Calculus Notebook
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بأفكارنا_نُبدع

Full material

CH. 1

No. Function
اقتران

R → Real number عدد حقيقي

Q → اعداد النسبية retional → $\{\frac{a}{b}, a \neq 0\}$
irretional اعداد غير نسبية → $\{\sqrt{2}, \pi, e\}$

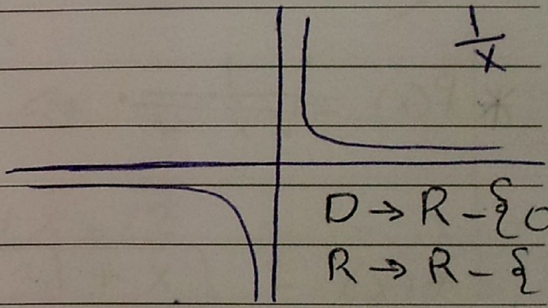
Z → اعداد الصحيحة → $\{\ldots, -3, \cdot, \ldots, 2, \ldots\}$

N → اعداد طبيعية → $\{\cdot, 1, 2, \ldots\}$

$$N \subset Z \subset Q \subset R$$

$\xrightarrow{\hspace{4cm}}$
اكبر احتواء

Domain مجال
Rang مدى



$D \to R - \{0\}$
$R \to R - \{0\}$

$f(x) = \frac{1}{x-1} \Rightarrow R - \{1\}$

$f(x) = \frac{1}{x^2-1} \Rightarrow R - \{1, -1\}$

$f(x) = \sqrt{x} \Rightarrow [\cdot, \infty)$

$f(x) = \sqrt{x-1} \Rightarrow [1, \infty)$

* $f(x) = \frac{1}{\sqrt{x}} \Rightarrow [1, \infty)$

$f(x) = x^2 + 5 \Rightarrow R \quad , (-\infty, \infty)$

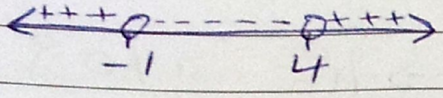
$f(x) = \frac{1}{\sqrt{4-x^2}} \Rightarrow 4 - x^2 = 0 \Rightarrow 4 = x^2 \Rightarrow x = \pm 2$

$\overset{---}{\longleftarrow}\underset{-2}{\circ}\overset{++\;++}{\longleftrightarrow}\underset{2}{\circ}\overset{---}{\longrightarrow}$ $(-2, 2)$

No. fuction

$f(x) = \frac{1}{x^2-3x-4} \Rightarrow x^2-3x-4=0$

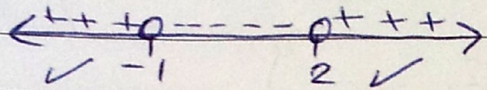
$(x-4)(x+1) \rightarrow x=+4, -1$



$R - \{-1, 4\}$

$f(x) = \frac{1}{\sqrt{x^2-x-2}} \Rightarrow x^2-x-2=0$

$(x-2)(x+1) \quad x = 2, -1$



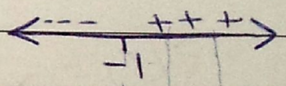
$(-\infty, -1) \cup (2, \infty).$

$f(x) = \frac{1}{x^2+1} \Rightarrow R$

$* f(x) = \frac{x+1}{x^2+x+1} \Rightarrow R$

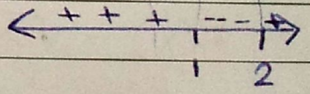
$* f(x) = \frac{1}{\sqrt[3]{x-4}} \Rightarrow R$

$f(x) = \sqrt{\frac{x+1}{x^2-3x+2}} \Rightarrow x+1 \rightarrow$

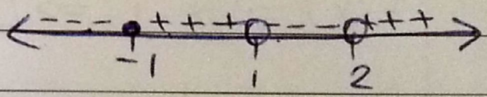


x^2-3x+2

$(x-2)(x-1)$



$[-1, 1) \cup (2, \infty)$



Equtíon معادلة

inequality متباينة

solve حل

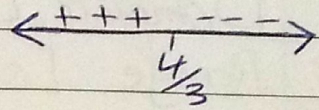
No. # Function

$f(x) = \frac{1}{x-1} > 3 \Rightarrow \frac{1}{x-1} - 3 = 0$

$\frac{1}{x-1} - \frac{3(x-1)}{x-1} = 0$

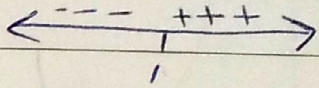
$\frac{1-3x-3}{x-1} = 0 \Rightarrow \frac{4-3x}{x-1} = 0$

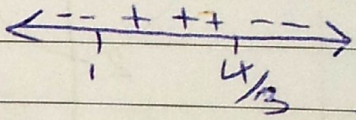
$4-3x \Rightarrow$



$(1, 4/3]$

$x-1 \Rightarrow$

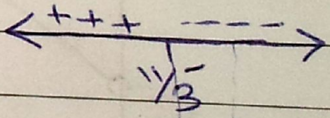




$f(x) = \frac{x+1}{x-3} > 4 \Rightarrow \frac{x+1}{x-3} - 4 = 0$

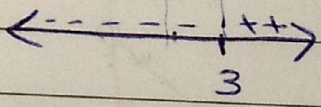
$\frac{x+1-4(x-3)}{x-3} = 0 \Rightarrow \frac{x+1-4x-12}{x-3} \Rightarrow \frac{-3x-11}{x-3} = 0$

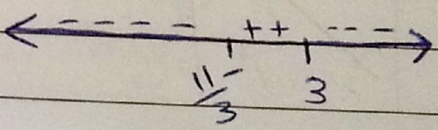
$-3x-11$



$[\frac{11}{3}, 3)$

$x-3$

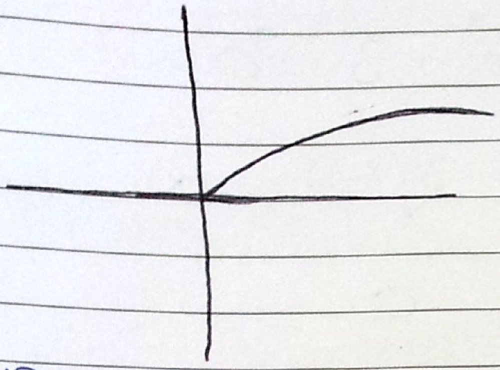


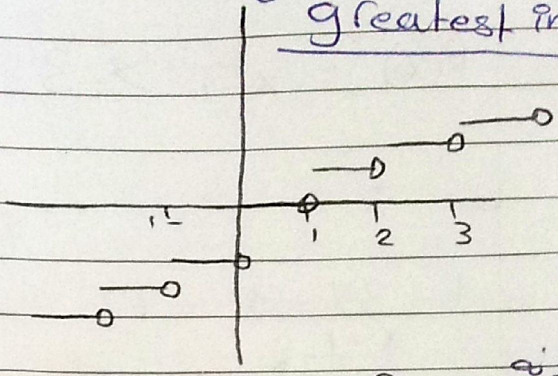


* $[1.01] = 1$
* $[-1.01] -2$

No.

اقتران اكبر عدد صحيح
greatest integ

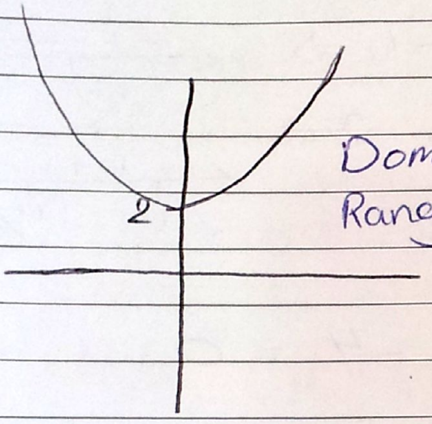




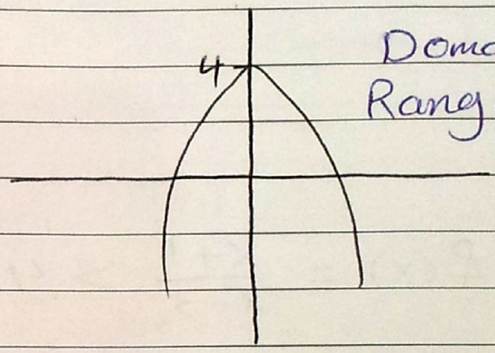
Domain $[0,\infty)$
Range $[0,\infty)$

محور الصادات

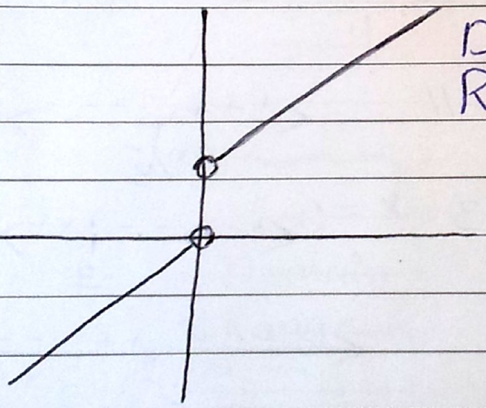
Domain R ⇒ لأنه لم يحدد الاعداد
Range Z



Domain R
Rang $[2,\infty)$



Domain R
Rang $(-\infty,4]$



Domain $R-\{0\}$
Rang $R-\{0,2\}$

* $f(x) = \sqrt{x-2}$
Domain $[2,\infty)$
Range $[0,\infty)$

* $f(x) = \sqrt{4-x^2}$
Domain $[-2,2]$
Range $[0,2]$

* $f(x) = \frac{x-1}{x+3}$

Domain $R-\{-3\}$
Range $R-\{1\}$ ⇒ $\frac{y}{1} = \frac{x-1}{\cancel{3}x+3}$

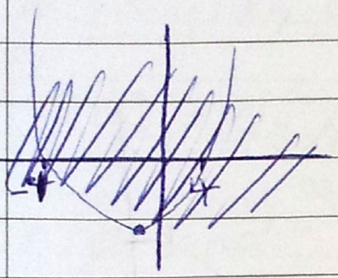
$x-1 = y(x+3)$
$x-1 = yx + y3$
$x = \frac{-1-3y}{y-1} = f(x) = \frac{-1-3x}{x-1}$

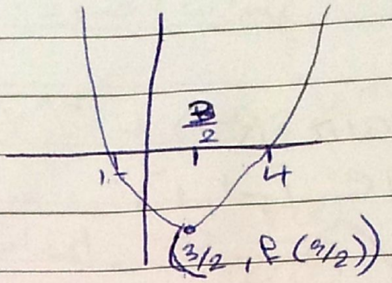
No.

polynomial كثيرات الحدود ⇒ Domain R ⇒ دائماً

Rational function اقترانات نسبية

$* f(x) = x^2 - 3x - 4 \Rightarrow (x-4)(x+1)$ طريقة ①





الرأس هو المسافة بين 4 و 1 ونقسمه على 2

Range $\Rightarrow [f(3/2), \infty)$.

