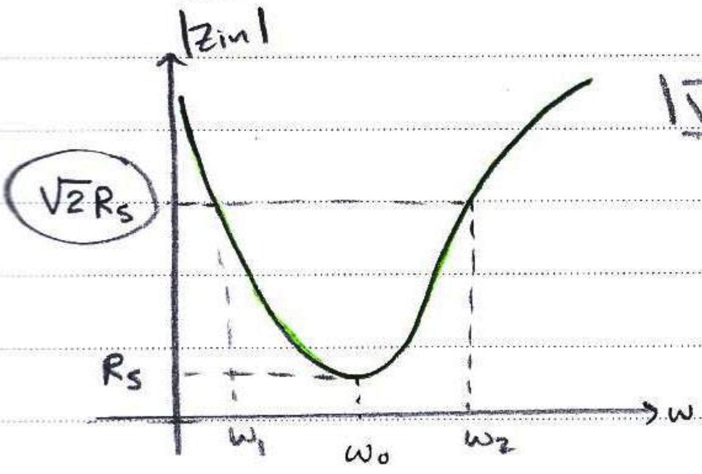
$$\& \boxed{\vec{Z}_{in}(j\omega_0) = R_s}$$

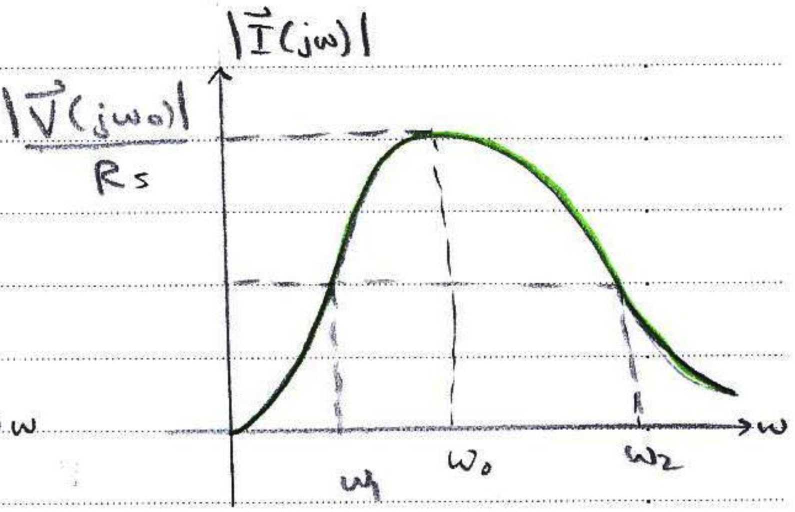
from ① & ② at ω_0 :-

$$\omega_0 L_s = \frac{1}{\omega_0 C_s}$$

$$\boxed{\omega_0 = \frac{1}{\sqrt{L_s C_s}}}$$ نفس الـ Parallel

$$\boxed{Q_0 = \frac{w_0 L_s}{R_s}}$$ from duality





مش شرط يكون في تماثل حول wo إلا إذا كان

$\underline{Q_0 \geq 5}$

at resonanse:- $\vec{V}_R = \vec{I}_s R_s = \vec{V}_s$

$$|\vec{V}_c| = |\vec{V}_L| = Q_0 |\vec{V}_s(jw_0)|$$

- if $Q_0 > 1 \rightarrow$ Voltage amplifier

exact: $w_{1,2} = w_0 \left[\sqrt{1 + \left(\frac{1}{2Q_0}\right)^2} \pm \frac{1}{2Q_0}\right]$

$$\beta = w_2 - w_1 = \frac{w_0}{Q_0} = 2\alpha$$

$$\boxed{\alpha = \frac{w_0}{2Q_0} = \frac{R_s}{2L_s}}$$ duality!

in paralel $\alpha = \frac{1}{2RC}$

$$\boxed{\begin{matrix} C \leftrightarrow L \\ R \leftrightarrow \frac{1}{R} \end{matrix}}$$

$$w_d = \sqrt{w_0^2 - \alpha^2}$$
$$= w_0 \sqrt{1 - \left(\frac{1}{2Q_0}\right)^2}$$
$\hookrightarrow (\zeta)$

$\boxed{w_0 = \sqrt{w_1 w_2}}$ same

approximation: $Q_o \geq 5$ & $0.9\omega_o \leq \omega \leq 1.1\,\omega_o$

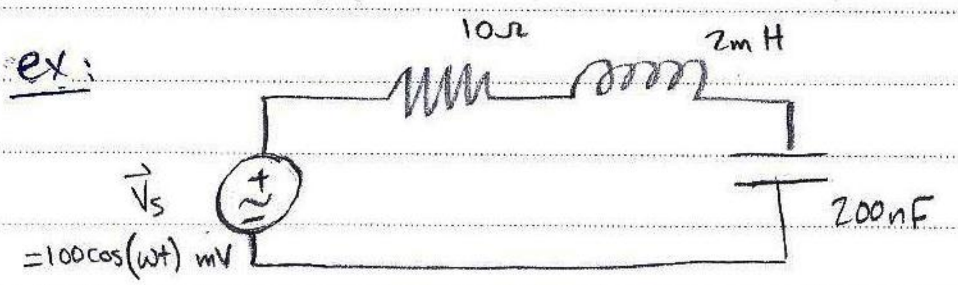
- $\omega_d \simeq \omega_o$
- $\omega_{1,2} \simeq \omega_o \mp \frac{1}{2}\beta$
- $\omega_o \simeq \frac{1}{2}(\omega_1 + \omega_2)$

Same as parallel

$$\vec{Z}_{in}(j\omega) \simeq R(1+jN) = R\sqrt{1+N^2}\ \angle \tan^{-1}(N)$$

$$N = \frac{\omega - \omega_o}{\frac{1}{2}\beta}$$

ex:



find $\vec{Z}_{in}(j\omega)$ & $\vec{I}$ at $\omega = 48$ K rad/s b) Exact & a) approx

sol: a) check for the approximation conditions:-

$$\omega_o = \frac{1}{\sqrt{L_s C_s}} = 50 \text{ K rad/s}$$

$$Q_o = \frac{\omega_o L_s}{R_s} = 10 \geq 5 \checkmark$$

$$\omega = 0.96\ \omega_o \rightarrow 0.9\omega_o \leq \omega \leq 1.1\ \omega_o \checkmark$$

use approx. → $\vec{Z}_{in} \approx R\sqrt{1+\underset{\downarrow}{\underline{N}}^2}\ \angle \tan^{-1} N$

$$N = \frac{\omega - \omega_o}{\frac{1}{2}\underset{\left(\frac{\omega_o}{Q_o}\right)}{\underline{\beta}}} = \frac{48-50}{\frac{1}{2}\left(\frac{50k}{10}\right)} = -0.8$$

$$\vec{Z}_{in} = 12.81\ \angle -38.60^\circ$$

$$|\vec{I}(j\omega)| = \frac{100m}{} = 7.806 \text{ mA}$$

b) <u>exact</u> :-

$$Z_{exact} = 10 + j(48)(2) - \frac{j}{48\times10^{-3}(200)\times10^{-9}} = 12.9 \angle{-39^\circ}$$

$$|I_{exact}| = \frac{|V|}{|Z_{exact}|} = \frac{100m}{12.9} = 7.75\ mA \quad *$$

∴ Differences between Parallel & Series :-

Parallel	Series
$Q_o = w_o RC$	$Q_o = \frac{w_o L}{R}$
$\alpha = \frac{1}{2RC}$	$\alpha = \frac{R}{2L}$

$$|I_L| = |I_c| = Q_o|I_R| \xleftrightarrow[\text{Resonance}]{\text{at}} |V_L| = |V_c| = Q_o|V_R|$$

* The series resonant CKT is characterized by a minimum impedance at resonance, whereas the parallel resonant CKT produces a maximum resonant impedance, the latter CKT provides inductor & capacitor currents @ resonance greater than the source current by the factor Q_o. while series resonant CKTs provides inductor & capacitor voltages at resonance greater than the voltage sorce by a factor of Q_o, so, when $Q_o > 1$ → series CKTs work as a voltage amplifiers

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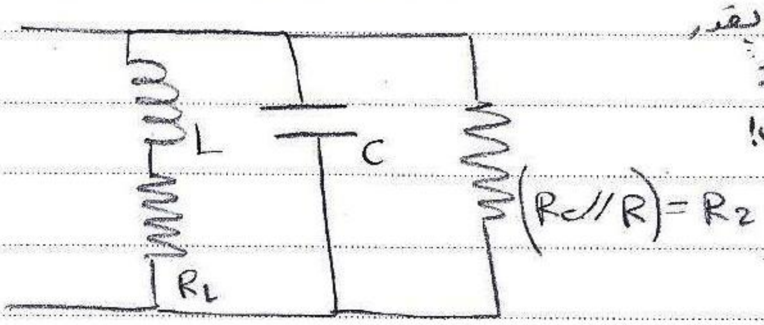
→ if the CKT is not purely series or parallel, what to do? & how to deal with it?

Previous RLC Parallel & series CKTs were ideal! without R_L, R_c

• Other Resonant CKTs :-



* بشكل عام، أي شكل الدارة بقدر احسبله Z_{eq} وأخلي $I_m\{Z_{eq}\}=0$ واحسب w_o وبعدها بكمل حل!

- Resistance of the inductor (R_L) is in series with L

- 〃 〃 〃 Capacitor (R_c) 〃 〃 parallel 〃 C

at Resonance :-

$$\vec{Y}(jw) = \frac{1}{R_2} + jwC + \frac{1}{R_L + jwL} \times \left(\frac{R_L - jwL}{R_L - jwL}\right)$$

$$= \frac{1}{R_2} + jwC + \frac{R_L - jwL}{R_L^2 + w^2 L^2}$$

$* \; I_m\{\vec{Y}\} = 0$ at w_o (resonance)

$* \; Re\{\vec{Y}\} = \frac{1}{R}$

$$I_m\{\vec{Y}(jw_o)\} = w_o C - \frac{w_o L}{R_L^2 + w_o^2 L^2} = 0$$

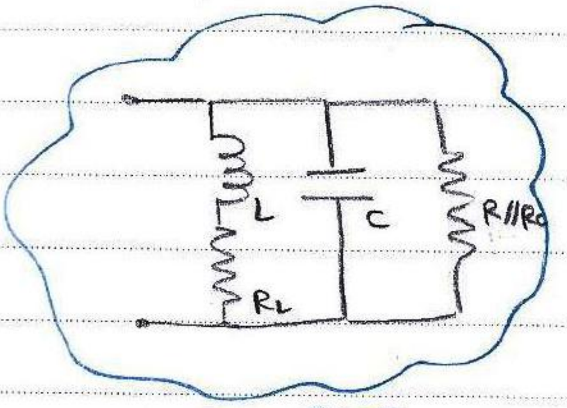
$$\therefore \; R_L^2 + W_o^2 L^2 = \frac{L}{C}$$

$$\therefore \; w_o^2 L^2 = \frac{L}{C} - R_L^2$$

$$\boxed{w_o = \sqrt{\frac{1}{LC} - \left(\frac{R_L}{L}\right)^2}} \xrightarrow{} \text{if } R_L = 0$$

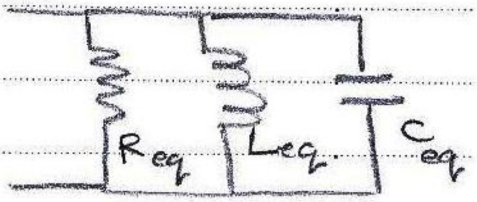
رجعنا لل w_o اللي بنعرفها (:

actually, we won't follow the previous method each time, we have to find a way to replace the previous CKT with an equivelant parallel RLC circuit.

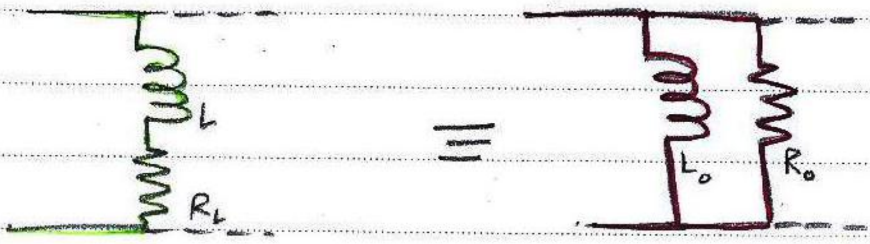


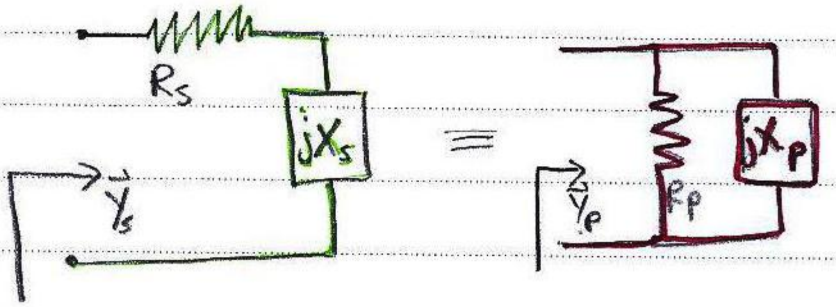
Our CKT

≡



equivelant parallel RLC Circuit.





$$Q_s = \frac{|X_s|}{R_s} \qquad Q_p = \frac{R_p}{|X_p|}$$

these 2 CKTs are equivelant if

$$\vec{Y}_s = \vec{Y}_p$$

$$\rightarrow \vec{Y}_s = \frac{1}{R_s + jX_s} = \frac{R_s - jX_s}{R_s^2 + X_s^2}$$

$\rightarrow \vec{Y}_p = \frac{1}{R_p} - j\frac{1}{X_p}$

but $\vec{Y}_s = \vec{Y}_p$

$\therefore \boxed{Re\{\vec{Y}_p\} = Re\{\vec{Y}_s\}}$ & $\boxed{Im\{\vec{Y}_p\} = Im\{\vec{Y}_s\}}$

$\boxed{\frac{R_s^2 + X_s^2}{R_s} = R_p}$ ....① $\boxed{\frac{R_s^2 + X_s^2}{X_s} = X_p}$ ...②

divide ① by ② :-

$\boxed{Q_s = Q_p} = Q$

Exact Values $Q_0 < 5$

- back to ①:-

$$R_p = \frac{R_s^2 + X_s^2}{R_z} = R_s + \frac{X_s^2}{R_s}$$

$$R_p = R_s\left(1 + \frac{X_s^2}{R_s^2}\right)$$

$\boxed{R_p = R_s\left(1 + Q^2\right)}$ ....①' $s \rightarrow p$

↳ $\boxed{R_s = \frac{R_p}{(1+Q^2)}}$ $p \rightarrow s$

- from ②:-

$$X_p = \frac{R_s^2}{X_s} + X_s = X_s\left(1 + \frac{R_s^2}{X_s^2}\right) = X_s\left(1 + \left(\frac{1}{Q}\right)^2\right)$$

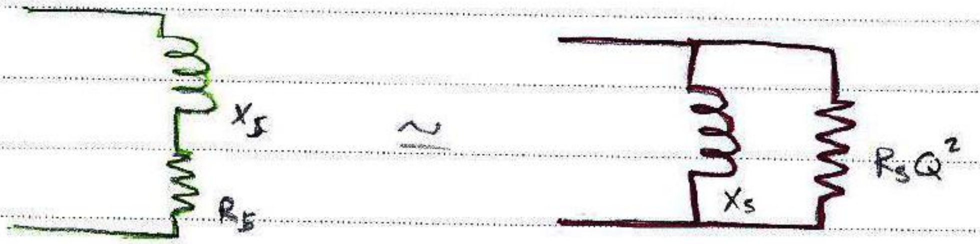
$\boxed{X_p = X_s\left(1 + \left(\frac{1}{Q}\right)^2\right)}$ ....②' $s \rightarrow p$

↳ $\boxed{X_s = \frac{X_p}{1 + \left(\frac{1}{Q}\right)^2}}$ $s \rightarrow p$

approximation $Q_o \geq 5$

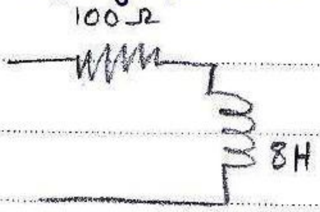
$\boxed{R_p = R_s Q^2}$; $R_p = R_s \overset{\simeq Q^2}{\underline{(1+Q^2)}}$

$\boxed{X_p = X_s}$; $X_p = X_s \left(1 + \overset{\simeq \frac{1}{\infty} = 0}{\frac{1}{Q^2}}\right)$



ex:- at w= 1000 rad/s, find a parallel Network that's equivelant to the series combination of 8H & 100 Ω.

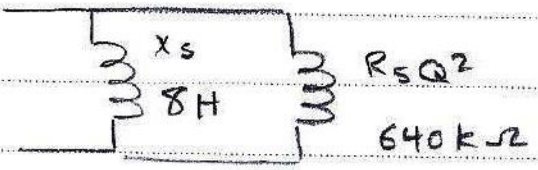
Sol:



* First we have to check approximation condition!

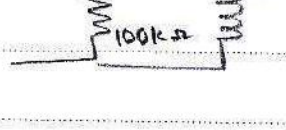
$$Q = \frac{\omega L_s}{R_s} = \frac{1000 \times 8}{100} = 80 \geq 5$$

→ Use approx.

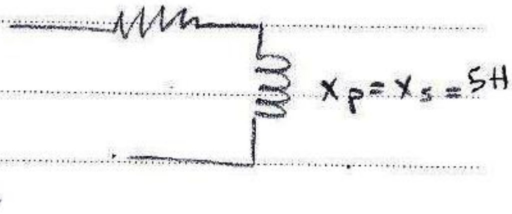


ex 2:- find the series combination equivelant to a parallel 100 KΩ & 5 H at w= 1000 rad/s

Sol:-

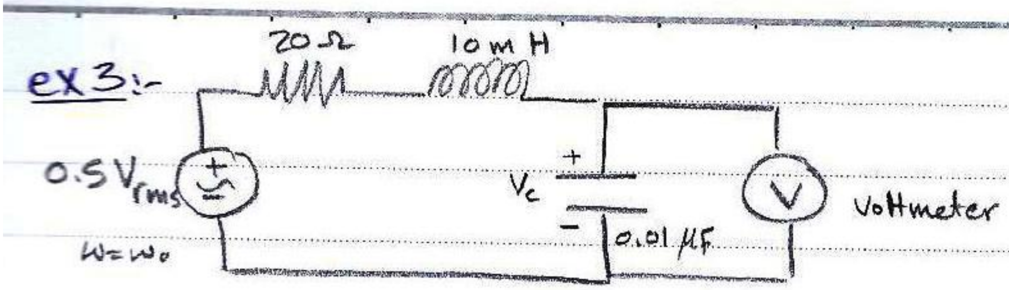


$Q = \frac{100k}{1000 \times 5} = 20 \geq 5 \xrightarrow{\text{Use approx.}}$



$R_s = R_p / Q^2$
$= 250 \Omega$

ex3:-



the Voltmeter has an internal impedance = $\overset{\text{pure } R}{100k\Omega}$

→ Before adding Ⓥ:-

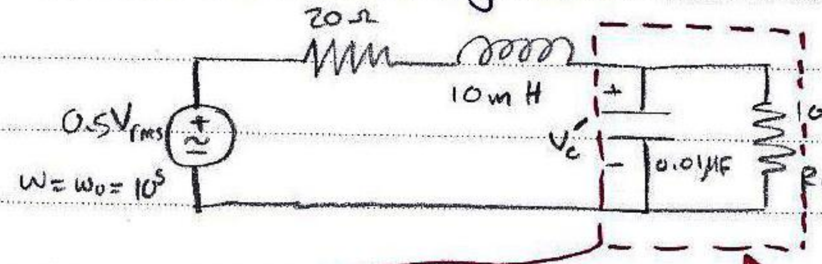
$$W_o = \frac{1}{\sqrt{LC}} = 10^5 \text{ rad/s}$$

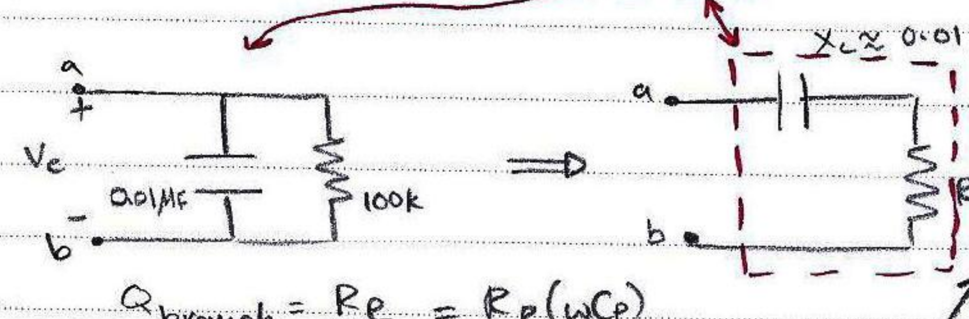
$$Q_o = \frac{w_o L}{R} = 50$$

$$|\vec{I}(jw_o)| = \frac{0.5}{20} = 25 m A_{rms}$$

$$|V_c(jw_o)| = |\vec{V}_s| \; Q_o = 0.5 \times 50 = 25 V_{rms}$$

→ After adding Ⓥ:-



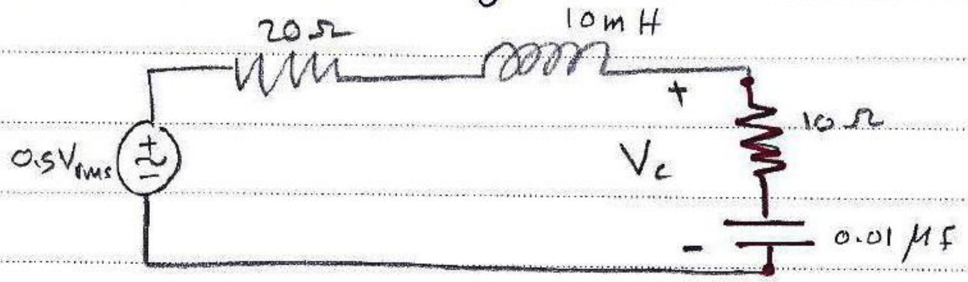


$$R_s \approx \frac{100k}{100^2} = 10\Omega$$

use approx.

$$Q_{branch} = \frac{R_p}{X_p} = R_p(wC_p)$$
$$= 100k(10^5 \times 0.01\mu) = 100 \geq 5$$

back to the original CKT:-



$$Q_{o\,new} = \frac{w_oL}{R_{eq}} = \frac{10^5 \times 10\,m}{(20+10)} = 33.33$$

$$\rightarrow Q_{old} = 50 \quad \& \quad \boxed{Q_{new} = 33.33}$$

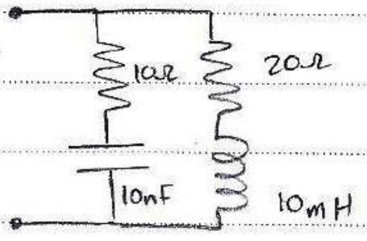
بعد إضافة ⓥ

$$|\vec{V}_c| = |\vec{I}| \left| 10 - \frac{1}{j(10^5 \times 0.01\mu)} \right| = \frac{0.5}{\underset{R_{eq}}{30}} \times \sqrt{10^2 + \left(\frac{1}{10^{-3}}\right)^2}$$

$$\boxed{|V_c'| = 16.67\ V_{rms}} \quad \text{while } |V_c| = 25V$$

هذا الفرق في القراءات يظهر عند اضافة ال ⓥ واهمال مقاومة الداخلية!

ex 3:



a) Find the approximated wo:

$$w_o = \frac{1}{\sqrt{LC}} = 100\ K\,rad/s$$

b) find Q_o for the RC branch:

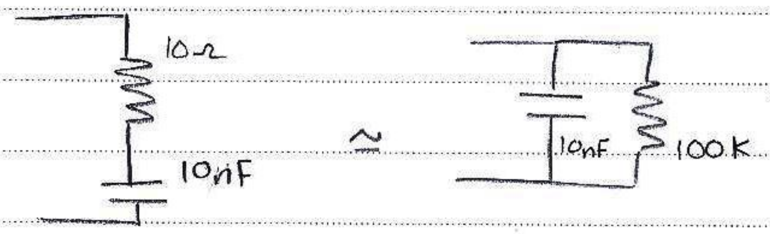
$$Q = \frac{|X_s|}{R_s} = \frac{1}{w_oCR_s} = 100 \geq 5$$

c) find Q_o for the RL branch

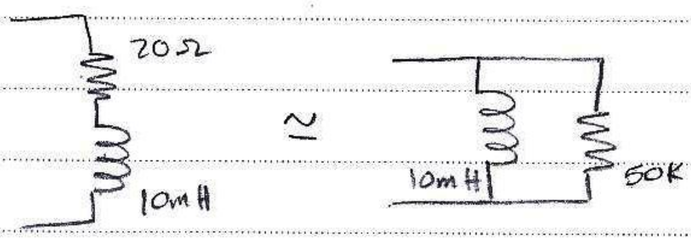
$$Q_o = \frac{|X_s|}{R_s} = \frac{w_oL}{R_s} = 50 \geq 5$$

d) find the 3-elements equivelant of the original network ?

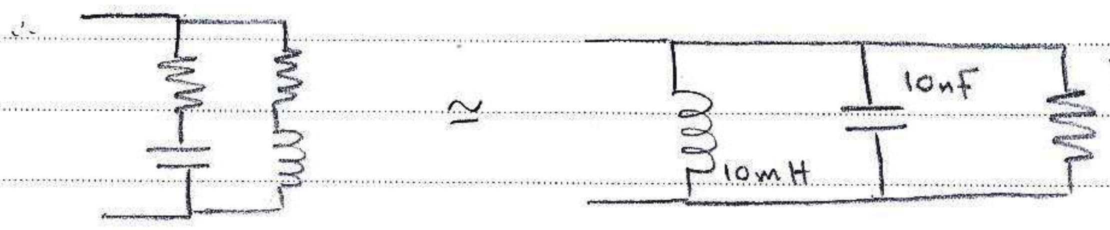
→



$Q = 100$



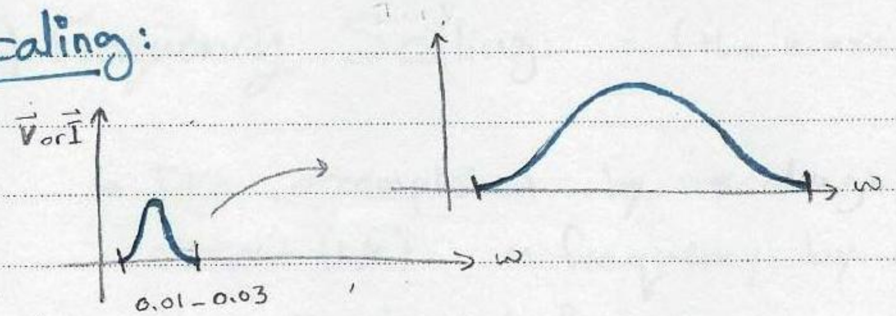
$Q = 50$



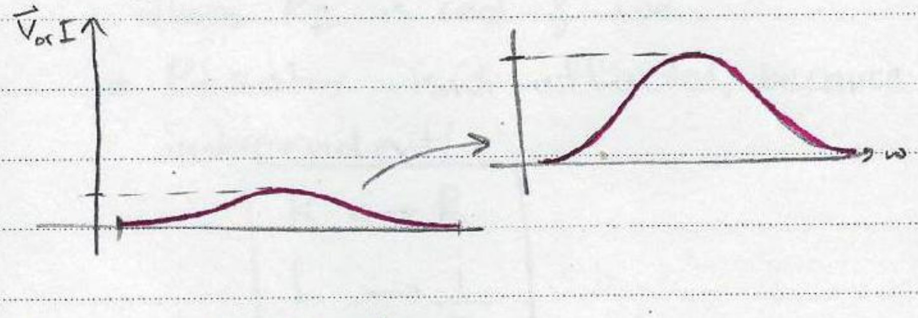
Req =
50k // 100k
= 33.33KΩ

18 / 4 / 2013

• Scaling:



frequency Scaling.

