CALCULUS 3 NOTEBOOK

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POWER UNIT
ELECTRICAL ENGINEERING

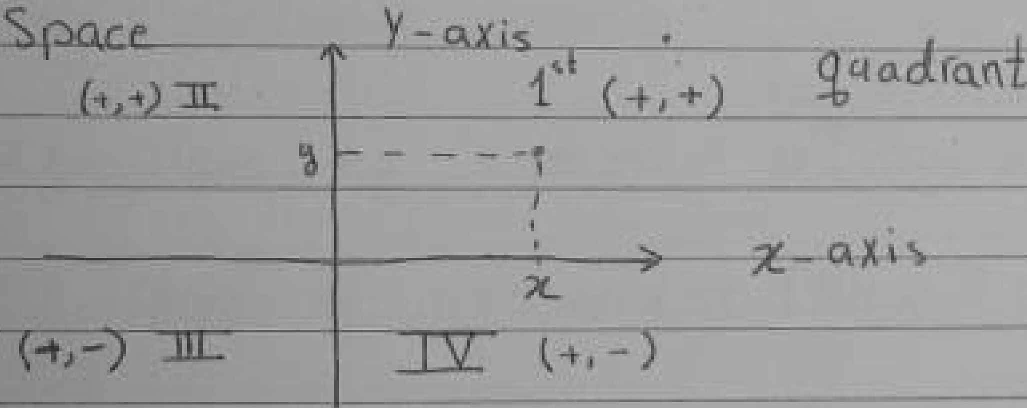
Subject Cal-3 Date 11/6/2012 No.

Ch # 12

Vectors & the Geometry of Space

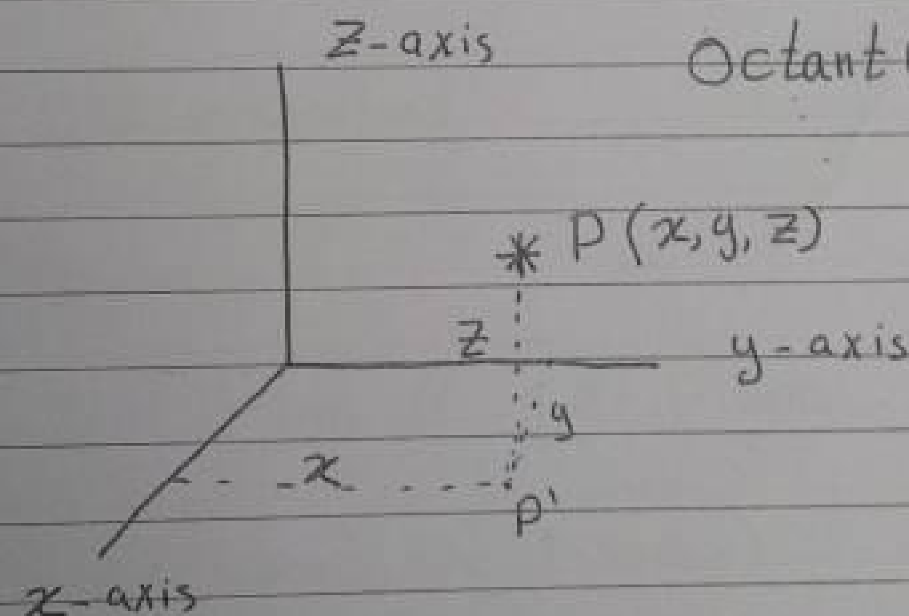
S. 12.1 the three Spaces

$\boxed{1}$ 2- Space



$\boxed{2}$ 3- Space

Octant (+,+,+)



$$(x\ ,\ y\ ,\ z)$$

بعد النقطة عن الـ (x,y) Plane

(x,y,0) اسقاط النقطة

بعد النقطة عن (y,z) Plane

بعد النقطة عن (x,z) Plane

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* The three spaces consists of:

[1] three coordinate axis:
x-axis, y-axis & z axis

[2] three coordinate planes:

① xy-plane ; eg: $z=0 \Rightarrow z=c$ [Plane // xy-plane]

② xz-plane; eg: $y=0 \Rightarrow y=c$ [Plane // xz-plane]

③ yz-plane ; eg: $x=0 \Rightarrow x=c$ [Plane // yz-plane]

· c = constant

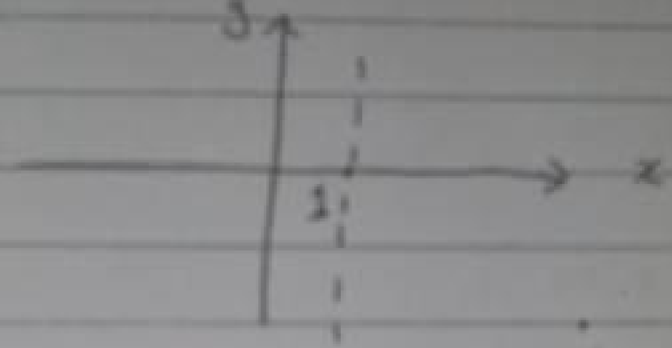
ex Identify:

1) $x=1$ plane // yz-plane

2) $x=1$, (in one space) (in 1-space)
∴ Point cuz

3) $x = 1$ (in two spaces) (in 2-space)

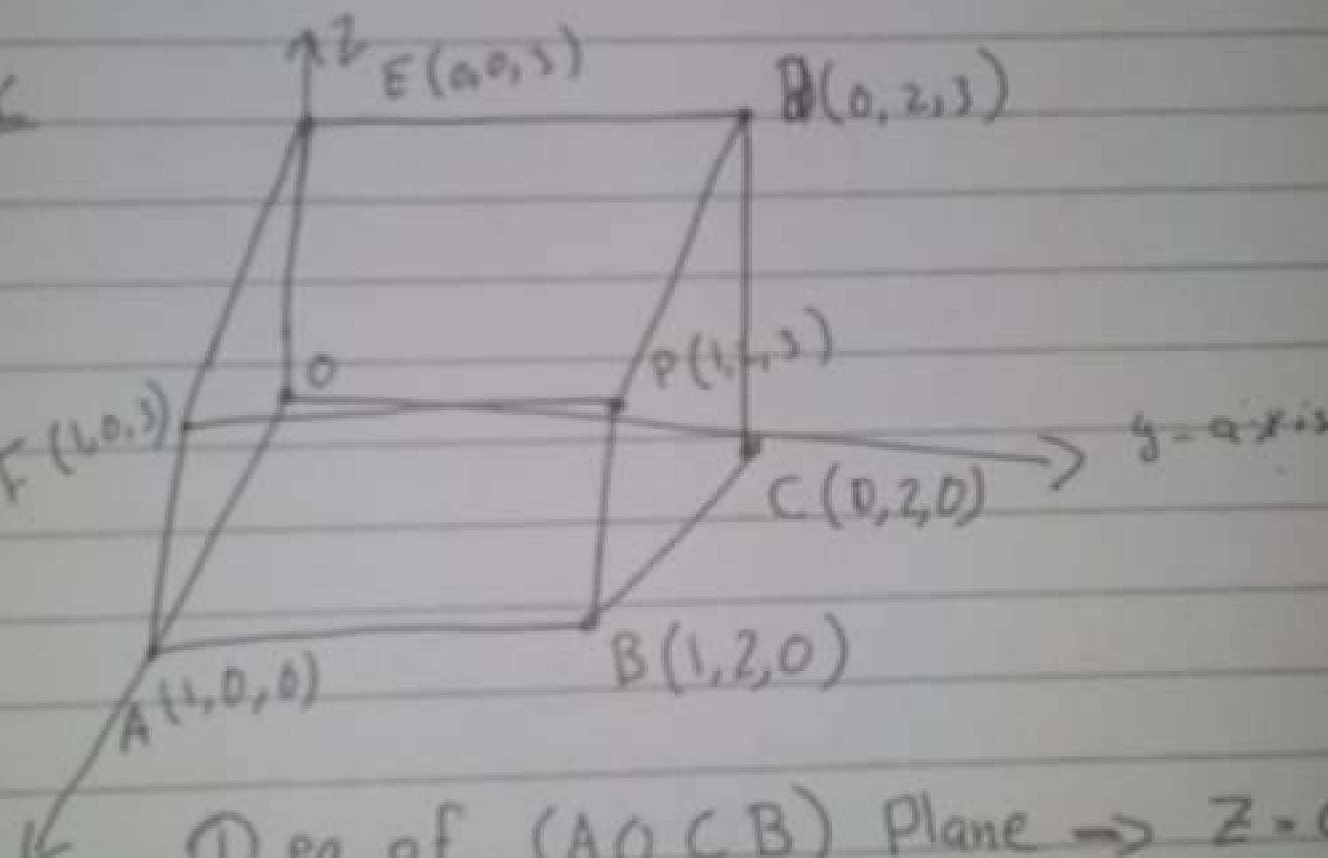
Line



V. Line

Line // y-axis

ex



① eq of (AOCB) Plane $\Rightarrow$ $z = 0$

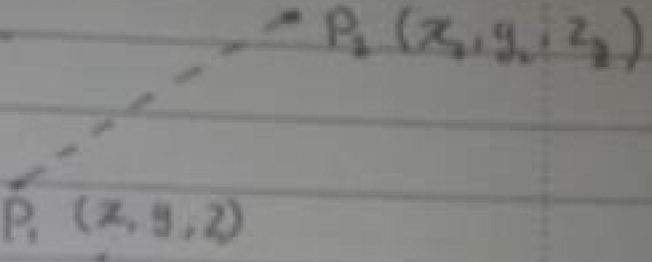
② eq of (FEOA) $\Rightarrow$ $y = 0$

③ eq of (BPDC) $\Rightarrow$ $y = 2$

④ eq of (EDCO) $\Rightarrow$ $x = 0$

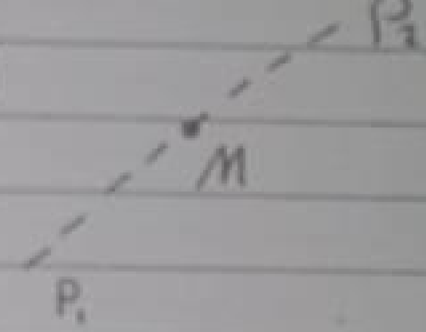
* Distance between 2-Points

$$d(P_1, P_2)=\sqrt{\Delta x^2+\Delta y^2+\Delta z^2}$$



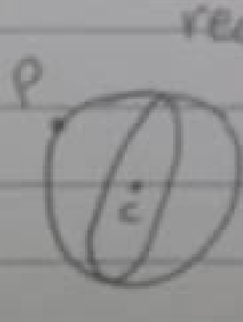
* Coordinate of the mid point

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$$



Defenetion of Sphere:-

the set of all points $p(x, y, z)$ in 3-space that are $\underbrace{\text{equidistant}}_{\text{red}}$ from a $\underbrace{\text{givin fixed point}}_{\text{center}}$



$$d(P,C) = r = \sqrt{(x-h)^2+(y-k)^2+(z-l)^2}$$

$r^2=(x-h)^2+(y-k)^2+(z-l)^2$ the eq of sphere

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ex: write an eqation of the sphere with end points of diameter points $(-3, 5, 0)$ & $(1, 3, 6)$.

$$C = \text{Mid point} = \left(\frac{-3+1}{2}, \frac{5+3}{2}, \frac{0+6}{2}\right)$$

$$= \left(\frac{-2}{2}, \frac{8}{2}, \frac{6}{2}\right) \rightarrow (-1, 4, 3)$$

$$r = \frac{\overline{P_1P_2}}{2} = \overline{P_1M} = \overline{P_2M}$$

$$\overline{P_1M} = \sqrt{(-3+1)^2 + (5-4)^2 + (0-3)^2}$$

Seq:

$$(x+1)^2 + (y-4)^2 + (z-3)^2 = 14$$

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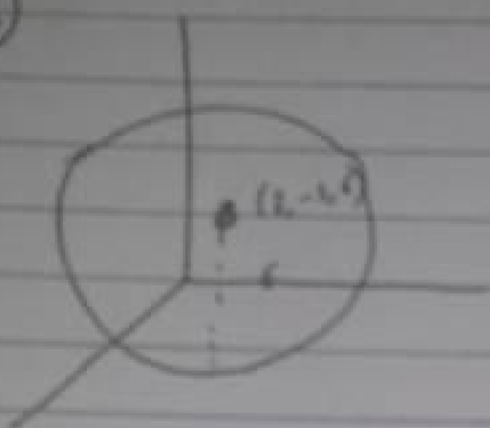
(Q)



i) the Sphere touches (x,y) Plane writ its eq.

reduis = 6

eq : $36 = (x-2)^2 + (y+3)^2 + (z-6)^2$

ii) touches xz-plane

redius is $|y| = 3$

$9 = (x-2)^2 + (y(+3))^2 + (z-6)^2$

(Q) write the eq of the Largest sphere $c\ (5,4,9)$ that is contained in the 1st octant

* Note : always take the smallest one.

$r = 4$

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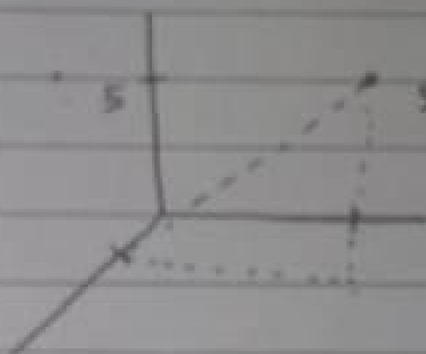
Q10 ? Distance between $P(3, 7, -5)$ &

i) xy-plane

$= |-5| = 5$

ii) x-axis



$= \sqrt{7^2 + 5^2}$

iii) y-axis

$= \sqrt{3^2 + 5^2}$

ex Find the Solution set: [Identify]

① $x^2 + (y-1)^2 + (z+1)^2 = 4$

Sphere, $C(0,1,-1)$, $r=2$

$\Rightarrow$ the solution set, the set of al points (x,y,z) on the sphere.

② $x^2 + (y-1)^2 + (z+1)^2 < 4$

$\Rightarrow$ the set of point, inside the sphere $C(0,1,-1)$, $r=2$

③ $x^2 + (y-1)^2 + (z+1)^2 > 4$

$\Rightarrow$ the set of points out side the Sphere.

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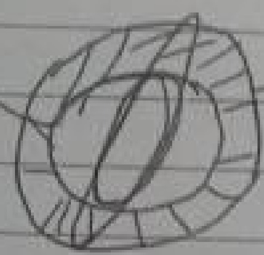
④ what region in $\mathbb{R}^3$ is reprented by these inqualities :-

$1 \leqslant x^2+y^2+z^2 \leqslant 4$, $z \leqslant 0$

① $1 \leqslant x^2+y^2+z^2$ ② $x^2+y^2+z^2 \leqslant 4$



$$x^2+y^2+z^2=4 \Longrightarrow z = \pm\sqrt{4-x^2-y^2}$$

Out side the smaller sphere & inside the Larger at xy-plane with z-direction

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* Quadratic eq in x, y, z :-

$$x^2+y^2+z^2+Ax+By+Cz+D=0$$

(*) Represents:

① Sphere ② single point, ③ empty, ϕ

* the eq represents "Sphere", if all ~~var~~ coefficients are equal.

ex (18) Identify or find the Solution :-

1) $4x^2+4y^2+4z^2+16y-8x=1$

$= 4(x^2-2x+1^2-1^2)+4(y^2+4y+2^2-2^2)+4z^2=1$

$= 4(x-1)^2+4(y+2)^2+4z^2-16+(-4)=1$

$\Rightarrow (x-1)^2+(y+2)^2+z^2=\frac{21}{4}$

So, it's sphere

$C(1,-2,0)$ / $r=\sqrt{\frac{21}{4}}$

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2) $4x^2 + 4y^2 + 4z^2 + 16y - 8x = -20$

$= 4(x-1)^2 + 4(y+2)^2 + 4z^2 = 0$

So, it's point $P(1, -2, 0)$

3) $4x^2 + 4y^2 + 4z^2 + 16y - 8x = -21$

$= 4(x-1)^2 + 4(y+2)^2 + 4z^2 = -1$

So, it's empty set, their is no point, ϕ

☆ Cylinders:

3-spaces

Def: any equation in $\mathbb{R}^3$ containing two variables out of three variables is a cylinder with axis is or parallel to the missing variable.

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a Identify:

① $x^2+y^2=4$

* cylinder with axis is $\underline{\underline{Z\text{-axis}}}$ · missing

② $x^2+y^2=4,\ z=0$

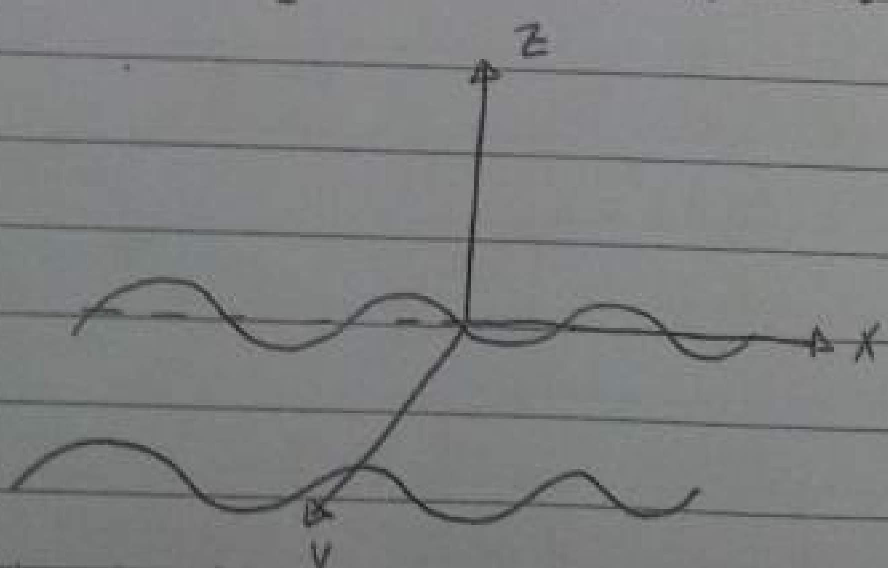
* $x^2+y^2=4$, cyLinder
* $Z=0$, xy-plane

the intersection is a circle → (curve)
$c\ (0,0)$, $r=2$ on xy-plane.

- sphere + plane = circle (curve)
- curve + curve = Point / Points
- Surface + surface = curve / Line

③ $Z = \sin x$

- cylinder with axis is y-axis



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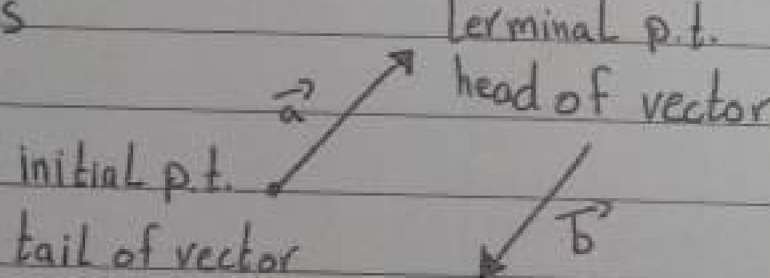
S 12.2 Vectors:-

- Def 1: Scaler: quantity with magnitude only.
  ((mass, distance, speed, ...))

- Def 2: Vector: quantity with magnitude and direction; ((weight, displacement, position, velocity, ...))

* Vector Representation:-

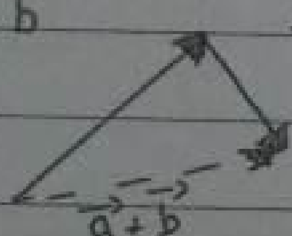
① arrows



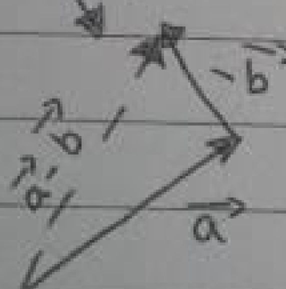
⊛ Vector addition:

1. head to tail addition

$\vec{a}+\vec{b} = \vec{b}+\vec{a}$

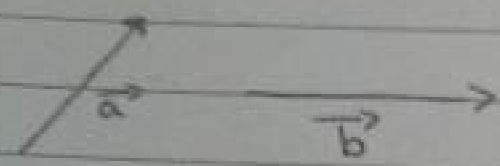


$\vec{a}-\vec{b}$
$= \vec{a}+(-\vec{b})$

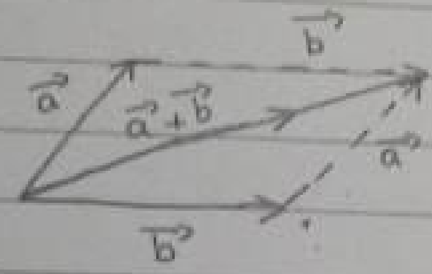


$\vec{a}+\vec{b}$ is a vector with In.p.t same as $\vec{a}$ & ter. p.t. same as $\vec{b}$

2. Parallelogram addition:



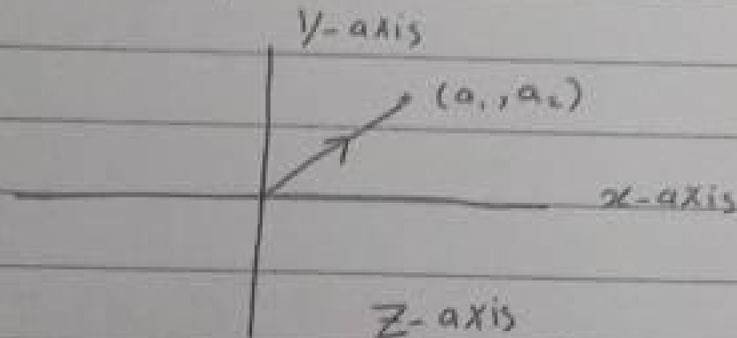
$\vec{a} + \vec{b} \Longrightarrow$



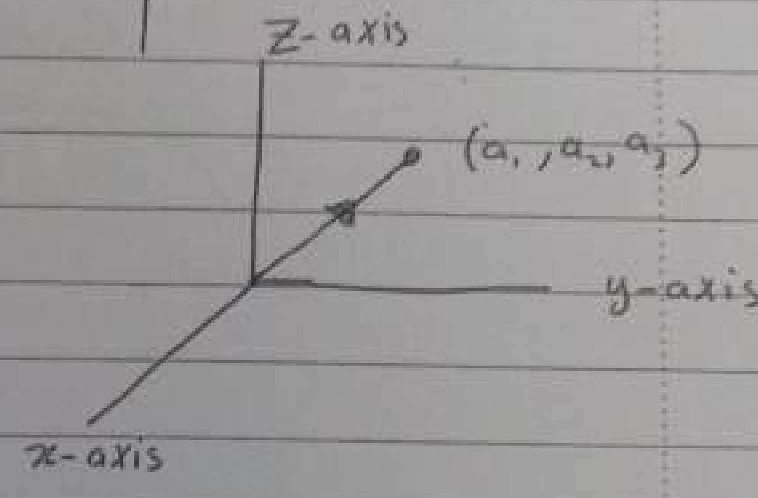
② Order pairs or triples

2-space 3-space

$\vec{a} = (a_1, a_2)$
$= \langle a_1, a_2 \rangle$



$\vec{a} = (a_1, a_2, a_3)$
$= \langle a_1, a_2, a_3 \rangle$



ex: $\vec{a} = \langle a_1, a_2 \rangle$
$\vec{b} = \langle b_1, b_2 \rangle$

Find $\vec{a} + \vec{b} = \langle \vec{a}_1 + \vec{b}_1 , \vec{a}_2 + \vec{b}_2 \rangle$

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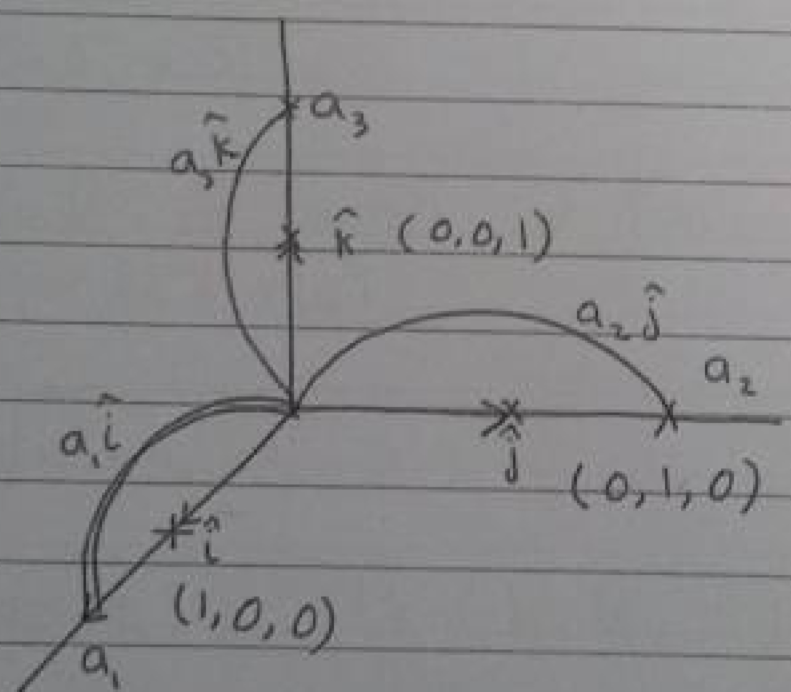
③ Vector Representation as a Linear combination of three Unit vectors.

$$\vec{a} = \langle a_1, a_2, a_3 \rangle$$

$$= \langle a_1, 0, 0 \rangle + \langle 0, a_2, 0 \rangle + \langle 0, 0, a_3 \rangle$$

$$= a_1 \langle 1, 0, 0 \rangle + a_2 \langle 0, 1, 0 \rangle + a_3 \langle 0, 0, 1 \rangle$$

$$\vec{a} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$$



$$\|\vec{a}\| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

ex:

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$
$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\vec{a} + \vec{b} = (a_1+b_1)\hat{i} + (a_2+b_2)\hat{j} + (a_3+b_3)\hat{k}$$
$$\vec{a} - \vec{b} = (a_1-b_1)\hat{i} + (a_2-b_2)\hat{j} + (a_3-b_3)\hat{k}$$

* $\vec{0} = 0\hat{i} + 0\hat{j} + 0\hat{k}$

Zero is a vector with no direction and its length is zero.

ex $\vec{a} - \vec{a} = \vec{0}$

* Magnitude or Length or Norm of a vector $\equiv \|\vec{a}\| = |\vec{a}|$

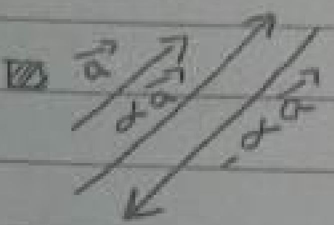
$$\|\vec{a}\| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

ex $P_1(a_1, a_2, a_3)$, $P_2(b_1, b_2, b_3)$

① $\overrightarrow{P_1P_2} = (b_1-a_1)\hat{i} + (b_2-a_2)\hat{j} + (b_3-a_3)\hat{k}$

② $\|\overrightarrow{P_1P_2}\| = \sqrt{(b_1-a_1)^2 + (b_2-a_2)^2 + (b_3-a_3)^2}$

* Scaler multiple of vector:- (Prallel Vector)



$\alpha \in \mathbb{R} \Longrightarrow \alpha\vec{a}$ is a vector such that $\alpha\vec{a}$ is prallel to $\vec{a}$

1) in a same direction if α is +ve
2) in opposit direction if α is -ve

- $\|\alpha\vec{a}\| = |\alpha|\|\vec{a}\|$

ex: $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$

① $\|-3\vec{a}\| = 3\|\vec{a}\| = 3\sqrt{4+1+9} = 3\sqrt{14}$

[Note: $-3\vec{a} = -6\hat{i} + 3\hat{j} - 9\hat{k}$]

② $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$
$\vec{b} = -4\hat{i} + 2\hat{j} - 6\hat{k}$

IS $a \parallel b$??

$\Longrightarrow \vec{b} = -2\vec{a}$

$\hat{i}$

$\frac{-4}{2} = \frac{2}{-1} = \frac{-6}{3} = -2$ ✓

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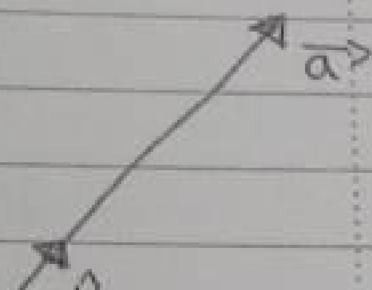
* Normalising a vector :-

"to get a vector of Length = 1"

☆ $\vec{a}$ is a vector

$\hat{u}_{\vec{a}} = \dfrac{\vec{a}}{\|\vec{a}\|}$ [at the Same direction]



to get a Unit vector in opposite direction Use $\dfrac{-\vec{a}}{\|\vec{a}\|}$

$\hat{u}_{\vec{a}}$ [unit vector in a direction of $\vec{a}$]

$\Longrightarrow \vec{a} = \|\vec{a}\| \cdot \hat{u}_{\vec{a}}$

$\boxed{\dfrac{24}{277}}$ Find a vector $\vec{a}$ that has the same direction as the Vector $\underbrace{\langle -2, 4, 2 \rangle}_{\vec{b}}$ but has Length 6

Sol: i) direction of $\vec{b} = \hat{u}_{\vec{b}}$

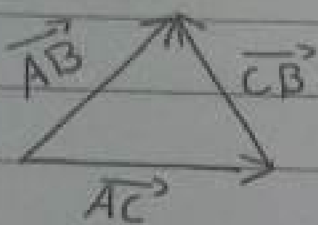
$= \dfrac{\langle -2, 4, 2 \rangle}{\sqrt{4+16+4}} = \dfrac{1}{\sqrt{24}} \langle -2, 4, 2 \rangle = \hat{u}_{\vec{a}}$

ii) $\vec{a} = 6\, \hat{u}_{\vec{a}}$

ex $\overrightarrow{AB} = 2\hat{i} + 3\hat{j} - \hat{k}$

$\overrightarrow{AC} = -\hat{i} + 2\hat{j} + 4\hat{k}$

Find $\overrightarrow{CB} = ??$



$\Rightarrow \overrightarrow{AC} + \overrightarrow{CB} = \overrightarrow{AB}$

$\overrightarrow{CB} = \overrightarrow{AB} - \overrightarrow{AC}$

$= (2 - -1)\hat{i} + (3-2)\hat{j} + (-1-4)\hat{k}$

$\overrightarrow{CB} = 3\hat{i} + \hat{j} - 5\hat{k}$

☆ equality of vectors :-

① $\vec{a} = \vec{b} \Longleftrightarrow$ same magnitude & same direction

OR

② $\vec{a} = \vec{b}$ $[\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}]$ $[\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}]$

$\Rightarrow a_1 = b_1 \,/\, a_2 = b_2 \,/\, a_3 = b_3$

$\vec{a} \parallel \vec{b} \Longleftrightarrow \vec{b} = \alpha\vec{a}$

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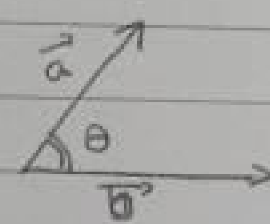
S : 12.3

Dot (Scaler) Product "$\cdot$"

Let $\vec{a}$ & $\vec{b}$ be 2-vectors (2/3 - spaces)

such that: $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$.