طريقة ② $y = x^2 - 3x - 4$ إكمال مربع

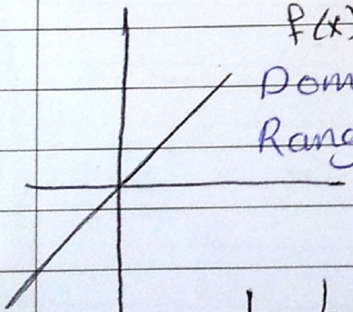
$$y = x^2 - 3x + \frac{9}{4} - \frac{9}{4} - 4$$

$$y = (x - 3/2)^2 - 25/4$$

Range $[-25/4, \infty)$.

~~~~~~~~ ✂

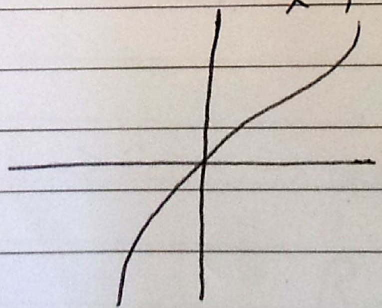
$f(x) = x$



Domain R

Range R

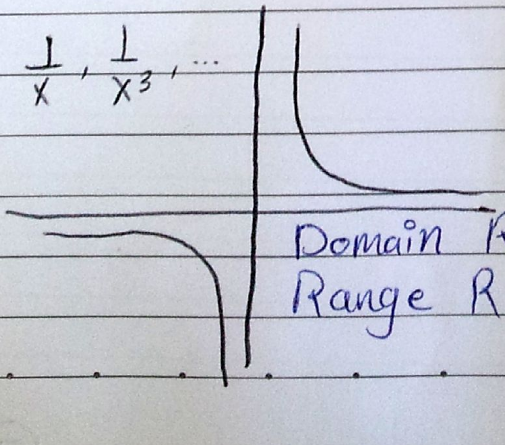
$x^3, x^5, ..$



Domain R

Range R.

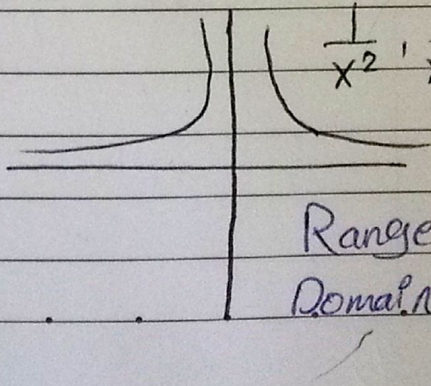
$\frac{1}{x}, \frac{1}{x^3}, ...$



Domain $R - \{0\}$

Range $R - \{0\}$

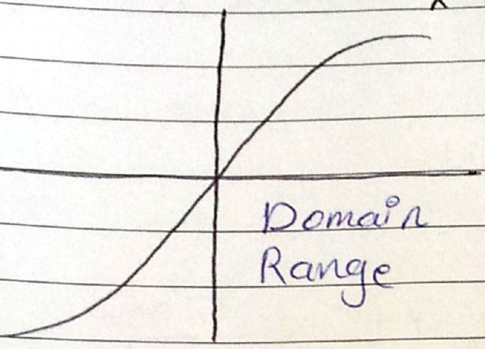
$\frac{1}{x^2}, \frac{1}{x^4}, ...$



Range $(0, \infty)$

Domain $R - \{0\}$
~~~~~~~~

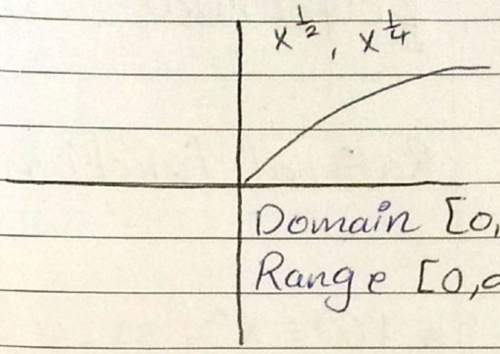
$x^{\frac{1}{3}}, x^{\frac{1}{5}}, \ldots$



Domain

Range

$x^{\frac{1}{2}}, x^{\frac{1}{4}}$



Domain $[0,\infty)$

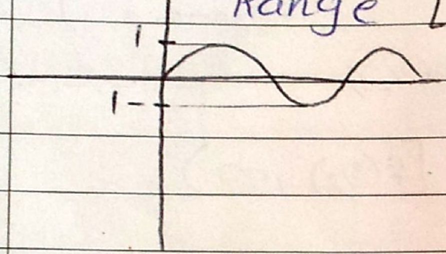
Range $[0,\infty)$

sin

ω

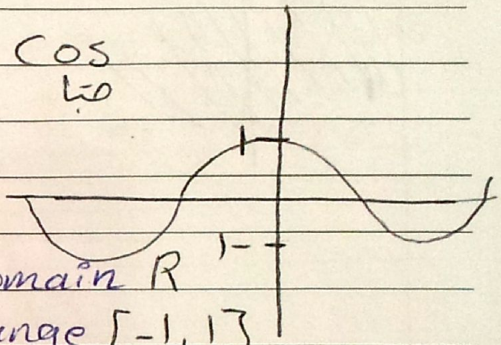
Domain R

Range $[-1,1]$



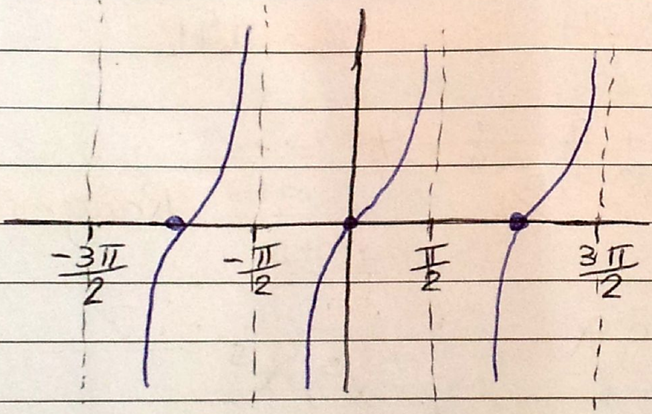
cos

ω



Domain R

Range $[-1,1]$



Domain $R-(2n+1)\frac{\pi}{2}$

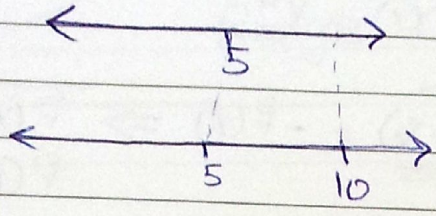
Range $\mathbb{R}$

No.

$f(x) = \sqrt{x-5} - \sqrt{10-x}$

$x-5=0$

$10-x=0$



Domain $\Rightarrow [5, 10]$

أن تقاطعهم

* Polynomials ⇐ كثيرات الحدود

الصورة العامة $f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + ax + a_0$

* Rational اعداد النسبية $\Rightarrow \frac{f(x)}{q(x)} \Rightarrow q(x) \neq 0$.

$\frac{x-1}{x^2+x+1}$	$\frac{x-1}{x^2 \mp 4x+4}$	$\frac{x+3}{x^2-3x-4}$
	$(x-2)(x-2)$	$(x-4)(x+1)$
اقل من صفر غيره لأنه لا يوجد له حل.	$(x-2)^2$ ⇐ يعني غيره صفر لأنه له حل واحد.	أكبر من صفر غيره لأن له حلان
R	$R - \{2\}$	$R - \{-1, 4\}$

$\frac{x+5}{x^2+2x-4} \Rightarrow b^2-4ac \Rightarrow 20 \Rightarrow \frac{-b \pm \sqrt{b^2-4ac}}{2a} \Rightarrow 1 \mp \sqrt{5}$

$R - \{1 \mp \sqrt{5}\}$

No.

Even function $\Rightarrow f(-x) = f(x)$

$f(x) = x^2$ ✱ $f(x) = \cos x$

$f(x) = x^4$

Odd function $\Rightarrow f(-x) = -f(x) \Rightarrow f(x) = x^3$

$f(x) = (-x^3) = -x^3$

$f(x) = \sin x$ / $f(-x) = \sin(-x) = -\sin(x)$.

Neither even Nor odd $\Rightarrow f(x) = x^2 + x^3$

$f(-x) = (-x)^2 + (-x)^3 =$

$x^2 - x^3 \neq f(x)$.

✱

Exponential functions :-

$\underline{\underline{e}}$ is an irrational number عدد غير نسبي

$e \cong 2.78$

$a^{x_1} . a^{x_2} = x_1 + x_2$

$a^{x_1} \div a^{x_2} = x_1 - x_2$

$(a^{x_1})^{x_2} = a^{x_1 x_2}$ $\Rightarrow$ ما ينطبق على

a ينطبق → على → $\underline{\underline{e}}$

$\longrightarrow$

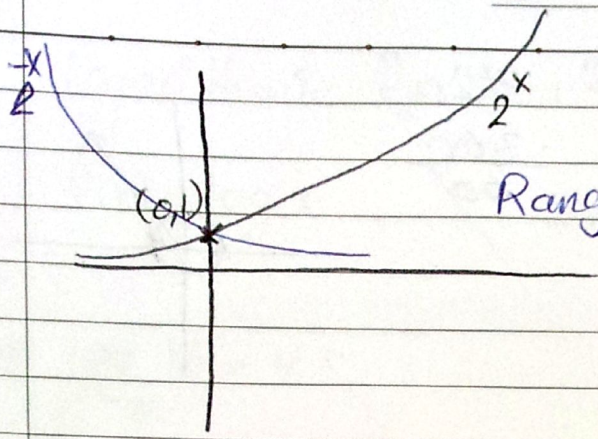
$(ab)^x = a^x b^x$

$a^{-x} = \frac{1}{a^x}$

$a^0 = 1$

$a^1 = a$

No.