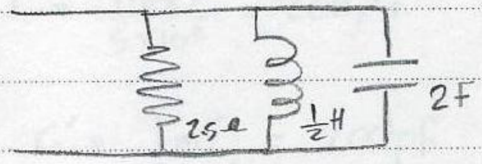


Magnitude Scaling

1 Magnitude Scaling :- ≡ (the Y-axis is scaled by k_m)

- frequency remains constant.
- The impedance of the two-terminals network is increased by a factor k_m, where k_m is real & possitive and may be greater or less than unity.

ex1:



$k_m = 2000$

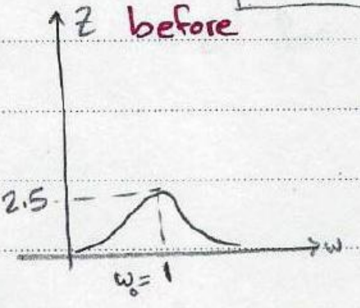
$$\omega_o' = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{\cancel{k_m} L \cdot \frac{C}{\cancel{k_m}}}}$$

Magnitude Scaling :

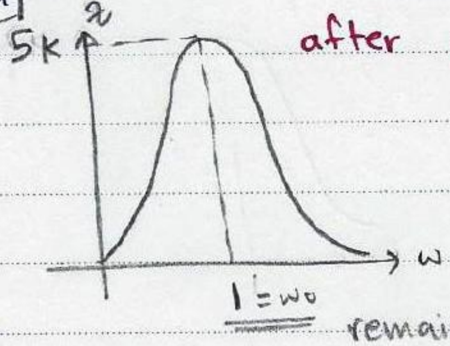
$$\boxed{\begin{aligned} R' &= k_m R \\ L' &= k_m L \\ C' &= \frac{C}{K_m} \end{aligned}} \begin{aligned} &= 5 k\Omega \\ &= 1000\, H \\ &= 1\, mF \end{aligned}$$

$$\boxed{\omega_o' = \frac{1}{\sqrt{LC}} = \omega_o}$$

Z before



after mag. Scaling!



remains constant

$\boxed{2}$ Frequency Scaling:- ≡ (the x-axis is scaled by K_f)

- It's accomplished by scaling each passive element (L/C) in frequency by a factor of K_f, where K_f is real & +ve.
- Resistor isn't affected, because it's a frequency independent!

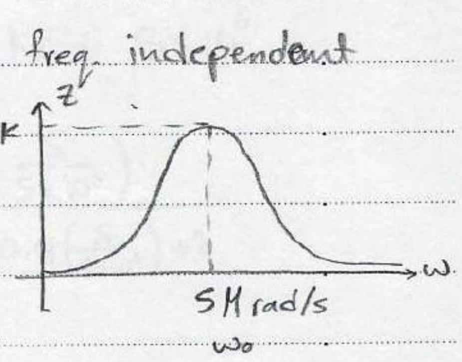
$$\boxed{\begin{array}{l} R \leftrightarrow R \\ L \rightarrow \frac{L}{K_f} \\ C \rightarrow \frac{C}{K_f} \end{array}}$$

ex2:- from the previous example, R=5K , L=1000H
C = 1mF , if $K_f = 5\times10^6$, find R', L', C'

Sol:- R' = 5K (no change). it's freq. independent

$$L' = \frac{1000}{5\times10^6} = 200\mu F$$

$$C' = \frac{1mF}{5\times10^6} = 200pF$$



Scaling can be made:-
1. element by element
2. for Zeq.

- Scaling of $Z(s)$:

  - Mag. Scaling:-

$$\boxed{Z'(s) = K_m Z(s)}$$

for the CKT in(ex1):-

$$\vec{Z}(s) = \frac{s}{2s^2+0.4s+2} \quad , \text{if } K_m = 2000$$

$$Z'(s) = \frac{2000\, s}{2s^2+0.4s+2} \quad \text{....①}$$

  - freq. Scaling:-

$$\boxed{Z'(s) = Z'\left(\frac{s}{K_f}\right)}$$

for $Z'(s)$ in ① , if $K_f = 5\times10^6$

$$Z''(s) = \frac{2000\left(\frac{s}{5\times10^6}\right)}{2\left(\frac{s}{5\times10^6}\right)^2 + 0.4\left(\frac{s}{5\times10^6}\right)+2}$$

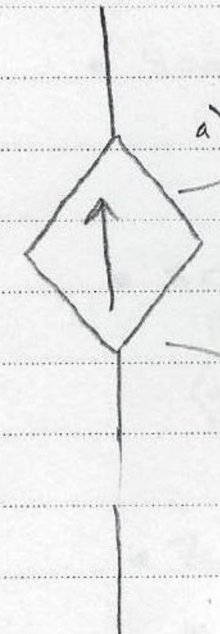
- Dealing with dependant sources:

* although scaling is a process normally applied to passive elements , dependent sources may also be scaled in magnitude & frequency.

* if the network containing the dependent source is scaled by Km, then it's neccesary only to treat k_x or k_y as if it was the type of element consistent with its dimensions; that is if k_x or k_y is dimensionless → it's left unchanged!
& if it is an admittance → divide by Km
~ ~~ ~ impedance → multiply ~ Km

* Frequency scaling doesn't affect the dependent sources!

ex:-



a) $0.02V_x \rightsquigarrow [0.02] = \frac{A}{V}$ (since it's a current source)
$= S$ (admittance)

$\therefore 0.02V_x \xrightarrow{\text{mag. scaling}} \frac{0.02}{K_m}V_x$

b) $0.02I_y \rightsquigarrow [0.02]$ = dimensionless → leave it as is!

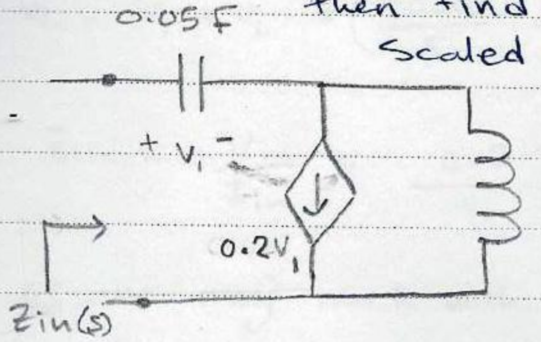
c) $0.02V_x \rightsquigarrow [0.02]$ = dimensionless
leave it as is!

d) $0.02I_y \rightsquigarrow [0.02] = \frac{V}{A} = \Omega$! impedance

$\therefore 0.02I_x \xrightarrow{\text{Mag. scaling}} 0.02K_mI_x$

ex 16.6 مع الكتاب

ex 2:- Scale the following Network with $\boxed{\begin{matrix} K_m = 20 \\ K_f = 50 \end{matrix}}$ then find $Z_{in}(s)$ for the Scaled Network:



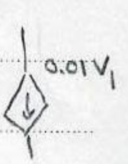
Sol:- • Scaling C:-

$$C' = \frac{C}{K_m * K_f} = \frac{0.05}{(20)(50)} = 50\,\mu F$$

• Scaling L:-

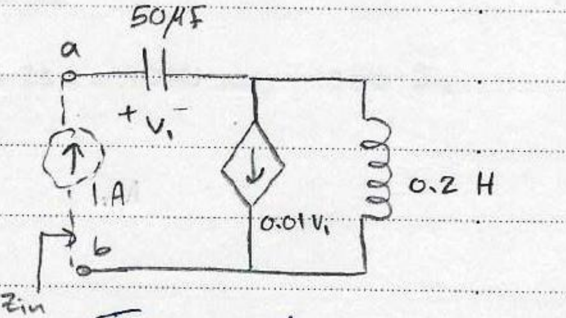
$$L' = \frac{L * K_m}{K_f} = \frac{0.5\,(20)}{50} = (0.2)\,H$$

• Scaling the dependant source:- (only mag. Scaling)

 $\underline{\underline{0.2}}\ V_1$ ↳ $\frac{A}{V}$ ↳ $\mho$ units of an admittance → $K_x' = \frac{0.2}{20} = \underline{\underline{0.01}}$ → 

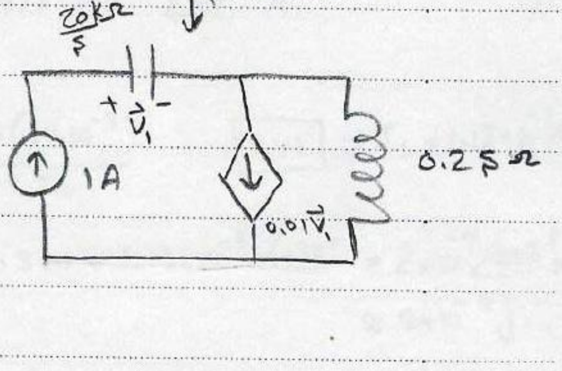
• $Z_{in}(s)$?

in order to calculate $Z_{in}(s)$, we need to apply a current source at the terminals a-b



The Scaled Network

$\downarrow$ to s-domain



$$\vec{V_1} = \frac{20\,000}{s}(1) = \frac{20000}{s}$$

$$I_L = 1 - 0.01\vec{V_1} = 1 - 0.01\left(\frac{20000}{s}\right)$$

$$I_L = 1 - \frac{200}{s}$$

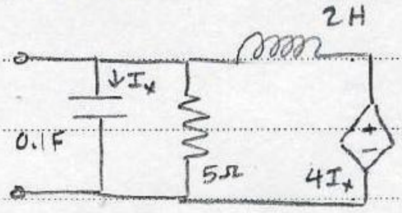
$\vec{V}_L = I_L(0.2S) = (0.2S - 40)$ V

$V_{in} = V_1 + V_L = \left(\frac{20000}{S} + 0.2S - 40\right)$ V

$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{0.2S^2 - 40S + 20000}{S}$ Ω

<u>or</u> we may work with the first (unscaled) network to calculate the (unscaled impedance) & then scale it (multiply it by k_m & replace each S with $\left(\frac{S}{k_f}\right)$)

++ Problem 53 page 688 :- Scale the following network by $k_m = 250$, $k_f = 400$, then find thevinin equivelant of the scaled network at ω = 1 k rad/s

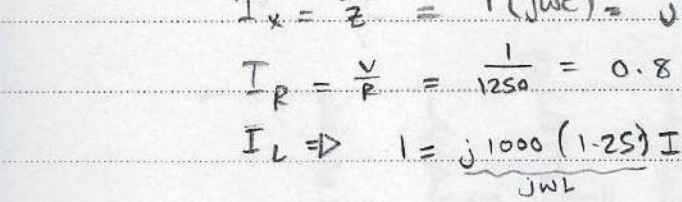


Sol: • $C' = \frac{C}{k_m \times k_f} = \frac{0.1}{(250)(400)} = 1\mu F$

• $L' = \frac{L * k_m}{k_f} = \frac{2 * 250}{400} = 1.25H$

• $R' = R \times k_m = 5 \times 250 = 1250\,\Omega$

• $k_y' = k_y \times k_m = 4(k_m) = 4 \times 250 = 1000 \rightarrow 1000\,I_x$



the scaled Network

$I_x = \frac{V}{Z} = 1(j\omega c) = j(1000)(1\mu F) = 10^{-3}\angle 90°$ A

$I_R = \frac{V}{R} = \frac{1}{1250} = 0.8$ mA

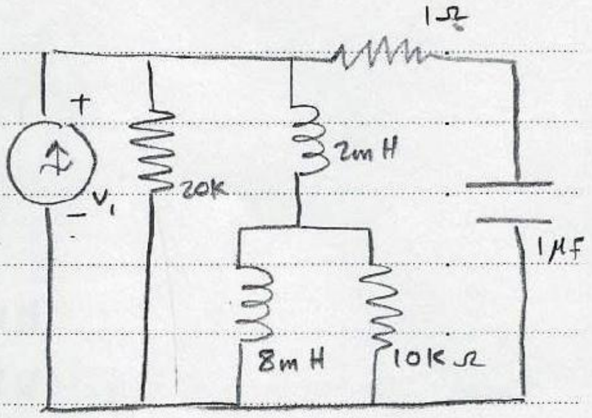
$I_L \Rightarrow 1 = \underbrace{j1000(1.25)}_{j\omega L} I_L + \underbrace{j1000(1 \times 10^{-3})}_{1000 I_x}$ $\boxed{KVL} \rightarrow I_L = 1.13 \times 10^{-3} \angle -13$

$I_{source} = I_x + I_R + I_L = 10^{-3}\angle 90° + 0.8m + 1.13 \times 10^{-3}\angle -135° = 2 \times 10^{-4}\angle 89.7°$ mA
$\approx 2 \times 10^{-4} j$

$Z_{th} = \frac{V_{source}}{I_{source}} = -j\,5000\Omega$

** Problem 52 page 688 :-

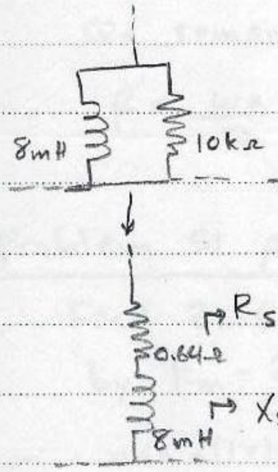
Use good approximation, find:-
a) ω_0, b) Q_0, c) Scale the network so it reaches resonance at $\omega = 1\ M\ rad/s$, then d) find ω_0 & β for the scaled network;



Sol:- first we have to find the equivelent parallel RLC CKT:-

مو تحسب الدارة كاملة!

1)



$$\omega_0 = \frac{1}{\sqrt{L_{eq}C_{eq}}} = \frac{1}{\sqrt{(8+2)\times10^{-3}\times(1\times10^{-6})}} = \boxed{10^4\ rad/s}\ ⓐ$$

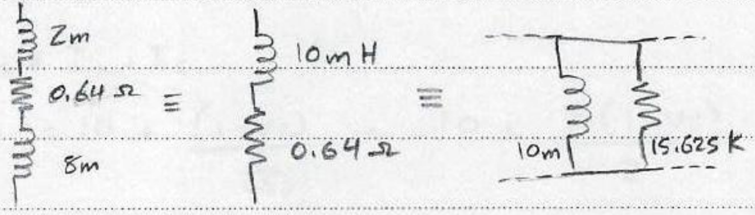
$$\rightarrow Q_p = \frac{R}{\omega_0 L} = \frac{10\,K}{10^4(8m)} = 125$$ ← تحسب ال Branch لوحدة!

$\boxed{Q_0 \geqslant 5}$ ← use approximation.

$\rightarrow R_s = \frac{R_p}{Q^2} = \frac{10\,k}{(125)^2} = 0.64\ \Omega$

$\rightarrow X_s = X_p = 8mH$

2)

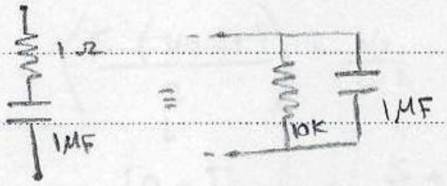


$Q_s = \frac{\omega_0 L}{R} = \frac{10^4 \times 10m}{0.64}$
$= 156.25$

$R_p = R_s Q^2 = 15.625\,K\Omega$

$X_p = X_s = 10mH$

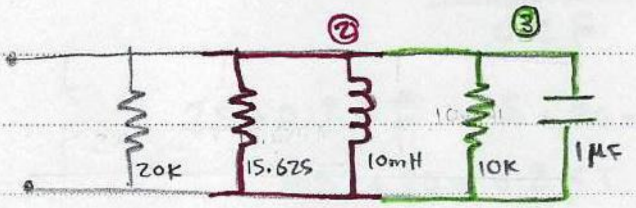
3)



$Q_s = 1/(\omega_0 RC) = 1/(10^4)(1)(1\times10^{-6}) = 100 \geqslant 5$

$R_p = R_s Q^2 = 1\,(100)^2 = 10\,K\,\Omega$

$X_p = X_s = 1\mu F$



$R_{eq} = 20K \,//\, 15.625K \,//\, 10K$
$= 4.673\,K\Omega$ ⓑ

$$Q_0 = R/\omega_0 L = (4.673K)/(10^4)(10\times10^{-3}) = \boxed{46.73}$$

c) Scaling :-

$$K_f = \frac{1M}{10^4} = 100 \quad , \; K_m = 1$$

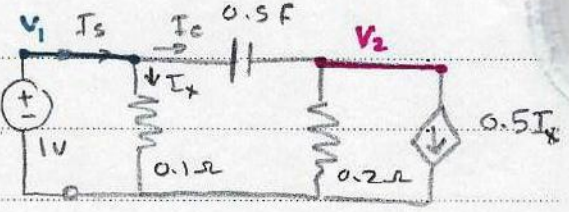
- $R' = R$
- $C_1' = (8\times10^{-3})/100 = 80\mu H$
- $C_2' = (2\times10^{-3})/100 = 20\mu H$
- $L' = (1\mu F)/100 = 10\,nF$

d) $\omega_0 = 1M$ rad/s

Q_0 remains the same

$$\beta' = \frac{\omega_0}{Q_0} = \frac{10^6}{47.73} = 21.4 \text{ k rad/s}$$

++ Problem 51 page 687 :-



Find $Z_{in}(s) \Rightarrow$ and scale it by $K_m = 2$, $K_f = 5$

Sol :- add a (1V) voltage source :-

$$I_x = \frac{V}{R} = \frac{1}{0.1} = 10A \quad / \; 0.5I_x = 5A$$

$$I_s = I_x + I_c$$

$$= 10 + \frac{(1-V_2)}{\frac{1}{0.5s}} = 10 + \frac{s(1-V_2)}{2} \Rightarrow \quad V_2 = \frac{s+20-2I_s}{s}$$

Using Nodal analysis at node 2:-

$$\left(\frac{s(V_2-1)}{2}\right) + \frac{V_2}{0.2} + 5 = 0$$

$$\downarrow$$

$$10 - I_s + \frac{s+20-2I_s}{0.2s} + 5 = 0$$

$$2s - 0.2sI + s + 20 - 2I + s = 0$$

$$4s + 20 = 0.2sI + 2I$$

$$I(0.2s+2) = 4s + 20$$

$$I = \left(\frac{4s+20}{0.2s+2}\right)$$

$$Z_{in}(s) = \frac{V}{I} = \frac{0.2s+2}{4s+20} = \frac{s+10}{20s+100}$$

Scaling: $Z'(s) = 2\left(\frac{s+10}{20s+10}\right)$ mag. scaling.

$$Z''(s) = Z'\left(\frac{s}{5}\right) = \frac{2\left(\frac{s}{5}\right)+20}{20\left(\frac{s}{5}\right)+100} = \frac{0.4s+20}{4s+100}$$

$$Z_{scaled} = \frac{\overset{0.1}{\cancel{0.4}}s+\overset{5}{\cancel{20}}}{\underset{1}{\cancel{4}}s+\underset{25}{\cancel{100}}} \times \frac{10}{10} = \boxed{\frac{s+50}{10s+250}} \quad \divideontimes$$

*

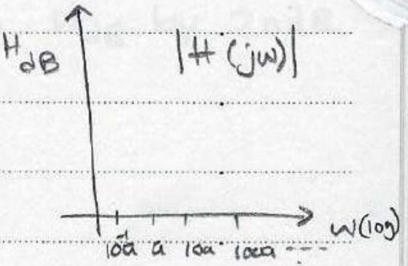
• Bode diagram:-

- It's an approximation picture of the amplitude & phase variation of a given transfer function as function of w.

- The Decible (dB) scale:-

- the approximate response curve we construct is called asymptotic plot / bode plot / bode diagram.
- both Magnitude & phase curves are shown using logarithmic frequency scale for the x-axis & logarithmic units called decibels (dB) for the y-axis.

$$\boxed{H_{dB} = 20 \log_{10} (H(j\omega))}$$



- log Properties:-
  - * $\log 1 = 0$
  - * $\log 2 = 0.3$
  - * $\log 10 = 1$
  - * $\log 10^n = n \log 10 = n$
  - * $\log \frac{1}{10^n} = -n$
  - * $\log a = x \longrightarrow \log \frac{1}{a} = -x$
  - * $\log(\frac{a}{b}) = \log a - \log b$
  - * $\log(ab) = \log a + \log b$
  - * $\log \sqrt{a} = \frac{1}{2} \log a$
  - * $\log a^x = x \log a$

- Some common HdB values:-

$|H(j\omega)| = 1 \longrightarrow H_{dB} = 0\,dB \quad \equiv 20\log 1 = 0$

$|H(j\omega)| = 2 \longrightarrow H_{dB} = 6dB \quad \equiv 20\log 2 = 6$

$|H(j\omega)| = 10 \longrightarrow H_{dB} = 20dB$

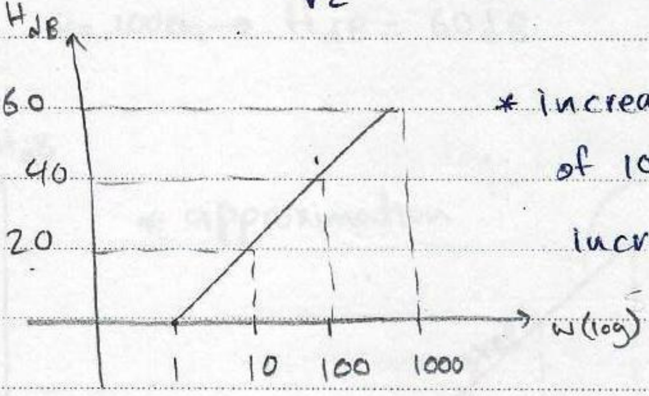
$|H(j\omega)| = 100 \longrightarrow H_{dB} = 40dB$

$|H(j\omega)| = 10^n \longrightarrow H_{dB} = 20n\,dB$

$|H(j\omega)| = 5 = \frac{10}{2} \longrightarrow H_{dB} = H_{dB}(10) - H_{dB}(2 = 20-6 = 14H_{dB}$

$|H(j\omega)| = \sqrt{2} \longrightarrow H_{dB} = 3dB$

$|H(j\omega)| = \frac{1}{\sqrt{2}} \longrightarrow H_{dB} = -3\,dB$



* increasing $|H(j\omega)|$ by a factor of 10, corresponds to an increasement in H_{dB} by 20dB

-Standard forms of plotting:

A) Assume $\boxed{H(s) = 1 + \frac{s}{a}}$, a is constant

$\hookrightarrow = \frac{a+s}{a}$

- Zero at $s = -a$
- to plot this form, we have to find what we'll call "the asymptotic curve (s)".
- we'll plot with respect to (ω)!

$$H(j\omega) = 1 + \frac{j\omega}{a} \Rightarrow |H(j\omega)| = \sqrt{1 + \frac{\omega^2}{a^2}}$$

$$\therefore H_{dB} = 20\log\sqrt{1 + \frac{\omega^2}{a^2}} = 10\log\left(1 + \frac{\omega^2}{a^2}\right)$$

/ /

$\rightarrow \quad H_{dB} = 10\log\left(1+\frac{w^2}{a^2}\right)$

Step 1 → to find the assym. curve we do the following approximation:- $w \ll a$. $\log\left(1+\overset{\rightarrow 0}{\frac{w^2}{a^2}}\right) = \log(1)$

$\therefore H_{dB} \simeq 10\log(1) = 0$

Step 2 → $w \gg a \rightarrow H_{dB} = 10\log\frac{w^2}{a^2} = 20\log\left(\frac{w}{a}\right)$

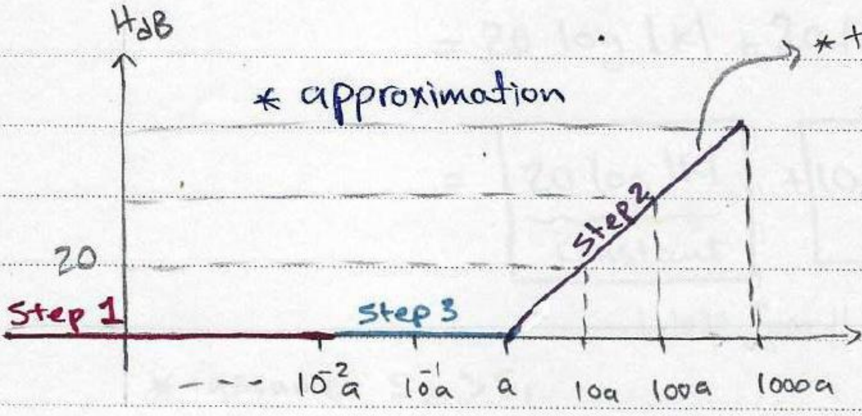
$\hookrightarrow 10\log\left(1+\underbrace{\frac{w^2}{a^2}}_{\gg 1}\right)$ ∴ we neglect (1)

& approximately

Step 3 → $w = a \rightarrow H_{dB} = 0$ نعوض في step 1 or 2

$w = 10a \rightarrow H_{dB} = 20dB$, $w = 100a \rightarrow H_{dB} = 40dB$

$w = 1000a \rightarrow H_{dB} = 60dB$



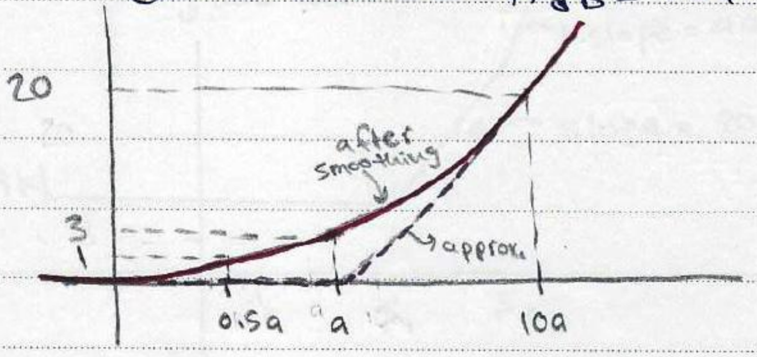
* this curve has a slope of 20dB every 10 fold. or we say: 20dB per decade! or $\frac{6dB}{\text{octave}}$!
↓
increase in 2 fold
a → 2a → 4a → 8a

* Since it's an approximation, we have to do smoothing :- راح نعمله دائماً ! عالأغلب بيزم ال approx.