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$



θ must be $0^\circ \leq \theta \leq \pi^\circ$

*Then :

① $\vec{a} \cdot \vec{b} = \|\vec{a}\| \, \|\vec{b}\| \cos\theta$

② $\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 \qquad \in R$

from ① & ② $\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \, \|\vec{b}\|}$

ex $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

$\vec{b} = 4\hat{i} - 10\hat{k}$

* find the angle between $\vec{a}$ & $\vec{b}$

$$\theta = \cos^{-1}\left(\frac{2(4) + (-3)(0) + (4)(-10)}{\sqrt{4+9+16}\,\sqrt{16+100}}\right) = \frac{-32}{\sqrt{29(16)}}$$

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* Properties of Dot Product:-

1) $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

* 2) $\vec{a} \cdot \vec{a} = (\|\vec{a}\|)^2$

3) $l\vec{a} \cdot m\vec{b} = (lm)(\vec{a} \cdot \vec{b})$

4) $\vec{a} \cdot (\vec{b} + \vec{c}) = (\vec{a} \cdot \vec{b}) + (\vec{a} \cdot \vec{c})$

5) $\vec{a} \cdot \vec{b} = 0 \Longrightarrow \underset{\text{in btwn}}{\theta} = 90 \Longrightarrow$ number cuz it's dot Product.

i) $\vec{a} = \vec{0}$ ii) $\vec{b} = \vec{0}$ iii) $\vec{a} \perp \vec{b}$ $\theta = \frac{\pi}{2}$

6) $\vec{a} \perp \vec{b} \Longleftrightarrow \vec{a} \cdot \vec{b} = 0$

they aren't Zero

7) $\hat{i} \cdot \hat{j} = 0$ $[\theta = 90^\circ]$

8) $\hat{i} \cdot \hat{i} = 1$

ex Given two vectors $\|\vec{a}\| = 5$, $\|\vec{b}\| = 8$, $\theta = \frac{2\pi}{3}$

* find $\|2\vec{a} - 3\vec{b}\| = ??$

Sol:

$\|2\vec{a} - 3\vec{b}\|^2 = (2\vec{a} - 3\vec{b})(2\vec{a} - 3\vec{b})$

$= 4\vec{a} \cdot \vec{a} - 6\vec{a} \cdot \vec{b} - 6\vec{a} \cdot \vec{b} + 9\vec{b} \cdot \vec{b}$

$= 4\|a\|^2 - 12\|\vec{a}\| \cdot \|\vec{b}\| \cdot \cos\frac{2\pi}{3} + 4\|\vec{b}\|^2$ $\Longrightarrow$ Cont

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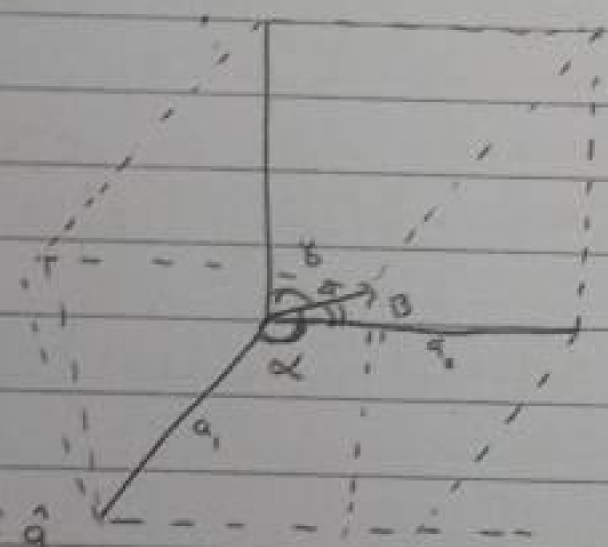
$$= 100 - (12)(5)(8)\left(\frac{-1}{2}\right) + 9(46)$$

$$\|2\vec{a} - 3\vec{b}\| = \sqrt{(100) - (12)(5)(8)\left(\frac{-1}{2}\right) + 9(64)}$$

* Direction Angles and Direction Cosines.



α, β, γ are direction angles

$\vec{a} = \|\vec{a}\| \hat{U}_{\vec{a}}$

↳ Unit vector in a direction of $\hat{a}$

$\vec{a} = \|\vec{a}\| (\cos\alpha \hat{i} + \cos\beta \hat{j} + \cos\gamma \hat{k})$

* $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

- $\cos\alpha = \frac{a_1}{\|a\|}$ • $\cos\beta = \frac{a_2}{\|a\|}$ • $\cos\gamma = \frac{a_3}{\|a\|}$

ex Can 45°, 45°, 60° For direction angel for any Vector?!

$\cos^2 45 + \cos^2 45 + \cos^2 60 \overset{?}{=} 1$

$\frac{1}{2} + \frac{1}{2} + \frac{1}{4} \overset{??}{=} 1$

$1 + \frac{1}{4} \neq 1$

ex: Find vector(s) $\vec{a}$: $\|\vec{a}\| = 10$ &
$\alpha = 60^\circ, \beta = 135^\circ$

$\vec{a} = \|\vec{a}\| \cos x + \cos \beta + \cos c$

$\Rightarrow \cos^2 60 + \cos^2 135 + \cos^2 \gamma = 1$

$\cos \gamma = \pm \frac{1}{2}$

- If $\cos \gamma = \frac{1}{2} \Rightarrow \vec{a} = 10\left(\left(\frac{1}{2}\right)\hat{i} + \left(\frac{-1}{\sqrt{2}}\right)\hat{j} + \frac{1}{2}\hat{k}\right)$

- If $\cos \gamma = \frac{-1}{2} \Rightarrow \vec{a} = 10\left(\left(\frac{1}{2}\right)\hat{i} + \left(\frac{-1}{2}\right)\hat{j} - \frac{1}{2}\hat{k}\right)$

* $\vec{a} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$

\# Find: ① $\|\vec{a}\| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

② $\vec{a} \cdot \hat{i} = a_1$

$\cos \alpha = \frac{a_1}{\|a\|}$

ex $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

Find:

① $\|\vec{a}\| = \sqrt{29}$

② the direction of $\vec{a}$ ((unit vector in its direc))

$= \frac{\vec{a}}{\|\vec{a}\|} = \frac{2}{\sqrt{29}}\hat{i} - \frac{3}{\sqrt{29}}\hat{j} + \frac{4}{\sqrt{29}}\hat{k}$

Subject Date No.

③ Find the cosin of the angel that $\vec{a}$ makes with z-axis:-

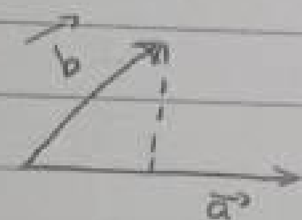
$\cos\gamma = \frac{4}{\sqrt{29}}$

④ Find the direct angel of $\vec{a}$:

$\alpha = \cos^{-1}\left(\frac{2}{\sqrt{29}}\right)$

Subject Cal-3 Date 18/6/2012 No.

✯ Scaler projection of $\vec{b}$ on to $\vec{a}$



$\equiv$ Component of $\vec{b}$ on to $\vec{a}$

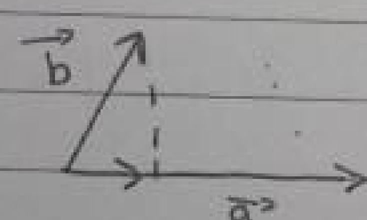
$= \text{Comp}_{\vec{a}} \vec{b} = \|\vec{b}\| \cos\theta$

$= \|\vec{b}\| \cos\theta \cdot \dfrac{\|\vec{a}\|}{\|\vec{a}\|}$

$\text{Comp}_{\vec{a}} \vec{b} = \dfrac{\vec{b} \cdot \vec{a}}{\|\vec{a}\|}$ [scaler]

Note : Scaler ●——● Vector ●——→ ●——→●

✯ Vector projection of $\vec{b}$ on to $\vec{a}$

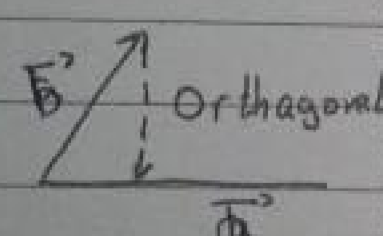


$\equiv \text{proj}_{\vec{a}} \vec{b}$

$= \dfrac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|} \cdot \dfrac{\vec{a}}{\|\vec{a}\|}$

$= \dfrac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|^2} \cdot a$ [Vector]

✯ Orthagonal projection of $\vec{b}$ on to $\vec{a}$



✶ Vector proj of $\vec{b}$ Orthagonal to $\vec{a}$

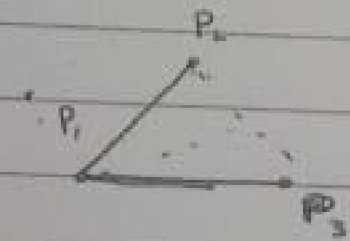
$= \text{Orth}_{\vec{a}} \vec{b} = \vec{b} - \text{Proj}_{\vec{a}} \vec{b}$

Subject Date No.

<u>ex</u> Given: $\overrightarrow{P_1P_2} = 2\hat{i} + 3\hat{j} + \hat{k}$

$\overrightarrow{P_1P_3} = -\hat{i} + 2\hat{j} - 3\hat{k}$

① find $\text{proj}_{\overrightarrow{P_2P_3}} \overrightarrow{P_1P_2} = \dfrac{\overrightarrow{P_1P_2} \cdot \overrightarrow{P_2P_3}}{\|P_2P_3\|^2} \cdot \overrightarrow{P_2P_3}$



?? $\overrightarrow{P_2P_3} = \overrightarrow{P_2P_1} + \overrightarrow{P_1P_3}$

$= -\overrightarrow{P_1P_2} + \overrightarrow{P_1P_3}$

$= -3\hat{i} - \hat{j} - 4\hat{k}$

$$\text{Proj}_{\overrightarrow{P_2P_3}} \overrightarrow{P_1P_2} = \frac{2(-3) + 3(-1) + (-4)}{(\sqrt{9+1+16})^2} \cdot \overrightarrow{P_2P_3} = \frac{-13}{26} \cdot \overrightarrow{P_2P_3}$$

$$= -\frac{1}{2} \overrightarrow{P_2P_3} = \frac{3}{2}\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k}$$

② find $\text{Orth}_{\overrightarrow{P_1P_3}} \overrightarrow{P_1P_2}$:

$$\text{Orth}_{\overrightarrow{P_1P_3}} \overrightarrow{P_1P_2} = \overrightarrow{P_1P_2} - \text{Pro}_{\overrightarrow{P_1P_3}} \overrightarrow{P_1P_2}$$

$$= \left(2 - \frac{3}{2}\right)\hat{i} + \left(3 - \frac{1}{2}\right)\hat{j} + (1-2)\hat{k}$$

③ $$\text{Comp}_{\overrightarrow{P_2P_3}} \overrightarrow{P_1P_2} = \frac{\overrightarrow{P_1P_2} \; \overrightarrow{P_2P_3}}{\|\overrightarrow{P_2P_3}\|} = \frac{-13}{\sqrt{26}}$$

Subject Date No.

43. If $\vec{a} = \langle 3, 0, -1 \rangle$, find a vector $\vec{b}$
$comp_{\vec{a}} \vec{b} = 2$

Sol:

Let $\vec{b} = \langle b_1, b_2, b_3 \rangle$

$$comp_{\vec{a}} \vec{b} = \frac{\vec{b} \cdot \vec{a}}{\|\vec{a}\|} = 2$$

$$= \frac{3b_1 + 0(b_2) + (-1) b_3}{\sqrt{10}} = 2$$

$$= 3b_1 - b_3 = 2\sqrt{10}$$

$$\Longrightarrow b_3 = 3b_1 - 2\sqrt{10}$$

$\vec{b} \ \langle b_1, b_2, 3b_1 - 2\sqrt{10} \rangle$ b_1, b_2 are arbitrary.

Sol. $b_1 = 0$, $b_2 = 5$ $\Longrightarrow \langle 0, 5, -2\sqrt{10} \rangle$

NOTEBOOK

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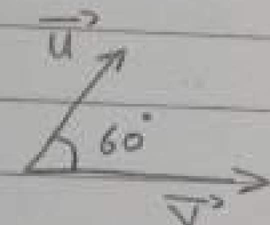
طول

28) Find 2-Unit vectors that make an angel of 60° with $\vec{v} = \langle 3, 4 \rangle$



Sol:

let $\vec{u} = \langle u_1, u_2 \rangle$

* $\|\vec{u}\| = 1 \Rightarrow \sqrt{u_1^2 + u_2^2} = 1 \Rightarrow u_1^2 + u_2^2 = 1 \Rightarrow$ ①
* $\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos 60°$

$3u_1 + 4u_2 = (1)(5)(\frac{1}{2}) \Rightarrow$ ②

from ① & ②

* $4u_2 = \frac{5}{2} - 3u_1$

$u_1^2 + \left(\frac{5}{8} - \frac{3}{4}u_1\right)^2 = 1$

$u^2 + \frac{25}{46} - \frac{15}{16}u_1 + \frac{9}{16}u_1^2 = 1$

$\frac{25}{16}u_1^2 - \frac{15}{16}u_1 + \frac{25}{64} - 1 = 0$

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60) Show that a+V & a-V are Orthagonal and the vector a & V have the same length:

$$?? \ \|\vec{a}\| = \|\vec{v}\|$$

Given $\vec{u}+\vec{v} \perp \vec{u}-\vec{v}$

$$\Rightarrow (\vec{u}+\vec{v}) \cdot (\vec{u}-\vec{v}) = 0$$

$$\vec{u}\cdot\vec{u} - \vec{u}\vec{v} + \vec{u}\cdot\vec{v} - \vec{v}\cdot\vec{v} = 0$$

$$\|\vec{u}\|^2 - \|\vec{v}\|^2 = 0$$

So, $\|\vec{u}\|^2 = \|\vec{v}\|$

Subject Cal-3 Date 19/6/2012 No.

S 12.4 Cross (vector) Product

* let $\vec{a}$ & $\vec{b}$ be 2-Vectors in 3-Space
θ is the angle between them &
$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$
$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$
Then $\Longrightarrow$ $\vec{a} \times \vec{b}$ is a vector with
• Magnitude $\|\vec{a} \times \vec{b}\| = \|\vec{a}\| \, \|\vec{b}\| \sin\theta$ & "the Direction is $\perp$ to both $\vec{a}$ & $\vec{b}$" (that means $\Rightarrow$ $\vec{a} \times \vec{b} \perp$ Plane that contains $\vec{a}$ & $\vec{b}$)

* Def: $\vec{a} \cdot \vec{a} \times \vec{b}$
• $\vec{a} \cdot \underbrace{\vec{a} \times \vec{b}} = 0$
since $\vec{a} \times \vec{b} \perp \vec{a}$
and $\cos 90^\circ = 0$
• $\vec{b} \cdot \vec{a} \times \vec{b} = 0$ since $\vec{a} \times \vec{b} \perp \vec{b}$

Note: تبدأ بالـ "X" أو "."
لأن الـ "." تنتج Scaler وبيصبح
Scaler X Vector وهذا ليس له معنى

ex $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = 2(5) - 3(4) = 10 - 12 = -2 \in R$

ex $\begin{array}{ccc} \oplus & \ominus & \oplus \end{array}$
$\begin{vmatrix} 2 & -3 & -4 \\ 0 & 1 & 5 \\ 4 & 1 & 3 \end{vmatrix} = 2\begin{vmatrix} 1 & 5 \\ 1 & 3 \end{vmatrix} - (-3)\begin{vmatrix} 0 & 5 \\ 4 & 3 \end{vmatrix} + (-4)\begin{vmatrix} 0 & 1 \\ 4 & 1 \end{vmatrix}$
$= 2(3-5) + 3(0-20) + (-4)0-4)$
$= -4 - 60 + 16 = -48$

* $\vec{a} \times \vec{b} =$ $\begin{array}{ccc} \oplus & \ominus & \oplus \end{array}$
$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = [(a_2 b_3) - (a_3 b_2)]\hat{i} - [(a_1 b_3) - (a_3$
$\hat{j} + [(a_1 b_2) - (a_2 b_1)]\hat{k}$

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① $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a} \rightsquigarrow \vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$

↳ in opposite direction

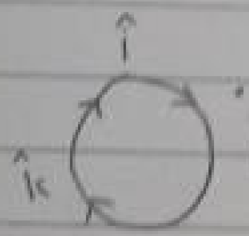
② $\vec{a} \times \vec{a} = \vec{0}$, $\vec{a} \times -\vec{a} = 0$

لأن الزاوية 0 أو 180

$\sin 0 = \sin 180 = 0$

ex $\hat{i} \times \hat{j} = \hat{k}$

Not



إذا مشينا مع عقارب الساعة تعطي (+) وعكس عقارب الساعة (-)

[المصفوفات تعطي نفس الحل] ⟶

Properties

③ $\vec{a} \times (\vec{b} + \vec{c}) = (\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c})$

بمعنى آخر نستطيع أخذ عامل مشترك

④ $(\vec{b} + \vec{c}) \times \vec{a} = (\vec{b} \times \vec{a}) + (\vec{c} \times \vec{a})$

انتبه الترتيب

⑤ $\vec{a} \parallel \vec{b} \Longrightarrow \vec{a} \times \vec{b} = \vec{0}$

لإثبات أن 2.Vec متوازيان نثبت أن "X" لهما = 0

⑥ $l\vec{a} \times m\vec{b} = lm\ \vec{a} \times \vec{b}$

ex $2\vec{a} \times -3\vec{b} = -6\ \vec{a} \times \vec{b}$

N O T E B O O K

Subject Cal-3 Date 20/6/2012 No.

* $\vec{a} \times (\vec{b} \times \vec{c})$

هنا العمليه ليست تجميعيه لذلك

لايجوز تغيير القوس

But $\Rightarrow$ $\left[\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} \right]$

ex $\vec{a} = 2\hat{i} + 3\hat{j} - k$, $\vec{b} = -4\hat{i} + 2\hat{k}$, $\vec{c} = 3\hat{i} + 2\hat{j} + 3\hat{k}$

i) Find : $\vec{a} \times (\vec{b} \times \vec{c})$

$$\begin{aligned}
\cdot\ \vec{a} \times (\vec{b} \times \vec{c}) &= (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} \\
&= (6 + 6 - 3)\vec{b} - (-8 - 2)\vec{c} \\
&= 9\vec{b} + 10\vec{c} \\
&= (-36\hat{i} + 18\hat{k}) + (30\hat{i} + 20\hat{j} + 30\hat{k}) \\
&= -6\hat{i} + 20\hat{j} + 48\hat{k}
\end{aligned}$$

ii) Find $(\vec{a} \times \vec{b}) \times \vec{c}$ [is it = to $\vec{a} \times (\vec{b} \times \vec{c})$]

No

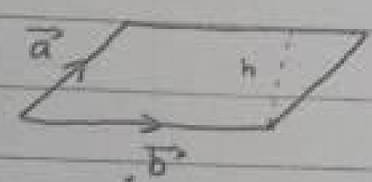
$$\begin{aligned}
&= -\vec{c} \times (\vec{a} \times \vec{b}) \\
&= -\left[(\vec{c} \cdot \vec{b})\vec{a} - (\vec{c} \cdot \vec{a})\vec{b} \right] \\
&=
\end{aligned}$$

iii) Find $2\vec{a} \times (3\vec{b} \times -\vec{c})$

$$\begin{aligned}
&= 2(3)(-1)\left[\vec{a} \times (\vec{b} \times \vec{c}) \right] \\
&= -6 \left[-6\hat{i} + 20\hat{j} + 48\hat{k} \right]
\end{aligned}$$

* Geometrical meaning of $\|\vec{a} \times \vec{b}\|$

$\Rightarrow |\vec{a} \times \vec{b}|$ is the area of the parallelogram with sides $\vec{a}$ & $\vec{b}$



Proof

$$\text{area} \,\square = \text{base} \cdot \text{height}$$
$$= |\vec{b}| \cdot h \qquad \text{but } h = |\vec{a}| \sin\theta$$
$$= |\vec{b}| \cdot |\vec{a}| \cdot \sin\theta$$
$$= |\vec{a} \times \vec{b}|$$

Note: $|\vec{a}| = |-\vec{a}|$

$|\vec{a} \times \vec{b}| = |-\vec{b} \times \vec{a}|$ لهما نفس الطول ولكن عكس

الاتجاه [Mag لها متساويات]

ex Find the Area of $\triangle$ with vertices $A(1,2,3)$, $B(-2,0,2)$, $C(2,3,5)$

$$\text{Area}\,\triangle = \frac{1}{2}\,\text{area}\,\square$$

$\Rightarrow \vec{AB} = (-2-1)\hat{i} + (0-2)\hat{j} + (1-3)\hat{k}$

$= -3\hat{i} - 2\hat{j} - 2\hat{k}$

$\vec{AC} = \hat{i} + \hat{j} + 2\hat{k}$

$\vec{AB} \times \vec{AC}$

$$\begin{vmatrix} i & j & k \\ -3 & -2 & -2 \\ 1 & 1 & 2 \end{vmatrix} = -2\hat{i} + 4\hat{j} - \hat{k}$$

Now: $A_{\square} = \frac{1}{2} |\vec{AB} \times \vec{AC}|$

$$= \frac{\sqrt{4+16+1}}{2} = \frac{\sqrt{21}}{2}$$

Subject Date No.

* Triple scaler product:-

$$\vec{a} \cdot \vec{b} \times \vec{c} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

ال "." قبل "X"

<u>ex</u> $\vec{a} = 2\hat{i} + 3\hat{k}$

$\vec{b} = -\hat{i} + 2\hat{j} - 4\hat{k}$ ① Find $\vec{b} \times \vec{c} \cdot \vec{a}$

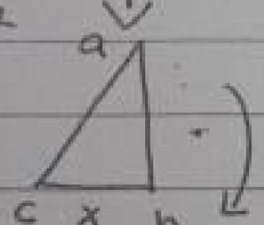
$\vec{c} = \hat{i} + 3\hat{j} + 2\hat{k}$

$$\vec{b} \times \vec{c} \cdot \vec{a} = \vec{a} \cdot \vec{b} \times \vec{c}$$

لا نضع (–) لأن (.) تبديلي

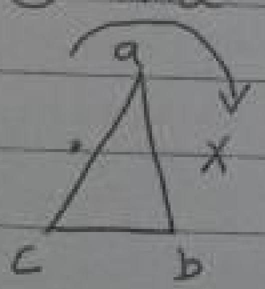
$$= \begin{vmatrix} \overset{+}{2} & \overset{-}{0} & \overset{+}{3} \\ -1 & 2 & -4 \\ 1 & 3 & 2 \end{vmatrix} = 2(16) - 0 + 3(-5)$$

$$= 32 - 15 = 17$$



إذا كان السؤال مع اتجاه السهم فبنوخذ نفس اشارة الجواب

② Find $\vec{c} \cdot \vec{a} \times \vec{b}$



$= 17$

③ Find $2\vec{a} \cdot \vec{c} \times 3\vec{b}$

$= 6 \cdot [\vec{a} \cdot \vec{c} \times \vec{b}]$

$= 6(-17)$

$=$

* Geometrical meaning of $|\vec{a}\cdot\vec{b}\times\vec{c}\,|$

Volume of the parallelepiped with adjacent sides $\vec{a},\vec{b},\vec{c}$

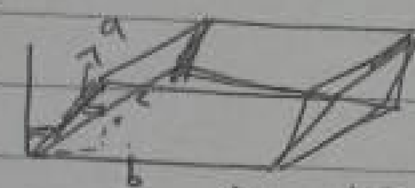
$V =$ area of the Base . hegh t (متجاورة (نفس النقطة

$= |\vec{b}\times\vec{c}\,|\,h \;;\; h = |\vec{a}|\cos\theta$

$= \|\vec{b}\times\vec{c}\,\|\,\|\vec{a}\|\,\cos\theta$

$= \|\vec{b}\times\vec{c}\cdot\vec{a}\,\|$

قيمة مطلقة لأنه الجواب ممكن يطلع سالب والـ Volume موجب



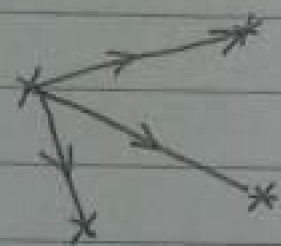
متوازي الأوجه

(*) θ بين $\vec{a}$, $\vec{b}\times\vec{c}$

ex $\vec{a} \cdot \vec{b} \times \vec{c} = 0 \iff \vec{a}, \vec{b}$ & $\vec{c}$ are Coplanner

[مشتركين بنفس الـ Plane]

$\frac{38}{793}$] Whatever the 4-point $A(1,3,2)$, $B(3,-1,6)$ $C(5,2,0)$ and $D(3,6,-4)$ are Coplanner or not:-



I want to check : $\overrightarrow{AB} \cdot \overrightarrow{AC} \times \overrightarrow{AD} \stackrel{?}{=} 0$

$\overrightarrow{AB} = 2\hat{i} - 4\hat{j} + 4\hat{k}$

$\overrightarrow{AC} = 4\hat{i} - 1\hat{j} - 2\hat{k}$

$\overrightarrow{AD} = 2\hat{i} + 3\hat{j} - 6\hat{k}$

$$\begin{array}{ccc} \oplus & \ominus & \oplus \end{array}$$

$$\begin{vmatrix} 2 & -4 & 4 \\ 4 & -1 & -2 \\ 2 & 3 & -6 \end{vmatrix} = 2(12) + 4(-20) + 4(14)$$

$$= 24 - 80 + 56$$

$$= 80 - 80 = 0$$

• So, they are Coplanner.

Subject Date No.

20 Find 2-Unit Vectors Orthagonal to both $\underbrace{\hat{i} + \hat{j} + \hat{k}}_{A}$ & $\underbrace{2\hat{i} + \hat{k}}_{B}$

$$\vec{A} \times \vec{B} \perp \vec{A} \text{ \& } \vec{B}$$

$$\mp \frac{\vec{A} \times \vec{B}}{\| \vec{A} \times \vec{b} \|}$$ [Unit] [Unit يعني الطول قسمة على]

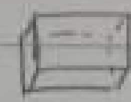
$$\vec{A} \times \vec{B} = \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{vmatrix} \end{array} = \hat{i} + \hat{j} - 2\hat{k}$$

$$\| \vec{A} \times \vec{B} \| = \sqrt{1+1+4} = \sqrt{6}$$

the two unit vectors are

$$\pm \frac{1}{\sqrt{6}} (\hat{i} + \hat{j} - 2\hat{k})$$

$\frac{51}{785}$



ضلع $\vec{e}$: edge $\quad \vec{d}$: diagonal قطر

$\vec{f}$: face

$$\vec{e} = a\hat{i} \;,\; \vec{d} = a\hat{i} + a\hat{j} + a\hat{k}$$

[الزاوية بين الـ e & d]

$$\cos\theta_1 = \frac{\vec{e}\cdot\vec{d}}{\|\vec{e}\|\,\|\vec{d}\|} = \frac{a^2}{a\sqrt{3a^2}} = \frac{1}{\sqrt{3}}$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

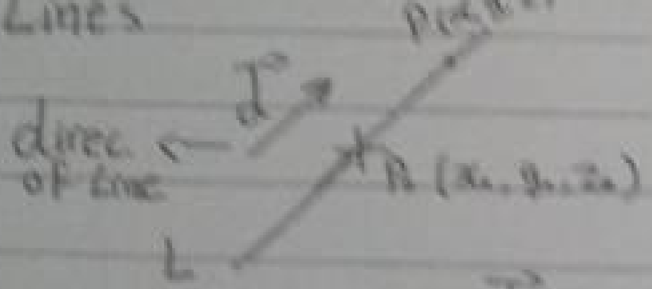
52) [الزاوية بين ~~القطر~~ قطر المكعب وقطر أحد الأوجه] angel between d & f

$$f = a\hat{i} + a\hat{j}$$

$$\cos\theta_2 = \frac{f\cdot d}{\|f\|\,\|d\|}$$

S. 12.5 Lines & Planes

* Lines



$P_0(x_0, y_0, z_0) \in L$ & $\vec{d} = d_1\hat{i} + d_2\hat{j} + d_3\hat{k}$

* $\vec{d} \parallel L$
* $\vec{P_0P} \parallel \vec{d} \iff \vec{P_0P} = t\vec{d}$, $t \in R$

(I) $\left[(x - x_0)\hat{i} + (y - y_0)\hat{j} + (z - z_0)\hat{k} = t(d_1\hat{i} + d_2\hat{j} + d_3\hat{k})\right]$

$\Rightarrow (x - x_0) = td_1$ / $x = x_0 + td_1$

$\Rightarrow (y - y_0) = td_2$ / $y = y_0 + td_2$ (II) Parametric equations of Line

$\Rightarrow (z - z_0) = td_3$ / $z = z_0 + td_3$

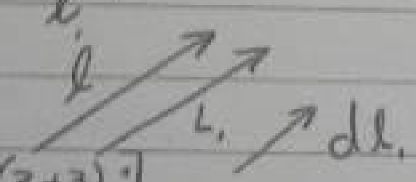
(III) Symmetric equations of Line:

$$t = \left[\frac{x - x_0}{d_1} = \frac{y - y_0}{d_2} = \frac{z - z_0}{d_3}\right]$$

ℓ_1

ex write $\underbrace{\text{Prammetric equations}}$ of Line passes through $P_1(1,-2,3)$ and Parallel to Line $\ell\left[\frac{x-1}{3}=\frac{1-y}{4}=\frac{2z+6}{10}\right]$

$\vec{d_{\ell_1}} \parallel \vec{d_\ell}$



$\ell_1\left[\frac{x-1}{3}=\frac{-(y-1)}{4}=\frac{2(z+3)}{10}\right]$

$d\ell_1 = 3\hat{i}-4\hat{j}+5\hat{k}$ اتجاه الخط المطلوب

ℓ:

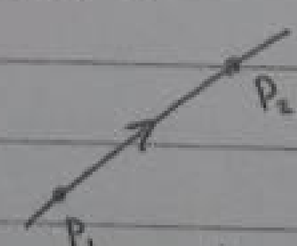
$[x=1+3t,\ y=-2-4t,\ z=3+5t] \quad t\in\mathbb{R}$

24/6/2012

* Eq.s of Line containing 2-Points

ex write prammetric eqs of Line passes through points, $P_1(1,2,-1)$ and $P_2(0,3,5)$

$\vec{d}\,(P_1P_2) = \langle -1,1,6\rangle$



ℓ: $x=1-t,\ y=2+t,\ z=-1+6t$

test P_2 on ℓ

$0=1-t \Rightarrow t=1$	$3=2+t \Rightarrow t=1$	$5=-1+6t \Rightarrow t=1$

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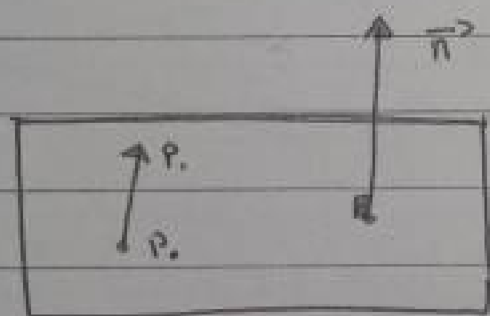
* Planes : ($\mathbb{P}$)

eq.s of a plane $P_0(x_0, y_0, z_0) \in \mathbb{P}$
and $\vec{n} = a\hat{i} + b\hat{j} + c\hat{k}$, normal to plane

* $\overrightarrow{P_0P} \perp \vec{n} \iff \overrightarrow{P_0P} \cdot \vec{n} = 0$
* $\overrightarrow{P_0P} = \langle x - x_0, y - y_0, z - z_0 \rangle$



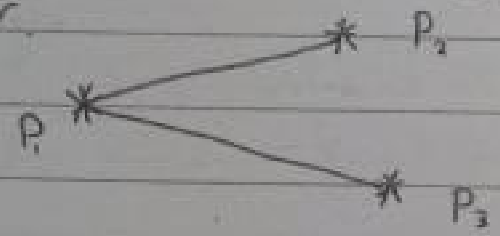
$$\Rightarrow \boxed{a(x - x_0) + b(y - y_0) + c(z - z_0) = 0}$$

$$\Rightarrow \boxed{ax + by + cz + d = 0}$$

ex: write an eq of plane passes through $P_1(-1,2,3)$ and normal is $\vec{n} = 2\hat{i} + 3\hat{j} - 5\hat{k}$

$$P: 2(x+1) + 3(y-2) - 5(z-3) = 0$$

~~ex~~ * Eqs of plane containing 3- Points not collinear.