Range $(0,\infty)$.

لا يجوز وضع $[1,\infty)$ ← لأنه
يوجد اعداد كثيرة بين 0 و 1

$$* \quad f(x) = \frac{5}{3+2\cos X}$$

Domain $\Rightarrow +1 \geq \cos X \geq -1 \Rightarrow R$

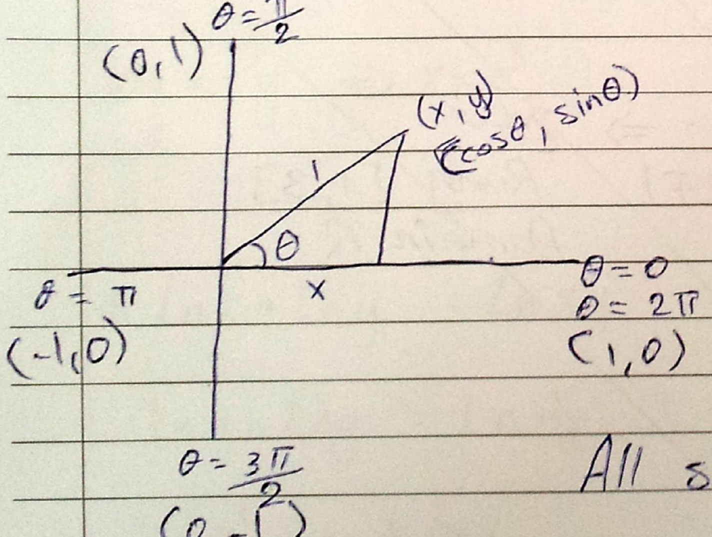
$$3+2\cos X = 0$$

$$\cos = -\frac{3}{2} \quad X$$

Range ⇒ الصورة ⇒ $[1,5]$.

لل $f(x)$

نعوض مرة (-1) ومره (1)



$\cos\theta = X$

$\sin\theta = y$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

All studentare talking calkulus.

	sin	cos	tan
$\pi/6 \to 30^\circ \to$	$1/2$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$\pi/3 \to 60^\circ \to$	$\sqrt{3}/2$	$\frac{1}{2}$	$\sqrt{3}$
$\pi/4 \to 45^\circ \to$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1

logarithmic function :

$f(x) = \log_a x$ logarithm x to the base a

⊛ $\log_2 16 = 4$

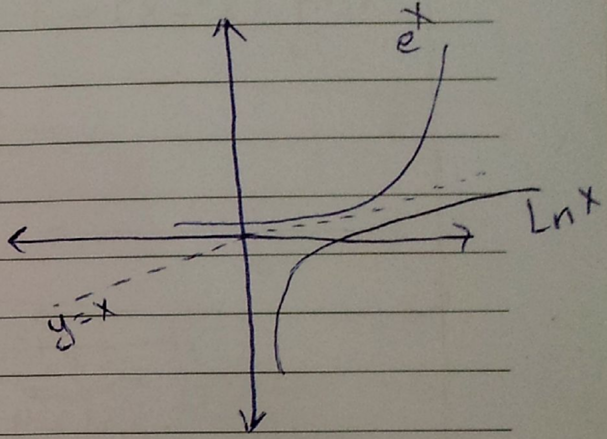
⊛ $\log_3 27 = 3$

⊛ $\log_9 27 = y \Rightarrow 27 = 9^y$

$3^3 = (3^2)^y$

$2y = 3$

$y = 3/2$



⊛ $\log_e x = \text{Ln}\, x$

$\log_e x = y \Rightarrow x = e^y$

⊛ $\text{Ln} x + \text{Ln} y = \text{Ln}\, xy$

$\text{Ln} x - \text{Ln} y = \text{Ln} \frac{x}{y}$

$\text{Ln}\, x^k = k\, \text{Ln} x$

$\text{Ln}\, e = 1$

$\text{Ln}\, 1 = 0$

* $f(x) = \text{Ln}\, x$
  Domain $= (0, \infty)$
  Rang $= (-\infty, \infty)$

$x - 1 =$

$x = 1$

* $f(x) = \text{Ln}(x-1) \Rightarrow$
  Domain $= (1, \infty)$

* $f(x) = \text{Ln}(x^2 - x - 2)$
  Domain $= R - [-1, 2]$

$x^2 - x - 2 = 0$

$(x-2)(x+1)$

Solve :=

① $\log_3 (x-1) = 2$

$x - 1 = 3^2$

$x - 1 = 9$

$x = 10$ ✓

② $\log_2 x + \log_2 (x-1) = 1$

Domain $x = (0, \infty)$

Domain $x-1 = (1, \infty)$

$\log_2 (x^2 - x) = 1 \Rightarrow x^2 - x = 2^1$

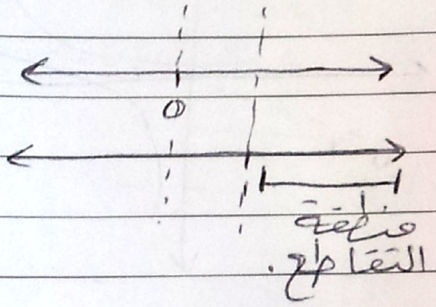
$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2, -1$

Domain العام $\Rightarrow = (0, \infty) \cap (1, \infty)$:.

$= (1, \infty)$



$(f \circ g)(x) = f(g(x))$

$(g \circ f)(x) = g(f(x))$

مثال

$f(x) = x^2 + 5 \qquad g(x) = \sqrt{x+1}$

$(f \circ g)(x) = f(g(x)) = f(\sqrt{x+1}) = (\sqrt{x+1})^2 + 5 = x + 6$

find (i) $(f \circ g)(x) = x + 6$

Domain $(f \circ g)(x) = \underline{\mathbb{R}} \cap \underline{\underline{[-1, \infty)}}$

Domain تابع f

$x + 6 \Leftarrow$ الجواب

Domain تابع $g(x)$

No.

* ① $(f o g)(x) = \sin(3x+5)$

احل $\Downarrow$

$f(x) = \sin(x)$

$g(x) = (3x+5)$

② $(f o g) = (x+1)^3 + 5$

$f(x) = x^3 + 5$

$g(x) = x + 1$

③ $f(x) = 3x+1$, $g(x) = 5 - x^2$

fined x such that $f o g = g o f$

$(f o g) = f(5-x) = 3(5-x)+1 = \boxed{16-3x}$

$(g o f) = g(3x+1) = 5-(3x+1) = \boxed{4-3x}$.

نساويهم ببعض ونحل

(*) $f(x) = \dfrac{x+3}{x-1}$, $f^{-1}(x)$

$\dfrac{y}{1} = \dfrac{x+3}{x-1} \Rightarrow x(y-1) = y+3 \Rightarrow x = \dfrac{y+3}{y-1}$

$\text{Domai}_{f(x)} \Rightarrow R - \{1\}$

$f^{-1}(x) = g(y) = \dfrac{y+3}{y-1}$

$\text{Range}_{f(x)} \Rightarrow R - \{1\}$

(*) $f(x) = x^2 - 3x - 4$

$y = x^2 - 3x - 4$

$x^2 - 3x = y + 4$

$\left(\dfrac{x \text{ معامل}}{2}\right)^2$

اكمال مربع

$\Rightarrow x^2 - 3x + \left(\frac{3}{2}\right)^2 = y + 4 + \left(\frac{3}{2}\right)^2$

$x = \sqrt{y + \frac{25}{4}} + \frac{3}{2}$

$g(y) = \sqrt{y + \frac{25}{4}} + \frac{3}{2}$

No.

$f^{-1}(x) = \sqrt{x+\frac{25}{4}+\frac{3}{2}}$

I will go to buy
tooth brush
for you only

Domain $f(x)$

Range $\Rightarrow x+\frac{25}{4} \geq 0$

$x+\frac{25}{4} = 0 \Rightarrow x = -\frac{25}{4} \Rightarrow [-\frac{25}{4}, \infty)$

B

* $f(x) = x^2 - 4x + 1$, $x \leq 2$
find f^{-1}

Domain $\Rightarrow (-\infty, 2]$
Range $\Rightarrow [-3, \infty)$

$y = x^2 - 4x + 1$
$x^2 - 4x + 4 = y - 1 + 4$
$(x-2)^2 = y+3$
$(x-2) = +\sqrt{y+3}$, $-\sqrt{y+3}$