- exact values:-

@ $w = a \rightarrow H_{dB} = 20\log\sqrt{1+\frac{a^2}{a^2}} = 20\log\sqrt{2} = 3dB$

@ $w = 0.5a \rightarrow H_{dB} = 20\log\sqrt{1+0.25} \simeq 1\ dB$



Standard form of plotting (cont)

B) * Multiple terms:

$$H(s) = k\left(1+\frac{s}{s_1}\right)\left(1+\frac{s}{s_2}\right)$$

- zeros at $s = -s_1 \;, -s_2$
- $H_{dB} = 20\log|H(j\omega)|$

$$= 20\log\left|k\left(1+\frac{s}{s_1}\right)\left(1+\frac{s}{s_2}\right)\right|$$

→ in terms of $j\omega$

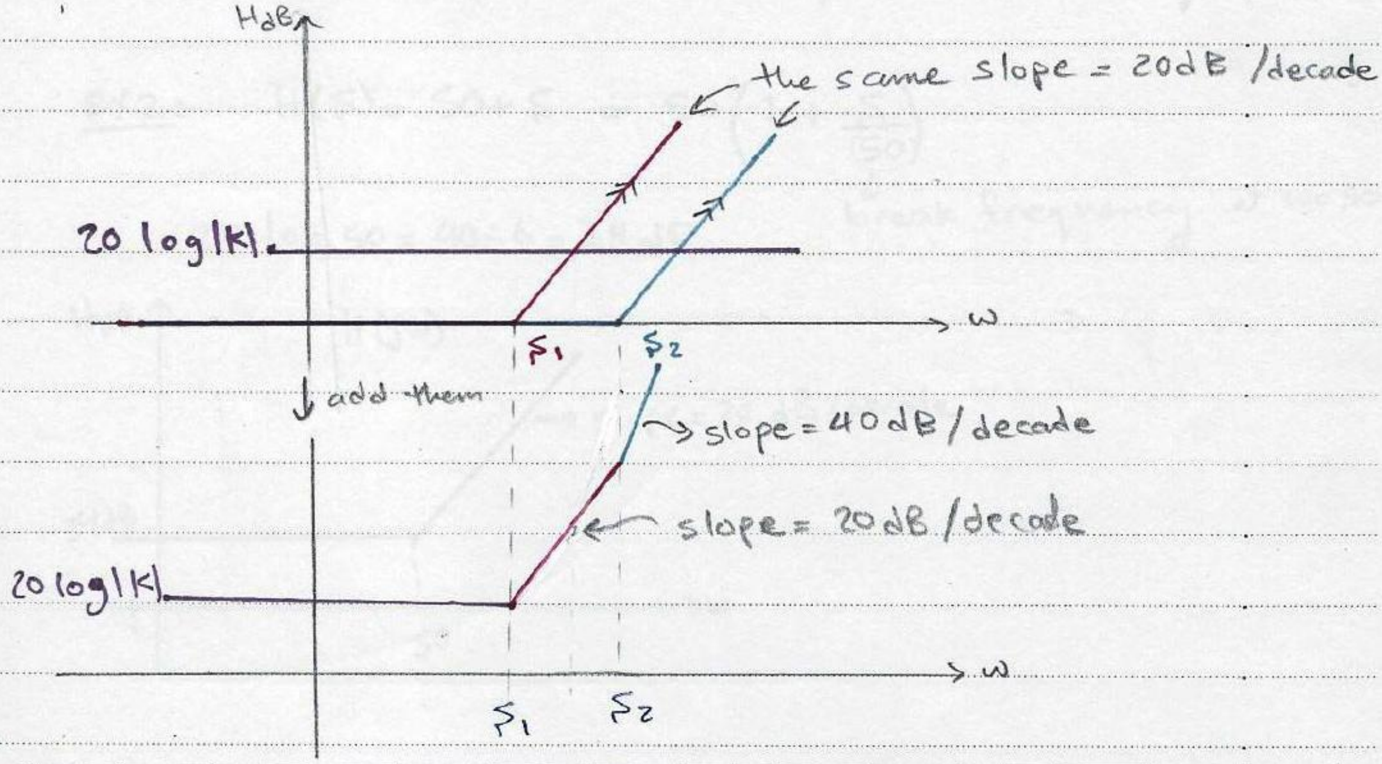
$$H_{dB} = 20\log\left|k\left(1+\frac{j\omega}{s_1}\right)\left(1+\frac{j\omega}{s_2}\right)\right|$$

$$= 20\log|k| + 20\log\sqrt{1+\frac{\omega^2}{s_1^2}} + 20\log\sqrt{1+\frac{\omega^2}{s_2^2}}$$

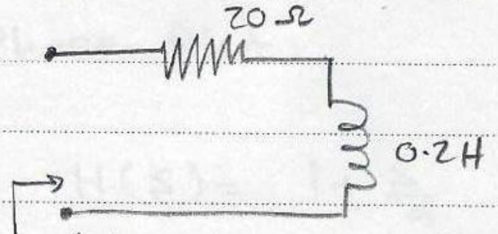
$$= \boxed{\underbrace{20\log|k|}_{\text{constant}}} + \boxed{10\log\left(1+\frac{\omega^2}{s_1^2}\right)} + \boxed{10\log\left(1+\frac{\omega^2}{s_2^2}\right)}$$

نفس شكل الـ form الأول!

* assume $s_2 > s_1$



ex :-

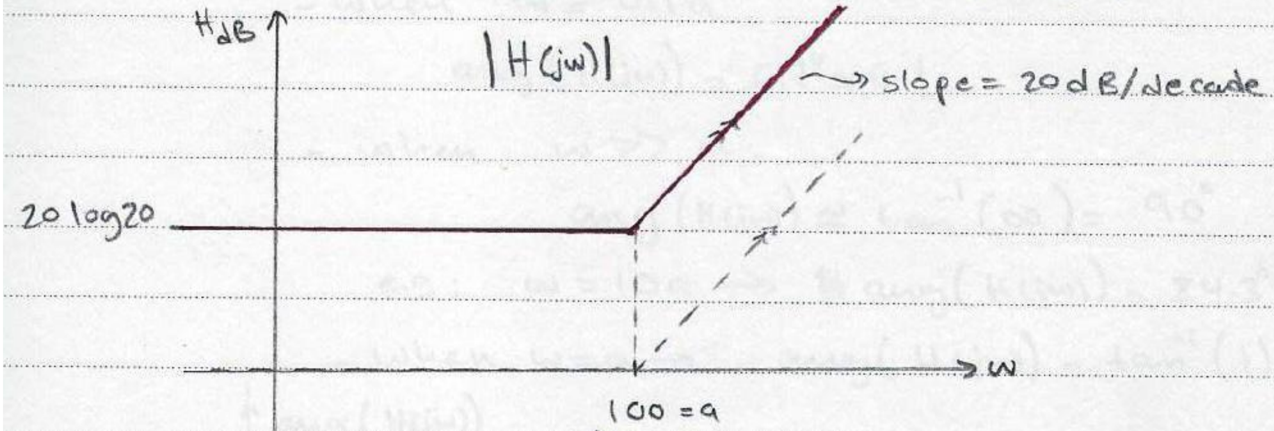


$H(s) = Z_{in}(s)$

$$H(s) = Z_{in}(s) = 20 + 0.2s = 20\left(1 + \frac{0.2}{20} s\right)$$

* بعوض jw بدالها ~ s

$$H_{dB} = 20\log(20) + 20\log\sqrt{1+\left(\frac{w}{100}\right)^2}$$

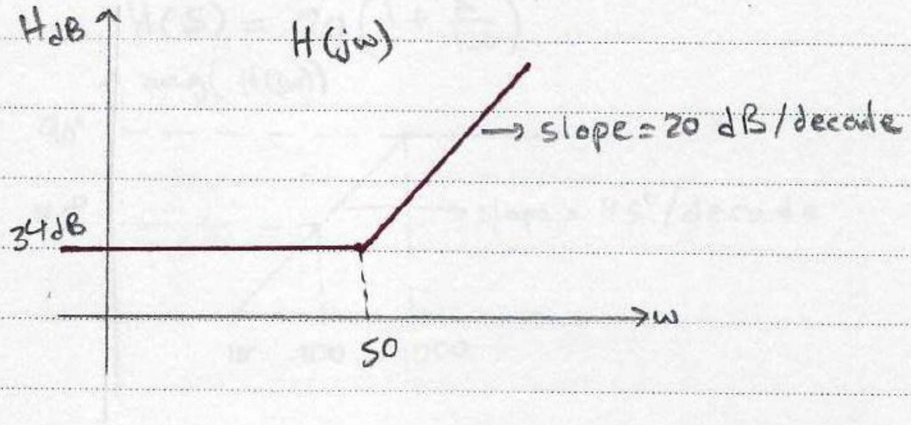


it's called : "corner frequency"/ "half power freq."/ "break freq."/ "cutoff freq."/ "3dB freq."

ex2 :- $H(s) = 50 + s = 50\left(1 + \frac{s}{\boxed{50}}\right)$

break frequency at w=50

$20\log 50 = 40 - 6 = 34\,dB$

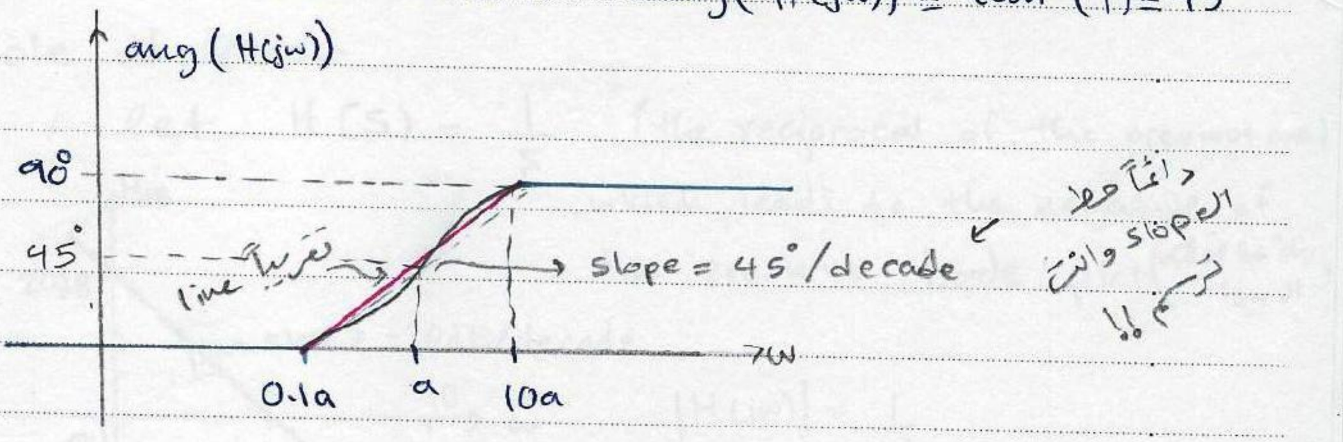


→ Phase Plot :

$$H(s) = 1 + \frac{s}{a}$$

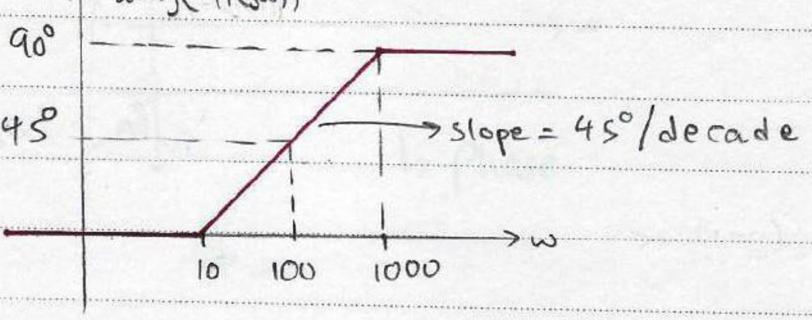
$$\text{ang}(H(j\omega)) = \text{ang}\left(1 + j\frac{\omega}{a}\right) = \tan^{-1}\left(\frac{\omega}{a}\right)$$

- when $\omega \ll a \Rightarrow \text{ang}(H(j\omega)) \simeq \tan^{-1}(0) = 0$
- when $\omega = 0.1a$
  $\text{ang}(H(j\omega)) = 5.7^\circ \approx 0$!
- when $\omega \gg a$,
  $\text{ang}(H(j\omega)) \simeq \tan^{-1}(\infty) = 90^\circ$
  e.g : $\omega = 10a \rightarrow \text{ang}(H(j\omega)) = 84.3^\circ \approx 90^\circ$
- when $\omega = a \rightarrow \text{ang}(H(j\omega)) = \tan^{-1}(1) = 45^\circ$



ex: Draw the Bode phase plot for:-

$$H(s) = 20\left(1 + \frac{s}{100}\right)$$



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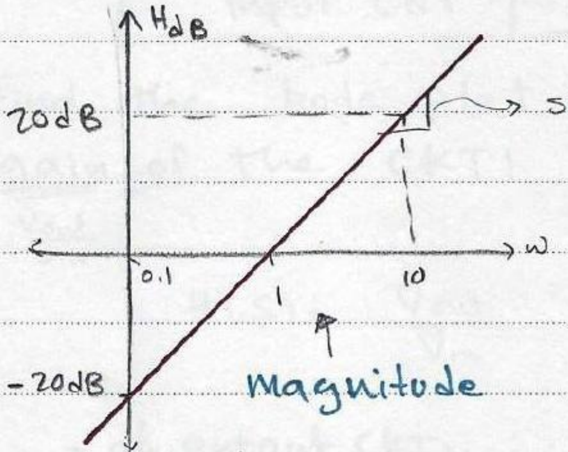
Zero at zero:-

let $H(s) = s$ $\longrightarrow$ $|H(j\omega)| = \omega$

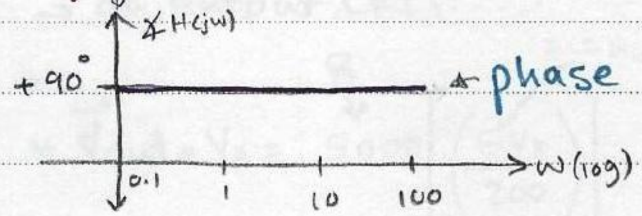
↳ $H(j\omega) = j\omega$ $\quad H_{dB} = 20\log(\omega)$



at $\omega = 0.1 \rightarrow H_{dB} = 20 \times -1 = -20dB$

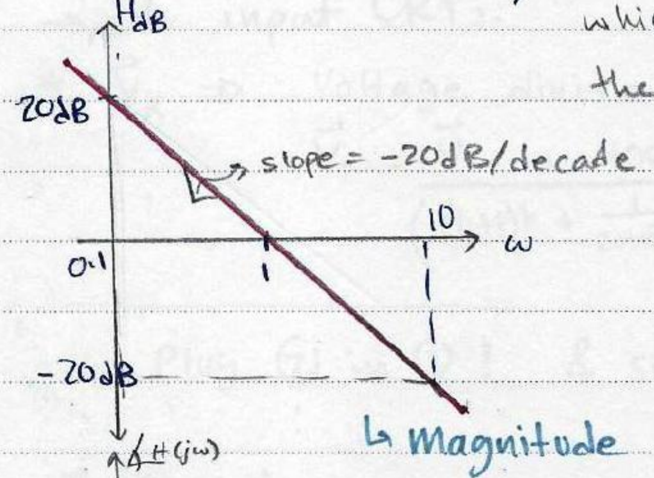
at $\omega = 1 \rightarrow H_{dB} = 0$ dB

at $\omega = 10 \rightarrow H_{dB} = 20 \times 1 = 20dB$



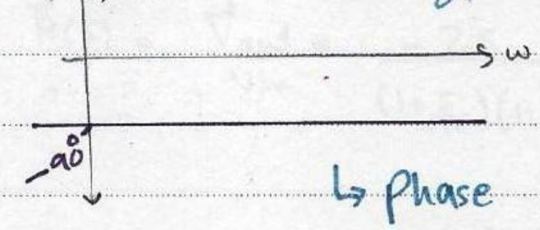
Pole at zero:-

let $H(s) = \frac{1}{s}$ (the reciprocal of the previous one) which leads to the negative of the previous bode plot (بنعكسها حول ال log)



$|H(j\omega)| = \frac{1}{\omega}$

$H_{dB} = -20\log(\omega)$



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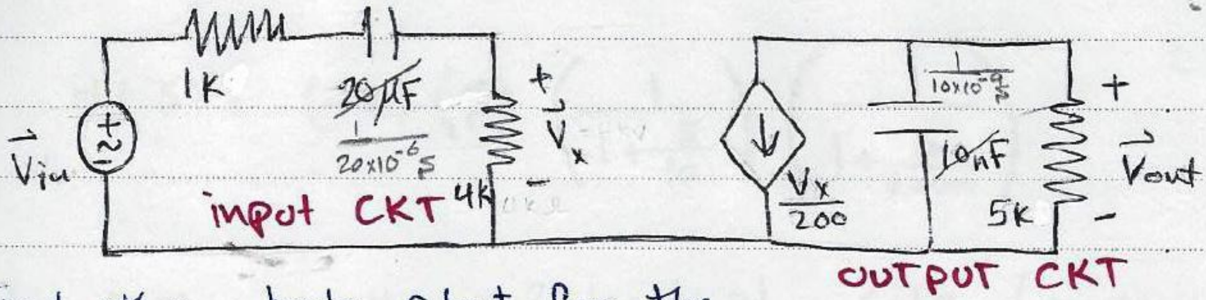
ex:-



Find the bode plot for the gain of the CKT!

$\frac{V_{out}}{V_{in}}$

$$H(s) = \frac{\vec{V}_{out}}{\vec{V}_{in}}$$

→ at output CKT:-

بسبب عكس الاشارة

$$* \ \vec{V}_{out} = V_R = \overset{R}{5000} \underbrace{\left[\left(\frac{\ominus V_x}{200} \right)_{I_{total}} \cdot \left(\frac{\frac{1}{10\times10^{-9}s}}{\frac{1}{10\times10^{-9}s} + 5000} \right) \right]}_{I_R \text{ (current division)}} \quad \cdots ①$$

→ at input CKT:-

$* \ \vec{V}_x \Rightarrow$ Voltage division:

$$\vec{V}_x = \frac{\vec{V}_{in} \times 4000}{\left((4+1)k + \frac{1}{20\times10^{-6}s} \right)} \quad \cdots ② \quad \rightarrow Zeq$$

Plug ② in ①! & simplify!

$$\vec{H}(s) = \frac{\vec{V}_{out}}{V_{in}} = \frac{-2s}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{20000}\right)}$$

$$= -2\,(s) \left(\frac{1}{1+\frac{s}{10}} \right) \left(\frac{1}{1+\frac{s}{20000}} \right) \longrightarrow$$

$$H(s) = (-2)(s)\left(\frac{1}{1+\frac{s}{10}}\right)\left(\frac{1}{1+\frac{s}{20000}}\right)$$

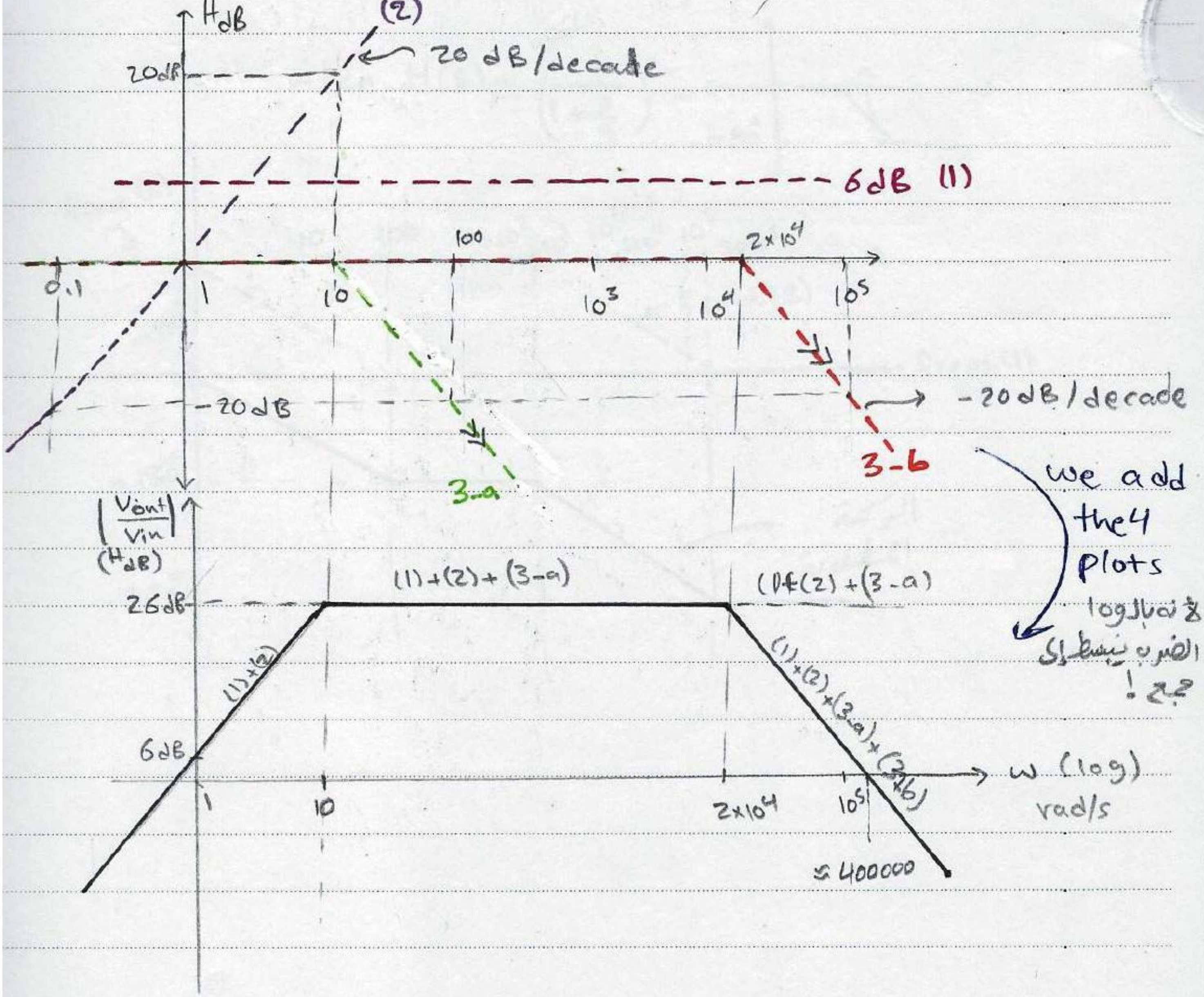
1) $(-2) \Rightarrow$ $H_{dB} = 20\log|-2| = 6\,dB$ (constant)

2) $(s) \Rightarrow$ $H_{dB} = 20\log(w)$ linear factor

↳ zero at w = 1! slope = 20 dB/decade

3) we have break points at w = 10 rad/s

& w = 20 000 rad/s



we add the4 plots

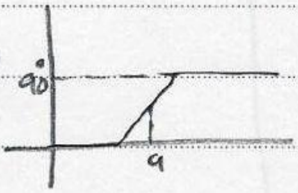
& نه بالlog الضرب بيسط إلى جمع !