* $\vec{n} = \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3}$

ex $P_1(1,2,-1)$, $P_2(0,1,3)$ and $P_3(2,1,4)$

$$\vec{n} = \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -1 & 4 \\ 1 & -1 & 5 \end{vmatrix} = -\hat{i} + 9\hat{j} + 2\hat{k}$$

$$\Rightarrow P: -(x-1) + 9(y-2) + 2(z+1) = 0$$

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ex $\ell_1: x = -3t$, $y = 1+2t$, $z = 3-t$; $\ell_2: x = 1+6s$, $y = -4s$, $z = 2+2s$

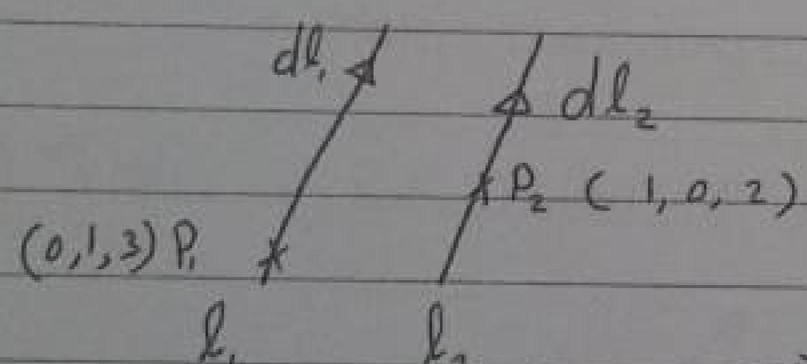
Write plane containing ℓ_1 & ℓ_2

Sol:

$\overrightarrow{d\ell_1} <-3, 2, -1>$

$\overrightarrow{d\ell_2} <6, -4, 2>$

$\overrightarrow{d\ell_1} \parallel \overrightarrow{d\ell_2} \iff \ell_1 \parallel \ell_2$



$\vec{n} = \overrightarrow{P_1P_2} \times \overrightarrow{d\ell_1} = \overrightarrow{P_1P_2} \times \overrightarrow{d\ell_2}$

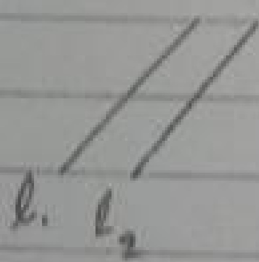
$$= \begin{array}{c} \begin{matrix} \oplus & \ominus & \oplus \end{matrix} \\ \begin{vmatrix} i & j & k \\ 1 & -1 & -1 \\ -3 & 2 & -1 \end{vmatrix} \end{array} = 3\hat{i} + 4\hat{j} - \hat{k}$$

$\Rightarrow P: 3(x-0) + 4(y-1) - (z-3) = 0$

Subject Cal-3 Date 25/6/2012 No.

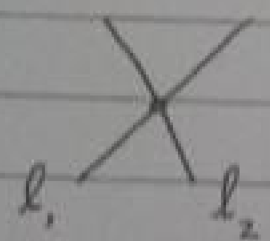
* lines in Space are either:

① Prallel



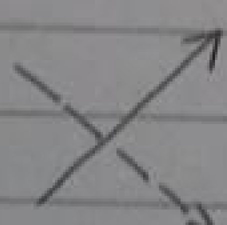
* plane containg both.

② Intersected



* plane containing both

③ Skew (Not prallel & not intersected).



* No plane contains them
* In a different planes.

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ex. $l_1: x = 1+2t,\ y = 3+2t,\ z = 2-t$

$l_2: x = 2+s,\ y = 6-s,\ z = -2+3s$

* Are these Lines prallel, intersected or skew :

$\vec{dl_1} < 2, 2, -1 >$, $\vec{dl_2} : < 1, -1, 3 >$

Part (I)

① $\vec{d_1} \parallel d_2 \Rightarrow ?? \Rightarrow \frac{1}{2} \neq \frac{-1}{2} \neq -3$

② the intersection

$$x_1 = x_2 \Rightarrow 1+2t = 2+s \Rightarrow t = 1$$
$$y_1 = y_2 \Rightarrow 3+2t = 6-s \Rightarrow s = 1.$$
$$4 + 4t = 8$$
$$t = 1$$

$$\left.\begin{array}{l} t=1 \xRightarrow{l_1} z = 2-1 = 1 \\ s=1 \xRightarrow{l_2} z = -2+3(1) = 1 \end{array}\right] \checkmark$$

③ Find the point of intersection :-

$t = 1 \xRightarrow{l_1} x = 3$ $(3, 5, 1)$ P. of I.

$\xRightarrow{l_1} y = 5$

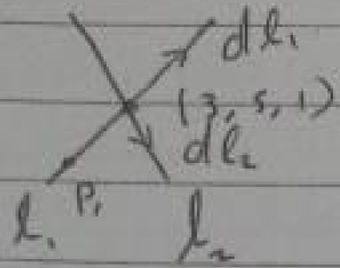
$\xRightarrow{l_1} z = 1$

NOTEBOOK

Part ②

Point & normal

Find the equation of plane containing l_1 & l_2



$t = 0 \Rightarrow P_1\,(1,3,2) \in l_1$

$\vec{n} = \vec{dl_1} \times \vec{dl_2}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & -1 \\ 1 & -1 & 3 \end{vmatrix} = 5\hat{i} - 7\hat{j} - 4\hat{k}$$

$P:\ 5(x-1) - 7(y-3) - 4(z-2) = 0$

Part ③:

Find the cute angle between l_1 & l_2

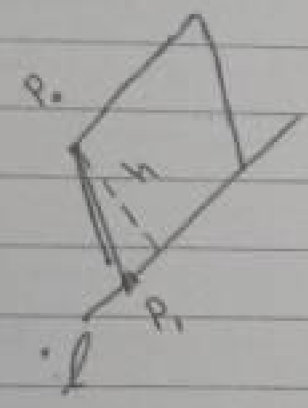
$\equiv$ ~~anget~~ angle between $\vec{d_1}$ & $\vec{d_2}$

$$\cos\theta = \frac{|\vec{d_1} \cdot \vec{d_2}|}{\|\vec{d_1}\|\,\|\vec{d_2}\|} = \frac{|2(1) + (2)(-1) + (-1)(3)|}{\sqrt{4+4+1}\ \sqrt{1+1+9}}$$

$$= \frac{3}{3\sqrt{11}} = \frac{1}{\sqrt{11}}$$

* distance between a point and a line :

$d(P_0, \ell)$



$$h = \frac{\text{Area}}{\text{base}} = \frac{\| \overrightarrow{P_0P_1} \times \overrightarrow{d\ell} \|}{\| \overrightarrow{d\ell} \|}$$

ex

$$\ell : \frac{x-1}{2} = \frac{3-y}{4} = \frac{z+1}{1}$$

Find the distance between $P_0\ (2, 3, -1)$ & line ℓ

$P_1 : (1, 3, -1) \in$ line أعمل نقطة أصفار البسط

$$\overrightarrow{P_0P_1} \times \overrightarrow{d\ell_1} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 0 & 0 \\ 2 & -4 & 1 \end{vmatrix} = \hat{j} + 4\hat{k}$$

$\overrightarrow{d\ell} = \langle 2, -4, 1 \rangle$ مقام الـx على معامل الـx وهكذا

$\| \overrightarrow{d\ell} \| = \sqrt{4+16+1} = \sqrt{21}$

$\| \overrightarrow{P_0P_1} \times \overrightarrow{d\ell_1} \| = \sqrt{16+1} = \sqrt{17}$

$$d(P_0, \ell) = \frac{\sqrt{17}}{\sqrt{21}}$$

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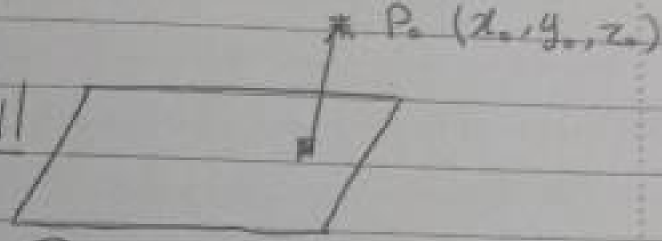
* distance between a point and plane :-

$$d(P_0, P) = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$



$P = ax + by + cz + d$

* المسافه بين خطين متوازيين : خذ نقطة على أيا خط وحلها مسافه بين نقطة وخط

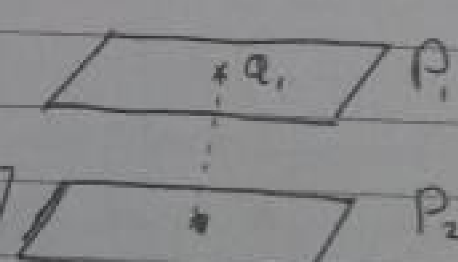
* المسافه بين مستويين متوازيين خذ نقطة على أيا مستوى وحلها بين نقطة ومستوى

ex Find the distance between ;
the two planes

$P_1 : x - y + 2z = 1$

$P_2 : -2x + 2y - 4z = 7$

$P_1 \parallel P_2 \Longleftrightarrow \vec{n_1} \parallel \vec{n_2}$



Q_1 in P_1 $(1, 0, 0)$ [عوض اي نقطتين وطلع الثالثه]

$d(P_1, P_2) = d(P_0, P_2)$

$$= \frac{|-2(1) + (2)(0) + (4)(0) - 7|}{\sqrt{4 + 4 + 16}} = \frac{9}{\sqrt{24}}$$

NOTEBOOK

Subject Cal-3 Date 26/6/2012 No.

ex Given : $P_1 : 5x-2y-2z=1 \Rightarrow n_1 \langle 5,-2,-2 \rangle$
& $P_2 : 4x+y+z=6 \Rightarrow n_2 \langle 4,1,1 \rangle$

1) write symenetric eqs of Line of intersection of planes P_1 & P_2 [P.t on Line of intersection]

$Z=0 \xRightarrow{P_1} 5x-2y=1$

$Z=0 \xRightarrow{P_2} (4x+y=6)\times 2$

$13x = 13 \Rightarrow \boxed{x=1}$ & $\boxed{y=2}$

$P_t(1,2,0) \in$ Line of intersection

$$\vec{n_1} \times \vec{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -2 & -2 \\ 4 & 1 & 1 \end{vmatrix} = 0\hat{i} - 13\hat{j} + 13\hat{k}$$

Line has this eq:

$$\left(x=1, \ \frac{y-2}{-13} = \frac{z-0}{13} \right)$$

- $\vec{n_1} \times \vec{n_2} \perp \vec{n_1} \perp P_1 \Rightarrow \vec{n_1} \times \vec{n_2} \parallel P_1$
- $\vec{n_1} \times \vec{n_2} \perp \vec{n_2} \perp P_2 \Rightarrow \vec{n_1} \times \vec{n_2} \parallel P_2$
- $\vec{n_1} \times \vec{n_2} \parallel$ Line of int of P_1 & P_2

ii) Find the angle between P_1 & P_2

[angle between 2-planes]

a cute angle between $\vec{n_1}$ & $\vec{n_2}$

$$\cos\theta = \frac{|n_1 \cdot n_2|}{\|\vec{n_1}\| \, \|\vec{n_2}\|} = \frac{|(4)(5)+(-2)(1)+(-2)(1)|}{\sqrt{5^2+2^2+2^2}\sqrt{4^2+1^2+1^2}} = \frac{16}{\sqrt{(33)(18)}}$$

ex $l_1 : x = 1+t \ , \ y = -2+3t \ , \ z = 4-t$

$l_2 : x = 2s \ , \ y = 3+s \ , \ z = -3+4s$

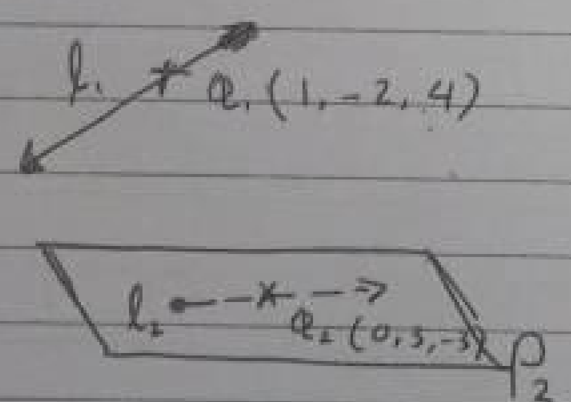
i) show that l_1 & l_2 are skew

* Not prallel and not intersected

ii) Find the distance between l_1 & l_2

① step①: Find P_2 : P_2 contains l_2 & prallel to l_1

$l_1 \parallel P_2$



② $d(l_1, l_2) = d(Q_1, P_2)$

* normal to P_2

$dl_1 \parallel P_2$ & $dl_2 \parallel P_2$

$\vec{n} = \vec{dl_1} \times \vec{dl_2}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -1 \\ 2 & 1 & 4 \end{vmatrix} = 13\hat{i} - (6)\hat{j} - 5\hat{k}$$

$P_2 : 13(x-0) - 6(y-3) - 5(z+3) = 0$

$d(l_1, l_2) = d(Q_1, P_2)$

$$= \frac{|13(1-0) - 6(-2-3) - 5(4+3)|}{\sqrt{13^2 + 6^2 + 5^2}}$$

Subject

Date No.

P

38 write an equation of the plane passes through the line of intersection of the plane $P_1: x - z = 1$ & $P_2: y + 2z = 3$ and is perpendicular to the plane $P_3: x + y - 2z = 1$

* Point on line of int.

$z = 0 \xrightarrow{P_1} \boxed{x = 1}$
$\xrightarrow{P_2} \boxed{y = 3}$ $[(1, 3, 0) \in \text{line of int}]$

$(1, 3, 0) \in P$

$\vec{n_1} \times \vec{n_2}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -1 \\ 0 & 1 & 2 \end{vmatrix} = \hat{i} - 2\hat{j} + \hat{k}$$

اتجاه خط التقاطع

ناتج الـ X بين الـ normals

• $\vec{n_3} \parallel P$

$\Rightarrow \vec{n_3} \times (\vec{n_1} \times \vec{n_2}) \perp P$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ 1 & -2 & 1 \end{vmatrix} = -3\hat{i} - 3\hat{j} - 3\hat{k}$$

$P: -3(x - 1) - 3(y - 3) - 3(z - 0) = 0$

NOTEBOOK

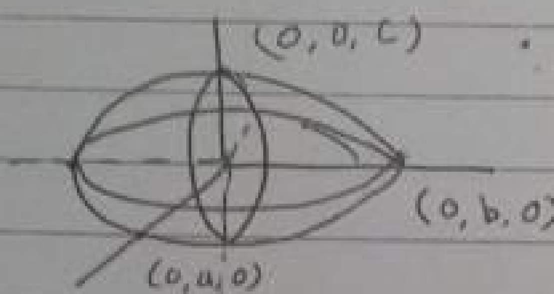
S: 12.6

Quadric Surfaces

[1] Ellipsoid:

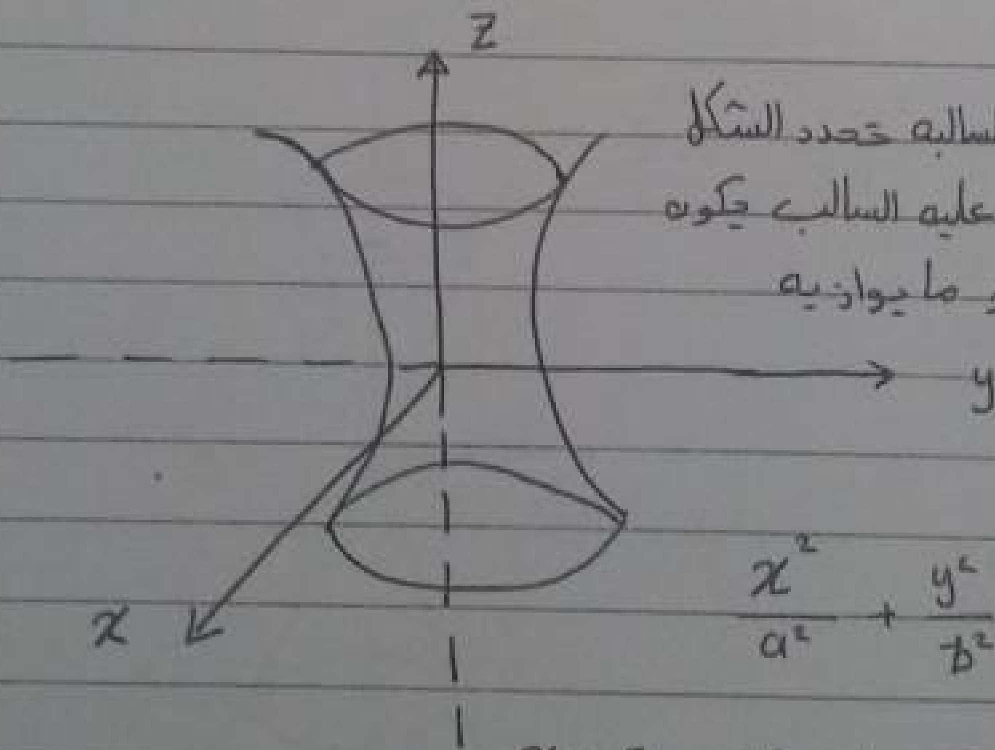
القطع الناقص (البيضاوي)

$z = 0 \rightarrow$ elips



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

[2] Hyperboloid of one sheet:



الإشارة السالبة تحدد الشكل
والمتغير الذي عليه السالب يكون
هو المحور أو ما يوازيه

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

$x = 0$ or $y = 0$
hyperbola

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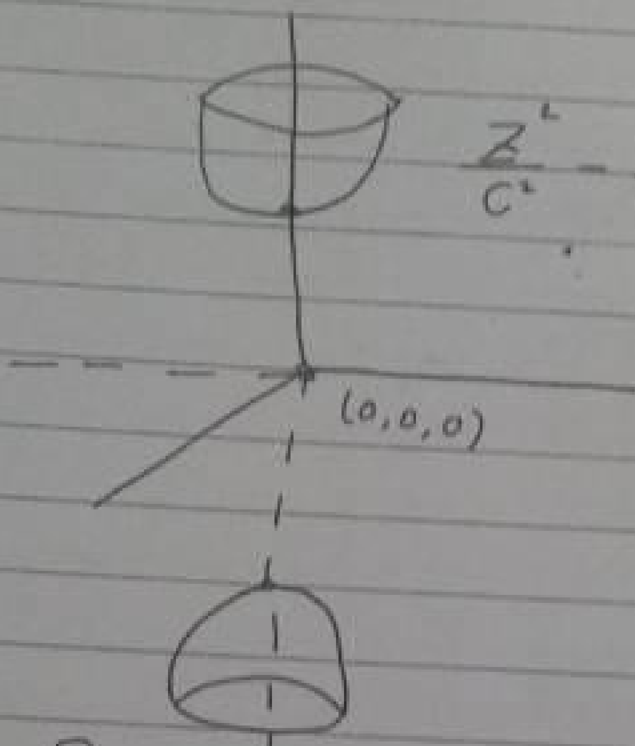
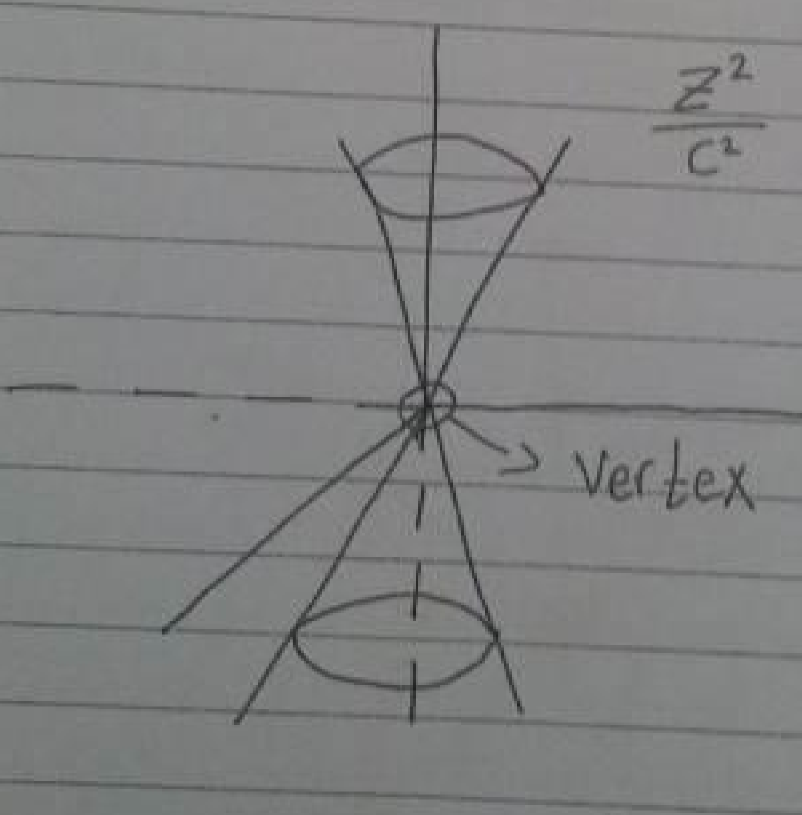
$\boxed{3}$ Hyperboloid of 2-Sheets:



$$\frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$\boxed{4}$ Double Cone:

$$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

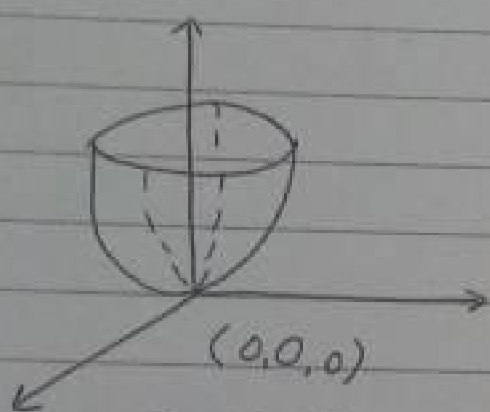


5 Papboloid :

Function:

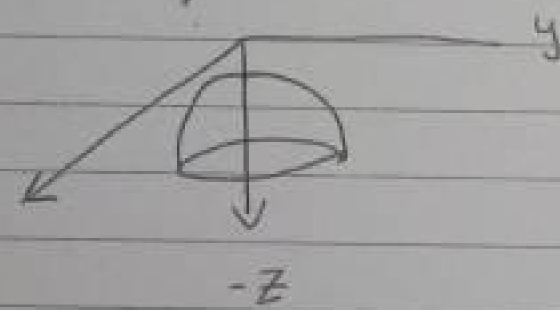
$$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

+C



$z = x^2 + y^2$

−C

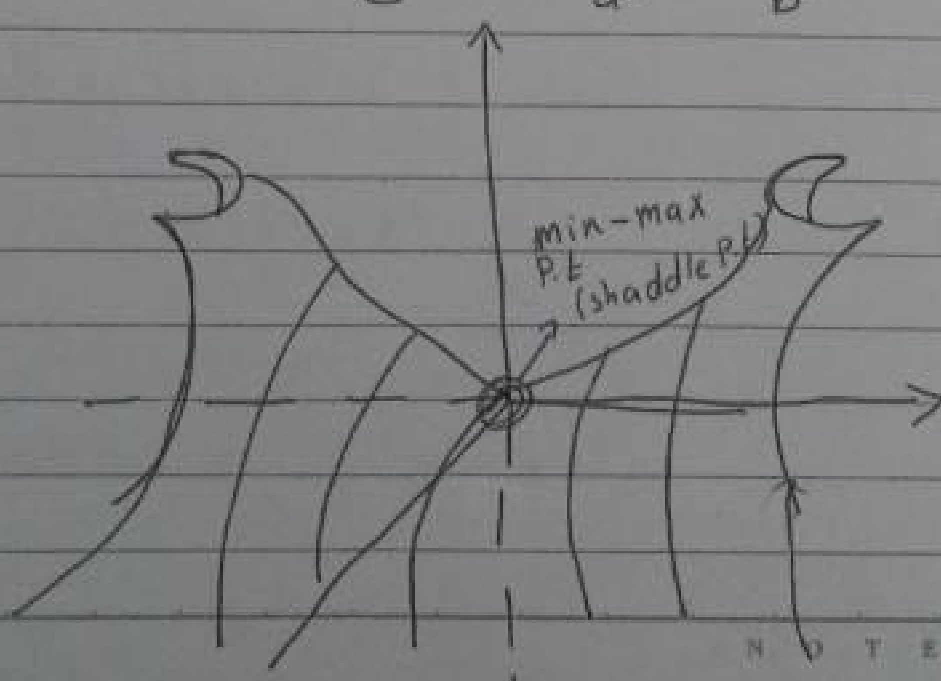


$z = -x^2 - y^2$

6 Hyperbolic Paraboloid:

Function

$$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$



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ex Identify:

① $z^2 = x^2 - 2z + 3$

it's cylinder

axis // y-axis

② $z^2 = x^2 - 2z + y^2 + 3$

$z^2 + 2z - x^2 - y^2 = 3$

$(z + 1)^2 - x^2 - y^2 = 4$

Hyperboloid of two-sheets

③ $(z+1)^2 - x^2 - y^2 = -2$

$-(z+1)^2 + x^2 + y^2 = 2$

Hyperboloid of 1-sheet

④ $x^2 - 2x + (-y^2) - z^2 = 0$

$(x-1)^2 - y^2 - z^2 = 0$

Hyperboloid of 2-sheets

⑤ $x + z = 3$

plane // y-axis

⑥ $z - x^2 - y^2 = 4$

$z = 4 + x^2 + y^2$

Paraboloid ↑

⑦ $z - 3 + x^2 + y^2 = 0$

$z = 3 - x^2 - y^2$ ↓

Paraboloid

⑧ $y + 4 - x^2 - z^2 = 0$

$y = +4 + x^2 + z^2$

Paraboloid

عدد المتغيرات	
1	plane
2	cylinder

⑨ $y + 4 - z^2 = 0$

Cylinder

⑩ $y + 4 = 0$

plane

Subject Date No.

$\frac{60}{803}$ write an equation of the plane consisting of all points equidistant from the two points

$Q_1 (2, 5, 5)$ Q_2 $(-6, 5, 1)$

Sol,

Mid Point $\in P$

$$\left(\frac{-6+2}{2}, \frac{3+5}{2}, \frac{1+5}{2}\right) = (-2, 4, 3) \in P$$

$$\overrightarrow{Q_1Q_2} \perp P \quad , \quad \vec{n} \parallel \overrightarrow{Q_1Q_2}$$

$$\overrightarrow{Q_1Q_2} = (-6-2)\hat{i} + (3-5)\hat{j} + (1-5)\hat{k}$$

$$= (-8\hat{i} - 2\hat{j} - 4\hat{k}) \times \frac{-1}{2}$$

$\vec{n} = \quad = 4\hat{i} + \hat{j} + 2\hat{k}$ normal

Point $(-2, 4, 3)$

$$P: \ 4(x+2) + 1(y-4) + 2(z-3) = 0$$

N O T E B O O K

ch * 13 Vector Function

S. 13. 1 "Vector Functions and Space curves"

$C: \vec{r}_{(t)} = x_{(t)}\,\hat{i} + y_{(t)}\,\hat{j} + z_{(t)}\,\hat{k}$

* Domain: $D_{x_{(t)}} \cap D_{y_{(t)}} \cap D_{z_{(t)}}$

* Range is set of vectors.

* Graph is an Oriented space curve
[with increasing "t"] له اتجاه محدد

ex $C: \vec{r}_{(t)} = 2\cos t\,\hat{i} + 2\sin t\,\hat{j}$, $t \in [0, 2\pi]$
graph this function ??