لأن x أقل من 2 نختار السالب

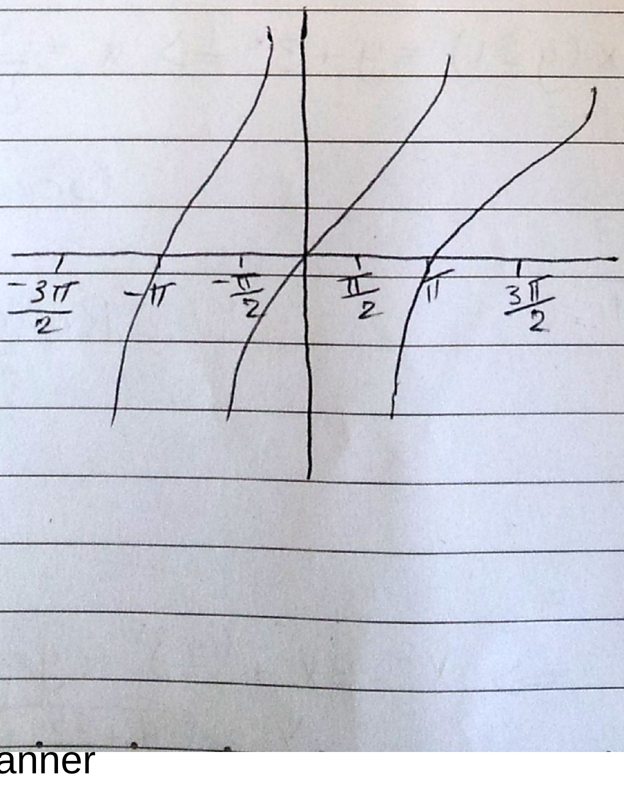
$-\sqrt{y+3}$

Domain $f^{-1} = [-3, \infty)$

asymptotes
خطوط التقارب

*

* $f(x)$



Domain $\Rightarrow$ tanx
Range $\Rightarrow$ R

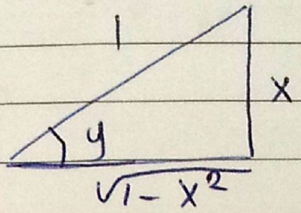
* find $f^{-1}(x)$ where

$(-\frac{\pi}{2}, \frac{\pi}{2})$

$f^{-1}(x) = \tan^{-1}(x)$

Domain $(-\infty, \infty)$

* $y = \sin x^{-1} \Leftrightarrow x = \sin y$



simplify $\cos(\sin^{-1}(x)) = \cos y$

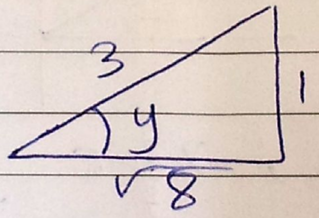
$\cos y = \sqrt{1-x^2}$

* simplify $\cos(\sin^{-1}(\frac{1}{3}))$

$\sin^{-1}(1/3) = y$

$\frac{1}{3} = \sin y$

$\cos(y) = \frac{\sqrt{8}}{3}$



* simplify $\Rightarrow \cos(\tan^{-1}(\frac{1}{2}))$

$\tan^{-1}(\frac{1}{2}) = y$

$\frac{1}{2} = \tan y$

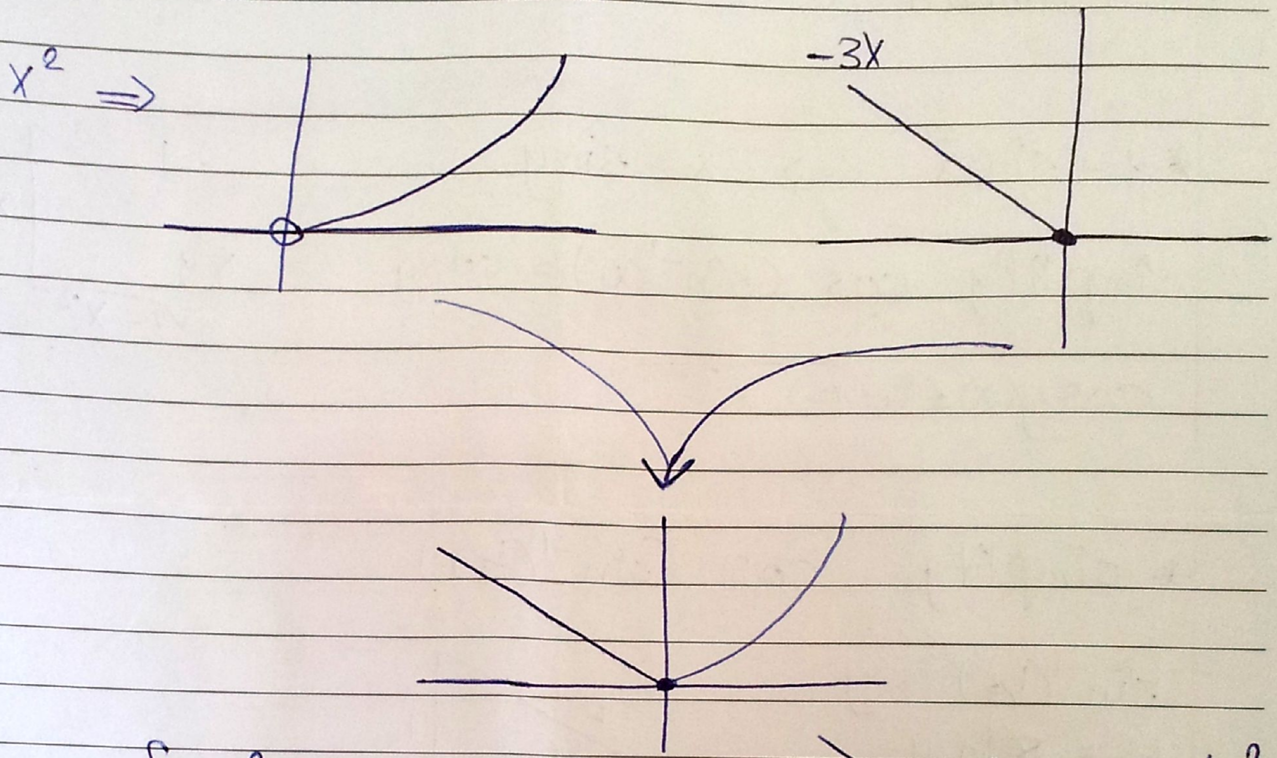
$\cos(y) = 3\sqrt{5}$

$\cos(\cos^{-1}(x)) = x$

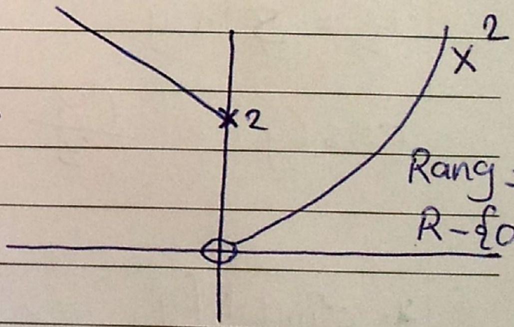
$\sin(\sin^{-1}(x)) = x$

* piecewise function اقتران متشعب

$$f(x) = \begin{cases} x^2 & x > 0 \\ -3x & x \leq 0 \end{cases} \Rightarrow \text{Domain} = R$$



* $$f(x) = \begin{cases} x^2 & , x > 0 \\ 2-x & , x \leq 0 \end{cases} \Rightarrow$$



Rang = R-{0}

* Domain → محور السينات

Range → محور الصادات

مدرج واحد
مجمع الانسانية
١١ — ١٢
يوم السبت

No. CH.1 مراجعة

$\boxed{1}$

$f(x) = \frac{1}{x}$ ⇒ domain $R - \{0\}$

$g(x) = x^2 - 2x \pm 3$ ⇒ R

$h(x) = \sqrt{x}$ ⇒ $[0, \infty)$

* مجال النهائي للثلاث
مجال نهائي ∩ مجال g ∩ مجال h

* find the domain of $(f \circ g \circ h)(x)$

$\Rightarrow (f \circ g \circ h)(x) =$
$(f \circ g)(h(x)) =$
$f \circ g\,(x^2 - 2x - 3) =$
$f(g(x^2 - 2x - 3) =$
$f(\sqrt{x^2 - 2x - 3})$
$\Downarrow$

$\frac{1}{\sqrt{x^2 - 2x - 3}}$ $\xLeftarrow{\text{قبل}}$

⇐ لكن تقاطعهم هو الجواب النهائي وهو $[3, \infty)$

$\frac{1}{\sqrt{x^2 - 2x - 3}}$ ⟶ domain $= (-\infty, -1)$
$(3, \infty$

$\boxed{2}$ $f(x) = x^2$ $g(x) = 5x + 1$

Find x at which $(f \circ g)(x) = (g \circ f)(x)$

$f(g(x)) = g(f(x))$

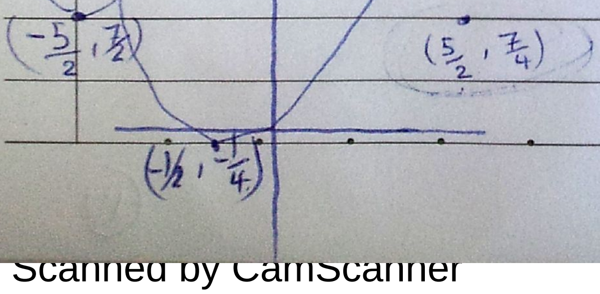
$f(5x+1) = g(x^2)$
$(5x+1)^2 = 5x^2 + 1$
$25x^2 + 10x + \cancel{1} = 5x^2 + \cancel{1}$
$20x^2 - 5x^2 + 10x = 0$
$20x^2 + 10x = 0$
$10x(2x+1) = 0$

$x = 0$
$x = -\frac{1}{2}$

let g(x) be the function obtained from $f(x) = x^2 + x$ by shifting 3 units to the right, 2unit up reflected about y-axis find g(x). ??

$f(x) = x^2 + x + \frac{1}{4} - \frac{1}{4}$ ⇒ $f(x) = (x + \frac{1}{2})^2 - \frac{1}{4}$

⇒ $g(x) = (x + \frac{5}{2})^2 + \frac{7}{4}$



طريقة ثانية لآخر سؤال

No. CH. 1 مراجعة

$g(x) = f(x-3) + 2 \Rightarrow$ y-axis بعدها نعكس على

$g(-x) = ?$

$g(x) = (x-3)^2 + x - 3 + 2$

$g(x) = x^2 - 6x + 9 + x - 3 + 2$

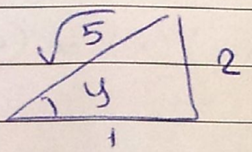
$= x^2 - 5x + 8$

$g(-x) = x^2 + 5x + 8 \Rightarrow$ نفس الجواب لكنه محلل .