$H(s) = \dfrac{-2s}{\left[1+\frac{s}{10}\right]\left[1+\frac{s}{20000}\right]}$, draw the phase bode plot for $H(s)$

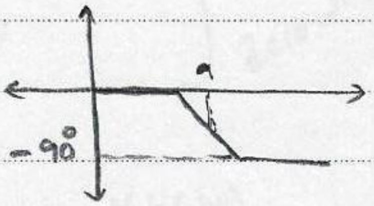
$\rightarrow H(j\omega) = \dfrac{-2j\omega}{\left(1+\frac{j\omega}{10}\right)\left(1+\frac{j\omega}{20000}\right)}$

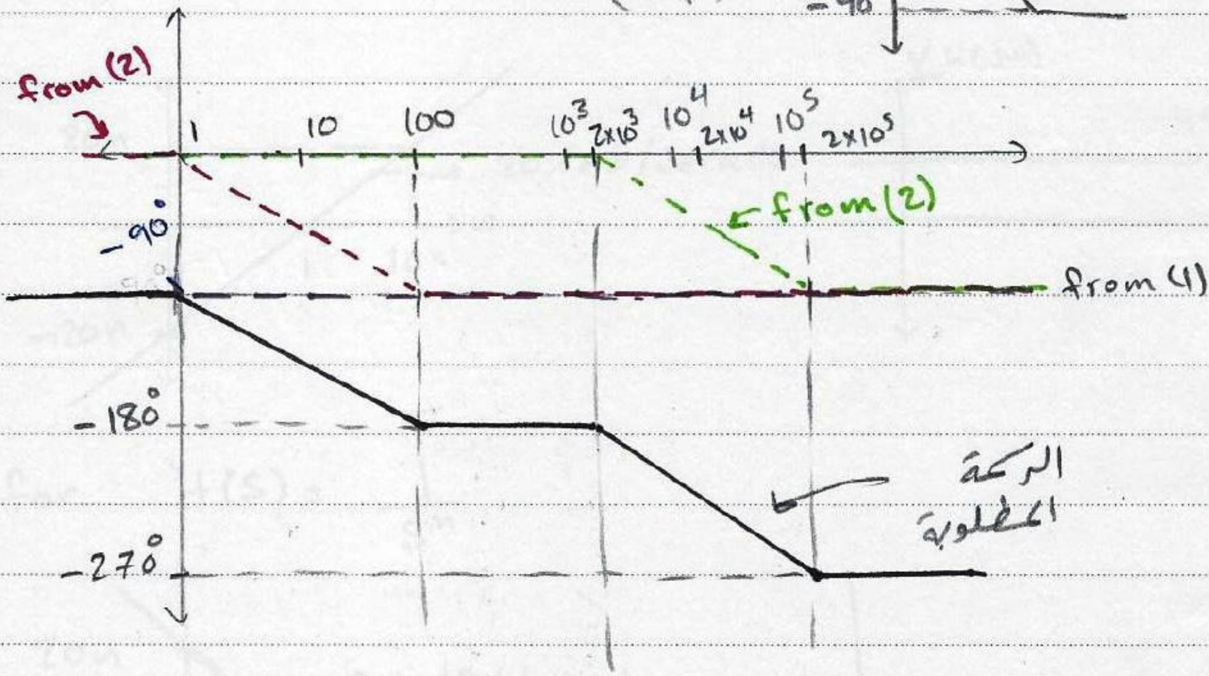
1) the angle of the numerator is constant $= -90°$

2) when $H(s) = 1 + \frac{s}{a} \rightarrow$



∴ when $H(s) = \overset{\text{المقلوب}}{\left(\dfrac{1}{1+\frac{s}{a}}\right)}$





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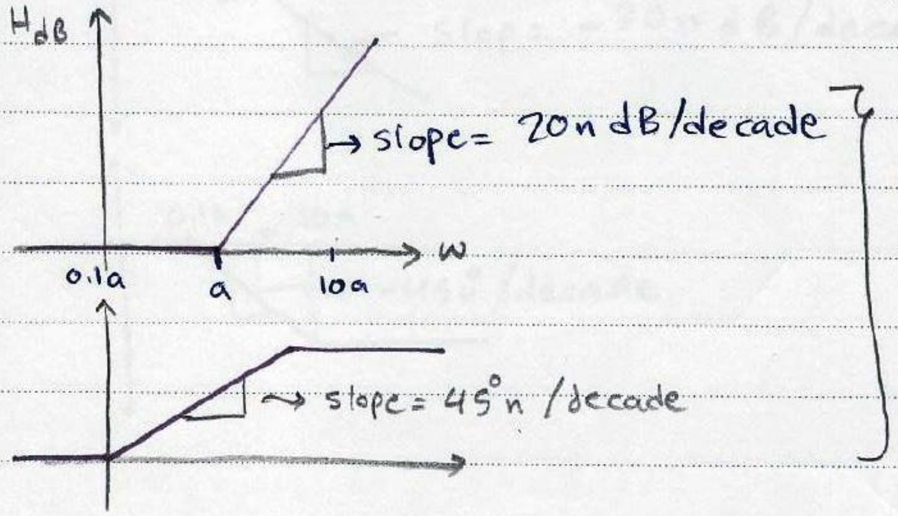
Higher order terms:

all the poles & zeros we've considered till now were from the first order e.g $(s^{\pm1}), (1+0.2s)^{\pm1}$ ---

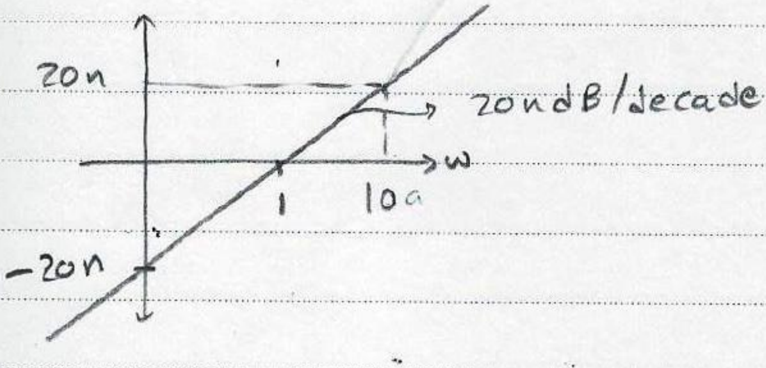
let's take $H(s) = s^n \rightarrow H_{dB} = 20 \log w^n$

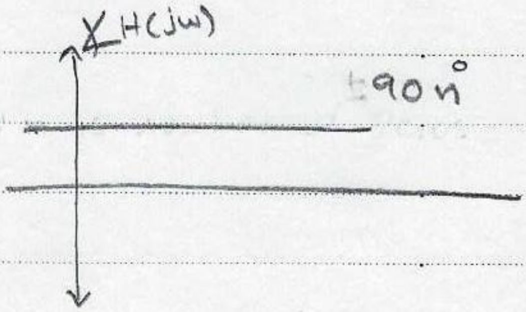
$= 20 n \log|w|$



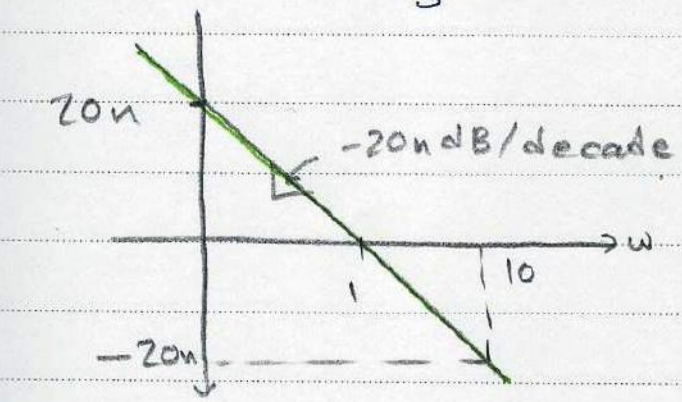
X

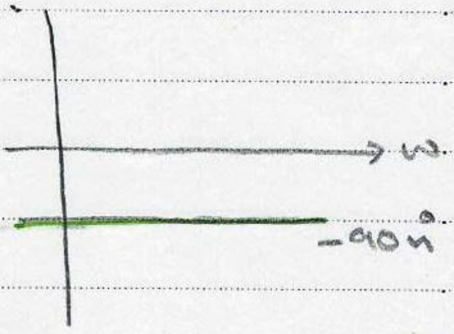
فقط إذا كان ال Breakpoint هي فقط ال Zero



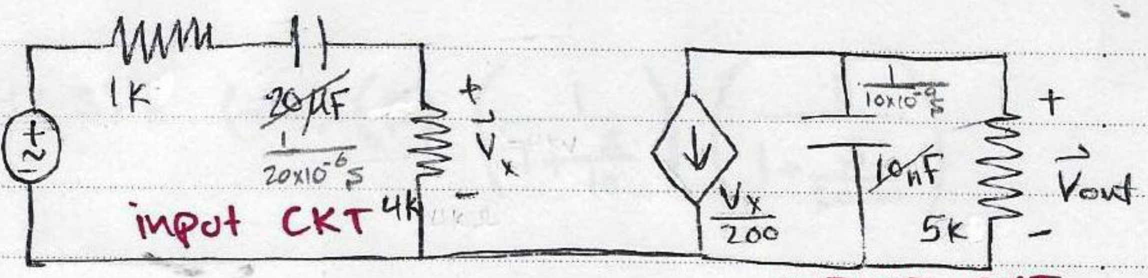


for $H(s) = \frac{1}{s^n}$





ex:-



Find the bode plot for the gain of the CKT!

$\frac{V_{out}}{V_{in}}$

$$H(s) = \frac{\vec{V}_{out}}{\vec{V}_{in}}$$

→ at output CKT:-

بسبب عكس الاشارة

$$* \vec{V}_{out} = V_R = \overset{R}{5000}\underbrace{\left[\left(\frac{\ominus V_x}{\underset{I_{total}}{200}}\right)\right]\cdot\left(\frac{\frac{1}{10\times10^{-9}s}}{\frac{1}{10\times10^{-9}s}+5000}\right)}_{I_R \text{ (current division)}} \cdots ①$$

→ at input CKT:-

$*\ \vec{V}_X \Rightarrow$ Voltage division:

$$\vec{V}_x = \frac{\vec{V}_{in} \times 4000}{\left((4+1)k + \frac{1}{20\times10^{-6}s}\right)} \rightarrow Z_{eq} \cdots ②$$

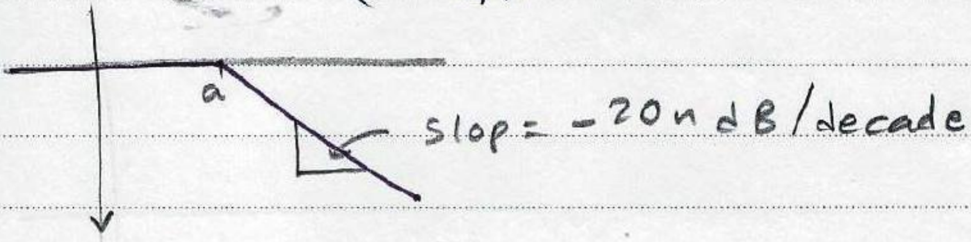
Plug ② in ① ! & simplify!

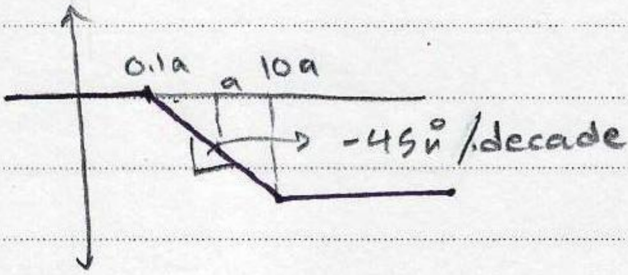
$$\vec{H}(s) = \frac{\vec{V}_{out}}{V_{in}} = \frac{-2s}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{20000}\right)}$$

$$= -2\ (s)\left(\frac{1}{1+\frac{s}{10}}\right)\left(\frac{1}{1+\frac{s}{20000}}\right) \longrightarrow$$

multi poles at $w = -a$

$$H(s) = \frac{1}{\left(1 + \frac{s}{a}\right)^n}$$





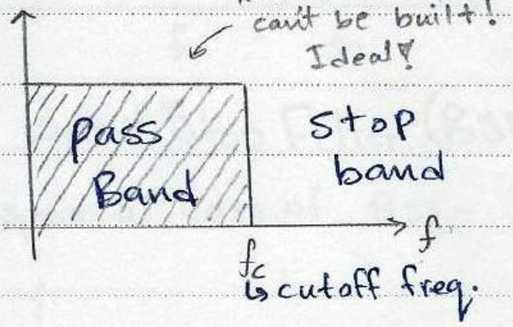
25/4/2013

Filters :-

- Filter : it's a device used to select certain frequency band

Ideal Filters

Non-ideal / Practical Filters

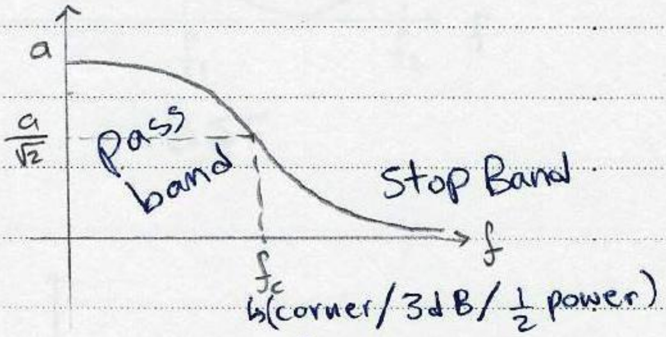


non-causal system can't be built! Ideal!

fc is cutoff freq.

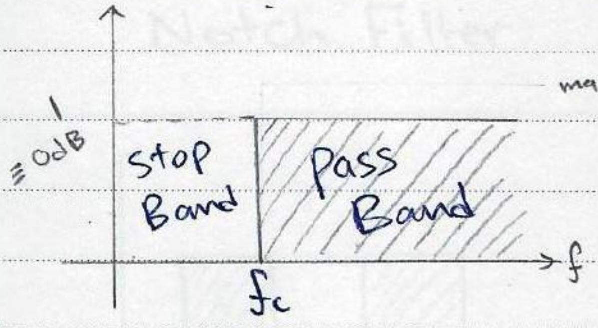
for the -ve part we apply even symmetry.

Low Pass Filter (LPF)

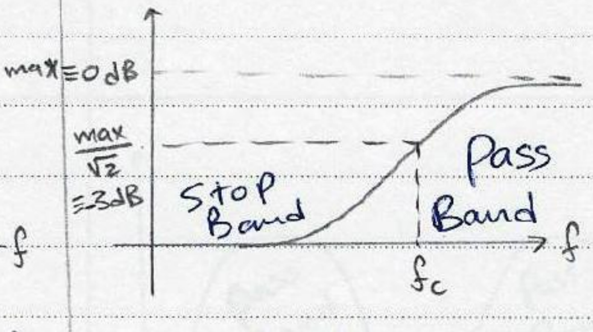


fc is (corner / 3dB / $\frac{1}{2}$ power) frequency.

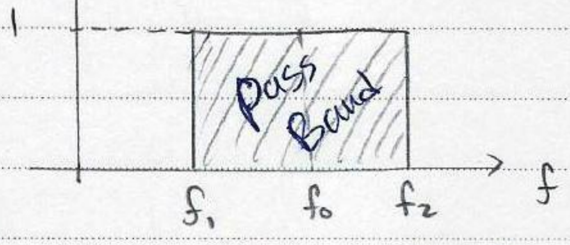
LPF



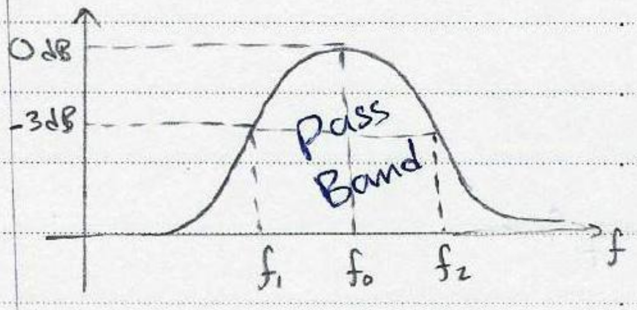
High Pass Filter (HPF)



HPF

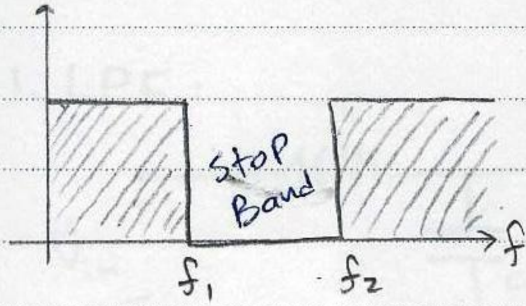


Band Pass Filter (BPF)



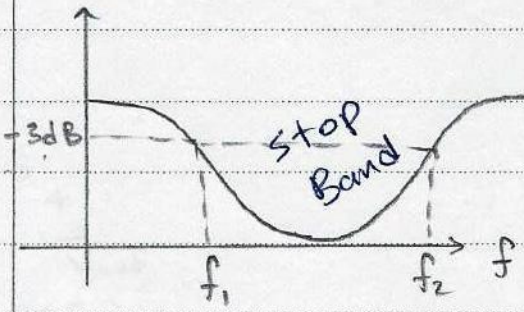
BPF

Ideal



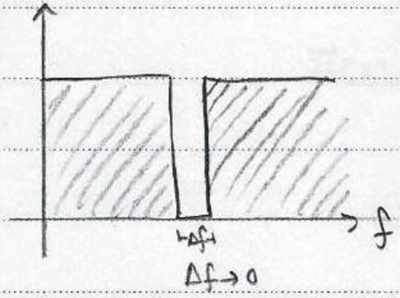
Band Stop Filter (BSF)

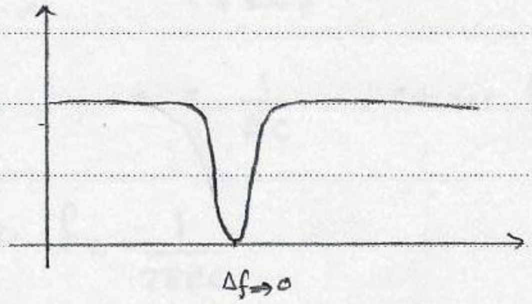
Practical



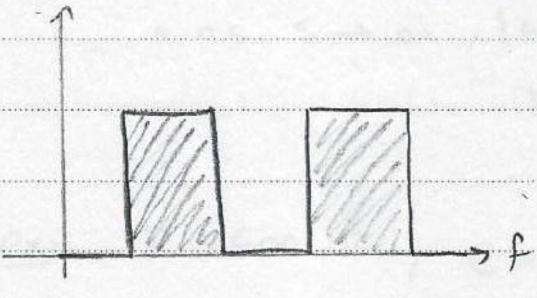
BSF

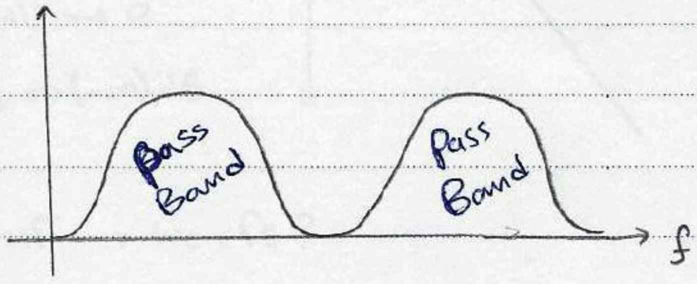
↳ speacial case of BSF:-





Notch Filter

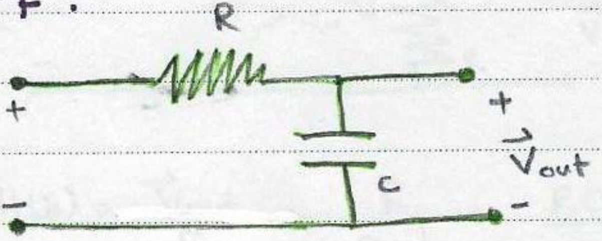




Multi-Band Pass Filter

Designing Filters:-

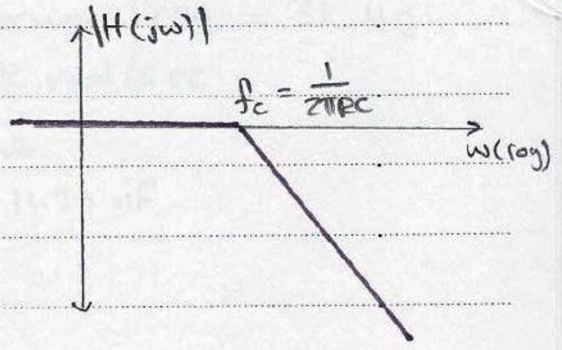
1. LPF :



$$\vec{H} = \frac{\vec{V}_{out}}{\vec{V}_{in}} = \frac{1/CS}{R+1/CS} = \frac{1}{1+RCS}$$

$$\vec{H}(S) = \frac{1}{1+\frac{S}{a}} \quad , \quad a = \frac{1}{RC}$$; corner freq.

$$\omega_c = \frac{1}{Rc} \Rightarrow f_c = \frac{1}{2\pi RC}$$



* as $S \to 0$, $|H(j\omega)| \to 1$

$H_{dB} = 0\,dB$

* as $S \to \infty$, $|H(j\omega)| \to 0$

$H_{dB} \Rightarrow (-\infty)\,dB$

ex:- $R = 500\,\Omega$, $C = 2\,nF$, ω_c , f_c?

Sol:-

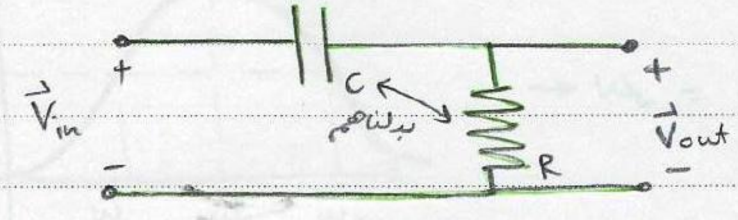
$$\omega_c = \frac{1}{RC} = 1\,Mrad/s$$

$$f_c = \frac{1}{2\pi RC} = 159\,KHz$$

** $|H(j\omega_c)| = \frac{1}{\sqrt{2}}$

$H_{dB} = -3\,dB$

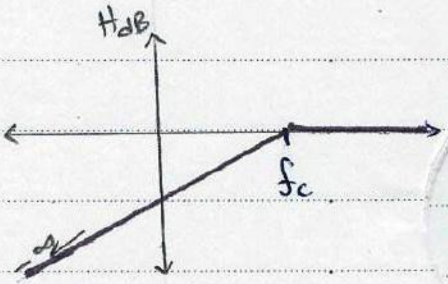
2. HPF



$H(s) = \dfrac{\vec{V}_{out}}{\vec{V}_{in}} = \dfrac{R}{R+\frac{1}{Cs}} = \dfrac{RCs}{1+RCs}$, $w_c = \frac{1}{RC}$; cutoff freq.

as $s \to 0$, $|H(jw)| \to 0$
$H_{dB} \to -\infty$ dB

as $s \to \infty$, $|H(jw)| \to 1$
$H_{dB} \to 0$ dB



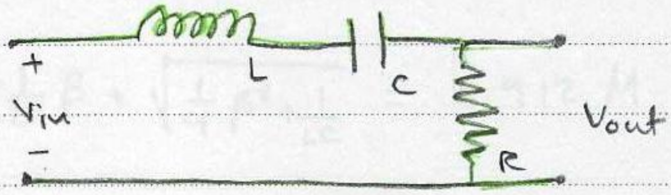
ex:- design a HPF with a corner freq = 3K Hz

$w_c = 2\pi f_c = 2\pi(3000) = 18.85$ K rad/sec.

$w_c = \frac{1}{RC} \longrightarrow$ choose $R = 4.7 k\Omega$

$C = \frac{1}{Rw_c} = 11.29$ nF

3. Band-Pass Filter BPF:-



$\vec{H} = \dfrac{\vec{V}_{out}}{\vec{V}_{in}} = A_v = \dfrac{R}{R+Ls+\frac{1}{Cs}} = \dfrac{sRC}{RCs+LCs^2+1}$

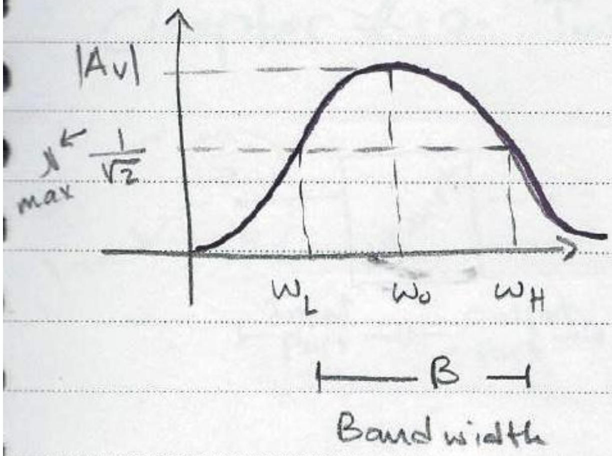
- $|\vec{A_v}| = \dfrac{wRC}{\sqrt{(1-w^2LC)^2+w^2R^2C^2}}$

as $w \to 0 \Rightarrow |\vec{A_v}| \approx wRC \to 0$

as $w \to \infty \Rightarrow |\vec{A_v}| \approx \frac{R}{wL} \to 0$

$* w_0 = \frac{1}{\sqrt{LC}}$; it's called the center freq. Sometimes

/ /



$$\beta = \omega_H - \omega_L$$

$$\rightarrow \omega_L = \frac{-R}{2L} + \frac{\sqrt{R^2C^2+4LC}}{2LC}$$

$$\rightarrow \omega_H = \frac{+R}{2L} + \frac{\sqrt{R^2C^2+4LC}}{2LC}$$

$$\beta = \frac{R}{L}$$

$$\therefore \omega_L = -\frac{\beta}{2} + \sqrt{\frac{1}{4}\beta^2 + \frac{1}{LC}}$$

$$\omega_H = +\frac{\beta}{2} + \sqrt{\frac{1}{4}\beta^2 + \frac{1}{LC}}$$

ex:- design a Band pass filter characterized by a bandwidth of 1 MHz, $f_H = 1.1\,MHz$,

Sol:- $\beta = 2\pi(1\times10^6) = 6.283\times10^6$ rad/s

$\omega_L = 2\pi \underline{f_L} = 628.3K$ rad/s $\quad \beta = F_H - F_L \quad 1.1\times10^6 - 10^6 = 100K\,Hz$

$\omega_H = 2\pi f_H = 2\pi(1.1\times10^6) = 6.912\,M$ rad/s

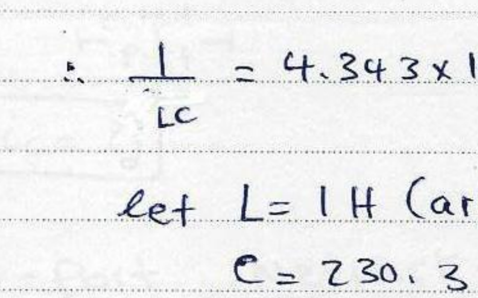
$$+\frac{1}{2}\beta + \sqrt{\frac{1}{4}\beta^2 + \frac{1}{LC}} = 6.912\,M$$

$$\therefore \frac{1}{LC} = 4.343\times10^{12}$$

let L = 1 H (arbitrarily)

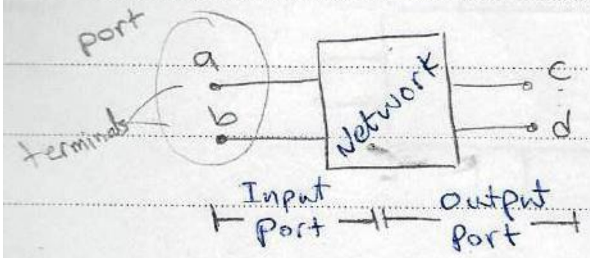
C = 230.3 fF

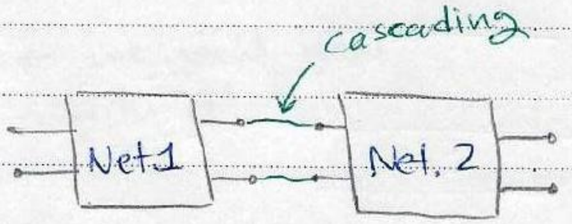
$R = 6.283\,M\Omega \quad \left(\beta = \frac{R}{L}\right)$ *



Sunday
5/5 /2013

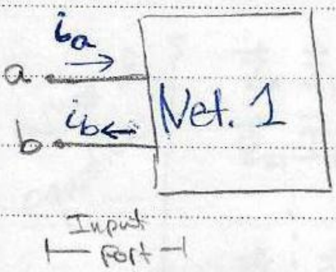
Chapter #17: Two-Ports Networks:

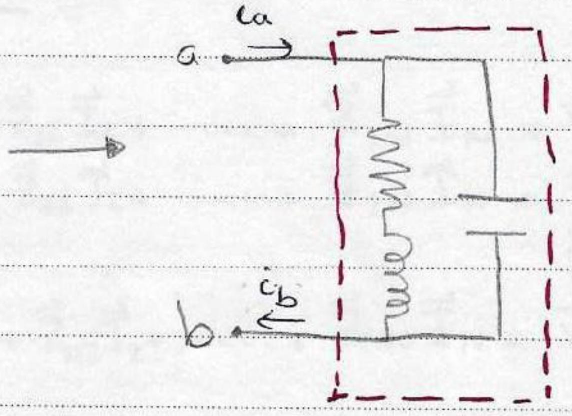




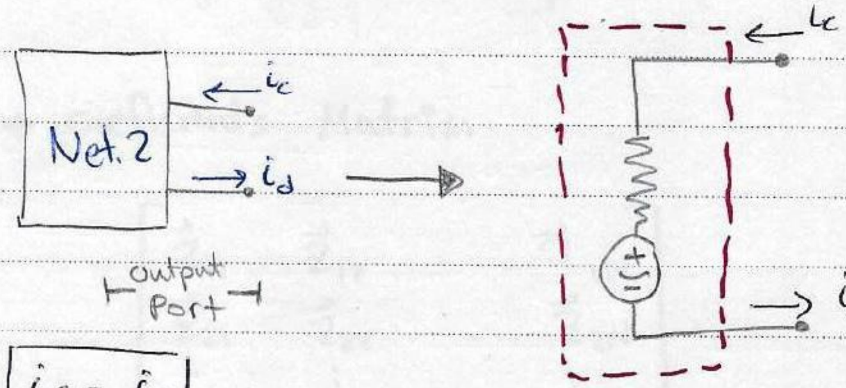
- Parameters.