Sol:
Domain $[0, 2\pi]$

① $\Longrightarrow$ $\underbrace{x = 2\cos t,\ y = 2\sin t}$ $(t \in [0, 2\pi])$
prametric equation for the same curve

② $\Longrightarrow$ you can Eliminate t:

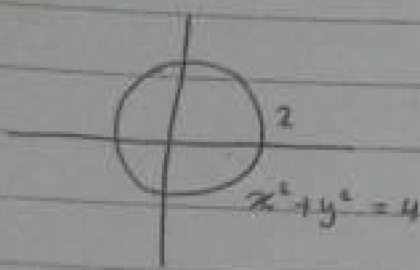
$\frac{x}{2} = \cos t$

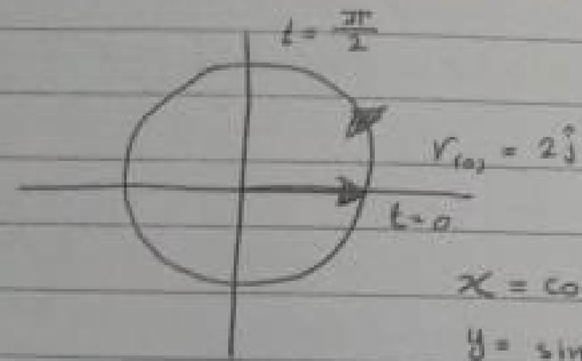
$\frac{y}{2} = \sin t$

$\frac{x^2}{4} + \frac{y^2}{4} = \cos^2 t + \sin^2 t = 1$

$\frac{x^2}{4} + \frac{y^2}{4} = 1$

$x^2 + y^2 = 4$ [circil curve]





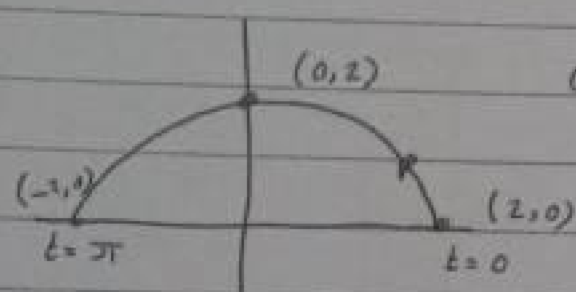
$x=\cos t$

$y=\sin t$

at $t=0$ $[x=2,\ x=0]$

at $t=\frac{\pi}{2}$ $[x=0,\ y=2]$

$0 \leq t \leq 2\pi$



$0 \leq t \leq \pi$

□ المتجه الذي يمثل الـ Function بدايته الـ Origin
ونهايته إلى النقطة التي تنتج من تعويض (x,y,z)

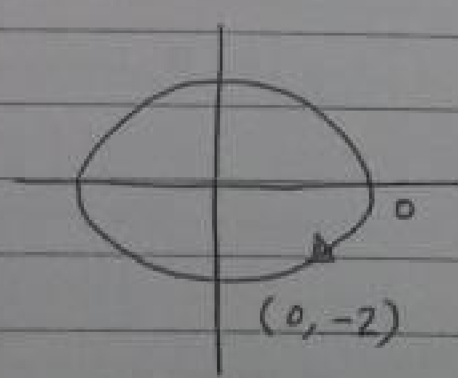
ex $\vec{r}_{(t)} = 2\cos t\,\hat{i} - 2\sin t\,\hat{j}$, $t \in [0, 2\pi]$

$C:\ x = 2\cos t,\ y = 2\sin t \quad t \in (0, 2\pi)$

$\frac{x}{2} = \cos t\,\hat{i}$

$\frac{y}{2} = -\sin t\,\hat{j}$

$\frac{x}{4} + \frac{y}{4} = 1$



<u>ex</u> $\vec{r}_{(t)} = 2\cos t\,\hat{i} + 2\sin t\,\hat{j} + 3\hat{k}$; $0 \leq t \leq 2\pi$

graph:

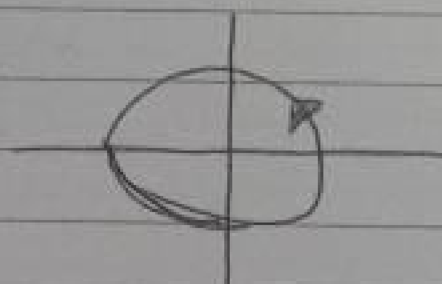
① C: $x = 2\cos t\,\hat{i}$

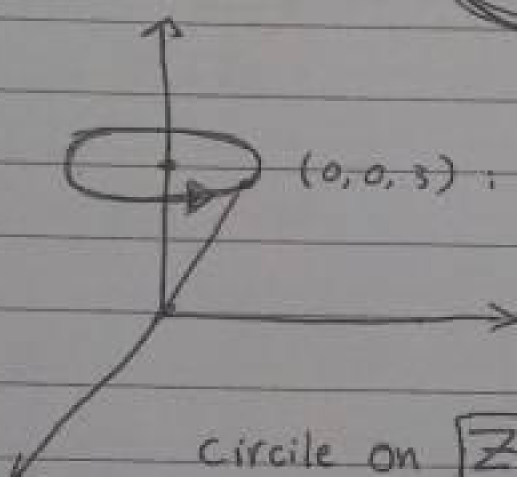
$y = 2\sin t\,\hat{j}$

$z = 3$

② Eliminate t:

$x^2 + y^2 = 4$, $z = 3$





(0, 0, 3) : Center

circle on $\boxed{z = 3}$

☆ Helix: Space Curve (No plane can containe t)

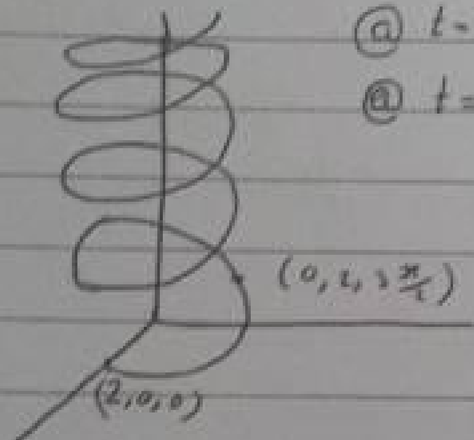
☆ $\vec{r}_{(t)} = a\cos t\,\hat{i} + b\sin t\,\hat{j} + ct\,\hat{k}$

axis is the Z-axis.

ex $C: \vec{r}_{(t)} = 2\cos t\,\hat{i} + 2\sin t\,\hat{j} + 3t\,\hat{k}$

@ $t=0$ $[x=2, y=0, z=0]$

@ $t=\frac{\pi}{2}$ $[0, 2, 3\frac{\pi}{2}]$



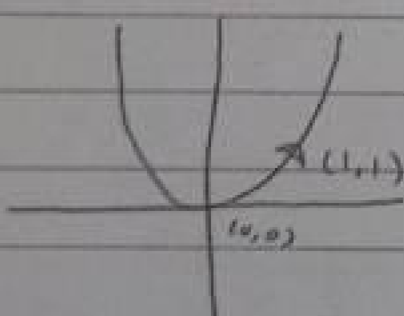
ex $C: \vec{r}_{(t)} = t\,\hat{i} + t^2\,\hat{j} + 2\hat{k}$

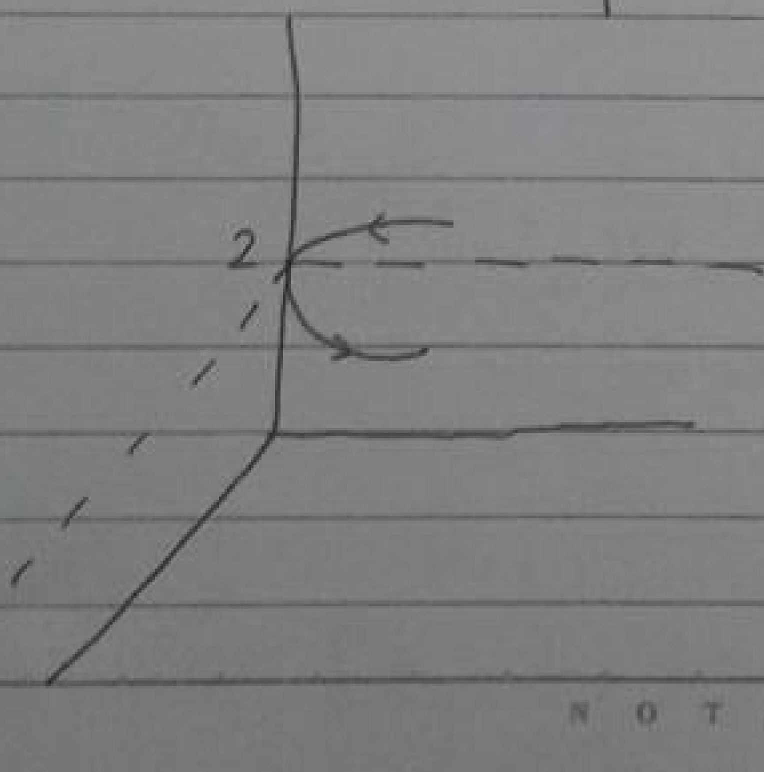
$C: x = t$

$y = t^2$

$z = 2$

$y = x^2,\ z = 2$





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ex Find a vector function that represents the curve of intersection of the cylinder: $x^2+y^2=9$ & the plane $y+z=2$

cylinder

⊛ $x = 3\cos t$

$y = 3\sin t$

plane $z = 2 - y$

$z = 2 - 3\sin t$

prametric eq.: $x = 3\cos t$, $y = 3\sin t$, $z = 2 - 3\sin t$

$$\vec{r}_{(t)} = 3\cos t\,\hat{i} + 3\sin t\,\hat{j} + (2 - 3\sin t)\,\hat{k}$$

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② Find the domain:

$$\vec{r}_{(t)} = \underbrace{\frac{t-2}{t+2}}_{t \neq -2}\,\hat{i} + \underbrace{\sin t}_{\mathbb{R}}\,\hat{j} + \underbrace{\ln(9-t^2)}_{9-t^2>0}\,\hat{k}$$

$-3, 3$

$-3 \quad 0 \quad 3$

$\therefore D: (-3,3)/\{-2\}$

$(-3,-2) \cup (-2,3)$

④ $$\lim_{t\to 0} \left\langle \frac{e^t-1}{t}, \frac{\sqrt{1+t}-1}{t}, \frac{3}{1+t} \right\rangle$$

① $\frac{0}{0}$, ② $\frac{0}{0}$, 3

① $\lim_{t\to 0} \frac{e^t-1}{t} \overset{LH}{=} \frac{e^t}{1} = 1$

② $\lim_{t\to 0} \frac{\sqrt{1+t}-1}{t} \cdot \frac{\sqrt{1+t}+1}{\sqrt{1+t}+1} = \lim_{t\to 0} \frac{1+t-1}{t(\sqrt{1+t}+1)} = \lim_{t\to 0} \frac{\cancel{t}}{\cancel{t}(\sqrt{1+t}+1)} = \frac{1}{2}$

Constan vector $[\langle 1, \frac{1}{2}, 3 \rangle$

NOTEBOOK

27) At what pt(s) does the curve
$\vec{r}(t) = t\hat{i} + (2t - t^2)\hat{k}$ intersect the paraboloid $z = x^2 + y^2$

① C: $x = t$, $y = 0$, $z = 2t - t^2$

$2t - t^2 = t^2 + 0$

$2t(t-1) = 0$

$t = 0$ or $t = 1$

@ $t = 0$ $(0,0,0)$

@ $t = 1$ $(1,0,1)$

(37) Find a vector function that represents the curve of intersection between the two surfaces *or Parametric*

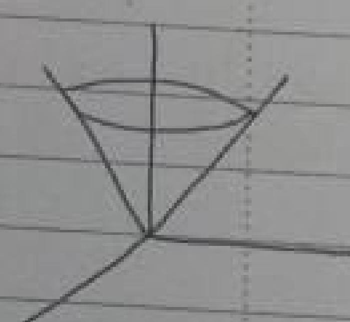
$\boxed{\text{Cone}} \Rightarrow z = \sqrt{x^2+y^2}$ & plane $z = 1 + y$

Sol: $z = z$

$1 + y = \sqrt{x^2+y^2}$

$1 + 2y + \cancel{y^2} = x^2 + \cancel{y^2}$

$1 + 2y = x^2 \Rightarrow y = \frac{x^2-1}{2}$



Let $x = t$

$\therefore y = \frac{t^2-1}{2} \Longleftrightarrow z = \frac{2 + t^2 - 1}{2} = \frac{1+t^2}{2}$

C: $\left(x = t,\ y = \frac{t^2-1}{2},\ z = \frac{1+t^2}{2}\right)$

$\therefore \vec{r}(t) = \left\langle t, \frac{t^2-1}{2}, \frac{1+t^2}{2} \right\rangle$

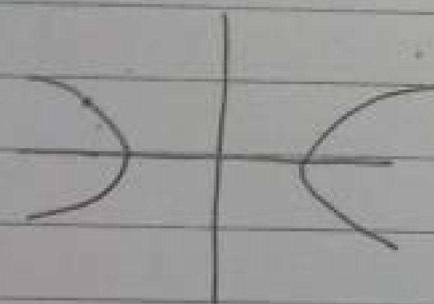
ex ① Circle: $x^2+y^2=4$

param:

$C: \ x = 2\cos t$

$\quad y = 2\sin t$

② Hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$



Param: $x = 2\cosh t$

$\quad y = 3\sinh t$

$\cosh^2 t - \sinh^2 t = 1$

Reminder:

$$\cosh x = \frac{e^x + e^{-x}}{2} \quad ; \quad \sinh = \frac{e^x - e^{-x}}{2}$$

- Surface + Surface $\longrightarrow$ curve
- plane + plane $\longrightarrow$ line

S. 13.2 Differentiation & integration of vector function:-

$$C: \ \vec{r}_{(t)} = x_{(t)}\,\hat{i} + y_{(t)}\,\hat{j} + z_{(t)}\,\hat{k}$$

$\boxed{1}$ $$\frac{d}{dt}\vec{r}_{(t)} = x'_{(t)}\hat{i} + y'_{(t)}\,\hat{j} + z'_{(t)}\,\hat{k}$$

$\boxed{2}$ $$\int \vec{r}_{(t)}\,dt = \underbrace{\hat{i}\int x_{(t)}\,dt + C_1\hat{i} + \hat{j}\int y_{(t)}\,dt + C_2\hat{j} + \hat{k}\int z_{(t)}\,dt + C_3\hat{k}}_{+C}$$

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$\Longrightarrow$ Differentiation & Integration are componentwise:

ex $\vec{r}_{(t)} = \cos e^{-t}\,\hat{i} + 2^{t}\,\hat{j} + \ln(t^2+3)\,\hat{k}$

Find $\vec{r}\,'_{(t)}$; $\vec{r}\,'_{(t)} = (-e^{-t})(-\sin e^{-t})\hat{i} + 2^{t}\ln(2)\,\hat{j} + \frac{2t}{t^2+3}\,\hat{k}$

ex $\int(\underbrace{t e^{t}\,\hat{i}}_{\text{by Parts}} + \cos t\,\hat{j} + \frac{1}{t}\,\hat{k})\,dt$

$= (t e^{t} - e^{t})\hat{i} + \sin t\,\hat{j} + \ln|t|\,\hat{k} + c$

☆ Rules: $f_{(t)}$, $\vec{r}_{1(t)}$ & $\vec{r}_{2(t)}$ are function,

① $\frac{d}{dt}(f_{(t)}\,\vec{r}_{1(t)}) = f_{(t)}\,\vec{r}_1\,'_{(t)} + f'_{(t)}\,\vec{r}_{1(t)}$

② $\frac{d}{dt}(\vec{r}_1 \cdot \vec{r}_2) = \vec{r}_1 \cdot \vec{r}_2\,' + \vec{r}_1\,' \cdot \vec{r}_2$

③ $\frac{d}{dt}(\vec{r}_1 \times \vec{r}_2) = \vec{r}_1 \times \vec{r}_2\,' + \vec{r}_1\,' \times \vec{r}_2$

ex $(t^2 < \sinh t^2, \frac{1}{t}, \cosh e^{t} >)'$

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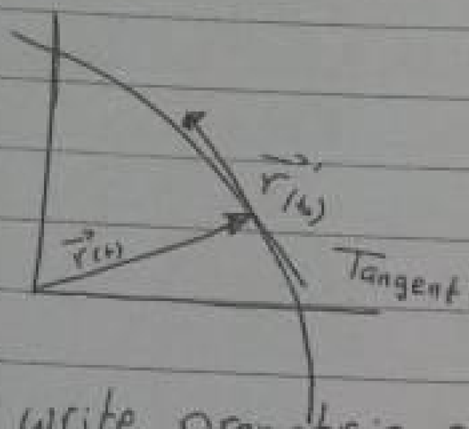
* Geometrical meaning of $\vec{r}\,'$ at t_0

$\vec{r}\,'_{(t_0)}$ = direction of the tangent line at t_0



25/829 write prametric eq. for the tangent line to the curve.

$$C:\ \vec{r}_{(t)} = e^{-t}\cos t\,\hat{i} + e^{-t}\sin t\,\hat{j} + e^{-t}\hat{k}$$

at the point $(1, 0, 1)$
$\quad\quad\quad\quad\quad\ \ x\ \ y\ \ z$

$\Rightarrow e^{-t}\cos t = 1$

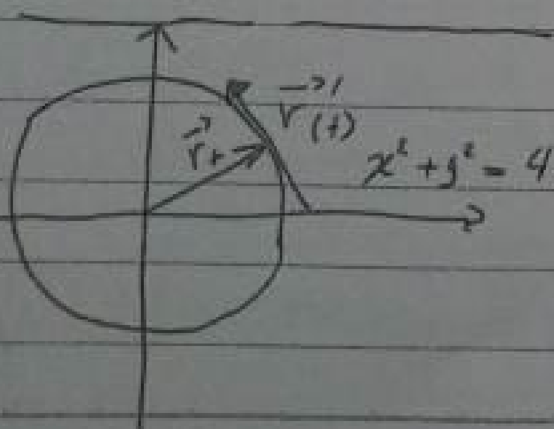
$e^{-t}\sin t = 0$

$e^{-t} = 1 \quad \Rightarrow t = 0$

$$\vec{r}\,'_{(t)} = [e^{-t}(-\sin t) + (\cos t)(-e^{-t})]\,\hat{i} + [(e^{-t}\cos t) + (-e^{-t}\sin t)]\,\hat{j} - e^{-t}\hat{k}$$

$\vec{r}\,'_{(0)} = -\hat{i} + \hat{j} - \hat{k}$ Direction of the tangent line

The Tangent Line: $x = 1 - t,\ y = t,\ z = 1 - t$



$C:\ \vec{r}_{(t)} = 2\cos t\,\hat{i} + 2\sin t\,\hat{j}$

* Them ; if then $\|\vec{r}\| = c$ [c is const], then
[illegible] Then $\vec{r}\,'$ is perpendecular to $\vec{r}_{(t)}$

proof: $\vec{r}\,' \perp \vec{r}$?? OR $\vec{r} \cdot \vec{r}\,' = 0$??

$\vec{r} \cdot \vec{r} = \|\vec{r}\|^2$

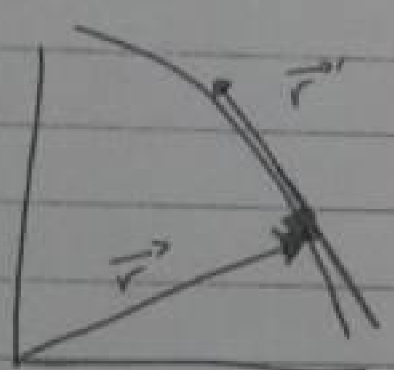
$\vec{r} \cdot \vec{r} = c$

$\vec{r} \cdot \vec{r}\,' + \vec{r}\,' \cdot \vec{r} = 0$

$2\vec{r} \cdot \vec{r}\,' = 0$ So, $\vec{r} \cdot \vec{r}\,' = 0$ Then $\vec{r}\,' \perp \vec{r}$

S: 13.3 Arc-length & Curveture

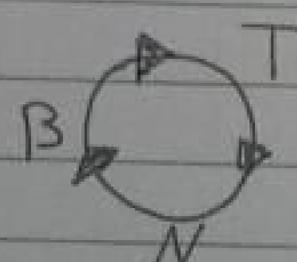
* Three Unit Vectors $\hat{T}, \hat{N}, \hat{B}$



$C = \vec{r}_{(t)}$

① Unit Tangent - $\hat{T}$

$\hat{T}_{(t)} = \dfrac{\vec{r}\,'_{(t)}}{\|\vec{r}\,'\|}$, $\|\hat{T}_{(t)}\| = 1$



② Normal, $\hat{N}$

$\hat{N} = \dfrac{\hat{T}'_{(t)}}{\|\hat{T}'_{(t)}\|}$ $\hat{N} \perp \hat{T}$

③ Binormal, $\hat{B} = \hat{T} \times \hat{N}$

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☒ cerveture $K_{(t)}$

$C: \vec{r}_{(t)}$

$$K_{(t)} = \frac{\|\vec{r}\,' \times \vec{r}\,''\|}{\|\vec{r}\,'\|^3}$$

$\boxed{\frac{44}{83}}$ $C: \vec{r}_{(t)} = < \cos t, \sin^2 t, \ln(\cos t) >$

Find $\hat{T}, \hat{N}$ & $\hat{B}$ at the point $(1, 0, 0)$

* t = [the first t]

$\left.\begin{array}{l}\ln \cos t = 0 \\ \cos t = 1\end{array}\right] \Rightarrow \boxed{t = 0}$

* $\vec{r}\,'_{(t)} = < -\sin t, \cos t, \frac{-\sin t}{\cos t} >$ $\quad -\tan t$

$(0, 1, 0) \Leftarrow \boxed{t = 0}$

a) $\hat{T}_{(0)} = \frac{\vec{r}\,'_{(0)}}{\|\vec{r}\,'\|} = \frac{<0, 1, 0>}{1} = \hat{j}$

b) i) $\hat{T}_{(t)} = \frac{\vec{r}\,'_{(t)}}{\|\vec{r}\,'_{(t)}\|} =$

$= \frac{< -\sin t, \cos t, -\tan t >}{\sec \cdot t}$

$= < -\sin t \cos t, \cos^2 t, -\sin t >$

$= < -\frac{1}{2} \sin 2t, \cos^2 t, -\sin t >$

$\vec{r}\,'_{(t)} = < -\sin t, \cos t, -\tan$

$\|\vec{r}\,'_{(t)}\| = \sqrt{\sin^2 t + \cos^2 t + \tan^2 t}$

$= \sqrt{1 + \tan^2 t}$

$= \sqrt{\sec^2 t} = \sec t$

• $\sin t \cos t = \frac{1}{2} \sin 2t$

$\hat{T}\,' = < -\cos 2t, -2 \cos t \sin t, -\cos t >$

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ii) $\frac{\hat{T}'}{\|T'\|} = \hat{N}$ • $2\cos t \sin t = \sin 2t$

$= \frac{T'}{\sqrt{(\cos 2t)^2 + (\sin 2t)^2 + (\cos t)^2}} = \frac{\langle -1, 0, -1 \rangle}{\sqrt{1 + \cos^2 t}}$ عوض t بصفر

$= \frac{\langle -1, 0, -1 \rangle}{\sqrt{2}} = \left\langle \frac{-1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right\rangle$

4/7/2012

$C: \vec{r}_{(t)}$

* ① $\hat{T} = \frac{\vec{r}\,'}{\|\vec{r}\,'\|}$

② $\hat{N} = \frac{\vec{T}'}{\|\vec{T}'\|}$

③ $\hat{B} = \vec{T} \times \vec{N}$

* Curveture

$$K_{(t)} = \frac{\|\vec{r}\,' \times \vec{r}\,''\|}{\|\vec{r}\,'\|^3}$$

ex $\vec{C}: r_{(t)} = \langle \underset{x}{t}, \underset{y}{t^2}, \underset{z}{t^3} \rangle$; Find the curveture at $\underset{x,\,y,\,z}{(1,1,1)}$

$$\left.\begin{aligned} t = 1 &\Longrightarrow t = 1 \\ t^2 = 1 &\Longrightarrow t = \pm 1 \\ t^3 = 1 &\Longrightarrow t = 1 \end{aligned}\right\} \to t = 1$$

$\vec{r}\,' = \langle 1, 2t, 3t^2 \rangle$, $\vec{r}\,'' \langle 0, 2, 6t \rangle$

$\vec{r}\,'_{(1)} = \langle 1, 2, 3 \rangle$, $\vec{r}\,''_{(1)} \langle 0, 2, 6 \rangle$

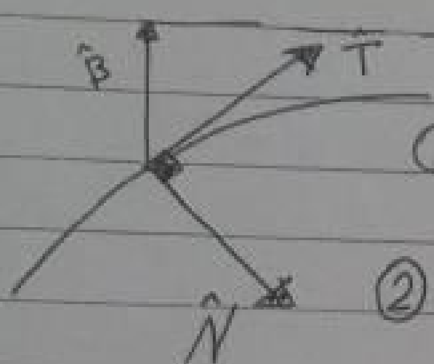
$$\vec{r}\,'_{(1)} \times \vec{r}\,''_{(1)} = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 0 & 2 & 6 \end{vmatrix} = \langle 6,-6,2 \rangle$$

$$\| \vec{r}\,'_{(1)} \times \vec{r}\,''_{(1)} \| = \sqrt{6^2+6^2+2^2} = \sqrt{76}$$

$$\| \vec{r}\,' \| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$$

$$K_{(1)} = \frac{\sqrt{76}}{(\sqrt{14})^3}$$

$\rho = \frac{1}{k}$ rad of carviture



① Normal plane $\Rightarrow \hat{N}$ & $\hat{B}$

the Normal to this plane // $\hat{T}$

② Osculating plane (contains $\hat{T}$ & $\hat{N}$)

the Normal to this plane // $\hat{B}$

② write an eq for the Normal plane. (contains $\hat{B}$ & $\hat{N}$) at the point $(1,1,1)$

the normal is prallel to $\hat{T}$

$\vec{r}_{(t)} = \langle t, t^2, t^3 \rangle \Rightarrow t = 1$

$\vec{r}\,'_{(t)} = \langle 1, 2t, 3t^2 \rangle$ normal to this plane

$\vec{r}\,'_{(t)} \,//\, \hat{T}$

$\vec{r}\,'_{(1)} = \langle 1,2,3 \rangle \,//\, \hat{T}$

the eq of P: $1(x-1)+2(y-1)+3(z-1)=0$

$$\hat{T} = \frac{\vec{r}\,'_{(1)}}{\|\vec{r}\,'_{(1)}\|} = \frac{\langle 1,2,3 \rangle}{\sqrt{14}}$$

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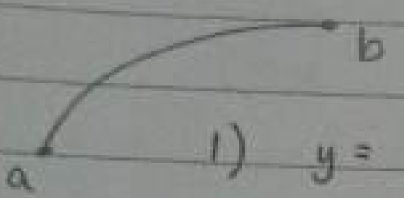
* Arc-Length : L



1) $\left.\begin{array}{l} y = f_{(x)} \\ a \leq x \leq b \end{array}\right\} \rightarrow L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \; dx$

2) $\left.\begin{array}{l} x = g_{(y)} \\ a \leq y \leq b \end{array}\right\} \rightarrow L = \int_a^b \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \; dy$

3) $\left.\begin{array}{l} x = x_t \\ y = y_t \\ z = z_t \end{array}\right\} \rightarrow L = \int_{t_1}^{t_2} \sqrt{x'^2 + y'^2 + z'^2} \; dt$

$C : \vec{r}(t) = x_{(t)} \hat{i} + y_{(t)} \hat{j} + z_{(t)} \hat{k}$

* $C : x = x_{(t)}$ $\quad t_1 \leq t \leq t_2$

$y = y_{(t)}$

$z = z_{(t)}$

$$L = \int_{t_1}^{t_2} \sqrt{x'^2 + y'^2 + z'^2} \; dt = \int_{t_1}^{t_2} \| \vec{r}\,'_{(t)} \| \; dt$$

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ex $C: \vec{r}(t) = \hat{i} + t^2\hat{j} + t^3\hat{k}$ $0 \le t \le 1$

Find Arc-Length

* $\vec{r}'_{(t)} = 2t\hat{j} + 3t^2\hat{k} \Rightarrow \|\vec{r}'\| \sqrt{4t^2+9t^4}$

$L = \int_0^1 \sqrt{4t^2+9t^4}\, dt$ $\sqrt{t^2} = |t|$

$= \int_0^1 t\sqrt{4+9t^2}\, dt$ by sub:-

$= \frac{1}{18} \frac{2}{3} (4+9t^2)^{3/2} \Big]_0^1$

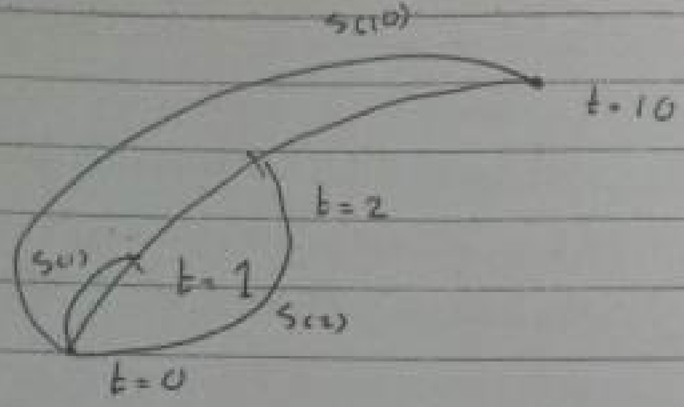
$= \frac{1}{27}\left[(13)^{3/2} - (4)^{3/2}\right]$

- $f_{(t)} = \int_1^t x\, dx = \frac{x^2}{2}\Big|_1^t = \frac{t^2}{2} - \frac{1}{2}$

- $f'_{(t)} = \frac{d}{dt}\int_1^t x\, dx = t$

* $L = \int_{t_0}^{t_1} \|\vec{r}'_{(t)}\|\, dt \in \mathbb{R}$

* $S = \int_{t_0}^{t_1} \|\vec{r}'_{(u)}\|\, du$ S: arc length prameter



S: القياس منه
ال Refrence
Point

* C: $\vec{r}_{(t)}$

Reparametrize this curve using Arc-length parameter "S"