$\boxed{4}$ $\sin(2\underbrace{\tan^{-1}(2)}_{x}) =$

$y = \tan^{-1}(2)$

$\tan y = 2$



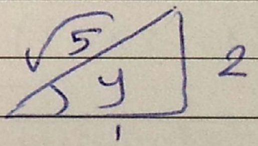
$\sin 2y = 2\sin y \cos y$

$= 2 \times \frac{2}{\sqrt{5}} \cdot \frac{1}{\sqrt{5}} = \frac{4}{5}$ //

$\boxed{5}$ $\cos(2\tan^{-1}(2))$

$\cos 2y = \cos^2 y - \sin y^2$

$= \frac{1}{5} - \frac{4}{5} = \frac{-3}{5}$



No. CH.1 مراجعة

- $\sin^2 x + \cos^2 x = 1$
- $\sin 2x = 2\sin x \cos x$
- $\cos 2x = \cos^2 x - \sin^2 x$

$= [illegible]\cos^2 x - 1$

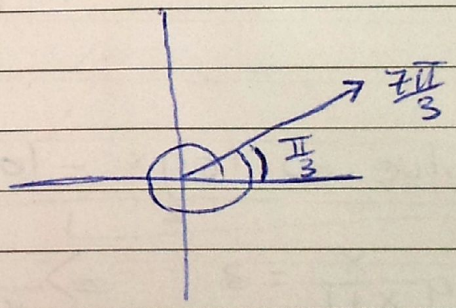
$= 1 - 2\sin^2 x$

- $\tan(A \mp B) = \dfrac{\tan A \mp \tan B}{1 \pm \tan A \tan B}$

- $\sin(A \mp B) = \sin A \cos B \mp \cos A \sin B)$
- $\cos(A + B) = \cos A \cos B + \sin A \sin B$

- $\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

$\sin^{-1}\left(\sin \frac{7\pi}{3}\right)$



$y = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \Rightarrow \sin y = \frac{\sqrt{3}}{2}$

$\sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3} + 2n\pi$

$\frac{2\pi}{3} + 2n\pi$

سؤال → $f(x) = \ln(x+3)$ → Domain of $f(x) = (-3, \infty)$

$y = \ln(x+3$

$e^y = x + 3$

$x = e^y - 3$

$f^{-1}(x) = e^x - 3$

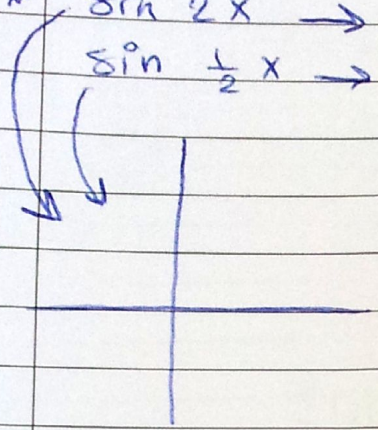
No. CH. 1 مراجعة

* $f(x) = f(-x) \rightarrow$ even $\rightarrow$ symeetric on y-axis

* $-f(x) = f(-x) \rightarrow$ odd $\rightarrow$ symeetric on x-axis

* $\sin 2x \rightarrow$ compress by 2 units

$\sin \frac{1}{2}x \rightarrow$ ~~compress~~ strech by $\frac{1}{2}$ unit



* Solve : $3^{2x} - 4(3)^x - 5 = 0$

$(3^x - 5)(3^x + 1) = 0$

$3^x = 5$ ✓ $\quad 3^x = -1$ ✗ $\quad \Rightarrow x = \log_3 5$

لا يجوز ان ندخل سالب الى log

* solve $\Rightarrow \log_2 x - \log_2 (x+1) = 3$

$\log_2 \frac{x}{x+1} = 3 \Rightarrow \frac{x}{x+1} = 2^3 \Rightarrow x = 8x + 8$

$-8 = 7x$

$\frac{-8}{7} = x$

$\rightarrow x = 0 \quad \longleftrightarrow \; 0 \quad (0, \infty) \Leftarrow$ ليس من فترة المجال

$\therefore$ No solution

$x = -1 \quad \longleftrightarrow \; -1 \quad (-1, \infty)$

* $f(x)$ with domain $[-1, 2]$, Range $[3, 4]$

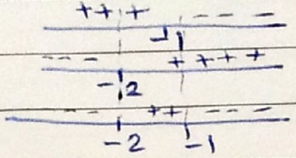
- Domain $f^{-1}(x) = [3, 4]$ — Domain $f^{-1}(3x) = [1, \frac{4}{3}]$
- Domain $f^{-1}(2x) = [\frac{3}{2}, \frac{4}{2}]$
- Domain $f^{-1}(\frac{1}{2}x) = [6, 8]$

$f(x) = \sqrt{x-1}$

$g(x) = \frac{1}{x+2}$

Domain of $(f \circ g)(x) = f\left(\frac{1}{x+2}\right) = \sqrt{\frac{1}{x+2} - 1}$

$\frac{1}{x+2} - \frac{1}{1} \geq 0 \quad = \quad \frac{-1-x}{x+2} > 0$



$[-2, -1] \cap \mathbb{R} - \{-2\}$

انتهى شابتر (1)

بحمد الله.

power unit ^^

CH. 2

No. Limits @ Continuity

* Lim f(x) exists if $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x)$

f(x) is continuose at x=a if $\lim_{x\to a} f(x) = f(a)$

سؤال.

* $f(x) = \begin{cases} ax^2+b & x>1 \\ 5 & x=1 \\ 3ax & x<1 \end{cases}$

Find a,b if f(x) is continuous at x=1 ??

$$\lim_{x\to 1^+} f(x) = a+b$$

$$\lim_{x\to 1^-} f(x) = 3a$$

$$f(1)=5$$

$3a=5 \Rightarrow a=\frac{5}{3}$ | $a+b=5$

$$\frac{5}{3}+b=5$$

$$b=\frac{10}{3}$$

سؤال

* $$\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = 4$$

سؤال

* $$\lim_{x\to 2} \frac{\sqrt{x^2+5}-3}{x-2} \cdot \frac{\sqrt{x^2+5}+3}{\sqrt{x^2+5}+3}$$

نضرب بالمرافق

conjugate

$$\frac{x^2+5-9}{x-2} \cdot \frac{1}{\sqrt{x^2+5}+3} =$$

* $x^2-y^2=(x-y)(x+y)$

$x^3-y^3=(x-y)(x^2+xy+y^2)$

$x^3+y^3=(x+y)(x^2-xy+y^2)$

$$\frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} \cdot \frac{1}{\sqrt{x^2+5}+3} = \frac{4}{6}$$

سؤال *

$$\lim_{x \to 2} \frac{|x-2|}{x-2} = \lim \begin{cases} \frac{x-2}{x-2} & , x > 2 \\ \frac{-(x-2)}{x-2} & , x < 2 \end{cases}$$

$$\lim \begin{cases} 1 & , x > 2 \\ -1 & , x < 2 \end{cases} \Rightarrow \lim_{x \to 2^+} = 1 \quad / \quad \underbrace{\lim_{x \to 2^-} = -1}$$

dose not exist.

قواعد *

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \to 0} \frac{\tan x}{x} = 1$$

$$\lim_{x \to 0} \frac{\cos x}{x} \Rightarrow \text{does not exist.}$$

$$\lim_{x \to 0} \frac{\sin ax}{bx} = \frac{a}{b}$$

$$\lim_{x \to 0} \frac{\sin ax}{\tan bx} = \frac{a}{b}$$

سؤال *

$$\lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi - x} \Rightarrow \qquad \pi - x = y$$

استبدال .

$$\lim_{y \to 0} \frac{\sin y}{y} = 1$$

سؤال *

$$\lim_{x \to \frac{\pi}{2}} \frac{(x - \frac{\pi}{2})}{2x - \pi} \Rightarrow \qquad x - \frac{\pi}{2} = y$$

$$2x - \pi = 2y$$

$$\lim_{y \to 0} \frac{\sin y}{2y} = \frac{1}{2}.$$

No. CH. 2

* قاعدة

$$\lim_{x \to \infty} \frac{1}{x} = 0 \quad , \quad \lim_{x \to \infty} \frac{1}{x^k} = 0 \quad , k > 0$$

* سؤال

$$\lim_{x \to \infty} \frac{3x^2+1}{4x^2+5} \Rightarrow \lim_{x \to \infty} \frac{\frac{3x^2}{x^2} + \frac{1}{x^2}}{\frac{4x^2}{x^2} + \frac{5}{x^2}} = \frac{3}{4}$$

* سؤال

$$\lim_{x \to \infty} \frac{3x+5}{x^7+1} = 0 \text{ zero}$$

دائماً اذا كانت درجة البسط < درجة المقام

الجواب يكون ——zero

* سؤال

$$\lim_{x \to \infty} \sqrt{x^2+3x} - x \Rightarrow \lim_{x \to \infty} (\sqrt{x^2+3x} - 3) \cdot \frac{(\sqrt{x^2+3x} + 3)}{\sqrt{x^2+3x} + 3}$$

$$\lim_{x \to \infty} \frac{\cancel{x^2} + 3x - \cancel{x^2}}{\sqrt{x^2+3x} + x} = \lim_{x \to \infty} \frac{3x}{\sqrt{x^2(1+\frac{3}{x}} + x}$$

$\sqrt{x^2} = |x|$

$$\lim_{x \to \infty} \frac{3\cancel{x}}{\cancel{x}(\sqrt{1+\frac{3}{x}}+1)} = \frac{3}{2}$$

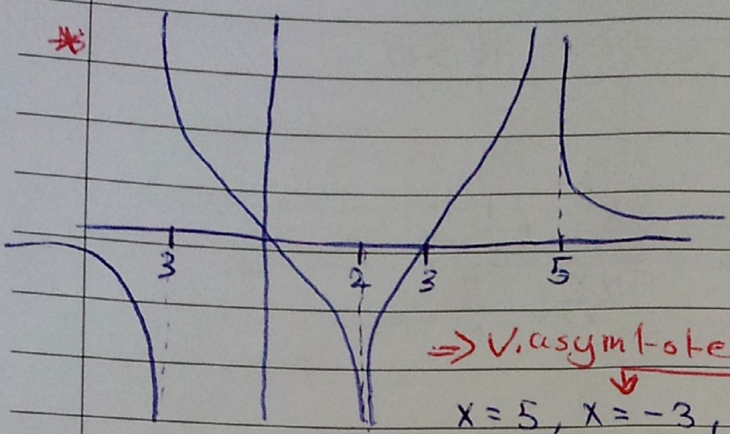
* سؤال

$$\lim_{x \to \infty} \sqrt{x^2+ax} - x$$

درجة البسط < درجة المقام ⇐ $\pm\infty$

درجة البسط = درجة المقام ⇐ $\frac{\text{معامل أكبر اس}}{\text{معامل أكبر اس}}$

No. CH. 2

* 

المطلوب: $\Rightarrow \lim_{x \to 2} R(x) = \boxed{-\infty}$ $\lim_{x \to ^+2} = \lim_{x \to ^-2}$
$-\infty = -\infty$

$\lim_{x \to 5} R(x) = \infty$ $\lim_{x \to 5^+} = \lim_{x \to ^-5}$
$\infty = \infty$

$\lim_{x \to -3^-} R(x) = -\infty$
$\lim_{x \to -3^+} R(x) = \infty$
does not exit

$\Rightarrow$ V. asymtotes
$x = 5,\ x = -3,\ x = 2$

$\Rightarrow$ h. asymtotes
$y = 0$

horizantal Asymptote
خط تقارب افقي.