→ Single-port Network:



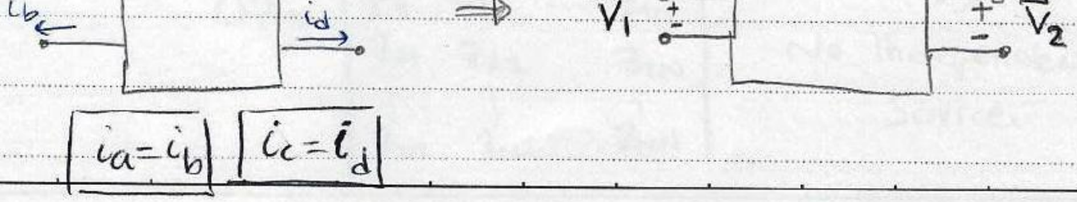


$\boxed{i_a = i_b}$



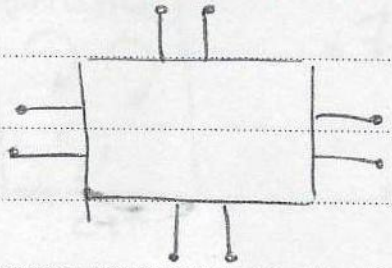
$\boxed{i_c = i_d}$

→ two-port Network:



$\boxed{i_a = i_b}$ $\boxed{i_c = i_d}$

→ Multi-port Network:



← we won't deal with it!

→ back to the single-port network:-

• **Mesh Analysis:**

$$\text{(N) \# of equations}\left\{\begin{array}{l} \vec{Z}_{11}\vec{I}_1+\vec{Z}_{12}\vec{I}_2+\cdots+\vec{Z}_{1N}\vec{I}_N=\vec{V}_1 \\ \vec{Z}_{21}\vec{I}_1+\vec{Z}_{22}\vec{I}_2+\cdots+\vec{Z}_{2N}\vec{I}_N=\vec{V}_2 \\ \vdots \qquad\qquad\qquad\qquad\qquad\qquad \vdots \\ \vec{Z}_{N1}\vec{I}_1+\vec{Z}_{N2}\vec{I}_2+\cdots+\vec{Z}_{NN}\vec{I}_N=\vec{V}_N \end{array}\right.$$

$Z_{ij}(S)$ is the impedance.

→ coeficients Matrix:

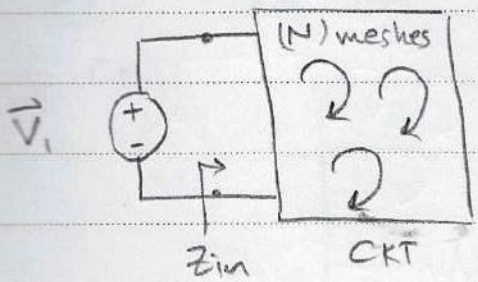
$$\begin{bmatrix} \vec{Z}_{11} & \vec{Z}_{12} & \cdots & \vec{Z}_{1N} \\ \vec{Z}_{21} & \vec{Z}_{22} & \cdots & \vec{Z}_{2N} \\ \vdots & \vdots & & \vdots \\ \vec{Z}_{N1} & Z_{N2} & & \vec{Z}_{NN} \end{bmatrix}$$

→ determinade:

$$\Delta Z=\begin{vmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & & Z_{2N} \\ \vdots & \vdots & & \vdots \\ Z_{N1} & Z_{N2} & & Z_{NN} \end{vmatrix}$$

*شرط:

No independent Source.



$$* \ \vec{Z}_{in} = \frac{\vec{V}_1}{\vec{I}_1}$$

$$* \ \vec{V}_2 = \vec{V}_3 = \vec{V}_4 = \cdots = \vec{V}_N = 0$$

using cramer's rule:-

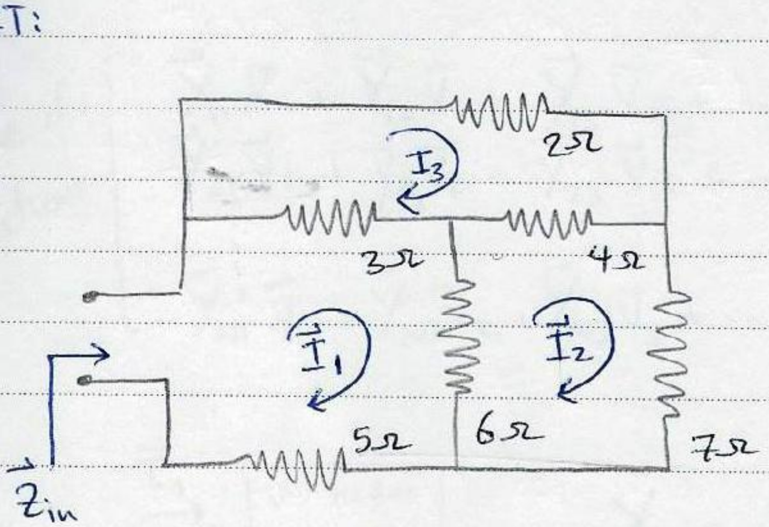
$$\vec{I}_1 = \frac{\begin{vmatrix} \vec{V}_1 & \vec{Z}_{12} & \vec{Z}_{13} & \cdots & \vec{Z}_{1N} \\ 0 & \vec{Z}_{22} & \vec{Z}_{23} & \cdots & \vec{Z}_{2N} \\ \vdots & \vdots & & & \vdots \\ 0 & Z_{N2} & \vec{Z}_{N3} & & \vec{Z}_{NN} \end{vmatrix}}{\Delta Z}$$

$$\vec{I}_1 = \frac{\vec{V}_1 \ \underbrace{\Delta_{11}}_{} }{\Delta Z} \xrightarrow{} \text{minor}$$

$$\Rightarrow \vec{Z}_{in} = \frac{\vec{I}_1}{\vec{V}_1} = \frac{\Delta Z}{\Delta_{11}}$$

$$\hookrightarrow \Delta_{11} = \begin{vmatrix} \vec{Z}_{22} & \vec{Z}_{23} & \cdots & \vec{Z}_{2N} \\ \vec{Z}_{32} & \vec{Z}_{33} & \cdots & \vec{Z}_{3N} \\ \vdots & & & \\ \vec{Z}_{N2} & \vec{Z}_{N3} & \cdots & \vec{Z}_{NN} \end{vmatrix}$$

ex:- find the input impedance for the following CKT:



$$14\vec{I}_1 - 6\vec{I}_2 - 3\vec{I}_3 = \vec{V}_1$$

$$-6\vec{I}_1 + 17\vec{I}_2 - 4\vec{I}_3 = 0 \leftarrow \vec{V}_2 = 0 = \vec{V}_3 = \dots = \vec{V}_N$$

$$-3\vec{I}_1 - 4\vec{I}_2 + 9\vec{I}_3 = 0$$

$$\Delta z = \begin{vmatrix} 14 & -6 & -3 \\ -6 & 17 & -4 \\ -3 & -4 & 9 \end{vmatrix} = 1297$$

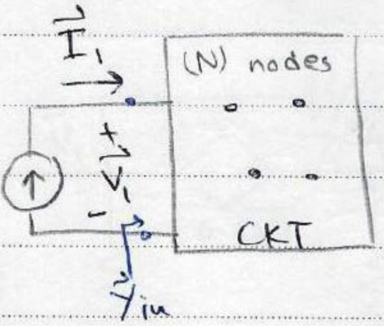
$$\Delta_{11} = \begin{vmatrix} 17 & -4 \\ -4 & 9 \end{vmatrix} = 137$$

$$\vec{Z}_{in} = \frac{\Delta z}{\Delta_{11}} = \frac{1297}{137} = 9.467\ \Omega$$

• Nodal analysis

(N) # of equations

$$\left[\begin{array}{l}\vec{Y}_{11}\vec{V}_1+\vec{Y}_{12}\vec{V}_2+\vec{Y}_{13}\vec{V}_3+\dots+\vec{Y}_{1N}\vec{V}_N=\vec{I}_1\\ \vec{Y}_{21}\vec{V}_1+\vec{Y}_{22}\vec{V}_2+\vec{Y}_{23}\vec{V}_3+\dots+\vec{Y}_{2N}\vec{V}_N=\vec{I}_2\\ \vdots\\ \vec{Y}_{N1}\vec{V}_1+\vec{Y}_{N2}\vec{V}_2+\vec{Y}_{N3}\vec{V}_3+\dots+\vec{Y}_{NN}\vec{V}_N=\vec{I}_N\end{array}\right.$$



$$\vec{Y}_{in}=\frac{\vec{I}_1}{\vec{V}_1}$$

$$\longrightarrow I_2=I_3=\dots=I_N=0$$

$$\vec{Y}_{in}=\begin{bmatrix}\vec{Y}_{11} & \vec{Y}_{12} & \dots & \vec{Y}_{1N}\\ \vec{Y}_{21} & \vec{Y}_{22} & & \vec{Y}_{2N}\\ \vdots & \vdots & & \vdots\\ \vec{Y}_{N1} & \vec{Y}_{N2} & \dots & \vec{Y}_{NN}\end{bmatrix}$$

$$\Delta Y=\begin{vmatrix}\vec{Y}_{11} & \vec{Y}_{12} & \dots & \vec{Y}_{1N}\\ \vec{Y}_{21} & \vec{Y}_{22} & \dots & \vec{Y}_{2N}\\ \vdots & & \vdots & \\ \vec{Y}_{N1} & \vec{Y}_{N2} & \dots & \vec{Y}_{NN}\end{vmatrix}$$

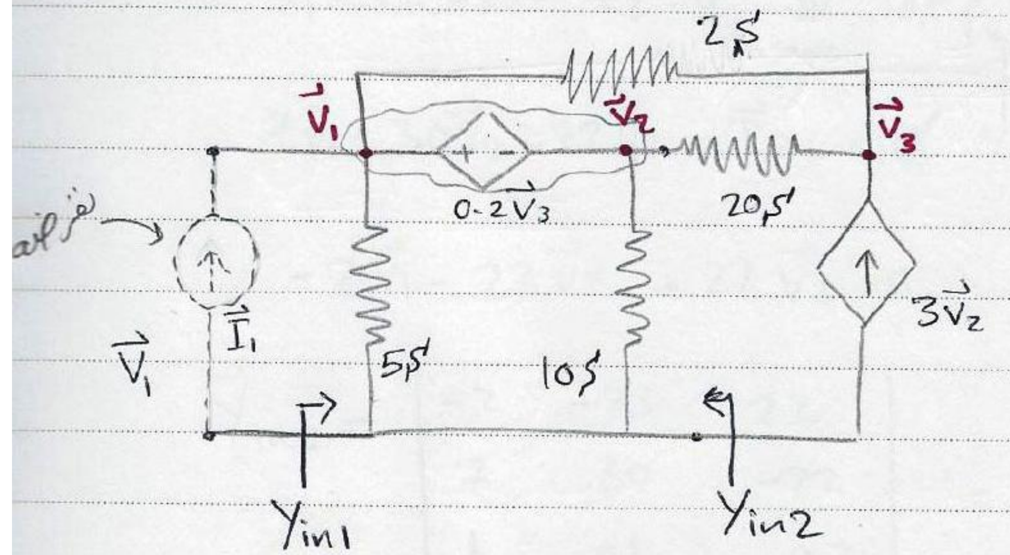
from cramer,

$$\vec{V}_1=\frac{\begin{vmatrix}\vec{I}_1 & \vec{Y}_{12} & \vec{Y}_{13} & \dots & \vec{Y}_{1N}\\ 0 & \vec{Y}_{22} & \vec{Y}_{23} & \dots & \vec{Y}_{2N}\\ \vdots & & \vdots & & \vdots\\ 0 & \vec{Y}_{N2} & \vec{Y}_{N3} & \dots & \vec{Y}_{NN}\end{vmatrix}}{\Delta Y}$$

$$Y_{in}=\frac{\Delta Y}{\Delta y_{11}}$$

$$\Delta_{y_{11}} = \begin{vmatrix} \vec{Y}_{22} & \vec{Y}_{23} & \cdots & \vec{Y}_{2N} \\ \vec{Y}_{32} & \vec{Y}_{33} & \cdots & \vec{Y}_{3N} \\ \vdots & \vdots & \vdots & \vdots \\ \vec{Y}_{N2} & Y_{N3} & \cdots & \vec{Y}_{NN} \end{vmatrix}$$

<u>ex:</u> find input admittance Y_{in1} & Y_{in2}



$\boxed{V_1 - V_2 = 0.2\, V_3}$ ... ① from the super node

$5\vec{V}_1 + 2\vec{V}_1 - 2\vec{V}_3 + 10\,\vec{V}_2 + 20\,\vec{V}_2 - 20\,\vec{V}_3 = \vec{I}_1$

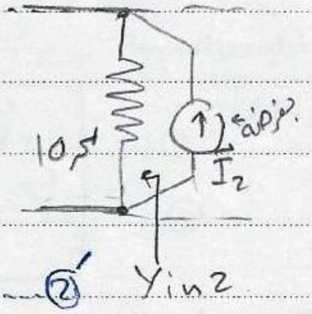
$\hookrightarrow \boxed{7\vec{V}_1 + 30\vec{V}_2 - 22\vec{V}_3 = \vec{I}_1}$ ... ②

$2\vec{V}_3 - 2\vec{V}_1 + 20\,\vec{V}_3 - 20\,\vec{V}_2 - 3\,\vec{V}_2 = 0$

$\hookrightarrow \boxed{-2\vec{V}_1 - 23\vec{V}_2 + 22\vec{V}_3 = 0}$ ... ③

$$\vec{Y}_{in1} = \frac{\begin{vmatrix} 7 & 30 & -22 \\ -2 & -23 & 22 \\ 1 & -1 & -0.2 \end{vmatrix}}{\begin{vmatrix} -23 & 22 \\ -1 & -0.2 \end{vmatrix}} = 10.684\ S$$

$\vec{Y}_{in2}$:



$\vec{V}_1 - \vec{V}_2 - 0.2\ \vec{V}_3 = 0$ ...①′

← same

$7\vec{V}_1 + 30\vec{V}_2 - 22\vec{V}_3 = \boxed{\vec{I}_2}$ ...②′

$-2\vec{V}_1 - 23\vec{V}_2 + 22\ \vec{V}_3 = 0$ ...③′

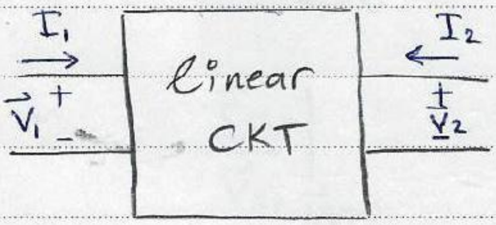
$$\vec{Y}_{in2} = \frac{\begin{vmatrix} -2 & -23 & 22 \\ 7 & 30 & -22 \\ 1 & -1 & -0.2 \end{vmatrix}}{\begin{vmatrix} -2 & 22 \\ 1 & -0.2 \end{vmatrix}} = 13.1574\ S$$

$S \equiv [\mho]$

9/5/2013

Admittance Parameters (Y-parameters)

or Short CKT addmittance par.



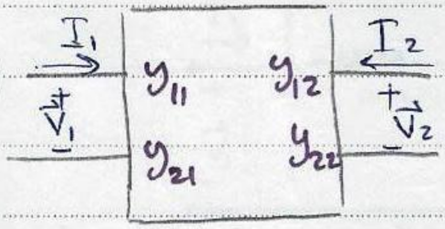
By super position:-

Proportionality constants [A/V]

$$\boxed{\begin{aligned}\vec{I}_1 &= y_{11}\vec{V}_1 + y_{12}\vec{V}_2 \quad \ldots ①\\ \vec{I}_2 &= y_{21}\vec{V}_1 + y_{22}\vec{V}_2 \quad \ldots ②\end{aligned}}$$

$[y_{ij}] = A/V = S$; short CKT admittance parameters or Y-parameters

deriving the eq.



$$\mathbb{I} = \begin{bmatrix}\vec{I}_1\\ \vec{I}_2\end{bmatrix} , \quad \mathbb{V} = \begin{bmatrix}\vec{V}_1\\ \vec{V}_2\end{bmatrix}$$

$$\mathbb{Y} = \begin{bmatrix} y_{11} & y_{12}\\ y_{21} & y_{22}\end{bmatrix}$$

$$\mathbb{I} = \mathbb{Y}.\mathbb{V}$$

$$\begin{bmatrix}\vec{I}_1\\ \vec{I}_2\end{bmatrix} = \begin{bmatrix} y_{11} & y_{12}\\ y_{21} & y_{22}\end{bmatrix}\begin{bmatrix}\vec{V}_1\\ \vec{V}_2\end{bmatrix} \rightarrow \begin{aligned}\vec{I}_1 &= y_{11}\vec{V}_1 + y_{12}\vec{V}_2 \ldots ①\\ \vec{I}_2 &= y_{21}\vec{V}_1 + y_{22}\vec{V}_2 \ldots ②\end{aligned}$$

* finding Y- parameters:-

from eq①:-

- $y_{11} = \left.\dfrac{\vec{I}_1}{\vec{V}_1}\right|_{\vec{V}_2=0}$; the Short Circuit input admittance

output is short-circuited

/ /

from eq ②:

- $y_{21} = \left.\frac{\vec{I}_{②}}{\vec{V}_{①}}\right|_{\vec{V}_2=0}$; Short CKT transfer admittance.

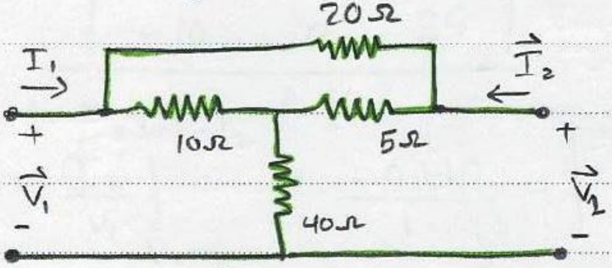
- $y_{12} = \left.\frac{\vec{I}_{①}}{\vec{V}_{②}}\right|_{\vec{V}_1=0}$; Short CKT transfer admittance.

- $y_{22} = \left.\frac{\vec{I}_{②}}{\vec{V}_{2}}\right|_{\vec{V}_1=0}$; Short CKT output admittance

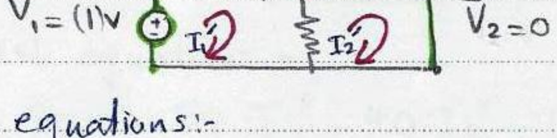
Input is short-circuited

≡ Y-parameters

ex:- Find the Short CKT admittance Parameters for the following resistive 2-ports network



sol:- step 1 to find y_{11} & y_{21}, the output most be short-circuited $\therefore V_2 = 0$, then we assume a voltage source at the input terminals $(\vec{V}_1 = (1)V)$



$I_1 = I_1'$

$I_2 = -I_2'$ ... ⊛

V (V matrix)

mesh equations:-

$$50\, I_1' - 40\, I_2' - 10\, I_3' = \vec{V}_1 \quad \text{...①}$$
$$-40\, I_1' + 45\, I_2' - 5\, I_3' = 0 \quad \text{...②}$$
$$-10\, I_1' - 5\, I_2' + 35\, I_3' = 0 \quad \text{...③}$$

using cramer's rule:-

$$\bullet\ I_{①}' = \frac{\begin{vmatrix} 1 & -40 & -10 \\ 0 & 45 & -5 \\ 0 & -5 & 35 \end{vmatrix} \leftarrow \Delta 11}{\Delta y \rightarrow \begin{vmatrix} 50 & -40 & -10 \\ -40 & 45 & -5 \\ -10 & -5 & 35 \end{vmatrix}} = 0.1192\ A$$

[illegible] V

$$\rightarrow y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \boxed{0.1192\ S}$$

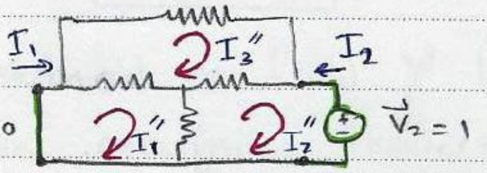
$\vec{V}_1 = 1V$

$$\bullet\ I_2' = \frac{\begin{vmatrix} 50 & 1 & -10 \\ -40 & 0 & -5 \\ -10 & 0 & 35 \end{vmatrix}}{\Delta y} = 0.1115\ A$$

from ⊛ $I_2 = -I_2'$

$$\rightarrow y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{-0.1115}{1} = \boxed{-0.1115\ S}$$

step 2 * let $V_1 = 0$ (short CKT) & $V_2 = (1)V$



$\vec{I}_1 = \vec{I}_1''$

$\vec{I}_2 = -\vec{I}_2''$ --- (**)

mesh equations:-

$$50 I_1'' - 40 I_2'' - 10 I_3'' = 0 \quad \dots ①'$$
$$-40 I_1'' + 45 I_2'' - 5 I_3'' = 1 \quad \dots ②'$$
$$-10 I_1'' - 5 I_2'' + 35 I_3'' = 0 \quad \dots ③'$$

- $I_1'' = \frac{\begin{vmatrix} 0 & -40 & -10 \\ -1 & 45 & -5 \\ 0 & -5 & 35 \end{vmatrix}}{\Delta y} = -0.1115 \text{ S}$

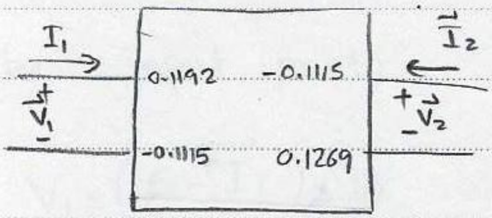
$\rightarrow y_{12} = \left.\frac{\vec{I}_2}{\vec{V}_2}\right|_{\vec{V}_1=0} = \frac{\vec{I}_1''}{1} = \boxed{-0.1115 \text{ S}}$

***Note:**

when $y_{12} = y_{21}$, it's called a "bilateral CKT"

- $I_2'' = \frac{\begin{vmatrix} 50 & 0 & -10 \\ -40 & -1 & -5 \\ -10 & 0 & 35 \end{vmatrix}}{\Delta y} = -0.1269 \text{ A}$

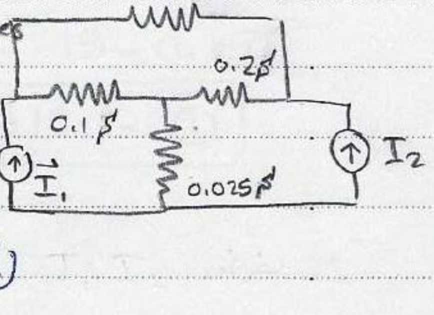
$\rightarrow y_{22} = \left.\frac{\vec{I}_2}{\vec{V}_2}\right|_{\vec{V}_1=0} = \frac{-\vec{I}_2''}{1} = \boxed{0.1269 \text{ S}}$



* another method to find $\mathbb{Y}$ (only for simple CKTs)

→ using Nodal analysis ! convert to admittances

Nodal equations:-



$0.15 \vec{V}_1 - 0.05 \vec{V}_2 - 0.1 \vec{V}_3 = \vec{I}_1$ ...①

$-0.05 \vec{V}_1 - 0.25 \vec{V}_2 + 0.2 \vec{V}_3 = \vec{I}_2$ ...②

$-0.1 \vec{V}_1 - 0.2 \vec{V}_2 + 0.325 \vec{V}_3 = 0$ ...③

3 eq → 2 eq

from ③ $\vec{V}_3 = \frac{0.1 \vec{V}_1}{0.325} + \frac{0.2}{0.325} \vec{V}_2$

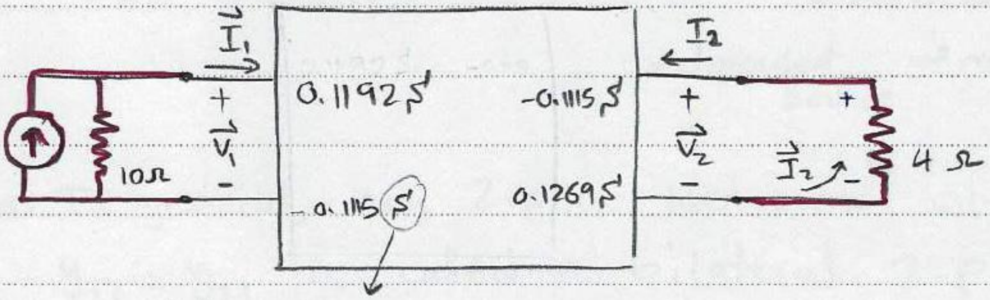
Substiute V_3 in ① & ②

in ①:- $\vec{I}_1 = \underset{y_{11}}{(0.1192)}\,\vec{V}_1 + \overset{y_{12}}{(-0.1115)}\,\vec{V}_2$

in ②:- $\vec{I}_2 = \underset{y_{21}}{(-0.1115)}\,\vec{V}_1 + \underset{y_{22}}{(0.1269)}\,\vec{V}_2$

* but, how to use Y-parameters?

ex تابع الـ السابق



صار ال Unit بعرف انهم Y-parameters

$$\boxed{\begin{aligned} 0.1192\,\vec{V}_1 - 0.1115\,\vec{V}_2 &= \vec{I}_1 \quad \text{...①} \\ -0.1115\,\vec{V}_1 + 0.1269\,\vec{V}_2 &= I_2 \quad \text{...②} \end{aligned}}$$

دائمًا اكتبهم قبل لا تحل !

- from the input part:

$V_1 = (15 - \vec{I}_1) \times 10$

$* \; I_1 = 15 - 0.1\,\vec{V}_1 \quad \text{--ⓐ}$

→ Sub. in ①

$0.1192\,\vec{V}_1 - 0.1115\,\vec{V}_2 = 15 - 0.1\,\vec{V}_1$

$$\boxed{0.2192\,\vec{V}_1 - 0.1115\,\vec{V}_2 = 15 \quad \text{---}①'}$$

- from the output part:

$\vec{V}_2 = -4\,I_2$

$* \; \vec{I}_2 = -0.25\,\vec{V}_2 \quad \text{--ⓑ}$

→ sub. in ②

$-0.1115\,\vec{V}_1 + 0.1269\,\vec{V}_2 = -0.25\,\vec{V}_2$

$$\boxed{-0.1115\,\vec{V}_1 + 0.3769\,\vec{V}_2 = 0 \quad \text{---}②'}$$

now I have 2 equations with 2 unknowns :-

Solve ①'&②' for v_1, v_2 :-

$\rightarrow \vec{V}_1 = 80.55\ V$

$\vec{V}_2 = 23.83\ V$

plug in ⓐ & ⓑ

$\rightarrow I_1 = 6.945 A$

$I_2 = -5.9575 A$

* In this ex, $y_{12} = y_{21}$

∴ it's a bilateral CKT

* & resistive CKTS are always bilateral!