① Find S

$$S = \int_{t_0}^{t} \| \vec{r}'_{(u)} \| \, du \quad ; \quad s = \dots t$$

② write t in term of S

③ $\vec{r}_{(s)}$

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Cal-3

* Arc-length parametric

ex $C: \vec{r}_{(t)} = \cos^3 t\,\hat{i} + \sin^3 t\,\hat{j} + 2\hat{k}$, Reparametrize this curve using arc-length parameter (S) with refrence point $t=0$

$$S = \int_0^t \|\vec{r}\,'_{(u)}\|\,du$$

$$\vec{r}\,'_{(t)} = 3\cos^2 t\,(-\sin t)\,\hat{i} + 3\sin^2 t\,(\cos t)\,\hat{j} + 0k$$

$$\|\vec{r}\,'_{(u)}\| = \sqrt{9\cos^4 u \cdot \sin^2 u + 9\sin^4 u \cdot \cos^2 u}\quad du$$

$$S_{(t)} = \int_0^t \sqrt{9\cos^4 u \cdot \sin^2 u + 9\sin^4 u \cdot \cos^2 u}\quad du$$

$$= \int_0^t 3\cos u \sin u \sqrt{\cos^2 u + \sin^2 u}\quad du$$

$\cos u \sin u = \frac{1}{2}\sin 2u$

$$= \int_0^t 3\cos u \sin u \sqrt{1}\quad du$$

$$= 3\int_0^t \frac{1}{2}\sin 2u\,du$$

$$= \frac{3}{2}\left(\frac{-\cos 2u}{2}\right)\Big]_0^t$$

$$S = \frac{-3}{4}(\cos 2t - 1)$$

$$\frac{-4}{3}S = \cos 2t - 1$$

$$1 - \frac{4}{3}S = \cos 2t$$

$$\cos^{-1}\left(1 - \frac{4}{3}S\right) = 2t$$

$$t = \frac{1}{2}\left(\cos^{-1}\left(1 - \frac{4}{3}S\right)\right)$$

$r : \vec{r} = \cos^3\left(\frac{1}{2}\cos^{-1}(1-\frac{4}{3}s)\right) + \sin^3(\ldots$

ex write a parametric equation for the line Segment from $P(1,2,0)$ to $Q(-1,3,4)$

direction $= \overrightarrow{PQ}$

$= (-1-1)\hat{i} + (3-2)\hat{j} + (4-0)\hat{k}$

$= -2\hat{i} + \hat{j} + 4\hat{k}$

$\ell: x = 1-2t \; , \; y = 2+t \; , \; z = 0+4t$

Line segment ; we must solve t ;

$\underline{\underline{0}} \leq t \leq \underline{1}$

عوض النقطة P

عوض النقطة Q

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ch: 14 Partial Derivatives

S: 14.1 Functions of Several variables

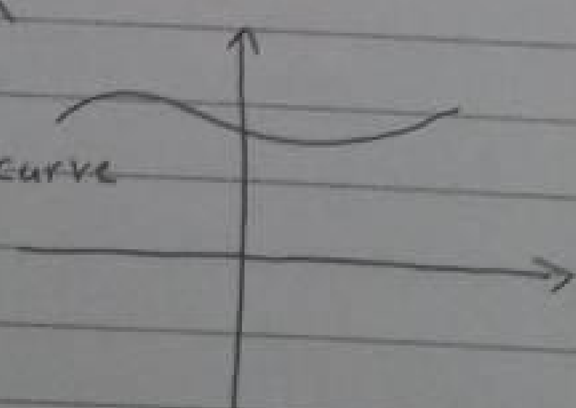
① Function of one-Variable

$y = f(x)$

D_f : Domain is interval $\subseteq R$ (real Line)

R_f : Range $\subseteq R$

graph is a curve



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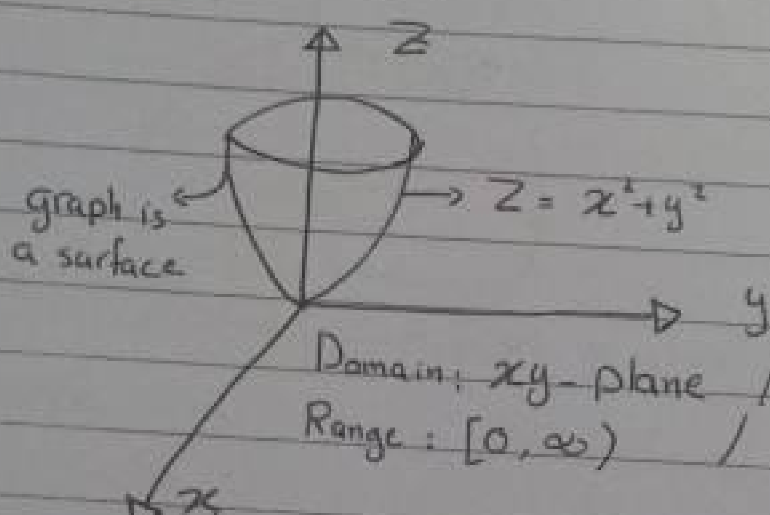
② Function of two-Variables

$$Z = f(x,y)$$

$D_f \subseteq R^{2}$ (3-D) $= \{(x,y) : x \in R,\ y \in R\}$

$\hookrightarrow$ is a region $\subseteq$ xy-plane.

$R_f \subseteq R$



Domain: xy-plane / R^2 منطقة

Range: $[0, \infty)$ / فترة

[Surface] المعادلة ذات 3-متغيرات.

اقتران من متغيرين.

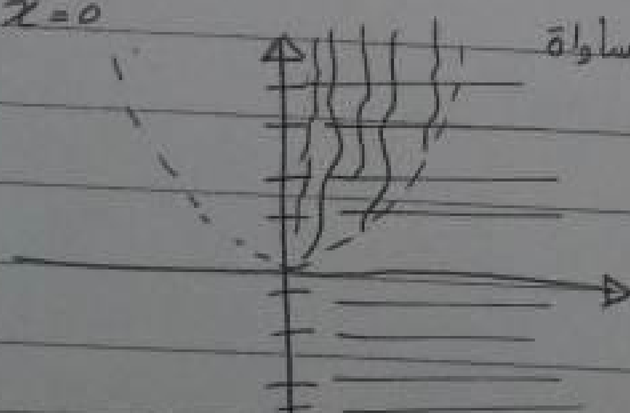
---

ex: $f(x,y) = \dfrac{\ln x}{\sqrt{y-x^2}}$; Find & sketch the domain of the function

$x > 0$, $y - x^2 > 0 \Rightarrow y > x^2 \Rightarrow y = x^2$

$\hookrightarrow x = 0$

عشان نعرف نرسم الحد مساواة



$D_f : \{(x,y) : y > x^2 \text{ \& } x > 0\}$

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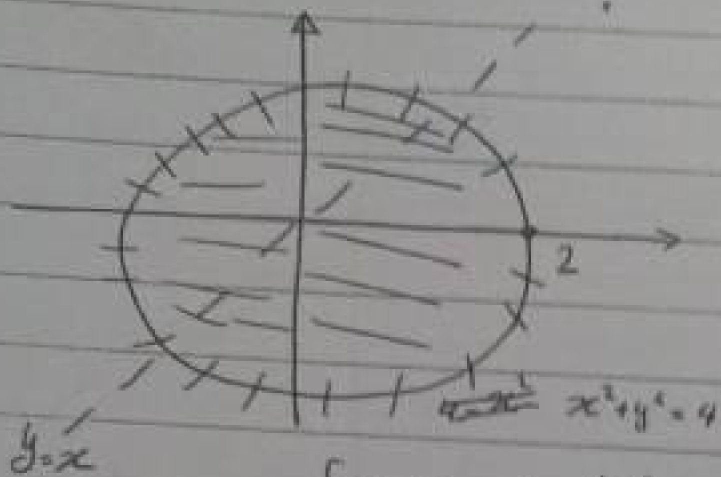
ex $f(x,y) = \frac{1}{y-x} + \frac{x}{\sqrt{4-x^2-y^2}}$

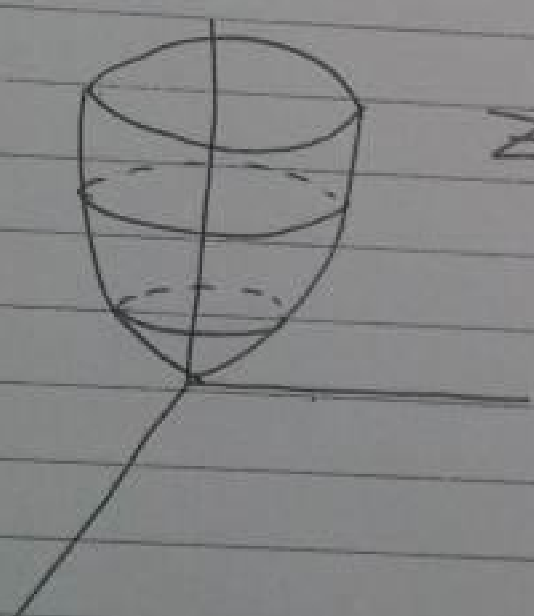
$y - x \neq 0$ & $4 - x^2 - y^2 > 0$

$y \neq x$ & $4 > x^2 + y^2$

$y = x$ & $4 = x^2 + y^2$ نعمل مساواة للرسم .



$f(2,2) = \text{undefined}$



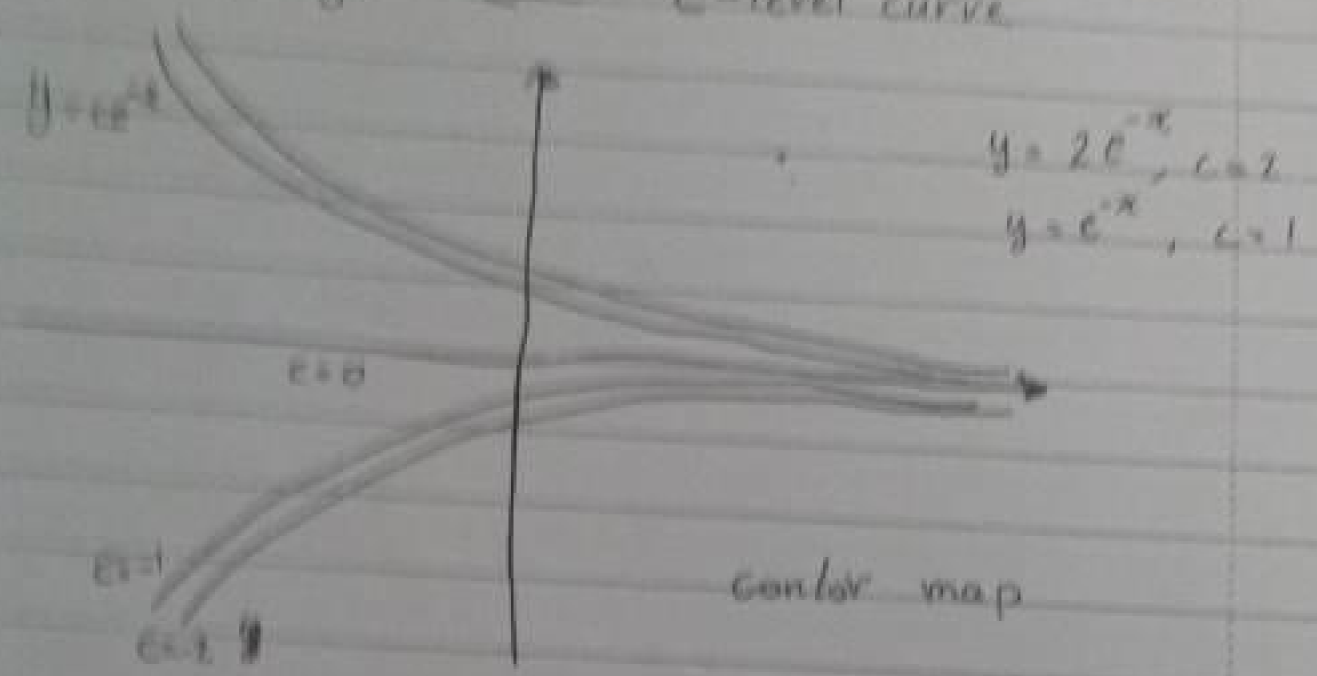
$Z = x^2 + y^2$

comtor map

(ii) $f(x,y) = ye^{x}$; sketch the contor map (it consists of c level curves)

$z = ye^{x}$; let $z = c$, $c \in$ Range

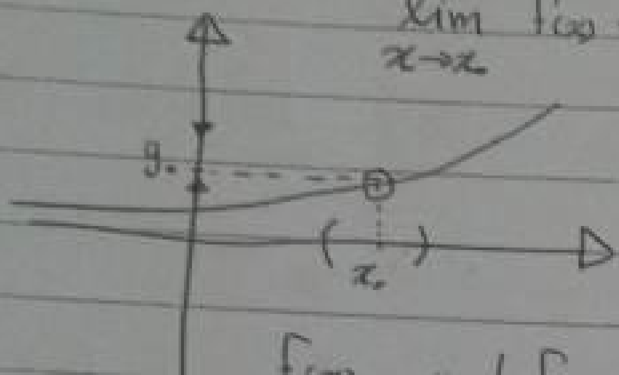
$c \in \mathbb{R}$ $\Rightarrow$ $c = ye^{x}$ ← c-level curve



Ch. 14 S: 14.2 limits & Continuity

i) 1-Variable

$\lim_{x \to x_0} f(x) = L$ exists: 1) Finite 2) unique 3) real number



$f(x_0)$ undefined

* $\lim_{x \to \infty} \frac{1}{x} = 0$

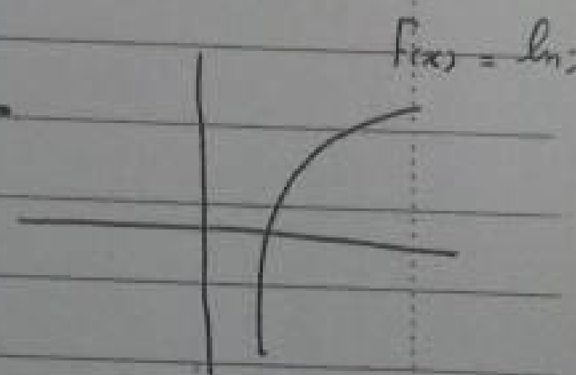
* $\lim_{x \to -\infty} \frac{1}{x} = \frac{1}{-\infty} = -0 = 0$

* $\lim_{x \to 0^+} \frac{1}{x} = \frac{1}{0^+} = +\infty$

* $\lim_{x \to 0^-} \frac{1}{x} = \frac{1}{0^-} = -\infty$

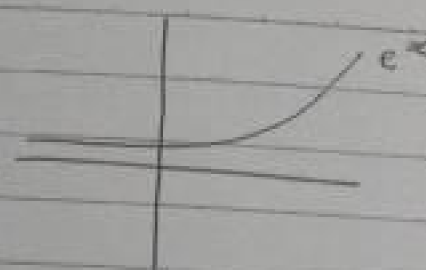
* $\lim_{x \to \infty} \ln x = \ln \infty = \infty$

* $\lim_{x \to 0^+} \ln x = \ln 0^+ = -\infty$.



- $\lim_{x\to\infty} e^x = e^\infty = \infty$

- $\lim_{x\to-\infty} e^x = \frac{1}{e^\infty} = 0$



ex ① $\lim_{x\to\infty} \frac{\ln x}{x} = \frac{\infty}{\infty}$!!?

$\overset{L.H.}{=} \lim_{x\to\infty} \frac{1}{x} = 0$

② $\lim_{x\to\infty} \frac{e^x}{x} = \frac{\infty}{\infty}$!?

$\overset{L.H.}{=} \lim_{x\to\infty} \frac{e^x}{1} = \infty$

* Indeterminales:

$\frac{0}{0}$, $\frac{\infty}{\infty}$, $0 \cdot \infty$, $\infty - \infty$, 1^∞, 0^0, ∞^0

L.H.

$\frac{\infty}{\frac{1}{0}} = \frac{\infty}{\infty}$

$\infty + \infty = \infty$

$-\infty - \infty = -\infty$

$\infty - \infty =$!!

$-\infty + \infty =$!!

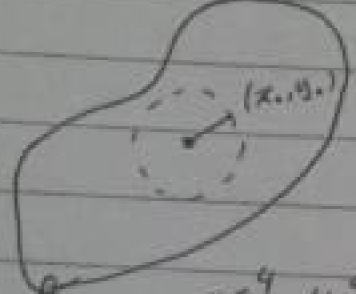
$\lim_{x\to\infty} \left(1 + \frac{2}{x}\right)^x \; e^2$

Subject Date No.

ii) 2-Variables

$\lim\limits_{(x,y)\to(x_0,y_0)} f(x,y) = L$, Exist, if for all parts that leads to (x_0,y_0) you get the same answer $= L$

$* \lim\limits_{(x,y)\to(x_0,y_0)} F$ doesn't Exist "Two diffrent pathes to get two different answer

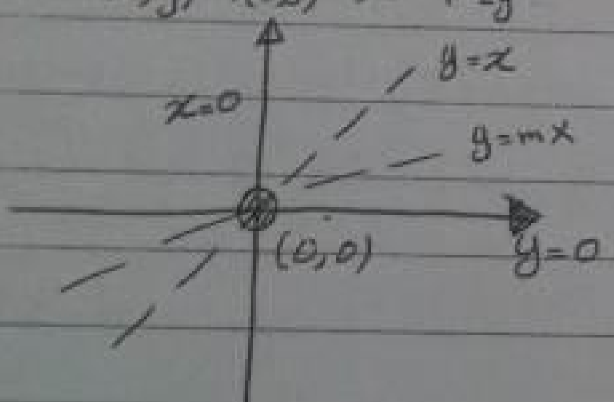


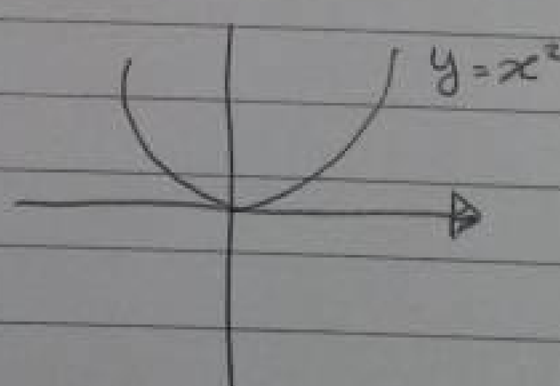
ex ① $\lim\limits_{(x,y)\to(1,2)} \dfrac{x^4-y^4}{x^2+y^2} = \dfrac{1-2^4}{1+2^4} = \dfrac{-15}{5} = -3$

② $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^4-y^4}{x^2+y^2} = \dfrac{0}{0}$!??

$= \lim\limits_{(x,y)\to(0,0)} \dfrac{(x^2-y^2)\cancel{(x^2+y^2)}}{\cancel{(x^2+y^2)}} = 0-0=0$

③ $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy^3}{x^4+2y^4} = \dfrac{0}{0}$!!?





$\Longrightarrow$ cont

N O T E B O O K

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i) a long path $y=x$

$$\Rightarrow \lim_{x\to 0} \frac{x \cdot x^3}{x^4+2x^4} = \lim_{x\to 0} \frac{x^4}{3x^4} = \frac{1}{3}$$

ii) a long path $y=2x$

$$\Rightarrow \lim_{x\to 0} \frac{x \cdot 8x^3}{x^4+2.2^4x^4} = \lim_{x\to 0} \frac{8x^4}{33x^4} = \frac{8}{33}$$

two different pathes give two different answers; ∴ limit doesn't exist

---

cal-3　　　　4/7/2012

* Evaluate the limit if exists OR doesn't exist:-

17/877

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1} = \frac{0}{0}!!$$

$$= \lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1} \cdot \frac{\sqrt{x^2+y^2+1}+1}{\sqrt{x^2+y^2+1}+1}$$

$$= \lim_{(x,y)\to(0,0)} \frac{\cancel{x^2+y^2}(\sqrt{x^2+y^2+1}+1)}{\cancel{(x^2+y^2)+1-1}} = 2$$

15

$$\lim_{(x,y)\to(0,0)} \frac{x^2 y e^y}{x^4+4y^2} = \frac{0}{0}!!$$

i) a long $y=x^2$

$$\Rightarrow \lim_{(x)\to 0} \frac{x^2 \cdot x^2 e^{x^2}}{x^4+4x^4} = \lim_{x\to 0} \frac{x^4 e^{x^2}}{x^4(1+4)} = \frac{1}{5}$$

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ii) a long $y = 2x^2$

$$\lim_{x\to 0} \frac{x^2 \cdot 2x^2 e^{2x^2}}{x^4 + 4(4x^4)} = \lim_{x\to 0} \frac{2x^4 e^{2x^2}}{x^4(1+16)} = \frac{2}{17}$$

two different pathes $\Longrightarrow$ two differenb answers

$\therefore$ limit doesn't exist .

③ $$\lim_{(x,y)\to(0,0)} \frac{y \sin xy}{x} = \frac{0}{0} \;!!$$

$$\boxed{\lim_{h\to 0} \frac{\sin h}{h} = 1}$$

$$= \lim_{(x,y)\to(0,0)} \frac{y \sin xy}{x} \cdot \frac{y}{y}$$

$$= \lim_{(x,y)\to(0,0)} \frac{y^2 \sin xy}{xy} = 0 \cdot \lim_{(x,y)\to(0,0)} \overbrace{\frac{\sin xy}{xy}}^{1} = 0$$

④ $$\lim_{(x,y)\to(0,0)} \sin \frac{1}{x^2+y^2} = \sin \infty$$

• doesn't exist because it has many answers between $[-1, 1]$

ال Cos نفس الحل

ex $f(x,y) = \sin \dfrac{1}{x^2+y^2}$

$D_f : R^2 / \{(0,0)\}$

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⑤ $\lim_{(x,y)\to(0,0)} x \sin\frac{1}{x^2+y^2}$

$= 0$. doesn't exist

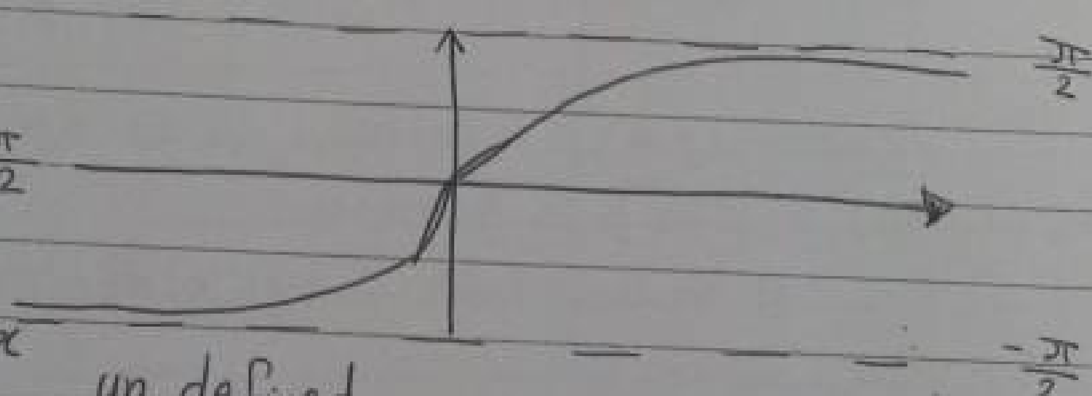
$-1 \leqslant \sin\frac{1}{x^2+y^2} \leqslant 1$

$-x \leqslant \underbrace{x\sin\frac{1}{x^2+y^2}}_{\downarrow\ 0} \leqslant x$

$\downarrow$ 0 $\qquad$ $\downarrow$ 0 by squeezing thrm

* $\tan^{-1}x$

$\lim_{x\to\infty} \tan^{-1}x = \frac{\pi}{2}$



* $f(x) = (-1)^x$ un defined

$f(\frac{1}{2}) = (-1)^{\frac{1}{2}} = \sqrt{-1}$ X

ex $\lim_{(x,y)\to(0,0)} \frac{x^{1/3}\,y}{x + y^{3/2}} = \frac{0}{0}$!!

i) $y = x^{2/3}$

$\lim_{x\to0}\frac{x^{1/3}x^{2/3}}{x+(x^{2/3})^{3/2}} = \lim_{x\to0}\frac{x}{x+x} = \frac{1}{2}$ $\Longrightarrow$ conb.

N O T E B O O K

34/40/41

ii) $y = 2x^{2/3}$

$$\lim_{x\to 0} \frac{x^{1/3}\, 2x^{2/3}}{(x)+(2x^{2/3})^{3/2}} = \lim_{x\to 0} \frac{2x}{x(2^{3/2}+1)} = \frac{2}{(2^{3/2}+1)}$$

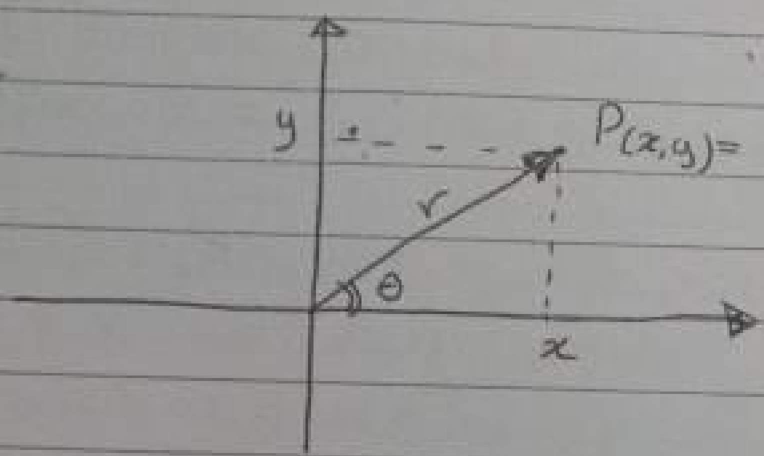
Polar coordinate

- $x = r\cos\theta$

  $y = r\sin\theta$

  $x^2+y^2 = r^2$

  $\tan\theta = \frac{y}{x}$



* Origin (0,0) cartesin $[(x,y)\to(0,0)]$

polar $(r=0)$ [only r (goes to zero)]

(40) $\lim_{(x,y)\to(0,0)} (x^2+y^2)\ln(x^2+y^2) = 0\ln 0 = 0.\infty$!!

Polar

$= \lim_{r\to 0} r^2 \ln r^2 = \lim_{r\to 0} 2r^2 \ln r = 0.\infty$!!

$= 2\lim \frac{\ln r}{r^{-2}} \xRightarrow{L.H.}$

Subject

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S. 14.3 Partial derivatives (for a function of several variables)

$Z = f(x,y)$

$\frac{\partial z}{\partial x} = f(x)$ = differentiate partially with respect to x & treat y as const

$\frac{\partial z}{\partial y} = f(y)$ = differentiate partially with respect to y & treat x as const

ex $f(x,y) = \cos xy + x^2 + xy^3 + e^{xy^2}$

i) $f_x = y(-\sin xy) + 2x + y^3 + y^2 e^{xy^2}$

ii) $f_y = x(-\sin xy) + 0 + 3xy^2 + 2xy\, e^{xy^2}$

$\boxed{P. 884}$ ① $f(x,y) = \frac{x-y}{x+y}$ 10/7/2012

$f_x = \frac{\partial f}{\partial x} = ??$

$f_x = \frac{(x+y)(1) - (x-y)(1)}{(x+y)^2}$

$f_y = \frac{(x+y)(-1) - (x-y)(1)}{(x+y)^2}$

② $f(x,y) = \frac{x}{y}$

$f_x = \frac{1}{y}$ $f_y = \frac{-x}{y^2}$

NOTEBOOK

☒ Reminder :

$(x^2)' = 2x$

$(2^x)' = 2^x \ln 2$

③ $f(x,y) = x^y$

$f_x = y x^{y-1} \quad , \quad f_y = x^y \ln x$

④ $u = xy \sin^{-1} yz$

$u_x = y \sin^{-1} yz \quad , \quad u_y = \dfrac{(xy)\cdot z}{\sqrt{1-(yz)^2}} + x \cdot \sin^{-1} yz$

$u_z = \dfrac{xy \cdot y}{\sqrt{1-(yz)^2}}$

⑤ $f(x,t) = \tan^{-1} x\sqrt{t}$

$f_x = \dfrac{\sqrt{t}}{1+(x\sqrt{t})^2} = \dfrac{\sqrt{t}}{1+x^2 t}$

$f_t = \cancel{\dfrac{1}{1+\cos x}} \; \dfrac{1}{1+x^2 t} \cdot \dfrac{x}{2\sqrt{t}}$

ex $f(x) = \displaystyle\int_{x^2}^{\cos x} \frac{1}{1+t^3}\, dt$

$f'(x) = \dfrac{d}{dx} \displaystyle\int_{x^2}^{\cos x} \frac{1}{1+t^3}\, dt$

مشتقة الحد

مشتقة اد

$= \dfrac{1}{1+\cos^3 x} \cdot (-\sin x) - \dfrac{2x}{1+(x^2)^3}$

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(28) $f(x,y) = \int_y^x \cos t^2 \, dt$

$f_x = [\cos x^2 \cdot 1] - [\cos y^2 \cdot 0]$ مشتقة حد ثابت

$f_y = -\cos y^2 \cdot 1$

ex $f(x,y) = xy^2 + e^{xy}$

① $\frac{\partial f}{\partial x} = f_x = y^2 + y e^{xy}$

② $\frac{\partial f}{\partial y} = f_y = 2xy + x e^{xy}$

☒ 2^{nd} derv of $\boxed{1}$

i) $\frac{\partial^2 f}{\partial x^2} = f_{xx} = 0 + yy e^{xy}$

ii) $\underset{\leftarrow}{\frac{\partial^2 f}{\partial y \partial x}} = \underset{\rightarrow}{f_{xy}} = 2y + y \cdot x e^{xy} + e^{xy}$ [2^{nd} mixed partial derivater]

☒ 2^{nd} derv of $\boxed{2}$

i) $\frac{\partial^2 f}{\partial y^2} = f_{yy}$

ii) $\frac{\partial^2 f}{\partial x \partial y} = f_{yx}$

ex $f(x,y) = x^3 y^6$

find $\frac{\partial^5 f}{\partial x^2 \partial y^3} = f_{yyyxx}$

$\frac{\partial f}{\partial y} = 6x^3y^5$

$\frac{\partial^2 f}{\partial y^2} = 6.(5)\, x^3 y^4$

$\frac{\partial^3 f}{\partial y^3} = 6.(5)(4)\, x^3 y^3$

$\frac{\partial^4 f}{\partial x \partial y^3} = 120\,(3)\, x^2 y^3$

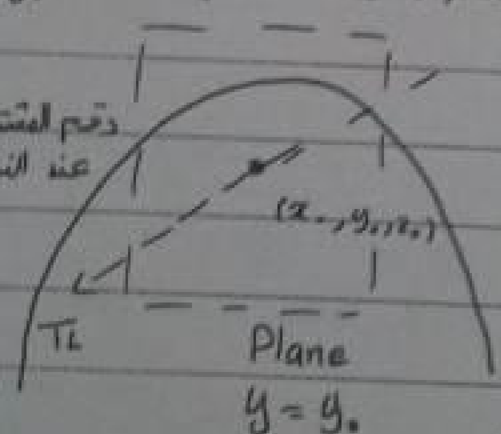
$\frac{\partial^5 f}{\partial x^2 \partial y^3} = 120(3)(2)\, x\, y^3 = 6!\; x y^3$

* Geometrical meaning of f_x at (x,y)

$Z = f_{(x,y)} \longleftarrow$ Surface

Note:
surface + plane = Curve

$f_x(x_0,y_0) =$ رسم المشتقة عند النقطة
$y = y_0$



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* $f_x(x_0, y_0)$ = slope of the tangent line to the curve of intersection between the surface $z = f(x,y)$ with the plane $y = y_0$ at the point (x_0, y_0, z_0).