* $F(x) = \frac{3x-1}{x+2}$ $\Rightarrow$ horizonbal Asymptote $y = \lim_{\substack{x \to \infty \\ x \to -\infty}} F(x)$

vertical Asymptote

* $\lim_{x \to \infty} \frac{x^2+1}{x^2-1}$ $\Rightarrow$ H. Asym $= 1$
$\underbrace{\text{V. Asym} = \mp 1}_{\text{اصفار المقام}}$

* $\lim_{x \to -3^+} \frac{x+2}{x+3} = \frac{-1}{0^+} = -\infty$

$\frac{+}{0^-} = -\infty$

$\lim_{x \to -3^-} \frac{x+2}{x-3} = \frac{-1}{0^-} = +\infty$

$\frac{-}{0^-} = +\infty$

$\frac{+}{0^+} = +\infty$

$\lim_{x \to 1} \frac{2-x}{(x-1)^2}$
$\rightarrow \lim_{x \to 1^+} = \frac{1}{0^+} = +\infty$
$\rightarrow \lim_{x \to 1^-} = \frac{1}{0^+} = \infty$

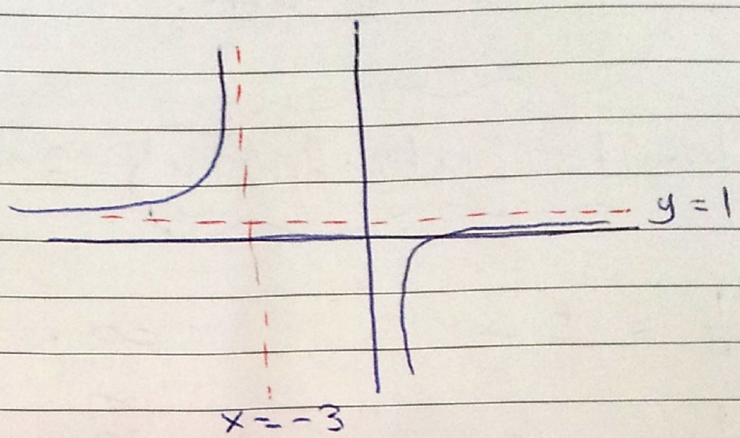
$\frac{-}{0^+} = -\infty$

$\lim_{x \to 5^-} \frac{e^x}{(x-5)^3} = \frac{e^5}{0^-} = -\infty$

* $\frac{x+2}{x+3}$

H. Asym = $\lim_{x\to\infty} \frac{x+2}{x+3} = 1$ $\boxed{y=1}$

V. Asym ⇒ $x=-3$



* $\lim_{x\to 3^+} \ln(x^2-9) = -\infty$

* $y = \frac{x^2+1}{3x-2x2}$ ⇒ V. asym ⇒ $0, \frac{3}{2}$

H. asym ⇒ $\lim_{x\to\infty} = -\frac{1}{2}$ ⇒ $y = -\frac{1}{2}$

find H.A $f(x) = \tan^{-1}(x)$??

sol: $\lim_{x\to\infty} \tan^{-1}(x) = \tan^{-1}(\infty) = \frac{\pi}{2}$

$\lim_{x\to-\infty} \tan^{-1}(x) = \tan^{-1}(-\infty) = -\frac{\pi}{2}$

No.

* find V.A for $f(x) = \ln(x-3)$

الشئ إلي يصفر فاداخل Ln يكون U.A

$\therefore x = 3$ is U.A.

* Limits at infinity :-

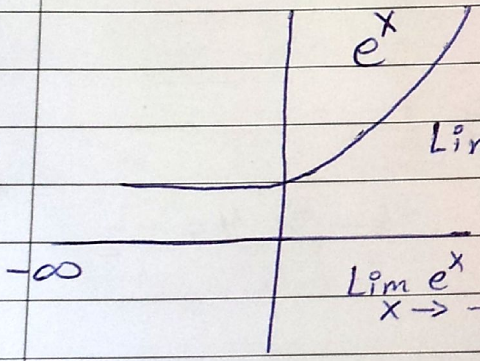
$\frac{1}{0} = \pm\infty$ $\qquad$ $\infty \cdot \infty = \infty$

$\frac{\infty}{\infty}$ او $\frac{0}{0}$ $\Rightarrow$!

$\frac{1}{\pm\infty} = 0$

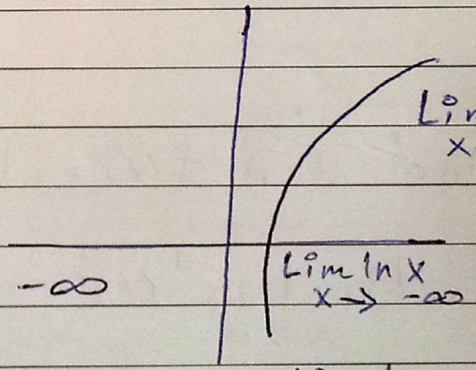
$\infty + \infty = \infty$

$\infty - \infty =$!



$\lim_{x \to \infty} e^x = \infty$

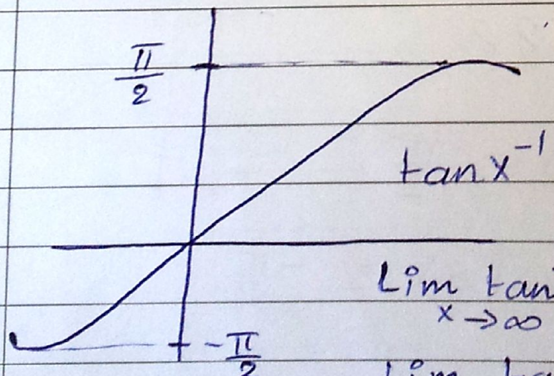
$\lim_{x \to -\infty} e^x = 0$



$\lim_{x \to \infty} \ln x = \infty$

$\lim_{x \to -\infty} \ln x =$ dosn't exist

$\lim_{x \to 0^+} \ln x = -\infty$



$\lim_{x \to \infty} \tan^{-} x = \frac{\pi}{2}$

$\lim_{x \to -\infty} \tan^{-} x = \frac{-\pi}{2}$

No.

الاتصال على فترة $\Leftarrow$ domain هو نفسه ال

$f(x) = \sqrt{x-2}$ find interal of continuity

نفس ال domain لكن الفترات دائماً مفتوحة .

$x - 2 = 0 \rightarrow x = 2 \rightarrow \underset{2}{\overset{--\quad\;++}{\overline{\quad\;|\quad\;}}}$ $(2, \infty)$.

* Types of discontinuty $\rightarrow$ نقاط عدم الاتصال . -

1) Jumb $\rightarrow \lim\limits_{x^+} \neq \lim\limits_{x^-}$

2) Removable dis cont (hole)

$\lim\limits_{x} \neq f(x)$ but $\lim\limits_{x^+} = \lim\limits_{x^-}$

* find Removable discont points for $f(x) = \frac{x-2}{x^2-4}$

$\lim\limits_{x \to 2^+} \frac{x-2}{x^2-4} = \frac{1}{4}$

$\lim\limits_{x \to 2^-} \frac{x-2}{x^2-4} = \frac{1}{4}$

$\rightarrow$ Removable discont (hole) $x=2$ is a hole $\boxed{or}$ Removable discont point.

$\lim\limits_{x \to -2^+} \frac{x-2}{x^2-4} = +\infty$

$\lim\limits_{x \to -2^-} \frac{x-2}{x^2-4} = -\infty$

$\rightarrow$ $x = -2$ is infinite discont point.

* $\lim e^{-2x} \cos x$:

$-1 \leq \cos x \leq 1$

$-e^{-2x} \leq -e^{-2x} \cos x \leq e^{-2x}$

$\lim\limits_{x \to \infty} -e^{-2x} \leq \lim\limits_{x \to \infty} e^{-2x} \cos x \leq \lim\limits_{x \to \infty} e^{-2x}$

$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$

$\qquad\qquad 0 \qquad\qquad 0$

No.

* if $\lim\limits_{x \to 1} \frac{f(x) - 10}{x - 1} = 8$, find f(1) ??

$f(1) - 10 = 0 \Rightarrow f(1) = 10$

* $\lim\limits_{x \to -\infty} \frac{2x + \sqrt{x^2 + 1}}{4x - 3} = \lim\limits_{x \to -\infty} \frac{2x + \sqrt{x^2}}{4x}$

$\Downarrow$

$\frac{\text{الكبر قوة}}{\text{الكبر قوة}}$

$\lim\limits_{x \to -\infty} \frac{2x - x}{4x} = \lim\limits_{x \to -\infty} \frac{x}{4x} = \frac{1}{4}$

انتهى شابتر (٣)

بحمد الله

power Unit ^^

CH. 3

No.

* $f`(x) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to \text{[illegible]}} \frac{f(h+a) - f`(a)}{h}$

- If $f(x) = \sqrt{x}$, fin $f`(x)$??