* dependent source شرط ما يكون في

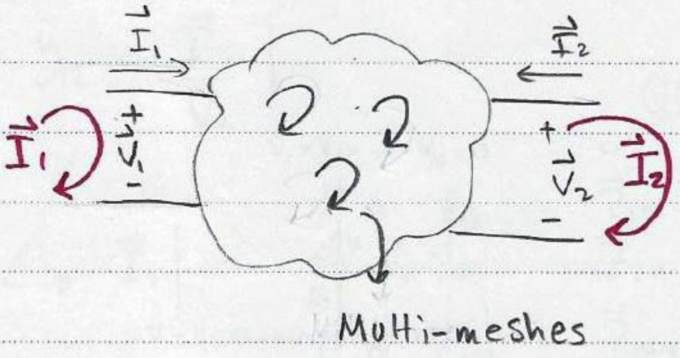
⇒ In general; any 2-port Network with $y_{12} = y_{21}$ is called a bilateral 2-port Network!

11/5/2013 →

محاضرة تعويضية

Bilateral Networks:

$$\boxed{y_{12} = y_{21}}$$



$$\vec{Z}_{11}\vec{I}_1 + \vec{Z}_{12}\vec{I}_2 + \vec{Z}_{13}\vec{I}_3 + --- + \vec{Z}_{1N}\vec{I}_N = \vec{V}_1 \quad ...①$$
$$\vec{Z}_{21}\vec{I}_1 + \vec{Z}_{22}\vec{I}_2 + \vec{Z}_{23}\vec{I}_3 + \cdots + \vec{Z}_{2N}\vec{I}_N = -\vec{V}_2 \quad ...②$$
$$\vec{Z}_{31}\vec{I}_1 + \vec{Z}_{32}\vec{I}_2 + \vec{Z}_{33}\vec{I}_3 + \cdots + \vec{Z}_{3N}\vec{I}_N = \vec{V}_3 \quad ...③$$
$$\vdots$$
$$\vec{Z}_{N1}\vec{I}_1 + \vec{Z}_{N2}\vec{I}_2 + \vec{Z}_{N3}\vec{I}_3 + \cdots + \vec{Z}_{NN}\vec{I}_N = \vec{V}_N \quad ...Ⓝ$$

/ /

* assume, it's a one-port network $\therefore$ $V_2 = V_3 = V_4 = \dots = V_N = 0$

$$\rightarrow \vec{I}_1 = \vec{V}_1 \frac{\Delta_{11}}{\Delta Z} \longrightarrow \vec{Z}_{in}\Big|_{\vec{V}_2=\vec{V}_3=\dots=\vec{V}_N=0} = \frac{\Delta Z}{\Delta_{11}}$$

- $$y_{11} = \frac{\vec{I}_1}{\vec{V}_1}\Big|_{V_2=0} = \boxed{\frac{\Delta_{11}}{\Delta Z}} = Y_{in} = \frac{1}{Z_{in}}!$$

- $$y_{22} = \frac{\vec{I}_2}{\vec{V}_2}\Big|_{V_1=V_3=V_4=\dots=V_N=0} \Rightarrow ?\ \boxed{y_{22} = \frac{\Delta_{22}}{\Delta Z}}$$

بشطب column 2 & row 2

$$\vec{I}_2\Big|_{V_1=V_3=\dots=0} = \frac{\begin{array}{c} \text{col. } ② \\ \begin{vmatrix} Z_{11} & 0 & Z_{13} & \cdots & Z_{1N} \\ Z_{21} & -V_2 & Z_{23} & & Z_{2N} \\ \vdots & 0 & \vdots & & \vdots \\ Z_{N1} & 0 & Z_{N3} & & Z_{NN} \end{vmatrix}\end{array}}{\Delta Z} = \frac{-(-\vec{V}_2)\,\Delta_{22}}{\Delta Z}$$

- $$y_{12} = \frac{\vec{I}_1}{\vec{V}_2}\Big|_{V_1=V_3=\dots=V_N=0} =$$

بشطب السطر(2) مع العمود①

$$\vec{I}_1\Big|_{V_1=V_3=\dots=0} = \frac{\begin{vmatrix} \oplus\ 0 & \vec{Z}_{12} & \cdots & \vec{Z}_{1N} \\ \ominus\ -\vec{V}_2 & \vec{Z}_{22} & \cdots & \vec{Z}_{2N} \\ \oplus\ 0 & \vdots & & \vdots \\ 0 & \vec{Z}_{N2} & \cdots & \vec{Z}_{NN} \end{vmatrix}}{\Delta Z} = \frac{-(-\vec{V}_2)\,\Delta_{21}}{\Delta Z}$$

plug in y_{12}:

$$\boxed{y_{12} = \frac{\Delta_{21}}{\Delta Z}} \longrightarrow \Delta_{21} = \begin{vmatrix} \vec{Z}_{12} & Z_{13} & \cdots & Z_{1N} \\ Z_{32} & Z_{33} & \cdots & Z_{3N} \\ Z_{42} & Z_{43} & \cdots & Z_{4N} \\ \vdots & & & \\ Z_{N2} & Z_{N3} & \cdots & Z_{NN} \end{vmatrix}$$

- $y_{21} = \frac{I_2}{V_1}\Big|_{\vec{V}_2 = \vec{V}_3 = \dots = \vec{V}_N = 0}$

$$\hookrightarrow I_2\Big|_{V_2=V_3=\dots=V_N=0} = \frac{\begin{vmatrix} Z_{11} & \vec{V}_1 & Z_{13} & \cdots & Z_{1N} \\ Z_{21} & 0 & Z_{23} & \vdots & Z_{2N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{N1} & 0 & Z_{N3} & \vdots & Z_{NN} \end{vmatrix}}{\Delta Z} = \frac{V_1 \Delta 12}{\Delta Z}$$

plug in y_{21}:

$$y_{21} = \frac{\Delta_{12}}{\Delta Z} \longrightarrow \Delta_{12} = \begin{vmatrix} Z_{21} & Z_{23} & Z_{24} & \cdots & Z_{2N} \\ Z_{31} & Z_{33} & Z_{34} & \cdots & Z_{3N} \\ \vdots & \vdots & \vdots & & \vdots \\ Z_{N1} & Z_{N3} & Z_{N4} & \cdots & Z_{NN} \end{vmatrix}$$

since it's bila~~te~~teral

$$\therefore y_{12} = y_{21}$$

$$\boxed{\frac{\Delta_{12}}{\cancel{\Delta Z}} = \frac{\Delta_{21}}{\cancel{\Delta Z}}}$$

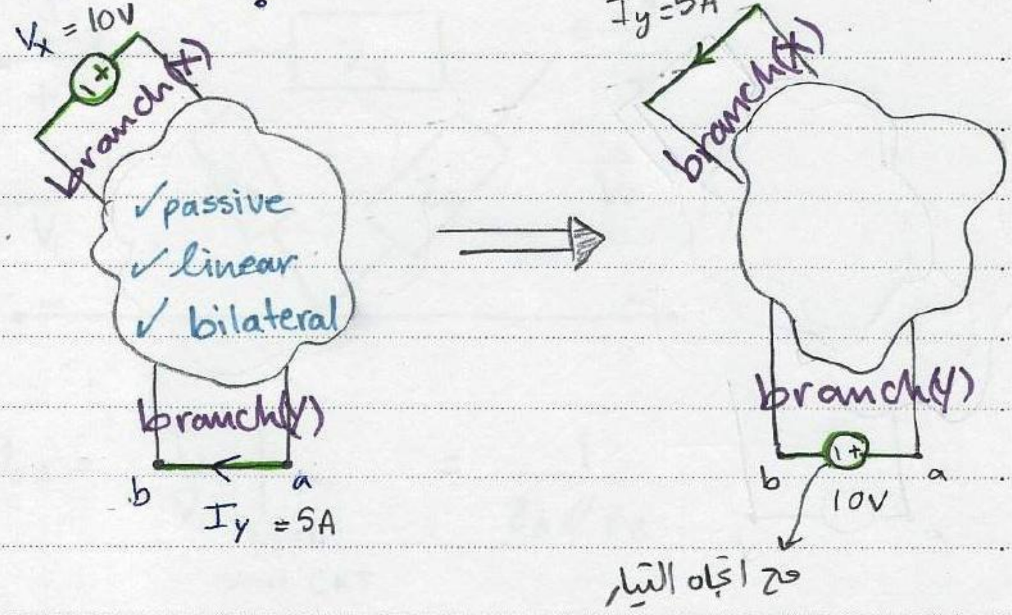
$$\Delta_{12} = \Delta_{21} \longrightarrow \therefore \boxed{Z_{ij} = Z_{ji}} \rightarrow \text{after transposing } \Delta_{21}$$

* if this is true for an element, it's called "a bilateral element", and if all the elements in the CKT are bilateral, then it's called a "bilateral CKT."

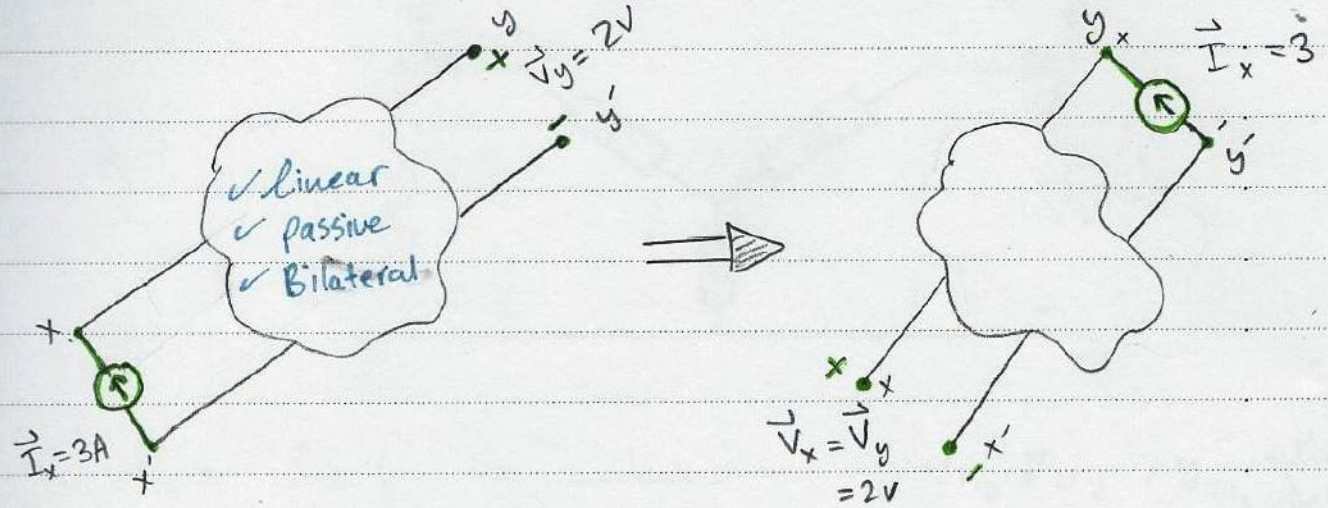
* Resiprocity Theorem :

- In any *linear* , *Passive* , *bilateral* network:-

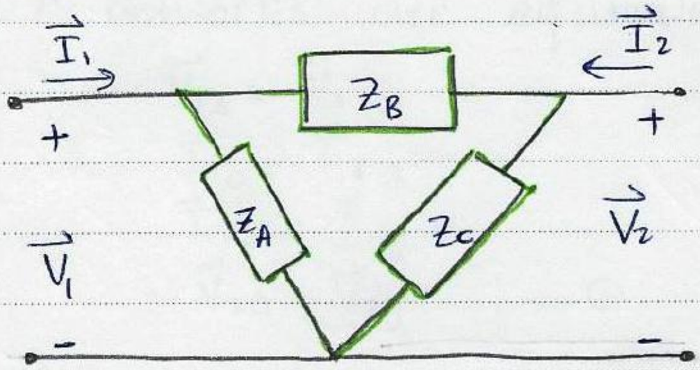
1. if the single voltage source (V_x) in branch (X) produces the current response (I_y) in branch (Y), then the removal of (V_x) from branch (X) and it's insertion in branch (Y) will Produce the current response $\vec{I_y}$ in branch (X) !



2. if the single current source (I_x) between nodes x & x' produces the voltage response $\vec{V}_x$ between nodes Y & Y' , then the removal of the current source from nodes x & x' and it's insertion between nodes Y & Y', will produce the voltage V_y between x & x'

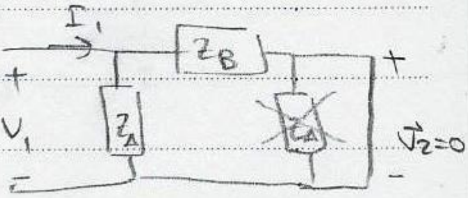


Δ↔Y transformation!

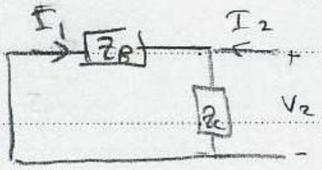


$$y_{11\Delta} = \frac{\vec{I}_1}{\vec{V}_1}\Bigg|_{V_2=0} = \frac{1}{Z_A // Z_B}$$

Short CKT



$$y_{12\Delta} = \frac{\vec{I}_1}{\vec{V}_2}\Bigg|_{V_1=0} = \frac{-1}{Z_B+Z_C}$$



$I_1 = \frac{-I_2 Z_C}{Z_B+Z_C}$

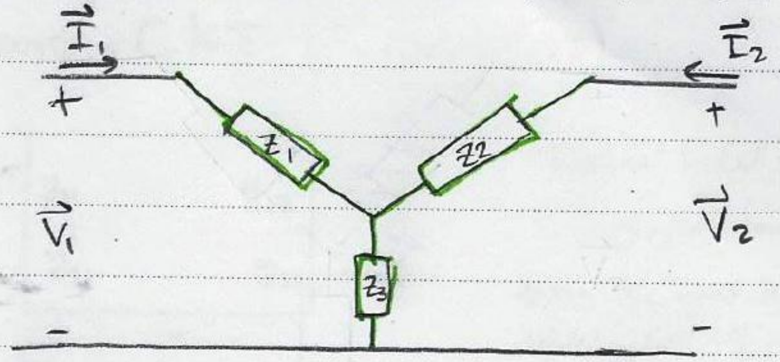
& $I_2 = \frac{V_2}{Z_C}$

$I_1 = -\frac{V_2}{Z_B+Z_C}$

$\therefore \frac{I_1}{V_1} = -\frac{1}{Z_B+Z_C}$

$$y_{21\Delta} = \frac{\vec{I}_2}{\vec{V}_1}\Bigg|_{\vec{V}_2=0} = \frac{-1}{Z_A+Z_B}$$

$$y_{22\Delta} = \frac{\vec{I}_2}{\vec{V}_2}\Bigg|_{V_1=0} = \frac{1}{Z_B // Z_C}$$



$$y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0} = \frac{1}{Z_2 \parallel Z_3 + Z_1}$$, y_{12y}, y_{21y}, y_{22} تدربوا عليهم!

* Y & Δ networks are equivalent

$$\therefore \vec{V}_{1\Delta} = \vec{V}_{1y}$$
$$\vec{I}_{1\Delta} = \vec{I}_{1y}$$
$$\vec{I}_{2\Delta} = \vec{I}_{2y}$$
$$\vec{V}_{2\Delta} = \vec{V}_{2y} \quad ---(*)$$

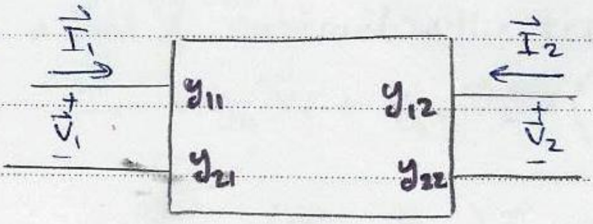
for Δ:-

$$I_{1\Delta} = y_{11\Delta}\vec{V}_{1\Delta} + y_{12\Delta}\vec{V}_{2\Delta} \quad ---1$$
$$I_{2\Delta} = y_{21\Delta}\vec{V}_{1\Delta} + y_{22\Delta}\vec{V}_{2\Delta} \quad ---2$$

the same is done for (y)!

↳ $\therefore$ $y_{11\Delta} = y_{11y}$
$y_{12\Delta} = y_{12y}$
$y_{21\Delta} = y_{21y}$
$y_{22\Delta} = y_{22y}$

- Equivelant CKT (Y-parameters)



* المطلوب ايجاد الدارة المكافئة
بدلالة Y-parameters
* الدارة الناتجة ليست بالضرورة
مساوية للدارة الأصلية لكنها
مكافئة لها!

for this network:-

$$\boxed{\begin{aligned}\vec{I}_1 &= y_{11}\vec{V}_1 + y_{12}\vec{V}_2 \\ \vec{I}_2 &= y_{21}\vec{V}_1 + y_{22}\vec{V}_2\end{aligned}} \quad \begin{aligned}&\ldots ①\\ &\ldots ②\end{aligned}$$

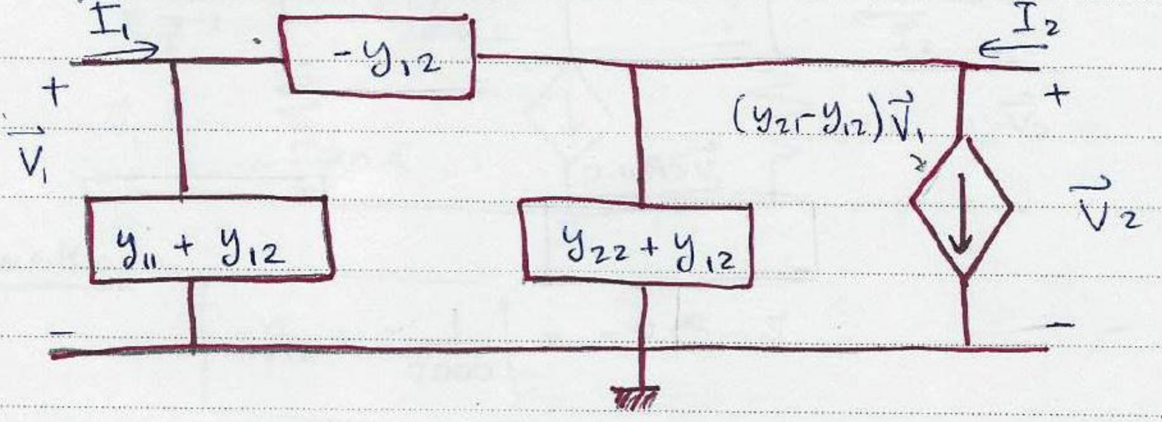
- The 1st equivelant CKT:-
  - add & subtract the term $(y_{12}\vec{V}_1)$ from ②

$$\vec{I}_2 = y_{21}\vec{V}_1 + y_{22}\vec{V}_2 + (y_{12}\vec{V}_1 - y_{12}\vec{V}_1)$$

$$\vec{I}_2 - (y_{21} - y_{12})\vec{V}_1 = y_{12}\vec{V}_1 + y_{22}\vec{V}_2 \quad \ldots ②'$$

$$\vec{I}_1 = y_{11}\vec{V}_1 + y_{12}\vec{V}_2 \quad \ldots ①'$$ ← stays the same

from ①' & ②' and from nodal analysis the equivelant CKT will be:-



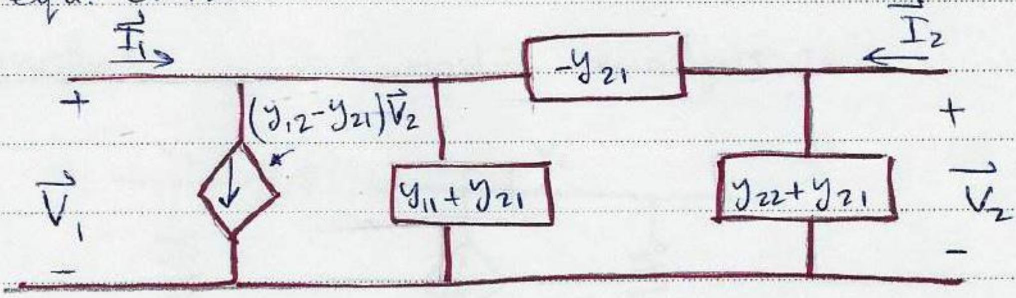
- 2^{nd} equivelant CKT:-
  - Add & subtract the term $(y_{21}\vec{V}_2)$ to ①

$$\vec{I}_1 = y_{11}\vec{V}_1 + y_{12}\vec{V}_2 + (y_{21}\vec{V}_2 - y_{21}\vec{V}_2)$$

$$\vec{I}_1 - (y_{12} - y_{21})\vec{V}_2 = y_{11}\vec{V}_1 + y_{12}\vec{V}_2 \quad \dots ①'$$

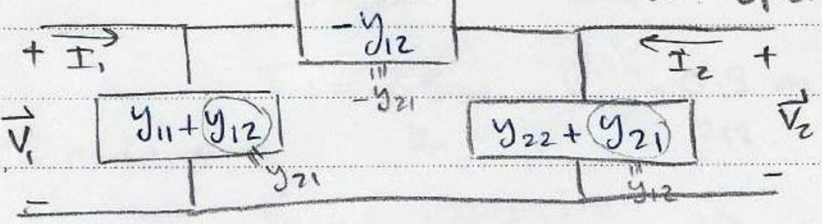
$$\vec{I}_2 = y_{21}\vec{V}_1 + y_{22}\vec{V}_2 \quad \dots ②$$ stays the same

equ: CKT:-

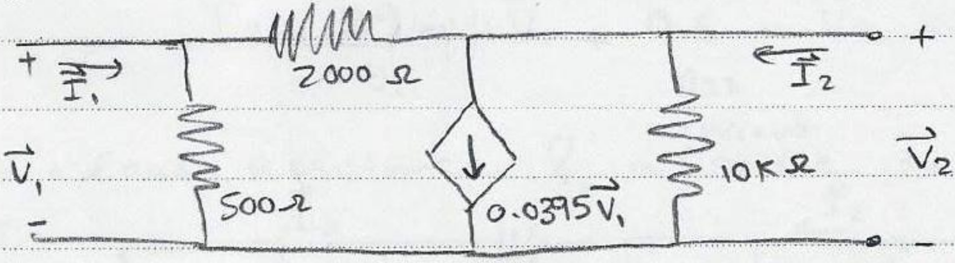


- for bilateral CKTs:- $\boxed{y_{21} = y_{12}}$

current source will become an open CKT



ex:- A) Determine the y-parameters:-



1^{st} method :-

$$\boxed{-y_{12} = -\frac{1}{2000}} = -0.5 mS$$

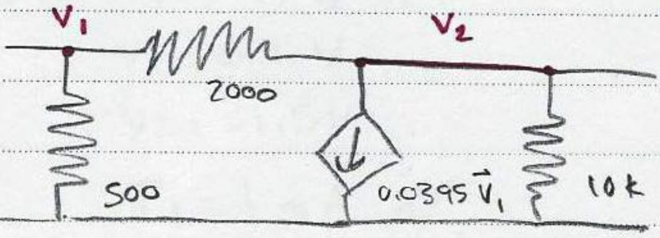
$$\boxed{y_{11} + y_{12} = \frac{1}{500}} \rightarrow y_{11} = 2.5 mS$$

$$\boxed{y_{22} + y_{12} = \frac{1}{10000}} \Rightarrow y_{22} = 0.6\,mS$$

$$\boxed{(y_{21} - y_{12}) = 0.0395} \Rightarrow y_{21} = 39\,mS$$

$$\therefore \boxed{\begin{aligned} \vec{I}_1 &= 2.5\,m\,\vec{V}_1 - 0.5\,m\,\vec{V}_2 \\ \vec{I}_2 &= 39\,m\,\vec{V}_1 + 0.6\,m\,\vec{V}_2 \end{aligned}} \begin{aligned} &\ldots ① \\ &\ldots ② \end{aligned}$$

2nd method, use nodal analysis :



@ node 1:

$$\vec{I}_1 = \frac{\vec{V}_1}{500} + \frac{V_1 - V_2}{2000}$$

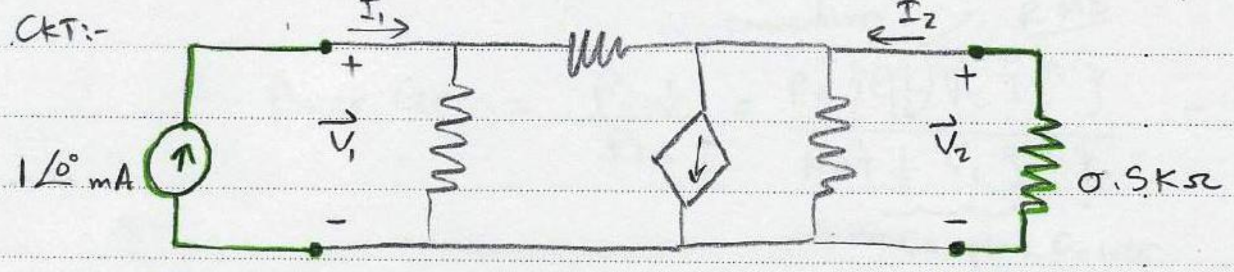
$$I_1 = \underset{y_{11}}{2.5\,m}\,\vec{V}_1 \underset{y_{12}}{-0.5\,m}\,\vec{V}_2 \quad \ldots ①$$

@ node 2:

$$\vec{I}_2 - 0.0395\,\vec{V}_1 = \frac{\vec{V}_2}{10000} + \frac{\vec{V}_2 - \vec{V}_1}{2000}$$

$$I_2 = \underset{y_{21}}{39\,m}\,\vec{V}_1 + \underset{y_{22}}{0.6\,m}\,\vec{V}_2 \quad \ldots ②$$

B) add a load Resistance & a current source to the previous CKT:-



$\vec{I}_1 = 1\angle 0°\,mA$ for any value of $\vec{V}_1$!