* $f_y(x_0, y_0)$ = $x = x_0$

ex $f(x,y) = x^2 y^3$

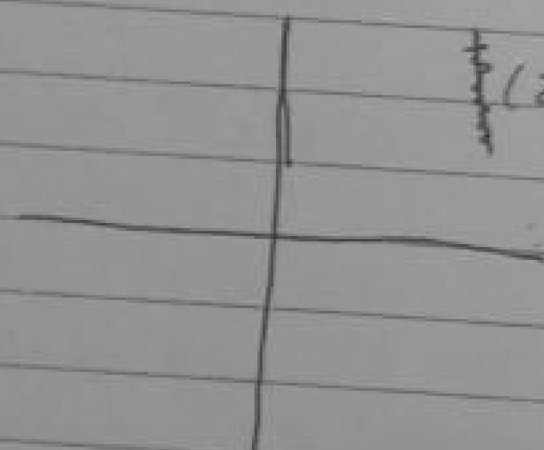
write the eq of the tangent line to the curve of intersection between this surface & plane $y=2$ at the point $(1, 2, 8)$

∴ slope $f_x(1,2)$ beause it's intersect with the plane $y = y_0$

$f_x = 2xy^3 \Rightarrow$

$\Rightarrow 2(1)(2)^3 = 16$

$(z-8) = 16(x-1)$



$(z - z_0) = m(x - x_0)$

* Laplace Eq :

$$u_{xx} + u_{yy} = 0$$

ex Is $u(x,y) = \ln\sqrt{x^2+y^2} = \frac{1}{2}\ln(x^2+y^2)$ a solution for Laplace or Not.

$u_x = \frac{1}{2}\,\frac{2x}{x^2+y^2} = \frac{x}{x^2+y^2}$

$u_y = \frac{y}{x^2+y^2}$; $u_{yy} = \frac{x^2-y^2}{(y^2+x^2)^2}$

$u_{xx} = \frac{(x^2+y^2)-(x)(2x)}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$

$u_{xx} + u_{yy} =$

$\frac{y^2-x^2+x^2-y^2}{(y^2+x^2)^2} = 0$

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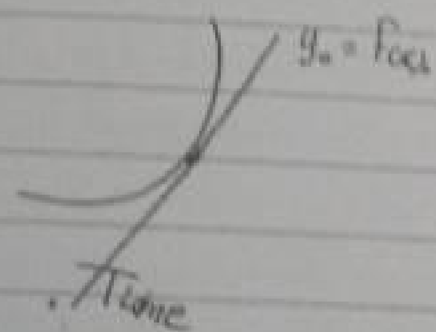
Subject Cal-3

Date 11/7/2012 No.

14.4 Tangent planes & Linear approximation

i) 1-Variable

$y = f(x)$



T: $y - y_0 = f'(x_0)(x - x_0)$

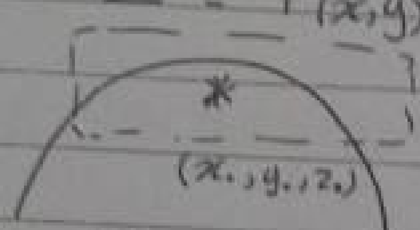
$y = y_0 + f'(x_0)(x - x_0)$

$y = f(x_0) + f'(x_0)(x - x_0)$

ii) 2-Variables

$Z = f(x, y) \Longrightarrow$ Surface

$Z_0 = f(x_0, y_0)$



T. P.

$$Z - Z_0 = f_x(x - x_0) + f_y(y - y_0)$$

$$Z = \underbrace{Z_0 + f_x(x - x_0) + f_y(y - y_0)}_{L(x,y)}$$

$\Longleftarrow$ Linearization of f

- $f(x, y) \cong L(x, y)$
- $f(x, y) \cong Z$ tangent

$f(x,y) \cong f(x_0, y_0) + f_x(x - x_0) + f_y(y - y_0)$

Linerization of f

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ex $f_{(x,y)} = x e^{xy}$

① find the Linearization of "f" at (2,0)

$* \ L_{(x,y)} = f_{(x_0,y_0)} + f_x (x - x_0) + f_y (y - y_0)$

$\cdot f_{(2,0)} = 2e^{2\times 0} = 2$

$\cdot f_x = x \cdot y \cdot e^{xy} \Big|_{(2,0)} = e^0 = 1$

$\cdot f_y = x^2 e^{xy} \Big|_{(2,0)} = 4e^0 = 4$

$\therefore L_{(x,y)} = 2 + 1(x-2) + 4(y-0)$

② write an eq of tangent plane at (2,0)

$$Z = 2 + (x-2) + 4(y-0)$$

③ Use this app to approximate $f_{(2.01, -0.3)}$

$f_{(2.01, -0.3)} \cong 2 + 1(2.01 - 2) + 4(-0.3 - 0)$

$\cong 2 + 0.01 - (1.2)$

$\cong 0.80 + 0.01$

$\cong 0.81$

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ex approximate Linearly:

$$\sqrt{(3.02)^2+(1.97)^2+(5.99)^2}$$

let $f(x,y,z) = \sqrt{x^2+y^2+z^2}$, at $(3,2,6)$; Linearization

* $f(3,2,6) = \sqrt{3^2+2^2+6^2} = \sqrt{49} = 7$

$$f_x = \frac{\partial f}{\partial x} = \frac{2x}{2\sqrt{x^2+y^2+z^2}} = \left.\frac{x}{\sqrt{x^2+y^2+z^2}}\right|_{(3,2,6)} = \frac{3}{7}$$

$$f_y = \frac{\partial f}{\partial y} = \frac{2y}{2\sqrt{x^2+y^2+z^2}} = \left.\frac{y}{\sqrt{x^2+y^2+z^2}}\right|_{(3,2,6)} = \frac{2}{7}$$

$$f_z = \frac{\partial f}{\partial z} = \frac{6}{7}$$

$$f(x,y,z) \cong 7 + f_x(x-3) + f_y(y-2) + f_z(z-6)$$

$$f(x,y,z) \cong 7 + \frac{3}{7}(x-3) + \frac{2}{7}(y-2) + \frac{6}{7}(z-6)$$

$$\sqrt{3.02^2+1.97^2+5.99^2} \cong 7 + \frac{3}{7}(0.02) + \frac{2}{7}(-0.03) + \frac{6}{7}(-0.01)$$

$$\cong 7 + \frac{0.06}{7} - \frac{0.06}{7} - \frac{0.06}{7}$$

$$\cong 7 - \frac{0.06}{7}$$

$\boxed{17}$ Verify this Linear app.

$$\frac{2x+3}{4y+1} \cong 3+2x-12y \quad \text{at } (0,0)$$

let $f(x,y) = \frac{2x+3}{4y+1}$, app at $(0,0)$

- $f(0,0) = 3$
- $f_x = \left.\frac{2}{4y+1}\right|_{(0,0)} = 2$
- $f_y = \left.\frac{-4(2x+3)}{(4y+1)^2}\right|_{(0,0)} = -12$ سالب مشتقة المقام في البسط على المقام تربيع

$$f(x,y) \cong 3 + 2(x-0) + (-12)(y-0)$$

$$\frac{2x+3}{4y+1} \cong 3+2x-12y$$

☆ 1-Variable $(y=f(x))$

- $dy = f'(x)\,dx$

$\Delta x = x_2 - x_1 = dx$

$\Delta y = y_2 - y_1 \neq dy$ But $\Delta y \cong dy$

2-Variables $z = f(x,y)$

$\Delta x = x_2 - x_1 = dx$

$\Delta y = y_2 - y_1 = dy$ $\quad dz = f_x\,dx + f_y\,dy$

$\Delta z = z_2 - z_1 = dz$

Subject

Date No.

$\boxed{31}$ $Z = 5x^2 + y^2$; (x,y) changes from $(1,2)$ to $(1.05, 2.10)$, Compare the Values of ΔZ & dZ

① $x_1 = 1$, $y_1 = 2$

$x_2 = 1.05$, $y_2 = 2.10$

i) $\Delta Z = Z_2 - Z_1$

$= (f(1.05, 2.10)) - (f(1,2))$

$= 5(1.05)^2 + (2.1)^2 - 5 + 4$

$= 9.9225 - 9 = 0.9225$

ii) $dz = f_x \cdot dx + f_y \cdot dy$

$f_x = 10x \Big|_{(1,2)} = 10$

$f_y = 2y \Big|_{(1,2)} = 4$

$dx = \Delta x = (1.05 - 1) = 0.05$

$dy = \Delta y = (2.1 - 2) = 0.1$

$dz = 10(0.05) + 4(0.1)$

$= 0.5 + 0.4 = 0.9$

② find the app. Value of ΔZ

نحسب ال dz

$\Delta Z \cong dz$

$\cong 0.9$

N O T E B O O K

S, 14.5 Chain Rule

i) 1 Variable,

$$y = f(x), \quad x = x_{(t)}$$

$$\begin{tikzcd} y \arrow[r] \arrow[rr, bend left] & x \arrow[r] & t \end{tikzcd}$$

$$\frac{dy}{dx} \qquad \frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

ii) 2-Variable

① $Z = f_{(x,y)}, \quad x = x_{(t)} \ \& \ y = y_{(t)}$

$$\begin{tikzcd} & x \arrow[r] & t \\ Z = f \arrow[ur] \arrow[dr, "①"'] & & \\ & y \arrow[r, "②"'] & t \end{tikzcd}$$

$$\frac{dz}{dt} = f_x \frac{dx}{dt} + f_y \frac{dy}{dt}$$

② $Z = f_{(x,y)}, \quad x = x(t,s) \ \& \ y = y_{(t,s)}$

$$\begin{tikzcd} & & t \\ & x \arrow[ur] \arrow[r] & s \\ Z \arrow[ur] \arrow[dr] & & t \\ & y \arrow[ur] \arrow[r] & s \end{tikzcd}$$

find $\frac{\partial Z}{\partial t} = \frac{\partial Z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial Z}{\partial y} \cdot \frac{\partial y}{\partial t}$

<u>ex</u> $Z = f(x, y)$; $x = t^3 S$, $y = t^4$

① find $\frac{\partial z}{\partial t} = ??$

$$\begin{tikzcd}[row sep=small] & x \arrow[r] \arrow[dr] & t \\ Z \arrow[ur] \arrow[dr] & & s \\ & y \arrow[r] & t \end{tikzcd}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

$$= Z_x \cdot 3t^2 S + Z_y \cdot 4t^3$$

② find $\frac{\partial z}{\partial s} = ??$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial s} = Z_x \cdot t^3$$

<u>ex</u> $Z = x^2 y + 3xy^4$, $x = \sin 2t$, $y = \cos t$

find $\left.\frac{dz}{dt}\right|_{t=0} = ??$

$$\begin{tikzcd}[row sep=small] & x \arrow[r] & t \\ Z \arrow[ur] \arrow[dr] & & \\ & y \arrow[r] & t \end{tikzcd}$$

$$\frac{dz}{dt} = Z_x \cdot \frac{dx}{dt} + Z_y \frac{dy}{dt}$$

$$= (2xy + 3y^4)(2\cos 2t) + (x^2 + 12xy^3)(-\sin$$

$$= 3(2) + 0 = 6$$

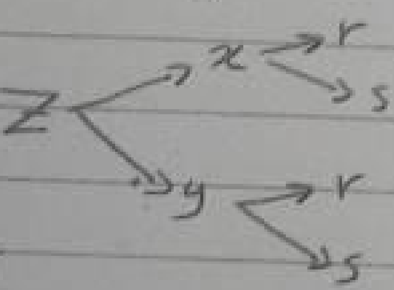
$$\begin{tikzcd}[row sep=small] & x \arrow[r] & t \\ Z_x \arrow[ur] \arrow[dr] & & \\ & y \arrow[r] & t \end{tikzcd}$$

$t =$
$x =$
$y =$

* 2^{nd} derivative (chain Rule)

ex $Z = f(x,y)$, $x = r^2 + s^2$, $y = 2r^3 s$

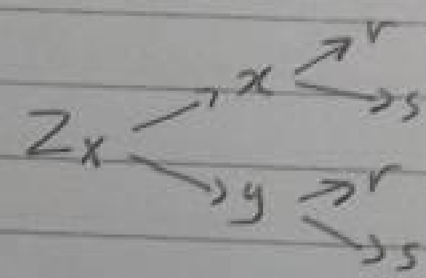
find $\dfrac{\partial^2 Z}{\partial r^2}$

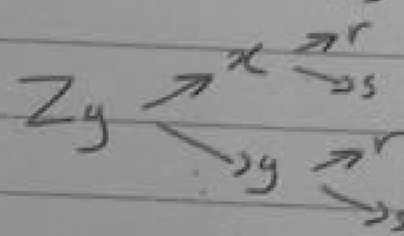


i) $\dfrac{\partial Z}{\partial r} = Z_x \cdot x_r + Z_y \cdot y_r$

$= Z_x \cdot 2r + Z_y \, 6r^2 s$

ii) $\dfrac{\partial^2 Z}{\partial r^2} = \left[(Z_x \cdot 2) + (2r \, Z_{xr})\right] + \left[(Z_y \cdot 12rs) + (6r^2 s \cdot Z_{yr})\right]$





$Z_{xr} = Z_{xx} \dfrac{\partial x}{\partial r} + Z_{xy} \dfrac{\partial y}{\partial r}$

$Z_{yr} = Z_{yx} \dfrac{\partial x}{\partial r} + Z_{yy} \dfrac{\partial y}{\partial r}$

$\dfrac{\partial^2 Z}{\partial r^2} = 2Z_x + 2r\left[Z_{xx} \dfrac{\partial x}{\partial r} + Z_{xy} \dfrac{\partial y}{\partial r}\right] + Z_y \, 12rs$
$+ \, 6r^2 s \cdot \left[Z_{yx} \dfrac{\partial x}{\partial r} + Z_{yy} \dfrac{\partial y}{\partial r}\right]$

$\dfrac{\partial x}{\partial r} = 2r$, $\dfrac{\partial y}{\partial r} = 6r^2 s$

Subject Date No.

* Emplicit differentiation (Equation)

ex $x^2y^3 + x^3 + y^2 = 4$

find $\frac{dy}{dx}$??

$$x^2y^3 + x^3 + y^2 - 4 = 0$$

$F(x,y) = 0$ (eq)

$0 = f(x,y) \to x,\ y \to x$

$\Rightarrow 0 = F_x + F_y \frac{dy}{dx}$

$$\therefore \frac{dy}{dx} = \frac{-F_x}{F_y}$$

$$\frac{dy}{dx} = -\left[\frac{2xy^3 + 3x^2}{3y^2x^2 + 2y}\right]$$

* $F(x,y,z) = 0$

Find $\frac{\partial z}{\partial x}$; ??

$$\frac{\partial z}{\partial x} = \frac{-F_x}{F_z} \quad \Bigg| \quad \frac{\partial z}{\partial y} = \frac{-F_z}{F_y}$$

NOTEBOOK

ex $Z = \frac{\ln(x+z)}{y}$

Find $\frac{\partial z}{\partial x} = ??$

$\Rightarrow Zy = \ln(x+z)$

$\Rightarrow Zy - \ln(x+z) = 0$

$F(x,y,z)$

$$\frac{\partial z}{\partial x} = \frac{-F_x}{F_z} = \frac{-\left[\frac{1}{x+z}\right]}{y - \frac{1}{x+z}}$$

Cal-3 15/7

ex Find $\frac{\partial z}{\partial x}$

① $Z = \sqrt{x^2 + y^2} + \frac{1}{x}$ "function"

i) $\frac{\partial z}{\partial x} = \frac{2x}{2\sqrt{x^2+y^2}} - \frac{1}{x^2}$

ii) $\frac{\partial z}{\partial y} = \frac{2y}{2\sqrt{x^2+y^2}}$

Subject Date No.

② $Z = \sqrt{y^2+z^2} + \frac{1}{x}$ "equation"

هذه ليست اقتران وهذه معادلة

$Z - \sqrt{y^2+z^2} - \frac{1}{x} = 0$

$F(x,y,z) = 0$ [Implicit].

i) $\frac{\partial z}{\partial x} = \frac{-F_x}{F_z}$

$= \frac{-\left[\frac{1}{x^2}\right]}{1-\left[\frac{2z}{2\sqrt{y^2+z^2}}\right]}$

ii) $\frac{\partial z}{\partial y} = \frac{-F_y}{F_z}$

$= \frac{-\left[\frac{2y}{2\sqrt{y^2+z^2}}\right]}{1-\left[\frac{2z}{2\sqrt{y^2+z^2}}\right]}$

ex $Z = y + f(x^2-y^2) \Longrightarrow Z = y + f(u)$, $u = x^2-y^2$ منفرض

$f \to u \to x, y$

show that

$x\frac{\partial z}{\partial y} + y\frac{\partial z}{\partial x} = x$

i) $\left(\frac{\partial z}{\partial y} = 1 + f'(u)\frac{\partial u}{\partial y} = [1 + f'(u)(-2y)]\right) \times x$

ii) $\left(\frac{\partial z}{\partial x} = 0 + f'(u)\frac{\partial u}{\partial x} = [f'(u)(2x)]\right) \times y$

$x\frac{\partial z}{\partial y} + y\frac{\partial z}{\partial x} = x \cancel{-2xyf'(u)} + \cancel{2xyf'(u)}$

Subject Date No.

S: 14.6 Directional derivatives & the gradient Vec

* Gredient Vector: $\Rightarrow \vec{\nabla}$

① $\vec{\nabla} f_{(x,y)} = f_x \hat{i} + f_y \hat{j}$
② $\vec{\nabla} f_{(x,y,z)} = f_x \hat{i} + f_y \hat{j} + f_z \hat{k}$

ex $f_{(x,y,z)} = x^2 y + y^3 x + z^2$

$\vec{\nabla} f = f_x \hat{i} + f_y \hat{j} + f_z \hat{k}$
$= [2xy + y^3] \hat{i} + [x^2 + 3y^2 x] \hat{j} + 2z \hat{k}$

$\vec{\nabla} f_{(2,-1,3)} = -5 \hat{i} + 6 \hat{j} + 6 \hat{k}$

* Directional Derivative: $D_{\vec{a}} f$

→ directional derivative of the function f in the direction of the vector $(\vec{a})$ → (Rate of change)

$D_{\vec{a}} f = \vec{\nabla} f \cdot \hat{u}_{\vec{a}}$, $\hat{u}_{\vec{a}} \equiv$ Unit Vector in the direction of $\vec{a}$

$= \frac{\vec{a}}{\|\vec{a}\|}$

ex $f_{(x,y)} = xy$

① $D_{(3\hat{i}+4\hat{j})} f = \vec{\nabla} f \cdot \hat{u}_{\vec{a}}$

$\hat{u}_{\vec{a}} = \frac{3\hat{i} + 4\hat{j}}{5}$

$\vec{\nabla} f = f_x \hat{i} + f_y \hat{j} = y \hat{i} + x \hat{j}$

$Df = \frac{3}{5} \hat{i} + \frac{4}{5} \hat{j}$

Next ⇒

Subject

Date No.

② $D_{(3\hat{i}-4\hat{j})}(-2,3) = \vec{\nabla} f_{(-2,3)} \cdot \hat{u}_{\vec{a}}$

$\hat{u}_{\vec{a}} = \frac{3}{5}\hat{i} - \frac{4}{5}\hat{j}$

$\vec{\nabla} f = y\hat{i} + x\hat{j}$

$\nabla f = 3\hat{i} - 2\hat{j}$

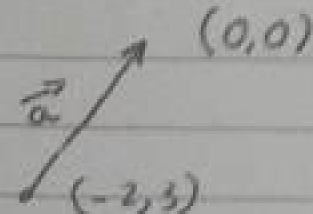
$D = \frac{3}{5}(3) - \frac{4}{5}(-2)$

$= \frac{9}{5} + \frac{8}{5} = \frac{17}{5}$

Subject:

③ $D_{\vec{f}}f_{(-2,3)}$ towards the Origin

$\vec{a} = 2\hat{i} - 3\hat{j}$



$u_{\vec{a}} = \dfrac{2\hat{i} - 3\hat{j}}{\sqrt{13}}$

$D_{(2\hat{i}-3\hat{j})}f_{(-2,3)} = \vec{\nabla}f(-2,3) \cdot \hat{u}_{\vec{a}}$

$= (3\hat{i} - 2\hat{j}) \cdot \dfrac{(2\hat{i} - 3\hat{j})}{\sqrt{13}}$

$= \dfrac{6+6}{\sqrt{13}} = \dfrac{12}{\sqrt{13}}$

④ $D_{-2\hat{i}}f_{(x,y)}$

$\hat{u}_{-2\hat{i}} = -i$

$= \vec{\nabla}f \cdot (-\hat{i})$

$= -f_x$

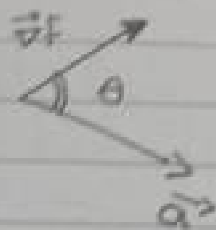
⑤ $D_{3\hat{j}}f_{(-2,3)} = \vec{\nabla}f \cdot \hat{j}$

$= f_y$

$\hat{a} = \hat{j}$ $\quad = x\Big|_{(-2,3)} = -2$

Subject:

$* \; D_{\vec{a}} f = \vec{\nabla} f \cdot \hat{u}_{\vec{a}}$

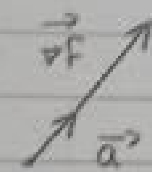


$= \|\vec{\nabla} f\| \, \|\hat{u}_{\vec{a}}\| \cos\theta$

$= \|\vec{\nabla} f\| \cdot 1 \cdot \cos\theta$

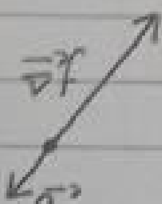
i) max value of $D_{\vec{a}} f$; $\theta = 0$

$\vec{a}$ in the direction of



ii) min Value of $D f$; $\theta = \pi$

$-\|\vec{\nabla} f\| \leq D_{\vec{a}} f$



in the opposite direction of $\vec{\nabla} f$

at a point:-

$\boxed{1}$ Df increases most rapidly in the direction of $\vec{\nabla} f$ & its max. value $= \|\vec{\nabla} f\|$

$\boxed{2}$ Df decreases most rapidly in the opposite direction of $\vec{\nabla} f$ & its min value $= -\|\vec{\nabla} f\|$

ex:

$D_{(2\hat{i}-\hat{j}-2\hat{k})} f_{(3,-2,1)} = -5$ & $\|\vec{\nabla} f\|_{(3,-2,1)} = 5$

Find $\vec{\nabla} f_{(3,-2,1)} = ??$

direction $\vec{\nabla} f = -\vec{a}$

$= -2\hat{i} + \hat{j} + 2\hat{k}$

$\vec{\nabla} f = \|\vec{\nabla} f\| \, \hat{u}_{\vec{\nabla} f} = 5 \, \hat{u}_{\vec{\nabla} f}$

$\hat{u} = \frac{-2\hat{i} + \hat{j} + 2\hat{k}}{3}$

$\hat{u}_{\vec{\nabla} f} = \frac{-2\hat{i} + \hat{j} + 2\hat{k}}{3}$

$= \frac{-2}{3}\hat{i} + \frac{\hat{j}}{3} + \frac{2}{3}\hat{k}$

$\vec{\nabla} f = 5\left(\frac{-2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}\right)$

ZOOM Notebook

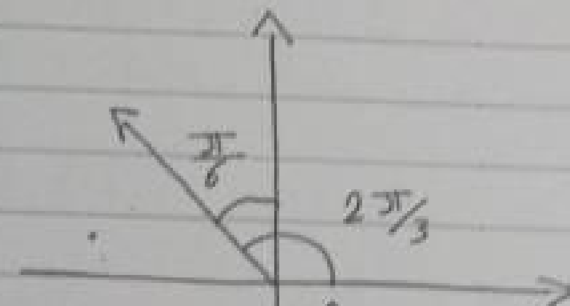
Subject: Cal-3

16.7.2012

ex: Directional Derivative :-

① Find directional derivative of $f(x,y) = ye^{-x}$ at the point $(0,4)$ in the direction that makes an angel $\frac{2\pi}{3}$ with the x-axis

$$D_{\hat{u}} f = \vec{\nabla} f \cdot \hat{u}$$



$\hat{u} = \cos\alpha \,\hat{i} + \cos\beta$

$\hat{u} = \cos\frac{2\pi}{3}\hat{i} + \cos\frac{\pi}{6}$

$= \frac{-1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}$

$\vec{\nabla} f = f_x \hat{i} + f_y \hat{j}$

$= -ye^{-x}\hat{i} + e^{-x}\hat{j}$

$\vec{\nabla} f_{(0,4)} = -4\hat{i} + \hat{j}$

$D_{\hat{u}} f_{(0,4)} = -4\left(-\frac{1}{2}\right)\hat{i} + \frac{\sqrt{3}}{2}\hat{j} = \frac{4+\sqrt{3}}{2}$

$\boxed{28}$ Find the directions $\hat{u}$ in which the directional derivative of $f(x,y) = ye^{-xy}$ at $(0,2)$ has value "one 1"

$u = u_1\hat{i} + u_2\hat{j}$ & $\|\hat{u}\| = 1$

$u_1^2 + u_2^2 = 1$

Given: $D_{\hat{u}} f_{(0,2)} = 1$

$\vec{\nabla} f = f_x\hat{i} + f_y\hat{j} = -y^2 e^{-xy}\hat{i} + \left[(y)(-xe^{-xy}) + (e^{-xy})\right]\hat{j}$

$\vec{\nabla} f_{(0,2)} = -4\hat{i} + \hat{j}$

$1 = D_{\hat{u}} f = \vec{\nabla} f \cdot \hat{u}$

$1 = -4u_1 + u_2$

$u_2 = 4u_1 + 1$

$u_1^2 + (1 + 4u_1)^2 = 1$

$u_1^2 + 1 + 8u_1 + 16u_1^2 = 1$

$17u_1^2 + 8u_1 = 0$

$u_1(17u_1 + 8) = 0$

$u_1 = 0$ OR $u_1 = \frac{-8}{17}$

$u_2 = 1$

$\hat{u} = \hat{j}$

$u_2 = 1 - \frac{32}{17} = \frac{-15}{17}$

$\hat{u} = -\frac{8}{17}\hat{i} - \frac{15}{17}\hat{j}$

ZOOM Notebook

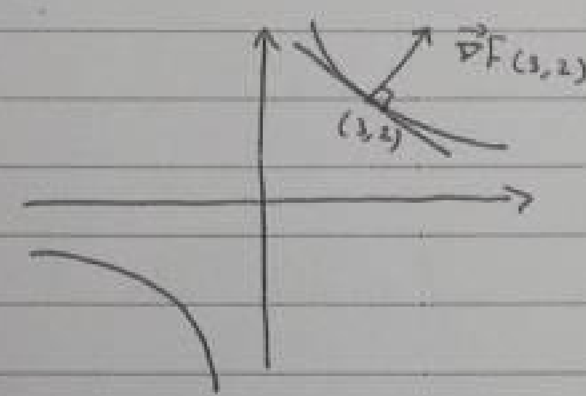
Subject:

* $\vec{\nabla} f_{(x_0, y_0)}$ is the direction of normal to the tangent Line to the level curve $f_{(x,y)} = f_{(x_0,y_0)}$

* $\vec{\nabla} f_{(x_0, y_0, z_0)}$ is the direction of the normal to the tangent plane to the level surface $F_{(x,y,z)} = f_{(x_0,y_0,z_0)}$

ex $\boxed{47}$ $f(x,y) = xy$; ① Find $\vec{\nabla} f_{(3,2)}$??

$\vec{\nabla} f_{(3,2)} = 2\hat{i} + 3\hat{j}$



$f_{(3,2)} = 6 \Rightarrow$ level curve

$\Rightarrow xy = 6 \Rightarrow y = \frac{x}{6}$

② Normal to the tangent Line:

$\frac{x-3}{2} = \frac{y-2}{3}$, $6, 0 = z$

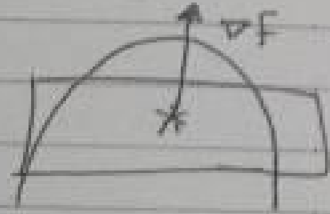
③ Tangent Line

$\frac{x-3}{-3} = \frac{y-2}{2}$, 6 | $\frac{x-3}{3} = \frac{y-2}{-2}$, $6 = z$

Subject:

ex $x^2-2y^2+z^2+yz=2$ "equaition"
"Surface"

Write an eq of tangent plane and the normal line to this surface at the point $(2,1,-1)$



1) $x^2-2y^2+z^2+yz-2=0$
$F(x,y,z)=0$

$\vec{\nabla} f \perp$ Tangent Plane
$=2x\,i+(-4y+z)\,\hat{j}+(2z+y)\,\hat{k}$

$\vec{\nabla} f_{(2,1,-1)} = 4\hat{i}-5\hat{j}-\hat{k}$ [normal to the plane]

- Tplane : $4(x-2)-5(y-1)-(z+1)=0$
- Tplane : $f_x(x-x_0)+f_y(y-y_0)+f_z(z-z_0)$

Normal Line $= \dfrac{x-x_0}{f_x} = \dfrac{y-y_0}{f_y} = \dfrac{z-z_0}{f_z}$

$= \dfrac{x-2}{4} = \dfrac{y-1}{5} = \dfrac{z+1}{-1}$

$\boxed{*}$ $S_1 : f(x,y,z)=0$ $S_2 : g(x,y,z)=0$

$\vec{\nabla} f \times \vec{\nabla} g$ // Tangent Line to the curve of intersection between S_1 & S_2
at (x_0,y_0,z_0)
$\in S_1$ & S_2

ZOOM Notebook

Subject:

59 find parametric equations of the tangent line to the curve of intersection of the paraboloid $z = x^2+y^2$ & the ellipsoid, $4x^2+y^2+z^2=9$ at $(-1,1,2)$

$S_1: z - x^2 - y^2 = 0$ $\quad F$

$S_2: 4x^2+y^2+z^2-9=0$ $\quad G$

$\vec{\nabla}F = -2x\hat{i} - 2y\hat{j} + \hat{k} \xRightarrow{(-1,1,2)} 2\hat{i} - 2\hat{j} + \hat{k}$

$\vec{\nabla}G = 8x\hat{i} + 2y\hat{j} + 2z\hat{k} \Longrightarrow -8\hat{i} + 2\hat{j} + 4\hat{k}$

$$\vec{\nabla}F \times \vec{\nabla}G = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 1 \\ -8 & 2 & 4 \end{vmatrix} = -10\hat{i} - 16\hat{j} - 12\hat{k}$$

direction of the T.line

T.L: $x = -1 - 10t$

$y = 1 - 16t$

$z = 2 - 12t$

Subject: CaL-3

17/7/2012

S: 14.7 Maximum and Minimum Values.