$\lim_{x \to 0} \frac{\sqrt{x} - \sqrt{a}}{x - a} \cdot \frac{\sqrt{x} + \sqrt{a}}{\sqrt{x} + \sqrt{a}} = \lim_{x \to a} \frac{\cancel{x - a}}{\cancel{x - a}} \cdot \frac{1}{\sqrt{a} + \sqrt{a}}$

$= \frac{1}{2\sqrt{a}}$

* $f(x)$	$f`(x)$
c	0
x^n	$n x^{n-1}$
$f(x) g(x)$	$f(x) g'(x) + g(x) f`(x)$
$\frac{f(x)}{g(x)}$	$\frac{g(x) f`(x) - f(x) g`(x)}{(g(x))^2}$
$\frac{a}{x}$	$\frac{-a}{x^2}$
e^x	$e^x \cdot \underline{1} \longrightarrow$ مشتقة الاس
a^x	$a^x \cdot 1 \cdot \ln a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\csc^2 x$
$\sec x$	$\sec x \tan x$
$\csc x$	$-\text{cse}x \cot x$
$\ln f(x)$	$\frac{f`(x)}{f(x)}$
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$

No.

* $2x^{100} + \sqrt{2x} + 4$, find $f'(x)$??

$(2)(100)\,x^{99} + \frac{1}{2}\,x^{-\frac{1}{2}} \cdot 2$

* $\log_a f(x) = \dfrac{f'(x)}{f(x)\,\mathrm{Ln}\,a}$

* $f(x) = \begin{cases} x & , x > 0 \\ 3x^2 & , x \leq 0 \end{cases}$

find $f(x)$ is diff on $x = 0$ ⇒ يعني قابل للاشتقاق عند $x = 0$

الحل ①+②

⇓

① $f(x)$ is diff on $x = a$ ⇒ $\lim_{x \to 0^+} x = 0$ $\lim_{x \to 0^-} x = 0$

1- $f(x)$ cont at $x = a$

2- $f'_{+}(x) = f'_{-}(x)$ $f(0) = 0$ ∴ $f(x)$ cont at $x = 0$ #

② $f'(x) \begin{cases} 1 & , x > 0 \\ 6x & , x < 0 \end{cases}$ ⇒ عندما نشتق لا نضع اشارة المساواة

$f'(0)^+ = 1 \neq f'(0)^- = 0$

∴ $f(x)$ is not diff at $x = 0$

* find $'\sin^{(99)} x$

$$\begin{array}{r} 24 \\ 4\overline{)99} \\ 96 \\ \hline \boxed{3} \end{array}$$

$\boxed{3} \rightarrow$ يمثل رقم المشتقة

↓

لو كان الباقي صفر يبقى الاقتران نفسه

$\Rightarrow \therefore \sin^{(99)} x = \sin^{(3)} x$

$= -\cos x$

* $f(x) = 2^x \Rightarrow$ find $f^{(40)}_x$

$f'(x) = 2^x \ \text{Ln}\ 2$

$f''(x) = \text{Ln}\ 2\ (2^x)\ \text{Ln}\ 2 \qquad \Rightarrow \ \therefore\ 2^x (\text{Ln}\ 2)^{40}$

$= 2^x (\text{Ln}\ 2)^2$

* Chain Rule :-

1) $f(g(x)) \Rightarrow f'(x) = f'(g(x)) \cdot g'(x)$

* $\sin(x^2) \longrightarrow \cos(x^2) \cdot 2x$

2) $y = f(u)$, $u = f(x)$ $\Rightarrow$ find $\frac{dy}{dx} \rightarrow \overset{\text{هي نفسها}}{f'(x)} \Rightarrow$ قاعدة السلسلة

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{dy}{dx} \ \#$$

* $y = \sin u$, $u = x^2$, find $\frac{dy}{dx}$??

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \cos \underset{\text{لازم تكون بدلالة } X}{\underline{u}} \cdot 2x = 2x \cdot \cos x^2$$

3) $(f(x))^n \Rightarrow f'(x) = n\,(f(x))^{n-1} \cdot f'(x)$

* $f(x) = (3x+1)^2 \Rightarrow 2\,(3x+1)^1 \cdot 3$

No.

* Impliet differentiation : الاشتقاق الضمني

- $x^3 + y^3 = xy$, find $\frac{dy}{dx}$ --- ??

$$3x^2 + 3y^2 \frac{dy}{dx} = x \frac{dy}{dx} + y$$

$$3y^2 \frac{dy}{dx} - x \frac{dy}{dx} = y - 3x^2$$

$$\frac{dy}{dx} = \frac{y - 3x^2}{3y^2 - x}$$

* Derivative of logarithm :-

- $\ln f(x) \rightarrow \frac{f'(x)}{f(x)}$ $\qquad$ $\log_a (f(x)) \rightarrow \frac{f'(x)}{f(x) \ln a}$

- $y = \ln (x^3 + 1)$ find $\frac{dy}{dx}$??

$$y' = \frac{3x^2}{x^3 + 1}$$

- $y = (\ln x)^{3x} \rightarrow \ln y = \ln (\ln x)^{3x}$

$$\ln y = 3x \ln(\ln x) \Rightarrow \frac{y'}{y} = 3x \cdot \frac{1}{x \ln x} + 3 \ln(\ln x)$$

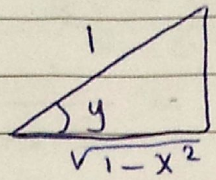
$$y' = y \left(3x \cdot \frac{1}{x \ln x} + 3 \ln(\ln x)\right)$$

$$= (\ln x)^{3x} \left(\frac{3x}{x \ln x} + 3 \ln(\ln x)\right).$$

* Inverse :-

$y = \sin^{-} x \Rightarrow x = \sin y \Rightarrow 1 = \cos y \cdot y'$

$y' = \frac{1}{\cos y}$



* $f(x)$	$f'(x)$
$\sin^{-} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-} x$	$\frac{-1}{\sqrt{1-x^2}}$
$\tan^{-} x$	$\frac{1}{x^2+1}$
$\cot^{-} x$	$\frac{-1}{x^2+1}$
$\sec^{-} x$	$\frac{1}{\sqrt{x^2-1}}$
$\csc^{-} x$	$\frac{-1}{\sqrt{x^2-1}}$

* $f(x) = \sin^{-} 2x$, find $f'(x)$??

$f'(x) = \frac{1}{\sqrt{1-(2x)^2}} \cdot 2$

* $(f^{-}(x))' = \frac{1}{f'(f^{-}(x))}$

- If $f(4) = 5$, $f'(4) = \frac{2}{3}$, find $(f^{-}(5))'$??

$(f^{-}(5))' = \frac{1}{f'(f^{-}(5))} = \frac{1}{f'(4)} = \frac{3}{2}$.

* Lhopital Rule $\Rightarrow$ قاعدة نستخدمها لايجاد النهايات .

$\lim \frac{f(x)}{g(x)} = \lim \frac{f'(x)}{g'(x)}$

$e^{\infty} = \infty$

$e^{-\infty} = 0$

$\ln \infty = \infty$

$\ln 0^{+} = -\infty$

- $\lim\limits_{x \to \infty} \frac{x}{e^{x}} = \frac{\infty}{\infty} = !$

الحل $\Downarrow$

$\lim\limits_{x \to \infty} \frac{x}{e^{x}} = \lim\limits_{x \to \infty} \frac{1}{e^{x}} = 0$.

- $\lim\limits_{x \to 0^{+}} \frac{\ln x}{\csc x} = \frac{-\infty}{\infty} \Rightarrow \lim \frac{\frac{1}{x}}{(-\csc x \cot x)}$

$= \lim\limits_{x \to 0^{+}} \underbrace{\frac{-\sin x}{x}}_{-1} \tan x = (-1)(0) = 0$

$$* \lim_{x \to 0^+} x \ln x = 0 \cdot \infty = ! \Rightarrow \lim_{x \to 0^+} \frac{\ln x}{1/x} = \frac{-\infty}{\infty}$$

$$= \lim_{x \to 0^+} \frac{1 \cdot x^2}{x \cdot (-1)} = \lim_{x \to 0^+} -x = 0$$

$$* \lim_{x \to \frac{\pi}{2}} \sec x - \tan x = \infty - \infty \ !$$

$$\frac{1}{\cos x} - \frac{\sin x}{\cos x} = \frac{1 - \sin x}{\cos x} = \frac{0}{0} \ !$$

$$\lim_{x \to \frac{\pi}{2}} \frac{-\cos x}{-\sin x} = \frac{0}{1} = 0$$

$$* \lim_{x \to 0^+} x^x \Rightarrow 0^0 , 1^\infty , \infty^0 \ !$$

$$y = x^x \to \ln y = x \ln x \Rightarrow \lim_{x \to 0^+} \ln y = \lim_{x \to 0^+} x \ln x = 0$$

$$\lim y = e^0 = 1$$

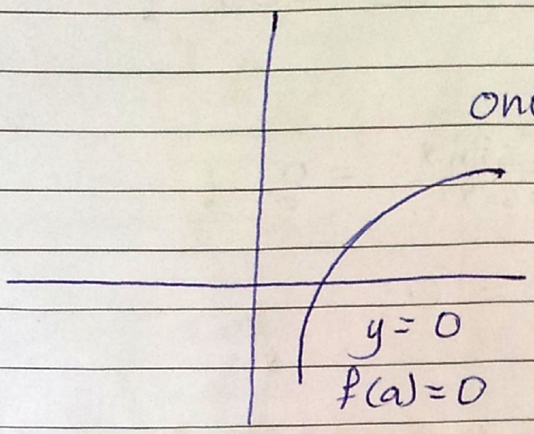
$$* \lim_{x \to \infty} \left(1 + \frac{a}{x}\right) = e^a \Rightarrow y = \left(1 + \frac{a}{x}\right)^x \Rightarrow \ln y = x \ln \left(1 + \frac{a}{x}\right)$$

$$\lim_{x \to \infty} \ln y = \lim_{x \to \infty} \frac{\ln\left(1 + \frac{a}{x}\right)}{\frac{1}{x}} = \frac{0}{0} \ !$$

$$= \lim \frac{-a \cdot x^2}{x^2 \left(1 + \frac{a}{x}\right) \cdot -1} .$$

* Intermediate value Thurm ⇒

\- show that $x^3 - x^2 - x - 2 = 0$ has at Leat one root ??