$$\vec{I}_2 = \frac{-\vec{V}_2}{0.5K} = -2m\,\vec{V}_2$$

plug in ① & ②

$$1\cancel{m} = 2.5\cancel{m}\,\vec{V}_1 - 0.5\cancel{m}\,\vec{V}_2 \quad \ldots ①'$$

$$-2\cancel{m}\,\vec{V}_2 = 39\cancel{m}\,\vec{V}_1 + 0.6\cancel{m}\,\vec{v}_2$$

$$39\vec{V}_1 + 2.6\vec{V}_2 = 0 \quad \ldots ②'$$

solve for v_1 & v_2 :-

$V_1 = 0.1\ V$

$V_2 = -1.5V$

$I_1 = 1\ mA\ \angle 0°$

$I_2 = 3mA\ \angle 0°$

c) find the Voltage Gain G_V

$\frac{\text{output}}{\text{input}}$ = Gain

$$\boxed{G_V = \frac{\vec{V}_2}{\vec{V}_1}} = -15$$

d) find the Current Gain G_I

$$\boxed{G_I = \frac{\vec{I}_2}{\vec{I}_1}} = 3$$

e) 〃 〃 Power Gain

from the passive sign convention

لأن القيم ليست RMS

$$\text{Power Gain} = \frac{P_{out}}{P_{in}} = \frac{Re\left\{-\frac{1}{2}\vec{V}_2\vec{I}_2^*\right\}}{Re\left\{\underbrace{\frac{1}{2}\vec{V}_1\vec{I}_1^*}_{\text{Complex Power}}\right\}} = 45$$

f) find the input impedance:-

$$Z_{in} = \frac{\vec{V}_1}{\vec{I}_1} = \frac{0.1}{1m} = 0.1 \ K\Omega$$

g) find the input impedance with the output short circuited : $(Z_{11}) = \frac{1}{y_{11}}$

$\hookrightarrow \vec{V}_2 = 0$

$$\vec{Z}_{in}\Big|_{\vec{V}_2=0} = \frac{\vec{V}_1}{\vec{I}_1}\Big|_{V_2=0} = ?$$

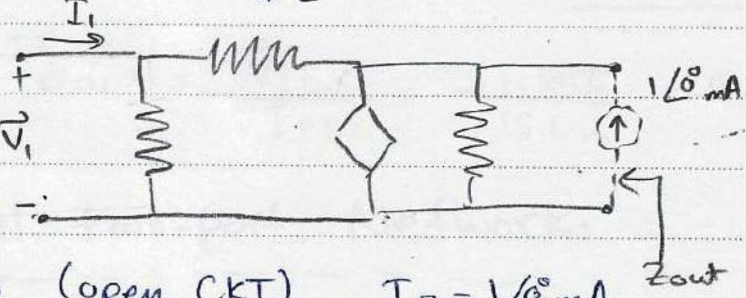
$V_2 = 0$ plug in ①

$$\xrightarrow{} \quad I_1 = 2.5 \, m \, \vec{V}_1 + 0$$

$$\frac{\vec{V}_1}{\vec{I}_1} = \frac{1}{2.5m} = 400 \, \Omega$$

h) the output impedance: (للدارة الأصلية قبل إضافة ال load & source)

1st method: $Z_{out} = \frac{\vec{V}_2}{I_2}$



assume a test current source $I_2 = 1\angle 0^\circ \, mA$

$\vec{I}_1 = 0$ (open CKT) , $I_2 = 1\angle 0^\circ mA$

plug in ① & ②

$$0 = 2.5 \, \vec{V}_1 - 0.5 \, \vec{V}_2 \quad \dots ①'$$

$$1 = 39 \, \vec{V}_1 + 0.6 \, \vec{V}_2 \quad \dots ②'$$

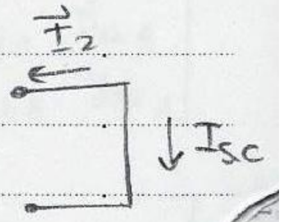
solve for V_1 & V_2

$$\vec{V}_2 = 0.1190 \, V$$

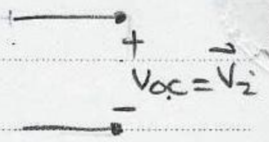
$$\vec{V}_1 = 0.024 \, V$$

$$Z_{out} = \frac{\vec{V}_2}{\vec{I}_2} = 0.1190 \ K\Omega$$

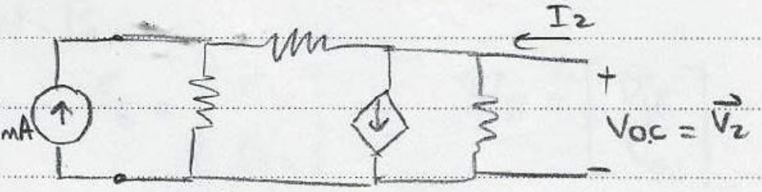
OR 2nd method:- $Z_{out} = Z_{th} = \frac{\vec{V}_{2\,o.c}}{\vec{I}_{2\,s.c}}$



$I_{sc} = -I_2$



assume a test current source $\vec{I}_1 = 1m\angle 0^\circ$ A



$I_1 = 1\angle 0^\circ$ mA
$I_2 = 0$

plug in ①&②

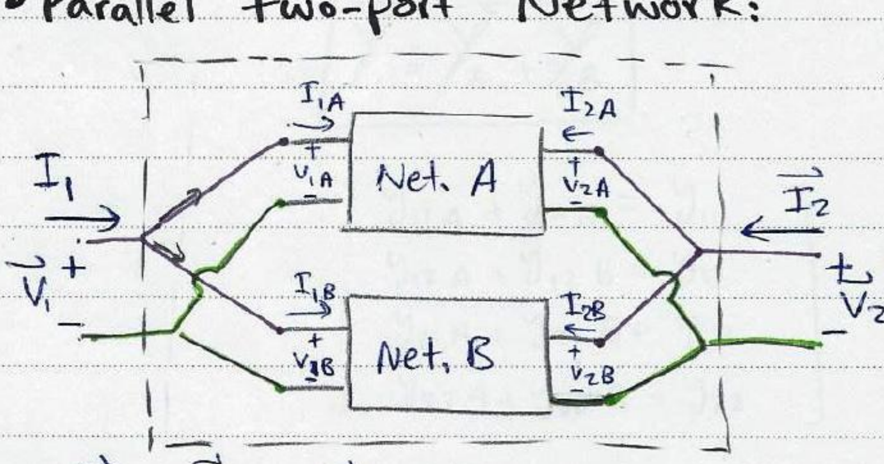
$$1 = 2.5\vec{V}_1 - 0.5\vec{V}_{2o.c}$$
$$0 = 39\vec{V}_1 + 0.6\vec{V}_{2o.c}$$
$$\downarrow$$
$$\boxed{V_{2o.c} = -1.857 \text{ V}}$$

$I_{2\,sc}$? $V_2 = 0$ (s.c)

$$1 = 2.5 V_1 - 0$$
$$I_{2sc} = 39m V_1 + 0 \Rightarrow \boxed{I_{2\,s.c} = 15.6 \text{ mA}}$$

$$\vec{Z}_{out} = \frac{\vec{V}_{2o.c}}{\vec{I}_{2sc}} = \frac{-1.857}{15.6m} = 0.1190 \text{ K}\Omega \text{ ✱}$$

• Parallel two-port Network:



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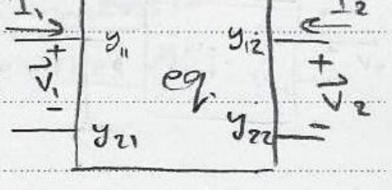


equivelant y-paramete

$$\mathbf{Y} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

- $\vec{V}_1 = \vec{V}_{1B} = \vec{V}_{1A}$
- $\vec{V}_2 = \vec{V}_{2A} = \vec{V}_{2B}$

] because the are parallel

- $\vec{I}_1 = \vec{I}_{1A} + \vec{I}_{1B}$
- $\vec{I}_2 = \vec{I}_{2A} + \vec{I}_{2B}$

- for net. A :-

$$\mathbb{I}_A = \begin{bmatrix} \vec{I}_{1A} \\ \vec{I}_{2A} \end{bmatrix}, \quad \mathbb{V}_A = \begin{bmatrix} \vec{V}_{1A} \\ \vec{V}_{2A} \end{bmatrix}, \quad \mathbb{Y}_A = \begin{bmatrix} y_{11A} & y_{12A} \\ y_{21A} & y_{22A} \end{bmatrix}$$

- for net. B :-

$$\mathbb{I}_B = \begin{bmatrix} \vec{I}_{1B} \\ \vec{I}_{2B} \end{bmatrix}, \quad \mathbb{V}_B = \begin{bmatrix} \vec{V}_{1B} \\ \vec{V}_{2B} \end{bmatrix}, \quad \mathbb{Y}_B = \begin{bmatrix} y_{11B} & y_{12B} \\ y_{21B} & y_{22B} \end{bmatrix}$$

- for the overall network :-

$$\mathbb{V} = \begin{bmatrix} \vec{V}_1 \\ \vec{V}_2 \end{bmatrix}, \quad \mathbb{I} = \begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \end{bmatrix}$$

$$\mathbb{I}_A = \mathbb{Y}_A \mathbb{V}_A = \mathbb{Y}_A \textcircled{\mathbb{V}} \rightarrow \mathbb{V}_A = \mathbb{V}_B = \mathbb{V}$$

$$\mathbb{I}_B = \mathbb{Y}_B \mathbb{V}_B = \mathbb{Y}_B \mathbb{V}$$

$$\mathbb{I} = \mathbb{I}_A + \mathbb{I}_B = \mathbb{Y}_A \mathbb{V} + \mathbb{Y}_B \mathbb{V}$$

$$= \mathbb{V} \underline{(\mathbb{Y}_A + \mathbb{Y}_B)} \equiv \mathbb{Y}$$

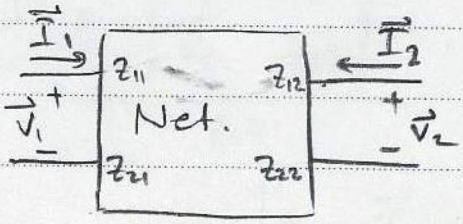
$$\therefore \boxed{\mathbb{Y} = \mathbb{Y}_A + \mathbb{Y}_B}$$

$$\therefore \left.\begin{aligned} y_{11A} + y_{11B} &= y_{11} \\ y_{12A} + y_{12B} &= y_{12} \\ y_{21A} + y_{21B} &= y_{21} \\ y_{22A} + y_{22B} &= y_{22} \end{aligned}\right\}$$

لأنه عم نساوي
ال matrices!

II. Impedance (Z) Parameters:

(open CKT impedance Parameters).



← from mesh analysis

$$\boxed{\begin{aligned}\vec{V}_1 &= Z_{11}\vec{I}_1 + Z_{12}\vec{I}_2 \\ \vec{V}_2 &= Z_{21}\vec{I}_1 + Z_{22}\vec{I}_2\end{aligned}}$$ ...① ...②

matrices!

$$\mathbb{V} = \mathbb{Z}\,\mathbb{I}$$

; $\mathbb{V} = \begin{bmatrix}\vec{V}_1 \\ \vec{V}_2\end{bmatrix}$, $\mathbb{I} = \begin{bmatrix}\vec{I}_1 \\ \vec{I}_2\end{bmatrix}$, $\mathbb{Z} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22}\end{bmatrix}$

from ①

- $Z_{11} = \left.\dfrac{\vec{V}_1}{\vec{I}_1}\right|_{I_2=0}$

o.c. that's why it's called o.c Z-parameters.

- $Z_{12} = \left.\dfrac{\vec{V}_1}{\vec{I}_2}\right|_{I_1=0}$ o.c

- $Z_{22} = \left.\dfrac{\vec{V}_2}{\vec{I}_2}\right|_{I_1=0}$ o.c

- $Z_{21} = \left.\dfrac{\vec{V}_2}{\vec{I}_1}\right|_{I_2=0}$

* transformation from Z → Y parameters:

using cramer:

1. $$\vec{I}_1 = \frac{\begin{vmatrix} \vec{V}_1 & Z_{12} \\ \vec{V}_2 & Z_{22} \end{vmatrix}}{\Delta Z} = \frac{Z_{22}\vec{V}_1}{Z_{11}Z_{22}-Z_{21}Z_{12}} - \frac{Z_{12}\vec{V}_2}{Z_{11}Z_{22}-Z_{12}Z_{21}}$$

$Z_{22} \equiv \Delta_{11}$ $\quad$ $Z_{12} \equiv \Delta_{21}$

$$\hookrightarrow \vec{I}_1 = \underbrace{\left(\frac{Z_{22}}{\Delta Z}\right)}_{y_{11}}\vec{V}_1 \underbrace{\left(-\frac{Z_{12}}{\Delta Z}\right)}_{y_{12}}\vec{V}_2 \quad \dots ①$$

- $y_{11} = \frac{\Delta_{11}}{\Delta Z} \equiv \frac{Z_{22}}{\Delta Z}$ —ⓐ
- $y_{12} = -\frac{\Delta_{21}}{\Delta Z} = \frac{-Z_{12}}{\Delta Z}$ —ⓑ

2. $$\vec{I}_2 = \frac{\begin{vmatrix} \vec{Z}_{11} & \vec{V}_1 \\ Z_{21} & \vec{V}_2 \end{vmatrix}}{\Delta Z} = \underbrace{\left(\frac{-Z_{21}}{\Delta Z}\right)}_{y_{21}}\vec{V}_1 + \underbrace{\left(\frac{Z_{11}}{\Delta Z}\right)}_{y_{22}}\vec{V}_2 \quad \dots ②$$

- $y_{21} = \frac{-Z_{21}}{\Delta Z} = -\frac{\Delta_{12}}{\Delta Z}$ —ⓒ
- $y_{22} = \frac{Z_{11}}{\Delta Z} = \frac{\Delta_{22}}{\Delta Z}$ —ⓓ

* transform from Z → Y parameters:

I have y-parameters:

$$I_1 = y_{11}\vec{V}_1 + y_{12}\vec{V}_2 \quad \dots ①$$
$$\vec{I}_2 = y_{21}\vec{V}_1 + y_{22}\vec{V}_2 \quad \dots ②$$

$$\vec{V}_1 = \frac{\begin{vmatrix} \vec{I}_1 & y_{12} \\ \vec{I}_2 & y_{22} \end{vmatrix}}{\Delta y} \quad \& \quad \vec{V}_2 = \frac{\begin{vmatrix} y_{11} & \vec{I}_1 \\ y_{21} & \vec{I}_2 \end{vmatrix}}{\Delta y}$$

∴ • $Z_{11} = \frac{y_{22}}{\Delta y} = \frac{\Delta_{11}}{\Delta y}$ ...ⓐ • $Z_{21} = -\frac{y_{21}}{\Delta y} = -\frac{\Delta_{12}}{\Delta y}$ ...ⓒ

• $Z_{12} = \frac{-y_{12}}{\Delta y} = \frac{-\Delta_{21}}{\Delta y}$ ...ⓑ • $Z_{22} = +\frac{y_{11}}{\Delta y} = \frac{\Delta_{11}}{\Delta y}$ ...ⓓ

if it's a bilateral CKT → $Z_{12} = Z_{21}$!

/ /

* Equivelant CKT : (Z-parameters)

$$\vec{V}_1 = Z_{11}\vec{I}_1 + Z_{12}\vec{I}_2 \quad \ldots ①$$

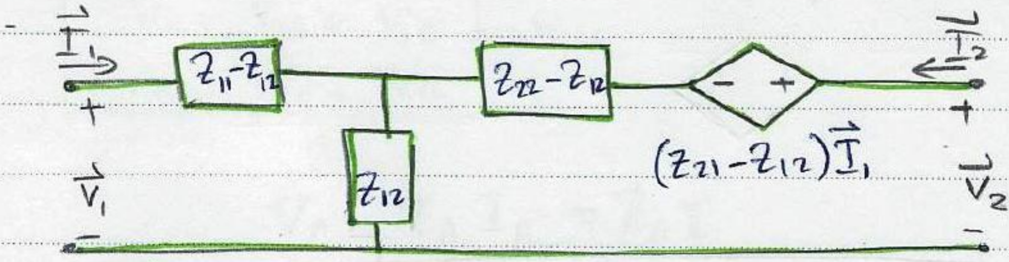
$$\vec{V}_2 = Z_{21}\vec{I}_1 + Z_{22}\vec{I}_2 \quad \ldots ②$$

- 1st equivelant CKT:-
  - add & subtract $(Z_{12}\vec{I}_1)$ from ②

$$\vec{V}_1 = Z_{11}\vec{I}_1 + Z_{12}\vec{I}_2 \quad \ldots ①'$$ stays the same.

$$\vec{V}_2 = Z_{12}\vec{I}_1 + Z_{22}\vec{I}_2 + (Z_{21} - Z_{12})\vec{I}_1 \quad \ldots ②'$$

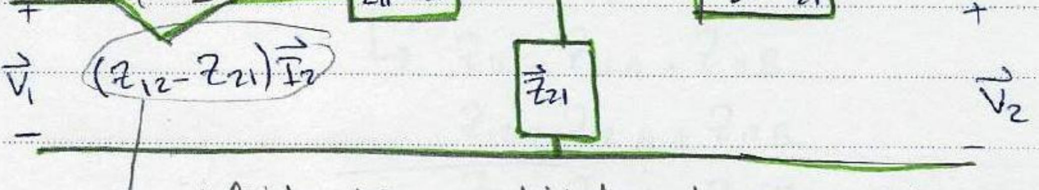
from mesh analysis:-



- 2nd equivelant CKT:-
  - add & subtract $(Z_{21}\vec{I}_2)$ from ①:-

$$\vec{V}_1 = Z_{11}\vec{I}_1 + Z_{12}\vec{I}_2 + (Z_{12} - Z_{21})\vec{I}_2 \quad \ldots ①'$$

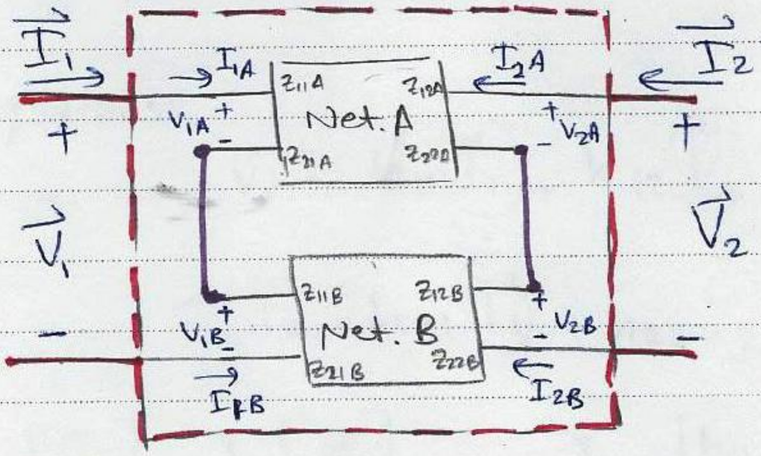
$$\vec{V}_2 = Z_{21}I_1 + Z_{22}\vec{I}_2 \quad \ldots ②'$$ stays the same

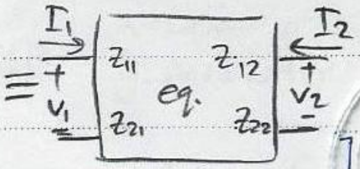


↳ if it is a bilateral CKT, this source will be short-circuited watch out !

series ≠ cascade

* Series 2Port network:





$$\boxed{\begin{array}{l} I_{1A} = I_{1B} = I_1 \\ I_{2A} = I_{2B} = I_2 \\ V_1 = V_{1A} + V_{1B} \quad \text{---} \circledast \\ V_2 = V_{2A} + V_{2B} \end{array}}$$

→ in series [illegible]

$$\mathbb{V}_A = \mathbb{Z}_A \mathbb{I}_A = \mathbb{Z}_A \mathbb{I}$$

$$\mathbb{V}_B = \mathbb{Z}_B \mathbb{I}_B = \mathbb{Z}_B \mathbb{I}$$

$$\mathbb{V} = \mathbb{V}_A + \mathbb{V}_B \qquad \text{from } \circledast$$

$$\mathbb{V} = \mathbb{I} \underbrace{(\mathbb{Z}_A + \mathbb{Z}_B)}_{\mathbb{Z}}$$

$$\boxed{\mathbb{Z}_{total} = \mathbb{Z}_A + \mathbb{Z}_B}$$

↳ $Z_{11} = Z_{11A} + Z_{11B}$

$Z_{12} = Z_{12A} + Z_{12B}$

$Z_{21} = Z_{21A} + Z_{21B}$

$Z_{22} = Z_{22A} + Z_{22B}$

III. Hybrid Parameters (h)

حلّي I_1 فيطلعلي قانون

$$I_1 = \frac{1}{h_{11}}\vec{V}_1 - \frac{h_{12}}{h_{11}}\vec{V}_2$$

كأني مشابهه y-parameters

- equations:

$$\vec{V}_1 = h_{11}\vec{I}_1 + h_{12}\vec{V}_2 \quad \dots ①$$

$$\vec{I}_2 = h_{21}\vec{I}_1 + h_{22}\vec{V}_2 \quad \dots ②$$

→ like the z-paramet.

$$\begin{bmatrix} \vec{V}_1 \\ \vec{I}_2 \end{bmatrix} = h \begin{bmatrix} \vec{I}_1 \\ \vec{V}_2 \end{bmatrix} \quad , \quad h = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$

← تحسين ال Amplifier !

- $h_{11} = \left.\frac{\vec{V}_1}{\vec{I}_1}\right|_{V_2=0}$; Short CKT input impedance $= h_i$

  Output S.C

- $h_{21} = \left.\frac{\vec{I}_2}{\vec{I}_1}\right|_{V_2=0}$; $\overset{\frac{Out}{In}}{\text{Forward}}$ Current Gain (G_I) $= h_f$

  Output S.C

- $h_{12} = \left.\frac{\vec{V}_1}{\vec{V}_2}\right|_{I_1=0}$; Open CKT $\overset{\frac{in}{out}}{\text{reverse}}$ Voltage Gain $= h_r$

- $h_{22} = \left.\frac{\vec{I}_2}{\vec{V}_2}\right|_{I_1=0}$; Open CKT output admittance $= h_o$

* **NOTE:** for bilateral CKTs : $\boxed{h_{12} = -h_{21}}$

* transform from h → Z:

my (h) equations:

$$\boxed{\begin{aligned}\vec{V}_1 &= h_{11}\vec{I}_1 + h_{12}\vec{V}_2 \\ \vec{I}_2 &= h_{21}\vec{I}_1 + h_{22}\vec{V}_2\end{aligned}} \quad \begin{aligned}&\ldots ① \\ &\ldots ②\end{aligned}$$

from ②:

$$\vec{V}_2 = \underbrace{\left(\frac{-h_{21}}{h_{22}}\right)}_{z_{21}} I_1 + \underbrace{\left(\frac{1}{h_{22}}\right)}_{z_{22}} \vec{I}_2 \quad \ldots ②'$$

substitute $\overset{\vec{V}_2}{\uparrow}$ in ①

$$\vec{V}_1 = h_{11}\vec{I}_1 + \frac{h_{12}}{h_{22}}\left(I_2 - h_{21}\vec{I}_1\right)$$

$$\vec{V}_1 = \underbrace{\left(h_{11} - \frac{h_{12}h_{21}}{h_{22}}\right)}_{z_{11}} \vec{I}_1 + \underbrace{\left(\frac{h_{12}}{h_{22}}\right)}_{z_{12}} I_2$$

* transform from h → Y:

You can do it h→Z then Z→Y

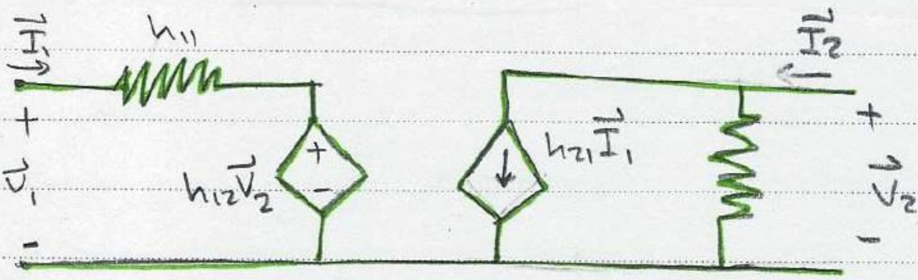
from ① $\vec{I}_1 = \underbrace{\left(\frac{1}{h_{11}}\right)}_{y_{11}} V_1 + \underbrace{\left(\frac{-h_{12}}{h_{11}}\right)}_{y_{12}} \vec{V}_2$

plug $\vec{I}_1$ in ②:

$$I_2 = h_{21}\left(\frac{1}{h_{11}}\vec{V}_1 - \frac{h_{12}}{h_{11}}\vec{V}_2\right) + h_{22}\vec{V}_2$$

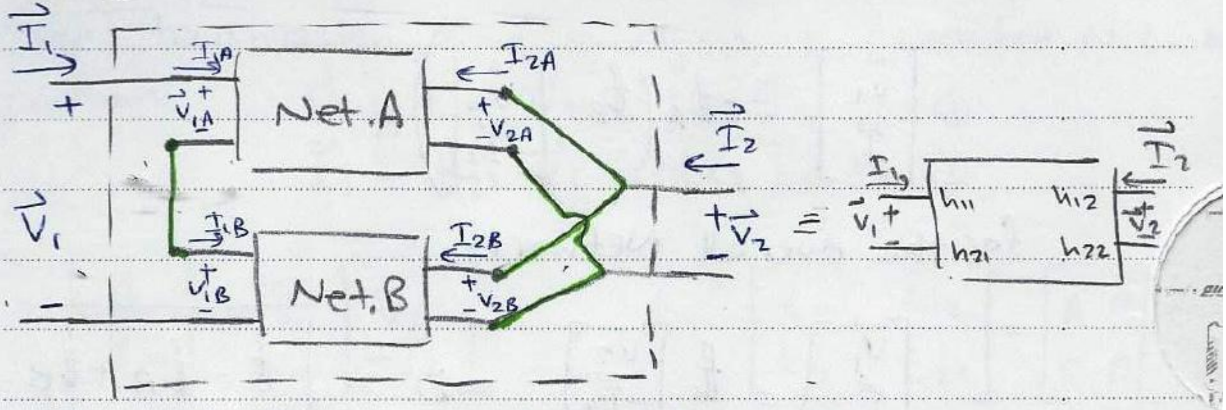
$$\vec{I}_2 = \underbrace{\left(\frac{h_{21}}{h_{11}}\right)}_{y_{21}} \vec{V}_1 + \underbrace{\left(\frac{-h_{21}h_{12}}{h_{11}} + h_{22}\right)}_{y_{22}} \vec{V}_2$$

* equivelant CKT:



→ take two meshes to proove their equivelance.