$z = f(x,y)$ ← Surface

* critical point :-

① $f_x = 0$ & $f_y = 0$ (stationary)

② f_x or f_y or both doesn't exist.

* to classify the criticals (stationary) for local maximum, minimum or sadle Use 2nd partial derivative test.

* 2nd Partial derivative test

(x_0, y_0) is critical (stationary)

Find $\underset{\text{real number}}{D} = f_{xx} \cdot f_{yy} - (f_{xy})^2$

i) if $D > 0$ & $f_{xx} > 0 \Longrightarrow$ Local minimum

ii) if $D > 0$ & $f_{xx} < 0 \Longrightarrow$ Local maximum

iii) if $D < 0$ saddle

IV) $D = 0$ test Fails

Subject:

ex $Z = \sqrt{x^2+y^2}$; find the critical point

$f_x = \dfrac{x}{\sqrt{x^2+y^2}}$

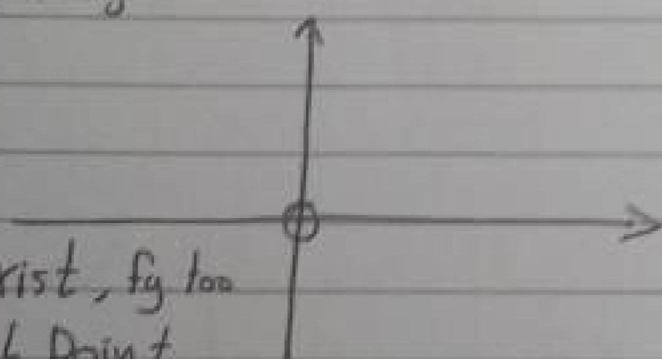
$D_f : x^2+y^2 \geq 0$
x-y plane
R^2

$f_y = \dfrac{y}{\sqrt{x^2+y^2}}$

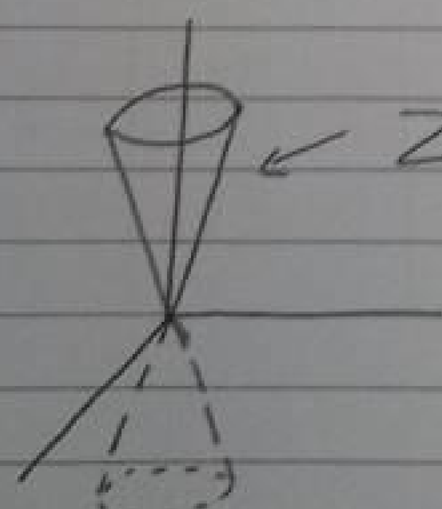
$f_x = 0 \Longrightarrow \dfrac{x}{\sqrt{x^2+y^2}} = 0 \Longrightarrow x = 0$

$f\ (0,0)$

$f_y = 0 \Longrightarrow \dfrac{y}{\sqrt{x^2+y^2}} = 0 \Longrightarrow y = 0$



f_x doesn't exist, f_y too
at $(0,0)$ critical point



$Z = \sqrt{x^2+y^2}$

Subject:

ex $f(x,y) = x^4 + y^4 - 4xy + 1$

Find & classify the critical:

- $f_x = 4x^3 - 4y \Rightarrow f_x = 0 \Rightarrow 4x^3 - 4y = 0 \Rightarrow x^3 = y$ ~ ①
- $f_y = 4y^3 - 4x \Rightarrow f_y = 0 \Rightarrow 4y^3 - 4x = 0$ ~ ②

$y^3 - x = 0$

- $(x^3)^3 - x = 0$

$x^9 - x = 0$

$x(x^8 - 1) = 0$

$x = 0$ or $x = \pm 1$

at $x = 0 \Rightarrow y = 0$ $(0,0)$

at $x = 1 \Rightarrow y = 1$ $(1,1)$

at $x = -1 \Rightarrow y = -1$ $(-1,-1)$

P.t.	$f_{xx} = 12x^2$	$f_{yy} = 12y^2$	$f_{xy} = -4$	$D = f_{xx} f_{yy} - (f_{xy})^2$
$(0,0)$	0	0	-4	$-16 < 0$ sad.
$(1,1)$	12	12	-4	$144 - 16 > 0$ & $f_{xx} > 0$ minimum $f_{(1,1)}$
$(-1,-1)$	12	12	-4	$144 - 16 > 0$ & $f_{xx} > 0$ minimum $f_{(-1,-1)}$

Subject:

ex $f_{(x,y)} = e^{y}(y^2 - x^2)$

$D_f = R^2$

$f_x = -2x e^{y}$

$f_y = e^{y}(2y) + (y^2 - x^2) e^{y}$

$f_x = 0 \Longrightarrow -2x e^{y} = 0 \Longrightarrow x = 0$

& $f_y = 0 \Longrightarrow 2y e^{y} + e^{y}(y^2 - x^2) = 0$

$2y e^{y} + e^{y} y^2 = 0$

$y e^{y}(2+y) = 0$

$y e^{y} = 0$ or $y = -2$

$y = 0$

P.t.	$F_{xx} = -2e^{y}$	$f_{yy} = ye^{y} + 2e^{y} + 2ye^{y} + e^{y}(y^2 - x^2)$	$f_{xy} = -2xe^{y}$	D
$(0,0)$	-2	2	0	$-4 < 0$ saddle
$(0,-2)$	$-2e^{-2}$	$-4e^{-2} + 2e^{-2} =$ $2e^{-2}$	0	$4e^{-4} > 0$ & f_{xx} is $-ve$ maximum $f_{(0,-2)}$

Subject:

ex $f(x,y) = xy + \frac{1}{x} + \frac{1}{y}$; $x \neq 0$ & $y \neq 0$

$f_x = y - \frac{1}{x^2} \Rightarrow f_x = 0 \rightarrow y = \frac{1}{x^2}$

$f_y = x - \frac{1}{y^2} \longrightarrow f_y = 0$

$$x - \frac{1}{\left(\frac{1}{x^2}\right)^2} = 0$$

$x - x^4 = 0$

$x(1 - x^3) = 0$

$x = 0$ or $x = 1 \Rightarrow y = 1$

مهمل

S. 14.8 Lagrange Multipliers : λ

Lag problem:

① Maximize and/or Minimize the function

$Z = f(x,y)$ subject to the constraint $g(x,y) = 0$

اقتران [Curve] معادلة

② Maximize OR Minimize $\omega = f(x,y,z)$ اقتران

subject to constraint $g(x,y,z) = 0$

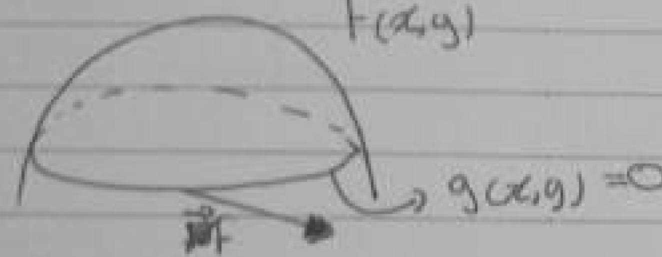
معادلة [Surface]

Subject

☆ Therom of Langrange:

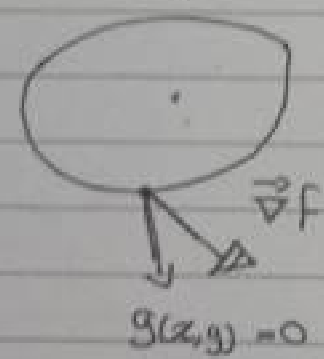
At mim or max

$\vec{\nabla} f \parallel \vec{\nabla} g$



$\Longrightarrow \vec{\nabla} f = \lambda \vec{\nabla} g$

; λ lag multiplier



$\Longrightarrow$ lag eqas:

$f_x = \lambda g_x$

$f_y = \lambda g_y$

& $g(x,y) = 0$ constraint

Subject: Cal-3 18/7/2012

S: 14.8

Lag problem $f(x,y)$ subject to constraint $g(x,y)=0$

معادلة

Lag eqs:

$$f_x = \lambda g_x$$
$$f_y = \lambda g_y$$
$$g(x,y) = 0$$

---

ex $\frac{15}{940}$

Maximize and Minimize $\overbrace{f(x,y) = x^2y}^{f}$ subject to $\underbrace{x^2+2y^2=6}_{g}$

Lag eqs:

- $2xy = \lambda 2x$ ①
- $x^2 = \lambda 4y$ ②
- $x^2+2y^2=6$ ③

$$\frac{①}{②} = \frac{2xy}{x^2} = \frac{\lambda 2x}{\lambda 4y} \xRightarrow{x,y \neq 0} 4y^2 = x^2 \quad ④$$

③ $\longrightarrow 4y^2 + 2y^2 = 6$ ⑤ $\Longrightarrow 6y^2 = 6 \Longrightarrow y = \pm 1$

⑤ @ $y=1 \Longrightarrow x^2 = 4 \Longrightarrow x = \pm 2$ $(2,1)(-2,1)$

@ $y=-1 \Longrightarrow x^2 = 4 \Longrightarrow x = \pm 2$ $(2,-1)(-2,-1)$

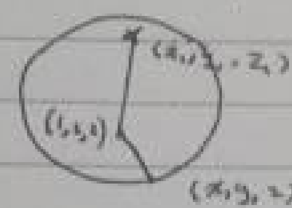
P.t	$f(x,y) = x^2y$	
(2, 1)	4	Maximum
(2, −1)	−4	Minimum
(−2, 1)	4	
(−2, −1)	−4	

ZOOM Notebook

Subject:

ex find the points on the sphere $x^2+y^2+z^2=36$ that are clossest to and farthest from the p.t. $(1,2,2)$

- $1^2+2^2+2^2=9$ so its in the sphere



- distance function

$$d=\sqrt{(x-1)^2+(y-2)^2+(z-2)^2}$$

$$f(x,y,z)=(x-1)^2+(y-2)^2+(z-2)^2=d^2$$

- max & min

- $f(x,y,z)=(x-1)^2+(y-2)^2+(z-2)^2$
- $x^2+y^2+z^2=36$

Lag equ:

$f_x=\lambda g_x \qquad f_z=\lambda g_z$

$f_y=\lambda g_y \qquad x^2+y^2+z^2=36$

- $2(x-1)=\lambda 2x$ ---- ①
- $2(y-2)=\lambda 2y$ ---- ②
- $2(z-2)=\lambda 2z$ ---- ③
- $x^2+y^2+z^2=36$ ---- ④

① $\Rightarrow x-x\lambda=1 \Rightarrow x=\dfrac{1}{1-\lambda}$

② $\Rightarrow \dfrac{2}{1-\lambda}=y$ ③ $\Rightarrow \dfrac{2}{1-\lambda}=z$

④ $\Rightarrow \dfrac{1}{(1-\lambda)^2}+\dfrac{4}{(1-\lambda)^2}+\dfrac{4}{(1-\lambda)^2}=36$

$\Rightarrow$ Cont.

ZOOM Notebook

Subject:

$$\frac{9}{(1-\lambda)^2} = 36 \Longrightarrow \frac{1}{(1-\lambda)^2} = 4$$

$$(1-\lambda)^2 = \frac{1}{4} \Rightarrow 1-\lambda = \pm\frac{1}{2} \Rightarrow \lambda = \pm\frac{1}{2} + 1$$

$$\lambda = \frac{1}{2} \text{ or } \frac{3}{2}$$

@ $\lambda = \frac{1}{2} \Longrightarrow x = 2,\ y = 4,\ z = 4$ clossest

@ $\lambda = \frac{3}{2} \Longrightarrow x = -2,\ y = -4,\ z = -4$ farthest

$f(2,4,4) = 9 \quad d = \sqrt{9} = 3$

$f(-2,-4,-4) = 81 \quad d = \sqrt{81} = 9$

ZOOM Notebook

Subject:

(44) Find three possitive numbers where sum is 12 & the sum of those square is as small as possible.

$x, y, z = ??$ $\quad x>0,\ y>0,\ z>0$

s. to, $x+y+z=12$ $\quad g$
$f(x,y,z) = x^2+y^2+z^2$ $\quad f$

Lag eq :

$2x = \lambda$
$2y = \lambda$
$2z = \lambda$
$x+y+z=12$

Method II 14.7

$z = 12 - x - y$
$f(x,y,z) = x^2 + y^2 + (12-x-y)^2$

$F_x = 0$ $\quad (x, y)$
$F_y = 0$

Subject: Cal - 3

19 / 7 /

Ch. 15 Multiple Integrals

* Double Integrals

S: 15.3

① $\int_1^2 x \, dx = \frac{x^2}{2}\Big|_1^2$ $\quad {}_1[\ \ ]_2 \quad \in \mathbb{R}$

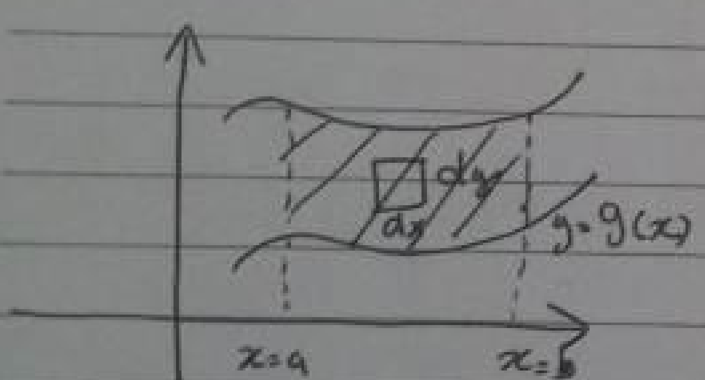
▫ Double Integral : $\in \mathbb{R}$

$$\underbrace{\iint}_{R} f(x,y)\, dA$$

$R =$ Region $\subseteq \mathbb{R}^2$ (xy-plane)

* $dA = dx\,dy$ OR $dy\,dx$

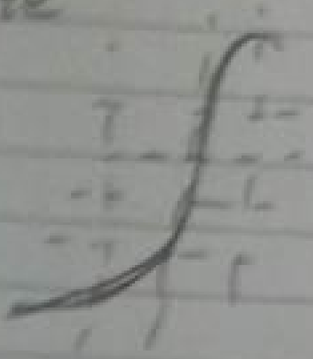
Case 1:



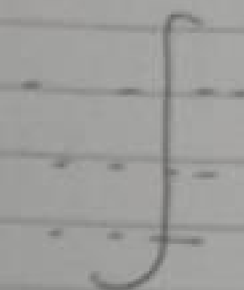
$$= \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\, dy\, dx$$

Subject   /   /

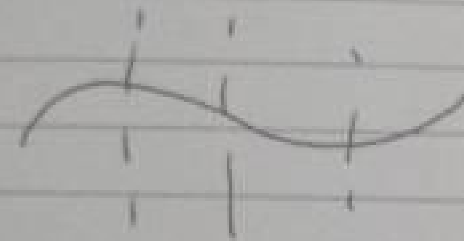
Note



both

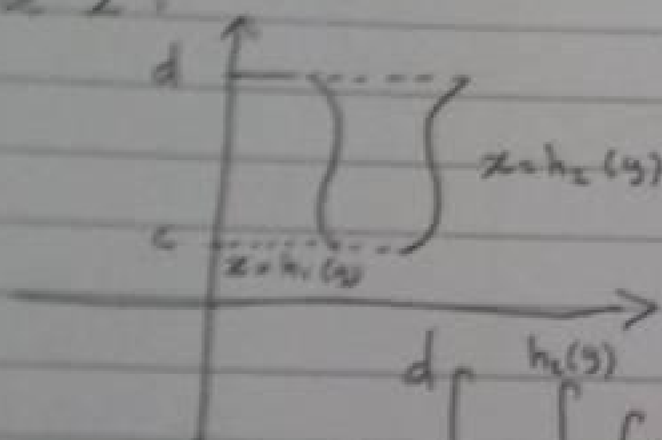


$x = g(x)$



$y = f(x)$

Case 2 :



$$= \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\, dx\, dy$$

ex
$$\int_0^1 \int_x^{x^2} (xy + x^2)\, dy\, dx$$

$$= \int_0^1 \left[x\left(\frac{y^2}{2}\Big|_x^{x^2}\right) + x^2\left(y\Big|_x^{x^2}\right)\right] dx$$

$$= \int_0^1 \cancel{\tfrac{1}{2}x^3}\ \frac{x}{2}\left(\frac{x^4}{x} - \frac{x^2}{1}\right) + x^2(x^2 - x)\, dx$$

$$= \int_0^1 \frac{1}{2}(x^5 - x^3) + (x^4 - x^3)\, dx$$

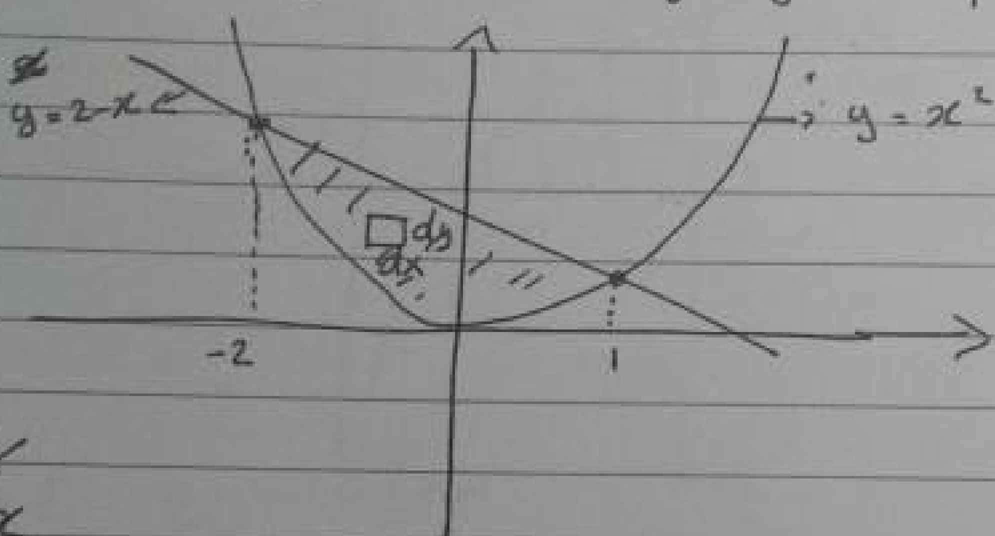
Subject:

② Evaluate

$$\iint_R f(x,y)\,dA$$

R is the region bounded by $y = x^2$ & $x + y = 2$

$\Rightarrow y = 2 - x$



$y = y$

$x^2 = 2 - x$

$(x+2)(x-1) = 0$

$x = -2$ or $x = 1$

$$= \int_{-2}^{1}\int_{x^2}^{2-x} f(x,y)\,dy\,dx$$

Revers the order of Integration

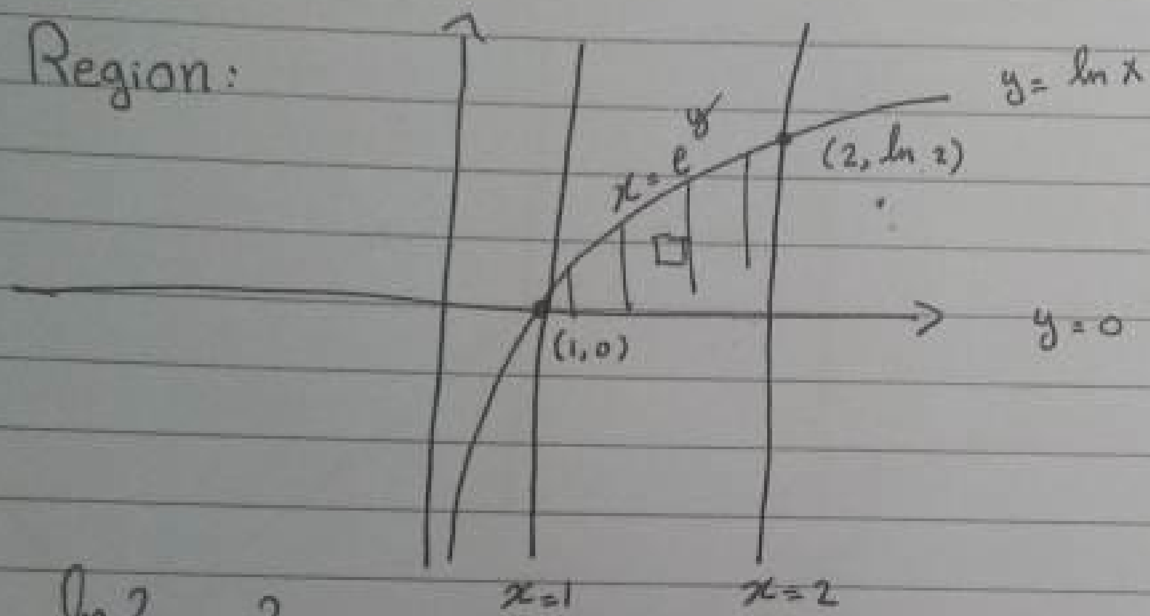
Subject:

ex

$\ln x = y$

1) $$\int_{x=1}^{x=2}\int_{0=y}^{} f(x,y)\,dy\,dx \xrightarrow{??} \int\int f(x,y)\,dx\,dy$$

Region:



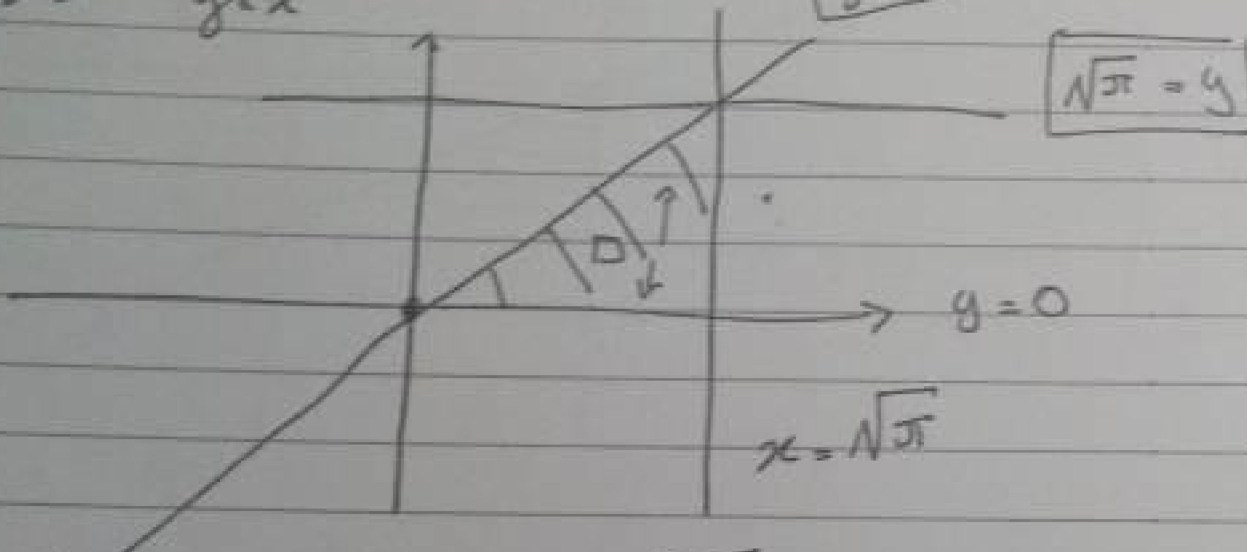
$$\int_{0}^{\ln 2}\int_{e^y}^{2} f(x,y)\,dx\,dy$$

- $\int e^x\,dx = e^x$
- $\int x e^{x^2}\,dx = \frac{e^{x^2}}{2} + C$ $\quad u = x^2$
- $\int e^{x^2}\,dx$?? ✗ حل
- $\int \cos x^2\,dx$, $\int \frac{\cos x}{x}\,dx$ ✗ حل

Subject

$\boxed{46}$ Evaluate

$$\int_{y=0}^{y=\sqrt{\pi}} \int_{y=x}^{\sqrt{\pi}=x} \cos x^2 \, dx \, dy$$



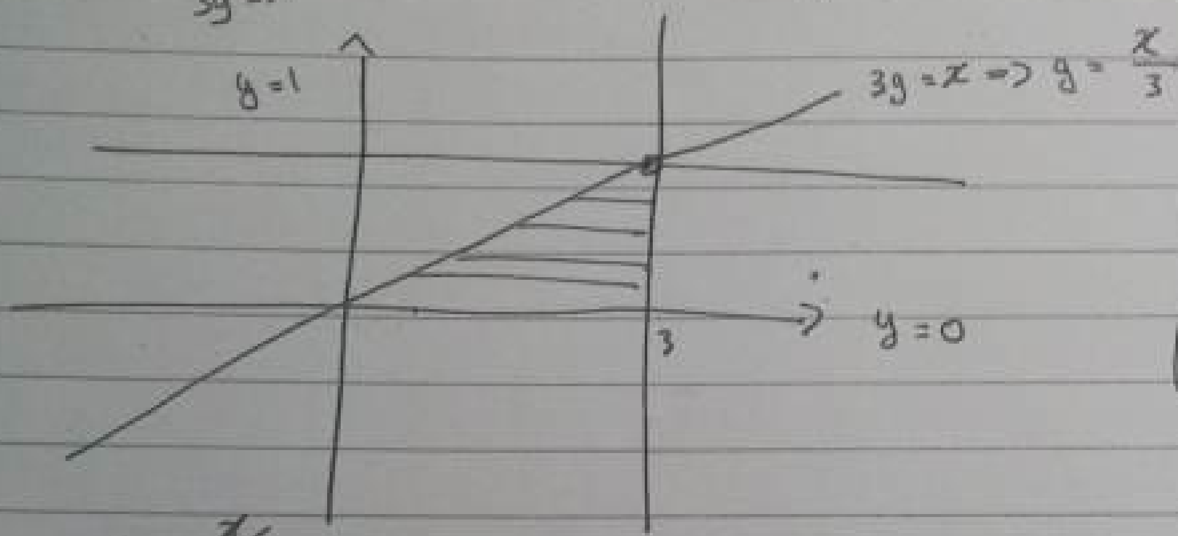
$$\int_0^{\sqrt{\pi}} \int_0^{x} \cos x^2 \, dy \, dx = \int_0^{\sqrt{\pi}} \cos x^2 \, (x) \, dy$$

$$= \frac{\sin x^2}{2} \Big|_0^{\sqrt{\pi}}$$

Subject: Double Integrals 22/7/2012

$\frac{45}{973}$

$$\int_0^1 \int_{3y=x}^{3=x} e^{x^2}\,dx\,dy \Longrightarrow \int\int e^{x^2}\,dy\,dx$$



- $\int dy = y + c$
- $\int_a^b dy = b - a$

$$= \int_0^3 \int_0^{\frac{x}{3}} e^{x^2}\,dy\,dx$$

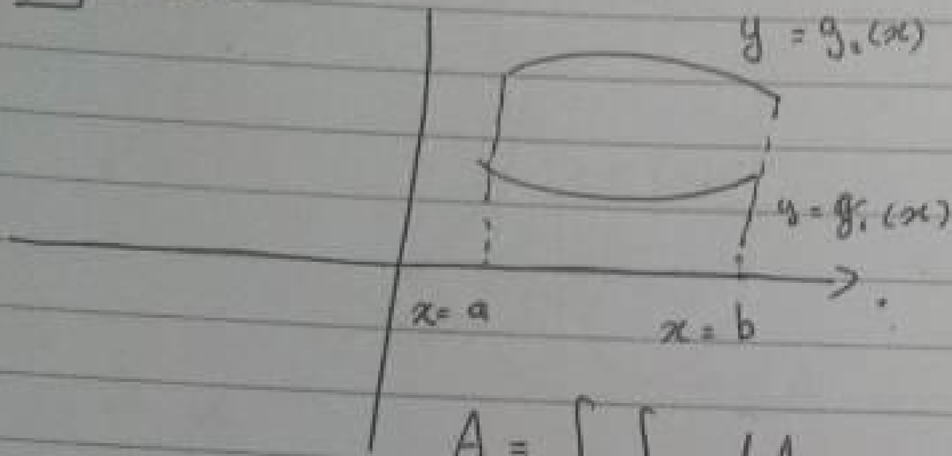
$$= \int_0^3 \frac{x}{3} e^{x^2}\,dx \quad \text{by sub.}$$

$$= \frac{1}{3.2} e^{x^2}\Big]_0^3 = \frac{1}{6}\left[e^9 - e^0\right]$$

Subject:

* Applications of double Integrals 15.5

1 Area

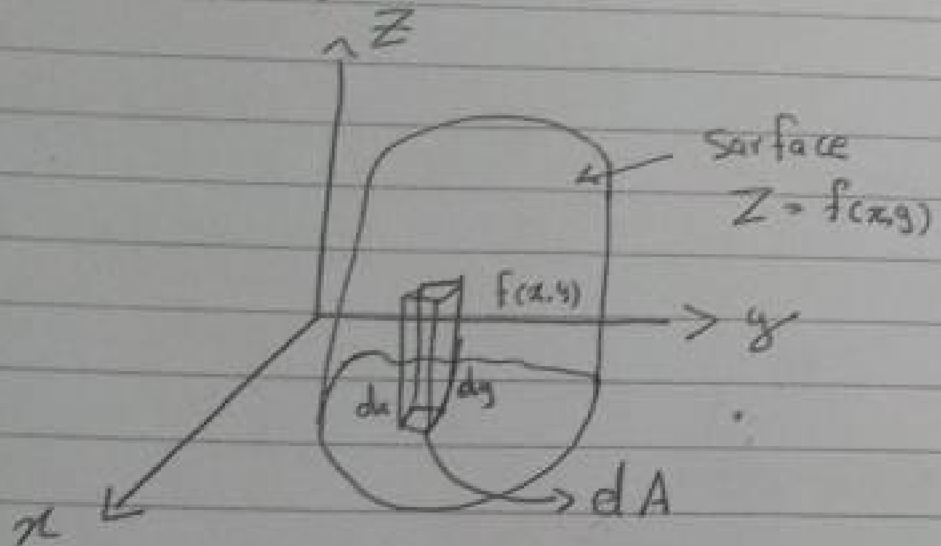


$$A = \iint_R dA$$

$$= \int_a^b \int_{g_1(x)}^{g_2(x)} dy\, dx$$

$$= \int_a^b \left(g_2(x) - g_1(x) \right) dx$$

$\boxed{2}$ Volumes :



$$dV = \underbrace{f(x,y)}_{\text{الارتفاع}} \; dA$$

$$V = \iint_{R} \underbrace{f(x,y)}_{\text{الارتفاع}} \, dA$$

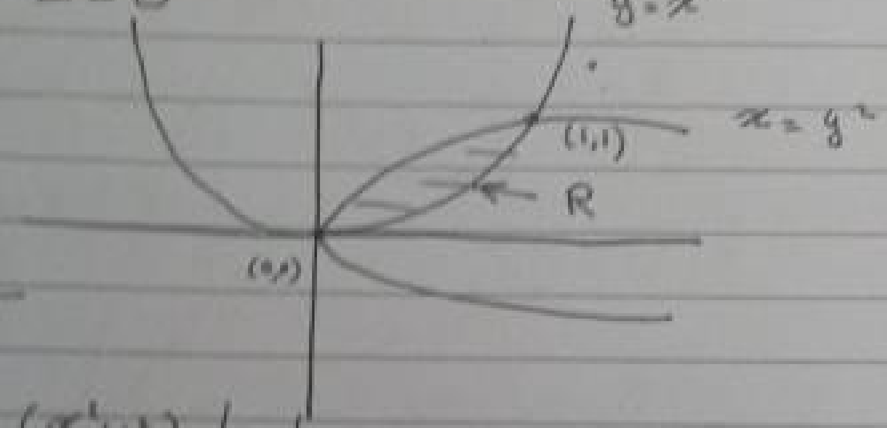
R → القاعدة

Region of Integration

Subject: / /

[17] Find the Volume of the solid b.d.d by the paraboloid $z = x^2+y^2$, $\underbrace{y = x^2 ,\ x = y^2}_{\text{cylinder}}$ جوانبه

& the $\underbrace{xy\text{-plane}}_{Z=0}$.