$f(0) = -2 < 0$

$f(3) > 0$

$f(x)$ is cont $(0,3)$.

انتهى شابتر (٣)

بحمد الله

power

Unit ^U^

CH. 4

1) I.V.T Inter mediat value Theorm

show that $x^3 - x^2 - x - 2 = 0$ has at least one root?

1) $f(0) = -2 < 0$ ① نختار رقم ناتج تعويضه اقل من صفر
2) $f(3) = 13 > 0$ ② نختار رقم ناتج تعويضه اكبر من صفر
3) $f(x)$ is cont $(0,3)$ ③ نثبت ان الاقتران متصل في الفترة ما بين الرقمين

$\boxed{\text{By I.V.T } \exists\, c \text{ such that } f(c)=0}$ ← نكتب بالامتحان

2) M.V.T

1. $f(x)$ is conts on $[a,b]$
2. $f(x)$ is diff on (a,b)

* Rulles Theorem MVT حالة خاصة من

1. $f(x)$ is conts $[a,b]$
2. $f(x)$ is diff (a,b)
3. $f(a) = f(b)$

* Ex: M.U.T

$f(x) = x^3 + x - 1$, $[1,3]$ find values of c that salisfies M.U.T ??

1. $f(x)$ is cont $c \ni [1,3]$ (poly)
2. $f(x)$ is diff $c \ni (1,3)$ (poly) $\Rightarrow$

$f(x) = x^3 + x - 1$
$f'(x) = 3x^2 + 1$
$f'(c) = 3c^2 + 1$

تحقق الشرطين إذن

$$f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{29 - 1}{3 - 1} = 14$$

$3c^2 + 1 = 14$
$c = \pm\sqrt{\frac{13}{3}}$

Ex: show that $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

1) prove that f(x) is constant ⟸ : طريقة الحل

2) prove that constant $= \frac{\pi}{2}$

$f(x) = 3 \Longrightarrow f'(x) = 0$ ⟸ لاثبات ان الاقتران = constant
يجب ان تكون مشتقته = صفر.

sol: ⟹ $f(x) = \tan^{-1}(x) + \cot^{-1}(x)$

① $f'(x) = \frac{1}{1+x^2} + \frac{-1}{1+x^2} = 0$

$f'(x) = 0 \Rightarrow$ so that $f(x) =$ constant

② $\tan^{-1}x + \cot^{-1}x$ تعويض اي رقم نعطي ناتج تعويضه ⟸ فلك الرقم (1)

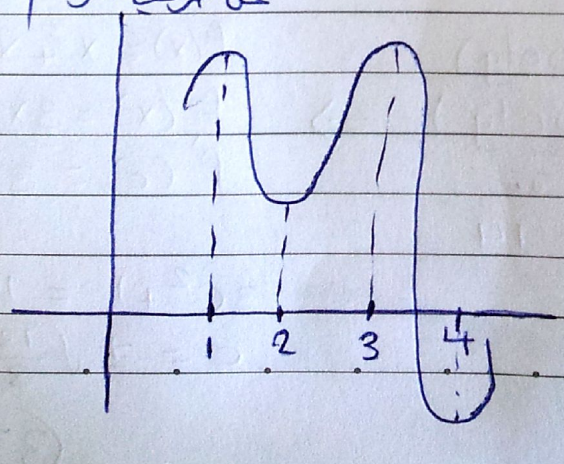
$\tan^{-1}1 + \cot^{-1}1$

$\frac{\pi}{4} + \frac{\pi}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$ ⋕

* Miximum and Minimum

local max — local min — abs max — abs min

① عن طريق الرسم



* abs max → اكبر قيمة موجودة بالرسم
at $x = 1 = f(1)$

* abs min → اصغر قيمة موجودة بالرسم
at $x = 4 = f(4)$

* local max اكبر قيمة بين النقاط الموجودة حولها
at $x = 3 = f(3)$

* local min اصغر قيمة بين النقاط الموجودة حولها
at $x = 2 = f(2)$

No.

* ملاحظة ⇒ end can be called only abs
الأطراف يمكن أن تكون abs max او abs min او لا شيء.
ولكنها لا تكون local أبداً.

② عن طريق اختبار المشتقة الثانية

1- find (critical numbers) → النقاط الحرجة.
2- find values of critical number
3- largest value is abs max
smallest value is abs min

** Critical numeber :- النقاط الحرجة.

is a number (c) in the domain
$f'(c) = 0$ or $f'(c)$ does not exist

الأطراف

اصفار مقام المشتقة.

Ex: $f(x) = x^3 - 3x^2 + 1$, $D = [-\frac{1}{2}, 4]$
find abs max and abs min ??

$f(x) = x^3 - 3x^2 + 1 \rightarrow f'(x) = 3x^2 - 6x \rightarrow f'(x) = 3x(x-2)$

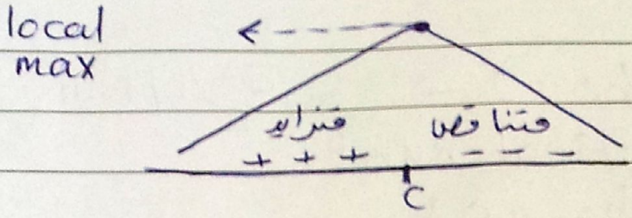
$x = 0$, $x = 2$ → الاصفار
$x = -\frac{1}{2}$, $x = 4$ → الأطراف
⇒ جميع القيم موجودة في Domain

$x=0$, $x = 2$, $x = -\frac{1}{2}$, $x = 4$ نوجد صور النقاط الحرجة
$f(0) = 1$, $f(2) = -3$, $f(-\frac{1}{2}) = \frac{1}{8}$, $f(4) = 17$

abs max = 17
abs min = −3

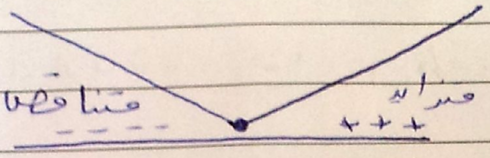
* Increasing and Decreasing :⊃

if $f'(x) > 0 \rightarrow$ then $f(x)$ is inc ↗ متزايد
if $f'(x) < 0 \rightarrow$ then $f(x)$ is dec ↘ متناقص



$f'(x) \rightarrow$ نبحث اشارة المشتقة

c → critical number   at c local number

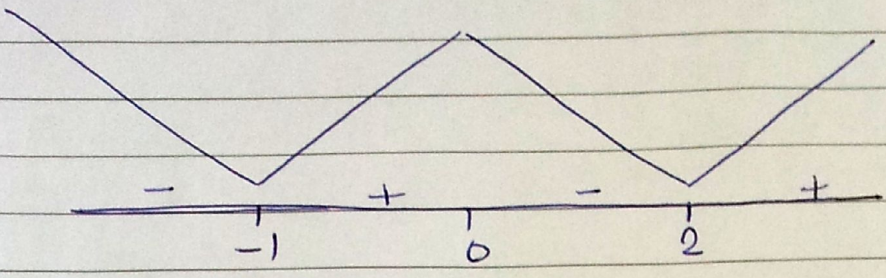


Ex : $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$
1) - find Inervals at Increasing and deacreasing?!
2) - find local max and min ?!

sol :- اولاً نجد المجال
ثم critical num ثم نعوضها على خط
الاعداد.

$f(x) = 3x^4 - 4x^3 - 12x^2 + 5$
$D = R$
$f'(x) = 12x^3 - 12x^2 - 24x$
$0 = 12x^3 - 12x^2 - 24x$
$12x\,(x-2)\,(x+1)$
$x = 0\ ,\ x = 2\ ,\ x = -1$ → D كلهم في،

No.



inc ⇒ $(-1,0)$, $(2,\infty)$

dec ⇒ $(-\infty,-1)$, $(0,2)$

at $x=-1$ local min $= f(-1)=0$

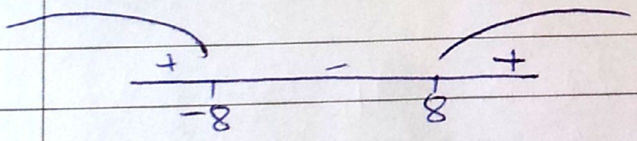
at $x=0$ local max $= f(0)=5$

at $x=2$ local min $= -27$

Ex: $f(x)=\sqrt{x^2-64}$, find Increasing and decreasing
find local min, max.

- Domain :-

$x^2-64=0$ $x=8$, $x=-8$



$D=(-\infty,-8],[8,\infty)$

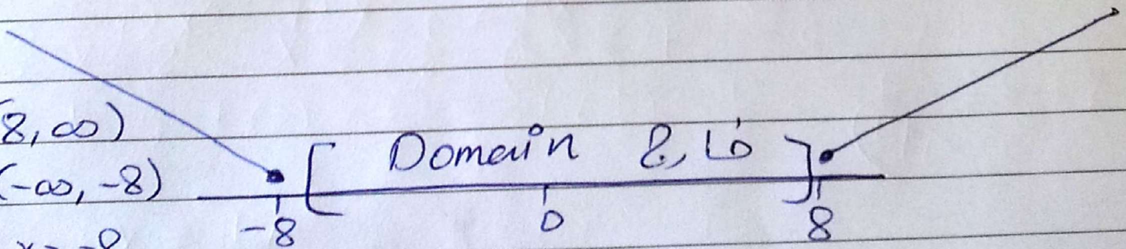
$$f'(x)=\frac{2x}{2\sqrt{x^2-64}}=\frac{x}{\sqrt{x^2-64}}$$

$x=0$ $f'(x)=0$

$\sqrt{x^2-64}=0$ $x=8$, $x=-8$ $f'(x)$ dose not exist

inc $(8,\infty)$

dec $(-\infty,-8)$



$x=8$, $x=-8$

abs min $= f(8) = f(-8) \Rightarrow f(8) = f(-8) = 0$

absmax ⇒ لايوجد