* Series - Parallel Interconnection :



$\vec{V}_1 = \vec{V}_{1A} + \vec{V}_{1B}$, $\vec{I}_1 = \vec{I}_{1A} = \vec{I}_{1B}$

$\vec{V}_2 = \vec{V}_{2A} + \vec{V}_{2B}$, $\vec{I}_2 = \vec{I}_{2A} + \vec{I}_{2B}$

$$\boxed{h_{overall} = h_A + h_B}$$

proof:-

- for Net. A $\begin{bmatrix}\vec{V}_{1A}\\ \vec{I}_{2A}\end{bmatrix} = h_A \begin{bmatrix}\vec{I}_{1A}\\ \vec{V}_{2A}\end{bmatrix} = h_A \begin{bmatrix}\vec{I}_1\\ \vec{V}_2\end{bmatrix}$ ....ⓐ

- for Net. B $\begin{bmatrix}\vec{V}_{1B}\\ \vec{I}_{2B}\end{bmatrix} = h_B \begin{bmatrix}\vec{I}_{1B}\\ \vec{V}_{2B}\end{bmatrix} = h_B \begin{bmatrix}\vec{I}_1\\ \vec{V}_2\end{bmatrix}$ ---ⓑ

- for overall Net. $\begin{bmatrix}\vec{V}_1\\ \vec{I}_2\end{bmatrix} = h \begin{bmatrix}\vec{I}_1\\ \vec{V}_2\end{bmatrix}$ ---⊛

plug ⓐ & ⓑ

$$\begin{bmatrix}\vec{V}_{1A}+\vec{V}_{1B}\\ \vec{I}_{2A}+\vec{I}_{2B}\end{bmatrix} = \begin{bmatrix}V_{1A}\\ I_{2A}\end{bmatrix} + \begin{bmatrix}\vec{V}_{1B}\\ \vec{I}_{2B}\end{bmatrix}$$

$$= (h_A + h_B)\begin{bmatrix}\vec{I}_1\\ \vec{V}_1\end{bmatrix}$$

from ⊛:

$$\boxed{h = h_A + h_B}$$ ##

/ /

IV Transmission Parameters: (t $\overset{or}{/}$ ABCD - Parameters)

⇒ Usage: transmission line analysis & cascaded Networks

equations:- $\vec{V}_1 = t_{11}\ \vec{V}_2 - t_{12}\vec{I}_2$ ... ①

$\vec{I}_1 = t_{21}\ \vec{V}_2 - t_{22}\ \vec{I}_2$ ... ②

$$\begin{bmatrix}\vec{V}_1\\ \vec{I}_1\end{bmatrix} = t\begin{bmatrix}\vec{V}_2\\ -\vec{I}_2\end{bmatrix} \quad , \quad t = \begin{bmatrix}t_{11} & t_{12}\\ t_{21} & t_{22}\end{bmatrix} = \begin{bmatrix}A & B\\ C & D\end{bmatrix}$$

- $t_{11} = \dfrac{\vec{V}_1}{\vec{V}_2}\Big|_{I_2=0}$; open circuit Output
- $t_{21} = \dfrac{\vec{I}_1}{\vec{V}_2}\Big|_{I_2=0}$; O.C output
- $t_{12} = -\dfrac{\vec{V}_1}{\vec{I}_2}\Big|_{V_2=0}$; short CKT output
- $t_{22} = \dfrac{-I_1}{I_2}\Big|_{V_2=0}$; S.C o/p

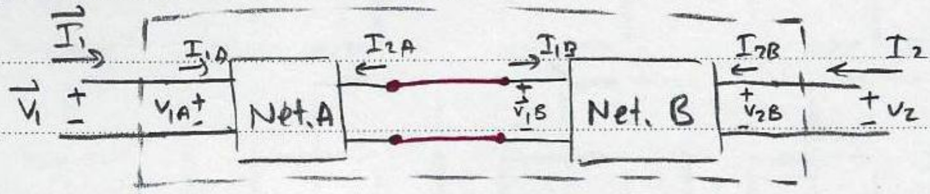
ما بهمني غير ال output!

*** NOTE:-** for bilateral Networks:-

$$\Delta t = t_{11}t_{22} - t_{12}t_{21} = 1$$

* this Net. doesn't have an equivelant CKT !! :)

* Cascaded Networks:-



in the input side:- $\vec{I}_1 = \vec{I}_{1A}$, $\vec{V}_1 = \vec{V}_{1A}$

" " output " :- $\vec{I}_2 = \vec{I}_{2B}$, $\vec{V}_2 = \vec{V}_{2B}$

" " middle :- $\vec{V}_{2A} = \vec{V}_{1B}$, $I_{2A} = -I_{1B}$

* I'm looking for the overall t :

$\boxed{t = t_A \cdot t_B}$ be careful! it's matrix Multiplication

$t_A \cdot t_B \neq t_B \cdot t_A$

for Net. A: $\begin{bmatrix}\vec{V}_1\\ \vec{I}_1\end{bmatrix} = t_A\begin{bmatrix}\vec{V}_{2A}\\ -\vec{I}_{2A}\end{bmatrix} = t_A\begin{bmatrix}\vec{V}_{1B}\\ \vec{I}_{1B}\end{bmatrix}$ ... ⓐ

for Net. B: $\begin{bmatrix}\vec{V}_{1B}\\ I_{1B}\end{bmatrix} = t_B\begin{bmatrix}\vec{V}_{2B}\\ -\vec{I}_{2B}\end{bmatrix} = t_B\begin{bmatrix}\vec{V}_2\\ -\vec{I}_2\end{bmatrix}$ ... ⓑ

→ plug ⓑ in ⓐ :-

$$\begin{bmatrix}\vec{V}_1\\ \vec{I}_1\end{bmatrix} = t_A\left(t_B\begin{bmatrix}\vec{V}_2\\ -\vec{I}_2\end{bmatrix}\right)$$

for the overall Network:-

$$\begin{bmatrix}\vec{V}_1\\ \vec{I}_1\end{bmatrix} = t\begin{bmatrix}\vec{V}_2\\ -\vec{I}_2\end{bmatrix} \Longrightarrow \therefore \boxed{t = t_A + t_B} \ \#$$

* transform $t \rightarrow Z$

$$\vec{V}_1 = t_{11}\vec{V}_2 - t_{12}\vec{I}_2 \quad —①$$

$$\vec{I}_1 = t_{21} V_2 - t_{22}\vec{I}_2 \quad --②$$

from ② :

$$\vec{V}_2 = \overset{Z_{21}}{\left(\frac{1}{t_{21}}\right)} I_1 + \overset{Z_{22}}{\left(\frac{t_{22}}{t_{21}}\right)} \vec{I}_2$$

sub. V_2 in ①.

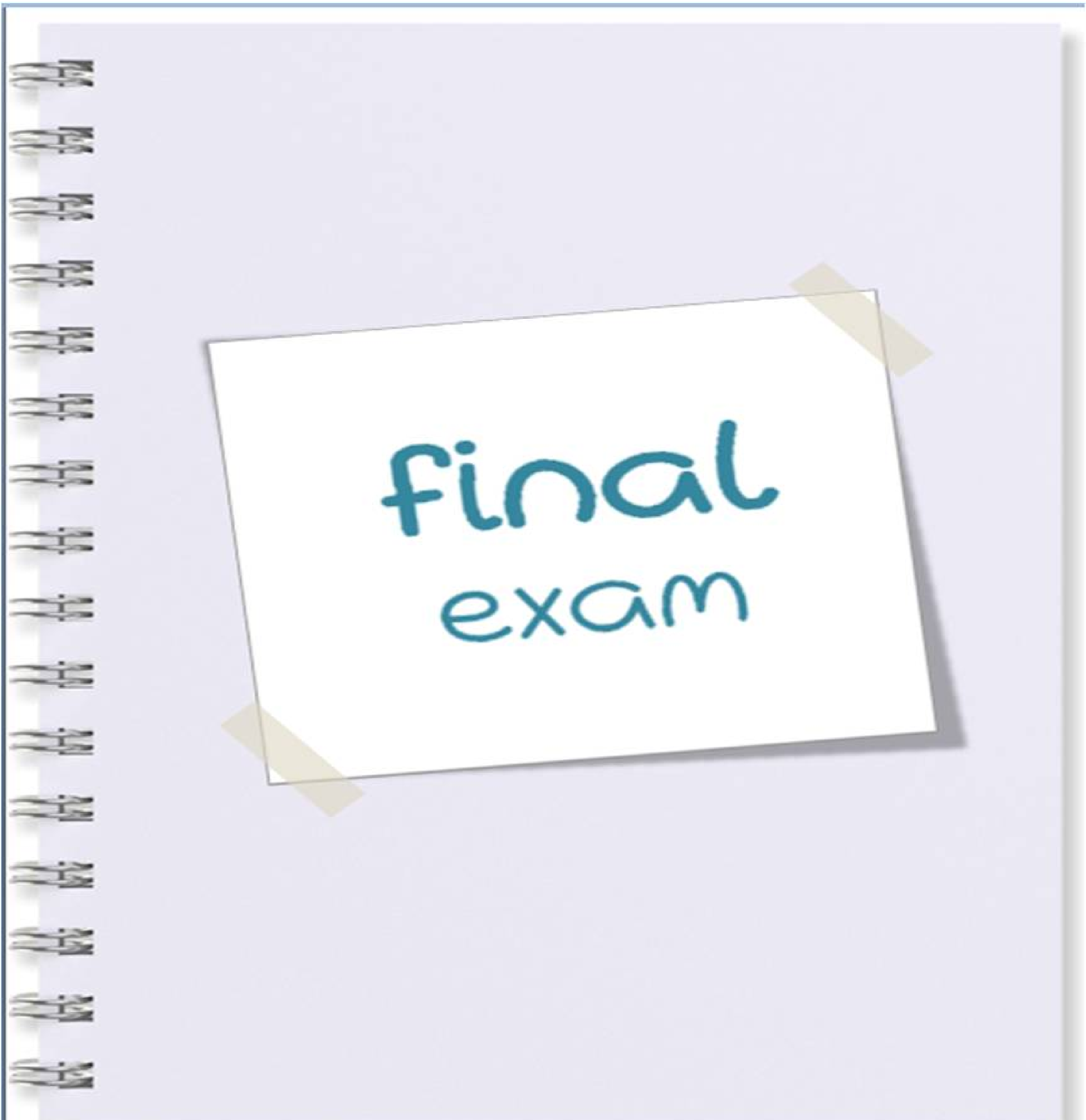
$$\vec{V}_1 = \overset{Z_{11}}{\left(\frac{t_{11}}{t_{21}}\right)} \vec{I}_1 + \overset{Z_{12}}{\left(\frac{t_{11}t_{22}}{t_{21}} - t_{12}\right)} \vec{I}_2$$

now I can do

$t \rightarrow y$; $t \rightarrow Z$ then $Z \rightarrow y$

$t \rightarrow h$; $t \rightarrow Z$ then $Z \rightarrow h$

** مهم تعرف تحول من parameter لآخر مهما كانت ال CKT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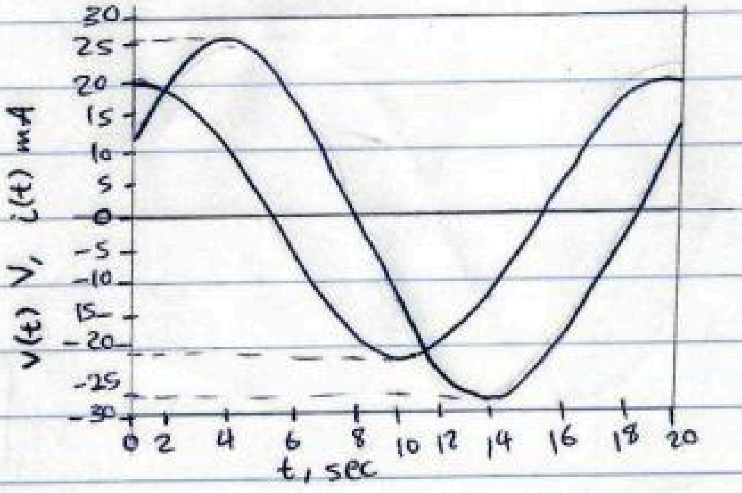


Figure: Q1

Question 1: In Fig. Q1:

a. The minimum power P_{min} =

b. The average power P =

c. The reactive power Q =

d. The peak power P_{max} =

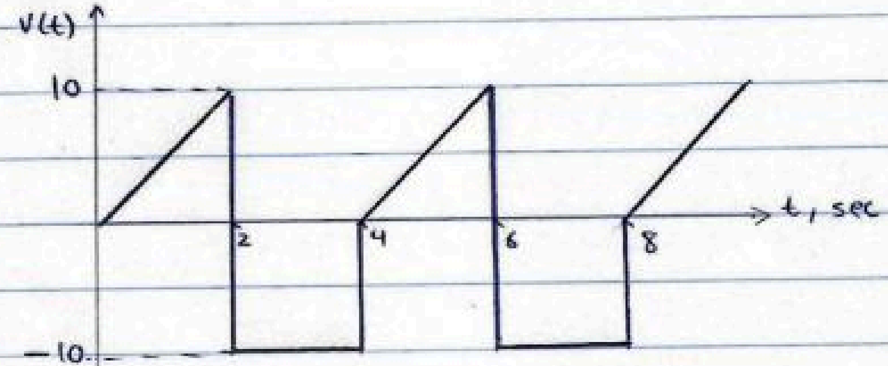


Figure : Q2

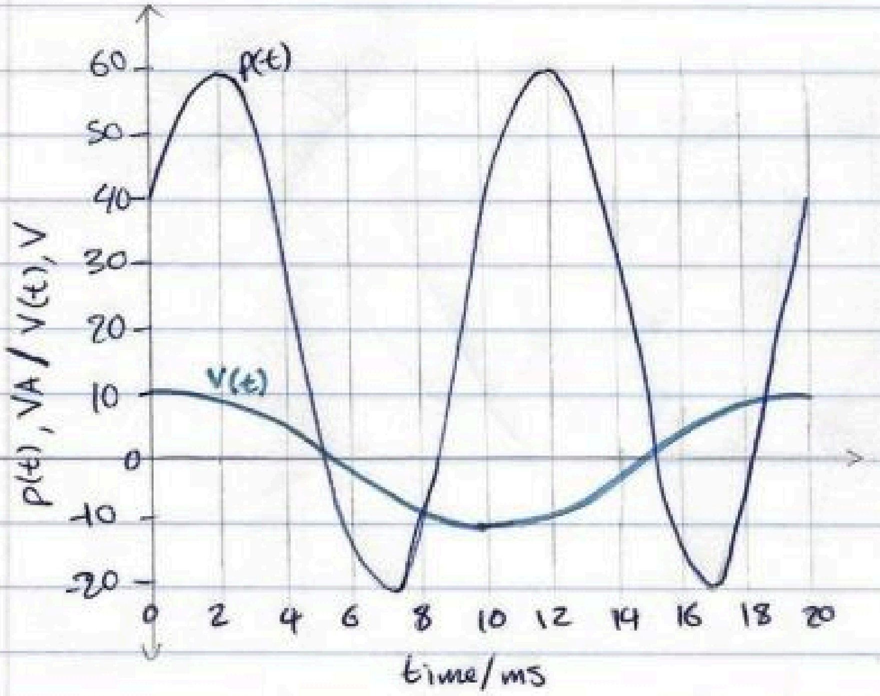
Question 2: The RMS value of the signal shown in Fig. Q2 is:

a. 86.7 V.

b. 8.166 V.

c. 8.615 V.

d. 6.867 V.



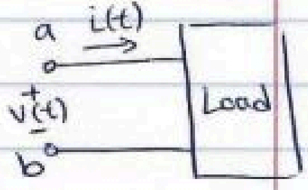


Figure: Q3

Question 3: If the load is composed of two elements in series, then the current i(t) flows to the load is:

a. $8\cos(100\pi t - 60^\circ)$

b. $15\cos(100\pi t + 30^\circ)$

c. $10\cos(100\pi t - 30^\circ)$

d. $12\cos(100\pi t + 60^\circ)$

Question 7: Fig.Q7, shows the instantanoous power $p(t)$, where the voltage is given in the form

$$v(t) = 100 \cos(2\pi f t + 0^\circ)\ V.$$

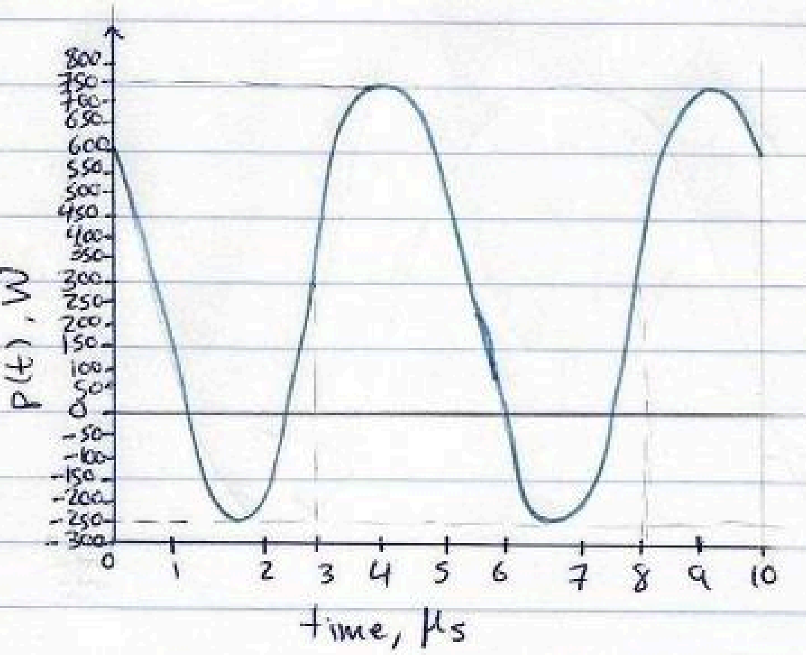


Figure: Q7

The frequency of the voltage f is:-

a. 10 KHz

b. 200 KHz

c. 50 KHz

d. 100 KHz

One of the numerical expressions of the instantaneous power $p(t)$ is wrong:-

a. $400 - 500 \cos 143.13^\circ \cos 2\omega t + 500 \sin 143.13^\circ \sin 2\omega t$

b. none of these

c. $400 - 500 \cos(2\omega t + 143.13^\circ)$

d. $400 + 500 \cos(2\omega t - 36.87^\circ)$

e. $400 + 400 \cos(2\omega t) + 300 \sin(2\omega t)$

Question 7:- ~~In~~ Fig. Q 7 shows the instantaneous power p(t) absorbed by the load & the voltage v(t) applied to it. The load composed of 2 elements in series. The current i(t) is:-

a. $15 \cos(100\pi t + 30^\circ)$

b. none of these.

c. $12 \cos(100\pi t + 60^\circ)$

d. $8 \cos(100\pi t - 60^\circ)$

e. $10 \cos(100\pi t - 30^\circ)$

Question 5:- a Three-phase balanced voltage source feeds the load shown in Fig. Q5, the $V_{AB} = 400 \angle 30^\circ$. The $|Z_{AB}| = |\vec{Z}_{BC}| = |\vec{Z}_{CA}|$ & the three-phase load absorbs 4000 W.

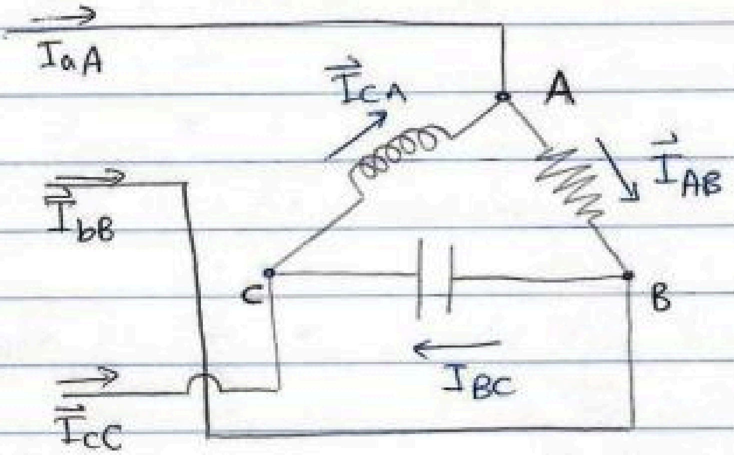


Figure: Q5

The line current $\vec{I}_{aA}$ is:

a. none of these

b. $19.32 \angle -45^\circ$ A

c. $10.0 \angle -30^\circ$ A

d. $19.32 \angle 45^\circ$ A

e. $10.0 \angle -45.0^\circ$ A

Q: A three-phase balanced voltage source feeds the load shown in fig. Q4 , $|Z_Y| = 5\angle 30^\circ\ \Omega$, $V_{AN} = 400\angle 60^\circ\ V_{rms}$

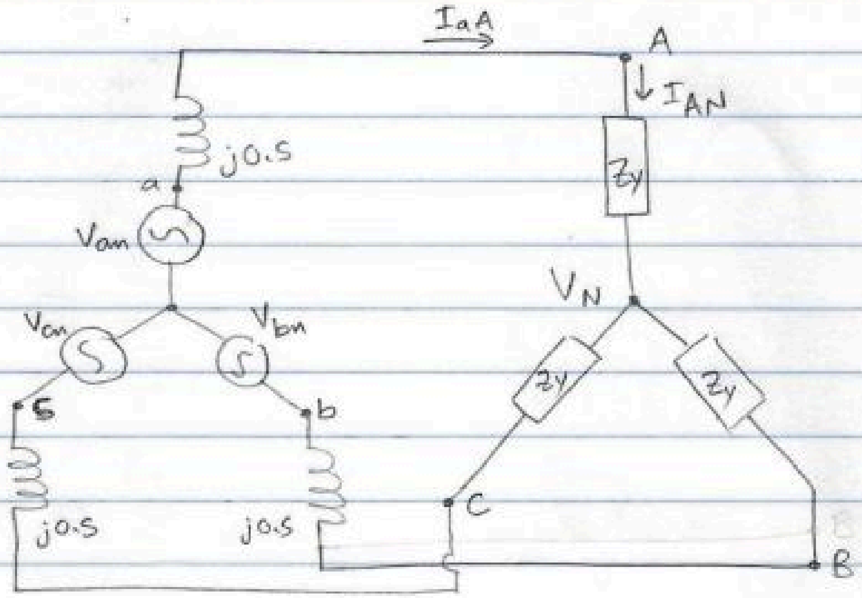


Figure: Q4

The $V_{bn} \simeq$:

a. $421.43\angle -55.3^\circ$ A

b. $252.0\ \angle -98$ A.

c. $250.32\angle -30^\circ$ A

d. $267\ 5.0\ \angle -132^\circ$ A

e. $243.73\ \angle -186^\circ$ A

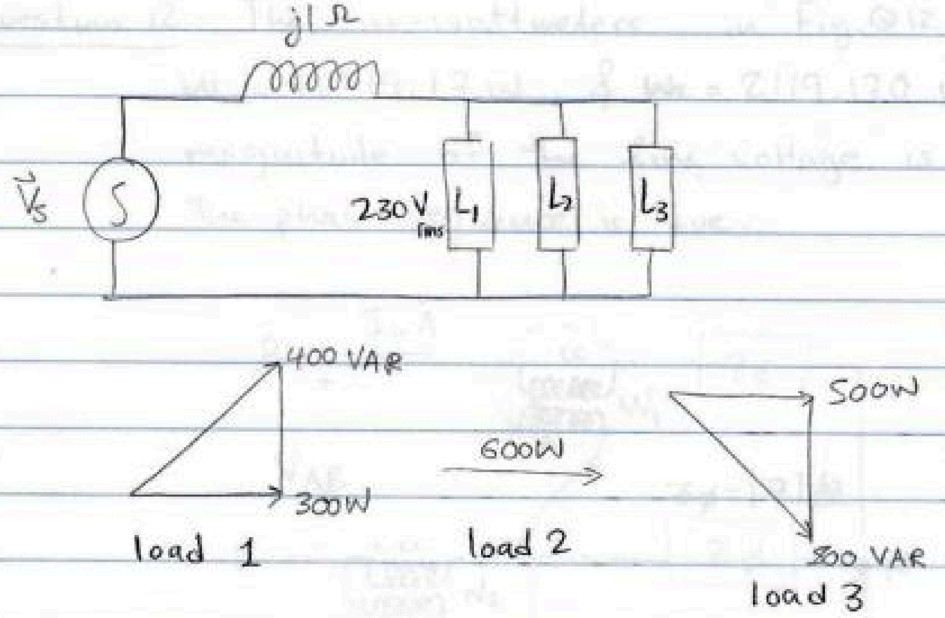


Figure: Q11

<u>Question 11</u>:- In fig. Q 11, the reactive power delivered by the source to the network is:-

a. j440 VAR
b. j359.9 VAR
c. none of these
d. j35.59 VAR
e. -j359.9 VAR

Question 12: The two-wattmeters in Fig. Q12, read as: $W_1 = 16519.17$ W, & $W_2 = 2119.170$ W. The magnitude of the line voltage is 415 V_{rms}. The phase sequence is +ve.

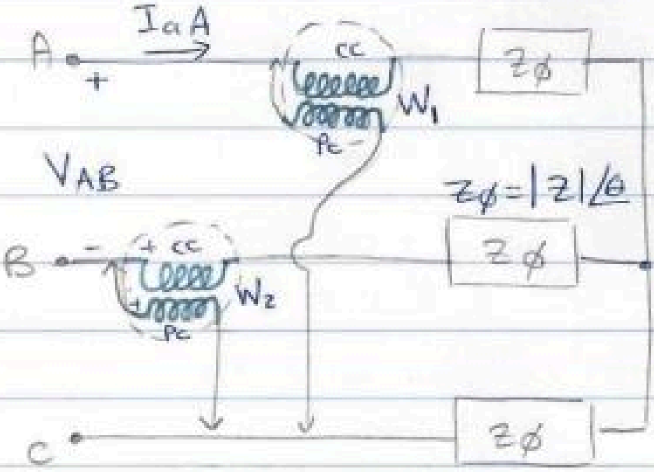


Figure: Q12

The power factor of the load P.F is:-

a. none of these.

b. 0.9823 Lagging

c. 0.866 Lagging

d. 0.8923 Lagging

e. 0.5986 Lagging

Question 8: In Fig. Q12, The line Voltage $V_{AB} = 416\ V_{rms}$ & possitive sequence, The load impedance $Z_\phi = 5\angle 45°$

The two-wattmeters read as:-

a. $W_1 = 2781.6$ W & $W_2 = 11172.5$ W

b. $W_1 = 2981.6$ W & $W_2 = 16519.17$ W.

c. $W_1 = 11127.5$ W & $W_2 = 2981.6$ W.

d. None of these

e. $W_1 = 2981.6$ W & $W_2 = 11127.5$ W

Question 9:- In Fig. Q9, the source delivers 7500 VA at $\vec{V}_S = 250 \angle 30^\circ\ V_{rms}$ with power factor of 0.8660205 lagging.

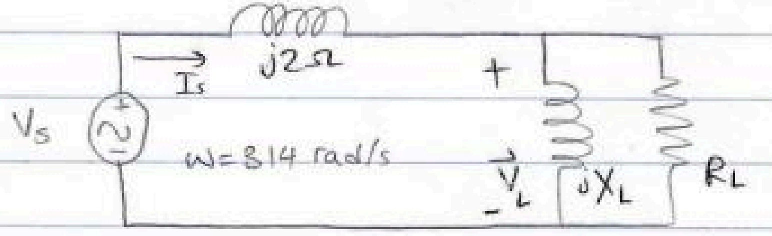


Figure : Q9

The value of X_L is:-

a. 32.38 Ω

b. 18.70 Ω

c. none of these

d. 11.0 Ω

e. 12.47 Ω