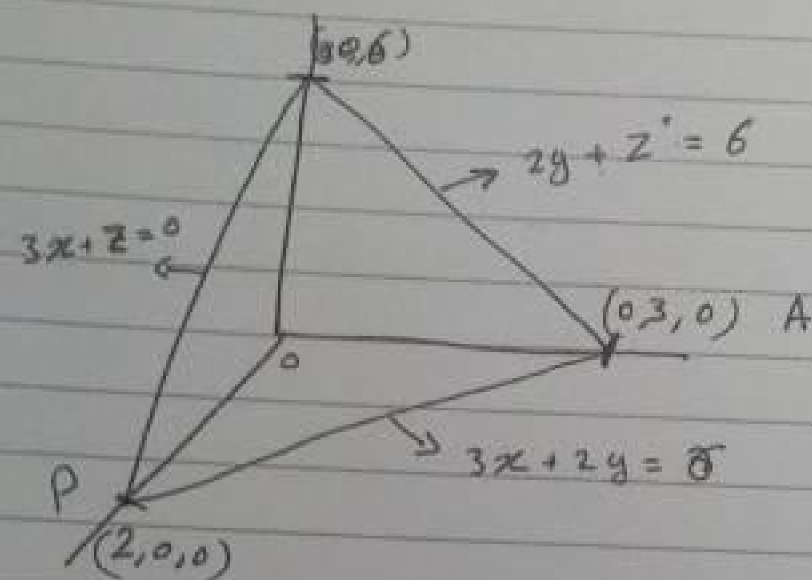
$$V = \int_0^1 \int_{x^2}^{\sqrt{x}} (x^2+y^2)\, dy\, dx$$

$$= \int_0^1 x^2 y + \frac{y^3}{3} \Big|_{x^2}^{\sqrt{x}} \, dx$$

Subject:

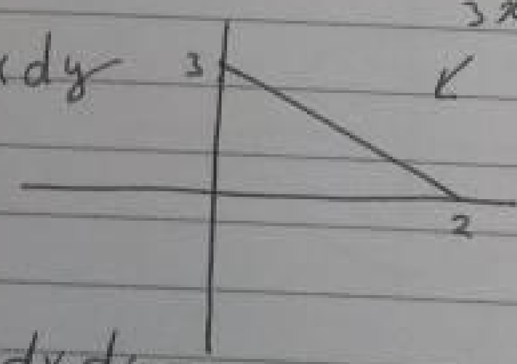
23 Find Volume bdd by the plane $3x+2y+z=6$ & the coordinat planes

- planes $(x=0,\ y=0,\ z=0)$



* $Z = 6-3x-2y$ الارتفاع

$$V = \iint\limits_{OAP} (6-3x-2y)\,dx\,dy$$



$3x+2y=6$

$x = \frac{6-2y}{3}$

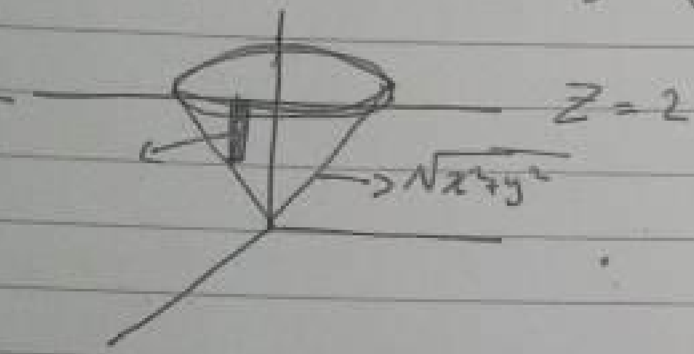
$$= \int_0^3 \int_0^{\frac{6-2y}{3}} (6-3x-2y)\,dx\,dy$$

Subject: Cal-3

23/7/2012

<u>ex</u> Find the Volume of solid region

$Z = \sqrt{x^2+y^2}$ & $Z = 2$ Using (double integral)

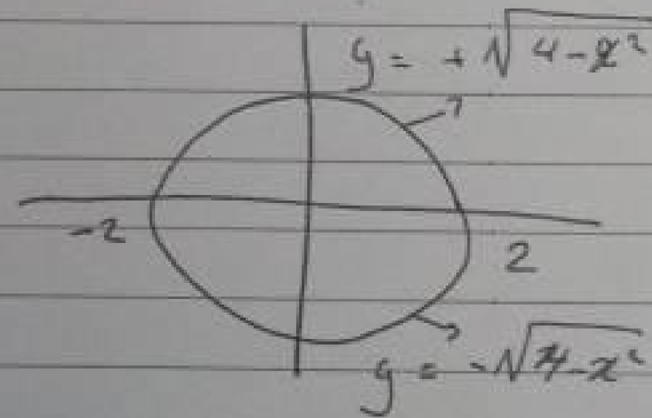


• Region of intersection

$Z = Z$

$2 = \sqrt{x^2+y^2} \Rightarrow x^2+y^2=4$

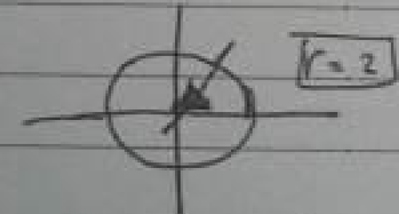
$$V = \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{+\sqrt{4-x^2}} 2-\sqrt{x^2+y^2} \; dy \, dx$$



☑ Use polar coordinat:

$dA = r\,dr\,d\theta$

$$V = \int_0^{2\pi} \int_0^{2} (2-r)\,r\,dr\,d\theta$$



$$= 2r - r^2 \, dr \, d\theta = \int_0^{2\pi} r^2 - \frac{r^3}{3} \Big|_0^2 \, d\theta = 2\pi\left(2^2 - \frac{2^3}{3}\right)$$

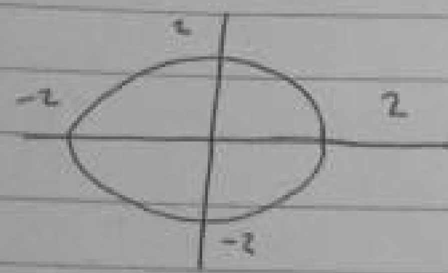
ZOOM Notebook

Subject: Cal-3

24 / 7 / 2012

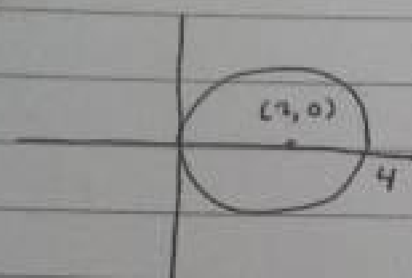
Circles in polar

① $r = 2$



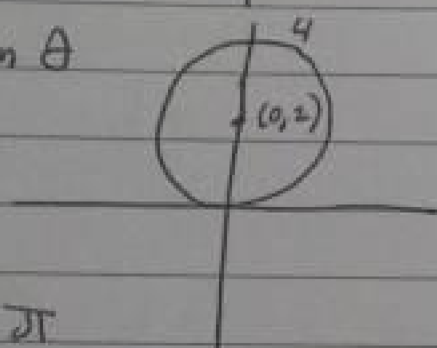
$0 < \theta < 2\pi$

② $r = 4\cos\theta$



$0 \leq \theta \leq \pi$

③ $r = 4\sin\theta$



$0 \leq \theta \leq \pi$

Subject:

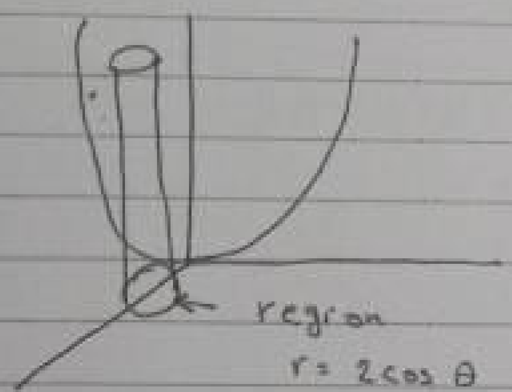
ex Find the Volume (of the Solid). Unde the paraboloid $Z = x^2 + y^2$, a bove xy-plane & inside the cylinder $x^2+y^2 = 2x$

جواب + قاعدة $\Rightarrow r^2 = 2r\cos\theta$

$r = 2\cos\theta$

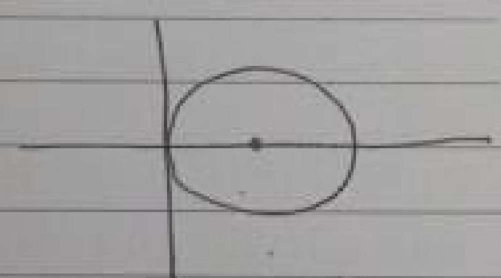
Double :

$$V = \iint_R (x^2+y^2)\,dA$$



$$= \int_0^{\pi}\int_0^{2\cos\theta} r^2 r\,dr\,d\theta$$

$$= \int_0^{\pi} \frac{r^4}{4}\Big|_0^{2\cos\theta} d\theta$$



$$= \int_0^{\pi} \frac{2^4\cos^4\theta}{4}\,d\theta$$

$$= \frac{1}{4} \times 2^4 \int (\cos^2\theta)^2\,d\theta$$

$$= \frac{2^4}{4}\int\left(\frac{1+\cos 2\theta}{2}\right)^2 d\theta$$

ZOOM Notebook

Subject: ____________ / /

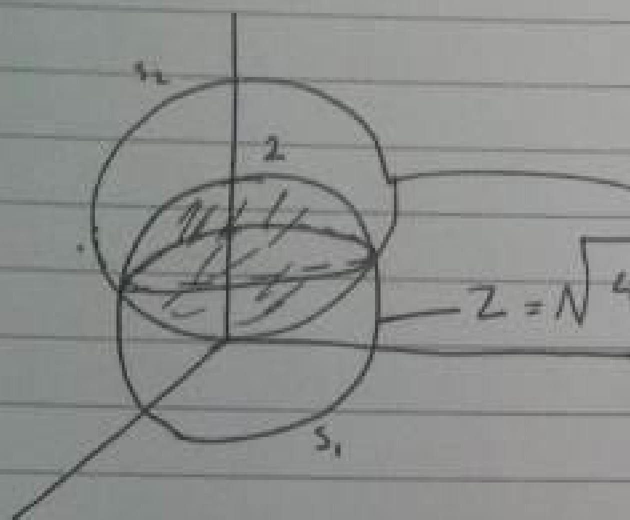
* Find the Volume common between two spheres

$S_1 : x^2+y^2+z^2=4$ & $S_2 : x^2+y^2+z^2=4z$

$x^2+y^2+z^2-4z=0$

$x^2+y^2+(z-2)^2=4$

$(0,0,2)$



$z = 2-\sqrt{4-(x^2+y^2)}$

$z=\sqrt{4-(x^2+y^2)}$

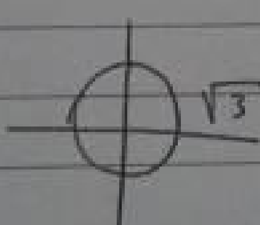
Intersection

$4=4z \Rightarrow z=1$ مستوي التقاطع

$S_1 \Rightarrow x^2+y^2+1=4$

$x^2+y^2=3$ curve of intersection

$$V=\iint_R \left(\sqrt{4-(x^2+y^2)} - 2+\sqrt{4-(x^2+y^2)}\right) dA$$



Pollar

$$=\int_0^{2\pi}\int_0^{\sqrt{3}} \left(\sqrt{4-r^2} - 2+\sqrt{4-r^2}\right) r\, dr\, d\theta$$

Subject: / /

☆ Coordinate syten in 3-spaces

① cartesion (x, y, z)

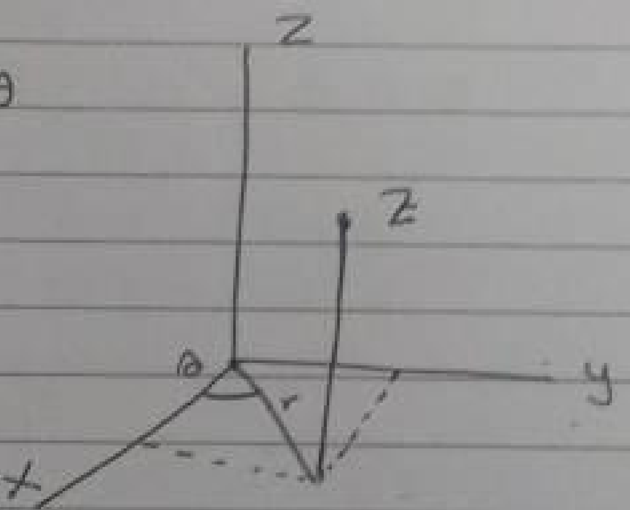
$dv = dz\, dA$

$= dz\, dy\, dx$

$= dz\, dx\, dy$

② cylindrical (r, θ, z)

$dv = r\, dz\, dr\, d\theta$



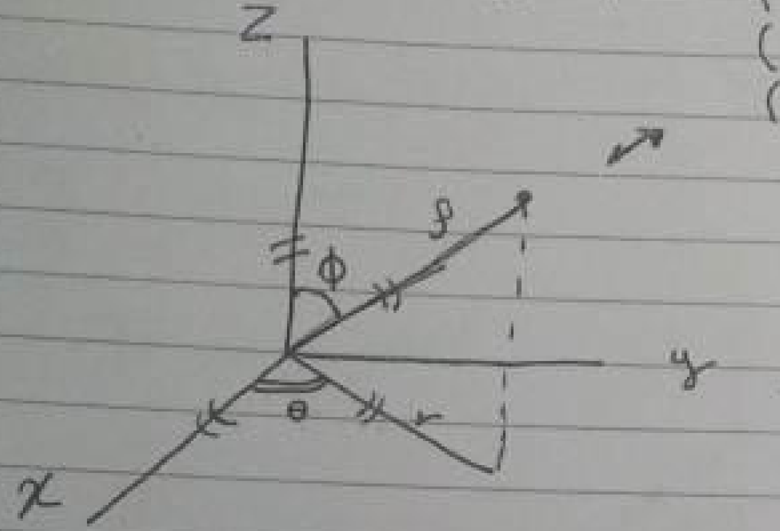
- r = constant cylinder
- θ = constant Line ⟹ half plane
- z = constant plane // xy-plane

Subject: Cal-3

25/7/2012

- Spherical coordinates :

(x, y, z)
(r, θ, z)
(ρ, θ, ϕ)



$\rho \geq 0, \quad 0 \leq \theta \leq 2\pi, \quad 0^\circ \leq \phi \leq \pi$

- $Z = \rho \cos \phi$
- $r = \rho \sin \phi$
- $x = r \cos \theta$
- $y = r \sin \phi$
- $x = \rho \sin \phi \cos \theta$ ((spherical with cartesian))
- $y = \rho \sin \phi \sin \theta$ // // //
- $\rho^2 = r^2 + Z^2$
- $\rho^2 = x^2 + y^2 + z^2$

① ρ = constant

sphere Center (origin), rad = constant

② θ = constant [half plane]

③ ϕ = constant

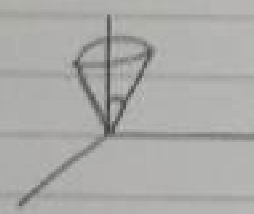
i) $\phi = 0$ +Z-axis $\qquad \phi = \frac{\pi}{2} \Rightarrow xy$-plane

$\phi = \pi$ -Z axis

Subject:

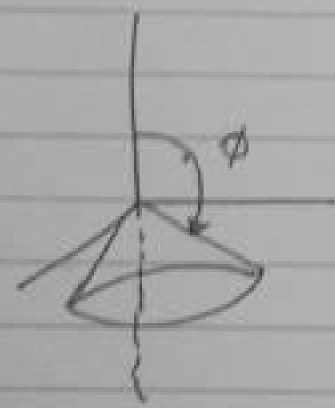
ii) $0 < \phi < \frac{\pi}{2}$

Cone >>> upper part



iii) $\frac{\pi}{2} < \phi < \pi$

Cone >>> lower part



ex Convert to spherical

1) $\sqrt{3}\, z = \sqrt{x^2+y^2}$

$z = \frac{\sqrt{x^2+y^2}}{\sqrt{3}} = \frac{1}{\sqrt{3}}\sqrt{x^2+y^2}$

$\rho \cos\phi = \frac{1}{\sqrt{3}} r$

$\rho \cos\phi = \frac{1}{\sqrt{3}} \rho \sin\phi$

$\sqrt{3} = \frac{\sin\phi}{\cos\phi} = \tan\phi \quad [\phi = 60^\circ = \frac{\pi}{3}]$

2) $z = -\sqrt{3x^2+3y^2}$

$= -\sqrt{3}\sqrt{x^2+y^2}$

$\tan\phi = \frac{-1}{\sqrt{3}} \qquad \phi = \frac{5\pi}{6}$

Subject:

ex $\phi = \frac{2\pi}{3}$ [cone Lower]

convert to cartesian

$\tan\phi = \tan\frac{2\pi}{3} = -\sqrt{3}$

$Z = \frac{-1}{\sqrt{3}}\left(\sqrt{x^2+y^2}\right)$

* $dV = \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

▫ triple Integral

$\iiint\limits_{D \subseteq R^3} F(x,y,z)\, dV$ يعطي جواب (عدد حقيقي)

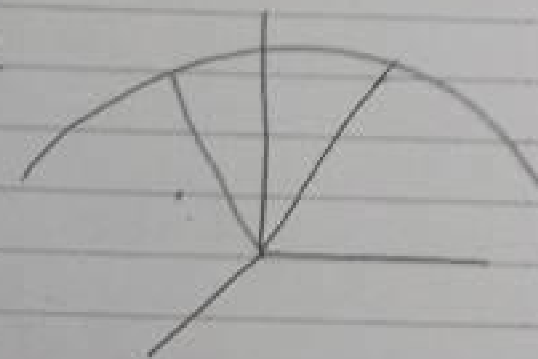
▫ In double Volume

$V = \iint \underbrace{f(x,y)}_{\to\ \Delta Z\ \text{الارتفاع}} dA$

Subject:

□ Find the Volume of the Solid bdd below by the curve $z = \sqrt{3x^2+3y^2}$ & bdd above by sphere $x^2+y^2+z^2=4$

✩ Using Triple Integral



□ Spherical

$$\tan\phi = \frac{1}{\sqrt{3}}$$

Cone $\phi = \frac{\pi}{6}$

Sphere : $\rho = 2$ sphere مركزها الOrigin ونصف قطرها 2

$$V = \int_0^{2\pi}\int_0^{\pi/6}\int_0^{2} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi/6} \frac{\rho^3}{3}\Big|_0^2 \sin\phi \, d\phi \, d\theta$$

$$= \frac{8}{3}\iint \sin\phi \, d\phi \, d\theta$$

$$= \frac{8}{3}(-\cos\phi)\Big|_0^{\pi/6} \int_0^{2\pi} d\theta$$

$$= \frac{2\pi 8}{3}\left(-\cos\phi\right)\Big|_0^{\pi/6}$$

✩ In the first octant $[0, \frac{\pi}{2}]$

ZOOM Notebook

Subject: Cal-3

26 / 7 / 2017

ex Identify these surfaces:-

① $\rho = 2\sin\theta \sin\phi$

$\rho^2 = 2\rho \sin\theta \sin\phi$

$x^2 + y^2 + z^2 = 2y$

sphere c $(0,1,0)$, rad $= 1$

② $\rho^2(\sin^2\phi \sin^2\theta + \cos^2\phi) = 9$

$\rho^2 \sin^2\phi \sin^2\theta + \rho^2\cos^2\phi = 9$

$y^2 + z^2 = 9$ cylinder / x-axis

③ $\rho = \csc\phi$

$\rho = \frac{1}{\sin\phi} \Rightarrow \rho \sin\phi = 1$

$r = 1$

$\sqrt{x^2+y^2} = 1$

$x^2 + y^2 = 1$ cylinder / z-axis

ZOOM Notebook

Subject: /   /

(4) $x^2+y^2+z^2=2z$

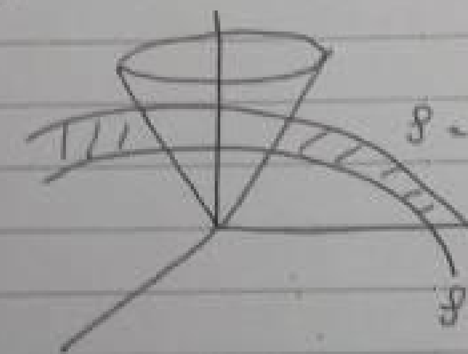
$\xrightarrow{\text{cyli}} r^2+z^2=2z$

$\xrightarrow{\text{sph}} \rho^2\sin^2\phi+\rho^2\cos^2\phi=2\rho\cos\phi$

$\rho^{\cancel{2}}=2\rho\cos\phi$

$\rho=2\cos\phi$

ex Voleume outside the cone $z=\sqrt{x^2+y^2}$
& inside the sphere $x^2+y^2+z^2=9$
outside the other sphere $x^2+y^2+z^2=4$
a bove zy-plane



* spherical نحول لل sph

$z=\sqrt{x^2+y^2}=\phi=\frac{\pi}{4}$

$\rho=3$

$\rho=2$

xy-plan $=\frac{\pi}{2}$

$$V=\int_0^{2\pi}\int_{\pi/4}^{\pi/2}\int_2^3 \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

ZOOM Notebook

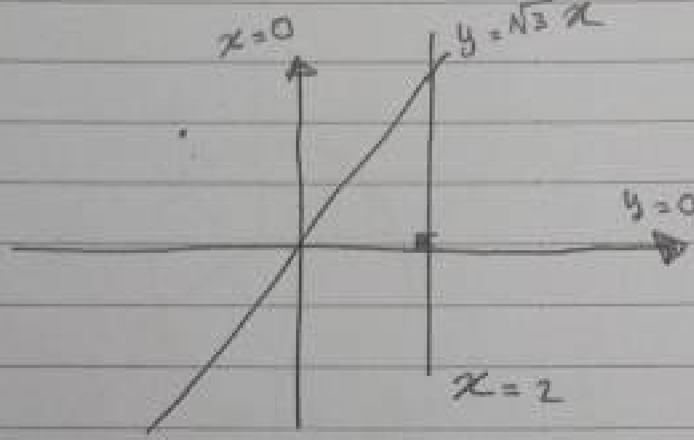
Subject: / /

ex

$$\int_{0}^{2}\int_{0=y}^{\sqrt{3}x=y} e^{-x^2-y^2}\,dy\,dx$$ convert to polar

$$e^{-(x^2+y^2)}$$

$$=\int\int e^{-r^2}\,r\,dr\,d\theta$$



the converting:

- $y=\sqrt{3}\,x$

$r\sin\theta=\sqrt{3}\,r\cos\theta$

$\Longrightarrow \theta=\frac{\pi}{3}$

- $y=0$

$r\sin\theta=0$

$\theta=0$

- $x=0$

$r\cos\theta=0$

- $x=2$

$r\cos\theta=2$

$r=\frac{2}{\cos\theta}$

$$\Longrightarrow \int_{0}^{\pi/3}\int_{0}^{2/\cos\theta} e^{-r^2}\,r\,dr\,d\theta$$

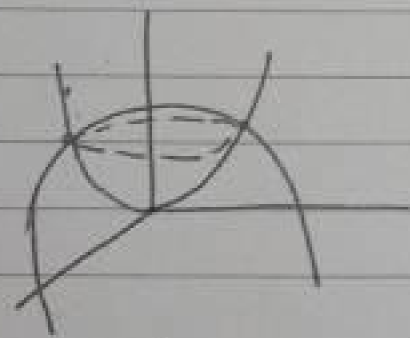
Subject:

ex Convert to cylindrical

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{x^2+y^2=z}^{8-x^2-y^2=z}(x+y+z)\,dz\,dy\,dx$$

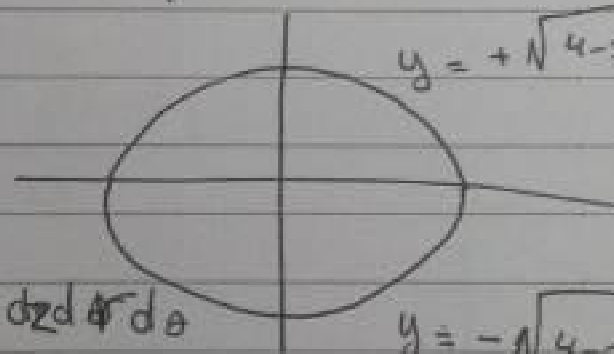
منطقة ال xy-plane

تقاطع ال Praboloid.



$z = z$

☆ $x^2+y^2=4$



$y = +\sqrt{4-x^2}$

$y = -\sqrt{4-x^2}$

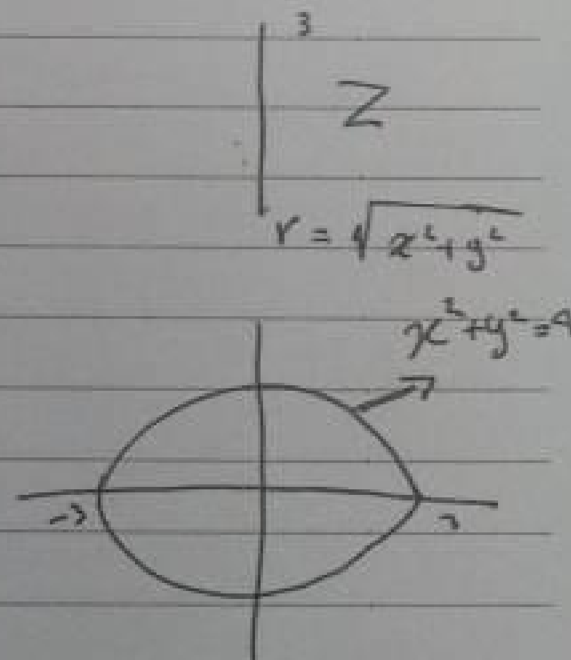
$$= \int_0^{2\pi}\int_0^{2}\int_{r^2}^{8-r^2}(r\cos\theta + r\sin\theta + z)\,r\,dz\,dr\,d\theta$$

ex $\displaystyle\int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=3}\int_{r=z}^{3=z} dz\,dr\,d\theta$

$\Rightarrow$ Cartesian

$$\left(\iiint dz\,dr\,d\theta\right) \times \frac{r}{r}$$

$$\int_{-3}^{3}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_{\sqrt{x^2+y^2}}^{3} \frac{dz\,dx\,dy}{\sqrt{x^2+y^2}}$$



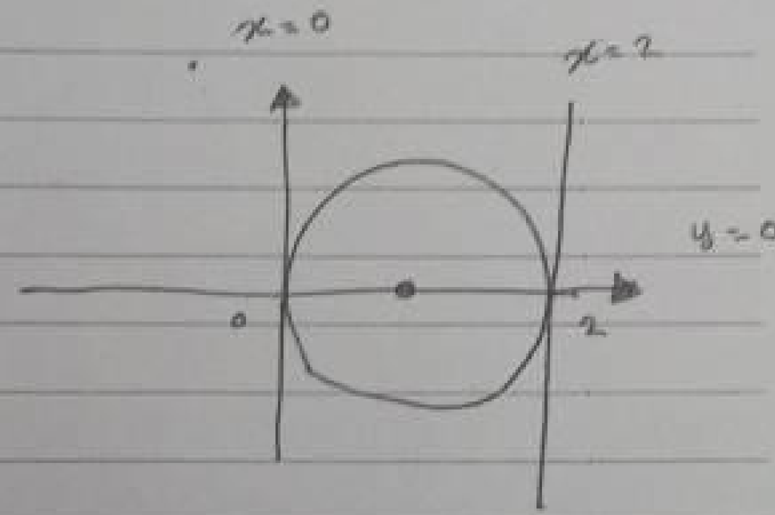
Subject: $\sqrt{2x-x^2} = y$

32) $$\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\, dy\, dx$$

$= y$

Polar

$$\int_0^{\pi/2} \int_0^{2\cos\theta} r \cdot r\, dr\, d\theta$$



* $y = \sqrt{2x - x^2}$

$x^2 + y^2 = 2x$

$r^2 = 2r\cos\theta$

$r = 2\cos\theta$

Note:
الدائرة كلها
$[0, \pi]$